Elementary Particles I

1 – Symmetries

Mostly *C,P,T*

Classical Particle Dynamics

Lagrangian formalism

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{q}_i}\right)-\frac{\partial L}{\partial q_i}=0$$ Lagrange's equations

$$\frac{\partial L}{\partial q_i}=0\rightarrow\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{q}_i}\right)=0\rightarrow p_i\equiv\frac{\partial L}{\partial \dot{q}_i}=\text{const}$$

Hamiltonian formalism

$$\dot{p}_i=-\frac{\partial H}{\partial q_i}$$ Hamilton's equations

$\rightarrow H$ not depending on $q_i \Rightarrow p_i=$ const

Symmetry = Irrelevance of … to the dynamics of a closed system

Simmetry	Invariance	Conservation
Frame origin	**Space translation**	**Total *momentum***
Time origin	**Time translation**	**Total *energy***
Frame orientation	**Space rotation**	**Total *angular momentum***
Frame velocity	**Lorentz transformation**	***CM velocity***

Extension to Classical Fields

Take any physical system, *including fields*: Can describe its motion by the same methods (Lagrangian or Hamiltonian) in terms of a *Principle of Minimum Action*

Then Noether's Theorem states that

For every continuous transformation of the field functions and coordinates which leaves the action unchanged, there is a definite combination of the field functions and their derivatives which is conserved

This is called a *conserved current*

Main point to stress:

Conservation laws must include contributions from the fields, besides particles

Continuous vs Discrete Symmetry

Previous examples: *Continuous* symmetry operations

Any given operation *S* depends on a number of *continuous parameters*
Any given operation *S* can be thought as *continuosly evolving* from the identity operation

Example: S = Shift by ***a*** of the frame origin
Depends on 3 parameters a_x, a_y, a_z

$$S(\mathbf{a}) \underset{a_x, a_y, a_z \to 0}{\longrightarrow} I$$

What are *discrete* symmetry operations?

They do *not* depend on any parameter
They are *intrinsically separated* from the identity operation

Example: S = Axis inversion $x \to -x, y \to -y, z \to -z$
No parameter, no continuous evolution from identity

Quantum Mechanics - I

Link between invariance and conservation in QM

For any observable Q:

$$\langle Q\rangle = \langle\psi|Q|\psi\rangle \rightarrow \frac{d\langle Q\rangle}{dt} = \left\langle\frac{\partial\psi}{\partial t}\right|Q|\psi\rangle + \langle\psi|\frac{\partial Q}{\partial t}|\psi\rangle + \langle\psi|Q\left|\frac{\partial\psi}{\partial t}\right\rangle$$

From Schrodinger equation:

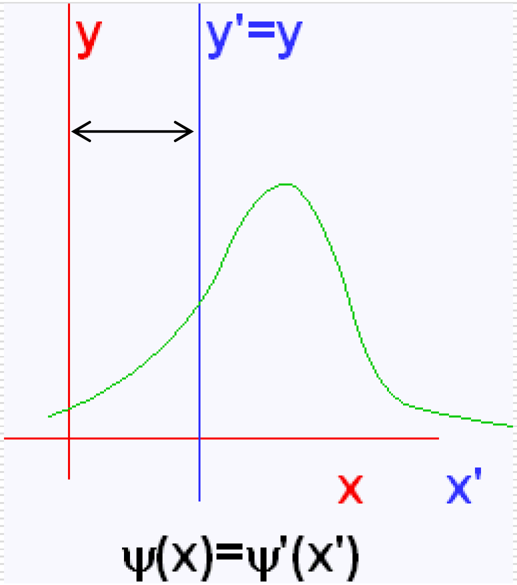
$$i\left|\frac{\partial\psi}{\partial t}\right\rangle = H|\psi\rangle, -i\left\langle\frac{\partial\psi}{\partial t}\right| = \langle\psi|H^\dagger$$

$$\frac{d\langle Q\rangle}{dt} = \left\langle\frac{\partial\psi}{\partial t}\right|Q|\psi\rangle + \langle\psi|\frac{\partial Q}{\partial t}|\psi\rangle + \langle\psi|Q\left|\frac{\partial\psi}{\partial t}\right\rangle$$

$$= i\langle\psi|H^\dagger Q|\psi\rangle + \langle\psi|\frac{\partial Q}{\partial t}|\psi\rangle - iQH|\psi\rangle$$

$$H^\dagger = H \rightarrow \frac{d\langle Q\rangle}{dt} = i\langle\psi|\left([H,Q] + \frac{\partial Q}{\partial t}\right)|\psi\rangle \rightarrow \left.\begin{array}{l}[H,Q]=0\\ \dfrac{\partial Q}{\partial t}=0\end{array}\right\} \rightarrow \langle Q\rangle = \text{constant}$$

Quantum Mechanics - II



Coordinate transformation *S* induces a transformation in function space

Example:

$$S: x \to x' = S(x) = x - a$$

$$S^{-1}: x' \to x = S^{-1}(x') = x' + a$$

$$\psi'(x') = \psi(x) \leftrightarrow \psi'[S(x)] = \psi(x)$$

$$\to \psi'[S^{-1}(S(x))] = \psi(S^{-1}(x))$$

$$\to \psi'(x) = \psi[S^{-1}(x)] \neq \psi(x)$$

Indeed, for example:

$$\psi(x) = Ne^{-\frac{x^2}{2\sigma^2}}$$

$$\psi'(x') = \psi[S^{-1}(x')] = Ne^{-\frac{(x'+a)^2}{2\sigma^2}} \left(= Ne^{-\frac{(x-a+a)^2}{2\sigma^2}} = Ne^{-\frac{x^2}{2\sigma^2}} \right)$$

$$\psi'(x') = Ne^{-\frac{(x'+a)^2}{2\sigma^2}} \neq Ne^{-\frac{x'^2}{2\sigma^2}} = \psi(x')$$

Just meaning:
One and same state
Different coordinates *x,x′* in Σ,Σ′

-> $\psi(x)=\psi'(x')$

One state, two wave functions in Σ,Σ′

Not the only possible point of view...

Quantum Mechanics - III

Unitary transformation

$U:\psi(x)\rightarrow\psi'(x)=\psi\left[S^{-1}(x)\right]=U\left[\psi(x)\right]$

Because scalar products must be preserved by *U*.
Actually too restrictive:
U can also be *anti-unitary*
(see later)

In term of state vectors:

$|\psi'\rangle=U|\psi\rangle\rightarrow U$ unitary: $U^{\dagger}=U^{-1}$

Take A = Any operator

$\langle\psi|A|\psi\rangle=\langle\psi'|A'|\psi'\rangle$ defining transformed operator

$\langle\psi'|A'|\psi'\rangle=\langle\psi|U^{\dagger}A'U|\psi\rangle$

$\langle\psi|A|\psi\rangle=\langle\psi'|A'|\psi'\rangle=\langle\psi|U^{\dagger}A'U|\psi\rangle\rightarrow A=U^{\dagger}A'U\rightarrow A'=UAU^{\dagger}$

When A invariant wrt U

$A=A'$

$\rightarrow UAU^{\dagger}=A$

$\rightarrow[U,A]=0$ when U is a symmetry operator for A

By taking $A=H$: $[U,H]=0\rightarrow\langle U\rangle=\text{const}$

Symmetry operators are *constants*

Quantum Field Theory

Take FT Course!

Summary

Coordinate transformation *S* (possibly including *internal* coordinates)

Continuous, Discrete

Examples:

Space translation (C)

Axis inversion (D)

Corresponding 'Relativity Principles'

Example:

The laws of physics are the same for all the observers, irrespective of the chosen origin of their reference frames

Induced transformation U on state space

Unitary, Antiunitary

$\langle U \rangle$ = constant of motion

Discrete Symmetries

Coordinate transformations not evolving continuously from identity

Very important in Particle Physics:

Space Inversion
Time Reversal
Charge Conjugation

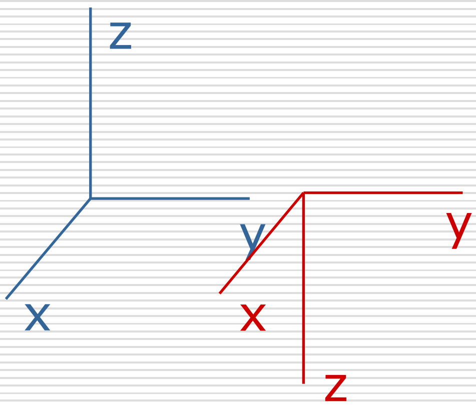
Leading to:

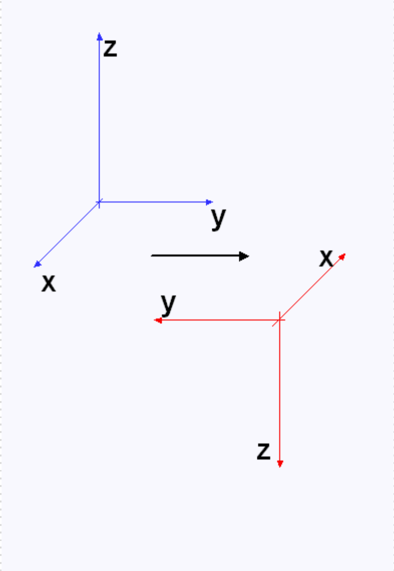
Quantum Numbers
Laws of Conservation
Selection Rules
Constraints on Properties of Interactions

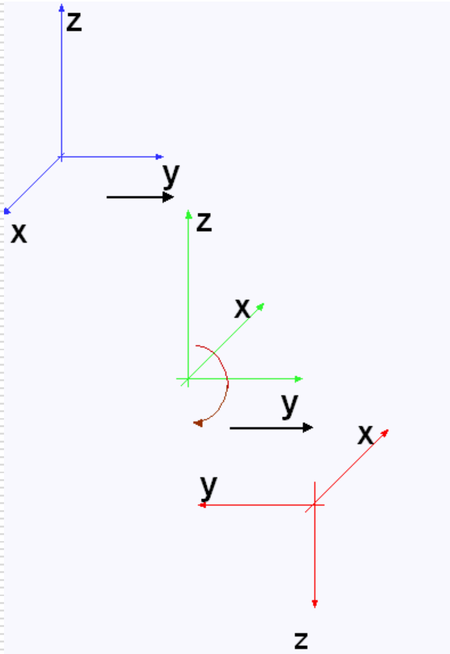
Parity in Classical Physics - I

Two, non equivalent ways of choosing a Cartesian frame in 3D

In 3D: 3 axis inversions equivalent to 1 axis (= mirror) inversion * Rotation







Non equivalent = Not connected by a rotation
Connected by 3 axis inversions

Parity in Classical Physics - II

Corresponding 'Relativity Principle'
The laws of physics are the same for all the observers, irrespective of the chosen chirality of their reference frames

Define coordinate symmetry operation

$$P : \mathbf{r} \rightarrow \mathbf{r}' = -\mathbf{r}$$

Transformations of physical quantities follow:

$\mathbf{r} \rightarrow -\mathbf{r}$	position
$t \rightarrow t$	time
$\mathbf{p} \rightarrow -\mathbf{p}$	3-momentum
$E \rightarrow E$	energy
$\mathbf{L} \rightarrow \mathbf{L}$	angular momentum

Hamilton's equations: form invariant wrt inversions $\leftrightarrow$ $H(q,p) = H(-q,-p)$

No conservation law!
P not connected to identical transformation

Parity in Classical Physics - III

General taxonomy of physical quantities (observables):

- With respect to rotations
- With respect to reflections

'Parity' of corresponding Hermitian operator:

	True	Pseudo
Scalar	$+1$	-1
Vector	-1	$+1$
Rank N Tensor	$(-1)^N$	$(-1)^{N+1}$

Parity in Classical Physics - IV

Parity behavior of selected electromagnetic quantities:

$$\mathbf{j}(\mathbf{r},t) \to -\mathbf{j}(-\mathbf{r},t) \quad \text{current density}$$

$$\rho(\mathbf{r},t) \to \rho(-\mathbf{r},t) \quad \text{charge density}$$

$$\mathbf{E}(\mathbf{r},\mathrm{t}) \to -\mathbf{E}(-\mathbf{r},t) \quad \text{electric field}$$

$$\mathbf{B}(\mathbf{r},\mathrm{t}) \to \mathbf{B}(-\mathbf{r},t) \quad \text{magnetic field}$$

Note

All of this is indeed conventional, as based on our definition of the electric charge as a *scalar*.

