Elementary Particles I

2 – Structure I: Partons

Leptons

Leptons

1st family	2nd family	3rd family	
$\nu_e, \overline{\nu}_e$	$\nu_\mu, \overline{\nu}_\mu$	$\nu_\tau, \overline{\nu}_\tau$	Neutral, 'Massless'
e^-, e^+	μ^-, μ^+	τ^-, τ^+	Charged, Massive

"Pointlike", spin ½ Fermions
Electromagnetic and weak interactions

Lepton scattering by several targets as a powerful tool to probe constituents:

Electromagnetic (and weak) coupling to leptons simple, well understood

Small coupling constant→Perturbative expansion reliable

Electromagnetic Interaction

Try to find transition amplitude for electromagnetic scattering

1st order perturbative contribution

$T_{fi} = -i \langle f | \int d^4x H' | i \rangle$ H': Interaction Hamiltonian density

$H' = j^{\mu} A_{\mu}$ Classical analogy, j_{μ} current 4-density

Reminder

For any system of charges and currents:

$u_E = \frac{1}{2} \rho \varphi$ Electrostatic potential energy density

$u_B = \frac{1}{2} \mathbf{j} \cdot \mathbf{A}$ Magnetostatic potential energy density

$j^{\mu} = (\rho, \mathbf{j})$ 4-current density

$A_{\mu} = (\varphi, \mathbf{A})$ 4-potential

Spin 0 - I

First take a simple example:

Spinless, pointike “pion” scattering off a fixed, Coulomb potential

$$A_\mu = \left(\frac{eZ}{r}, \mathbf{0}\right)$$

$$j^\mu = (\rho, \mathbf{j}) = ie\left(\varphi^*\left(\frac{\partial\varphi}{\partial t}\right) - \left(\frac{\partial\varphi^*}{\partial t}\right)\varphi, \left((\nabla\varphi^*)\varphi - \varphi^*(\nabla\varphi)\right)\right)$$

Usual definition of current density
ϕ: Stationary state

$$j^\mu = (\rho, \mathbf{j}) = ie\left(\left(\varphi'^*\left(\frac{\partial\varphi}{\partial t}\right) - \left(\frac{\partial\varphi'^*}{\partial t}\right)\varphi\right), \left((\nabla\varphi'^*)\varphi - \varphi'^*(\nabla\varphi)\right)\right)$$

$$\rightarrow j^\mu = eNN'e^{-i\left((E-E')t-(\mathbf{p}-\mathbf{p}')\cdot\mathbf{r}\right)}\left((E+E'), (\mathbf{p}'+\mathbf{p})\right)$$

$$\rightarrow j^\mu A_\mu = NN'(E+E')e^{-i\left((E-E')t-(\mathbf{p}-\mathbf{p}')\cdot\mathbf{r}\right)}\frac{e^2Z}{r}$$

Generalize to a scattering state
ϕ,ϕ': Stationary states, plane waves

Integrate over time:

$$\int_{-\infty}^{+\infty} NN'e^{i(E-E')t}dt = NN'2\pi\delta(E-E')$$

Energy conservation; momentum *not conserved* by fixed Coulomb potential

Spin 0 - II

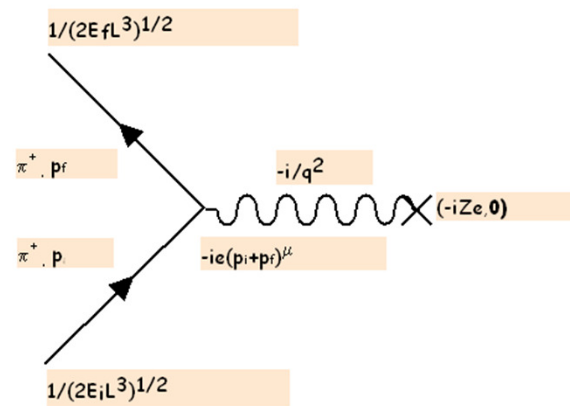
Integrate over space:

$$\int e^{+i((\mathbf{p}-\mathbf{p}')\cdot\mathbf{r})} \frac{e^2 Z}{r} d^3\mathbf{r} = \frac{4\pi Z e^2}{|\mathbf{q}|^2}, \mathbf{q} = \mathbf{p} - \mathbf{p}'$$

Matrix element:

$$T_{fi} = -i\langle f | \int d^4 x H' | i \rangle = -NN' 2\pi i \delta(E - E') \frac{4\pi Z e^2}{|\mathbf{q}|^2}$$

$$E = E' \rightarrow |\mathbf{q}|^2 = -q^2 \rightarrow T_{fi} = NN' 2\pi i \delta(E - E') \frac{4\pi Z e^2}{q^2}$$



Virtual photon:
Coupling fixed source to current

Spin 0 - III

Evaluate transition probability

$$w = \frac{|T_{fi}|^2}{T} \quad \text{Transition probability/Time}$$

$$|\delta(E' - E)|^2 = \lim_{T\to\infty} \left| \frac{1}{2\pi} \int_{-T/2}^{+T/2} e^{i(E'-E)t} dt \right|^2 = \lim_{T\to\infty} \left| \frac{\sin[(E'-E)T/2]}{\pi(E'-E)} \right|^2 = \frac{T}{2\pi} \delta(E'-E)$$

$$w = N^2 N'^2 \frac{4\pi^2}{T} |\delta(E-E')|^2 \frac{16\pi^2 Z^2 e^4}{|\mathbf{q}|^4} = N^2 N'^2\, 2\pi\delta(E-E') \frac{Z^2 e^4}{|\mathbf{q}|^4}$$

$$\rightarrow d\sigma = w \cdot \frac{\text{phase space}}{\text{incident flux}} = w \frac{\frac{V d^3 p'}{(2\pi)^3}}{\frac{p}{EV}} = w \frac{d^3 p'}{(2\pi)^3} \frac{E}{p} V^2$$

$$d\sigma = N^2 N'^2\, 2\pi\delta(E-E') \frac{16\pi^2 Z^2 e^4}{|\mathbf{q}|^4} \frac{d^3 p'}{(2\pi)^3} \frac{E}{p} V^2$$

$$N = N' = \frac{1}{\sqrt{V}} \rightarrow d\sigma = \frac{1}{V}\frac{1}{V} 2\pi\delta(E-E') \frac{16\pi^2 Z^2 e^4}{|\mathbf{q}|^4} \frac{d^3 p'}{(2\pi)^3} \frac{E}{p} V^2 = 2\pi\delta(E-E') \frac{16\pi^2 Z^2 e^4}{|\mathbf{q}|^4} \frac{d^3 p'}{(2\pi)^3} \frac{E}{p}$$

Spin 0 - IV

Calculate differential cross-section

$$\int d\sigma = \int 2\pi\delta(E-E')\frac{16\pi^2 Z^2 e^4}{|\mathbf{q}|^4}\frac{d^3p'}{(2\pi)^3}\frac{E}{p}$$

$$d^3p' = p'^2\,dp'\,d\Omega$$

$$p'dp' = E'dE' \to d^3p' = p'E'dE'd\Omega$$

$$\to \int 2\pi\delta(E-E')\frac{16\pi^2 Z^2 e^4}{|\mathbf{q}|^4}\frac{d^3p'}{(2\pi)^3}\frac{E}{p} = \int 2\pi\delta(E-E')\frac{16\pi^2 Z^2 e^4}{|\mathbf{q}|^4}\frac{dE'}{(2\pi)^3}\frac{E}{p}p'E'd\Omega$$

$$= \frac{1}{(2\pi)^2}\frac{Z^2e^4}{|\mathbf{q}|^4}\frac{E}{p}p'\int\delta(E-E')dE'E'd\Omega = \frac{1}{(2\pi)^2}\frac{Z^2e^4}{|\mathbf{q}|^4}E^2d\Omega$$

$$q = 2p\sin\frac{\theta}{2} \to q^4 = 16p^4\sin^4\frac{\theta}{2}$$

$$\to \frac{d\sigma}{d\Omega} = \frac{1}{(2\pi)^2}\frac{16\pi^2 Z^2 e^4}{16p^4\sin^4\theta/2}E^2 = \frac{1}{4}Z^2\alpha^2\frac{E^2}{p^4}\frac{1}{\sin^4\theta/2}$$

Useful to remember

$$E^2 = \mathbf{p}^2 + m^2$$

$$\to 2EdE = 2|\mathbf{p}|d|\mathbf{p}|$$

$$\to EdE = |\mathbf{p}|d|\mathbf{p}|$$

Compare to non-relativistic Rutherford cross-section:

$$\frac{d\sigma}{d\Omega} = \frac{1}{4}Z^2\alpha^2\frac{1}{T^2}\frac{1}{\sin^4\theta/2}$$

Spin ½ - I

Take now a spin ½ Dirac electron scattering off the same, static Coulomb potential

$$j^\mu = e\overline{\psi}'\gamma^\mu\psi$$ Dirac transition current

$$\rightarrow \psi = \frac{1}{\sqrt{V}}\sqrt{\frac{m}{E}}u(s,p)e^{ipx}, \overline{\psi}' = \frac{1}{\sqrt{V}}\sqrt{\frac{m}{E'}}\overline{u}(s',p')e^{-ip'x}$$

$$\rightarrow j^\mu = e\frac{1}{V}\frac{m}{E}\overline{u}(s',p')e^{-ip'x}\gamma^\mu u(s,p)e^{ipx} = e\,\frac{1}{V}\frac{m}{E}e^{-i(p-p')x}\overline{u}\gamma^\mu u$$

$$j^\mu A_\mu = e\frac{1}{V}\frac{m}{E}\; e^{-i((E-E')t-(\mathbf{p}-\mathbf{p}')\cdot\mathbf{r})}\overline{u}(s',p')\gamma^\mu u(s,p)\left(\frac{eZ}{r},\mathbf{0}\right)$$

$$j^\mu A_\mu = e\frac{1}{V}\frac{m}{E}\; e^{-i((E-E')t-(\mathbf{p}-\mathbf{p}')\cdot\mathbf{r})}\overline{u}\gamma^0 u\frac{eZ}{r}$$ Dirac matrices

$$\underbrace{\overline{u}(s',p')}_{=u^\dagger\gamma^0}\gamma^0 u(s,p) = u^\dagger(s',p')\overbrace{\gamma^0\gamma^0}^{=1}u(s,p) = u^\dagger(s',p')u(s,p)$$

$$\frac{d\sigma}{d\Omega} = \frac{Z^2e^4}{16(2\pi)^2}\frac{E^2}{p^4}\frac{1}{E^2}\frac{1}{\sin^4\theta/2}\left|u^\dagger(s',p')u(s,p)\right|^2 = \frac{1}{4}\frac{1}{p^4}Z^2\alpha^2\frac{1}{\sin^4\theta/2}\left|u^\dagger(s',p')u(s,p)\right|^2$$

m^2 factor drops out due to Dirac flux factor

Spin ½ - II

$$\frac{1}{2}\sum_{s=-1/2}^{+1/2}\sum_{s'=-1/2}^{+1/2}\left|u^{\dagger}(s',p')u(s,p)\right|^2 = 4E^2\left(1-\beta^2\sin^2\theta/2\right)$$

$$\rightarrow \frac{d\sigma}{d\Omega} = \frac{1}{4}Z^2\alpha^2\frac{4E^2}{p^4}\frac{1}{\sin^4\theta/2}\left(1-\beta^2\sin^2\theta/2\right) = Z^2\alpha^2\frac{E^2}{p^4}\frac{\left(1-\beta^2\sin^2\theta/2\right)}{\sin^4\theta/2}$$

$$\frac{d\sigma}{d\Omega}\underset{E\gg m}{\simeq} Z^2\alpha^2\frac{E^2}{p^4}\frac{\cos^2\theta/2}{\sin^4\theta/2}$$

Mott cross section (high energy limit)

New factor, important at high speed
Reducing cross section at large angles (= *0* for $\theta\rightarrow\pi$)

Matrix element - I

Matrix element the dull way: use Dirac spinors

$$u^\dagger(1,p')u(1,p)=\sqrt{E'+m}\begin{bmatrix}\cos\frac{\theta}{2} & \sin\frac{\theta}{2} & \frac{|\mathbf{p}|\cos\frac{\theta}{2}}{E'+m} & \frac{|\mathbf{p}|\sin\frac{\theta}{2}}{E'+m}\end{bmatrix}\sqrt{E+m}\begin{bmatrix}1\\0\\\frac{|\mathbf{p}|}{E+m}\\0\end{bmatrix}$$

$$\rightarrow u^\dagger(1,p')u(1,p)=(E+m)\left(\cos\frac{\theta}{2}+0+\frac{|\mathbf{p}|^2}{(E+m)^2}\cos\frac{\theta}{2}+0\right)=\left[(E+m)+\frac{|\mathbf{p}|^2}{E+m}\right]\cos\frac{\theta}{2}$$

$$u^\dagger(2,p')u(1,p)=\sqrt{E'+m}\begin{bmatrix}-\sin\frac{\theta}{2} & \cos\frac{\theta}{2} & \frac{|\mathbf{p}|}{E+m}\sin\frac{\theta}{2} & -\frac{|\mathbf{p}|}{E+m}\cos\frac{\theta}{2}\end{bmatrix}\sqrt{E+m}\begin{bmatrix}1\\0\\\frac{|\mathbf{p}|}{E+m}\\0\end{bmatrix}$$

$$\rightarrow u^\dagger(2,p')u(1,p)=(E+m)\left(-\sin\frac{\theta}{2}+0+\frac{|\mathbf{p}|^2}{(E+m)^2}\sin\frac{\theta}{2}+0\right)=\left(-(E+m)+\frac{|\mathbf{p}|^2}{E+m}\right)\sin\frac{\theta}{2}$$

Matrix element - II

$$u^\dagger(1,p')u(2,p)=\sqrt{E'+m}\begin{bmatrix}\cos\frac{\theta}{2} & \sin\frac{\theta}{2} & \frac{|\mathbf{p}|}{E'+m}\cos\frac{\theta}{2} & \frac{|\mathbf{p}|}{E'+m}\sin\frac{\theta}{2}\end{bmatrix}\sqrt{E+m}\begin{bmatrix}0\\1\\0\\-\frac{|\mathbf{p}|}{E+m}\end{bmatrix}$$

$$\rightarrow u^\dagger(1,p')u(2,p)=(E+m)\left(0+\sin\frac{\theta}{2}+0-\frac{|\mathbf{p}|^2}{(E+m)^2}\sin\frac{\theta}{2}\right)=\left[(E+m)-\frac{|\mathbf{p}|^2}{(E+m)^2}\right]\sin\frac{\theta}{2}$$

$$u^\dagger(2,p')u(2,p)=\sqrt{E'+m}\begin{bmatrix}-\sin\frac{\theta}{2} & \cos\frac{\theta}{2} & \frac{|\mathbf{p}|}{E'+m}\sin\frac{\theta}{2} & -\frac{|\mathbf{p}|}{E'+m}\cos\frac{\theta}{2}\end{bmatrix}\sqrt{E+m}\begin{bmatrix}0\\1\\0\\-\frac{|\mathbf{p}|}{E+m}\end{bmatrix}$$

$$\rightarrow u^\dagger(2,p')u(2,p)=(E+m)\left(0+\cos\frac{\theta}{2}+0+\frac{|\mathbf{p}|^2}{(E+m)^2}\cos\frac{\theta}{2}\right)=\left[E+m+\frac{|\mathbf{p}|^2}{E+m}\right]\cos\frac{\theta}{2}$$

Matrix element - III

$$\frac{1}{2}\sum_{s=-1/2}^{+1/2}\sum_{s'=-1/2}^{+1/2}\left|u^{\dagger}(s',p')u(s,p)\right|^2=\frac{1}{2}\left\{\begin{array}{l}\left[(E+m)+\frac{|\mathbf{p}|^2}{E+m}\right]^2\cos^2\frac{\theta}{2}+\left(-(E+m)+\frac{|\mathbf{p}|^2}{E+m}\right)^2\sin^2\frac{\theta}{2}+\\+\left[(E+m)-\frac{|\mathbf{p}|^2}{E+m}\right]^2\sin^2\frac{\theta}{2}+\left[E+m+\frac{|\mathbf{p}|^2}{E+m}\right]^2\cos^2\frac{\theta}{2}\end{array}\right\}$$

$$=\frac{1}{2}\left\{2\left[(E+m)+\frac{|\mathbf{p}|^2}{E+m}\right]^2\cos^2\frac{\theta}{2}+2\left[(E+m)-\frac{|\mathbf{p}|^2}{E+m}\right]^2\sin^2\frac{\theta}{2}\right\}$$

$$=\frac{1}{2}\left\{2\left[(E+m)+\frac{E^2-m^2}{E+m}\right]^2\cos^2\frac{\theta}{2}+2\left[(E+m)-\frac{E^2-m^2}{E+m}\right]^2\sin^2\frac{\theta}{2}\right\}$$

$$=\left[(E+m)+E-m\right]^2\cos^2\frac{\theta}{2}+\left[(E+m)-E+m\right]^2\sin^2\frac{\theta}{2}$$

$$=(2E)^2\cos^2\frac{\theta}{2}+4m^2\sin^2\frac{\theta}{2}=(2E)^2\cos^2\frac{\theta}{2}+4\left(E^2-|\mathbf{p}|^2\right)\sin^2\frac{\theta}{2}$$

Matrix element - IV

Unpolarized cross section:

Average initial spin projections

Add final spin spin projections

$$\frac{1}{2}\sum_{s=-1/2}^{+1/2}\sum_{s'=-1/2}^{+1/2}\left|u^{\dagger}(s',p')u(s,p)\right|^2=(2E)^2\cos^2\frac{\theta}{2}+(2E)^2\sin^2\frac{\theta}{2}-4|\mathbf{p}|^2\sin^2\frac{\theta}{2}$$

$$=(2E)^2-(2|\mathbf{p}|)^2\sin^2\frac{\theta}{2}$$

$$\rightarrow\frac{1}{2}\sum_{s=-1/2}^{+1/2}\sum_{s'=-1/2}^{+1/2}\left|u^{\dagger}(s',p')u(s,p)\right|^2=(2E)^2\left(1-\beta^2\sin^2\frac{\theta}{2}\right)$$

Form Factors - I

Take now a spin ½ Dirac electron scattering off a *distributed*, static source (like a *(A,Z)* nucleus)

$$A_\mu = (\varphi, \mathbf{0})$$

$$\varphi(r) = \int d^3\mathbf{r}' \frac{\rho(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}, \qquad \int \rho(\mathbf{r}') d^3\mathbf{r}' = Ze$$

Only change: the space integral

$$\int e^{+i((\mathbf{p}\text{-}\mathbf{p}')\cdot\mathbf{r})} \frac{e^2 Z}{r} d^3\mathbf{r} = \frac{4\pi Z e^2}{|\mathbf{q}|^2}, \mathbf{q} = \mathbf{p} - \mathbf{p}' \to \int d^3\mathbf{r} \int e^{+i((\mathbf{p}\text{-}\mathbf{p}')\cdot\mathbf{r})} \frac{\rho(\mathbf{r}')}{|\mathbf{r}'-\mathbf{r}|} d^3\mathbf{r}'$$

$$= \int d^3\mathbf{r}' \frac{\rho(\mathbf{r}')}{Ze} e^{+i(\mathbf{p}\text{-}\mathbf{p}')\cdot\mathbf{r}'} \int e^{+i((\mathbf{p}\text{-}\mathbf{p}')\cdot(\mathbf{r}-\mathbf{r}'))} \frac{Ze}{|\mathbf{r}'-\mathbf{r}|} d^3\mathbf{r}$$

$$F(\mathbf{q}) = \frac{\int \rho(\mathbf{r}) e^{+i\mathbf{q}\cdot\mathbf{r}} d^3\mathbf{r}}{Ze}$$ Form factor of the charge distribution

$$\Rightarrow \frac{Z^2 e^2}{|\mathbf{q}|^4} \to \frac{Z^2 e^2}{|\mathbf{q}|^4} |F(\mathbf{q})|^2$$

Form Factors - II

$$F(\mathbf{q}) = \frac{\int \rho(\mathbf{r}) e^{+i\mathbf{q}\cdot\mathbf{r}} d^3\mathbf{r}}{Ze} = \frac{1}{Ze}\int \rho(r) e^{+iqr\cos\theta} r^2 dr d\Omega$$

$$e^{+i|\mathbf{q}|r\cos\theta} = 1 + i|\mathbf{q}| r\cos\theta - \frac{1}{2}|\mathbf{q}|^2 r^2 \cos^2\theta ...$$

$$\rightarrow \frac{1}{Ze}\int \rho(r) e^{+i|\mathbf{q}|r\cos\theta} r^2 dr d\Omega \approx$$

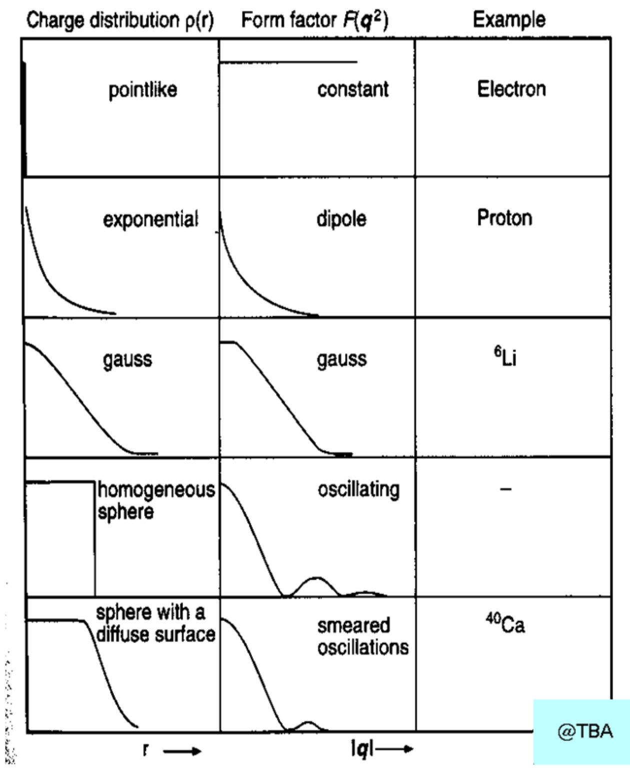
$$\approx \frac{1}{Ze}\left[\int \rho(r) r^2 dr d\Omega + i|\mathbf{q}|\int \rho(r) r^3 \cos\theta dr d\Omega - \frac{|\mathbf{q}|^2}{2}\int \rho(r) r^4 \cos^2\theta dr d\Omega\right]$$

$$\rightarrow F(\mathbf{q}) \approx 1 + \frac{i|\mathbf{q}|}{Ze}\int \rho(r) r^3 dr \underbrace{\int \cos\theta d\Omega}_{=\frac{2\pi}{2}\left(-\cos^2\theta\right)\Big|_0^\pi = 0} - \frac{|\mathbf{q}|^2}{2Ze}\underbrace{\int \rho(r) r^4 dr}_{=\frac{Ze\langle r^2\rangle}{4\pi}} \underbrace{\int \cos^2\theta d\Omega}_{=\frac{2\pi}{3}\left(-\cos^3\theta\right)\Big|_0^\pi = \frac{4\pi}{3}} = 1 - \frac{|\mathbf{q}|^2\langle r^2\rangle}{6}$$

$$\left.\begin{aligned} F\left(|\mathbf{q}|^2\right) &= F(0) + \frac{\partial F}{\partial |\mathbf{q}|^2}|\mathbf{q}|^2 + ... \\ F\left(|\mathbf{q}|^2\right) &\approx 1 - \frac{1}{6}|\mathbf{q}|^2\langle r^2\rangle \end{aligned}\right\} \rightarrow\rightarrow \left\{\begin{aligned} &F(0) = 1 \\ &\langle r^2\rangle = -6\left.\frac{\partial F}{\partial |\mathbf{q}|^2}\right|_{|\mathbf{q}|^2=0} \end{aligned}\right.$$

showing that measuring the form factor yields the rms charge radius

Form Factors - III



$$\rho(r)=\frac{1}{8\pi a^3}e^{-ar}\to F\left(|\mathbf{q}|^2\right)=\left(\frac{1}{1+|\mathbf{q}|^2/a^2}\right)^2$$

$$\rho(r)=\begin{cases}\text{constant r<R}\\ 0 \qquad \text{r>R}\end{cases}\to F\left(|\mathbf{q}|^2\right)=\frac{3}{R^3|\mathbf{q}|^3}\left(\sin\left(R|\mathbf{q}|\right)-R|\mathbf{q}|\cos\left(R|\mathbf{q}|\right)\right)$$

Nuclear Form Factors - I

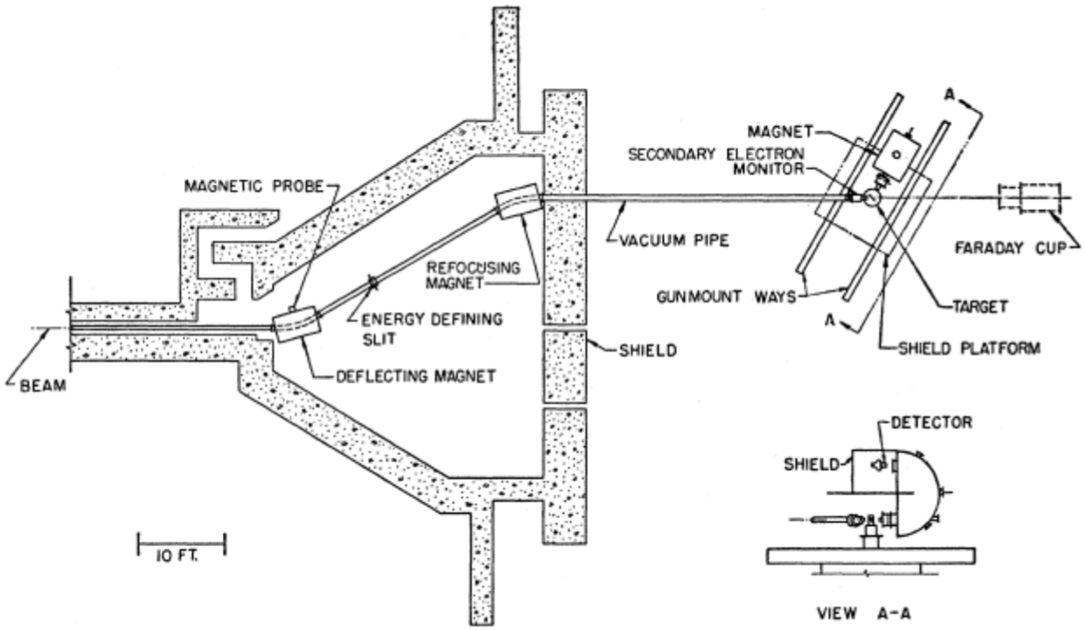


FIG. 18. The experimental installation of the 550-Mev spectrometer.

