Polymer Heat Transport Enhancement in Thermal Convection:
The Case of Rayleigh-Taylor Turbulence

G. Boffetta,[1] A. Mazzino,[2] S. Musacchio,[3] and L. Vozella[2]

[1]*Dipartimento di Fisica Generale and INFN, Università di Torino, via P. Giuria 1, 10125 Torino, Italy*
[2]*Dipartimento di Fisica, Università di Genova, INFN and CNISM, via Dodecaneso 33, 16146 Genova, Italy*
[3]*CNRS, Laboratoire J.A. Dieudonné UMR 6621, Parc Valrose, 06108 Nice, France*

We study the effects of polymer additives on turbulence generated by the ubiquitous Rayleigh-Taylor instability. Numerical simulations of complete viscoelastic models provide clear evidence that the heat transport is enhanced up to 50% with respect to the Newtonian case. This phenomenon is accompanied by a speed-up of the mixing layer growth. We give a phenomenological interpretation of these results based on small-scale turbulent reduction induced by polymers.

　　　　　PACS numbers: 47.27.te, 47.27.E−, 47.57.Ng

Controlling transport properties in a turbulent flow is an issue of paramount importance in a variety of situations ranging from pure science to technological applications [1–3]. One of the most spectacular ways to achieve this goal consists in adding inside the fluid solvent a small amount of long-chain polymers (parts per million by weight). The resulting fluid solution acquires a non-Newtonian character and the most interesting dynamical effect played by polymers is encoded in the drag coefficient, a dimensionless measure of the power needed to maintain a given throughput in a pipe. With respect the Newtonian case (i.e., in the absence of polymers), it can be reduced up to 80% [4,5].

In many relevant situations (e.g., atmospheric convection) the velocity field is two-way coupled to the temperature field with the result that, together with mass, also heat is transported by the flow. Because the drag reduction is associated with mass transport enhancement, an intriguing question is whether or not this is accompanied by a similar variation in the heat transport.

In this Letter we demonstrate the simultaneous occurrence of mass transport enhancement and heat transport enhancement induced by polymers in a three-dimensional buoyancy-driven turbulent flow originated by the ubiquitous Rayleigh–Taylor (RT) instability. This instability arises at the interface between a layer of light fluid and a layer of heavy fluid placed above and develops in a turbulent mixing layer (see Fig. 1) which grows accelerated in time. Heuristically, the RT system can be assimilated to a channel inside which vertical motion of thermal plumes is maintained by the available potential energy. In analogy with drag reduction phenomena recently observed in systems without boundaries (see, e.g., [6–9]), we argue that polymer additives could reduce the turbulent drag between uprising and downfalling plumes. This is also suggested by recent analytical results which show a speed-up of RT instability due to polymer additives [10].

Direct numerical simulations of primitive equations show that thermal plumes are faster in the presence of polymers (see Fig. 1); therefore, the mixing layer accelerates (up to 30% at final observation time) with respect to the Newtonian case and complete mixing is achieved in a shorter time. A second and more dramatic effect, also clearly detectable in Fig. 1, is that polymers reduce small-scale turbulence [6–8]. As a consequence, thermal
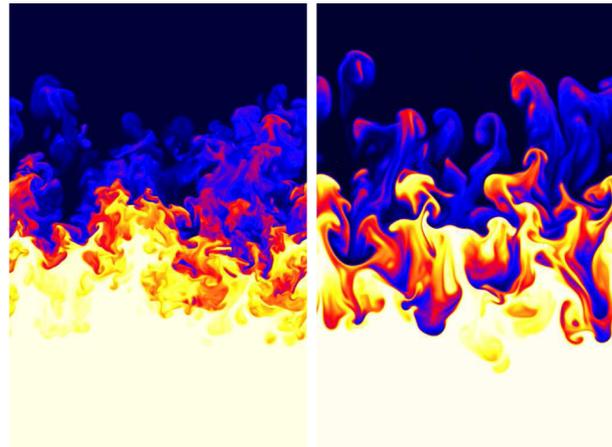

