Elementary Particles I

1 – Symmetries

Symmetry, Invariance, Conservation

Deep connection between:

Physical Symmetry
(Geometrical, Kinematical, Dynamical, …)

Invariance vs Coordinate Transformations
(Space-time, Internal, ..)

Conservation Laws
(Physical Observables)

Old subject, finally cast into modern framework by Emmy Noether (1918)

Cyclic Coordinates - I

Ignorable coordinates:

Lagrangian formalism

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{q}_i}\right)-\frac{\partial L}{\partial q_i}=0 \quad \text{Lagrange's equations}$$

$$\frac{\partial L}{\partial q_i}=0 \rightarrow \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{q}_i}\right)=0 \rightarrow p_i \equiv \frac{\partial L}{\partial \dot{q}_i}=\text{const}$$

Hamiltonian formalism

$$\dot{p}_i=-\frac{\partial H}{\partial q_i} \quad \text{Hamilton's equations}$$

$\rightarrow H$ not depending on $q_i \Rightarrow p_i=\text{const}$

q_i cyclic $\leftrightarrow$ p_i conjugate momentum = constant

Es:

$$L=\frac{1}{2}mv^2 \rightarrow \frac{\partial L}{\partial x}=0$$

$$\rightarrow \frac{\partial L}{\partial v}=p=mv=\text{const}$$

Cyclic Coordinates - II

Example: Particle in a cylindrical potential

Write L in cylindrical coordinates:

$$L = \frac{1}{2} m\left(\dot{\rho}^2 + \rho^2\dot{\varphi}^2 + \dot{z}^2\right) - V(\rho)$$

$$z,\varphi \text{ ignorable} \rightarrow \begin{cases} p_z = \dfrac{\partial L}{\partial \dot{z}} = m\dot{z} = \text{const} & \text{Linear momentum along } z \\ p_\varphi = \dfrac{\partial L}{\partial \dot{\varphi}} = m\rho^2\dot{\varphi} = \text{const} & \text{Angular momentum along } z \end{cases}$$

Write L in Cartesian coordinates:

$$L = \frac{1}{2} m\left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right) - V\left(\sqrt{x^2 + y^2}\right)$$

$$z \text{ ignorable} \rightarrow p_z = \frac{\partial L}{\partial \dot{z}} = m\dot{z} = \text{const} \qquad \text{Linear momentum along } z$$

$\rightarrow$ Only one constant ???

Total number of constants must be independent of coordinates...

Noether's Theorem - I

Coordinate transformation:

$$q_i \rightarrow q_i' = q_i + \delta q_i = q_i + \varepsilon f_i(q,\dot{q}), \quad \varepsilon \text{ parameter}$$

$$L = L(q,\dot{q},t) \rightarrow L' = L(q',\dot{q}',t)$$

$$L(q',\dot{q}',t) = L(q+\delta q,\dot{q}+\delta\dot{q},t) = L\left(q+\varepsilon f(q,\dot{q}),\dot{q}+\varepsilon\dot{f}(q,\dot{q}),t\right)$$

$$L' \simeq L(q,\dot{q},t) + \frac{dL}{d\varepsilon}\varepsilon \qquad \varepsilon \rightarrow 0 \text{ Infinitesimal transformation}$$

$$\frac{dL}{d\varepsilon} = \sum_i \left(\frac{\partial L}{\partial q_i}\frac{\partial q_i}{\partial \varepsilon} + \frac{\partial L}{\partial \dot{q}_i}\frac{\partial \dot{q}_i}{\partial \varepsilon} \right) = \sum_i \left(\frac{\partial L}{\partial q_i} f_i + \frac{\partial L}{\partial \dot{q}_i}\dot{f}_i \right)$$

$$\rightarrow \frac{dL}{d\varepsilon} = \sum_i \left(\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) f_i + \frac{\partial L}{\partial \dot{q}_i}\dot{f}_i \right) = \frac{d}{dt}\left(\sum_i \frac{\partial L}{\partial \dot{q}_i} f_i \right)$$

$$L \text{ invariant } \rightarrow \frac{dL}{d\varepsilon} = 0 \rightarrow \sum_i \frac{\partial L}{\partial \dot{q}_i} f_i = \text{const}$$

Noether's Theorem - II

$$L = \frac{1}{2} m\left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right) - V\left(\sqrt{x^2 + y^2}\right)$$

$$z \text{ ignorable} \rightarrow p_z = \frac{\partial L}{\partial \dot{z}} = m\dot{z} = \text{const} \qquad \text{Linear momentum along } z$$

L: invariant wrt rotations around z

$$\begin{cases} x' = x\cos\varepsilon - y\sin\varepsilon \\ y' = x\sin\varepsilon + y\cos\varepsilon \end{cases} \rightarrow \begin{cases} x' \simeq x - y\varepsilon \\ y' \simeq x\varepsilon + y \end{cases} \text{ Infinitesimal rotation}$$

$$\rightarrow f_x = -y, f_y = +x$$

$$\sum_i \frac{\partial L}{\partial \dot{q}_i} f_i = -m\dot{x}y + m\dot{y}x = m\left(x\dot{y} - y\dot{x}\right) = L_z = \text{const}$$

Classical Particle Dynamics

Symmetry = Irrelevance of … to the dynamics of a closed system

Simmetry	Invariance	Conservation
Frame origin	Space translation	Total *momentum*
Time origin	Time translation	Total *energy*
Frame orientation	Space rotation	Total *angular momentum*
Frame velocity	Lorentz transformation	*CM velocity*

Extension to Fields - I

Take any physical system, *including fields*:
Can describe its motion by the same methods (Lagrangian or Hamiltonian) in terms of a *Principle of Minimum Action*

Then Noether's Theorem states that

For every continuous transformation of the field functions and coordinates which leaves the action unchanged, there is a definite combination of the field functions and their derivatives which is conserved

This is called a *conserved current*

Main point:

Conservation laws must include contributions from the fields, besides particles

Spring 2012 E.Menichetti - Universita' di Torino 8

Extension to Fields - II

$$S = \int dt \sum_{r=1}^{N^3} L(q_r, \dot{q}_r)$$

$$V = L^3, \quad \Delta V = (L/N)^3$$

$$\lim_{\frac{L}{N}\to 0} S$$

many-particle system	$\rightarrow$	field
q_r	$\rightarrow$	$\phi(x)$
numbering by r	$\rightarrow$	numbering by x
$\sum_{r=1}^{N^3} \Delta V$	$\rightarrow$	$\int d^3x$

$$S = \int dt \left[\int d^3x \mathcal{L}\left(\phi(x), \dot{\phi}(x)\right) \right] \equiv \int d^4x \mathcal{L}(\phi(x), \partial_t \phi(x))$$

$$\mathcal{L}(\phi(x), \dot{\phi}(x)) \longrightarrow \mathcal{L}(\phi(x), \partial_\mu \phi(x)) \quad \partial_\mu = (\partial_0, \nabla)$$

Extension to Fields - III

$$\delta \mathcal{L} = \frac{\delta \mathcal{L}}{\delta \phi_r} \delta \phi_r + \frac{\delta \mathcal{L}}{\delta(\partial_\mu \phi_r)} \delta(\partial_\mu \phi_r)$$

$$\delta(\partial_\mu \phi_r) = \partial_\mu \delta \phi_r$$

$$\delta \mathcal{L} = \left[\frac{\delta \mathcal{L}}{\delta \phi_r} - \partial_\mu \frac{\delta \mathcal{L}}{\delta(\partial_\mu \phi_r)} \right] \delta \phi_r + \partial_\mu \left\{ \frac{\delta \mathcal{L}}{\delta(\partial_\mu \phi_r)} \delta \phi_r \right\}$$

$\mathcal{L}$ invariant:

$$J^\mu = \frac{\delta \mathcal{L}}{\delta(\partial_\mu \phi_r)} \delta \phi_r , \quad \partial_\mu J^\mu = 0$$

$$\frac{dQ}{dt} = 0, \quad Q = \int d^3x \, J^0$$

Extension to Fields - IV

As a consequence of Noether's theorem:

Field contribution to mechanical observables:

Energy
Momentum
Angular momentum

Must be included in conservation laws

After field quantization, carried by field quanta (= particles)

Non-mechanical, interaction dependent conservation laws:
Also tied to symmetries

Intrinsically quantum, no classical analogy

Quantum Mechanics - I

For any observable Q:

$$\langle Q\rangle = \langle\psi|Q|\psi\rangle \rightarrow \frac{d\langle Q\rangle}{dt} = \left\langle\frac{\partial\psi}{\partial t}\right|Q|\psi\rangle + \langle\psi|\frac{\partial Q}{\partial t}|\psi\rangle + \langle\psi|Q\left|\frac{\partial\psi}{\partial t}\right\rangle$$

From Schrodinger equation:

$$i\left|\frac{\partial\psi}{\partial t}\right\rangle = H|\psi\rangle, -i\left\langle\frac{\partial\psi}{\partial t}\right| = \langle\psi|H^{\dagger}$$

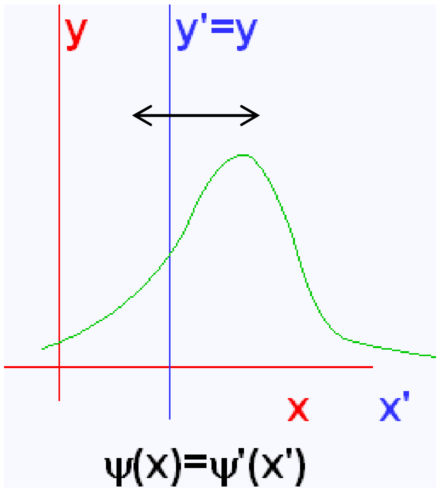
$$\frac{d\langle Q\rangle}{dt} = \left\langle\frac{\partial\psi}{\partial t}\right|Q|\psi\rangle + \langle\psi|\frac{\partial Q}{\partial t}|\psi\rangle + \langle\psi|Q\left|\frac{\partial\psi}{\partial t}\right\rangle$$

$$= i\langle\psi|H^{\dagger}Q|\psi\rangle + \langle\psi|\frac{\partial Q}{\partial t}|\psi\rangle - i\langle\psi|QH|\psi\rangle$$

$$H^{\dagger} = H \rightarrow \frac{d\langle Q\rangle}{dt} = i\langle\psi|\left([H,Q] + \frac{\partial Q}{\partial t}\right)|\psi\rangle \rightarrow \left.\begin{array}{c}[H,Q]=0\\ \dfrac{\partial Q}{\partial t}=0\end{array}\right\} \rightarrow \langle Q\rangle = \text{constant}$$

Quantum Mechanics - II

Coordinate transformation *S* induces a transformation in state (e.g. wave function) space



Just meaning:
One and same state
Different coordinates *x,x'* in S,S'

–> $\psi(x)=\psi'(x')$

<u>*One physical state, two wave functions in S,S'*</u>

Example:

$$S : x \to x' = S(x) = x - a$$

$$S^{-1} : x' \to x = S^{-1}(x') = x' + a$$

$$\psi'(x') = \psi(x) \leftrightarrow \psi'[S(x)] = \psi(x)$$

$$\to \psi'\left[S^{-1}(S(x))\right] = \psi\left(S^{-1}(x)\right)$$

$$\to \psi'(x) = \psi\left[S^{-1}(x)\right] \neq \psi(x)$$

Indeed, for example:

$$\psi(x) = Ne^{-\frac{x^2}{2\sigma^2}}$$

$$\psi'(x') = \psi\left[S^{-1}(x')\right] = Ne^{-\frac{(x'+a)^2}{2\sigma^2}} \left(= Ne^{-\frac{(x-a+a)^2}{2\sigma^2}} = Ne^{-\frac{x^2}{2\sigma^2}} \right)$$

$$\psi'(x') = Ne^{-\frac{(x'+a)^2}{2\sigma^2}} \neq Ne^{-\frac{x'^2}{2\sigma^2}} = \psi(x')$$

