Elementary Particles I

2 – Electromagnetic Interaction

Form Factors, Structure Functions, Scaling, Partons

Leptons

Leptons

1st family	2nd family	3rd family	
$\nu_e, \bar{\nu}_e$	$\nu_\mu, \bar{\nu}_\mu$	$\nu_\tau, \bar{\nu}_\tau$	Neutral, 'Massless'
e^-, e^+	μ^-, μ^+	τ^-, τ^+	Charged, Massive

"Pointlike", spin ½ Fermions
Electromagnetic and weak interactions

Lepton scattering by several targets as a powerful tool to probe constituents:

Electromagnetic (and weak) coupling to leptons simple, well understood

Small coupling constant→Perturbative expansion reliable

Electromagnetic Interaction

Try to find transition amplitude for electromagnetic scattering

1st order perturbative contribution

$T_{fi} = -i\langle f|\int d^4xH'|i\rangle$ H': Interaction Hamiltonian density

$H' = j^\mu A_\mu$ Classical analogy, j_μ current 4-density

Reminder

For any system of charges and currents:

$u_E = \frac{1}{2}\rho\varphi$ Electrostatic potential energy density

$u_B = \frac{1}{2}\mathbf{j}\cdot\mathbf{A}$ Magnetostatic potential energy density

$j^\mu = (\rho, \mathbf{j})$ 4-current density

$A_\mu = (\varphi, \mathbf{A})$ 4-potential

An oversimplified example - I

First take a simple example:

Spinless, pointike "pion" scattering off a fixed, Coulomb potential

$$A_\mu = \left(\frac{eZ}{r}, \mathbf{0}\right)$$

$$j^\mu = (\rho, \mathbf{j}) = ie\left(\varphi^*\left(\frac{\partial\varphi}{\partial t}\right) - \left(\frac{\partial\varphi^*}{\partial t}\right)\varphi, \left((\nabla\varphi^*)\varphi - \varphi^*(\nabla\varphi)\right)\right)$$

$$j^\mu = (\rho, \mathbf{j}) = ie\left(\left(\varphi'^*\left(\frac{\partial\varphi}{\partial t}\right) - \left(\frac{\partial\varphi'^*}{\partial t}\right)\varphi\right), \left((\nabla\varphi'^*)\varphi - \varphi'^*(\nabla\varphi)\right)\right)$$

$$\rightarrow j^\mu = eNN'e^{-i\left((E-E')t-(\mathbf{p}-\mathbf{p}')\cdot\mathbf{r}\right)}\left((E+E'), (\mathbf{p}'+\mathbf{p})\right)$$

$$\rightarrow j^\mu A_\mu = NN'(E+E')e^{-i\left((E-E')t-(\mathbf{p}-\mathbf{p}')\cdot\mathbf{r}\right)}\frac{e^2 Z}{r}$$

Usual definition of current density

ϕ: Stationary state

Generalize to a scattering state

ϕ, ϕ': Stationary states, plane waves

Integrate over time:

$$\int_{-\infty}^{+\infty} NN'e^{i(E-E')t}dt = NN'2\pi\delta(E-E')$$

Energy conservation; momentum *not conserved* by fixed Coulomb potential

An oversimplified example - II

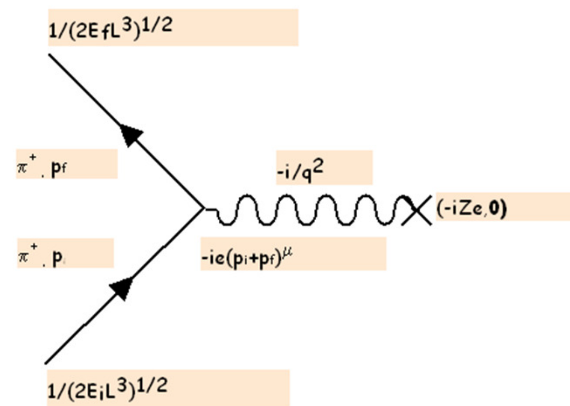
Integrate over space:

$$\int e^{+i((\mathbf{p}-\mathbf{p}')\cdot\mathbf{r})}\frac{e^2 Z}{r}d^3\mathbf{r}=\frac{4\pi Ze^2}{|\mathbf{q}|^2},\mathbf{q}=\mathbf{p}-\mathbf{p}'$$

Matrix element:

$$T_{fi}=-i\langle f|\int d^4xH'|i\rangle=-NN'2\pi i\delta(E-E')\frac{4\pi Ze^2}{|\mathbf{q}|^2}$$

$$E=E'\to|\mathbf{q}|^2=-q^2\to T_{fi}=NN'2\pi i\delta(E-E')\frac{4\pi Ze^2}{q^2}$$



Virtual photon:
Coupling fixed source to current

An oversimplified example - III

Evaluate transition probability

$$w = \frac{\left|T_{fi}\right|^2}{T}$$ Transition probability/Time

$$\left|\delta\left(E'-E\right)\right|^2 = \lim_{T\to\infty}\left|\frac{1}{2\pi}\int_{-T/2}^{+T/2} e^{i(E'-E)t}dt\right|^2 = \lim_{T\to\infty}\left|\frac{\sin\left[\left(E'-E\right)T/2\right]}{\pi\left(E'-E\right)}\right|^2 = \frac{T}{2\pi}\delta\left(E'-E\right)$$

$$w = N^2N'^2\frac{4\pi^2}{T}\left|\delta\left(E-E'\right)\right|^2\frac{16\pi^2Z^2e^4}{\left|\mathbf{q}\right|^4} = N^2N'^2\,2\pi\delta\left(E-E'\right)\frac{Z^2e^4}{\left|\mathbf{q}\right|^4}$$

$$\to d\sigma = w\cdot\frac{\text{phase space}}{\text{incident flux}} = w\frac{\frac{Vd^3p'}{(2\pi)^3}}{\frac{p}{EV}} = w\frac{d^3p'}{(2\pi)^3}\frac{E}{p}V^2$$

$$d\sigma = N^2N'^2\,2\pi\delta\left(E-E'\right)\frac{16\pi^2Z^2e^4}{\left|\mathbf{q}\right|^4}\frac{d^3p'}{(2\pi)^3}\frac{E}{p}V^2$$

$$N = N' = \frac{1}{\sqrt{V}} \to d\sigma = \frac{1}{V}\frac{1}{V}2\pi\delta\left(E-E'\right)\frac{16\pi^2Z^2e^4}{\left|\mathbf{q}\right|^4}\frac{d^3p'}{(2\pi)^3}\frac{E}{p}V^2 = 2\pi\delta\left(E-E'\right)\frac{16\pi^2Z^2e^4}{\left|\mathbf{q}\right|^4}\frac{d^3p'}{(2\pi)^3}\frac{E}{p}$$

An oversimplified example - IV

Calculate differential cross-section

$$\int d\sigma = \int 2\pi\delta(E-E')\frac{16\pi^2 Z^2 e^4}{|\mathbf{q}|^4}\frac{d^3p'}{(2\pi)^3}\frac{E}{p}$$

$$d^3p' = p'^2\,dp'\,d\Omega$$

$$p'dp' = E'dE' \to d^3p' = p'E'dE'd\Omega$$

$$\to \int 2\pi\delta(E-E')\frac{16\pi^2 Z^2 e^4}{|\mathbf{q}|^4}\frac{d^3p'}{(2\pi)^3}\frac{E}{p} = \int 2\pi\delta(E-E')\frac{16\pi^2 Z^2 e^4}{|\mathbf{q}|^4}\frac{dE'}{(2\pi)^3}\frac{E}{p}p'E'd\Omega$$

$$= \frac{1}{(2\pi)^2}\frac{Z^2e^4}{|\mathbf{q}|^4}\frac{E}{p}p'\int\delta(E-E')dE'E'd\Omega = \frac{1}{(2\pi)^2}\frac{Z^2e^4}{|\mathbf{q}|^4}E^2 d\Omega$$

$$q = 2p\sin\frac{\theta}{2} \to q^4 = 16p^4\sin^4\frac{\theta}{2}$$

$$\to \frac{d\sigma}{d\Omega} = \frac{1}{(2\pi)^2}\frac{16\pi^2 Z^2 e^4}{16p^4\sin^4\theta/2}E^2 = \frac{1}{4}Z^2\alpha^2\frac{E^2}{p^4}\frac{1}{\sin^4\theta/2}$$

Useful to remember

$$E^2 = \mathbf{p}^2 + m^2$$

$$\to 2EdE = 2|\mathbf{p}|d|\mathbf{p}|$$

$$\to EdE = |\mathbf{p}|d|\mathbf{p}|$$

Compare to non-relativistic Rutherford cross-section:

$$\frac{d\sigma}{d\Omega} = \frac{1}{4}Z^2\alpha^2\frac{1}{T^2}\frac{1}{\sin^4\theta/2}$$

Theme Variation: Spin ½ - I

Take now a spin ½ Dirac electron scattering off the same, static Coulomb potential

$$j^{\mu} = e\overline{\psi}'\gamma^{\mu}\psi$$

Dirac transition current

$$\rightarrow \psi = \sqrt{\frac{m}{E}}u(s,p)e^{ipx}, \overline{\psi}' = \sqrt{\frac{m}{E'}}\overline{u}(s',p')e^{-ip'x}$$

$$\rightarrow j^{\mu} = e\sqrt{\frac{m}{E'}}\overline{u}(s',p')e^{-ip'x}\gamma^{\mu}\sqrt{\frac{m}{E}}u(s,p)e^{ipx} = e\frac{m}{E}e^{-i(p-p')x}\overline{u}\gamma^{\mu}u$$

$$j^{\mu}A_{\mu} = e\frac{m}{E}e^{-i((E-E')t-(\mathbf{p}-\mathbf{p}')\cdot\mathbf{r})}\overline{u}(s',p')\gamma^{\mu}u(s,p)\left(\frac{eZ}{r},\mathbf{0}\right) = e\frac{m}{E}e^{-i((E-E')t-(\mathbf{p}-\mathbf{p}')\cdot\mathbf{r})}\overline{u}\gamma^{0}u\frac{eZ}{r}$$

$$\underbrace{\overline{u}(s',p')}_{=u^{\dagger}\gamma^{0}}\gamma^{0}u(s,p) = u^{\dagger}(s',p')\overbrace{\gamma^{0}\gamma^{0}}^{=1}u(s,p) = u^{\dagger}(s',p')u(s,p)$$

Dirac matrices

$$\frac{d\sigma}{d\Omega} = \frac{m^2}{E^2}\frac{Z^2e^4}{16(2\pi)^2}\frac{E^2}{p^4}\frac{1}{\sin^4\theta/2}\left|u^{\dagger}(s',p')u(s,p)\right|^2 = \frac{1}{4}\frac{m^2}{p^4}Z^2\alpha^2\frac{1}{\sin^4\theta/2}\left|u^{\dagger}(s',p')u(s,p)\right|^2$$

Theme Variation: Spin ½ - II

Unpolarized cross-section:

Sum over final / *Average over initial* *spin projections*

$$\frac{1}{2}\sum_{s=-1/2}^{+1/2}\sum_{s'=-1/2}^{+1/2}\left|u^{\dagger}(s',p')u(s,p)\right|^2 = 4\frac{E^2}{m^2}\left(1-\beta^2\sin^2\theta/2\right)$$

$$\rightarrow \frac{d\sigma}{d\Omega} = \frac{1}{4}Z^2\alpha^2\frac{m^2}{p^4}\frac{1}{\sin^4\theta/2}\frac{4E^2}{m^2}\left(1-\beta^2\sin^2\theta/2\right) = \frac{1}{4}Z^2\alpha^2\frac{E^2}{p^4}\frac{\left(1-\beta^2\sin^2\theta/2\right)}{\sin^4\theta/2}$$

$$\frac{d\sigma}{d\Omega} \underset{E\gg m}{\simeq} \frac{1}{4}Z^2\alpha^2\frac{E^2}{p^4}\frac{\cos^2\theta/2}{\sin^4\theta/2}$$

New factor, important at high speed
Reducing cross section at large angles (= *0* for $\theta\rightarrow\pi$)

Helicity Conservation

Dirac equation: High energy limit

$$E\psi = (\boldsymbol{\alpha}\cdot\mathbf{p} + \beta m)\psi$$

$u = \begin{pmatrix}\phi \\ \chi\end{pmatrix}$ Generic spinor; ϕ, χ 2-components spinors

$\boldsymbol{\alpha} = \begin{pmatrix}\boldsymbol{\sigma} & 0 \\ 0 & -\boldsymbol{\sigma}\end{pmatrix}, \beta = \begin{pmatrix}0 & 1 \\ 1 & 0\end{pmatrix}$ Dirac matrices, chiral representation, "2×2" block format

$$\begin{cases} E\phi = (\boldsymbol{\sigma}\cdot\mathbf{p})\phi + m\chi \\ E\chi = -(\boldsymbol{\sigma}\cdot\mathbf{p})\chi + m\phi \end{cases}$$

$$E \gg m \to \left.\begin{matrix} E \approx |\mathbf{p}| \\ m \approx 0 \end{matrix}\right\} \to \begin{cases} (\boldsymbol{\sigma}\cdot\mathbf{p})\phi \approx |\mathbf{p}|\phi \\ (\boldsymbol{\sigma}\cdot\mathbf{p})\chi \approx -|\mathbf{p}|\chi \end{cases} \to \begin{cases} \phi \simeq u_R \\ \chi \simeq u_L \end{cases} \to u \approx \begin{pmatrix} u_R \\ u_L \end{pmatrix} \to u^\dagger(s', p')u(s, p) \approx u'^\dagger_R u_R + u'^\dagger_L u_L$$

No mixed terms $\to$ *Helicity is conserved at high energy*

Explains the $\left(1 - \beta^2 \sin^2 \theta/2\right)$ factor, cutting off the cross-section $\theta \to \pi$:
Solves conflicting helicity/angular momentum conservation

Always true for Dirac currents coupling to vector fields

Another Step into a Realistic Model

Take now a spin ½ Dirac electron scattering off a *distributed*, static source (like a *(A,Z)* nucleus)

$$A_\mu = (\varphi, \mathbf{0})$$

$$\varphi(r) = \int d^3\mathbf{r}' \frac{\rho(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}, \qquad \int \rho(\mathbf{r}') d^3\mathbf{r}' = Ze$$

Only change: the space integral

$$\int e^{+i((\mathbf{p}\text{-}\mathbf{p}')\cdot\mathbf{r})} \frac{e^2 Z}{r} d^3\mathbf{r} = \frac{4\pi Z e^2}{|\mathbf{q}|^2}, \mathbf{q} = \mathbf{p} - \mathbf{p}' \rightarrow \int d^3\mathbf{r} \int e^{+i((\mathbf{p}\text{-}\mathbf{p}')\cdot\mathbf{r})} \frac{\rho(\mathbf{r}')}{|\mathbf{r}'-\mathbf{r}|} d^3\mathbf{r}'$$

$$= \int d^3\mathbf{r}' \frac{\rho(\mathbf{r}')}{Ze} e^{+i(\mathbf{p}\text{-}\mathbf{p}')\cdot\mathbf{r}'} \int e^{+i((\mathbf{p}\text{-}\mathbf{p}')\cdot(\mathbf{r}-\mathbf{r}'))} \frac{Ze}{|\mathbf{r}'-\mathbf{r}|} d^3\mathbf{r}$$

$$F(\mathbf{q}) = \frac{\int \rho(\mathbf{r}) e^{+i\mathbf{q}\cdot\mathbf{r}} d^3\mathbf{r}}{Ze}$$ Form factor of the charge distribution

$$\Rightarrow \frac{Z^2 e^2}{|\mathbf{q}|^4} \rightarrow \frac{Z^2 e^2}{|\mathbf{q}|^4} |F(\mathbf{q})|^2$$

The Form Factor

$$F(\mathbf{q}) = \frac{\int \rho(\mathbf{r}) e^{+i\mathbf{q}\cdot\mathbf{r}} d^3\mathbf{r}}{Ze} = \frac{1}{Ze}\int \rho(r) e^{+iqr\cos\theta} r^2 dr d\Omega$$

$$e^{+i|\mathbf{q}|r\cos\theta} = 1 + i|\mathbf{q}| r\cos\theta - \frac{1}{2}|\mathbf{q}|^2 r^2 \cos^2\theta \ldots$$

$$\rightarrow \frac{1}{Ze}\int \rho(r) e^{+i|\mathbf{q}|r\cos\theta} r^2 dr d\Omega \approx$$

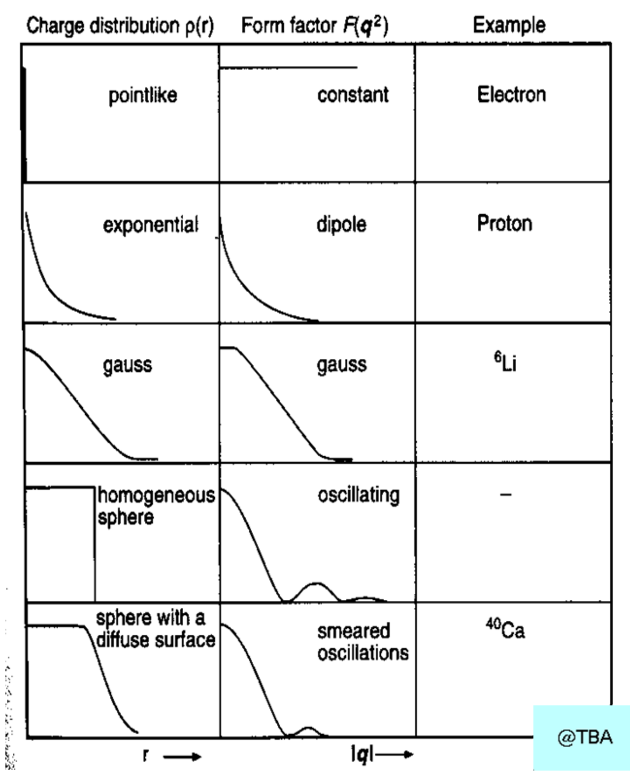
$$\approx \frac{1}{Ze}\left[\int \rho(r) r^2 dr d\Omega + i|\mathbf{q}|\int \rho(r) r^3 \cos\theta dr d\Omega - \frac{|\mathbf{q}|^2}{2}\int \rho(r) r^4 \cos^2\theta dr d\Omega\right]$$

$$\rightarrow F(\mathbf{q}) \approx 1 + \frac{i|\mathbf{q}|}{Ze}\int \rho(r) r^3 dr \underbrace{\int \cos\theta d\Omega}_{=\frac{2\pi}{2}\left(-\cos^2\theta\right)\Big|_0^\pi = 0} - \frac{|\mathbf{q}|^2}{2Ze}\underbrace{\int \rho(r) r^4 dr}_{=\frac{Ze\langle r^2\rangle}{4\pi}} \underbrace{\int \cos^2\theta d\Omega}_{=\frac{2\pi}{3}\left(-\cos^3\theta\right)\Big|_0^\pi = \frac{4\pi}{3}} = 1 - \frac{|\mathbf{q}|^2\langle r^2\rangle}{6}$$

$$\left.\begin{array}{l} F\left(|\mathbf{q}|^2\right) = F(0) + \dfrac{\partial F}{\partial |\mathbf{q}|^2}|\mathbf{q}|^2 + \ldots \\ F\left(|\mathbf{q}|^2\right) \approx 1 - \dfrac{1}{6}|\mathbf{q}|^2\langle r^2\rangle \end{array}\right\} \rightarrow\rightarrow \left\{\begin{array}{l} F(0) = 1 \\ \langle r^2\rangle = -6\dfrac{\partial F}{\partial |\mathbf{q}|^2}\Bigg|_{|\mathbf{q}|^2=0} \end{array}\right.$$

showing that measuring the form factor yields the rms charge radius

FF: The Mathematical Game



$$\rho(r) = \frac{1}{8\pi a^3} e^{-ar} \rightarrow F\left(|\mathbf{q}|^2\right) = \left(\frac{1}{1+|\mathbf{q}|^2/a^2}\right)^2$$

$$\rho(r) = \begin{cases} \text{constant} & \text{r<R} \\ 0 & \text{r>R} \end{cases} \rightarrow F\left(|\mathbf{q}|^2\right) = \frac{3}{R^3|\mathbf{q}|^3}\left(\sin\left(R|\mathbf{q}|\right) - R|\mathbf{q}|\cos\left(R|\mathbf{q}|\right)\right)$$

Nuclear Form Factors - I

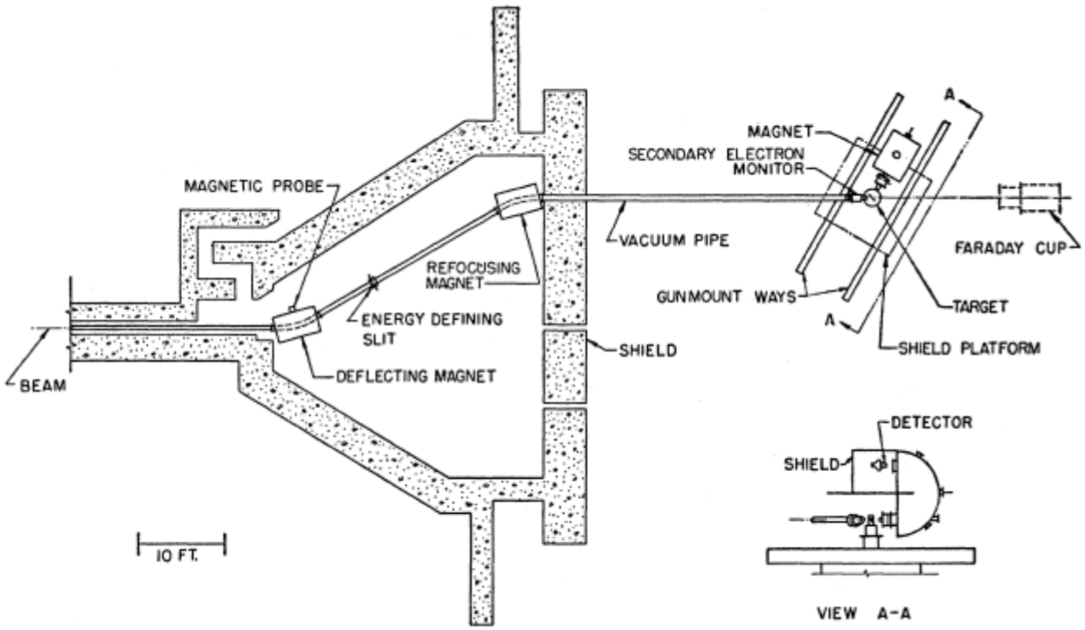


FIG. 18. The experimental installation of the 550-Mev spectrometer.

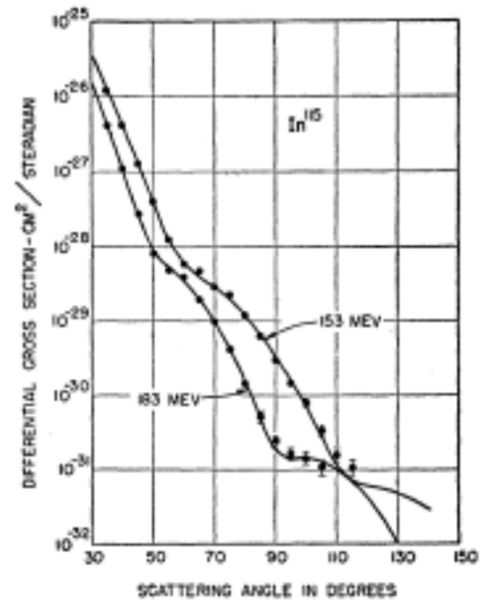


FIG. 46. Theoretical and experimental curves for In^{115} at two energies.

