Elementary Particles I

4 – Quarks

Quark Model, Light and Heavy Quarks

The Quark Model

Fundamental hypothesis:

Mesons = Bound states $q\bar{q}$

Baryons, Antibaryons = Bound states $qqq, \overline{qqq}$

What are states $q, \bar{q}$? They are called *quark, antiquark*
Building blocks of ordinary hadrons:
A new level of structure for the hadronic matter

Quarks fill the fundamental representation of *SU(3)*
Quarks are spin *1/2*, point-like fermions
Guess:

$$\left.\begin{array}{l}\textit{They are never observed as free particles}\\ \textit{The only bound states observed are } q\bar{q}, qqq, \overline{qqq}\end{array}\right\}\ \text{Why ?}$$

Predicting New Particles

Not a new game in town…

In the Thirties:

Pauli: *Neutrino*

Required in order to save energy, angular momentum conservation in nuclear β decay
Observed in 1956 (Reines et al., Nuclear reactor experiment)

Yukawa: *Pion*

Welcome in order to explain the general features of nuclear force
Observed in 1947 (Blackett et al., Cosmic radiation)

Quarks

Fundamental and conjugate irr.rep. of *SU(3)*: ***3***, ***3****
Each made of 3 states

Quantum numbers: From Gell-Mann – Nishijima & SU(3) $Q = I_3 + Y/2$

Symbol	Flavor	Spin	Q	B	S	Y	I	I_3
u	*Up*	½	*2/3*	*1/3*	*0*	*1/3*	*1/2*	*+1/2*
d	*Down*	½	*-1/3*	*1/3*	*0*	*1/3*	*1/2*	*-1/2*
s	*Strange*	½	*-1/3*	*1/3*	*-1*	*-2/3*	*0*	*0*

} isospin doublet

isospin singlet

Quarks are predicted to carry fractional charge, baryon number!

Should they show up as free particles, would be easy to detect :

Expect unusual electromagnetic rates $\propto Q^2$

Expect bound states with fractional mass numbers $\propto B$

Mesons and Baryons

Hadrons: Expected to fill product representations

From our group theory rudiments:

Mesons $3 \otimes 3^* = 1 \oplus 8$

Baryons $3 \otimes 3 \otimes 3 = 1 \oplus 8 \oplus 8 \oplus 10$

Expect:

Nonets of mesons with given spin, parity
Singlets, octets, decuplets of baryons, as above

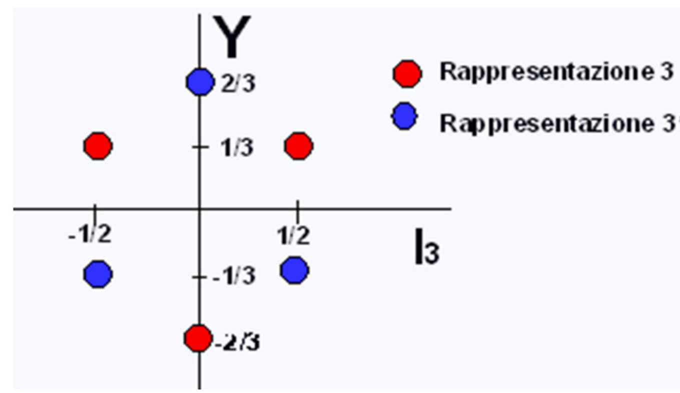
Quarks & Antiquarks: **3 & 3***

More quantum numbers

Relative space parity = -1 (Fermions)
Absolute space parity = +1 quarks, -1 antiquarks (Conventional)

Flavor	Spin	Q	B	S	Y	I	I_3
Up	½	*2/3*	*1/3*	*0*	*1/3*	*1/2*	*+1/2*
Down	½	*-1/3*	*1/3*	*0*	*1/3*	*1/2*	*-1/2*
Strange	½	*-1/3*	*1/3*	*-1*	*-2/3*	*0*	*0*

Flavor	Spin	Q	B	S	Y	I	I_3
Anti-Up	½	*-2/3*	*-1/3*	*0*	*-1/3*	*1/2*	*-1/2*
Anti-Down	½	*+1/3*	*-1/3*	*0*	*-1/3*	*1/2*	*+1/2*
Anti-Strange	½	*+1/3*	*-1/3*	*+1*	*+2/3*	*0*	*0*



A Couple of Subtle Points - I

Q: *Why are isospin 3rd components swapped for antiquarks?*

A: *Want to stick to Gell-Mann – Nishijima for them too*

Required in order to deal with $qqq,\ q\overline{q}, \overline{qqq}$

E.g. all present in the same process

$$u = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, d = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$Q = I_3 + \frac{Y}{2} = I_3 + \frac{B}{2}$$

$$Q(\overline{u}) = -\frac{2}{3} = I_3(\overline{u}) + \frac{B(\overline{u})}{2} = I_3(\overline{u}) - \frac{1}{6} \to I_3(\overline{u}) = -\frac{2}{3} + \frac{1}{6} = -\frac{1}{2}$$

$$Q(\overline{d}) = +\frac{1}{3} = I_3(\overline{d}) + \frac{B(\overline{d})}{2} = I_3(\overline{d}) - \frac{1}{6} \to I_3(\overline{d}) = +\frac{1}{3} + \frac{1}{6} = +\frac{1}{2}$$

A Couple of Subtle Points - II

Q: *Why there is a -1 extra phase for u antiquark?* $\begin{pmatrix} \bar{d} \\ -\bar{u} \end{pmatrix}$

A: *Want to stick to same C-G coefficient for both quarks and antiquarks*

Same C-G $\leftrightarrow$ Same I-spin rotation matrices

Indeed, required because mesons *are* made of quark-antiquark pairs

$$\begin{pmatrix} u' \\ d' \end{pmatrix} = e^{-i\tau_2\theta/2}\begin{pmatrix} u \\ d \end{pmatrix} = \begin{pmatrix} \cos(\theta/2)u - \sin(\theta/2)d \\ \sin(\theta/2)u + \cos(\theta/2)d \end{pmatrix}$$ Rotation of generic state

$$\rightarrow \begin{pmatrix} \bar{d}' \\ \bar{u}' \end{pmatrix} = \begin{pmatrix} \sin(\theta/2)\bar{u} + \cos(\theta/2)\bar{d} \\ \cos(\theta/2)\bar{u} - \sin(\theta/2)\bar{d} \end{pmatrix} = \begin{pmatrix} \cos(\theta/2)\bar{d} + \sin(\theta/2)\bar{u} \\ -\sin(\theta/2)\bar{d} + \cos(\theta/2)\bar{u} \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} \bar{d}' \\ \bar{u}' \end{pmatrix} = \begin{pmatrix} \cos(\theta/2)\bar{d} - \sin(\theta/2)(-\bar{u}) \\ -\sin(\theta/2)\bar{d} - \cos(\theta/2)(-\bar{u}) \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} \bar{d}' \\ -\bar{u}' \end{pmatrix} = \begin{pmatrix} \cos(\theta/2)\bar{d} - \sin(\theta/2)(-\bar{u}) \\ \sin(\theta/2)\bar{d} + \cos(\theta/2)(-\bar{u}) \end{pmatrix} = e^{-i\tau_2\theta/2}\begin{pmatrix} \bar{d} \\ -\bar{u} \end{pmatrix}$$

The Light Mesons - I

Combine 3 quarks with 3 antiquarks: Get 9 combinations

Quark content $u\bar{d}, u\bar{s}, u\bar{u}, d\bar{u}, d\bar{s}, d\bar{d}, s\bar{u}, s\bar{d}, s\bar{s}$

Identified mesons

'State'	Q	I_3	I	S	Remarks	$J^{PC}=0^{-+}$	$J^{PC}=1^{--}$	$J^{PC}=2^{++}$
$u\bar{d}$	+1	+1	1	0		π^+	ρ^+	a_2^+
$u\bar{s}$	+1	+1/2	½	+1		K^+	K^{+*}	K^{+**}
$u\bar{u}$	0	0	0,1	0	*I-spin undefined*	π^0,η,η'	ρ^0,ω,φ	a_2^0, f_2, f_2'
$d\bar{u}$	-1	-1	1	0		π^-	ρ^-	a_2^-
$d\bar{s}$	0	-1/2	1/2	+1		K^0	K^{0*}	K^{0**}
$d\bar{d}$	0	0	0,1	0	*I-spin undefined*	π^0,η,η'	ρ^0,ω,φ	a_2^0, f_2, f_2'
$s\bar{u}$	-1	-1/2	1/2	-1		K^-	K^{-*}	K^{-**}
$s\bar{d}$	0	+1/2	1/2	-1		$\bar{K}^0$	$\bar{K}^{0*}$	$\bar{K}^{0**}$
$s\bar{s}$	0	0	0	0		π^0,η,η'	ρ^0,ω,φ	a_2^0, f_2, f_2'

L # 0

The Light Mesons - II

Physical particles must have *I* defined: *I*-spin is a good symmetry
Build isospin eigenstates from $S=0, I_3=0$ states:

$$\frac{1}{\sqrt{2}}\left(u\bar{u}-d\bar{d}\right),\frac{1}{\sqrt{6}}\left(u\bar{u}+d\bar{d}-2s\bar{s}\right),\frac{1}{\sqrt{3}}\left(u\bar{u}+d\bar{d}+s\bar{s}\right)$$

6 unambiguous states are octet members
Left with 3 ambiguous states: $I_3=0 \rightarrow 2$ octets, 1 singlet ambiguous
SU(3) singlet: Invariant wrt SU(3) rotations

$$\frac{1}{\sqrt{3}}\left(u\bar{u}+d\bar{d}+s\bar{s}\right): \quad \eta_1$$

SU(3) Octets: 1 *SU(2)* triplet, 1 *SU(2)* singlet

$$\frac{1}{\sqrt{2}}\left(u\bar{u}-d\bar{d}\right): \quad \pi^0; \qquad \frac{1}{\sqrt{6}}\left(u\bar{u}+d\bar{d}-2s\bar{s}\right): \quad \eta_8$$

η_1, η_8 cannot be identified with physical particles

The Light Mesons - III

$$\mathbf{J} = \mathbf{L} + \mathbf{S}$$

$$P = (-1)^{l+1}$$

$$C = (-1)^{l+s}$$

Ground state $L = 0 \to J = S$

Singlets $\to J = 0 \to P = -1, C = +1 \to J^{PC} = 0^{-+}$

Triplets $\to J = 1 \to P = -1, C = -1 \to J^{PC} = 1^{--}$

Remark 1:
Very simple and clear, but: Not covariant!
J separation into ***L,S*** contributions is frame dependent
$\to$We are assuming small quark speed: Is this correct?

