Elementary Particles I

3 – Strong Interaction

Resonances, Isospin, Strangeness, Unitary Symmetries

Strong Interaction

Originally pictured as an attractive force between nucleons, required to overcome repulsive Coulomb interaction in nuclei
Main features:

- *Strength*
- *Short range*
- *Charge independence*

For a long time, difficult to understand: lot of guesswork, many models
Today, believed to be a *residual force* between ‘color neutral’ particles (hadrons), a remnant of color interaction between quarks and gluons
Somewhat similar to Van der Waals/Covalent bond between ‘neutral’ molecules, coming from electromagnetic interaction between charged electrons and nuclei

Yukawa Theory

First attempt to model strong interaction after the electromagnetic:
Exchange of mediator particles $\rightarrow$ Prediction of *pion*

Mass >0 — Limited range

Spin $\neq 1$ — Vector particle would yield repulsive forces between identical particle

Charged,Neutral — Same force for $pp,\ nn,\ pn$

Electromagnetism		Yukawa	
$\frac{\partial^2 \varphi}{\partial t^2} - \nabla^2 \varphi = -\rho$	Wave equation	$\frac{\partial^2 \varphi}{\partial t^2} - \nabla^2 \varphi - m^2 = -\rho$	Wave equation
	Scalar potential		Pion field
$\nabla^2 \varphi = \rho$	Static case	$\nabla^2 \varphi + m^2 = \rho$	Static case
$\rho_G(\mathbf{r}) = e\delta(\mathbf{r})$	Point source at the origin	$\rho_G(\mathbf{r}) = g\delta(\mathbf{r})$	Point source at the origin
$\rightarrow \varphi_G(\mathbf{r}) = \frac{e}{r}$	Green's function $\equiv$ Coulomb potential	$\rightarrow \varphi_G(\mathbf{r}) = \frac{g\ e^{-mr}}{r}$	Green's function $\equiv$ Yukawa potential

Pions

Discovered after the II World War (Cosmic Rays, Accelerators)
Properties

Mass	135 MeV	Neutral
	139 MeV	Charged
Spin	0	
Parity	-	
Charge parity	+	
Lifetime	25 10^{-9} s	Charged
	10^{-16} s	Neutral
Decay modes (Dominant)	$\mu\nu$	Charged
	$\gamma\gamma$	Neutral

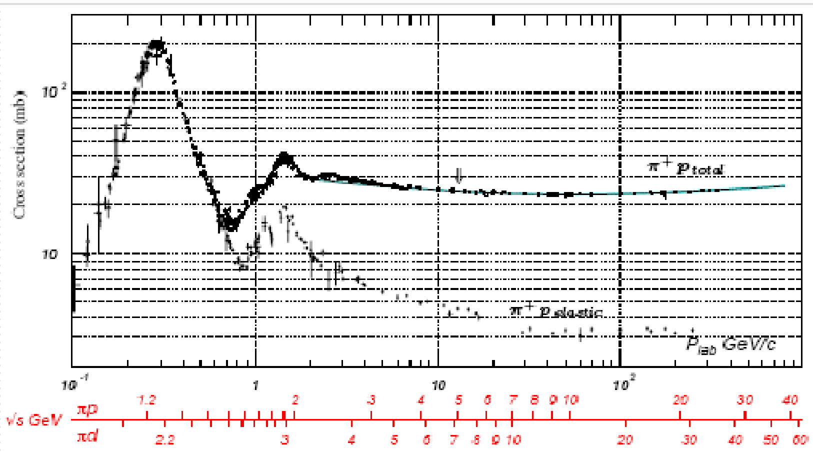
Stable vs. strong decays, as the *lightest hadron*
Copiously produced at first accelerators (synchrocylotrons)
Charged pions easily focused into collimated, high energy beams

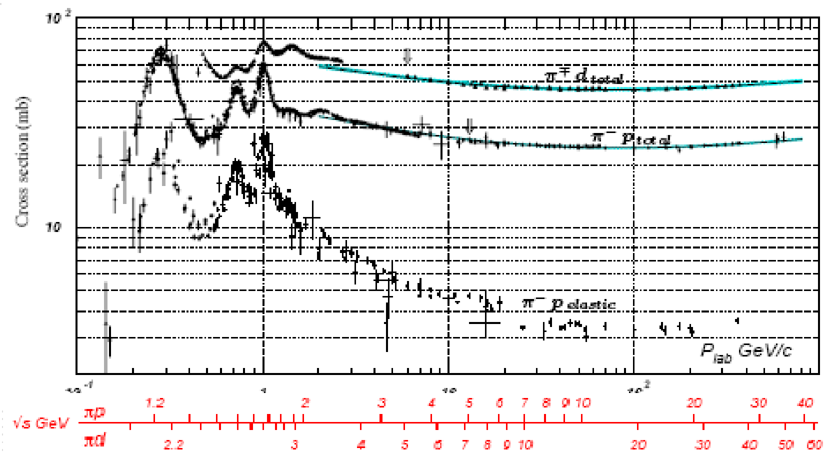
Scattering

As for electromagnetic, strong interaction can be investigated by scattering experiments
Perform experiments like

$$p+p, \quad p+n, \quad \pi^{\pm}+p, \quad \pi^{\pm}+n$$

Pion: Spinless $\rightarrow$ Understanding πN scattering easier than NN





Total cross section plots - Observe lot of structure

Propagators in the *s*-channel - I

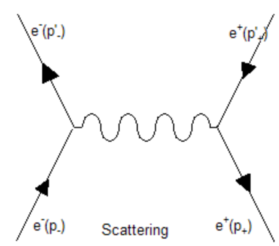
Take first a QED example: Bhabha scattering at $\sqrt{s} \ll M_{Z^0}$

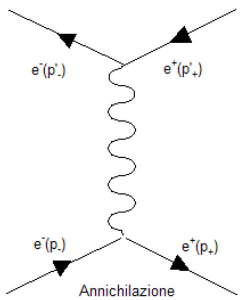
$$e^- + e^+ \to e^- + e^+$$

Two one-photon diagrams

Virtual photon propagator $= \dfrac{1}{q^2}$

t-channel: Virtual photon has $q^2 < 0$ space-like
s-channel: Virtual photon has $q^2 > 0$ time-like



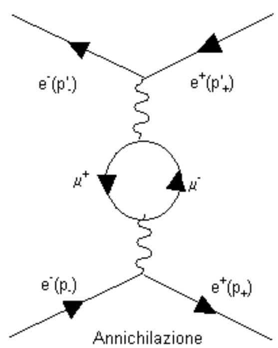


t-channel *s*-channel

Taking radiative corrections to one loop

Virtual photon propagator $= \dfrac{1}{q^2\left(1-\bar{\Pi}_{\gamma}^{(2)}\left(q^2\right)\right)}$

Correction resulting from fermion e.m. currents circulating in the loop, after renormalization



Propagators in the *s*-channel - II

Among all fermion currents circulating in the loop, take a muon pair.
Now, a $\mu^+\mu^-$ pair has bound states, like a hydrogen atom.
For these, total energy is < $2m_\mu$: Binding energy<*0*

This term tied to bound state being *unstable*: $\Gamma > 0 \rightarrow \tau = 1/\Gamma < \infty$
Unlike the *H* atom, muonic atom annihilates into various channels

When $\sqrt{s} = E_{CM} \sim M_{bound} \equiv M$

$$q^2 = s = E_{CM}^2$$

$$\rightarrow \frac{1}{q^2\left(1-\overline{\Pi}_\gamma^{(2)}\left(q^2\right)\right)} \approx \frac{1}{q^2 - M\left(M - i\Gamma\right)} = \frac{1}{E_{CM}^2 - M^2 + iM\Gamma}$$

$$\rightarrow \frac{1}{q^2\left(1-\overline{\Pi}_\gamma^{(2)}\left(q^2\right)\right)} \approx \frac{1}{\left(E_{CM} - M\right)\underbrace{\left(E_{CM} + M\right)}_{\approx 2M} + iM\Gamma} \approx \frac{1}{2M}\frac{1}{\left(E_{CM} - M\right) + i\,\Gamma/2}$$

Breit-Wigner form

The existence of bound states for the current coupled to the photon is reflected into resonant behavior of the s-channel scattering amplitude

NB Resonant peaks in total, elastic $e^+ e^-$ cross-section not observed because of their exceedingly small width

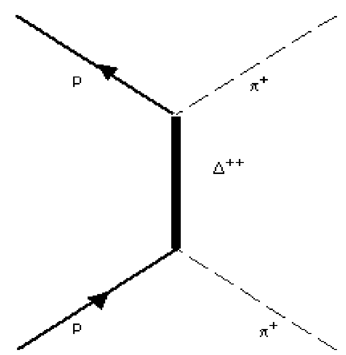
Propagators in the *s*-channel - III

General rule:

Every time the intermediate state can couple to an unstable state (excited bound state, genuine elementary particle coupled to decay channels, …), the s-channel propagator shows resonant behavior when the total energy is close to the mass of the unstable state

Δ-Resonance: Formation

First observed by Fermi and collaborators in πN scattering (1951)

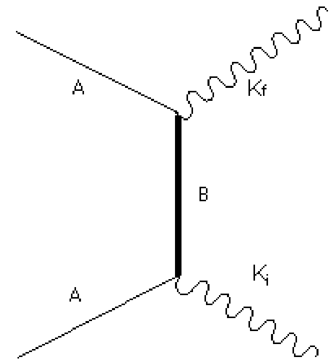


<u>With some caveats</u>, can be considered as a kind of excited nucleon state

$$\pi^+ + p \to \Delta^{++} \to \pi^+ + p$$

Different spin, quark content

Also observed in other charge states Δ^+, Δ^-, Δ^0 and in many different processes (strong, e.m. and weak)



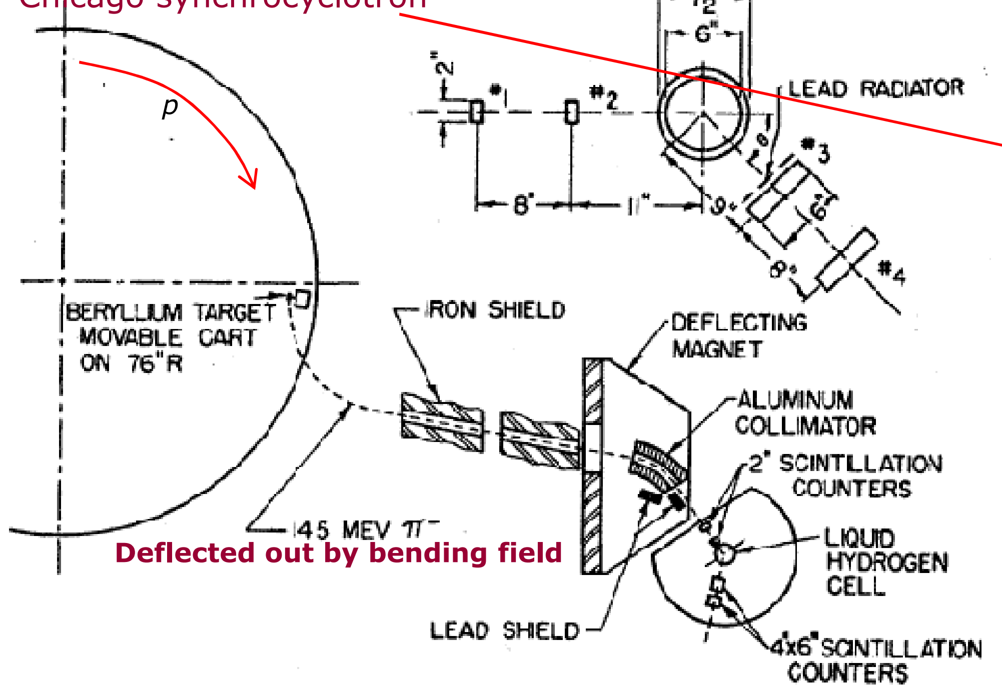
Some analogy with photon excitation of atomic levels

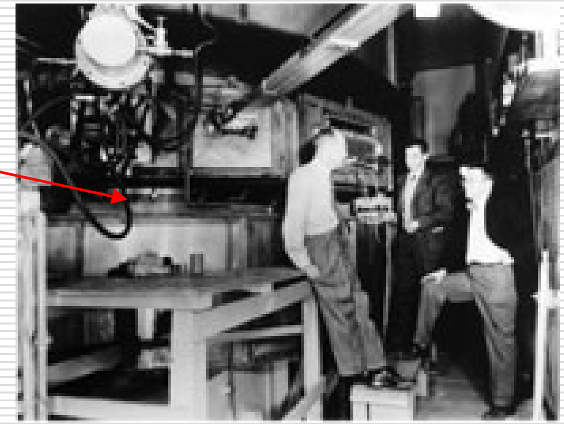
$\gamma + A \to B \to \gamma + A,$ A ground state, B excited level

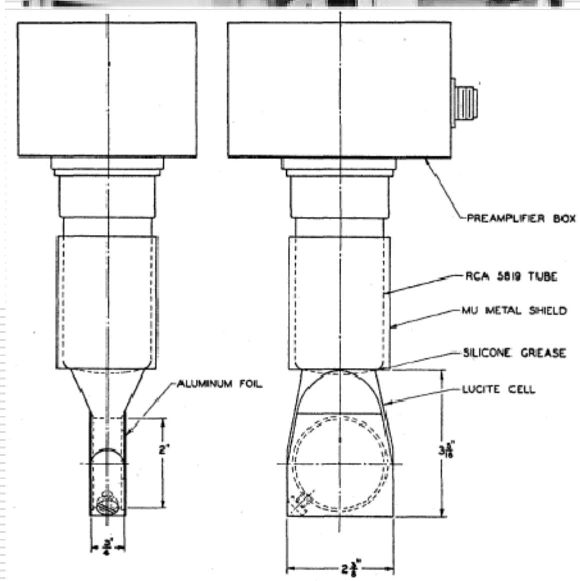
Good indication that the nucleon is a *composite* object

Discovery of Δ - 1951

Chicago synchrocyclotron







Plastic scintillators

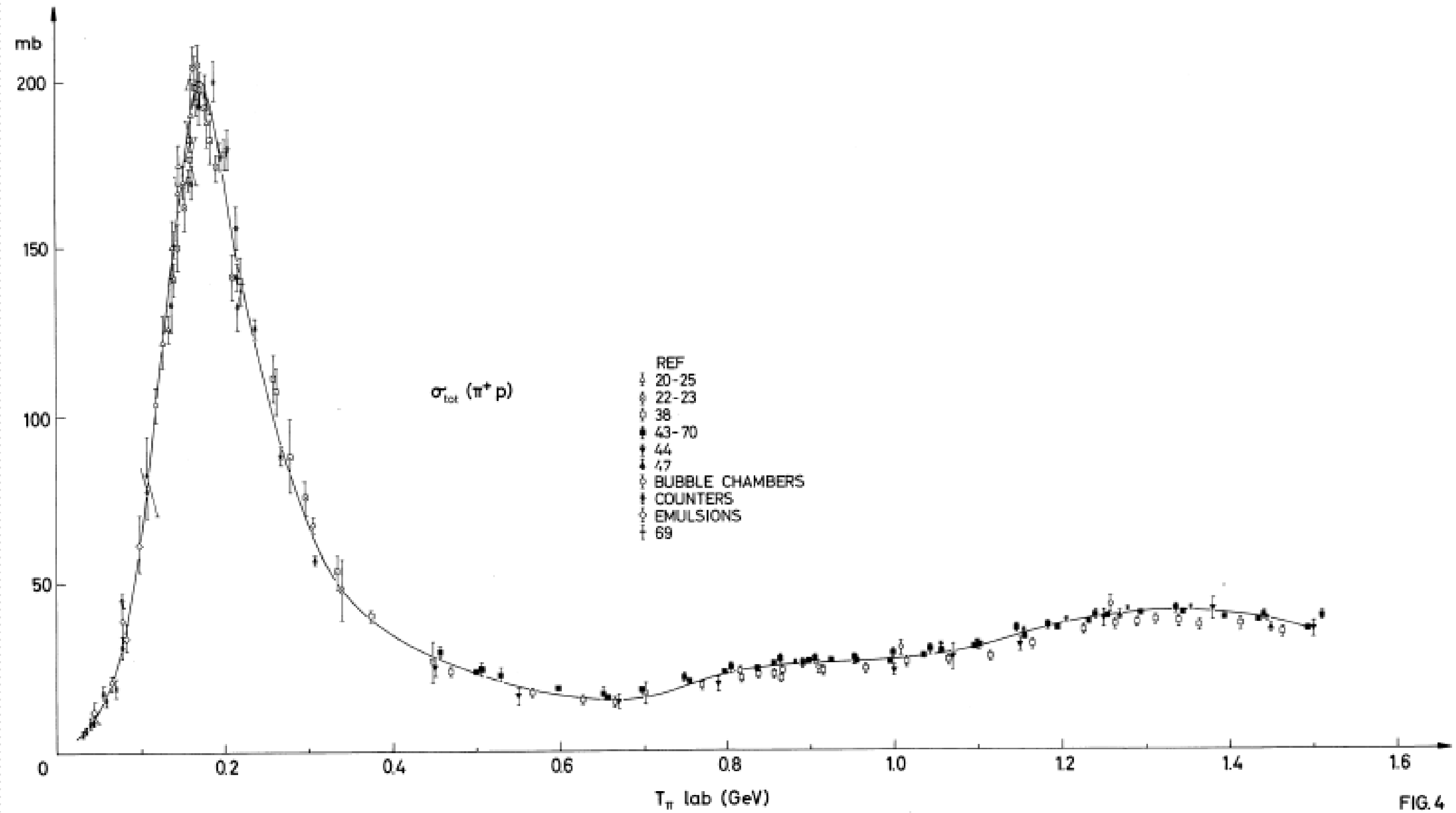
Collect first data on 2-body reactions:

$$\pi^+ + p \rightarrow \pi^+ + p$$

$$\pi^- + p \rightarrow \pi^- + p$$

$$\pi^- + p \rightarrow \pi^0 + n$$

Δ^{++} Resonance



Propagators in the *t*-channel - I

The same propagator describes the *t*-channel amplitude, $t=q^2<0$:

$$\frac{1}{q^2\left(1-\overline{\Pi}_\gamma^{(2)}\left(q^2\right)\right)} \approx \frac{1}{q^2 - M\left(M - i\Gamma\right)} \underset{\Gamma \ll M}{\approx} \frac{1}{q^2 - M^2}$$ 'Pole' amplitude

In this case, there is *no* resonant behavior: $q^2 - M^2 < 0$ strictly

Rather, the amplitude can be seen as an extension of the virtual photon idea, corresponding to the exchange of a *virtual particle*, with mass M and width Γ , or lifetime $1/\Gamma$. In the previous example, it would be a *virtual muonic atom.*

As for the virtual photon , the virtual particle exchanged is said to be *off mass-shell*:

$$q^2 \neq M^2$$

Propagators in the *t*-channel - II

Besides being very appealing as a qualitative visualization of processes, this interpretation also appears to be superficially consistent with perturbation theory. But…

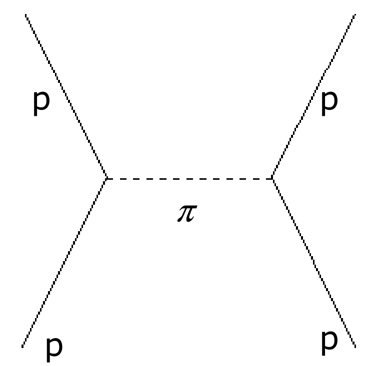
…*It is unfortunately not very useful as a tool for quantitative work in strong interactions physics, just because perturbative expansion cannot be maintained for strong coupling constant*.