Quantum Mechanics - III

Transformation in state space:

$$U : \psi(x) \rightarrow \psi'(x) = \psi\left[S^{-1}(x)\right] = U\left[\psi(x)\right]$$

Unitary transformation

Scalar products must be preserved by U.
Unitary is actually too restrictive:
U *anti-unitary* is also OK (see later)

In term of state vectors:

$|\psi'\rangle = U|\psi\rangle \rightarrow U$ unitary: $U^{\dagger} = U^{-1}$

Take A = Any operator

$\langle\psi|A|\psi\rangle = \langle\psi'|A'|\psi'\rangle$ defining transformed operator

$$\langle\psi'|A'|\psi'\rangle = \langle\psi|U^{\dagger}A'U|\psi\rangle$$

$$\langle\psi|A|\psi\rangle = \langle\psi'|A'|\psi'\rangle = \langle\psi|U^{\dagger}A'U|\psi\rangle \rightarrow A = U^{\dagger}A'U \rightarrow A' = UAU^{\dagger}$$

When A invariant wrt U:

$$A = A' \rightarrow UAU^{\dagger} = A \rightarrow [U, A] = 0$$

$\rightarrow [U, A] = 0$ when U is a symmetry operator for A

By taking A=H: $[U, H] = 0 \rightarrow \langle U \rangle$=const

Summary

Coordinate transformation S (possibly including *internal* coordinates)

Continuous, Discrete

Examples:

Space translation (C)   -   Axis inversion (D)

External, Internal

Examples:

Space Translation (E)   -   Isospin (I)

Corresponding ‘Relativity Principles’

Example:

The laws of physics are the same for all the observers, irrespective of the chosen origin of their reference frames

Symmetry vs Invariance vs Conservation:

S is a symmetry of a physical system $\leftrightarrow [U_S, H] = 0 \quad \rightarrow \langle U_S \rangle = \text{const}$

Continuous vs Discrete Symmetry

Previous examples: *Continuous* symmetry operations

Any given operation S depends on a number of *continuous parameters*
Any given operation S can be thought as *continuosly evolving* from the identity operation

Example: S = Shift by $\boldsymbol{a}$ of the frame origin
Depends on 3 parameters a_x, a_y, a_z

$$S(\mathbf{a}) \underset{a_x, a_y, a_z \to 0}{\longrightarrow} I$$

What are *discrete* symmetry operations?

They do *not* depend on any parameter
They are *intrinsically separated* from the identity operation

Example: S = Axis inversion
No parameter, no continuous evolution from identity

$$x \to -x, \; y \to -y, \; z \to -z$$

Discrete Symmetries

Coordinate transformations not evolving continuously from identity

Very important in Particle Physics:

Space Inversion
Time Reversal
Charge Conjugation

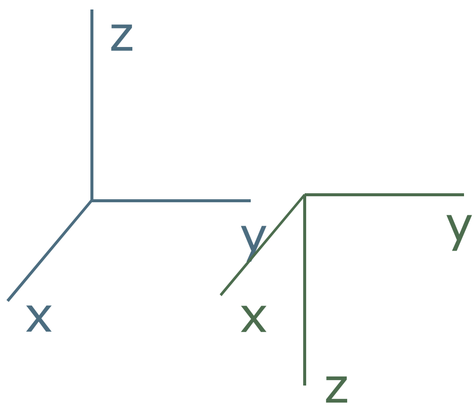
Leading to:

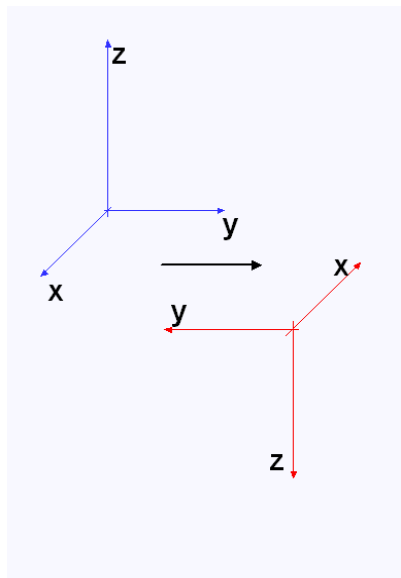
Quantum Numbers
Laws of Conservation
Selection Rules
Constraints on Interactions

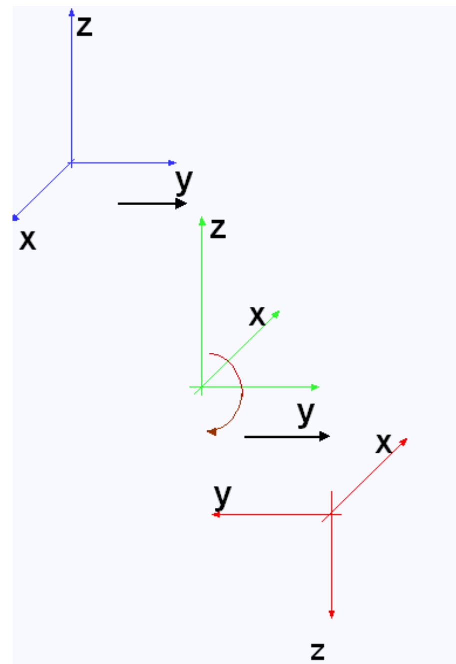
Parity in Classical Physics - I

Two, non equivalent ways of choosing a Cartesian frame in 3D

In 3D: 3 axis inversions equivalent to 1 axis (= mirror) inversion * Rotation







Non equivalent = Not connected by a rotation
Connected by 3 axis inversions

Parity in Classical Physics - II

Define coordinate symmetry operation

$$P : \mathbf{r} \to \mathbf{r}' = -\mathbf{r}$$

Corresponding ‘Relativity Principle’

The laws of physics are the same for all the observers, irrespective of the chosen chirality of their reference frames

Transformations of physical quantities follow:

$\mathbf{r} \to -\mathbf{r}$	position
$t \to t$	time
$\mathbf{p} \to -\mathbf{p}$	3-momentum
$E \to E$	energy
$\mathbf{L} \to \mathbf{L}$	angular momentum

No conservation law!
P not connected to identical transformation

Parity in Classical Physics - III

General taxonomy of physical quantities (observables):

With respect to rotations

With respect to reflections

'Parity' of corresponding Hermitian operator:

	True	Pseudo
Scalar	$+1$	-1
Vector	-1	$+1$
Rank N Tensor	$(-1)^N$	$(-1)^{N+1}$

Parity in Classical Physics - IV

Parity behavior of selected electromagnetic quantities:

$\mathbf{j}(\mathbf{r},t) \to -\mathbf{j}(-\mathbf{r},t)$ current density

$\rho(\mathbf{r},t) \to \rho(-\mathbf{r},t)$ charge density

$\mathbf{E}(\mathbf{r},\mathrm{t}) \to -\mathbf{E}(-\mathbf{r},t)$ electric field

$\mathbf{B}(\mathbf{r},\mathrm{t}) \to \mathbf{B}(-\mathbf{r},t)$ magnetic field

Note

All of this is indeed conventional, as based on our definition of the electric charge as a *scalar*.

What actually matters is that *force* is a polar vector

Then

$$\left.\begin{array}{l} \boldsymbol{F} = q\boldsymbol{E} \\ \boldsymbol{F} = q\boldsymbol{v}\times\boldsymbol{B} \\ \boldsymbol{F},\boldsymbol{v}\ \text{polar} \end{array}\right\} \to \left\{ \begin{array}{l} \text{either}\quad q\ \text{scalar},\ \boldsymbol{E}\ \text{polar},\ \boldsymbol{B}\ \text{axial} \\ \text{or}\ q\ \text{pseudoscalar},\ \boldsymbol{E}\ \text{axial},\ \boldsymbol{B}\ \text{polar} \end{array}\right.$$

Parity in Quantum Mechanics - I

Coordinate transformation as before: $P:\mathbf{r}\rightarrow\mathbf{r}'=P(\mathbf{r})=-\mathbf{r}$

Induced transformation in state space: unitary operator U_P

A) Commutation relations with position, momentum, angular momentum

$$U_P\mathbf{r}|\psi\rangle=U_P\mathbf{r}\sum|\mathbf{r}\rangle\langle\mathbf{r}|\psi\rangle=U_P\sum\mathbf{r}|\mathbf{r}\rangle\langle\mathbf{r}|\psi\rangle=\sum U_P\mathbf{r}|\mathbf{r}\rangle\langle\mathbf{r}|\psi\rangle$$
$$=\sum(-\mathbf{r})|-\mathbf{r}\rangle\langle-\mathbf{r}|\psi\rangle=(-\mathbf{r})U_P|\psi\rangle=-\mathbf{r}U_P|\psi\rangle$$
$$\rightarrow U_P\mathbf{r}U_P^{-1}=-\mathbf{r}\rightarrow U_P\mathbf{r}=-\mathbf{r}U_P$$

$$U_PU_T(\delta\mathbf{r})=U_P(1-\mathbf{p}\cdot\delta\mathbf{r})$$
$$U_P(1-\mathbf{p}\cdot\delta\mathbf{r})=(1+\mathbf{p}\cdot\delta\mathbf{r})U_P$$
$$\rightarrow -U_P\mathbf{p}\cdot\delta\mathbf{r}=\mathbf{p}\cdot\delta\mathbf{r}U_P\rightarrow -U_P\mathbf{p}=\mathbf{p}U_P$$

$$U_P\hat{\mathbf{L}}=\hat{\mathbf{L}}U_P$$

Summary:

U_P anticommutes with ***r,p,*** commutes with ***L***

Parity in Quantum Mechanics - II

B) Action of U_P on states

$U_P : |\psi\rangle \to |\psi'\rangle \equiv U_P|\psi\rangle$

Same physical state described by two different kets in the two frames

Take position eigenstate:

$U_P : |\mathbf{r}\rangle \to |\mathbf{r}'\rangle \equiv U_P|\mathbf{r}\rangle = \eta|-\mathbf{r}\rangle$, η arbitrary phase $= e^{i\alpha}$

Take generic state:

$U_P : |\psi\rangle \to |\psi'\rangle \equiv U_P|\psi\rangle$

Expand into position eigenstates: $|\psi\rangle = \sum_{\boldsymbol{r}} |\boldsymbol{r}\rangle\langle\boldsymbol{r}|\psi\rangle$

$$U_P|\psi\rangle = U_P\sum_{\boldsymbol{r}}|\boldsymbol{r}\rangle\langle\boldsymbol{r}|\psi\rangle = \sum_{\boldsymbol{r}} U_P|\boldsymbol{r}\rangle\langle\boldsymbol{r}|U_P^{-1}U_P\,\psi\rangle = \sum_{\boldsymbol{r}}\left[U_P|\boldsymbol{r}\rangle\langle\boldsymbol{r}|U_P^{-1}\right]U_P\,\psi\rangle$$

$$= \sum_{\boldsymbol{r}'}|\boldsymbol{r}'\rangle\langle\boldsymbol{r}'|U_P\,\psi\rangle = \sum_{\boldsymbol{r}'}|\boldsymbol{r}'\rangle\langle\boldsymbol{r}'|\psi'\rangle = |\psi'\rangle = \sum_{\boldsymbol{r}}|-\boldsymbol{r}\rangle\langle-\boldsymbol{r}|\psi'\rangle$$

$$\to \langle\boldsymbol{r}'|\psi'\rangle = \langle\boldsymbol{r}|\psi\rangle \to \langle\boldsymbol{r}|\psi'\rangle = \langle-\boldsymbol{r}|\psi\rangle$$

