Elementary Particles I

1 – Symmetries

Mostly *C,P,T*

Classical Particle Dynamics

Lagrangian formalism

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{q}_i}\right)-\frac{\partial L}{\partial q_i}=0$$ Lagrange's equations

$$\frac{\partial L}{\partial q_i}=0 \rightarrow \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{q}_i}\right)=0 \rightarrow p_i \equiv \frac{\partial L}{\partial \dot{q}_i}=\text{const}$$

Hamiltonian formalism

$$\dot{p}_i=-\frac{\partial H}{\partial q_i}$$ Hamilton's equations

$\rightarrow H$ not depending on $q_i \Rightarrow p_i = \text{const}$

Symmetry = Irrelevance of … to the dynamics of a closed system

Simmetry	**Invariance**	**Conservation**
Frame origin	**Space translation**	**Total *momentum***
Time origin	**Time translation**	**Total *energy***
Frame orientation	**Space rotation**	**Total *angular momentum***
Frame velocity	**Lorentz transformation**	***CM velocity***

Extension to Classical Fields

Take any physical system, *including fields*: Can describe its motion by the same methods (Lagrangian or Hamiltonian) in terms of a *Principle of Minimum Action*

Then Noether's Theorem states that

For every continuous transformation of the field functions and coordinates which leaves the action unchanged, there is a definite combination of the field functions and their derivatives which is conserved

This is called a *conserved current*

Main point to stress:

Conservation laws must include contributions from the fields, besides particles

Continuous vs Discrete Symmetry

Previous examples: *Continuous* symmetry operations

Any given operation *S* depends on a number of *continuous parameters*
Any given operation *S* can be thought as *continuosly evolving* from the identity operation

Example: S = Shift by ***a*** of the frame origin
Depends on 3 parameters a_x, a_y, a_z

$$S(\mathbf{a}) \underset{a_x, a_y, a_z \to 0}{\longrightarrow} I$$

What are *discrete* symmetry operations?

They do *not* depend on any parameter
They are *intrinsically separated* from the identity operation

Example: S = Axis inversion $x \to -x, y \to -y, z \to -z$
No parameter, no continuous evolution from identity

Quantum Mechanics - I

Link between invariance and conservation in QM

For any observable Q:

$$\langle Q\rangle = \langle\psi|Q|\psi\rangle \rightarrow \frac{d\langle Q\rangle}{dt} = \left\langle\frac{\partial\psi}{\partial t}\right|Q|\psi\rangle + \langle\psi|\frac{\partial Q}{\partial t}|\psi\rangle + \langle\psi|Q\left|\frac{\partial\psi}{\partial t}\right\rangle$$

From Schrodinger equation:

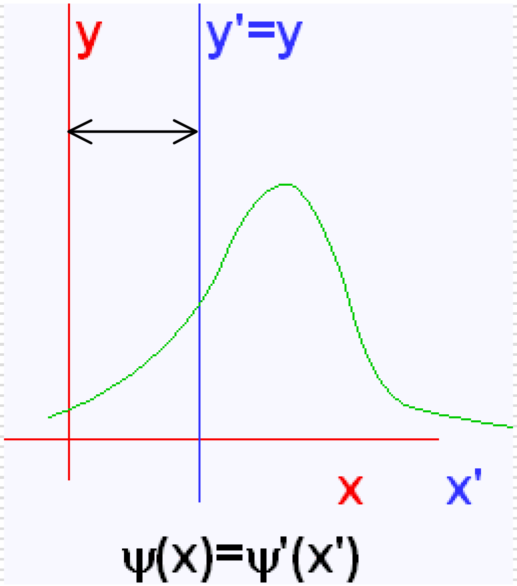
$$i\left|\frac{\partial\psi}{\partial t}\right\rangle = H|\psi\rangle, -i\left\langle\frac{\partial\psi}{\partial t}\right| = \langle\psi|H^\dagger$$

$$\frac{d\langle Q\rangle}{dt} = \left\langle\frac{\partial\psi}{\partial t}\right|Q|\psi\rangle + \langle\psi|\frac{\partial Q}{\partial t}|\psi\rangle + \langle\psi|Q\left|\frac{\partial\psi}{\partial t}\right\rangle$$

$$= i\langle\psi|H^\dagger Q|\psi\rangle + \langle\psi|\frac{\partial Q}{\partial t}|\psi\rangle - iQH|\psi\rangle$$

$$H^\dagger = H \rightarrow \frac{d\langle Q\rangle}{dt} = i\langle\psi|\left([H,Q] + \frac{\partial Q}{\partial t}\right)|\psi\rangle \rightarrow \left.\begin{matrix}[H,Q]=0 \\ \frac{\partial Q}{\partial t}=0\end{matrix}\right\} \rightarrow \langle Q\rangle = \text{constant}$$

Quantum Mechanics - II



Coordinate transformation *S* induces
a transformation in function space

Example:

$$S : x \to x' = S(x) = x - a$$

$$S^{-1} : x' \to x = S^{-1}(x') = x' + a$$

$$\psi'(x') = \psi(x) \leftrightarrow \psi'[S(x)] = \psi(x)$$

$$\to \psi'\left[S^{-1}(S(x))\right] = \psi\left(S^{-1}(x)\right)$$

$$\to \psi'(x) = \psi\left[S^{-1}(x)\right] \neq \psi(x)$$

Indeed, for example:

$$\psi(x) = Ne^{-\frac{x^2}{2\sigma^2}}$$

$$\psi'(x') = \psi\left[S^{-1}(x')\right] = Ne^{-\frac{(x'+a)^2}{2\sigma^2}} \left(= Ne^{-\frac{(x-a+a)^2}{2\sigma^2}} = Ne^{-\frac{x^2}{2\sigma^2}} \right)$$

$$\psi'(x') = Ne^{-\frac{(x'+a)^2}{2\sigma^2}} \neq Ne^{-\frac{x'^2}{2\sigma^2}} = \psi(x')$$

Just meaning:
One and same state
Different coordinates *x,x′* in Σ,Σ′

–> $\psi(x) = \psi'(x')$

One state, two wave functions in Σ,Σ′

Not the only possible point of view…

Quantum Mechanics - III

Unitary transformation

$$U:\psi(x)\to\psi'(x)=\psi\left[S^{-1}(x)\right]=U\left[\psi(x)\right]$$

Because scalar products must be preserved by *U*.
Actually too restrictive:
U can also be *anti-unitary* (see later)

In term of state vectors:

$|\psi'\rangle=U|\psi\rangle\to U$ unitary: $U^{\dagger}=U^{-1}$

Take A = Any operator

$\langle\psi|A|\psi\rangle=\langle\psi'|A'|\psi'\rangle$ defining transformed operator

$$\langle\psi'|A'|\psi'\rangle=\langle\psi|U^{\dagger}A'U|\psi\rangle$$

$$\langle\psi|A|\psi\rangle=\langle\psi'|A'|\psi'\rangle=\langle\psi|U^{\dagger}A'U|\psi\rangle\to A=U^{\dagger}A'U\to A'=UAU^{\dagger}$$

When A invariant wrt U

$$A=A'$$

$$\to UAU^{\dagger}=A$$

$\to[U,A]=0$ when U is a symmetry operator for A

By taking A=H: $[U,H]=0\to\langle U\rangle=\text{const}$

Symmetry operators are *constants*

Quantum Field Theory

Take FT Course!

Summary

Coordinate transformation *S* (possibly including *internal* coordinates)

- *Continuous, Discrete*
- Examples:
  - *Space translation (C)*
  - *Axis inversion (D)*

Corresponding ‘Relativity Principles’

- Example:
- *The laws of physics are the same for all the observers, irrespective of the chosen origin of their reference frames*

Induced transformation U on state space

- *Unitary, Antiunitary*

$\langle U \rangle$ = constant of motion

Discrete Symmetries

Coordinate transformations not evolving continuously from identity

Very important in Particle Physics:

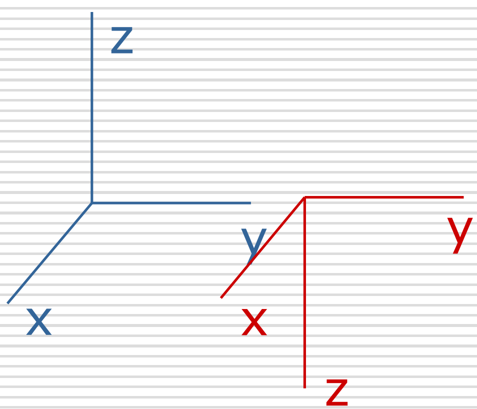
Space Inversion
Time Reversal
Charge Conjugation

Leading to:

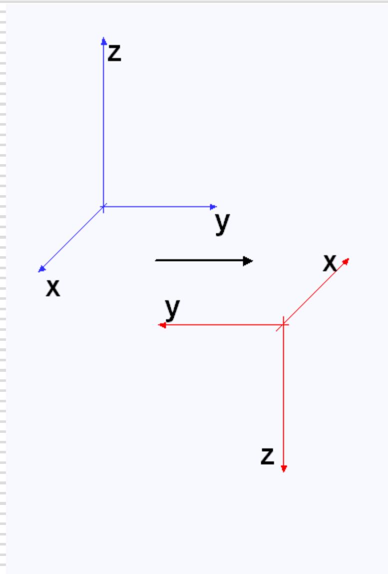
Quantum Numbers
Laws of Conservation
Selection Rules
Constraints on Properties of Interactions

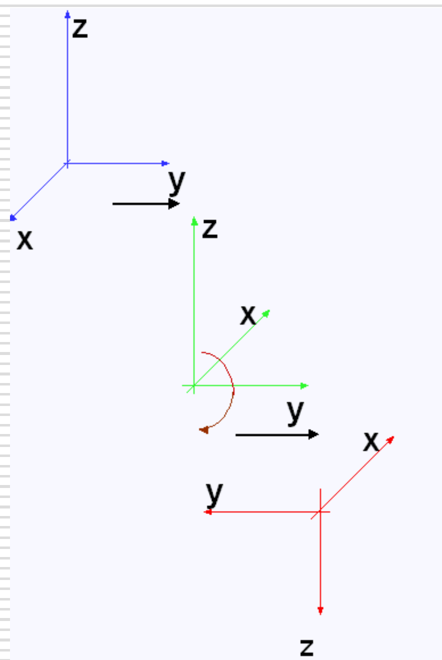
Parity in Classical Physics - I

Two, non equivalent ways of choosing a Cartesian frame in 3D



In 3D: 3 axis inversions equivalent to 1 axis (= mirror) inversion * Rotation





Non equivalent = Not connected by a rotation
Connected by 3 axis inversions

Parity in Classical Physics - II

Define coordinate symmetry operation

$$P:\mathbf{r}\to\mathbf{r}'=-\mathbf{r}$$

Corresponding 'Relativity Principle'
The laws of physics are the same for all the observers, irrespective of the chosen chirality of their reference frames

Transformations of physical quantities follow:

$\mathbf{r}\to-\mathbf{r}$	position
$t\to t$	time
$\mathbf{p}\to-\mathbf{p}$	3-momentum
$E\to E$	energy
$\mathbf{L}\to\mathbf{L}$	angular momentum

Hamilton's equations: form invariant wrt
inversions ↔ $H(q,p)=H(-q,-p)$

No conservation law!
P not connected to identical transformation

Parity in Classical Physics - III

General taxonomy of physical quantities (observables):

- With respect to rotations
- With respect to reflections

'Parity' of corresponding Hermitian operator:

	True	Pseudo
Scalar	$+1$	-1
Vector	-1	$+1$
Rank N Tensor	$(-1)^N$	$(-1)^{N+1}$

Parity in Classical Physics - IV

Parity behavior of selected electromagnetic quantities:

$$\mathbf{j}(\mathbf{r},t) \to -\mathbf{j}\left(-\mathbf{r},t\right) \quad \text{current density}$$

$$\rho(\mathbf{r},t) \to \rho\left(-\mathbf{r},t\right) \quad \text{charge density}$$

$$\mathbf{E}(\mathbf{r},\mathrm{t}) \to -\mathbf{E}(-\mathbf{r},t) \quad \text{electric field}$$

$$\mathbf{B}(\mathbf{r},\mathrm{t}) \to \mathbf{B}(-\mathbf{r},t) \quad \text{magnetic field}$$

Note

All of this is indeed conventional, as based on our definition of the electric charge as a *scalar*.

