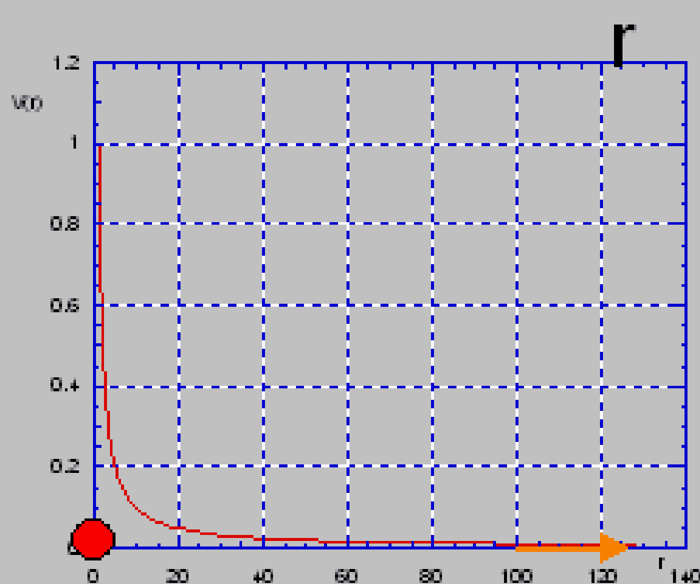
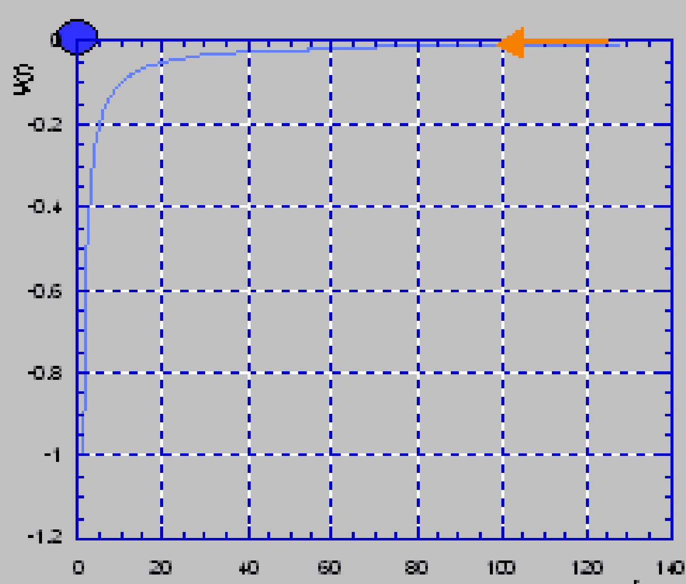
Campo elettrico e potenziale

$$\mathbf{E} = -\nabla V$$

Carica puntiforme



Carica +va:
V decrescente
Derivata -va
con segno - OK!



Carica -va:
V crescente
Derivata +va
con segno - OK!

Dal potenziale al campo

Es.: carica puntiforme

$$\boldsymbol{E} = -\nabla V$$

$$E_x = -\frac{\partial V}{\partial x},\ \text{etc}$$

$$V = \frac{q}{4\pi\varepsilon_0}\frac{1}{r}, r = \left(x^2 + y^2 + z^2\right)^{1/2}$$

$$\frac{\partial V}{\partial x} = \frac{dV}{dr}\frac{\partial r}{\partial x} = -\frac{q}{4\pi\varepsilon_0}\frac{1}{r^2}\frac{2x}{2\left(x^2 + y^2 + z^2\right)^{1/2}}$$