@TBA

One of the Hofstadter's spectrometers at SLAC

Results for Indium

Nuclear Form Factors - II

Details of one of Hofstadter's focusing spectrometers

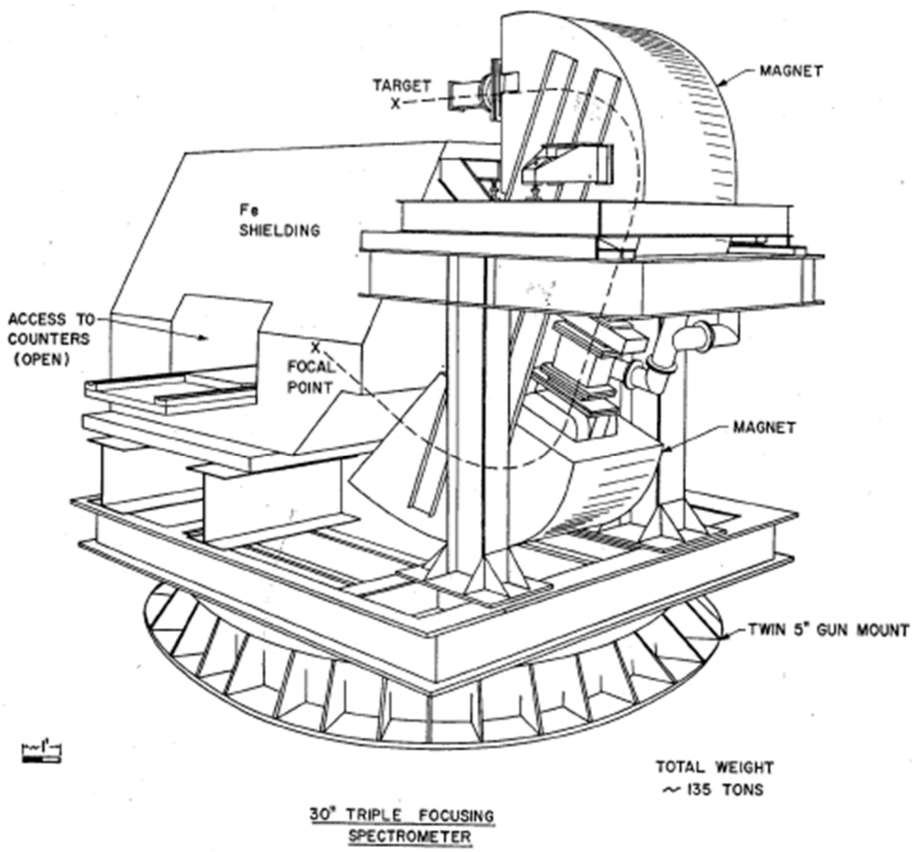
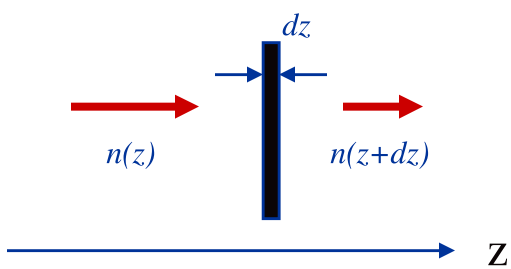


FIG. 4. Perspective drawing of the spectrometer in position on the gun mount, showing iron shielding in place around the counter house. The momentum slit box and part of the vacuum chamber are shown between the magnets.

Nuclear Form Factors - III

From counting rate to cross section:



$$dn = -n(z)\,n_T\sigma\,dz \rightarrow |\Delta n| \simeq n_0 n_T \sigma \Delta z, \Delta n \ll n_0$$

$$n_T = \frac{\rho_T}{A} N_A \rightarrow |\Delta n| \simeq n_0 \rho_T \frac{N_A}{A} \sigma \Delta z$$

$$\rightarrow \sigma = \frac{1}{N_A}\frac{A}{\rho_T \Delta z}\frac{\Delta n}{n_0}, \Delta n \ll n_0, \Delta z \text{ small}$$

What is known:

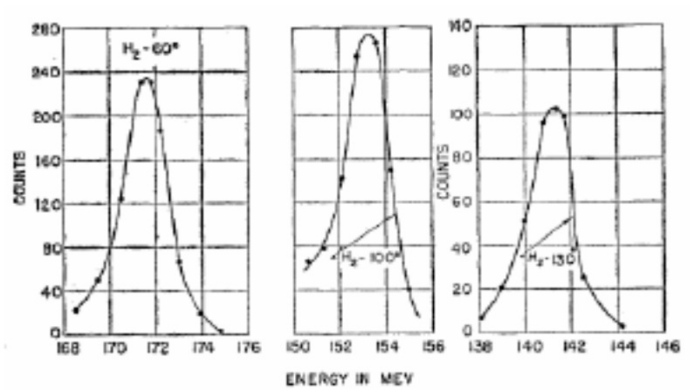
Beam energy
Scattering angle
# of incident beam particles, n_0
# of scattering events, Δn
Target thickness, Δz
Target mass density, ρ_T

Count scattering events, count beam particles, measure target
→ Get σ

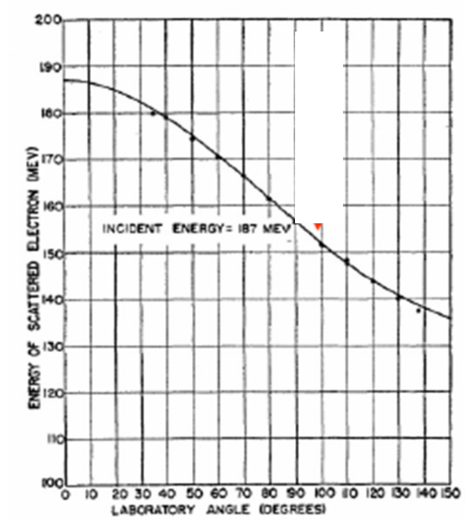
Nuclear Form Factors - IV

Elastic nuclear scattering

Count rate vs energy at different angles:
Elastic peak



Scattered electron energy vs angle:
2-body kinematics



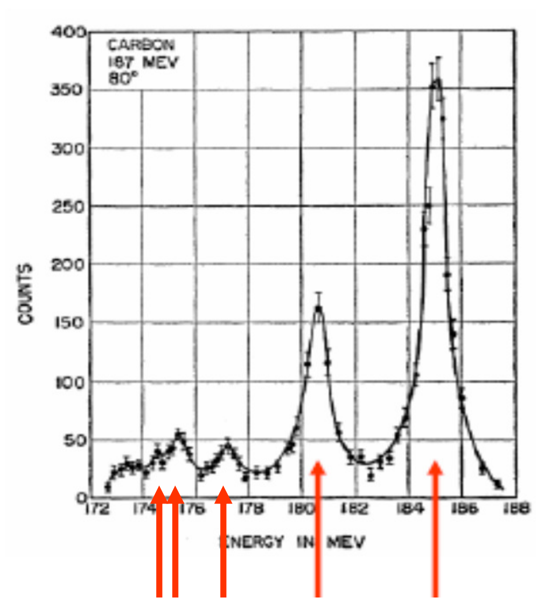
$$\frac{E_e'}{E_e} = \frac{1}{1 + 2\frac{E_e}{m_N}\sin^2\frac{\theta}{2}}$$

Nuclear Form Factors - V

Inelastic nuclear scattering:

Count rate vs energy at fixed angle
Excitation of ${}^{12}C$ nuclear levels

Scattered electron energy vs angle:
>2 body kinematics



Inelastic peaks Elastic peak

$$\frac{E_e^{'}}{E_e} = \frac{1 - \dfrac{\Delta m^2}{2 m_N E_e}}{1 + 2\dfrac{E_e}{m_N}\sin^2\dfrac{\theta}{2}}$$

Δm Excitation energy

Tracing Nuclear Constituents

Inelastic cross section: Providing evidence for nuclear constituents

Detect γ-rays from level de-excitation

Also:

Measurement of inclusive energy spectra of scattered electrons yields detailed information on nuclear structure

Snapshot of proton wave function within the nucleus:

Fermi motion
Radius and depth of potential well

At high energy, constituents seen as free particles upon nuclear breakup

The ‘Nuclear Parton Model’

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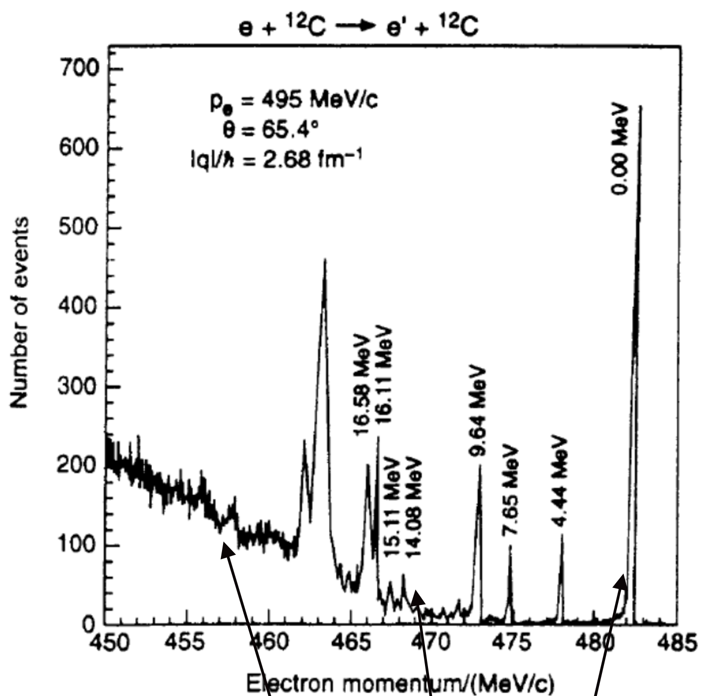
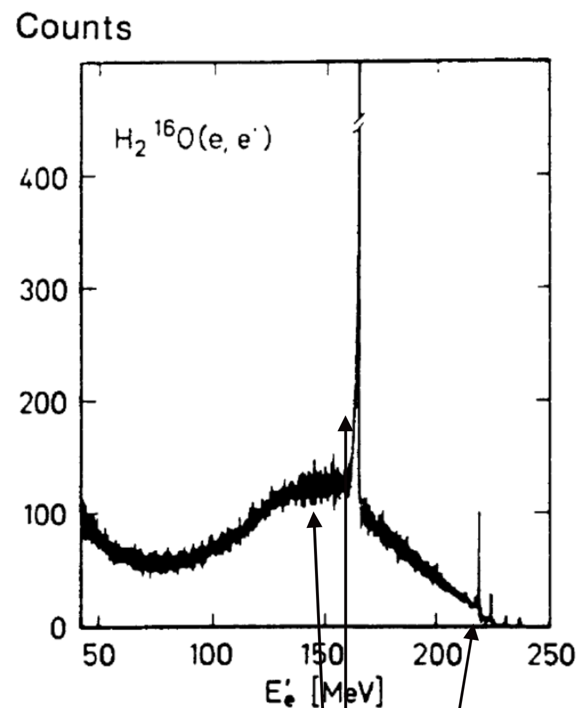


Scattering off ^{12}C
Elastic scattering off the *whole nucleus*
Nuclear levels excitation (inelastic)
Beginning of elastic scattering off *protons*



Scattering off water
Elastic scattering off the *whole ^{16}O nucleus*
Nuclear levels excitation (inelastic)

Elastic scattering off *^{16}O protons*
Elastic scattering off *free protons*

Particle-Particle Scattering

1st order Transition amplitude:

$$H' = j^{\mu} A_{\mu} \rightarrow H' = \left(j_1^{\mu} + j_2^{\mu} \right) A_{\mu}$$

$$\rightarrow M_{fi} = i(2\pi)^4 \delta\left(p_1 + p_2 - \left(p_1^{'} + p_2^{'}\right)\right) T_{fi} = i(2\pi)^4 \delta\left(p_1 + p_2 - \left(p_1^{'} + p_2^{'}\right)\right) j_{\mu}^{(1)} \frac{ig^{\mu\nu}}{q^2} j_{\nu}^{(2)}$$

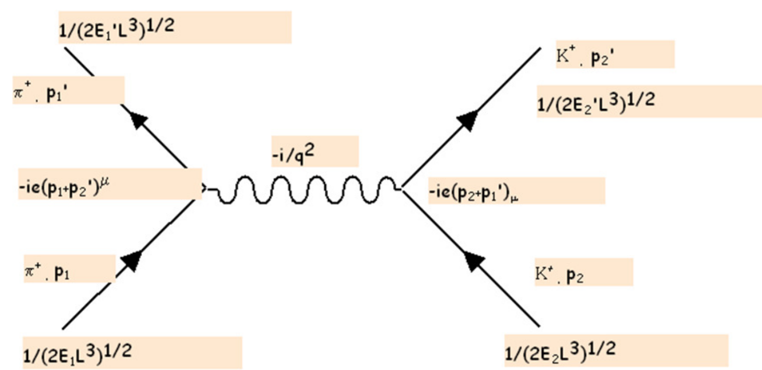
$q = p_1 - p_1' = p_2 - p_2'$ 4-momentum transfer

Transition currents:

$j^{\mu} = e\left(p_1 + p_3\right)^{\mu}$ scalar

$j^{\mu} = e\overline{u}_4 \gamma^{\mu} u_2$ fermion

....



Scattering Spin 0 – Spin ½

Just take the two currents as defined before: Another extension of Rutherford cross section
Includes *target recoil*

$$T_{fi} = e\bar{u}'\gamma^{\mu}u\frac{g_{\mu\nu}}{q^2}e\left(p+p'\right)^{\nu} \rightarrow d\sigma = \frac{1}{4EE'\mathrm{v}}\left|T_{fi}\right|^2 (2\pi)^4 \delta^4\left(p_1^{'}+p_2^{'}-p_1-p_2\right)\frac{1}{(2\pi)^3}\frac{1}{(2\pi)^3}\frac{d^3\mathbf{p}_1^{'}}{2E_1^{'}}\frac{d^3\mathbf{p}_2^{'}}{2E_2^{'}}$$

$$\frac{1}{2}\sum_{s=-1/2}^{+1/2}\sum_{s'=-1/2}^{+1/2}\left|T_{fi}\right|^2 = \left(\frac{e^2}{q^2}\right)^2\frac{1}{2}\sum_{s,s'=-1/2}^{+1/2}\bar{u}\left(p_1',s'\right)\gamma^{\mu}u\left(p_1,s\right)\bar{u}\left(p_1,s\right)\gamma^{\nu}u\left(p_1',s'\right)\left(p_2+p_2'\right)_{\mu}\left(p_2+p_2'\right)_{\nu}$$

By defining…

$$T_{\mu\nu} = \left(p_2+p_2'\right)_{\mu}\left(p_2+p_2'\right)_{\nu}$$

$$L^{\mu\nu} = 2\left[p_1^{'\mu}p_1^{\nu}+p_1^{'\nu}p_1^{\mu}+\frac{q^2}{2}g^{\mu\nu}\right]$$

…it can be shown that ⟵ ;-) Not really difficult, just a bit long

$$\frac{d\sigma}{dq^2} = \frac{2\alpha^2}{\left(p_1+p_2\right)^2 q^4}\left[2\left(p_1\cdot p_2\right)\left(p_1\cdot p_2'\right)+\frac{q^2}{2}M^2\right]$$ Invariant cross-section

$$\left.\frac{d\sigma}{d\Omega}\right|_{LAB} \underset{E\gg m}{\simeq} \frac{\alpha^2}{4\left|\mathbf{p}_1\right|^2\sin^4\theta/2}\cos^2\frac{\theta}{2}\frac{\left|\mathbf{p}_1'\right|}{\left|\mathbf{p}_1\right|}$$ LAB = "2" rest frame

The π Form Factor - I

Consider electron-pion scattering: The π is not a point-like object…
What are we to take for the pion current?
Must build a 4-vector operator
Some guesswork:
1) *Lorentz invariance*

$$\left.\begin{array}{ll} p_2, p_2', q & \text{Three 4-momentum vectors} \\ p_2' = p_2 + q & \text{Constraint} \end{array}\right\} \rightarrow \text{2 independent}$$

Choose:

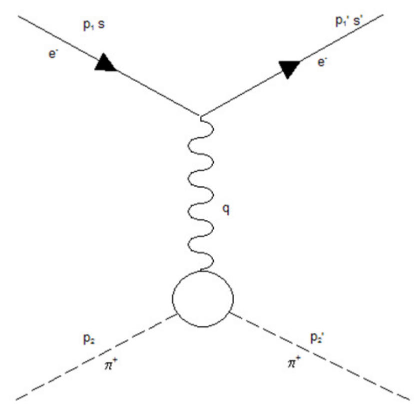
$$\left.\begin{array}{l} p_2' + p \\ p_2' - p = q \end{array}\right\} \text{Both can contribute to the current}$$

Only one independent 4-scalar:

E.g. $(p_2')^2 = (p_2)^2 = m^2 \rightarrow p_2 \cdot p_2'$

Choose instead q^2

$$\rightarrow j^{\mu}_{(\pi)} = e\left[F\left(q^2\right)\left(p' + p\right)^{\mu} + G\left(q^2\right)q^{\mu}\right]e^{-iq\cdot x}$$



Blob indicating a non-QED vertex:
The pion is an extended object

The π Form Factor - II

2) *Gauge Invariance*
Charge conservation ↔ Current must be *divergenceless*

$$\partial_\mu j^\mu = 0 \rightarrow \partial_\mu j_\pi^\mu = e\partial_\mu \left[F\left(q^2\right)\left(p'+p\right)^\mu + G\left(q^2\right)q^\mu \right] e^{-iq\cdot x}$$

$$= -iq_\mu e\left[F\left(q^2\right)\left(p'+p\right)^\mu + G\left(q^2\right)q^\mu \right] e^{-iq\cdot x} = 0$$

$$\rightarrow \partial_\mu j^\mu = 0 \Rightarrow q_\mu j^\mu = 0$$

$$q_\mu \left[F\left(q^2\right)\left(p_2 + p_2'\right)^\mu + G\left(q^2\right)q^\mu \right] = 0$$

$$\left.\begin{array}{l} q_\mu \left(p_2 + p_2'\right)^\mu = \left(p_2 - p_2'\right)_\mu \left(p_2 + p_2'\right)^\mu = 0 \\ q_\mu q^\mu \neq 0 \end{array}\right\} \rightarrow G\left(q^2\right) = 0$$

$$\rightarrow j^\mu = e\left(p_2 + p_2'\right)^\mu F\left(q^2\right)$$

Just *one* form factor for a scalar particle like the π

The π Form Factor - III

What is $F(q^2)$?
In the CM frame:

$$q^2 = (E' - E, \mathbf{p}' - \mathbf{p})^2 = (E' - E)^2 - (\mathbf{p}' - \mathbf{p})^2 = 0 - \mathbf{q}^2 = -|\mathbf{q}|^2$$

$$\rightarrow F_{scatt}(q^2) = F_{scatt}(|\mathbf{q}|^2)$$

Again, Fourier transform of the charge distribution

If crossing is good, can extend to the reaction

$$e^+ + e^- \rightarrow \pi^+ + \pi^-$$

$$q^2 = (E_1 + E_2, \boldsymbol{p}_1 + \boldsymbol{p}_2)^2 = (E_1 + E_2)^2 - (\boldsymbol{p}_1 + \boldsymbol{p}_2)^2$$

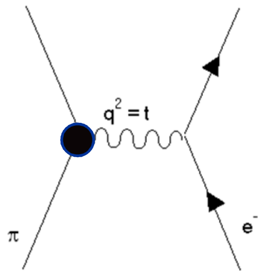
$$q^2 = E_{CM}^2$$

$$\rightarrow F_{annihil}(q^2) = F_{annihil}(E_{CM}^2)$$

$$\rightarrow F(q^2) = \begin{cases} F_{scatt}(q^2), q^2 < 0 \\ F_{annihil}(q^2), q^2 > 0 \end{cases}$$

Experiments – Space-like

π scattering off electrons

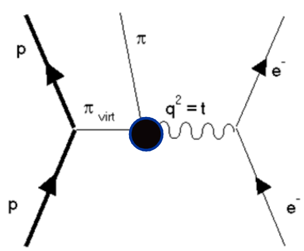


$$p^* = \frac{p_\pi m_e}{\sqrt{s}} \to t_{\max} = -4\frac{p_\pi^2 m_e^2}{s} = -4\frac{p_\pi^2 m_e^2}{m_\pi^2 + m_e^2 + 2E_\pi m_e} \approx -4\frac{p_\pi^2 m_e^2}{m_\pi^2 + 2E_\pi m_e}$$

$$\to t_{\max} \underset{p_\pi \to \infty}{\sim} -4\frac{p_\pi m_e^2}{2m_e} = -2p_\pi m_e$$

Unappealing, t_{max} too small

Electroproduction of one π



OK

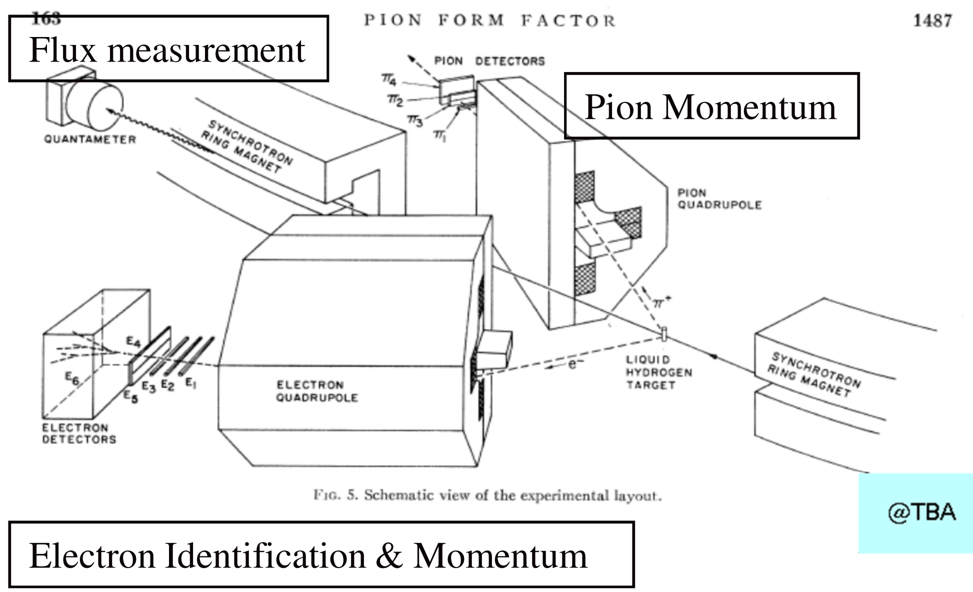
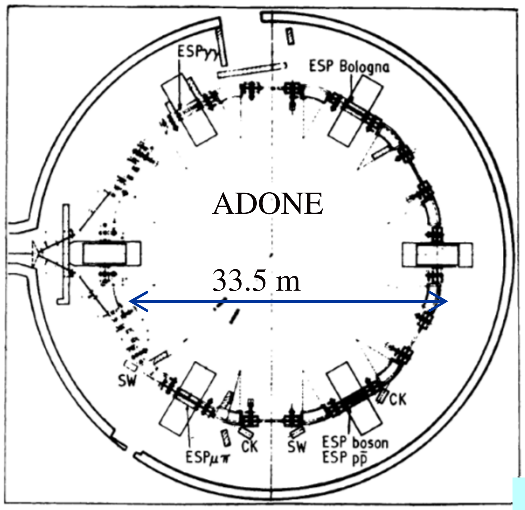


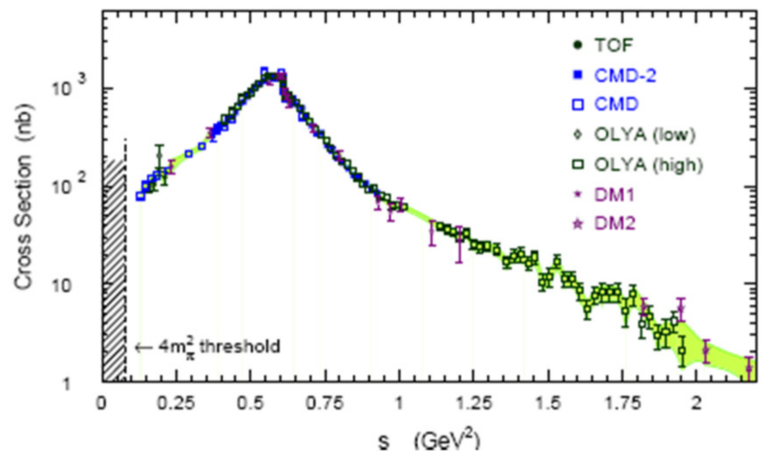
FIG. 5. Schematic view of the experimental layout.

