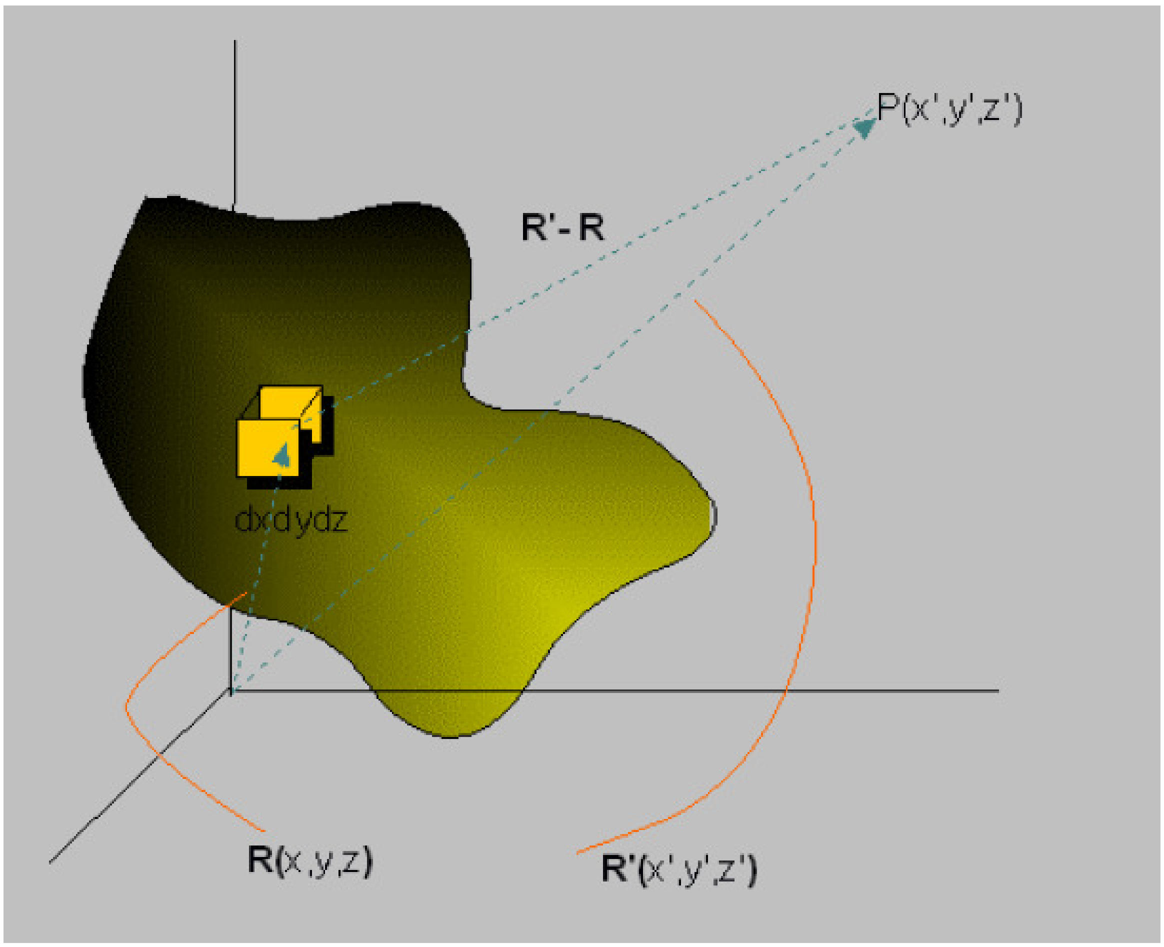
Caso di una distribuzione continua di carica:

$\rho(x,y,z) = \lim_{\Delta V \to 0} \frac{\Delta q}{\Delta V}$ densita' volumetrica di carica

Per sistemi in 2D e 1D:

$\sigma(x,y,z) = \lim_{\Delta S \to 0} \frac{\Delta q}{\Delta S}$ densita' superficiale di carica

$\lambda(x,y,z) = \lim_{\Delta l \to 0} \frac{\Delta q}{\Delta l}$ densita' lineare di carica



