

Elementary Particles I

1 – Symmetries

Mostly C, P, T

Classical Particle Dynamics

Lagrangian formalism

$$\frac{\partial}{\partial t} \left(\frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = 0 \quad \text{Lagrange's equations}$$

$$\frac{\partial L}{\partial q_i} = 0 \rightarrow \frac{\partial}{\partial t} \left(\frac{\partial L}{\partial \dot{q}_i} \right) = 0 \rightarrow p_i \equiv \frac{\partial L}{\partial \dot{q}_i} = \text{const}$$

Hamiltonian formalism

$$\dot{p}_i = - \frac{\partial H}{\partial q_i} \quad \text{Hamilton's equations}$$

$$\rightarrow H \text{ not depending on } q_i \Rightarrow p_i = \text{const}$$

Symmetry = Irrelevance of ... to the dynamics of a closed system

Simmetry	Invariance	Conservation
Frame origin	Space translation	Total momentum
Time origin	Time translation	Total energy
Frame orientation	Space rotation	Total angular momentum
Frame velocity	Lorentz transformation	CM velocity

Extension to Classical Fields

Take any physical system, *including fields*: Can describe its motion by the same methods (Lagrangian or Hamiltonian) in terms of a *Principle of Minimum Action*

Then Noether's Theorem states that

For every continuous transformation of the field functions and coordinates which leaves the action unchanged, there is a definite combination of the field functions and their derivatives which is conserved

This is called a *conserved current*

Main point to stress:

Conservation laws must include contributions from the fields, besides particles

Continuous vs Discrete Symmetry

Previous examples: Continuous symmetry operations

Any given operation S depends on a number of *continuous parameters*
Any given operation S can be thought as *continuously evolving* from the identity operation

Example: S = Shift by $\mathbf{a}$ of the frame origin

Depends on 3 parameters a_x, a_y, a_z

$$S(\mathbf{a}) \xrightarrow{a_x, a_y, a_z \rightarrow 0} I$$

What are discrete symmetry operations?

They do *not* depend on any parameter

They are *intrinsically separated* from the identity operation

Example: S = Axis inversion $x \rightarrow -x, y \rightarrow -y, z \rightarrow -z$

No parameter, no continuous evolution from identity

Quantum Mechanics - I

Link between invariance and conservation in QM

For any observable Q :

$$\langle Q \rangle = \langle \psi | Q | \psi \rangle \rightarrow \frac{d\langle Q \rangle}{dt} = \left\langle \frac{\partial \psi}{\partial t} \middle| Q | \psi \right\rangle + \langle \psi | \frac{\partial Q}{\partial t} | \psi \rangle + \langle \psi | Q \middle| \frac{\partial \psi}{\partial t} \right\rangle$$

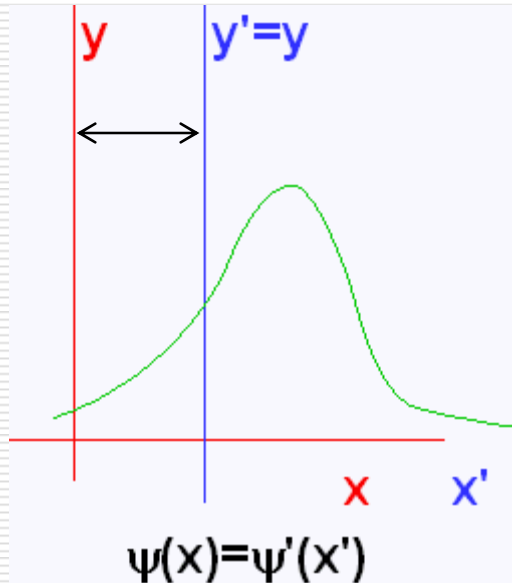
From Schrodinger equation:

$$i \left| \frac{\partial \psi}{\partial t} \right\rangle = H | \psi \rangle, -i \left\langle \frac{\partial \psi}{\partial t} \right| = \langle \psi | H^\dagger$$

$$\begin{aligned} \frac{d\langle Q \rangle}{dt} &= \left\langle \frac{\partial \psi}{\partial t} \middle| Q | \psi \right\rangle + \langle \psi | \frac{\partial Q}{\partial t} | \psi \rangle + \langle \psi | Q \middle| \frac{\partial \psi}{\partial t} \right\rangle \\ &= i \langle \psi | H^\dagger Q | \psi \rangle + \langle \psi | \frac{\partial Q}{\partial t} | \psi \rangle - i \langle \psi | Q H | \psi \rangle \end{aligned}$$

$$H^\dagger = H \rightarrow \frac{d\langle Q \rangle}{dt} = i \langle \psi | \left([H, Q] + \frac{\partial Q}{\partial t} \right) | \psi \rangle \rightarrow \left. \begin{aligned} [H, Q] &= 0 \\ \frac{\partial Q}{\partial t} &= 0 \end{aligned} \right\} \rightarrow \langle Q \rangle = \text{constant}$$

Quantum Mechanics - II



Just meaning:

One and same state

Different coordinates x, x' in Σ, Σ'

-> $\psi(x) = \psi'(x')$

One state, two wave functions
in Σ, Σ'

Coordinate transformation S induces
a transformation in function space

Example:

$$S: x \rightarrow x' = S(x) = x - a$$

$$S^{-1}: x' \rightarrow x = S^{-1}(x') = x' + a$$

$$\psi'(x') = \psi(x) \leftrightarrow \psi'[S(x)] = \psi(x)$$

$$\rightarrow \psi'[S^{-1}(S(x))] = \psi(S^{-1}(x))$$

$$\rightarrow \psi'(x) \neq \psi[S^{-1}(x)] \neq \psi(x)$$

Indeed, for example:

$$\psi(x) = Ne^{-\frac{x^2}{2\sigma^2}}$$

$$\psi'(x') = \psi[S^{-1}(x')] = Ne^{-\frac{(x'+a)^2}{2\sigma^2}} \left(= Ne^{-\frac{(x-a+a)^2}{2\sigma^2}} = Ne^{-\frac{x^2}{2\sigma^2}} \right)$$

$$\psi'(x') = Ne^{-\frac{(x'+a)^2}{2\sigma^2}} \neq Ne^{-\frac{x'^2}{2\sigma^2}} = \psi(x')$$

Not the only possible point of view...

Quantum Mechanics - III

Unitary transformation

$$U : \psi(x) \rightarrow \psi'(x) = \psi[S^{-1}(x)] = U[\psi(x)]$$

In term of state vectors:

$$|\psi'\rangle = U|\psi\rangle \rightarrow U \text{ unitary: } U^\dagger = U^{-1}$$

Take A = Any operator

$$\langle\psi|A|\psi\rangle = \langle\psi'|A'|\psi'\rangle \text{ defining transformed operator}$$

$$\langle\psi'|A'|\psi'\rangle = \langle\psi|U^\dagger A' U|\psi\rangle$$

$$\langle\psi|A|\psi\rangle = \langle\psi'|A'|\psi'\rangle = \langle\psi|U^\dagger A' U|\psi\rangle \rightarrow A = U^\dagger A' U \rightarrow A' = U A U^\dagger$$

When A invariant wrt U

$$A = A'$$

$$\rightarrow U A U^\dagger = A$$

$$\rightarrow [U, A] = 0 \text{ when } U \text{ is a symmetry operator for } A$$

By taking $A=H$: $[U, H] = 0 \rightarrow \langle U \rangle = \text{const}$

Symmetry operators
are *constants*

Quantum Field Theory

Take FT Course!

Summary

Coordinate transformation S (possibly including *internal* coordinates)

Continuous, Discrete

Examples:

Space translation (C)

Axis inversion (D)

Corresponding 'Relativity Principles'

Example:

The laws of physics are the same for all the observers, irrespective of the chosen origin of their reference frames

Induced transformation U on state space

Unitary, Antiunitary

$\langle U \rangle$ = constant of motion

Discrete Symmetries

Coordinate transformations not evolving continuously from identity

Very important in Particle Physics:

Space Inversion

Time Reversal

Charge Conjugation

Leading to:

Quantum Numbers

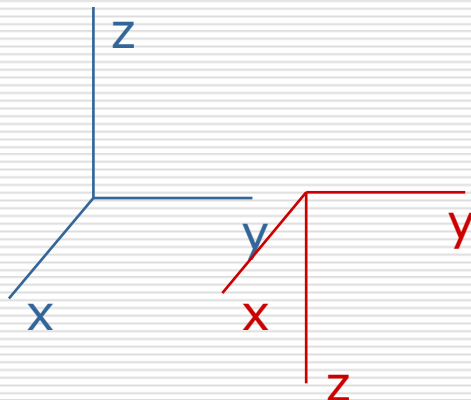
Laws of Conservation

Selection Rules

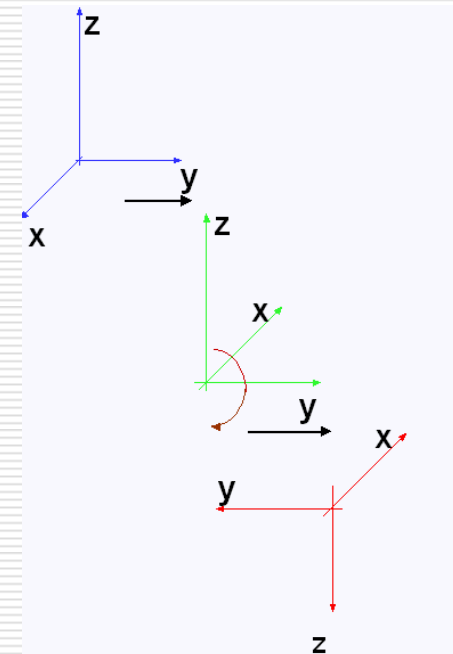
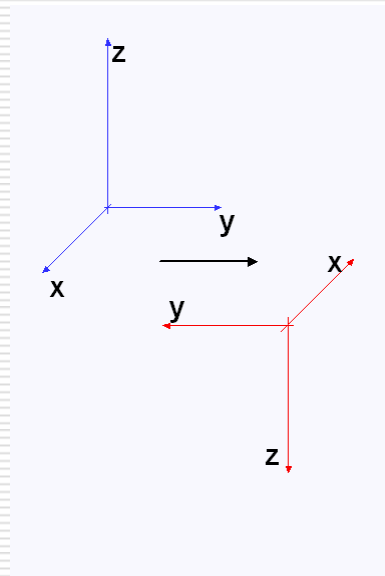
Constraints on Properties of Interactions

Parity in Classical Physics - I

Two, non equivalent ways of choosing a Cartesian frame in 3D



In 3D: 3 axis inversions equivalent to 1 axis (= mirror) inversion * Rotation



Non equivalent = Not connected by a rotation
Connected by 3 axis inversions

Parity in Classical Physics - II

Define coordinate symmetry operation

$$P : \mathbf{r} \rightarrow \mathbf{r}' = -\mathbf{r}$$

Corresponding 'Relativity Principle'
The laws of physics are the same for all the observers, irrespective of the chosen chirality of their reference frames

Transformations of physical quantities follow:

$\mathbf{r} \rightarrow -\mathbf{r}$	position
$t \rightarrow t$	time
$\mathbf{p} \rightarrow -\mathbf{p}$	3-momentum
$E \rightarrow E$	energy
$\mathbf{L} \rightarrow \mathbf{L}$	angular momentum

Hamilton's equations: form invariant wrt inversions $\leftrightarrow H(q, p) = H(-q, -p)$

No conservation law!

P not connected to identical transformation

Parity in Classical Physics - III

General taxonomy of physical quantities (observables):

- With respect to rotations
- With respect to reflections

'Parity' of corresponding Hermitian operator:

	True	Pseudo
Scalar	$+1$	-1
Vector	-1	$+1$
Rank N Tensor	$(-1)^N$	$(-1)^{N+1}$

Parity in Classical Physics - IV

Parity behavior of selected electromagnetic quantities:

$$\mathbf{j}(\mathbf{r}, t) \rightarrow -\mathbf{j}(-\mathbf{r}, t) \quad \text{current density}$$

$$\rho(\mathbf{r}, t) \rightarrow \rho(-\mathbf{r}, t) \quad \text{charge density}$$

$$\mathbf{E}(\mathbf{r}, t) \rightarrow -\mathbf{E}(-\mathbf{r}, t) \quad \text{electric field}$$

$$\mathbf{B}(\mathbf{r}, t) \rightarrow \mathbf{B}(-\mathbf{r}, t) \quad \text{magnetic field}$$

Note

All of this is indeed conventional, as based on our definition of the electric charge as a *scalar*.