Experiments – Time-like



@TBA

First e^+ - e^- colliding beams
ADONE – Frascati, 1967 etc.



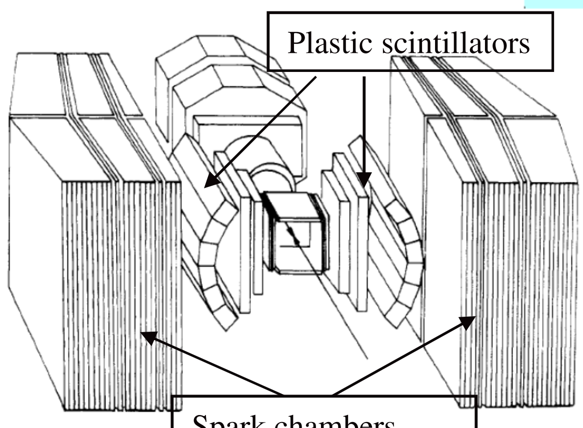
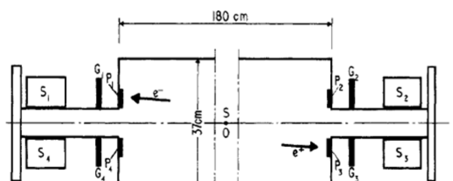


Figure 4. BCF apparatus.



Luminosity monitor measures Bhabha scattering rate
at small angles: α = acceptance

$Rate_{Bhabha} = \sigma_{Bhabha} \cdot \alpha \cdot Luminosity$

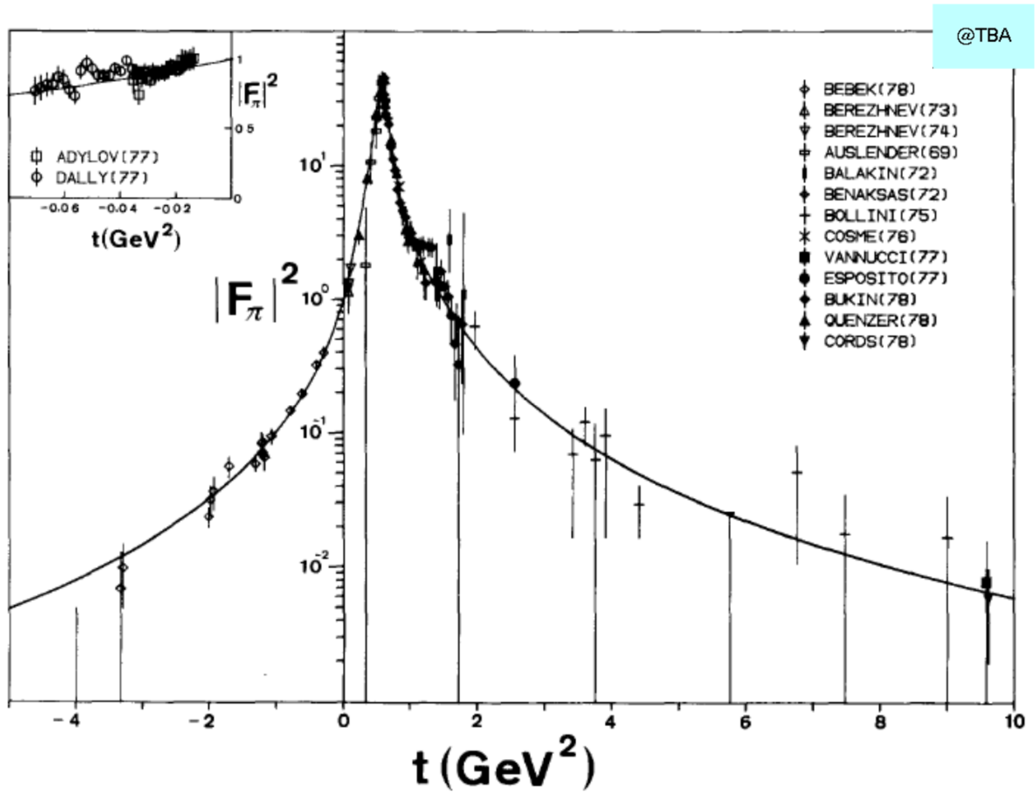
$e^+ + e^- \to e^+ + e^-$ pure QED at low energy, small angle

$\to$ accurate, reliable σ_{Bhabha} prediction

$\to Luminosity = Rate_{Bhabha} / (\sigma_{Bhabha} \cdot \alpha)$

The π Form Factor at Large

Is there a unique function $F(q^2)$?? Yes!
Good check of crossing



Electron Form Factors - I

$j^\mu = e\bar{\psi}\gamma^\mu\psi$ Dirac current

$\bar{u}(p')\gamma^\mu u(p) = \frac{1}{2m}\left[(p+p')^\mu + i\sigma^{\mu\nu}(p-p')_\nu\right]$ Gordon's identity

$$\begin{cases} \frac{e}{2m}u(p')(p+p')^\mu u(p) & \text{charge, like a scalar particle} \\ \frac{ie}{2m}\bar{u}(p')\sigma^{\mu\nu}\underbrace{(p-p')_\nu}_{=q_\nu}u(p) & \text{extra term} \end{cases}$$

Extra term due to *magnetic dipole current.*
Indeed, it contributes the interaction energy:

$$\frac{ie}{2m}\bar{u}(p')\sigma^{\mu\nu}u(p)q_\nu A_\mu \underset{\text{low speed}}{\rightarrow} -\frac{e}{2m}\phi'^\dagger\boldsymbol{\sigma}\cdot(\mathbf{q}\times\mathbf{A})\phi \qquad \text{Magnetic dipole interaction energy}$$

$$\frac{e}{2m}\phi'^\dagger\boldsymbol{\sigma}\cdot(\nabla\times\mathbf{A})\phi \equiv \frac{e}{2m}\phi'^\dagger(\boldsymbol{\sigma}\cdot\mathbf{B})\phi \Rightarrow \frac{ie}{2m}\bar{u}(p')\sigma^{\mu\nu}u(p)q_\nu A_\mu \underset{\text{low speed}}{\rightarrow} -\frac{e}{2m}\phi'^\dagger(\boldsymbol{\sigma}\cdot\mathbf{B})\phi$$

$\mu \approx \frac{e\hbar}{2mc}$ Magnetic moment , $j = \frac{1}{2}\hbar$ Spin, $\gamma \equiv \frac{\mu}{j}$ Gyromagnetic ratio

$\gamma \approx \frac{e\hbar}{2mc}\frac{2}{\hbar} \underset{\text{natural units}}{=} \frac{e}{2m}\cdot 2,$ Define $\gamma \equiv g\frac{e}{2m} \rightarrow g \approx 2$ Dirac g-factor

Electron Form Factors - II

Now: g-factor not exactly 2, as predicted by Dirac equation
Reason: *Radiative corrections*
Largest correction: Anomalous magnetic moment

$$\mu_{Dirac} = \frac{e}{2m} \to \mu = \frac{e}{2m}\left(1+\kappa_e\right)$$

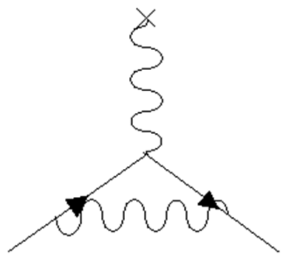
$$j^\mu = \frac{e}{2m}\bar{u}\left(p'\right)\left[\left(p+p'\right)^\mu + i\sigma^{\mu\nu}\left(1+\kappa_e\right)q_\nu\right]u\left(p\right)$$

$$\bar{u}\left(p'\right)\gamma^\mu u\left(p\right) = \frac{1}{2m}\bar{u}\left(p'\right)\left[\left(p+p'\right)^\mu + i\sigma^{\mu\nu}q_\nu\right]u\left(p\right)$$

$$\to \bar{u}\left(p'\right)\left(p+p'\right)^\mu u\left(p\right) = \bar{u}\left(p'\right)\left(\gamma^\mu - i\sigma^{\mu\nu}q_\nu\right)u\left(p\right)$$

$$\to j^\mu = \frac{e}{2m}\bar{u}\left(p'\right)\left[\gamma^\mu - i\sigma^{\mu\nu}q_\nu + i\sigma^{\mu\nu}\left(1+\kappa_e\right)q_\nu\right]u\left(p\right)$$

$$\to j^\mu = \frac{e}{2m}\bar{u}\left(p'\right)\left[\gamma^\mu + i\kappa_e\sigma^{\mu\nu}q_\nu\right]u\left(p\right)$$



In spite of the electron *being* a pointlike fermion, radiative corrections make it behaving like an extended object

Further radiative corrections lumped into 2 *form factors*

$$j^\mu = \frac{e}{2m}\bar{u}\left(p'\right)\left[f\left(q^2\right)\gamma^\mu + g\left(q^2\right)i\kappa_e\sigma^{\mu\nu}q_\nu\right]u\left(p\right)$$ Most general form

g-2 - Theory

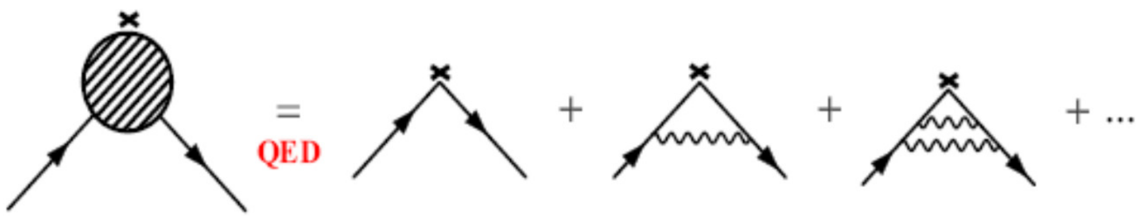
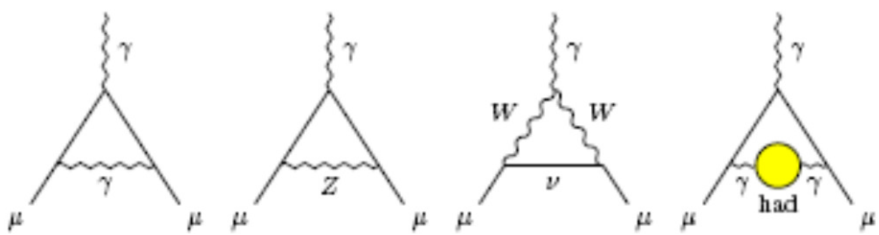


Figure 1: The perturbative expansion of $\Gamma^{\rho}(p', p)$ in single flavour QED. The tree graph gives $F_1 = 1$, $F_2 = F_3 = 0$. The one loop vertex correction graph gives the coefficient A_1 in Eq. (2.21). The cross denotes the insertion of the external field.

QED only…

@TBA



QED Electroweak QCD

Contributions from new physics ?

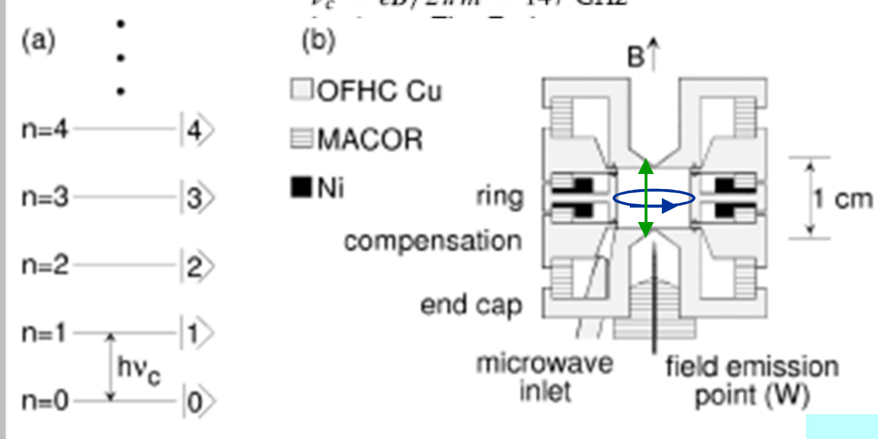
???

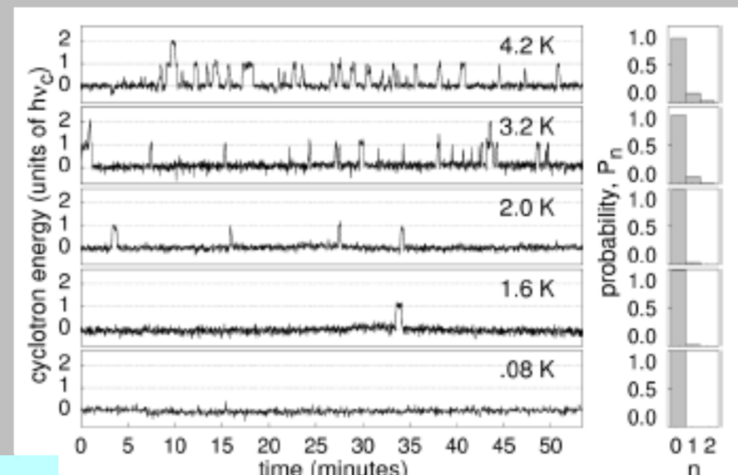
..and more

Electron *g-2*: Quantum Cyclotron

Clever use of magnetic and electric fields at low temperature: The *Penning Trap*

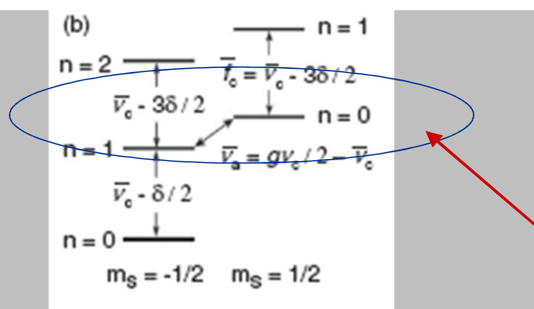
$\nu_c = eB/2\pi m = 147$ GHz

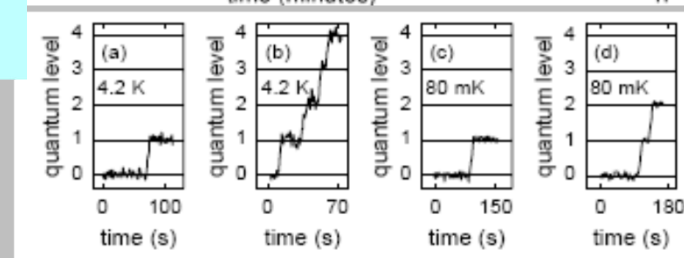




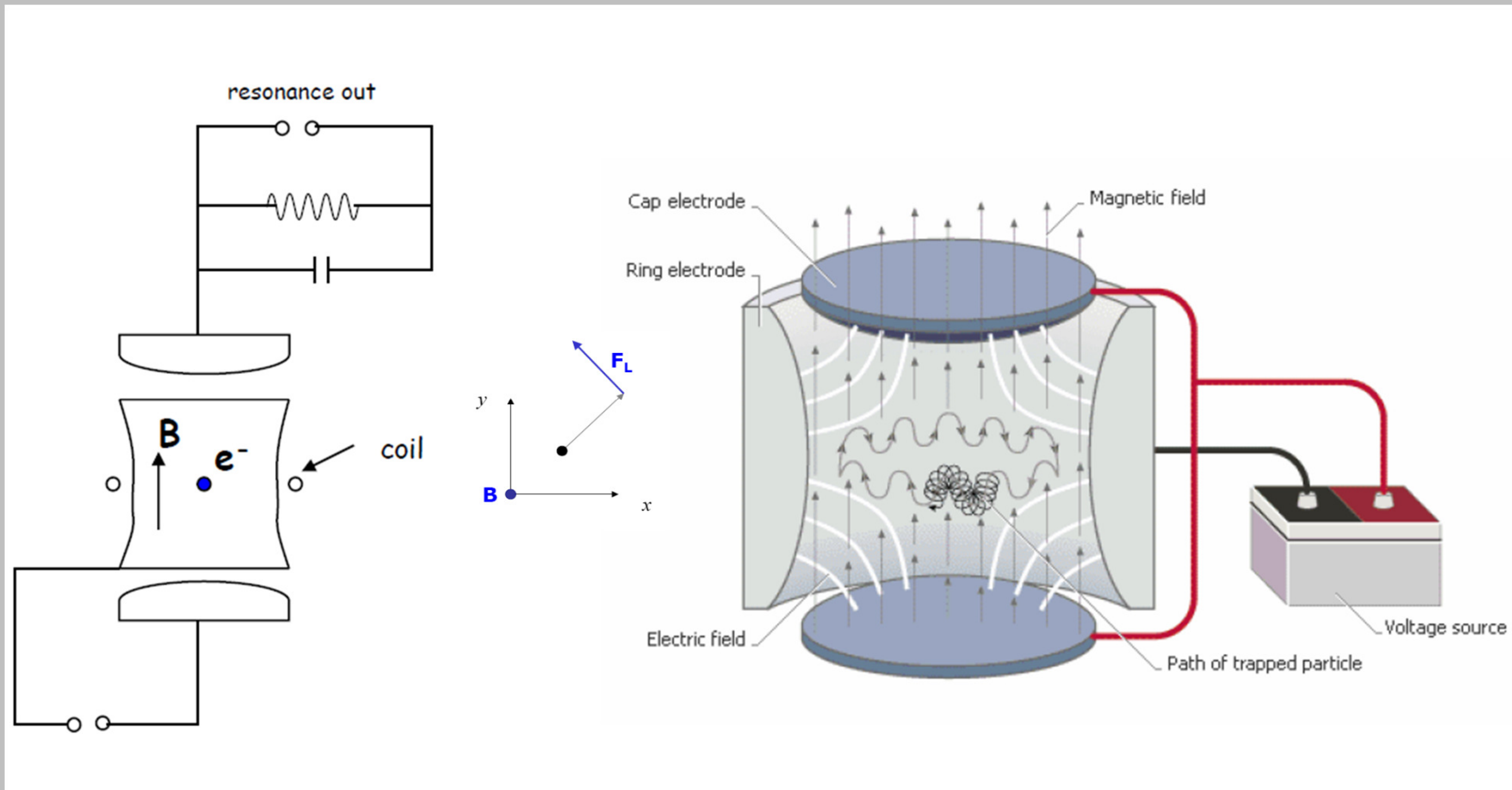
$$E(n, m_s) = \frac{g}{2}h\nu_c m_s + \left(n + \frac{1}{2}\right)h\bar{\nu}_c - \frac{1}{2}h\delta\left(n + \frac{1}{2} + m_s\right)^2$$

@TBA





The Penning Trap



Penning Trap - Electrostatics

$$V(x,y,z) = V_0\left(\frac{x^2+y^2}{r_0^2} - \frac{z^2}{z_0^2}\right)$$

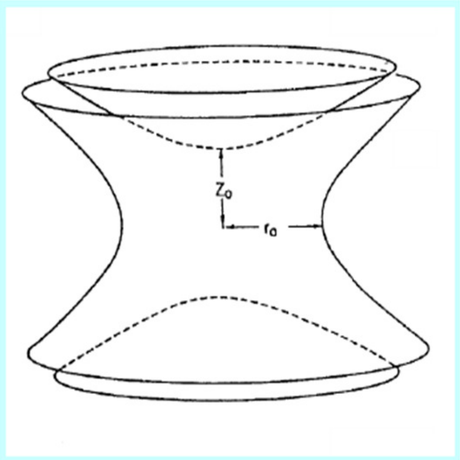
Equipotential surface:

$$\rightarrow \frac{x^2+y^2}{r_0^2} - \frac{z^2}{z_0^2} = \pm 1 \text{ Hyperboloid of one, two sheets}$$

$$\rightarrow \mathbf{E} = -\nabla V$$

$$z_0 = \frac{r_0}{\sqrt{2}}$$

$$U = -eV_0\left(\frac{r^2}{r_0^2} - \frac{z^2}{z_0^2}\right) \rightarrow \begin{matrix} r: \text{Inverted harmonic potential} \\ z: \text{Harmonic potential} \end{matrix}$$



Confining in *z*, not in *r:*

→ *Add uniform magnetic field along z*

Muon *g*-2: Experimental Method

$\mu \approx \frac{e\hbar}{2mc}$ Magnetic moment , $j = \frac{1}{2}\hbar$ Spin, $\gamma \equiv \frac{\mu}{j}$ Gyromagnetic ratio

$$\gamma \approx \frac{e\hbar}{2mc}\frac{2}{\hbar} \underset{\text{natural units}}{=} \frac{e}{2m}\cdot 2 \equiv g\,\frac{e}{2m} \to g \approx 2$$

$a \equiv \frac{g-2}{2} \ll 1$ Anomaly

For a charged particle moving in a uniform magnetic field:

$\frac{d\boldsymbol{p}}{dt} = e\left(\boldsymbol{p} \times \boldsymbol{B}\right) \to \omega_c = \frac{eB}{m}$ Cyclotron frequency

For a particle carrying a magnetic moment ***m***:

$\left.\begin{aligned}\boldsymbol{\mu} &= \gamma \boldsymbol{J}\\ \boldsymbol{\tau} &= \boldsymbol{\mu}\times\boldsymbol{B}\\ \boldsymbol{\tau} &= d\boldsymbol{J}/dt\end{aligned}\right\} \to \frac{d\boldsymbol{\mu}}{dt} = \gamma\left(\boldsymbol{\mu}\times\boldsymbol{B}\right)$ Precession of $\boldsymbol{\mu}$ around $\boldsymbol{B}$ in the particle rest frame

$\omega_s = \frac{2\mu B}{\hbar} = g\,\frac{eB}{2m} = \left(1+a\right)\frac{eB}{m}$ Precession frequency

Get *a* by measuring the *beat frequency* ω_s-ω_c Lorentz invariant!