Talking wave functions: $\psi'(\boldsymbol{r}) = \psi(-\boldsymbol{r})$ (as shown before)

Would yield similar results by expanding into momentum eigenstates

Parity in Quantum Mechanics - III

C) Fundamental property of parity operator

$P:\mathbf{r}\to\mathbf{r}'=P(\mathbf{r})=-\mathbf{r}, P:\mathbf{r}'\to\mathbf{r}''=P(\mathbf{r}')=-\mathbf{r}'=\mathbf{r}\to P^2=I$

$U_P|\psi\rangle=|\psi'\rangle, U_P|\psi'\rangle=U_P\left(U_P|\psi\rangle\right)=U_P^2|\psi\rangle\to U_P^2=\eta^2 I$

Take η real $\to \eta=\pm 1$

(Can be shown to be always possible)

$$\left.\begin{array}{r} U_P^\dagger=U_P^{-1},\ U_P \text{ unitary} \\ U_P^2=I \end{array}\right\}\to U_P^2=U_PU_P=I=U_PU_P^{-1}=U_PU_P^\dagger\to U_P^\dagger=U_P\quad U_P \text{ Hermitian}$$

$\eta\ \equiv\eta_{\mathrm{P}}$ parity quantum number

$\to U_P$ eigenstates: $U_P|a\rangle=\pm|a\rangle$

Consequences:

$[H,U_P]=0\to$ stationary states have definite η_P- when not degenerate

$[H,U_P]=0\to\eta_P=$ constant of motion

Parity in Quantum Mechanics - IV

Parity for a composite system (just meaning with several degrees of freedom)

$|a\rangle$ compound state of subsystems 1 and 2, parity eigenstate

$U_P^{(1)}, U_P^{(2)}$ parity operators for 1,2

$U_P^{(1)}|a\rangle = \eta_P^{(1)}|a\rangle, U_P^{(2)}|a\rangle = \eta_P^{(2)}|a\rangle$

$U_P^{(2)}\left[U_P^{(1)}|a\rangle\right] = U_P^{(2)}\left[\eta_P^{(1)}|a\rangle\right] = \eta_P^{(2)}\eta_P^{(1)}|a\rangle \rightarrow \eta_P$ *multiplicative* quantum number

Quantum numbers (i.e., conserved quantities) are usually *additive*: ???
Reason: For continuos symmetries the unitary operators are not Hermitian

U unitary $\rightarrow U = e^{iaH} \simeq 1 + iaH$, H Hermitian; $\lim_{a\to 0} U = 1 \rightarrow$ Use infinitesimal generators

Example: Translations

$$U_a^{(1)} \simeq 1 + ia\mathbf{p}^{(1)}, U_a^{(2)} \simeq 1 + ia\mathbf{p}^{(2)} \rightarrow U_a^{(2)}\left(U_a^{(1)}|\psi\rangle\right) \simeq \left(1 + ia\mathbf{p}^{(2)}\right)\left[\left(1 + ia\mathbf{p}^{(1)}\right)|\psi\rangle\right]$$

$$\rightarrow U_a^{(2)}\left(U_a^{(1)}|\psi\rangle\right) \simeq \left(1 + ia\mathbf{p}^{(2)} + ia\mathbf{p}^{(1)}\right)|\psi\rangle = \left[1 + ia\left(\mathbf{p}^{(2)} + \mathbf{p}^{(1)}\right)\right]|\psi\rangle$$

U Hermitian$\rightarrow$ Constant of motion = *U* $\rightarrow$ Multiplicative
U not Hermitian$\rightarrow$ Constant of motion ~ “Logarithm” of *U*…$\rightarrow$ Additive

The Parity Quantum Number

As a quantum number, parity may or may not be conserved

Experimental fact:

All interactions, except weak interaction, do conserve parity

To the extent we can neglect weak interaction, all *stationary, non degenerate* states are parity eigenstates

Scattering states: similar but not identical to stationary states
Being ~ momentum eigenstates $[\boldsymbol{p}, U_P] \neq 0 \rightarrow$ Parity not defined

Stable particles and resonances: bound states
Angular momentum eigenstates $[\boldsymbol{L}, U_P] = 0 \quad \rightarrow$ Parity defined

Orbital vs Intrinsic Parity

Parity quantum number introduced by making reference to orbital motion.

For angular momentum eigenstates, take position representation:

$$\psi(\mathbf{r}) = Nf(r)Y_l^m(\theta,\varphi)$$

$$P:\mathbf{r}\to-\mathbf{r}\Rightarrow Y_l^m(\theta,\varphi)\to Y_l^m(\theta-\pi,\varphi+\pi)=(-1)^l\,Y_l^m(\theta,\varphi)$$

Therefore

$$\eta_P^{(orb)}=(-1)^l$$

This is known as *orbital parity*.
Orbital parity, like angular momentum, is *frame dependent*

Arbitrary, extra phase factor: *State intrinsic parity*

Not relevant in non-relativistic regime:
Cannot be measured in non-relativistic processes, where particle numbers never change

Intrinsic Parity - I

Orbital parity not sufficient to deal with relativistic processes

Reason: *Particles are created and annihilated*

Best clarified first by a non relativistic example

Take a reaction between 2 nuclei, ignore nucleon spin

$$A + B \rightarrow C + D, \quad A, B \neq C, D$$

Angular momentum conservation:

$$\mathbf{L}_{TOT}^{(in)} = \mathbf{L}_A^{(CM-A)} + \mathbf{L}_B^{(CM-B)} + \mathbf{L}_{AB}^{(CM-AB)}$$

$$P_{TOT}^{in} = (-1)^{L_{TOT}^{(in)}} = \underbrace{(-1)^{L_A^{(CM-A)}}}_{P_A}\underbrace{(-1)^{L_B^{(CM-B)}}}_{P_B}\underbrace{(-1)^{L_{AB}^{(CM-AB)}}}_{P_{ORB}}$$

L_A, L_B originated by internal nuclear motion
L_{AB} orbital motion

$$\mathbf{L}_{TOT}^{(out)} = \mathbf{L}_C^{(CM-C)} + \mathbf{L}_D^{(CM-D)} + \mathbf{L}_{CD}^{(CM-CD)}$$

$$P_{TOT}^{out} = (-1)^{L_{TOT}^{(out)}} = \underbrace{(-1)^{L_C^{(CM-C)}}}_{P_C}\underbrace{(-1)^{L_B^{(CM-D)}}}_{P_D}\underbrace{(-1)^{L_{CD}^{(CM-CD)}}}_{P_{ORB}}$$

Same for final state

If we don't know *A,B,C,D* are composite systems, 'intrinsic' nuclear parity cannot be ignored, or the process would violate parity whenever L_{CD} # L_{AB}

Intrinsic Parity - II

Constituent intrinsic parity not originating from internal structure
Just a drop of QFT to show the origin of intrinsic parity

1) One component, <u>Boson</u> field $\varphi(\boldsymbol{r},t)$

Expand $\phi(\boldsymbol{r})$ into creation+annihilation operators

$$\varphi(\boldsymbol{r},t)=\sum_{\boldsymbol{p}}\frac{1}{\sqrt{2EV}}\left[a_{\boldsymbol{p}}e^{-ip\cdot r}+a_{\boldsymbol{p}}^{\dagger}e^{+ip\cdot r}\right],\quad p\cdot r=(E,\boldsymbol{p})\cdot(t,\boldsymbol{r})=Et-\boldsymbol{p}\cdot\boldsymbol{r}$$

Apply parity operator:

$$U_P\varphi U_P^{-1}=\eta_P\varphi(-\boldsymbol{r},t)=\sum_{\boldsymbol{p}}\frac{1}{\sqrt{2EV}}\left[U_Pa_{\boldsymbol{p}}U_P^{-1}e^{-i(Et-\boldsymbol{p}\cdot\boldsymbol{r})}+U_Pa_{\boldsymbol{p}}^{\dagger}U_P^{-1}e^{+i(Et-\boldsymbol{p}\cdot\boldsymbol{r})}\right]$$

η_P: Intrinsic parity of ϕ operator

Expand $\phi(-\boldsymbol{r})$ into creation+annihilation operators

$$\eta_P\varphi(-\boldsymbol{r},t)=\eta_P\sum_{\boldsymbol{p}}\frac{1}{\sqrt{2EV}}\left[a_{\boldsymbol{p}}e^{-i(Et-\boldsymbol{p}\cdot(-\boldsymbol{r}))}+a_{\boldsymbol{p}}^{\dagger}e^{+i(Et-\boldsymbol{p}\cdot(-\boldsymbol{r}))}\right]$$

$$=\sum_{\boldsymbol{p}}\frac{1}{\sqrt{2EV}}\left[\eta_Pa_{-\boldsymbol{p}}e^{-i(Et-(-\boldsymbol{p})\cdot\boldsymbol{r})}+\eta_Pa_{-\boldsymbol{p}}^{\dagger}e^{+i(Et-(-\boldsymbol{p})\cdot\boldsymbol{r})}\right]\rightarrow\begin{cases}U_Pa_{\boldsymbol{p}}U_P^{-1}=\eta_Pa_{-\boldsymbol{p}}\\U_Pa_{\boldsymbol{p}}^{\dagger}U_P^{-1}=\eta_Pa_{-\boldsymbol{p}}^{\dagger}\end{cases}$$

$$\rightarrow U_P\left|\boldsymbol{p}\right\rangle=U_Pa_{\boldsymbol{p}}^{\dagger}U_P^{-1}U_P\left|0\right\rangle=\eta_Pa_{-\boldsymbol{p}}^{\dagger}\left|0\right\rangle=\eta_P\left|-\boldsymbol{p}\right\rangle;U_P\left|\boldsymbol{p}=0\right\rangle=\eta_P\left|\boldsymbol{p}=0\right\rangle$$