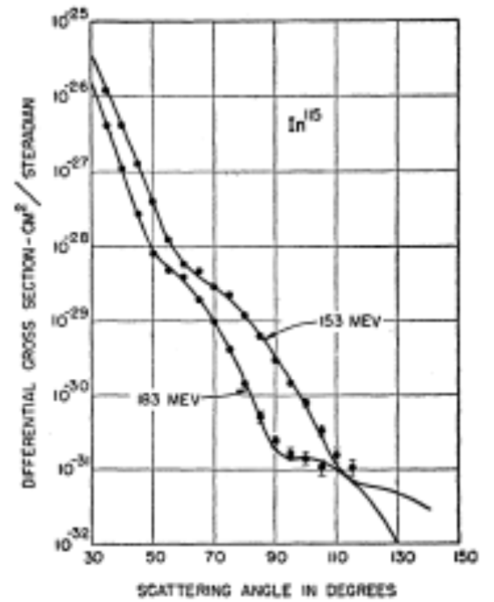


FIG. 46. Theoretical and experimental curves for In^{115} at two energies.

@TBA

One of the Hofstadter's spectrometers at SLAC

Results for Indium

Nuclear Form Factors - II

Details of one of Hofstadter's focusing spectrometers

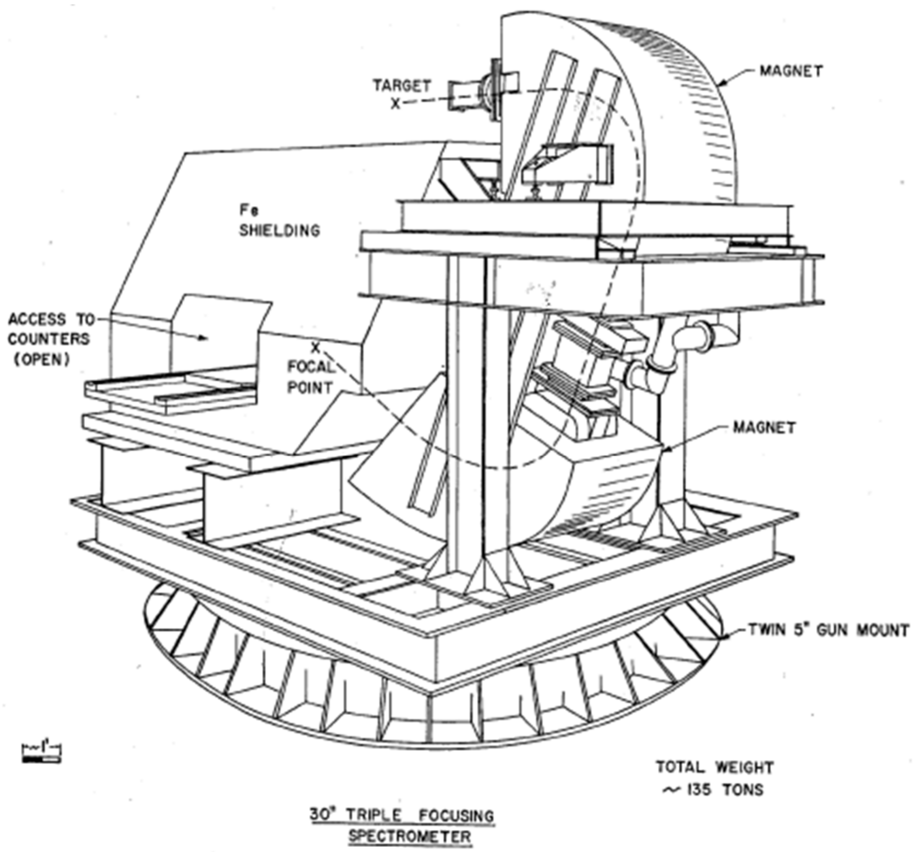
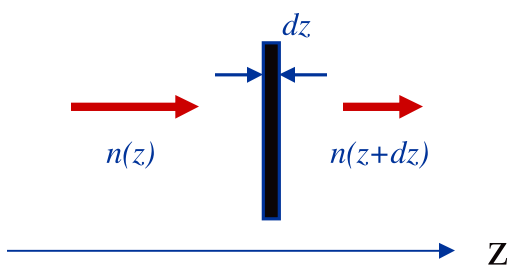


Fig. 4. Perspective drawing of the spectrometer in position on the gun mount, showing iron shielding in place around the counter house. The momentum slit box and part of the vacuum chamber are shown between the magnets.

Nuclear Form Factors - III

From counting rate to cross section:



$$dn = -n(z)\, n_T \sigma dz \rightarrow |\Delta n| \simeq n_0 n_T \sigma \Delta z, \Delta n \ll n_0$$

$$n_T = \frac{\rho_T}{A} N_A \rightarrow |\Delta n| \simeq n_0 \rho_T \frac{N_A}{A} \sigma \Delta z$$

$$\rightarrow \sigma = \frac{1}{N_A} \frac{A}{\rho_T \Delta z} \frac{\Delta n}{n_0}, \Delta n \ll n_0, \Delta z \text{ small}$$

Count scattering events, count beam particles, measure target

$\rightarrow$ Get σ

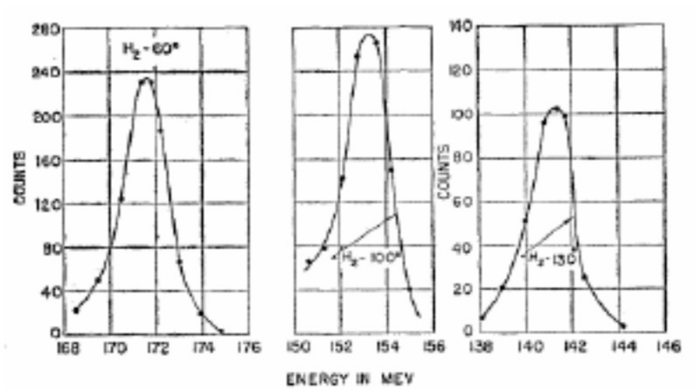
What is known:

Beam energy
Scattering angle
# of incident beam particles, n_0
# of scattering events, Δn
Target thickness, Δz
Target mass density, ρ_T

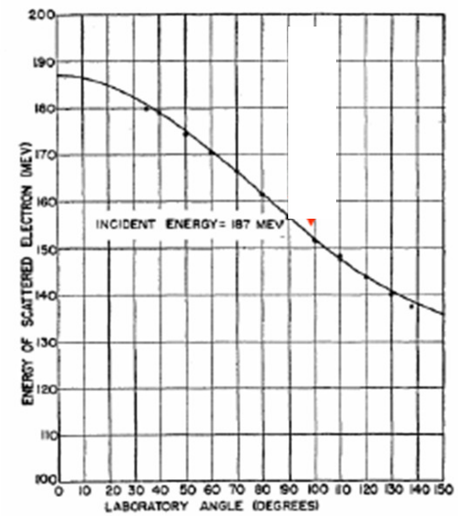
Nuclear Form Factors - IV

Elastic nuclear scattering

Count rate vs energy at different angles:
Elastic peak



Scattered electron energy vs angle:
2-body kinematics



$$\frac{E_e'}{E_e} = \frac{1}{1 + 2\frac{E_e}{m_N}\sin^2\frac{\theta}{2}}$$

Inelastic Scattering - I

Inelastic cross section: Providing evidence for nuclear constituents

Detect γ*-rays from level de-excitation*

Also:

Inclusive energy spectra of scattered electrons yields detailed information on nuclear structure

Snapshot of proton wave function within the nucleus:

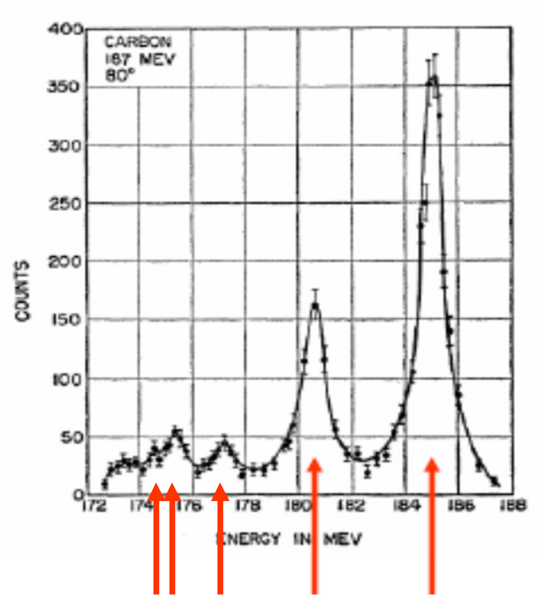
Fermi motion
Radius and depth of potential well

At high energy, constituents seen as *free particles* upon nuclear breakup

Inelastic Scattering - II

Inelastic nuclear scattering:

Count rate vs energy at fixed angle
Excitation of ^{12}C nuclear levels



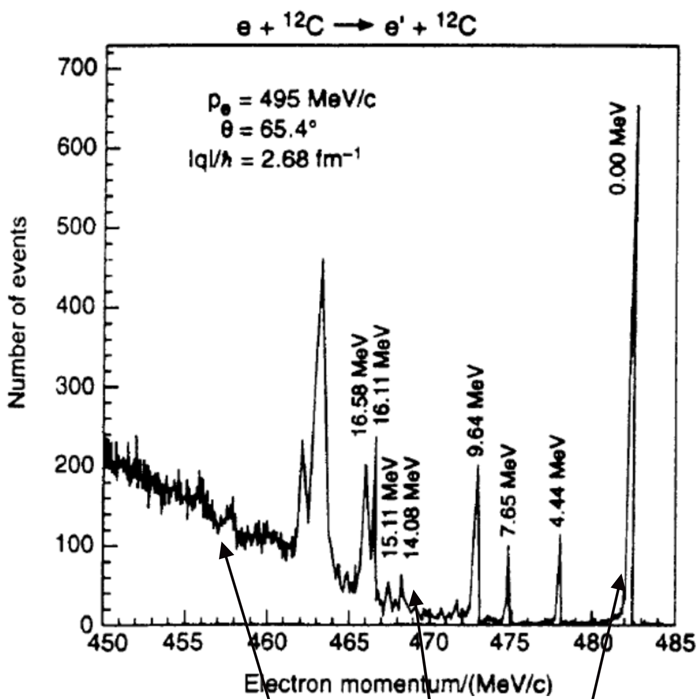
Inelastic peaks Elastic peak

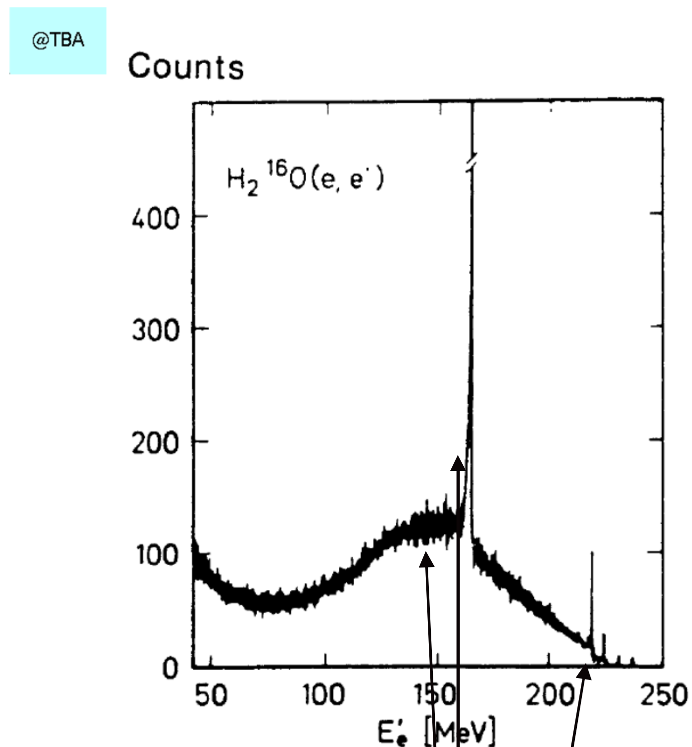
Scattered electron energy vs angle:
>2 body kinematics

$$\frac{E_e'}{E_e} = \frac{1 - \dfrac{\Delta m^2}{2 m_N E_e}}{1 + 2\dfrac{E_e}{m_N}\sin^2\dfrac{\theta}{2}}$$

Δm Excitation energy

Inelastic Scattering - III





Scattering off ^{12}C

Elastic scattering off the *whole nucleus*

Nuclear levels excitation (inelastic)

Beginning of elastic scattering off *protons*

Scattering off water

Elastic scattering off the *whole ^{16}O nucleus*

Nuclear levels excitation (inelastic)

Elastic scattering off *^{16}O protons*

Elastic scattering off *free protons*

Particle-Particle Scattering

1st order Transition amplitude:

$$H' = j^{\mu} A_{\mu} \rightarrow H' = \left(j_1^{\mu} + j_2^{\mu} \right) A_{\mu}$$

$$\rightarrow M_{fi} = i(2\pi)^4 \delta\left(p_1 + p_2 - \left(p_1^{'} + p_2^{'} \right) \right) T_{fi} = i(2\pi)^4 \delta\left(p_1 + p_2 - \left(p_1^{'} + p_2^{'} \right) \right) j_{\mu}^{(1)} \frac{ig^{\mu\nu}}{q^2} j_{\nu}^{(2)}$$

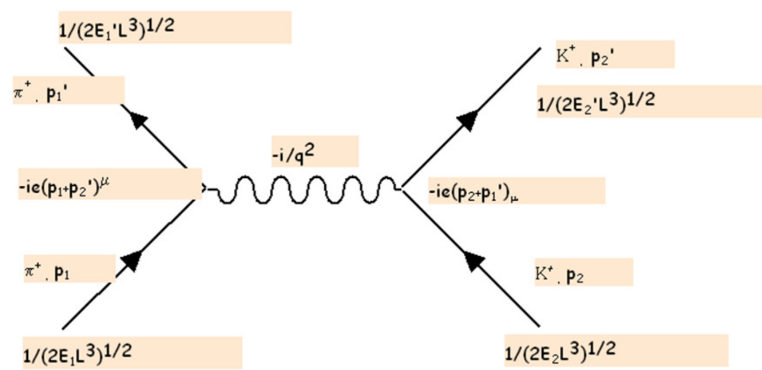
$q = p_1 - p_1' = p_2 - p_2'$ 4-momentum transfer

Transition currents:

$j^{\mu} = e\left(p_1 + p_3 \right)^{\mu}$ scalar

$j^{\mu} = e\overline{u}_4 \gamma^{\mu} u_2$ fermion

....



Spin ½ - Spin ½

Just to simplify things, take *different* spin ½ particles (e.g. electron-muon scattering)

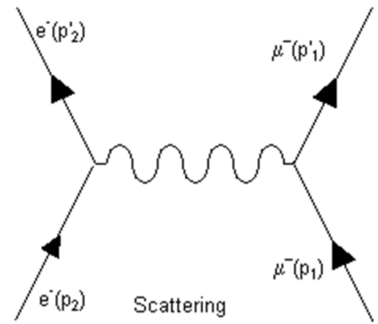
$$T_{fi}(s,s',r,r') = e\bar{u}'(p_2',s')\gamma^\mu u(p_2,s)\frac{g_{\mu\nu}}{q^2}\bar{u}'(p_1',r')\gamma^\nu u(p_1,r)$$

$$\frac{1}{4}\sum_{s,s',r,r'}\left|T_{fi}(s,s',r,r')\right|^2 = \frac{e^4}{q^4}L_{\mu\nu}M^{\mu\nu}$$

$$L^{\mu\nu} = 2\left[p_1'^{\mu}p_1^{\nu} + p_1'^{\nu}p_1^{\mu} + \frac{q^2}{2}g^{\mu\nu}\right]$$

$$M_{\mu\nu} = 2\left[p_{2\mu}'p_{2\nu} + p_{2\nu}'p_{2\mu} + \frac{q^2}{2}g_{\mu\nu}\right]$$

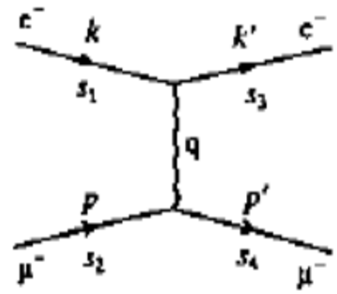
$$\rightarrow \left.\frac{d\sigma}{d\Omega}\right|_{LAB} \underset{E\gg m}{\simeq} \frac{\alpha^2\cos^2\theta/2}{4|\mathbf{p}_1|^2\sin^4\theta/2}\frac{|\mathbf{p}_1'|}{|\mathbf{p}_1|}\left(1-\frac{q^2\tan^2\theta/2}{2m_2^2}\right)$$



Yet another term…
Electron scattering off the muon *magnetic moment*

Matrix Element - I

$e^- + \mu^- \to e^- + \mu^-$ Unpolarized cross section



(Squared) Matrix element the smart way:

$$T_{fi} = \left[\bar{u}(k')\gamma^{\mu}u(k)\right]\frac{e^2}{q^2}\left[\bar{u}(p')\gamma_{\mu}u(p)\right]$$

$$\left|T_{fi}\right|^2 = \frac{1}{4}\sum_{s_1,s_2,s_3,s_4}\left|\left[\bar{u}(k',s_3)\gamma^{\mu}u(k,s_1)\right]\frac{e^2}{q^2}\left[\bar{u}(p',s_4)\gamma_{\mu}u(p,s_2)\right]\right|^2$$

$$= \frac{e^4}{4q^4}\sum_{s_1,s_2,s_3,s_4}\left[\bar{u}(k',s_3)\gamma^{\mu}u(k,s_1)\right]\left[\bar{u}(p',s_4)\gamma_{\mu}u(p,s_2)\right]\left[\bar{u}(k',s_3)\gamma^{\nu}u(k,s_1)\right]^*\left[\bar{u}(p',s_4)\gamma_{\nu}u(p,s_2)\right]^*$$

$$L^{\mu\nu} = \sum_{s_1,s_3}\left[\bar{u}(k',s_3)\gamma^{\mu}u(k,s_1)\right]\left[\bar{u}(k',s_3)\gamma^{\nu}u(k,s_1)\right]^*$$

$$M_{\mu\nu} = \sum_{s_1,s_3}\left[\bar{u}(p',s_4)\gamma_{\mu}u(p,s_2)\right]\left[\bar{u}(p',s_4)\gamma_{\nu}u(p,s_2)\right]^* \to \left|T_{fi}\right|^2 = \frac{e^4}{4q^4}L^{\mu\nu}M_{\mu\nu}$$

Matrix Element - II

$$\left[\bar{u}(k',s_3)\gamma^\nu u(k,s_1)\right]^* = \left[u^\dagger(k',s_3)\gamma^0\gamma^\nu u(k,s_1)\right]^*$$

Dirac algebra:

$$\left[\bar{u}(k',s_3)\gamma^\nu u(k,s_1)\right]^* = \left[u^\dagger(k,s_1)\gamma^{\nu\dagger}\gamma^0 u(k',s_3)\right] = \left[u^\dagger(k,s_1)\gamma^0\gamma^\nu u(k',s_3)\right]$$

$$= \left[\bar{u}(k,s_1)\gamma^\nu u(k',s_3)\right]$$

$$L^{\mu\nu} = \sum_{s_1,s_3}\left[\bar{u}(k',s_3)\gamma^\mu u(k,s_1)\right]\left[\bar{u}(k',s_3)\gamma^\nu u(k,s_1)\right]^*$$

$$\to L^{\mu\nu} = \sum_{s_1,s_3}\left[\bar{u}(k',s_3)\gamma^\mu u(k,s_1)\right]\left[\bar{u}(k,s_1)\gamma^\nu u(k',s_3)\right]$$

To write off matrix products in full:

Use Einstein convention on repeated indexes (= summed)

$$\to L^{\mu\nu} = \sum_{s_3}\bar{u}_\alpha(k',s_3)\gamma^\mu{}_{\alpha\beta}\sum_{s_1}u_\beta(k,s_1)\bar{u}_\gamma(k,s_1)\gamma^\nu{}_{\gamma\delta}u_\delta(k',s_3)$$

Matrix Element - III

Use Feynman convention:

$$\not{a} \equiv a_\mu \gamma^\mu (= \sum_{\mu=0}^{3} a_\mu \gamma^\mu)$$

$$\rightarrow \sum_{s_1} u_\beta(k, s_1) \bar{u}_\gamma(k, s_1) = (\not{k} + m)_{\beta\gamma}$$

$$\rightarrow L^{\mu\nu} = \sum_{s_3} \bar{u}_\alpha(k', s_3) \gamma^\mu{}_{\alpha\beta} (\not{k} + m)_{\beta\gamma} \gamma^\nu{}_{\gamma\delta} u_\delta(k', s_3)$$

$$= \sum_{s_3} u_\delta(k', s_3) \bar{u}_\alpha(k', s_3) \gamma^\mu{}_{\alpha\beta} (\not{k} + m)_{\beta\gamma} \gamma^\nu{}_{\gamma\delta}$$

$$\sum_{s_1} u_\delta(k', s_3) \bar{u}_\alpha(k', s_3) = (\not{k}' + m)_{\delta\alpha}$$

$$\rightarrow L^{\mu\nu} = (\not{k}' + m)_{\delta\alpha} \gamma^\mu{}_{\alpha\beta} (\not{k} + m)_{\beta\gamma} \gamma^\nu{}_{\gamma\delta} = Tr(\not{k}' + m) \gamma^\mu (\not{k} + m) \gamma^\nu$$

$$\rightarrow M_{\mu\nu} = Tr(\not{p}' + M) \gamma_\mu (\not{p} + M) \gamma_\nu$$

Matrix Element - IV

More Dirac algebra:

$$L^{\mu\nu} = Tr\left(\not{k}' + m\right)\gamma^{\mu}\left(\not{k} + m\right)\gamma^{\nu}$$

$$L^{\mu\nu} = Tr\left[\not{k}'\gamma^{\mu}\not{k}\gamma^{\nu}\right] + mTr\left[\gamma^{\mu}\not{k}\gamma^{\nu}\right] + mTr\left[\not{k}'\gamma^{\mu}\gamma^{\nu}\right] + m^2 Tr\left[\gamma^{\mu}\gamma^{\nu}\right]$$

$$\rightarrow L^{\mu\nu} = Tr\left[\not{k}'\gamma^{\mu}\not{k}\gamma^{\nu}\right] + m^2 g^{\mu\nu}$$

$$\rightarrow M_{\mu\nu} = Tr\left[\not{p}'\gamma_{\mu}\not{p}\gamma_{\nu}\right] + M^2 g_{\mu\nu}$$

$$a_{\mu}b_{\nu}Tr\left[\not{p}'\gamma_{\mu}\not{p}\gamma_{\nu}\right] = Tr\left[\not{p}'\not{a}\not{p}\not{b}\right]$$

$$\rightarrow Tr\left[\not{p}'\not{a}\not{p}\not{b}\right] = 4\left[\left(k'\cdot a\right)\left(k\cdot b\right) + \left(k'\cdot b\right)\left(k\cdot a\right) - \left(k'\cdot k\right)\left(a\cdot b\right)\right]$$

$$\rightarrow Tr\left[\not{p}'\not{a}\not{p}\not{b}\right] = 4a_{\mu}b_{\nu}\left[k'^{\mu}k^{\nu} + k'^{\nu}k^{\mu} - \left(k'\cdot k\right)g^{\mu\nu}\right]$$

$$\rightarrow Tr\left[\not{k}'\gamma^{\mu}\not{k}\gamma^{\nu}\right] = 4\left[k'^{\mu}k^{\nu} + k'^{\nu}k^{\mu} - \left(k'\cdot k\right)g^{\mu\nu}\right]$$

Matrix Element - V

$$\rightarrow L^{\mu\nu} = 4\left[k'^{\mu} k^{\nu} + k'^{\nu} k^{\mu} + \left(m^2 - (k' \cdot k)\right) g^{\mu\nu}\right]$$

$$\rightarrow M_{\mu\nu} = 4\left[p'_{\mu} p_{\nu} + p'_{\nu} p_{\mu} + \left(M^2 - (p' \cdot p)\right) g_{\mu\nu}\right]$$

$$\left|T_{fi}\right|^2 = \frac{e^4}{4q^4} L^{\mu\nu} M_{\mu\nu}$$

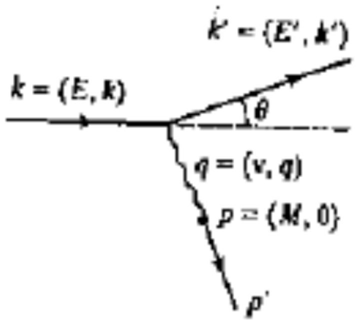
$$= \frac{8e^4}{q^4}\left[(k' \cdot p')(k \cdot p) + (k \cdot p')(k' \cdot p) - m^2 (p' \cdot p) - M^2 (k' \cdot k) + 2m^2 M^2\right]$$

$$= \frac{8e^4}{q^4}\left[(k' \cdot p')(k \cdot p) + (k \cdot p')(k' \cdot p) - M^2 (k' \cdot k)\right]$$

$$= \frac{8e^4}{q^4}\left[-\frac{1}{2} q^2 (k \cdot p - k' \cdot p) + 2(k \cdot p)(k' \cdot p) + \frac{1}{2} M^2 q^2\right]$$

Cross-Section

In the LAB frame: Muon at rest



$q = (\nu, \mathbf{q})$ virtual photon 4-momentum

$$|T_{fi}|^2 = \frac{8e^4}{q^4} 2EE'M^2 \left[\cos^2\left(\frac{\theta}{2}\right) - \frac{q^2}{2M^2}\sin^2\left(\frac{\theta}{2}\right)\right]$$

$$\rightarrow \frac{d\sigma}{d\Omega dE'} = \frac{4\alpha^2 E'^2}{q^4}\left[\cos^2\left(\frac{\theta}{2}\right) - \frac{q^2}{2M^2}\sin^2\left(\frac{\theta}{2}\right)\right]\delta\left(\nu + \frac{q^2}{2M}\right)$$

$$\rightarrow \frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2\sin^4\left(\frac{\theta}{2}\right)} \frac{1}{1 + \frac{2E}{M}\sin^2\left(\frac{\theta}{2}\right)}\left[\cos^2\left(\frac{\theta}{2}\right) - \frac{q^2}{2M^2}\sin^2\left(\frac{\theta}{2}\right)\right]$$

$$\rightarrow \frac{d\sigma}{d\Omega} = \left.\frac{d\sigma}{d\Omega}\right|_{Mott}\left[1 - \frac{q^2}{2M^2}\tan^2\left(\frac{\theta}{2}\right)\right]$$