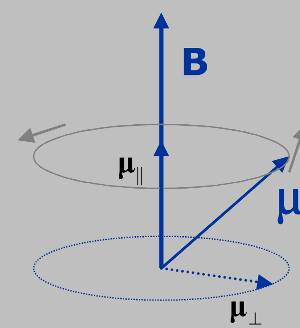
Larmor Precession

Just a short reminder of the classical result:

$$\frac{d\boldsymbol{\mu}}{dt}=\gamma\left(\boldsymbol{\mu}\times\boldsymbol{B}\right)\rightarrow\boldsymbol{B}\cdot\frac{d\boldsymbol{\mu}}{dt}=\gamma\boldsymbol{B}\cdot\left(\boldsymbol{\mu}\times\boldsymbol{B}\right)=0\rightarrow\boldsymbol{\mu}_{\parallel}=\text{const}$$

Say $\boldsymbol{B}=B\hat{\boldsymbol{k}}\rightarrow\ \boldsymbol{\mu}_{\perp}=\mu_x\hat{\boldsymbol{i}}+\mu_y\hat{\boldsymbol{j}}$

$$\rightarrow\frac{d\boldsymbol{\mu}_{\perp}}{dt}=\gamma\left(\boldsymbol{\mu}_{\perp}\times\boldsymbol{B}\right)\rightarrow\begin{cases}\dfrac{d\mu_x}{dt}=\gamma\left(\boldsymbol{\mu}_{\perp}\times\boldsymbol{B}\right)_x=+\gamma\mu_y B\\ \dfrac{d\mu_y}{dt}=\gamma\left(\boldsymbol{\mu}_{\perp}\times\boldsymbol{B}\right)_y=-\gamma\mu_x B\end{cases}$$

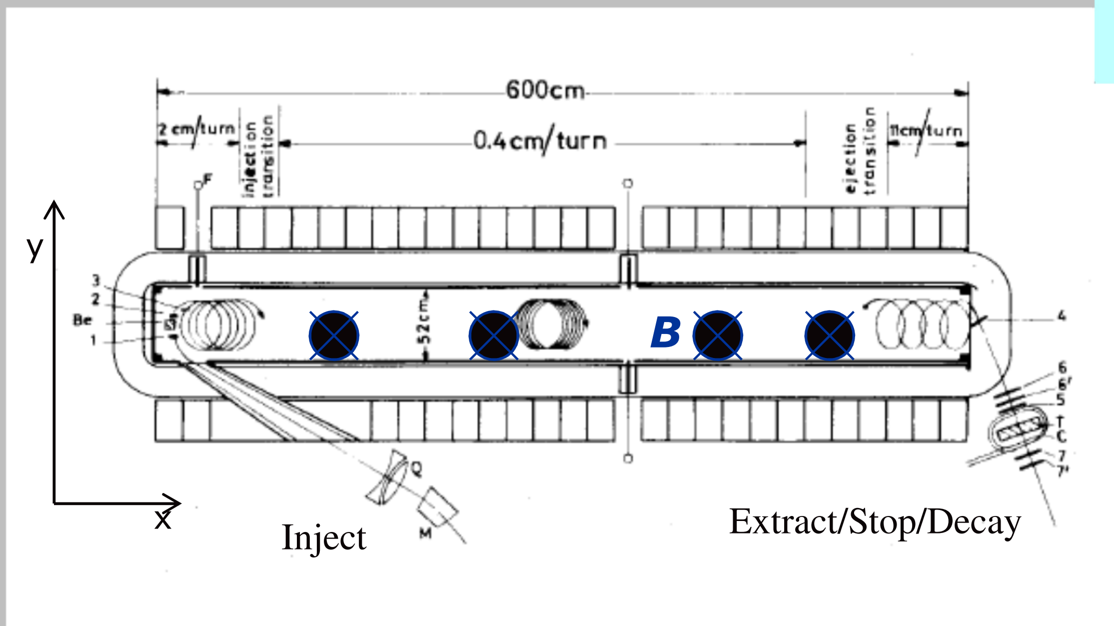


$$\tilde{\mu}\equiv\mu_x+i\mu_y\rightarrow\begin{cases}\dfrac{d\mu_x}{dt}=+\gamma\mu_y B\\ i\dfrac{d\mu_y}{dt}=-i\gamma\mu_x B\end{cases}\rightarrow\frac{d\mu_x}{dt}+i\frac{d\mu_y}{dt}=+\gamma B\left(\mu_y-i\mu_x\right)=-i\gamma B\left(\mu_x+i\mu_y\right)$$

$$\rightarrow\frac{d\tilde{\mu}}{dt}=-i\gamma B\tilde{\mu}\rightarrow\tilde{\mu}\left(t\right)=\tilde{\mu}_0 e^{-i\gamma Bt}\rightarrow\begin{cases}\mu_x\left(t\right)=\mu_{x0}\cos\gamma Bt\\ \mu_y\left(t\right)=-\mu_{y0}\sin\gamma Bt\end{cases}$$

The First Muon *g-2* at CERN

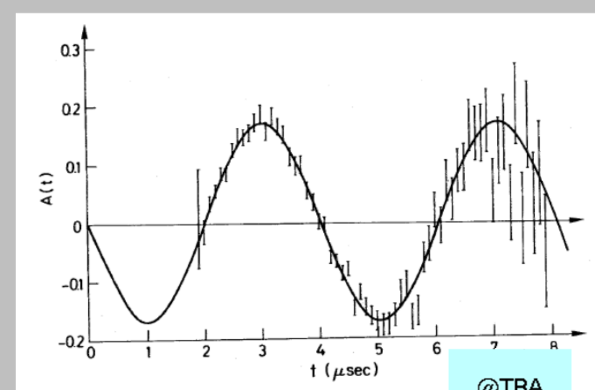
Small ***B*** *y*-gradient allowing for orbit drift along *x*



@TBA

Stopping μ^+ weakly decays to $\overline{\nu}_\mu e^+ \nu_e$ with $\tau = 2.2\ \mu s$
Parity violation enhances decays where positron is emitted along $s_\mu \rightarrow$ Polarization direction can be measured
By counting

$$\frac{N_{e^+}^{forward} - N_{e^+}^{backward}}{N_{e^+}^{forward} + N_{e^+}^{backward}} \propto \text{spin rotation angle } = (1+a)\frac{eB}{mc}\gamma t$$



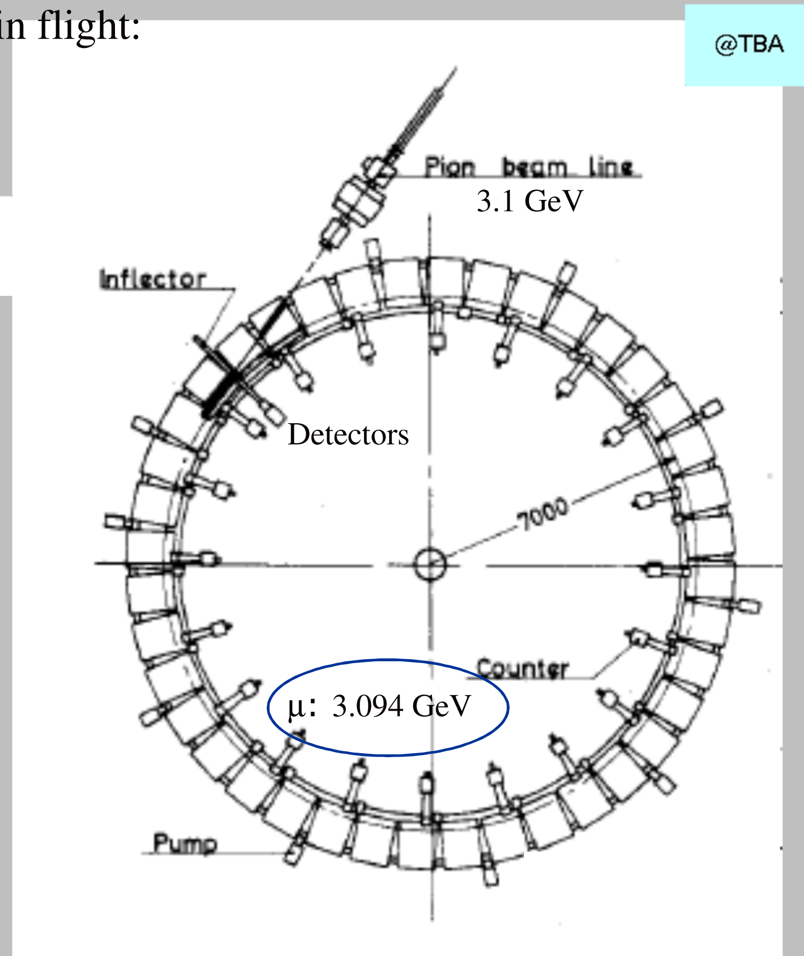
@TBA

An Improved *g-2* at CERN

Detect high energy electrons from μ's decaying in flight:
Forward-emitted in the μ rest frame

$$N(t) = N_0 e^{-t/\tau} \{1 - A \sin(2\pi f_a t + \phi)\}$$

$$\omega_a = a \frac{eB}{mc} \gamma$$



Muon momentum: 3.094 GeV

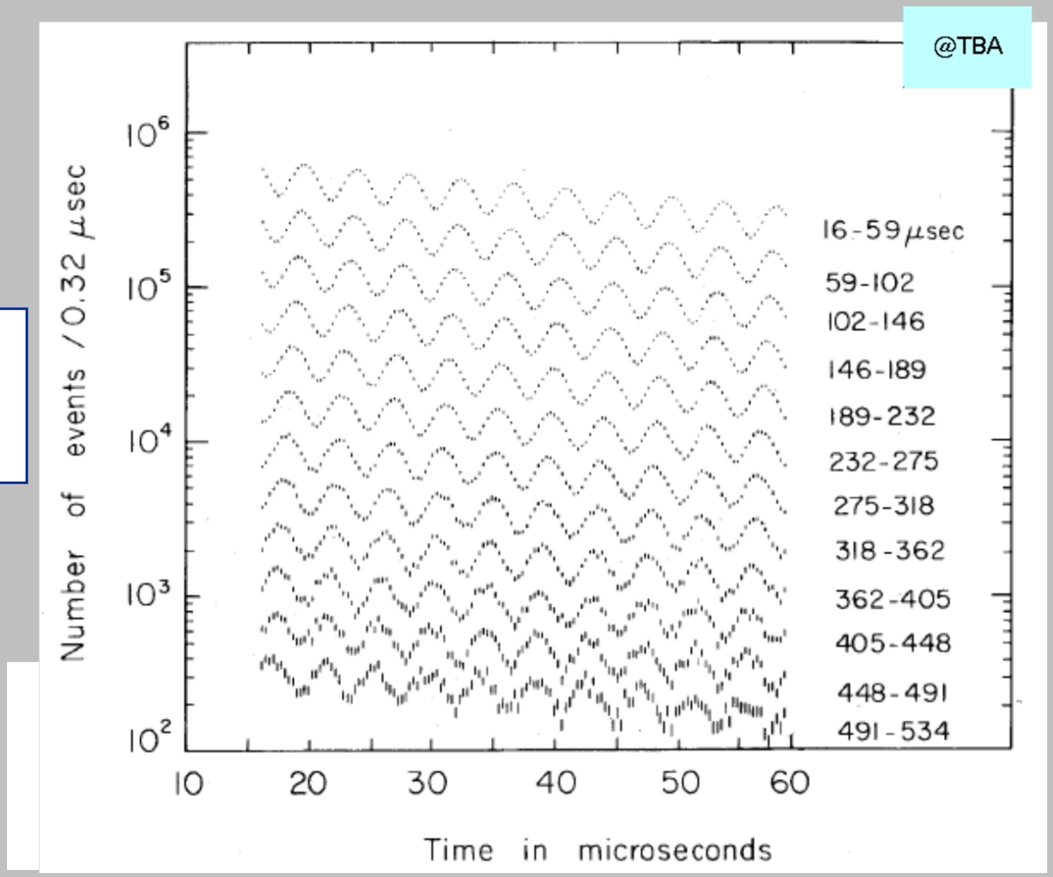
Why this momentum?

'Magic' cancellation of spurious effects (*E*-field appearing in the μ rest frame from Lorentz-transformed *B*-field in the LAB frame)

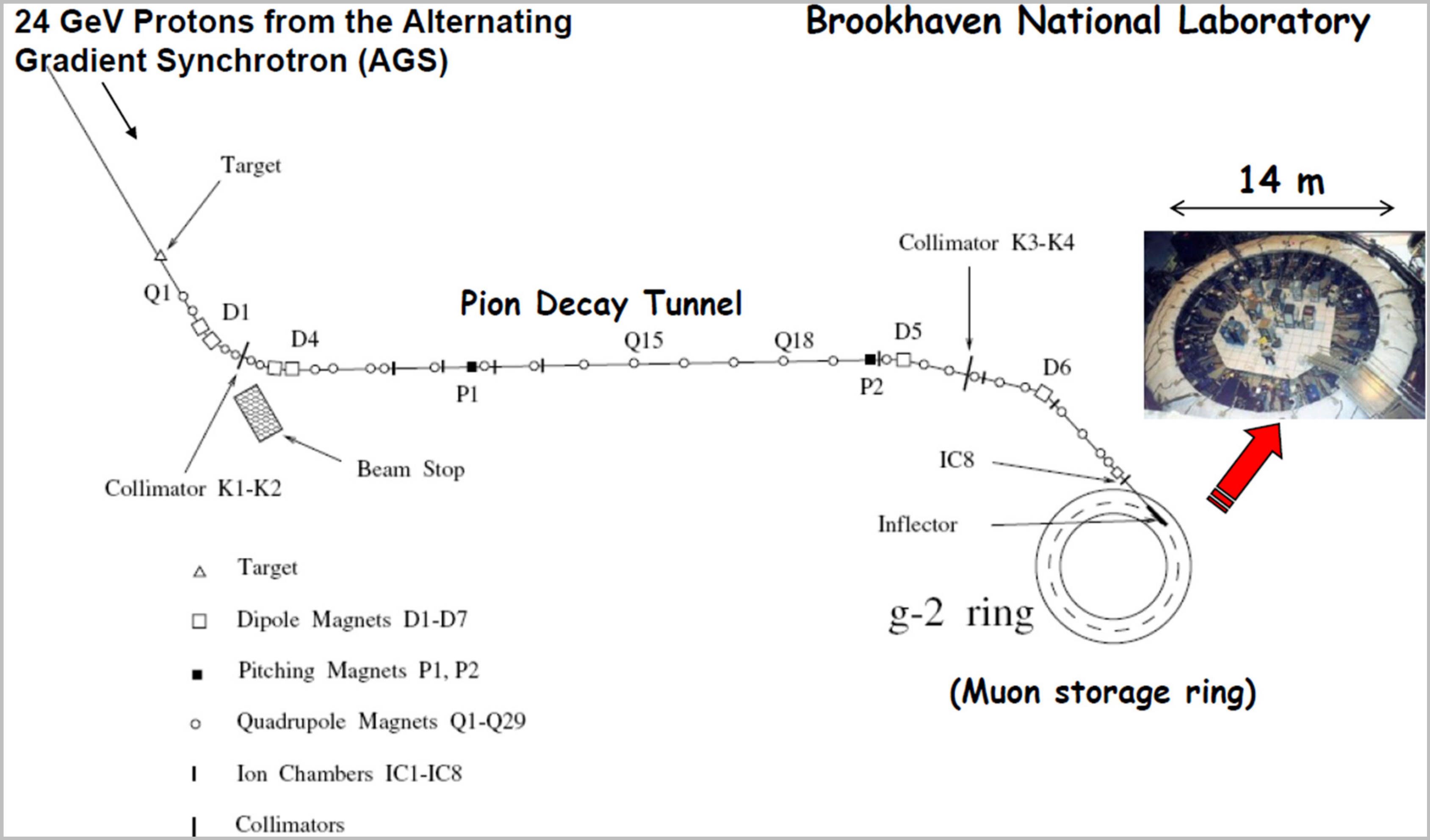
Astonishing Precision…

$$a_{\mu^+} = 1\ 165\ 911\ (11) \times 10^{-9} \quad (10\ \text{ppm})$$

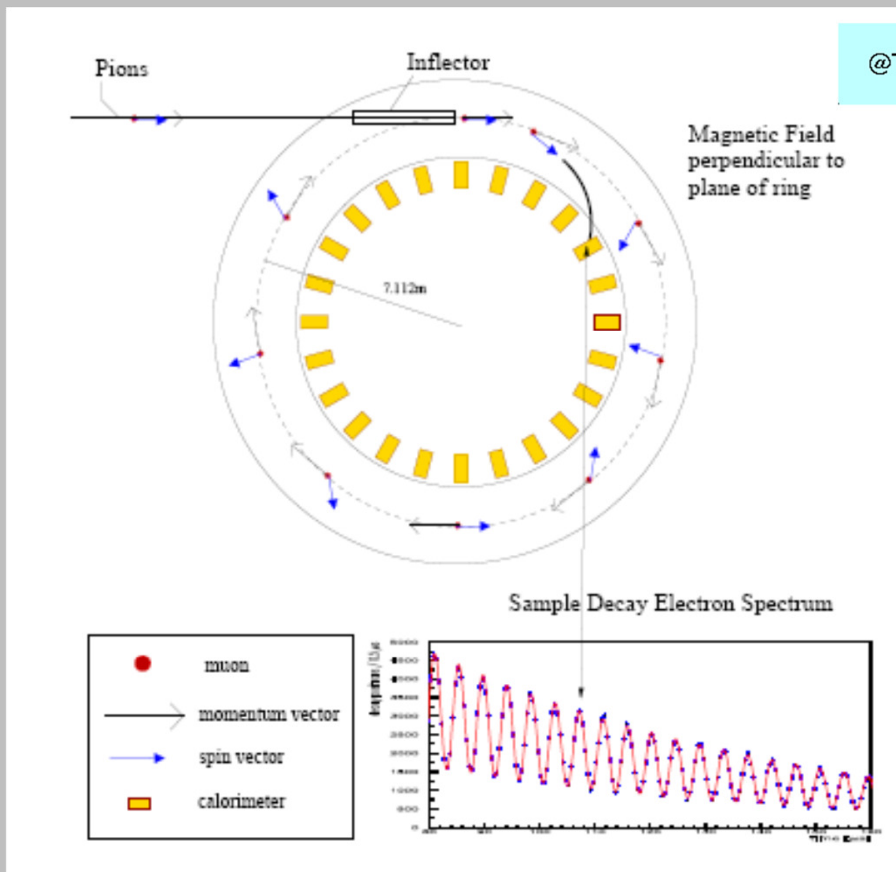
$$a_{\mu^-} = 1\ 165\ 937\ (12) \times 10^{-9} \quad (10\ \text{ppm})$$



The Latest Muon *g-2* at BNL - I



The Latest Muon *g-2* at BNL - II



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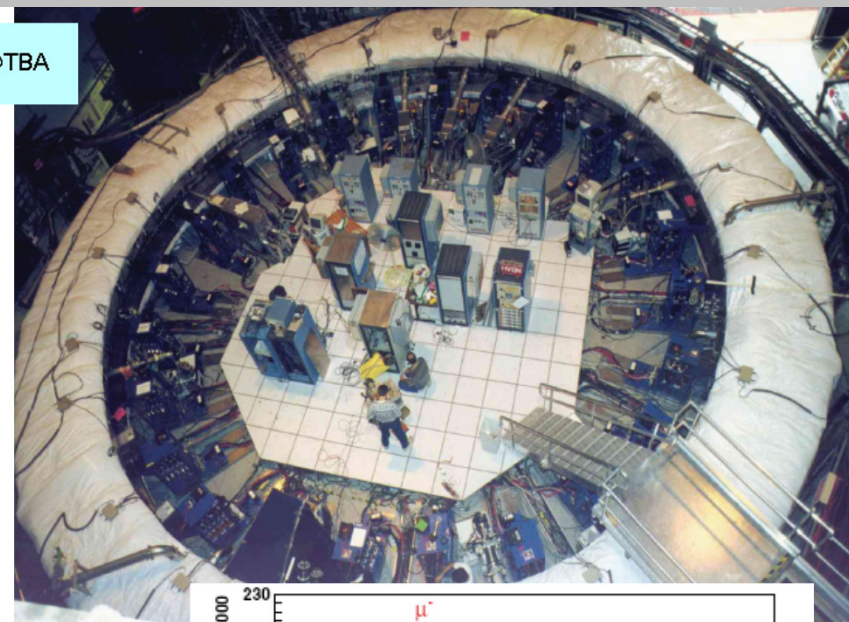
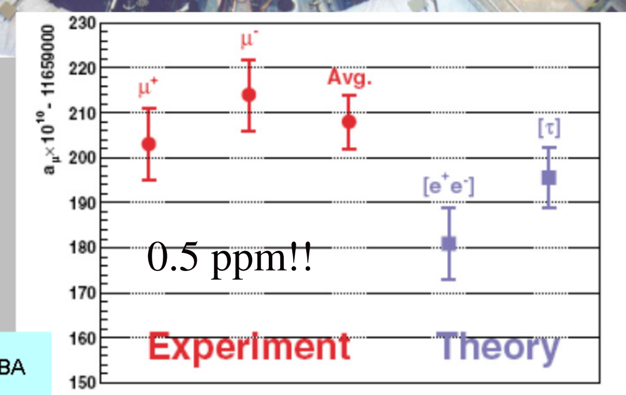




@TBA

Scattering Spin ½ - Spin ½

Just to simplify things, take *different* spin ½ particles (e.g. electron-muon scattering)

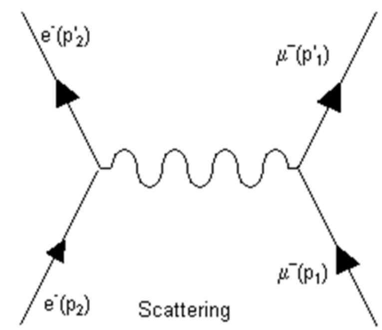
$$T_{fi}(s,s',r,r') = e\bar{u}'(p_2',s')\gamma^\mu u(p_2,s)\frac{g_{\mu\nu}}{q^2}\bar{u}'(p_1',r')\gamma^\nu u(p_1,r)$$

$$\frac{1}{4}\sum_{s,s',r,r'}\left|T_{fi}(s,s',r,r')\right|^2 = \frac{e^4}{q^4}L_{\mu\nu}M^{\mu\nu}$$

$$L^{\mu\nu} = 2\left[p_1'^{\mu}p_1^{\nu} + p_1'^{\nu}p_1^{\mu} + \frac{q^2}{2}g^{\mu\nu}\right]$$

$$M_{\mu\nu} = 2\left[p_{2\mu}'p_{2\nu} + p_{2\nu}'p_{2\mu} + \frac{q^2}{2}g_{\mu\nu}\right]$$

$$\rightarrow \left.\frac{d\sigma}{d\Omega}\right|_{LAB} \underset{E\gg m}{\simeq} \frac{\alpha^2\cos^2\theta/2}{4|\mathbf{p}_1|^2\sin^4\theta/2}\frac{|\mathbf{p}_1'|}{|\mathbf{p}_1|}\left(1-\frac{q^2\tan^2\theta/2}{2m_2^2}\right)$$



Yet another term…
Electron scattering off the muon *magnetic moment*

Interlude: Crossing Symmetry

Simple relationship between any pair of 2-body reactions

$a+b\to c+d$ Reaction A

$a+\underset{crossed}{[c]}\to\underset{crossed}{[b]}+d$ Reaction B

Define: Crossed particle $\equiv$ Antiparticle

By changing the 4-momentum sign of the crossed particle. the two amplitudes are identical

$$A\left[a\left(p_A\right)+b\left(p_B\right)\to c\left(p_C\right)+d\left(p_D\right)\right]=A\left[a\left(p_A\right)+\bar{c}\left(-p_C\right)\to\bar{b}\left(-p_B\right)+d\left(p_D\right)\right]$$