FIG. 1 (color online).　Vertical sections of temperature field for Newtonian (left) and viscoelastic (right) RT simulation at time $t = 3\tau$ starting from the same initial conditions. White (black) regions correspond to hot (cold) fluid. Boussinesq-Oldroyd-B equations (1) are integrated by a standard fully dealiased pseudospectral code on uniform grid at resolution $512 \times 512 \times 1024$ with periodic boundary conditions in the three directions. Physical parameters are $\text{Pr} = \nu/\kappa = 1$, $\text{Sc} = \nu/\kappa_p = 0.3$, $\eta = 0.2$ ($\eta = 0$ for Newtonian run), $\beta g = 0.5$, $\theta_0 = 1$ (Ag = 0.25). Deborah number $\text{De} = \tau_p/\tau$ is De = 0.2. The initial perturbation is seeded in both cases by adding a 10% of white noise (same realization for both runs) to the initial temperature profile in a small layer around the middle plane $z = 0$. Time evolution is stopped when the mixing layer invades 60% of the domain height.

plumes in the viscoelastic case are more coherent and transport heat more efficiently. Quantitatively, the enhancement of the heat transport corresponds to larger values (more than 50% at the final observation time) of the Nusselt number with respect to the Newtonian case.

We consider the incompressible RT system in the Boussinesq approximation generalized to a viscoelastic fluid using the standard Oldroyd-B model [11]

$$\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \nu \nabla^2 \mathbf{u} - \beta \mathbf{g} T + \frac{2\nu\eta}{\tau_p} \nabla \cdot \sigma,$$

$$\partial_t T + \mathbf{u} \cdot \nabla T = \kappa \nabla^2 T,$$

$$\partial_t \sigma + \mathbf{u} \cdot \nabla \sigma = (\nabla \mathbf{u})^T \cdot \sigma + \sigma \cdot (\nabla \mathbf{u}) - \frac{2}{\tau_p}(\sigma - \mathbb{I})$$

$$+ \kappa_p \nabla^2 \sigma, \qquad (1)$$

together with the incompressibility condition $\nabla \cdot \mathbf{u} = 0$. In (1) $T(\mathbf{x}, t)$ is the temperature field, proportional to the density via the thermal expansion coefficient β as $\rho = \rho_0[1 - \beta(T - T_0)]$ (ρ_0 and T_0 are reference values), $\sigma_{ij}(\mathbf{x}, t)$ is the positive symmetric conformation tensor of polymer molecules, $\mathbf{g} = (0, 0, -g)$ is gravity acceleration, ν is the kinematic viscosity, κ is the thermal diffusivity, η is the zero-shear polymer contribution to viscosity (proportional to polymer concentration) and τ_p is the (longest) polymer relaxation time, given, e.g., by the Zimm relation for a linear chain $\tau_p = \nu R_0^3/(\rho k_B T)$ with k_B Boltzmann constant and R_0 the radius of gyration [11]. As customary, the diffusive term $\kappa_p \nabla^2 \sigma$ is added to prevent numerical instabilities [12].

The initial condition for the RT problem is an unstable temperature jump $T(\mathbf{x}, 0) = -(\theta_0/2)\mathrm{sgn}(z)$ in a fluid at rest $\mathbf{u}(\mathbf{x}, 0) = 0$ and coiled polymers $\sigma(\mathbf{x}, 0) = \mathbb{I}$. The physical assumptions under which the set of equations (1) is valid are of small Atwood number $A = (1/2)\beta\theta_0$ (dimensionless density fluctuations) and dilute polymer solution. Experimentally, density fluctuations can also be obtained by some additives (e.g., salts) instead of temperature fluctuations: within the validity of Boussinesq approximation, these situations are described by the same set of equations (1). In the following, all physical quantities are made dimensionless using the vertical side L_z of the computational domain, the temperature jump θ_0, and the characteristic time $\tau = (L_z/Ag)^{1/2}$ as fundamental units. Elasticity of the polymer solution is measured by the Deborah number De, the ratio of polymer relaxation time to a characteristic time of the flow. In our unsteady case De grows in time starting from De $= 0$; therefore, viscoelastic effects are initially absent. An estimate of the largest Deborah number achievable is based on the large scale convective time as De $= \tau_p/\tau$.