What actually matters is that *force* is a polar vector

Then

$$\left. \begin{array}{l} \mathbf{F} = q\mathbf{E} \\ \mathbf{F} = q\mathbf{v} \times \mathbf{B} \\ \mathbf{F}, \mathbf{v} \text{ polar} \end{array} \right\} \rightarrow \left\{ \begin{array}{l} \text{either } q \text{ scalar, } \mathbf{E} \text{ polar, } \mathbf{B} \text{ axial} \\ \text{or } q \text{ pseudoscalar, } \mathbf{E} \text{ axial, } \mathbf{B} \text{ polar} \end{array} \right.$$

Parity in Quantum Mechanics - I

Coordinate transformation as before: $P: \mathbf{r} \rightarrow \mathbf{r}' = P(\mathbf{r}) = -\mathbf{r}$

Induced transformation in state space: U_P unitary operator

A) Commutation relations with position, momentum, angular momentum

$$\begin{aligned} U_P \mathbf{r} |\psi\rangle &= U_P \mathbf{r} \sum |\mathbf{r}\rangle \langle \mathbf{r} | \psi \rangle = U_P \sum \mathbf{r} |\mathbf{r}\rangle \langle \mathbf{r} | \psi \rangle = \sum U_P \mathbf{r} |\mathbf{r}\rangle \langle \mathbf{r} | \psi \rangle \\ &= \sum (-\mathbf{r}) |-\mathbf{r}\rangle \langle -\mathbf{r} | \psi \rangle = (-\mathbf{r}) U_P |\psi\rangle = -\mathbf{r} U_P |\psi\rangle \\ &\rightarrow U_P \mathbf{r} U_P^{-1} = -\mathbf{r} \rightarrow U_P \mathbf{r} = -\mathbf{r} U_P \end{aligned}$$

$$U_P U_T(\delta \mathbf{r}) = U_P (1 - \mathbf{p} \cdot \delta \mathbf{r})$$

$$U_P (1 - \mathbf{p} \cdot \delta \mathbf{r}) = (1 + \mathbf{p} \cdot \delta \mathbf{r}) U_P$$

$$\rightarrow -U_P \mathbf{p} \cdot \delta \mathbf{r} = \mathbf{p} \cdot \delta \mathbf{r} U_P \rightarrow -U_P \mathbf{p} = \mathbf{p} U_P$$

$$U_P \hat{\mathbf{L}} = \hat{\mathbf{L}} U_P$$

Summary:

U_P anticommutes with $\mathbf{r}, \mathbf{p}$
commutes with $\mathbf{L}$

Parity in Quantum Mechanics - II

B) Action of U_P on states

$$U_P : |\psi\rangle \rightarrow |\psi'\rangle \equiv U_P |\psi\rangle$$

Same physical state described by two different kets in the two frames

Take position eigenstate:

$$U_P : |\mathbf{r}\rangle \rightarrow |\mathbf{r}'\rangle \equiv U_P |\mathbf{r}\rangle = \eta |-\mathbf{r}\rangle, \quad \eta \text{ arbitrary phase} = e^{i\alpha}$$

Take generic state:

$$U_P : |\psi\rangle \rightarrow |\psi'\rangle \equiv U_P |\psi\rangle$$

Would yield similar results by expanding into momentum eigenstates

Expand into position eigenstates: $|\psi\rangle = \sum_{\mathbf{r}} |\mathbf{r}\rangle \langle \mathbf{r} | \psi \rangle$

$$U_P |\psi\rangle = U_P \sum_{\mathbf{r}} |\mathbf{r}\rangle \langle \mathbf{r} | \psi \rangle = \sum_{\mathbf{r}} U_P |\mathbf{r}\rangle \langle \mathbf{r} | U_P^{-1} U_P \psi \rangle = \sum_{\mathbf{r}} [U_P |\mathbf{r}\rangle \langle \mathbf{r} | U_P^{-1}] U_P \psi \rangle$$

$$= \sum_{\mathbf{r}'} |\mathbf{r}'\rangle \langle \mathbf{r}' | U_P \psi \rangle = \sum_{\mathbf{r}'} |\mathbf{r}'\rangle \langle \mathbf{r}' | \psi' \rangle = |\psi'\rangle = \sum_{\mathbf{r}} |-\mathbf{r}\rangle \langle -\mathbf{r} | \psi' \rangle$$

$$\rightarrow \langle \mathbf{r}' | \psi' \rangle = \langle \mathbf{r} | \psi \rangle \rightarrow \langle \mathbf{r} | \psi' \rangle = \langle -\mathbf{r} | \psi \rangle$$

Talking wave functions: $\psi'(\mathbf{r}) = \psi(-\mathbf{r})$ (as shown before)

Parity in Quantum Mechanics - III

C) Fundamental property of parity operator

$$P:\mathbf{r} \rightarrow \mathbf{r}' = P(\mathbf{r}) = -\mathbf{r}, P:\mathbf{r}' \rightarrow \mathbf{r}'' = P(\mathbf{r}') = -\mathbf{r}' = \mathbf{r} \rightarrow P^2 = I$$

$$U_P |\psi\rangle = |\psi'\rangle, U_P |\psi'\rangle = U_P (U_P |\psi\rangle) = U_P^2 |\psi\rangle \rightarrow U_P^2 = \eta^2 I$$

$$\left. \begin{array}{l} U_P^\dagger = U_P^{-1}, \quad U_P \text{ unitary} \\ U_P^2 = I \end{array} \right\} \rightarrow U_P^2 = U_P U_P = I = U_P U_P^{-1} = U_P U_P^\dagger \rightarrow U_P^\dagger = U_P \quad U_P \text{ Hermitian}$$

$\rightarrow U_P$ eigenvalues are real

$\rightarrow \eta = e^{i\alpha} = \pm 1 \equiv \eta_p$ parity quantum number

$\rightarrow U_P$ eigenstates: $U_P |a\rangle = \pm |a\rangle$

Consequences:

$[H, U_P] = 0 \rightarrow$ stationary states have definite η_p - when not degenerate

$[H, U_P] = 0 \rightarrow \eta_p = \text{constant of motion}$

Parity in Quantum Mechanics - IV

Parity for a composite system (just meaning with several degrees of freedom)

$|a\rangle$ compound state of subsystems 1 and 2, parity eigenstate

$U_P^{(1)}, U_P^{(2)}$ parity operators for 1,2

$$U_P^{(1)}|a\rangle = \eta_P^{(1)}|a\rangle, U_P^{(2)}|a\rangle = \eta_P^{(2)}|a\rangle$$

$$U_P^{(2)}[U_P^{(1)}|a\rangle] = U_P^{(2)}[\eta_P^{(1)}|a\rangle] = \eta_P^{(2)}\eta_P^{(1)}|a\rangle \rightarrow \eta_P \text{ multiplicative quantum number}$$

Quantum numbers (i.e., conserved quantities) are usually *additive*: ???

Reason: For continuous symmetries the unitary operators are not Hermitian

U unitary $\rightarrow U = e^{iaH} \simeq 1 + iaH$, H Hermitian; $\lim_{a \rightarrow 0} U = 1 \rightarrow$ Use infinitesimal generators

Example for translations:

$$U_a^{(1)} \simeq 1 + ia\mathbf{p}^{(1)}, U_a^{(2)} \simeq 1 + ia\mathbf{p}^{(2)} \rightarrow U_a^{(2)}(U_a^{(1)}|\psi\rangle) \simeq (1 + ia\mathbf{p}^{(2)})[(1 + ia\mathbf{p}^{(1)})|\psi\rangle]$$

$$\rightarrow U_a^{(2)}(U_a^{(1)}|\psi\rangle) \simeq (1 + ia\mathbf{p}^{(2)} + ia\mathbf{p}^{(1)})|\psi\rangle = [1 + ia(\mathbf{p}^{(2)} + \mathbf{p}^{(1)})]|\psi\rangle$$

U Hermitian $\rightarrow$ Constant of motion = $U \rightarrow$ Multiplicative

U not Hermitian $\rightarrow$ Constant of motion $\sim$ "Logarithm" of $U \dots \rightarrow$ Additive

The Parity Quantum Number

As a quantum number, parity may or may not be conserved

Experimental fact:

All interactions, except weak interaction, do conserve parity

To the extent we can neglect weak interaction, all *stationary, non degenerate* states must be parity eigenstates

Scattering states: similar but not identical to stationary states

Being $\sim$ momentum eigenstates $[\mathbf{p}, U_p] \neq 0 \rightarrow$ Parity not defined

Stable particles and resonances: bound states

Angular momentum eigenstates $[\mathbf{L}, U_p] = 0 \rightarrow$ Parity defined

Orbital Parity

Parity quantum number introduced by making reference to orbital motion.

For angular momentum eigenstates, take position representation:

$$\psi(\mathbf{r}) = Nf(r)Y_l^m(\theta, \varphi)$$

$$P:\mathbf{r} \rightarrow -\mathbf{r} \Rightarrow Y_l^m(\theta, \varphi) \rightarrow Y_l^m(\theta - \pi, \varphi + \pi) = (-1)^l Y_l^m(\theta, \varphi)$$

Therefore

$$\eta_P^{(orb)} = (-1)^l$$

This is known as *orbital parity*.

Orbital parity, like angular momentum, is *frame dependent*

Intrinsic Parity - I

Orbital parity not sufficient to deal with relativistic processes

Reason: *Particles are created and annihilated*

Best clarified first by a non relativistic example

Take a reaction between 2 nuclei, ignore nucleon spin

$$A + B \rightarrow C + D, \quad A, B \neq C, D$$

Angular momentum conservation:

$$\mathbf{L}_{TOT}^{(in)} = \mathbf{L}_A^{(CM-A)} + \mathbf{L}_B^{(CM-B)} + \mathbf{L}_{AB}^{(CM-AB)}$$

L_A, L_B originated by internal nuclear motion
 L_{AB} orbital motion

$$P_{TOT}^{in} = (-1)^{L_{TOT}^{(in)}} = \underbrace{(-1)^{L_A^{(CM-A)}}}_{P_A} \underbrace{(-1)^{L_B^{(CM-B)}}}_{P_B} \underbrace{(-1)^{L_{AB}^{(CM-AB)}}}_{P_{ORB}}$$

$$\mathbf{L}_{TOT}^{(out)} = \mathbf{L}_C^{(CM-C)} + \mathbf{L}_D^{(CM-D)} + \mathbf{L}_{CD}^{(CM-CD)}$$

Same for final state

$$P_{TOT}^{out} = (-1)^{L_{TOT}^{(out)}} = \underbrace{(-1)^{L_C^{(CM-C)}}}_{P_C} \underbrace{(-1)^{L_D^{(CM-D)}}}_{P_D} \underbrace{(-1)^{L_{CD}^{(CM-CD)}}}_{P_{ORB}}$$

If we don't know A, B, C, D are composite systems, 'intrinsic' nuclear parity cannot be ignored, or the process would violate parity whenever $L_{CD} \neq L_{AB}$

Intrinsic Parity - II

Just a drop of QFT to show the origin of intrinsic parity

1) One component (boson) field $\varphi(\mathbf{r}, t)$

Expand $\phi(r)$ into creation+annihilation operators

$$\varphi(\mathbf{r}, t) = \sum_{\mathbf{p}} \frac{1}{\sqrt{2EV}} [a_{\mathbf{p}} e^{-ip \cdot r} + a_{\mathbf{p}}^{\dagger} e^{+ip \cdot r}], \quad p \cdot r = (E, \mathbf{p}) \cdot (t, \mathbf{r}) = Et - \mathbf{p} \cdot \mathbf{r}$$

Intrinsic parity of ϕ operator

$$U_P \varphi U_P^{-1} = \eta_P \varphi(-\mathbf{r}, t) = \sum_{\mathbf{p}} \frac{1}{\sqrt{2EV}} [U_P a_{\mathbf{p}} U_P^{-1} e^{-i(Et - \mathbf{p} \cdot (-\mathbf{r}))} + U_P a_{\mathbf{p}}^{\dagger} U_P^{-1} e^{+i(Et - \mathbf{p} \cdot (-\mathbf{r}))}]$$