Remark 2:
Higher spin multiplets more difficult to explain, due to orbital degrees of freedom

The Light Mesons - IV

Particle identification with SU(3) eigenstates not always straightforward

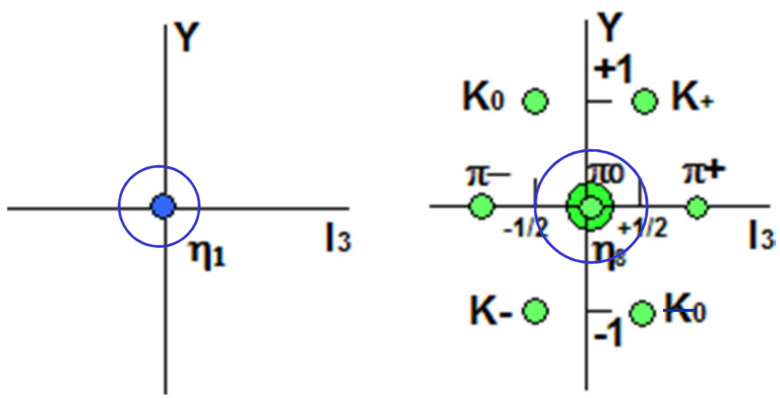
Example: Take pseudoscalars

$$\left|\mathbf{8};1,0\right\rangle = \frac{1}{\sqrt{2}}\left(u\bar{u} - d\bar{d}\right) \rightarrow \pi^0$$

Must be true because I-spin is a good symmetry

$$\left.\begin{array}{l} \left|\mathbf{8};0,0\right\rangle = \frac{1}{\sqrt{6}}\left(u\bar{u} + d\bar{d} - 2s\bar{s}\right) \\ \left|\mathbf{1};0,0\right\rangle = \frac{1}{\sqrt{2}}\left(u\bar{u} + d\bar{d} + s\bar{s}\right) \end{array}\right\}$$

Not identified
Get some insight
from decay modes



π^0, η_1, η_8 Central states, $I_3 = Y = Q = 0$

The Light Mesons - V

Use *SU(2)* shift operators: First, π^+

$I^- \left|\pi^+\right\rangle = \sqrt{2}\left|\pi^0\right\rangle$ From definition (and multiplet diagram)

From π^+ wave function:

$$I^- \left|\pi^+\right\rangle = I^- \left|u\bar{d}\right\rangle = \left|d\bar{d} - u\bar{u}\right\rangle \Rightarrow \pi^0 = -\frac{1}{\sqrt{2}}\left|d\bar{d} - u\bar{u}\right\rangle$$

Then re-define π^+ $as\ -u\bar{d} \rightarrow \pi^0 = \frac{1}{\sqrt{2}} I^- \pi^+$

Repeat for π^0:

$$I^- \pi^0 = \sqrt{2}\pi^- = I^- \frac{1}{\sqrt{2}}\left|u\bar{u} - d\bar{d}\right\rangle = \frac{1}{\sqrt{2}}\left|d\bar{u} + d\bar{u}\right\rangle \Rightarrow \pi^- = d\bar{u}$$

Isosinglet (with *u* and *d* only), is *h*:

$$I^- \eta = I^- \left(\frac{d\bar{d} + u\bar{u}}{\sqrt{2}}\right) = \frac{-d\bar{u} + d\bar{u}}{\sqrt{2}} = 0$$

Conclude the π^0 is an octet, don't know about η_1, η_8

The Bad News

Should *SU(3)* be exact, all particle states would fit to irr.reps

Try to apply mass formula to mesons: Use M^2 instead of M in the mass formula, for reasons not very convincing..

Free field Fermion Lagrangian: m Free field Boson Lagrangian: m^2

Assume the octet member is to be identified with a physical particle

Vectors

Predict

$$m_8^2 = \frac{1}{3}\left(4m_{K^*}^2 - m_\rho^2\right) \approx 0.859\ GeV^2 \quad \leftrightarrow \quad m_\omega^2 \simeq 0.613\ GeV^2, m_\varphi^2 \simeq 1.038\ GeV^2$$

Pseudoscalars

Predict

$$m_8^2 = \frac{1}{3}\left(4m_K^2 - m_\pi^2\right) \approx 0.321\ GeV^2 \quad \leftrightarrow \quad m_\eta^2 \simeq 0.299\ GeV^2,\ m_{\eta'}^2 \simeq 0.918\ GeV^2$$

All in all, not very brilliant…

Breaking Everywhere

Since *SU(3)* is broken, its eigenstates can mix:
Besides *intra*-multiplet (as before), consider *inter*-multiplet mixing
Call H_0 the *SU(3)* symmetric part of the Hamiltonian:

$$\langle 1|H_0|1\rangle = M_1, \ \langle 8|H_0|8\rangle = M_8$$

SU(3) breaking can manifest itself in a non-diagonal, singlet-octet mass matrix:

$$M^2 = \begin{pmatrix} M_1^2 & \Delta \\ \Delta & M_8^2 \end{pmatrix}$$

By standard diagonalization find the physical masses:

$$M_{a,b}^2 = \frac{M_1^2 + M_8^2}{2} \pm \sqrt{\frac{\left(M_1^2 - M_8^2\right)^2}{4} + \Delta^2}$$

Can infer M_1, Δ:

$$M_1^2 + M_8^2 = \frac{M_a + M_b}{2}$$

$$\Delta^2 = \frac{\left(M_a - M_b\right)^2 - \left(M_1 - M_8\right)^2}{4}$$

How Mixing is Measured

Try to make a sense out of *SU(3)* breaking
Simple idea: Central states of ***1,8*** just mix in physical particles

$$\begin{cases} |a\rangle = \sin\theta|1\rangle - \cos\theta|8\rangle \\ |b\rangle = \cos\theta|1\rangle + \sin\theta|8\rangle \end{cases}$$

'Rotation' of states: Must be unitary, phase preserving
→Just 1 angle

Find the mixing angle:

$$\left.\begin{array}{l} H|a\rangle = M_a|a\rangle \\ H|b\rangle = M_b|b\rangle \end{array}\right\} \rightarrow \left.\begin{array}{l} M_a^2 = M_1^2 - \Delta^2 \cot\theta = M_8^2 - \Delta^2 \tan\theta \\ M_b^2 = M_1^2 + \Delta^2 \cot\theta = M_8^2 + \Delta^2 \tan\theta \end{array}\right\} \rightarrow \tan^2\theta = \frac{M_b^2 - M_1^2}{M_b^2 - M_8^2} = \frac{M_8^2 - M_a^2}{M_1^2 - M_a^2}$$

$\theta_P = -11^0$ Pseudoscalars

$\theta_V = +38^0$ Vectors

$\theta_T = +32^0$ Tensors

Best observed in vector mesons:

$m_\omega \approx u\bar{u} + d\bar{d} \rightarrow \omega = 1/\sqrt{2}\left(u\bar{u} + d\bar{d}\right)$ ρ,ω only u, d quarks: OK mass degenerate

$m_\varphi \approx s\bar{s} \rightarrow \varphi = s\bar{s}$ φ only s quarks: OK decays modes

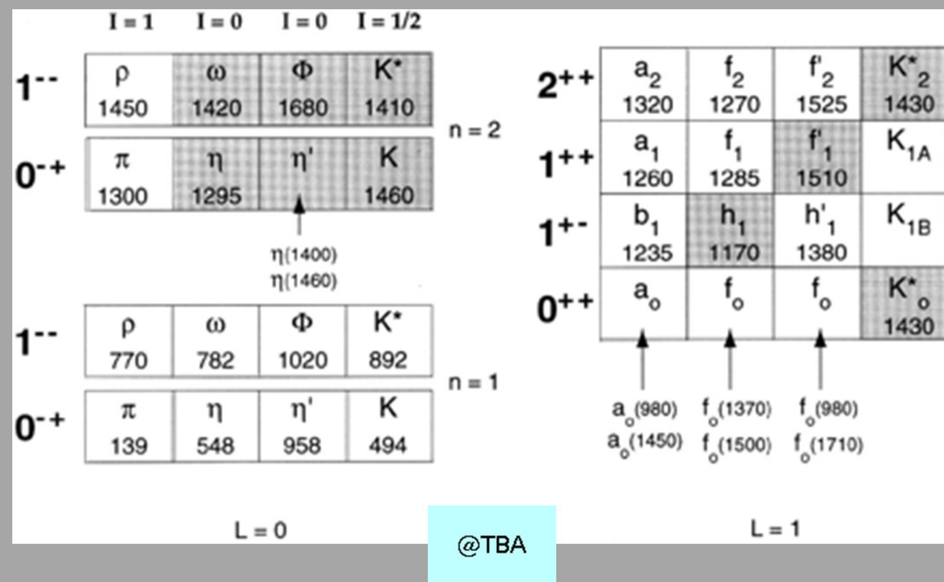
Higher Spin Mesons

Combine non relativistically ***L, S***:

L	*S*	*State*	J^{PC}
0	0	$^1S_0$	0^{-+}
0	1	$^3S_1$	1^{--}
1	0	$^1P_1$	1^{+-}
1	1	$^3P_0$	0^{++}
1	1	$^3P_1$	1^{++}
1	1	$^3P_2$	2^{++}
2	0	$^1D_2$	2^{-+}
2	1	$^3D_1$	1^{--}
2	1	$^3D_2$	2^{--}
2	1	$^3D_3$	3^{--}

Remarks:

States in grey can mix
C is meant for Q=S=0



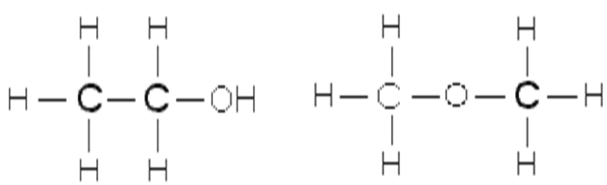
The Light Baryons - I

Combine 3 quarks: Get 3x3x3 = 27 combinations
But: Only 10 different quark contents

$$3+3\cdot 2+1=10: \quad uuu, ddd, sss, uud, uus, ddu, dds, ssu, ssd, uds$$

Remember:
Same composition does not imply same quantum state

Somewhat similar to difference between *raw* and *structural* formulae
Examples:



Same atomic content, 2 different chemicals

$$\frac{1}{\sqrt{2}}\left(|pn\rangle+|np\rangle\right) \quad \text{symmetric}$$

No bound states

Same nuclear content, 2 different states

$$\frac{1}{\sqrt{2}}\left(|pn\rangle-|np\rangle\right) \quad \text{antisymmetric}$$

One bound state

The Light Baryons - II

SU(3) Multiplets: ***1, 8, 8, 10***

What about different quark masses?
Well, that's all out of SU(3) *breaking*..

Reminder:

Quarks of different flavor to be taken as *different states of identical particles* (like electrons with spin up, down)
$\rightarrow$ Multi-quark states expected to have definite *exchange symmetry*

Can derive flavor exchange symmetry of each multiplet

1 – Singlet
Fully antisymmetric

8 – Two Octets
Undefined symmetry

10 – Decuplet
Fully symmetric

The Light Baryons - III

Now look at the remaining part of the wave function:

$|a\rangle = |space\rangle |spin\rangle | flavor\rangle$ NB: This expression is incomplete! See later

Space: Expect S-Wave → *Symmetric*
Difficult to guess an effective potential originating a ground state with L#0

Spin: Quarks are Fermions
Combine 3 spin ½:

$$1/2 \oplus 1/2 = \begin{cases} 0 \to 0 \oplus 1/2 = 1/2 & \text{2 sub-states} \\ 1 \to 1 \oplus 1/2 = 1/2, 3/2 & \text{2+4 sub-states} \end{cases}$$

→ Expect 1 quartet, 2 doublets

$$\left.\begin{array}{l} |3/2,+3/2\rangle = (\uparrow\uparrow\uparrow), \quad |3/2,-3/2\rangle = (\downarrow\downarrow\downarrow) \\ |3/2,+1/2\rangle = 1/\sqrt{3}(\uparrow\uparrow\downarrow + \uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow), \quad |3/2,-1/2\rangle = 1/\sqrt{3}(\downarrow\downarrow\uparrow + \downarrow\uparrow\downarrow + \uparrow\downarrow\downarrow) \end{array}\right\} \text{Quartet - Symmetric}$$

$$\left.|1/2,+1/2\rangle_A = 1/\sqrt{2}(\uparrow\downarrow - \downarrow\uparrow)\uparrow, \quad |1/2,-1/2\rangle_A = 1/\sqrt{2}(\uparrow\downarrow - \downarrow\uparrow)\downarrow\right\} \text{Doublet - Antisymmetric 1-2}$$

$$\left.|1/2,+1/2\rangle_S = 1/\sqrt{2}\uparrow(\uparrow\downarrow - \downarrow\uparrow), \quad |1/2,-1/2\rangle_S = 1/\sqrt{2}\downarrow(\uparrow\downarrow - \downarrow\uparrow)\right\} \text{Doublet - Antisymmetric 2-3}$$

The Light Baryons - IV

Can use another bit of group theory to write:

$$2 \otimes 2 \otimes 2 = 4 \oplus 2_S \oplus 2_A \quad \text{spin}$$

$$3 \otimes 3 \otimes 3 = 1 \oplus 8_S \oplus 8_A \oplus 10 \quad \text{flavor}$$

Summary of flavor, spin symmetry of different representations:

SU(3)

Flavor	Symmetry	Spin	Symmetry
$\boldsymbol{10_S}$	*S*	$\boldsymbol{4_S}$	*S*
$\boldsymbol{8_{M,S}}$	*n.a.; symmetric 1-2*	$\boldsymbol{2_{M,S}}$	*n.a.; symmetric 1-2*
$\boldsymbol{8_{M,A}}$	*n.a.; antisymmetric 1-2*	$\boldsymbol{2_{M,A}}$	*n.a.; antisymmetric 1-2*
$\boldsymbol{1_A}$	*A*		

SU(2)

Now combine flavor *and* spin:

S,A,M referring to *flavor*spin*

	$\boldsymbol{10_S}$	$\boldsymbol{8_{M,S}}$	$\boldsymbol{8_{M,A}}$	$\boldsymbol{1_A}$
$\boldsymbol{4_S}$	***(10,4) S***	***(8,4) M***	***(8,4) M***	***(1,4) A***
$\boldsymbol{2_{M,S}}$	***(10,2) M***	***(8,2) M***	***(8,2) M***	***(1,2) M***
$\boldsymbol{2_{M,A}}$	***(10,2) M***	***(8,2) M***	***(8,2) M***	***(1,2) M***

Singlet, Decuplet - I

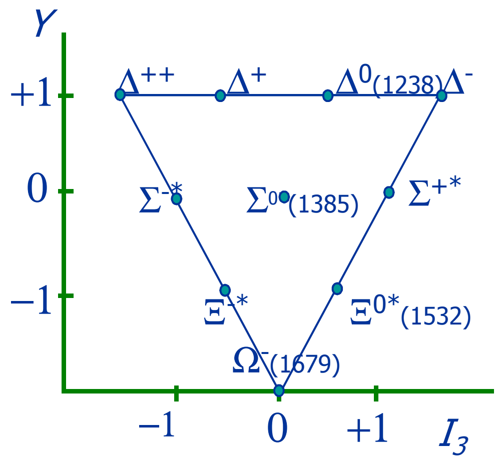
Observed multiplets

(SU(3), SU(2)) flavor spin	Flavor Wave-Function
Singlet: (***1***, ***?***) Tricky..	$\frac{1}{\sqrt{6}}(uds - usd + dsu - dus + sud - sdu)$
Decuplet: (***10, 4***) Astonishing..	$\begin{cases} uuu, ddd, sss, \frac{1}{\sqrt{6}}(uds + usd + dsu + dus + sud + sdu) \\ \frac{1}{\sqrt{3}}(ddu + dud + udd), \frac{1}{\sqrt{3}}(uud + udu + duu), \\ \frac{1}{\sqrt{3}}(dds + dsd + sdd), \frac{1}{\sqrt{3}}(uus + usu + suu), \\ \frac{1}{\sqrt{3}}(ssd + sds + dss), \frac{1}{\sqrt{3}}(ssu + sus + uss) \end{cases}$

Singlet, Decuplet - II

Most unexpected:

Total wave function appears to be exchange symmetric for decuplet!
Would expect it *anti-symmetric* for a bundle of identical fermions
Are we forgetting something in this game?



Baryon resonances, except Ω^-

Octet - I

Assume a globally *symmetric* wave-function for octet too:
Very difficult to account for a multiplet-dependent symmetry!
Guess the symmetric spin-flavor part:

Flavor: Two sets, 8 states each

$$\frac{1}{\sqrt{2}}(ud-du)d, \frac{1}{\sqrt{2}}(ud-du)u,$$
$$\frac{1}{\sqrt{2}}(ds-sd)d, \frac{1}{\sqrt{2}}(ds-sd)s,$$
$$\frac{1}{\sqrt{2}}(us-su)u, \frac{1}{\sqrt{2}}(us-su)s,$$
$$\frac{1}{2}\left[(us-su)d+(ds-sd)u\right],$$
$$\frac{1}{\sqrt{12}}\left[2(ud-du)s+(us-su)d-(ds-sd)u\right]$$
$$\varphi_{A12}^{(i)}, i=1,8$$

Antisymmetric 1↔2

$$\frac{1}{\sqrt{2}}d(ud-du), \frac{1}{\sqrt{2}}u(ud-du),$$
$$\frac{1}{\sqrt{2}}d(ds-sd), \frac{1}{\sqrt{2}}s(ds-sd),$$
$$\frac{1}{\sqrt{2}}u(us-su), \frac{1}{\sqrt{2}}s(us-su),$$
$$\frac{1}{2}\left[d(us-su)+u(ds-sd)\right],$$
$$\frac{1}{\sqrt{12}}\left[2s(ud-du)+d(us-su)-u(ds-sd)\right]$$
$$\varphi_{A23}^{(i)}, i=1,8$$

Antisymmetric 2↔3

Octet - II

Spin: Two sets, 2 states each

$$\boxed{\left.\begin{aligned} \left|1/2,+1/2\right\rangle_A &= 1\big/\sqrt{2}\left(\uparrow\downarrow-\downarrow\uparrow\right)\uparrow \\ \left|1/2,-1/2\right\rangle_A &= 1\big/\sqrt{2}\left(\uparrow\downarrow-\downarrow\uparrow\right)\downarrow \end{aligned}\right\}\chi_{A12}^{(j)},\ j=1,2}$$

$$\boxed{\left.\begin{aligned} \left|1/2,+1/2\right\rangle_S &= 1\big/\sqrt{2}\uparrow\left(\uparrow\downarrow-\downarrow\uparrow\right) \\ \left|1/2,-1/2\right\rangle_S &= 1\big/\sqrt{2}\downarrow\left(\uparrow\downarrow-\downarrow\uparrow\right) \end{aligned}\right\}\chi_{A23}^{(j)},\ j=1,2}$$

Can also write down a 3rd set of 8 (flavor) or 2 (spin) wave functions, antisymmetric wrt $1\leftrightarrow3$:

$\varphi_{A13}^{(i)}, i=1,8,\ \chi_{A13}^{(j)}, j=1,2$

Not independent from the former

Octet - III

Question:
What is the spin-flavor wave function of, say, a proton with spin up?
Answer:
Must consider all symmetric spin-flavor products with the proper quark content and s_z
The appropriate functions are n.*2* (flavor) and n.*1* (spin)

$$\varphi = \begin{cases} \varphi_{A12}^{(2)} = \frac{1}{\sqrt{2}}(ud - du)u \\ \varphi_{A23}^{(2)} = \frac{1}{\sqrt{2}}u(ud - du) \\ \varphi_{A13}^{(2)} = \frac{1}{\sqrt{2}}(uud - duu) \end{cases} \qquad \chi = \begin{cases} \chi_{A12}^{(1)} = \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)\uparrow \\ \chi_{A23}^{(1)} = \frac{1}{\sqrt{2}}\uparrow(\uparrow\downarrow - \downarrow\uparrow) \\ \chi_{A13}^{(1)} = \frac{1}{\sqrt{2}}(\uparrow\uparrow\downarrow - \downarrow\uparrow\uparrow) \end{cases}$$

Products:

$$\begin{cases} \varphi_{A12}^{(2)}\chi_{A12}^{(1)} = \frac{1}{\sqrt{2}}(ud - du)u\frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)\uparrow \\ \varphi_{A23}^{(2)}\chi_{A23}^{(1)} = \frac{1}{\sqrt{2}}u(ud - du)\frac{1}{\sqrt{2}}\uparrow(\uparrow\downarrow - \downarrow\uparrow) \\ \varphi_{A13}^{(2)}\chi_{A13}^{(1)} = \frac{1}{\sqrt{2}}(uud - duu)\frac{1}{\sqrt{2}}(\uparrow\uparrow\downarrow - \downarrow\uparrow\uparrow) \end{cases}$$

Octet - IV

In order to get a fully exchange-symmetric state, must take a linear combination of *all* contributions

$$\left|p,+1/2\right\rangle = \sum_{k=A12}^{A13} \varphi_k^{(2)} \chi_k^{(1)}$$

$$\left|p,+1/2\right\rangle = \frac{1}{\sqrt{3}}\left[\begin{array}{l} \frac{1}{\sqrt{2}}(ud-du)u\frac{1}{\sqrt{2}}(\uparrow\downarrow-\downarrow\uparrow)\uparrow \\ +\frac{1}{\sqrt{2}}u(ud-du)\frac{1}{\sqrt{2}}\uparrow(\uparrow\downarrow-\downarrow\uparrow) \\ +\frac{1}{\sqrt{2}}(uud-duu)\frac{1}{\sqrt{2}}(\uparrow\uparrow\downarrow-\downarrow\uparrow\uparrow) \end{array}\right]$$

Unconvinced? Still it's true, because any sum of 2 products is equivalent to the 3rd…

Octet - V

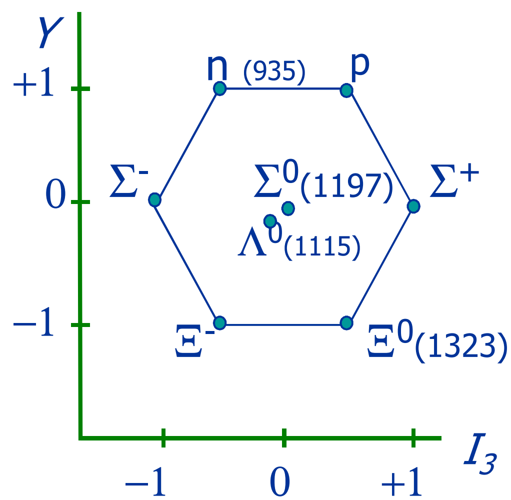
Finally: The *proton, spin up* wave function!

$$|p,+1/2\rangle = N\begin{pmatrix} 2u\uparrow d\downarrow u\uparrow + 2u\uparrow u\uparrow d\downarrow + 2d\downarrow u\uparrow u\uparrow \\ -u\downarrow d\uparrow u\uparrow - d\uparrow u\downarrow u\uparrow - u\uparrow u\downarrow d\uparrow - u\uparrow d\uparrow u\downarrow - u\downarrow u\uparrow d\uparrow - d\uparrow u\uparrow u\downarrow \end{pmatrix}$$

N = Normalization constant

$$N = \frac{1}{\sqrt{6\cdot 1^2 + 3\cdot 2^2}} = \frac{1}{\sqrt{6+12}} = \frac{1}{\sqrt{18}}$$

Baryon Octet



The Light Baryons: *SU(6)* - I

Flavor symmetry: *SU(3)*
Spin 1/2 : *SU(2)*

→Total symmetry: $SU(3)\otimes SU(2)$

Can think of extending to a larger group, *SU(6)*: Giant symmetry
SU(6) includes $SU(3)\otimes SU(2)$ as a *subgroup*

Just meaning *SU(6)* has *extra* transformations wrt $SU(3)\otimes SU(2)$:

Generic *SU(6)* operation can mix states sitting in *different* (flavor, spin) multiplets

Generic $SU(3)\otimes SU(2)$ operation only mixes states sitting in the *same* (flavor, spin) multiplet

The Light Hadrons: *SU(6)* - II

Observe: Situation similar to *SU(3) vs SU(2)⊗U(1)*

Different SU(2) multiplets grouped into a single SU(3) supermultiplet

Besides exchanging states within each *SU(2)* multiplet, *SU(6)* can exchange states among different *SU(2)* multiplets, within the same *SU(3)* representation

Mesons

$$\mathbf{6} \otimes \mathbf{6}^* = \mathbf{1} \oplus \mathbf{35}$$

$$SU(3) \otimes SU(2) \ \text{ content: } \ \mathbf{35} = \underbrace{\{\mathbf{1},\mathbf{3}\}}_{\text{3 states}} \oplus \underbrace{\{\mathbf{8},\mathbf{1}\}}_{\text{8 states}} \oplus \underbrace{\{\mathbf{8},\mathbf{3}\}}_{\text{24 states}}$$

Baryons

$$\mathbf{6} \otimes \mathbf{6} \otimes \mathbf{6} = \mathbf{20} \oplus \mathbf{56} \oplus \mathbf{70} \oplus \mathbf{70}$$

$$SU(3) \otimes SU(2) \ \text{ content: } \ \mathbf{56} = \underbrace{\{\mathbf{8},\mathbf{2}\}}_{\text{16 states}} \oplus \underbrace{\{\mathbf{10},\mathbf{4}\}}_{\text{40 states}} \ \ \text{Symmetric!}$$