Total energy [see Fig. 2(a)] of the solution has an additional elastic contribution to kinetic energy $E = K + \Sigma = (1/2)\langle u^2 \rangle + (\nu\eta/\tau_p)\langle \mathrm{tr}\sigma \rangle$ and the energy balance for (1) reads

$$-\frac{dP}{dt} = \beta g \langle wT \rangle = \frac{dE}{dt} + \varepsilon_\nu + \frac{2\nu\eta}{\tau_p^2}[\langle \mathrm{tr}\sigma \rangle - 3], \quad (2)$$

where $P = -\beta g \langle zT \rangle$ is the potential energy and $\varepsilon_\nu = \nu\langle(\partial_\alpha u_\beta)^2\rangle$ is the viscous dissipation and the last term represents elastic dissipation. Because this last term in (2) is not negative, one might expect that the presence of polymers accelerates the consumption of potential energy with respect to the Newtonian case ($\eta = 0$), as it is indeed observed in Fig. 2(c).

Of course, the speed-up of potential energy consumption due to polymers does not automatically imply the increase of kinetic energy growth. Indeed, part of potential energy is converted to elastic energy Σ by polymer elongation. Nonetheless, Fig. 2(b) shows that kinetic energy for viscoelastic runs is larger than in the Newtonian case (of about 40% at $t = 3.5\tau$). Together with the reduced viscous dissipation [Fig. 2(d)] this is the fingerprint of a "drag reduction" as defined for homogeneous-isotropic turbulence in the absence of a mean flow [6,7] (i.e., a reduction of turbulent energy dissipation which is equivalent to the friction drag reduction occurring in wall bounded flow).

The most important effect of polymers on turbulent velocity is to generate more coherent thermal plume with respect to the Newtonian case, as it is evident in the Fig. 1. The coherence is quantified in the inset of Fig. 3 where we plot the time evolution of the horizontal velocity correlation scale, which becomes larger in the presence of polymers. Another effect, also shown in Fig. 3, is a larger vertical component of the velocity with respect to the horizontal one, i.e., an increased anisotropy of the velocity field. The velocity ratio, which is around 1.8 for the Newtonian case [13], becomes larger than 2.5 for the viscoelastic run. More important, in the viscoelastic case anisotropy persists also at small scales (i.e., in the ratio of
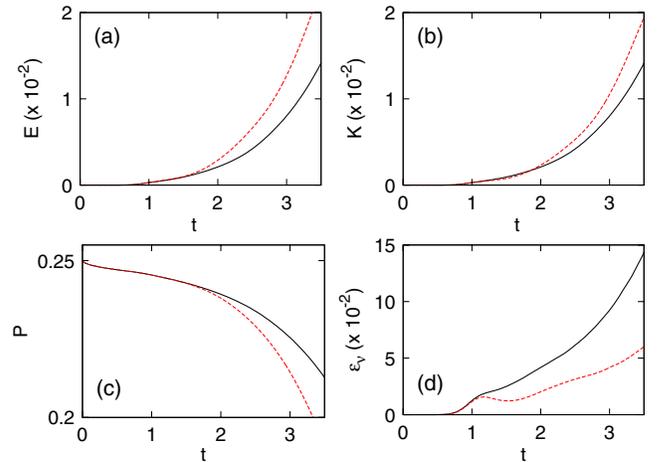

FIG. 2 (color online).   Time evolution of total energy $E = K + \Sigma$ (a), kinetic energy K (b), potential energy P (c), and viscous dissipation ε_ν (d) for the Newtonian run (De $= 0$, black continuous lines) and the viscoelastic run (De $= 0.2$, red dashed lines).