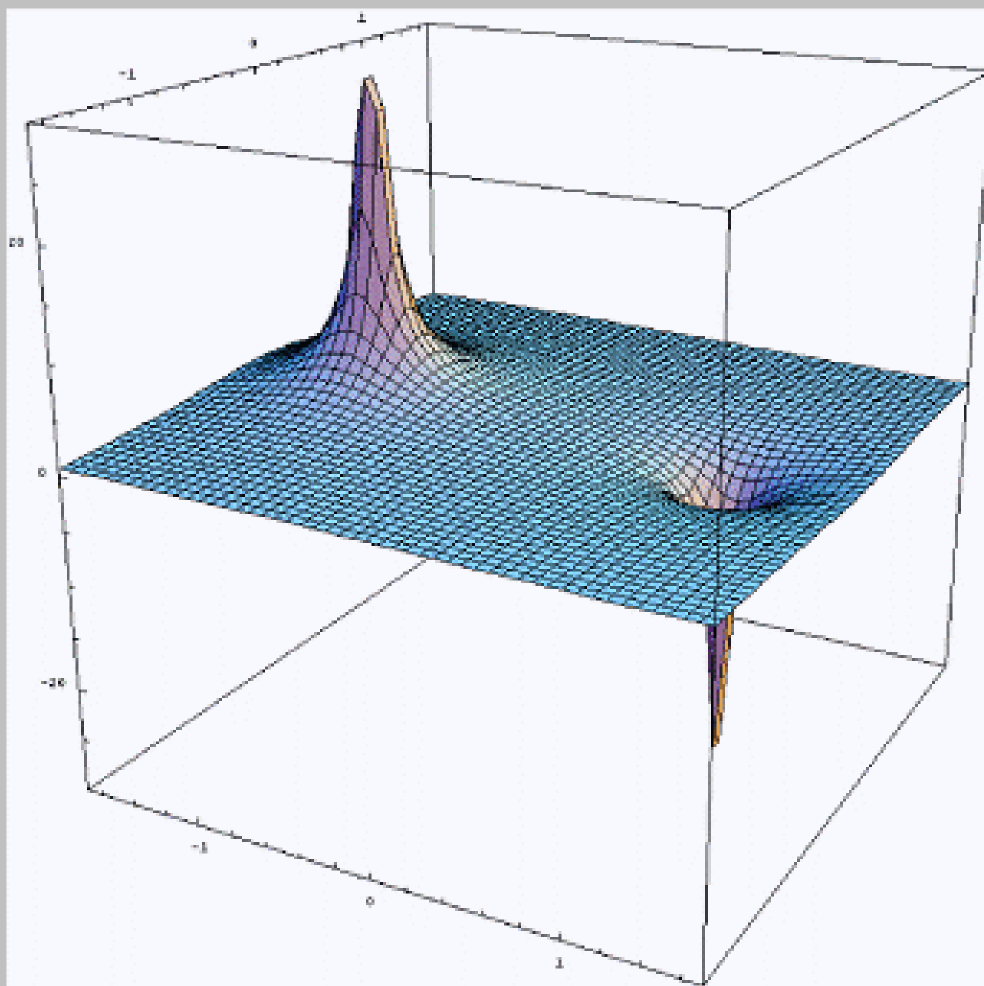
$$E_x = -\left(-\frac{q}{4\pi\varepsilon_0}\frac{x}{r^3}\right), E_y, E_z \ \text{ analoghi}$$

$$\mathbf{E} = E_x\hat{\mathbf{i}} + E_y\hat{\mathbf{j}} + E_z\hat{\mathbf{k}} = \frac{q}{4\pi\varepsilon_0}\frac{x}{r^3}\hat{\mathbf{i}} + \frac{q}{4\pi\varepsilon_0}\frac{y}{r^3}\hat{\mathbf{j}} + \frac{q}{4\pi\varepsilon_0}\frac{z}{r^3}\hat{\mathbf{k}}$$

$$\mathbf{E} = \frac{q}{4\pi\varepsilon_0}\frac{\mathbf{r}}{r^3} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{\mathbf{r}}$$