What actually matters is that *force* is a polar vector

Then

$$\left.\begin{array}{l} \boldsymbol{F} = q\boldsymbol{E} \\ \boldsymbol{F} = q\boldsymbol{v}\times\boldsymbol{B} \\ \boldsymbol{F},\boldsymbol{v} \text{ polar} \end{array}\right\} \to \left\{ \begin{array}{l} \text{either } q \text{ scalar, } \boldsymbol{E} \text{ polar, } \boldsymbol{B} \text{ axial} \\ \text{or } q \text{ pseudoscalar, } \boldsymbol{E} \text{ axial, } \boldsymbol{B} \text{ polar} \end{array}\right.$$

Parity in Quantum Mechanics - I

Coordinate transformation as before: $P : \mathbf{r} \to \mathbf{r}' = P(\mathbf{r}) = -\mathbf{r}$

Induced transformation in state space: U_P unitary operator

A) Commutation relations with position, momentum, angular momentum

$$U_P \mathbf{r} |\psi\rangle = U_P \mathbf{r} \sum |\mathbf{r}\rangle\langle\mathbf{r}|\psi\rangle = U_P \sum \mathbf{r} |\mathbf{r}\rangle\langle\mathbf{r}|\psi\rangle = \sum U_P \mathbf{r} |\mathbf{r}\rangle\langle\mathbf{r}|\psi\rangle$$

$$= \sum (-\mathbf{r}) |-\mathbf{r}\rangle\langle-\mathbf{r}|\psi\rangle = (-\mathbf{r}) U_P |\psi\rangle = -\mathbf{r} U_P |\psi\rangle$$

$$\to U_P \mathbf{r} U_P^{-1} = -\mathbf{r} \to U_P \mathbf{r} = -\mathbf{r} U_P$$

$$U_P U_T (\delta\mathbf{r}) = U_P (1 - \mathbf{p} \cdot \delta\mathbf{r})$$

$$U_P (1 - \mathbf{p} \cdot \delta\mathbf{r}) = (1 + \mathbf{p} \cdot \delta\mathbf{r}) U_P$$

$$\to -U_P \mathbf{p} \cdot \delta\mathbf{r} = \mathbf{p} \cdot \delta\mathbf{r} U_P \to -U_P \mathbf{p} = \mathbf{p} U_P$$

$$U_P \hat{\mathbf{L}} = \hat{\mathbf{L}} U_P$$

Summary:

U_P anticommutes with ***r,p***
commutes with ***L***

Parity in Quantum Mechanics - II

B) Action of U_P on states

$$U_P : |\psi\rangle \rightarrow |\psi'\rangle \equiv U_P |\psi\rangle$$

Same physical state described by two different kets in the two frames

Take position eigenstate:

$$U_P : |\mathbf{r}\rangle \rightarrow |\mathbf{r}'\rangle \equiv U_P |\mathbf{r}\rangle = \eta |-\mathbf{r}\rangle, \quad \eta \text{ arbitrary phase } = e^{i\alpha}$$

Take generic state:

$$U_P : |\psi\rangle \rightarrow |\psi'\rangle \equiv U_P |\psi\rangle$$

Would yield similar results by expanding into momentum eigenstates

Expand into position eigenstates: $|\psi\rangle = \sum_{\boldsymbol{r}} |\boldsymbol{r}\rangle\langle\boldsymbol{r}|\psi\rangle$

$$U_P|\psi\rangle = U_P \sum_{\boldsymbol{r}} |\boldsymbol{r}\rangle\langle\boldsymbol{r}|\psi\rangle = \sum_{\boldsymbol{r}} U_P|\boldsymbol{r}\rangle\langle\boldsymbol{r}|U_P^{-1}U_P\,\psi\rangle = \sum_{\boldsymbol{r}} \left[U_P|\boldsymbol{r}\rangle\langle\boldsymbol{r}|U_P^{-1}\right]U_P\,\psi\rangle$$

$$= \sum_{\boldsymbol{r}'} |\boldsymbol{r}'\rangle\langle\boldsymbol{r}'|U_P\,\psi\rangle = \sum_{\boldsymbol{r}'} |\boldsymbol{r}'\rangle\langle\boldsymbol{r}'|\psi'\rangle = |\psi'\rangle = \sum_{\boldsymbol{r}} |-\boldsymbol{r}\rangle\langle-\boldsymbol{r}|\psi'\rangle$$

$$\rightarrow \langle\boldsymbol{r}'|\psi'\rangle = \langle\boldsymbol{r}|\psi\rangle \rightarrow \langle\boldsymbol{r}|\psi'\rangle = \langle-\boldsymbol{r}|\psi\rangle$$

Talking wave functions: $\psi'(\boldsymbol{r}) = \psi(-\boldsymbol{r})$ (as shown before)

Parity in Quantum Mechanics - III

C) Fundamental property of parity operator

$$P:\mathbf{r}\rightarrow\mathbf{r}'=P(\mathbf{r})=-\mathbf{r},P:\mathbf{r}'\rightarrow\mathbf{r}''=P(\mathbf{r}')=-\mathbf{r}'=\mathbf{r}\rightarrow P^2=I$$

$$U_P|\psi\rangle=|\psi'\rangle,U_P|\psi'\rangle=U_P(U_P|\psi\rangle)=U_P^2|\psi\rangle\rightarrow U_P^2=\eta^2 I$$

$$\left.\begin{array}{r}U_P^\dagger=U_P^{-1},\ U_P\text{ unitary}\\ U_P^2=I\end{array}\right\}\rightarrow U_P^2=U_PU_P=I=U_PU_P^{-1}=U_PU_P^\dagger\rightarrow U_P^\dagger=U_P\quad U_P\text{ Hermitian}$$

$\rightarrow U_P$ eigenvalues are real

$\rightarrow\eta=e^{i\alpha}=\pm1\ \equiv\eta_P$ parity quantum number

$\rightarrow U_P$ eigenstates: $U_P|a\rangle=\pm|a\rangle$

Consequences:

$[H,U_P]=0\rightarrow$ stationary states have definite η_P- when not degenerate

$[H,U_P]=0\rightarrow\eta_P=$ constant of motion

Parity in Quantum Mechanics - IV

Parity for a composite system (just meaning with several degrees of freedom)

$|a\rangle$ compound state of subsystems 1 and 2, parity eigenstate

$U_P^{(1)}, U_P^{(2)}$ parity operators for 1,2

$$U_P^{(1)}|a\rangle = \eta_P^{(1)}|a\rangle, U_P^{(2)}|a\rangle = \eta_P^{(2)}|a\rangle$$

$U_P^{(2)}\left[U_P^{(1)}|a\rangle\right] = U_P^{(2)}\left[\eta_P^{(1)}|a\rangle\right] = \eta_P^{(2)}\eta_P^{(1)}|a\rangle \to \eta_P$ *multiplicative* quantum number

Quantum numbers (i.e., conserved quantities) are usually *additive*: ???
Reason: For continuos symmetries the unitary operators are not Hermitian
U unitary $\to U = e^{iaH} \simeq 1 + iaH,\ H$ Hermitian; $\lim_{a\to 0} U = 1 \to$ Use infinitesimal generators

Example for translations:

$$U_a^{(1)} \simeq 1 + ia\mathbf{p}^{(1)}, U_a^{(2)} \simeq 1 + ia\mathbf{p}^{(2)} \to U_a^{(2)}\left(U_a^{(1)}|\psi\rangle\right) \simeq \left(1 + ia\mathbf{p}^{(2)}\right)\left[\left(1 + ia\mathbf{p}^{(1)}\right)|\psi\rangle\right]$$

$$\to U_a^{(2)}\left(U_a^{(1)}|\psi\rangle\right) \simeq \left(1 + ia\mathbf{p}^{(2)} + ia\mathbf{p}^{(1)}\right)|\psi\rangle = \left[1 + ia\left(\mathbf{p}^{(2)} + \mathbf{p}^{(1)}\right)\right]|\psi\rangle$$

U Hermitian→ Constant of motion = *U* → Multiplicative
U not Hermitian→ Constant of motion ~ “Logarithm” of *U*…→ Additive

Parity in Quantum Field Theory

Take FT Course!

Just a few, oversimplified remarks:

A. Observables are (functions of) field operators
B. Just as in Quantum Mechanics, one finds different kinds of operators:
   (Pseudo-)Scalars, (Pseudo-)Vectors, …
C. Fields of each kind have their own ‘parity’ built-in
D. States of a given field do inherit field ‘parity’, which is called *intrinsic parity*

The Parity Quantum Number

As a quantum number, parity may or may not be conserved

Experimental fact:

All interactions, except weak interaction, do conserve parity

To the extent we can neglect weak interaction, all *stationary, non degenerate* states must be parity eigenstates

Scattering states: similar but not identical to stationary states
Being ~ momentum eigenstates $\left[\boldsymbol{p},U_P\right]\neq 0 \;\rightarrow$ Parity not defined

Stable particles and resonances: bound states
Angular momentum eigenstates $\left[\boldsymbol{L},U_P\right]=0 \;\rightarrow$ Parity defined

Orbital Parity

Parity quantum number introduced by making reference to orbital motion.

For angular momentum eigenstates, take position representation:

$$\psi(\mathbf{r}) = Nf(r)Y_l^m(\theta,\varphi)$$

$$P:\mathbf{r} \to -\mathbf{r} \Rightarrow Y_l^m(\theta,\varphi) \to Y_l^m(\theta-\pi,\varphi+\pi) = (-1)^l Y_l^m(\theta,\varphi)$$

Therefore

$$\eta_P^{(orb)} = (-1)^l$$

This is known as *orbital parity*.