Most simply, diagrams with more than one particle exchanged correspond to amplitudes *larger* than diagrams with just one…

One π Exchange $\leftrightarrow$ Yukawa Potential

Nevertheless, just as an interesting exercise:
Take *NN* scattering at small q^2 as due to *one pion exchange*: This *can* be maintained, to some extent (or so one believes).
Then

$$A \propto \frac{1}{q^2 - m_\pi^2}$$



In the static potential limit

$$E_C \approx E_A$$

$$q^2 = \left(E_C - E_A\right)^2 - \left(\mathbf{p}_C - \mathbf{p}_A\right)^2 \approx -\left(\mathbf{p}_C - \mathbf{p}_A\right)^2 = -|\mathbf{q}|^2$$

$$\rightarrow \frac{1}{q^2 - m_\pi^2} \approx \frac{1}{-|\mathbf{q}|^2 - m_\pi^2} = -\frac{1}{|\mathbf{q}|^2 + m_\pi^2}$$

Assuming Born approximation as valid here

$$V(r) \propto \int e^{i\mathbf{q}\cdot\mathbf{r}} \left(-\frac{1}{|\mathbf{q}|^2 + m_\pi^2} \right) d^3\mathbf{q} \propto -\frac{e^{-m_\pi r}}{r}$$ Yukawa potential

Potential Scattering

Formalism of potential scattering:
Not a proper tool to describe relativistic regime (particle creation/destruction) $\rightarrow$ Go for Field Theory

Nevertheless:
Believed to be somewhat useful to get insight into simplest (2-body) reactions, like elastic scattering, even at high energy

Phase shifts analysis:
Try to reconstruct the interaction structure from scattering data

Past: Lot of work spent in the attempt of modeling 'simplest' reactions (e.g. Mandelstam representation, Regge poles, …)

Now: The 'simplest' reactions finally understood to be quite complicated, much more than anticipated ($\leftarrow$ Non perturbative interaction regime)

Phase Shifts and Resonances - I

Partial waves expansion

$$d\sigma = v\frac{|f|^2}{v}d\Omega = |f|^2 d\Omega \rightarrow \frac{d\sigma}{d\Omega} = |f|^2$$

Scattering amplitude:

$$f(\theta) = \tfrac{1}{2ik}\sum_{l=0}^{\infty}(2l+1)\left(e^{2i\delta_l}-1\right)P_l(\cos\theta) = \tfrac{1}{k}\sum_{l=0}^{\infty}(2l+1)\frac{e^{2i\delta_l}-1}{2i}P_l(\cos\theta)$$

$$f_l = \frac{e^{2i\delta_l}-1}{2i} = e^{i\delta_l}\,\frac{e^{i\delta_l}-e^{-i\delta_l}}{2i} = e^{i\delta_l}\sin\delta_l$$

$$\rightarrow \frac{1}{f_l} = \frac{1}{\sin\delta_l}e^{-i\delta_l} = \frac{1}{\sin\delta_l}\left(\cos\delta_l - i\sin\delta_l\right) = \cot\delta_l - i$$

$$\rightarrow f_l = \frac{1}{\cot\delta_l - i}$$

$$\cot\delta_l\Big|_{\delta_l=\frac{\pi}{2}} = 0 - \frac{1}{\sin^2\delta_l}\Bigg|_{\delta_l=\frac{\pi}{2}}\left(\delta_l - \frac{\pi}{2}\right) + \ldots \approx -\left(\delta_l - \frac{\pi}{2}\right)$$

Phase Shifts and Resonances - II

For E_R such that $\delta_l(E_R)=\dfrac{\pi}{2}$, expand into power series around E_R :

$$\delta_l(E)=\delta_l(E_R)+\left.\frac{d\delta_l}{dE}\right|_{E=E_R}(E-E_R)+...,\qquad \frac{2}{\Gamma}\equiv\left.\frac{d\delta_l}{dE}\right|_{E=E_R}\rightarrow\delta_l\approx\frac{\pi}{2}+\frac{E-E_R}{\Gamma/2}$$

$$\rightarrow\cot\delta_l\underset{E\sim E_R}{\approx}-\left(\delta_l-\frac{\pi}{2}\right)=-\left(\frac{\pi}{2}+\frac{E-E_R}{\Gamma/2}-\frac{\pi}{2}\right)\approx-\frac{E-E_R}{\Gamma/2}=\frac{E_R-E}{\Gamma/2}$$

$$\rightarrow f_l\approx\frac{1}{\dfrac{(E_R-E)}{\Gamma/2}-i}=\frac{\Gamma/2}{E-E_R+i\Gamma/2}$$ Breit-Wigner resonant amplitude

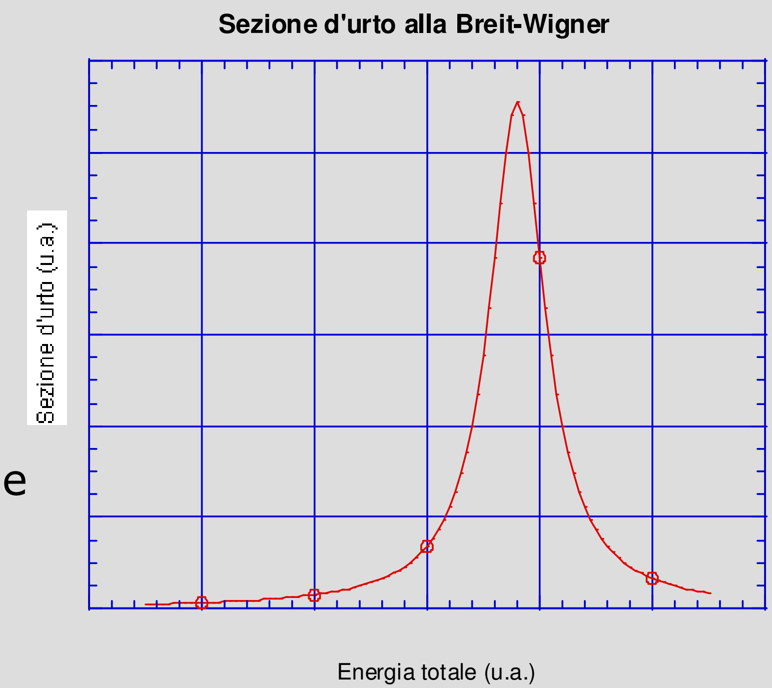
E_R: characteristic energy of the system

$1/\Gamma$: Phase variation at $E_R \rightarrow [1/\Gamma]$ = Time

Phase Shifts and Resonances - III

Partial cross-section for l wave:

$$\rightarrow |f_l|^2 = \sin^2 \delta_l = \frac{\Gamma^2/4}{(E-E_R)^2 + \Gamma^2/4},$$



Total cross-section= Sum of partial wave cross-sections

Often dominated by a resonance in one partial wave

Resonance 'symptoms':

a) Fast increasing phase shift, going through $\pi/2$ at maximum rate
b) $|f_l|^2$ strongly peaked
c) Wave function large
d) $d\delta/dk$, and delay, strongly peaked

Resonances - I

Generalize concept of stationary state:

$$\psi(\mathbf{r},t)=\varphi(\mathbf{r})e^{-iE_0t} \rightarrow \int_{-\infty}^{+\infty} e^{-iE_0t}e^{iEt}dt=\delta(E-E_0)$$

(Amplitude to find energy E when system is prepared in the state ψ)

to a kind of non-stationary, decaying state

Complex E: Just meaning "System is unstable"

$$e^{-iEt}=e^{-i(E_0-i\Gamma)t}=e^{-iE_0t}e^{-\Gamma t}, \quad t > 0$$

$$\int_0^{+\infty} e^{-i(E_0-i\Gamma)t}e^{iEt}dt=\int_0^{+\infty} e^{-i(E_0-E-i\Gamma)t}dt=-\frac{1}{i(E_0-E-i\Gamma)}e^{-i(E_0-E-i\Gamma)t}\Big|_0^{+\infty}=\frac{i}{(E-E_0+i\Gamma)}$$

(Breit-Wigner:
Amplitude to find energy E when system prepared in the state ψ)

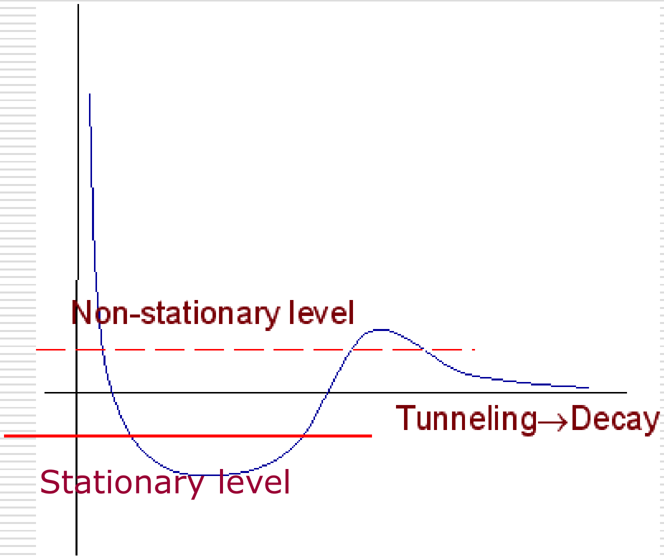
$$|\psi|^2 \propto \left|\frac{i}{E-E_0+i\Gamma}\right|^2=\left|\frac{E-E_0-i\Gamma}{(E-E_0)^2+\Gamma^2}\right|^2=\frac{(E-E_0-i\Gamma)(E-E_0+i\Gamma)}{\left[(E-E_0)^2+\Gamma^2\right]^2}=\frac{\left((E-E_0)^2+\Gamma^2\right)}{\left[(E-E_0)^2+\Gamma^2\right]^2}=\frac{1}{(E-E_0)^2+\Gamma^2}$$

Resonances - II

Non-stationary levels may result from a particular shape of the effective potential

Non stationary, scattering state
But: *Almost* stationary…

Long lifetime, sharp quantum numbers: Like a *stable* state



$$\Gamma = \begin{matrix} \text{1/time constant of decaying state} \approx \text{time uncertainty} \\ \text{Half width at half maximum} \approx \text{energy uncertainty} \end{matrix} \Bigg\} \rightarrow \Delta E \Delta t \sim \Gamma \frac{1}{\Gamma} = 1$$

Δ Resonance Formation - I

Take πp scattering at low energy: use phase shift analysis
Some complication arising from spin 1/2

$k \sim m, r \le R = \frac{1}{m} \to l = kr \le 1$ Limited range, low energy: just 2 waves S and P

$$J = 1/2 \oplus 0 \oplus l = 1/2 \oplus l = \begin{cases} 1/2 & S-wave \\ 1/2, 3/2 & P-wave \end{cases}$$

Expand first incident wave:

$$e^{ikz} \underbrace{\chi_{1/2}^{+1/2}}_{\text{spin eigenstate}} = \frac{1}{2ikr}\sum_{l=0}^{1}(2l+1)\left(e^{ikr} - (-1)^l e^{-ikr}\right)P_l(\cos\theta)\chi_{1/2}^{+1/2}$$

$$e^{ikz}\chi_{1/2}^{+1/2} = \frac{1}{2ikr}\sum_{l=0}^{1}\sqrt{4\pi(2l+1)}\left(e^{ikr} - (-1)^l e^{-ikr}\right)Y_l^0(\cos\theta)\chi_{1/2}^{+1/2}$$

$$Y_l^0\chi_{1/2}^{+1/2} = \sqrt{\frac{l+1}{2l+1}}y_{l+1/2}^{+1/2} - \sqrt{\frac{l}{2l+1}}y_{l-1/2}^{+1/2}$$

Spin spherical harmonics

$$y_{l+1/2}^{+1/2} = \sqrt{\frac{l+1}{2l+1}}Y_l^0\chi_{1/2}^{+1/2} + \sqrt{\frac{l}{2l+1}}Y_l^1\chi_{1/2}^{-1/2}, \quad y_{l-1/2}^{+1/2} = \sqrt{\frac{l+1}{2l+1}}Y_l^1\chi_{1/2}^{-1/2} - \sqrt{\frac{l}{2l+1}}Y_l^0\chi_{1/2}^{+1/2}$$

Δ Resonance Formation - II

$$\frac{1}{2ikr}\sum_{l=0}^{1}\sqrt{4\pi(2l+1)}\left(e^{ikr}-(-1)^{l}e^{-ikr}\right)Y_{l}^{0}(\cos\theta)\chi_{1/2}^{+1/2}$$

$$=\frac{1}{2ikr}\sum_{l=0}^{1}\sqrt{4\pi(2l+1)}\left(e^{ikr}-(-1)^{l}e^{-ikr}\right)\left(\sqrt{\frac{l+1}{2l+1}}y_{l+1/2}^{+1/2}-\sqrt{\frac{l}{2l+1}}y_{l-1/2}^{+1/2}\right)$$

$$=\frac{1}{2ikr}\sum_{l=0}^{1}\sqrt{4\pi}\left(e^{ikr}-(-1)^{l}e^{-ikr}\right)\left(\sqrt{l+1}y_{l+1/2}^{+1/2}-\sqrt{l}\,y_{l-1/2}^{+1/2}\right)$$

Scattering amplitude: Phase shifts only modify *outgoing* spherical wave

$$f(\theta)=\frac{1}{2ik}\sum_{l=0}^{\infty}(2l+1)(a_{l}-1)P_{l}(\cos\theta)$$

$$\rightarrow f(\theta)=\frac{\sqrt{4\pi}}{2ik}\sum_{l=0}^{\infty}\left(\sqrt{l+1}y_{l+1/2}^{+1/2}\left(a_{l}^{+}-1\right)-\sqrt{l}\,y_{l-1/2}^{+1/2}\left(a_{l}^{-}-1\right)\right)$$

$$a_{l}^{\pm}=e^{2i\delta_{l}^{\pm}}-1$$

Δ Resonance Formation - III

Re-arrange scattering amplitude:

$$f(\theta)=\frac{1}{2ik}\sum_{l=0}^{1}\left[(l+1)\left(a_l^+-1\right)+l\left(a_l^--1\right)\right]P_l^0(\cos\theta)\chi_{1/2}^{+1/2}+\left(a_l^+-a_l^-\right)P_l^{+1}(\cos\theta)e^{i\varphi}\chi_{1/2}^{-1/2}$$

$$=\underbrace{\frac{1}{2ik}\sum_{l=0}^{1}\left[(l+1)\left(a_l^+-1\right)+l\left(a_l^--1\right)\right]P_l^0(\cos\theta)}_{g(\theta)}\chi_{1/2}^{+1/2}+\underbrace{\frac{1}{2ik}\sum_{l=0}^{1}\left(a_l^+-a_l^-\right)P_l^{+1}(\cos\theta)e^{i\varphi}}_{h(\theta)}\chi_{1/2}^{-1/2}$$

Spin <u>non-flip</u> amplitude — Spin <u>flip</u> amplitude

Differential cross-section:

$$\frac{d\sigma}{d\Omega}=\frac{1}{k^2}\left|g(\theta)\right|^2+\left|h(\theta)\right|^2$$ g,h spin eigenfunctions orthogonal

$P_0^0=1,\ P_1^0=\cos\theta,\ P_1^{+1}=-\sin\theta$ Associate Legendre functions

$$\frac{d\sigma}{d\Omega}=\frac{1}{k^2}\left|\left(a_0^+-1\right)+\left[2\left(a_1^+-1\right)+\left(a_1^--1\right)\right]\cos\theta\right|^2+\left|\left(a_1^+-a_1^-\right)(-\sin\theta)\right|^2$$

$$\rightarrow\frac{d\sigma}{d\Omega}=\frac{1}{k^2}\left(A_0+A_1\cos\theta+A_2\cos^2\theta\right),$$ A_0,A_1,A_2 Energy dependent coefficients

Δ Resonance Formation - IV

Around $\sqrt{s} \sim 1230$ MeV find $\frac{d\sigma}{d\Omega} = \frac{1}{k^2}\left(1+3\cos^2\theta\right)$

consistent with the decay of a *J=3/2* state

Indeed, taking for example $J_z = +1/2$:

$$\left|3/2,+1/2\right\rangle = \sqrt{\frac{1}{3}}\left|1/2,-1/2\right\rangle Y_1^{+1} + \sqrt{\frac{2}{3}}\left|1/2,+1/2\right\rangle Y_1^0$$

$$\frac{dN}{d\Omega} \propto \frac{1}{3}\left|Y_1^{+1}\right|^2 + \frac{2}{3}\left|Y_1^0\right|^2 = \frac{1}{3}\frac{1}{2}\sin^2\theta + \frac{2}{3}\cos^2\theta = \frac{1}{6} + \frac{3}{6}\cos^2\theta \propto 1+3\cos^2\theta$$

Width:

ΔE = Breit-Wigner full width at half maximum $\sim$ 100 MeV

$\Delta t \sim 1/\Delta E = 1/100\ MeV^{-1}$

$\rightarrow \Delta t = 10^{-2} \cdot \hbar c \cdot 1/c = 10^{-2}$ 197 MeV fm $\cdot$ $1/\left(3\times10^{23}\ \text{fm s}\right) \sim 0.7\ 10^{-23}$ s

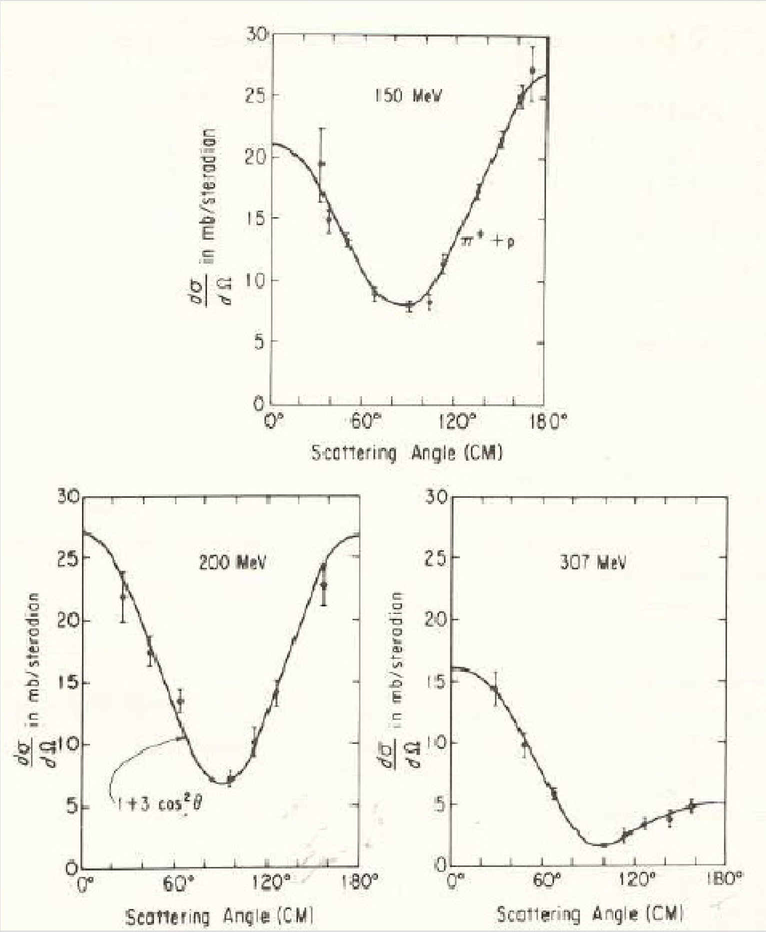
Parity $\eta_\Delta = \eta_p \eta_\pi \eta_{orb} = (+1)(-1)(-1)^{l=1} = +1$