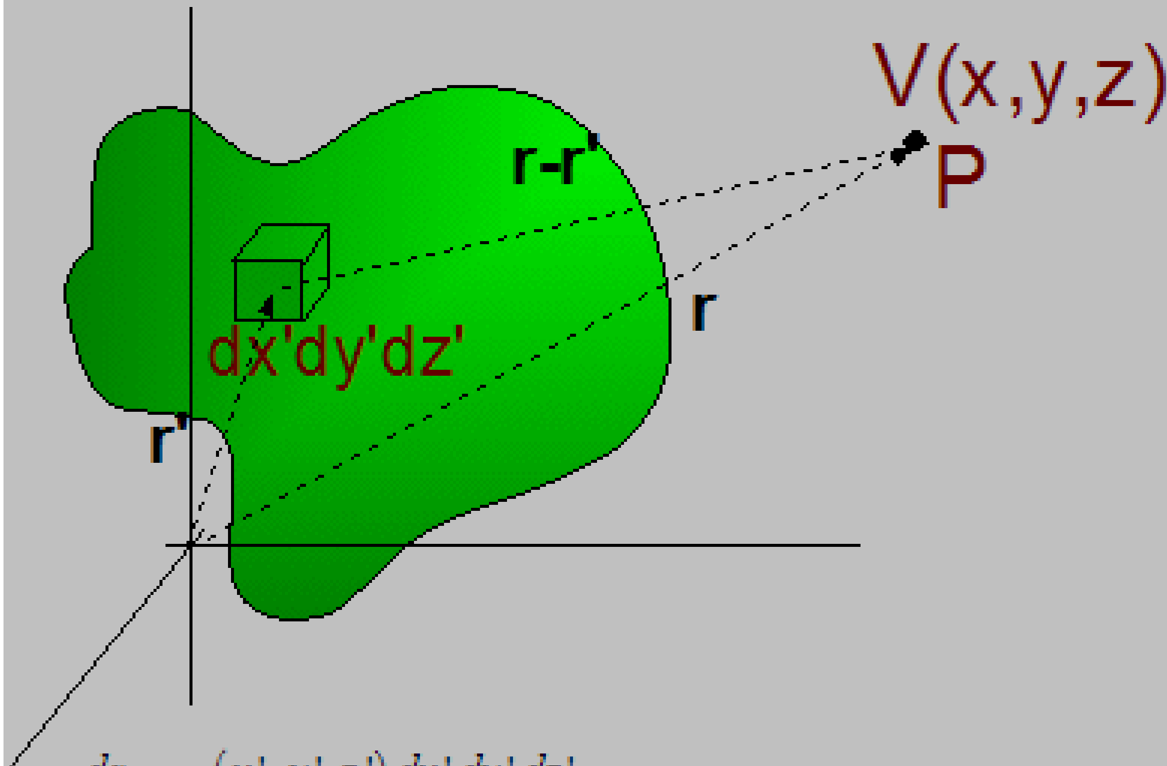
Campo coulombiano

Potenziale: 2 cariche opposte



Potenziale in funzione di (x,y)
(da V.Gracco, Fis. generale II)

Potenziale: distribuzione continua



$$dq = \rho(x', y', z')\,dx'dy'dz'$$

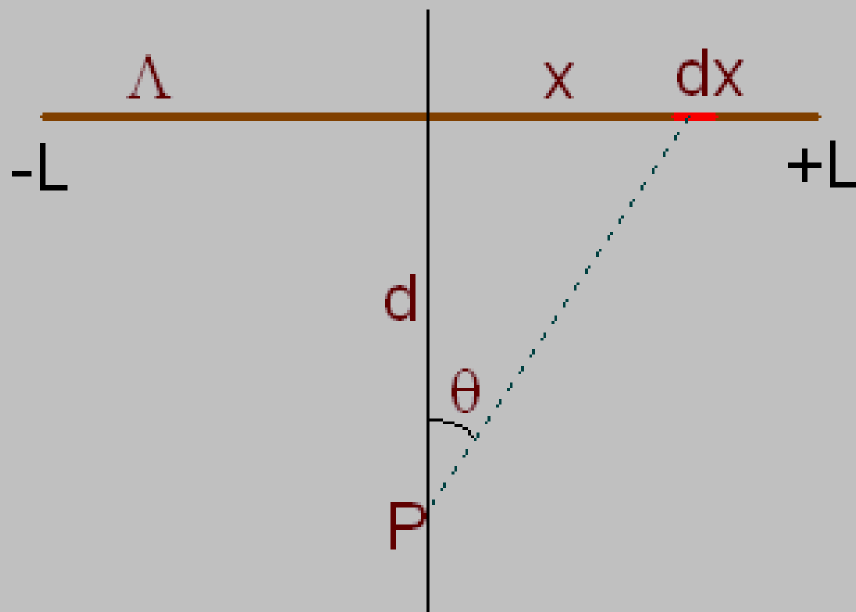
$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{|\mathbf{r}-\mathbf{r}'|} = \frac{1}{4\pi\varepsilon_0}\frac{\rho\,dx'dy'dz'}{|\mathbf{r}-\mathbf{r}'|}$$