Electron Form Factors - I

$j^\mu = e\bar{\psi}\gamma^\mu\psi$ Dirac current

$\bar{u}(p')\gamma^\mu u(p) = \frac{1}{2m}\left[(p+p')^\mu + i\sigma^{\mu\nu}(p-p')_\nu\right]$ Gordon's identity

$$\begin{cases} \frac{e}{2m}u(p')(p+p')^\mu u(p) & \text{charge, like a scalar particle} \\ \frac{ie}{2m}\bar{u}(p')\sigma^{\mu\nu}\underbrace{(p-p')_\nu}_{=q_\nu}u(p) & \text{extra term} \end{cases}$$

Extra term due to *magnetic dipole current.*
Indeed, it contributes the interaction energy:

$$\frac{ie}{2m}\bar{u}(p')\sigma^{\mu\nu}u(p)q_\nu A_\mu \underset{\text{low speed}}{\rightarrow} -\frac{e}{2m}\phi'^\dagger\boldsymbol{\sigma}\cdot(\mathbf{q}\times\mathbf{A})\phi \qquad \text{Magnetic dipole interaction energy}$$

$$\frac{e}{2m}\phi'^\dagger\boldsymbol{\sigma}\cdot(\nabla\times\mathbf{A})\phi \equiv \frac{e}{2m}\phi'^\dagger(\boldsymbol{\sigma}\cdot\mathbf{B})\phi \Rightarrow \frac{ie}{2m}\bar{u}(p')\sigma^{\mu\nu}u(p)q_\nu A_\mu \underset{\text{low speed}}{\rightarrow} -\frac{e}{2m}\phi'^\dagger(\boldsymbol{\sigma}\cdot\mathbf{B})\phi$$

$\mu \approx \frac{e\hbar}{2mc}$ Magnetic moment , $j = \frac{1}{2}\hbar$ Spin, $\gamma \equiv \frac{\mu}{j}$ Gyromagnetic ratio

$\gamma \approx \frac{e\hbar}{2mc}\frac{2}{\hbar} \underset{\substack{\text{natural}\\\text{units}}}{=} \frac{e}{2m}\cdot 2,$ Define $\gamma \equiv g\frac{e}{2m} \rightarrow g \approx 2$ Dirac g-factor

Electron Form Factors - II

Now: g-factor not exactly 2, as predicted by Dirac equation
Reason: *Radiative corrections*
Largest correction: Anomalous magnetic moment

$$\mu_{Dirac} = \frac{e}{2m} \to \mu = \frac{e}{2m}(1+\kappa_e)$$

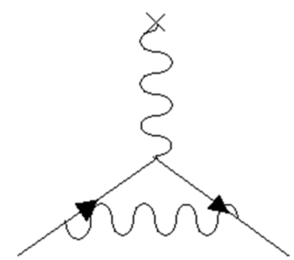
$$j^\mu = \frac{e}{2m}\bar{u}(p')\left[(p+p')^\mu + i\sigma^{\mu\nu}(1+\kappa_e)q_\nu\right]u(p)$$

$$\bar{u}(p')\gamma^\mu u(p) = \frac{1}{2m}\bar{u}(p')\left[(p+p')^\mu + i\sigma^{\mu\nu}q_\nu\right]u(p)$$

$$\to \bar{u}(p')(p+p')^\mu u(p) = \bar{u}(p')\left(\gamma^\mu - i\sigma^{\mu\nu}q_\nu\right)u(p)$$

$$\to j^\mu = \frac{e}{2m}\bar{u}(p')\left[\gamma^\mu - i\sigma^{\mu\nu}q_\nu + i\sigma^{\mu\nu}(1+\kappa_e)q_\nu\right]u(p)$$

$$\to j^\mu = \frac{e}{2m}\bar{u}(p')\left[\gamma^\mu + i\kappa_e\sigma^{\mu\nu}q_\nu\right]u(p)$$

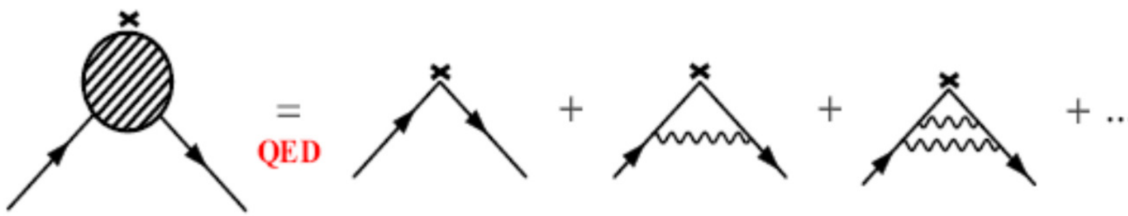


In spite of the electron *being* a pointlike fermion, radiative corrections make it behaving like an extended object

Further radiative corrections lumped into 2 *form factors*

$$j^\mu = \frac{e}{2m}\bar{u}(p')\left[f(q^2)\gamma^\mu + g(q^2)i\kappa_e\sigma^{\mu\nu}q_\nu\right]u(p)$$ Most general form

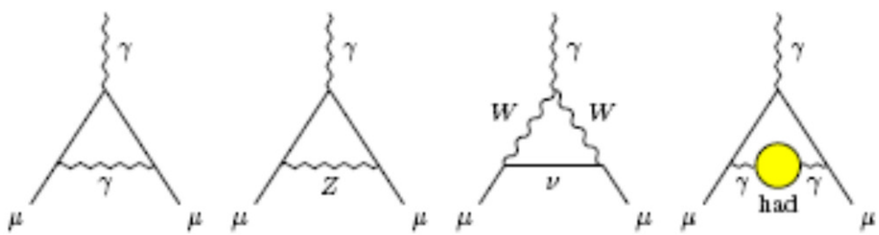
g - 2 - Theory



QED only…

Figure 1: The perturbative expansion of $\Gamma^\rho(p', p)$ in single flavour QED. The tree graph gives $F_1 = 1$, $F_2 = F_3 = 0$. The one loop vertex correction graph gives the coefficient A_1 in Eq. (2.21). The cross denotes the insertion of the external field.

@TBA



QED	Electroweak	QCD	Contributions from new physics ? ???	..and more

Nucleon Form Factors

Take the same current for the nucleon

$$j_p^\mu = e\bar{u}(p')\left(F(q^2)\gamma^\mu + G(q^2)i\kappa_p\sigma^{\mu\nu}q_\nu\right)u(p)$$

$$\kappa_p = ?$$

Anomalous magnetic moment well measured, not understood

Anomaly originating from the extended shape of the proton

$$F_1(q^2) = F(q^2)$$

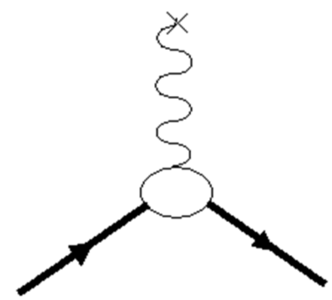
$$F_2(q^2) = 2MG(q^2)$$

$$\rightarrow j_p^\mu = e\bar{u}(p')\left(F_1(q^2)\gamma^\mu + \frac{i\kappa_p F_2(q^2)}{2M}\sigma^{\mu\nu}q_\nu\right)u(p)$$

Redefine:

$$G_E(q^2) = F_1 + \frac{\kappa_P q^2}{4M^2}F_2$$ Electric form factor

$$G_M(q^2) = F_1 + \kappa_P F_2$$ Magnetic form factor



Blob indicates a non-QED vertex

Nucleon Magnetic Moments

Electron-Proton comparison

$$\mu_B = \frac{e\hbar}{2m_e} = 5.78\times10^{-5} \;\; eV/T$$

$$\mu = g\,s\,\mu_B$$

μ_e/μ_B = 1.001 159 652 187 ± 0.000 000 000 004

$\frac{g}{2}$: Electron

$$\mu_N = \frac{e\hbar}{2m_p} = 3.15\times10^{-8} \;\; eV/T$$

$$\mu_p = g\,s\,\mu_N$$

μ_p/μ_N = 2.792847351 ± 0.000000028

$\frac{g}{2}$: Proton

Reminder: For a free Dirac particle $g=2$

Strong indication: *The nucleon is not a point-like particle*

Rosenbluth Formula - I

Consider elastic electron-nucleon scattering
Going through the same steps as for electron-muon scattering

$$\begin{cases} A(q^2) = F_1^2(q^2) - \kappa_p^2 \dfrac{q^2}{4M^2} F_2^2(q^2) \\ B(q^2) = -\dfrac{q^2}{2M^2}\left(F_1(q^2) + \kappa_p F_2(q^2)\right)^2 \end{cases}$$

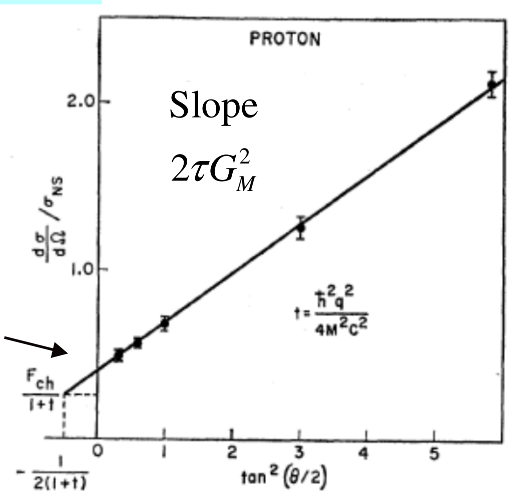
$$\left.\frac{d\sigma}{d\Omega}\right|_{LAB} = \frac{\alpha^2}{4|\mathbf{p}_1|^2 \sin^4\theta/2} \cos^2\theta/2 \frac{|\mathbf{p}_1'|}{|\mathbf{p}_1|}\left(A(q^2) + B(q^2)\tan^2\theta/2\right)$$

$$\left.\frac{d\sigma}{d\Omega}\right|_{LAB} = \frac{\alpha^2}{4|\mathbf{p}_1|^2 \sin^4\theta/2} \cos^2\theta/2 \frac{|\mathbf{p}_1'|}{|\mathbf{p}_1|}\left[\frac{G_E^2 - (q^2/4m^2)G_M^2}{1 - q^2/4m^2} - \frac{q^2}{m^2} G_M^2 \tan^2(\theta/2)\right]$$

$$\left.\frac{d\sigma}{d\Omega}\right|_{LAB} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott}\left[\frac{G_E^2 + \tau G_M^2}{1+\tau} + 4\tau G_M^2 \tan^2\theta/2\right], \tau = -\frac{q^2}{4m^2}$$

@TBA

Fixed q^2



Intercept
$\dfrac{G_E^2 + \tau G_M^2}{1+\tau}$

'Rosenbluth separation' gives G_E, G_M

Rosenbluth Formula - II

Rewrite electron-muon differential cross-section in LAB:

$$\frac{d^2\sigma}{d\Omega} \underset{E \gg m}{\simeq} \underbrace{\frac{\alpha^2 \cos^2\theta/2}{4E^2 \sin^4\theta/2}}_{\text{Mott}} \underbrace{\frac{E'}{E}}_{\text{Recoil}} \left(1 - \underbrace{\frac{q^2}{2m_\mu^2} \tan^2\theta/2}_{\text{Magnetic dipole}} \right)$$

as follows:

$$\frac{d^2\sigma}{d\Omega} = \int dE' \frac{d^3\sigma}{dE' d\Omega}$$

$$\frac{d^3\sigma}{dE' d\Omega} = \frac{\alpha^2 \cos^2\theta/2}{4E^2 \sin^4\theta/2} \left(1 - \frac{q^2}{2m_\mu^2} \tan^2\theta/2 \right) \delta\left(\nu + \frac{q^2}{2m_\mu} \right)$$

$$\left.\begin{array}{l} q^2 \simeq -4EE' \sin^2\theta/2 \\ \nu = E - E' \end{array}\right\} \rightarrow \delta\left(\nu + \frac{q^2}{2m_\mu} \right) = \delta\left(E - E' - \frac{2E}{m_\mu} E' \sin^2\theta/2 \right)$$

Namely,

$q^2, \nu \leftrightarrow E', \theta$ fully correlated by 4-momentum conservation

Rosenbluth Formula - III

Electron-proton, following the same path as before:

$$\frac{d^3\sigma}{dE'd\Omega}=\frac{\alpha^2\cos^2\theta/2}{4E^2\sin^4\theta/2}\left[\frac{G_E^2\left(q^2\right)-\frac{q^2}{4M^2}G_M^2\left(q^2\right)}{1-\frac{q^2}{4M^2}}-\frac{q^2}{2M^2}G_M^2\left(q^2\right)\tan^2\theta/2\right]\delta\left(\nu+\frac{q^2}{2M}\right)$$

$$\rightarrow\frac{d^3\sigma}{dE'd\Omega}=\frac{\alpha^2\cos^2\theta/2}{4E^2\sin^4\theta/2}\left(A+B\tan^2\theta/2\right)\delta\left(\nu+\frac{q^2}{2M}\right)$$

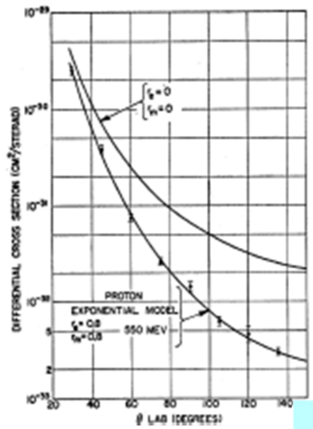
Scattering by a point-like source (e.g. muon) recovered by taking

$$\begin{cases}\dfrac{G_E^2\left(q^2\right)-\frac{q^2}{4M^2}G_M^2\left(q^2\right)}{1-\frac{q^2}{4M^2}}=1\\ G_M^2\left(q^2\right)=1\end{cases}$$

$\rightarrow$ Point-like scatterer:

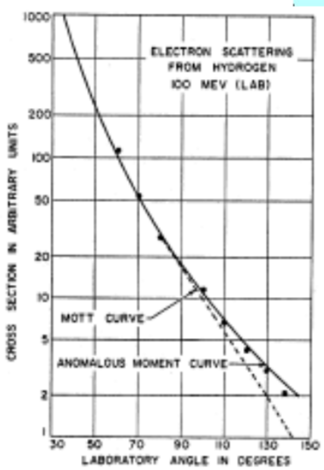
$$G_E^2\left(q^2\right)=G_M^2\left(q^2\right)=1$$

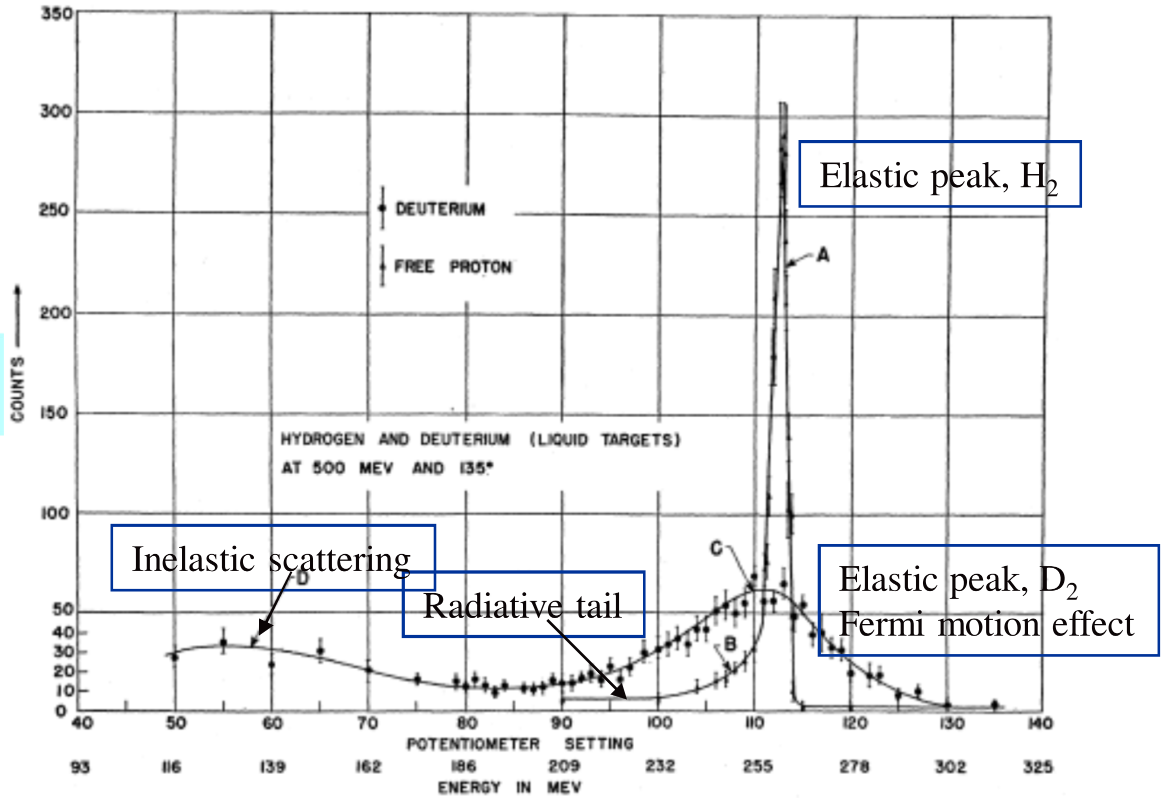
Experimental Results - I



Hydrogen

@TBA

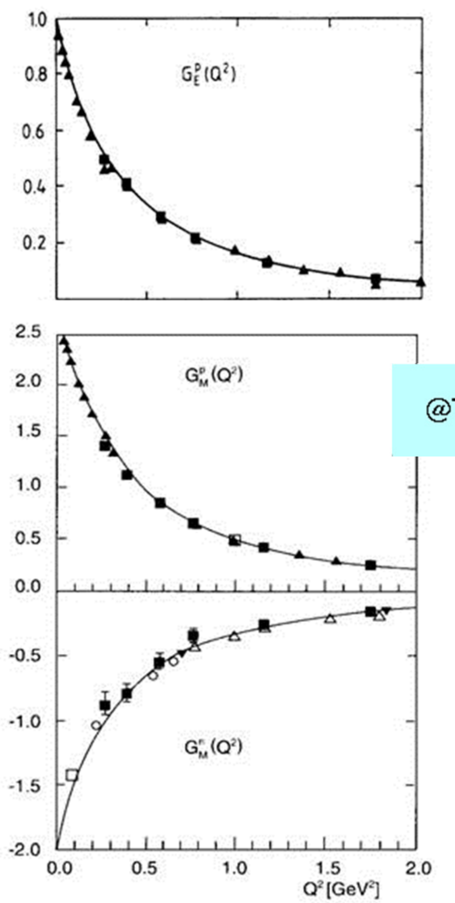




Electron energy at fixed angle for H_2, D_2

Experimental Results - II

Space-like experiments



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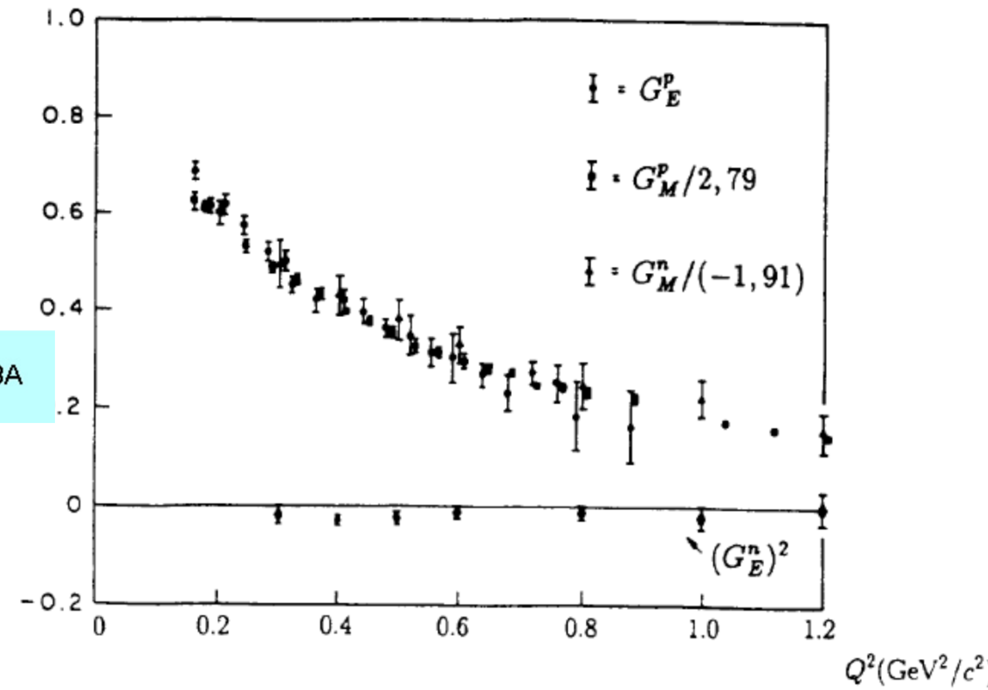
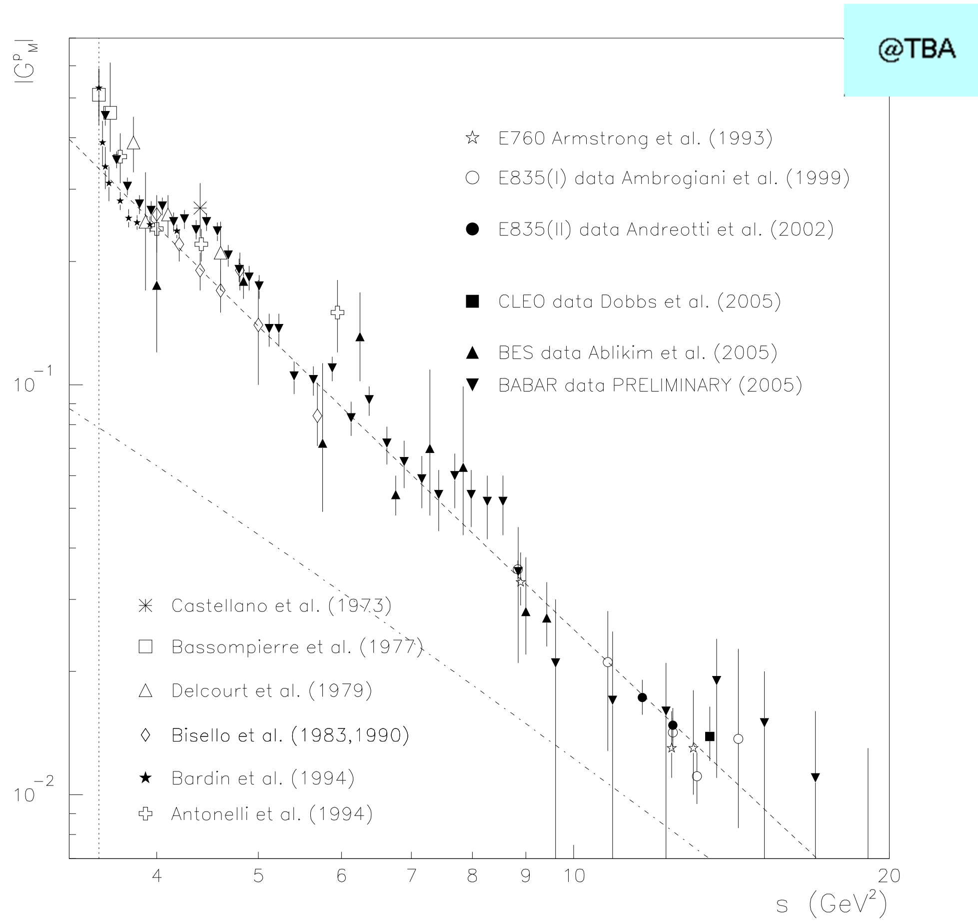


Same shape for the 3 non-zero FF

Probing the same structure at scale ≥~ 0.2 fm

Experimental Results - III

Time-like experiments



Inelastic Scattering – I

Elastic kinematics: 4-momentum conservation at the nucleon vertex

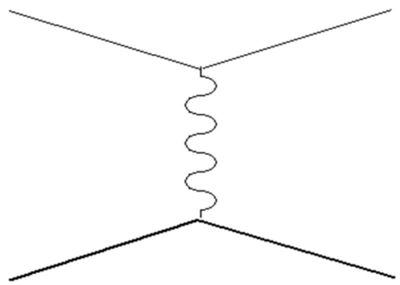
$$P+q=P'$$
$$\to (P+q)^2 = P'^2 \to P^2+q^2+2P\cdot q = P'^2$$
$$P^2 = P'^2 = M^2$$
$$\to 2P\cdot q = -q^2$$

Rewrite in the LAB frame, take massless lepton

$$P=(M,\mathbf{0})$$
$$p=(E,\mathbf{p})\approx(|\mathbf{p}|,\mathbf{p}),\ \ p'\approx(|\mathbf{p'}|,\mathbf{p}')$$
$$q=p-p'$$
$$\to q^2 \approx (|\mathbf{p}|-|\mathbf{p'}|,\mathbf{p}-\mathbf{p}')^2 = |\mathbf{p}|^2+|\mathbf{p'}|^2-2|\mathbf{p}||\mathbf{p'}|-\left(|\mathbf{p}|^2+|\mathbf{p'}|^2-2\mathbf{p}\cdot\mathbf{p'}\right)$$
$$\to q^2 \approx (|\mathbf{p}|-|\mathbf{p'}|,\mathbf{p}-\mathbf{p}')^2 = -2|\mathbf{p}||\mathbf{p'}|+2\mathbf{p}\cdot\mathbf{p'} = -2|\mathbf{p}||\mathbf{p'}|(1-\cos\theta) = -4|\mathbf{p}||\mathbf{p'}|\sin^2\theta/2$$
$$Q^2 \equiv -q^2$$
$$P\cdot q \approx (M,\mathbf{0})\cdot(|\mathbf{p}|-|\mathbf{p'}|,\mathbf{p}-\mathbf{p}') = M\left(|\mathbf{p}|-|\mathbf{p'}|\right)$$
$$\nu \equiv |\mathbf{p}|-|\mathbf{p'}| \to P\cdot q \approx M\nu$$