What actually matters is that *force* is a polar vector

Then

$$\left.\begin{array}{l} \boldsymbol{F} = q\boldsymbol{E} \\ \boldsymbol{F} = q\boldsymbol{v}\times\boldsymbol{B} \\ \boldsymbol{F},\boldsymbol{v} \text{ polar} \end{array}\right\} \to \left\{\begin{array}{l} \text{either } q \text{ scalar, } \boldsymbol{E} \text{ polar, } \boldsymbol{B} \text{ axial} \\ \text{or } q \text{ pseudoscalar, } \boldsymbol{E} \text{ axial, } \boldsymbol{B} \text{ polar} \end{array}\right.$$

Parity in Quantum Mechanics - I

Coordinate transformation as before: $P : \mathbf{r} \to \mathbf{r}' = P(\mathbf{r}) = -\mathbf{r}$

Induced transformation in state space: U_P unitary operator

A) Commutation relations with position, momentum, angular momentum

$$U_P \mathbf{r} |\psi\rangle = U_P \mathbf{r} \sum |\mathbf{r}\rangle\langle\mathbf{r}|\psi\rangle = U_P \sum \mathbf{r} |\mathbf{r}\rangle\langle\mathbf{r}|\psi\rangle = \sum U_P \mathbf{r} |\mathbf{r}\rangle\langle\mathbf{r}|\psi\rangle$$

$$= \sum (-\mathbf{r}) |-\mathbf{r}\rangle\langle -\mathbf{r}|\psi\rangle = (-\mathbf{r}) U_P |\psi\rangle = -\mathbf{r} U_P |\psi\rangle$$

$$\to U_P \mathbf{r} U_P^{-1} = -\mathbf{r} \to U_P \mathbf{r} = -\mathbf{r} U_P$$

$$U_P U_T(\delta\mathbf{r}) = U_P (1 - \mathbf{p}\cdot\delta\mathbf{r})$$

$$U_P (1 - \mathbf{p}\cdot\delta\mathbf{r}) = (1 + \mathbf{p}\cdot\delta\mathbf{r}) U_P$$

$$\to -U_P \mathbf{p}\cdot\delta\mathbf{r} = \mathbf{p}\cdot\delta\mathbf{r} U_P \to -U_P \mathbf{p} = \mathbf{p} U_P$$

$$U_P \hat{\mathbf{L}} = \hat{\mathbf{L}} U_P$$

Summary:

U_P anticommutes with ***r,p***
commutes with ***L***

Parity in Quantum Mechanics - II

B) Action of U_P on states

$$U_P : |\psi\rangle \to |\psi'\rangle \equiv U_P |\psi\rangle$$

Same physical state described by two different kets in the two frames

Take position eigenstate:

$$U_P : |\mathbf{r}\rangle \to |\mathbf{r}'\rangle \equiv U_P |\mathbf{r}\rangle = \eta |-\mathbf{r}\rangle, \ \eta \text{ arbitrary phase } = e^{i\alpha}$$

Take generic state:

$$U_P : |\psi\rangle \to |\psi'\rangle \equiv U_P |\psi\rangle$$

Would yield similar results by expanding into momentum eigenstates

Expand into position eigenstates: $|\psi\rangle = \sum_{\boldsymbol{r}} |\boldsymbol{r}\rangle\langle \boldsymbol{r}|\psi\rangle$

$$U_P|\psi\rangle = U_P \sum_{\boldsymbol{r}} |\boldsymbol{r}\rangle\langle \boldsymbol{r}|\psi\rangle = \sum_{\boldsymbol{r}} U_P|\boldsymbol{r}\rangle\langle \boldsymbol{r}|U_P^{-1}U_P\,\psi\rangle = \sum_{\boldsymbol{r}} \left[U_P|\boldsymbol{r}\rangle\langle \boldsymbol{r}|U_P^{-1}\right]U_P\,\psi\rangle$$

$$= \sum_{\boldsymbol{r}'} |\boldsymbol{r}'\rangle\langle \boldsymbol{r}'|U_P\,\psi\rangle = \sum_{\boldsymbol{r}'} |\boldsymbol{r}'\rangle\langle \boldsymbol{r}'|\psi'\rangle = |\psi'\rangle = \sum_{\boldsymbol{r}} |-\boldsymbol{r}\rangle\langle -\boldsymbol{r}|\psi'\rangle$$

$$\to \langle \boldsymbol{r}'|\psi'\rangle = \langle \boldsymbol{r}|\psi\rangle \to \langle \boldsymbol{r}|\psi'\rangle = \langle -\boldsymbol{r}|\psi\rangle$$

Talking wave functions: $\psi'(\boldsymbol{r}) = \psi(-\boldsymbol{r})$ (as shown before)

Parity in Quantum Mechanics - III

C) Fundamental property of parity operator

$$P:\mathbf{r}\to\mathbf{r}'=P(\mathbf{r})=-\mathbf{r},P:\mathbf{r}'\to\mathbf{r}''=P(\mathbf{r}')=-\mathbf{r}'=\mathbf{r}\to P^2=I$$

$$U_P|\psi\rangle=|\psi'\rangle,U_P|\psi'\rangle=U_P(U_P|\psi\rangle)=U_P^2|\psi\rangle\to U_P^2=\eta^2 I$$

$$\left.\begin{array}{r}U_P^\dagger=U_P^{-1},\ U_P\ \text{unitary}\\ U_P^2=I\end{array}\right\}\to U_P^2=U_PU_P=I=U_PU_P^{-1}=U_PU_P^\dagger\to U_P^\dagger=U_P\quad U_P\ \text{Hermitian}$$

$\to U_P$ eigenvalues are real

$\to\eta=e^{i\alpha}=\pm1\ \equiv\eta_P$ parity quantum number

$\to U_P$ eigenstates: $U_P|a\rangle=\pm|a\rangle$

Consequences:

$[H,U_P]=0\to$ stationary states have definite η_P- when not degenerate

$[H,U_P]=0\to\eta_P=$ constant of motion

Parity in Quantum Mechanics - IV

Parity for a composite system (just meaning with several degrees of freedom)

$|a\rangle$ compound state of subsystems 1 and 2, parity eigenstate

$U_P^{(1)}, U_P^{(2)}$ parity operators for 1,2

$U_P^{(1)}|a\rangle = \eta_P^{(1)}|a\rangle, U_P^{(2)}|a\rangle = \eta_P^{(2)}|a\rangle$

$U_P^{(2)}\left[U_P^{(1)}|a\rangle\right] = U_P^{(2)}\left[\eta_P^{(1)}|a\rangle\right] = \eta_P^{(2)}\eta_P^{(1)}|a\rangle \to \eta_P$ *multiplicative* quantum number

Quantum numbers (i.e., conserved quantities) are usually *additive*: ???
Reason: For continuos symmetries the unitary operators are not Hermitian
U unitary $\to U = e^{iaH} \simeq 1 + iaH,\ H$ Hermitian; $\lim_{a\to 0} U = 1 \to$ Use infinitesimal generators

Example for translations:

$$U_a^{(1)} \simeq 1 + ia\mathbf{p}^{(1)}, U_a^{(2)} \simeq 1 + ia\mathbf{p}^{(2)} \to U_a^{(2)}\left(U_a^{(1)}|\psi\rangle\right) \simeq \left(1 + ia\mathbf{p}^{(2)}\right)\left[\left(1 + ia\mathbf{p}^{(1)}\right)|\psi\rangle\right]$$

$$\to U_a^{(2)}\left(U_a^{(1)}|\psi\rangle\right) \simeq \left(1 + ia\mathbf{p}^{(2)} + ia\mathbf{p}^{(1)}\right)|\psi\rangle = \left[1 + ia\left(\mathbf{p}^{(2)} + \mathbf{p}^{(1)}\right)\right]|\psi\rangle$$

U Hermitian→ Constant of motion = *U* → Multiplicative
U not Hermitian→ Constant of motion ~ “Logarithm” of *U*…→ Additive

The Parity Quantum Number

As a quantum number, parity may or may not be conserved

Experimental fact:

All interactions, except weak interaction, do conserve parity

To the extent we can neglect weak interaction, all *stationary, non degenerate* states must be parity eigenstates

Scattering states: similar but not identical to stationary states
Being ~ momentum eigenstates $\left[\boldsymbol{p}, U_P\right] \neq 0 \rightarrow$ Parity not defined

Stable particles and resonances: bound states
Angular momentum eigenstates $\left[\boldsymbol{L}, U_P\right] = 0 \rightarrow$ Parity defined

Orbital Parity

Parity quantum number introduced by making reference to orbital motion.

For angular momentum eigenstates, take position representation:

$$\psi(\mathbf{r}) = Nf(r)Y_l^m(\theta,\varphi)$$

$$P:\mathbf{r} \to -\mathbf{r} \Rightarrow Y_l^m(\theta,\varphi) \to Y_l^m(\theta-\pi,\varphi+\pi) = (-1)^l Y_l^m(\theta,\varphi)$$

Therefore

$$\eta_P^{(orb)} = (-1)^l$$

This is known as *orbital parity*.

Orbital parity, like angular momentum, is *frame dependent*

Intrinsic Parity - I

Orbital parity not sufficient to deal with relativistic processes

Reason: *Particles are created and annihilated*

Best clarified first by a non relativistic example

Take a reaction between 2 nuclei, ignore nucleon spin

$A + B \to C + D, \quad A, B \neq C, D$

Angular momentum conservation:

$$\mathbf{L}_{TOT}^{(in)} = \mathbf{L}_A^{(CM-A)} + \mathbf{L}_B^{(CM-B)} + \mathbf{L}_{AB}^{(CM-AB)}$$

L_A, L_B originated by internal nuclear motion
L_{AB} orbital motion

$$P_{TOT}^{in} = (-1)^{L_{TOT}^{(in)}} = \underbrace{(-1)^{L_A^{(CM-A)}}}_{P_A} \underbrace{(-1)^{L_B^{(CM-B)}}}_{P_B} \underbrace{(-1)^{L_{AB}^{(CM-AB)}}}_{P_{ORB}}$$

$$\mathbf{L}_{TOT}^{(out)} = \mathbf{L}_C^{(CM-C)} + \mathbf{L}_D^{(CM-D)} + \mathbf{L}_{CD}^{(CM-CD)}$$

Same for final state

$$P_{TOT}^{out} = (-1)^{L_{TOT}^{(out)}} = \underbrace{(-1)^{L_C^{(CM-C)}}}_{P_C} \underbrace{(-1)^{L_B^{(CM-D)}}}_{P_D} \underbrace{(-1)^{L_{CD}^{(CM-CD)}}}_{P_{ORB}}$$

If we don't know A,B,C,D are composite systems, 'intrinsic' nuclear parity cannot be ignored, or the process would violate parity whenever L_{CD} # L_{AB}

Intrinsic Parity - II

Just a drop of QFT to show the origin of intrinsic parity

1) One component (boson) field $\varphi(\boldsymbol{r},t)$

Expand $\phi(r)$ into creation+annihilation operators

$$\varphi(\boldsymbol{r},t)=\sum_{\boldsymbol{p}}\frac{1}{\sqrt{2EV}}\left[a_{\boldsymbol{p}}e^{-ip\cdot r}+a_{\boldsymbol{p}}^{\dagger}e^{+ip\cdot r}\right],\qquad p\cdot r=(E,\boldsymbol{p})\cdot(t,\boldsymbol{r})=Et-\boldsymbol{p}\cdot\boldsymbol{r}$$

Intrinsic parity of ϕ operator

$$U_P\varphi U_P^{-1}=\eta_P\varphi(-\boldsymbol{r},t)=\sum_{\boldsymbol{p}}\frac{1}{\sqrt{2EV}}\left[U_Pa_{\boldsymbol{p}}U_P^{-1}e^{-i(Et-\boldsymbol{p}\cdot\boldsymbol{r})}+U_Pa_{\boldsymbol{p}}^{\dagger}U_P^{-1}e^{+i(Et-\boldsymbol{p}\cdot\boldsymbol{r})}\right]$$

Expand $\phi(-\boldsymbol{r})$ into creation+annihilation operators

$$\eta_P\varphi(-\boldsymbol{r},t)=\eta_P\sum_{\boldsymbol{p}}\frac{1}{\sqrt{2EV}}\left[a_{\boldsymbol{p}}e^{-i(Et-\boldsymbol{p}\cdot(-\boldsymbol{r}))}+a_{\boldsymbol{p}}^{\dagger}e^{+i(Et-\boldsymbol{p}\cdot(-\boldsymbol{r}))}\right]$$