DNA Markers : Δ Angular Distributions

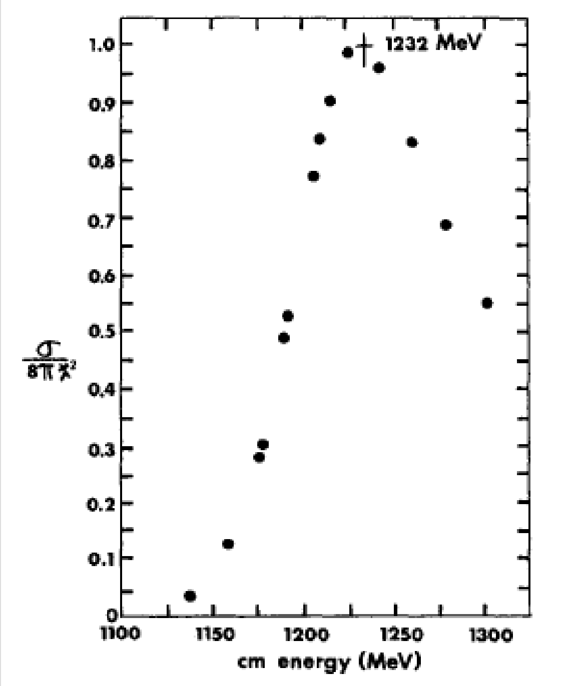
Experimental data nicely fit a simple picture where around T_π = *200* MeV
the dominant amplitude is *J=3/2,* namely:

The large peak observed in the total cross-section can be traced back to a resonant amplitude in the L=1, J=3/2 partial wave

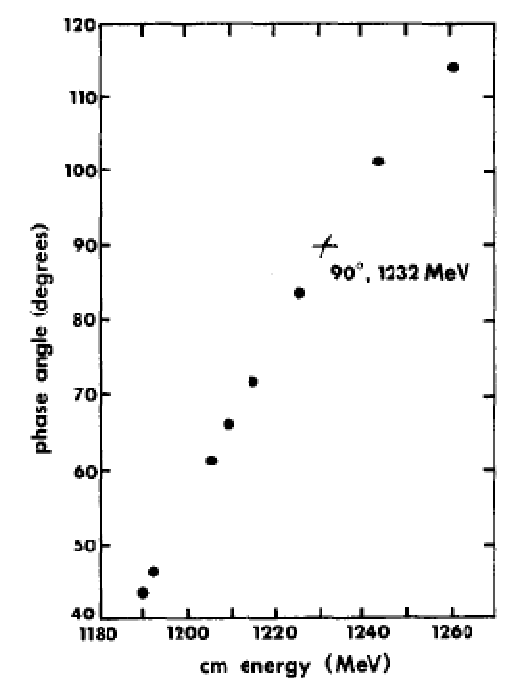
Several attempts to recover phase shifts from data in this energy range (Fermi, …):
Messy game, lots of ambiguities



Δ^{++} : More Fingerprints



Cross-section



Phase

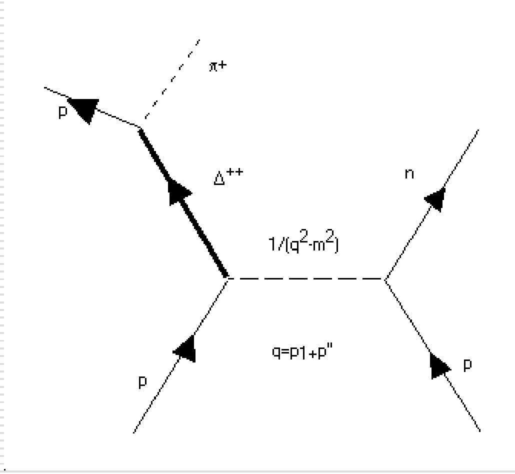
Production Resonances

With higher energy beams available, new processes become possible.
Use *virtual pions* to excite nucleon levels

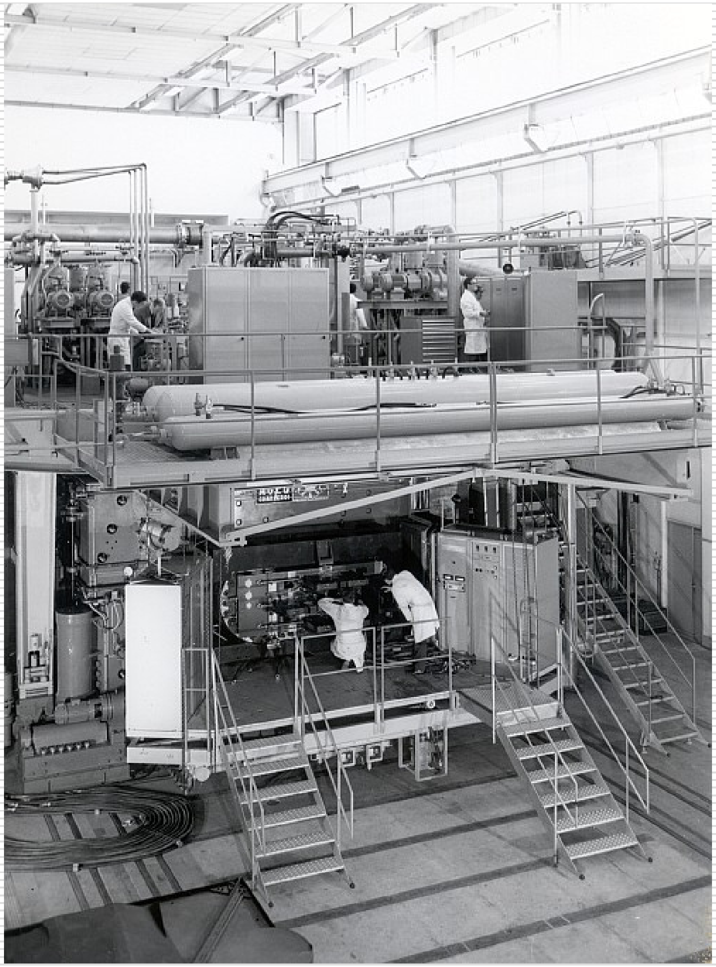
$$p + p \rightarrow n + \Delta^{++} \rightarrow n + p + \pi^{+}$$

Resonance is *produced* in the *t*-channel, rather than *formed* in the *s*-channel.
Not directly observed in the cross-section vs. energy plot.

But: Resonance mass and quantum numbers are *invariant* properties, like the corresponding quantities of a stable particle



The Bubble Chamber



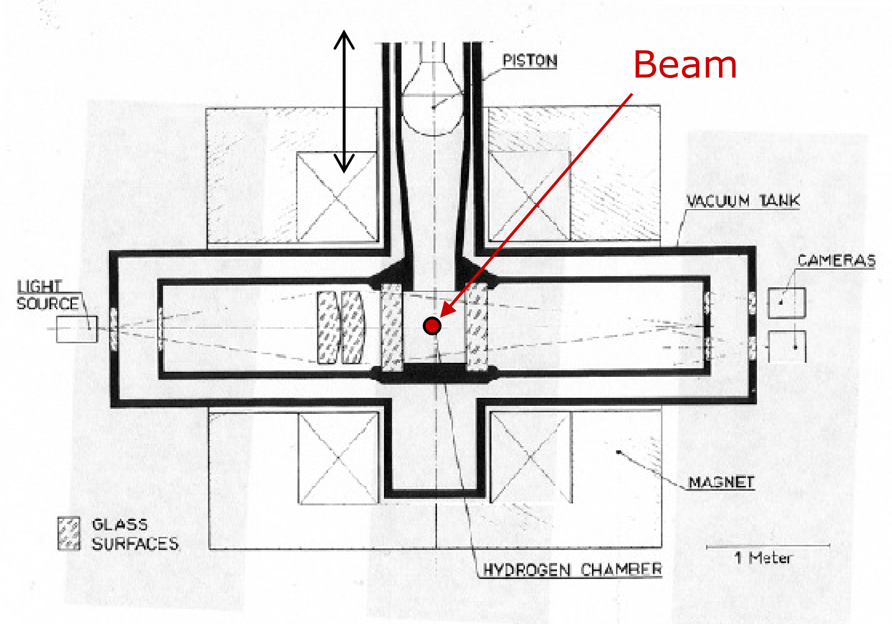
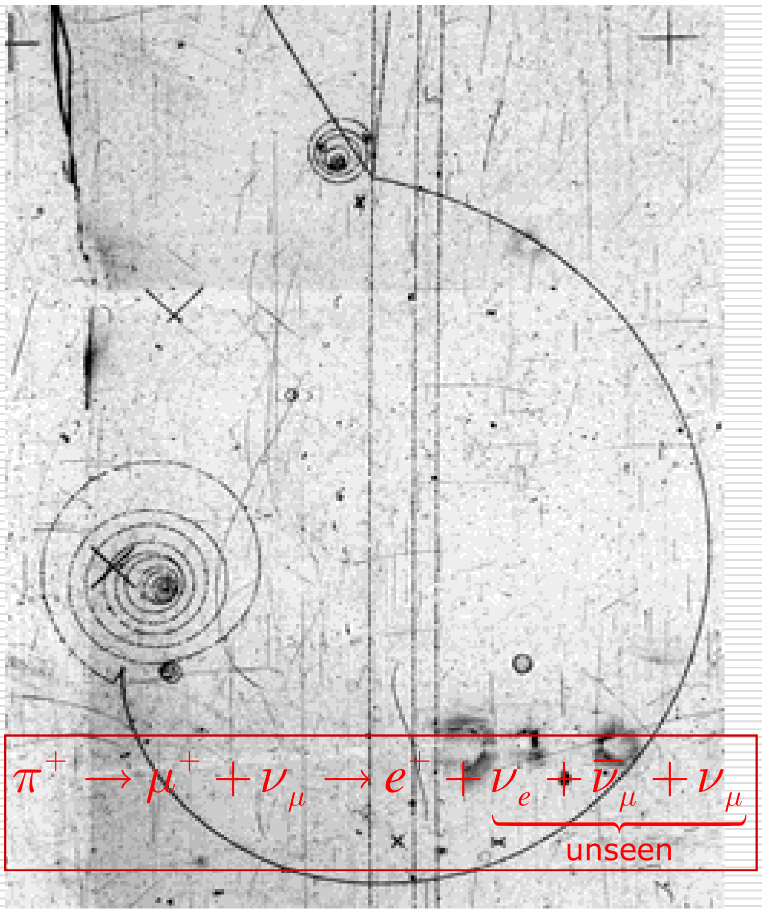
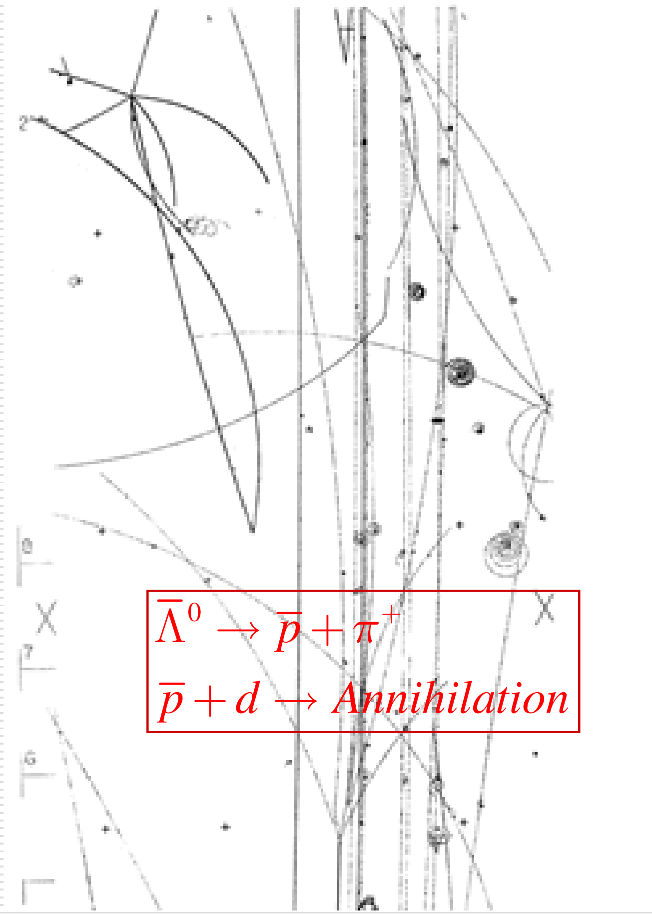


Fig. 2 Simplified sketch of a vertical section through the CERN 2 m bubble chamber. The beam enters from above, perpendicular to the plane of the figure. The liquid hydrogen is contained between the two thick, plane glass windows, the cameras are to the right and the flash lights to the left in the figure.

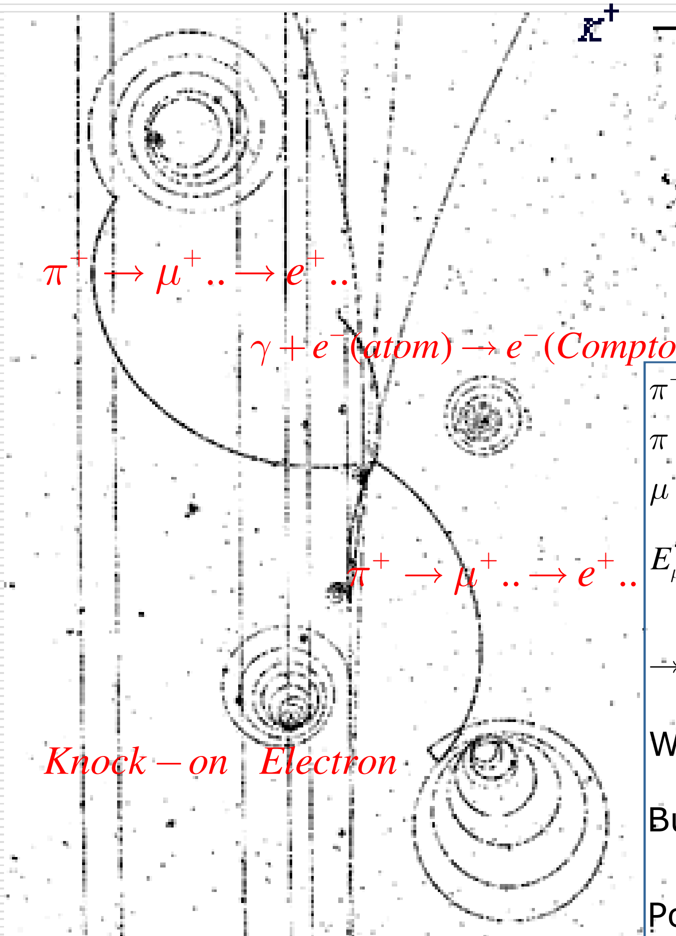
CERN 2m Bubble Chamber

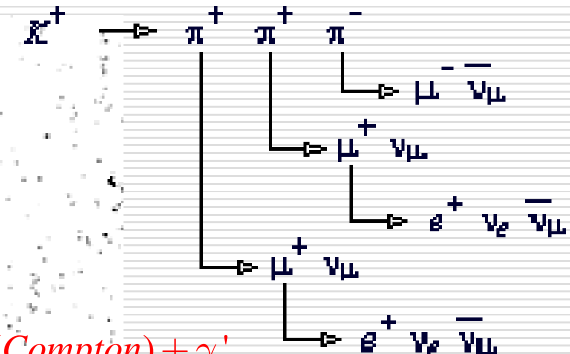
Bubble Chamber Events - I





Bubble Chamber Events - II





Knock-On electron

$Beam + Atomic\ Electron$

$\to Beam + \ Free\ Electron$

Usually modest energy

$\pi\mu\varepsilon$ kinematics

π^+ only: π^- is usually captured to a π-*mesic* atom

π decays after stopping: 'long' lifetime..

μ Energy, momentum:

$$E_\mu = \frac{1}{2m_\pi}\left(m_\pi^2 + m_\mu^2 - 0\right) \sim 109.9\ MeV \to p_\mu = \sqrt{109.9^2 - 106^2} \sim 29.1\ MeV$$

$\to \beta_\mu = \frac{p_\mu}{E_\mu} \sim \frac{29.1}{109.9} \sim 0.265, \gamma_\mu \sim 1.04$ when created

Would expect typical path length $\sim \beta_\mu \gamma_\mu c\tau_\mu \sim 182\ m$

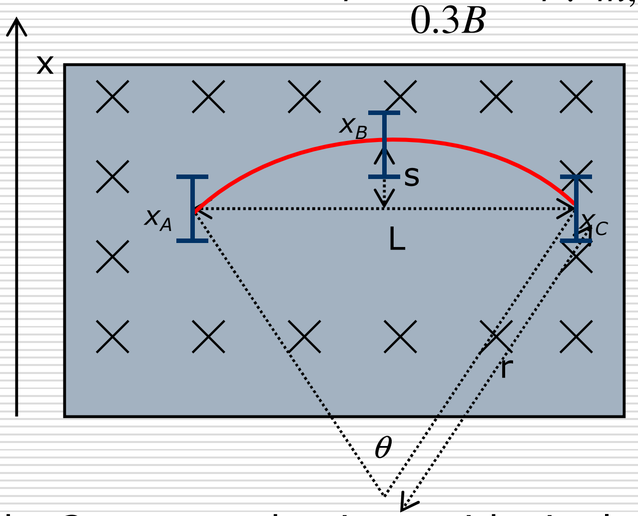
But: μ quickly slows down by $\frac{dE}{dx} \to$ Total path length $\sim$ few cm

Positron spiralling down: Energy loss by $\begin{cases} ionization \\ radiation \end{cases}$

Magnetic Analysis & Accuracy

Motion of a charged particle in a magnetic field: Cylindrical helix coaxial to ***B***

$$r = \frac{p_\perp}{0.3B} \qquad r:\ m,\ p_\perp:\ GeV,\quad B:\ T$$



Get p from s

$$\sin\frac{\theta}{2} = \frac{L}{2r} \underset{L \ll 2r}{\rightarrow} \frac{\theta}{2} \approx \frac{L}{2r} \rightarrow \theta \approx \frac{0.3BL}{p_\perp}$$

$$s = r - r\cos\frac{\theta}{2} \approx r\left[1 - \left(1 - \frac{\theta^2}{4}\right)\right] = r\frac{\theta^2}{8} \approx \frac{0.3BL^2}{8p_\perp}$$

$$\rightarrow p_\perp \approx \frac{0.3BL^2}{8s}$$

Take 3 measured points, with single point accuracy σ

Then:

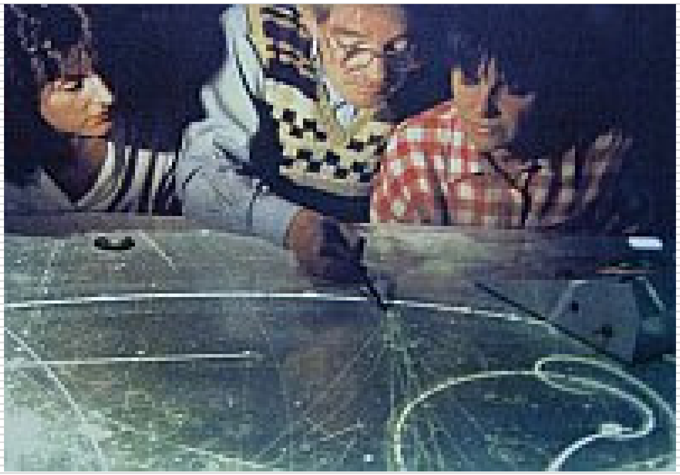
$$s = x_B - \frac{x_A + x_B}{2} \rightarrow \sigma_s^2 = \sigma^2 + \frac{1}{2}\sigma^2 = \frac{3}{2}\sigma^2$$

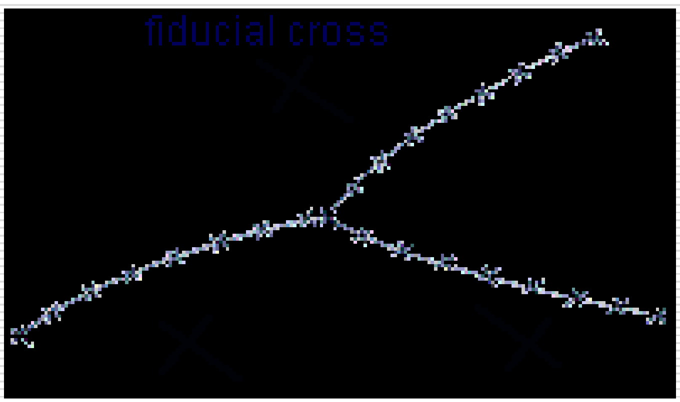
$$\frac{\sigma_{p_\perp}}{p_\perp} = \frac{\sigma_s}{s} = \sqrt{\frac{3}{2}}\frac{\sigma}{s} = \sqrt{\frac{3}{2}}\frac{\sigma 8 p_\perp}{0.3BL^2} = \sqrt{\frac{300\cdot 64}{18}}\frac{\sigma p_\perp}{BL^2} \approx 32.7\frac{\sigma p_\perp}{BL^2}$$

$N \geq 10$, uniformly spaced points:

$$\frac{\sigma_{p_\perp}}{p_\perp} \approx 28.3\frac{\sigma p_\perp}{BL^2\sqrt{N+4}}$$

Bubble Chamber Reconstruction





Particle	p_x	p_y	p_z	E
K-	8213.4	-248.3	15.2	8232
p	0	0	0	938.3
Sum	8213.4	-248.3	15.2	9170.3
K-	1481.8	27.8	224	1578.1
p-	149.7	-11.3	38.8	208.6
p+	37.9	-122.2	-22.7	190.7
p	1508.6	128.5	-70.5	1782.6
K0	3545.6	-162.9	-245	3592.4
Sum	6723.6	-140.1	-75.4	7352.4
Difference	-1489.8	108.2	-90.6	-1817.9

mass	1032.153

This mass doesn't correspond to a known particle - so there must be at least two neutral particles from the collision leaving the bubble chamber undetected.