Expand $\phi(-\mathbf{r})$ into creation+annihilation operators

$$\eta_P \varphi(-\mathbf{r}, t) = \eta_P \sum_{\mathbf{p}} \frac{1}{\sqrt{2EV}} [a_{\mathbf{p}} e^{-i(Et - \mathbf{p} \cdot (-\mathbf{r}))} + a_{\mathbf{p}}^{\dagger} e^{+i(Et - \mathbf{p} \cdot (-\mathbf{r}))}]$$

Creation/Annihilation operators for $-\mathbf{p}$

$$= \sum_{\mathbf{p}} \frac{1}{\sqrt{2EV}} [\eta_P a_{-\mathbf{p}} e^{-i(Et - (-\mathbf{p}) \cdot \mathbf{r})} + \eta_P a_{-\mathbf{p}}^{\dagger} e^{+i(Et - (-\mathbf{p}) \cdot \mathbf{r})}] \rightarrow \begin{cases} U_P a_{\mathbf{p}} U_P^{-1} = \eta_P a_{-\mathbf{p}} \\ U_P a_{\mathbf{p}}^{\dagger} U_P^{-1} = \eta_P a_{-\mathbf{p}}^{\dagger} \end{cases}$$

Intrinsic parity!
+1 Scalar
- 1 Pseudo

$$\rightarrow U_P |\mathbf{p}\rangle = U_P a_{\mathbf{p}}^{\dagger} U_P^{-1} U_P |0\rangle = \eta_P a_{-\mathbf{p}}^{\dagger} |0\rangle = \eta_P |-\mathbf{p}\rangle; U_P |\mathbf{p}=0\rangle = \eta_P |\mathbf{p}=0\rangle$$

Different states

Same state

Intrinsic Parity - III

2) Multi-component, Boson fields

Same as one component field

E.g. Vector/Pseudo-vector field

In all cases:

$$\eta_P^* = \eta_P \rightarrow \text{Same parity for particle, antiparticle}$$

3) Multi-component, Fermion fields

Field satisfying Dirac's equation:

$$\begin{cases} U_P \psi(\mathbf{r}, t) U_P^{-1} = \eta_P \gamma^0 \psi(-\mathbf{r}, t) \\ U_P \bar{\psi}(\mathbf{r}, t) U_P^{-1} = \eta_P^* \bar{\psi}(-\mathbf{r}, t) \gamma^0 \end{cases}$$

$$\rightarrow |\eta_P|^2 = 1 \rightarrow \eta_P = \pm 1, \pm i \rightarrow \text{Opposite parity for particle, antiparticle}$$

Intrinsic Parity - IV

Absolute intrinsic parity not defined for most particle states

1) Fermions

Cannot be singly created/annihilated

Even more trouble: Consider 2π rotation of a $J=1/2$ state

$$U_R(\hat{n}, \theta)|\psi\rangle = e^{i\mathbf{J}\cdot\hat{n}\theta}|\psi\rangle \rightarrow U_R(\hat{z}, 2\pi)|\psi\rangle = e^{iJ_z 2\pi}|\psi\rangle = e^{i\frac{1}{2}2\pi}|\psi\rangle = -|\psi\rangle$$

So:

$$\left. \begin{array}{l} I = U_R(\hat{z}, 4\pi) = U_R^2(\hat{z}, 2\pi) \\ U_P^2 = U_R(\hat{z}, 2\pi) \end{array} \right\} \rightarrow I = U_P^4 \text{ for Fermions} \rightarrow \eta_P = \pm 1, \pm i$$

2) Charged Bosons

As above $\rightarrow I = U_P^2$ for Bosons $\rightarrow \eta_P = \pm 1$

Funny result! Not relevant
in the Standard Model
(Maybe *relevant* beyond..)

3) Really neutral particles (*Photon*, π^0 , J/ψ , ...)

Have all additive quantum numbers (charge, baryonic/leptonic number, strangeness, charm, ...) = 0, like vacuum state

$\rightarrow$ Absolute parity *is* defined upon defining vacuum state parity
(usually = +1)

Intrinsic Parity of Dirac Particles

$$(\boldsymbol{\alpha} \cdot \mathbf{p} + \beta m) \psi = E \psi \xrightarrow{p} (\boldsymbol{\alpha} \cdot (-\mathbf{p}) + \beta m) \psi_p = E \psi_p$$

$$\beta (\boldsymbol{\alpha} \cdot (-\mathbf{p}) + \beta m) \psi_p = E \beta \psi_p$$

$$\beta \boldsymbol{\alpha} = -\boldsymbol{\alpha} \beta \rightarrow ((-\boldsymbol{\alpha}) \cdot (-\mathbf{p}) + \beta m) \beta \psi_p = E \beta \psi_p$$

$$\rightarrow (\boldsymbol{\alpha} \cdot \mathbf{p} + \beta m) \beta \psi_p = E \beta \psi_p \rightarrow \beta \psi_p \text{ satisfies Dirac eq.}$$

Reminder:

$$\beta = \gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$

Take plane wave solutions:

$$\psi_1(\mathbf{r}, t) = \begin{pmatrix} \varphi \\ \frac{\boldsymbol{\sigma} \cdot \mathbf{p}}{E + m} \varphi \end{pmatrix} e^{-i(Et - \mathbf{p} \cdot \mathbf{r})}, \psi_2(\mathbf{r}, t) = \begin{pmatrix} \chi \\ \frac{\boldsymbol{\sigma} \cdot \mathbf{p}}{E + m} \chi \end{pmatrix} e^{-i(Et - \mathbf{p} \cdot \mathbf{r})}, \psi_3(\mathbf{r}, t) = \begin{pmatrix} -\frac{\boldsymbol{\sigma} \cdot \mathbf{p}}{E + m} \varphi \\ \varphi \end{pmatrix} e^{-i(-Et - \mathbf{p} \cdot \mathbf{r})}, \psi_4(\mathbf{r}, t) = \begin{pmatrix} -\frac{\boldsymbol{\sigma} \cdot \mathbf{p}}{E + m} \chi \\ \chi \end{pmatrix} e^{-i(-Et - \mathbf{p} \cdot \mathbf{r})}$$

Go to rest frame: $\mathbf{p} = 0$, $E = m$

$$\left. \begin{aligned} \psi_1 &= \begin{pmatrix} \varphi \\ 0 \end{pmatrix} e^{-imt} \xrightarrow{p} \gamma^0 \begin{pmatrix} \varphi \\ 0 \end{pmatrix} e^{-imt} = \begin{pmatrix} \varphi \\ 0 \end{pmatrix} e^{-imt} = \psi_1 \\ \psi_2 &= \begin{pmatrix} \chi \\ 0 \end{pmatrix} e^{-imt} \xrightarrow{p} \gamma^0 \begin{pmatrix} \chi \\ 0 \end{pmatrix} e^{-imt} = \begin{pmatrix} \chi \\ 0 \end{pmatrix} e^{-imt} = \psi_2 \end{aligned} \right\} \text{Intrinsic parity} = +1$$

$$\left. \begin{aligned} \psi_3 &= \begin{pmatrix} 0 \\ \varphi \end{pmatrix} e^{-i(-m)t} \xrightarrow{p} \gamma^0 \begin{pmatrix} 0 \\ \varphi \end{pmatrix} e^{-i(-m)t} = \begin{pmatrix} 0 \\ -\varphi \end{pmatrix} e^{-i(-m)t} = -\psi_3 \\ \psi_4 &= \begin{pmatrix} 0 \\ \chi \end{pmatrix} e^{-i(-m)t} \xrightarrow{p} \gamma^0 \begin{pmatrix} 0 \\ \chi \end{pmatrix} e^{-i(-m)t} = \begin{pmatrix} 0 \\ -\chi \end{pmatrix} e^{-i(-m)t} = -\psi_4 \end{aligned} \right\} \text{Intrinsic parity} = -1$$

Remark:

Absolute parity *not*
defined for fermions
Only defined relative
fermion-antifermion

Intrinsic Parity - Photon

Classical EM field

Standard convention:

$$\left. \begin{array}{l} P: \mathbf{E} \rightarrow -\mathbf{E} \\ P: \mathbf{B} \rightarrow \mathbf{B} \\ \mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} - \nabla \phi \\ \mathbf{B} = \nabla \times \mathbf{A} \end{array} \right\} \rightarrow \left\{ \begin{array}{l} P: \mathbf{A} \rightarrow -\mathbf{A} \\ P: \phi \rightarrow \phi \end{array} \right.$$

$\mathbf{A}$: Vector field

ϕ : Not needed for radiation field

Quantum EM field

$\mathbf{A}$: Vector field

Photon spin = 1

→ *Negative intrinsic parity*

Interesting question: EM interactions conserve parity

→ *Photons from reactions, decays should have defined, total parity*

Photon Total Parity

What is the photon parity in a given state? Can expand any $\mathbf{A}$ into creation & annihilation operators

$$\mathbf{A}(\mathbf{r}, t) = \sum_{\mathbf{k}} \sum_{i=1}^2 \frac{1}{\sqrt{|\mathbf{k}|V}} \left[a_i(\mathbf{k}) \boldsymbol{\epsilon}_i e^{i(\mathbf{p}_\gamma \cdot \mathbf{x} - E_\gamma t)} + a_i^\dagger(\mathbf{k}) \boldsymbol{\epsilon}_i^* e^{-i(\mathbf{p}_\gamma \cdot \mathbf{x} - E_\gamma t)} \right]$$

But: $a_i(\mathbf{k})$ do not commute with parity. Can also expand into J, m C&A operators

$$\mathbf{A}(\mathbf{r}, t) = \sum_{J, m} \sum_{L=J-1}^{J+1} \sum_{i=1}^2 \frac{1}{\sqrt{|\mathbf{k}|V}} \left[a_i(J, L, m) \boldsymbol{\epsilon}_i Y_{Jm}^L(\theta, \varphi) + a_i^\dagger(J, L, m) \boldsymbol{\epsilon}_i^* Y_{Jm}^{L*}(\theta, \varphi) \right]$$

Photon states with defined total parity: states with given J, L, m . Indeed:

$$\mathbf{J} = \mathbf{L} + \underbrace{\mathbf{S}}_{=1} \rightarrow J = L, L \pm 1 \rightarrow L = J, J \pm 1$$

$$\rightarrow \eta_P = (-1)^L (-1) = (-1)^{L+1} = (-1)^J, (-1)^{J \pm 1}$$

$$\rightarrow \begin{cases} \eta_P = (-1)^J & \text{Electric} \\ \eta_P = (-1)^{J \pm 1} & \text{Magnetic} \end{cases}$$

2 kinds of photons
for any given J

Intrinsic Parity - Scalars

Really neutral states decaying into two photons

Example: π^0 , $spin = 0$

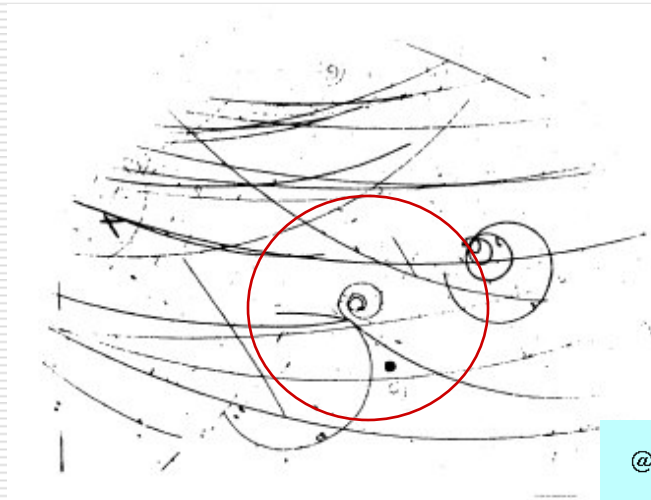
$$\pi^0 \rightarrow \gamma\gamma$$

Parity conserved by EM interaction

Final state: 2 creation operators $\propto \epsilon_1, \epsilon_2$

Can determine parity by looking at distribution of the angle between ϵ_1, ϵ_2

Observe: Double Dalitz decays $\pi^0 \rightarrow e^+e^-e^+e^-$



π^- stopping in liq. H_2

$$\pi^- + p \rightarrow \pi^0 + n \rightarrow e^+e^-e^+e^-n$$

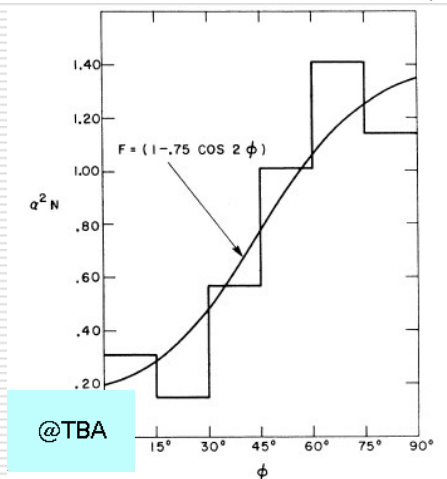
$$\epsilon_1 \perp plane_1(p_+, p_-)$$

$$\epsilon_2 \perp plane_2(p_+, p_-)$$

@TBA

Pseudoscalar $\sin^2 \varphi$

Scalar $\cos^2 \varphi$



@TBA

Intrinsic Parity- Vectors

Most interesting result:
Yang Theorem

A vector ($s=1$) state cannot decay to 2 photons

Consequence of:

Angular momentum conservation

Parity conservation

Bose symmetry for identical photons

Gauge invariance of QED (transversality of photons)

Examples:

$\rho, \omega, \phi, J/\psi, Y, Z^0, ..$

Intrinsic Parity – Baryons

States with different charge, baryon number (absolutely conserved quantities) cannot be superposed $\leftarrow$ Limitation to the principle of superposition, also called *Superselection Rule*

In other words: *No transition can violate Q, B conservation*

Relative parity unmeasurable, must be fixed by convention

Also true for states with different strangeness, charm, ...*when weak interaction is neglected.*

But: While weak transitions *can* connect states with different $S, C, \dots$, they *do* violate parity!

Remembering that a particle+antiparticle state has -ve intrinsic parity, state our general convention for Leptons & Quarks

<i>+ve parity to particles</i>	<i>-ve parity to antiparticles</i>
Quarks	Antiquarks
Leptons	Antileptons

NB -ve parity baryons actually exist, with higher mass and spin:
Include quark *orbital* degrees of freedom

Intrinsic Parity – Mesons

Particles singly produced in parity conserving reactions → Parity *can* be determined

Examples

Charged Pion

$\pi^- + d \rightarrow n + n$ at low energy

$$\eta_P^{\pi^-} (\eta_P^N)^2 (-1)^L = (\eta_P^N)^2 (-1)^{L'}$$

Initial state: $L=0 \rightarrow J=0 \oplus 1=1$

Final state: $J'=1$, two identical fermions

$L=0(\text{symm}) \rightarrow S=1(\text{symm}) \rightarrow \text{Symm}$ KO

$L=1(\text{antisymm}) \rightarrow S = \begin{cases} 0(\text{antisymm}) \rightarrow \text{Symm} & \text{KO} \\ 1(\text{symm}) \rightarrow \text{Antisymm} & \text{OK} \end{cases}$

→ P-wave, triplet → $L'=1$

$$\rightarrow \eta_P^{\pi^-} (-1)^0 = (-1)^1 \rightarrow \eta_P^{\pi^-} = -1$$

Charged Kaon

$K^- + {}^4\text{He} \rightarrow \pi^0 + {}^4_\Lambda\text{H}$ at low energy

$$\eta_P^{K^-} \eta_P^{4\text{He}} (-1)^L = \eta_P^{\pi^0} \eta_P^{4_\Lambda\text{H}} (-1)^{L'}$$

Initial state: $L=0, S_{K^-}=0 \rightarrow J=0$

Final state: $J'=0, S_{4_\Lambda\text{H}}=0 \rightarrow L'=0$

$$\rightarrow \eta_P^{K^-} \cdot 1 \cdot (-1)^0 = (-1) \cdot 1 \cdot (-1)^0 \rightarrow \eta_P^{K^-} = -1$$

$$= (+1)^4 (-1)^0 = +1$$

Ground state $L=0$

Hypernucleus
One neutron → Λ^0

Two-Particle States

Fermion-Antifermion: CM frame

$$\eta_P^{\bar{f}f} = \underbrace{(-1)}_{\text{intrinsic}} \underbrace{(-1)^l}_{\text{orbital}} = (-1)^{l+1}$$

Boson-Antiboson: CM frame

$$\eta_P^{\bar{b}b} = \underbrace{(\eta_P)^2}_{\text{intrinsic}} \underbrace{(-1)^l}_{\text{orbital}} = (-1)^l$$

Photon-photon: CM frame

Take single photon states with defined helicity:

$$U_P |\mathbf{k}, R; -\mathbf{k}, R\rangle = |-\mathbf{k}, L; \mathbf{k}, L\rangle \quad \text{not a } U_P \text{ eigenstate}$$

$$U_P |\mathbf{k}, L; -\mathbf{k}, L\rangle = |-\mathbf{k}, R; \mathbf{k}, R\rangle \quad \text{not a } U_P \text{ eigenstate}$$

$$U_P |\mathbf{k}, R; -\mathbf{k}, L\rangle = |-\mathbf{k}, L; \mathbf{k}, R\rangle \equiv |\mathbf{k}, R; -\mathbf{k}, L\rangle \quad U_P \text{ eigenstate, } \eta_P = +1, J_3 = +2$$

$$U_P |\mathbf{k}, L; -\mathbf{k}, R\rangle = |-\mathbf{k}, R; \mathbf{k}, L\rangle \equiv |\mathbf{k}, L; -\mathbf{k}, R\rangle \quad U_P \text{ eigenstate, } \eta_P = +1, J_3 = -2$$

$$|\mathbf{k}, R; -\mathbf{k}, R\rangle \pm |\mathbf{k}, L; -\mathbf{k}, L\rangle \quad U_P \text{ eigenstates, } \eta_P = \pm 1, J_3 = 0$$

Fall of Parity – I

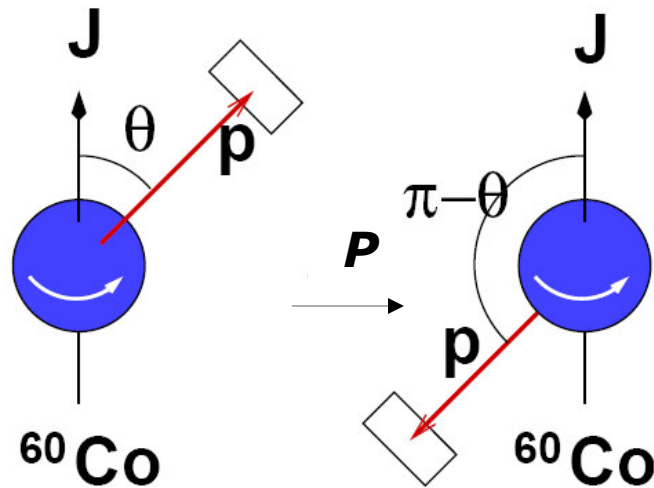
${}^{60}\text{Co} \rightarrow {}^{60}\text{Ni}^* + e^- + \bar{\nu}_e$ β^- decay: Weak process

If weak interaction is parity invariant,

$$\rightarrow \text{Ampl}(\theta) = \text{Ampl}(\pi - \theta)$$

Otherwise:

Expect β^- direction *anisotropy*



@TBA

Require nuclear polarization:
For an unpolarized sample, by
averaging over $\mathbf{J}$ z-projections any
possible anisotropy is washed out

Fall of Parity – II

The ^{60}Co Experiment: Polarization

Zeeman:

$$\mathcal{E}(M) = E_0 - \vec{\mu} \cdot \vec{B} = -g\mu_N BM$$

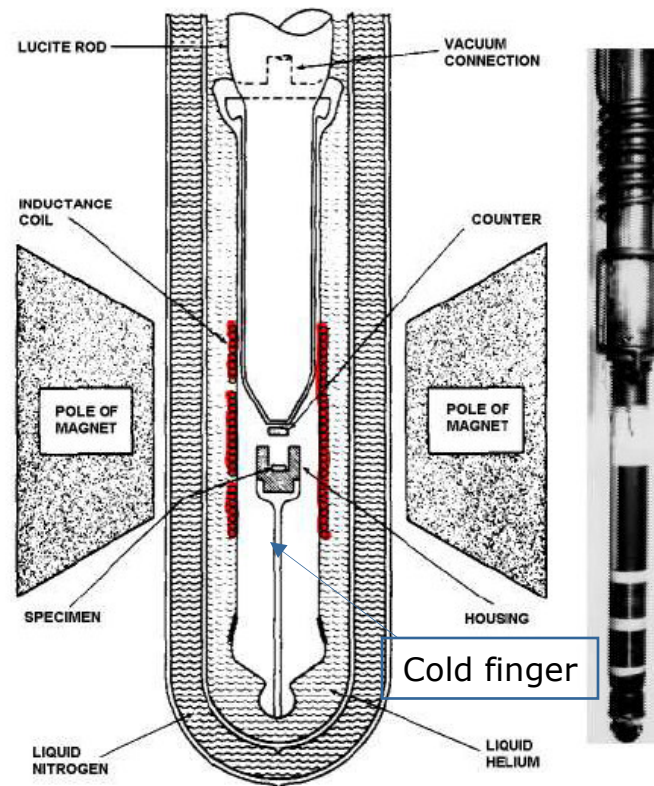
Boltzmann:

$$\frac{n(M')}{n(M)} = \frac{e^{\frac{\mathcal{E}(M)}{kT}}}{e^{\frac{\mathcal{E}(M')}{kT}}} = e^{\frac{(M-M')g\mu_N B}{kT}}$$

Magnetic field amplification in cerium-magnesium-nitrate crystal
 $0.05\text{ T} \rightarrow 10\text{--}100\text{ T}$

The ^{60}Co polarizes at a temperature of about 10 mK.

@TBA



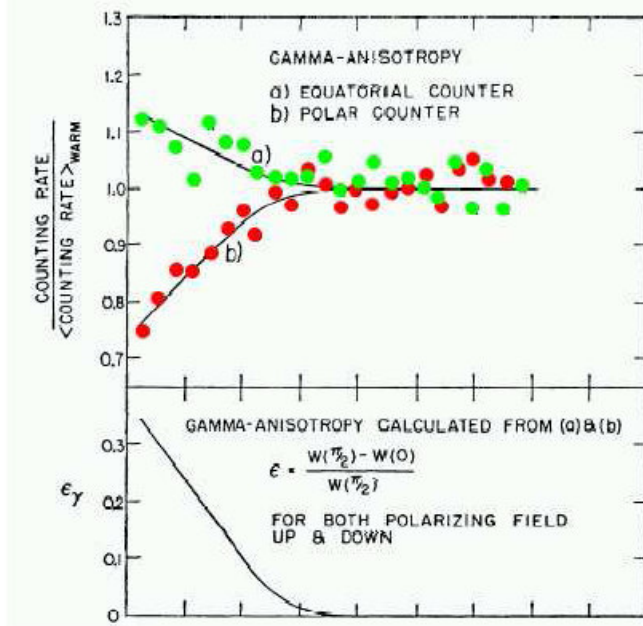
Fall of Parity – III

To measure nuclear polarization:

γ anisotropy in $^{60}\text{Ni}^* \rightarrow ^{60}\text{Ni} + \gamma(E2)$

Electric quadrupole transition

$$\epsilon_\gamma = \frac{W(\pi/2) - W(0)}{W(\pi/2)}$$



@TBA

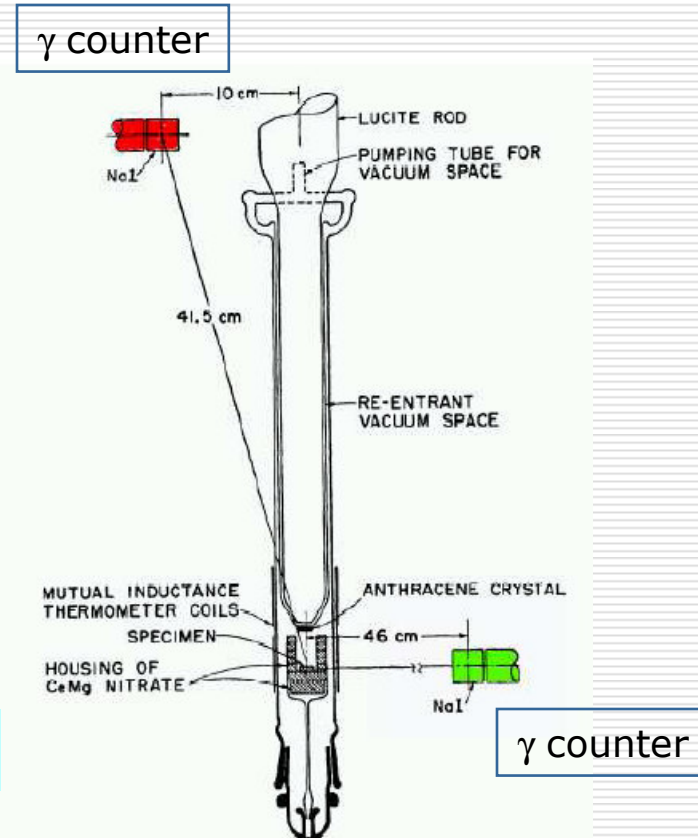
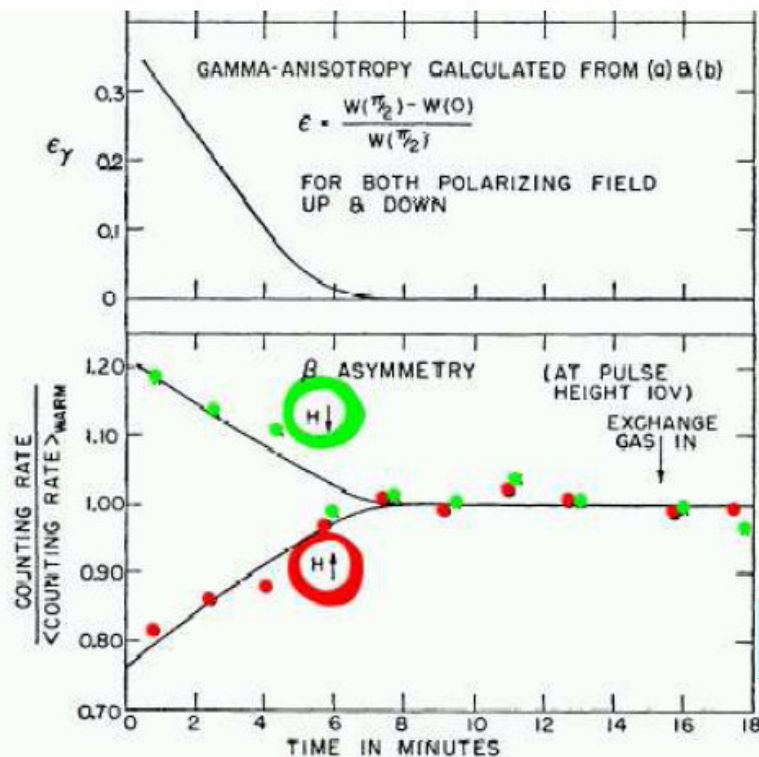
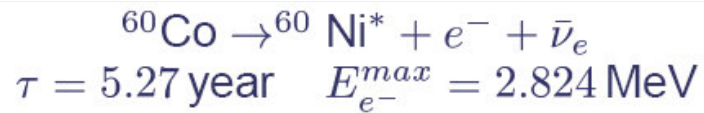


FIG. 1. Schematic drawing of the lower part of the cryostat

Fall of Parity – IV



@TBA

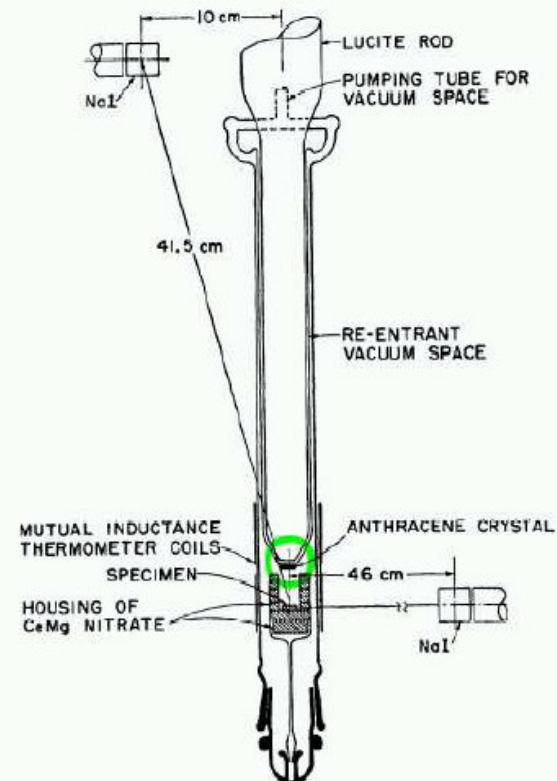


FIG. 1. Schematic drawing of the lower part of the cryostat

Another Fall of Parity

$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$

$$\sigma_\mu \cdot \mathbf{p}_\mu \xrightarrow{U_P} \sigma_\mu \cdot (-\mathbf{p}_\mu) = -\sigma_\mu \cdot \mathbf{p}_\mu \quad \text{Pseudoscalar observable}$$

$$\rightarrow P_\mu^{\text{long}} = \left\langle \frac{\sigma_\mu \cdot \mathbf{p}_\mu}{|\mathbf{p}_\mu|} \right\rangle = 0 \quad \text{if parity is a good symmetry}$$

$$\mu^+ \rightarrow e^+ + \bar{\nu}_\mu + \nu_e$$

$$\sigma_\mu \cdot \mathbf{p}_e \xrightarrow{U_P} \sigma_\mu \cdot (-\mathbf{p}_e) = -\sigma_\mu \cdot \mathbf{p}_e \quad \text{Pseudoscalar observable}$$

$$\rightarrow \left\langle \frac{\sigma_\mu \cdot \mathbf{p}_e}{|\mathbf{p}_e|} \right\rangle = 0 \quad \text{if parity is a good symmetry}$$

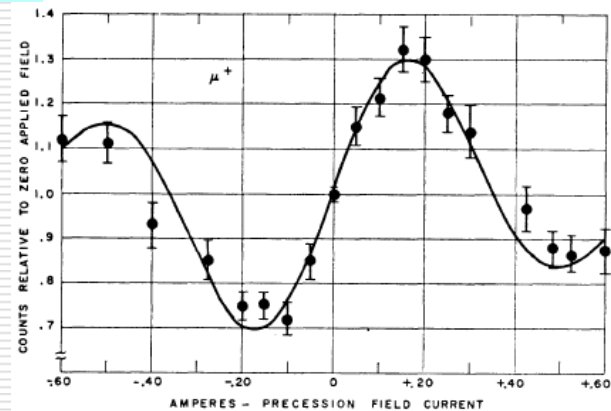
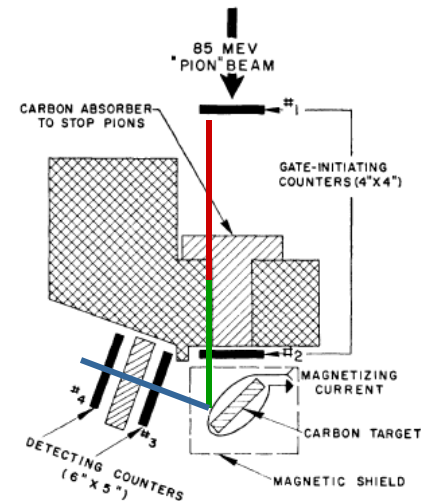
@TBA

If parity is violated:

$$\begin{cases} \pi^+ \rightarrow \mu^+ + \nu_\mu & \text{Expect } \mu \text{ polarization along } \mathbf{p}_\mu \\ \mu^+ \rightarrow e^+ + \bar{\nu}_\mu + \nu_e & \text{Expect } e^+ \text{ direction correlated with } \mathbf{s}_\mu \end{cases}$$

In order to detect e^+ correlation:

μ spin precession in $\mathbf{B}$



Charge Conjugation - I

Charge conjugation operation: similar to parity

But: Operating on 'internal', rather than spatial, coordinates

Change sign of all additive quantum numbers $Q, B, S, C, \dots$

Like parity, unitary transformation in state space:

$$C: |\psi\rangle \equiv |\alpha; Q, B, S, C, \dots\rangle \rightarrow |\psi'\rangle = U_C |\psi\rangle \equiv |\alpha; -Q, -B, -S, -C, \dots\rangle$$

[α : All other quantum numbers, like energy, position, momentum, angular momentum, ...]

Corresponding 'Relativity Principle' (to be checked by experiment)

The laws of physics are the same for all the observers, irrespective of the chosen positive direction of their 'charge axis'

Experimental fact:

All interactions, except weak interaction, do conserve charge parity

Charge Conjugation - II

A. Commutation relations with observables

U_C commutes with all observables, except those yielding additive quantum numbers :

$$[U_C, \mathbf{r}] = [U_C, \mathbf{p}] = [U_C, \mathbf{L}] = 0$$

$$U_C Q = -Q U_C, U_C B = -B U_C, U_C S = -S U_C, \dots$$

B. Action on states

$$U_C : |Q\rangle \rightarrow |Q'\rangle \equiv U_C |Q\rangle = \eta |-Q\rangle, \quad \eta \text{ arbitrary phase} = e^{i\alpha}$$

Charge Conjugation - III

C. Fundamental property of charge conjugation operator

$$C: Q \rightarrow Q' = C(Q) = -Q, C: Q' \rightarrow Q'' = C(Q') = -Q' = Q \rightarrow C^2 = I$$

$$U_C |\psi\rangle = |\psi'\rangle, U_C |\psi'\rangle = U_C (U_C |\psi\rangle) = U_C^2 |\psi\rangle \rightarrow U_C^2 = \eta^2 I$$

$$\left. \begin{array}{l} U_C^\dagger = U_C^{-1}, \quad U_C \text{ unitary} \\ U_C^2 = I \end{array} \right\} \rightarrow U_C^2 = U_C U_C = I = U_C U_C^{-1} = U_C U_C^\dagger \rightarrow U_C^\dagger = U_C$$

$\rightarrow U_C$ hermitian

$\rightarrow U_C$ eigenvalues are real

$\rightarrow \eta = e^{i\alpha} = \pm 1 \equiv \eta_C$ charge parity quantum number

$\rightarrow U_C$ eigenstates: $U_C |a\rangle = \pm |a\rangle$

Very few particles are U_C eigenstates: Must have

$$Q=B=S=C=\dots=0$$

Applications - I

$$j_\mu = (\rho, \mathbf{j}) \xrightarrow{C} (-\rho, -\mathbf{j}) = -j_\mu$$

$$A_\mu = (\varphi, \mathbf{A}) \xrightarrow{C} -(\varphi, \mathbf{A}) = -A_\mu \rightarrow U_C |\gamma\rangle = (-1) |\gamma\rangle$$

$$j^\mu A_\mu \xrightarrow{C} j^\mu A_\mu$$

$$U_C |n\gamma\rangle = (-1)^n |n\gamma\rangle$$

Boson-antiboson: CM frame

$$U_C |b\bar{b}\rangle = U_P |b\bar{b}\rangle = (-1)^l |b\bar{b}\rangle \rightarrow \eta_C = (-1)^l$$