Creation/Annihilation operators for $-\boldsymbol{p}$

$$=\sum_{\boldsymbol{p}}\frac{1}{\sqrt{2EV}}\left[\eta_Pa_{-\boldsymbol{p}}e^{-i(Et-(-\boldsymbol{p})\cdot\boldsymbol{r})}+\eta_Pa_{-\boldsymbol{p}}^{\dagger}e^{+i(Et-(-\boldsymbol{p})\cdot\boldsymbol{r})}\right]\rightarrow\begin{cases}U_Pa_{\boldsymbol{p}}U_P^{-1}=\eta_Pa_{-\boldsymbol{p}}\\U_Pa_{\boldsymbol{p}}^{\dagger}U_P^{-1}=\eta_Pa_{-\boldsymbol{p}}^{\dagger}\end{cases}$$

Intrinsic parity!
+1 Scalar
- 1 Pseudo

$$\rightarrow U_P|\boldsymbol{p}\rangle=U_Pa_{\boldsymbol{p}}^{\dagger}U_P^{-1}U_P|0\rangle=\eta_Pa_{-\boldsymbol{p}}^{\dagger}|0\rangle=\eta_P|-\boldsymbol{p}\rangle;U_P|\boldsymbol{p}=0\rangle=\eta_P|\boldsymbol{p}=0\rangle$$

Different states

Same state

Intrinsic Parity - III

2) Multi-component, Boson fields

Same as one component field

E.g. Vector/Pseudo-vector field

In all cases:

$\eta_P^* = \eta_P \rightarrow$ Same parity for particle,antiparticle

3) Multi-component, Fermion fields

Field satisfying Dirac's equation:

$$\begin{cases} U_P \psi(\boldsymbol{r},t) U_P^{-1} = \eta_P \gamma^0 \psi(-\boldsymbol{r},t) \\ U_P \overline{\psi}(\boldsymbol{r},t) U_P^{-1} = \eta_P^* \overline{\psi}(-\boldsymbol{r},t) \gamma^0 \end{cases}$$

$\rightarrow |\eta_P|^2 = 1 \rightarrow \eta_P = \pm 1, \pm i \rightarrow$ Opposite parity for particle,antiparticle

Intrinsic Parity - IV

Absolute intrinsic parity not defined for most particle states

1) Fermions

Cannot be singly created/annihilated

Even more trouble: Consider 2π rotation of a $J=1/2$ state

$$U_R(\hat{\boldsymbol{n}},\theta)|\psi\rangle = e^{i\boldsymbol{J}\cdot\hat{\boldsymbol{n}}\theta}|\psi\rangle \to U_R(\hat{\boldsymbol{z}},2\pi)|\psi\rangle = e^{iJ_z 2\pi}|\psi\rangle = e^{i\frac{1}{2}2\pi}|\psi\rangle = -|\psi\rangle$$

So:

$$\left.\begin{array}{l} I = U_R(\hat{\boldsymbol{z}},4\pi) = U_R^2(\hat{\boldsymbol{z}},2\pi) \\ U_P^2 = U_R(\hat{\boldsymbol{z}},2\pi) \end{array}\right\} \to I = U_P^4 \quad \text{for Fermions} \to \eta_P = \pm 1, \pm i$$

Funny result! Not relevant in the Standard Model (Maybe *relevant* beyond..)

2) Charged Bosons

As above $\to I = U_P^2$ for Bosons $\to \eta_P = \pm 1$

3) Really neutral particles (*Photon*, π^0, J/ψ, …)

Have all additive quantum numbers (charge, baryonic/leptonic number, strangeness, charm, …) = 0, like vacuum state

→Absolute parity *is* defined upon defining vacuum state parity (usually= +*1*)

Intrinsic Parity of Dirac Particles

$$(\boldsymbol{\alpha}\cdot\mathbf{p}+\beta m)\psi = E\psi \underset{P}{\rightarrow} (\boldsymbol{\alpha}\cdot(-\mathbf{p})+\beta m)\psi_P = E\psi_P$$

$$\beta(\boldsymbol{\alpha}\cdot(-\mathbf{p})+\beta m)\psi_P = E\beta\psi_P$$

$$\beta\boldsymbol{\alpha} = -\boldsymbol{\alpha}\beta \rightarrow ((-\boldsymbol{\alpha})\cdot(-\mathbf{p})+\beta m)\beta\psi_P = E\beta\psi_P$$

$\rightarrow (\boldsymbol{\alpha}\cdot\mathbf{p}+\beta m)\beta\psi_P = E\beta\psi_P \rightarrow \beta\psi_P$ satisfies Dirac eq.

Reminder:

$$\beta = \gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$

Take plane wave solutions:

$$\psi_1(\boldsymbol{r},t) = \begin{pmatrix} \varphi \\ \dfrac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{E+m}\varphi \end{pmatrix} e^{-i(Et-\boldsymbol{p}\cdot\boldsymbol{r})}, \psi_2(\boldsymbol{r},t) = \begin{pmatrix} \chi \\ \dfrac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{E+m}\chi \end{pmatrix} e^{-i(Et-\boldsymbol{p}\cdot\boldsymbol{r})}, \psi_3(\boldsymbol{r},t) = \begin{pmatrix} -\dfrac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{E+m}\varphi \\ \varphi \end{pmatrix} e^{-i(-Et-\boldsymbol{p}\cdot\boldsymbol{r})}, \psi_4(\boldsymbol{r},t) = \begin{pmatrix} -\dfrac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{E+m}\chi \\ \chi \end{pmatrix} e^{-i(-Et-\boldsymbol{p}\cdot\boldsymbol{r})}$$

Go to rest frame: $\boldsymbol{p} = 0,\ E = m$

$$\left.\begin{aligned}\psi_1 &= \begin{pmatrix} \varphi \\ 0 \end{pmatrix} e^{-imt} \underset{P}{\rightarrow} \gamma^0 \begin{pmatrix} \varphi \\ 0 \end{pmatrix} e^{-imt} = \begin{pmatrix} \varphi \\ 0 \end{pmatrix} e^{-imt} = \psi_1 \\ \psi_2 &= \begin{pmatrix} \chi \\ 0 \end{pmatrix} e^{-imt} \rightarrow \gamma^0 \begin{pmatrix} \chi \\ 0 \end{pmatrix} e^{-imt} = \begin{pmatrix} \chi \\ 0 \end{pmatrix} e^{-imt} = \psi_2\end{aligned}\right\} \text{Intrinsic parity} = +1$$

$$\left.\begin{aligned}\psi_3 &= \begin{pmatrix} 0 \\ \varphi \end{pmatrix} e^{-i(-m)t} \rightarrow \gamma^0 \begin{pmatrix} 0 \\ \varphi \end{pmatrix} e^{-i(-m)t} = \begin{pmatrix} 0 \\ -\varphi \end{pmatrix} e^{-i(-m)t} = -\psi_3 \\ \psi_4 &= \begin{pmatrix} 0 \\ \chi \end{pmatrix} e^{-i(-m)t} \rightarrow \gamma^0 \begin{pmatrix} 0 \\ \chi \end{pmatrix} e^{-i(-m)t} = \begin{pmatrix} 0 \\ -\chi \end{pmatrix} e^{-i(-m)t} = -\psi_4\end{aligned}\right\} \text{Intrinsic parity} = -1$$

Remark:
Absolute parity *not defined* for fermions
Only defined relative fermion-antifermion

Intrinsic Parity - Photon

Classical EM field

Standard convention:

$$\left.\begin{array}{l} P:\mathbf{E}\rightarrow-\mathbf{E} \\ P:\mathbf{B}\rightarrow\mathbf{B} \\ \mathbf{E}=-\dfrac{\partial\mathbf{A}}{\partial t}-\nabla\varphi \\ \mathbf{B}=\nabla\times\mathbf{A} \end{array}\right\}\rightarrow\begin{cases} P:\mathbf{A}\rightarrow-\mathbf{A} \\ P:\varphi\rightarrow\varphi \end{cases}$$

A: Vector field

ϕ: Not needed for radiation field

Quantum EM field

A: Vector field

Photon spin = 1

$\rightarrow$ *Negative intrinsic parity*

Interesting question: EM interactions conserve parity

$\rightarrow$ *Photons from reactions, decays should have defined, total parity*

Photon Total Parity

What is the photon parity in a given state? Can expand any ***A*** into creation & annihilation operators

$$\mathbf{A}(\mathbf{r},t)=\sum_{\mathbf{k}}\sum_{i=1}^{2}\frac{1}{\sqrt{|\mathbf{k}|V}}\left[a_i(\mathbf{k})\boldsymbol{\varepsilon}_i e^{i(\mathbf{p}_\gamma\bullet\mathbf{x}-E_\gamma t)}+a_i^\dagger(\mathbf{k})\boldsymbol{\varepsilon}_i^* e^{-i(\mathbf{p}_\gamma\bullet\mathbf{x}-E_\gamma t)}\right]$$

But: $a_i(\boldsymbol{k})$ do not commute with parity. Can also expand into *J,m* C&A operators

$$\mathbf{A}(\mathbf{r},t)=\sum_{J,m}\sum_{L=J-1}^{J+1}\sum_{i=1}^{2}\frac{1}{\sqrt{|\mathbf{k}|V}}\left[a_i(J,L,m)\boldsymbol{\varepsilon}_i Y_{Jm}^L(\theta,\varphi)+a_i^\dagger(J,L,m)\boldsymbol{\varepsilon}_i^* Y_{Jm}^{L*}(\theta,\varphi)\right]$$

Photon states with defined total parity: states with given *J,L,m*. Indeed:

$$\boldsymbol{J}=\boldsymbol{L}+\underbrace{\boldsymbol{S}}_{=1}\rightarrow J=L,L\pm1\rightarrow L=J,J\pm1$$

$$\rightarrow\eta_P=(-1)^L(-1)=(-1)^{L+1}=(-1)^J,(-1)^{J\pm1}$$

$$\rightarrow\begin{cases}\eta_P=(-1)^J & \text{Electric}\\ \eta_P=(-1)^{J\pm1} & \text{Magnetic}\end{cases}$$

2 kinds of photons for any given *J*

Intrinsic Parity - Scalars

Really neutral states decaying into two photons

Example: π^0, *spin = 0*

$$\pi^0 \longrightarrow \gamma\gamma$$

Parity conserved by EM interaction

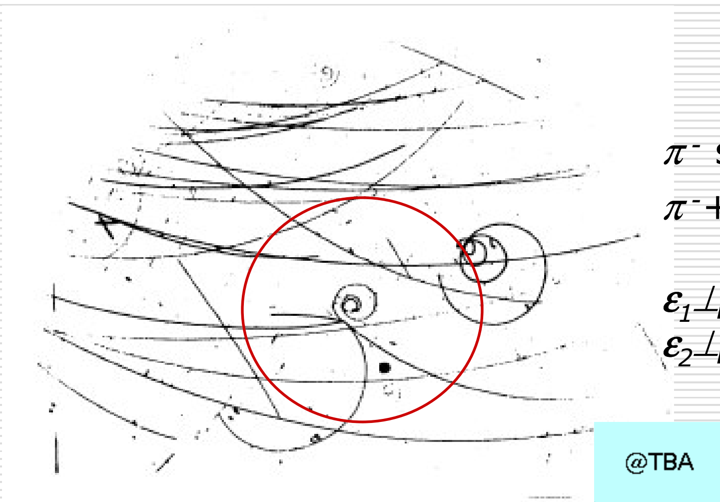
Final state: 2 creation operators $\propto \boldsymbol{\varepsilon_1}, \boldsymbol{\varepsilon_2}$

Can determine parity by looking at distribution of the angle between $\boldsymbol{\varepsilon_1}, \boldsymbol{\varepsilon_2}$

Observe: Double Dalitz decays $\pi^0 \longrightarrow e^+e^-e^+e^-$

Pseudoscalar $\sin^2\varphi$

Scalar $\cos^2\varphi$

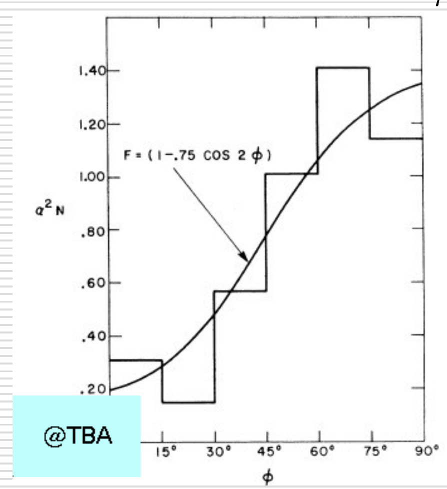


π^- stopping in liq. H_2

$\pi^- + p \rightarrow \pi^0 + n \rightarrow e^+e^-e^+e^- \, n$

$\varepsilon_1 \perp plane_1(\mathbf{p}_+, \mathbf{p}_-)$

$\varepsilon_2 \perp plane_2(\mathbf{p}_+, \mathbf{p}_-)$



Intrinsic Parity- Vectors

Most interesting result:
Yang Theorem

A vector (s=1) state cannot decay to 2 photons

Consequence of:

Angular momentum conservation
Parity conservation
Bose symmetry for identical photons
Gauge invariance of QED (transversality of photons)

Examples:

$\rho, \omega, \phi, J/\psi, \Upsilon, Z^0, ..$

Intrinsic Parity – Baryons

States with different charge, baryon number (absolutely conserved quantities) cannot be superposed ← Limitation to the principle of superposition, also called *Superselection Rule*

In other words: *No transition can violate Q,B conservation*

Relative parity unmeasurable, must be fixed by convention

Also true for states with different strangeness, charm, …*when weak interaction is neglected.*

But: While weak transitions *can* connect states with different *S,C*,.., they *do* violate parity!