Caso piu' semplice: ρ, σ, λ costanti

C. elettrico totale in un punto generico:

Somma di contributi coulombiani

$$\mathbf{E} \approx \frac{1}{4\pi\varepsilon_0} \sum_{volumetti} \frac{dq}{\left|\mathbf{R}' - \mathbf{R}\right|^2} \frac{\mathbf{R}' - \mathbf{R}}{\left|\mathbf{R}' - \mathbf{R}\right|}$$

$$dq = \rho\left(x, y, z\right) dxdydz = \rho dV$$

$$\rightarrow \mathbf{E} = \frac{1}{4\pi\varepsilon_0} \iiint \frac{\rho dV}{\left|\mathbf{R}' - \mathbf{R}\right|^2} \frac{\mathbf{R}' - \mathbf{R}}{\left|\mathbf{R}' - \mathbf{R}\right|}$$

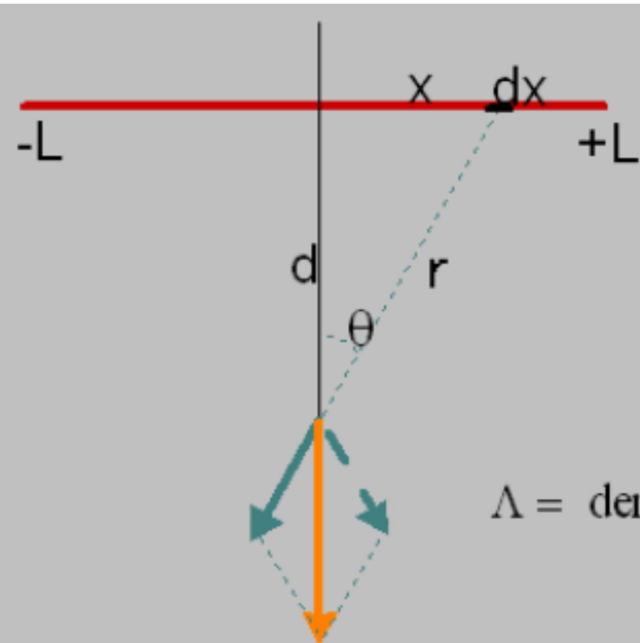
Distribuzione superficiale di carica:

$$\boldsymbol{E} = \frac{1}{4\pi\varepsilon_0} \iint \frac{\sigma dS}{\left|\boldsymbol{R} - \boldsymbol{R}'\right|^2} \frac{\boldsymbol{R} - \boldsymbol{R}'}{\left|\boldsymbol{R} - \boldsymbol{R}'\right|}$$ int. di superficie

Distribuzione lineare di carica:

$$\boldsymbol{E} = \frac{1}{4\pi\varepsilon_0} \int \frac{\lambda dl}{\left|\boldsymbol{R} - \boldsymbol{R}'\right|^2} \frac{\boldsymbol{R} - \boldsymbol{R}'}{\left|\boldsymbol{R} - \boldsymbol{R}'\right|}$$ int. di linea (NB non curvilineo!)

Es: Segmento carico



$\Lambda =$ densita' lineare di carica

$$d\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{d^2 + x^2}\hat{\mathbf{r}} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\hat{\mathbf{r}}$$

$$dq = \Lambda dx$$

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{\Lambda dx}{r^2}\cos\theta$$

$$x = d\tan\theta \rightarrow dx = d\frac{1}{\cos^2\theta}d\theta$$

$$d = r\cos\theta \rightarrow r^2 = \frac{d^2}{\cos^2\theta}$$

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{\Lambda d\frac{1}{\cos^2\theta}d\theta}{\frac{d^2}{\cos^2\theta}}\cos\theta = \frac{1}{4\pi\varepsilon_0}\frac{\Lambda}{d}\cos\theta d\theta$$

$$E = \int dE = \int_{-\theta_{max}}^{+\theta_{max}} \frac{1}{4\pi\varepsilon_0}\frac{\Lambda}{d}\cos\theta d\theta = \frac{1}{4\pi\varepsilon_0}\frac{2\Lambda}{d}\sin\theta_{max} = \frac{1}{2\pi\varepsilon_0}\frac{\Lambda}{d}\sin\theta_{max}$$