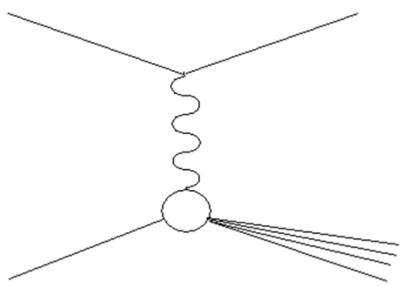
Inelastic Scattering - II

Elastic:

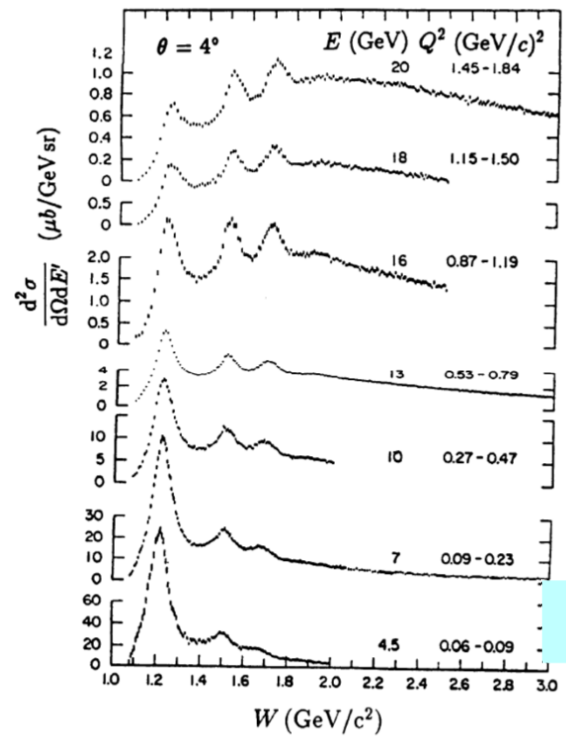


Invariant mass = M

Generalise to inelastic reactions:



Invariant mass = $W>M$

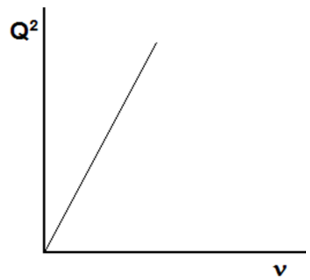


@TBA

Copious production of *resonances* (nucleon excited states), when q^2 not too big

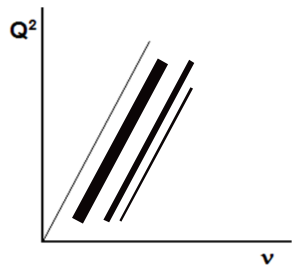
Inelastic Scattering - III

$$\nu = \frac{Q^2}{2M} \rightarrow 2M\nu = Q^2$$



Elastic

$$2M\nu = Q^2 + M'^2 - M^2$$



Inelastic

Generalise Rosenbluth cross-section to account for variable *W*:
Introduce inelastic *structure functions* W_1, W_2 to replace elastic form factors G_E, G_M

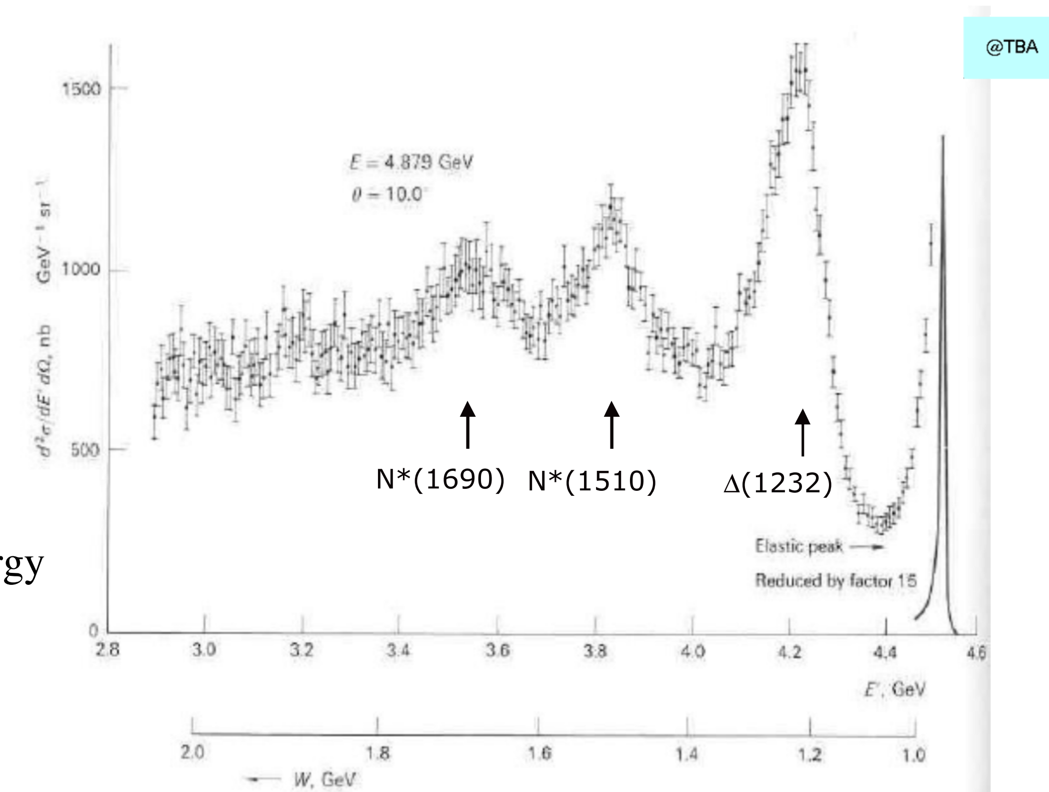
$$\frac{d\sigma}{d\Omega dE'} = \frac{\alpha^2 \cos^2 \theta/2}{4E^2 \sin^4 \theta/2}\left[W_2\left(\nu, q^2\right) + W_1\left(\nu, q^2\right)\tan^2 \theta/2\right]$$

$W_{1,2}$ depending on q^2 *and* ν

Inelastic Scattering - IV

First DESY machine

Electron synchrotron
$E_{max} = 6\ GeV$



$e + p \rightarrow e + X$

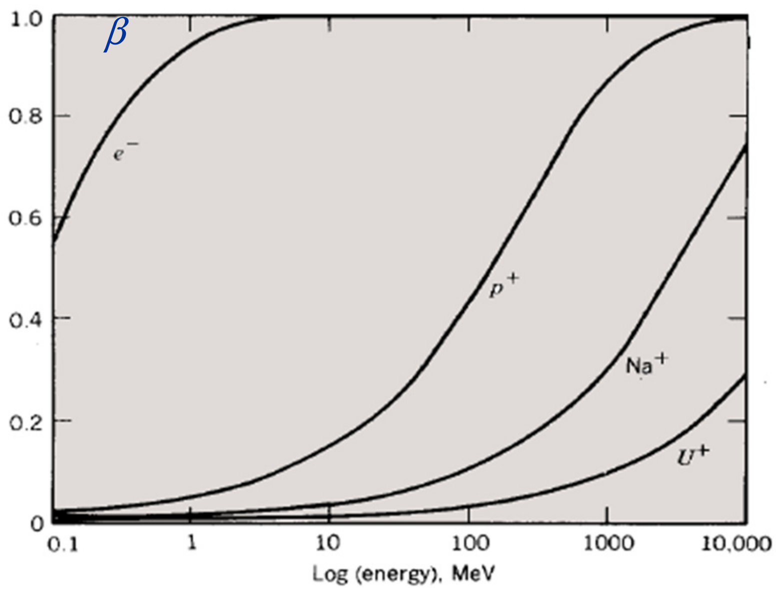
E, E': Incident, scattered electron energy

$W = M_X$

Excitation of resonant states

SLAC - I

Electron LINAC



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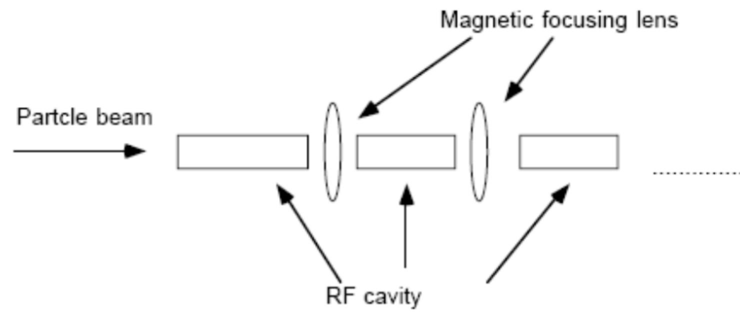
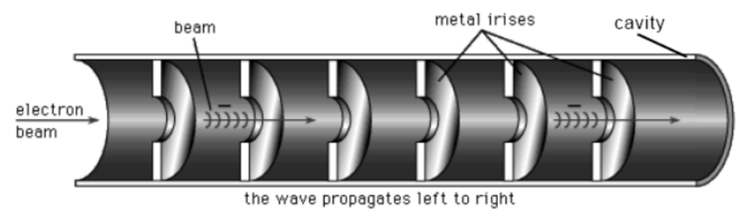
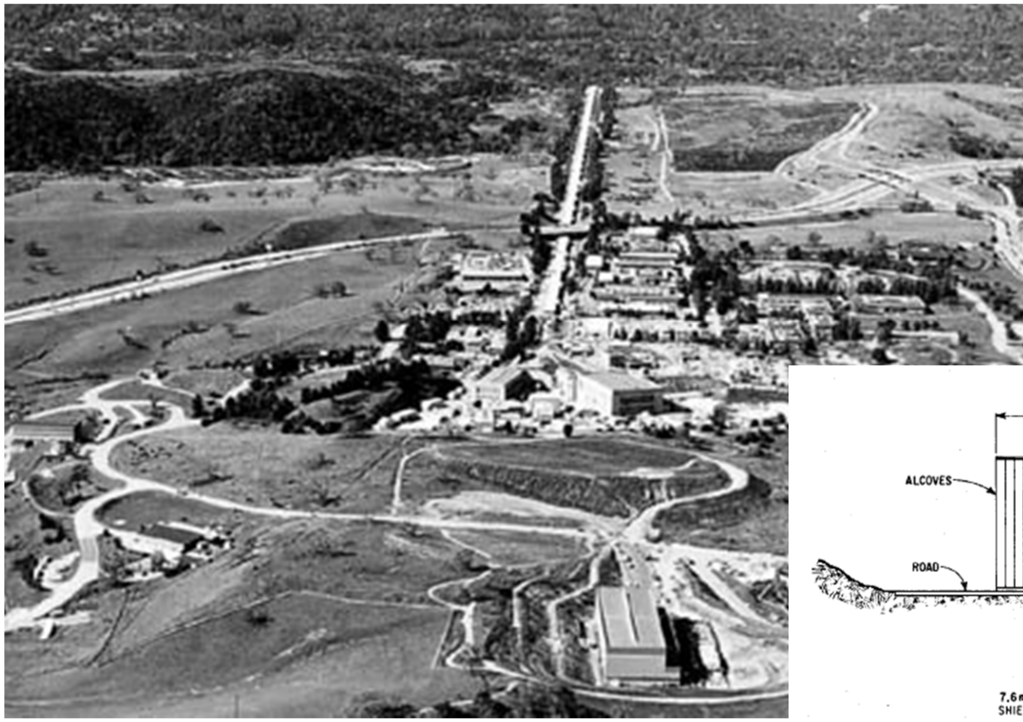
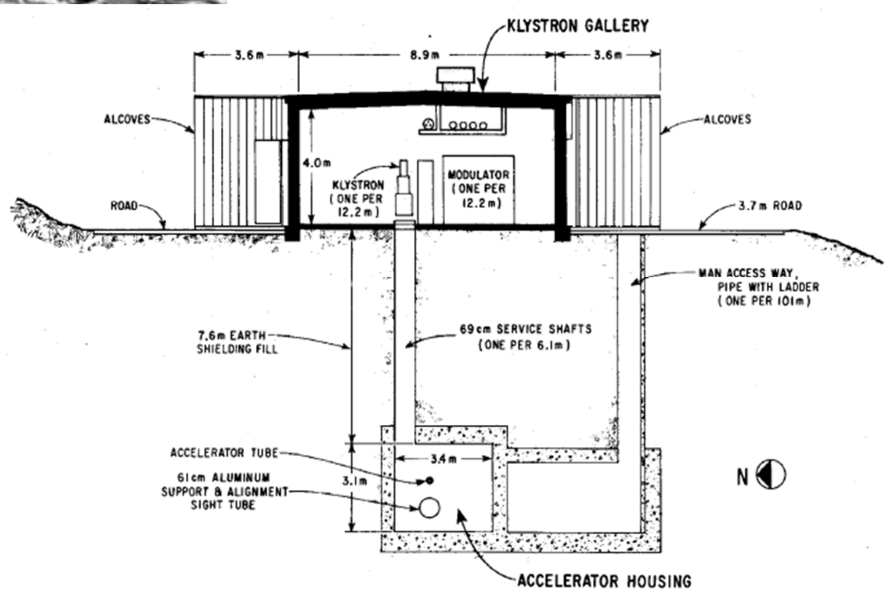




Traveling wave linear accelerator:
Electrons riding the traveling EM wave at constant phase

SLAC - II

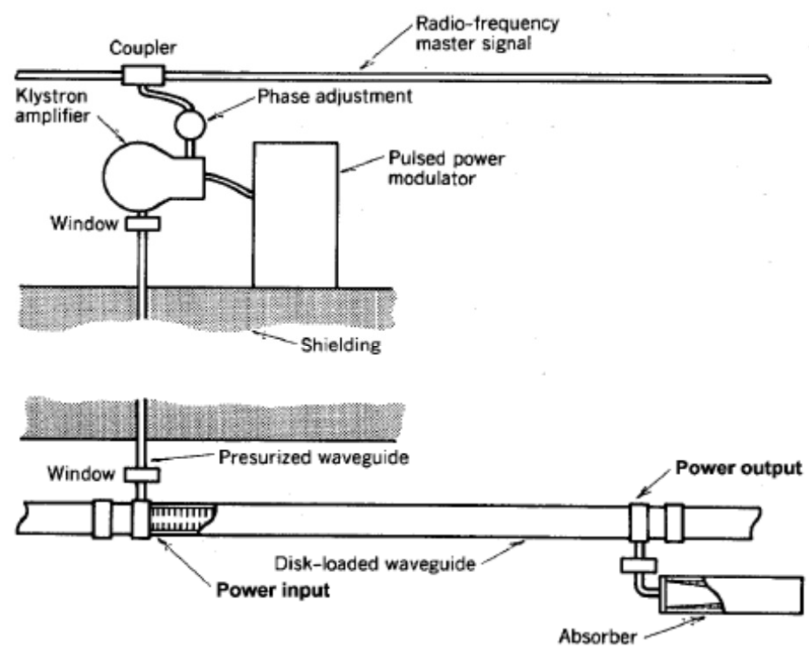



KLYSTRON GALLERY
3.6m
8.9m
3.6m
ALCOVES
ALCOVES
4.0m
KLYSTRON (ONE PER 12.2m)
MODULATOR (ONE PER 12.2m)
ROAD
3.7m ROAD
MAN ACCESS WAY, PIPE WITH LADDER (ONE PER 101m)
7.6m EARTH SHIELDING FILL
69cm SERVICE SHAFTS (ONE PER 6.1m)
ACCELERATOR TUBE
61cm ALUMINUM SUPPORT & ALIGNMENT SIGHT TUBE
3.4m
3.1m
N
ACCELERATOR HOUSING

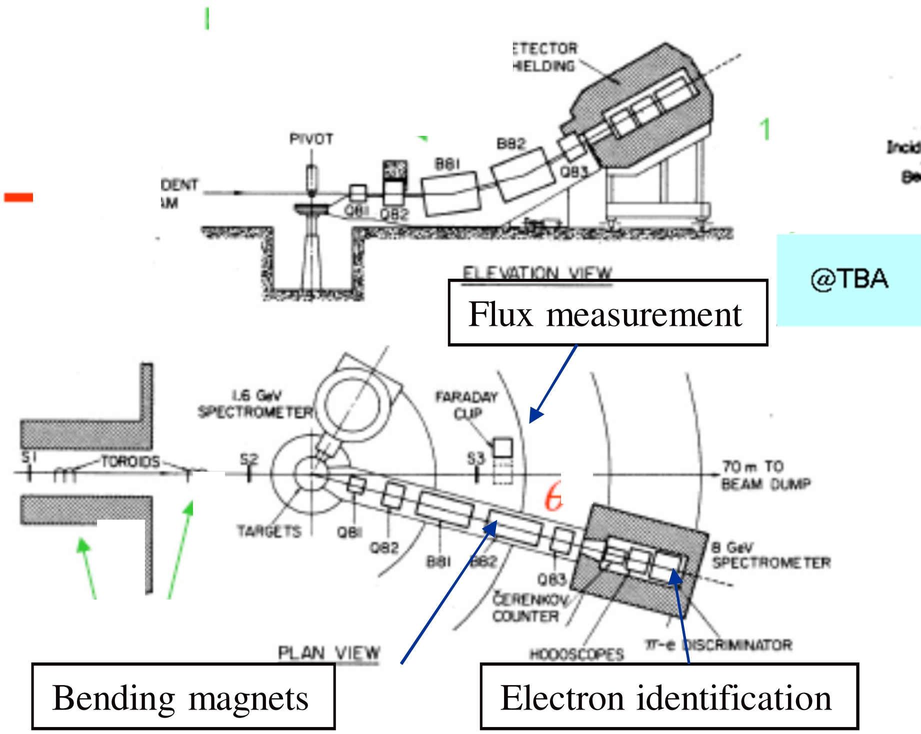
SLAC - III

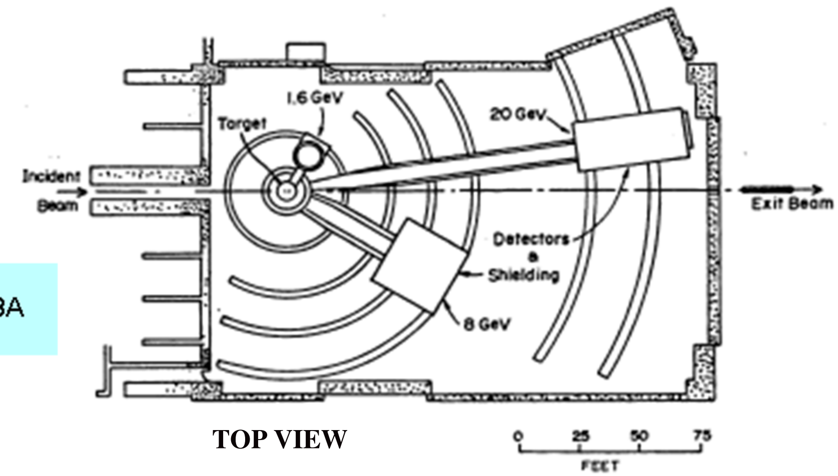
Accelerator length	3100 m
Length between power feeds	3.1 m
Number of accelerator sections	960
Number of klystrons	245
Peak power per klystron	6–24 MW
Beam pulse repetition rate	1–360 pulses/s
Radio-frequency pulse length	2.5 μs
Filling time	0.83 μs
Shunt impedance	53 MΩ/m
Electron energy (unloaded)	11.1–22.2 GeV
Electron energy (loaded)	10–20 GeV
Electron beam peak current	25–50 mA
Electron beam average current	15–30 μA
Average electron beam power	0.15–0.6 MW
Efficiency	4.3%
Positron energy	7.4–14.8 GeV
Positron average beam current	0.45 μA
Operating frequency	2.856 GHz
Accelerating structure	Iris-loaded waveguide
Waveguide outer diameter	10.5 cm
Aperture diameter	1.9 cm

@TBA



SLAC - IV

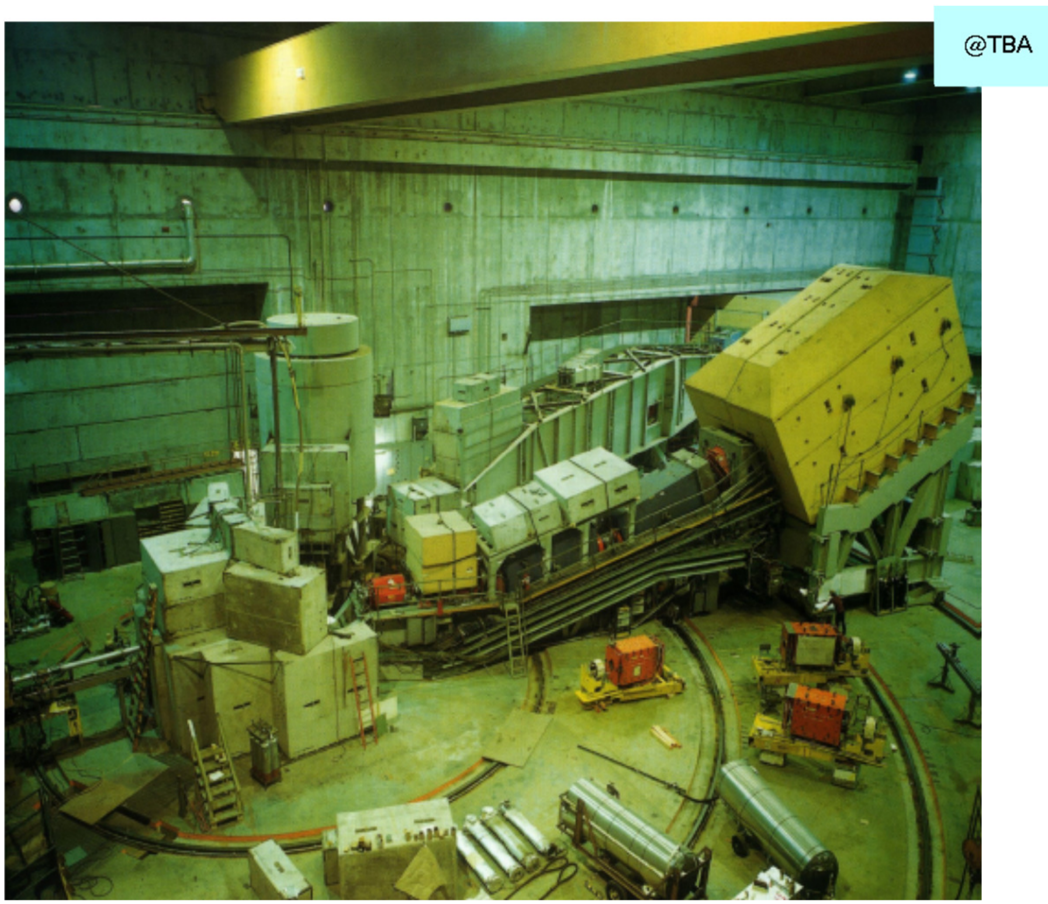




Measure E', θ of the scattered electron $\rightarrow$ Get q^2, ν

$$\frac{d^2\sigma}{dE'\,d\Omega} \rightarrow \frac{d^2\sigma}{dq^2 d\nu}$$

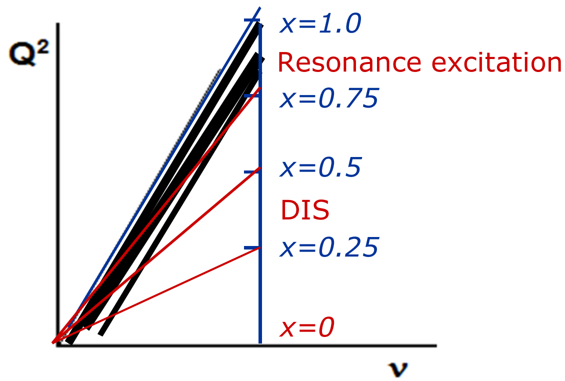
SLAC - V

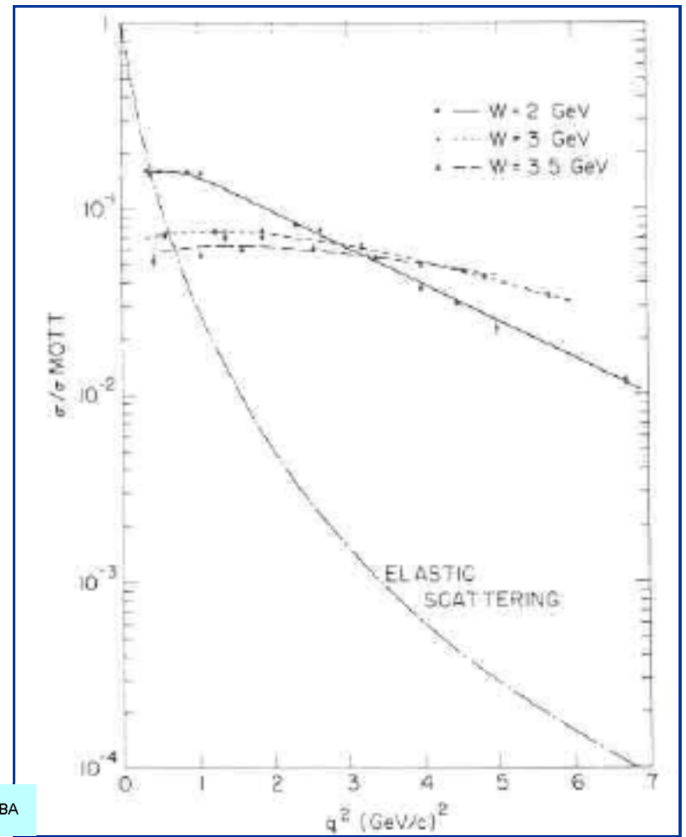


Deep Inelastic Scattering - I

Details of structure functions in the resonance region difficult to explain
But: Beyond small q^2, ν things are surprisingly simple!