Remembering that a particle+antiparticle state has –ve intrinsic parity, state our general convention for Leptons & Quarks

+ve parity to particles	*-ve parity to antiparticles*
Quarks	Antiquarks
Leptons	Antileptons

NB -ve parity baryons actually exist, with higher mass and spin:

Include quark *orbital* degrees of freedom

Intrinsic Parity – Mesons

Particles singly produced in parity conserving reactions → Parity *can* be determined

Examples

Hypernucleus
One neutron→Λ^0

Charged Pion

$\pi^- + d \to n + n$ at low energy

$\eta_P^{\pi^-} \left(\eta_P^N\right)^2 (-1)^L = \left(\eta_P^N\right)^2 (-1)^{L'}$

Initial state: $L = 0 \to J = 0 \oplus 1 = 1$

Final state: $J' = 1$, two identical fermions

$L = 0(symm) \to S = 1(symm) \to Symm$ KO

$L = 1(antisymm) \to S = \begin{cases} 0(antisymm) \to Symm & \text{KO} \\ 1(symm) \to Antisymm & \text{OK} \end{cases}$

→P-wave,triplet $\to L' = 1$

$\to \eta_P^{\pi^-} (-1)^0 = (-1)^1 \to \eta_P^{\pi^-} = -1$

Charged Kaon

$K^- + {}^4He \to \pi^0 + {}^4_\Lambda H$ at low energy

$\eta_P^{K^-} \eta_P^{{}^4He} (-1)^L = \eta_P^{\pi^0} \eta_P^{{}^4_\Lambda H} (-1)^{L'}$

Initial state: $L = 0, S_{K^-} = 0 \to J = 0$

Final state: $J' = 0,\ S_{{}^4_\Lambda H} = 0 \to L' = 0$

$\to \eta_P^{K^-} \cdot 1 \cdot (-1)^0 = (-1) \cdot 1 \cdot (-1)^0 \to \eta_P^{K^-} = -1$

= (+1)⁴ (-1)⁰ = +1
Ground state *L=0*

Two-Particle States

Fermion-Antifermion: CM frame

$$\eta_P^{f\bar{f}} = \underbrace{(-1)}_{\text{intrinsic}}\underbrace{(-1)^l}_{\text{orbital}} = (-1)^{l+1}$$

Boson-Antiboson: CM frame

$$\eta_P^{b\bar{b}} = \underbrace{(\eta_P)^2}_{\text{intrinsic}}\underbrace{(-1)^l}_{\text{orbital}} = (-1)^l$$

Photon-photon: CM frame

Take single photon states with defined helicity:

$U_P|\mathbf{k},R;-\mathbf{k},R\rangle = |-\mathbf{k},L;\mathbf{k},L\rangle$ not a U_P eigenstate

$U_P|\mathbf{k},L;-\mathbf{k},L\rangle = |-\mathbf{k},R;\mathbf{k},R\rangle$ not a U_P eigenstate

$U_P|\mathbf{k},R;-\mathbf{k},L\rangle = |-\mathbf{k},L;\mathbf{k},R\rangle \equiv |\mathbf{k},R;-\mathbf{k},L\rangle$ U_P eigenstate, $\eta_P = +1,\ J_3 = +2$

$U_P|\mathbf{k},L;-\mathbf{k},R\rangle = |-\mathbf{k},R;\mathbf{k},L\rangle \equiv |\mathbf{k},L;-\mathbf{k},R\rangle$ U_P eigenstate, $\eta_P = +1\ \ J_3 = -2$

$|\mathbf{k},R;-\mathbf{k},R\rangle \pm |\mathbf{k},L;-\mathbf{k},L\rangle$ U_P eigenstates, $\eta_P = \pm 1, J_3 = 0$

Fall of Parity – I

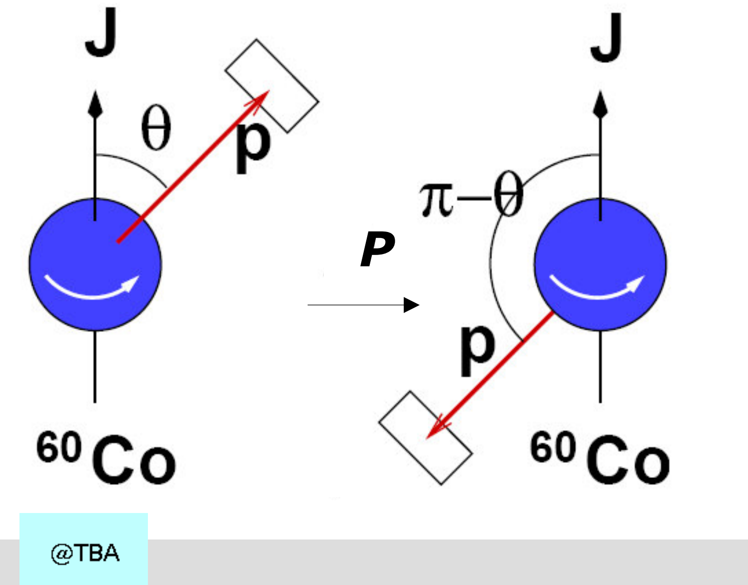
$^{60}Co \to {}^{60}Ni^{*} + e^{-} + \bar{\nu}_e \quad \beta^{-}$ decay: Weak process

If weak interaction is parity invariant,

$\to Ampl(\theta) = Ampl(\pi - \theta)$

Otherwise:

Expect β^{-} direction *anisotropy*



Require nuclear polarization:
For an unpolarized sample, by averaging over ***J*** z-projections any possible anisotropy is washed out

Fall of Parity – II

The ^{60}Co Experiment: Polarization

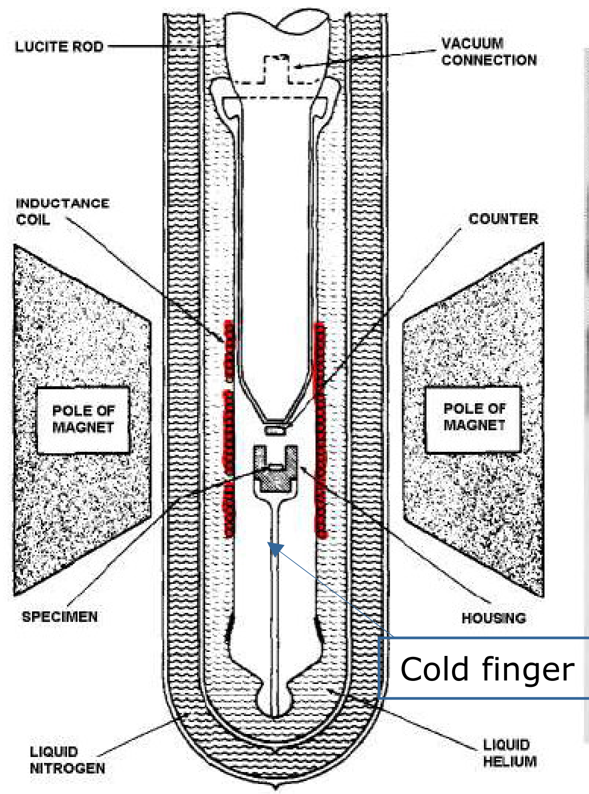
Zeeman:
$\mathcal{E}(M) = E_0 - \vec{\mu} \cdot \vec{B} = -g\mu_N BM$

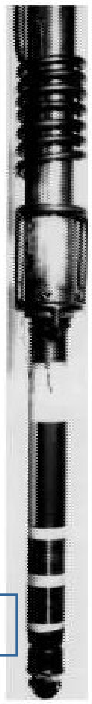
Boltzmann:
$$\frac{n(M')}{n(M)} = \frac{e^{\frac{\mathcal{E}(M)}{kT}}}{e^{\frac{\mathcal{E}(M')}{kT}}} = e^{\frac{(M-M')g\mu_N B}{kT}}$$

Magnetic field amplification in cerium-magnesium-nitrate crystal $0.05\,\mathrm{T} \to 10\text{–}100\,\mathrm{T}$

The ^{60}Co polarizes at a temperature of about 10 mK.

@TBA





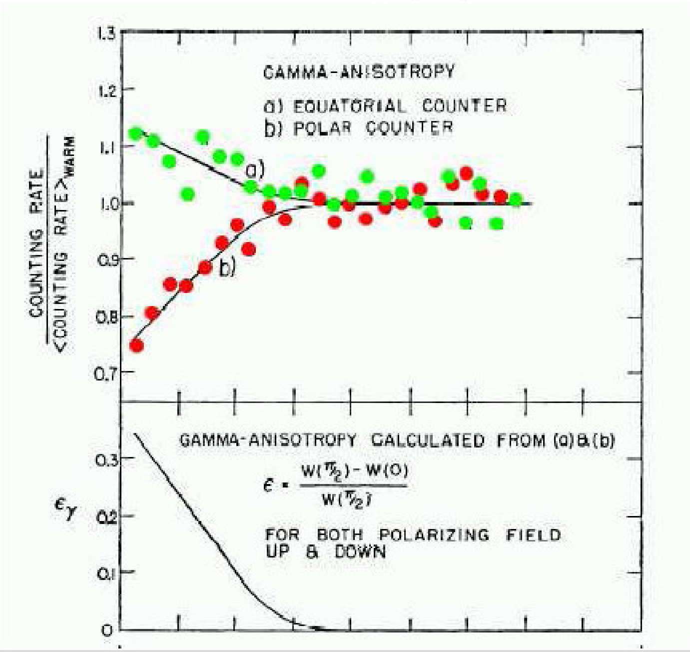
Fall of Parity – III

To measure nuclear polarization:

γ anisotropy in ${}^{60}\text{Ni}^* \rightarrow {}^{60}\text{Ni} + \gamma(E2)$

Electric quadrupole transition

$$\epsilon_\gamma = \frac{W(\pi/2) - W(0)}{W(\pi/2)}$$



@TBA

γ counter

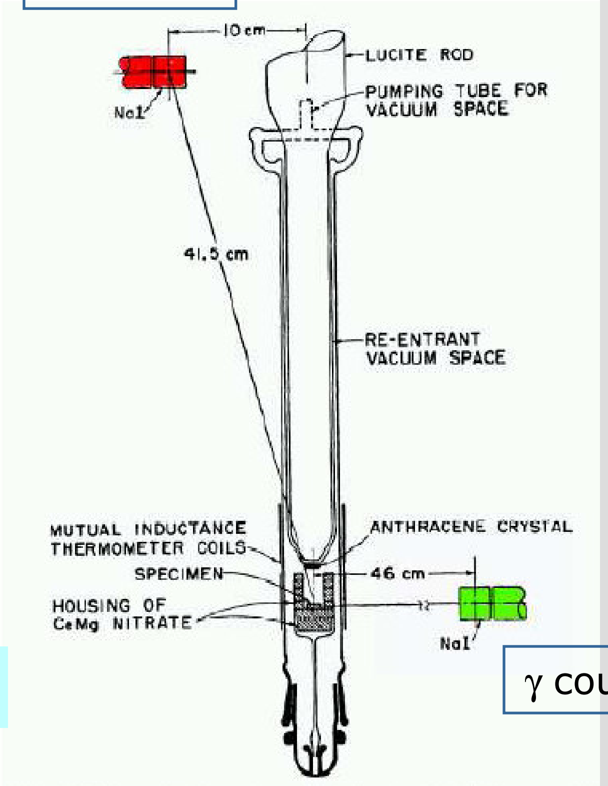
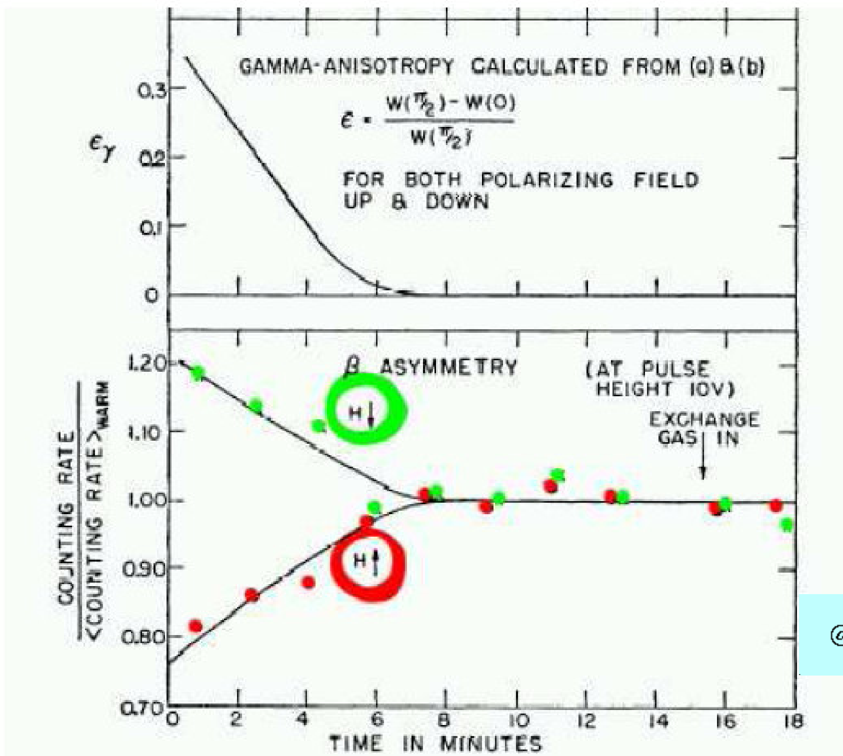


Fig. 1. Schematic drawing of the lower part of the cryostat

γ counter

Fall of Parity – IV

$$^{60}\text{Co} \to ^{60}\text{Ni}^* + e^- + \bar{\nu}_e$$
$$\tau = 5.27\,\text{year} \quad E_{e^-}^{max} = 2.824\,\text{MeV}$$



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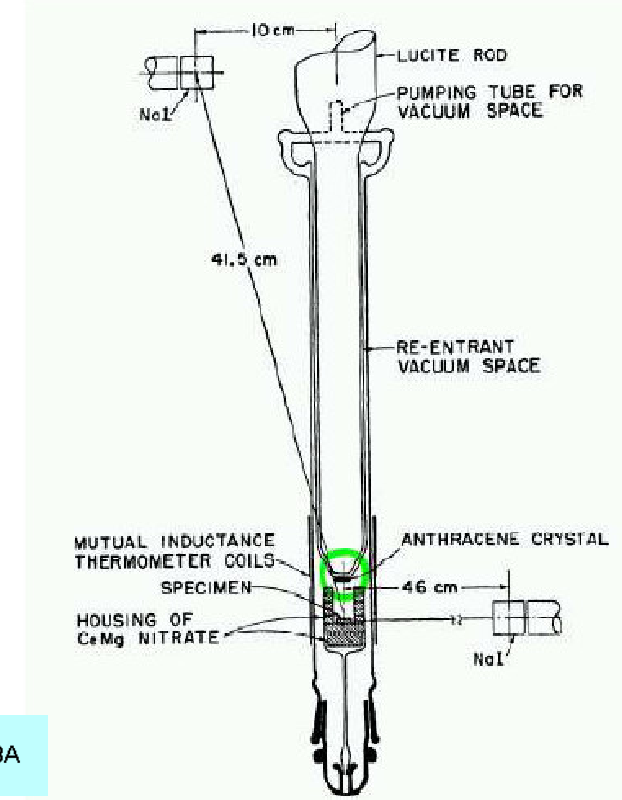


Fig. 1. Schematic drawing of the lower part of the cryostat

Another Fall of Parity

$\pi^+ \to \mu^+ + \nu_\mu$

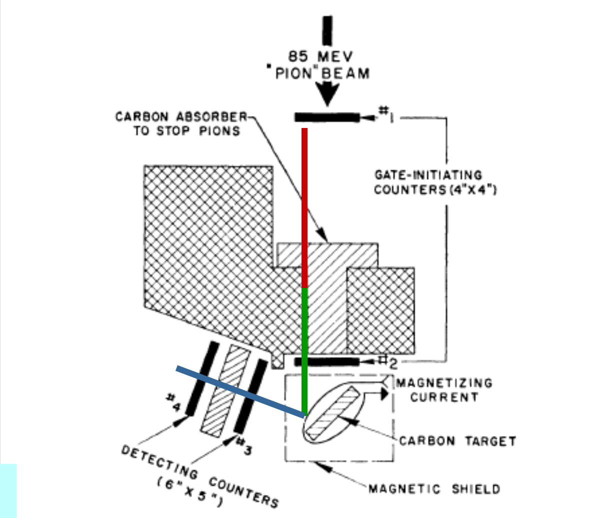
$\boldsymbol{\sigma}_\mu \cdot \mathbf{p}_\mu \underset{U_P}{\to} \boldsymbol{\sigma}_\mu \cdot (-\mathbf{p}_\mu) = -\boldsymbol{\sigma}_\mu \cdot \mathbf{p}_\mu$ Pseudoscalar observable

$\to P_\mu^{long} = \left\langle \frac{\boldsymbol{\sigma}_\mu \cdot \mathbf{p}_\mu}{|\mathbf{p}_\mu|} \right\rangle = 0$ if parity is a good symmetry

$\mu^+ \to e^+ + \bar{\nu}_\mu + \nu_e$

$\boldsymbol{\sigma}_\mu \cdot \mathbf{p}_e \underset{U_P}{\to} \boldsymbol{\sigma}_\mu \cdot (-\mathbf{p}_e) = -\boldsymbol{\sigma}_\mu \cdot \mathbf{p}_e$ Pseudoscalar observable

$\to \left\langle \frac{\boldsymbol{\sigma}_\mu \cdot \mathbf{p}_e}{|\mathbf{p}e|} \right\rangle = 0$ if parity is a good symmetry



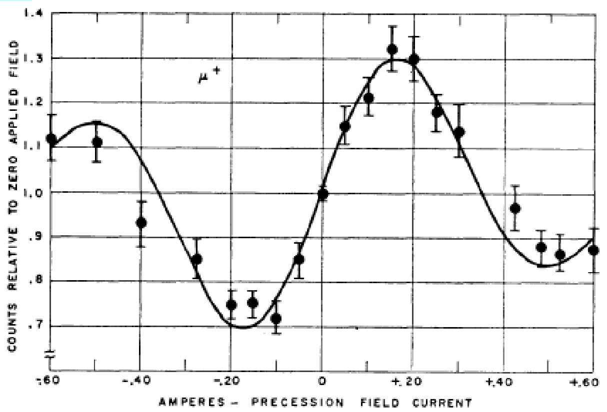
@TBA

If parity is violated:

$\begin{cases} \pi^+ \to \mu^+ + \nu_\mu & \text{Expect } \mu \text{ polarization along } \mathbf{p}_\mu \\ \mu^+ \to e^+ + \bar{\nu}_\mu + \nu_e & \text{Expect } e^+ \text{ direction correlated with } \mathbf{s}_\mu \end{cases}$

In order to detect e^+ correlation:

μ spin precession in $\mathbf{B}$



Charge Conjugation - I

Charge conjugation operation: similar to parity
But: Operating on 'internal', rather than spatial, coordinates

Change sign of all additive quantum numbers $Q, B, S, C, ...$

Like parity, unitary transformation in state space:

$$C: |\psi\rangle \equiv |\alpha; Q, B, S, C, ..\rangle \to |\psi'\rangle = U_C |\psi\rangle \equiv |\alpha; -Q, -B, -S, -C, ..\rangle$$

[α: All other quantum numbers, like energy, position, momentum, angular momentum, …]
Corresponding 'Relativity Principle' (<u>to be checked by experiment</u>)

The laws of physics are the same for all the observers, irrespective of the chosen positive direction of their 'charge axis'

Experimental fact:
All interactions, <u>except weak interaction</u>, do conserve charge parity

Charge Conjugation - II

A. Commutation relations with observables

U_C commutes with all observables, except those yielding additive quantum numbers :

$$[U_C, \boldsymbol{r}] = [U_C, \boldsymbol{p}] = [U_C, \boldsymbol{L}] = 0$$

$$U_C Q = -QU_C, U_C B = -BU_C, U_C S = -SU_C, \ldots$$

B. Action on states

$U_C : |Q\rangle \to |Q'\rangle \equiv U_C |Q\rangle = \eta |-Q\rangle$, η arbitrary phase $= e^{i\alpha}$

Charge Conjugation - III

C. Fundamental property of charge conjugation operator

$$C:Q\rightarrow Q'=C\left(Q\right)=-Q, C:Q'\rightarrow Q''=C\left(Q'\right)=-Q'=Q\rightarrow C^2=I$$

$$U_C\left|\psi\right\rangle=\left|\psi'\right\rangle, U_C\left|\psi'\right\rangle=U_C\left(U_C\left|\psi\right\rangle\right)=U_C^2\left|\psi\right\rangle\rightarrow U_C^2=\eta^2 I$$

$$\left.\begin{array}{r}U_C^\dagger=U_C^{-1},\ U_C\text{ unitary}\\ U_C^2=I\end{array}\right\}\rightarrow U_C^2=U_CU_C=I=U_CU_C^{-1}=U_CU_C^\dagger\rightarrow U_C^\dagger=U_C$$

$\rightarrow U_C$ hermitian

$\rightarrow U_P$ eigenvalues are real

$\rightarrow \eta=e^{i\alpha}=\pm 1\ \equiv\eta_C$ charge parity quantum number

$\rightarrow U_C$ eigenstates: $U_C\left|a\right\rangle=\pm\left|a\right\rangle$

Very few particles are U_C eigenstates: Must have

$$Q=B=S=C=...=0$$

Applications - I

$$j_\mu = (\rho, \mathbf{j}) \underset{C}{\rightarrow} (-\rho, -\mathbf{j}) = -j_\mu$$

$$A_\mu = (\varphi, \mathbf{A}) \underset{C}{\rightarrow} -(\varphi, \mathbf{A}) = -A_\mu \rightarrow U_C |\gamma\rangle = (-1)|\gamma\rangle$$

$$j^\mu A_\mu \underset{C}{\rightarrow} j^\mu A_\mu$$

$$U_C |n\gamma\rangle = (-1)^n |n\gamma\rangle$$

Boson-antiboson: CM frame

$$U_C |b\overline{b}\rangle = U_P |b\overline{b}\rangle = (-1)^l |b\overline{b}\rangle \rightarrow \eta_C = (-1)^l$$

Fermion-antifermion: CM frame

$U_C = U_P U_S$ Parity * Spin exchange

$$U_P |f\ \overline{f}\rangle = (-1)^{l+1} |f\overline{f}\rangle$$

$$U_S |\uparrow\downarrow - \downarrow\uparrow\rangle = |\downarrow\uparrow - \uparrow\downarrow\rangle = -|\uparrow\downarrow - \downarrow\uparrow\rangle,\ U_S |\uparrow\downarrow + \downarrow\uparrow\rangle = |\downarrow\uparrow + \uparrow\downarrow\rangle = |\uparrow\downarrow + \downarrow\uparrow\rangle,$$

$$U_S |\uparrow\uparrow\rangle = |\uparrow\uparrow\rangle, U_S |\downarrow\downarrow\rangle = |\downarrow\downarrow\rangle$$

$$\rightarrow U_C |f\ \overline{f}\rangle = (-1)^{l+s} |f\overline{f}\rangle$$

Applications - II

$\pi^0, \eta, \ldots: \quad \eta_C = (-1)^2 = +1$ 2 photons decays

States $f\,\bar{f}$ decaying to n photons:

$$C:(-1)^{l+s} \rightarrow S \text{ state} \begin{cases} \eta_C = +1 \text{ singlet} \\ \eta_C = -1 \text{ triplet} \end{cases}$$

$$n \text{ photons: } \eta_C = (-1)^l \rightarrow \begin{cases} 2 \text{ photons } \eta_C = +1 \\ 3 \text{ photons } \eta_C = -1 \end{cases}$$

$$\rightarrow \begin{cases} \text{singlet} \rightarrow 2 \text{ photons} \\ \text{triplet} \rightarrow 3 \text{ photons} \end{cases}$$