Summary: Decuplet

State	*Q*	I_3	*I*	*S*	$J^{PC}=3/2^+$
uuu	+2	+3/2	3/2	0	Δ^{++}
$1/\sqrt{3}\left(uud+udu+duu\right)$	+1	+1/2	3/2	0	Δ^{+}
$1/\sqrt{3}\left(udd+dud+duu\right)$	0	-1/2	3/2	0	Δ^{0}
ddd	-1	-1/2	3/2	0	Δ^{-}
$1/\sqrt{3}\left(uus+usu+suu\right)$	+1	+1	1	-1	Σ^{*+}
$1/\sqrt{6}\left(uds+sud+dsu+sdu+dus+usd\right)$	0	0	1	-1	Σ^{*0}
$1/\sqrt{3}\left(dds++dsd+sdd\right)$	-1	-1	1	-1	Σ^{*-}
$1/\sqrt{3}\left(uss+sus+ssu\right)$	0	+1/2	1/2	-2	Ξ^{*0}
$1/\sqrt{3}\left(dss+sds+ssd\right)$	-1	-1/2	1/2	-2	Ξ^{*-}
sss	-1	0	0	-3	Ω^{-}

Wave functions

Summary: Octet

Quarks	Q	I_3	I	S	$J^{PC}=1/2^+$
uud	+1	+1/2	1/2	0	p
udd	0	-1/2	1/2	0	n
dds	-1	-1	1	-1	Σ^-
uds	0	0	1,0	-1	Σ^0, Λ^0
uus	+1	+1	1	-1	Σ^-
dss	-1	-1/2	1/2	-2	Ξ^-
uss	0	+1/2	1/2	-2	Ξ^0

Quark content only
(no wave function)

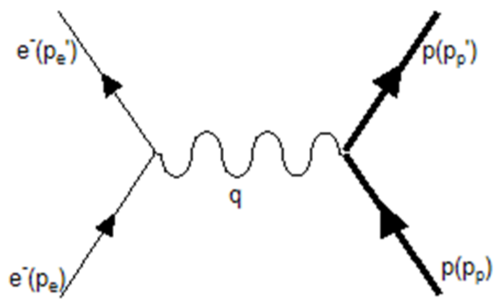
The *e-p* Effective Interaction - I

Go for some dynamics…

Examine first electron-positron bound states: *Positronium*
Somewhat similar to mesons: *Particle-antiparticle bound state*

Can be dealt with by use of non-relativistic potential models
Useful insight by connecting to perturbation theory..

Start first from electron-proton interaction:



$$T_{fi} = e^2 \frac{\left(\overline{u}\left(p_e^{'}\right)\gamma^{\mu}u\left(p_e\right)\right)\left(\overline{u}\left(p_p{'}\right)\gamma_{\mu}u\left(p\right)\right)}{q^2}$$

The *e-p* Effective Interaction - II

Expand matrix element to low speed approximation

Get a non-relativistic matrix element, where χ, χ' are 2-dimensional (Pauli) spinors for electron and proton

The Bottom Line:

At low speed/energy we can neglect radiation, pair production (real & virtual)

$\rightarrow$Left with corrections:

Relativistic Energy/Momentum
Magnetic Moments
More

The *e-p* Effective Interaction - III

Transition matrix element:

$$T_{fi} \simeq -\frac{e^2}{q^2}\left[1-\frac{\mathbf{p}_e^2+\mathbf{p}_e^{'2}}{8m_e^2}\right]\left[1-\frac{\mathbf{p}_p^2+\mathbf{p}_p^{'2}}{8m_p^2}\right]\cdot$$

$$\left\{\underbrace{\tilde{\chi}^{'\dagger}\left[1+\frac{\mathbf{p}_p^{'}\cdot\mathbf{p}_p+i\boldsymbol{\sigma}\cdot\left(\mathbf{p}_p^{'}\times\mathbf{p}_p\right)}{4m_p^2}\right]\tilde{\chi}}_{\text{time section, p 4-current}}\underbrace{\chi^{'\dagger}\left[1+\frac{\mathbf{p}_e^{'}\cdot\mathbf{p}_e+i\boldsymbol{\sigma}\cdot\left(\mathbf{p}_e^{'}\times\mathbf{p}_e\right)}{4m_e^2}\right]\chi}_{\text{time section, e 4-current}}+\right.$$

$$\left.-\underbrace{\tilde{\chi}^{'\dagger}\left[\frac{\mathbf{p}_p^{'}+\mathbf{p}_p-i\boldsymbol{\sigma}\times\left(\mathbf{p}_p^{'}-\mathbf{p}_p\right)}{2m_p}\right]\tilde{\chi}}_{\text{space section. p 4-current}}\cdot\underbrace{\chi^{'\dagger}\left[\frac{\mathbf{p}_e^{'}+\mathbf{p}_e-i\boldsymbol{\sigma}\times\left(\mathbf{p}_e^{'}-\mathbf{p}_e\right)}{2m_e}\right]\chi}_{\text{space section, e 4-current}}\right\}$$

The *e-p* Effective Interaction - IV

Reminder from potential scattering:
Transition amplitude = Fourier transform of the effective potential
→Get effective *e-p* potential by anti-transforming the amplitude

$$V_C = -\frac{e^2}{r}\left(\tilde{\chi}'^{\dagger}\,\tilde{\chi}\right)\left(\chi'^{\dagger}\,\chi\right) \qquad \text{Coulomb term}$$

$$\left.\begin{aligned} V_{SO} &= \frac{e^2}{4m_e^2 r^3}\left(\tilde{\chi}'^{\dagger}\,\tilde{\chi}\right)\left(\chi'^{\dagger}\,\boldsymbol{\sigma}\cdot\mathbf{L}\chi\right) \qquad \text{Spin-orbit} \\ V_D &= \frac{e^2}{8m_e^2}4\pi\delta^{(3)}(\mathbf{r})\left(\tilde{\chi}'^{\dagger}\,\tilde{\chi}\right)\left(\chi'^{\dagger}\,\chi\right) \qquad \text{'Darwin term'} \end{aligned}\right\} \text{Fine structure terms}$$

$$V_{dip-dip} = \underbrace{\frac{8\pi e^2}{3m_e m_p} g_p \mathbf{s}_e\cdot\mathbf{s}_p\delta^{(3)}(\mathbf{r})}_{\text{Hyperfine structure}} + \underbrace{\frac{e^2}{m_e m_p r^3} g_p\left[3\left(\mathbf{s}_e\cdot\mathbf{r}\right)\left(\mathbf{s}_p\cdot\mathbf{r}\right)-\mathbf{s}_e\cdot\mathbf{s}_p\right]}_{\text{Tensor interaction}} \qquad \text{Dipole-dipole interaction}$$

Valid for *S* states

Astonishing: Everything included in our modest 1-photon diagram…

The *e-p* Effective Interaction - V

Effect of hyperfine interaction on ground state energy:

$$\left\langle \frac{8\pi e^2}{3m_e m_p} g_p \mathbf{s}_e \cdot \mathbf{s}_p \delta^{(3)}(\mathbf{r}) \right\rangle_{\text{ground state}}$$

$$\rightarrow \Delta E_{hyp} = \frac{8\pi e^2}{3m_e m_p} g_p \cdot \frac{1}{2}\left[j(j+1) - s_e(s_e+1) - s_p(s_p+1) \right] \cdot |\psi(0)|^2$$

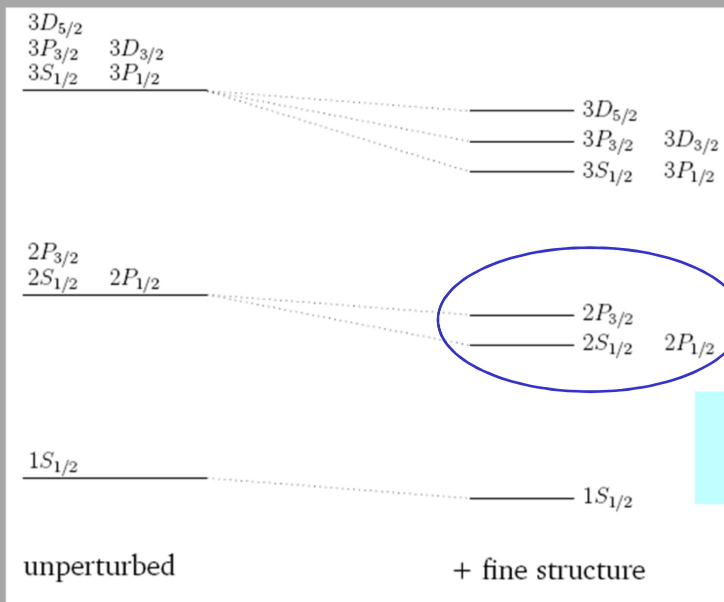
$$|\psi(0)|^2 = \frac{(m_e\alpha)^3}{\pi} \rightarrow \Delta E_{hyp} = \frac{8\pi e^2}{3m_e m_p} g_p \frac{1}{2} \frac{(m_e\alpha)^3}{\pi}\left[j(j+1) - \frac{3}{4} - \frac{3}{4} \right]$$

$$\rightarrow \Delta E_{hyp} = \frac{4}{3} g_p \frac{e^2}{m_e m_p}\left[j(j+1) - \frac{3}{2} \right](m_e\alpha)^3$$

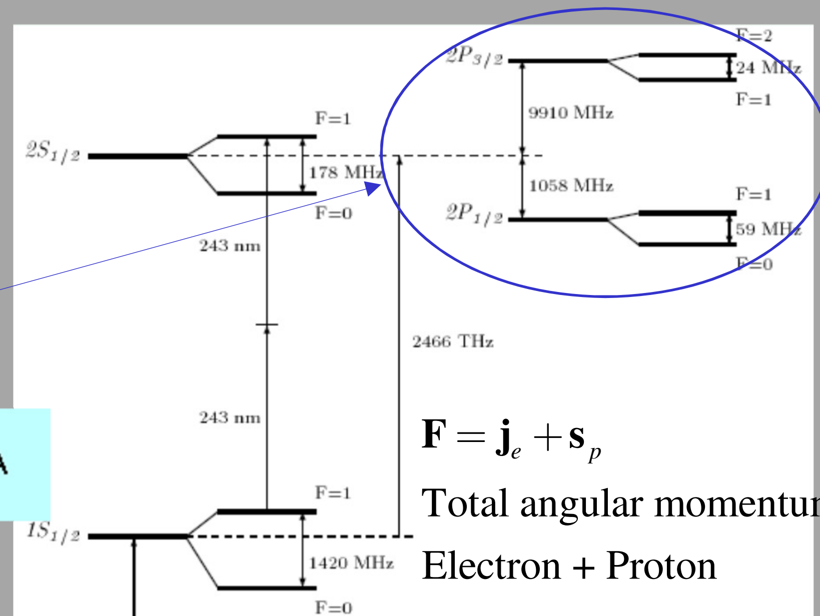
$$\Delta E_{hyp} = \begin{cases} \dfrac{4}{3} g_p \dfrac{e^2 m_e}{m_p}(2 - 3/2)\left(m_e\alpha^3\right) = \dfrac{4}{3} g_p \dfrac{m_e}{m_p}\dfrac{1}{2}\left(m_e\alpha^4\right) & \text{hyperfine shift -triplet} \\ \dfrac{4}{3} g_p \dfrac{2e^2 m_e}{m_p}(0 - 3/2)\left(m_e\alpha^3\right) = -\dfrac{12}{3} g_p \dfrac{1}{2}\dfrac{m_e}{m_p}\left(m_e\alpha^4\right) & \text{hyperfine shift - singlet} \end{cases}$$

$$\rightarrow \Delta\left(\Delta E_{hyp}\right)_{\text{triplet-singlet}} = \frac{8}{3} g_p \frac{m_e}{m_p}\left(m_e\alpha^4\right)$$

H: Fine & Hyperfine Structure



@TBA



$\mathbf{F} = \mathbf{j}_e + \mathbf{s}_p$

Total angular momentum

Electron + Proton

$$\Delta E_{l,1/2;j,m_j} = E_n \frac{\alpha^2}{n^2}\left(\frac{n}{j+1/2} - \frac{3}{4}\right).$$