$$|\mathbf{r}-\mathbf{r}'|^2 = (x-x')^2 + (y-y')^2 + (z-z')^2$$

$$V = \int dV = \int_{\substack{\text{volume}\\\text{della}\\\text{carica}}} \frac{1}{4\pi\varepsilon_0}\frac{\rho\,dx'dy'dz'}{\left[(x-x')^2+(y-y')^2+(z-z')^2\right]^{1/2}}$$

Simile a c.elettrico, ma piu' semplice: una sola funzione (V= scalare)

Esempio: filo finito



$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\frac{\Lambda dx}{r}$$

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{\Lambda dx}{\left(d^2 + x^2\right)^{1/2}}$$

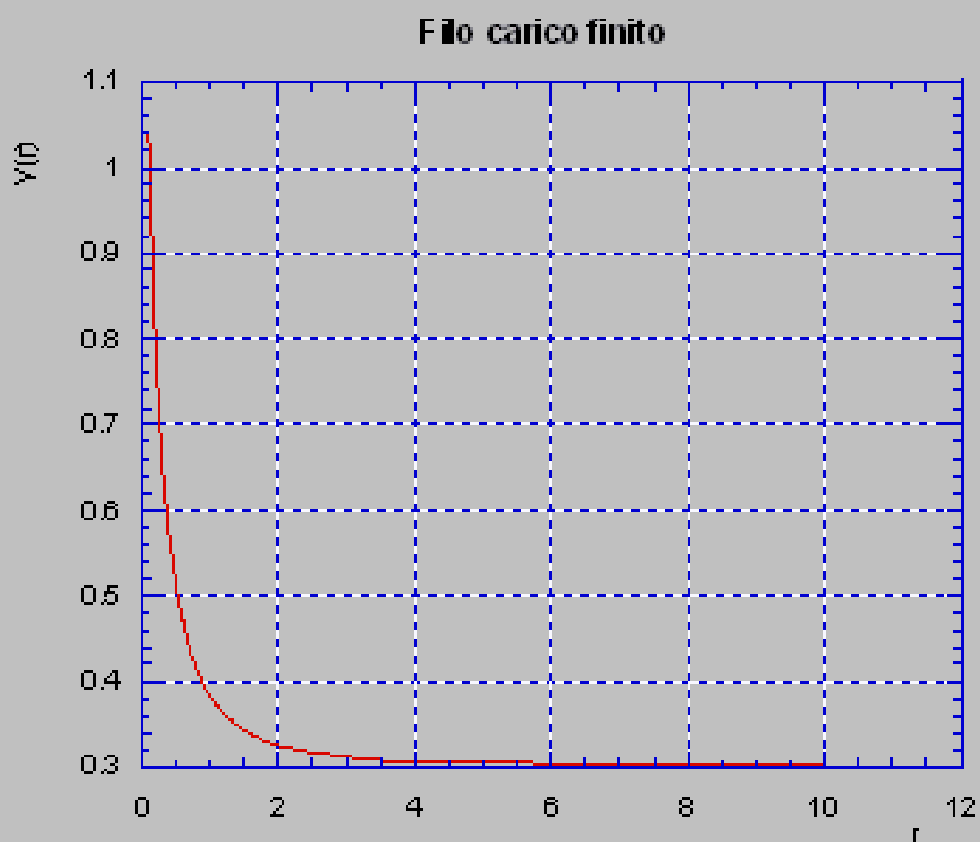
$$\rightarrow V = \frac{2\Lambda}{4\pi\varepsilon_0}\int_0^{+L}\frac{dx}{\left(d^2 + x^2\right)^{1/2}} = \frac{2\Lambda}{4\pi\varepsilon_0}\mathbf{ln}\left(x + \sqrt{x^2 + d^2}\right)\Big|_0^L$$

$$\rightarrow V = \frac{\Lambda}{2\pi\varepsilon_0}\left[\mathbf{ln}\left(L + \sqrt{L^2 + d^2}\right) - \mathbf{ln}\, d\right]$$