Orbital parity, like angular momentum, is *frame dependent*

Intrinsic Parity - I

Orbital parity not sufficient to deal with relativistic processes

Reason: *Particles are created and annihilated*

Best clarified first by a non relativistic example

Take a reaction between 2 nuclei, ignore nucleon spin

$A+B \rightarrow C+D, \quad A,B \neq C,D$

Angular momentum conservation:

$$\mathbf{L}_{TOT}^{(in)} = \mathbf{L}_A^{(CM-A)} + \mathbf{L}_B^{(CM-B)} + \mathbf{L}_{AB}^{(CM-AB)}$$

L_A, L_B originated by internal nuclear motion
L_{AB} orbital motion

$$P_{TOT}^{in} = (-1)^{L_{TOT}^{(in)}} = \underbrace{(-1)^{L_A^{(CM-A)}}}_{P_A} \underbrace{(-1)^{L_B^{(CM-B)}}}_{P_B} \underbrace{(-1)^{L_{AB}^{(CM-AB)}}}_{P_{ORB}}$$

$$\mathbf{L}_{TOT}^{(out)} = \mathbf{L}_C^{(CM-C)} + \mathbf{L}_D^{(CM-D)} + \mathbf{L}_{CD}^{(CM-CD)}$$

Same for final state

$$P_{TOT}^{out} = (-1)^{L_{TOT}^{(out)}} = \underbrace{(-1)^{L_C^{(CM-C)}}}_{P_C} \underbrace{(-1)^{L_B^{(CM-D)}}}_{P_D} \underbrace{(-1)^{L_{CD}^{(CM-CD)}}}_{P_{ORB}}$$

If we don't know A,B,C,D are composite systems, 'intrinsic' nuclear parity cannot be ignored, or the process would violate parity whenever L_{CD} # L_{AB}

Intrinsic Parity - II

Just a drop of QFT to show the origin of intrinsic parity

1) One component (boson) field $\varphi(\boldsymbol{r},t)$

Expand $\phi(r)$ into creation+annihilation operators

$$\varphi(\boldsymbol{r},t)=\sum_{\boldsymbol{p}}\frac{1}{\sqrt{2EV}}\left[a_{\boldsymbol{p}}e^{-ip\cdot r}+a_{\boldsymbol{p}}^{\dagger}e^{+ip\cdot r}\right], \quad p\cdot r=(E,\boldsymbol{p})\cdot(t,\boldsymbol{r})=Et-\boldsymbol{p}\cdot\boldsymbol{r}$$

Intrinsic parity of ϕ operator

$$U_P\varphi U_P^{-1}=\eta_P\varphi(-\boldsymbol{r},t)=\sum_{\boldsymbol{p}}\frac{1}{\sqrt{2EV}}\left[U_P a_{\boldsymbol{p}}U_P^{-1}e^{-i(Et-\boldsymbol{p}\cdot\boldsymbol{r})}+U_P a_{\boldsymbol{p}}^{\dagger}U_P^{-1}e^{+i(Et-\boldsymbol{p}\cdot\boldsymbol{r})}\right]$$

Expand $\phi(-\boldsymbol{r})$ into creation+annihilation operators

$$\eta_P\varphi(-\boldsymbol{r},t)=\eta_P\sum_{\boldsymbol{p}}\frac{1}{\sqrt{2EV}}\left[a_{\boldsymbol{p}}e^{-i(Et-\boldsymbol{p}\cdot(-\boldsymbol{r}))}+a_{\boldsymbol{p}}^{\dagger}e^{+i(Et-\boldsymbol{p}\cdot(-\boldsymbol{r}))}\right]$$

Creation/Annihilation operators for $-\boldsymbol{p}$

$$=\sum_{\boldsymbol{p}}\frac{1}{\sqrt{2EV}}\left[\eta_P a_{-\boldsymbol{p}}e^{-i(Et-(-\boldsymbol{p})\cdot\boldsymbol{r})}+\eta_P a_{-\boldsymbol{p}}^{\dagger}e^{+i(Et-(-\boldsymbol{p})\cdot\boldsymbol{r})}\right] \rightarrow \begin{cases} U_P a_{\boldsymbol{p}}U_P^{-1}=\eta_P a_{-\boldsymbol{p}} \\ U_P a_{\boldsymbol{p}}^{\dagger}U_P^{-1}=\eta_P a_{-\boldsymbol{p}}^{\dagger}\end{cases}$$

Intrinsic parity!
+1 Scalar
- 1 Pseudo

$$\rightarrow U_P\left|\boldsymbol{p}\right\rangle=U_P a_{\boldsymbol{p}}^{\dagger}U_P^{-1}U_P\left|0\right\rangle=\eta_P a_{-\boldsymbol{p}}^{\dagger}\left|0\right\rangle=\eta_P\left|-\boldsymbol{p}\right\rangle; U_P\left|\boldsymbol{p}=0\right\rangle=\eta_P\left|\boldsymbol{p}=0\right\rangle$$

Different states — Same state

Intrinsic Parity - III

2) Multi-component, Boson fields

Same as one component field

E.g. Vector/Pseudo-vector field

In all cases:

$\eta_P^* = \eta_P \rightarrow$ Same parity for particle,antiparticle

3) Multi-component, Fermion fields

Field satisfying Dirac's equation:

$$\begin{cases} U_P \psi(\mathbf{r},t) U_P^{-1} = \eta_P \gamma^0 \psi(-\mathbf{r},t) \\ U_P \overline{\psi}(\mathbf{r},t) U_P^{-1} = \eta_P^* \overline{\psi}(-\mathbf{r},t) \gamma^0 \end{cases}$$

$\rightarrow |\eta_P|^2 = 1 \rightarrow \eta_P = \pm 1, \pm i \rightarrow$ Opposite parity for particle,antiparticle

Intrinsic Parity of Dirac Particles

$(\boldsymbol{\alpha}\cdot\mathbf{p}+\beta m)\psi = E\psi \underset{P}{\rightarrow} (\boldsymbol{\alpha}\cdot(-\mathbf{p})+\beta m)\psi_P = E\psi_P$

$\beta(\boldsymbol{\alpha}\cdot(-\mathbf{p})+\beta m)\psi_P = E\beta\psi_P$

$\beta\boldsymbol{\alpha} = -\boldsymbol{\alpha}\beta \rightarrow ((-\boldsymbol{\alpha})\cdot(-\mathbf{p})+\beta m)\beta\psi_P = E\beta\psi_P$

$\rightarrow (\boldsymbol{\alpha}\cdot\mathbf{p}+\beta m)\beta\psi_P = E\beta\psi_P \rightarrow \beta\psi_P$ satisfies Dirac eq.

Reminder:

$\beta = \gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$

Take plane wave solutions:

$$\psi_1(\mathbf{r},t) = \begin{pmatrix} \varphi \\ \frac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{E+m}\varphi \end{pmatrix} e^{-i(Et-\boldsymbol{p}\cdot\boldsymbol{r})}, \psi_2(\mathbf{r},t) = \begin{pmatrix} \chi \\ \frac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{E+m}\chi \end{pmatrix} e^{-i(Et-\boldsymbol{p}\cdot\boldsymbol{r})}, \psi_3(\mathbf{r},t) = \begin{pmatrix} -\frac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{E+m}\varphi \\ \varphi \end{pmatrix} e^{-i(-Et-\boldsymbol{p}\cdot\boldsymbol{r})}, \psi_4(\mathbf{r},t) = \begin{pmatrix} -\frac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{E+m}\chi \\ \chi \end{pmatrix} e^{-i(-Et-\boldsymbol{p}\cdot\boldsymbol{r})}$$

Go to rest frame: $\boldsymbol{p}=0,\ E=m$

$$\left.\begin{aligned} \psi_1 &= \begin{pmatrix} \varphi \\ 0 \end{pmatrix} e^{-imt} \underset{P}{\rightarrow} \gamma^0 \begin{pmatrix} \varphi \\ 0 \end{pmatrix} e^{-imt} = \begin{pmatrix} \varphi \\ 0 \end{pmatrix} e^{-imt} = \psi_1 \\ \psi_2 &= \begin{pmatrix} \chi \\ 0 \end{pmatrix} e^{-imt} \rightarrow \gamma^0 \begin{pmatrix} \chi \\ 0 \end{pmatrix} e^{-imt} = \begin{pmatrix} \chi \\ 0 \end{pmatrix} e^{-imt} = \psi_2 \end{aligned}\right\} \text{Intrinsic parity} = +1$$

$$\left.\begin{aligned} \psi_3 &= \begin{pmatrix} 0 \\ \varphi \end{pmatrix} e^{-i(-m)t} \rightarrow \gamma^0 \begin{pmatrix} 0 \\ \varphi \end{pmatrix} e^{-i(-m)t} = \begin{pmatrix} 0 \\ -\varphi \end{pmatrix} e^{-i(-m)t} = -\psi_3 \\ \psi_4 &= \begin{pmatrix} 0 \\ \chi \end{pmatrix} e^{-i(-m)t} \rightarrow \gamma^0 \begin{pmatrix} 0 \\ \chi \end{pmatrix} e^{-i(-m)t} = \begin{pmatrix} 0 \\ -\chi \end{pmatrix} e^{-i(-m)t} = -\psi_4 \end{aligned}\right\} \text{Intrinsic parity} = -1$$

Remark:
Absolute parity *not defined* for fermions
Only defined relative fermion-antifermion

Intrinsic Parity-IV

Absolute intrinsic parity not defined for most particle states

1) Fermions

Cannot be singly created/annihilated

Even more trouble: Consider 2π rotation of a $J=1/2$ state

$$U_R(\hat{\boldsymbol{n}},\theta)|\psi\rangle = e^{i\boldsymbol{J}\cdot\hat{\boldsymbol{n}}\theta}|\psi\rangle \to U_R(\hat{\boldsymbol{z}},2\pi)|\psi\rangle = e^{iJ_z 2\pi}|\psi\rangle = e^{i\frac{1}{2}2\pi}|\psi\rangle = -|\psi\rangle$$

So:

$$\left.\begin{array}{l} I = U_R(\hat{\boldsymbol{z}},4\pi) = U_R^2(\hat{\boldsymbol{z}},2\pi) \\ U_P^2 = U_R(\hat{\boldsymbol{z}},2\pi) \end{array}\right\} \to I = U_P^4 \quad \text{for Fermions} \to \eta_P = \pm 1, \pm i$$

Funny result! Not relevant in the Standard Model (Maybe *relevant* beyond..)