Striking elastic/deep inelastic comparison:
no q^2 dependence in DIS





@TBA

Deep Inelastic Scattering - II

$$\frac{d^2\sigma}{dE'd\Omega} \to \frac{d^2\sigma}{dq^2 d\nu}$$

Also introduce:

$$\begin{cases} x = \dfrac{Q^2}{2M\nu} \\ y = \dfrac{\nu}{E_1} \end{cases} \to \frac{d^2\sigma}{dxdy} = \frac{4\pi\alpha^2 M E_1}{Q^4}\left[2xF_1\left(\frac{1+(1-y)^2}{2}\right)+(1-y)(F_2-2xF_1)-\frac{M^2 xyF_2}{s-M^2}\right]$$

Bjorken scaling hypothesis:

$$\frac{d^2\sigma}{dQ^2 d\nu} \underset{\substack{q^2,\nu\to\infty \\ \frac{q^2}{\nu}\ \text{finite}}}{\longrightarrow} f(x), \quad x = \frac{Q^2}{2M\nu}$$

f(x) Universal function of x

f(x) Q^2 independent

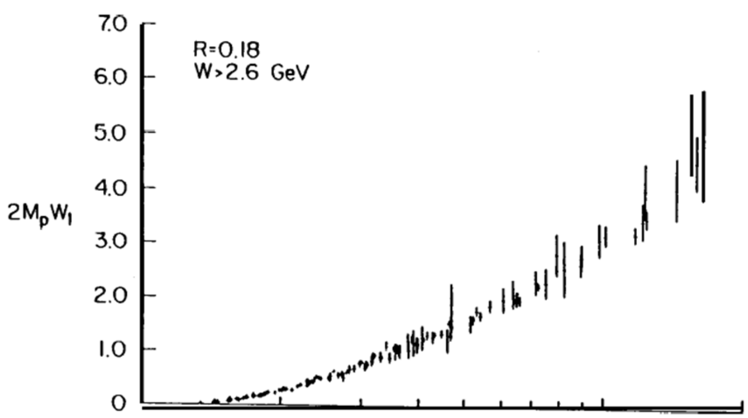
Deep Inelastic Scattering - III

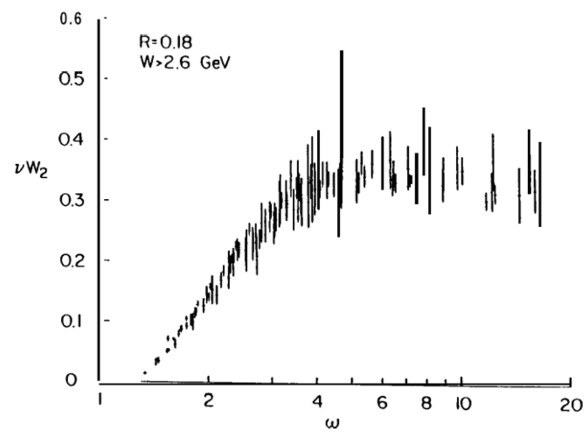
Expect:

$$\begin{cases} F_1\left(x,Q^2\right) \to F_1\left(x\right) \\ F_2\left(x,Q^2\right) \to F_2\left(x\right) \end{cases} \text{ if scaling is good}$$

Observe ($\omega = 1/x$):

@TBA



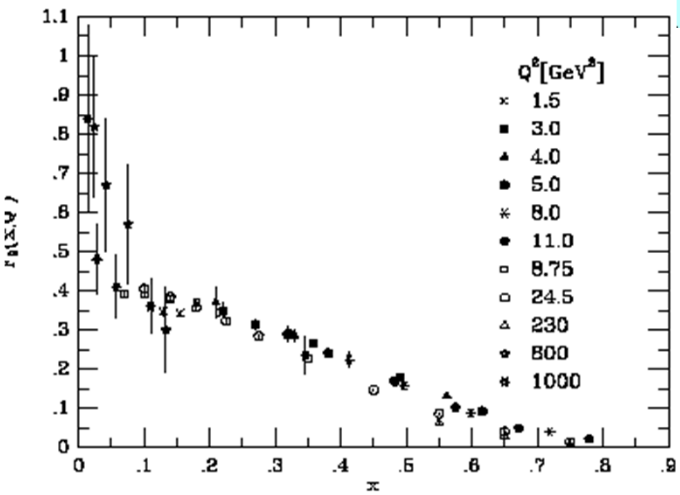


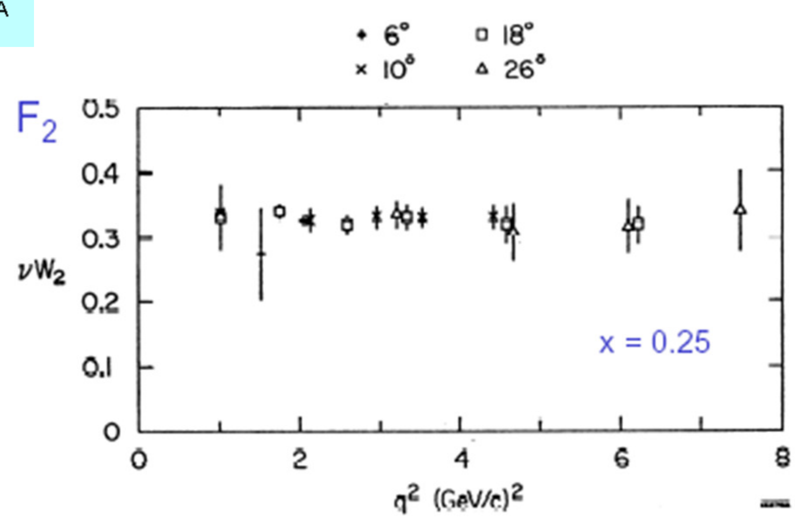
Scaling at high energy is indeed well verified!

Deep Inelastic Scattering - IV

W_2: Universal function of x …. q^2-independent!

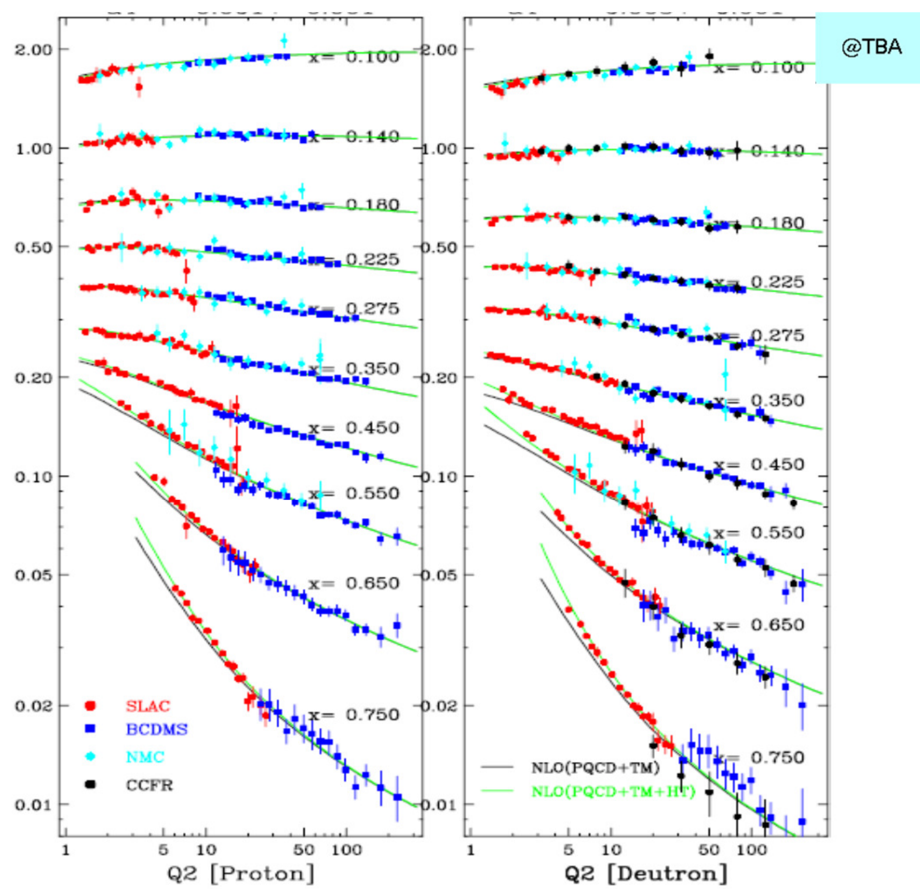
@TBA





As compared to fast varying elastic f.f., just astonishing…

Deep Inelastic Scattering - V



$e^- p$, $\mu^- p$

$e^- d$, $\mu^- d$, (νd)

Extensive compilation
Data from fixed target experiments
Observe:

Electron vs. Muon
Red points are from electron DIS
Blue points are from muon DIS
Muon merits:

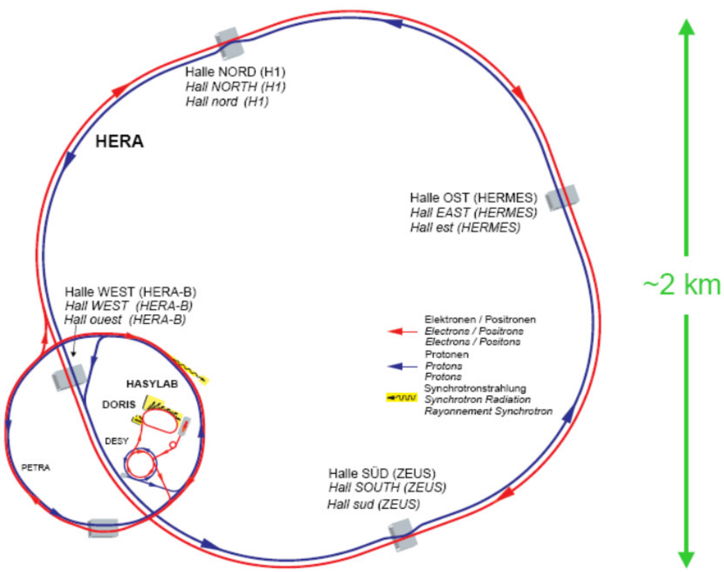
Easier to get high energy
Reduced radiative corrections

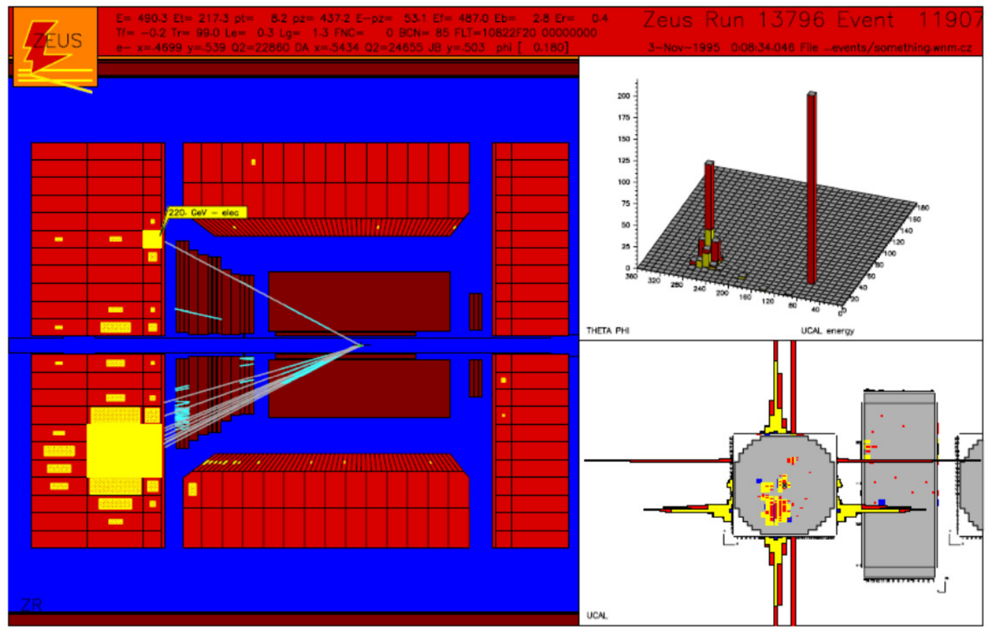
Muon drawbacks:
Intensity

Proton (L) vs. Deuteron (R)
Get *neutron* structure function

Deep Inelastic Scattering - VI

First example of *asymmetric* collider





@TBA

Deep Inelastic Scattering - VII

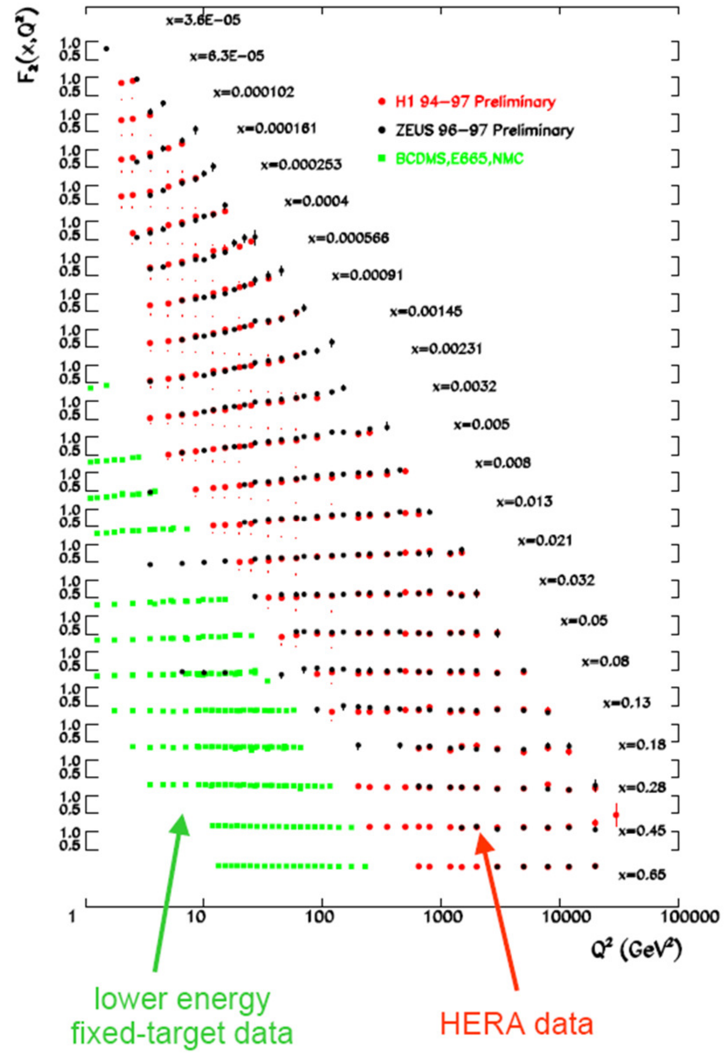
Results from several experiments:

muon DIS NMC, BCDMS, E665
CERN FNAL
electron DIS at HERA collider
DESY

Huge q^2, x range

Small, measurable scaling violation
Interesting features at small x

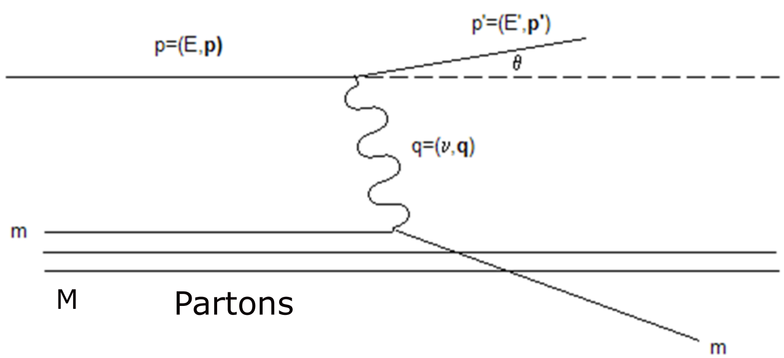
→ QCD !



Parton Model - I

Structure functions generalize form factors
Form factor of a point source = *constant*
Feynman suggestion:

Maybe deep inelastic scaling just indicates elastic scattering off free, pointlike constituents



Kinematical constraint:

$$m = (m + \nu, \mathbf{q})^2 = m^2 + 2m\nu + \underbrace{\nu^2 - |\mathbf{q}|^2}_{=q^2}$$

$$\rightarrow \nu + \frac{q^2}{2m} = 0$$

Parton Model - II

Elastic scattering off a parton: Energy and angle of the scattered electron *fully correlated*

Differential cross-section for elastic scattering off a free, pointlike constituent of mass m

$$\frac{d\sigma}{d\Omega} = \int dE' \frac{d^3\sigma}{dE' d\Omega} = \int dE' \frac{\alpha^2 z^2}{4E^2 \sin^4 \theta/2} \left(\cos^2 \theta/2 - \frac{q^2}{2m^2} \sin^2 \theta/2 \right) \delta\left(\nu + \frac{q^2}{2m} \right)$$

z: Parton charge, units e

Full E', θ correlation

$$\begin{cases} \nu = E - E' \\ q^2 = -4EE' \sin^2 \theta/2 \end{cases} \rightarrow E - E' = \frac{4EE' \sin^2 \theta/2}{2m} \rightarrow E' \left(1 + \frac{4E}{2m} \sin^2 \theta/2 \right) = E$$

$$\rightarrow E' = \frac{E}{1 + \frac{4E}{2m} \sin^2 \theta/2}$$

$$\nu + \frac{q^2}{2m} = 0, x = -\frac{q^2}{2M\nu} \rightarrow x = \frac{m}{M}$$

$$\frac{d^2\sigma}{dE' d\Omega} = \frac{\alpha^2 z^2}{4E^2 \sin^2 \theta/2} \left(\cos^2 \theta/2 - \frac{q^2}{2M^2 x^2} \sin^2 \theta/2 \right) \delta\left(\nu + \frac{q^2}{2Mx} \right)$$

Parton Model - III

Summing over all types of partons

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{\alpha^2 \sum_i z_i^2 n_i}{4E^2 \sin^2\theta/2}\left(\cos^2\theta/2 - \frac{q^2}{2M^2x^2}\sin^2\theta/2\right)\delta\left(\nu + \frac{q^2}{2Mx}\right)$$

Compare to inelastic cross-section

$$\frac{d\sigma}{d\Omega dE'} = \frac{\alpha^2}{4E^2 \sin^4\theta/2}\left[W_2\left(\nu, q^2\right)\cos^2\theta/2 + W_1\left(\nu, q^2\right)\sin^2\theta/2\right]$$

Then predict structure functions:

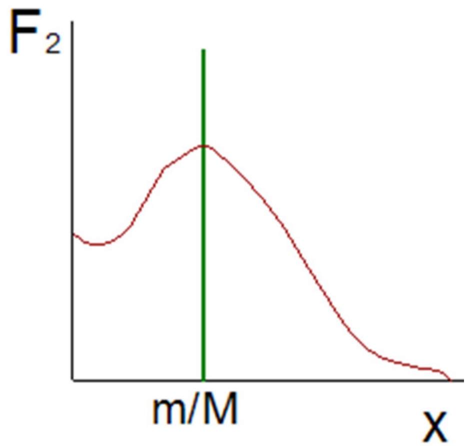
$$\rightarrow \begin{cases} W_2 = \dfrac{F_2}{\nu} = \left(\sum_i z_i^2 n_i\right)\delta\left(\nu + \dfrac{q^2}{2Mx}\right) \\ W_1 = \dfrac{2F_1}{M} = \left(\sum_i z_i^2 n_i\right)\left(\dfrac{-q^2}{2M^2x^2}\right)\delta\left(\nu + \dfrac{q^2}{2Mx}\right) \end{cases}$$

Parton Model - IV

$$\to F_2 = \nu\left(\sum_i z_i^2 n_i\right)\delta\left(\nu + \frac{q^2}{2Mx}\right) = \left(\sum_i z_i^2 n_i\right)\delta\left(1 + \frac{q^2}{2Mx\nu}\right)$$

$$\to F_2 = \left(\sum_i z_i^2 n_i\right)x\delta\left(x + \frac{q^2}{2M\nu}\right) = \left(\sum_i z_i^2 n_i\right)x\delta\left(x - \frac{m}{M}\right)$$

Parton model prediction
Actual shape



Parton Model - V

The true meaning of x

P, q: proton, virtual photon 4-momenta

$$x = \frac{Q^2}{2M\nu} = \frac{Q^2}{2P \cdot q} \quad \text{invariant}$$

$$x = \frac{m}{M}$$

$$m^2 = x^2 M^2 = x^2 \left(E^2 - \mathbf{p}^2 \right)$$

$$m^2 = \left(E_{parton}, \mathbf{p}_{parton} \right)^2 = E_{parton}{}^2 - \mathbf{p}_{parton}{}^2$$

$$\left(E_{parton}, \mathbf{p}_{parton} \right) = x \cdot \left(E, \mathbf{P} \right)$$

$$\rightarrow \mathbf{p}_{parton} \approx x \cdot \mathbf{P} \quad \text{when } m \ll |\mathbf{p}|$$

Therefore, in the *(Proton) Infinite Momentum Frame* :

x is the momentum fraction carried by the struck parton

Parton Model - VI

Lot of insight in this limit (Feynman):

$\beta \to 1 \Rightarrow \gamma \to \infty$ Large time dilation

Time constants of internal motions: $\tau \to \infty$ in the IMF

Constituents seen as *still* by the DIS virtual photon

Use time-energy indeterminacy relation:

$$P = \Big(E_p \underbrace{\underline{\mathbf{0}}}_{comp.trasversa}, |\mathbf{P}| \Big), \quad E_p = \sqrt{M^2 + |\mathbf{P}|^2} \approx |\mathbf{P}|$$

$$q = \left(E_\gamma, \mathbf{q_T}, 0 \right)$$

$$\to P \cdot q \approx |\mathbf{P}| E_\gamma \to E_\gamma \approx \frac{P \cdot q}{|\mathbf{P}|} = \frac{Q^2}{2x|\mathbf{P}|}$$

$$\tau_0 \sim \frac{1}{E_\gamma} \approx \frac{2x|\mathbf{P}|}{Q^2} \quad \text{DIS time scale}$$

$$\tau \sim \frac{1}{\Delta E} \approx \frac{2x|\mathbf{P}|}{p_T^2} \quad \text{Constituents motion time scale}$$

p_T: parton transverse momentum scale

$$\to \frac{\tau_0}{\tau} \approx \frac{p_T^2}{Q^2} \sim 0$$

No binding effects, free constituents OK

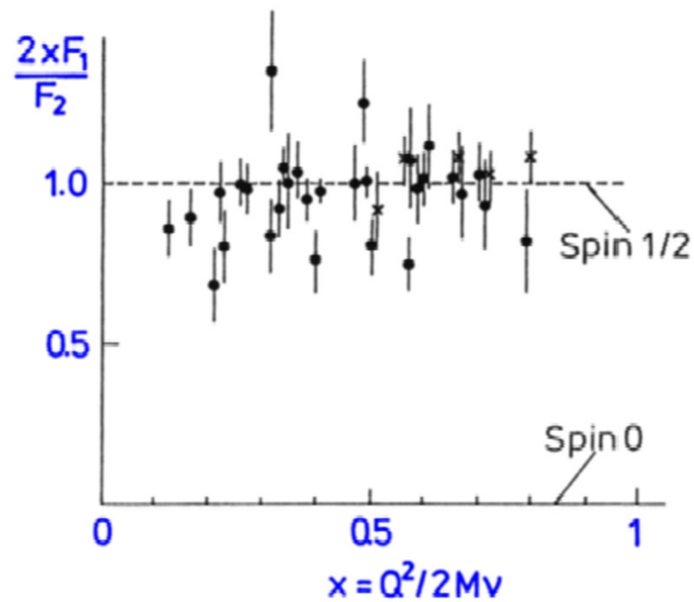
Parton Model - VII

Callan-Gross relation for spin 1/2 partons

$$\begin{cases} \dfrac{F_2}{\nu} = \left(\sum_i z_i^2 n_i\right)\delta\left(\nu + \dfrac{q^2}{2Mx}\right) \\ \dfrac{2F_1}{M} = \left(\sum_i z_i^2 n_i\right)\left(\dfrac{-q^2}{2M^2x^2}\right)\delta\left(\nu + \dfrac{q^2}{2Mx}\right) \end{cases}$$

$$\rightarrow \frac{F_2}{\nu} = \frac{2F_1}{M}\frac{2M^2x^2}{-q^2} = \frac{2F_1}{M}\frac{2M^2x^2}{2M\nu x} = \frac{2F_1 x}{\nu}$$

$$\rightarrow F_2 = 2F_1 x$$



@TBA

Parton Model - VIII

Several unanswered questions…

Most important issues:

One does not observe any free constituent out of the collision

Constituents seem to be essentially <u>free</u> (as partons) and <u>tightly bound</u> (as never observed free outside the nucleon) at the same time

For some time, these points were believed to rule out any constituent model

Spin 0 – Spin ½

$$T_{fi} = e\bar{u}'\gamma^{\mu}u\frac{g_{\mu\nu}}{q^2}e(p+p')^{\nu} \rightarrow d\sigma = \frac{1}{4EE'\mathrm{v}}\left|T_{fi}\right|^2(2\pi)^4\delta^4\left(p_1^{'}+p_2^{'}-p_1-p_2\right)\frac{1}{(2\pi)^3}\frac{1}{(2\pi)^3}\frac{d^3\mathbf{p}_1^{'}}{2E_1^{'}}\frac{d^3\mathbf{p}_2^{'}}{2E_2^{'}}$$

$$\frac{1}{2}\sum_{s=-1/2}^{+1/2}\sum_{s'=-1/2}^{+1/2}\left|T_{fi}\right|^2 = \left(\frac{e^2}{q^2}\right)^2\frac{1}{2}\sum_{s,s'=-1/2}^{+1/2}\bar{u}(p_1',s')\gamma^{\mu}u(p_1,s)\bar{u}(p_1,s)\gamma^{\nu}u(p_1',s')(p_2+p_2')_{\mu}(p_2+p_2')_{\nu}$$

By defining…

$$T_{\mu\nu} = (p_2+p_2')_{\mu}(p_2+p_2')_{\nu}$$

$$L^{\mu\nu} = 2\left[p_1^{'\mu}p_1^{\nu} + p_1^{'\nu}p_1^{\mu} + \frac{q^2}{2}g^{\mu\nu}\right]$$

…it can be shown that ⟵ ;-) Not really difficult, just a bit long

$$\frac{d\sigma}{dq^2} = \frac{2\alpha^2}{(p_1+p_2)^2q^4}\left[2(p_1\cdot p_2)(p_1\cdot p_2') + \frac{q^2}{2}M^2\right]$$ Invariant cross-section

$$\left.\frac{d\sigma}{d\Omega}\right|_{LAB} \underset{E\gg m}{\simeq} \frac{\alpha^2}{4\left|\mathbf{p}_1\right|^2\sin^4\theta/2}\cos^2\frac{\theta}{2}\frac{\left|\mathbf{p}_1'\right|}{\left|\mathbf{p}_1\right|}$$ LAB = "2" rest frame

π Form Factor - I

Consider electron-pion scattering: The π is not a point-like object…
What are we to take for the pion current?
Must build a 4-vector operator
Some guesswork:
1) *Lorentz invariance*

$$\left.\begin{array}{ll} p_2, p_2', q & \text{Three 4-momentum vectors} \\ p_2' = p_2 + q & \text{Constraint} \end{array}\right\} \to \text{2 independent}$$