Different states

Same state

Intrinsic Parity - III

Non-Hermitian field:

$$\varphi^\dagger(\boldsymbol{r},t)=\sum_{\boldsymbol{p}}\frac{1}{\sqrt{2EV}}\left[b_{\boldsymbol{p}}e^{-ip\cdot r}+b_{\boldsymbol{p}}^\dagger e^{+ip\cdot r}\right]$$

$$\rightarrow\begin{cases}U_P\varphi^\dagger U_P^{-1}=\eta_P^*\varphi^\dagger(-\boldsymbol{r},t)\\ U_P\varphi^\dagger U_P^{-1}=\sum_{\boldsymbol{p}}\frac{1}{\sqrt{2EV}}\left[U_P b_{\boldsymbol{p}}U_P^{-1}e^{-i(Et-\boldsymbol{p}\cdot\boldsymbol{r})}+U_P b_{\boldsymbol{p}}^\dagger U_P^{-1}e^{+i(Et-\boldsymbol{p}\cdot\boldsymbol{r})}\right]\end{cases}$$

$$\rightarrow\eta_P^*\sum_{\boldsymbol{p}}\frac{1}{\sqrt{2EV}}\left[b_{\boldsymbol{p}}e^{-i(Et-\boldsymbol{p}\cdot(-\boldsymbol{r}))}+b_{\boldsymbol{p}}^\dagger e^{+i(Et-\boldsymbol{p}\cdot(-\boldsymbol{r}))}\right]=\sum_{\boldsymbol{p}}\frac{1}{\sqrt{2EV}}\left[\eta_P b_{-\boldsymbol{p}}e^{-i(Et-(-\boldsymbol{p})\cdot\boldsymbol{r})}+\eta_P b_{-\boldsymbol{p}}^\dagger e^{+i(Et-(-\boldsymbol{p})\cdot\boldsymbol{r})}\right]$$

$$\rightarrow\begin{cases}U_P b_{\boldsymbol{p}}U_P^{-1}=\eta_P^* b_{-\boldsymbol{p}}\\ U_P b_{\boldsymbol{p}}^\dagger U_P^{-1}=\eta_P^* b_{-\boldsymbol{p}}^\dagger\end{cases}$$

$$\rightarrow U_P\left|\boldsymbol{p}\right\rangle=U_P b_{\boldsymbol{p}}^\dagger U_P^{-1}U_P\left|0\right\rangle=\eta_P^* b_{-\boldsymbol{p}}^\dagger\left|0\right\rangle=\eta_P^*\left|-\boldsymbol{p}\right\rangle;U_P\left|\boldsymbol{p}=0\right\rangle=\eta_P^*\left|\boldsymbol{p}=0\right\rangle$$

Therefore, for boson particle-antiparticle:

$$\rightarrow\eta_b\eta_{\bar{b}}=\eta_P\eta_P^*=1$$

Intrinsic Parity - IV

Intrinsic parity of a Dirac particle:

$$(\boldsymbol{\alpha}\cdot\mathbf{p}+\beta m)\psi = E\psi \underset{P}{\rightarrow} (\boldsymbol{\alpha}\cdot(-\mathbf{p})+\beta m)\psi_P = E\psi_P$$

$$\beta(\boldsymbol{\alpha}\cdot(-\mathbf{p})+\beta m)\psi_P = E\beta\psi_P$$

$$\beta\boldsymbol{\alpha} = -\boldsymbol{\alpha}\beta \rightarrow ((-\boldsymbol{\alpha})\cdot(-\mathbf{p})+\beta m)\beta\psi_P = E\beta\psi_P$$

$\rightarrow (\boldsymbol{\alpha}\cdot\mathbf{p}+\beta m)\beta\psi_P = E\beta\psi_P \rightarrow \beta\psi_P$ satisfies Dirac eq.

$$\beta = \gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$

Plane wave solutions:

$$\psi_1(\mathbf{r},t) = \begin{pmatrix} \varphi \\ \dfrac{\boldsymbol{\sigma}\cdot\mathbf{p}}{E+m}\varphi \end{pmatrix} e^{-i(Et-\mathbf{p}\cdot\mathbf{r})}, \psi_2(\mathbf{r},t) = \begin{pmatrix} \chi \\ \dfrac{\boldsymbol{\sigma}\cdot\mathbf{p}}{E+m}\chi \end{pmatrix} e^{-i(Et-\mathbf{p}\cdot\mathbf{r})},$$

$$\psi_3(\mathbf{r},t) = \begin{pmatrix} -\dfrac{\boldsymbol{\sigma}\cdot\mathbf{p}}{E+m}\varphi \\ \varphi \end{pmatrix} e^{-i(-Et-\mathbf{p}\cdot\mathbf{r})}, \psi_4(\mathbf{r},t) = \begin{pmatrix} -\dfrac{\boldsymbol{\sigma}\cdot\mathbf{p}}{E+m}\chi \\ \chi \end{pmatrix} e^{-i(-Et-\mathbf{p}\cdot\mathbf{r})}$$

Intrinsic Parity - V

Rest frame: $\mathbf{p} = 0,\ E = m$

$$\psi_1 = \begin{pmatrix} \varphi \\ 0 \end{pmatrix} e^{-imt} \to \gamma^0 \begin{pmatrix} \varphi \\ 0 \end{pmatrix} e^{-imt} = \begin{pmatrix} \varphi \\ 0 \end{pmatrix} e^{-imt} = \psi_1$$

$$\psi_2 = \begin{pmatrix} \chi \\ 0 \end{pmatrix} e^{-imt} \to \gamma^0 \begin{pmatrix} \chi \\ 0 \end{pmatrix} e^{-imt} = \begin{pmatrix} \chi \\ 0 \end{pmatrix} e^{-imt} = \psi_2$$

$$\psi_3 = \begin{pmatrix} 0 \\ \varphi \end{pmatrix} e^{-i(-m)t} \to \gamma^0 \begin{pmatrix} 0 \\ \varphi \end{pmatrix} e^{-i(-m)t} = \begin{pmatrix} 0 \\ -\varphi \end{pmatrix} e^{-i(-m)t} = -\psi_3$$

$$\psi_4 = \begin{pmatrix} 0 \\ \chi \end{pmatrix} e^{-i(-m)t} \to \gamma^0 \begin{pmatrix} 0 \\ \chi \end{pmatrix} e^{-i(-m)t} = \begin{pmatrix} 0 \\ -\chi \end{pmatrix} e^{-i(-m)t} = -\psi_4$$

$\to$ Generic frame:

$$\begin{cases} U_P \psi(\boldsymbol{r}, t) U_P^{-1} = \eta_P \gamma^0 \psi(-\boldsymbol{r}, t) \\ U_P \overline{\psi}(\boldsymbol{r}, t) U_P^{-1} = \eta_P^* \overline{\psi}(-\boldsymbol{r}, t) \gamma^0 \end{cases}$$

Therefore, for fermion particle-antiparticle:

$$\to \eta_f \eta_{\bar{f}} = \eta_P \left(-\eta_P^*\right) = -1$$

Intrinsic Parity - VI

Absolute intrinsic parity *not defined* for most particle states

Baryon particles: Baryonic number exactly conserved by all interactions
Charged particles: Electric charge as above

→*Superselection rules*, limiting the quantum Superposition Principle:

Cannot superpose states with different values of exactly conserved quantum numbers

(Also true for integer/half integer spin; *almost* true for other quantum numbers as well, *by neglecting weak interactions*: Strangeness, Charm, Leptonic family numbers, ..)

Then phase factor unconstrained, can be fixed by convention

→ Take it *real*

Intrinsic Parity - VII

$\rightarrow U_P^2 = I$ for bosons $\rightarrow \eta_P = \pm 1 = \eta_P^*$

$\rightarrow$ Boson, Antiboson : Same intrinsic parity

Can't take $U_P^2 = I$ for fermions:

Funny behavior of fermion states under rotations

$$U_R(\hat{\boldsymbol{n}},\theta)|\psi\rangle = e^{i\boldsymbol{J}\cdot\hat{\boldsymbol{n}}\theta}|\psi\rangle \rightarrow U_R(\hat{z},2\pi)|\psi\rangle = e^{iJ_z 2\pi}|\psi\rangle = e^{i\frac{1}{2}2\pi}|\psi\rangle = -|\psi\rangle$$

$(\leftarrow$ Compare integer/half integer superselection rule$)$

$$\left.\begin{array}{l} I = U_R(\hat{z},4\pi) = U_R^2(\hat{z},2\pi) \\ U_P^2 = U_R(\hat{z},2\pi) \end{array}\right\} \rightarrow I = U_P^4 \text{ for Fermions} \rightarrow \eta_P = \pm 1, \pm i$$

Funny result! Not relevant in the Standard Model (Maybe *relevant* beyond..)

Take $\eta_P = \pm 1$ by convention

$\rightarrow$ Fermion,Antifermion: Opposite intrinsic parity

General convention for Leptons & Quarks

+ve parity particles	*-ve parity antiparticles*
Quarks	Antiquarks
Leptons	Antileptons

Intrinsic Parity - VIII

Observe:

φ Hermitian $\rightarrow \varphi = \varphi^\dagger \rightarrow \eta_P^* = \eta_P \rightarrow \eta_P$ real $\rightarrow \eta_P = \pm 1$

True for *really neutral* particles (*Photon, π^0, J/ψ*, …):

All additive quantum numbers = 0, like vacuum state

(Charge, Baryonic/Leptonic number, Strangeness, …)

Indeed, such particles can be singly emitted and absorbed without violating any conservation law

Absolute parity *is* defined and measurable upon defining vacuum state parity (usually= +*1*)

Photon - I

Classical EM field

Standard convention:

$$\left.\begin{aligned} &P:\mathbf{E}\to-\mathbf{E}\\ &P:\mathbf{B}\to\mathbf{B}\\ &\mathbf{E}=-\frac{\partial\mathbf{A}}{\partial t}-\nabla\varphi\\ &\mathbf{B}=\nabla\times\mathbf{A}\end{aligned}\right\}\to\begin{cases}P:\mathbf{A}\to-\mathbf{A}\\ P:\varphi\to\varphi\end{cases}$$

A: Vector field

ϕ: Not needed for radiation field

Quantum EM field

A: Vector field

Photon spin = 1

$\to$ *Negative intrinsic parity*

Interesting question: EM interactions conserve parity
$\to$ *Photons from reactions, decays should have defined, total parity*

Photon - II

What is the photon parity in a given state?
Can expand any ***A*** into creation & annihilation operators

$$\mathbf{A}(\mathbf{r},t) = \sum_{\mathbf{k}} \sum_{i=1}^{2} \frac{1}{\sqrt{|\mathbf{k}|V}} \left[a_i(\mathbf{k}) \boldsymbol{\varepsilon}_i e^{i(\mathbf{p}_\gamma \bullet \mathbf{x} - E_\gamma t)} + a_i^\dagger(\mathbf{k}) \boldsymbol{\varepsilon}_i^* e^{-i(\mathbf{p}_\gamma \bullet \mathbf{x} - E_\gamma t)} \right]$$

But: $a_i(\boldsymbol{k})$ do not commute with parity. Expand instead into J,m C&A operators

$$\mathbf{A}(\mathbf{r},t) = \sum_{J,m} \sum_{L=J-1}^{J+1} \sum_{i=1}^{2} \frac{1}{\sqrt{|\mathbf{k}|V}} \left[a_i(J,L,m) \boldsymbol{\varepsilon}_i Y_{Jm}^{L}(\theta,\varphi) + a_i^\dagger(J,L,m) \boldsymbol{\varepsilon}_i^* Y_{Jm}^{L\,*}(\theta,\varphi) \right]$$

Photon states with defined total parity: states with given J,L,m.
Indeed:

$$\boldsymbol{J} = \boldsymbol{L} + \underbrace{\boldsymbol{S}}_{=1} \rightarrow J = L, L \pm 1 \rightarrow L = J, J \pm 1$$

$$\rightarrow \eta_P = (-1)^L(-1) = (-1)^{L+1} = (-1)^J, (-1)^{J\pm1}$$

$$\rightarrow \begin{cases} \eta_P = (-1)^J & \text{Electric} \\ \eta_P = (-1)^{J\pm1} & \text{Magnetic} \end{cases}$$