Δ-Resonance: Production

Observe Δ^{++} resonance production as a peak in the invariant (p,π^+) mass distribution

Take reaction

$$\pi^+ p \to \pi^+ p \pi^+ \pi^-$$

$$m_{p\pi_1}{}^2 = \left(p_p + p_{\pi_1}\right)^2 = \left(E_p + E_{\pi_1}\right)^2 - \left(\mathbf{p}_p + \mathbf{p}_{\pi_1}\right)^2$$

$$m_{p\pi_2}{}^2 = \left(p_p + p_{\pi_2}\right)^2 = \left(E_p + E_{\pi_1}\right)^2 - \left(\mathbf{p}_p + \mathbf{p}_{\pi_2}\right)^2$$

2 entries per reconstructed event: Count everything

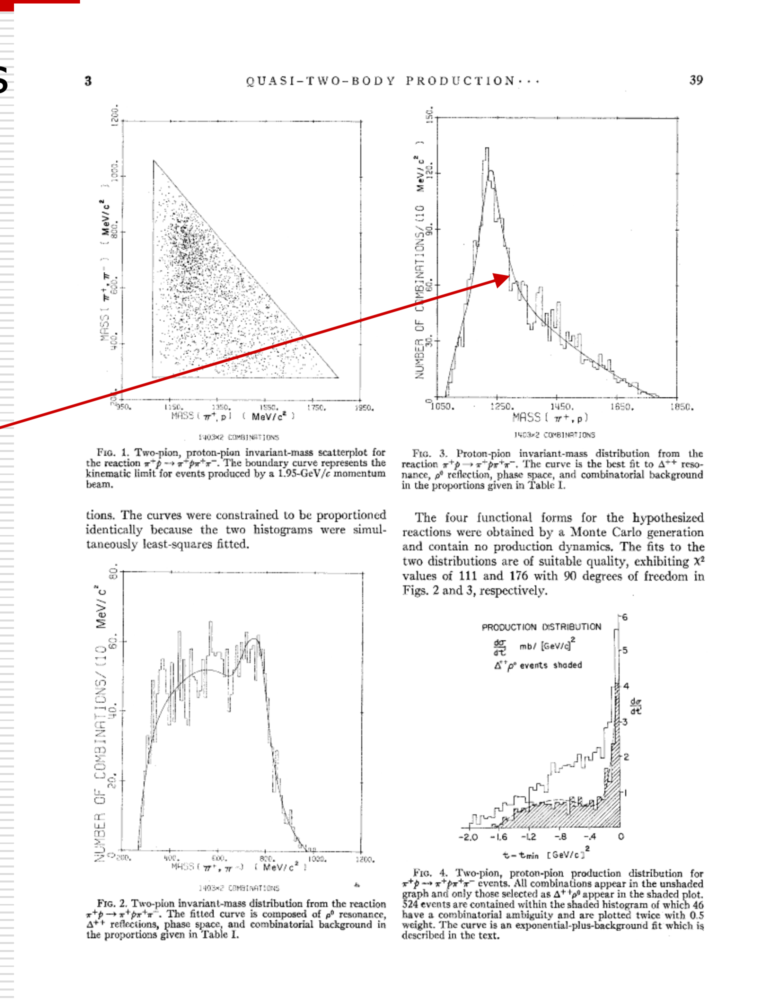


FIG. 1. Two-pion, proton-pion invariant-mass scatterplot for the reaction $\pi^+p \to \pi^+p\pi^+\pi^-$. The boundary curve represents the kinematic limit for events produced by a 1.95-GeV/c momentum beam.

FIG. 3. Proton-pion invariant-mass distribution from the reaction $\pi^+p \to \pi^+p\pi^+\pi^-$. The curve is the best fit to Δ^{++} resonance, ρ^0 reflection, phase space, and combinatorial background in the proportions given in Table I.

tions. The curves were constrained to be proportioned identically because the two histograms were simultaneously least-squares fitted.

The four functional forms for the hypothesized reactions were obtained by a Monte Carlo generation and contain no production dynamics. The fits to the two distributions are of suitable quality, exhibiting χ^2 values of 111 and 176 with 90 degrees of freedom in Figs. 2 and 3, respectively.

FIG. 2. Two-pion invariant-mass distribution from the reaction $\pi^+p \to \pi^+p\pi^+\pi^-$. The fitted curve is composed of ρ^0 resonance, Δ^{++} reflections, phase space, and combinatorial background in the proportions given in Table I.

FIG. 4. Two-pion, proton-pion production distribution for $\pi^+p \to \pi^+p\pi^+\pi^-$ events. All combinations appear in the unshaded graph and only those selected as $\Delta^{++}\rho^0$ appear in the shaded plot. 524 events are contained within the shaded histogram of which 46 have a combinatorial ambiguity and are plotted twice with 0.5 weight. The curve is an exponential-plus-background fit which is described in the text.

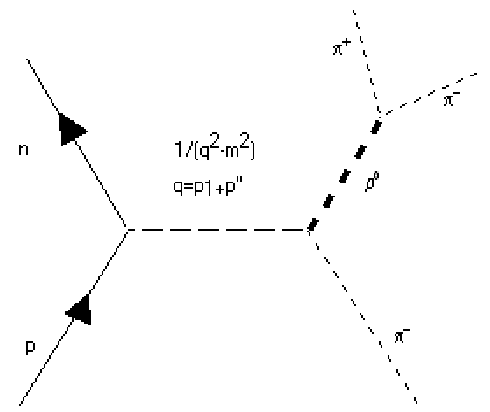
Meson Resonances - I

Expect resonant behavior also for mesonic systems, e.g. $\pi\pi$:
Virtual and real pion coupled at the strong vertex

Observation of meson resonances possible in production experiments

Remark:

Taking baryon resonances only, possible isospin:
Minimum coupling is between nucleon and pion
$\rightarrow$Expect $1 \oplus 1/2 = 1/2, 3/2$ as observed
Take meson resonances:
Minimum coupling is between pion and pion
$\rightarrow$Expect $1 \oplus 1 = 0,1,2$ I=2 mesons not observed



Meson Resonances - II

Take reaction

$$\pi^- + p \rightarrow n + \pi^+ + \pi^-$$

Observe strong enhancements for

$$m_{\pi\pi} \sim 760,\ 1260,\ 1550 \quad MeV$$
$$m_{\pi n} \sim 1230 - 1550 \quad MeV$$

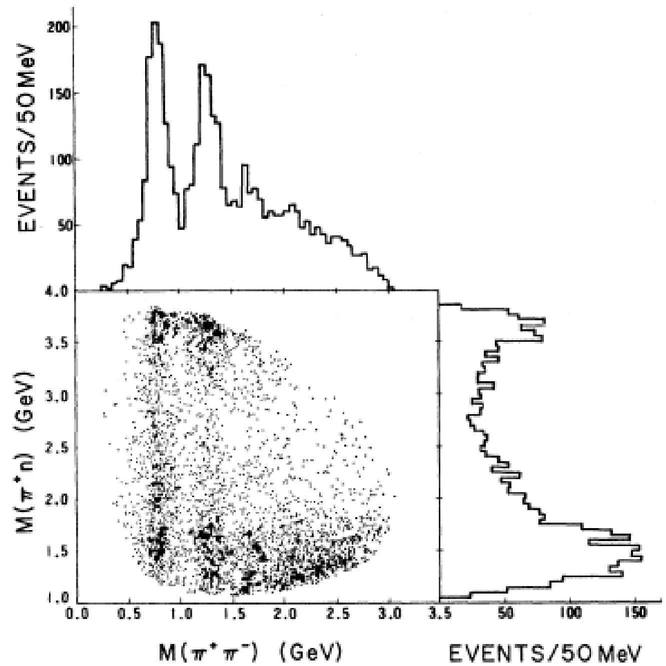


FIG. 2. Scatter plot of $M(\pi^+\pi^-)$ versus $M(\pi^+ n)$ with the projections on both axes.

Interpretation:

Meson Baryon Resonances

$$\left.\begin{matrix}\rho(760)\\ f_0(1250)\\ g(1550)\end{matrix}\right\} \rightarrow \pi^\pm\pi^\mp, \quad \Delta^{+,-}(1232) \rightarrow n\pi^\pm$$

Spin-parity of the ρ Meson - I

Use angular distributions to investigate ρ spin, parity

$$S_\pi = 0 \rightarrow J_\rho = L_{\pi\pi} \equiv L$$

$$\rightarrow \psi_{final} \propto Y_l^m(\theta, \varphi)$$

$$\eta_P^{(\rho)} = \eta_P^{(\pi)} \eta_P^{(\pi)} (-1)^l = (-1)^l$$

Suppose the produced ρ mesons uniformly populate the *2l+1* J_3 substates: Then, by a property of spherical harmonics

$$\frac{dP}{d\Omega} = \frac{1}{2J+1} \sum_{m=-l}^{+l} Y_l^m(\theta,\varphi) Y_l^{*m}(\theta,\varphi); \quad \sum_{m=-l}^{+l} Y_l^m Y_l^{*m} = \frac{2l+1}{4\pi}$$

$$\rightarrow \frac{dP}{d\Omega} = \frac{1}{2J+1} \frac{2J+1}{4\pi} = \frac{1}{4\pi}$$ Uniform distribution

So a non-uniform angular distribution indicates some *polarization* of the decaying state, useful to perform spin-parity analysis

Spin-parity of the ρ Meson - II

Observe CM angular distribution for different $\pi\pi$ mass 'slices'

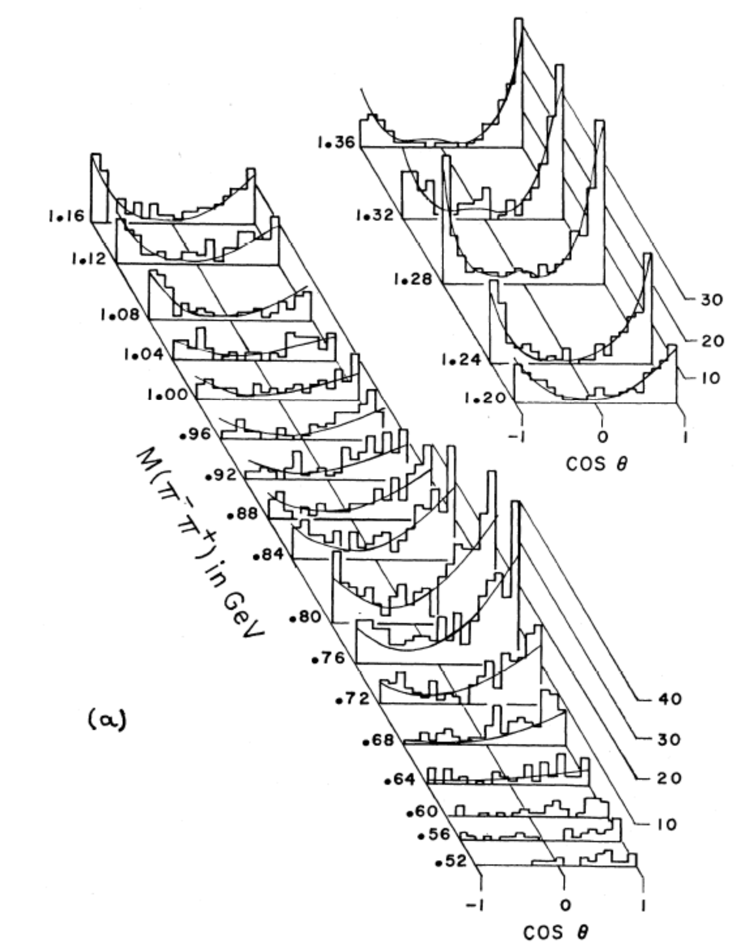
In the ρ resonance mass region (about 700-800 MeV)

$$\frac{dP}{d\Omega} \propto \cos^2\theta \propto \left|Y_1^0\left(\cos\theta\right)\right|^2 \rightarrow l = 1$$

$\rightarrow$ The ρ is a *vector* particle

Interestingly, in the f_0 mass region (about 1250-1350 MeV) observe some indication of spin 2

$$\frac{dP}{d\Omega} \propto \left(3\cos^2\theta - 1\right)^2 \propto \left|Y_2^0\right|^2 \rightarrow l = 2$$



Isospin - I

Charge independence leads to a new classification scheme:
All hadrons cast into *isospin multiplets*
Strong interaction identical for all members of each multiplet

$$\left.\begin{array}{ll}\text{proton} & p\\ \text{neutron} & n\end{array}\right\}\ 2\ \text{ states of the } \textit{nucleon} \quad N=\begin{pmatrix}\alpha\\ \beta\end{pmatrix} \quad 2 \text{ states system - isospinor}$$

$$\text{Base } \begin{pmatrix}1\\0\end{pmatrix}\equiv p,\quad \begin{pmatrix}0\\1\end{pmatrix}\equiv n \qquad \text{Base states: } \textit{doublet}$$

$$\left.\begin{array}{l}\pi^+\\ \pi^0\\ \pi^-\end{array}\right\}\ 3\ \text{ states of the } \textit{pion} \quad \pi=\begin{pmatrix}\alpha\\ \beta\\ \gamma\end{pmatrix} \quad 3 \text{ state system - isovector}$$

$$\text{Base } \begin{pmatrix}1\\0\\0\end{pmatrix}\equiv \pi^+,\begin{pmatrix}0\\1\\0\end{pmatrix}\equiv \pi^0,\begin{pmatrix}0\\0\\1\end{pmatrix}\equiv \pi^- \qquad \text{Base states: } \textit{triplet}$$

Isospin - II

Isospins add up as angular momenta (Astonished? More on this later…)

For πN system obtain:

$$\left.\begin{array}{l}\pi : I = 1 \\ N : I = 1/2\end{array}\right\} \rightarrow \pi N : I = 1 \oplus 1/2 = \begin{cases} 1/2 & \text{doublet} \\ 3/2 & \text{quadruplet} \end{cases}$$

By using Clebsch-Gordan coefficients, expand physical (particle) states into total isospin eigenstates

Single particle: Base states

$$I_N = 1/2 \;\; ; \;\; \left|\mathrm{p}\right\rangle = \left|1/2, +1/2\right\rangle \;\; , \;\; \left|\mathrm{n}\right\rangle = \left|1/2, -1/2\right\rangle$$

$$I_\pi = 1 \;\; ; \;\; \left|\pi^+\right\rangle = \left|1, +1\right\rangle \;\; , \;\; \left|\pi^0\right\rangle = \left|1, 0\right\rangle \;\; , \;\; \left|\pi^-\right\rangle = \left|1, -1\right\rangle$$

Isospin - III

Expand physical, 2 particle states into total isospin eigenstates:

$$\left|\pi^- p\right\rangle = \left|1,-1,1/2,+1/2\right\rangle = \sqrt{\frac{1}{3}}\left|3/2,-1/2\right\rangle - \sqrt{\frac{2}{3}}\left|1/2,-1/2\right\rangle$$

$$\left|\pi^+ n\right\rangle = \left|1,+1,1/2,-1/2\right\rangle = \sqrt{\frac{1}{3}}\left|3/2,+1/2\right\rangle + \sqrt{\frac{2}{3}}\left|1/2,+1/2\right\rangle$$

$$\left|\pi^+ p\right\rangle = \left|1,+1,1/2,+1/2\right\rangle = \left|3/2,+3/2\right\rangle$$

$$\left|\pi^- n\right\rangle = \left|1,-1,1/2,-1/2\right\rangle = \left|3/2,-3/2\right\rangle$$

$$\left|\pi^0 p\right\rangle = \left|1,0,1/2,+1/2\right\rangle = \sqrt{\frac{2}{3}}\left|3/2,+1/2\right\rangle - \sqrt{\frac{1}{3}}\left|1/2,+1/2\right\rangle$$

$$\left|\pi^0 n\right\rangle = \left|1,0,1/2,-1/2\right\rangle = \sqrt{\frac{2}{3}}\left|3/2,-1/2\right\rangle + \sqrt{\frac{1}{3}}\left|1/2,-1/2\right\rangle$$

Isospin - IV

Guess isospin is a new *symmetry* for hadrons: connect to some *invariance* property (like angular momentum).
Non-trivial conservation rule follows:

Total isospin conserved by all strong processes

Interesting predictions for πN scattering and reactions:

$$\begin{cases}(A)\,\pi^+ p \to \pi^+ p \\ (B)\,\pi^- n \to \pi^- n\end{cases} \to A_A = A_B = A_{3/2}$$ pure I = 3/2

$$\begin{cases}(A)\,\pi^+ n \to \pi^+ n \\ (B)\,\pi^- n \to \pi^- n\end{cases} \to A_A = \frac{1}{3}A_{3/2} + \frac{2}{3}A_{1/2},\ A_B = A_{3/2}$$

$$\begin{cases}(A)\,\pi^+ p \to \pi^+ p \\ (B)\,\pi^- p \to \pi^- p\end{cases} \to A_A = A_{3/2},\ A_B = \frac{1}{3}A_{3/2} - \frac{2}{3}A_{1/2}$$

$$\begin{cases}(A)\,\pi^+ p \to \pi^+ p \\ (B)\,\pi^- p \to \pi^0 n\end{cases} \to A_A = A_{3/2},\ A_B = \sqrt{\frac{2}{9}}A_{3/2} - \sqrt{\frac{2}{9}}A_{1/2}$$

Isospin - V

If $A_{3/2} >> A_{1/2}$

$$\begin{cases} (A)\,\pi^+ p \to \pi^+ p \\ (B)\,\pi^- n \to \pi^- n \end{cases} \to \sigma_A = \sigma_B$$

$$\begin{cases} (A)\,\pi^+ n \to \pi^+ n \\ (B)\,\pi^- n \to \pi^- n \end{cases} \to \sigma_A \simeq \frac{1}{9}\sigma_B$$

$$\begin{cases} (A)\,\pi^+ p \to \pi^+ p \\ (B)\,\pi^- p \to \pi^- p \end{cases} \to \sigma_A \simeq 9\sigma_B$$

$$\begin{cases} (A)\,\pi^+ p \to \pi^+ p \\ (B)\,\pi^- p \to \pi^0 n \end{cases} \to \sigma_A \simeq \frac{9}{2}\sigma_B$$

Still lacking: *What exactly is isospin?*

What is Spin? - I

For any physical system with *m>0*, we are allowed to choose CM as a reference frame.