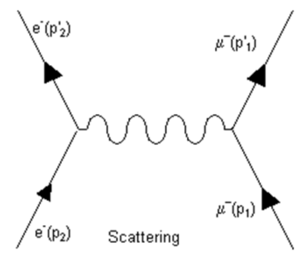
e^+e^- Annihilation to $\mu^+\mu^-$

Apply crossing symmetry to electron-muon scattering

$$e^- + \mu^- \to e^- + \mu^- \qquad \text{A: Scattering}$$

$$e^- + \underbrace{\left[e^-\right]}_{crossed} \to \underbrace{\left[\mu^-\right]}_{crossed} + \mu^- \equiv e^- + e^+ \to \mu^+ + \mu^- \qquad \text{B: Annihilation}$$

Amplitude for scattering:

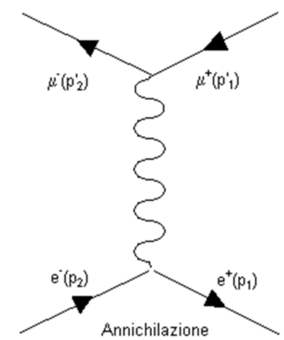


$$T_{fi}\left(s,s',r,r'\right) = (-e)\bar{u}_{(\mu)}\left(p_2',s'\right)\gamma^\mu u_{(\mu)}\left(p_2,s\right)\frac{-ig_{\mu\nu}}{q^2}(-e)\bar{u}_{(e)}\left(p_1',r'\right)\gamma^\nu u_{(e)}\left(p_1,r\right)$$

$$q = p_1 - p_1' \to q^2 = \left(p_1 - p_1'\right)^2 = p_1^2 + p_1'^2 - 2p_1 \cdot p_1'$$

$$q^2 = 2m_e^2 - 2\left(E_1E_1' - \mathbf{p}_1 \cdot \mathbf{p}_1'\right) \underset{E \gg m}{\simeq} -2\left(E_1E_1' - \mathbf{p}_1 \cdot \mathbf{p}_1'\right) < 0 \qquad q\text{=4-momentum transfer}$$

Amplitude for annihilation:



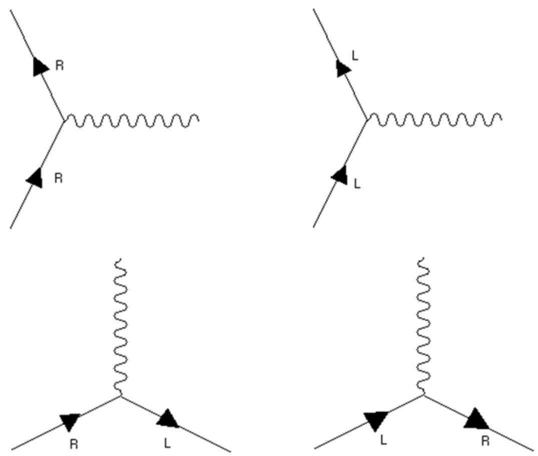
$$T_{fi}\left(s,s',r,r'\right) = (-e)\bar{u}_{(\mu)}\left(p_2',s'\right)\gamma^\mu v_{(\mu)}\left(p_1',r'\right)\frac{-ig_{\mu\nu}}{q^2}(-e)\bar{v}_{(e)}\left(p_1,s\right)\gamma^\nu u_{(e)}\left(p_2,r\right)$$

$$q = p_1 + p_2 \to q^2 = \left(p_1 + p_2\right)^2 = p_1^2 + p_2^2 + 2p_1 \cdot p_2$$

$$q^2 = 2m_e^2 + 2\left(E_1E_2 - \mathbf{p}_1 \cdot \mathbf{p}_2\right) \underset{E \gg m}{\simeq} 2\left(E_1E_2 - \mathbf{p}_1 \cdot \mathbf{p}_2\right) > 0 \qquad q\text{=total 4-momentum}$$

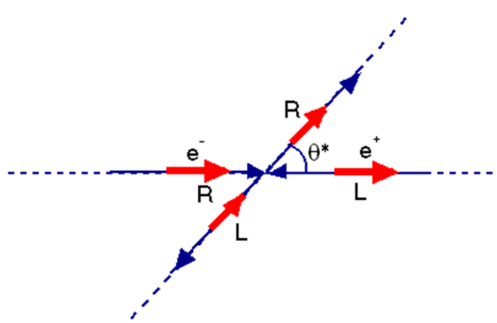
The Annihilation Cross-Section - I

Helicity conservation at high energy:
Consequence of electromagnetic field being a *vector*



Scattering: $R \rightarrow R,\ L \rightarrow L$

Annihilation: $R{+}L,\ L{+}R$



For both initial and final state:
Particle and antiparticle must have *opposite* helicity
Decompose differential cross-section into 4 pieces:

$$\frac{d\sigma}{d\Omega^*} = \frac{1}{4} \cdot 2 \left[\left.\frac{d\sigma}{d\Omega^*}\right|_{LR\rightarrow LR} + \left.\frac{d\sigma}{d\Omega^*}\right|_{RL\rightarrow RL} + \left.\frac{d\sigma}{d\Omega^*}\right|_{LR\rightarrow RL} + \left.\frac{d\sigma}{d\Omega^*}\right|_{RL\rightarrow LR} \right]$$

<u>Average</u> over initial states, <u>Sum</u> over final states

The Annihilation Cross-Section - II

Transition amplitude = Amplitude to find final particles at angle θ^* wrt to initial direction

Phase space, incident flux and normalization factors just cancel out at high energy
Matrix element:

$$T_{fi} = \frac{\alpha}{q^2 \equiv s} \cdot \text{Amplitude to find } J = 1 \text{ state rotated by } \theta^*$$

Use rotation matrices for a $J=1$ state: Take y-axis $\perp$ reaction plane

$$e^{-i\theta^* J_2} \left| J, m \right\rangle = \sum_{m'} d^J_{m,m'}\left(\theta^*\right) \left| J, m' \right\rangle, \quad d^J_{m,m'}\left(\theta^*\right) = \left\langle J, m \right| e^{-i\theta^* J_2} \left| J, m' \right\rangle$$

$$d^1_{+1,+1}\left(\theta^*\right) = d^1_{-1,-1}\left(\theta^*\right) = \frac{1}{2}\left(1 + \cos\theta^*\right)$$

$$d^1_{+1,-1}\left(\theta^*\right) = d^1_{-1,+1}\left(\theta^*\right) = \frac{1}{2}\left(1 - \cos\theta^*\right)$$

$$\left.\begin{array}{l} \left.\frac{d\sigma}{d\Omega^*}\right|_{LR\to LR} = \left.\frac{d\sigma}{d\Omega^*}\right|_{RL\to RL} = \frac{\alpha^2}{s}\left(\frac{1}{2}\right)^2 \left(1 + \cos\theta^*\right)^2 \\ \left.\frac{d\sigma}{d\Omega^*}\right|_{LR\to RL} = \left.\frac{d\sigma}{d\Omega^*}\right|_{RL\to LR} = \frac{\alpha^2}{s}\left(\frac{1}{2}\right)^2 \left(1 - \cos\theta^*\right)^2 \end{array}\right\} \to \frac{d\sigma}{d\Omega^*} = \frac{\alpha^2}{4s}\left(1 + \cos^2\theta^*\right)$$

$$\sigma = \int_{4\pi} \frac{\alpha^2}{4s}\left(1 + \cos^2\theta^*\right) d\Omega^* = \frac{4\pi\alpha^2}{3s}$$

Nucleon Form Factors

Take the same current for the nucleon

$$j_p^\mu = e\bar{u}(p')\left(F(q^2)\gamma^\mu + G(q^2)i\kappa_p\sigma^{\mu\nu}q_\nu\right)u(p)$$

$$\kappa_p = ?$$

Anomalous magnetic moment well measured, not understood

Anomaly originating from the extended shape of the proton, rather than radiative corrections

$$F_1(q^2) = F(q^2)$$

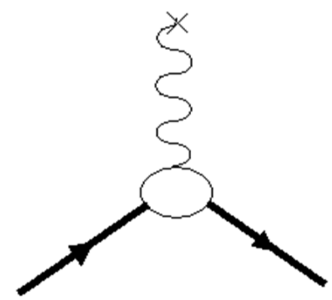
$$F_2(q^2) = 2MG(q^2)$$

$$\rightarrow j_p^\mu = e\bar{u}(p')\left(F_1(q^2)\gamma^\mu + \frac{i\kappa_p F_2(q^2)}{2M}\sigma^{\mu\nu}q_\nu\right)u(p)$$

Redefine:

$$G_E(q^2) = F_1 + \frac{\kappa_P q^2}{4M^2}F_2 \quad \text{Electric form factor}$$

$$G_M(q^2) = F_1 + \kappa_P F_2 \quad \text{Magnetic form factor}$$



Blob indicates a non-QED vertex

Nucleon Magnetic Moments

Electron-Proton comparison

$$\mu_B = \frac{e\hbar}{2m_e} = 5.78\times10^{-5} \quad eV/T$$

$$\mu = g\, s\, \mu_B$$

$$\mu_e/\mu_B = \underline{1.001\ 159}\ 652\ 187 \pm 0.000\ 000\ 000\ 004$$

$\frac{g}{2}$: Electron

$$\mu_N = \frac{e\hbar}{2m_p} = 3.15\times10^{-8} \quad eV/T$$

$$\mu_p = g\, s\, \mu_N$$

$$\mu_p/\mu_N = \underline{2.792}847351 \pm 0.000000028$$

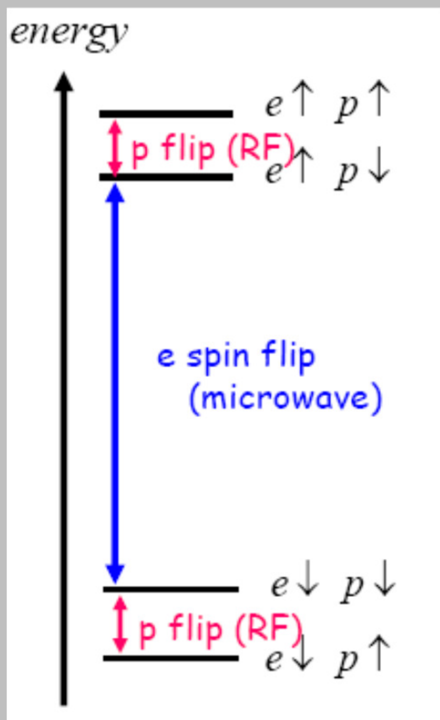
$\frac{g}{2}$: Proton

Reminder: For a free Dirac particle $g=2$

Strong indication: *The nucleon is not a point-like particle*

Proton Magnetic Moment - I

Modern method: RF+Microwave atomic spectroscopy



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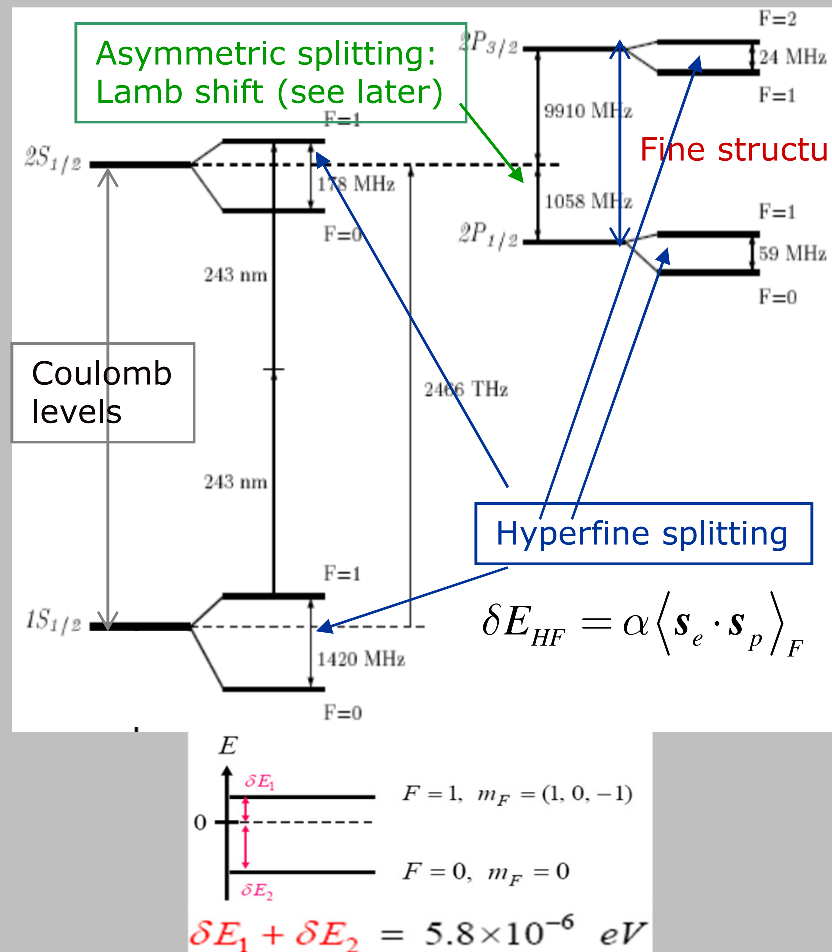


Proton Magnetic Moment - II

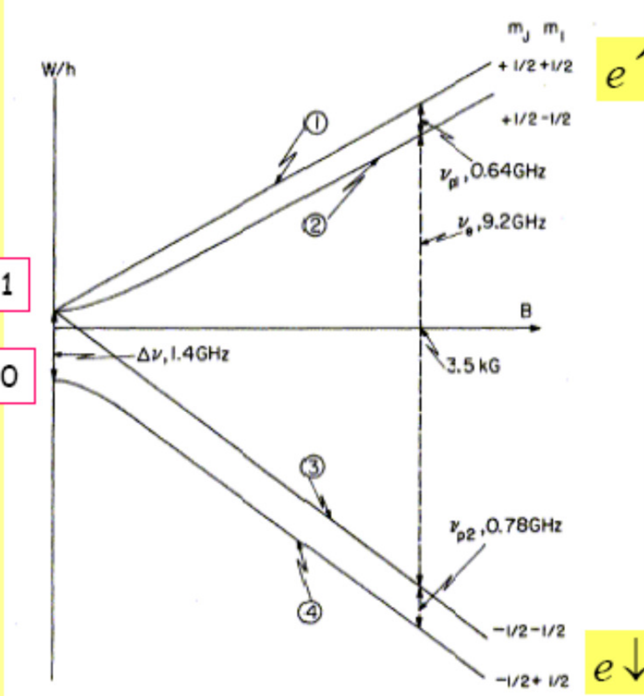


FIG. 1. Energy levels of hydrogen in the ground electronic state in an applied magnetic field.

F=1

F=0

$e\uparrow$ $p\uparrow,\ m_F = 1\ \therefore\ F = 1$

$p\downarrow,\ m_F = 0$

The electron's magnetic moment is almost 2000x larger than that of the proton and of opposite sign. The main splitting between states at large B is due to the electron.

The hyperfine interaction between the electron and proton spins still causes the F = 1 state to have higher energy. This explains the order of states in the diagram.

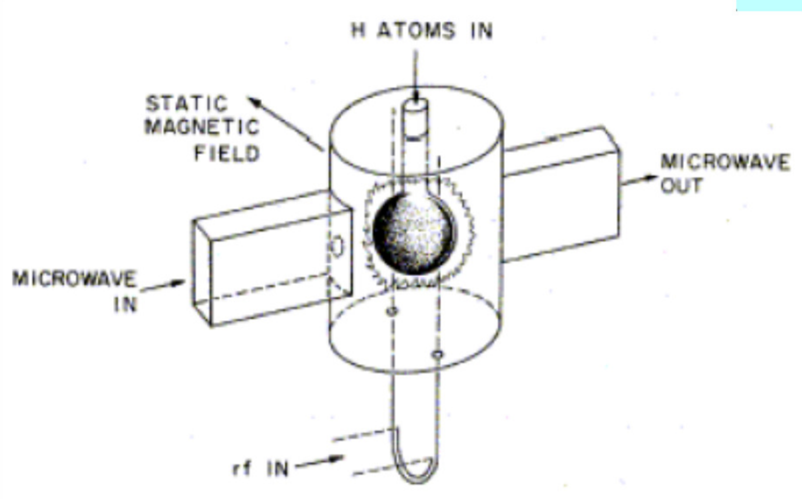
$e\downarrow$ $p\downarrow,\ m_F = -1\ \therefore\ F = 1$

$p\uparrow,\ m_F = 0$

@TBA

Proton Magnetic Moment - III

@TBA



$$\frac{\mu_{e^-}(H)}{\mu_p(H)} = -658.210\,7058(66) \quad [1.0 \times 10^{-8}],$$

Correct for atomic effects

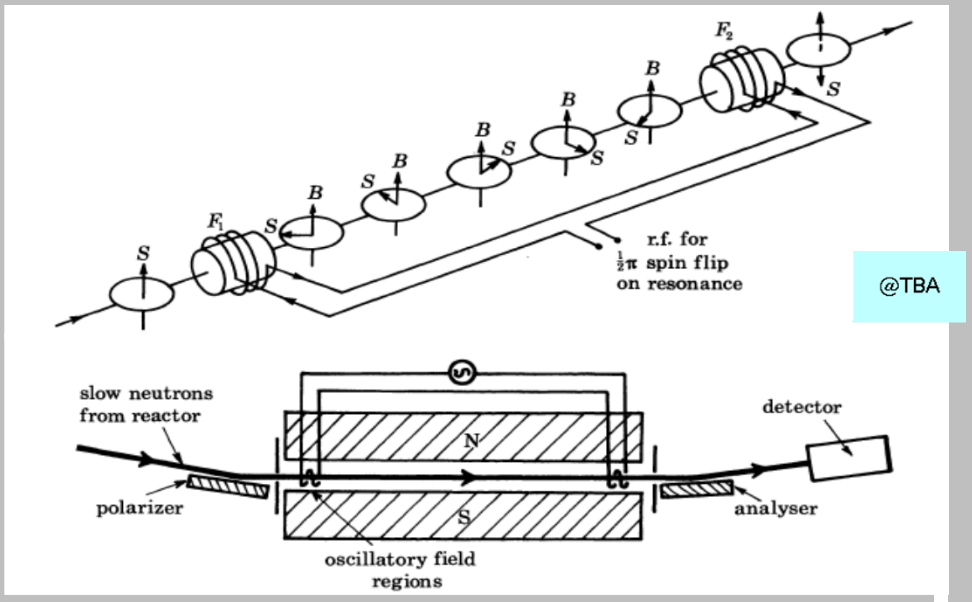
$$\frac{\mu_{e^-}}{\mu_p} = \frac{g_p(H)}{g_p}\left(\frac{g_{e^-}(H)}{g_{e^-}}\right)^{-1}\frac{\mu_{e^-}(H)}{\mu_p(H)}$$
$$= -658.210\,6860(66) \quad [1.0 \times 10^{-8}],$$

μ_p/μ_N = $\underline{2.7928}47351 \pm 0.000000028$

Neutron Magnetic Moment - I

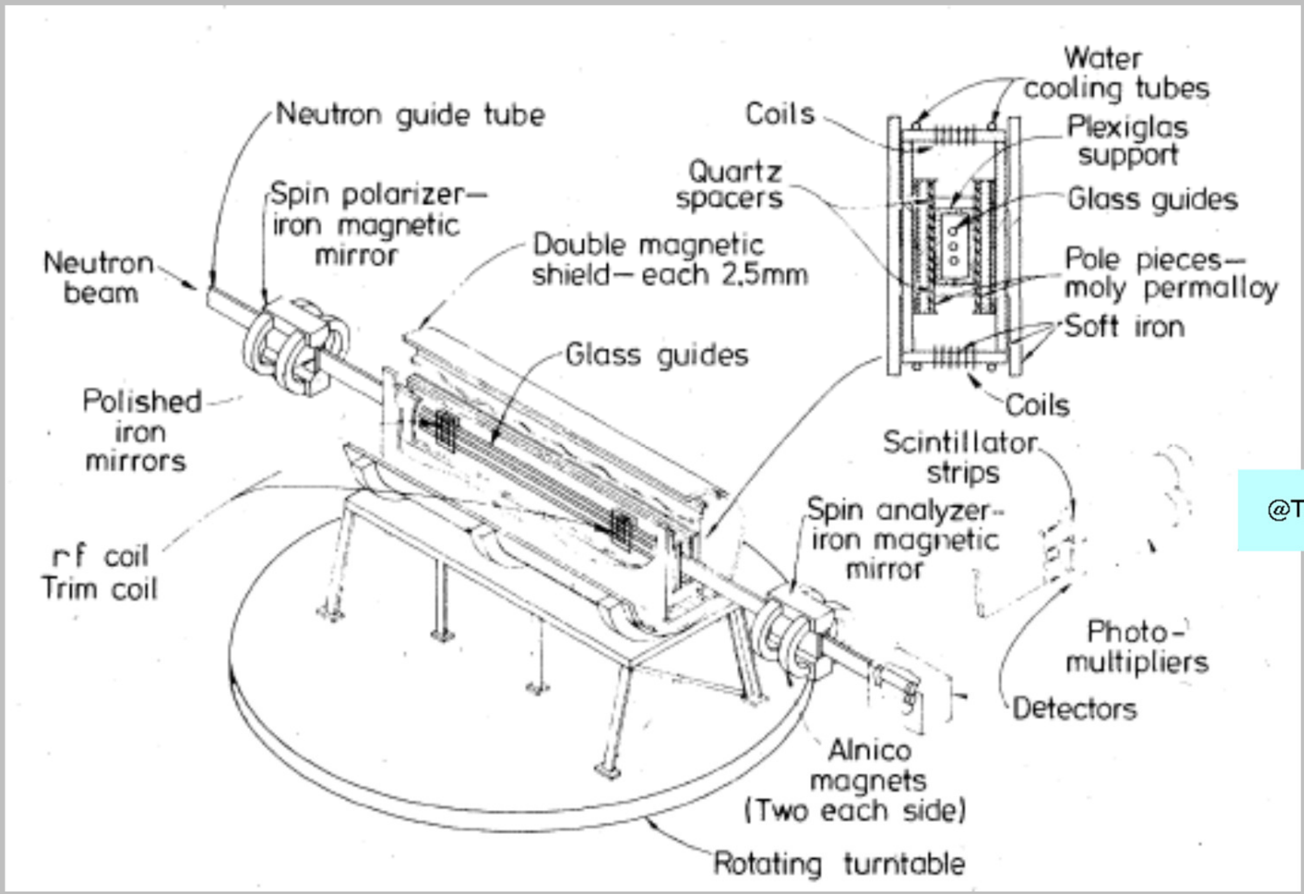
Modern measurements: Thermal neutrons from a reactor

RF pulses in *F1,F2*
Periodic spin rotation in *F1,F2* : Get $\mathbf{s} \perp \mathbf{B}$
Time of flight between F_1,F_2
Spin precession in ***B*** field
Count analyzed neutrons
'Resonance':
Count rate vs. frequency of spin flip