2) Charged Bosons

As above $\to I = U_P^2$ for Bosons $\to \eta_P = \pm 1$

3) Really neutral particles (*Photon*, π^0, J/ψ, …)

Have all additive quantum numbers (charge, baryonic/leptonic number, strangeness, charm, …) = 0, like vacuum state

$\to$Absolute parity *is* defined upon defining vacuum state parity (usually= $+1$)

Intrinsic Parity- Photon

Classical EM field

Standard convention:

$$\left.\begin{array}{l} P:\mathbf{E}\to-\mathbf{E} \\ P:\mathbf{B}\to\mathbf{B} \\ \mathbf{E}=-\dfrac{\partial\mathbf{A}}{\partial t}-\nabla\varphi \\ \mathbf{B}=\nabla\times\mathbf{A} \end{array}\right\}\to\begin{cases} P:\mathbf{A}\to-\mathbf{A} \\ P:\varphi\to\varphi \end{cases}$$

A: Vector field

ϕ: Not needed for radiation field

Quantum EM field

A: Vector field

Photon spin = 1

$\to$ *Negative intrinsic parity*

Interesting question: EM interactions conserve parity

$\to$ *Photons from reactions, decays should have defined, total parity*

Photon Total Parity

What is the photon parity in a given state? Can expand any ***A*** into creation & annihilation operators

$$\mathbf{A}(\mathbf{r},t)=\sum_{\mathbf{k}}\sum_{i=1}^{2}\frac{1}{\sqrt{|\mathbf{k}|V}}\left[a_i(\mathbf{k})\boldsymbol{\varepsilon}_i e^{i(\mathbf{p}_\gamma\bullet\mathbf{x}-E_\gamma t)}+a_i^\dagger(\mathbf{k})\boldsymbol{\varepsilon}_i^* e^{-i(\mathbf{p}_\gamma\bullet\mathbf{x}-E_\gamma t)}\right]$$

But: $a_i(\boldsymbol{k})$ do not commute with parity. Can also expand into J,m C&A operators

$$\mathbf{A}(\mathbf{r},t)=\sum_{J,m}\sum_{L=J-1}^{J+1}\sum_{i=1}^{2}\frac{1}{\sqrt{|\mathbf{k}|V}}\left[a_i(J,L,m)\boldsymbol{\varepsilon}_i Y_{Jm}^{L}(\theta,\varphi)+a_i^\dagger(J,L,m)\boldsymbol{\varepsilon}_i^* Y_{Jm}^{L*}(\theta,\varphi)\right]$$

Photon states with defined total parity: states with given J,L,m. Indeed:

$$\boldsymbol{J}=\boldsymbol{L}+\underbrace{\boldsymbol{S}}_{=1}\rightarrow J=L,L\pm 1\rightarrow L=J,J\pm 1$$

$$\rightarrow \eta_P=(-1)^L(-1)=(-1)^{L+1}=(-1)^J,(-1)^{J\pm 1}$$

$$\rightarrow \begin{cases}\eta_P=(-1)^J & \text{Electric}\\ \eta_P=(-1)^{J\pm 1} & \text{Magnetic}\end{cases}$$

2 kinds of photons for any given J

Intrinsic Parity - Scalars

Really neutral states decaying into two photons

Example: π^0, *spin = 0*

$$\pi^0 \longrightarrow \gamma\gamma$$

Parity conserved by EM interaction

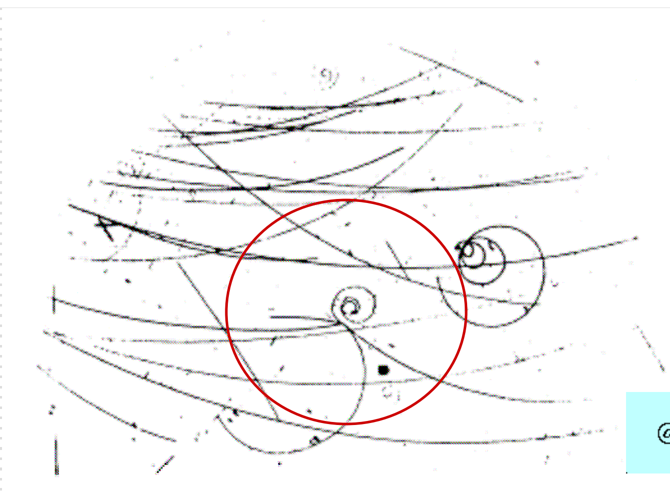
Final state: 2 creation operators $\propto \boldsymbol{\varepsilon_1}, \boldsymbol{\varepsilon_2}$

Can determine parity by looking at distribution of the angle between $\boldsymbol{\varepsilon_1}, \boldsymbol{\varepsilon_2}$

Pseudoscalar $\sin^2\varphi$

Scalar $\cos^2\varphi$

Observe: Double Dalitz decays $\pi^0 \longrightarrow e^+e^-e^+e^-$



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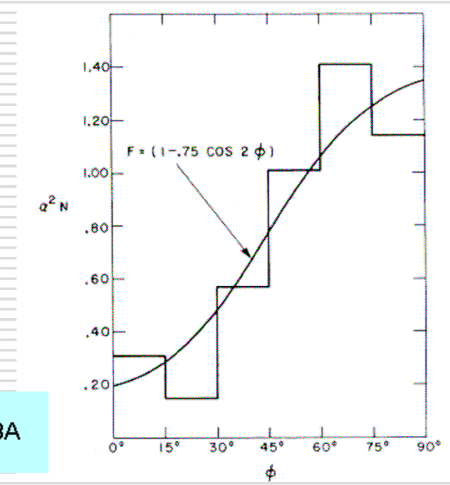


Intrinsic Parity- Vectors

Most interesting result:
Yang Theorem

A vector (s=1) state cannot decay to 2 photons

Consequence of:

Angular momentum conservation
Parity conservation
Bose symmetry for identical photons
Gauge invariance of QED (transversality of photons)

Intrinsic Parity – Baryons

States with different charge, baryon number (absolutely conserved quantities) cannot be superposed ← Limitation to the principle of superposition, also called *Superselection Rule*

In other words: *No transition can violate Q,B conservation*

Relative parity unmeasurable, must be fixed by convention

Also true for states with different strangeness, charm, …*when weak interaction is neglected.*

But: While weak transitions *can* connect states with different *S,C*,.., they *do* violate parity!

Remembering that a particle+antiparticle state has –ve intrinsic parity, state our general convention for Leptons & Quarks

+ve parity to particles	*-ve parity to antiparticles*
Quarks	Antiquarks
Leptons	Antileptons

NB -ve parity baryons actually exist, with higher mass and spin

Intrinsic Parity – Mesons

Particles singly produced in parity conserving reactions $\rightarrow$ Parity *can* be determined

Examples

Hypernucleus
One neutron$\rightarrow\Lambda^0$

Charged Pion

$\pi^- + d \rightarrow n + n$ at low energy

$\eta_P^{\pi^-}\left(\eta_P^N\right)^2(-1)^L = \left(\eta_P^N\right)^2(-1)^{L'}$

Initial state: $L = 0 \rightarrow J = 0 \oplus 1 = 1$

Final state: $J' = 1$, two identical fermions

$\rightarrow$P-wave,triplet $\rightarrow L' = 1$

$\rightarrow \eta_P^{\pi^-}(-1)^0 = (-1)^1 \rightarrow \eta_P^{\pi^-} = -1$

Charged Kaon

$K^- + {}^4He \rightarrow \pi^0 + {}^4_\Lambda H$ at low energy

$\eta_P^{K^-}\eta_P^{^4He}(-1)^L = \eta_P^{\pi^0}\eta_P^{^4_\Lambda H}(-1)^{L'}$

Initial state: $L = 0, S_{K^-} = 0 \rightarrow J = 0$

Final state: $J' = 0,\ S_{^4_\Lambda H} = 0 \rightarrow L' = 0$

$\rightarrow \eta_P^{K^-}\cdot 1\cdot(-1)^0 = (-1)\cdot 1\cdot(-1)^0 \rightarrow \eta_P^{K^-} = -1$

$= (+1)^4\,(-1)^0 = +1$
Ground state $L=0$

Two-Particle States

Fermion-Antifermion: CM frame

$$\eta_P^{f\bar{f}} = \underset{\text{intrinsic}}{(-1)}\underset{\text{orbital}}{(-1)^l} = (-1)^{l+1}$$

Boson-Antiboson: CM frame

$$\eta_P^{b\bar{b}} = \underset{\text{intrinsic}}{(\eta_P)^2}\underset{\text{orbital}}{(-1)^l} = (-1)^l$$

Photon-photon: CM frame

Take single photon states with defined helicity:

$U_P|\mathbf{k},R;-\mathbf{k},R\rangle = |-\mathbf{k},L;\mathbf{k},L\rangle$ not a U_P eigenstate

$U_P|\mathbf{k},L;-\mathbf{k},L\rangle = |-\mathbf{k},R;\mathbf{k},R\rangle$ not a U_P eigenstate

$U_P|\mathbf{k},R;-\mathbf{k},L\rangle = |-\mathbf{k},L;\mathbf{k},R\rangle \equiv |\mathbf{k},R;-\mathbf{k},L\rangle$ U_P eigenstate, $\eta_P = +1,\ J_3 = +2$

$U_P|\mathbf{k},L;-\mathbf{k},R\rangle = |-\mathbf{k},R;\mathbf{k},L\rangle \equiv |\mathbf{k},L;-\mathbf{k},R\rangle$ U_P eigenstate, $\eta_P = +1\ \ J_3 = -2$

$|\mathbf{k},R;-\mathbf{k},R\rangle \pm |\mathbf{k},L;-\mathbf{k},L\rangle$ U_P eigenstates, $\eta_P = \pm 1, J_3 = 0$

Fall of Parity – I

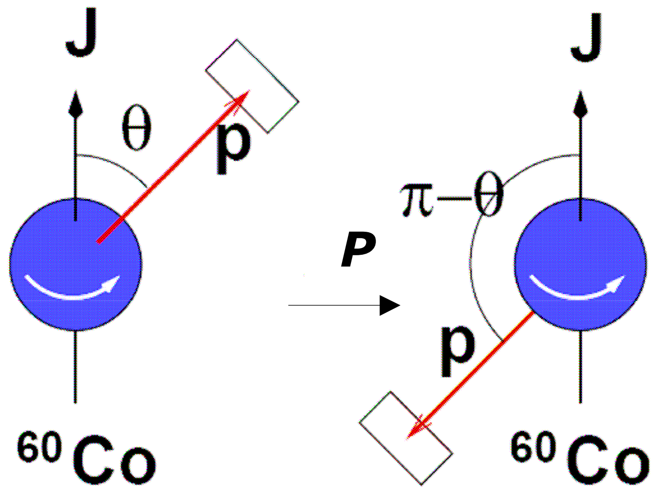
$^{60}Co \rightarrow {}^{60}Ni^{*} + e^{-} + \bar{\nu}_e \quad \beta^{-}$ decay: Weak process

If weak interaction is parity invariant,

$\rightarrow Ampl(\theta) = Ampl(\pi - \theta)$

Otherwise:

Expect β^{-} direction *anisotropy*



Require nuclear polarization:
For an unpolarized sample, by averaging over ***J*** z-projections any possible anisotropy is washed out

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Fall of Parity – II

The ^{60}Co Experiment: Polarization

Zeeman:

$$\mathcal{E}(M) = E_0 - \vec{\mu} \cdot \vec{B} = -g\mu_N BM$$

Boltzmann:

$$\frac{n(M')}{n(M)} = \frac{e^{\frac{\mathcal{E}(M)}{kT}}}{e^{\frac{\mathcal{E}(M')}{kT}}} = e^{\frac{(M-M')g\mu_N B}{kT}}$$

Magnetic field amplification in cerium-magnesium-nitrate crystal 0.05 T $\rightarrow$ 10–100 T

The ^{60}Co polarizes at a temperature of about 10 mK.