Riscrittura utile:

$$E = \frac{1}{2\pi\varepsilon_0}\frac{\Lambda}{d}\sin\theta_{max}$$

$$\sin\theta_{max} = \frac{L}{\sqrt{d^2 + L^2}}$$

$$\Lambda = \frac{Q}{2L}$$

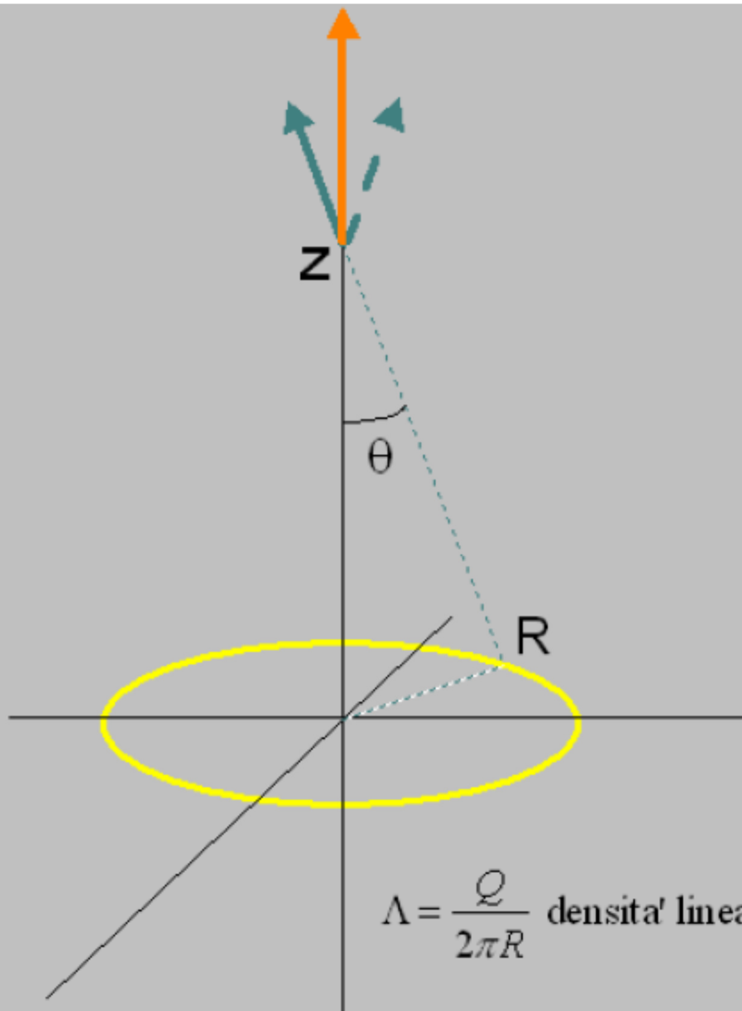
$$\rightarrow E = \frac{1}{2\pi\varepsilon_0}\frac{Q}{2Ld}\frac{L}{\sqrt{d^2 + L^2}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{d\sqrt{d^2 + L^2}}$$

Andamenti limite:

$$\lim_{d \gg L} E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{d^2}$$ coulombiano

$$\lim_{L \gg d} E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{dL} = \frac{1}{2\pi\varepsilon_0}\frac{\Lambda}{d}$$ filo infinito carico

Es: Anello carico



$$\Lambda = \frac{Q}{2\pi R} \quad \text{densita' lineare di carica}$$

$$d\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{R^2+z^2}\hat{\mathbf{r}}$$

$$dq = \Lambda ds = \frac{Q}{2\pi R}ds$$

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{Q}{2\pi R\left(R^2+z^2\right)}\cos\theta ds = \frac{1}{4\pi\varepsilon_0}\frac{Q}{2\pi R\left(R^2+z^2\right)}\frac{R}{\left(R^2+z^2\right)^{1/2}}ds$$

$$E = \int dE = \int_0^{2\pi R}\frac{1}{4\pi\varepsilon_0}\frac{Q}{2\pi R\left(R^2+z^2\right)}\frac{R}{\left(R^2+z^2\right)^{1/2}}ds = \frac{1}{4\pi\varepsilon_0}\frac{QR}{\left(R^2+z^2\right)^{3/2}}$$