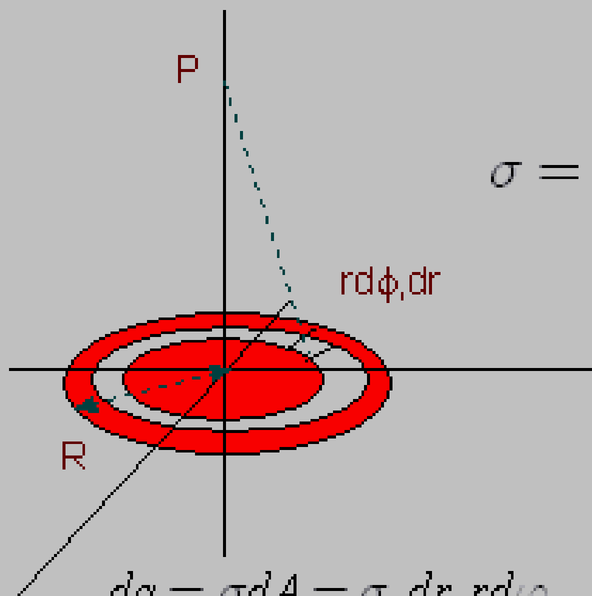
$$\rightarrow V = \frac{\Lambda}{2\pi\varepsilon_0}\left[\mathbf{ln}\frac{L + \sqrt{L^2 + d^2}}{d}\right]$$

Filo carico finito

Grafico del potenziale: *V(r)* vs *r*



Esempio: disco carico



$$\sigma = \frac{Q}{\pi R^2}$$

$$dq = \sigma dA = \sigma \ dr \ rd\varphi$$

$$u^2 = r^2 + z^2$$

$$dV = \frac{1}{4\pi\varepsilon_0} \frac{dq}{u} = \frac{1}{4\pi\varepsilon_0} \frac{\sigma \ dr \ rd\varphi}{\left(r^2 + z^2\right)^{1/2}}$$

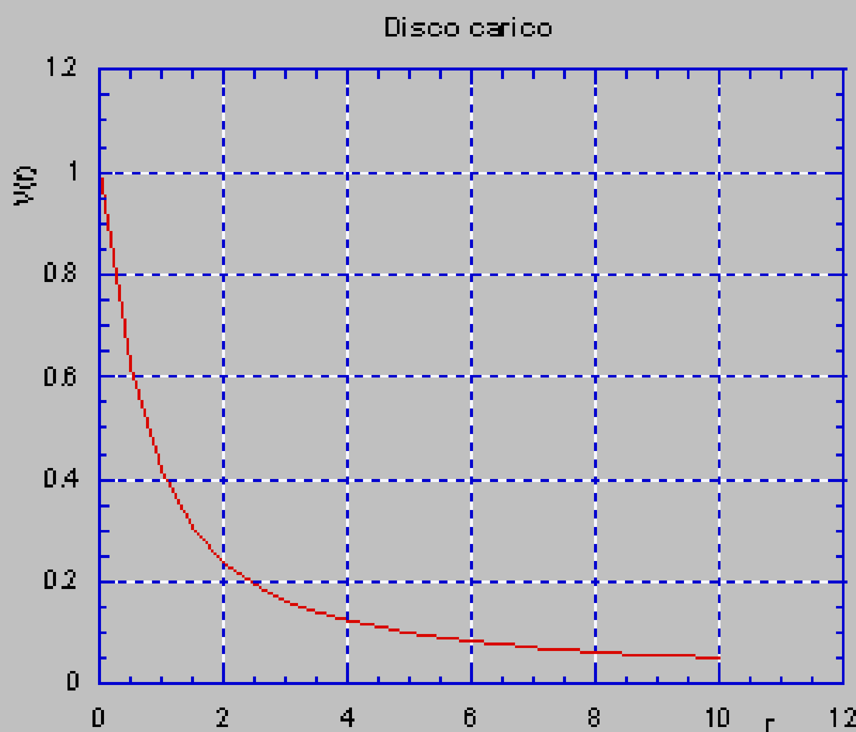
$$\rightarrow V = \frac{1}{4\pi\varepsilon_0} \iint_{disco} \frac{\sigma \ dr \ rd\varphi}{\left(r^2 + z^2\right)^{1/2}}$$