Fermion-antifermion: CM frame

$$U_C = U_P U_S \quad \text{Parity * Spin exchange}$$

$$U_P |f \bar{f}\rangle = (-1)^{l+1} |\bar{f} f\rangle$$

$$U_S |\uparrow\downarrow - \downarrow\uparrow\rangle = |\downarrow\uparrow - \uparrow\downarrow\rangle = -|\uparrow\downarrow - \downarrow\uparrow\rangle, \quad U_S |\uparrow\downarrow + \downarrow\uparrow\rangle = |\downarrow\uparrow + \uparrow\downarrow\rangle = |\uparrow\downarrow + \downarrow\uparrow\rangle,$$

$$U_S |\uparrow\uparrow\rangle = |\uparrow\uparrow\rangle, U_S |\downarrow\downarrow\rangle = |\downarrow\downarrow\rangle$$

$$\rightarrow U_C |f \bar{f}\rangle = (-1)^{l+s} |\bar{f} f\rangle$$

Applications - II

$\pi^0, \eta, \dots: \eta_C = (-1)^2 = +1$ 2 photons decays

States $f \bar{f}$ decaying to n photons:

$$C: (-1)^{l+s} \rightarrow S \text{ state} \begin{cases} \eta_C = +1 \text{ singlet} \\ \eta_C = -1 \text{ triplet} \end{cases}$$

$$n \text{ photons: } \eta_C = (-1)^l \rightarrow \begin{cases} 2 \text{ photons } \eta_C = +1 \\ 3 \text{ photons } \eta_C = -1 \end{cases}$$

$$\rightarrow \begin{cases} \text{singlet} \rightarrow 2 \text{ photons} \\ \text{triplet} \rightarrow 3 \text{ photons} \end{cases}$$

Meson: Fermion-Antifermion bound state

L	$S=0$	$S=1$
0	0^+	1^{--}
1	1^{+-}	$0^{++}, 1^{++}, 2^{++}$
2	2^{++}	$1^{--}, 2^{--}, 3^{--}$

States decaying to 2 pions

J	$PC = ++$	$PC = +-, --, --$
0	OK	KO
1	KO	KO, KO, OK
2	OK	KO

Fall of Charge Conjugation

Fundamental question:

Is charge parity conserved by all interactions?

$$[H, U_C] = 0 \quad ?$$

Answer: Experiment

Same experiments probing spatial parity:

Charge conjugation *is* violated by weak interaction

First predicted by Lee, Oehme and Yang in 1957:

CPT symmetry requires *both* CT and P violation

Since T is $\sim$ good, C must be violated

Observed by Garwin, Lederman and Weinrich in their parity violation experiment with muons

Time Reversal

Need to make clear:

These considerations are relevant to physical systems with a (very) small number of degrees of freedom

Do not apply to complex systems, whose time evolution is driven by the II Law of thermodynamics. For them, a *time arrow* can be defined with no ambiguities. Or so we believe... (A very hard subject)

Can state another 'relativity principle', about the choice of the positive direction of the time coordinate of physical events:

The laws of physics are the same for all the observers, irrespective of the chosen positive direction of their 'time axis'

Experimental fact:

All interactions, except weak interaction, are invariant wrt to time reversal

Classical Physics

General behavior of physical quantities wrt to time reversal:

$$t \rightarrow -t:$$

$$\mathbf{r} \rightarrow \mathbf{r}$$

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} \rightarrow \frac{d\mathbf{r}}{d(-t)} = -\mathbf{v}, \mathbf{p} \rightarrow -\mathbf{p}$$

$$\mathbf{a} = \frac{d^2\mathbf{r}}{dt^2} \rightarrow \mathbf{a}, \mathbf{F} \rightarrow \mathbf{F}$$

$$\mathbf{L} = \mathbf{r} \times \mathbf{p} \rightarrow -\mathbf{L}$$

$$E \rightarrow E$$

$$\rho \rightarrow \rho$$

$$\mathbf{j} = \rho \mathbf{v} \rightarrow -\mathbf{j}$$

$$\varphi \rightarrow \varphi$$

$$\mathbf{A} \rightarrow -\mathbf{A}$$

$$\mathbf{E} \rightarrow \mathbf{E}$$

$$\mathbf{B} \rightarrow -\mathbf{B}$$

Observe: Ohm's law is *not* TR invariant
 $\mathbf{j} = \sigma \mathbf{E} \rightarrow -\mathbf{j} = \sigma \mathbf{E}$, take σ as TR invariant
Example of a *macroscopic* law...

Schrodinger Equation

$$T : t \rightarrow t' = -t$$

First guess...

$$\langle t' | \psi' \rangle = \langle t | \psi \rangle = \langle -t' | \psi \rangle$$

$$\langle t | \psi' \rangle = \langle -t | \psi \rangle$$

$$\rightarrow \psi'(t) = \psi(-t)$$

$$U_T : |\psi\rangle \rightarrow |\psi'\rangle \equiv U_T |\psi\rangle$$

$$U_T : |t\rangle \rightarrow |t'\rangle \equiv U_T |t\rangle = \eta | -t \rangle \quad \text{Wrong!}$$

Indeed:

$$H |\psi(t)\rangle = i \frac{\partial |\psi(t)\rangle}{\partial t} \xrightarrow{U_T} H |\psi(-t)\rangle = -i \frac{\partial |\psi(-t)\rangle}{\partial t}$$

Redefine:

$$\left. \begin{array}{l} T : t \rightarrow -t \\ K : i \rightarrow -i \end{array} \right\}, \quad U_T = KT$$

$$|\psi(t)\rangle \rightarrow U_T |\psi(t)\rangle = |\psi^T(t)\rangle = |\psi^*(-t)\rangle \quad \text{OK}$$

Time Reversal

Take a plane wave:

$$e^{-i(Et-\mathbf{p}\cdot\mathbf{r})} \xrightarrow{U_T} e^{+i(E(-t)-\mathbf{p}\cdot\mathbf{r})} = e^{-i(Et+\mathbf{p}\cdot\mathbf{r})} = e^{-i(Et-(-\mathbf{p})\cdot\mathbf{r})}$$

Quite natural....:

U_T sends a progressive plane wave into a regressive one

Take a particle with $s=1/2$:

$$\begin{pmatrix} \psi_+(t) \\ \psi_-(t) \end{pmatrix} \rightarrow \begin{pmatrix} \psi_+^T(t) \\ \psi_-^T(t) \end{pmatrix} = M \begin{pmatrix} \psi_+^*(-t) \\ \psi_-^*(-t) \end{pmatrix} \quad \text{Must take into account that } s \text{ is reversed by } T$$

$$\begin{pmatrix} \psi_+(t) \\ \psi_-(t) \end{pmatrix} \xrightarrow{U_T} \begin{pmatrix} \psi_+^T(t) \\ \psi_-^T(t) \end{pmatrix} = -i\sigma_y \begin{pmatrix} \psi_+^*(-t) \\ \psi_-^*(-t) \end{pmatrix} = \begin{pmatrix} \psi_-^*(-t) \\ -\psi_+^*(-t) \end{pmatrix} \quad \text{OK} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ +i & 0 \end{pmatrix} \rightarrow M = -i\sigma_y = \begin{pmatrix} 0 & +1 \\ -1 & 0 \end{pmatrix}$$

Spin-reversing operator

Apply U_T a second time:

$$\begin{pmatrix} \psi_+^T(t) \\ \psi_-^T(t) \end{pmatrix} \xrightarrow{U_T} -i\sigma_y \begin{pmatrix} \psi_-^*(t) \\ -\psi_+^*(t) \end{pmatrix} = \begin{pmatrix} -\psi_+(t) \\ -\psi_-(t) \end{pmatrix} = - \begin{pmatrix} \psi_+(t) \\ \psi_-(t) \end{pmatrix} \quad \text{Similar to parity for spin } 1/2$$

Puzzled? Think of it in this way:

U_T inverts the positive direction of time for the 2nd observer. Therefore, the 2nd observer has a clock which is running *backwards*.

Then the same wave which is seen as *progressive* by the first observer, is seen as *regressive* by the second..

Applications

Basic remark:

U_T is *not* a unitary operator
 U_T is *not* a Hermitian operator

Therefore:

No real eigenvalues
No (conserved) observables
→ No 'time parity'

Nevertheless:

Quite useful to establish
properties of fundamental
interactions

Because is not linear, rather it is antilinear

$$U_T(a|\psi_1\rangle + b|\psi_2\rangle) = a^*U_T|\psi_1\rangle + b^*U_T|\psi_2\rangle$$

Complex conjugation

$$K: \alpha z \rightarrow (\alpha z)^* = \alpha^* z^*$$

is a non-linear operation. Proof:

$$\alpha = ae^{i\varphi}, z = \rho e^{i\psi} \rightarrow \alpha^* = ae^{-i\varphi}, z^* = \rho e^{-i\psi}$$

$$\alpha z = ae^{i\varphi} \rho e^{i\psi} = a\rho e^{i(\varphi+\psi)} \rightarrow (\alpha z)^* = a\rho e^{-i(\varphi+\psi)} = \alpha^* z^*$$

But:

$$\alpha^* z^* \neq \alpha z^* \rightarrow \text{Non linear whenever } \alpha \neq \alpha^* !!$$

Indeed, a linear operator must satisfy:

$$A: (\alpha z) \rightarrow A(\alpha z) = \alpha A(z)$$

Kramers Degeneracy etc

Wave functions:

$$\psi_T(x) = \langle x | \psi_T \rangle = \langle x | U_T | \psi \rangle = \langle x | \psi \rangle^* = \psi^*(x) \quad \text{C.Conjugate}$$

If $H|\psi\rangle = E|\psi\rangle$, $[H, U_T] = 0$, $|\psi\rangle$ non-degenerate

$$\rightarrow H|\psi_T\rangle = HU_T|\psi\rangle = U_TH|\psi\rangle = EU_T|\psi\rangle = E|\psi_T\rangle \rightarrow |\psi_T\rangle = |\psi\rangle$$

$$\rightarrow \psi(x) \text{ real}$$

Degeneracy of fermion states:

$$U_T^2|\psi\rangle = -|\psi\rangle$$

$$\rightarrow \langle \psi | \psi_T \rangle = \langle \psi_T | \psi \rangle^* = \langle \psi | U_T^\dagger | \psi \rangle = -\langle \psi | U_T^\dagger U_T^2 | \psi \rangle = -\langle \psi | U_T | \psi \rangle = -\langle \psi | \psi_T \rangle$$

$$\rightarrow \langle \psi | \psi_T \rangle = 0$$

$$\rightarrow |\psi\rangle, |\psi_T\rangle \text{ orthogonal, independent}$$

Example (trivial): Spin $\frac{1}{2}$ particle in rest frame

$\rightarrow$ 2 independent states (up & down)

Effect on Scalar Products

$\{|i\rangle, i=1, \dots\}$ complete set of states

$$\langle i | j \rangle = \delta_{ij}$$

$\rightarrow \langle U_T i | U_T j \rangle = \delta_{ij}$ norm is U_T -invariant

$$|\psi_1\rangle = \sum_i |i\rangle \langle i | \psi_1 \rangle \rightarrow |U_T \psi_1\rangle = \sum_i |U_T i\rangle \langle i | \psi_1 \rangle^*$$

$$\langle \psi_2 | = \sum_j \langle \psi_2 | j \rangle \langle j | \rightarrow \langle U_T \psi_2 | = \sum_j \langle \psi_2 | j \rangle^* \langle U_T j |$$

$$\rightarrow \langle U_T \psi_2 | U_T \psi_1 \rangle = \sum_j \langle \psi_2 | j \rangle^* \langle U_T j | \sum_i | U_T i \rangle \langle i | \psi_1 \rangle^*$$

$$= \sum_{i,j} \langle \psi_2 | j \rangle^* \underbrace{\langle U_T j | U_T i \rangle}_{\delta_{ij}} \langle i | \psi_1 \rangle^* = \sum_i \langle \psi_2 | i \rangle^* \langle i | \psi_1 \rangle^*$$

$$= \sum_i \langle i | \psi_2 \rangle \langle \psi_1 | i \rangle = \langle \psi_1 | \psi_2 \rangle \quad U_T \text{ swaps states in any scalar product}$$

Matrix Elements

Matrix element for a transition: Initial $\leftrightarrow$ Final

H T -invariant $\rightarrow$ Same matrix element for direct and reversed transition