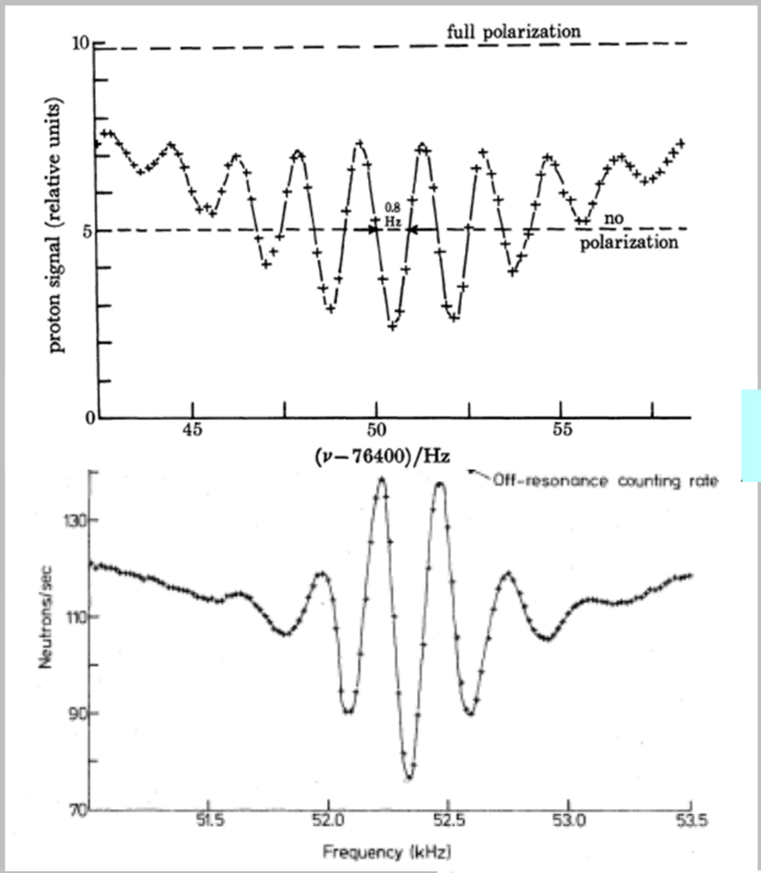
Polarizer,Analyzer:
Neutron Scattering from magnetized iron crystals, sensitive to spin orientation

Neutron Magnetic Moment - II



@TBA

Neutron Magnetic Moment - III



@TBA

Proton resonance (Calibration)

Neutron resonance

$$\mu_n/\mu_N = -1.913\,041\,84\,(88)\;(0.45\text{ ppm})\,.$$

The Rosenbluth Formula

Consider elastic electron-nucleon scattering
Going through the same steps as for electron-muon scattering

$$\begin{cases} A(q^2) = F_1^2(q^2) - \kappa_p^2 \dfrac{q^2}{4M^2} F_2^2(q^2) \\ B(q^2) = -\dfrac{q^2}{2M^2}\left(F_1(q^2) + \kappa_p F_2(q^2)\right)^2 \end{cases}$$

$$\left.\frac{d\sigma}{d\Omega}\right|_{LAB} = \frac{\alpha^2}{4|\mathbf{p}_1|^2 \sin^4\theta/2}\cos^2\theta/2 \frac{|\mathbf{p}_1'|}{|\mathbf{p}_1|}\left(A(q^2) + B(q^2)\tan^2\theta/2\right)$$

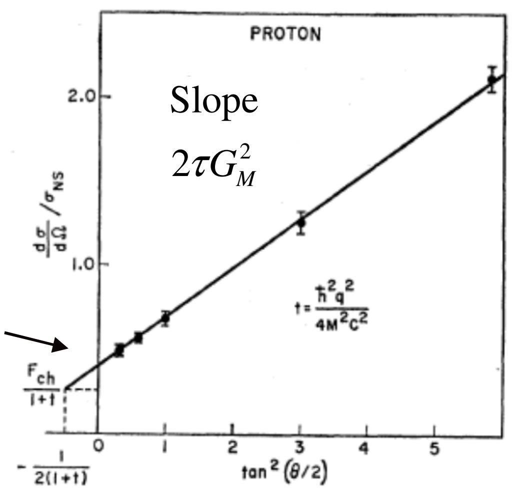
$$\left.\frac{d\sigma}{d\Omega}\right|_{LAB} = \frac{\alpha^2}{4|\mathbf{p}_1|^2 \sin^4\theta/2}\cos^2\theta/2 \frac{|\mathbf{p}_1'|}{|\mathbf{p}_1|}\left[\frac{G_E^2 - (q^2/4m^2)G_M^2}{1 - q^2/4m^2} - \frac{q^2}{m^2}G_M^2 \tan^2(\theta/2)\right]$$

$$\left.\frac{d\sigma}{d\Omega}\right|_{LAB} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott}\left[\frac{G_E^2 + \tau G_M^2}{1+\tau} + 4\tau G_M^2 \tan^2\theta/2\right], \tau = -\frac{q^2}{4m^2}$$

‘Rosenbluth separation’ gives G_E, G_M

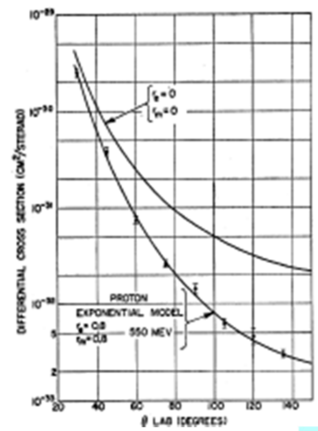
@TBA

Fixed q^2



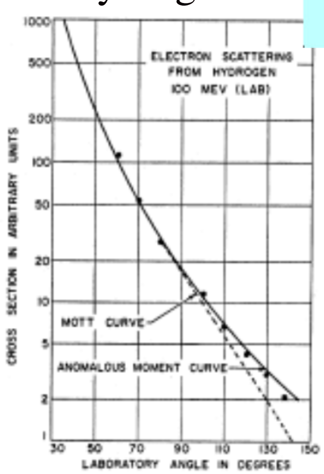
Intercept
$$\frac{G_E^2 + \tau G_M^2}{1+\tau}$$

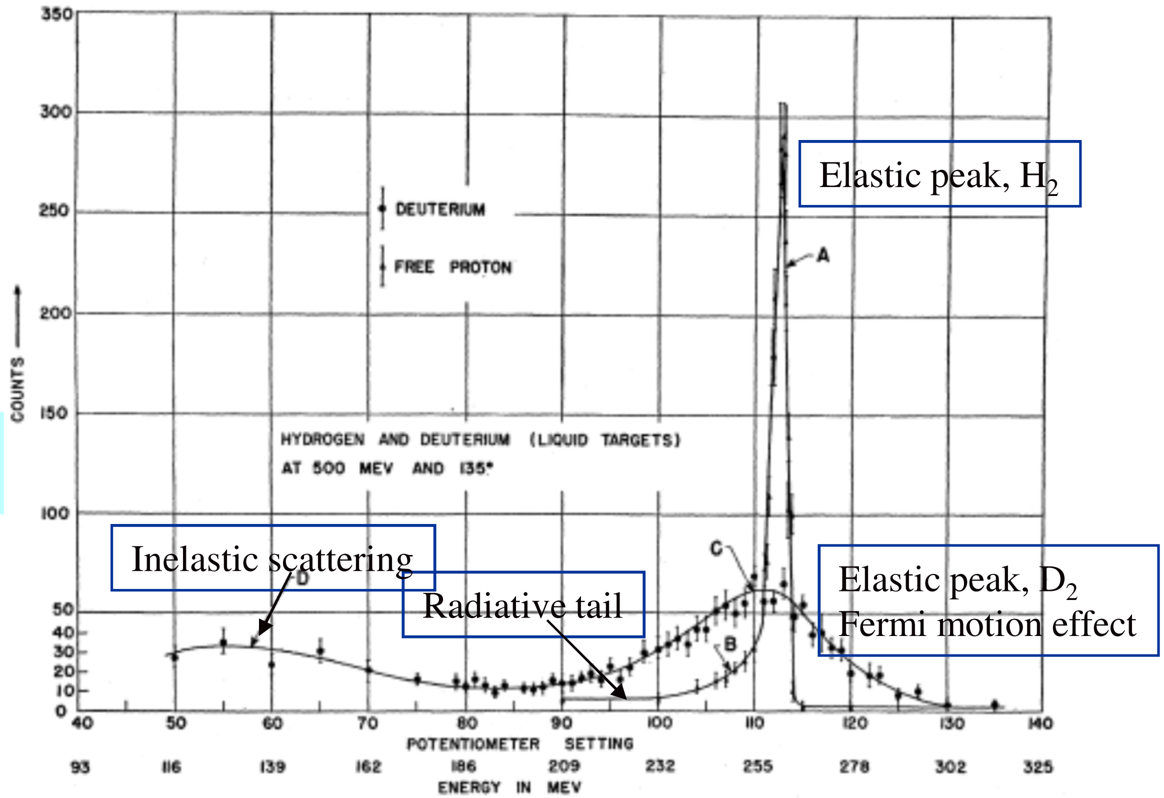
Measurements



Hydrogen

@TBA

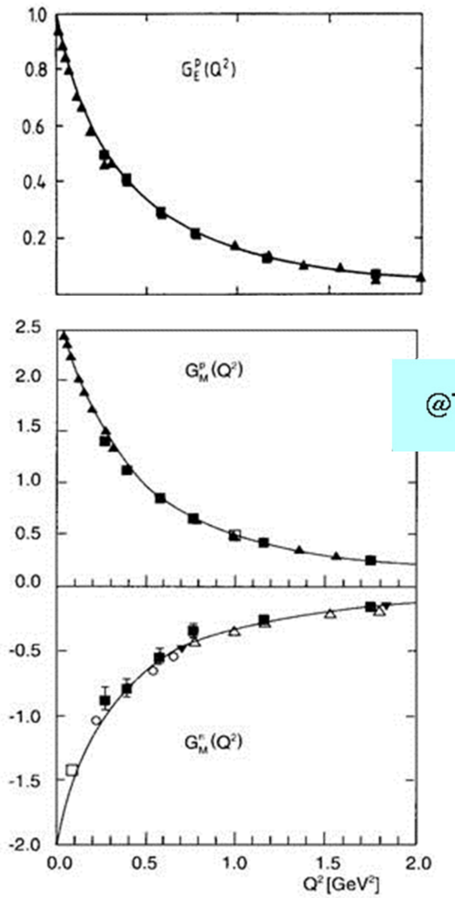


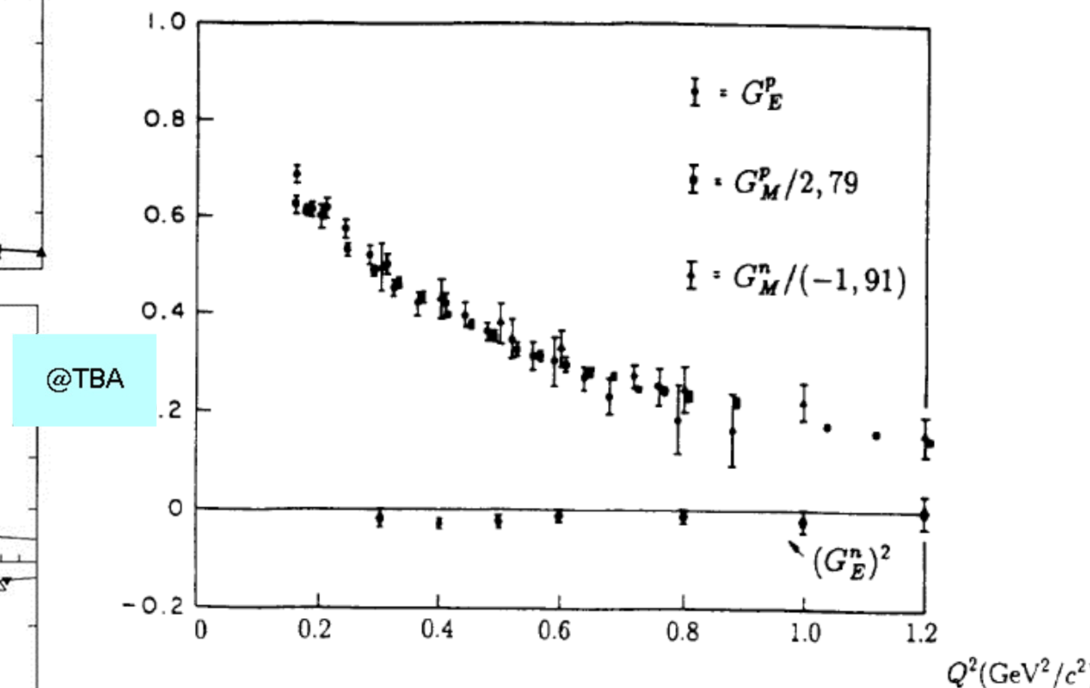


Electron energy at fixed angle for H_2, D_2

Experimental Results

Space-like experiments



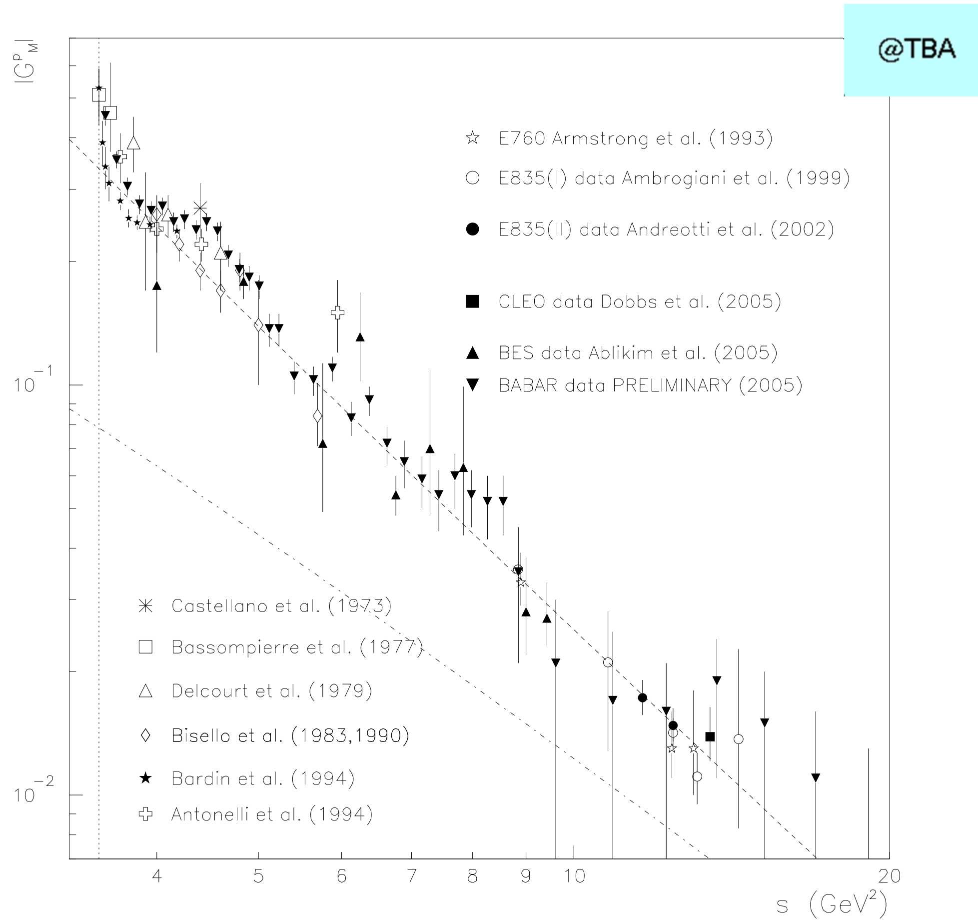


Same shape for the 3 non-zero FF

Probing the same structure at scale ≥~ 0.2 fm

Experimental Results

Time-like experiments



Elastic Scattering Kinematics

4-momentum conservation at the nucleon vertex

$$P + q = P'$$
$$\to (P+q)^2 = P'^2 \to P^2 + q^2 + 2P\cdot q = P'^2$$
$$P^2 = P'^2 = M^2$$
$$\to 2P\cdot q = -q^2$$

Rewrite in the LAB frame, take massless lepton

$$P = (M, \mathbf{0})$$
$$p = (E, \mathbf{p}) \approx (|\mathbf{p}|, \mathbf{p}), \;\; p' \approx (|\mathbf{p}'|, \mathbf{p}')$$
$$q = p - p'$$
$$\to q^2 \approx (|\mathbf{p}| - |\mathbf{p}'|, \mathbf{p} - \mathbf{p}')^2 = |\mathbf{p}|^2 + |\mathbf{p}'|^2 - 2|\mathbf{p}||\mathbf{p}'| - \left(|\mathbf{p}|^2 + |\mathbf{p}'|^2 - 2\mathbf{p}\cdot\mathbf{p}'\right)$$
$$\to q^2 \approx (|\mathbf{p}| - |\mathbf{p}'|, \mathbf{p} - \mathbf{p}')^2 = -2|\mathbf{p}||\mathbf{p}'| + 2\mathbf{p}\cdot\mathbf{p}' = -2|\mathbf{p}||\mathbf{p}'|(1 - \cos\theta) = -4|\mathbf{p}||\mathbf{p}'|\sin^2\theta/2$$
$$Q^2 \equiv -q^2$$
$$P\cdot q \approx (M, \mathbf{0})\cdot(|\mathbf{p}| - |\mathbf{p}'|, \mathbf{p} - \mathbf{p}') = M\,(|\mathbf{p}| - |\mathbf{p}'|)$$
$$\nu \equiv |\mathbf{p}| - |\mathbf{p}'| \to P\cdot q \approx M\nu$$

Rosenbluth Again - I

Rewrite electron-muon differential cross-section in LAB:

$$\frac{d^2\sigma}{d\Omega} \underset{E\gg m}{\simeq} \underbrace{\frac{\alpha^2 \cos^2\theta/2}{4E^2 \sin^4\theta/2}}_{\text{Mott}} \underbrace{\frac{E'}{E}}_{\text{Recoil}} \left(1 - \underbrace{\frac{q^2}{2m_\mu^2} \tan^2\theta/2}_{\text{Magnetic dipole}} \right)$$

as follows:

$$\frac{d^2\sigma}{d\Omega} = \int dE' \frac{d^3\sigma}{dE'd\Omega}$$

$$\frac{d^3\sigma}{dE'd\Omega} = \frac{\alpha^2 \cos^2\theta/2}{4E^2 \sin^4\theta/2} \left(1 - \frac{q^2}{2m_\mu^2} \tan^2\theta/2 \right) \delta\left(\nu + \frac{q^2}{2m_\mu} \right)$$

$$\left. \begin{array}{l} q^2 \simeq -4EE'\sin^2\theta/2 \\ \nu = E - E' \end{array} \right\} \to \delta\left(\nu + \frac{q^2}{2m_\mu} \right) = \delta\left(E - E' - \frac{2E}{m_\mu} E'\sin^2\theta/2 \right)$$

Namely,

$q^2, \nu \leftrightarrow E', \theta$ fully correlated by 4-momentum conservation

Rosenbluth Again - II

Electron-proton, following the same path as before:

$$\frac{d^3\sigma}{dE'd\Omega}=\frac{\alpha^2\cos^2\theta/2}{4E^2\sin^4\theta/2}\left[\frac{G_E^2\left(q^2\right)-\frac{q^2}{4M^2}G_M^2\left(q^2\right)}{1-\frac{q^2}{4M^2}}-\frac{q^2}{2M^2}G_M^2\left(q^2\right)\tan^2\theta/2\right]\delta\left(\nu+\frac{q^2}{2M}\right)$$

$$\rightarrow\frac{d^3\sigma}{dE'd\Omega}=\frac{\alpha^2\cos^2\theta/2}{4E^2\sin^4\theta/2}\left(A+B\tan^2\theta/2\right)\delta\left(\nu+\frac{q^2}{2M}\right)$$

Scattering by a point-like source (e.g. muon) recovered by taking

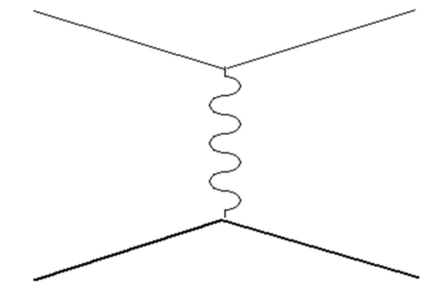
$$\begin{cases}\dfrac{G_E^2\left(q^2\right)-\frac{q^2}{4M^2}G_M^2\left(q^2\right)}{1-\frac{q^2}{4M^2}}=1\\ G_M^2\left(q^2\right)=1\end{cases}$$

$\rightarrow$ Point-like scatterer:

$$G_E^2\left(q^2\right)=G_M^2\left(q^2\right)=1$$

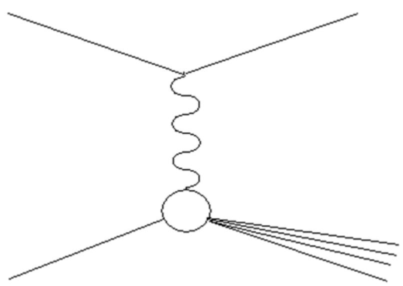
Inelastic Scattering

Elastic:

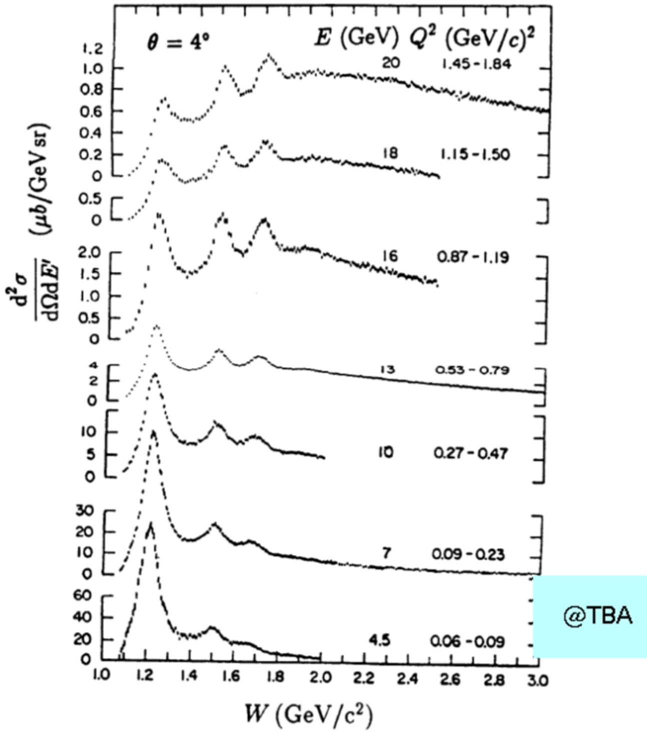


Invariant mass = M

Generalise to inelastic reactions:



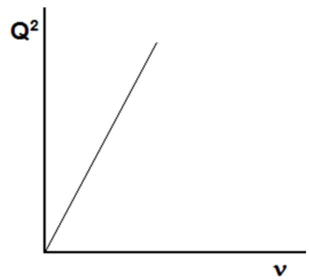
Invariant mass = $W>M$



Copious production of *resonances* (nucleon excited states), when q^2 not too big

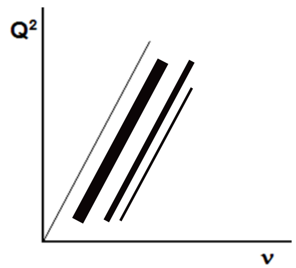
Inelastic Scattering Kinematics

$$\nu = \frac{Q^2}{2M} \rightarrow 2M\nu = Q^2$$



Elastic

$$2M\nu = Q^2 + M'^2 - M^2$$



Inelastic

Generalise Rosenbluth cross-section to account for variable *W*:
Introduce inelastic *structure functions* W_1, W_2 to replace elastic form factors G_E, G_M

$$\frac{d\sigma}{d\Omega dE'} = \frac{\alpha^2 \cos^2\theta/2}{4E^2\sin^4\theta/2}\left[W_2\left(\nu,q^2\right) + W_1\left(\nu,q^2\right)\tan^2\theta/2\right]$$