Choose:

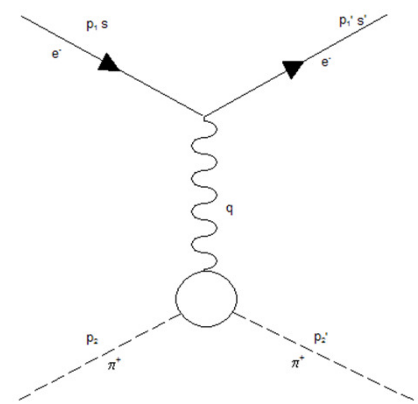
$$\left.\begin{array}{l} p_2' + p \\ p_2' - p = q \end{array}\right\} \text{Both can contribute to the current}$$

Only one independent 4-scalar:

E.g. $(p_2')^2 = (p_2)^2 = m^2 \to p_2 \cdot p_2'$

Choose instead q^2

$$\to j^{\mu}_{(\pi)} = e\left[F(q^2)(p'+p)^{\mu} + G(q^2)q^{\mu}\right]e^{-iq\cdot x}$$



Blob indicating a non-QED vertex:
The pion is an extended object

π Form Factor - II

2) *Gauge Invariance*

Charge conservation ↔ Current must be *divergenceless*

$$\partial_\mu j^\mu = 0 \rightarrow \partial_\mu j_\pi^\mu = e\partial_\mu \left[F\left(q^2\right)\left(p'+p\right)^\mu + G\left(q^2\right)q^\mu \right] e^{-iq\cdot x}$$

$$= -iq_\mu e\left[F\left(q^2\right)\left(p'+p\right)^\mu + G\left(q^2\right)q^\mu \right] e^{-iq\cdot x} = 0$$

$$\rightarrow \partial_\mu j^\mu = 0 \Rightarrow q_\mu j^\mu = 0$$

$$q_\mu \left[F\left(q^2\right)\left(p_2 + p_2'\right)^\mu + G\left(q^2\right)q^\mu \right] = 0$$

$$\left.\begin{array}{l} q_\mu \left(p_2 + p_2'\right)^\mu = \left(p_2 - p_2'\right)_\mu \left(p_2 + p_2'\right)^\mu = 0 \\ q_\mu q^\mu \neq 0 \end{array}\right\} \rightarrow G\left(q^2\right) = 0$$

$$\rightarrow j^\mu = e\left(p_2 + p_2'\right)^\mu F\left(q^2\right)$$

Just *one* form factor for a scalar particle like the π

π Form Factor - III

What is $F(q^2)$?
In the CM frame:

$$q^2 = \left(E' - E, \mathbf{p}' - \mathbf{p}\right)^2 = \left(E' - E\right)^2 - \left(\mathbf{p}' - \mathbf{p}\right)^2 = 0 - \mathbf{q}^2 = -\left|\mathbf{q}\right|^2$$

$$\rightarrow F_{scatt}\left(q^2\right) = F_{scatt}\left(\left|\mathbf{q}\right|^2\right)$$

Again, Fourier transform of the charge distribution

If crossing is good, can extend to the reaction

$$e^+ + e^- \rightarrow \pi^+ + \pi^-$$

$$q^2 = \left(E_1 + E_2, \boldsymbol{p}_1 + \boldsymbol{p}_2\right)^2 = \left(E_1 + E_2\right)^2 - \left(\boldsymbol{p}_1 + \boldsymbol{p}_2\right)^2$$

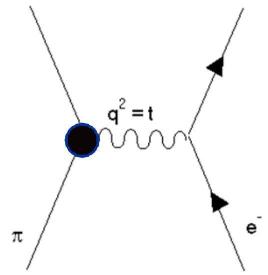
$$q^2 = E_{CM}^2$$

$$\rightarrow F_{annihil}\left(q^2\right) = F_{annihil}\left(E_{CM}^2\right)$$

$$\rightarrow F\left(q^2\right) = \begin{cases} F_{scatt}\left(q^2\right), q^2 < 0 \\ F_{annihil}\left(q^2\right), q^2 > 0 \end{cases}$$

Experiments: Space-like - I

π scattering off electrons

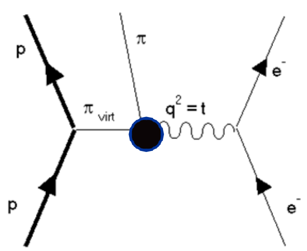


$$p^* = \frac{p_\pi m_e}{\sqrt{s}} \to t_{\max} = -4\frac{p_\pi^2 m_e^2}{s} = -4\frac{p_\pi^2 m_e^2}{m_\pi^2 + m_e^2 + 2E_\pi m_e} \approx -4\frac{p_\pi^2 m_e^2}{m_\pi^2 + 2E_\pi m_e}$$

$$\to t_{\max} \underset{p_\pi \to \infty}{\sim} -4\frac{p_\pi m_e^2}{2m_e} = -2p_\pi m_e$$

Unappealing, t_{max} too small

Electroproduction of one π



OK

Experiments: Space-like - II

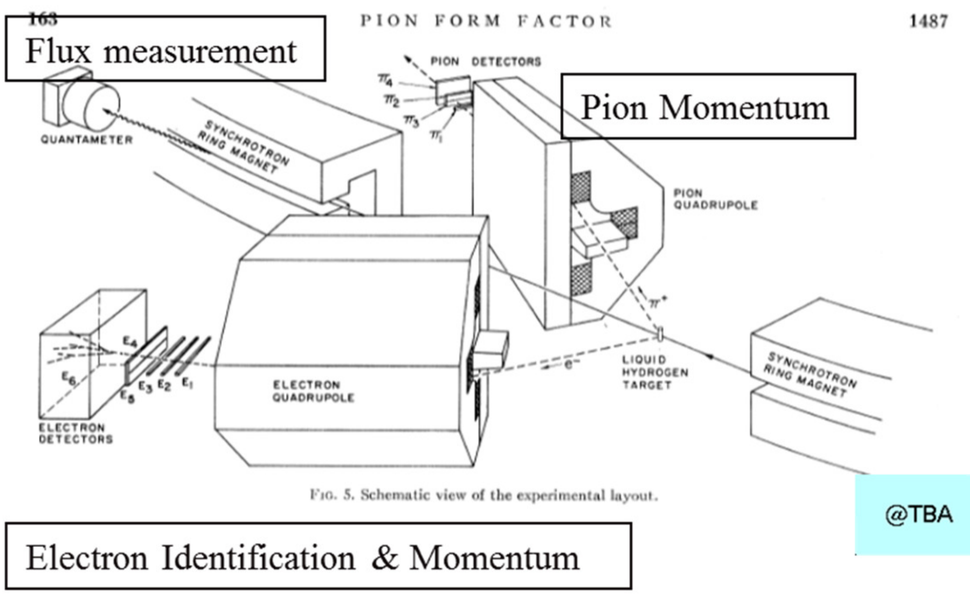
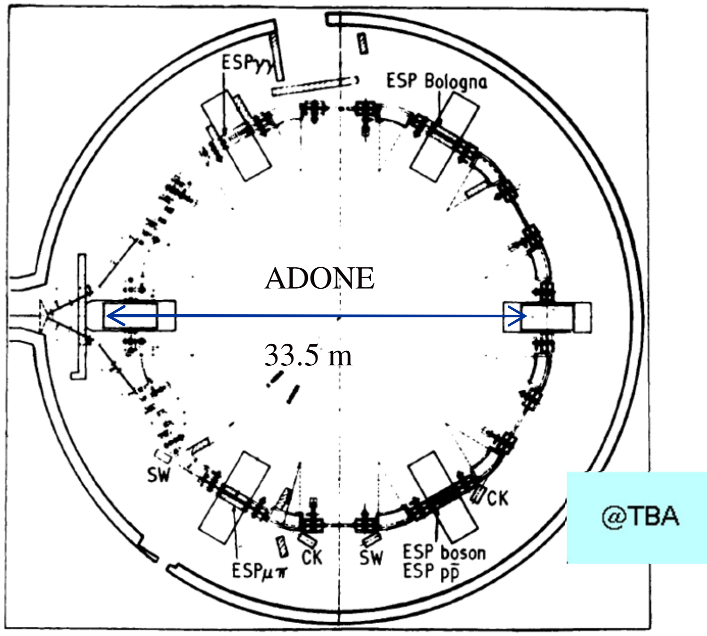


FIG. 5. Schematic view of the experimental layout.

Experiments: Time-like - I



First e^+ - e^- colliding beams
ADONE – Frascati, 1967 etc.

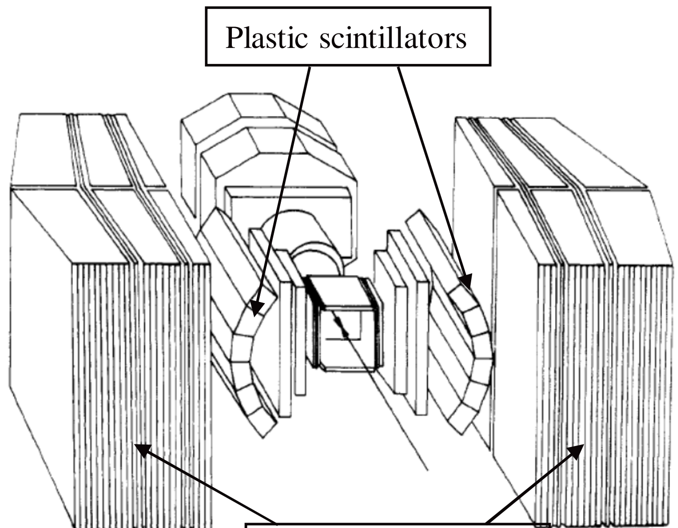
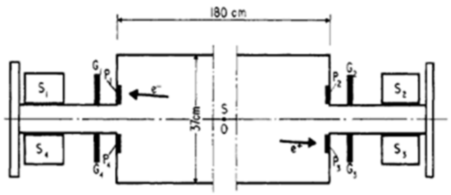


Figure 4. BCF apparatus.

Experiments: Time-like - II



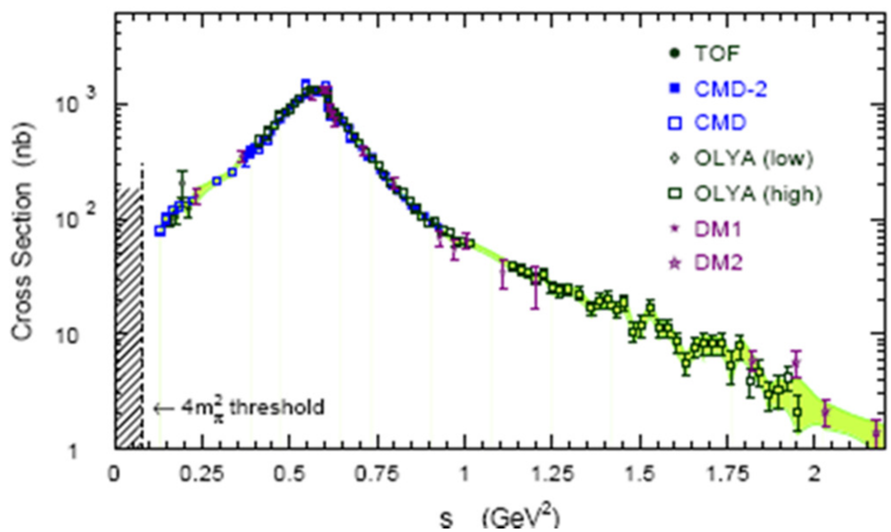
Luminosity monitor measures Bhabha scattering rate at small angles: α = acceptance

$$Rate_{Bhabha} = \sigma_{Bhabha} \cdot \alpha \cdot Luminosity$$

$e^+ + e^- \rightarrow e^+ + e^-$ pure QED at low energy, small angle

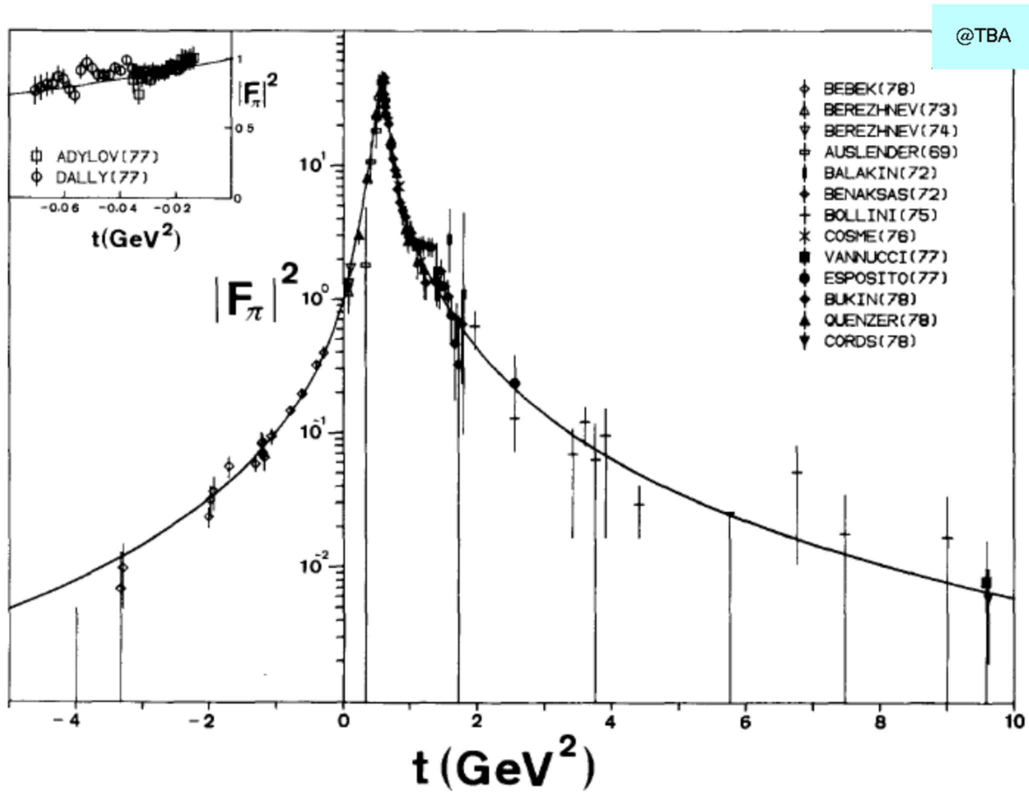
$\rightarrow$ accurate, reliable σ_{Bhabha} prediction

$$\rightarrow Luminosity = Rate_{Bhabha} / (\sigma_{Bhabha} \cdot \alpha)$$



π Form Factor at Large

Is there a unique function $F(q^2)$?? Yes!



Crossing Symmetry

Simple relationship between any pair of 2-body reactions

$a+b \rightarrow c+d$ Reaction A

$a+\underbrace{[c]}_{crossed} \rightarrow \underbrace{[b]}_{crossed}+d$ Reaction B

Define: Crossed particle $\equiv$ Antiparticle

By changing the 4-momentum sign of the crossed particle. the two amplitudes are identical

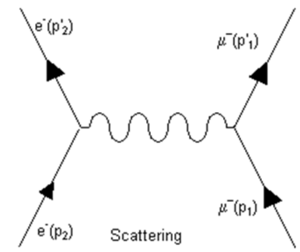
$$A\left[a(p_A)+b(p_B) \rightarrow c(p_C)+d(p_D)\right] = A\left[a(p_A)+\bar{c}(-p_C) \rightarrow \bar{b}(-p_B)+d(p_D)\right]$$

e^+e^- Annihilation into $\mu^+\mu^-$

Apply crossing symmetry to electron-muon scattering

$e^- + \mu^- \to e^- + \mu^-$ A: Scattering

$e^- + \underbrace{\left[e^-\right]}_{crossed} \to \underbrace{\left[\mu^-\right]}_{crossed} + \mu^- \equiv e^- + e^+ \to \mu^+ + \mu^-$ B: Annihilation



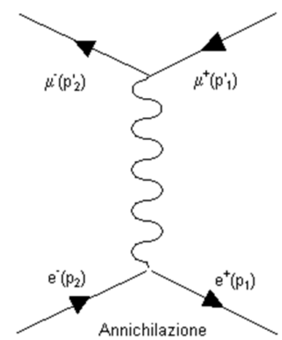
Amplitude for scattering:

$$T_{fi}\left(s,s',r,r'\right) = (-e)\overline{u}_{(\mu)}\left(p_2',s'\right)\gamma^\mu u_{(\mu)}\left(p_2,s\right)\frac{-ig_{\mu\nu}}{q^2}(-e)\overline{u}_{(e)}\left(p_1',r'\right)\gamma^\nu u_{(e)}\left(p_1,r\right)$$

$$q = p_1 - p_1' \to q^2 = \left(p_1 - p_1'\right)^2 = p_1^2 + p_1'^2 - 2p_1 \cdot p_1'$$

$$q^2 = 2m_e^2 - 2\left(E_1E_1' - \mathbf{p}_1\cdot\mathbf{p}_1'\right) \underset{E\gg m}{\simeq} -2\left(E_1E_1' - \mathbf{p}_1\cdot\mathbf{p}_1'\right) < 0$$ q=4-momentum transfer

Amplitude for annihilation:

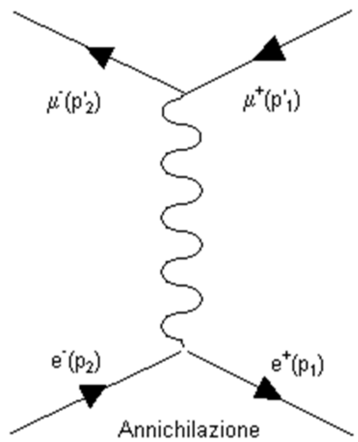


$$T_{fi}\left(s,s',r,r'\right) = (-e)\overline{u}_{(\mu)}\left(p_2',s'\right)\gamma^\mu v_{(\mu)}\left(p_1',r'\right)\frac{-ig_{\mu\nu}}{q^2}(-e)\overline{v}_{(e)}\left(p_1,s\right)\gamma^\nu u_{(e)}\left(p_2,r\right)$$

$$q = p_1 + p_2 \to q^2 = \left(p_1 + p_2\right)^2 = p_1^2 + p_2^2 + 2p_1 \cdot p_2$$

$$q^2 = 2m_e^2 + 2\left(E_1E_2 - \mathbf{p}_1\cdot\mathbf{p}_2\right) \underset{E\gg m}{\simeq} 2\left(E_1E_2 - \mathbf{p}_1\cdot\mathbf{p}_2\right) > 0$$ q=total 4-momentum

Annihilation Cross-Section - I



$$T_{fi} = \overline{v}(p_1, s')\gamma^\mu u(p_2, s)\frac{e^2}{q^2}\overline{u}(p_2', r)\gamma_\mu v(p_1', r')$$

$$\left|T_{fi}\right|^2 = \frac{e^4}{q^4}\left[\overline{v}(p_1, s')\gamma^\mu u(p_2, s)\overline{u}(p_2', s)\gamma^\nu v(p_1', s')\right]\left[\overline{v}(p_1, r)\gamma_\mu u(p_2, r')\overline{u}(p_2', r')\gamma_\nu v(p_1', r)\right]$$

Annihilation Cross-Section - II

$$\left|T_{fi}\right|^2 = \frac{e^4}{q^4}\left[\overline{v}(p_1,s')\gamma^\mu u(p_2,s)\overline{u}(p_2',s)\gamma^\nu v(p_1',s')\right]\left[\overline{v}(p_1,r)\gamma_\mu u(p_2,r')\overline{u}(p_2',r')\gamma_\nu v(p_1',r)\right]$$

$$\left\langle\left|T_{fi}\right|^2\right\rangle = \frac{1}{4}\sum_{s,s',r,r'}\left|T_{fi}\right|^2 = \frac{e^4}{4q^4}Tr\left[(\not{p}_1 - m)\gamma^\mu(\not{p}_2 + m)\gamma^\nu\right]Tr\left[(\not{p}_2' + M)\gamma_\mu(\not{p}_1' - M)\gamma_\nu\right]$$

$$Tr\left[(\not{p}_1 - m)\gamma^\mu(\not{p}_2 + m)\gamma^\nu\right] = 4\left[p_1^\mu p_2^\nu + p_1^\nu p_2^\mu - g^{\mu\nu}\left(p_2\cdot p_1 + m^2\right)\right]$$

$$Tr\left[(\not{p}_2' + M)\gamma_\mu(\not{p}_1' - M)\gamma_\nu\right] = 4\left[p_1^\mu{}' p_2^\nu{}' + p_1^\nu{}' p_2^\mu{}' - g^{\mu\nu}\left(p_2'\cdot p_1' + M^2\right)\right]$$

$$\rightarrow \left\langle\left|T_{fi}\right|^2\right\rangle = \frac{8e^4}{q^4}\left[(p_1\cdot p_1')(p_2\cdot p_2') + (p_1\cdot p_2')(p_2\cdot p_1') + M^2(p_1\cdot p_2)\right] \quad m \approx 0$$

$$CM: \begin{cases} p_1 = \begin{pmatrix}E & 0 & 0 & E\end{pmatrix}, p_2 = \begin{pmatrix}E & 0 & 0 & -E\end{pmatrix} \\ \left|\mathbf{p}'\right| = \left|\mathbf{p}_1'\right| = \left|\mathbf{p}_2'\right| = \sqrt{E^2 - M^2} \end{cases}$$

$$p_1' = \begin{pmatrix}E & \left|\mathbf{p}'\right|\sin\theta & 0 & \left|\mathbf{p}'\right|\cos\theta\end{pmatrix}, p_2' = \begin{pmatrix}E & -\left|\mathbf{p}'\right|\sin\theta & 0 & -\left|\mathbf{p}'\right|\cos\theta\end{pmatrix}$$

$$\rightarrow \left\langle\left|T_{fi}\right|^2\right\rangle = e^4\left[1 + \frac{M^2}{E^2} + \left(1 - \frac{M^2}{E^2}\right)\cos^2\theta\right]$$

Annihilation Cross-Section - III

$$\left.\frac{d\sigma}{d\Omega}\right|_{CM}=\frac{\alpha^2}{4s}\sqrt{1-\frac{M^2}{E^2}}\left[1+\frac{M^2}{E^2}+\left(1-\frac{M^2}{E^2}\right)\cos^2\theta\right]$$

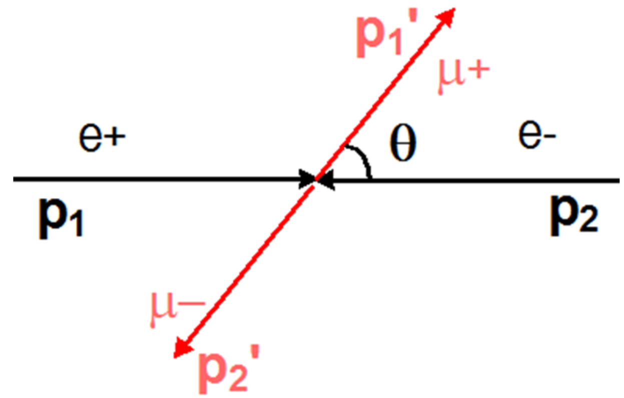
$$\rightarrow\left.\frac{d\sigma}{d\Omega}\right|_{CM}\approx\frac{\alpha^2}{4s}\left(1+\cos^2\theta\right),\qquad E\gg M$$

$$\sigma=\int_\Omega\frac{d\sigma}{d\Omega}d\Omega=\frac{4\pi\alpha^2}{3s}\sqrt{1-\frac{M^2}{E^2}}\left(1+\frac{1}{2}\frac{M^2}{E^2}\right)$$

$$\gamma=\frac{E}{M}=\frac{1}{\sqrt{1-\beta^2}}$$

$$\rightarrow\sigma=\frac{4\pi\alpha^2}{3s}\underbrace{\sqrt{\frac{\gamma^2-1}{\gamma^2}}}_{\beta}\left(\frac{1+2\gamma^2}{2\gamma^2}\right)=\frac{4\pi\alpha^2}{3s}\beta\left(\frac{1}{2\gamma^2}+1\right)=\frac{4\pi\alpha^2}{3s}\beta\left(\frac{3-\beta^2}{2}\right)$$

$$\rightarrow\sigma\approx\frac{4\pi\alpha^2}{3s}=\frac{87}{s\left[GeV^2\right]}nb,\qquad E\gg M$$



Annihilation Cross-Section - IV

Dirac equation: High energy limit

$$E\psi = (\boldsymbol{\alpha}\cdot\mathbf{p} + \beta m)\psi$$

$$u = \begin{pmatrix}\phi \\ \chi\end{pmatrix}$$ Generic spinor; ϕ, χ 2-components spinors

$$\boldsymbol{\alpha} = \begin{pmatrix}\boldsymbol{\sigma} & 0 \\ 0 & -\boldsymbol{\sigma}\end{pmatrix}, \beta = \begin{pmatrix}0 & 1 \\ 1 & 0\end{pmatrix}$$ Dirac matrices, chiral representation, "2×2" block format

$$\begin{cases} E\phi = (\boldsymbol{\sigma}\cdot\mathbf{p})\phi + m\chi \\ E\chi = -(\boldsymbol{\sigma}\cdot\mathbf{p})\chi + m\phi \end{cases}$$

$$E \gg m \rightarrow \left.\begin{matrix} E \approx |\mathbf{p}| \\ m \approx 0 \end{matrix}\right\} \rightarrow \begin{cases} (\boldsymbol{\sigma}\cdot\mathbf{p})\phi \approx |\mathbf{p}|\phi \\ (\boldsymbol{\sigma}\cdot\mathbf{p})\chi \approx -|\mathbf{p}|\chi \end{cases} \rightarrow \begin{cases} \phi \simeq u_R \\ \chi \simeq u_L \end{cases} \rightarrow u \approx \begin{pmatrix} u_R \\ u_L \end{pmatrix} \rightarrow u^\dagger(s',p')u(s,p) \approx u'^\dagger_R u_R + u'^\dagger_L u_L$$