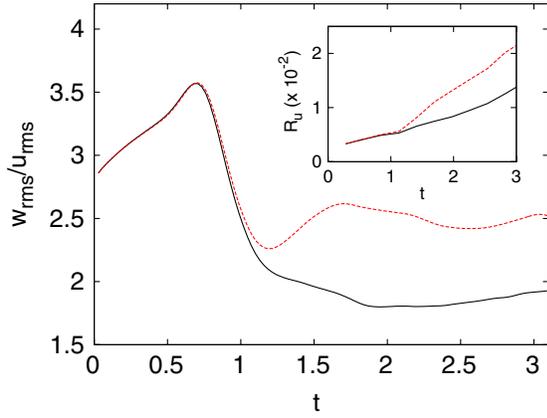FIG. 3 (color online).   Time evolution of velocity anisotropy w_{rms}/u_{rms} (w and u are the vertical and one horizontal velocity components). Inset: time evolution of the horizontal correlation scale R_u defined as the half-width of the velocity correlation function [17]. Black line: Newtonian; red dashed line: viscoelastic.
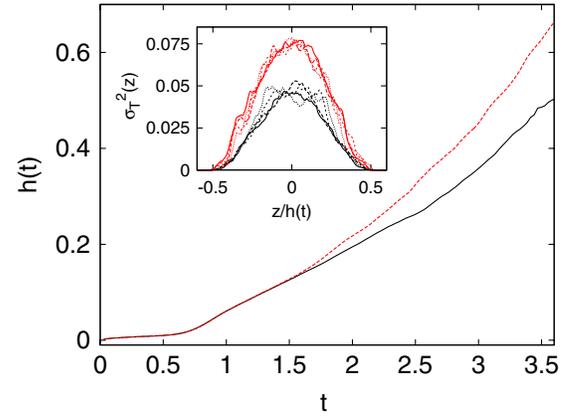

FIG. 4 (color online).   Growth of the mixing layer thickness $h_{0.98}(t)$ defined as the vertical range for which $|\bar{T}(z)| \leq 0.98\theta_0/2$ as a function of dimensionless time t/τ for Newtonian run (black line) and the viscoelastic run (dashed red line) starting from the same initial condition. Inset: temperature variance profiles $\sigma_T^2(z)$ at different times vs $z/h(t)$.

velocity gradients), while it is almost absent in the Newtonian case.

Despite the fact that RT turbulence has vanishing mean flow, a natural mean velocity is provided by the growth of the width $h(t)$ of the turbulent mixing layer where heavy and light fluids are well mixed. For ordinary fluids at small viscosity, as a consequence of constant acceleration, one expects $h(t) = \alpha Agt^2$ where α is a dimensionless parameter to be determined empirically [14–16]. Several definitions of $h(t)$ have been proposed, based on either local or global properties of the mean temperature profile $\bar{T}(z, t)$ (the overbar indicates the average over the horizontal directions) [17–20]. The simplest measure h_r is based on the threshold value of z at which $\bar{T}(z, t)$ reaches a fraction r of the maximum value, i.e., $\bar{T}(\pm h_r(t)/2, t) = \mp r\theta_0/2$.

Figure 4 shows the growth of the mixing layer thickness for both Newtonian and viscoelastic RT turbulence. As already suggested by Fig. 1, in the viscoelastic solution the growth of the mixing layer is faster than in the Newtonian case [i.e. larger $h(t)$, up to 30% at $t = 3.5\tau$]; therefore, we have an effect of polymer drag reduction; i.e., polymers make the transfer of mass more efficient or equivalently, they enhance the large scale mixing.

In Fig. 4 we also plot the variance profiles of temperature field computed at different times for both Newtonian and viscoelastic turbulence. In both cases, $\sigma_T^2(z)$ at different times collapse when plotted as a function of rescaled variable $z/h(t)$. Therefore as turbulence develops in the domain, the level of temperature fluctuations within the mixing layer remains constant as a consequence of new fluctuations introduced by plumes entering from unmixed regions. As Fig. 4 indicates, the level of fluctuations is larger in the viscoelastic case, as a consequence of the reduced mixing at small scales (already observed in Fig. 1). We remark that all together these results are consistent with the accepted phenomenology of viscoelastic

homogeneous-isotropic turbulence where polymers simultaneously reduce energy at small scales and enhance energy contained at large scales [6–8].