2 kinds of photons for any given J

Scalars

Really neutral states decaying into two photons

Example: π^0, $spin = 0$

$\pi^0 \to \gamma\gamma$

Parity conserved by EM interaction

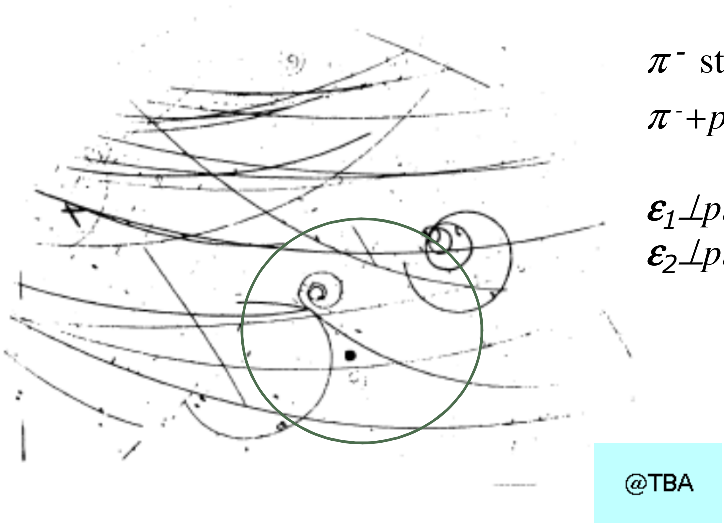
Final state: 2 creation operators $\propto \boldsymbol{\varepsilon_1}, \boldsymbol{\varepsilon_2}$

Can determine parity by looking at distribution of the angle between $\boldsymbol{\varepsilon_1}, \boldsymbol{\varepsilon_2}$

Observe: Double Dalitz decays $\pi^0 \to e^+e^-e^+e^-$

Pseudoscalar $\sin^2\varphi$

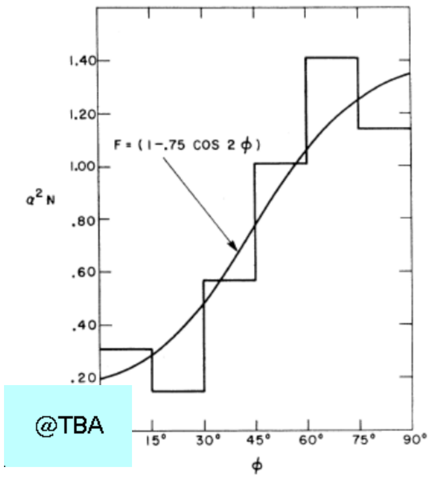
Scalar $\cos^2\varphi$



π^- stopping in liq. H_2

$\pi^- + p \to \pi^0 + n \to e^+e^-e^+e^- n$

$\boldsymbol{\varepsilon_1} \perp plane_1(\boldsymbol{p_+}, \boldsymbol{p_-})$
$\boldsymbol{\varepsilon_2} \perp plane_2(\boldsymbol{p_+}, \boldsymbol{p_-})$



Vectors

Most interesting result:
Yang Theorem

A vector (s=1) state cannot decay into 2 photons

Consequence of:

Angular momentum conservation
Parity conservation
Bose symmetry for identical photons
Gauge invariance of QED (transversality of photons)

Examples:

$\rho, \omega, \phi, J/\psi, \Upsilon, Z^0, ..$

Mesons

Examples

Charged Pion

$\pi^- + d \to n + n$ at low energy

$\eta_P^{\pi^-} \left(\eta_P^N\right)^2 (-1)^L = \left(\eta_P^N\right)^2 (-1)^{L'}$

Initial state: $L = 0 \to J = 0 \oplus 1 = 1$

Final state: $J' = 1$, two identical fermions

$L = 0(symm) \to S = 1(symm) \to Symm$ KO

$L = 1(antisymm)$

$\to S = \begin{cases} 0(antisymm) \to Symm & \text{KO} \\ 1(symm) \to Antisymm & \text{OK} \end{cases}$

$\to$ P-wave,triplet $\to L' = 1$

$\to \eta_P^{\pi^-} (-1)^0 = (-1)^1 \to \eta_P^{\pi^-} = -1$

by taking $\eta_P^n = \eta_P^p = +1$ by convention

Charged Kaon

Hypernucleus
One neutron$\to\Lambda^0$

$K^- + {}^4He \to \pi^0 + {}^4_\Lambda H$ at low energy

$\eta_P^{K^-} \eta_P^{{}^4He} (-1)^L = \eta_P^{\pi^0} \eta_P^{{}^4_\Lambda H} (-1)^{L'}$

$= (+1)^4 (-1)^0 = +1$
Ground state $L=0$

Initial state: $L = 0, S_{K^-} = 0 \to J = 0$

Final state: $J' = 0,\ S_{{}^4_\Lambda H} = 0 \to L' = 0$

$\to \eta_P^{K^-} \cdot 1 \cdot (-1)^0 = (-1) \cdot 1 (-1)^0 \to \eta_P^{K^-} = -1$

by taking $\eta_P^{\Lambda^0} = \eta_P^p = +1$ by convention

Two-Particle States

Fermion-Antifermion: CM frame

$$\eta_P^{\bar{f}f} = \underset{\text{intrinsic}}{(-1)}\underset{\text{orbital}}{(-1)^l} = (-1)^{l+1}$$

Boson-Antiboson: CM frame

$$\eta_P^{\bar{b}b} = \underset{\text{intrinsic}}{(\eta_P)^2}\underset{\text{orbital}}{(-1)^l} = (-1)^l$$

Photon-photon: CM frame

Take single photon states with defined helicity:

$U_P\left|\mathbf{k},R;-\mathbf{k},R\right\rangle = \left|-\mathbf{k},L;\mathbf{k},L\right\rangle$ not a U_P eigenstate

$U_P\left|\mathbf{k},L;-\mathbf{k},L\right\rangle = \left|-\mathbf{k},R;\mathbf{k},R\right\rangle$ not a U_P eigenstate

$U_P\left|\mathbf{k},R;-\mathbf{k},L\right\rangle = \left|-\mathbf{k},L;\mathbf{k},R\right\rangle \equiv \left|\mathbf{k},R;-\mathbf{k},L\right\rangle$ U_P eigenstate, $\eta_P = +1,\ J_3 = +2$

$U_P\left|\mathbf{k},L;-\mathbf{k},R\right\rangle = \left|-\mathbf{k},R;\mathbf{k},L\right\rangle \equiv \left|\mathbf{k},L;-\mathbf{k},R\right\rangle$ U_P eigenstate, $\eta_P = +1\ \ J_3 = -2$

$\left|\mathbf{k},R;-\mathbf{k},R\right\rangle \pm \left|\mathbf{k},L;-\mathbf{k},L\right\rangle$ U_P eigenstates, $\eta_P = \pm 1, J_3 = 0$

Charge Conjugation - I

Charge conjugation operation: similar to parity
But: Operating on 'internal', rather than spatial, coordinates

Change sign of all additive quantum numbers $Q, B, S, C, ...$

Like parity, unitary transformation in state space:

$$C : |\psi\rangle \equiv |\alpha; Q, B, S, C, ..\rangle \rightarrow |\psi'\rangle = U_C |\psi\rangle \equiv \eta_C |\alpha; -Q, -B, -S, -C, ..\rangle$$

[a: All other quantum numbers, like energy, position, momentum, angular momentum, …]

Corresponding 'Relativity Principle' (<u>to be checked by experiment</u>)

The laws of physics are the same for all the observers, irrespective of the chosen positive direction of their 'charge axis'

Experimental fact:
All interactions, <u>except weak interaction</u>, do conserve charge parity

Charge Conjugation - II

Commutation relations with observables

U_C commutes with all observables, except those yielding additive quantum numbers :

$$[U_C, \boldsymbol{r}] = [U_C, \boldsymbol{p}] = [U_C, \boldsymbol{L}] = 0$$
$$U_C Q = -Q U_C, U_C B = -B U_C, U_C S = -S U_C, \ldots$$

Action on states

$$U_C : |Q\rangle \to |Q'\rangle \equiv U_C |Q\rangle = \eta_C |-Q\rangle, \ \eta_C \text{ arbitrary phase } = e^{i\alpha}$$

Charge Conjugation - III

C. Fundamental property of charge conjugation operator

$$C: Q \to Q' = C(Q) = -Q, C: Q' \to Q'' = C(Q') = -Q' = Q \to C^2 = I$$

$$U_C|\psi\rangle = |\psi'\rangle, U_C|\psi'\rangle = U_C(U_C|\psi\rangle) = U_C^2|\psi\rangle \to U_C^2 = \eta^2 I$$

$$\left.\begin{array}{r} U_C^\dagger = U_C^{-1},\ U_C \text{ unitary} \\ U_C^2 = I \end{array}\right\} \to U_C^2 = U_C U_C = I = U_C U_C^{-1} = U_C U_C^\dagger \to U_C^\dagger = U_C$$

$\to U_C$ hermitian

$\to U_P$ eigenvalues are real

$\to \eta = e^{i\alpha} = \pm 1 \equiv \eta_C$ charge parity quantum number

$\to U_C$ eigenstates: $U_C|a\rangle = \pm|a\rangle$

Very few particles are U_C eigenstates:
As before, must have

$Q=B=S=C=\ldots=0$

Applications - I

$$j_\mu = (\rho, \mathbf{j}) \underset{C}{\rightarrow} (-\rho, -\mathbf{j}) = -j_\mu$$

$$A_\mu = (\varphi, \mathbf{A}) \underset{C}{\rightarrow} -(\varphi, \mathbf{A}) = -A_\mu \rightarrow U_C |\gamma\rangle = (-1)|\gamma\rangle$$

$$j^\mu A_\mu \underset{C}{\rightarrow} j^\mu A_\mu$$

$$U_C |n\gamma\rangle = (-1)^n |n\gamma\rangle$$

Boson-antiboson: CM frame

$$U_C |b\bar{b}\rangle = U_P |b\bar{b}\rangle = (-1)^l |b\bar{b}\rangle \rightarrow \eta_C = (-1)^l$$

Fermion-antifermion: CM frame

$U_C = U_P U_S$ Parity * Spin exchange

$$U_P |f\,\bar{f}\rangle = (-1)^{l+1} |f\bar{f}\rangle$$

$$U_S |\uparrow\downarrow - \downarrow\uparrow\rangle = |\downarrow\uparrow - \uparrow\downarrow\rangle = -|\uparrow\downarrow - \downarrow\uparrow\rangle,\; U_S |\uparrow\downarrow + \downarrow\uparrow\rangle = |\downarrow\uparrow + \uparrow\downarrow\rangle = |\uparrow\downarrow + \downarrow\uparrow\rangle,$$

$$U_S |\uparrow\uparrow\rangle = |\uparrow\uparrow\rangle, U_S |\downarrow\downarrow\rangle = |\downarrow\downarrow\rangle$$

$$\rightarrow U_C |f\,\bar{f}\rangle = (-1)^{l+s} |f\bar{f}\rangle$$

Applications - II

$\pi^0, \eta, \ldots: \quad \eta_C = (-1)^2 = +1 \quad$ 2 photons decays

States $f\,\bar{f}$ decaying to n photons:

$$C : (-1)^{l+s} \rightarrow \; S \text{ state} \begin{cases} \eta_C = +1 \text{ singlet} \\ \eta_C = -1 \text{ triplet} \end{cases}$$

$$n \text{ photons: } \eta_C = (-1)^l \rightarrow \begin{cases} 2 \text{ photons } \eta_C = +1 \\ 3 \text{ photons } \eta_C = -1 \end{cases}$$

$$\rightarrow \begin{cases} \text{singlet} \rightarrow 2 \text{ photons} \\ \text{triplet} \rightarrow 3 \text{ photons} \end{cases}$$