$\underbrace{1+2}_i \rightarrow \underbrace{3+4}_f$ 2-body process

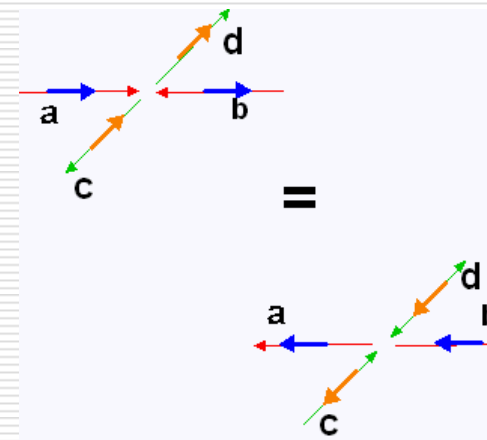
Reciprocity Theorem:

$$\langle \mathbf{p}_f, \mathbf{s}_f | S | \mathbf{p}_i, \mathbf{s}_i \rangle = \frac{\langle -\mathbf{p}_i, -\mathbf{s}_i | S | -\mathbf{p}_f, -\mathbf{s}_f \rangle^*}{T|\mathbf{p}_i, \mathbf{s}_i \rangle T\langle \mathbf{p}_f, \mathbf{s}_f |}$$

Detailed Balance Theorem:

$$\frac{d\sigma_{if}}{d\Omega} = \left(\frac{p_f}{p_i} \right)^2 \frac{(2s_3 + 1)(2s_4 + 1)}{(2s_1 + 1)(2s_2 + 1)}$$

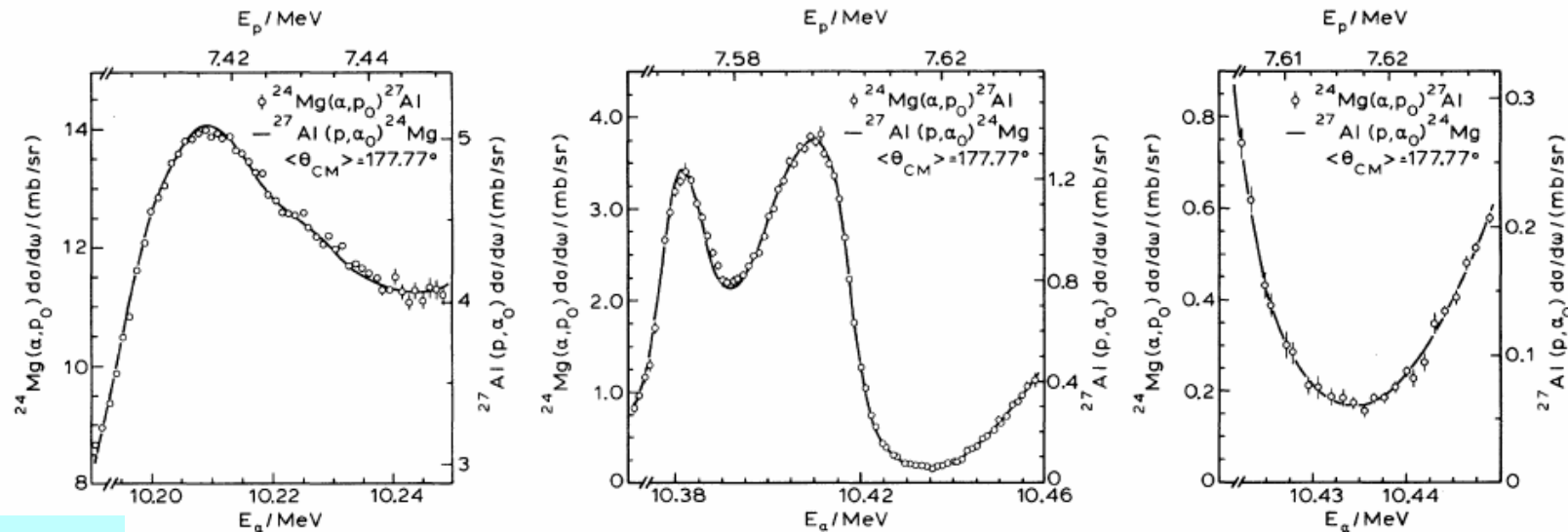
OK for Strong, E.M., KO for Weak



Example: Test of the Detailed Balance

Nuclear reactions:

- (1) $\alpha + {}^{24}\text{Mg} \rightarrow p + {}^{27}\text{Al}$
- (2) $p + {}^{27}\text{Al} \rightarrow \alpha + {}^{24}\text{Mg}$



@TBA

Full line: Reaction (1)
Circles: Reaction (2)

T-Violating Correlations

Example (many more exist):

$$\Lambda^0 \rightarrow p + \pi^-$$

$$\left. \begin{array}{l} \frac{1}{2}^+ = \frac{1}{2}^+ \oplus 0^- \oplus L^{(-1)^L} = \frac{1}{2} \oplus L \\ + \quad = + \quad \cdot \quad - \quad \cdot \quad (-1)^L = (-1)^{L+1} \end{array} \right\} \rightarrow L = 0, 1$$

Decay amplitude = Sum of S,P waves
S,P waves do interfere because of
 parity violation. So:
 Angular distribution has a term $\propto \cos \theta$

Most general form of angular distribution:

$$\frac{d\Gamma}{d\Omega_\pi} = 1 + A(\mathbf{J}_\Lambda + \mathbf{J}_p) \cdot \hat{\mathbf{p}}_\pi + B(\mathbf{J}_\Lambda \times \mathbf{J}_p) \cdot \hat{\mathbf{p}}_\pi + C(\mathbf{J}_\Lambda \cdot \mathbf{J}_p) + (1-C)(\mathbf{J}_\Lambda \cdot \hat{\mathbf{p}}_\pi)(\mathbf{J}_p \cdot \hat{\mathbf{p}}_\pi)$$

$$(\mathbf{J}_\Lambda \times \mathbf{J}_p) \cdot \hat{\mathbf{p}}_\pi \xrightarrow{U_T} ((-\mathbf{J}_\Lambda) \times (-\mathbf{J}_p)) \cdot (-\hat{\mathbf{p}}_\pi) = -(\mathbf{J}_\Lambda \times \mathbf{J}_p) \cdot \hat{\mathbf{p}}_\pi \quad \text{T-Violating term}$$

→ Expect $B = 0$

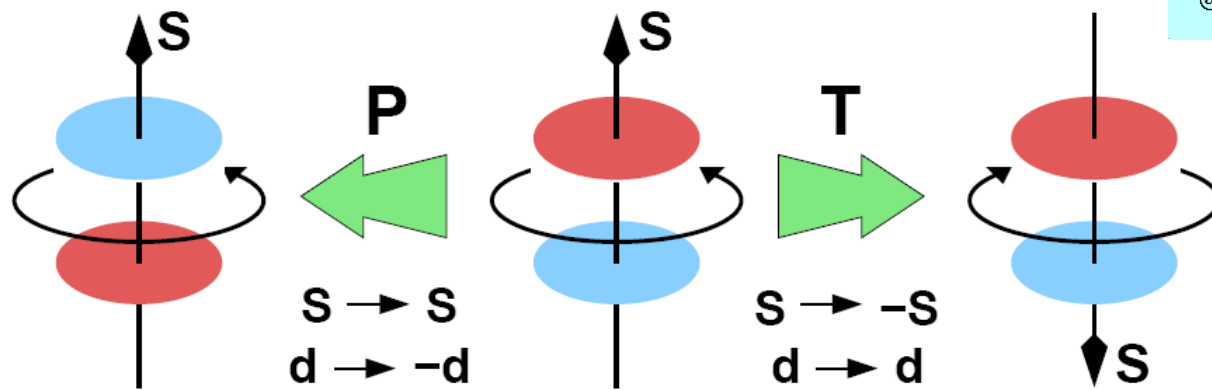
Must correct for final state interaction: $p\pi$ scattering

→ Get an upper limit of a few %

Electric Dipole Moments - I

Warning: This is true in the absence of a degenerate ground state
E.g., polar molecules *do* have degenerate ground states...

Elementary particle can only have a permanent electric dipole moment
if both **parity** and **time reversal** symmetries are broken:



$$\vec{\mu} = g \frac{e}{mc} \vec{S}$$

$$\vec{d} = \eta \frac{e}{2mc} \vec{S}$$

$\vec{d} // \vec{S}$, otherwise there would be an extra degree of freedom which is not seen

Electric Dipole Moments - II

For a particle with both magnetic and electric dipole moments:

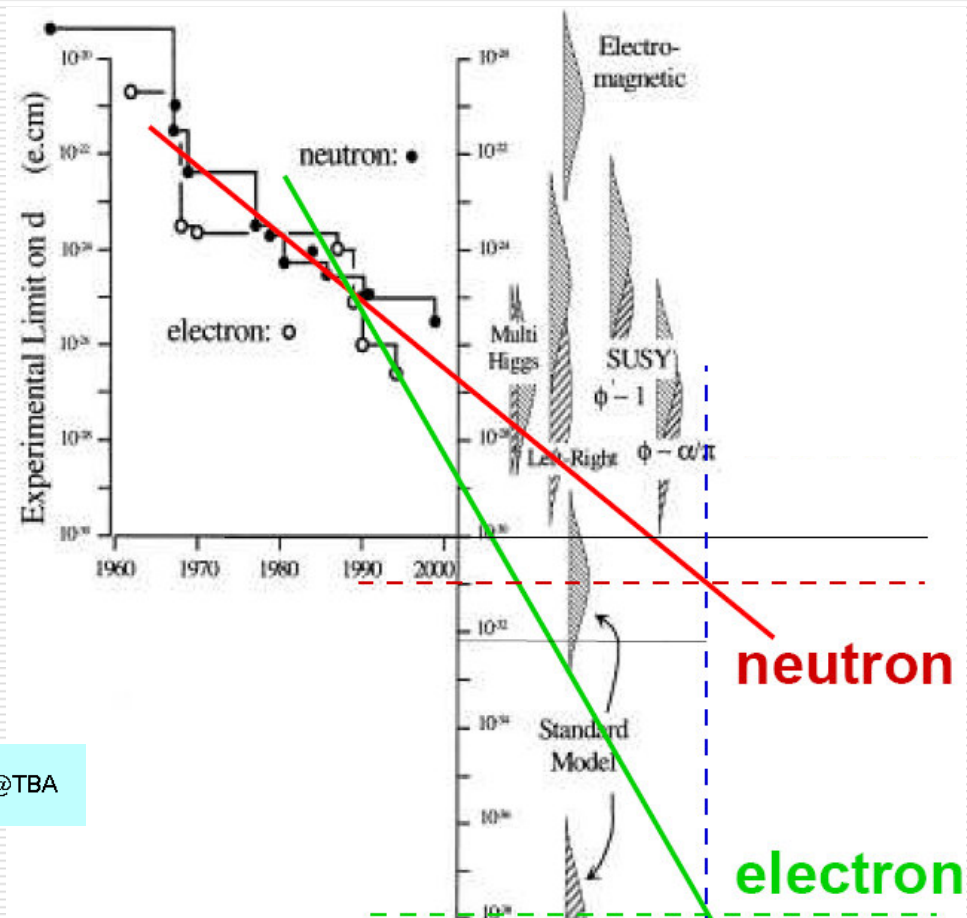
$$H' = -\boldsymbol{\mu} \cdot \mathbf{B} - \mathbf{d} \cdot \mathbf{E} \quad \text{Interaction energy}$$

Can observe in external $\mathbf{E}$:

Level shift

Spin precession

Status of the EDM



C, P and T in Quantum Field Theory

Just a few, oversimplified remarks:

- A. Observables are (functions of) field operators
- B. Just as in Quantum Mechanics, one finds different kinds of operators:
Scalars, Spinors, Vectors, ...
- C. Fields of each kind have their own 'parity' built-in
- D. States of a given field do inherit field 'parity', which is called *intrinsic parity*