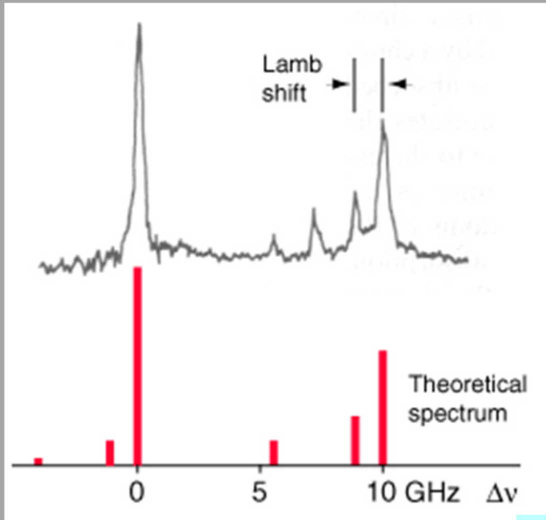
$$\Delta E = \frac{\mu_0 \, g_p \, e^2}{8\pi \, m_p \, m_e}\left\langle \frac{3\,(\mathbf{S}_p\cdot\hat{\mathbf{r}})\,(\mathbf{S}_e\cdot\hat{\mathbf{r}}) - \mathbf{S}_p\cdot\mathbf{S}_e}{r^3}\right\rangle + \frac{\mu_0 \, g_p \, e^2}{3\, m_p \, m_e}\langle \mathbf{S}_p\cdot\mathbf{S}_e\rangle \, |\psi(0)|^2.$$

Fine structure:
Spin-Orbit+Relativistic+Darwin
Splits *j* sublevels

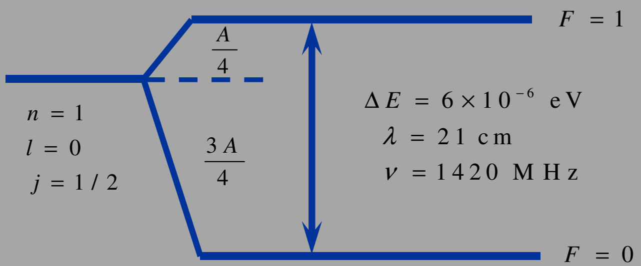
Hyperfine structure:
Dipole-Dipole
Splits *F* sublevels

Hyperfine Splitting of Hydrogen

Ground state

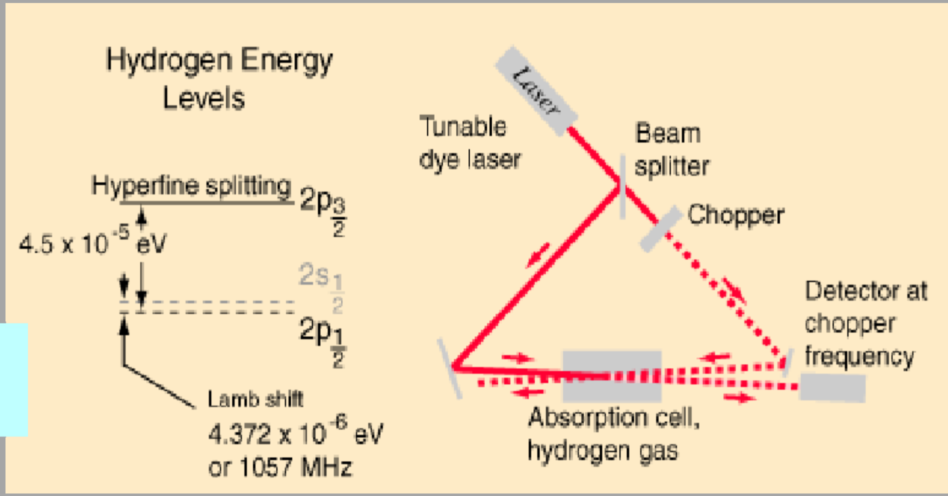


Hydrogen fine structure and hyperfine structure for the n=3 ► 2 transition.
(After Ohanian, Modern Physics, Ch 7., spectrum from T. W. Hansch, Stanford Univ.)



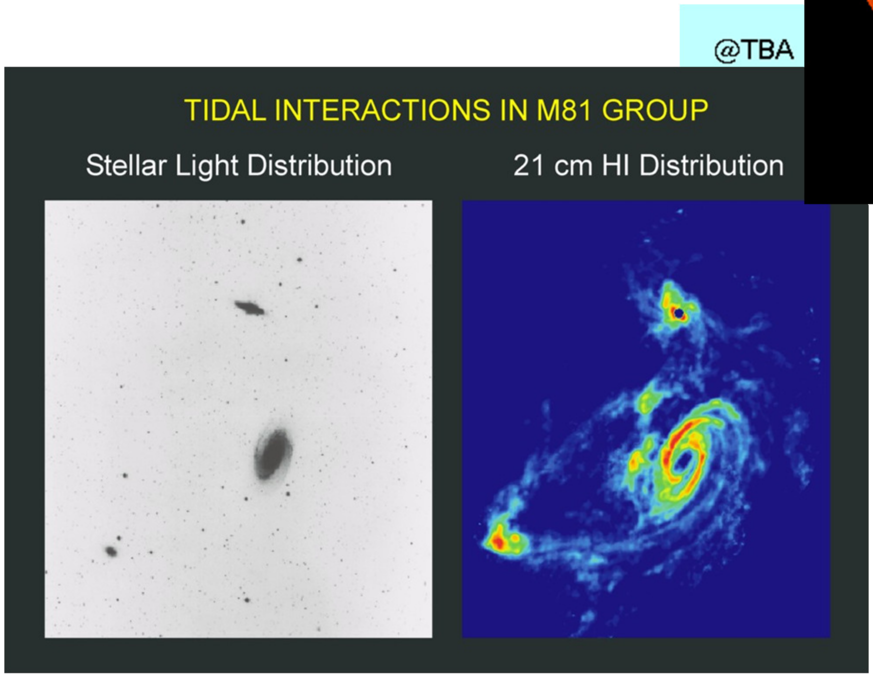
Ground state, ***L=0***

@TBA



The 21 cm Line: A Cosmic Tune

Important radio-astronomical tool: *Mapping the Hydrogen content of the Universe*

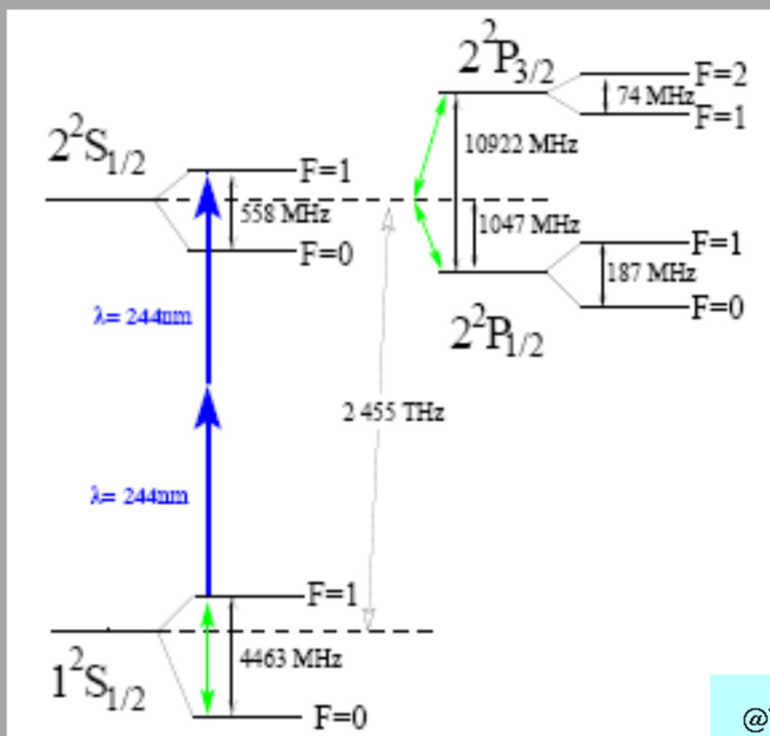


Lots of physics and cosmology..
Example:
How is the transition excited?

A measurement of the galactic/ intergalactic temperature

Muonium

μ^+-e^- 'atom'

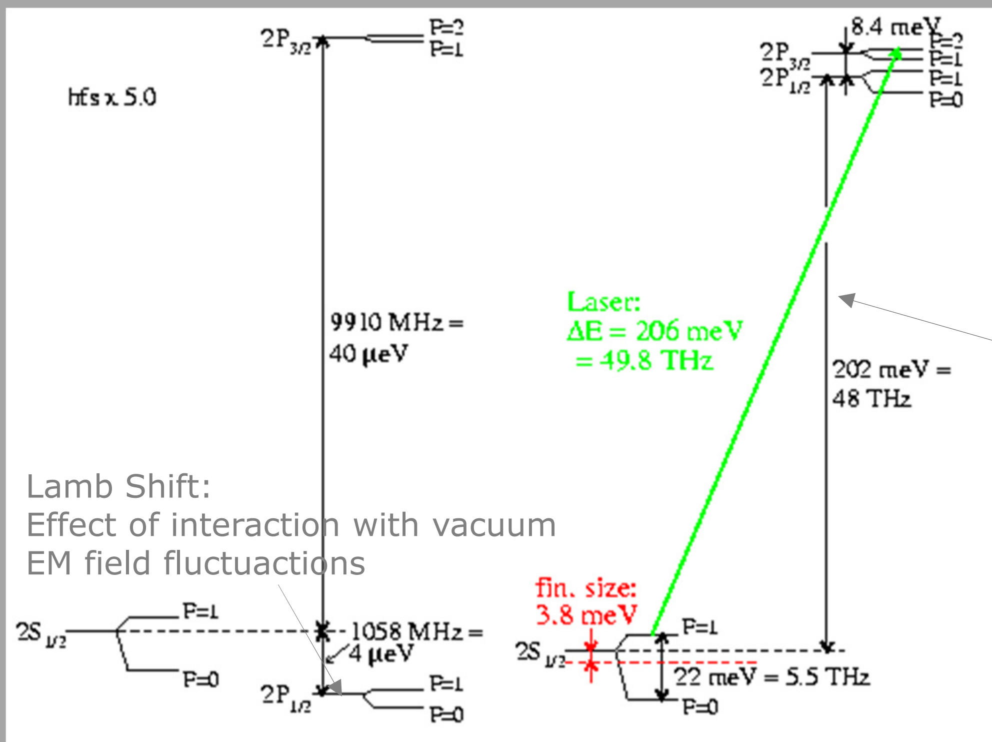


@TBA

Similar to Hydrogen

Different:
Reduced mass
Muon magnetic moment

Muonic Hydrogen



Standard *H*

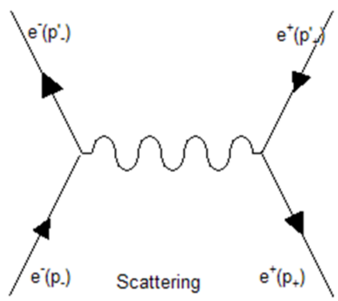
Muonic *H*
Replace e^- by μ^-

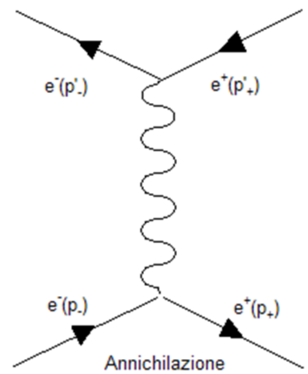
Simplest μ-mesic atom
Made by stopping μ^- in hydrogenated matter

Huge Lamb shift:
~ 45000 x Hydrogen!
~ $(m_m/m_e)^2$

Positronium - I

There are now 2 diagrams:



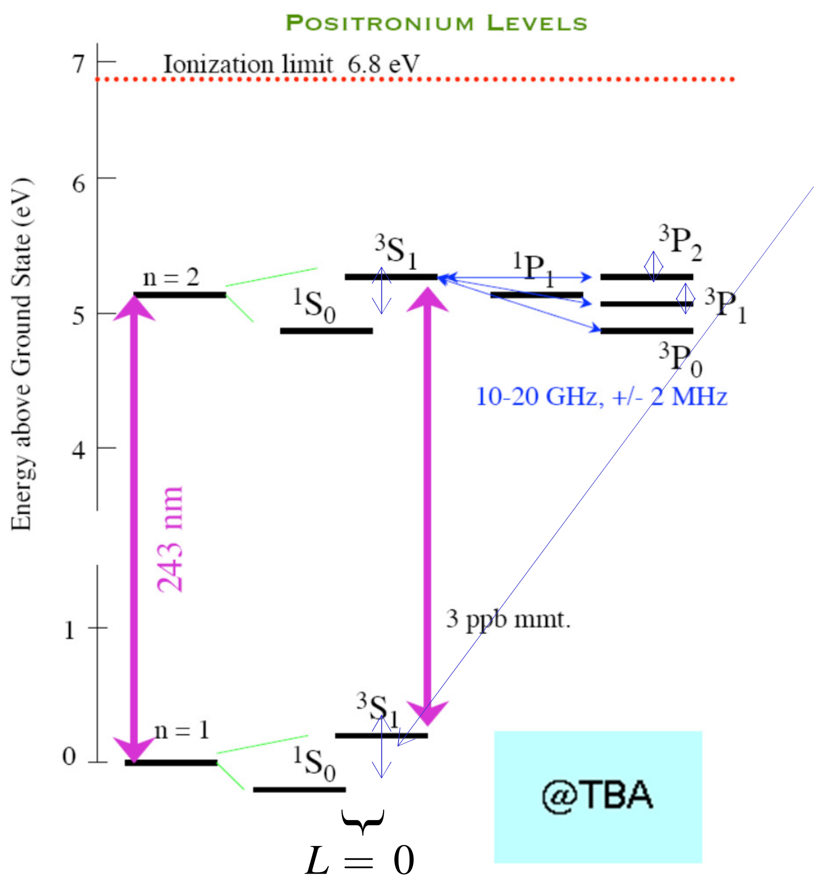


$$\mathcal{T}_{fi} = e^2 \left[-\frac{\left(\overline{u}\left(p_-^{'}\right)\gamma^\mu u\left(p_-\right)\right)\left(\overline{v}\left(p_+\right)\gamma_\mu v\left(p_+^{'}\right)\right)}{\left(p_- - p_-'\right)^2} + \frac{\left(\overline{v}\left(p_+\right)\gamma^\mu u\left(p_-\right)\right)\left(\overline{u}\left(p_-^{'}\right)\gamma_\mu v\left(p_+^{'}\right)\right)}{\left(p_+ + p_-\right)^2} \right]$$

Net effect: Add another term to the effective interaction

$V_A = \frac{e^2\pi}{2m^2}\left(3 + \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2\right)\delta^{(3)}\left(\mathbf{r}\right)$ Same structure as hyperfine term

Positronium - II



Form of hyperfine term:

$$\Delta E_{hyp} = \left\langle \frac{8\pi e^2}{3m_e^2} \mathbf{s}_{e^-} \cdot \mathbf{s}_{e^+} \delta^{(3)}(\mathbf{r}) \right\rangle$$

Ground state

More complicated for *n>1, l>0*

Observe:
Levels labeled by ${}^S L_J$
S: *Total* spin

Previous pictures:
Levels labeled by ${}^S L_J$
S: *Electron* spin
Proton spin only in hyperfine term

Masses - I

Observe large mass splitting between singlet and triplet mesons:

Guess effective strong interaction has some term similar to hyperfine electromagnetic

$$\Delta E = \frac{A}{m_1 m_2}\left(\mathbf{S}_1 \cdot \mathbf{S}_2\right)$$

Then expect for the hadron mass:

$$M = m_1 + m_2 + A\frac{\left(\mathbf{S}_1 \cdot \mathbf{S}_2\right)}{m_1 m_2}$$

$$\mathbf{J} = \mathbf{S}_1 + \mathbf{S}_2 \rightarrow J^2 = S_1^2 + S_2^2 + 2\mathbf{S}_1 \cdot \mathbf{S}_2$$

$$\rightarrow \mathbf{S}_1 \cdot \mathbf{S}_2 = 1/2\left(J^2 - S_1^2 - S_2^2\right) = 1/2\left(J\left(J+1\right) - 2S\left(S+1\right)\right)$$

$$\mathbf{S}_1 \cdot \mathbf{S}_2 = \begin{cases} +1/4 & \text{triplets} \\ \text{-}3/4 & \text{singlets} \end{cases}$$

Masses - II

1) About the expected, large hyperfine splitting:

Can be shown to be true, to some extent..

When perturbative expansion can be granted, color quark-(anti)quark interaction in the static limit yields a *chromomagnetic term* with the proper hyperfine structure

2) About the quark masses:

m_1, m_2 *constituent* quark mass

Somewhat difficult idea, basically similar to *effective mass* for electrons bound in a crystal
Different from the *current*, i.e. the free quark mass
Will be (somewhat) clarified when discussing QCD.

Masses - III

Free parameter counting:
3 quark masses (m_u,m_d,m_s)+1 constant A
Hope to fit 7 meson masses:
Pseudoscalars + Vectors

$\rightarrow$ *Go for a 3 constraints fit*

Meson	ΔE_{HF}	Fitted mass (MeV)
p	$-\frac{3a}{m_u^2}$	140
K	$-\frac{3a}{m_u m_s}$	485
η	$-\frac{a}{m_u^2}-\frac{2a}{m_s^2}$	559
ρ,ω	$\frac{a}{m_u^2}$	780
K^*	$\frac{a}{m_u m_s}$	896
ϕ	$\frac{a}{m_s^2}$	1032

Results:

$$m_u = m_d \simeq 310 \quad MeV$$
$$m_s \simeq 483 \quad MeV$$
$$A \simeq 160 \; m_{u,d}^2 \quad MeV^3$$

Masses - IV

Extend the idea to baryons: Sum over 3 quark pairs

$$m(q_1,q_2,q_3)=\sum_{i=1}^{3} m_i + A'\frac{1}{2}\sum_{i,j=1}^{3}\frac{\mathbf{S}_i\cdot\mathbf{S}_j}{m_i m_j}$$

As an exercise, first neglect differences between quark masses:

$$\mathbf{J}=\mathbf{S}_1+\mathbf{S}_2+\mathbf{S}_3 \to J^2=\left(\mathbf{S}_1+\mathbf{S}_2+\mathbf{S}_3\right)^2$$

$$=S_1^2+S_2^2+S_3^2+2\left(\mathbf{S}_1\cdot\mathbf{S}_2+\mathbf{S}_1\cdot\mathbf{S}_3+\mathbf{S}_2\cdot\mathbf{S}_3\right)$$

$$S^2=S\left(S+1\right)=3/4\to S_1^2+S_2^2+S_3^2=9/4$$

$$\to \mathbf{S}_1\cdot\mathbf{S}_2+\mathbf{S}_1\cdot\mathbf{S}_3+\mathbf{S}_2\cdot\mathbf{S}_3=1/2\left[J^2-9/4\right]=1/2\,J\left(J+1\right)-9/4$$

$$\to \frac{1}{2}\sum_{i,j=1}^{3}\mathbf{S}_i\cdot\mathbf{S}_j=\begin{cases}+3/4 & j=3/2 \text{ decuplet}\\ -3/4 & j=1/2 \text{ octet}\end{cases}$$

Masses - V

Now take into account different quark masses:

$$m(q_1, q_2, q_3) = \sum_{i=1}^{3} m_i + A' \frac{1}{2} \sum_{i,j=1}^{3} \frac{\mathbf{S}_i \cdot \mathbf{S}_j}{m_i m_j}$$

Must consider quark pairs in order to evaluate the hyperfine contribute

$$J_{ik}^2 = \left(\mathbf{S}_i + \mathbf{S}_k\right)^2 = S_i^2 + S_k^2 + 2\mathbf{S}_i \cdot \mathbf{S}_k$$

$$\rightarrow \mathbf{S}_i \cdot \mathbf{S}_k = 1/2\left[J_{ik}\left(J_{ik}+1\right) - S_i\left(S_i+1\right) - S_k\left(S_k+1\right)\right]$$

Quarks i, k in a spin triplet state:

$$\mathbf{S}_i \cdot \mathbf{S}_k = 1/2\left[1(1+1) - 1/2(1/2+1) - 1/2(1/2+1)\right]$$

$$\rightarrow \mathbf{S}_i \cdot \mathbf{S}_k = 1/4$$

Quarks i, k in a spin singlet state:

$$\mathbf{S}_i \cdot \mathbf{S}_k = 1/2\left[0(0+1) - 1/2(1/2+1) - 1/2(1/2+1)\right]$$

$$\rightarrow \mathbf{S}_i \cdot \mathbf{S}_k = -3/4$$

Masses - VI

N: Only u, d quarks $\rightarrow$ Same mass

$$\rightarrow m_N = 3m_u - \frac{3A'}{4m_u^2}, \quad m_u = m_d$$

Λ: u, d spin & isospin singlet

$$m_\Lambda = 2m_u + m_s + A'\left(\frac{\mathbf{S}_u \cdot \mathbf{S}_d}{m_u m_d} + \frac{\mathbf{S}_u \cdot \mathbf{S}_s}{m_u m_s} + \frac{\mathbf{S}_s \cdot \mathbf{S}_d}{m_s m_d}\right)$$

$$m_\Lambda = 2m_u + m_s + A'\left(\frac{\mathbf{S}_u \cdot \mathbf{S}_d}{m_u^2} + \frac{\mathbf{S}_u \cdot \mathbf{S}_s + \mathbf{S}_s \cdot \mathbf{S}_d}{m_u m_s}\right)$$

$$\mathbf{S}_u \cdot \mathbf{S}_s + \mathbf{S}_s \cdot \mathbf{S}_d = \frac{1}{2}\sum_{i,j=1}^{3} \mathbf{S}_i \cdot \mathbf{S}_j - \mathbf{S}_u \cdot \mathbf{S}_d = -3/4 - \left(-3/4\right) = 0$$

$$\rightarrow m_\Lambda = 2m_u + m_s - \frac{3A'}{4m_u^2}$$

Masses - VII

Σ: u,d spin & isospin triplet

$$m_\Sigma = 2m_u + m_s + A'\left(\frac{\mathbf{S}_u\cdot\mathbf{S}_d}{m_u m_d} + \frac{\mathbf{S}_u\cdot\mathbf{S}_s}{m_u m_s} + \frac{\mathbf{S}_s\cdot\mathbf{S}_d}{m_s m_d}\right)$$

$$m_\Sigma = 2m_u + m_s + A'\left(\frac{\mathbf{S}_u\cdot\mathbf{S}_d}{m_u^2} + \frac{\mathbf{S}_u\cdot\mathbf{S}_s + \mathbf{S}_s\cdot\mathbf{S}_d}{m_u m_s}\right)$$

$$\mathbf{S}_u\cdot\mathbf{S}_s + \mathbf{S}_s\cdot\mathbf{S}_d = \frac{1}{2}\sum_{i,j=1}^{3}\mathbf{S}_i\cdot\mathbf{S}_j - \mathbf{S}_u\cdot\mathbf{S}_d = -3/4 - (+1/4) = -1 \rightarrow m_\Sigma = 2m_u + m_s + A'\left(\frac{1}{4m_u^2} - \frac{1}{m_u m_s}\right)$$

Ξ: $s1, s2$ spin triplet

Why? Flavor = ss Symmetric $\rightarrow$ Spin must be symmetric too

$$m_\Xi = 2m_s + m_u + A'\left(\frac{\mathbf{S}_u\cdot\mathbf{S}_{s1}}{m_u m_s} + \frac{\mathbf{S}_u\cdot\mathbf{S}_{s2}}{m_u m_s} + \frac{\mathbf{S}_{s1}\cdot\mathbf{S}_{s2}}{m_s^2}\right)$$

$$m_\Xi = 2m_s + m_u + A'\left(\frac{\mathbf{S}_{s1}\cdot\mathbf{S}_{s2}}{m_s^2} + \frac{\mathbf{S}_u\cdot\mathbf{S}_{s1} + \mathbf{S}_u\cdot\mathbf{S}_{s2}}{m_u m_s}\right)$$

$$\mathbf{S}_u\cdot\mathbf{S}_{s1} + \mathbf{S}_u\cdot\mathbf{S}_{s2} = \frac{1}{2}\sum_{i,j=1}^{3}\mathbf{S}_i\cdot\mathbf{S}_j - \mathbf{S}_{s1}\cdot\mathbf{S}_{s2} = -3/4 - 1/4 = -1 \rightarrow m_\Xi = 2m_s + m_u + A'\left(\frac{1}{4m_s^2} - \frac{1}{m_u m_s}\right)$$

Masses - VIII

Fit all octet + decuplet:
8 masses $\rightarrow$ 4 constraints

Interesting questions:
Is A = A' ?
Are the quark masses the same in mesons as in baryons?

$$m_u = m_d \simeq 363 \quad MeV$$

$$m_s \simeq 538 \quad MeV$$

$$A' \simeq 50 \; m_{u,d}^2 \quad MeV^3$$

Baryons vs. Mesons:

Masses ~*+50 MeV ~10%* higher
Constant *~1/3* Hyperfine splitting reduced

Baryon	ΔE^{HF}	Fitted mass (MeV)
N(938)	$-\frac{3a'}{m_{u,d}^2}$	939
Λ(1116)	$-\frac{3a'}{m_{u,d}^2}$	1114
Σ(1193)	$\frac{a'}{m_{u,d}^2}-\frac{4a'}{m_u m_s}$	1179
Ξ(1318)	$\frac{a'}{m_{u,d}^2}-\frac{4a'}{m_u m_s}$	1327
Δ(1232)	$+\frac{3a'}{m_{u,d}^2}$	1239
Σ(1384)*	$\frac{a'}{m_{u,d}^2}+\frac{4a'}{m_u m_s}$	1381
Ξ(1533)*	$\frac{a'}{m_{u,d}^2}-\frac{4a'}{m_u m_s}$	1529
Ω(1672)	$+\frac{3a'}{m_s^2}$	1682

Magnetic Moments - I

Take total dipole moment operator as the sum of the single quark operators:

NB: Can this be really granted??

$$\boldsymbol{\mu} = \sum_{i=1}^{3} \boldsymbol{\mu}_i \rightarrow \mu_p = \langle p,+1/2 | \mu | p,+1/2 \rangle = \langle p,+1/2 | (\mu_1 + \mu_2 + \mu_3) | p,+1/2 \rangle$$

Each operator acting on the corresponding factor of the wave function

$$|p,+1/2\rangle = \frac{1}{\sqrt{18}} \begin{pmatrix} 2u\uparrow u\uparrow d\downarrow +2d\downarrow u\uparrow u\uparrow +2u\uparrow d\downarrow u\uparrow - \\ -u\downarrow d\uparrow u\uparrow -u\uparrow u\downarrow d\uparrow -u\downarrow u\uparrow d\uparrow - \\ -d\uparrow u\downarrow u\uparrow -u\uparrow d\uparrow u\downarrow -d\uparrow u\uparrow u\downarrow \end{pmatrix}$$

Magnetic Moments - II

Some really dull algebra:

$$4\langle u\uparrow u\uparrow d\downarrow|(\mu_1+\mu_2+\mu_3)|u\uparrow u\uparrow d\downarrow\rangle = 4\left[\langle u|\mu_1|u\rangle+\langle u|\mu_2|u\rangle-\langle d|\mu_3|d\rangle\right]$$

$$= 4[\mu_u+\mu_u-\mu_d] = 8\mu_u-4\mu_d$$

$$4\langle d\downarrow u\uparrow u\uparrow|(\mu_1+\mu_2+\mu_3)|d\downarrow u\uparrow u\uparrow\rangle$$

$$= 4\langle u\uparrow d\downarrow u\uparrow|(\mu_1+\mu_2+\mu_3)|u\uparrow d\downarrow u\uparrow\rangle = 8\mu_u-4\mu_d$$

$$\langle u\downarrow d\uparrow u\uparrow|(\mu_1+\mu_2+\mu_3)|u\downarrow d\uparrow u\uparrow\rangle = \langle u\uparrow u\downarrow d\uparrow|(\mu_1+\mu_2+\mu_3)|u\uparrow u\downarrow d\uparrow\rangle$$

$$\langle d\uparrow u\downarrow u\uparrow|(\mu_1+\mu_2+\mu_3)|d\uparrow u\downarrow u\uparrow\rangle = \ldots = \mu_d$$

$$\rightarrow \langle p,+1/2|(\mu_1+\mu_2+\mu_3)|p,+1/2\rangle = \frac{1}{18}\left[3(8\mu_u-4\mu_d)+6\mu_d\right] = \frac{1}{18}[24\mu_u-6\mu_d]$$

$$\rightarrow \mu_p \equiv \frac{1}{3}(4\mu_u-\mu_d)$$

Then take neutron: Just swap $u \leftrightarrow d$

$$|n,+1/2\rangle = \frac{1}{\sqrt{18}}\begin{pmatrix} 2d\uparrow d\uparrow u\downarrow+2u\downarrow d\uparrow d\uparrow+2d\uparrow u\downarrow d\uparrow- \\ -d\downarrow u\uparrow d\uparrow-d\uparrow d\downarrow u\uparrow-d\downarrow d\uparrow u\uparrow- \\ -u\uparrow d\downarrow d\uparrow-d\uparrow u\uparrow d\downarrow-u\uparrow d\uparrow d\downarrow \end{pmatrix} \rightarrow \mu_n = \frac{1}{3}(4\mu_d-\mu_u)$$

Magnetic Moments - III

Take quarks as Dirac particles:Can this be really granted??

$$\mu = \frac{e}{2m}$$

$$\rightarrow \mu_u = \frac{2}{3}\frac{e}{2m_u}, \mu_d = -\frac{1}{3}\frac{e}{2m_d}, \mu_s = -\frac{1}{3}\frac{e}{2m_s}$$

Predict moment ratio:

$$\frac{\mu_n}{\mu_p} = \frac{4\mu_d - \mu_u}{4\mu_u - \mu_d} = \frac{4\left(-\frac{e}{3\cdot 2m_d}\right) - \frac{2e}{3\cdot 2m_u}}{4\frac{2e}{3\cdot 2m_u} - \left(-\frac{e}{3\cdot 2m_d}\right)} \simeq \frac{-4e-2e}{8e+e} = -\frac{2}{3} \approx -0.667$$

Experimental ratio:

$$\frac{\mu_n}{\mu_p} \approx -0.685 \quad \text{Amazingly close!}$$

Absolute moments difficult to estimate, as involving unknown quark mass.
Nevertheless..

Magnetic Moments - Octet

…If one insists in believing the constituent quark masses have something to do with reality, can compute the expected magnetic moments for octet:

Baryon	*Moment*	*Predicted*	*Observed*
p	$\frac{4}{3}\mu_u - \frac{1}{3}\mu_d$	2.79	2.793
n	$\frac{4}{3}\mu_d - \frac{1}{3}\mu_u$	-1.86	-1.913
Λ^0	μ_s	-0.58	-0.614
Σ^+	$\frac{4}{3}\mu_u - \frac{1}{3}\mu_s$	2.68	2.33
Σ^0	$\frac{2}{3}(\mu_u + \mu_d) - \frac{1}{3}\mu_s$	0.82	Unstable
Σ^-	$\frac{4}{3}\mu_d - \frac{1}{3}\mu_s$	-1.05	-1.41
Ξ^0	$\frac{4}{3}\mu_s - \frac{1}{3}\mu_u$	-1.40	-1.253
Ξ^-	$\frac{4}{3}\mu_s - \frac{1}{3}\mu_d$	-0.47	-0.69

Not too bad for such a simple attempt…

We are taking baryons as composed only by valence quarks, which is wildly *incomplete*

Hyperon Magnetic Moments - I

Dipole rotation along a variable path length in a uniform magnetic field

$$\phi_{rot} \propto \mu_{\Lambda^0} \int Bdl$$

Collect Λ^0 decays at different distances
Measure Λ^0 energy

Λ^0 produced polarized at high energy: $\mathbf{s}_\Lambda \perp \Lambda^0$ production plane

$\rightarrow \mathbf{s}_\Lambda$ known at production

$\Lambda^0 \rightarrow p + \pi^0$ weak decay: Parity violation

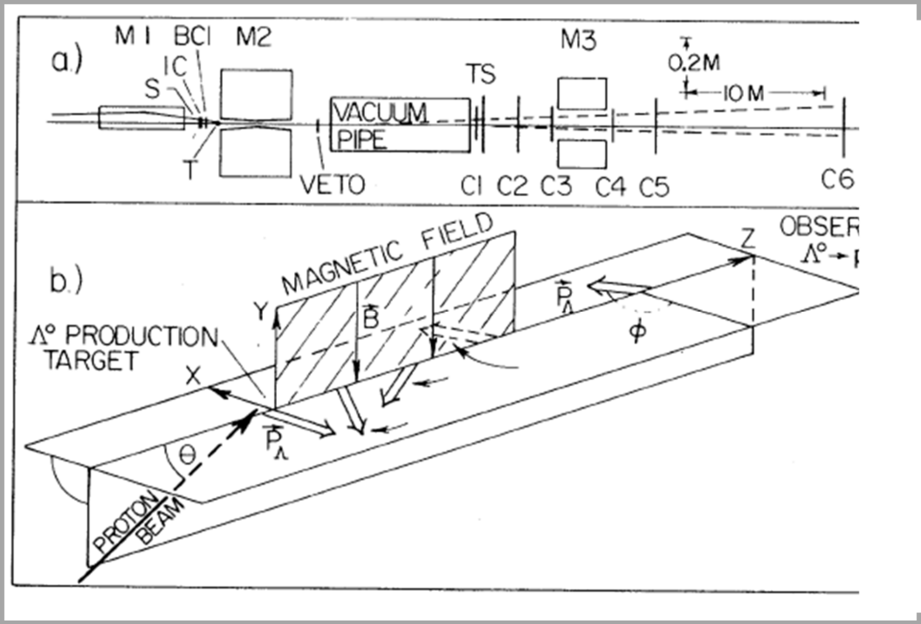
$\rightarrow p$ momentum preferentially $\angle\, \mathbf{s}_\Lambda$

$\rightarrow \mathbf{s}_\Lambda$ at decay known from $\mathbf{p}_p$ direction

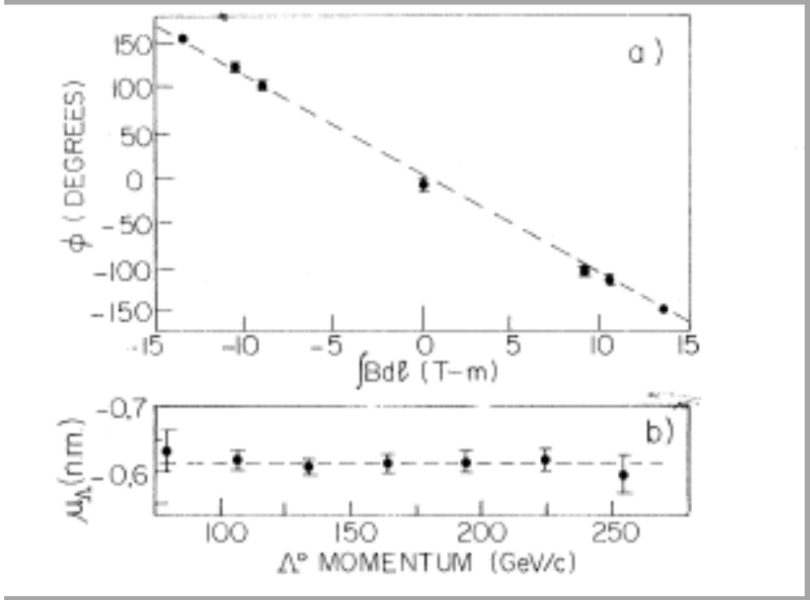
$\boldsymbol{\mu}_\Lambda \parallel \mathbf{s}_\Lambda : \boldsymbol{\mu}_\Lambda^{fin} \cdot \boldsymbol{\mu}_\Lambda^{in} \propto \cos\phi_{rot}$

Hyperon Magnetic Moments - II

Experimental setup



Results



@TBA

Mag. Moments: Unstable Particles

Cannot measure them with the same technique discussed for stable particles
$\rightarrow$Consider a different approach
Take as an example the Σ^0 decay (octet):

$\Sigma^0 \rightarrow \Lambda^0 + \gamma$ Parity conserving, electromagnetic decay

$\eta_P\left(\Sigma^0\right) = + = \eta_P\left(\Lambda^0\right) \rightarrow \eta_P\left(\gamma\right)$ must be $+$

$$\eta_P\left(\gamma\right) = \begin{cases} \left(-1\right)^{j+1} & \text{magnetic} \\ \left(-1\right)^{j} & \text{electric} \end{cases}, \ j = 1 \rightarrow magnetic$$

Just meaning:
This photon has
total angular momentum = 1,
total parity = +1

Transition is *M1* (magnetic dipole)

Σ^0, Λ^0 : Same quark content *uds*

Σ^0 : I-spin triplet $\rightarrow u, d$ Spin triplet

$\rightarrow$ Wave function=Sum of Permutations of $\left[\left(ud + du\right)s\right]\uparrow\uparrow\downarrow$