When the system is rotationally invariant, states are observed to group into multiplets of size *n*, *n=1,2,3,… (size n = number of states)*

States of a multiplet: *Same energy*

States belonging to different multiplets must be distinguished by some internal quantum number: Provisionally call the corresponding observable the particle <u>*spin*</u>

States of any given multiplet must be identified by some internal quantum number: Provisionally call the corresponding observable the <u>*3rd component of the particle spin*</u>

What is Spin? - II

Question: *What is the observable we have called spin?*

Answer: *Get some insight from conservation laws*.

Discover spin is just another kind of (non-orbital) angular momentum

$\mathbf{J} = \mathbf{L} + \mathbf{S}$ Total angular momentum

For any system: Extend to $\boldsymbol{S}$ known properties of $\boldsymbol{L}$

- (S_x, S_y, S_z) analogue to (L_x, L_y, L_z): Hermitian operators, infinitesimal generators of rotations around x, y, z
- Commutators: $\left[S_x, S_y\right] = iS_z$ + Cyclical permutations
- By assuming rotational invariance, in the CM H and $\boldsymbol{S}^2$ commute $\rightarrow \boldsymbol{S}^2, S_3$ are conserved

What is Spin? - III

- Besides other quantum numbers, in the CM reference frame all possible stationary states are then labeled by S^2, S_3 according to angular momentum algebra:

S^2 Eigenvalues: $s(s+1)$, $s = 0, 1/2, 1, 3/2, 2, ...$

Sequence of multiplets

S_3 Eigenvalues: $\underbrace{-s ... + s}_{2j+1}$, $2s+1 \equiv$ Multiplet size $1, 2, 3, 4, 5, ..$

- Each multiplet understood to realize an *irreducible representation* of some (unknown) symmetry group in the Hilbert space

NB Multiplets with even multiplicity *are* observed $\rightarrow 2j+1 = 2, 4, \ldots$
Implies j can be *integer* or *half-integer*

What is Spin? - IV

Representation:

A set of matrices acting on some kind of 'vectors', labeled by the integer $\mathbf{2s+1}$

→*Must have* 3 *independent matrices* $(= S_x, S_y, S_z)$ *for each rep.*
→*Must have* $2j+1$ *independent 'vectors' (= base states) for each rep*

Size of matrices: $(2s+1) \; x \; (2s+1)$

Each matrix correspond to a *specific rotation*

→*Must depend on* 3 *parameters (= rotation angles)*

What is Spin? - V

Integer s: Like l

- $\boldsymbol{L}$ eigenvalues are integer only $0,1,2,\ldots \rightarrow 2l+1 = 1,3,5,..$ odd integer
- l identifies an *irreducible representation* of the rotation group $SO(3)$
- (L_x,L_y,L_z): 3 matrices of size $1x1$, $3x3$, $5x5$, … operating on different objects of size $1, 3, 5, \ldots$: *Spherical Tensors* (e.g. Spherical Harmonics)

Half-integer s: Minimum size is for $s=1/2 \rightarrow 2 \times 2$

- 2-component 'vectors' acted upon by *2 x 2* matrices called *spinors*

Not really like ordinary vectors..

From the algebraic properties of ***S***:
Spin symmetry group must be a close relative of $SO(3)$
Just including extra values for s as compared to l

Matrix Fun - I

Take $j=1/2$:
Must represent rotations of 2-component spinors by $2x2$ matrices

1) Naive attempt: Try with orthogonal matrices

$M \equiv \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ orthogonal

$\rightarrow MM^T = 1$

$$\rightarrow \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} a & c \\ b & d \end{pmatrix} = \begin{pmatrix} a^2+b^2 & ac+bd \\ ac+bd & c^2+d^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\rightarrow \begin{cases} a^2+b^2=1 \\ c^2+d^2=1 \quad \& \ a,b,c,d \text{ real} \\ ac+bd=0 \end{cases}$$

$\rightarrow$ 1 free parameter

$\rightarrow$ KO to represent a 3D rotation

Matrix Fun - II

2) Better approach: Unitary matrices

$$U \equiv \begin{pmatrix} a & b \\ c & d \end{pmatrix} \text{ unitary} \rightarrow UU^{\dagger} = 1 \rightarrow \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} \bar{a} & \bar{c} \\ \bar{b} & \bar{d} \end{pmatrix} = \begin{pmatrix} a\bar{a} + b\bar{b} & a\bar{c} + b\bar{d} \\ c\bar{a} + d\bar{b} & c\bar{c} + d\bar{d} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\rightarrow \begin{cases} a\bar{a} + b\bar{b} = 1 \\ c\bar{c} + d\bar{d} = 1 \;\; \& \; a,b,c,d \text{ complex} \rightarrow 4 \text{ free parameters} \\ a\bar{c} + b\bar{d} = 0 \\ c\bar{a} + d\bar{b} = 0 \end{cases}$$

Require extra condition:

$\det M = 1 \rightarrow ad - bc = 1 \rightarrow$ 3 free parameters

$\rightarrow$ OK to represent a 3D rotation

Possible because absolute phase of states is irrelevant

Matrix Fun - III

Set of all $2x2$ matrices satisfying the 4 conditions above:

A group, called the *Special Unitary group of dimension 2*, or $SU(2)$.

$SU(2)$ *vs* $SO(3)$:

3 parameters → 3 generators

Commutators identical→They share the same *algebra*

The moral:

$O(3)$ and $SU(2)$ are *more or less* the same group

→ All the irr.reps of $SO(3)$ are also good for $SU(2)$

SU(2) - I

Instead of starting from rotations, just start from *SU(2)* defined as the set of all the *2x2*, unitary matrices (with *det=1)*

Not bound to understand this transformation of states as induced by a rotation of axis in the physical, 3D space.

Free to interpret any SU(2) matrix as representing a unitary, unimodular transformation in the Hilbert space of any two-state, degenerate system.

Do not need to specify what is the physical system whose two independent states we take as base vectors in the Hilbert space.

SU(2) - II

Some matrix fun:

4 complex parameters $\rightarrow 8$ real parameters

4 unitarity conditions: $\left.\begin{array}{l} UU^{\dagger} = 1 \\ \left(U^{\dagger}\right)_{ij} = U^{*}_{ji} \end{array}\right\} \rightarrow \sum_{j=1}^{2} a_{ij} a^{*}_{jk} = \delta_{ik}, \quad i,k = 1,2$

1 unimodularity condition: $\det U = 1$

$\rightarrow 8 - 5 = 3$ free parameters

One diagonal generator, σ_3

$\rightarrow$*Rank 1 group*

$\rightarrow$ *One* invariant function of generators
Quadratic: $\boldsymbol{\sigma}^2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2$

SU(2) - III

Some insight into *SU(2)* generators:

U unitary $\rightarrow U = e^{iH}, H$ Hermitian

$\det U = 1 \rightarrow \det e^{iH} = 1 \rightarrow e^{itr(H)} = 1 \rightarrow tr(H) = 0$

3 free parameters $\rightarrow$ 3 generators

3 Hermitian, traceless 2×2 matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Any *SU(2)* matrix can be written as a linear combination of the 3 generators, the *Pauli matrices*

What is Isospin? - I

When looking at strongly interacting particles, observe particle states similarly grouping themselves into multiplets of size *1,2,3,4*

States of a multiplet ≅ Same mass

→States belonging to different multiplets must be distinguished by some internal quantum number: By analogy, call the corresponding observable the particle *isospin*

→States of any given multiplet must be identified by some *internal* quantum number: Call the corresponding observable the *3rd component of the particle isospin*

What is Isospin? - II

Notice: Isospin symmetry is not exact (broken), still is quite good
Indeed, looking at symmetry breaking mass splittings:

$$\frac{m_n - m_p}{m_n} \simeq \frac{939.57 - 938.27}{939.57} \simeq 0.0014 \quad \text{Nucleon doublet}$$

$$\frac{m_{\pi^\pm} - m_{\pi^0}}{m_{\pi^\pm}} \simeq \frac{139.6 - 135.0}{139.6} \simeq 0.011 \quad \text{Pion triplet}$$

For a long time: Breaking entirely blamed on electromagnetic effects, which is only partially true (e.g. neutral and charged members indeed have quite different e.m. interactions contributing to their mass).

Today: Isospin taken as an 'accidental' symmetry, not due to some fundamental property of hadron constituents or strong interaction

What is Isospin? - III

Question: What is the observable we have called *isospin*?

Answer: *There is no classical analogy!*

Simply, as we observe the neutron and proton to be almost degenerate in mass, we can state they are just two states of the same physical system, the *nucleon.*

We guess the two nucleon states are the ‘vectors’ spanning the fundamental representation of a symmetry group, which we identify with $SU(2)$.

What is Isospin? - IV

Guess: $SU(2)$ is a symmetry of all the strongly interacting particles.

Therefore:

All strong interacting particles should fill some $SU(2)$ representation

This is actually true, after neglecting small symmetry breaking effects within each multiplet (see later)

As for any other symmetry, expect the invariance property to yield a conservation law

What is Isospin? - V

What is conserved in this case?
Since there is no classical analogy, stick to our algebraic skills to get insight

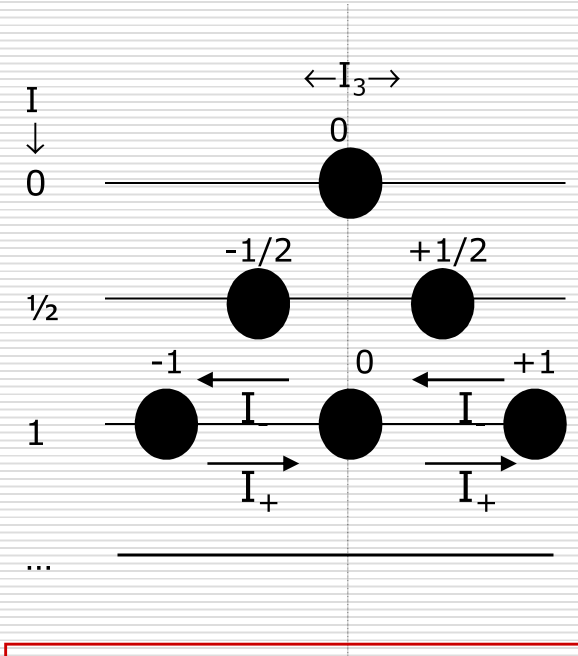
SU(2) algebra is just the same as *O(3)*, so we can expect the same conserved observables for a closed system of strongly interacting particles:

$$\boldsymbol{J}^2, J_3 \leftrightarrow \boldsymbol{I}^2, I_3$$

This is the origin of the common wisdom '*Isospin is like Angular Momentum'*

SU(2) Multiplet Graphics

Within any given *SU(2)* multiplet, states can be represented as points on a straight line



Reason is the group structure of *SU(2):*

3 parameters $\rightarrow$ *3 generators*

Just 1 invariant function of generators:
I$^2 \rightarrow$ *Multiplets identified just by I*

Generators do not commute with each other
$\rightarrow$ *States in any multiplet identified just by* I_3

Observe:
I_3 eigenvalues *symmetric* wrt *0*

Define 2 *ladder operators*: $I_\pm = I_1 \pm i I_2$

Action: Shift states right or left on the multiplet line, i.e. increment/ decrement I_3 by *1*

Conjugate Representation - I

More fun with matrices…

D : Any representation

$\psi^{'} = D(\alpha)\psi$

$\rightarrow D(\alpha) = e^{i\alpha F}$, F hermitian $\leftarrow$ True because D is unitary

Take complex conjugate of equations

$\psi^{'*} = D^{*}\psi^{*}$

Get another representation

$D^{*} = e^{-i\alpha(F)^{*}} = e^{i\alpha\left[-(F)^{*}\right]} \equiv e^{i\alpha\tilde{F}}$

Relation bewteen new and old generators

$\rightarrow \tilde{F} = -\left(F^{*}\right)$

Conjugate Representation - II

Take D of $SU(2)$ fundamental representation:

F Hermitian $\rightarrow \tilde{F}$ Hermitian

$\rightarrow$ Real eigenvalues for both $F, \tilde{F}$, and $f_i = -f_i^*$

$\rightarrow$ Since f_i are symmetric wrt 0, so are f_i^*

$\rightarrow \{f_i\} \equiv \{f_i^*\}$ $\tilde{F}$ eigenvalues are just a re-labeling of F's

Direct and conjugate representations are said to be *equivalent*

True for $SU(2)$, *generally false*

Product of Representations - I

Take a system made of 2 nucleons: *What is the total isospin?*
$SU(2)$ is equivalent to $O(3) \rightarrow$ *Can use Clebsch-Gordan coefficients*

But: Can also re-formulate the problem in a different way
Each nucleon spans the fundamental representation of *SU(2)*, ***2***

Then a 2 nucleon system span the *direct product rep.* $\mathbf{2} \otimes \mathbf{2}$

Question:

What are the irreducible representations of SU(2) contained in any state of 2 nucleons?

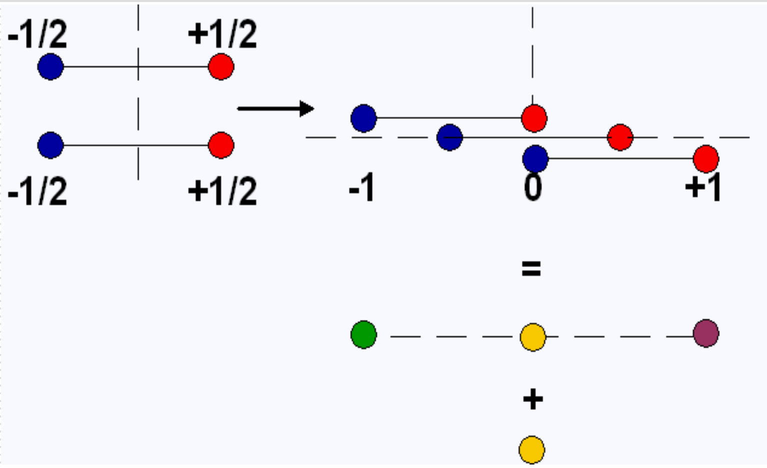
Need to decompose $\mathbf{2} \otimes \mathbf{2}$ into a *direct sum* of irr.rep.

Product of Representations - II

Answer (After a little group theory):

$$\mathbf{2} \otimes \mathbf{2} = \mathbf{1} \oplus \mathbf{3}$$

Answer (Graphical):
*Center the segment carrying the 2 states of representation **2** (1st nucleon) over the 2 states of representation **2** (2nd nucleon)*
$\rightarrow$ *Get a set of 4 states, decomposing into 2 sets of 1 and 3 states*



I-Spin Multiplets: The Nonstrange Zoo

Amazingly *large* number of resonant states

p, n	P_{11}	****	$\Delta(1232)$	P_{33}	****
$N(1440)$	P_{11}	****	$\Delta(1600)$	P_{33}	***
$N(1520)$	D_{13}	****	$\Delta(1620)$	S_{31}	****
$N(1535)$	S_{11}	****	$\Delta(1700)$	D_{33}	****
$N(1650)$	S_{11}	****	$\Delta(1750)$	P_{31}	*
$N(1675)$	D_{15}	****	$\Delta(1900)$	S_{31}	**
$N(1680)$	F_{15}	****	$\Delta(1905)$	F_{35}	****
$N(1700)$	D_{13}	***	$\Delta(1910)$	P_{31}	****
$N(1710)$	P_{11}	***	$\Delta(1920)$	P_{33}	***
$N(1720)$	P_{13}	****	$\Delta(1930)$	D_{35}	***
$N(1900)$	P_{13}	**	$\Delta(1940)$	D_{33}	*
$N(1990)$	F_{17}	**	$\Delta(1950)$	F_{37}	****
$N(2000)$	F_{15}	**	$\Delta(2000)$	F_{35}	**
$N(2080)$	D_{13}	**	$\Delta(2150)$	S_{31}	*
$N(2090)$	S_{11}	*	$\Delta(2200)$	G_{37}	*
$N(2100)$	P_{11}	*	$\Delta(2300)$	H_{39}	**
$N(2190)$	G_{17}	****	$\Delta(2350)$	D_{35}	*
$N(2200)$	D_{15}	**	$\Delta(2390)$	F_{37}	*
$N(2220)$	H_{19}	****	$\Delta(2400)$	G_{39}	**
$N(2250)$	G_{19}	****	$\Delta(2420)$	$H_{3,11}$	****
$N(2600)$	$I_{1,11}$	***	$\Delta(2750)$	$I_{3,13}$	**
$N(2700)$	$K_{1,13}$	**	$\Delta(2950)$	$K_{3,15}$	**

$$L_{2J+1,2I+1} \qquad L = S, P, D, \ldots$$

Baryons

I=1/2 *I=3/2*

LIGHT UNFLAVORED ($S = C = B = 0$)

	$I^G(J^{PC})$
• $\pi^{\pm}$	$1^-(0^-)$
• π^0	$1^-(0^{-+})$
• η	$0^+(0^{-+})$
• $f_0(400\text{–}1200)$	$0^+(0^{++})$
• $\rho(770)$	$1^+(1^{--})$
• $\omega(782)$	$0^-(1^{--})$
• $\eta'(958)$	$0^+(0^{-+})$
• $f_0(980)$	$0^+(0^{++})$
• $a_0(980)$	$1^-(0^{++})$
• $\phi(1020)$	$0^-(1^{--})$
• $h_1(1170)$	$0^-(1^{+-})$
• $b_1(1235)$	$1^+(1^{+-})$
• $a_1(1260)$	$1^-(1^{++})$
• $f_2(1270)$	$0^+(2^{++})$
• $f_1(1285)$	$0^+(1^{++})$
• $\eta(1295)$	$0^+(0^{-+})$
• $\pi(1300)$	$1^-(0^{-+})$
• $a_2(1320)$	$1^-(2^{++})$
• $f_0(1370)$	$0^+(0^{++})$
$h_1(1380)$	$?^-(1^{+-})$
$\pi_1(1400)$	$1^-(1^{-+})$
• $f_1(1420)$	$0^+(1^{++})$
• $\omega(1420)$	$0^-(1^{--})$
$f_2(1430)$	$0^+(2^{++})$
• $\eta(1440)$	$0^+(0^{-+})$
• $a_0(1450)$	$1^-(0^{++})$
• $\rho(1450)$	$1^+(1^{--})$
• $f_0(1500)$	$0^+(0^{++})$
$f_1(1510)$	$0^+(1^{++})$
• $f'_2(1525)$	$0^+(2^{++})$
$f_2(1565)$	$0^+(2^{++})$
$\pi_1(1600)$	$1^-(1^{-+})$
$X(1600)$	$2^+(2^{++})$
$a_1(1640)$	$1^+(1^{++})$
$f_2(1640)$	$0^+(2^{++})$
$\eta_2(1645)$	$0^+(2^{-+})$
• $\omega(1650)$	$0^-(1^{--})$
$X(1650)$	$0^-(?^{?-})$
$a_2(1660)$	$1^-(2^{++})$
• $\omega_3(1670)$	$0^-(3^{--})$
• $\pi_2(1670)$	$1^-(2^{-+})$
• $\phi(1680)$	$0^-(1^{--})$
• $\rho_3(1690)$	$1^+(3^{--})$
• $\rho(1700)$	$1^+(1^{--})$
• $f_0(1710)$	$0^+(0^{++})$
$a_2(1750)$	$1^-(2^{++})$
$\eta(1760)$	$0^+(0^{-+})$
$X(1775)$	$1^-(?^{-+})$
• $\pi(1800)$	$1^-(0^{-+})$
$f_2(1810)$	$0^+(2^{++})$
• $\phi_3(1850)$	$0^-(3^{--})$
$\eta_2(1870)$	$0^+(2^{-+})$
$X(1910)$	$0^+(?^{?+})$
$f_2(1950)$	$0^+(2^{++})$
$X(2000)$	$1^-(?^{?+})$
• $f_2(2010)$	$0^+(2^{++})$
$f_0(2020)$	$0^+(0^{++})$
• $a_4(2040)$	$1^-(4^{++})$
• $f_4(2050)$	$0^+(4^{++})$
$f_0(2060)$	$0^+(0^{++})$
$\pi_2(2100)$	$1^-(2^{-+})$
$f_2(2150)$	$0^+(2^{++})$
$\rho(2150)$	$1^+(1^{--})$
$f_0(2200)$	$0^+(0^{++})$
$f_J(2220)$	$0^+(2^{++}$ or $4^{++})$
$\eta(2225)$	$0^+(0^{-+})$
$\rho_3(2250)$	$1^+(3^{--})$
• $f_2(2300)$	$0^+(2^{++})$
$f_4(2300)$	$0^+(4^{++})$
• $f_2(2340)$	$0^+(2^{++})$
$\rho_5(2350)$	$1^+(5^{--})$
$a_6(2450)$	$1^-(6^{++})$
$f_6(2510)$	$0^+(6^{++})$
$X(3250)$	$?^?(?^{??})$

I=2 ???