C,P and T in QFT - I

1) Spin = 0

$$\phi(t, \mathbf{r}) = N \int d^3 \mathbf{p} [b(\mathbf{p}) e^{-ipx} + d^\dagger(\mathbf{p}) e^{+ipx}]$$

$$\phi^\dagger(t, \mathbf{r}) = N \int d^3 \mathbf{p} [d(\mathbf{p}) e^{-ipx} + b^\dagger(\mathbf{p}) e^{+ipx}]$$

$$\begin{cases} U_P \phi(t, \mathbf{r}) U_P^\dagger = \eta_P \phi(t, -\mathbf{r}) \\ U_P \phi^\dagger(t, \mathbf{r}) U_P^\dagger = \eta_P \phi^\dagger(t, -\mathbf{r}) \end{cases} \leftrightarrow \begin{cases} U_P \begin{pmatrix} b(\mathbf{p}) \\ d^\dagger(\mathbf{p}) \end{pmatrix} U_P^\dagger = \eta_P \begin{pmatrix} b(-\mathbf{p}) \\ d^\dagger(-\mathbf{p}) \end{pmatrix} \\ U_P \begin{pmatrix} b^\dagger(\mathbf{p}) \\ d(\mathbf{p}) \end{pmatrix} U_P^\dagger = \eta_P \begin{pmatrix} b^\dagger(\mathbf{p}) \\ d(\mathbf{p}) \end{pmatrix} \end{cases}, e^{\pm ipx} \rightarrow e^{\pm ipx}$$

$$\begin{cases} U_C \phi(t, \mathbf{r}) U_C^\dagger = \eta_C \phi^\dagger(t, \mathbf{r}) \\ U_C \phi^\dagger(t, \mathbf{r}) U_C^\dagger = \eta_C^* \phi(t, \mathbf{r}) \end{cases} \leftrightarrow \begin{cases} U_C \begin{pmatrix} b(\mathbf{p}) \\ d^\dagger(\mathbf{p}) \end{pmatrix} U_C^\dagger = \eta_C \begin{pmatrix} d(\mathbf{p}) \\ b^\dagger(\mathbf{p}) \end{pmatrix} \\ U_C \begin{pmatrix} b^\dagger(\mathbf{p}) \\ d(\mathbf{p}) \end{pmatrix} U_C^\dagger = \eta_C^* \begin{pmatrix} d^\dagger(\mathbf{p}) \\ b(\mathbf{p}) \end{pmatrix} \end{cases}, e^{\pm ipx} \rightarrow e^{\pm ipx}$$

$$\begin{cases} \bar{U}_T \phi(t, \mathbf{r}) \bar{U}_T^\dagger = \underbrace{\eta_T}_{=1} \phi(-t, \mathbf{r}) \\ \bar{U}_T \phi^\dagger(t, \mathbf{r}) \bar{U}_T^\dagger = \phi^\dagger(-t, \mathbf{r}) \end{cases} \leftrightarrow \begin{cases} \bar{U}_T \begin{pmatrix} b(\mathbf{p}) \\ d^\dagger(\mathbf{p}) \end{pmatrix} \bar{U}_T^\dagger = \begin{pmatrix} b(-\mathbf{p}) \\ d^\dagger(-\mathbf{p}) \end{pmatrix} \\ \bar{U}_T \begin{pmatrix} d(\mathbf{p}) \\ b^\dagger(\mathbf{p}) \end{pmatrix} \bar{U}_T^\dagger = \begin{pmatrix} d(-\mathbf{p}) \\ b^\dagger(-\mathbf{p}) \end{pmatrix} \end{cases}, e^{\pm ipx} \rightarrow e^{\mp ipx}$$

General:

Observables depend on $\phi \phi^\dagger$

Any observable yields a factor

$$\eta_P \eta_P^* = 1, \eta_C \eta_C^* = 1$$

$\rightarrow \eta_P, \eta_C$ unobservable

ϕ Hermitian:

$$\phi = \phi^\dagger$$

$$\rightarrow \eta_C = \eta_C^*, \eta_P = \pm 1$$

$$\rightarrow \begin{cases} \eta_P = \pm 1 \\ \eta_C = \pm 1 \end{cases} \text{ Observable}$$

η_T unobservable

C,P and T in QFT - II

2) Spin 1/2

$$\psi(t, \mathbf{r}) = \sum_s \int N d^3 \mathbf{p} \left[b(\mathbf{p}, s) u(\mathbf{p}, s) e^{+ipx} + d^\dagger(\mathbf{p}, s) v(\mathbf{p}, s) e^{-ipx} \right]$$

$$\psi^\dagger(t, \mathbf{r}) = \sum_s \int N d^3 \mathbf{p} \left[b^\dagger(\mathbf{p}, s) \bar{u}(\mathbf{p}, s) e^{-ipx} + d(\mathbf{p}, s) \bar{v}(\mathbf{p}, s) e^{+ipx} \right]$$

$$\begin{cases} U_P \psi(t, \mathbf{r}) U_P^\dagger = \eta_P \gamma^0 \psi(t, -\mathbf{r}) \\ U_P \bar{\psi}(t, \mathbf{r}) U_P^\dagger = \eta_P^* \bar{\psi}(t, -\mathbf{r}) \gamma^0 \end{cases} \leftrightarrow \begin{cases} U_P b(\mathbf{p}, s) U_P^\dagger = \eta_P b(-\mathbf{p}, s) \\ U_P d(\mathbf{p}, s) U_P^\dagger = \eta_P d(-\mathbf{p}, s) \\ U_P d^\dagger(\mathbf{p}, s) U_P^\dagger = \eta_P^* d^\dagger(-\mathbf{p}, s) \\ U_P b^\dagger(\mathbf{p}, s) U_P^\dagger = \eta_P^* b^\dagger(-\mathbf{p}, s) \end{cases}, e^{\pm ipx} \rightarrow e^{\pm ipx}$$

$$\begin{cases} U_C \psi(t, \mathbf{r}) U_C^\dagger = \eta_C i \gamma_2 \gamma_0 \bar{\psi}(t, \mathbf{r}) \\ U_C \bar{\psi}(t, \mathbf{r}) U_C^\dagger = \eta_C^* \psi(t, \mathbf{r}) i \gamma_2 \gamma_0 \end{cases} \leftrightarrow \begin{cases} U_C b(\mathbf{p}, s) U_C^\dagger = \eta_C d(\mathbf{p}, s) \\ U_C b^\dagger(\mathbf{p}, s) U_C^\dagger = \eta_C d^\dagger(\mathbf{p}, s) \\ U_C d(\mathbf{p}, s) U_C^\dagger = \eta_C^* b(\mathbf{p}, s) \\ U_C d^\dagger(\mathbf{p}, s) U_C^\dagger = \eta_C^* b^\dagger(\mathbf{p}, s) \end{cases}, e^{\pm ipx} \rightarrow e^{\pm ipx}$$

$$\begin{cases} \bar{U}_T \psi(t, \mathbf{r}) \bar{U}_T^\dagger = \underbrace{\eta_T}_{=1} \gamma_1 \gamma_3 \psi(-t, \mathbf{r}) \\ \bar{U}_T \bar{\psi}(t, \mathbf{r}) \bar{U}_T^\dagger = \bar{\psi}(-t, \mathbf{r}) (-\gamma_1 \gamma_3) \end{cases} \leftrightarrow \begin{cases} \bar{U}_T b(\mathbf{p}, s) \bar{U}_T^\dagger = b(-\mathbf{p}, -s) \\ \bar{U}_T d^\dagger(\mathbf{p}, s) \bar{U}_T^\dagger = d^\dagger(-\mathbf{p}, -s) \\ \bar{U}_T b^\dagger(\mathbf{p}, s) \bar{U}_T^\dagger = b^\dagger(-\mathbf{p}, -s) \\ \bar{U}_T d(\mathbf{p}, s) \bar{U}_T^\dagger = d(-\mathbf{p}, -s) \end{cases}, e^{\pm ipx} \rightarrow e^{\mp ipx}$$

CPT

Fundamental theorem applying to all field theories, including our beloved Standard Model:

Lorentz invariance

Micro-causality (whatever it means...)

Spin-statistics

→ *The product transformation CPT is a good symmetry*

Consequences:

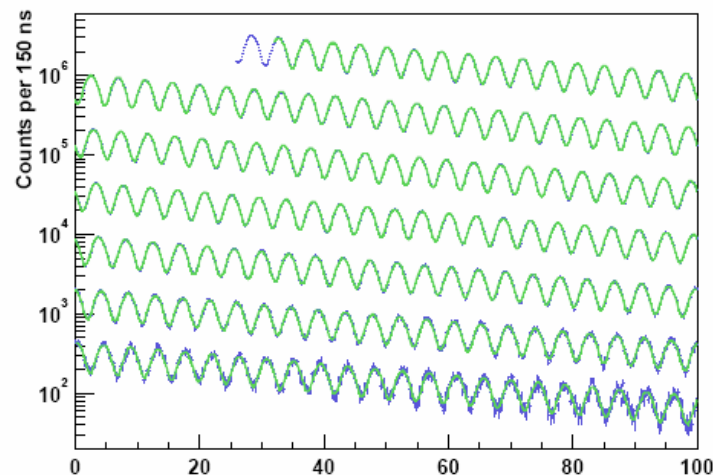
$$m_{particle} = m_{antiparticle}$$

$$\tau_{particle} = \tau_{antiparticle}$$

$$|\mu|_{particle} = |\mu|_{antiparticle}$$

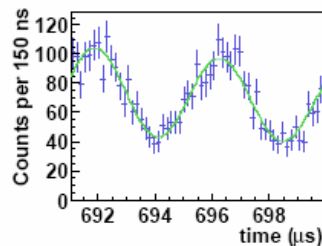
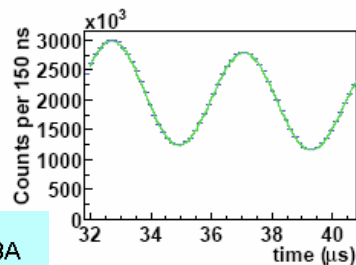
....more quantum numbers

Good Old QED Test: Muon Anomaly



Also an excellent test
of CPT in EM interaction

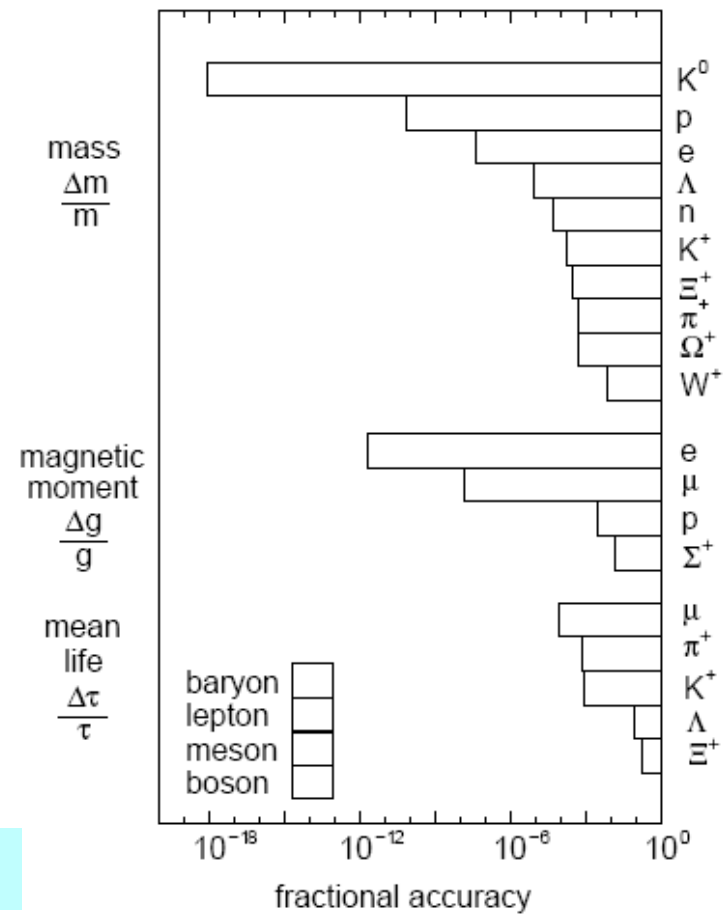
$$\left. \begin{aligned} a_{\mu^+} &= (11659203 \pm 8) \times 10^{-10} \\ a_{\mu^-} &= (11659214 \pm 9) \times 10^{-10} \end{aligned} \right\} \Delta a_{\mu} = (11 \pm 12) \times 10^{-10}$$



@TBA

CPT Tests

Just a flash on the general
status of CPT tests...
...very comfortable



Symmetries: Summary

Conserved quantity	Interaction		
	Strong	E.M.	Weak
4-momentum	OK	OK	OK
Charge	OK	OK	OK
Ang. Momentum, CM speed	OK	OK	OK
Baryonic number	OK	OK	OK
Leptonic numbers (3)	OK	OK	~OK
Parity	OK	OK	KO
Charge parity	OK	OK	KO
(Time reversal)	OK	OK	KO
CP	OK	OK	KO
(CPT)	OK	OK	OK
Flavor	OK	OK	KO