Meson: Fermion-Antifermion bound state

L	S=0	S=1
0	0^{-+}	1^{--}
1	1^{+-}	0^{++}, 1^{++}, 2^{++}
2	2^{-+}	1^{--}, 2^{--}, 3^{--}

States decaying to 2 pions

J	PC = ++	PC = +-,-+,--
0	OK	KO
1	KO	KO,KO,OK
2	OK	KO

Time Reversal

Need to make clear at the outset:

These considerations are relevant to physical systems with a (very) small number of degrees of freedom

Do not apply to complex systems, whose time evolution is driven by the II Law of thermodynamics. For them, a *time arrow* can be defined with no ambiguities. Or so we believe… (A *very* hard subject)

Can state another 'relativity principle', about the choice of the positive direction of the time coordinate of physical events:

The laws of physics are the same for all the observers,
irrespective of the chosen positive direction of their 'time axis'

Experimental fact:

All interactions, <u>except weak interaction</u>, are invariant wrt to time reversal

Classical Physics

General behavior of physical quantities wrt to time reversal:

$$t \to -t:$$

$$\mathbf{r} \to \mathbf{r}$$

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} \to \frac{d\mathbf{r}}{d(-t)} = -\mathbf{v}, \mathbf{p} \to -\mathbf{p}$$

$$\mathbf{a} = \frac{d^2\mathbf{r}}{dt^2} \to \mathbf{a}, \mathbf{F} \to \mathbf{F}$$

$$\mathbf{L} = \mathbf{r} \times \mathbf{p} \to -\mathbf{L}$$

$$E \to E$$

$$\rho \to \rho$$

$$\mathbf{j} = \rho\mathbf{v} \to -\mathbf{j}$$

$$\varphi \to \varphi$$

$$\mathbf{A} \to -\mathbf{A}$$

$$\mathbf{E} \to \mathbf{E}$$

$$\mathbf{B} \to -\mathbf{B}$$

Observe: Ohm's law is *not* TR invariant

$\mathbf{j} = \sigma\mathbf{E} \to -\mathbf{j} = \sigma\mathbf{E}$, take σ as TR invariant

Example of a *macroscopic* law…

Schrodinger Equation

$T: t \rightarrow t' = -t$

First guess...

$\langle t' | \psi' \rangle = \langle t | \psi \rangle = \langle -t' | \psi \rangle$

$\langle t | \psi' \rangle = \langle -t | \psi \rangle$

$\rightarrow \psi'(t) = \psi(-t)$

$U_T : |\psi\rangle \rightarrow |\psi'\rangle \equiv U_T |\psi\rangle$

$U_T : |t\rangle \rightarrow |t'\rangle \equiv U_T |t\rangle = \eta |-t\rangle$ Wrong!

Indeed:

$$H|\psi(t)\rangle = i\frac{\partial|\psi(t)\rangle}{\partial t} \underset{U_T}{\longrightarrow} H|\psi(-t)\rangle = -i\frac{\partial|\psi(-t)\rangle}{\partial t}$$

Redefine:

$$\left.\begin{array}{l} T: t \rightarrow -t \\ K: i \rightarrow -i \end{array}\right\}, \; U_T = KT$$

$|\psi(t)\rangle \rightarrow U_T|\psi(t)\rangle = |\psi^T(t)\rangle = |\psi^*(-t)\rangle$ OK

Time Reversal - I

Take a plane wave:

$$e^{-i(Et-\mathbf{p}\cdot\mathbf{r})} \underset{U_T}{\longrightarrow} e^{+i(E(-t)-\mathbf{p}\cdot\mathbf{r})} = e^{-i(Et+\mathbf{p}\cdot\mathbf{r})} = e^{-i(Et-(-\mathbf{p})\cdot\mathbf{r})}$$

Quite natural...:

U_T sends a progressive plane wave

into a regressive one

Puzzled? Think of it in this way:

U_T inverts the positive direction of time for the 2nd observer. Therefore, the 2nd observer has a clock which is running *backwards*.

Then the same wave which is seen as *progressive* by the first observer, is seen as *regressive* by the second.

Rather simple for spinless particles..

Time Reversal - II

Take a particle with *s=1/2*:

$$\begin{pmatrix}\psi_+(t)\\ \psi_-(t)\end{pmatrix} \to \begin{pmatrix}\psi_+^T(t)\\ \psi_-^T(t)\end{pmatrix} = M\begin{pmatrix}\psi_+^*(-t)\\ \psi_-^*(-t)\end{pmatrix}$$

Now, must take into account that **s** is *reversed* by T

$$\sigma_y = \begin{pmatrix}0 & -i\\ +i & 0\end{pmatrix} \to M = -i\sigma_y = \begin{pmatrix}0 & -1\\ +1 & 0\end{pmatrix}$$

Spin-reversing operator

$$\begin{pmatrix}\psi_+(t)\\ \psi_-(t)\end{pmatrix} \underset{U_T}{\to} \begin{pmatrix}\psi_+^T(t)\\ \psi_-^T(t)\end{pmatrix} = -i\sigma_y\begin{pmatrix}\psi_+^*(-t)\\ \psi_-^*(-t)\end{pmatrix} = \begin{pmatrix}-\psi_-^*(-t)\\ +\psi_+^*(-t)\end{pmatrix} \quad \text{OK}$$

Apply U_T a second time:

$$\begin{pmatrix}\psi_+^T(t)\\ \psi_-^T(t)\end{pmatrix} \underset{U_T}{\to} -i\sigma_y\begin{pmatrix}-\psi_-^*(t)\\ +\psi_+^*(t)\end{pmatrix}^* = \begin{pmatrix}-\psi_+(t)\\ -\psi_-(t)\end{pmatrix} = -\begin{pmatrix}\psi_+(t)\\ \psi_-(t)\end{pmatrix} \quad \text{Similar to parity for spin 1/2}$$

Applications

Basic remark:

U_T is *not* a unitary operator
U_T is *not* a Hermitian operator

Therefore:
No real eigenvalues
No (conserved) observables
$\rightarrow$*No 'time parity'*

Nevertheless:

Quite useful to establish properties of fundamental interactions

Because is not linear, rather it is antilinear

$$U_T\left(a|\psi_1\rangle + b|\psi_2\rangle\right) = a^* U_T|\psi_1\rangle + b^* U_T|\psi_2\rangle$$

Complex conjugation

$$K : \alpha z \rightarrow (\alpha z)^* = \alpha^* z^*$$

is a non-linear operation.

Proof:

$$\alpha = ae^{i\varphi}, z = \rho e^{i\psi} \rightarrow \alpha^* = ae^{-i\varphi}, z^* = \rho e^{-i\psi}$$

$$\alpha z = ae^{i\varphi}\rho e^{i\psi} = a\rho e^{i(\varphi+\psi)} \rightarrow (\alpha z)^* = a\rho e^{-i(\varphi+\psi)} = \alpha^* z^*$$

But: $\alpha^* z^* \neq \alpha z^*$ $\rightarrow$ Non linear whenever $\alpha \neq \alpha^*$!!

Indeed, a linear operator must satisfy:

$$A : (\alpha z) \rightarrow A(\alpha z) = \alpha A(z)$$

Kramers Degeneracy etc

Wave functions:

$$\psi_T(x) = \langle x|\psi_T\rangle = \langle x|U_T|\psi\rangle = \langle x|\psi\rangle^* = \psi^*(x) \quad \text{C.Conjugate}$$

$$\text{If } H|\psi\rangle = E|\psi\rangle, [H, U_T] = 0, \ |\psi\rangle \text{ non-degenerate}$$

$$\rightarrow H|\psi_T\rangle = HU_T|\psi\rangle = U_T H|\psi\rangle = EU_T|\psi\rangle = E|\psi_T\rangle \rightarrow |\psi_T\rangle = |\psi\rangle$$

$$\rightarrow \psi(x) \ \text{ real}$$

Degeneracy of fermion states:

$$U_T^2|\psi\rangle = -|\psi\rangle$$

$$\rightarrow \langle\psi|\psi_T\rangle = \langle\psi_T|\psi\rangle^* = \langle\psi|U_T^\dagger|\psi\rangle = -\langle\psi|U_T^\dagger U_T^2|\psi\rangle = -\langle\psi|U_T|\psi\rangle = -\langle\psi|\psi_T\rangle$$

$$\rightarrow \langle\psi|\psi_T\rangle = 0$$

$$\rightarrow |\psi\rangle, |\psi_T\rangle \ \text{ orthogonal, independent}$$

Example (trivial):
Spin ½ particle in rest frame $\rightarrow$ 2 independent states (up & down)

Effect on Scalar Products

$\{|i\rangle, i=1,...\}$ complete set of states

$\langle i | j\rangle = \delta_{ij}$

$\rightarrow \langle U_T i | U_T j\rangle = \delta_{ij}$ norm is $U_T -$ invariant

$$|\psi_1\rangle = \sum_i |i\rangle\langle i|\psi_1\rangle \rightarrow |U_T\psi_1\rangle = \sum_i |U_T i\rangle\langle i|\psi_1\rangle^*$$

$$\langle\psi_2| = \sum_j \langle\psi_2|j\rangle\langle j| \rightarrow \langle U_T\psi_2| = \sum_j \langle\psi_2|j\rangle^* \langle U_T j|$$

$$\rightarrow \langle U_T\psi_2|U_T\psi_1\rangle = \sum_j \langle\psi_2|j\rangle^* \langle U_T j|\sum_i |U_T i\rangle\langle i|\psi_1\rangle^*$$

$$= \sum_{i,j} \langle\psi_2|j\rangle^* \underbrace{\langle U_T j|U_T i\rangle}_{\delta_{ij}} \langle i|\psi_1\rangle^* = \sum_i \langle\psi_2|i\rangle^* \langle i|\psi_1\rangle^*$$

$= \sum_i \langle i|\psi_2\rangle\langle\psi_1|i\rangle = \langle\psi_1|\psi_2\rangle$ U_T swaps states in any scalar product

Matrix Elements

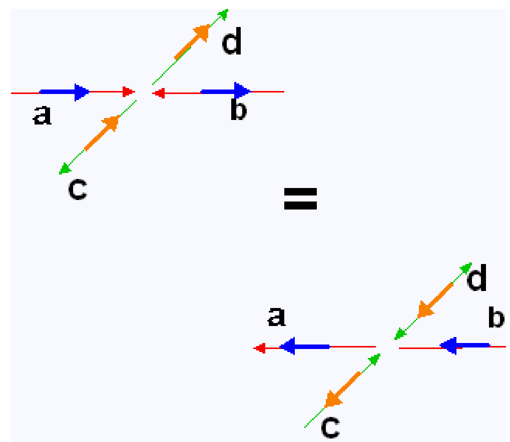
Matrix element for a transition: Initial $\leftrightarrow$ Final

H T-invariant $\rightarrow$ Same matrix element for direct and reversed transition

