

1) Scattering a 2 corpi: $a + b \rightarrow c + d$

Dimostrazione della relazione:

$$s + t + u = m_A^2 + m_B^2 + m_C^2 + m_D^2$$

$$s = (p_A + p_B)^2$$

$$t = (p_A - p_C)^2$$

$$u = (p_A - p_D)^2$$

$$s + t + u = (p_A + p_B)^2 + (p_A - p_C)^2 + (p_A - p_D)^2$$

$$s + t + u = p_A^2 + p_B^2 + 2p_A \cdot p_B + p_A^2 + p_C^2 - 2p_A \cdot p_C + p_A^2 + p_D^2 - 2p_A \cdot p_D$$

$$s + t + u = 3m_A^2 + m_B^2 + m_C^2 + m_D^2 + 2(p_A \cdot p_B - p_A \cdot p_C - p_A \cdot p_D)$$

$$s + t + u = 3m_A^2 + m_B^2 + m_C^2 + m_D^2 + 2p_A \cdot (p_B - p_C - p_D)$$

$$p_A + p_B = p_C + p_D \rightarrow p_B - p_C - p_D = -p_A$$

$$\rightarrow s + t + u = 3m_A^2 + m_B^2 + m_C^2 + m_D^2 + 2p_A \cdot (-p_A) = m_A^2 + m_B^2 + m_C^2 + m_D^2$$

2) Decadimento in 3 corpi: $M \rightarrow m_1 + m_2 + m_3$

Dimostrazione della relazione:

$$m_{12}^2 + m_{13}^2 + m_{23}^2 = m_1^2 + m_2^2 + m_3^2 + M^2$$

$$m_{12}^2 + m_{13}^2 + m_{23}^2 = (p_1 + p_2)^2 + (p_1 + p_3)^2 + (p_2 + p_3)^2$$

$$2(p_1^2 + p_2^2 + p_3^2) + 2(p_1 \cdot p_2 + p_1 \cdot p_3 + p_2 \cdot p_3) =$$

$$2(m_1^2 + m_2^2 + m_3^2) + 2(p_2 \cdot (P - p_2 - p_3) + p_3 \cdot (P - p_1 - p_3) + p_1 \cdot (P - p_1 - p_2)) =$$

$$2(m_1^2 + m_2^2 + m_3^2) + 2(p_2 \cdot P - m_2^2 - p_2 \cdot p_3 + p_3 \cdot P - m_3^2 - p_1 \cdot p_3 + p_1 \cdot P - m_1^2 - p_1 \cdot p_2) =$$

$$2 \left(\underbrace{P \cdot (p_1 + p_2 + p_3)}_{=P} - p_1 \cdot p_2 - p_1 \cdot p_3 - p_2 \cdot p_3 \right) =$$

$$m_{12}^2 + m_{13}^2 + m_{23}^2 = 2(M^2 - p_1 \cdot p_2 - p_1 \cdot p_3 - p_2 \cdot p_3)$$

$$p_1 \cdot p_2 + p_1 \cdot p_3 + p_2 \cdot p_3 = p_2 \cdot (P - p_2 - p_3) + p_3 \cdot (P - p_1 - p_3) + p_1 \cdot (P - p_1 - p_2)$$

$$p_1 \cdot p_2 + p_1 \cdot p_3 + p_2 \cdot p_3 = -(m_1^2 + m_2^2 + m_3^2) + M^2 - p_1 \cdot p_2 + p_1 \cdot p_3 + p_2 \cdot p_3$$

$$\rightarrow p_1 \cdot p_2 + p_1 \cdot p_3 + p_2 \cdot p_3 = \frac{M^2 - (m_1^2 + m_2^2 + m_3^2)}{2}$$

$$\rightarrow m_{12}^2 + m_{13}^2 + m_{23}^2 = 2 \left[M^2 - \frac{M^2 - (m_1^2 + m_2^2 + m_3^2)}{2} \right]$$

$$\rightarrow m_{12}^2 + m_{13}^2 + m_{23}^2 = M^2 + m_1^2 + m_2^2 + m_3^2$$