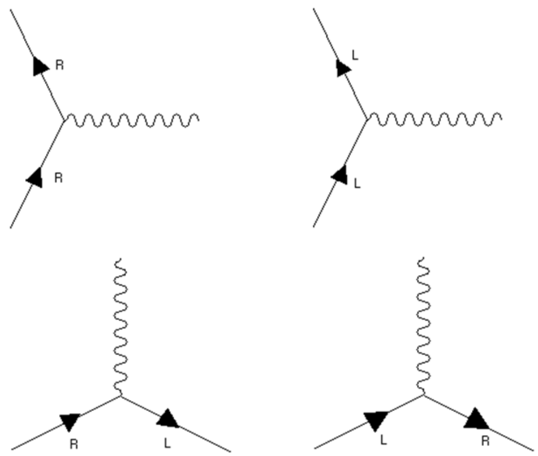
No mixed terms $\rightarrow$ *Helicity is conserved at high energy*

Explains the $\left(1-\beta^2 \sin^2 \theta/2\right)$ factor, cutting off the cross-section $\theta \rightarrow \pi$:
Solves conflicting helicity/angular momentum conservation

Always true for Dirac currents coupling to vector fields

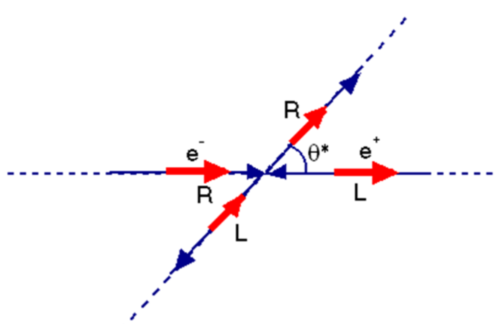
Annihilation Cross-Section - V

Helicity conservation at high energy:
Consequence of electromagnetic field being a *vector*



Scattering: $R \to R,\ L \to L$

Annihilation: $R+L,\ L+R$



For both initial and final state:
Particle and antiparticle must have *opposite* helicity
Decompose differential cross-section into 4 pieces:

$$\frac{d\sigma}{d\Omega^*} = \frac{1}{4} \cdot 2 \left[\left.\frac{d\sigma}{d\Omega^*}\right|_{LR\to LR} + \left.\frac{d\sigma}{d\Omega^*}\right|_{RL\to RL} + \left.\frac{d\sigma}{d\Omega^*}\right|_{LR\to RL} + \left.\frac{d\sigma}{d\Omega^*}\right|_{RL\to LR} \right]$$

<u>Average</u> over initial states, <u>Sum</u> over final states

Annihilation Cross-Section - VI

Transition amplitude = *Amplitude to find final particles at angle θ* wrt to initial direction*

Phase space, incident flux and normalization factors just cancel out at high energy
Matrix element:

$$T_{fi} = \frac{\alpha}{q^2 \equiv s} \cdot \text{Amplitude to find } J = 1 \text{ state rotated by } \theta^*$$

Use rotation matrices for a $J=1$ state: Take y-axis $\perp$ reaction plane

$$e^{-i\theta^* J_2} |J,m\rangle = \sum_{m'} d^J_{m,m'}(\theta^*) |J,m'\rangle, \quad d^J_{m,m'}(\theta^*) = \langle J,m| e^{-i\theta^* J_2} |J,m'\rangle$$

$$d^1_{+1,+1}(\theta^*) = d^1_{-1,-1}(\theta^*) = \frac{1}{2}(1+\cos\theta^*)$$

$$d^1_{+1,-1}(\theta^*) = d^1_{-1,+1}(\theta^*) = \frac{1}{2}(1-\cos\theta^*)$$

$$\left.\begin{aligned} \left.\frac{d\sigma}{d\Omega^*}\right|_{LR\to LR} &= \left.\frac{d\sigma}{d\Omega^*}\right|_{RL\to RL} = \frac{\alpha^2}{s}\left(\frac{1}{2}\right)^2 (1+\cos\theta^*)^2 \\ \left.\frac{d\sigma}{d\Omega^*}\right|_{LR\to RL} &= \left.\frac{d\sigma}{d\Omega^*}\right|_{RL\to LR} = \frac{\alpha^2}{s}\left(\frac{1}{2}\right)^2 (1-\cos\theta^*)^2 \end{aligned}\right\} \to \frac{d\sigma}{d\Omega^*} = \frac{\alpha^2}{4s}(1+\cos^2\theta^*)$$

$$\sigma = \int_{4\pi} \frac{\alpha^2}{4s}(1+\cos^2\theta^*) d\Omega^* = \frac{4\pi\alpha^2}{3s}$$

Annihilation Cross-Section - VII

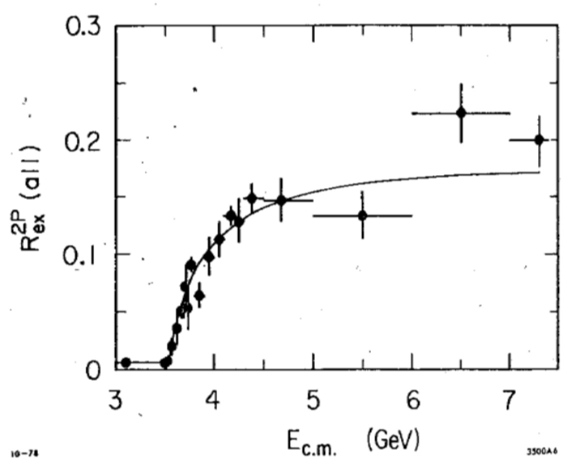
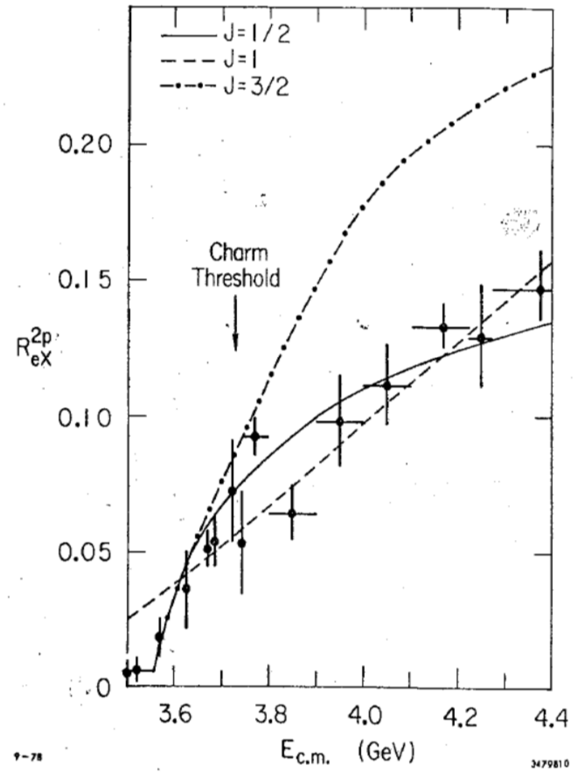


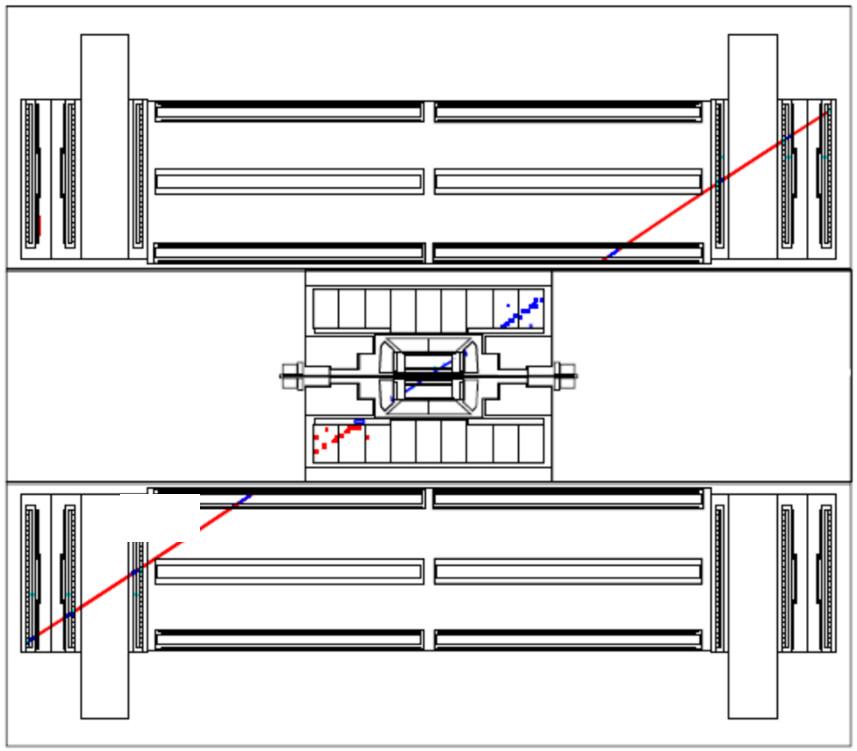
Figure 12b)



τ lepton discovery, mass & spin determination:

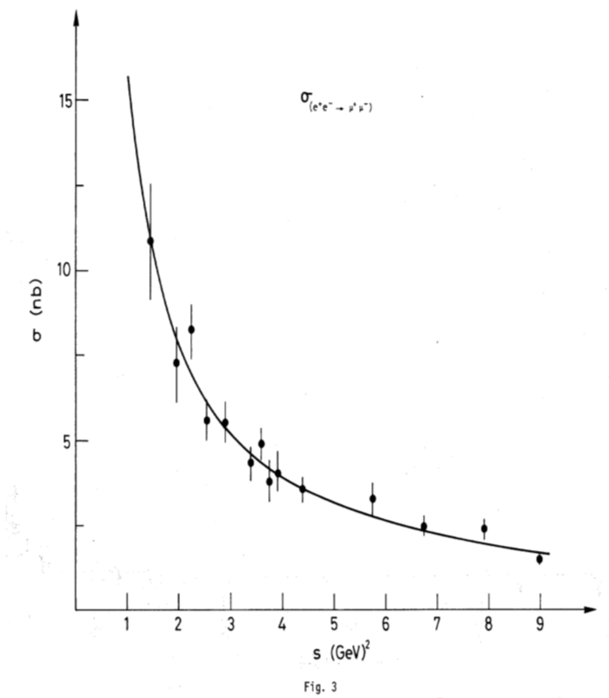
$$R_{eX}^{2p} \simeq \frac{\sigma\left(\tau^+\tau^-\right)}{\sigma\left(\mu^+\mu^-\right)} \simeq \sqrt{1-\frac{M_\tau^{\ 2}}{E^2}}\left(1+\frac{1}{2}\frac{M_\tau^{\ 2}}{E^2}\right), M_\mu \approx 0$$

Annihilation Cross-Section - VIII

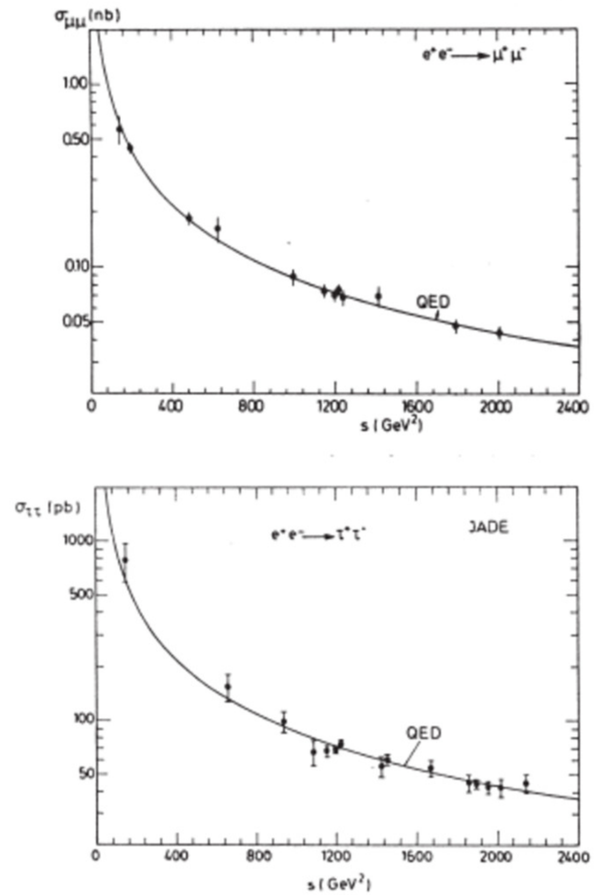


$\mu^+ \mu^-$ event: L3 detector at LEP

Annihilation Cross-Section - IX



ADONE



PETRA

Annihilation Cross-Section - X

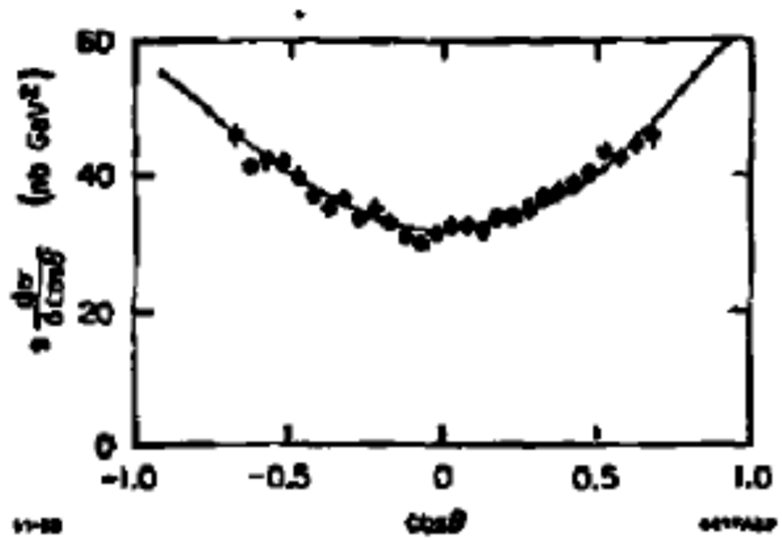
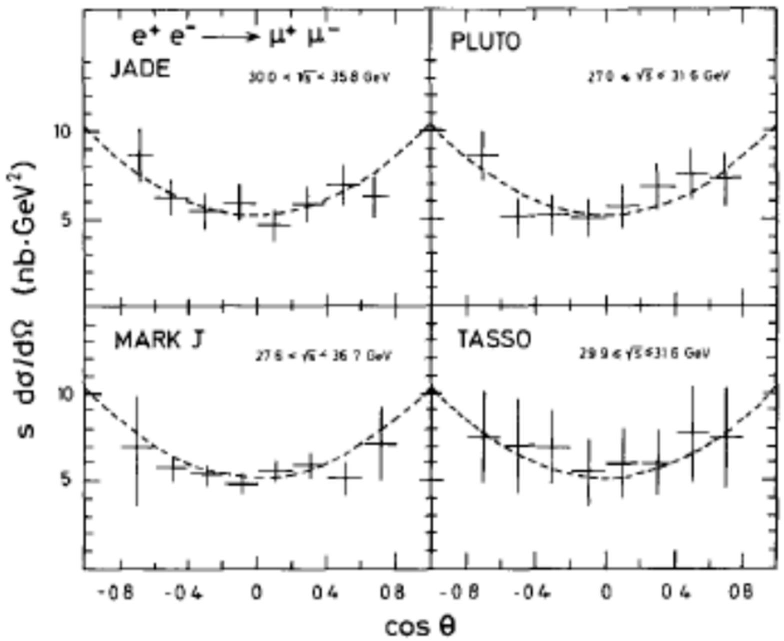
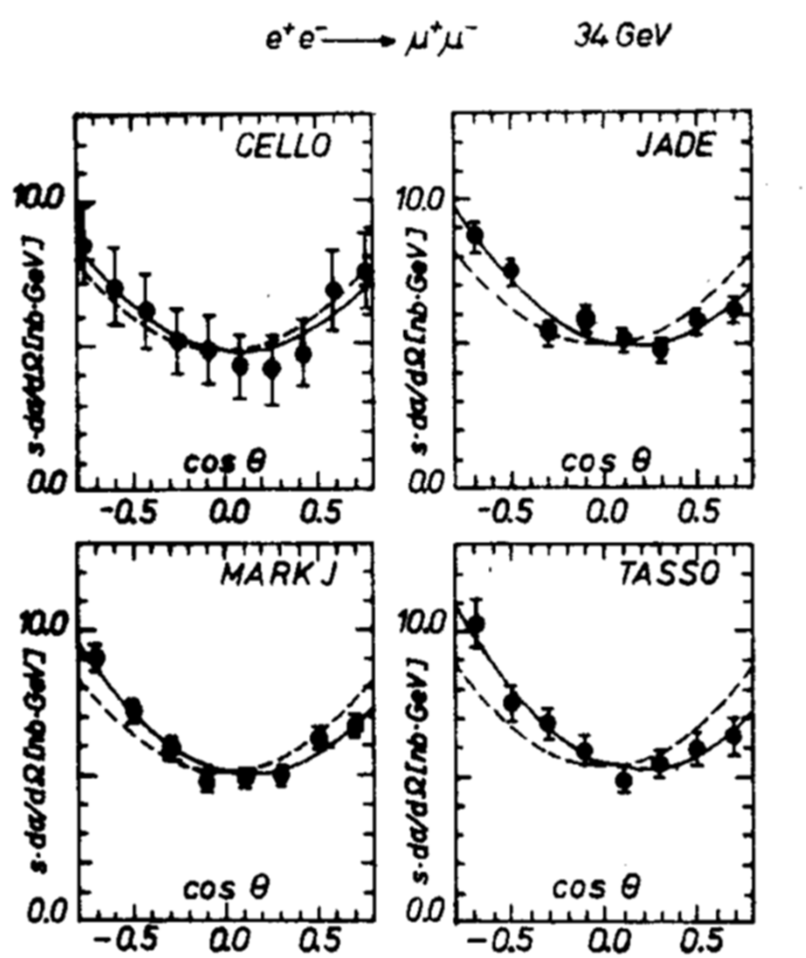


Fig. 15. MARK II $e^+e^- \to \mu^+\mu^-$ at $\langle E_{c.m.}\rangle$ = 5.847 compared to $1+\cos^2\theta$.

PEP: Angular distribution

Annihilation Cross-Section - XI



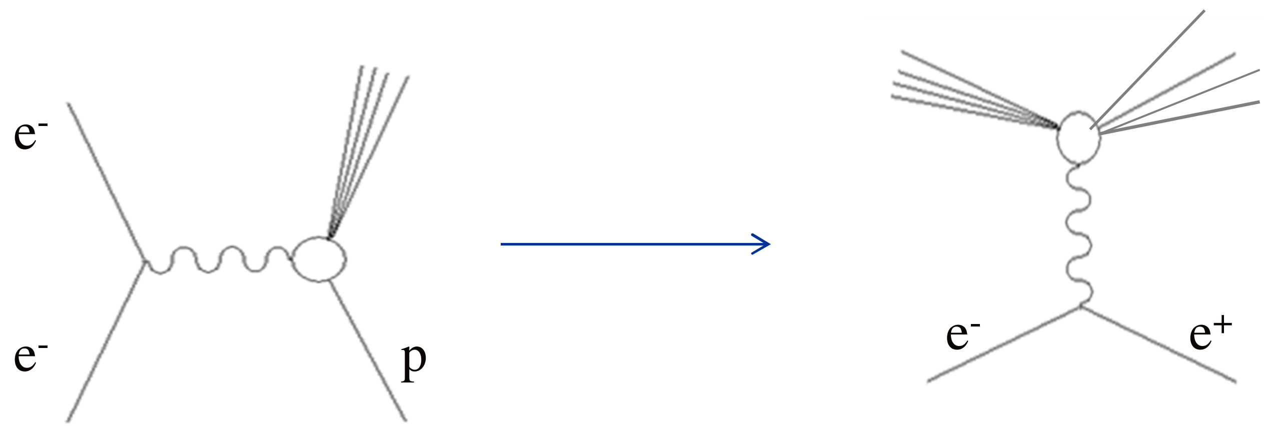


PETRA: Electroweak interference

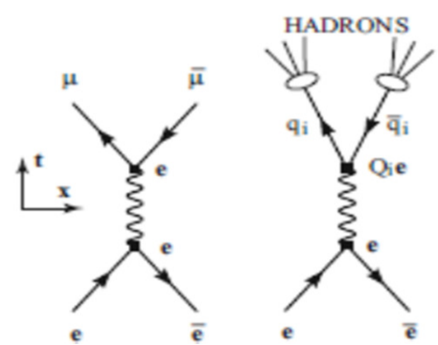
Annihilation into $q\bar{q}$ - I

e^+e^- annihilation into hadrons:

At the parton level = Crossed Deep Inelastic Scattering

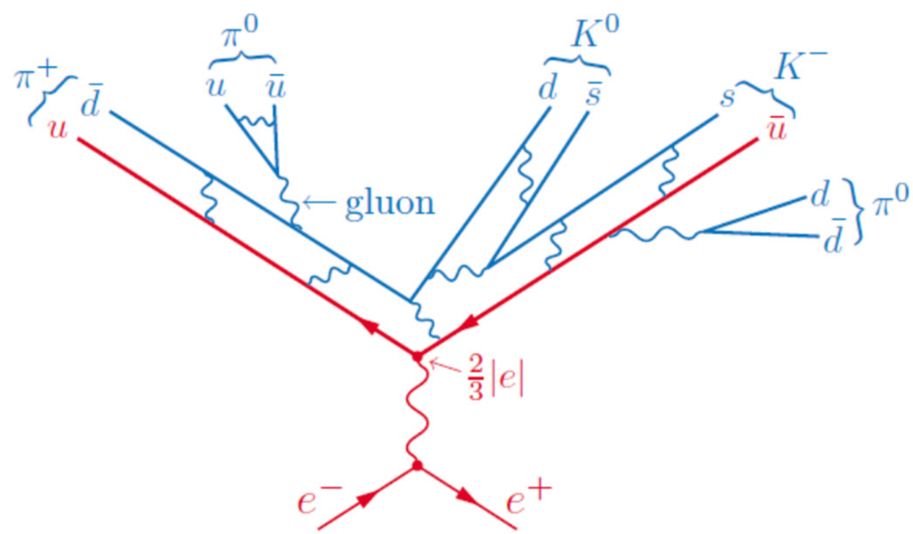


Interpreted as annihilation into a $q\bar{q}$ pair,
followed by quark fragmentation into hadrons

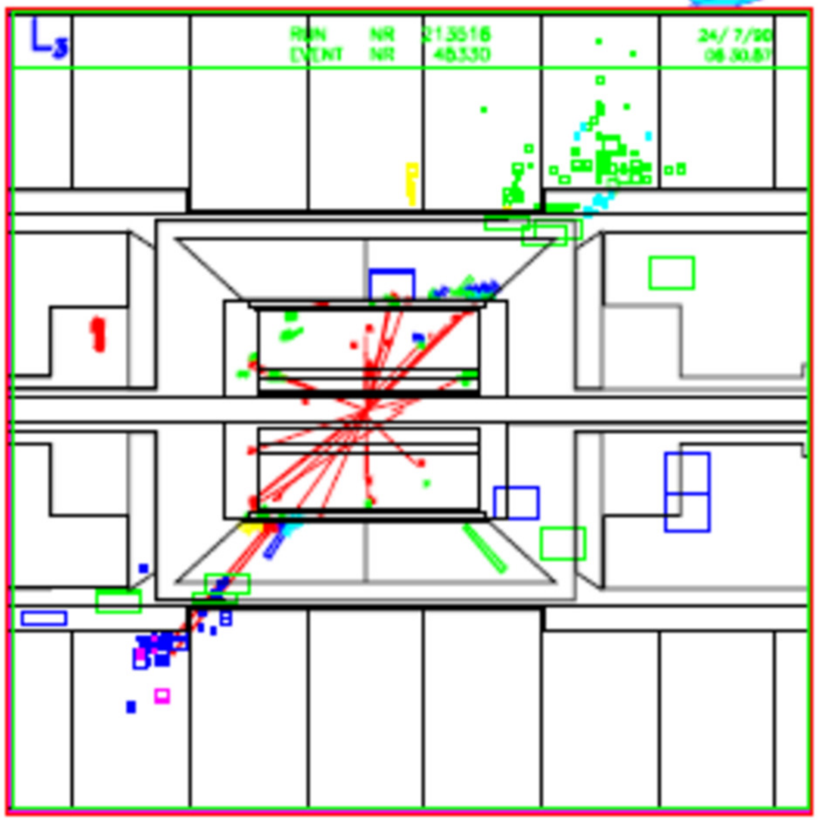


Annihilation into $q\bar{q}$ - II

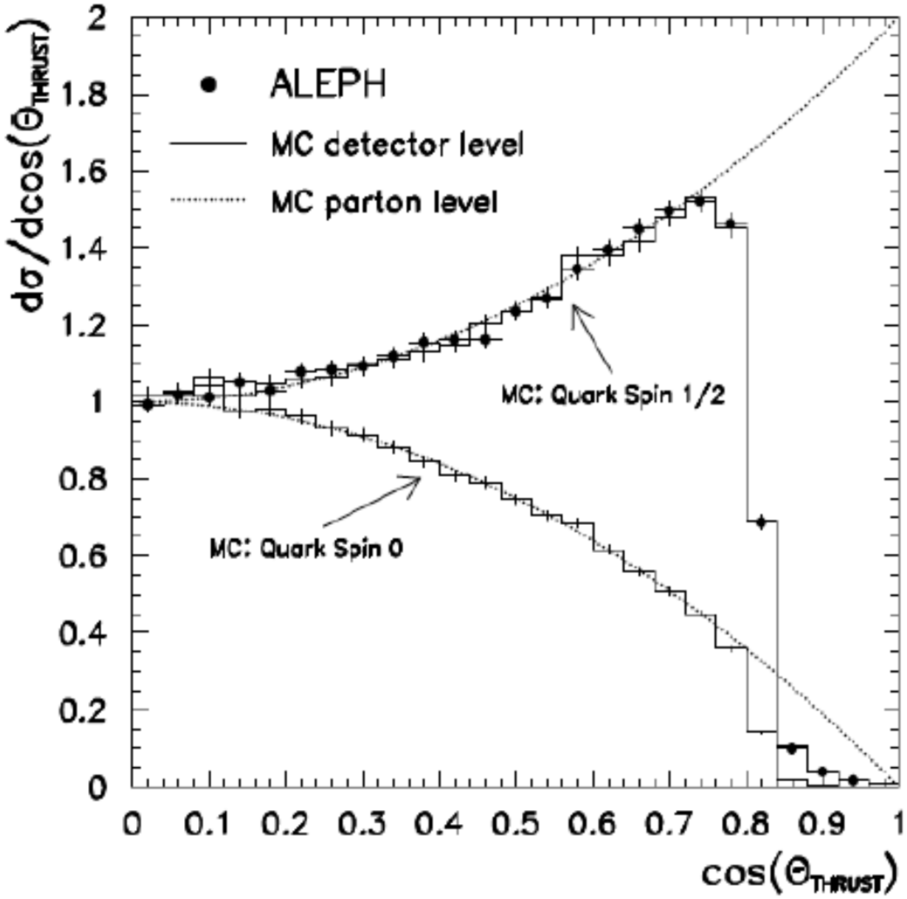
Picture of quark fragmentation



Annihilation into $q\bar{q}$ - III



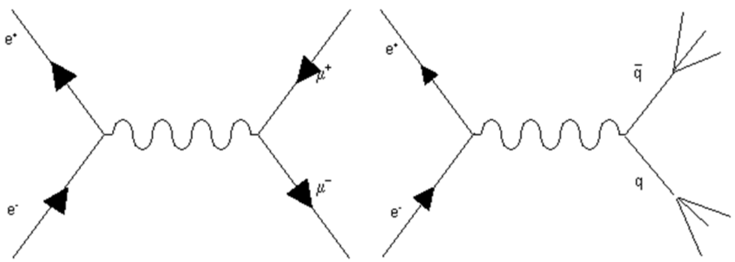
Annihilation into $q\bar{q}$ - IV



R Ratio - I

Assume the process $e^+e^- \to hadrons$ to proceed at the lowest order through

$e^+e^- \to q\,\bar{q} \to hadrons$



As for DIS:
Don't care about quark *hadronization*, assume the time scales for hard and soft sub-processes to be wildly different

$$\sigma\left(e^+e^- \to \mu^+\mu^-\right) = \frac{4\pi\alpha^2}{3s}$$

$$\sigma\left(e^+e^- \to q\,\bar{q}\right) = \frac{4\pi\alpha^2 Q_q^2}{3s}, \quad Q_q = \text{quark charge in } e \text{ units}$$

$$R\left(E_{CM}\right) = \frac{\sigma\left(e^+e^- \to adroni\right)}{\sigma\left(e^+e^- \to \mu^+\mu^-\right)} = \frac{\sum_q \sigma\left(e^+e^- \to q\bar{q}\right)}{\sigma\left(e^+e^- \to \mu^+\mu^-\right)} = \sum_q Q_q^2$$