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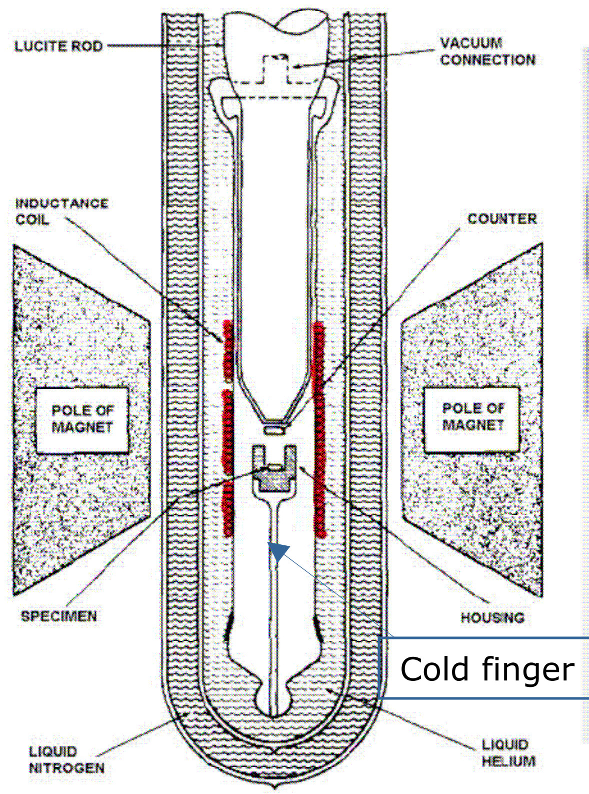
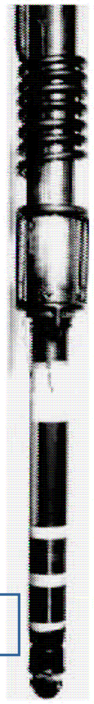




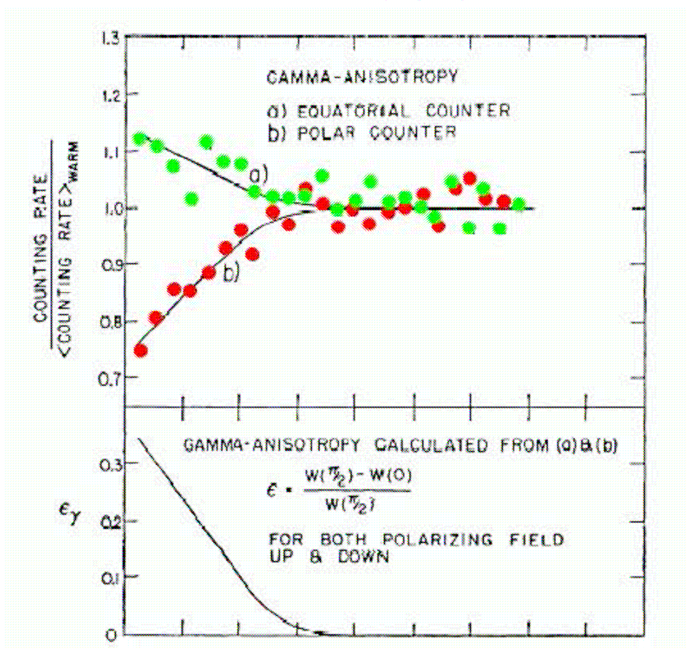
Fall of Parity – III

To measure nuclear polarization:

γ anisotropy in $^{60}\mathrm{Ni}^* \to {}^{60}\mathrm{Ni} + \gamma(E2)$

Electric quadrupole transition

$$\epsilon_\gamma = \frac{W(\pi/2) - W(0)}{W(\pi/2)}$$



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γ counter

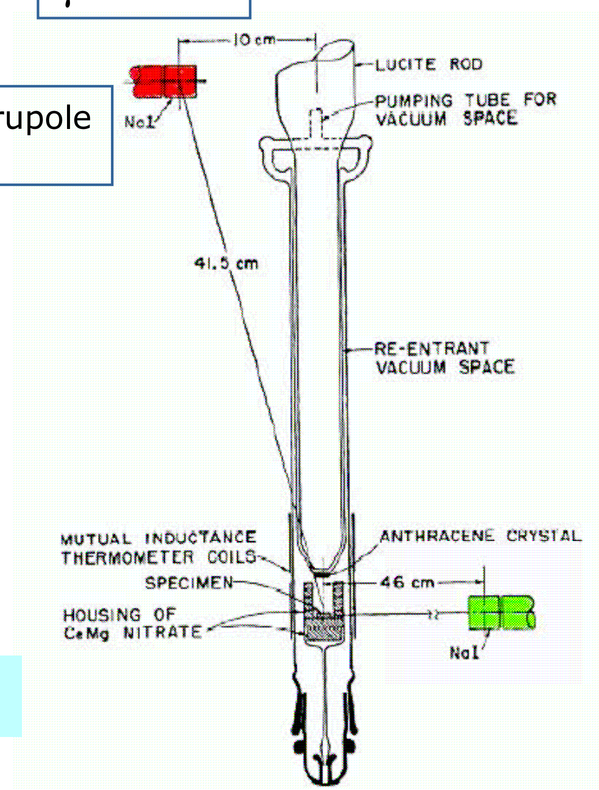
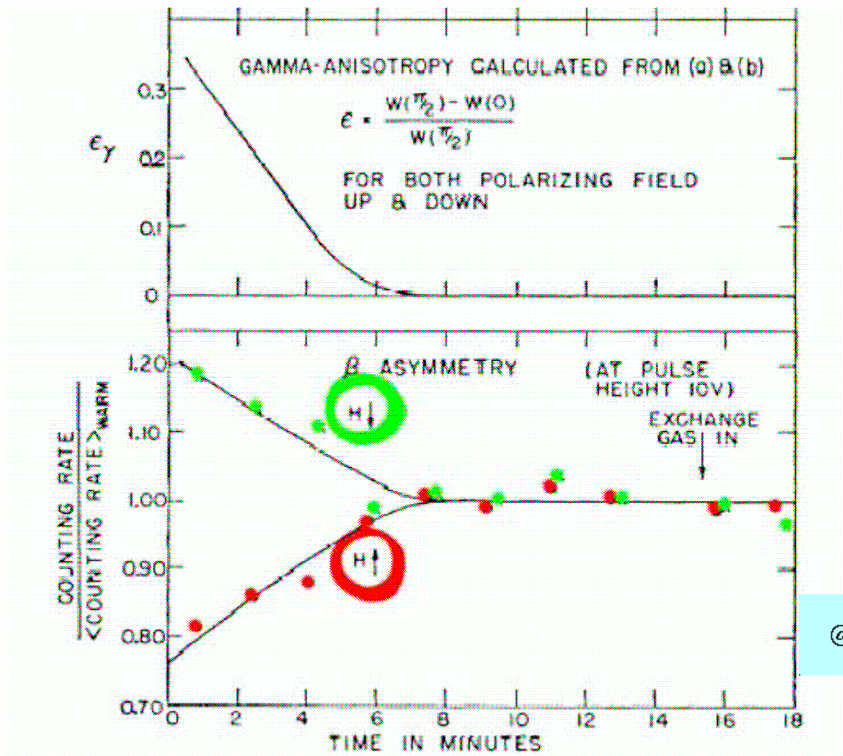


FIG. 1. Schematic drawing of the lower part of the cryostat

Fall of Parity – IV

$$^{60}\text{Co} \to ^{60}\text{Ni}^* + e^- + \bar{\nu}_e$$

$$\tau = 5.27\,\text{year} \qquad E_{e^-}^{max} = 2.824\,\text{MeV}$$



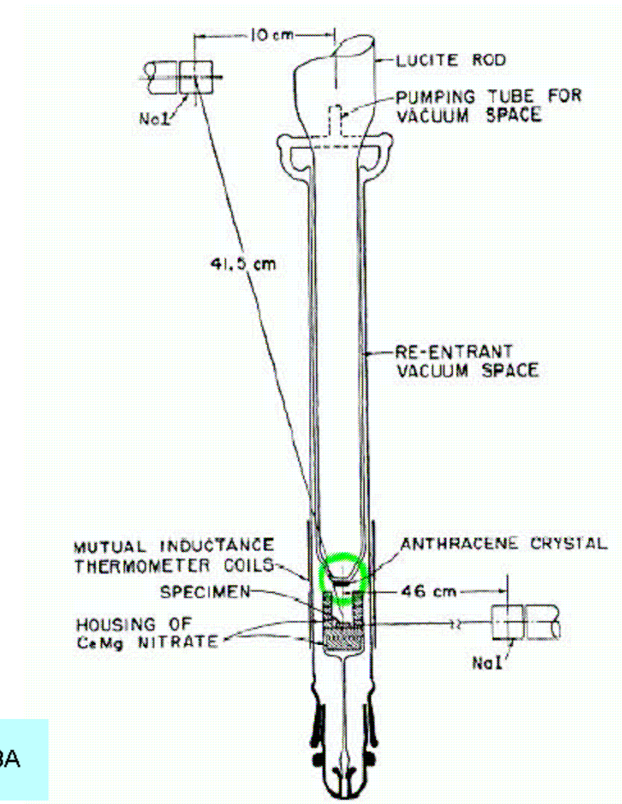


Fig. 1. Schematic drawing of the lower part of the cryostat

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Another Fall of Parity

$\pi^+ \to \mu^+ + \nu_\mu$

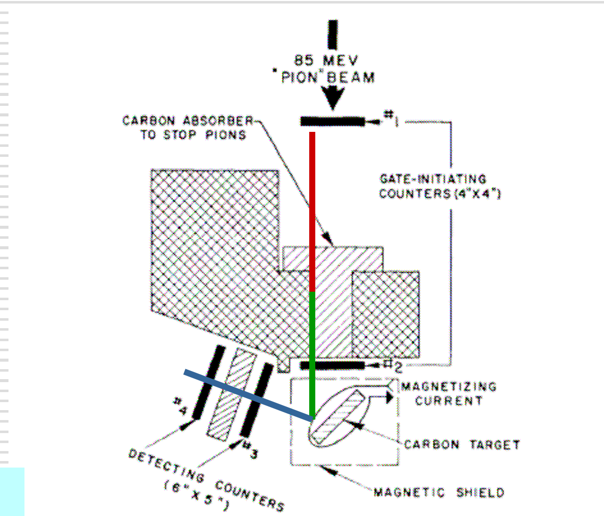
$\boldsymbol{\sigma}_\mu \cdot \mathbf{p}_\mu \underset{U_P}{\to} \boldsymbol{\sigma}_\mu \cdot (-\mathbf{p}_\mu) = -\boldsymbol{\sigma}_\mu \cdot \mathbf{p}_\mu$

$\to P_\mu^{long} = \left\langle \dfrac{\boldsymbol{\sigma}_\mu \cdot \mathbf{p}_\mu}{|\mathbf{p}_\mu|} \right\rangle = 0$ if parity is a good symmetry