$W_{1,2}$ depending on q^2 *and* ν

Resonance Excitation at DESY

@TBA

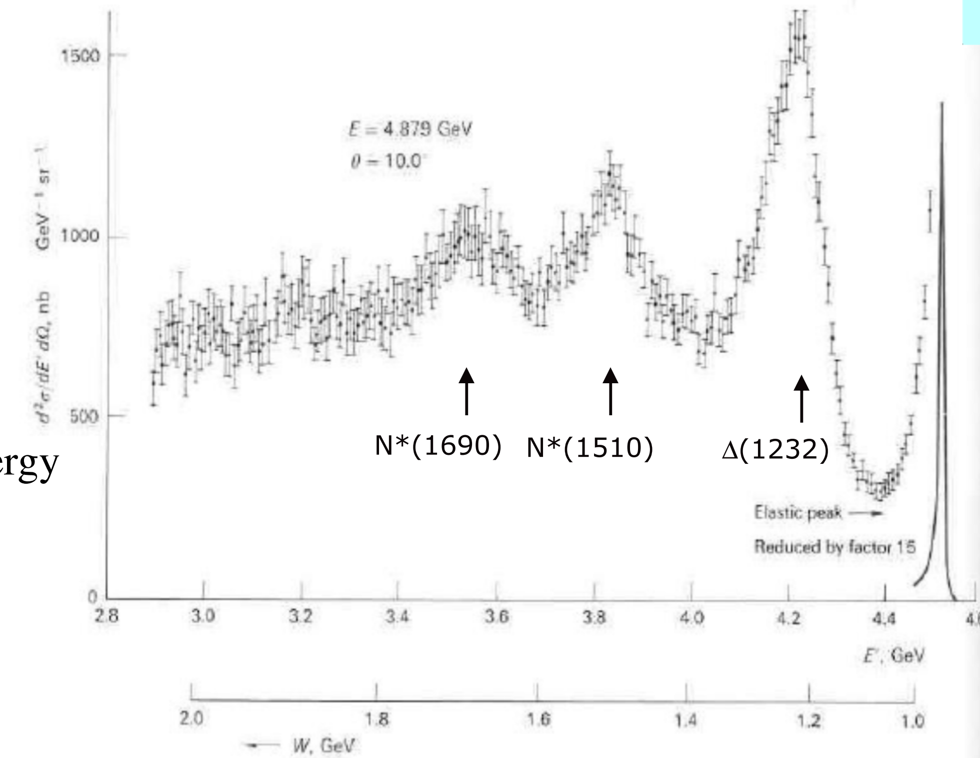
First DESY machine

Electron synchrotron
$E_{max} = 6\ GeV$

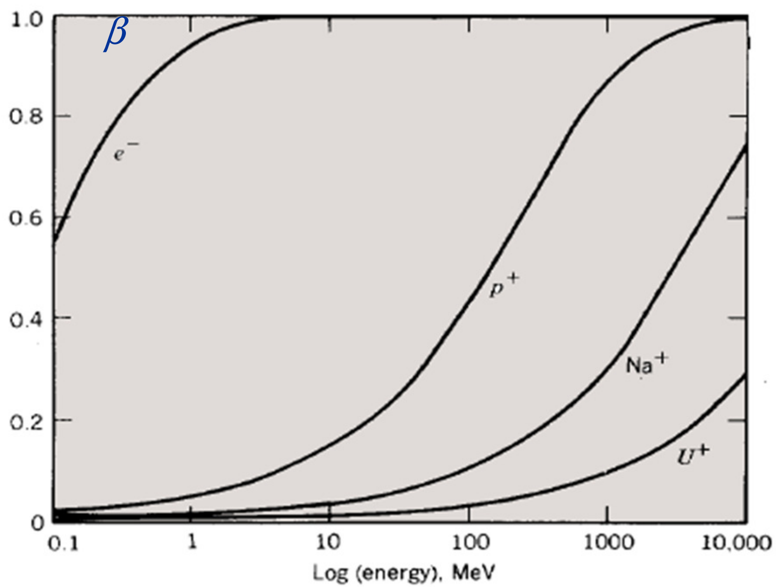
$e + p \rightarrow e + X$

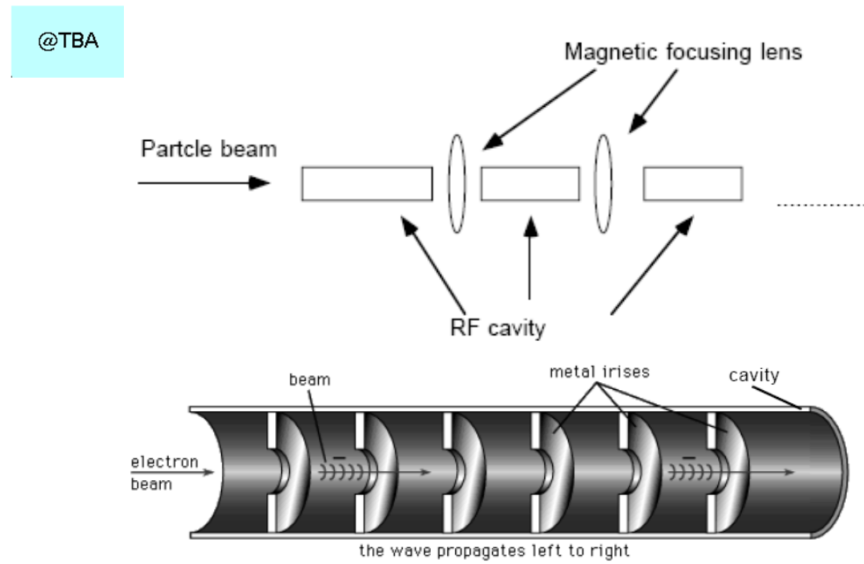
E, E': Incident, scattered electron energy

$W = M_X$



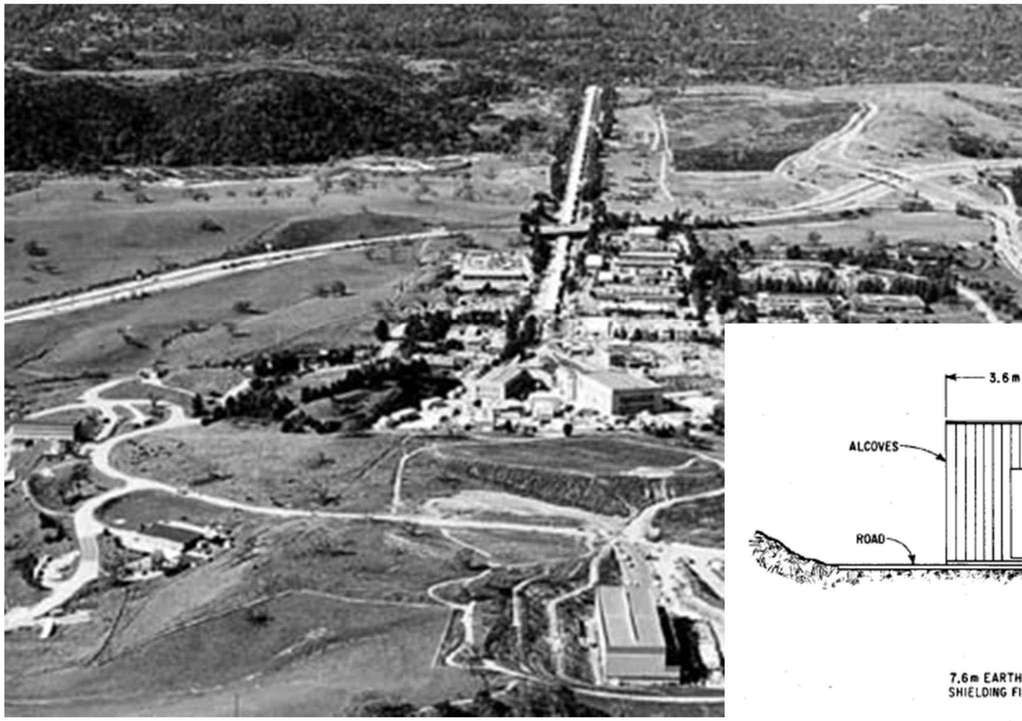
The Electron LINAC

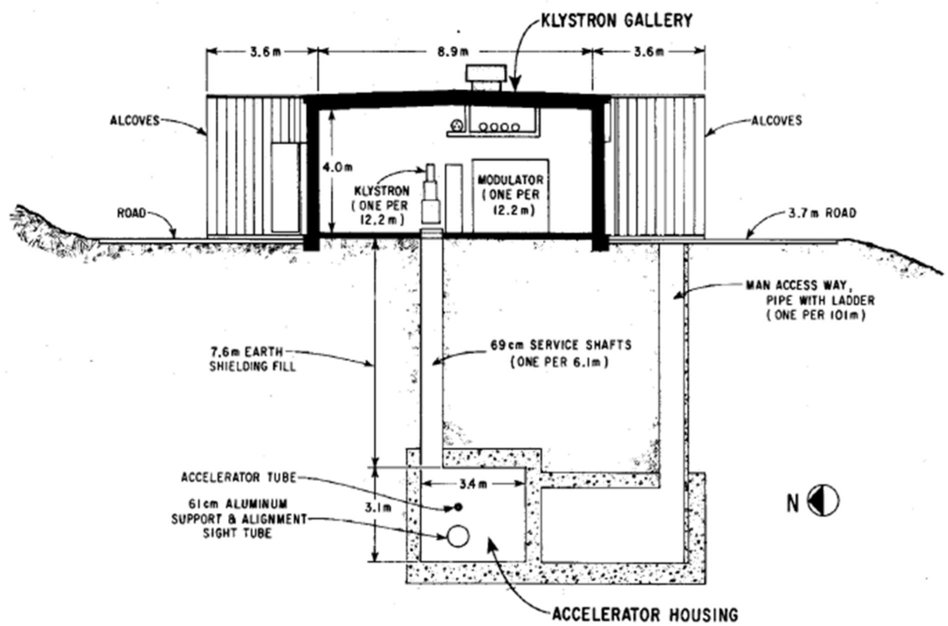




Traveling wave linear accelerator:
Electrons riding the traveling EM wave at constant phase

SLAC - I



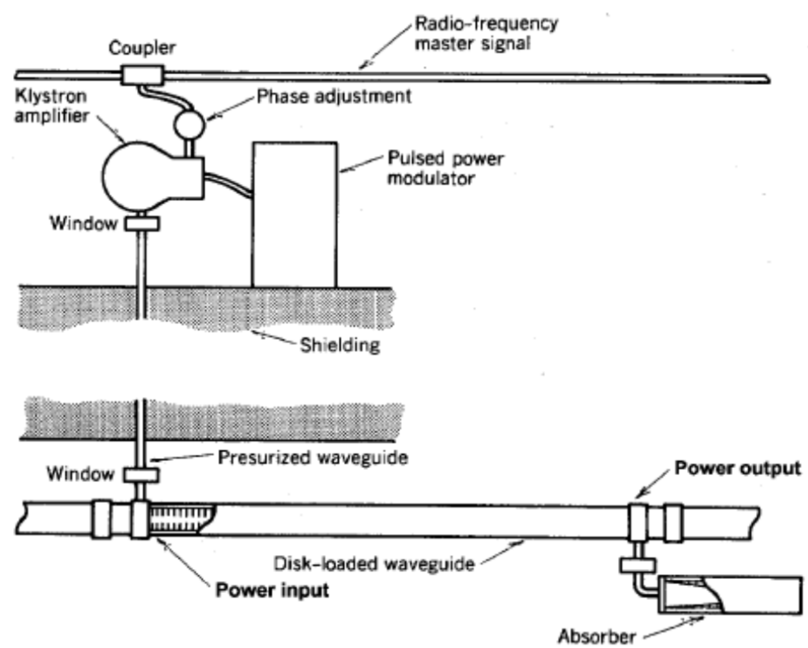


KLYSTRON GALLERY
3.6m
8.9m
3.6m
ALCOVES
ALCOVES
4.0m
KLYSTRON (ONE PER 12.2m)
MODULATOR (ONE PER 12.2m)
ROAD
3.7m ROAD
MAN ACCESS WAY, PIPE WITH LADDER (ONE PER 101m)
7.6m EARTH SHIELDING FILL
69cm SERVICE SHAFTS (ONE PER 6.1m)
ACCELERATOR TUBE
61cm ALUMINUM SUPPORT & ALIGNMENT SIGHT TUBE
3.4m
3.1m
N
ACCELERATOR HOUSING

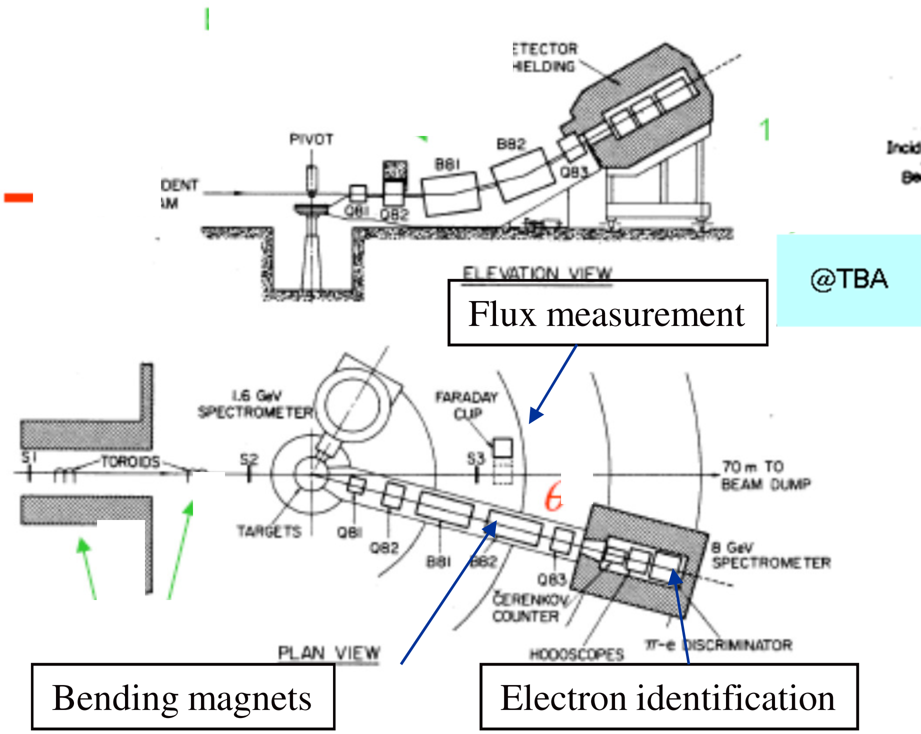
SLAC - II

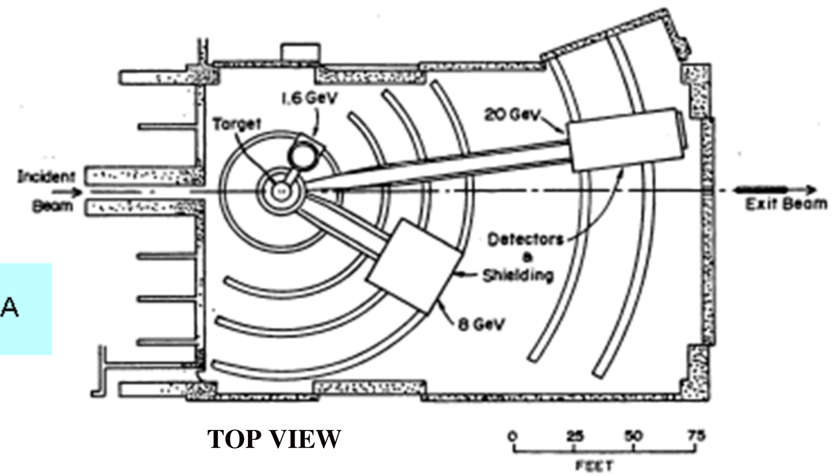
@TBA

Accelerator length	3100 m
Length between power feeds	3.1 m
Number of accelerator sections	960
Number of klystrons	245
Peak power per klystron	6–24 MW
Beam pulse repetition rate	1–360 pulses/s
Radio-frequency pulse length	2.5 μs
Filling time	0.83 μs
Shunt impedance	53 MΩ/m
Electron energy (unloaded)	11.1–22.2 GeV
Electron energy (loaded)	10–20 GeV
Electron beam peak current	25–50 mA
Electron beam average current	15–30 μA
Average electron beam power	0.15–0.6 MW
Efficiency	4.3%
Positron energy	7.4–14.8 GeV
Positron average beam current	0.45 μA
Operating frequency	2.856 GHz
Accelerating structure	Iris-loaded waveguide
Waveguide outer diameter	10.5 cm
Aperture diameter	1.9 cm



The SLAC Experiments



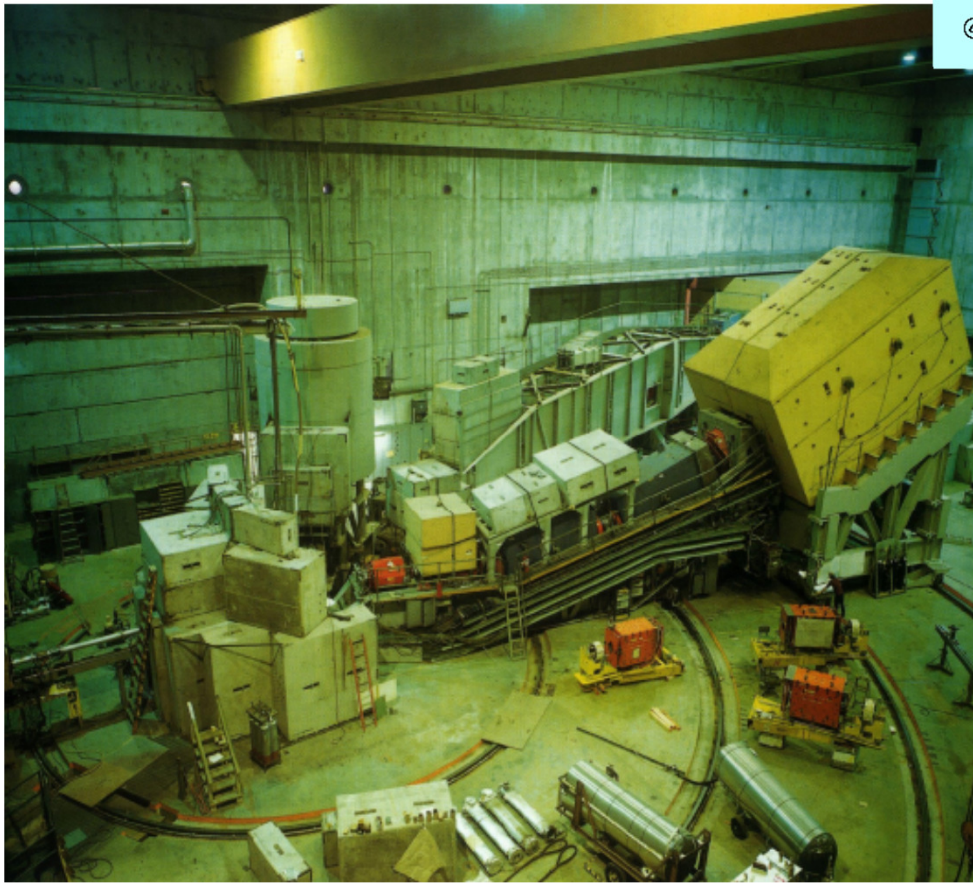


TOP VIEW

Measure E', θ of the scattered electron → Get q^2, ν

$$\frac{d^2\sigma}{dE'd\Omega} \to \frac{d^2\sigma}{dq^2 d\nu}$$

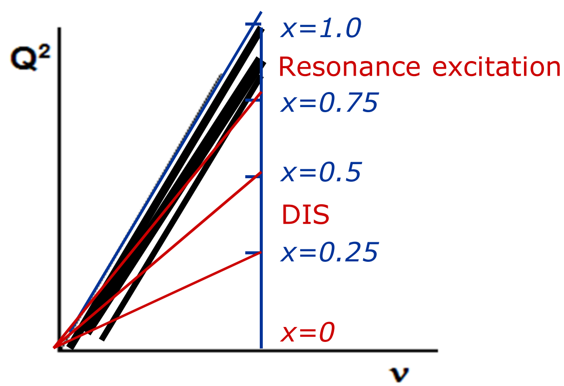
SLAC End Station A

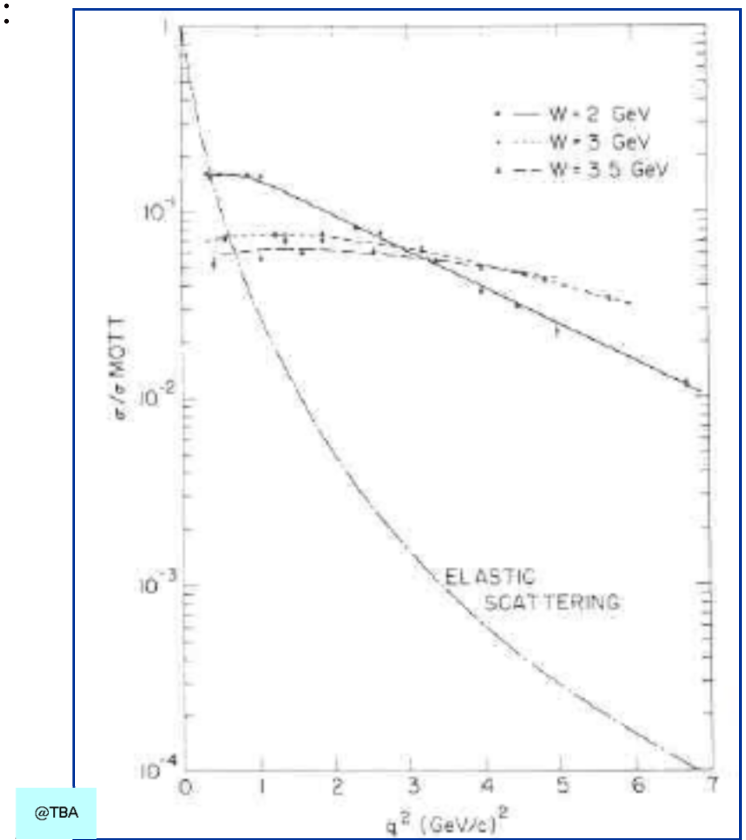

@TBA

Deep Inelastic Scattering - I

Details of structure functions in the resonance region difficult to explain
But: Beyond small q^2, ν things are surprisingly simple!