$\underbrace{1+2}_{i} \rightarrow \underbrace{3+4}_{f}$ 2-body process

Reciprocity Theorem:

$$\langle \mathbf{p}_f, \mathbf{s}_f | S | \mathbf{p}_i, \mathbf{s}_i \rangle = \underbrace{\langle -\mathbf{p}_i, -\mathbf{s}_i |}_{T|\mathbf{p}_i, \mathbf{s}_i\rangle} S \underbrace{| -\mathbf{p}_f, -\mathbf{s}_f \rangle^*}_{T\langle \mathbf{p}_f, \mathbf{s}_f|}$$



Detailed Balance Theorem:

$$\frac{\dfrac{d\sigma_{if}}{d\Omega}}{\dfrac{d\sigma_{fi}}{d\Omega}} = \left(\frac{p_f}{p_i}\right)^2 \frac{(2s_3+1)(2s_4+1)}{(2s_1+1)(2s_2+1)}$$

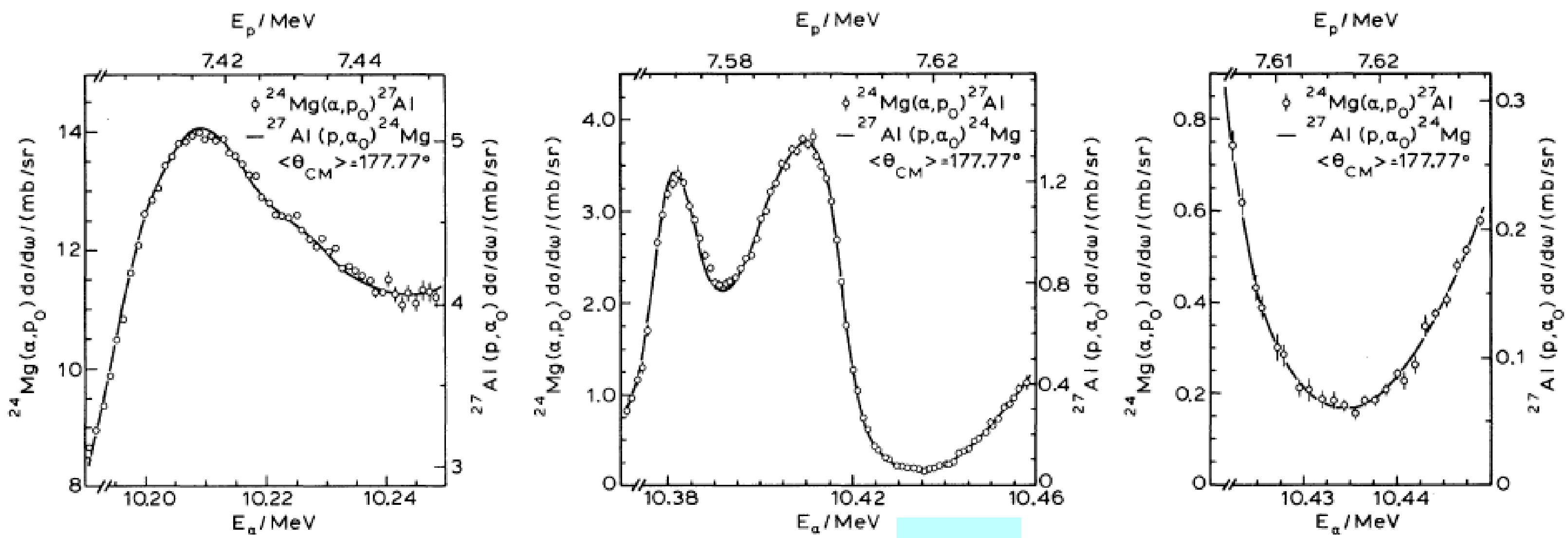
OK for Strong, E.M., KO for Weak

Example: Test of Detailed Balance

Nuclear reactions:

$$(1)\quad \alpha + {}^{24}Mg \rightarrow p + {}^{27}Al$$

$$(2)\quad p + {}^{27}Al \rightarrow \alpha + {}^{24}Mg$$



@TBA

Full line: Reaction (1)
Circles: Reaction (2)

T-Violating Correlations

Example (many more exist):

$$\Lambda^0 \to p + \pi^-$$

$$\left.\begin{array}{l} \dfrac{1}{2}^+ = \dfrac{1}{2}^+ \oplus 0^- \oplus L^{(-1)^L} = \dfrac{1}{2} \oplus L \\ + \quad = + \quad \cdot \quad - \quad \cdot \quad (\text{-}1)^{\text{L}} = (\text{-}1)^{\text{L+1}} \end{array}\right\} \to L = 0,1$$

Decay amplitude = Sum of S,P waves
S,P waves *do* interfere because of parity violation. So:
Angular distribution has a term $\propto \cos\,\theta$

Most general form of angular distribution:

$$\frac{d\Gamma}{d\Omega_\pi} = 1 + A\left(\boldsymbol{J}_\Lambda + \boldsymbol{J}_p\right)\cdot\hat{\boldsymbol{p}}_\pi + B\left(\boldsymbol{J}_\Lambda \times \boldsymbol{J}_p\right)\cdot\hat{\boldsymbol{p}}_\pi + C\left(\boldsymbol{J}_\Lambda \cdot \boldsymbol{J}_p\right) + (1-C)\left(\boldsymbol{J}_\Lambda\cdot\hat{\boldsymbol{p}}_\pi\right)\left(\boldsymbol{J}_p\cdot\hat{\boldsymbol{p}}_\pi\right)$$

$$\left(\boldsymbol{J}_\Lambda \times \boldsymbol{J}_p\right)\cdot\hat{\boldsymbol{p}}_\pi \underset{U_T}{\to} \left(\left(-\boldsymbol{J}_\Lambda\right)\times\left(-\boldsymbol{J}_p\right)\right)\cdot\left(-\hat{\boldsymbol{p}}_\pi\right) = -\left(\boldsymbol{J}_\Lambda \times \boldsymbol{J}_p\right)\cdot\hat{\boldsymbol{p}}_\pi \quad \text{T-Violating term}$$

$\to$ Expect $B = 0$

Must correct for final state interaction: $p\pi$ scattering

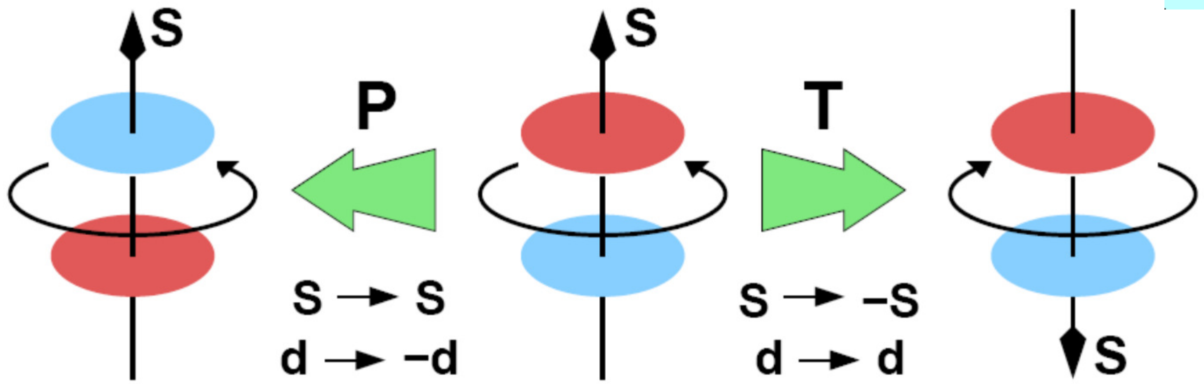
$\to$ Get an upper limit of a few %

Electric Dipole Moments - I

Electric Dipole Moments

Elementary particle can only have a permanent electric dipole moment
if both parity and time reversal symmetries are broken:

@TBA



$$\vec{\mu} = g\,\frac{e}{mc}\vec{S} \qquad \vec{d} = \eta\,\frac{e}{2mc}\vec{S}$$

$\boldsymbol{d}//\boldsymbol{S}$, otherwise there would be an extra degree of freedom which is not seen
Warning: True in the absence of a degenerate ground state
E.g., polar molecules like H_2O *do* have degenerate ground states…

Electric Dipole Moments - II

For a particle with both magnetic and electric dipole moments:

$H' = -\boldsymbol{\mu}\cdot\mathbf{B} - \mathbf{d}\cdot\mathbf{E}$ Interaction energy

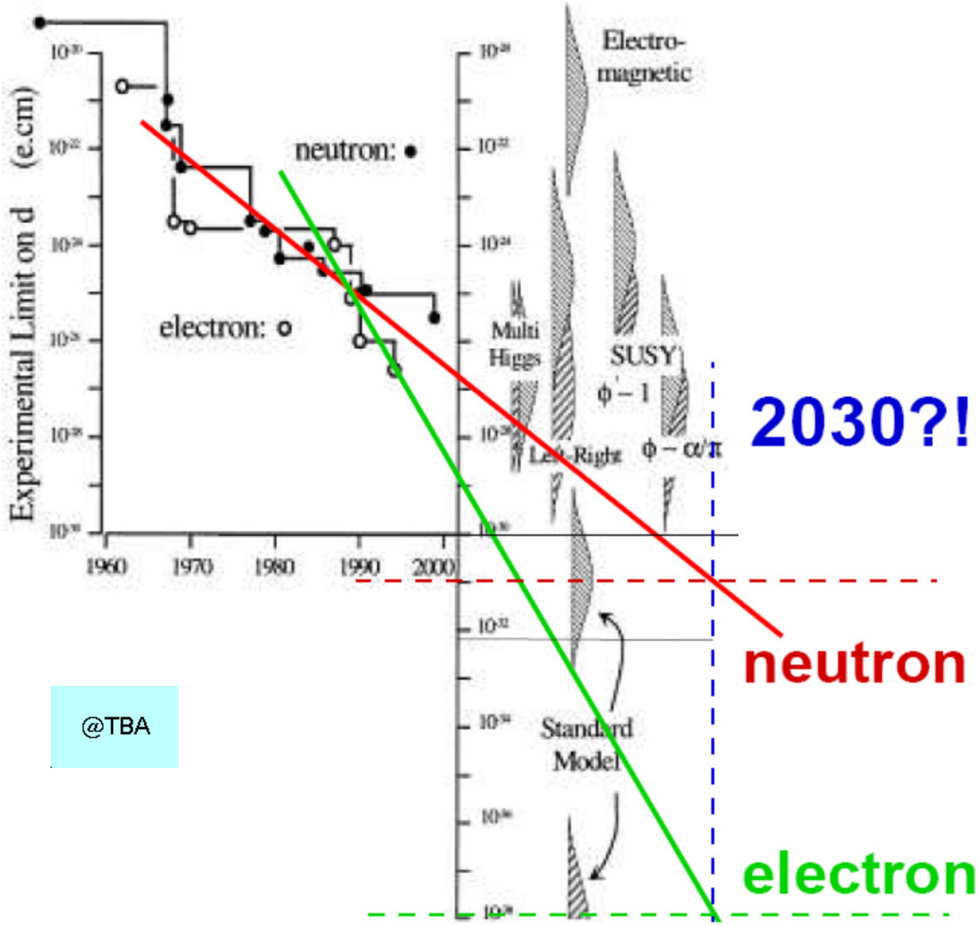
Similar to magnetic dipole in **B**:

Can observe in external **E**:

Level shift

Spin precession

Status of the EDM



C, P ,T in Quantum Field Theory

Just a few, oversimplified remarks:

Observables are (functions of) field operators

Just as in Quantum Mechanics, one finds different kinds of operators:

Scalars, Spinors, Vectors, …

Fields of each kind have their own ‘parity' built-in

States of a given field do inherit field ‘parity', which is called *intrinsic parity*

C,P and T in QFT - I

1) Spin = 0

$$\phi(t,\mathbf{r}) = N\int d^3\mathbf{p}\left[b(\mathbf{p})e^{-ipx} + d^\dagger(\mathbf{p})e^{+ipx}\right]$$

$$\phi^\dagger(t,\mathbf{r}) = N\int d^3\mathbf{p}\left[d(\mathbf{p})e^{-ipx} + b^\dagger(\mathbf{p})e^{+ipx}\right]$$

$$\begin{cases} U_P\phi(t,\mathbf{r})U_P^\dagger = \eta_P\phi(t,-\mathbf{r}) \\ U_P\phi^\dagger(t,\mathbf{r})U_P^\dagger = \eta_P\phi(t,-\mathbf{r}) \end{cases} \leftrightarrow \begin{cases} U_P \begin{matrix} b(\mathbf{p}) \\ d^\dagger(\mathbf{p}) \end{matrix} U_P^\dagger = \eta_P \begin{matrix} b(-\mathbf{p}) \\ d^\dagger(-\mathbf{p}) \end{matrix} \\ U_P \begin{matrix} b^\dagger(\mathbf{p}) \\ d(\mathbf{p}) \end{matrix} U_P^\dagger = \eta_P \begin{matrix} b^\dagger(\mathbf{p}) \\ d(\mathbf{p}) \end{matrix} \end{cases}, e^{\pm ipx} \to e^{\pm ipx}$$

$$\begin{cases} U_C\phi(t,\mathbf{r})U_C^\dagger = \eta_C\phi^\dagger(t,\mathbf{r}) \\ U_C\phi^\dagger(t,\mathbf{r})U_C^\dagger = \eta_C^*\phi(t,\mathbf{r}) \end{cases} \leftrightarrow \begin{cases} U_C \begin{matrix} b(\mathbf{p}) \\ d^\dagger(\mathbf{p}) \end{matrix} U_C^\dagger = \eta_C \begin{matrix} d(\mathbf{p}) \\ b^\dagger(\mathbf{p}) \end{matrix} \\ U_C \begin{matrix} b^\dagger(\mathbf{p}) \\ d(\mathbf{p}) \end{matrix} U_C^\dagger = \eta_C^* \begin{matrix} d^\dagger(\mathbf{p}) \\ b(\mathbf{p}) \end{matrix} \end{cases}, e^{\pm ipx} \to e^{\pm ipx}$$

$$\begin{cases} \bar{U}_T\phi(t,\mathbf{r})\bar{U}_T^\dagger = \underbrace{\cancel{\eta_T}}_{=1}\phi(-t,\mathbf{r}) \\ \bar{U}_T\phi^\dagger(t,\mathbf{r})\bar{U}_T^\dagger = \phi^\dagger(-t,\mathbf{r}) \end{cases} \leftrightarrow \begin{cases} \bar{U}_T \begin{matrix} b(\mathbf{p}) \\ d^\dagger(\mathbf{p}) \end{matrix} \bar{U}_T^\dagger = \begin{matrix} b(-\mathbf{p}) \\ d^\dagger(-\mathbf{p}) \end{matrix} \\ \bar{U}_T \begin{matrix} d(\mathbf{p}) \\ b^\dagger(\mathbf{p}) \end{matrix} \bar{U}_T^\dagger = \begin{matrix} d(-\mathbf{p}) \\ b^\dagger(-\mathbf{p}) \end{matrix} \end{cases}, e^{\pm ipx} \to e^{\mp ipx}$$

General:

Observables depend on $\phi\phi^\dagger$

Any observable yields a factor

$\eta_P\eta_P^* = 1, \eta_C\eta_C^* = 1$

$\to \eta_P, \eta_C$ unobservable

ϕ Hermitian:

$\phi = \phi^\dagger$

$\to \eta_C = \eta_C^*, \eta_P = \pm 1$

$\to \begin{cases} \eta_P = \pm 1 \\ \eta_C = \pm 1 \end{cases}$ Observable

η_T unobservable

C,P and T in QFT - II

2) Spin 1/2

$$\psi(t,\mathbf{r}) = \sum_s \int N d^3\mathbf{p}\left[b(\mathbf{p},s)u(\mathbf{p},s)e^{+ipx} + d^\dagger(\mathbf{p},s)v(\mathbf{p},s)e^{-ipx}\right]$$

$$\psi^\dagger(t,\mathbf{r}) = \sum_s \int N d^3\mathbf{p}\left[b^\dagger(\mathbf{p},s)\overline{u}(\mathbf{p},s)e^{-ipx} + d(\mathbf{p},s)\overline{v}(\mathbf{p},s)e^{+ipx}\right]$$

$$\begin{cases} U_P\psi(t,\mathbf{r})U_P^\dagger = \eta_P\gamma^0\psi(t,-\mathbf{r}) \\ U_P\overline{\psi}(t,\mathbf{r})U_P^\dagger = \eta_P^*\overline{\psi}(t,-\mathbf{r})\gamma^0 \end{cases} \leftrightarrow \begin{cases} U_P \begin{matrix} b(\mathbf{p},s) \\ d(\mathbf{p},s) \end{matrix} U_P^\dagger = \eta_P \begin{matrix} b(-\mathbf{p},s) \\ d(-\mathbf{p},s) \end{matrix} \\ U_P \begin{matrix} d^\dagger(\mathbf{p},s) \\ b^\dagger(\mathbf{p},s) \end{matrix} U_P^\dagger = \eta_P^* \begin{matrix} d^\dagger(-\mathbf{p},s) \\ b^\dagger(-\mathbf{p},s) \end{matrix} \end{cases}, e^{\pm ipx} \to e^{\pm ipx}$$

$$\begin{cases} U_C\psi(t,\mathbf{r})U_C^\dagger = \eta_C i\gamma_2\gamma_0\overline{\psi}(t,\mathbf{r}) \\ U_C\overline{\psi}(t,\mathbf{r})U_C^\dagger = \eta_C^*\psi(t,\mathbf{r})i\gamma_2\gamma_0 \end{cases} \leftrightarrow \begin{cases} U_C \begin{matrix} b(\mathbf{p},s) \\ b^\dagger(\mathbf{p},s) \end{matrix} U_C^\dagger = \eta_C \begin{matrix} d(\mathbf{p},s) \\ d^\dagger(\mathbf{p},s) \end{matrix} \\ U_C \begin{matrix} d(\mathbf{p},s) \\ d^\dagger(\mathbf{p},s) \end{matrix} U_C^\dagger = \eta_C^* \begin{matrix} b(\mathbf{p},s) \\ b^\dagger(\mathbf{p},s) \end{matrix} \end{cases}, e^{\pm ipx} \to e^{\pm ipx}$$

$$\begin{cases} \overline{U}_T\psi(t,\mathbf{r})\overline{U}_T^\dagger = \underbrace{\cancel{\eta_T}}_{=1}\gamma_1\gamma_3\psi(-t,\mathbf{r}) \\ \overline{U}_T\overline{\psi}(t,\mathbf{r})\overline{U}_T^\dagger = \overline{\psi}(-t,\mathbf{r})(-\gamma_1\gamma_3) \end{cases} \leftrightarrow \begin{cases} \overline{U}_T \begin{matrix} b(\mathbf{p},s) \\ d^\dagger(\mathbf{p},s) \end{matrix} \overline{U}_T^\dagger = \begin{matrix} b(-\mathbf{p},-s) \\ d^\dagger(-\mathbf{p},-s) \end{matrix} \\ \overline{U}_T \begin{matrix} b^\dagger(\mathbf{p},s) \\ d(\mathbf{p},s) \end{matrix} \overline{U}_T^\dagger = \begin{matrix} b^\dagger(-\mathbf{p},-s) \\ d(-\mathbf{p},-s) \end{matrix} \end{cases}, e^{\pm ipx} \to e^{\mp ipx}$$

CPT

Fundamental theorem applying to all field theories, including our beloved Standard Model:

Lorentz invariance

Micro-causality (whatever it means…)

Spin-statistics

→ *The product transformation CPT is a good symmetry*

Consequences:

$$m_{particle} = m_{antiparticle}$$

$$\tau_{particle} = \tau_{antiparticle}$$

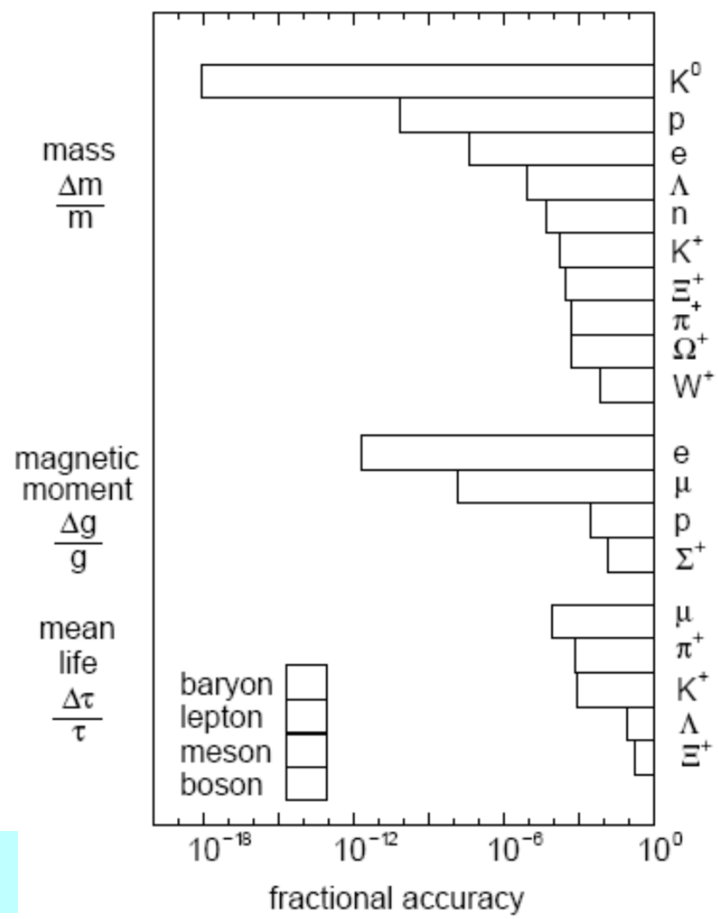
$$|\boldsymbol{\mu}|_{particle} = |\boldsymbol{\mu}|_{antiparticle}$$

....more quantum numbers

CPT Tests

Just a flash on the general status of CPT tests…

…very comfortable



@TBA

Symmetries: Summary

Conserved quantity	Interaction		
	Strong	E.M.	Weak
4-momentum	OK	OK	OK
Charge	OK	OK	OK
Ang. Momentum, CM speed	OK	OK	OK
Baryonic number	OK	OK	OK
Leptonic numbers (3)	OK	OK	~OK
Parity	OK	OK	KO
Charge parity	OK	OK	KO
(Time reversal)	OK	OK	KO
CP	OK	OK	KO
(CPT)	OK	OK	OK
Flavor	OK	OK	KO