Meson: Fermion-Antifermion bound state

L	S=0	S=1
0	0^{-+}	1^{--}
1	1^{+-}	0^{++}, 1^{++}, 2^{++}
2	2^{-+}	1^{--}, 2^{--}, 3^{--}

States decaying to 2 pions

J	PC = ++	PC = +-,-+,--
0	OK	KO
1	KO	KO,KO,OK
2	OK	KO

Fall of Charge Conjugation

Fundamental question:
Is charge parity conserved by all interactions?

$$[H, U_C] = 0 \quad ?$$

Answer: Experiment

Same experiments probing spatial parity:

Charge conjugation *is* violated by weak interaction

First predicted by Lee, Oheme and Yang in 1957:
CPT symmetry requires *both* CT and P violation
Since T is ~ good, C must be violated

Observed by Garwin, Lederman and Weinrich in their parity violation experiment with muons

Time Reversal

Need to make clear:
These considerations are relevant to physical systems with a (very) small number of degrees of freedom
Do not apply to complex systems, whose time evolution is driven by the II Law of thermodynamics. For them, a *time arrow* can be defined with no ambiguities. Or so we believe… (A *very* hard subject)

Can state another 'relativity principle', about the choice of the positive direction of the time coordinate of physical events:

The laws of physics are the same for all the observers, irrespective of the chosen positive direction of their 'time axis'

Experimental fact:

All interactions, except weak interaction, are invariant wrt to time reversal

Classical Physics

General behavior of physical quantities wrt to time reversal:

$$t \to -t:$$

$$\mathbf{r} \to \mathbf{r}$$

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} \to \frac{d\mathbf{r}}{d(-t)} = -\mathbf{v}, \mathbf{p} \to -\mathbf{p}$$

$$\mathbf{a} = \frac{d^2\mathbf{r}}{dt^2} \to \mathbf{a}, \mathbf{F} \to \mathbf{F}$$

$$\mathbf{L} = \mathbf{r} \times \mathbf{p} \to -\mathbf{L}$$

$$E \to E$$

$$\rho \to \rho$$

$$\mathbf{j} = \rho\mathbf{v} \to -\mathbf{j}$$

$$\varphi \to \varphi$$

$$\mathbf{A} \to -\mathbf{A}$$

$$\mathbf{E} \to \mathbf{E}$$

$$\mathbf{B} \to -\mathbf{B}$$

Observe: Ohm’s law is *not* TR invariant

$\mathbf{j} = \sigma\mathbf{E} \to -\mathbf{j} = \sigma\mathbf{E},$ take σ as TR invariant

Example of a *macroscopic* law…

Schrodinger Equation

$T: t \to t' = -t$

First guess...

$\langle t' | \psi' \rangle = \langle t | \psi \rangle = \langle -t' | \psi \rangle$

$\langle t | \psi' \rangle = \langle -t | \psi \rangle$

$\to \psi'(t) = \psi(-t)$

$U_T : |\psi\rangle \to |\psi'\rangle \equiv U_T |\psi\rangle$

$U_T : |t\rangle \to |t'\rangle \equiv U_T |t\rangle = \eta |-t\rangle$ Wrong!

Indeed:

$$H|\psi(t)\rangle = i \frac{\partial |\psi(t)\rangle}{\partial t} \underset{U_T}{\to} H|\psi(-t)\rangle = -i \frac{\partial |\psi(-t)\rangle}{\partial t}$$

Redefine:

$$\left.\begin{array}{l} T: t \to -t \\ K: i \to -i \end{array}\right\}, \; U_T = KT$$

$|\psi(t)\rangle \to U_T |\psi(t)\rangle = |\psi^T(t)\rangle = |\psi^*(-t)\rangle$ OK

Time Reversal

Take a plane wave:

$$e^{-i(Et-\mathbf{p}\cdot\mathbf{r})} \underset{U_T}{\longrightarrow} e^{+i(E(-t)-\mathbf{p}\cdot\mathbf{r})} = e^{-i(Et+\mathbf{p}\cdot\mathbf{r})} = e^{-i(Et-(-\mathbf{p})\cdot\mathbf{r})}$$

Quite natural...:

U_T sends a progressive plane wave into a regressive one

Puzzled? Think of it in this way:
U_T inverts the positive direction of time for the 2nd observer. Therefore, the 2nd observer has a clock which is running *backwards*.
Then the same wave which is seen as *progressive* by the first observer, is seen as *regressive* by the second..

Take a particle with *s=1/2*:

$$\begin{pmatrix} \psi_+(t) \\ \psi_-(t) \end{pmatrix} \rightarrow \begin{pmatrix} \psi_+^T(t) \\ \psi_-^T(t) \end{pmatrix} = M \begin{pmatrix} \psi_+^*(-t) \\ \psi_-^*(-t) \end{pmatrix}$$

Must take into account that $\mathbf{s}$ is *reversed* by T

$$\begin{pmatrix} \psi_+(t) \\ \psi_-(t) \end{pmatrix} \underset{U_T}{\longrightarrow} \begin{pmatrix} \psi_+^T(t) \\ \psi_-^T(t) \end{pmatrix} = -i\sigma_y \begin{pmatrix} \psi_+^*(-t) \\ \psi_-^*(-t) \end{pmatrix} = \begin{pmatrix} \psi_-^*(-t) \\ -\psi_+^*(-t) \end{pmatrix}$$

OK

$$\sigma_y = \begin{pmatrix} 0 & -i \\ +i & 0 \end{pmatrix} \rightarrow M = -i\sigma_y = \begin{pmatrix} 0 & +1 \\ -1 & 0 \end{pmatrix}$$

Spin-reversing operator

Apply U_T a second time:

$$\begin{pmatrix} \psi_+^T(t) \\ \psi_-^T(t) \end{pmatrix} \underset{U_T}{\longrightarrow} -i\sigma_y \begin{pmatrix} \psi_-^*(t) \\ -\psi_+^*(t) \end{pmatrix}^* = \begin{pmatrix} -\psi_+(t) \\ -\psi_-(t) \end{pmatrix} = -\begin{pmatrix} \psi_+(t) \\ \psi_-(t) \end{pmatrix}$$

Similar to parity for spin 1/2

Applications

Basic remark:

U_T is *not* a unitary operator
U_T is *not* a Hermitian operator

Therefore:
No real eigenvalues
No (conserved) observables
$\rightarrow$*No 'time parity'*

Nevertheless:

Quite useful to establish properties of fundamental interactions

Because is not linear, rather it is antilinear

$$U_T\left(a|\psi_1\rangle + b|\psi_2\rangle\right) = a^* U_T|\psi_1\rangle + b^* U_T|\psi_2\rangle$$

Complex conjugation

$$K:\alpha z \rightarrow (\alpha z)^* = \alpha^* z^*$$

is a non-linear operation. Proof:

$$\alpha = ae^{i\varphi}, z = \rho e^{i\psi} \rightarrow \alpha^* = ae^{-i\varphi}, z^* = \rho e^{-i\psi}$$

$$\alpha z = ae^{i\varphi}\rho e^{i\psi} = a\rho e^{i(\varphi+\psi)} \rightarrow (\alpha z)^* = a\rho e^{-i(\varphi+\psi)} = \alpha^* z^*$$

But:

$\alpha^* z^* \neq \alpha z^*$ $\rightarrow$Non linear whenever $\alpha \neq \alpha^*$!!

Indeed, a linear operator must satisfy:

$$A:(\alpha z) \rightarrow A(\alpha z) = \alpha A(z)$$

Kramers Degeneracy etc

Wave functions:

$$\psi_T(x) = \langle x|\psi_T\rangle = \langle x|U_T|\psi\rangle = \langle x|\psi\rangle^* = \psi^*(x) \quad \text{C.Conjugate}$$

If $H|\psi\rangle = E|\psi\rangle, [H, U_T] = 0, \ |\psi\rangle$ non-degenerate

$$\rightarrow H|\psi_T\rangle = HU_T|\psi\rangle = U_T H|\psi\rangle = EU_T|\psi\rangle = E|\psi_T\rangle \rightarrow |\psi_T\rangle = |\psi\rangle$$

$\rightarrow \psi(x)$ real

Degeneracy of fermion states:

$$U_T^2|\psi\rangle = -|\psi\rangle$$

$$\rightarrow \langle\psi|\psi_T\rangle = \langle\psi_T|\psi\rangle^* = \langle\psi|U_T^\dagger|\psi\rangle = -\langle\psi|U_T^\dagger U_T^2|\psi\rangle = -\langle\psi|U_T|\psi\rangle = -\langle\psi|\psi_T\rangle$$

$$\rightarrow \langle\psi|\psi_T\rangle = 0$$

$\rightarrow |\psi\rangle, |\psi_T\rangle$ orthogonal, independent

Example (trivial): Spin ½ particle in rest frame
→ 2 independent states (up & down)

Effect on Scalar Products

$\{|i\rangle, i=1,...\}$ complete set of states

$\langle i | j \rangle = \delta_{ij}$

$\rightarrow \langle U_T i | U_T j \rangle = \delta_{ij}$ norm is $U_T -$invariant

$$|\psi_1\rangle = \sum_i |i\rangle\langle i|\psi_1\rangle \rightarrow |U_T\psi_1\rangle = \sum_i |U_T i\rangle\langle i|\psi_1\rangle^*$$

$$\langle\psi_2| = \sum_j \langle\psi_2|j\rangle\langle j| \rightarrow \langle U_T\psi_2| = \sum_j \langle\psi_2|j\rangle^* \langle U_T j|$$

$$\rightarrow \langle U_T\psi_2|U_T\psi_1\rangle = \sum_j \langle\psi_2|j\rangle^* \langle U_T j|\sum_i |U_T i\rangle\langle i|\psi_1\rangle^*$$

$$= \sum_{i,j} \langle\psi_2|j\rangle^* \underbrace{\langle U_T j|U_T i\rangle}_{\delta_{ij}} \langle i|\psi_1\rangle^* = \sum_i \langle\psi_2|i\rangle^* \langle i|\psi_1\rangle^*$$

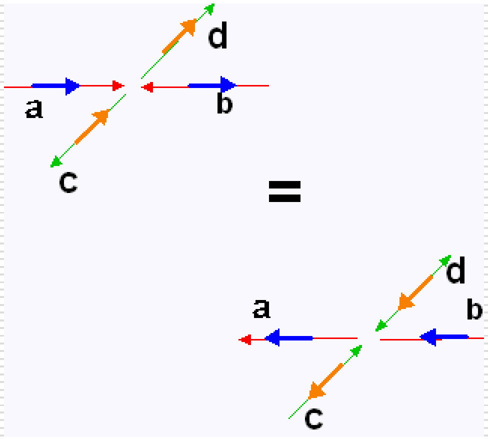
$$= \sum_i \langle i|\psi_2\rangle\langle\psi_1|i\rangle = \langle\psi_1|\psi_2\rangle$$ U_T swaps states in any scalar product

Matrix Elements

Matrix element for a transition: Initial $\leftrightarrow$ Final

H T-invariant $\rightarrow$ Same matrix element for direct and reversed transition

$\underbrace{1+2}_{i} \rightarrow \underbrace{3+4}_{f}$ 2-body process



Reciprocity Theorem:

$$\langle \mathbf{p}_f, \mathbf{s}_f | S | \mathbf{p}_i, \mathbf{s}_i \rangle = \underbrace{\langle -\mathbf{p}_i, -\mathbf{s}_i |}_{T|\mathbf{p}_i, \mathbf{s}_i\rangle} S \underbrace{| -\mathbf{p}_f, -\mathbf{s}_f \rangle}_{T\langle \mathbf{p}_f, \mathbf{s}_f |}{}^{*}$$