Riscrittura utile:

$$E = \frac{1}{4\pi\varepsilon_0} \frac{Qz}{\left(R^2 + z^2\right)^{3/2}}$$

$$\rightarrow E = \frac{1}{2\varepsilon_0} \frac{QRz}{2\pi R\left(R^2 + z^2\right)^{3/2}}$$

$$\Lambda = \frac{Q}{2\pi R}$$

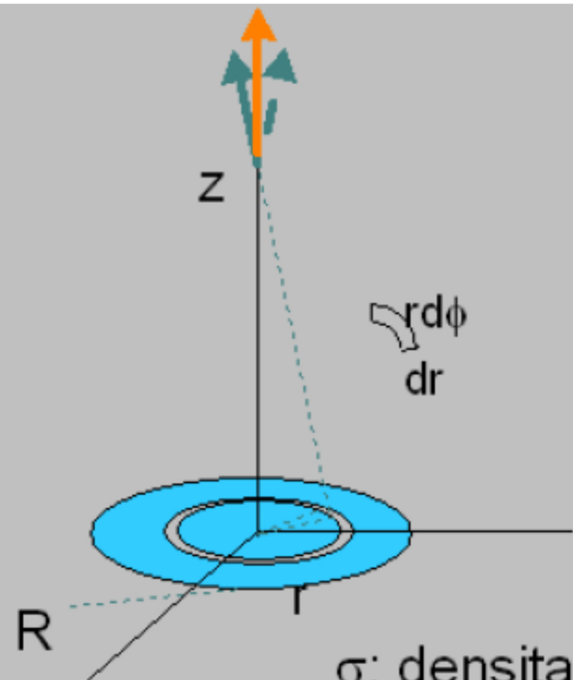
$$\rightarrow E = \frac{\Lambda}{2\varepsilon_0} \frac{Rz}{\left(R^2 + z^2\right)^{3/2}}$$

Andamenti limite:

$$\lim_{z \gg R} E = \lim_{z \gg R} \frac{\Lambda}{2\varepsilon_0} \frac{Rz}{z^3\left(1 + \frac{R^2}{z^2}\right)^{3/2}} = \frac{\Lambda}{2\varepsilon_0} \frac{R}{z^2} = \frac{Q}{4\pi\varepsilon_0 R} \frac{R}{z^2} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{z^2}$$ coulombiano

$$\lim_{R \gg z} E = \frac{\Lambda}{2\varepsilon_0} \frac{Rz}{R^3} = \frac{\Lambda}{2\varepsilon_0} \frac{z}{R^2}$$ lineare in z

Es: Disco carico



σ: densita' superficiale di carica

$$\sigma = \frac{Q}{\pi R^2}$$

$$dq = \sigma r dr d\varphi$$

$$d\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{d^2}\hat{\mathbf{r}}$$

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{\sigma r dr d\varphi}{d^2}\cos\theta$$

$$d^2 = r^2 + \mathbf{z}^2, \cos\theta = \frac{\mathbf{z}}{d}$$

$$\rightarrow dE = \frac{1}{4\pi\varepsilon_0}\frac{\sigma r dr d\varphi}{r^2+\mathbf{z}^2}\frac{\mathbf{z}}{\left(r^2+\mathbf{z}^2\right)^{1/2}}$$

$$E = \int dE = \iint \frac{1}{4\pi\varepsilon_0}\frac{\sigma r dr d\varphi}{r^2+\mathbf{z}^2}\frac{\mathbf{z}}{\left(r^2+\mathbf{z}^2\right)^{1/2}}$$

$$E = \frac{1}{4\pi\varepsilon_0}\int_0^{2\pi} d\varphi \int_o^R \frac{\sigma r dr}{r^2+\mathbf{z}^2}\frac{\mathbf{z}}{\left(r^2+\mathbf{z}^2\right)^{1/2}} = \frac{\sigma\mathbf{z}}{2\varepsilon_0}\int_o^R \frac{r dr}{\left(r^2+\mathbf{z}^2\right)^{3/2}}$$