$$\rightarrow V = \frac{2\pi\sigma}{4\pi\varepsilon_0} \int_0^R \frac{r \ dr}{\left(r^2 + z^2\right)^{1/2}} =$$

$$V = \frac{\sigma}{2\varepsilon_0} \left(r^2 + z^2\right)^{1/2} \Big|_0^R = \frac{\sigma}{2\varepsilon_0} \left[\left(R^2 + z^2\right)^{1/2} - z \right]$$

Disco carico

Grafico del potenziale: *V(r)* vs *r*

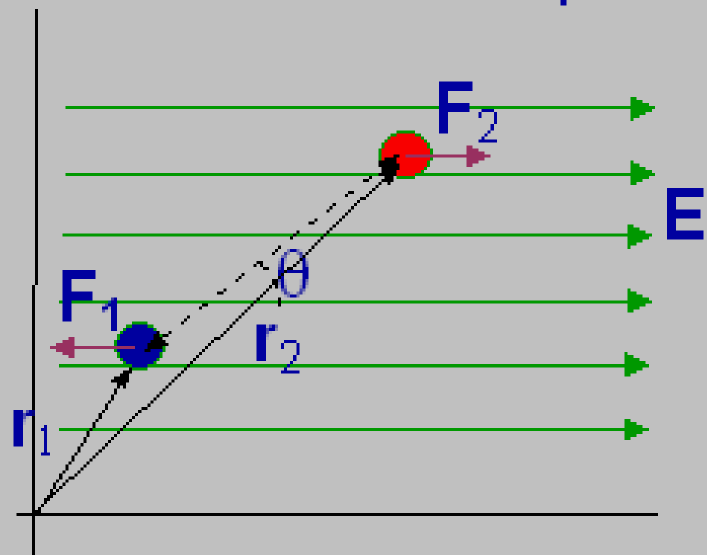


Campo di un dipolo elettrico:
gia' visto a inizio lezioni per il caso particolare di punti sull'asse di simmetria
Tuttavia, interessante calcolarlo in modo generale

Per questo: v.nota Potenziali e Campi di Dipolo

Forza su un dipolo elettrico

Dipolo immerso in campo uniforme:



Forza totale = 0

Coppia: momento meccanico

$$\mathbf{M} = \mathbf{r}_1 \times \mathbf{F}_1 + \mathbf{r}_2 \times \mathbf{F}_2 = \mathbf{r}_1 \times (-\mathbf{F}) + \mathbf{r}_2 \times \mathbf{F}$$

$$\rightarrow \mathbf{M} = (\mathbf{r}_2 - \mathbf{r}_1) \times \mathbf{F} = \mathbf{a} \times (q\mathbf{E}) = (q\mathbf{a}) \times \mathbf{E} = \mathbf{p} \times \mathbf{E}$$

Posizione di equilibrio: **M**=0

$$\rightarrow \mathbf{p} \parallel \mathbf{E}$$

Energia potenziale di un dipolo in un campo esterno

Spostamento angolare infinitesimo lavoro del momento meccanico:

$$dL = \mathbf{F}_1 \cdot d\mathbf{s}_1 + \mathbf{F}_2 \cdot d\mathbf{s}_2$$
$$= \mathbf{F}_1 \cdot \mathbf{r}_1 d\theta + \mathbf{F}_2 \cdot \mathbf{r}_2 d\theta$$
$$= \mathbf{F} \cdot \left(\mathbf{r}_1 d\theta - \mathbf{r}_2 d\theta \right) = M d\theta$$