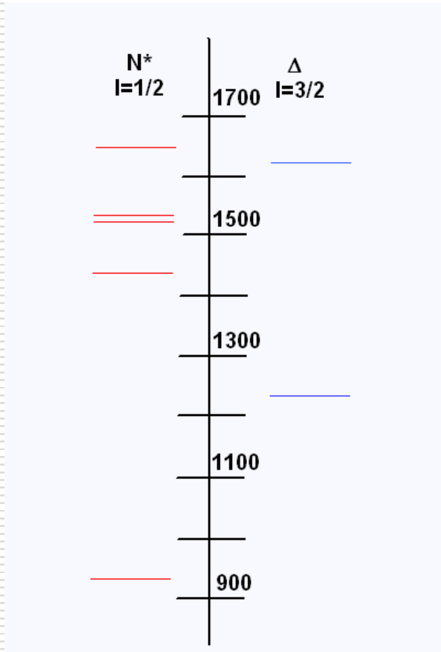
Mesons

I=0,1

Baryon Resonances Systematics

Two families of nucleon excited states: First, lightest states

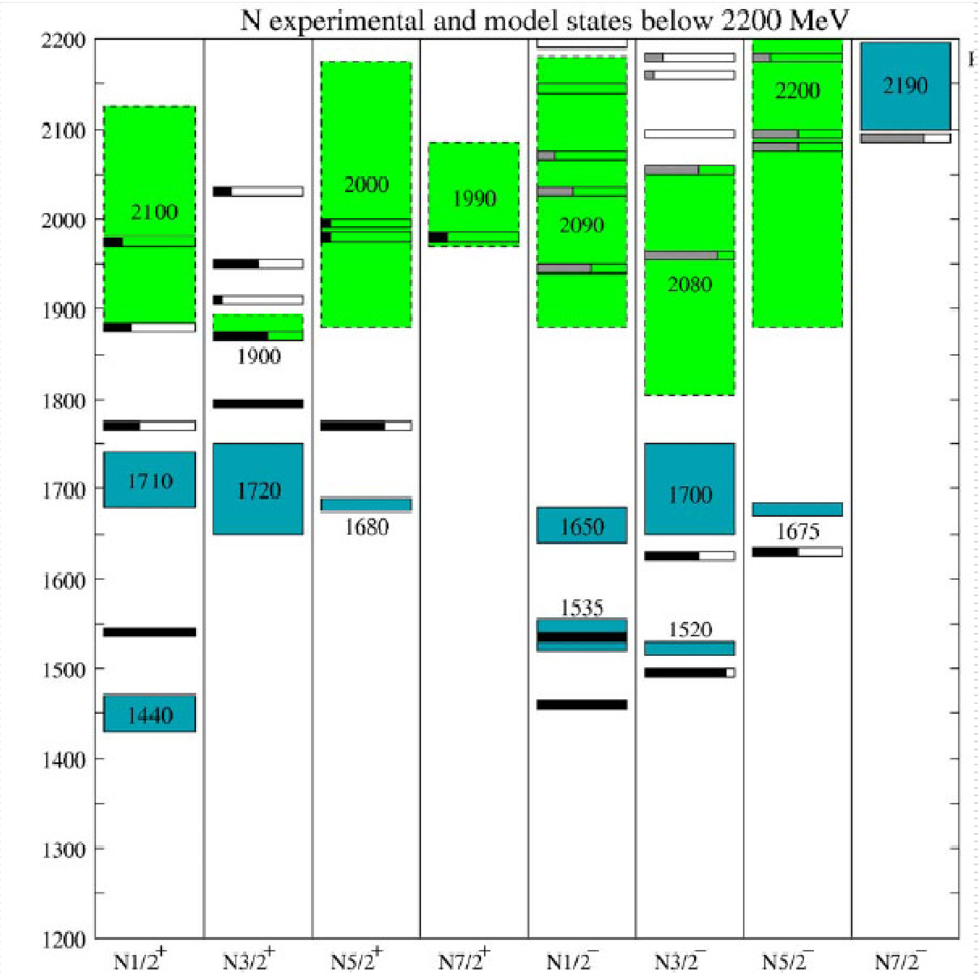
N* isospin =1/2



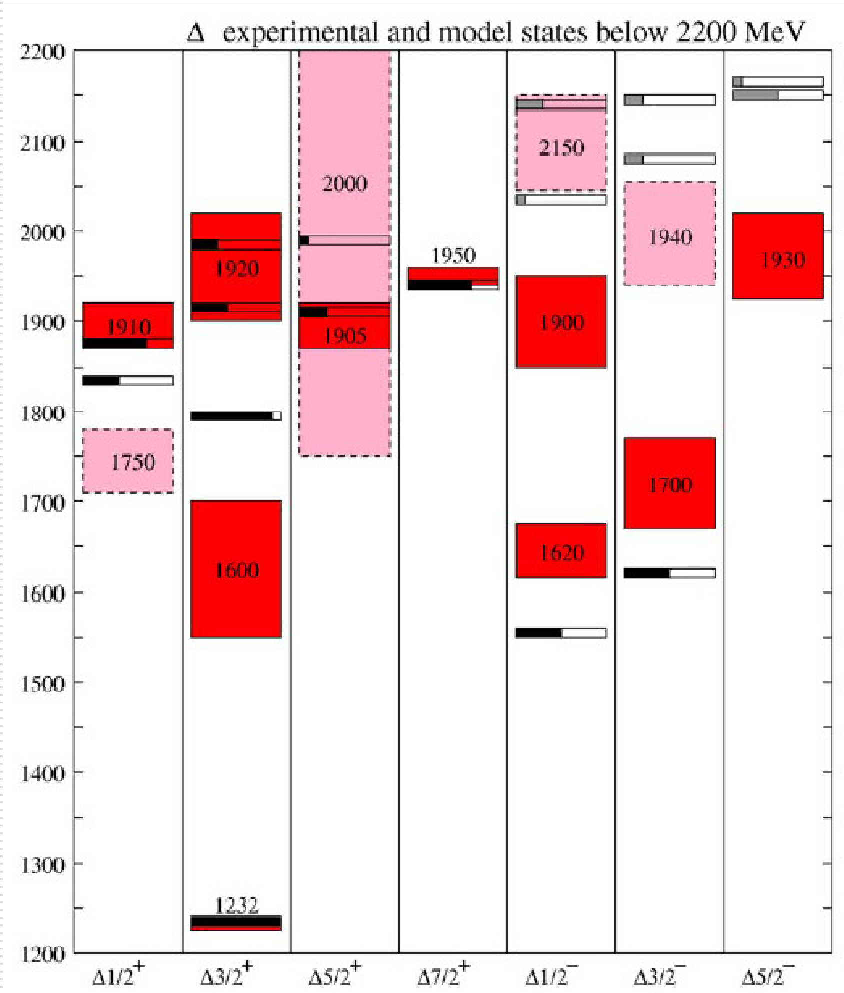
Δ isospin = 3/2

Many sub-families for each one (increasing J, parity + or -)

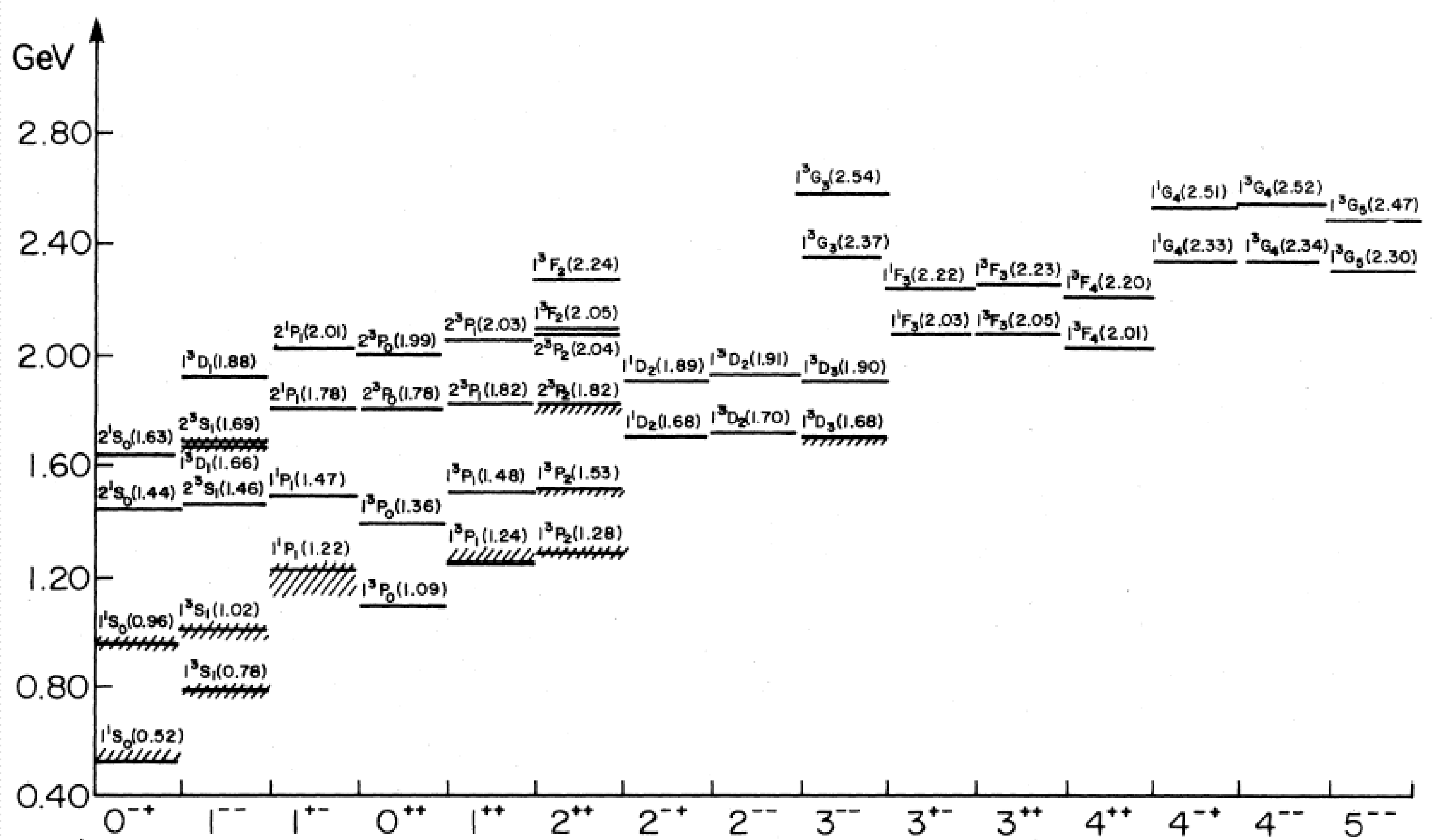
Non-strange Baryons – *I* =*1/2*



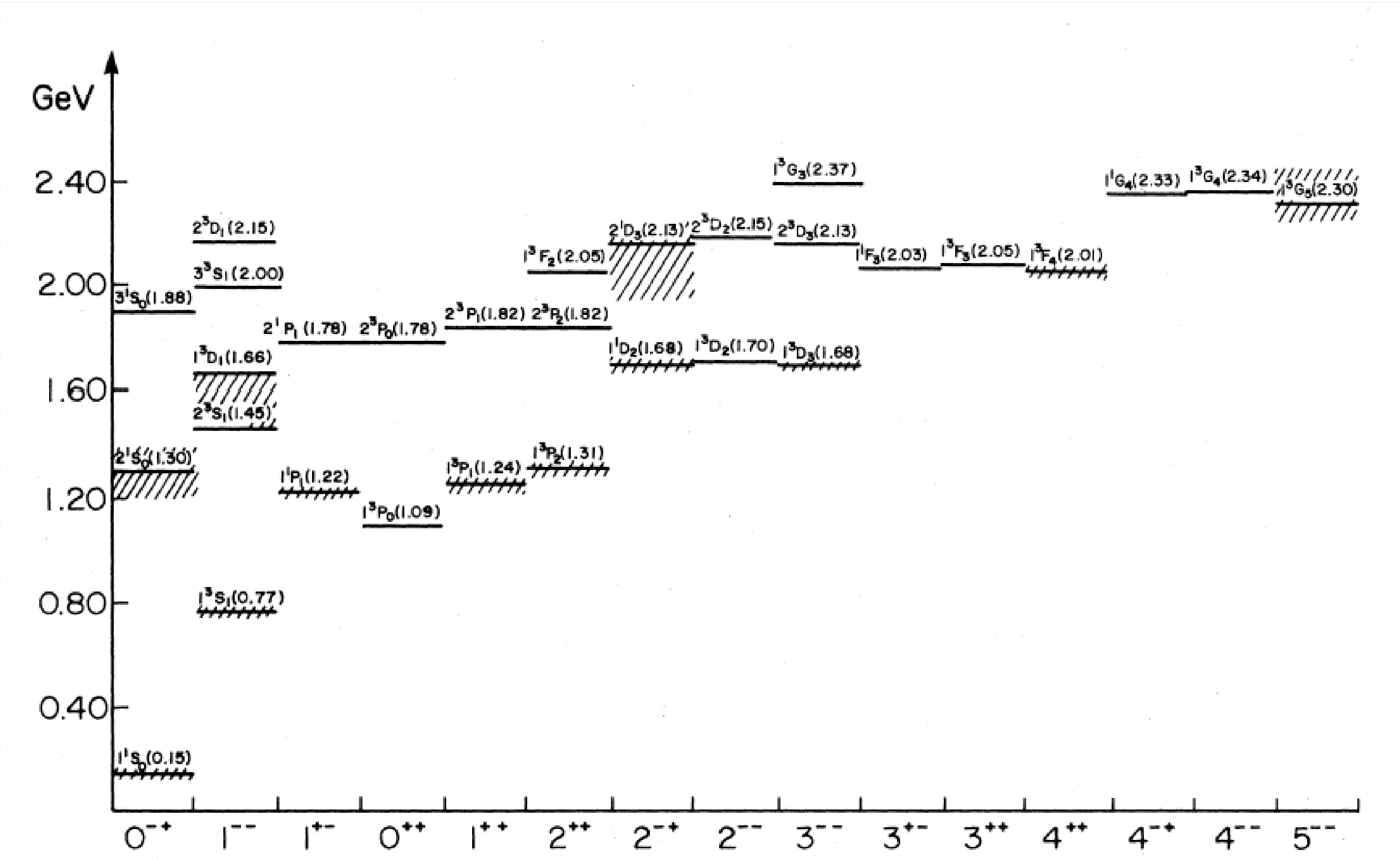
Non-strange Baryons – $I=3/2$



Non-Strange Mesons – *I=0*



Non-Strange Mesons – *I=1*



Gell-Mann – Nishijima Rule

B = Baryon number
Q = Charge in *e* units
I_3 = Isospin 3rd component

Empirical relationship for pions:

$$Q = I_3$$

Linking electromagnetic and strong properties of pions:
Electric charge as *3rd component* of isospin vector

Extend to nucleons:

$$Q = I_3 + B/2$$ Gell-Mann - Nishijima relation

More complicated properties:
Electric charge as both *scalar* and *3rd component*

Strangeness - I

Strange particles discovered in cosmic rays at the end of the '40s, and then quicky observed at the first GeV accelerators
Why strange?

Large production cross section $\rightarrow$ *Like ordinary hadrons*
Long lifetime $\rightarrow$ *Like weak decays*

Understood as carriers of a new quantum number: *Strangeness*

Ordinary hadrons $S = 0$
Strange particles $S \# 0$

Strangeness conserved by strong, e.m. processes, violated by weak
Explain funny behavior, also predicting *associated production* to guarantee *S* conservation in strong & EM processes:

Strange particles always produced in pairs

Strangeness - II

For strong processes, S similar to electric charge and to baryon or lepton numbers
But:

S not absolutely conserved
S not the source of a physical field

Large variety of strange particles, both baryons and mesons, including many strange resonances

Generalize Gell-Mann Nishijima relation to

$$Q = I_3 + \frac{B+S}{2} = I_3 + \frac{Y}{2}$$

Hypercharge

The Lightest Strange Particles

I_3	S=+1	S=-1
+1/2	K^+	K^0
-1/2	$\bar{K}^0$	K^-

Spin 0

I_3	S=+1	S=-1
+1/2	K^{*+}	$\bar{K}^{*0}$
-1/2	K^{*0}	K^{*-}

Spin 1

I_3	S	nome
0	-1	Λ^0
+1,0,-1	-1	$\Sigma^+, \Sigma^-, \Sigma^0$
+1/2,-1/2	-2	Ξ^0, Ξ^-
0	-3	Ω^-

Baryons

I_3	S	nome
0	+1	$\bar{\Lambda}^0$
+1,0,-1	+1	$\bar{\Sigma}^+, \bar{\Sigma}^0, \bar{\Sigma}^-$
+1/2,-1/2	+2	$\bar{\Xi}^0, \bar{\Xi}^-$
0	+3	$\bar{\Omega}^-$

Antibaryons

Isospin of Strange Particles

Isospin conservation in

$$\pi^- + p \to \pi^- + p$$

leads in a natural way to extend to virtual states like

$$\pi^- + p \to \left(K^0 + \Lambda^0\right)^* \to \pi^- + p$$

Therefore strange particles should group into I-spin multiplets.