$\mu^+ \to e^+ + \bar{\nu}_\mu + \nu_e$

$\boldsymbol{\sigma}_\mu \cdot \mathbf{p}_e \underset{U_P}{\to} \boldsymbol{\sigma}_\mu \cdot (-\mathbf{p}_e) = -\boldsymbol{\sigma}_\mu \cdot \mathbf{p}_e$

$\to \left\langle \dfrac{\boldsymbol{\sigma}_\mu \cdot \mathbf{p}_e}{|\mathbf{pe}|} \right\rangle = 0$ if parity is a good symmetry



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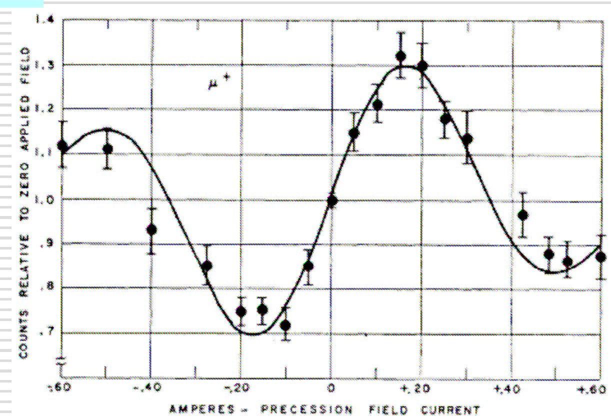


If parity is violated:

$\begin{cases} \pi^+ \to \mu^+ + \nu_\mu & \text{Expect } \mu \text{ polarization along } \mathbf{p}_\mu \\ \mu^+ \to e^+ + \bar{\nu}_\mu + \nu_e & \text{Expect } e^+ \text{ direction correlated with } \mathbf{s}_\mu \end{cases}$

In order to detect e^+ correlation:

μ spin precession in $\mathbf{B}$

Charge Conjugation - I

Charge conjugation operation: similar to parity
But: Operating on ‘internal’, rather than spatial, coordinates

Change sign of all additive quantum numbers $Q, B, S, C, \ldots$

Like parity, unitary transformation in state space:

$$C : |\psi\rangle \equiv |\alpha; Q, B, S, C, ..\rangle \rightarrow |\psi'\rangle = U_C |\psi\rangle \equiv |\alpha; -Q, -B, -S, -C, ..\rangle$$

[α: All other quantum numbers, like energy, position, momentum, angular momentum, …]
Corresponding ‘Relativity Principle’ (to be checked by experiment)

The laws of physics are the same for all the observers, irrespective of the chosen positive direction of their ‘charge axis’

Experimental fact:
All interactions, except weak interaction, do conserve charge parity

Charge Conjugation - II

A. Commutation relations with observables

U_C commutes with all observables, except those yielding additive quantum numbers :

$$[U_c, \boldsymbol{r}] = [U_c, \boldsymbol{p}] = [U_c, \boldsymbol{L}] = [U_c, H] = 0$$

$$U_C Q = -Q U_C, U_C B = -B U_C, U_C S = -S U_C, \ldots$$

B. Action on states

$U_P : |Q\rangle \rightarrow |Q'\rangle \equiv U_P |Q\rangle = \eta |-Q\rangle$, η arbitrary phase $= e^{i\alpha}$

Charge Conjugation - III

C. Fundamental property of charge conjugation operator

$$C:Q\rightarrow Q'=C\left(Q\right)=-Q, C:Q'\rightarrow Q''=C\left(Q'\right)=-Q'=Q\rightarrow C^2=I$$

$$U_C\left|\psi\right\rangle=\left|\psi'\right\rangle, U_C\left|\psi'\right\rangle=U_C\left(U_C\left|\psi\right\rangle\right)=U_C^2\left|\psi\right\rangle\rightarrow U_C^2=\eta^2 I$$

$$\left.\begin{array}{r}U_C^\dagger=U_C^{-1},\ U_C \text{ unitary}\\ U_C^2=I\end{array}\right\}\rightarrow U_C^2=U_CU_C=I=U_CU_C^{-1}=U_CU_C^\dagger\rightarrow U_C^\dagger=U_C$$

$\rightarrow U_C$ hermitian

$\rightarrow U_P$ eigenvalues are real

$\rightarrow \eta=e^{i\alpha}=\pm1\ \equiv\eta_C$ charge parity quantum number

$\rightarrow U_C$ eigenstates: $U_C\left|a\right\rangle=\pm\left|a\right\rangle$

Very few particles are U_C eigenstates: Must have

Q=B=S=C=…=0

Applications - I

$$j_\mu = (\rho, \mathbf{j}) \underset{C}{\rightarrow} (-\rho, -\mathbf{j}) = -j_\mu$$

$$A_\mu = (\varphi, \mathbf{A}) \underset{C}{\rightarrow} -(\varphi, \mathbf{A}) = -A_\mu \rightarrow U_C |\gamma\rangle = (-1)|\gamma\rangle$$

$$j^\mu A_\mu \underset{C}{\rightarrow} j^\mu A_\mu$$

$$U_C |n\gamma\rangle = (-1)^n |n\gamma\rangle$$

Boson-antiboson: CM frame

$$U_C |b\bar{b}\rangle = U_P |b\bar{b}\rangle = (-1)^l |b\bar{b}\rangle \rightarrow \eta_C = (-1)^l$$

Fermion-antifermion: CM frame

$U_C = U_P U_S$ Parity * Spin exchange

$$U_P |f\ \bar{f}\rangle = (-1)^{l+1} |f\bar{f}\rangle$$

$$U_S |\uparrow\downarrow - \downarrow\uparrow\rangle = |\downarrow\uparrow - \uparrow\downarrow\rangle = -|\uparrow\downarrow - \downarrow\uparrow\rangle,\ U_S |\uparrow\downarrow + \downarrow\uparrow\rangle = |\downarrow\uparrow + \uparrow\downarrow\rangle = |\uparrow\downarrow + \downarrow\uparrow\rangle,$$

$$U_S |\uparrow\uparrow\rangle = |\uparrow\uparrow\rangle, U_S |\downarrow\downarrow\rangle = |\downarrow\downarrow\rangle$$

$$\rightarrow U_C |f\ \bar{f}\rangle = (-1)^{l+s} |f\bar{f}\rangle$$

Applications - II

$\pi^0, \eta, \ldots: \quad \eta_C = (-1)^2 = +1$ 2 photons decays

States $f\ \bar{f}$ decaying to n photons:

$$C : (-1)^{l+s} \rightarrow S \text{ state} \begin{cases} \eta_C = +1 \text{ singlet} \\ \eta_C = -1 \text{ triplet} \end{cases}$$

$$n \text{ photons: } \eta_C = (-1)^l \rightarrow \begin{cases} 2 \text{ photons } \eta_C = +1 \\ 3 \text{ photons } \eta_C = -1 \end{cases}$$

$$\rightarrow \begin{cases} \text{singlet} \rightarrow 2 \text{ photons} \\ \text{triplet} \rightarrow 3 \text{ photons} \end{cases}$$

Meson: Fermion-Antifermion bound state

L	S=0	S=1
0	0^{-+}	1^{--}
1	1^{+-}	0^{++}, 1^{++}, 2^{++}
2	2^{-+}	1^{--}, 2^{--}, 3^{--}

States decaying to 2 pions

J	PC = ++	PC = +-, -+, --
0	OK	KO
1	KO	KO,KO,OK
2	OK	KO

Time Reversal

Need to make clear:
These considerations are relevant to physical systems with a (very) small number of degrees of freedom
Do not apply to complex systems, whose statistical evolution is driven by the II Law of Thermodynamics. For them, a *time arrow* can be defined with no ambiguities. Or so we believe… (A *very* hard subject)

Can state another 'relativity principle', about the choice of the positive direction of the time coordinate of physical events:

The laws of physics are the same for all the observers, irrespective of the chosen positive direction of their 'time axis'

Experimental fact:

All interactions, <u>except weak interaction</u>, are invariant wrt to time reversal

Classical Physics

General behavior of physical quantities wrt to time reversal:

$$t \to -t:$$

$$\mathbf{r} \to \mathbf{r}$$

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} \to \frac{d\mathbf{r}}{d(-t)} = -\mathbf{v}, \mathbf{p} \to -\mathbf{p}$$

$$\mathbf{a} = \frac{d^2\mathbf{r}}{dt^2} \to \mathbf{a}, \mathbf{F} \to \mathbf{F}$$

$$\mathbf{L} = \mathbf{r} \times \mathbf{p} \to -\mathbf{L}$$

$$E \to E$$

$$\rho \to \rho$$

$$\mathbf{j} = \rho\mathbf{v} \to -\mathbf{j}$$

$$\varphi \to \varphi$$

$$\mathbf{A} \to -\mathbf{A}$$

$$\mathbf{E} \to \mathbf{E}$$

$$\mathbf{B} \to -\mathbf{B}$$

Observe: Ohm's law is *not* TR invariant

$\mathbf{j} = \sigma\mathbf{E} \to -\mathbf{j} = \sigma\mathbf{E},$ take σ as TR invariant

Example of a *macroscopic* law…

Schrodinger Equation

$T : t \rightarrow t' = -t$

First guess...

$\langle t' | \psi' \rangle = \langle t | \psi \rangle = \langle -t' | \psi \rangle$

$\langle t | \psi' \rangle = \langle -t | \psi \rangle$

$\rightarrow \psi'(t) = \psi(-t)$

$U_T : |\psi\rangle \rightarrow |\psi'\rangle \equiv U_T |\psi\rangle$

$U_T : |t\rangle \rightarrow |t'\rangle \equiv U_T |t\rangle = \eta |-t\rangle$ Wrong!

Indeed:

$$H|\psi(t)\rangle = i\frac{\partial |\psi(t)\rangle}{\partial t} \underset{U_T}{\rightarrow} H|\psi(-t)\rangle = -i\frac{\partial |\psi(-t)\rangle}{\partial t}$$

Redefine:

$$\left.\begin{array}{l} T : t \rightarrow -t \\ K : i \rightarrow -i \end{array}\right\}, \ U_T = KT$$

$|\psi(t)\rangle \rightarrow U_T |\psi(t)\rangle = |\psi^T(t)\rangle = |\psi^*(-t)\rangle$ OK

Time Reversal

Take a plane wave:

$$e^{-i(Et-\mathbf{p}\cdot\mathbf{r})} \underset{U_T}{\longrightarrow} e^{+i(E(-t)-\mathbf{p}\cdot\mathbf{r})} = e^{-i(Et+\mathbf{p}\cdot\mathbf{r})} = e^{-i(Et-(-\mathbf{p})\cdot\mathbf{r})}$$

Quite natural...:

U_T sends a progressive plane wave into a regressive one

Puzzled? Think of it in this way:
U_T inverts the positive direction of time for the 2nd observer. Therefore, the 2nd observer has a clock which is running *backwards*.
Then the same wave which is seen as *progressive* by the first observer, is seen as *regressive* by the second..