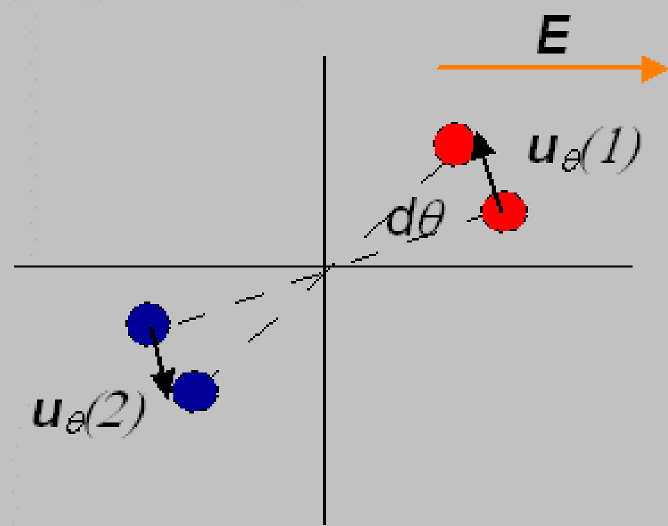
Spostamento finito da θ_0 a θ:

$$L = \int dL = \int_{\theta_0}^{\theta} M d\theta = \int_{\theta_0}^{\theta} -pE \sin\theta d\theta$$
$$\rightarrow L = pE\left(\cos\theta - \cos\theta_0 \right)$$

Variazione en. potenziale:

$$L = pE\left(\cos\theta - \cos\theta_0 \right)$$
$$L = -\Delta U = -\left(U(\theta) - U(\theta_0) \right)$$
$$\rightarrow U(\theta) = -pE\cos\theta = -\mathbf{p} \cdot \mathbf{E}$$

Calcolo lavoro elementare



$$dL = dL_1 + dL_2 = \boldsymbol{F}_1 \cdot d\boldsymbol{s}_1 + \boldsymbol{F}_2 \cdot d\boldsymbol{s}_2$$

$$\boldsymbol{F}_1 = -\boldsymbol{F}_2 = \boldsymbol{F} = q\boldsymbol{E}$$

$$d\boldsymbol{s}_1 = |\boldsymbol{r}_1| d\theta \hat{\boldsymbol{u}}_\theta = \frac{d}{2} d\theta \hat{\boldsymbol{u}}_\theta(1)$$

$$d\boldsymbol{s}_2 = |\boldsymbol{r}_2| d\theta \hat{\boldsymbol{u}}_\theta = \frac{d}{2} d\theta \hat{\boldsymbol{u}}_\theta(2)$$

$$\rightarrow dL = \boldsymbol{F} \cdot |\boldsymbol{r}_1| d\theta \hat{\boldsymbol{u}}_\theta(1) - \boldsymbol{F} \cdot |\boldsymbol{r}_2| d\theta \hat{\boldsymbol{u}}_\theta(2) = \boldsymbol{F} \cdot \left[\hat{\boldsymbol{u}}_\theta(1) - \hat{\boldsymbol{u}}_\theta(2)\right] \frac{d}{2} d\theta$$

$$\rightarrow dL = q\boldsymbol{E} \cdot \left[\hat{\boldsymbol{u}}_\theta(1) - \hat{\boldsymbol{u}}_\theta(2)\right] \frac{d}{2} d\theta$$

$$\boldsymbol{E} \cdot \hat{\boldsymbol{u}}_\theta(1) = |\boldsymbol{E}| \cos(\theta + \pi/2) = -|\boldsymbol{E}| \sin\theta$$

$$\boldsymbol{E} \cdot \hat{\boldsymbol{u}}_\theta(2) = |\boldsymbol{E}| \cos(\theta + 3\pi/2) = +|\boldsymbol{E}| \sin\theta$$

$$\rightarrow dL = q\left(-2|\boldsymbol{E}|\sin\theta\right)\frac{d}{2} d\theta = -|\boldsymbol{p}||\boldsymbol{E}|\sin\theta d\theta = -|\boldsymbol{p} \times \boldsymbol{E}| d\theta$$

$$\rightarrow \boldsymbol{dL = -(p \times E) d\theta = |M| d\theta}$$