Λ^0 only observed as a neutral state $\to$ Singlet , $I = 0$

Observe 3 charge states for K: Triplet?

$\pi^- + p : I = 1/2, 3/2 \to K$ must be $I = 1/2, 3/2$

Quartets not observed $\to$ 2 Doublets! Predict *two* neutral *K* states, with opposite S

Would imply charge +2

$$\pi^- + p \to K^0 + \Lambda^0$$

$$p + \overline{p} \to K^0 + \overline{K}^0$$

Must be different particles!

Bubble Chambers & Particle Zoology

Example: Historical Picture

$$K^- + p \rightarrow K^0 + K^+ + \Omega^-$$

$$K^0 \rightarrow \pi^+ + \pi^-$$

$$K^+ \rightarrow \pi^+ + \pi^0 \left(unseen\right)$$

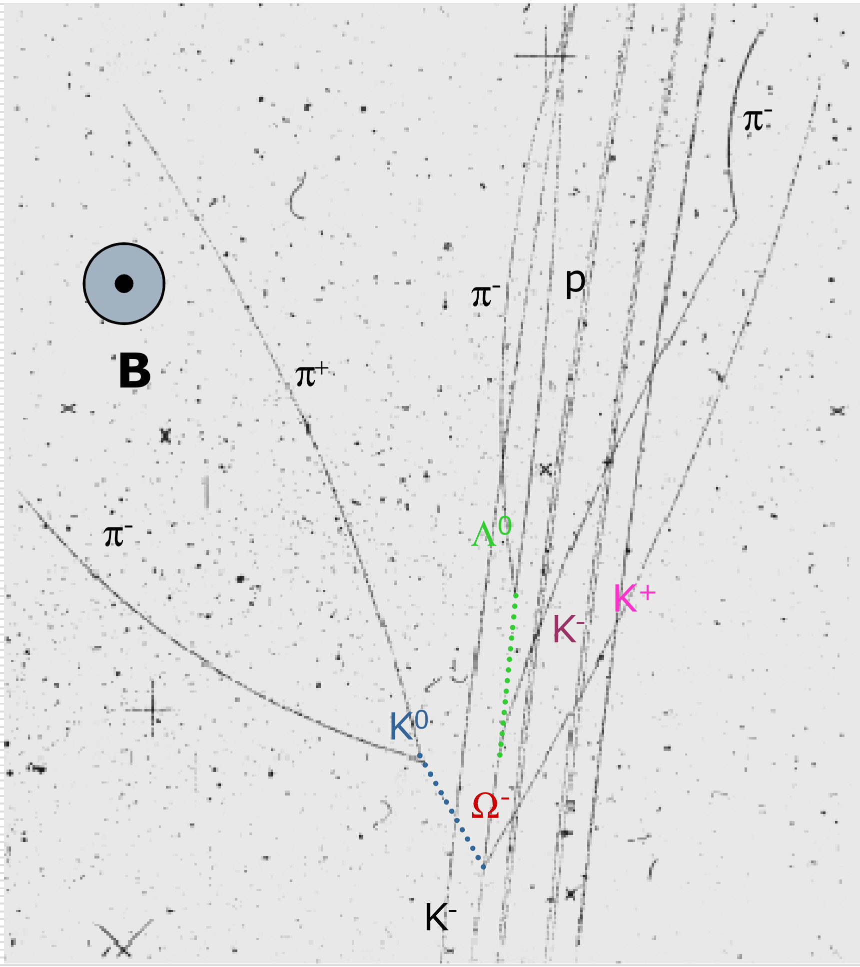
$$\Omega^- \rightarrow \Lambda^0 + K^-$$

$$\Lambda^0 \rightarrow p + \pi^-$$

$$K^- \rightarrow \pi^- + \pi^0 \left(unseen\right)$$

Beam momentum $4.2 \quad GeV$

Magnetic field $2 \quad T$



Old Hyperon Beam & Spectrometer

FNAL – '70s Beam & Detector of Hyperon Experiment

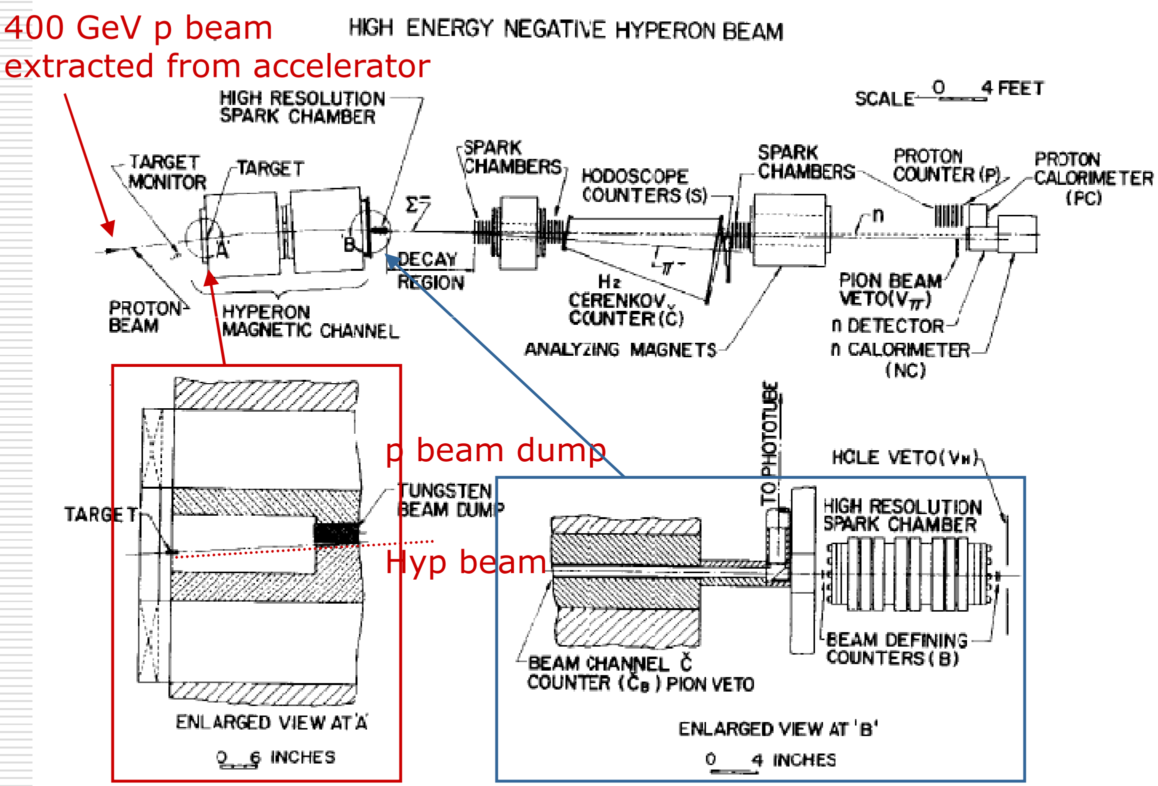


Fig. 1

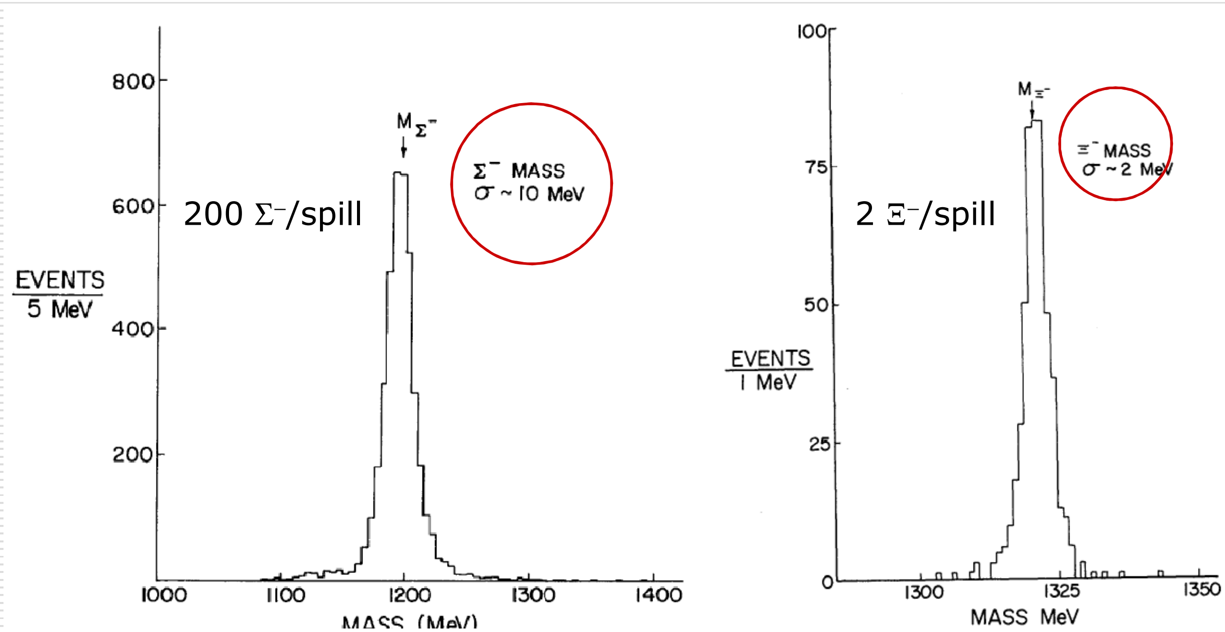
Figure 1
The Hyperon Magnet under construction

Hyperon Gymnastics

Old Hyperon Beam & Spectrometer

Reconstruct decays: $\Sigma^- \to n + \pi^-, \quad \Xi^- \to \Lambda^0 + \pi^-$

π: Identification (Threshold Cherenkov) + Magnetic Analysis
n: Calorimeter
p: Identification (Cherenkov π Veto) + Magnetic Analysis + Calorimeter
$\Lambda^0 \to p + \pi$: Identification + Magnetic Analysis



Particle Identification: Cherenkov - I

Fast, charged particle passing through a dielectric medium

Cherenkov radiation emitted for $\beta > \frac{1}{n}$, n refractive index

Main features:

Emission angle:

$\cos\theta_c = \frac{1}{\beta n}$ Cherenkov angle

For ultrarelativistic particles:

$\lim_{\beta\to 1}(\cos\theta_c) = \frac{1}{n}$ Asymptotic angle

Representative radiators

Medium	n	θ_{min} deg	$P_{thresh}(\pi)$ GeV	N_{ph} $eV^{-1}cm^{-1}$
Air	1.00028	1.36	5.9	0.21
Isobutane	1.00217	3.77	2.12	0.94
Aerogel	1.0065	6.51	1.3	4.7
Water	1.33	41.2	0.16	160.8
Quartz	1.46	46.7	0.13	196.4

Spectrum:

1/λ^2 spectrum: Blue/Near UV *very* important…

$$\frac{d^2N}{dx\,d\lambda} = 2\pi\alpha z^2 \frac{1}{\lambda^2}\sin^2\theta_c \quad photons/cm^2,$$ z particle charge in e units

$$\frac{d^2N}{dx\,dE} = \frac{\alpha}{\hbar c} z^2 \sin^2\theta_c \approx 365 z^2 \sin^2\theta_c \quad photons/(cm\cdot eV)$$

Number of photons/cm small…

Particle Identification: Cherenkov - II

Translate light signal into an electric charge: *Photomultiplier*, or similar

Typical result with a PM:

$$N_{pe} \approx 365 \int_{E_{\min}}^{E_{\max}} \varepsilon_{coll}(E)\varepsilon_{\det}(E)\sin^2\theta_c(E)\,dE$$ N. of photoelectrons/cm obtained

Collection efficiency

Conversion efficiency

Cherenkov angle depending on *E*:

$$\cos\theta_c = \frac{1}{\beta n(\lambda)} = \frac{1}{\beta n(E)}$$ Dispersion of refractive index

Typically:

$N_{pe} \le 100\sin^2\theta_c$ Photoelectrons/cm

Threshold counter

$$\beta > \frac{1}{n} \to \frac{p}{E} > \frac{1}{n} \to \frac{p}{\sqrt{p^2+m^2}} > \frac{1}{n} \to p^2 > \frac{1}{n^2}\left(p^2+m^2\right)$$

$$\to p^2\left(1-\frac{1}{n^2}\right) > \frac{m^2}{n^2} \to p^2 > \frac{m^2}{n^2-1} \to p > \frac{m}{\sqrt{n^2-1}}$$ Threshold momentum

Can discriminate among different masses with the same momentum

The Strange Zoo

Λ	P_{01}	****
$\Lambda(1405)$	S_{01}	****
$\Lambda(1520)$	D_{03}	****
$\Lambda(1600)$	P_{01}	***
$\Lambda(1670)$	S_{01}	****
$\Lambda(1690)$	D_{03}	****
$\Lambda(1800)$	S_{01}	***
$\Lambda(1810)$	P_{01}	***
$\Lambda(1820)$	F_{05}	****
$\Lambda(1830)$	D_{05}	****
$\Lambda(1890)$	P_{03}	****
$\Lambda(2000)$		*
$\Lambda(2020)$	F_{07}	*
$\Lambda(2100)$	G_{07}	****
$\Lambda(2110)$	F_{05}	***
$\Lambda(2325)$	D_{03}	*
$\Lambda(2350)$	H_{09}	***
$\Lambda(2585)$		**

Ω^-	****
$\Omega(2250)^-$	***
$\Omega(2380)^-$	**
$\Omega(2470)^-$	**

Ξ^0	P_{11}	****
Ξ^-	P_{11}	****
$\Xi(1530)$	P_{13}	****
$\Xi(1620)$		*
$\Xi(1690)$		***
$\Xi(1820)$	D_{13}	***
$\Xi(1950)$		***
$\Xi(2030)$		***
$\Xi(2120)$		*
$\Xi(2250)$		**
$\Xi(2370)$		**
$\Xi(2500)$		*

Σ^+	P_{11}	****
Σ^0	P_{11}	****
Σ^-	P_{11}	****
$\Sigma(1385)$	P_{13}	****
$\Sigma(1480)$		*
$\Sigma(1560)$		**
$\Sigma(1580)$	D_{13}	*
$\Sigma(1620)$	S_{11}	**
$\Sigma(1660)$	P_{11}	***
$\Sigma(1670)$	D_{13}	****
$\Sigma(1690)$		**
$\Sigma(1750)$	S_{11}	***
$\Sigma(1770)$	P_{11}	*
$\Sigma(1775)$	D_{15}	****
$\Sigma(1840)$	P_{13}	*
$\Sigma(1880)$	P_{11}	**
$\Sigma(1915)$	F_{15}	****
$\Sigma(1940)$	D_{13}	***
$\Sigma(2000)$	S_{11}	*
$\Sigma(2030)$	F_{17}	****
$\Sigma(2070)$	F_{15}	*
$\Sigma(2080)$	P_{13}	**
$\Sigma(2100)$	G_{17}	*
$\Sigma(2250)$		***
$\Sigma(2455)$		**
$\Sigma(2620)$		**
$\Sigma(3000)$		*
$\Sigma(3170)$		*

Baryons, *S=-1,-2,-3*
(Antibaryons not shown)

Mesons, *S=±1*

• K^-	$1/2(0^-)$
• K^0	$1/2(0^-)$
• K^0_S	$1/2(0^-)$
• K^0_L	$1/2(0^-)$
$K^*_0(800)$	$1/2(0^+)$
• $K^*(892)$	$1/2(1^-)$
• $K_1(1270)$	$1/2(1^+)$
• $K_1(1400)$	$1/2(1^+)$
• $K^*(1410)$	$1/2(1\ )$
• $K^*_0(1430)$	$1/2(0^+)$
• $K^*_2(1430)$	$1/2(2^+)$
$K(1460)$	$1/2(0^-)$
$K_2(1580)$	$1/2(2^-)$
$K(1630)$	$1/2(?^?)$
$K_1(1650)$	$1/2(1^+)$
• $K^*(1680)$	$1/2(1\ )$
• $K_2(1770)$	$1/2(2^-)$
• $K^*_3(1780)$	$1/2(3^-)$
• $K_2(1820)$	$1/2(2^-)$
$K(1830)$	$1/2(0^-)$
$K^*_0(1950)$	$1/2(0^+)$
$K^*_2(1980)$	$1/2(2^+)$
• $K^*_4(2045)$	$1/2(4^+)$
$K_2(2250)$	$1/2(2^-)$
$K_3(2320)$	$1/2(3^+)$
$K^*_5(2380)$	$1/2(5^-)$
$K_4(2500)$	$1/2(4^-)$
$K(3100)$	$?^?(?^{??})$

Higher Symmetry

Experimental evidence for several 'multiplets of multiplets'

$J^P=0^-$

I	S=+1	S=0	S=-1
0		η,η'	
1/2	K		$\bar{K}$
1		π	

$J^P=1^-$

I	S=+1	S=0	S=-1
0		ω,φ	
1/2	K^*		$\bar{K}^*$
1		ρ	

$J^P=2^+$

I	S=+1	S=0	S=-1
0		f_0, f_1	
1/2	K^{**}		$\bar{K}^{**}$
1		a_2	

Mesons

$J^P=1/2^+$

I	S=-2	S=-1	S=0
0		Λ^0	
1/2	Ξ		N
1		Σ	

$J^P=3/2^+$

I	S=-3	S=-2	S=-1	S=0
0	Ω^-			
1/2		Ξ^*		
1			Σ^*	
3/2				Δ

Baryons

Remember:
Each square is a I-spin multiplet, with size 2I+1
Total of 45 particle states in this page!

SU(3) - I

Try to find a larger group to encompass both strangeness and isospin into a unified symmetry scheme.