The turbulent mixing layer is responsible for the huge enhancement of the heat exchange with respect to the steady conductive case. The dimensionless measure of heat transport efficiency is usually given by the Nusselt number $Nu = \langle wT \rangle h/(\kappa\theta_0)$, the ratio between convective and conductive heat transport. For a convective flow in the fully developed turbulent regime, the Nusselt number is expected to behave as a simple scaling law with respect to the dimensionless temperature jump which defines the Rayleigh number $Ra = Agh^3/(\nu\kappa)$ [21]. For a flow in which boundary layers are irrelevant, as in our case, Kraichnan predicted many years ago the so-called ultimate state of thermal convection for which (a part logarithmic corrections) [21,22]

$$Nu = CPr^{1/2}Ra^{1/2}, \qquad Re = DPr^{-1/2}Ra^{1/2}, \quad (3)$$

where C and D are numerical coefficients. The ultimate state regime has indeed recently been observed in numerical simulations of RT turbulence both in two and three dimensions [13,23–25].

In Fig. 5 we show the evolution of the Rayleigh number Ra, the Nusselt number Nu, and the Reynolds number $Re = u_{rms}h/\nu$ as a function of time. For $t \geq \tau$, when turbulence is developed, all these dimensionless quantities grow following dimensional predictions, i.e., $Ra \sim t^6$ and $Nu \sim Re \sim t^3$. Moreover, it is evident that the effect of polymers is to increase the values attained by those quantities at late time. Of course, most of this effect is due to the enhanced value, for the viscoelastic solution, of the width $h(t)$ of the mixing layer which enters in the definition of all the quantities. Because polymers also reduce small-scale turbulence in thermal plumes, the Nusselt number for viscoelastic turbulence is expected to increase with respect
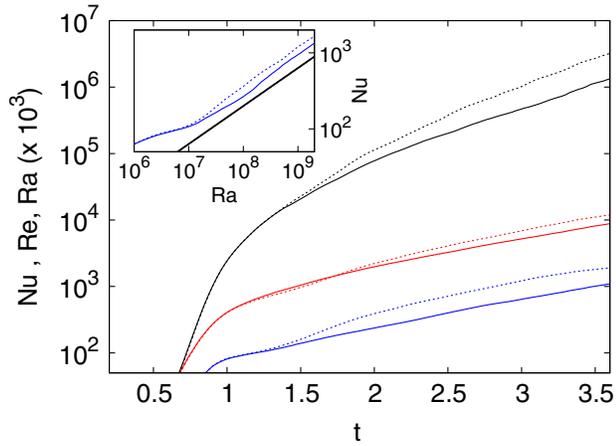
FIG. 5 (color online).   Time evolution of Nusselt number Nu $=$ $\langle wT \rangle/(\kappa \theta_0)$ (blue, lower lines), Reynolds number Re $= u_{\mathrm{rms}}h/\nu$ (red, intermediate lines) and Rayleigh number Ra $= Agh^3/(\nu\kappa)$ (black, upper lines) for the Newtonian run (continuous lines) and the viscoelastic run (dotted lines).

to the Newtonian case also when it is observed as a function of Ra. Indeed, as shown in the inset of Fig. 5, both in the Newtonian and in the viscoelastic cases, Nu $\simeq$ Ra$^{1/2}$ in agreement with the ultimate state regime (3) but with different coefficients, $C_N = 0.022 \pm 0.002$ and $C_{VE} = 0.028 \pm 0.002$, respectively, corresponding to an increase of 27%.

In conclusion, we have exploited high resolution direct numerical simulations to investigate the effects of polymer additives on Rayleigh-Taylor turbulence. There are several advantages in using the present buoyancy-driven turbulence system. The presence of a time evolving mixing layer allows us to quantify the acceleration induced by polymers on a natural (nonzero) mean velocity (the mixing layer growth velocity) at fixed buoyancy forcing, exactly in the same spirit of usual drag reduction in bounded flows. The relatively simple and well understood phenomenology of the heat transport (which follows the Kraichnan's ultimate state regime) allows us to quantify the effects of polymers on the heat transport. While the former feature is specific of the present configuration, the latter occurs in the bulk of the mixing region and therefore we conjecture that our findings hold in situations more general than the specific setup we studied, as indeed a recent investigation seems to indicate [26]. It has been argued that the presence of boundary layers could be responsible for the opposite phenomenon, i.e., heat transfer reduction, which has been recently observed in experiments of Rayleigh–Bénard (RB) convection [27]. Laboratory experiments of RT turbulence, not influenced by boundary layers effects, could be crucial to addressing this issue and to achieving a better understanding of the role of polymer additive on buoyancy-driven turbulent systems.