Striking elastic/deep inelastic comparison:
no q^2 dependence in DIS





Deep Inelastic Scattering - II

$$\frac{d^2\sigma}{dE'd\Omega} \to \frac{d^2\sigma}{dq^2 d\nu}$$

Also introduce:

$$\begin{cases} x = \dfrac{Q^2}{2M\nu} \\ y = \dfrac{\nu}{E_1} \end{cases} \to \frac{d^2\sigma}{dxdy} = \frac{4\pi\alpha^2 ME_1}{Q^4}\left[2xF_1\left(\frac{1+(1-y)^2}{2}\right)+(1-y)(F_2-2xF_1)-\frac{M^2 xyF_2}{s-M^2}\right]$$

Bjorken scaling hypothesis:

$$\frac{d^2\sigma}{dQ^2 d\nu} \underset{\substack{q^2,\nu\to\infty \\ \frac{q^2}{\nu}\ \text{finite}}}{\longrightarrow} f(x), \quad x = \frac{Q^2}{2M\nu}$$

f(x) Universal function of x

f(x) Q^2 independent

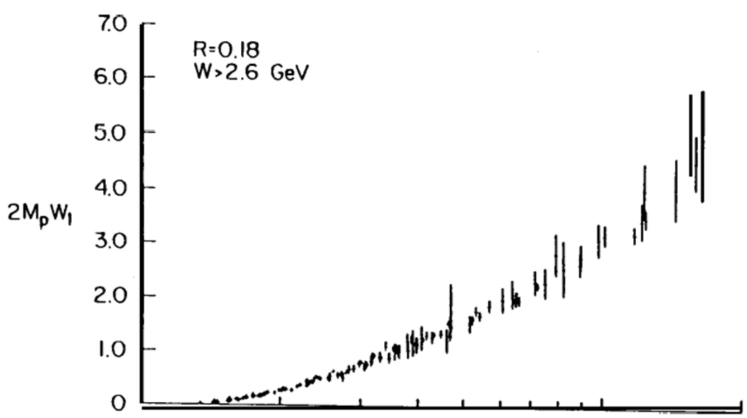
Deep Inelastic Scattering - III

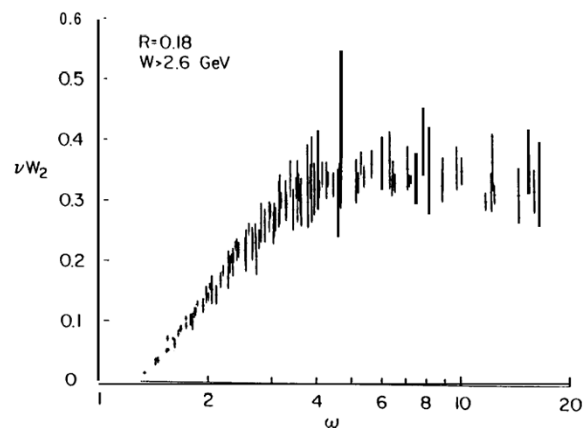
Expect:

$$\begin{cases} F_1\left(x,Q^2\right) \to F_1\left(x\right) \\ F_2\left(x,Q^2\right) \to F_2\left(x\right) \end{cases} \text{ if scaling is good}$$

Observe ($\omega = 1/x$):

@TBA

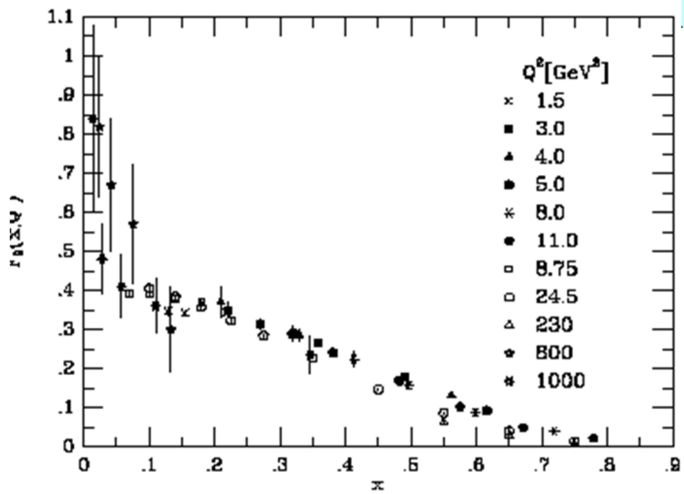




Scaling at high energy is indeed well verified!

Deep Inelastic Scattering - IV

W_2: Universal function of x …. q^2-independent!



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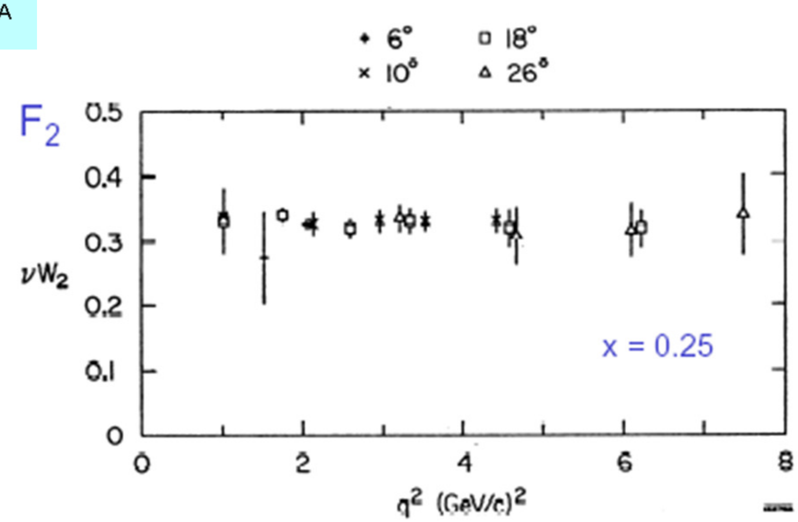
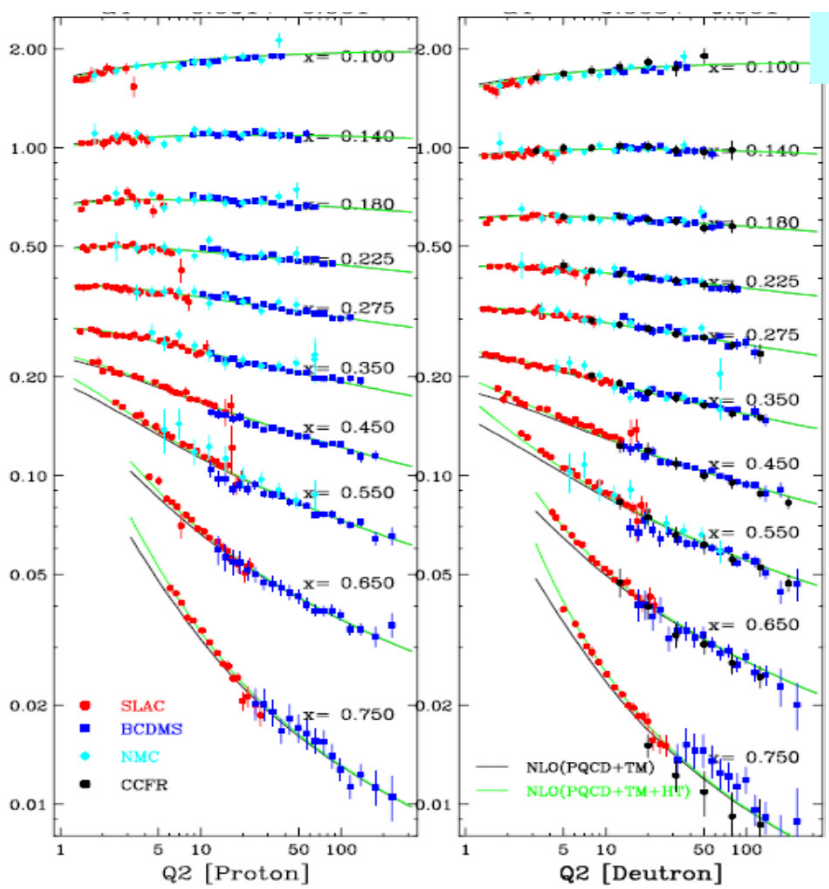


As compared to fast varying elastic f.f., just astonishing…

Scaling Violations



$e^- p$, $\mu^- p$ | $e^- d$, $\mu^- d$, (νd)

@TBA

Extensive compilation
Data from fixed target experiments
Observe:

Electron vs. Muon
Red points are from electron DIS
Blue points are from muon DIS
Muon merits:

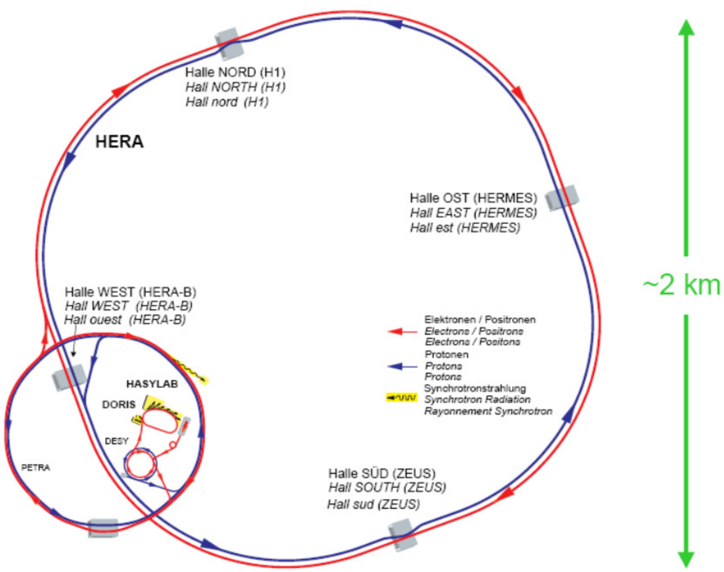
Easier to get high energy
Reduced radiative corrections

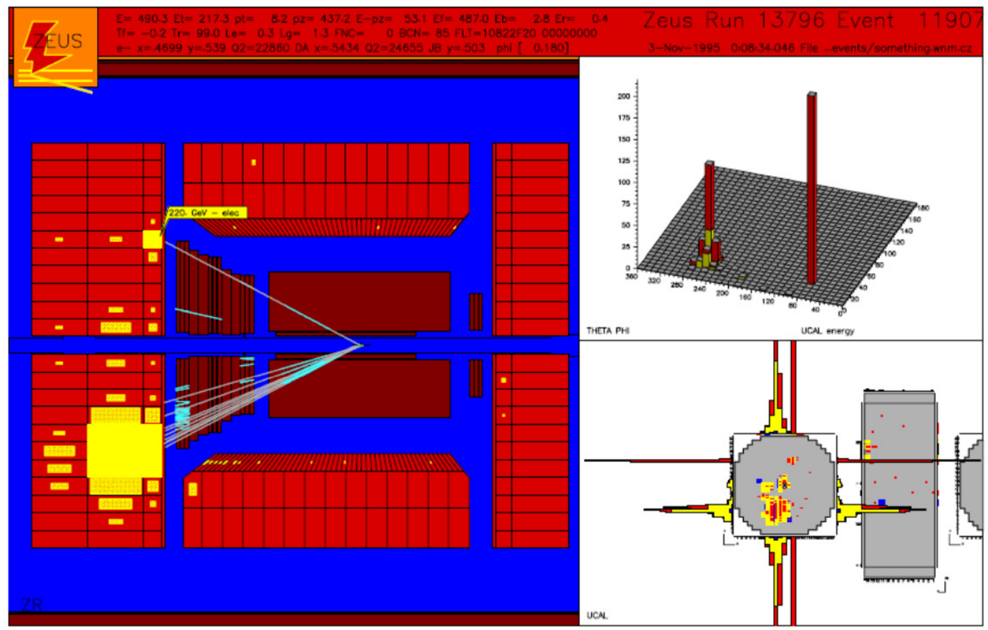
Muon drawbacks:
Intensity

Proton (L) vs. Deuteron (R)
Get *neutron* structure function

HERA

First example of *asymmetric* collider





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A Recent Compilation

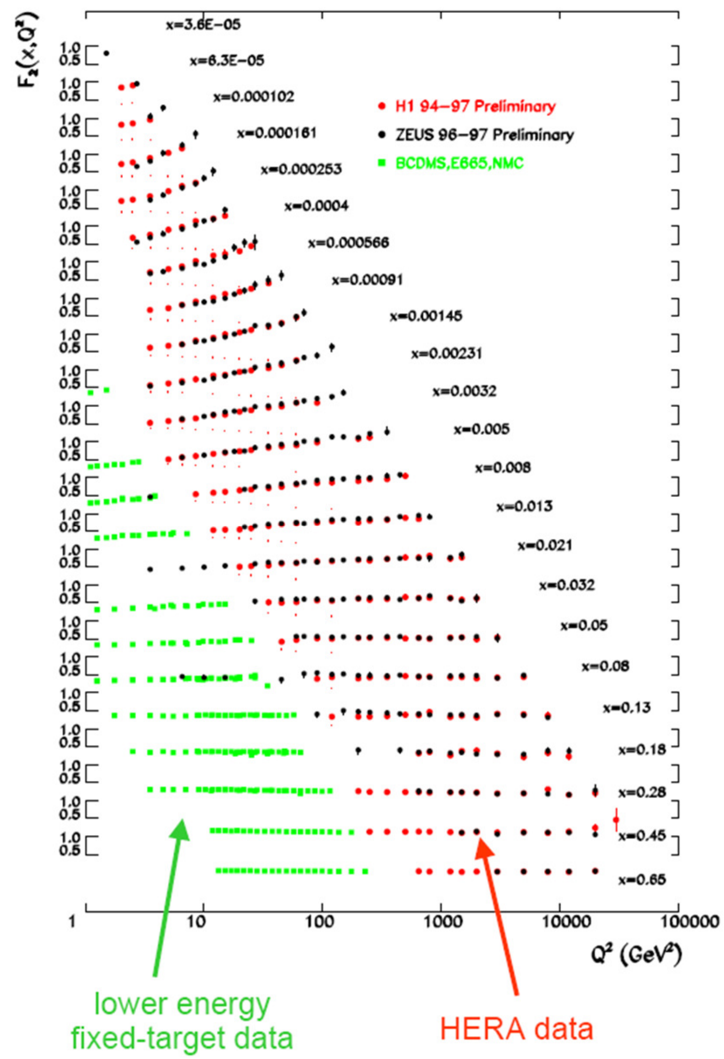
Results from several experiments:

muon DIS NMC, BCDMS, E665
CERN FNAL
electron DIS at HERA collider
DESY

Huge q^2, x range

Small, measurable scaling violation
Interesting features at small x

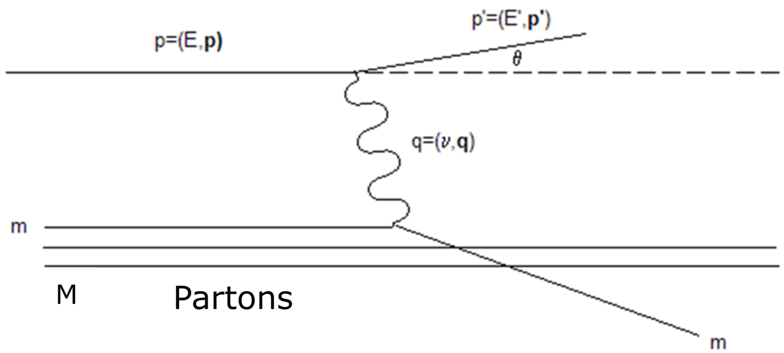
→ QCD !



Parton Model - I

Structure functions generalise form factors
Form factor of a point source = *constant*
Feynman suggestion:

Maybe deep inelastic scaling just indicates elastic scattering off free, pointlike constituents



Kinematical constraint:

$$m = (m + \nu, \mathbf{q})^2 = m^2 + 2m\nu + \underbrace{\nu^2 - |\mathbf{q}|^2}_{=q^2}$$

$$\rightarrow \nu + \frac{q^2}{2m} = 0$$

Parton Model - II

Elastic scattering off a parton: Energy and angle of the scattered electron *fully correlated*

Differential cross-section for elastic scattering off a free, pointlike constituent of mass m

$$\frac{d\sigma}{d\Omega} = \int dE' \frac{d^3\sigma}{dE'd\Omega} = \int dE' \frac{\alpha^2 z^2}{4E^2 \sin^4 \theta/2}\left(\cos^2\theta/2 - \frac{q^2}{2m^2}\sin^2\theta/2\right)\delta\left(\nu + \frac{q^2}{2m}\right)$$

z: Parton charge, units e

Full E', θ correlation

$$\begin{cases} \nu = E - E' \\ q^2 = -4EE'\sin^2\theta/2 \end{cases} \rightarrow E - E' = \frac{4EE'\sin^2\theta/2}{2m} \rightarrow E'\left(1 + \frac{4E}{2m}\sin^2\theta/2\right) = E$$

$$\rightarrow E' = \frac{E}{1 + \frac{4E}{2m}\sin^2\theta/2}$$

$$\nu + \frac{q^2}{2m} = 0, x = -\frac{q^2}{2M\nu} \rightarrow x = \frac{m}{M}$$

$$\frac{d^2\sigma}{dE'd\Omega} = \frac{\alpha^2 z^2}{4E^2 \sin^2\theta/2}\left(\cos^2\theta/2 - \frac{q^2}{2M^2x^2}\sin^2\theta/2\right)\delta\left(\nu + \frac{q^2}{2Mx}\right)$$

Parton Model - III

Summing over all types of partons

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{\alpha^2 \sum_i z_i^2 n_i}{4E^2 \sin^2\theta/2}\left(\cos^2\theta/2 - \frac{q^2}{2M^2x^2}\sin^2\theta/2\right)\delta\left(\nu + \frac{q^2}{2Mx}\right)$$

Compare to inelastic cross-section

$$\frac{d\sigma}{d\Omega dE'} = \frac{\alpha^2}{4E^2 \sin^4\theta/2}\left[W_2\left(\nu, q^2\right)\cos^2\theta/2 + W_1\left(\nu, q^2\right)\sin^2\theta/2\right]$$

Then predict structure functions:

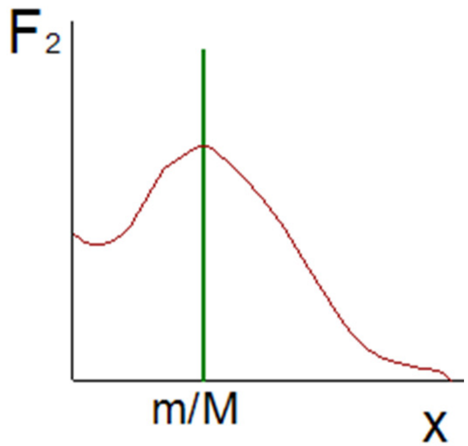
$$\rightarrow \begin{cases} W_2 = \dfrac{F_2}{\nu} = \left(\sum_i z_i^2 n_i\right)\delta\left(\nu + \dfrac{q^2}{2Mx}\right) \\ W_1 = \dfrac{2F_1}{M} = \left(\sum_i z_i^2 n_i\right)\left(\dfrac{-q^2}{2M^2x^2}\right)\delta\left(\nu + \dfrac{q^2}{2Mx}\right) \end{cases}$$

Parton Model - IV

$$\rightarrow F_2 = \nu\left(\sum_i z_i^2 n_i\right)\delta\left(\nu + \frac{q^2}{2Mx}\right) = \left(\sum_i z_i^2 n_i\right)\delta\left(1 + \frac{q^2}{2Mx\nu}\right)$$

$$\rightarrow F_2 = \left(\sum_i z_i^2 n_i\right)x\delta\left(x + \frac{q^2}{2M\nu}\right) = \left(\sum_i z_i^2 n_i\right)x\delta\left(x - \frac{m}{M}\right)$$

Parton model prediction
Actual shape



Parton Model - V

The true meaning of x

P, q: proton, virtual photon 4-momenta

$$x = \frac{Q^2}{2M\nu} = \frac{Q^2}{2P\cdot q} \quad \text{invariant}$$

$$x = \frac{m}{M}$$

$$m^2 = x^2 M^2 = x^2\left(E^2 - \mathbf{p}^2\right)$$

$$m^2 = \left(E_{parton}, \mathbf{p}_{parton}\right)^2 = E_{parton}{}^2 - \mathbf{p}_{parton}{}^2$$

$$\left(E_{parton}, \mathbf{p}_{parton}\right) = x\cdot\left(E, \mathbf{P}\right)$$

$$\rightarrow \mathbf{p}_{parton} \approx x\cdot\mathbf{P} \quad \text{when } m \ll |\mathbf{p}|$$

Therefore, in the *(Proton) Infinite Momentum Frame* :

x is the momentum fraction carried by the struck parton

Parton Model - VI

Lot of insight in this limit (Feynman):

$\beta \rightarrow 1 \Rightarrow \gamma \rightarrow \infty$ Large time dilation

Time constants of internal motions: $\tau \rightarrow \infty$ in the IMF

Constituents seen as *still* by the DIS virtual photon

Use time-energy indeterminacy relation:

$$P = \left(E_p \underbrace{\mathbf{0}}_{comp.trasversa}, |\mathbf{P}| \right), \quad E_p = \sqrt{M^2 + |\mathbf{P}|^2} \approx |\mathbf{P}|$$

$$q = \left(E_\gamma, \mathbf{q_T}, 0 \right)$$

$$\rightarrow P \cdot q \approx |\mathbf{P}| E_\gamma \rightarrow E_\gamma \approx \frac{P \cdot q}{|\mathbf{P}|} = \frac{Q^2}{2x|\mathbf{P}|}$$

$$\tau_0 \sim \frac{1}{E_\gamma} \approx \frac{2x|\mathbf{P}|}{Q^2} \quad \text{DIS time scale}$$

$$\tau \sim \frac{1}{\Delta E} \approx \frac{2x|\mathbf{P}|}{p_T^2} \quad \text{Constituents motion time scale}$$

p_T: parton transverse momentum scale

$$\rightarrow \frac{\tau_0}{\tau} \approx \frac{p_T^2}{Q^2} \sim 0$$

No binding effects, free constituents OK

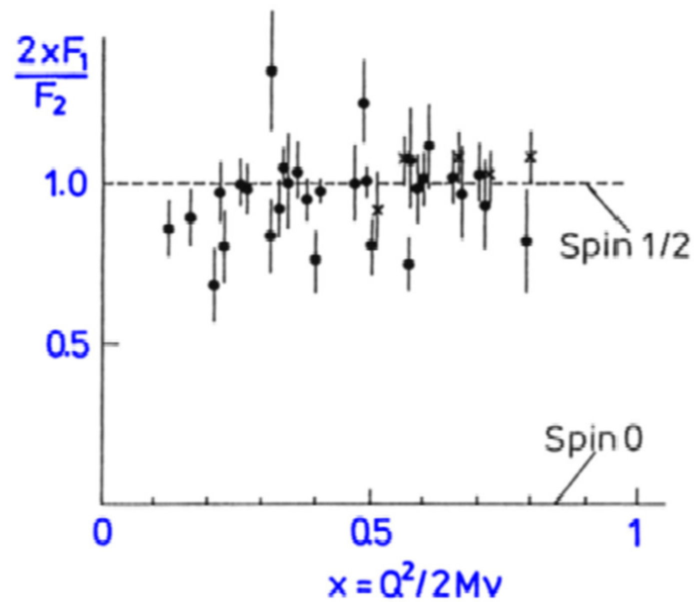
Parton Model - VII

Callan-Gross relation for spin 1/2 partons

$$\begin{cases} \dfrac{F_2}{\nu} = \left(\sum_i z_i^2 n_i\right)\delta\left(\nu + \dfrac{q^2}{2Mx}\right) \\ \dfrac{2F_1}{M} = \left(\sum_i z_i^2 n_i\right)\left(\dfrac{-q^2}{2M^2x^2}\right)\delta\left(\nu + \dfrac{q^2}{2Mx}\right) \end{cases}$$

$$\rightarrow \frac{F_2}{\nu} = \frac{2F_1}{M}\frac{2M^2x^2}{-q^2} = \frac{2F_1}{M}\frac{2M^2x^2}{2M\nu x} = \frac{2F_1 x}{\nu}$$

$$\rightarrow F_2 = 2F_1 x$$



@TBA

Parton Model - VIII

Several unanswered questions…

Most important issues:

One does not observe any free constituent out of the collision

Constituents seem to be essentially free (as partons) and tightly bound (as never observed free outside the nucleon) at the same time

For some time, these points were believed to rule out any constituent model