Requirements:

2 commuting generators, since both S and I_3 are defined within any observed supermultiplet

($\mathrm{SU}(2)$ has just one, I_3)

Multiplet structure matching experimental data

SU(3) - II

Take *SU(3)* as candidate to extend *SU(2):*

Group of unitary, unimodular 3x3 matrices

9 complex parameters $\rightarrow$ 18 real parameters

9 unitarity conditions: $\left.\begin{array}{r} UU^{\dagger}=1 \\ \left(U^{\dagger}\right)_{ij}=U^{*}_{ji} \end{array}\right\} \rightarrow \sum_{j=1}^{3} a_{ij}a^{*}_{jk}=\delta_{ik}, \quad i,k=1,..,3$

1 unimodularity condition: $\det U = 1$

$\rightarrow 18-10=8$ free, real parameters

SU(3) - III

As usual, for any unitary matrix

$U = e^{iH}, \quad H$ Hermitian

$\det U = 1 \rightarrow \det e^{iH} = 1 \rightarrow e^{itr(H)} = 1 \rightarrow tr(H) = 0$

8 parameters → 8 generators

Generalize Pauli matrices to *Gell-Mann matrices*

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix},$$

$$\lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

SU(3) - IV

Commutators:

$\left[\lambda_i, \lambda_j\right] = f_{ijk}\lambda_k, \quad f_{ijk}$ structure constants

Two diagonal generators, λ_3 and λ_8

→ *Rank 2 group*

→ *2* invariant functions of generators

Quadratic: $C^{(2)} = \sum_{i,j=1}^{8} \delta_{ij}\lambda_i\lambda_j$

Cubic: $C^{(3)} = \sum_{i,j,k=1}^{8} f_{ijk}\lambda_i\lambda_j\lambda_k$

$F_i \equiv \frac{\lambda_i}{2}$ Definition

Identify: $\begin{cases} I_3 = F_3 & \text{Isospin 3rd component} \\ Y = \frac{2}{\sqrt{3}} F_8 & \text{Hypercharge} \end{cases}$

Compare to *SU(2)*:

$\left[\sigma_i, \sigma_j\right] = i\varepsilon_{ijk}\sigma_k$

One diagonal generator, σ_3

→*Rank 1 group*

→*1 invariant function of generators*

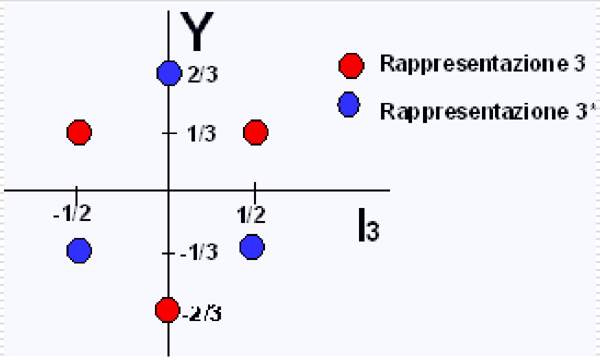
Quadratic: $C^{(2)} = \sum_{i,j=1}^{3} \delta_{ij}\sigma_i\sigma_j$

SU(3) Surprises

Fundamental representation (3 x 3 matrices): ***3***

Find eigenvalues & eigenvectors for **3**:

$$\begin{pmatrix}1\\0\\0\end{pmatrix} \to \begin{cases} I_3 = \frac{1}{2} \\ Y = \frac{1}{3} \end{cases}; \quad \begin{pmatrix}0\\1\\0\end{pmatrix} \to \begin{cases} I_3 = -\frac{1}{2} \\ Y = \frac{1}{3} \end{cases}; \quad \begin{pmatrix}0\\0\\1\end{pmatrix} \to \begin{cases} I_3 = 0 \\ Y = -\frac{2}{3} \end{cases}$$



$\to 3$ independent base states

$\to I_3, Y$ eigenvalues not symmetrical wrt origin

$\to$ Conjugate representation: $\mathbf{3}^*$ different from $\mathbf{3}$

$\to$ For both $\mathbf{3}, \mathbf{3}^*$ hypercharge eigenvalues fractionary $Y = B + S$

$\to Q = I_3 + Y/2$ fractionary!!!

SU(3) Multiplets - I

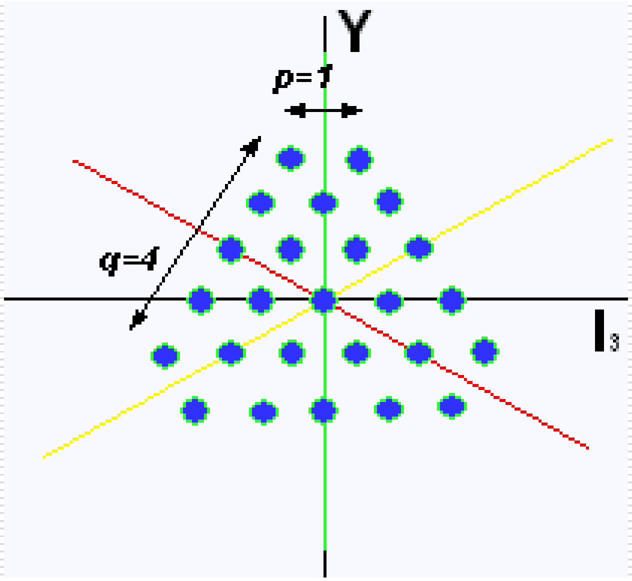
States identified by Y, I_3 eigenvalues
$\rightarrow$Points in a plane

Hexagonal/Triangular symmetry

Specified by *2* integers (p,q)

Multiplicity (i.e. size)

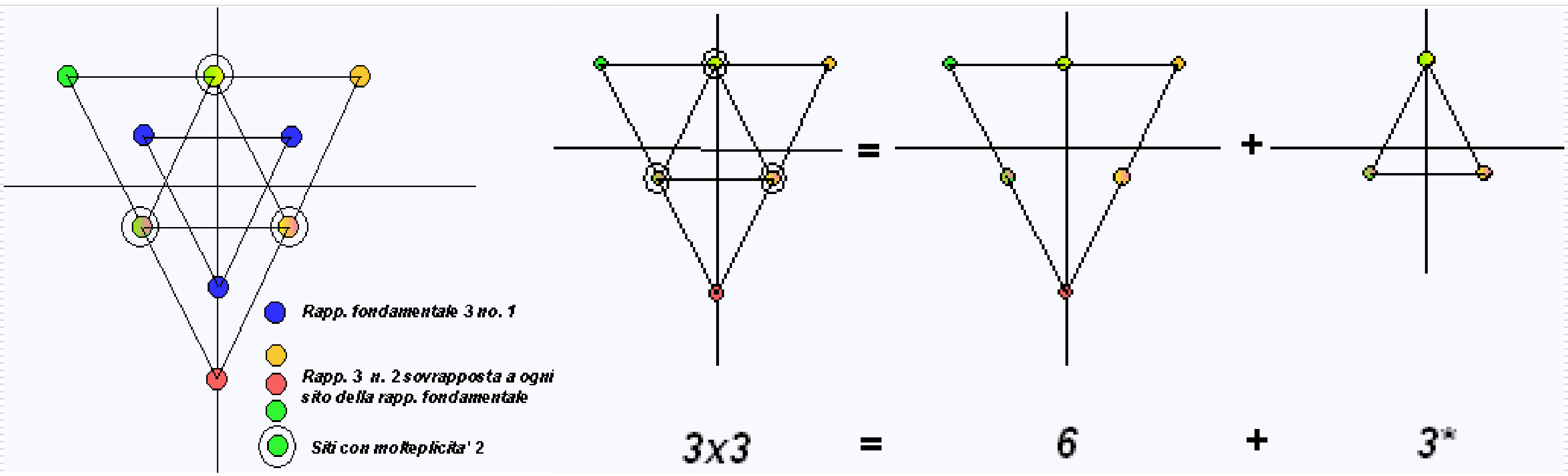
$$n = \frac{1}{2}(p+1)(q+1)(p+q+2)$$



Multiplet $(1,4)$
Frequently indicated by ***n****=35*

SU(3) Multiplets - II

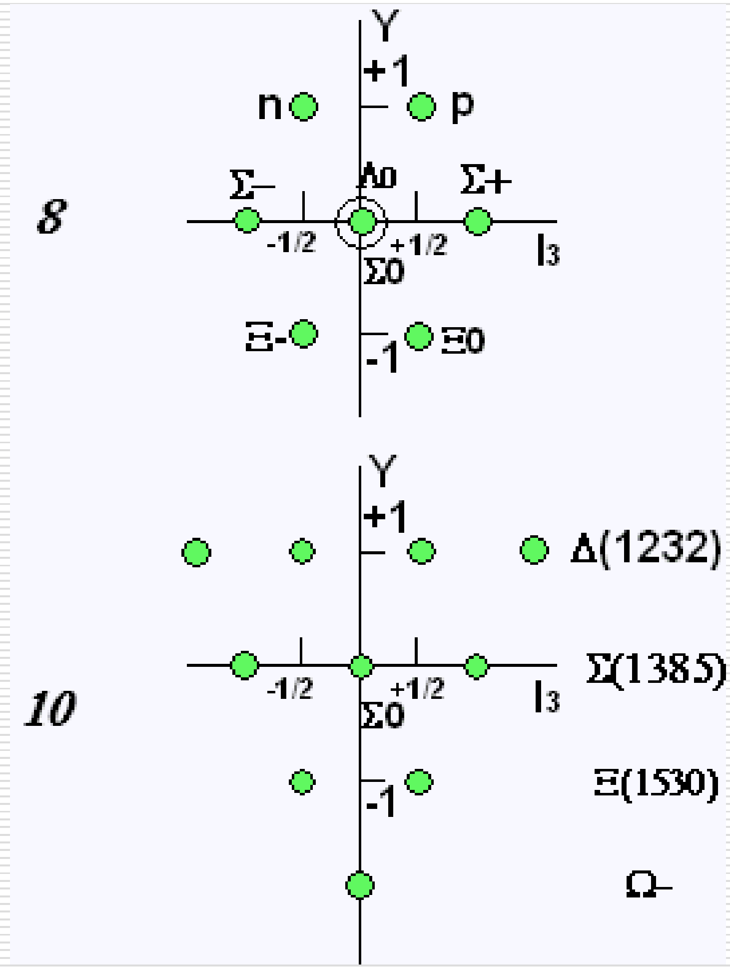
Products and decomposition into irr.rep.:
Proceed graphically as for *SU(2)*



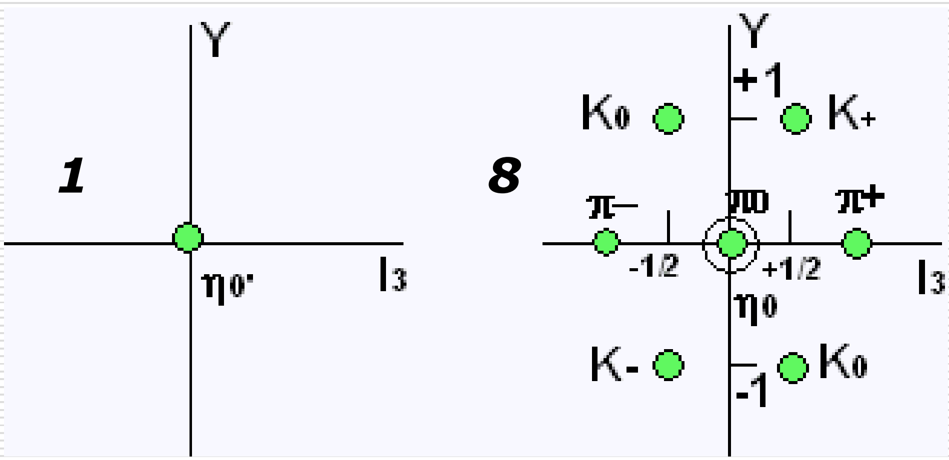
Hadrons and *SU(3):* The Eightfold Way

- *All the hadronic multiplets nicely fit some SU(3) representation*
- *No hadron found which does not fit*

Baryons $J^P = 1/2^+, 3/2^+$



Mesons $J^{PC} = 0^{-+}$



The Hard Facts: *SU(3)* Breaking

$J^P=0^-$

I	S=-1	S=0	S=+1
0		$\eta(547), \eta'(958)$	
1/2	$\bar{K}(496)$		$K(496)$
1		$\pi(137)$	

$J^P=1/2^+$

I	S=-2	S=-1	S=0
0		$\Lambda^0(1116)$	
1/2	$\Xi(1317)$		$N(938)$
1		$\Sigma(1192)$	

$J^P=1^-$

I	S=-1	S=0	S=+1
0		$\omega(782), \varphi(1020)$	
1/2	$\bar{K}^*(892)$		$K^*(892)$
1		$\rho(770)$	

$J^P=3/2^+$

I	S=-3	S=-2	S=-1	S=0
0	$\Omega^-(1672)$			
1/2		$\Xi^*(1530)$		
1			$\Sigma^*(1385)$	
3/2				$\Delta(1232)$

$J^P=2^+$

I	S=-1	S=0	S=+1
0		$f_2(1270), f_2'(1525)$	
1/2	$\bar{K}^{**}(1430)$		$K^{**}(1430)$
1		$a_2(1320)$	

As before, but including masses:
SU(3) is not an exact symmetry

Mass differences within a multiplet are large, typ. $\Delta m/m \sim 10\text{-}20\%$

SU(3) Breaking: Mass Formulas - I

Since SU(3) is a broken symmetry, try to find a sensible breaking scheme

Take an *effective Hamiltonian*:
Part SU(3)-Invariant + Part non SU(3)-Invariant

$$m_{hadron} \simeq \langle hadron | H_S | hadron \rangle, \quad H_S = H_0 + H'$$

$$\langle a | H_s | a \rangle \to \underbrace{\langle a | U^{-1}}_{\substack{SU(3)-transformed \\ state}} \; H \; \underbrace{U | a \rangle}_{\substack{SU(3)-transformed \\ state}}$$

$$\to \langle a | U^{-1} (H_0 + H') U | a \rangle = \langle a | U^{-1} H_0 U | a \rangle + \langle a | U^{-1} H' U | a \rangle$$

$$H_0: \ \text{invariant} \quad \to U^{-1} H_0 U = H_0$$

$$H': \ \text{non invariant} \ \to \ U^{-1} H' U \neq H'$$

$$\to \langle a | H | a \rangle = \langle a | U^{-1} H_0 U | a \rangle + \langle a | U^{-1} H' U | a \rangle = \langle a | H_0 | a \rangle + \langle a | U^{-1} H' U | a \rangle$$

Must guess SU(3) properties of H’

SU(3) Breaking: Mass Formulas - II

Must guess SU(3) properties of *H'*
Since the largest breaking concerns strange particles, suppose

$$\rightarrow H' \propto F_8 \propto Y$$

Reminder: $I_3 = F_3,\ \ Y = \dfrac{2}{\sqrt{3}} F_8$

According to *SU(3)* algebra:

Gell-Mann Okubo mass formula

$$\langle a|H'|a\rangle \propto \langle a|F_8|a\rangle \propto A + BY + C\left[Y^2/4 - I(I+1)\right]$$

$$m(Y,I) = m_0 + bY + C\left[Y^2/4 - I(I+1)\right]$$

A,B,C:
constants, rep. dependent

SU(3) Breaking: Mass Formulas - III

S = -3 decuplet member not observed.

What is the mass?

Take mass differences between decuplet members:

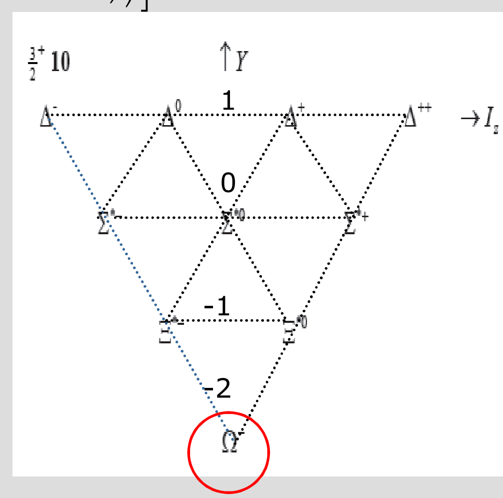
$$\Delta m_{ij} = m_i - m_j = b\left(\Delta Y\right)_{ij} + C\left[\left(Y_i^2 - Y_j^2\right)/4 - \left(I_i\left(I_i+1\right) - I_j\left(I_j+1\right)\right)\right]$$

From $\Delta(1232),\quad \Sigma^*(1385),\quad \Xi^*(1530):$

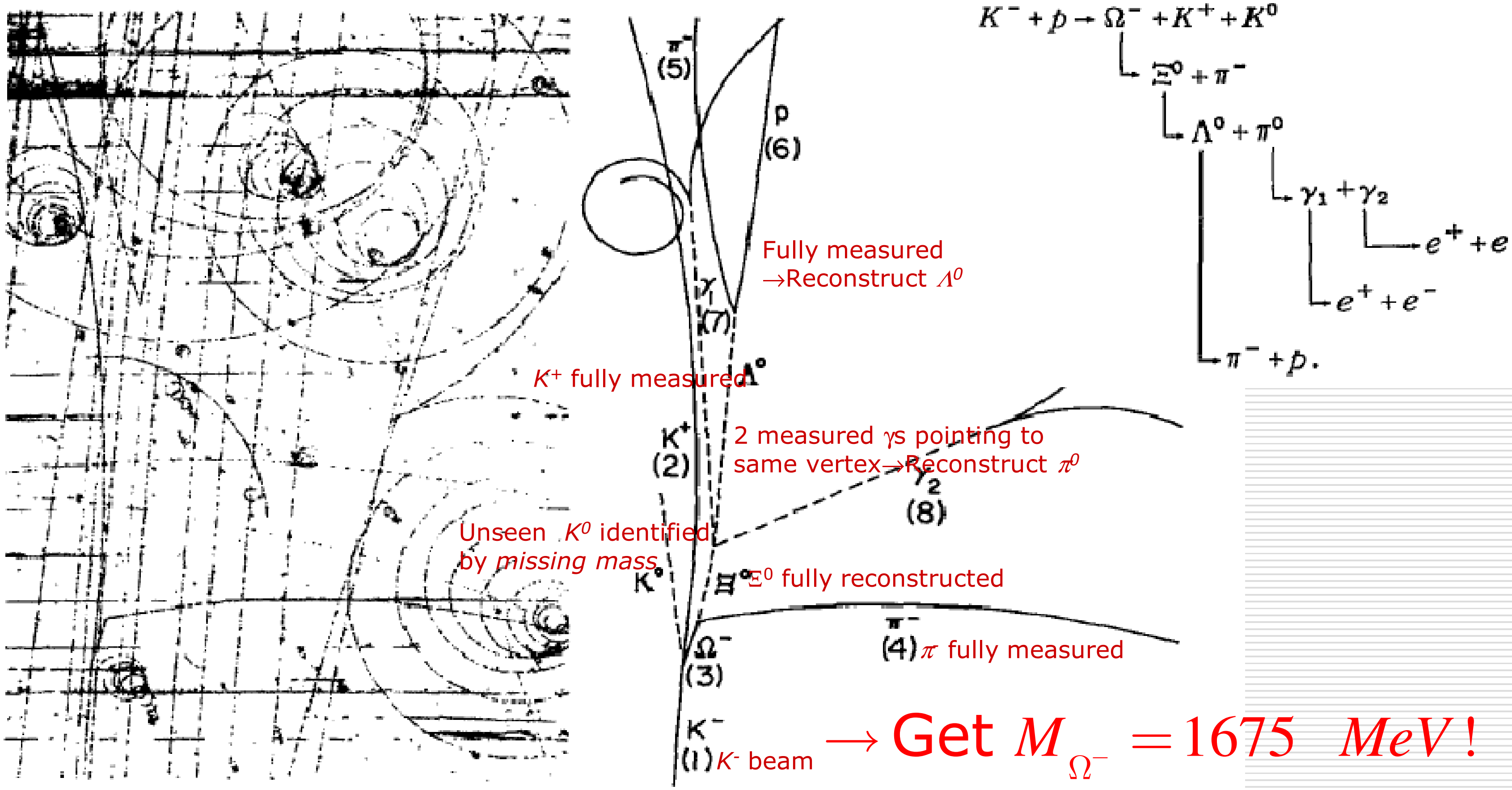
$$m_\Sigma - m_\Delta \approx m_\Xi - m_\Sigma \approx 150\quad MeV$$

$\rightarrow$ Predict missing $S = -3,\ J = 3/2$ decuplet baryon

Named Ω^-, predicted mass $m_\Omega \simeq 1672\ MeV$



The Ω^- Discovery at BNL



$\rightarrow$ Get $M_{\Omega^-} = 1675\ \ MeV\,!$