[1] M. Gad-el-Hak, *Flow Control: Passive, Active, and Reactive Flow Management* (Cambridge University Press, Cambridge, U.K., 2000).
[2] E. Siggia, Annu. Rev. Fluid Mech. **26**, 137 (1994).
[3] J. Warnatz, U. Maas, and R. W. Dibble, *Combustion: Physical and Chemical Fundamentals, Modeling and Simulation, Experiments, Pollutant Formation* (Springer, New York, 2001).
[4] J. L. Lumley, Annu. Rev. Fluid Mech. **1**, 367 (1969).
[5] K. R. Sreenivasan and M. C. White, J. Fluid Mech. **409**, 149 (2000).
[6] R. Benzi, E. De Angelis, R. Govindarajan, and I. Procaccia, Phys. Rev. E **68**, 016308 (2003).
[7] E. De Angelys, C. M. Casciola, R. Benzi, and R. Piva, J. Fluid Mech. **531**, 1 (2005).
[8] S. Berti, A. Bistagnino, G. Boffetta, A. Celani, and S. Musacchio, Europhys. Lett. **76**, 63 (2006).
[9] G. Boffetta, A. Celani, and A. Mazzino, Phys. Rev. E **71**, 036307 (2005).
[10] G. Boffetta, A. Mazzino, S. Musacchio, and L. Vozella, J. Fluid Mech. **643**, 127 (2010).
[11] R. B. Bird, O. Hassager, R. C. Armstrong, and C. F. Curtiss, *Dynamics of Polymeric Liquids* (Wiley-Interscience, New York, 1987).
[12] R. Sureshkumar and A. Beris, J. Non-Newtonian Fluid Mech. **60**, 53 (1995).
[13] G. Boffetta, A. Mazzino, S. Musacchio, and L. Vozella, Phys. Rev. E **79**, 065301(R) (2009).
[14] P. Ramaprabhu and M. Andrews, Phys. Fluids **16**, L59 (2004).
[15] G. Dimonte *et al.*, Phys. Fluids **16**, 1668 (2004).
[16] K. Kadau, C. Rosenblatt, J. L. Barber, T. C. Germann, Z. Huang, P. Carls, and B. J. Alder, Proc. Natl. Acad. Sci. U.S.A. **104**, 7741 (2007).
[17] N. Vladimirova and M. Chertkov, Phys. Fluids **21**, 015102 (2009).
[18] M. J. Andrews and D. B. Spalding, Phys. Fluids A **2**, 922 (1990).
[19] S. Dalziel, P. Linden, and D. Youngs, J. Fluid Mech. **399**, 1 (1999).
[20] W. H. Cabot and A. W. Cook, Nature Phys. **2**, 562 (2006).
[21] S. Grossmann and D. Lohse, J. Fluid Mech. **407**, 27 (2000).
[22] R. H. Kraichnan, Phys. Fluids **5**, 1374 (1962).
[23] D. Lohse and F. Toschi, Phys. Rev. Lett. **90**, 034502 (2003).
[24] A. Celani, A. Mazzino, and L. Vozella, Phys. Rev. Lett. **96**, 134504 (2006).
[25] G. Boffetta, F. De Lillo, and S. Musacchio, Phys. Rev. Lett. **104**, 034505 (2010).
[26] R. Benzi, E. S. C. Ching, and E. De Angelis, Phys. Rev. Lett. **104**, 024502 (2010).
[27] G. Ahlers and A. Nikolaenko, Phys. Rev. Lett. **104**, 034503 (2010).