Sum extended to all accessible quark flavors $\to 2m_q < E_{CM}$

R Ratio - II

R counts the number of different quark species created at any given E_{CM} . Expect:

$$u,d \rightarrow R = \left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 = \frac{5}{9} \qquad \textit{Low energy}$$

$$u,d,s \rightarrow R = \left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 = \frac{6}{9} \qquad E > 1\text{-}1.5\ GeV$$

$$u,d,s,c \rightarrow R = \left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 = \frac{10}{9} \qquad E > 3\ GeV$$

By taking 3 quark species
of any flavor:

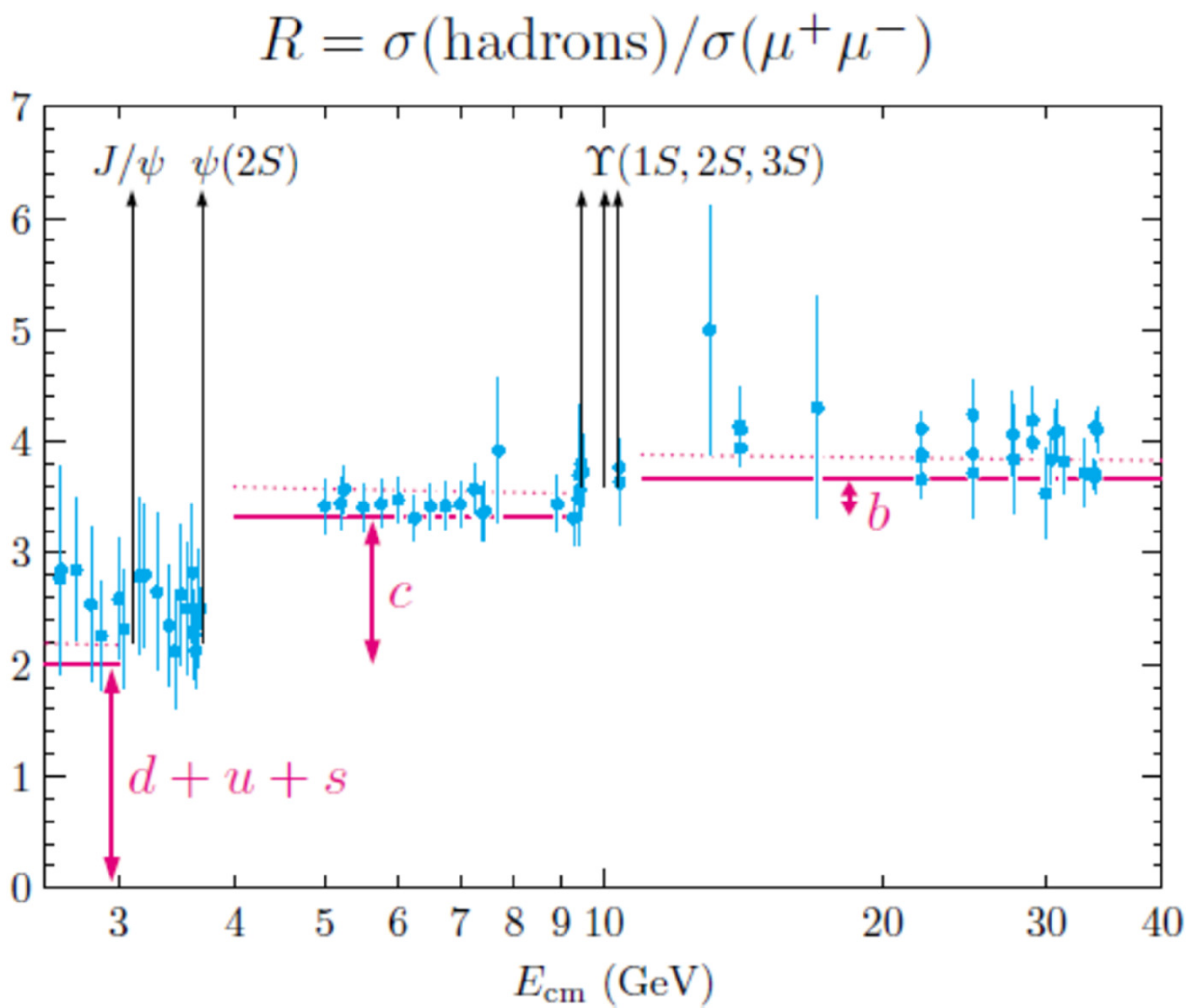
$$u,d \rightarrow R = \frac{15}{9}$$

$$u,d,s \rightarrow R = \frac{18}{9}$$

$$u,d,s,c \rightarrow R = \frac{30}{9}$$

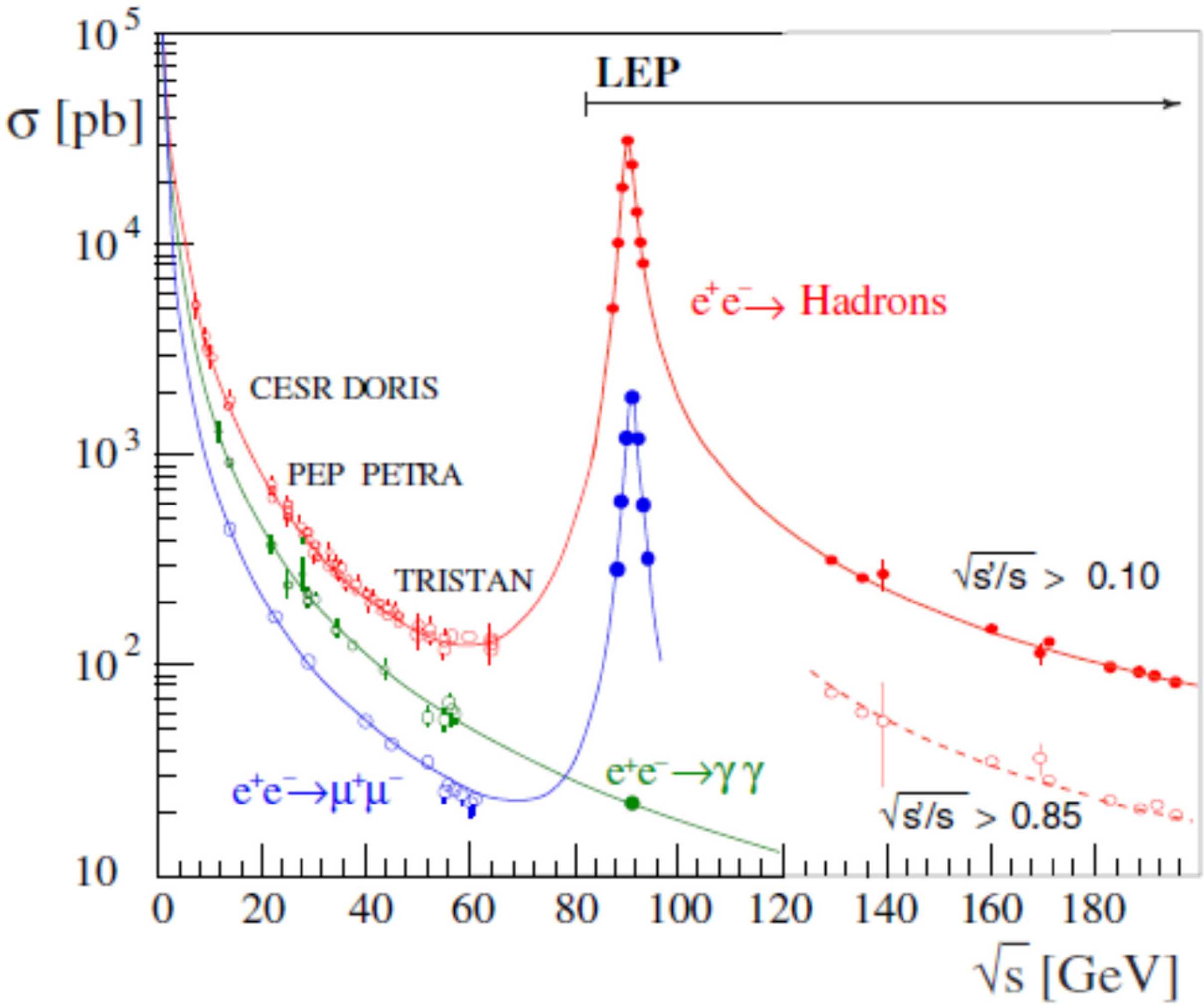
@TBA

R Ratio - III



$R = \sigma(\text{hadrons})/\sigma(\mu^+\mu^-)$
J/ψ
$\psi(2S)$
$\Upsilon(1S, 2S, 3S)$
c
b
$d + u + s$
0
1
2
3
4
5
6
7
3
4
5
6
7
8
9
10
20
30
40
E_{cm} (GeV)

$e^+ e^-$ Cross Sections

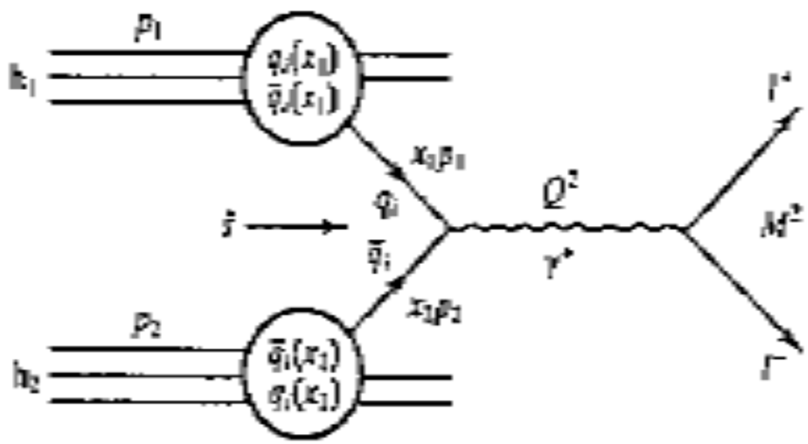


Drell – Yan - I

Reverse $e^+e^- \rightarrow q\bar{q}$ process: $q\bar{q} \rightarrow e^+e^-$

Obtained by any reaction

hadron + hadron $\rightarrow l^+ \; l^- + X$



Ignore parton transverse momentum

Ignore non-annihilating partons ($\rightarrow$ "spectators")

Ignore parton fragmentation

Drell – Yan - II

$e^+e^- \to \mu^+\mu^-$:

$$\frac{d\sigma}{d\Omega^*} = \frac{\alpha^2}{4s}\left(1+\cos^2\theta^*\right)$$

$$\sigma = \frac{4\pi\alpha^2}{3s}$$

$q\bar{q} \to \mu^+\mu^-$:

$$\frac{d\sigma_q}{d\Omega^*} = \frac{Q_q^2\alpha^2}{4M^2}\left(1+\cos^2\theta^*\right)\cdot\frac{1}{3}$$

$Q_q e$: Quark charge

$\frac{1}{3}$: Color factor

M^2 : $\mu^+\mu^-$ invariant mass = Total energy in partonic CM

Drell – Yan - III

$$p_1 = x_1 P_1 \simeq x_1 (E,0,0,P)$$

$$p_2 = x_2 P_2 \simeq x_2 (E,0,0,-P)$$

$$q = x_1 P_1 + x_2 P_2 \simeq \left[(x_1 + x_2)E,0,0,(x_1 - x_2)P\right]$$

$$M^2 = q^2 \simeq (x_1 + x_2)^2 E^2 - (x_1 - x_2)^2 P^2 \simeq 4x_1 x_2 E^2 = x_1 x_2 s \equiv \tau s$$

Kinematical variables: Either

$$\begin{cases} x_F \equiv x_1 - x_2 = \pm \dfrac{2|\mathbf{q}|}{\sqrt{s}} & \text{Feynman } x \text{ of parton pair} \\ M^2 = s x_1 x_2 \end{cases}$$

Or:

$$\begin{cases} y = \dfrac{1}{2}\ln\dfrac{E + p_3}{E - p_3} \simeq \dfrac{1}{2}\ln\dfrac{x_1}{x_2} & \text{Rapidity of parton pair (neglecting rest mass)} \\ M = \sqrt{s x_1 x_2} \end{cases}$$

Drell – Yan - IV

Inclusive cross-section:

Contribution by parton pair with (x_1, x_2) fractional momenta

$$d^2\sigma\left(pp \to \mu^+\mu^- + X\right) = \frac{4\pi\alpha^2}{9M^2}\sum_q Q_q^2\left[f_q\left(x_1\right)f_{\bar{q}}\left(x_2\right) + f_q\left(x_2\right)f_{\bar{q}}\left(x_1\right)\right]dx_1 dx_2$$

$$\to sx_2^{\,2} + sx_F x_2 - M^2 = 0 \to x_2 = \frac{-x_F \pm \sqrt{x_F^{\,2} + 4\frac{M^2}{s}}}{2} = \frac{-x_F \pm \sqrt{x_F^{\,2} + 4\tau}}{2}$$

$$\to x_2 = \frac{-x_F + \sqrt{x_F^{\,2} + 4\tau}}{2}$$

$$\to x_1 = x_F + x_2 = x_F + \frac{-x_F + \sqrt{x_F^{\,2} + 4\tau}}{2} = \frac{x_F + \sqrt{x_F^{\,2} + 4\tau}}{2}$$

$$x_2 = \frac{-x_F + \sqrt{x_F^{\,2} + 4\tau}}{2}, x_1 = \frac{x_F + \sqrt{x_F^{\,2} + 4\tau}}{2} \to x_1 + x_2 = \sqrt{x_F^{\,2} + 4\tau}$$

Drell – Yan - V

$$dx_1 dx_2 = J dx_F d\tau$$

$$J = \frac{\partial(x_1, x_2)}{\partial(x_F, \tau)} = \begin{vmatrix} \frac{\partial x_1}{\partial x_F} & \frac{\partial x_1}{\partial \tau} \\ \frac{\partial x_2}{\partial x_F} & \frac{\partial x_2}{\partial \tau} \end{vmatrix} = \frac{\partial x_1}{\partial x_F}\frac{\partial x_2}{\partial \tau} - \frac{\partial x_1}{\partial \tau}\frac{\partial x_2}{\partial x_F}$$

$$J = \frac{1}{4}\left(1 + \frac{x_F}{\sqrt{x_F^{\ 2} + 4\tau}}\right)\left(\frac{2}{\sqrt{x_F^{\ 2} + 4\tau}}\right) - \frac{1}{4}\left(\frac{2}{\sqrt{x_F^{\ 2} + 4\tau}}\right)\left(-1 + \frac{x_F}{\sqrt{x_F^{\ 2} + 4\tau}}\right)$$

$$\rightarrow J = \frac{1}{\sqrt{x_F^{\ 2} + 4\tau}} = \frac{1}{x_1 + x_2}$$

$$\rightarrow dx_1 dx_2 = \frac{1}{x_1 + x_2} dx_F d\tau$$

$$\tau = \frac{M^2}{s} \rightarrow d\tau = \frac{dM^2}{s} = \frac{dM^2}{M^2} x_1 x_2$$

$$\rightarrow dx_1 dx_2 = \frac{x_1 x_2}{M^2 (x_1 + x_2)} dM^2 dx_F = \frac{1}{s(x_1 + x_2)} dM^2 dx_F$$

Drell – Yan - VI

$$\rightarrow \frac{d^2\sigma}{dM^2 dx_F} = \frac{4\pi\alpha^2}{9M^2}\frac{1}{(x_1+x_2)s}\sum_q Q_q^2\left[f_q(x_1)f_{\bar q}(x_2)+f_q(x_2)f_{\bar q}(x_1)\right]$$

$$\rightarrow \frac{d^2\sigma}{dM^2 dx_F} = \frac{4\pi\alpha^2}{9M^4}\frac{x_1x_2}{x_1+x_2}\sum_q Q_q^2\left[f_q(x_1)f_{\bar q}(x_2)+f_q(x_2)f_{\bar q}(x_1)\right]$$

Central events:

$$x_F = 0 \rightarrow x_1 = x_2 = \sqrt{\tau} \rightarrow x_1 + x_2 = 2\sqrt{\tau}$$

$$\rightarrow \left.\frac{d^2\sigma}{dM^2 dx_F}\right|_{x_F=0} = \frac{2\pi\alpha^2}{9M^4}\sqrt{\tau}\sum_q Q_q^2\left[f_q(\sqrt{\tau})f_{\bar q}(\sqrt{\tau})+f_q(\sqrt{\tau})f_{\bar q}(\sqrt{\tau})\right]$$

$$\rightarrow s^2\left.\frac{d^2\sigma}{dM^2 dx_F}\right|_{x_F=0} = \frac{2\pi\alpha^2}{9\left(\sqrt{\tau}\right)^3}\sum_q Q_q^2\left[f_q(\sqrt{\tau})f_{\bar q}(\sqrt{\tau})+f_q(\sqrt{\tau})f_{\bar q}(\sqrt{\tau})\right]$$

Drell – Yan - VII

$$\begin{cases} y = \frac{1}{2}\ln\frac{E+p_3}{E-p_3} \simeq \frac{1}{2}\ln\frac{x_1}{x_2} \\ M = \sqrt{s x_1 x_2} \end{cases} \rightarrow \begin{matrix} x_1 = x_2 e^{2y} \\ M = \sqrt{s}\, x_2 e^{y} \end{matrix}$$

$$\rightarrow x_2 = \frac{M}{\sqrt{s}} e^{-y}, x_1 = \frac{M}{\sqrt{s}} e^{+y}$$

$$dx_1 dx_2 = J dM dy$$

$$J = \frac{\partial(x_1, x_2)}{\partial(M, y)} = \begin{vmatrix} \frac{\partial x_1}{\partial M} & \frac{\partial x_1}{\partial y} \\ \frac{\partial x_2}{\partial M} & \frac{\partial x_2}{\partial y} \end{vmatrix} = \frac{\partial x_1}{\partial M}\frac{\partial x_2}{\partial y} - \frac{\partial x_1}{\partial y}\frac{\partial x_2}{\partial M}$$

$$\rightarrow J = \frac{1}{\sqrt{s}} e^{+y}\left(-\frac{M}{\sqrt{s}} e^{-y}\right) - \frac{M}{\sqrt{s}} e^{+y} \frac{1}{\sqrt{s}} e^{-y} = -\frac{2M}{s} = -2\sqrt{\frac{x_1 x_2}{s}}$$

$$\rightarrow dx_1 dx_2 = \left|-2\sqrt{\frac{x_1 x_2}{s}}\right| dM dy$$

Drell – Yan - VIII

$$\rightarrow \frac{d^2\sigma}{dMdy} = \frac{4\pi\alpha^2}{9M^2}\left|-2\sqrt{\frac{x_1 x_2}{s}}\right|\sum_q Q_q^2\left[f_q(x_1)f_{\bar{q}}(x_2)+f_q(x_2)f_{\bar{q}}(x_1)\right]dMdy$$

$$s\tau = M^2 \rightarrow M = \sqrt{s\tau}$$

$$\rightarrow \frac{d^2\sigma}{dMdy} = \frac{8\pi\alpha^2}{9M^2}\frac{\tau}{M}\sum_q Q_q^2\left[f_q(x_1)f_{\bar{q}}(x_2)+f_q(x_2)f_{\bar{q}}(x_1)\right]$$

$$\rightarrow \frac{d^2\sigma}{dMdy} = \frac{8\pi\alpha^2}{9Ms}\sum_q Q_q^2\left[f_q(x_1)f_{\bar{q}}(x_2)+f_q(x_2)f_{\bar{q}}(x_1)\right]$$

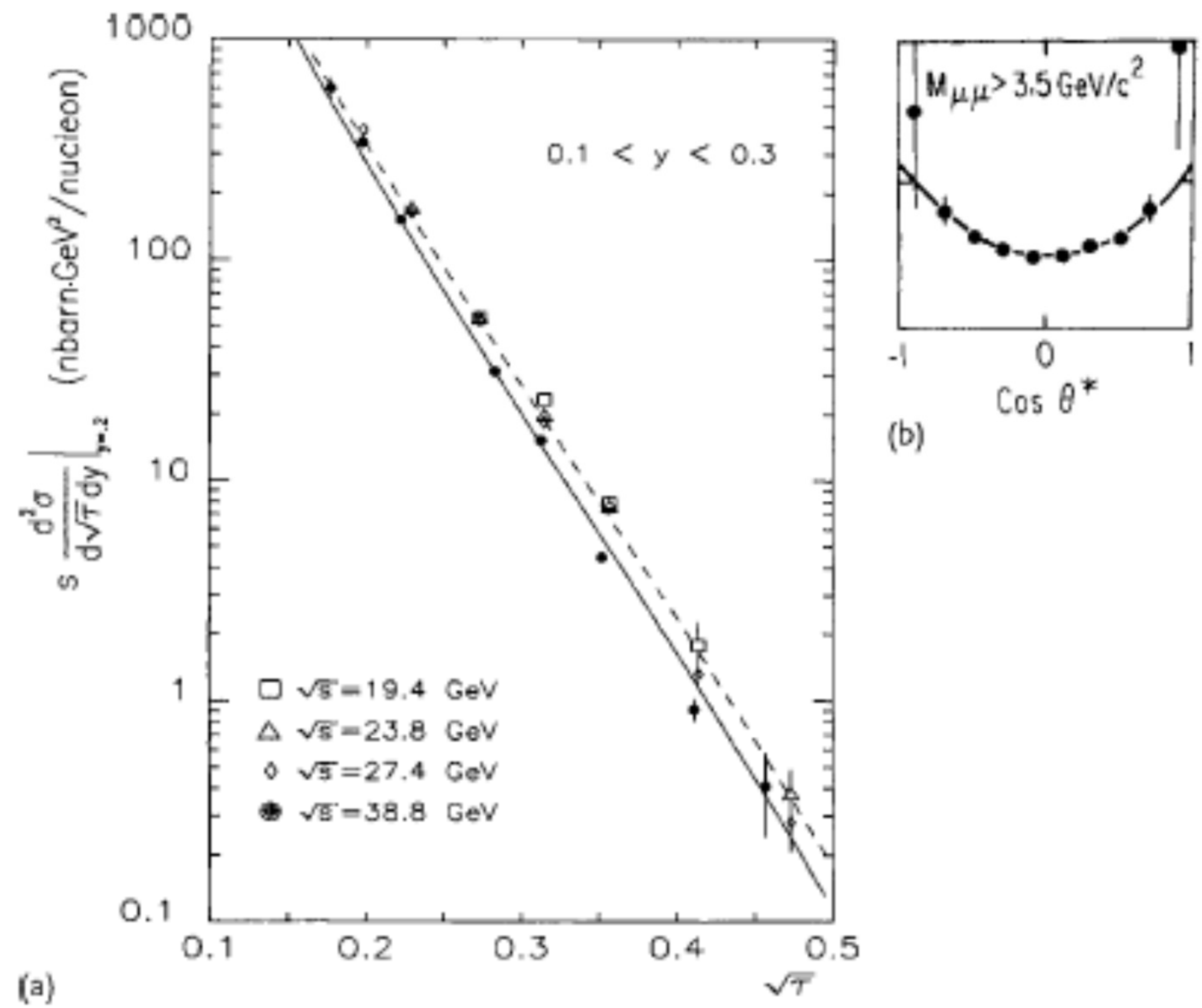
Central events:

$$y = 0, x_1 = x_2 = \sqrt{\tau}$$

$$\rightarrow \left.\frac{d^2\sigma}{dMdy}\right|_{y=0} = \frac{8\pi\alpha^2}{9Ms}\sum_q Q_q^2\left[f_q(\sqrt{\tau})f_{\bar{q}}(\sqrt{\tau})+f_q(\sqrt{\tau})f_{\bar{q}}(\sqrt{\tau})\right]$$

$$\rightarrow s^{3/2}\left.\frac{d^2\sigma}{dMdy}\right|_{y=0} = \frac{8\pi\alpha^2}{9}\frac{1}{\sqrt{\tau}}\sum_q Q_q^2\left[f_q(\sqrt{\tau})f_{\bar{q}}(\sqrt{\tau})+f_q(\sqrt{\tau})f_{\bar{q}}(\sqrt{\tau})\right]$$

Drell – Yan - IX



1000
100
10
1
0.1
0.1
0.2
0.3
0.4
0.5
0.1 < y < 0.3
√s=19.4 GeV
√s=23.8 GeV
√s=27.4 GeV
√s=38.8 GeV
(nbarn·GeV²/nucleon)
(a)
Mμμ> 3.5 GeV/c²
-1
0
1
Cos θ*
(b)