$$E = \frac{\sigma\mathbf{z}}{2\varepsilon_0}\frac{1}{2}\int_o^R \frac{2r dr}{\left(r^2+\mathbf{z}^2\right)^{3/2}} = \frac{\sigma\mathbf{z}}{4\varepsilon_0}(-2)\left(r^2+\mathbf{z}^2\right)^{-1/2}\Big|_0^R = \frac{\sigma}{2\varepsilon_0}\left[1-\frac{\mathbf{z}}{\sqrt{R^2+\mathbf{z}^2}}\right]$$

Limite: Piano carico

Caso del disco

$$E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{z}{\sqrt{R^2 + z^2}}\right]$$

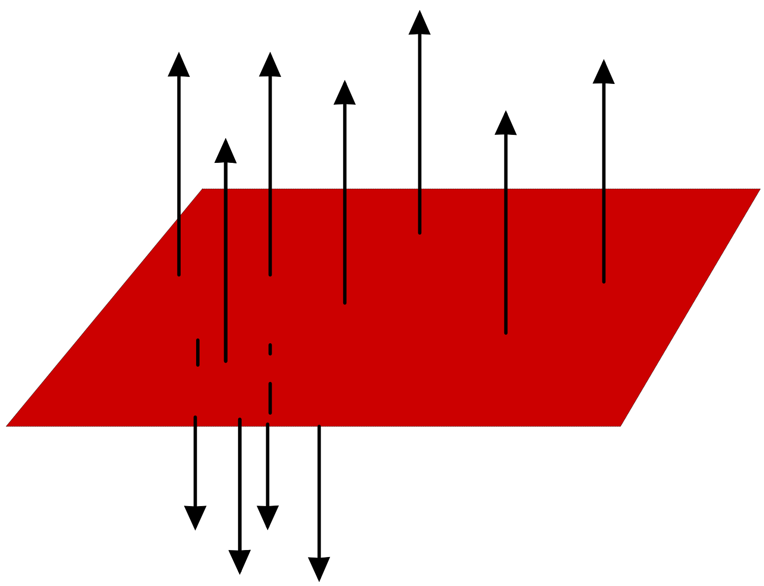
Se $R \to \infty, \frac{z}{R} \to 0$

Disco $\to$ Piano infinito

$$E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{z}{\sqrt{R^2 + z^2}}\right] = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{\frac{z}{R}}{\sqrt{1 + \left(\frac{z}{R}\right)^2}}\right] \xrightarrow[\frac{z}{R}\to 0]{} \frac{\sigma}{2\varepsilon_0}$$

Campo elettrostatico uniforme

Piano infinito carico +vamente



E costante anche all'∞: Carica totale infinita

Limite: grande distanza dal disco

$$E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{z}{\sqrt{R^2 + z^2}}\right]$$

$$\lim_{z \gg R} E = \lim_{z \gg R} \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{z}{z\sqrt{\frac{R^2}{z^2} + 1}}\right] = \lim_{z \gg R} \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{1}{\sqrt{\frac{R^2}{z^2} + 1}}\right]$$

Sviluppo in serie di Taylor di radice del binomio + inverso del binomio:

$$\sqrt{1+x} \simeq 1 + \frac{x}{2}$$

$$\frac{1}{1+x} \simeq 1 - x$$

$$\rightarrow \frac{1}{\sqrt{1+x}} \simeq 1 - \frac{x}{2}$$

$$\rightarrow \lim_{z \gg R} E \approx \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{1}{1 + \frac{R^2}{2z^2}}\right] = \frac{\sigma}{2\varepsilon_0}\left[1 - 1 + \frac{R^2}{2z^2}\right]$$

$$\rightarrow \lim_{z \gg R} E \approx \frac{\sigma}{4\varepsilon_0}\frac{R^2}{z^2} = \frac{Q}{4\pi\varepsilon_0 R^2}\frac{R^2}{z^2} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{z^2} \quad \text{coulombiano}$$