Take a particle with *s=1/2*:

$$\begin{pmatrix}\psi_+(t)\\ \psi_-(t)\end{pmatrix} \rightarrow \begin{pmatrix}\psi_+^T(t)\\ \psi_-^T(t)\end{pmatrix} = M\begin{pmatrix}\psi_+^*(-t)\\ \psi_-^*(-t)\end{pmatrix}$$ Must take into account that $\mathbf{s}$ is *reversed* by T

$$\begin{pmatrix}\psi_+(t)\\ \psi_-(t)\end{pmatrix} \underset{U_T}{\longrightarrow} \begin{pmatrix}\psi_+^T(t)\\ \psi_-^T(t)\end{pmatrix} = -i\sigma_y\begin{pmatrix}\psi_+^*(-t)\\ \psi_-^*(-t)\end{pmatrix} = \begin{pmatrix}\psi_-^*(-t)\\ -\psi_+^*(-t)\end{pmatrix}$$ OK

$$\sigma_y = \begin{pmatrix}0 & -i\\ +i & 0\end{pmatrix} \rightarrow M = -i\sigma_y = \begin{pmatrix}0 & +1\\ -1 & 0\end{pmatrix}$$

Spin-reversing operator

Apply U_T a second time:

$$\begin{pmatrix}\psi_+^T(t)\\ \psi_-^T(t)\end{pmatrix} \underset{U_T}{\longrightarrow} -i\sigma_y\begin{pmatrix}\psi_-^*(t)\\ -\psi_+^*(t)\end{pmatrix}^* = \begin{pmatrix}-\psi_+(t)\\ -\psi_-(t)\end{pmatrix} = -\begin{pmatrix}\psi_+(t)\\ \psi_-(t)\end{pmatrix}$$

Similar to parity for spin 1/2

Time Reversal in QFT

Take FT course!

Applications

Basic remark:

U_T is *not* a unitary operator
U_T is *not* a Hermitian operator

Therefore:
No real eigenvalues
No (conserved) observables
$\rightarrow$*No 'time parity'*

Nevertheless:

Quite useful to establish properties of fundamental interactions

Because is not linear, rather it is antilinear

$$U_T\left(a|\psi_1\rangle + b|\psi_2\rangle\right) = a^* U_T|\psi_1\rangle + b^* U_T|\psi_2\rangle$$

Complex conjugation

$$K : \alpha z \rightarrow (\alpha z)^* = \alpha^* z^*$$

is a non-linear operation. Proof:

$$\alpha = a e^{i\varphi}, z = \rho e^{i\psi} \rightarrow \alpha^* = a e^{-i\varphi}, z^* = \rho e^{-i\psi}$$

$$\alpha z = a e^{i\varphi} \rho e^{i\psi} = a\rho e^{i(\varphi+\psi)} \rightarrow (\alpha z)^* = a\rho e^{-i(\varphi+\psi)} = \alpha^* z^*$$

But:

$\alpha^* z^* \neq \alpha z^*$ $\rightarrow$Non linear whenever $\alpha \neq \alpha^*$!!

Indeed, a linear operator must satisfy:

$$A : (\alpha z) \rightarrow A(\alpha z) = \alpha A(z)$$

Effect on Scalar Products

$\{|i\rangle, i=1,...\}$ complete set of states

$\langle i | j\rangle = \delta_{ij}$

$\rightarrow \langle U_T i | U_T j\rangle = \delta_{ij}$ norm is $U_T -$invariant

$$|\psi_1\rangle = \sum_i |i\rangle\langle i|\psi_1\rangle \rightarrow |U_T\psi_1\rangle = \sum_i |U_T i\rangle\langle i|\psi_1\rangle^*$$

$$\langle\psi_2| = \sum_j \langle\psi_2 | j\rangle\langle j| \rightarrow \langle U_T\psi_2| = \sum_j \langle\psi_2 | j\rangle^* \langle U_T j|$$

$$\rightarrow \langle U_T\psi_2 | U_T\psi_1\rangle = \sum_j \langle\psi_2 | j\rangle^* \langle U_T j| \sum_i |U_T i\rangle\langle i|\psi_1\rangle^*$$

$$= \sum_{i,j} \langle\psi_2 | j\rangle^* \underbrace{\langle U_T j | U_T i\rangle}_{\delta_{ij}} \langle i|\psi_1\rangle^* = \sum_i \langle\psi_2 | i\rangle^* \langle i|\psi_1\rangle^*$$

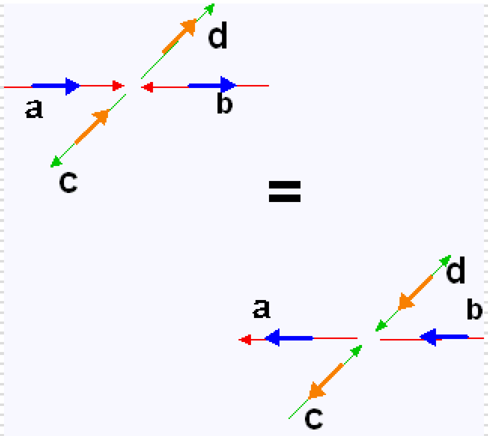
$$= \sum_i \langle i|\psi_2\rangle\langle\psi_1|i\rangle = \langle\psi_1|\psi_2\rangle$$ U_T swaps states in any scalar product

Matrix Elements

Matrix element for a transition: Initial $\leftrightarrow$ Final

H T-invariant $\rightarrow$ Same matrix element for direct and reversed transition

$\underbrace{1+2}_{i} \rightarrow \underbrace{3+4}_{f}$ 2-body process



Reciprocity Theorem:

$$\langle \mathbf{p}_f, \mathbf{s}_f | S | \mathbf{p}_i, \mathbf{s}_i \rangle = \langle \underbrace{-\mathbf{p}_i, -\mathbf{s}_i}_{T|\mathbf{p}_i, \mathbf{s}_i\rangle} | S | \underbrace{-\mathbf{p}_f, -\mathbf{s}_f}_{T\langle \mathbf{p}_f, \mathbf{s}_f|} \rangle^*$$

Detailed Balance Theorem:

$$\frac{\dfrac{d\sigma_{if}}{d\Omega}}{\dfrac{d\sigma_{fi}}{d\Omega}} = \left(\frac{p_f}{p_i}\right)^2 \frac{(2s_3+1)(2s_4+1)}{(2s_1+1)(2s_2+1)}$$

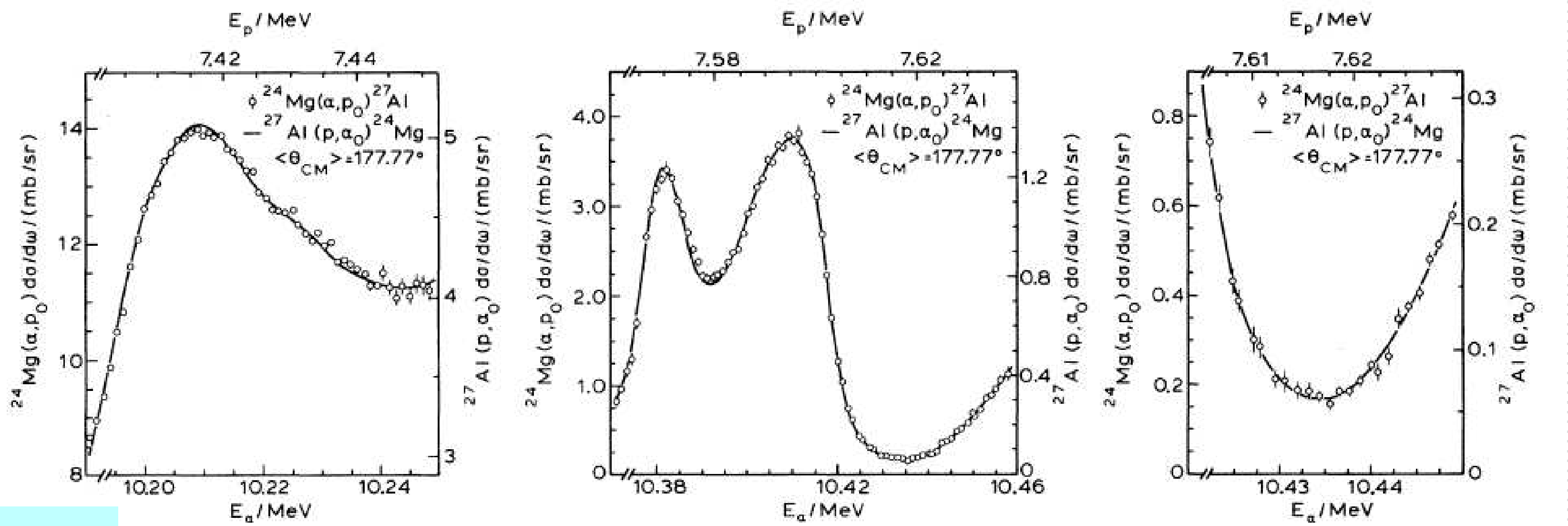
OK for Strong, E.M., KO for Weak

Example: Test of the Detailed Balance

Nuclear reactions:

$$(1) \quad \alpha + {}^{24}Mg \rightarrow p + {}^{27}Al$$

$$(2) \quad p + {}^{27}Al \rightarrow \alpha + {}^{24}Mg$$



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Full line: Reaction (1)
Circles: Reaction (2)

T-Violating Correlations

Example (many more exist):

$$\Lambda^0 \to p + \pi^-$$

$$\left.\begin{array}{l} \frac{1}{2}^+ = \frac{1}{2}^+ \oplus 0^- \oplus L^{(-1)^L} = \frac{1}{2} \oplus L \\ + \quad = + \quad \cdot \quad - \quad \cdot \quad (-1)^L = (-1)^{L+1} \end{array}\right\} \to L = 0,1$$

Decay amplitude = Sum of S,P waves
S,P waves *do* interfere because of parity violation. So:
Angular distribution has a term $\propto$ *cos* θ

Most general form of angular distribution:

$$\frac{d\Gamma}{d\Omega_\pi} = 1 + A\left(\boldsymbol{J}_\Lambda + \boldsymbol{J}_p\right)\cdot \hat{\boldsymbol{p}}_\pi + B\left(\boldsymbol{J}_\Lambda \times \boldsymbol{J}_p\right)\cdot \hat{\boldsymbol{p}}_\pi + C\left(\boldsymbol{J}_\Lambda \cdot \boldsymbol{J}_p\right) + (1-C)\left(\boldsymbol{J}_\Lambda \cdot \hat{\boldsymbol{p}}_\pi\right)\left(\boldsymbol{J}_p \cdot \hat{\boldsymbol{p}}_\pi\right)$$

$$\left(\boldsymbol{J}_\Lambda \times \boldsymbol{J}_p\right)\cdot \hat{\boldsymbol{p}}_\pi \underset{U_T}{\longrightarrow} \left(\left(-\boldsymbol{J}_\Lambda\right)\times\left(-\boldsymbol{J}_p\right)\right)\cdot\left(-\hat{\boldsymbol{p}}_\pi\right) = -\left(\boldsymbol{J}_\Lambda \times \boldsymbol{J}_p\right)\cdot \hat{\boldsymbol{p}}_\pi$$ T-Violating term