Detailed Balance Theorem:

$$\frac{\dfrac{d\sigma_{if}}{d\Omega}}{\dfrac{d\sigma_{fi}}{d\Omega}} = \left(\frac{p_f}{p_i}\right)^2 \frac{(2s_3+1)(2s_4+1)}{(2s_1+1)(2s_2+1)}$$

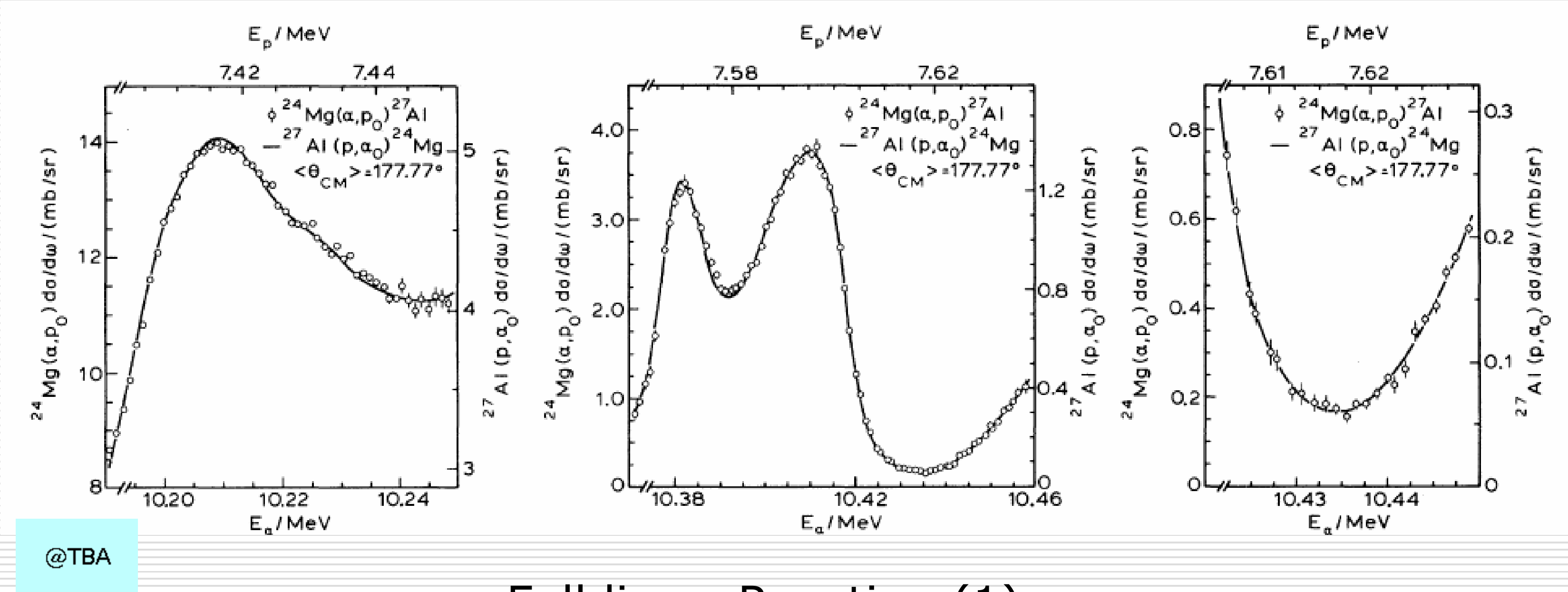
OK for Strong, E.M., KO for Weak

Example: Test of the Detailed Balance

Nuclear reactions:

$$(1)\quad \alpha + {}^{24}Mg \rightarrow p + {}^{27}Al$$

$$(2)\quad p + {}^{27}Al \rightarrow \alpha + {}^{24}Mg$$



Full line: Reaction (1)

Circles: Reaction (2)

T-Violating Correlations

Example (many more exist):

$$\Lambda^0 \rightarrow p + \pi^-$$

$$\left.\begin{array}{l} \frac{1}{2}^+ = \frac{1}{2}^+ \oplus 0^- \oplus L^{(-1)^L} = \frac{1}{2} \oplus L \\ + \quad = + \quad \cdot \; - \; \cdot \quad (-1)^L = (-1)^{L+1} \end{array}\right\} \rightarrow L = 0,1$$

Decay amplitude = Sum of S,P waves
S,P waves *do* interfere because of parity violation. So:
Angular distribution has a term $\propto$ *cos θ*

Most general form of angular distribution:

$$\frac{d\Gamma}{d\Omega_\pi} = 1 + A\left(\boldsymbol{J}_\Lambda + \boldsymbol{J}_p\right)\cdot\hat{\boldsymbol{p}}_\pi + B\left(\boldsymbol{J}_\Lambda \times \boldsymbol{J}_p\right)\cdot\hat{\boldsymbol{p}}_\pi + C\left(\boldsymbol{J}_\Lambda \cdot \boldsymbol{J}_p\right) + (1-C)\left(\boldsymbol{J}_\Lambda\cdot\hat{\boldsymbol{p}}_\pi\right)\left(\boldsymbol{J}_p\cdot\hat{\boldsymbol{p}}_\pi\right)$$

$$\left(\boldsymbol{J}_\Lambda \times \boldsymbol{J}_p\right)\cdot\hat{\boldsymbol{p}}_\pi \underset{U_T}{\rightarrow} \left(\left(-\boldsymbol{J}_\Lambda\right)\times\left(-\boldsymbol{J}_p\right)\right)\cdot\left(-\hat{\boldsymbol{p}}_\pi\right) = -\left(\boldsymbol{J}_\Lambda \times \boldsymbol{J}_p\right)\cdot\hat{\boldsymbol{p}}_\pi$$ T-Violating term

$\rightarrow$ Expect $B = 0$

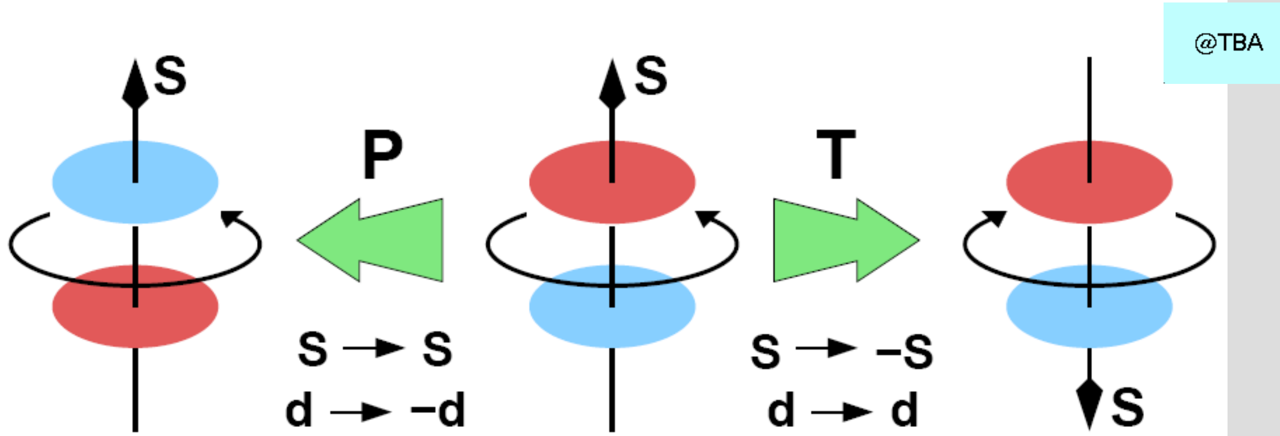
Must correct for final state interaction: $p\pi$ scattering

$\rightarrow$ Get an upper limit of a few %

Electric Dipole Moments - I

Warning: This is true in the absence of a degenerate ground state
E.g., polar molecules *do* have degenerate ground states…

Elementary particle can only have a permanent electric dipole moment if both parity and time reversal symmetries are broken:



$$\vec{\mu} = g\,\frac{e}{mc}\vec{S} \qquad \vec{d} = \eta\,\frac{e}{2mc}\vec{S}$$

d*//*S, otherwise there would be an extra degree of freedom which is not seen

Electric Dipole Moments - II

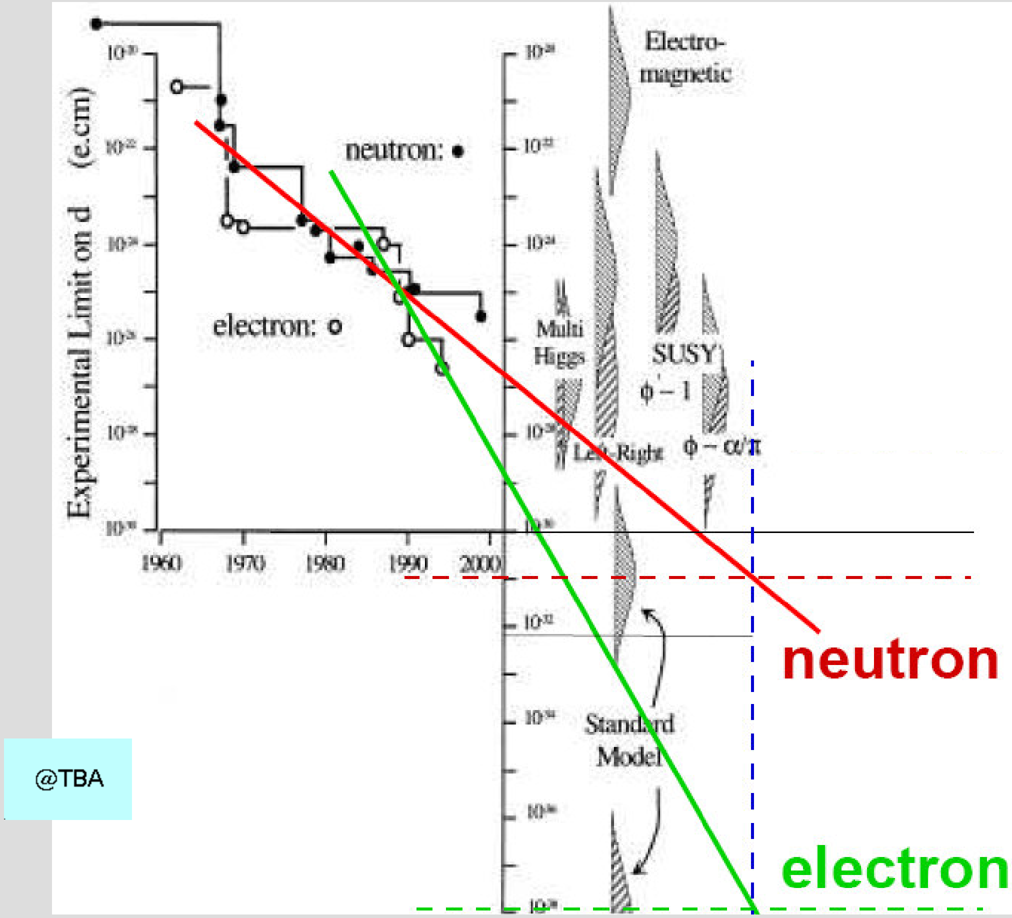
For a particle with both magnetic and electric dipole moments:

$H' = -\boldsymbol{\mu}\cdot\mathbf{B} - \mathbf{d}\cdot\mathbf{E}$ Interaction energy

Can observe in external $\mathbf{E}$:

Level shift
Spin precession

Status of the EDM



C, P and T in Quantum Field Theory

Just a few, oversimplified remarks:

A. Observables are (functions of) field operators
B. Just as in Quantum Mechanics, one finds different kinds of operators:
   Scalars, Spinors, Vectors, …
C. Fields of each kind have their own 'parity' built-in
D. States of a given field do inherit field 'parity', which is called *intrinsic parity*

