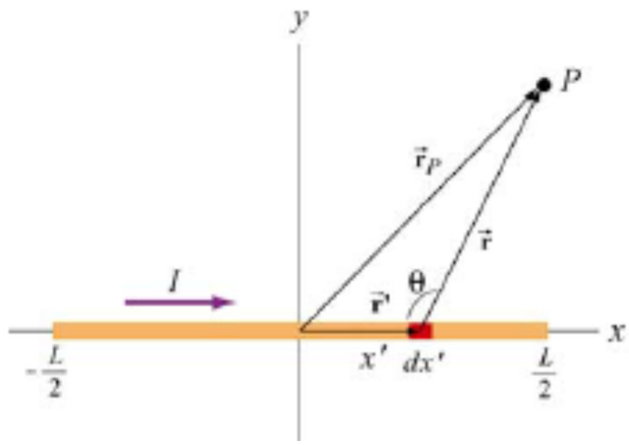
C. magnetico di un segmento finito di conduttore rettilineo percorso da corrente in un punto generico



Nel piano definito da segmento e punto P:

$$d\mathbf{B} = \frac{\mu_0 I}{4\pi}\frac{d\mathbf{s}\times\hat{\mathbf{r}}}{r^2}, \quad \text{Laplace I}$$

$$r = \left[(x-x')^2 + y^2\right]^{1/2}$$

$$\hat{\mathbf{r}} = \frac{(x-x')\hat{\mathbf{i}} + y\hat{\mathbf{j}}}{\left[(x-x')^2 + y^2\right]^{1/2}}$$

$$d\mathbf{s} = dx'\hat{\mathbf{i}}$$

$$\to d\mathbf{s}\times\hat{\mathbf{r}} = dx'\hat{\mathbf{i}}\times\frac{(x-x')\hat{\mathbf{i}} + y\hat{\mathbf{j}}}{\left[(x-x')^2 + y^2\right]^{1/2}} = \frac{ydx'\hat{\mathbf{k}}}{\left[(x-x')^2 + y^2\right]^{1/2}}$$

$$\to d\mathbf{B} = \frac{\mu_0 I}{4\pi}\frac{ydx'\hat{\mathbf{k}}}{\left[(x-x')^2 + y^2\right]^{1/2}}\frac{1}{\left[(x-x')^2 + y^2\right]} = \frac{\mu_0 I}{4\pi}\frac{ydx'\hat{\mathbf{k}}}{\left[(x-x')^2 + y^2\right]^{3/2}}$$

$$\rightarrow \mathbf{B} = \int d\mathbf{B} = \int_{-\frac{L}{2}}^{+\frac{L}{2}} \frac{\mu_0 I}{4\pi} \frac{y dx' \hat{\mathbf{k}}}{\left[(x-x')^2 + y^2\right]^{3/2}} = \frac{\mu_0 I y}{4\pi} \hat{\mathbf{k}} \int_{-\frac{L}{2}}^{+\frac{L}{2}} \frac{dx'}{\left[(x-x')^2 + y^2\right]^{3/2}}$$

$$u = x - x' \rightarrow dx' = -du$$

$$\int \frac{dx'}{\left[(x-x')^2 + y^2\right]^{3/2}} = -\int \frac{du}{\left[u^2 + y^2\right]^{3/2}} = -\int \frac{du}{y^3 \left[\frac{u^2}{y^2} + 1\right]^{3/2}}$$

$$v = \frac{u}{y} \rightarrow du = y dv$$

$$-\int \frac{du}{y^3 \left[\frac{u^2}{y^2} + 1\right]^{3/2}} = -\int \frac{y dv}{y^3 \left[v^2 + 1\right]^{3/2}} = -\frac{1}{y^2} \int \frac{dv}{\left[v^2 + 1\right]^{3/2}}$$

$$v = \tan\varphi \rightarrow dv = \frac{d\varphi}{\cos^2\varphi}$$

$$\rightarrow -\frac{1}{y^2} \int \frac{dv}{\left[v^2+1\right]^{3/2}} = -\frac{1}{y^2} \int \frac{d\varphi}{\cos^2\varphi \left[\tan^2\varphi + 1\right]^{3/2}} = -\frac{1}{y^2} \int \frac{\cos^3\varphi d\varphi}{\cos^2\varphi} = -\frac{1}{y^2} \int \cos\varphi d\varphi$$

$$\rightarrow -\frac{1}{y^2} \int \frac{dv}{\left[v^2+1\right]^{3/2}} = -\frac{1}{y^2} \sin\varphi = -\frac{1}{y^2} \frac{v}{\left[v^2+1\right]^{1/2}} = -\frac{1}{y^2} \frac{\frac{u}{y}}{\left[\frac{u^2}{y^2}+1\right]^{1/2}} = -\frac{1}{y^2} \frac{u}{\left[u^2+y^2\right]^{1/2}}$$

$$\rightarrow -\frac{1}{y^2} \frac{u}{\left[u^2+y^2\right]^{1/2}} = -\frac{1}{y^2} \frac{x-x'}{\left[(x-x')^2+y^2\right]^{1/2}}$$

$$\rightarrow \mathbf{B} = -\frac{\mu_0 I}{4\pi y} \hat{\mathbf{k}} \left. \frac{x-x'}{\left[(x-x')^2+y^2\right]^{1/2}} \right|_{-\frac{L}{2}}^{+\frac{L}{2}} = -\frac{\mu_0 I}{4\pi y} \hat{\mathbf{k}} \left[\frac{x-\frac{L}{2}}{\left[\left(x-\frac{L}{2}\right)^2+y^2\right]^{1/2}} - \frac{x+\frac{L}{2}}{\left[\left(x+\frac{L}{2}\right)^2+y^2\right]^{1/2}} \right]$$

Casi limite:

a) $x = 0 \quad \rightarrow \frac{\mu_0 I}{4\pi y} \hat{\mathbf{k}} \left[\frac{L}{\left[\left(\frac{L}{2}\right)^2 + y^2\right]^{1/2}} \right] = \frac{\mu_0 I}{4\pi y} \hat{\mathbf{k}} \frac{L}{L\left(\frac{1}{4} + \frac{y^2}{L^2}\right)^{1/2}} = \frac{\mu_0 I}{2\pi y} \frac{L}{\left(L^2 + 4y^2\right)^{1/2}} \hat{\mathbf{k}}$

b) $L \rightarrow \infty \quad \rightarrow \frac{\mu_0 I}{2\pi y} \hat{\mathbf{k}}$ Biot-Savart