$\to$ Expect $B = 0$

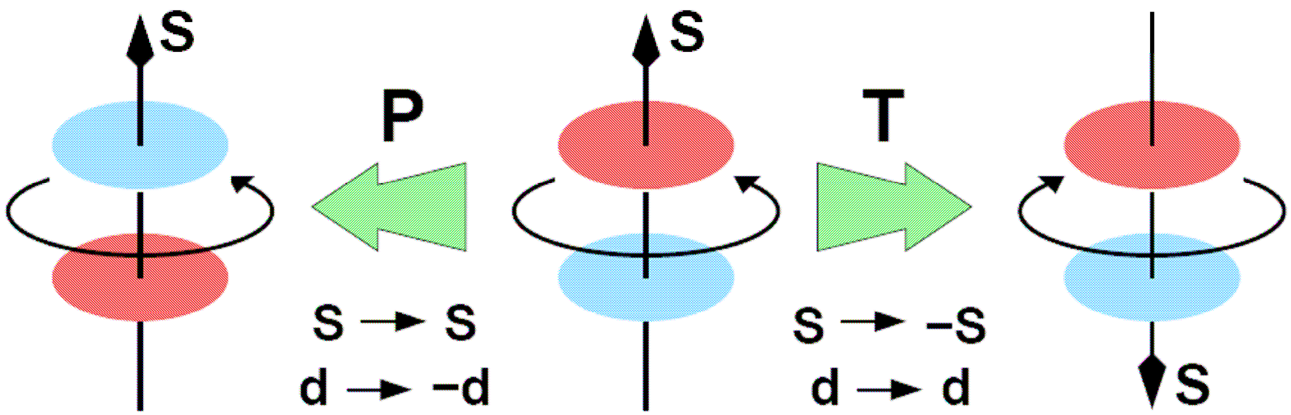
Must correct for final state interaction: $p\pi$ scattering

$\to$ Get an upper limit of a few %

Electric Dipole Moments - I

Warning: This is true in the absence of a degenerate ground state
Polar molecules *do* have degenerate ground states…

Elementary particle can only have a permanent electric dipole moment if both parity and time reversal symmetries are broken:



$$\vec{\mu} = g\,\frac{e}{mc}\vec{S} \qquad \vec{d} = \eta\,\frac{e}{2mc}\vec{S}$$

@TBA

Electric Dipole Moments - II

For a particle with both magnetic and electric dipole moments:

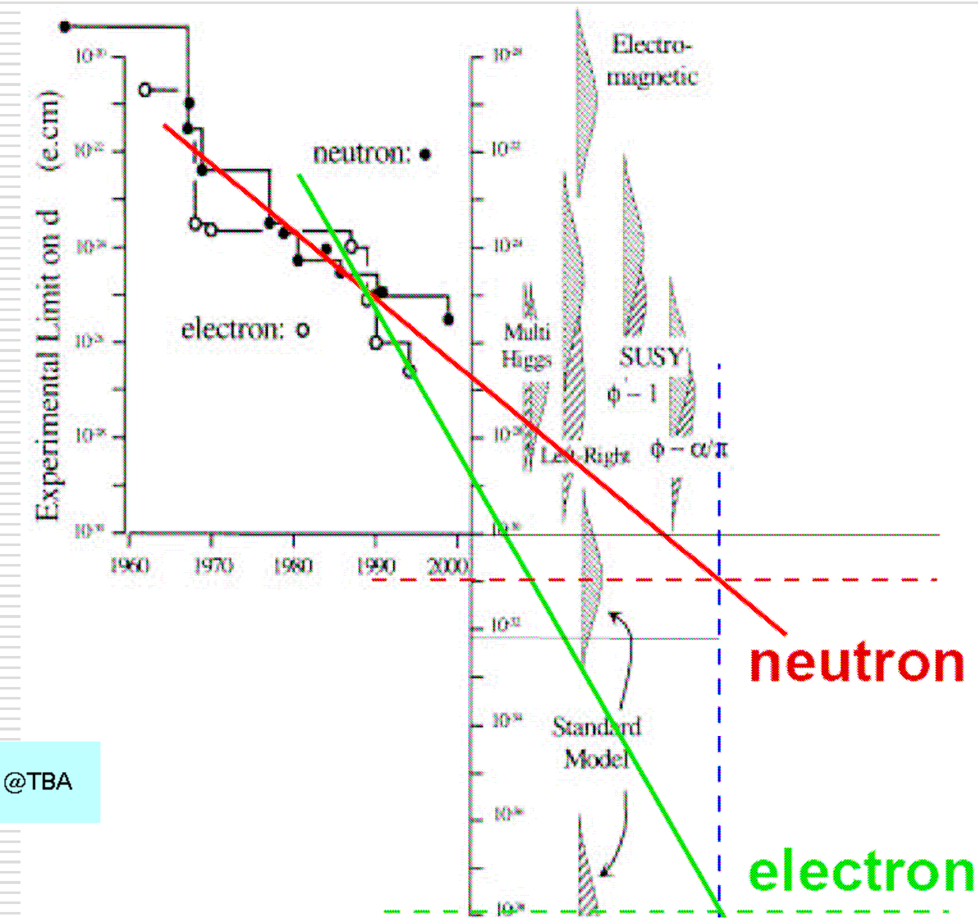
$H' = -\boldsymbol{\mu}\cdot\mathbf{B} - \mathbf{d}\cdot\mathbf{E}$ Interaction energy

Can observe in external $\mathbf{E}$:

Level shift

Spin precession

Status of the EDM



@TBA

CPT

Fundamental theorem applying to all field theories, including our beloved Standard Model:

Lorentz invariance

Micro-causality (whatever it means…)

Spin-statistics

$\rightarrow$ *The product transformation CPT is a good symmetry*

Consequences:

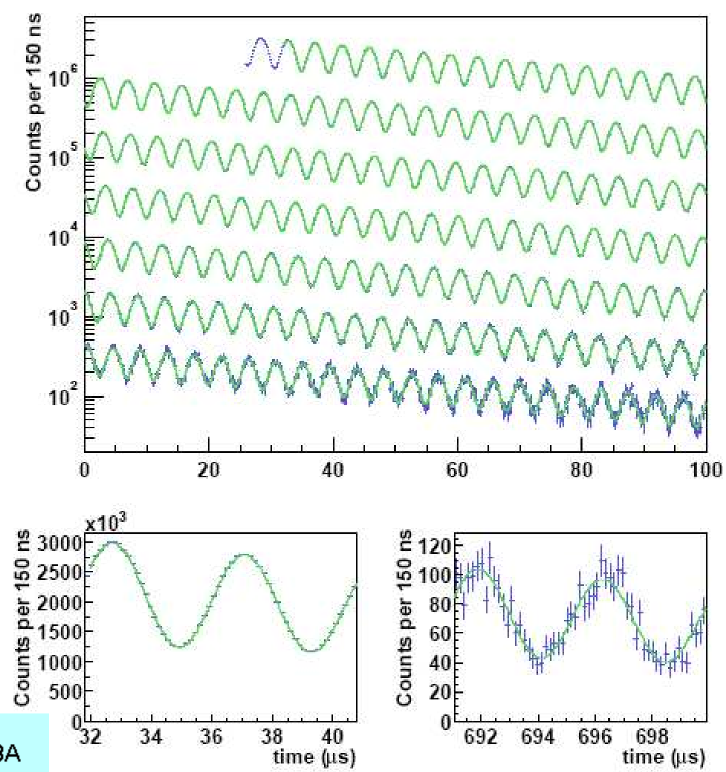
$$m_{particle} = m_{antiparticle}$$

$$\tau_{particle} = \tau_{antiparticle}$$

$$|\boldsymbol{\mu}|_{particle} = |\boldsymbol{\mu}|_{antiparticle}$$

....more quantum numbers

Good Old QED Test: Muon Anomaly



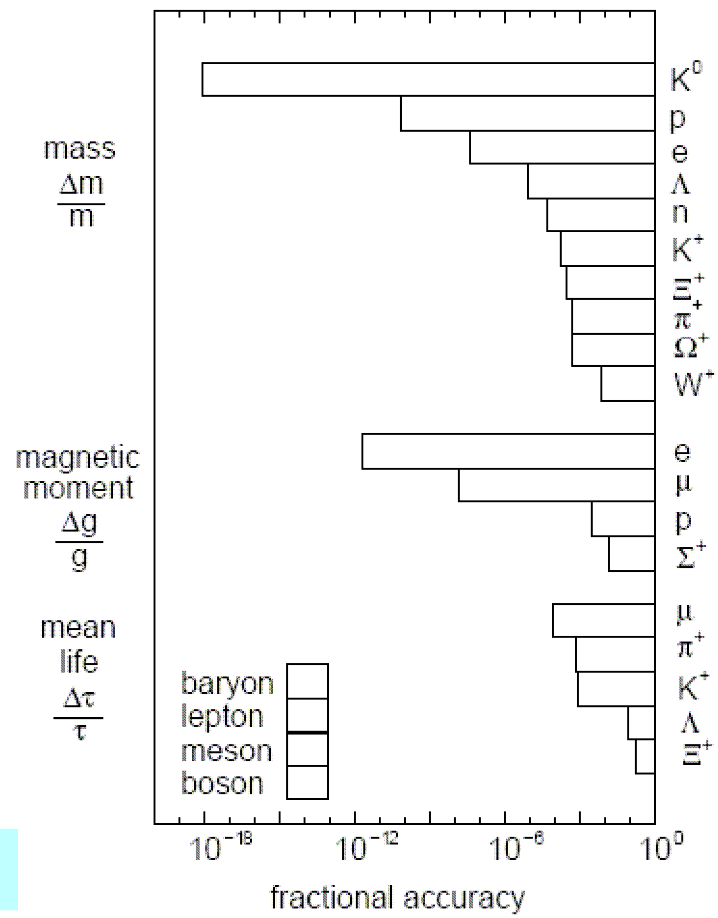
Also an excellent test of CPT in EM interaction

$$\left.\begin{aligned} a_{\mu^+} &= (11659203 \pm 8) \times 10^{-10} \\ a_{\mu^-} &= (11659214 \pm 9) \times 10^{-10} \end{aligned}\right\} \Delta a_\mu = (11 \pm 12) \times 10^{-10}$$

@TBA

CPT Tests

Just a flash on the general status of CPT tests…
…very comfortable



@TBA

Symmetries: Summary

Conserved quantity	Interaction		
	Strong	E.M.	Weak
4-momentum	OK	OK	OK
Charge	OK	OK	OK
Ang. Momentum, CM speed	OK	OK	OK
Baryonic number	OK	OK	OK
Leptonic numbers (3)	OK	OK	~OK
Parity	OK	OK	KO
Charge parity	OK	OK	KO
(Time reversal)	OK	OK	KO
CP	OK	OK	KO
(CPT)	OK	OK	OK
Flavor	OK	OK	KO