C,P and T in QFT - I

1) Spin = 0

$$\phi(t,\mathbf{r}) = N\int d^3\mathbf{p}\left[b(\mathbf{p})e^{-ipx} + d^\dagger(\mathbf{p})e^{+ipx}\right]$$

$$\phi^\dagger(t,\mathbf{r}) = N\int d^3\mathbf{p}\left[d(\mathbf{p})e^{-ipx} + b^\dagger(\mathbf{p})e^{+ipx}\right]$$

$$\begin{cases} U_P\phi(t,\mathbf{r})U_P^\dagger = \eta_P\phi(t,-\mathbf{r}) \\ U_P\phi^\dagger(t,\mathbf{r})U_P^\dagger = \eta_P\phi(t,-\mathbf{r}) \end{cases} \leftrightarrow \begin{cases} U_P \begin{matrix} b(\mathbf{p}) \\ d^\dagger(\mathbf{p}) \end{matrix} U_P^\dagger = \eta_P \begin{matrix} b(-\mathbf{p}) \\ d^\dagger(-\mathbf{p}) \end{matrix} \\ U_P \begin{matrix} b^\dagger(\mathbf{p}) \\ d(\mathbf{p}) \end{matrix} U_P^\dagger = \eta_P \begin{matrix} b^\dagger(\mathbf{p}) \\ d(\mathbf{p}) \end{matrix} \end{cases}, e^{\pm ipx} \to e^{\pm ipx}$$

$$\begin{cases} U_C\phi(t,\mathbf{r})U_C^\dagger = \eta_C\phi^\dagger(t,\mathbf{r}) \\ U_C\phi^\dagger(t,\mathbf{r})U_C^\dagger = \eta_C^*\phi(t,\mathbf{r}) \end{cases} \leftrightarrow \begin{cases} U_C \begin{matrix} b(\mathbf{p}) \\ d^\dagger(\mathbf{p}) \end{matrix} U_C^\dagger = \eta_C \begin{matrix} d(\mathbf{p}) \\ b^\dagger(\mathbf{p}) \end{matrix} \\ U_C \begin{matrix} b^\dagger(\mathbf{p}) \\ d(\mathbf{p}) \end{matrix} U_C^\dagger = \eta_C^* \begin{matrix} d^\dagger(\mathbf{p}) \\ b(\mathbf{p}) \end{matrix} \end{cases}, e^{\pm ipx} \to e^{\pm ipx}$$

$$\begin{cases} \bar{U}_T\phi(t,\mathbf{r})\bar{U}_T^\dagger = \underbrace{\cancel{\eta_T}}_{=1}\phi(-t,\mathbf{r}) \\ \bar{U}_T\phi^\dagger(t,\mathbf{r})\bar{U}_T^\dagger = \phi^\dagger(-t,\mathbf{r}) \end{cases} \leftrightarrow \begin{cases} \bar{U}_T \begin{matrix} b(\mathbf{p}) \\ d^\dagger(\mathbf{p}) \end{matrix} \bar{U}_T^\dagger = \begin{matrix} b(-\mathbf{p}) \\ d^\dagger(-\mathbf{p}) \end{matrix} \\ \bar{U}_T \begin{matrix} d(\mathbf{p}) \\ b^\dagger(\mathbf{p}) \end{matrix} \bar{U}_T^\dagger = \begin{matrix} d(-\mathbf{p}) \\ b^\dagger(-\mathbf{p}) \end{matrix} \end{cases}, e^{\pm ipx} \to e^{\mp ipx}$$

General:

Observables depend on $\phi\phi^\dagger$

Any observable yields a factor

$\eta_P\eta_P^* = 1, \eta_C\eta_C^* = 1$

$\to \eta_P, \eta_C$ unobservable

ϕ Hermitian:

$\phi = \phi^\dagger$

$\to \eta_C = \eta_C^*, \eta_P = \pm 1$

$\to \begin{cases} \eta_P = \pm 1 \\ \eta_C = \pm 1 \end{cases}$ Observable

η_T unobservable

C,P and T in QFT - II

2) Spin 1/2

$$\psi(t,\mathbf{r})=\sum_s\int Nd^3\mathbf{p}\left[b(\mathbf{p},s)u(\mathbf{p},s)e^{+ipx}+d^\dagger(\mathbf{p},s)v(\mathbf{p},s)e^{-ipx}\right]$$

$$\psi^\dagger(t,\mathbf{r})=\sum_s\int Nd^3\mathbf{p}\left[b^\dagger(\mathbf{p},s)\overline{u}(\mathbf{p},s)e^{-ipx}+d(\mathbf{p},s)\overline{v}(\mathbf{p},s)e^{+ipx}\right]$$

$$\begin{cases}U_P\psi(t,\mathbf{r})U_P^\dagger=\eta_P\gamma^0\psi(t,-\mathbf{r})\\ U_P\overline{\psi}(t,\mathbf{r})U_P^\dagger=\eta_P^*\overline{\psi}(t,-\mathbf{r})\gamma^0\end{cases}\leftrightarrow\begin{cases}U_P\begin{matrix}b(\mathbf{p},s)\\ d(\mathbf{p},s)\end{matrix}U_P^\dagger=\eta_P\begin{matrix}b(-\mathbf{p},s)\\ d(-\mathbf{p},s)\end{matrix}\\ U_P\begin{matrix}d^\dagger(\mathbf{p},s)\\ b^\dagger(\mathbf{p},s)\end{matrix}U_P^\dagger=\eta_P^*\begin{matrix}d^\dagger(-\mathbf{p},s)\\ b^\dagger(-\mathbf{p},s)\end{matrix}\end{cases},e^{\pm ipx}\to e^{\pm ipx}$$

$$\begin{cases}U_C\psi(t,\mathbf{r})U_C^\dagger=\eta_C i\gamma_2\gamma_0\overline{\psi}(t,\mathbf{r})\\ U_C\overline{\psi}(t,\mathbf{r})U_C^\dagger=\eta_C^*\psi(t,\mathbf{r})i\gamma_2\gamma_0\end{cases}\leftrightarrow\begin{cases}U_C\begin{matrix}b(\mathbf{p},s)\\ b^\dagger(\mathbf{p},s)\end{matrix}U_C^\dagger=\eta_C\begin{matrix}d(\mathbf{p},s)\\ d^\dagger(\mathbf{p},s)\end{matrix}\\ U_C\begin{matrix}d(\mathbf{p},s)\\ d^\dagger(\mathbf{p},s)\end{matrix}U_C^\dagger=\eta_C^*\begin{matrix}b(\mathbf{p},s)\\ b^\dagger(\mathbf{p},s)\end{matrix}\end{cases},e^{\pm ipx}\to e^{\pm ipx}$$

$$\begin{cases}\overline{U}_T\psi(t,\mathbf{r})\overline{U}_T^\dagger=\underbrace{\cancel{\eta_T}}_{=1}\gamma_1\gamma_3\psi(-t,\mathbf{r})\\ \overline{U}_T\overline{\psi}(t,\mathbf{r})\overline{U}_T^\dagger=\overline{\psi}(-t,\mathbf{r})(-\gamma_1\gamma_3)\end{cases}\leftrightarrow\begin{cases}\overline{U}_T\begin{matrix}b(\mathbf{p},s)\\ d^\dagger(\mathbf{p},s)\end{matrix}\overline{U}_T^\dagger=\begin{matrix}b(-\mathbf{p},-s)\\ d^\dagger(-\mathbf{p},-s)\end{matrix}\\ \overline{U}_T\begin{matrix}b^\dagger(\mathbf{p},s)\\ d(\mathbf{p},s)\end{matrix}\overline{U}_T^\dagger=\begin{matrix}b^\dagger(-\mathbf{p},-s)\\ d(-\mathbf{p},-s)\end{matrix}\end{cases},e^{\pm ipx}\to e^{\mp ipx}$$

CPT

Fundamental theorem applying to all field theories, including our beloved Standard Model:

Lorentz invariance

Micro-causality (whatever it means…)

Spin-statistics

$\rightarrow$ *The product transformation CPT is a good symmetry*

Consequences:

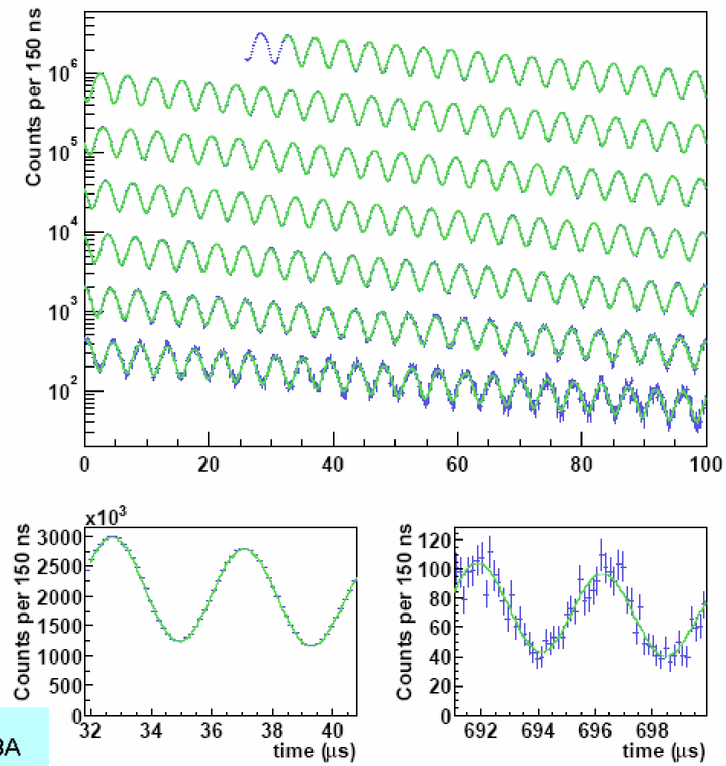
$$m_{particle} = m_{antiparticle}$$

$$\tau_{particle} = \tau_{antiparticle}$$

$$|\boldsymbol{\mu}|_{particle} = |\boldsymbol{\mu}|_{antiparticle}$$

....more quantum numbers

Good Old QED Test: Muon Anomaly



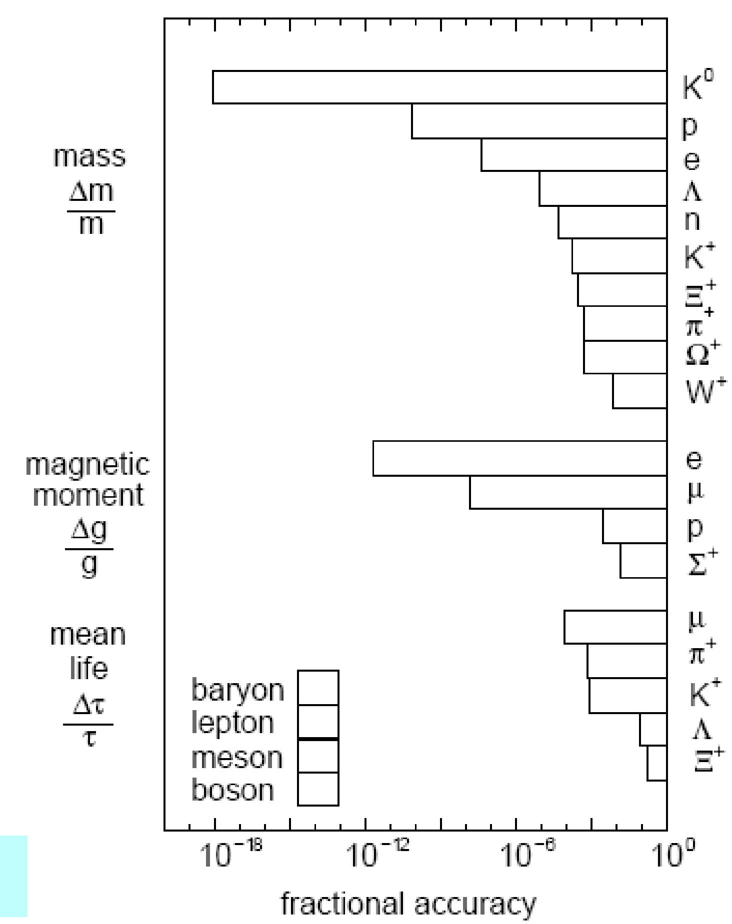
@TBA

Also an excellent test
of CPT in EM interaction

$$\left.\begin{array}{l} a_{\mu^+} = \left(11659203 \pm 8\right) \times 10^{-10} \\ a_{\mu^-} = \left(11659214 \pm 9\right) \times 10^{-10} \end{array}\right\} \Delta a_\mu = \left(11 \pm 12\right) \times 10^{-10}$$

CPT Tests

Just a flash on the general status of CPT tests…
…very comfortable



@TBA

Symmetries: Summary

Conserved quantity	Interaction		
	Strong	E.M.	Weak
4-momentum	OK	OK	OK
Charge	OK	OK	OK
Ang. Momentum, CM speed	OK	OK	OK
Baryonic number	OK	OK	OK
Leptonic numbers (3)	OK	OK	~OK
Parity	OK	OK	KO
Charge parity	OK	OK	KO
(Time reversal)	OK	OK	KO
CP	OK	OK	KO
(CPT)	OK	OK	OK
Flavor	OK	OK	KO