Λ^0 : I-spin singlet $\rightarrow u, d$ Spin singlet

$\rightarrow$ Wave function=Sum of Permutations of $\left[\left(ud - du\right)s\right]\left(\uparrow\downarrow - \downarrow\uparrow\right)\uparrow$

$Amplitude = A\left(Spin\ flip\right)\ for\ \left[u\ or\ d\right]$

The Transition Magnetic Moment

Neutron electromagnetic current

$$j_n^\mu = e\bar{u}_n(p')\left(F_n(q^2)\gamma^\mu + G_n(q^2)i\kappa_n\frac{\sigma^{\mu\nu}}{2m}q_\nu\right)u_n(p)$$

Take the Σ^0 as a kind of neutron…(Well, that's *SU(3)*…)

This would be the current involved e.g. in electron DIS off a Σ^0

$$j_{\Sigma^0}^\mu = e\bar{u}_{\Sigma^0}(p')\left(F_{\Sigma^0}(q^2)\gamma^\mu + G_{\Sigma^0}(q^2)i\kappa_{\Sigma^0}\frac{\sigma^{\mu\nu}}{2m}q_\nu\right)u_{\Sigma^0}(p)$$

$\kappa_{\Sigma^0} = ?$ Not observable, the Σ^0 is unstable

Define an e.m. transition current for our process

$$j_{\Sigma^0\Lambda^0}^\mu = e\bar{u}_{\Lambda^0}(p')\left(F_{\Sigma^0\Lambda^0}(q^2)\gamma^\mu + G_{\Sigma^0\Lambda^0}(q^2)i\kappa_{\Sigma^0\Lambda^0}\frac{\sigma^{\mu\nu}}{2m}q_\nu\right)u_{\Sigma^0}(p)$$

$\kappa_{\Sigma^0\Lambda^0} = ?$ Can be determined by the observed rate

In the static ($q^2=0$) limit (actually never reached in the transition) analog to the static magnetic dipole moment

Vector Mesons Radiative Decays

Take radiative decays of vector mesons to pseudoscalars:

$V \to P + \gamma$

$1^{--} \to 0^{-+} + \gamma$

$\to \gamma : 1^{+} \to magnetic \quad dipole$

For any magnetic dipole transition:

$Rate \propto \omega^3, \ \omega :$ Photon energy

From quark model perspective: *Triplet* $\to$ *Singlet, S-wave*
As before: Spin flip of one quark
(I = Fraction of space overlap between initial/final meson wave functions)

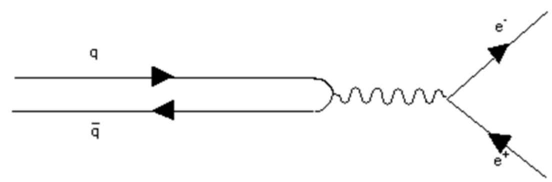
Process	Rate in units of $\omega^3\|I\|^2/3\pi$	Prediction (keV)	Experiment (keV)	$\|I\|^2$
$\omega \to \pi^0\gamma$	$(\mu_u - \mu_d)^2$	$1390\|I\|^2$	890 ± 50	0.64 ± 0.04
$\rho \to \pi\gamma$	$((\mu_u + \mu_d)^2$	$148\|I\|^2$	67 ± 7	0.45 ± 0.05
$\omega \to \eta\gamma$	$(\mu_u + \mu_d)^2/2$	$11\|I\|^2$	$3^{+2.5}_{-1.8}$	$0.27^{+0.23}_{-0.16}$
$\rho \to \eta\gamma$	$(\mu_u - \mu_d)^2/2$	$92\|I\|^2$	50 ± 13	0.54 ± 0.14
$\eta' \to \omega\gamma$	$3(\mu_u + \mu_d)^2/2$	$17\|I\|^2$	7.6 ± 3	0.45 ± 0.18
$\eta' \to \rho\gamma$	$3(\mu_u - \mu_d)^2/2$	$171\|I\|^2$	83 ± 30	0.48 ± 0.18
$\phi \to \eta\gamma$	$2\mu_s^2$	$110\|I\|^2$	62 ± 9	0.56 ± 0.08
$\phi \to \pi^0\gamma$	0	0	5.7 ± 2	
$K^{*+} \to K^+\gamma$	$(\mu_u + \mu_s)^2$	$153\|I\|^2$	60 ± 15	0.39 ± 0.10
$K^{*0} \to K^0\gamma$	$(\mu_d - \mu_s)^2$	$224\|I\|^2$	75 ± 35	0.34 ± 0.16

@TBA

Quite consistent with simple *SU(3)* symmetry: Same space wave function

Decays of Vector Mesons to $e^+ e^-$

$$\rho^0 \to e^+ + e^-$$
$$\omega \to e^+ + e^-$$
$$\varphi \to e^+ + e^-$$



$$\Gamma_{e^+e^-} = \frac{16\pi\alpha^2}{q^2\left(= M_V^2\right)} \left|\psi(0)\right|^2 \left|\sum_i a_i Q_i\right|^2$$ Van Royen-Weisskopf formula

$$\rho^0 : \frac{1}{\sqrt{2}}\left(u\bar{u} - d\bar{d}\right) \to \left|\sum_i a_i Q_i\right|^2 = \left|\frac{1}{\sqrt{2}}\left(\frac{2}{3} - \left(-\frac{1}{3}\right)\right)\right|^2 = \frac{1}{2}$$

$$\omega : \frac{1}{\sqrt{2}}\left(u\bar{u} + d\bar{d}\right) \to \left|\sum_i a_i Q_i\right|^2 = \left|\frac{1}{\sqrt{2}}\left(\frac{2}{3} + \left(-\frac{1}{3}\right)\right)\right|^2 = \frac{1}{18}$$

$$\varphi : \quad s\bar{s} \to \left|\sum_i a_i Q_i\right|^2 = \left|\left(-\frac{1}{3}\right)\right|^2 = \frac{1}{9}$$

$$\to \Gamma_{e^+e^-}\left(\rho^0\right) : \Gamma_{e^+e^-}\left(\omega\right) : \Gamma_{e^+e^-}\left(\varphi\right) = 9:1:2$$

Van-Royen - Weisskopf - I

Attempting to calculate the vector meson decay rate:

$\Gamma_V = |A_V|^2, \quad A_V = \langle f|T|V\rangle$ Transition amplitude between V (initial), f (final) state

The meson is a bound state $\rightarrow$ Initial state *not* a plane wave!

Then expand the amplitude into plane waves:

$$A_V = \sum_p \underbrace{\langle f|T|p\rangle}_{A(\mathbf{p})} \underbrace{\langle p|V\rangle}_{\psi(\mathbf{p})}$$

$A(\mathbf{p})$ plane wave amplitude, $\psi(\mathbf{p})$ momentum space wave function

$$\rightarrow A_V = \int d^3\mathbf{p} A(\mathbf{p})\psi(\mathbf{p}) = \frac{1}{(2\pi)^{3/2}} \int d^3\mathbf{p} A(\mathbf{p}) \int \psi(\mathbf{r}) e^{i\mathbf{p}\cdot\mathbf{r}} d^3\mathbf{r} = \frac{1}{(2\pi)^{3/2}} \int d^3\mathbf{r}\,\psi(\mathbf{r}) \int A(\mathbf{p}) e^{i\mathbf{p}\cdot\mathbf{r}} d^3\mathbf{r}$$

$$\text{Take } A(\mathbf{p}) \approx A = const \rightarrow A_V \approx \frac{A}{(2\pi)^{3/2}} \int d^3\mathbf{r}\,\psi(\mathbf{r}) \underbrace{\int e^{i\mathbf{p}\cdot\mathbf{r}} d^3\mathbf{p}}_{(2\pi)^3\delta^3(\mathbf{r})} = (2\pi)^{3/2} A\psi(0)$$

$$\rightarrow \Gamma_V = |A_V|^2 \approx (2\pi)^3 |A|^2 |\psi(0)|^2$$

Why is $A(p) \approx const$?

Van-Royen - Weisskopf - II

Consider the process involving free quarks (plane wave):

$q\bar{q} \rightarrow e^+e^-$

Cross section, averaged over initial, summed over final spin projections:

$$\sigma_{q\bar{q}\rightarrow e^+e^-}(p) = |A(p)|^2 \frac{1}{\underbrace{\frac{v}{(2\pi)^3}}_{\text{flux}}}, v\ q,\ \bar{q}\ \text{relative velocity} \rightarrow \sigma_{q\bar{q}\rightarrow e^+e^-}(p) = |A(p)|^2 \frac{(2\pi)^3}{v}$$

Take 1-photon annihilation QED diagram:

$$\sigma_{q\bar{q}\rightarrow e^+e^-}(p) = \frac{\pi\alpha^2 Q^2}{s}\frac{p_e}{p_q}\left(1+\frac{v^2}{3}+\frac{4m_q^2}{s}\right)$$

Just the same as $e^+ + e^- \rightarrow \mu^+ + \mu^-$
But: Do not neglect rest mass

For small initial velocity:

$$s \approx (2m_q)^2, p_q \approx m_q \frac{v}{2}$$

$$\sigma_{q\bar{q}\rightarrow e^+e^-}(p) = \frac{\pi\alpha^2 Q^2}{s}\frac{p_e}{p_q}\left(1+\frac{v^2}{3}+\frac{4m_q^2}{s}\right) \approx \frac{\pi\alpha^2 Q^2}{4m_q^2}\frac{p_e}{m_q\frac{v}{2}}\left(1+\frac{v^2}{3}+1\right) \approx \frac{\pi\alpha^2 Q^2}{4m_q^2}\frac{p_e}{m_q v}4 \approx \frac{\pi\alpha^2 Q^2}{m_q^3}\frac{p_e}{v}$$

Van-Royen - Weisskopf - III

Obtain the decay rate:

$$\rightarrow \sigma_{q\bar{q}\rightarrow e^+e^-}(p) = |A(p)|^2 \frac{(2\pi)^3}{v} = \frac{\pi\alpha^2 Q^2}{m_q^3}\frac{p_e}{v}$$

$$\rightarrow |A(p)|^2 = \frac{\pi\alpha^2 Q^2}{m_q^3}\frac{p_e}{v}\frac{v}{(2\pi)^3} = \frac{\pi\alpha^2 Q^2}{m_q^3}\frac{p_e}{(2\pi)^3}$$

$$p_e \approx m_q \rightarrow |A(p)|^2 \approx \frac{\pi\alpha^2 Q^2}{(2\pi)^3 m_q^2}$$ Neglect quark momentum, electron mass

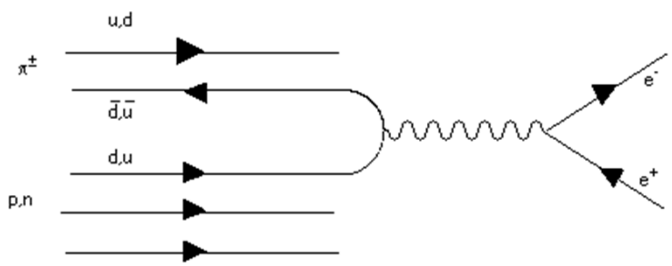
$$m_q \approx \frac{M_V}{2} \rightarrow |A(p)|^2 \approx \frac{4\pi\alpha^2 Q^2}{(2\pi)^3 M_V^2}$$ p-independent OK

We took the average over initial spins for our plane wave cross section, resulting in 3 (triplet) + 1 (singlet) = 4 states
Vector mesons have spin 1, so we should not count spin 0
$\rightarrow$Get a further factor *4/3*:

$$\Gamma_V \approx (2\pi)^3 |A|^2 |\psi(0)|^2 \approx (2\pi)^3 \frac{4}{3}\frac{4\pi\alpha^2 Q^2}{(2\pi)^3 M_V^2}|\psi(0)|^2 = \frac{16}{3}\frac{\pi\alpha^2 Q^2}{M_V^2}|\psi(0)|^2$$ Formula still incomplete...

Drell-Yan from Isoscalar Targets

Take production of electron pairs from pion beams: *Drell-Yan*



Rate $\propto Q^2_{quark}$

Cross section: Electromagnetic, counting antiquark content in π

For isoscalar targets: $N_p = N_n \rightarrow N_u = N_d$

$$\left.\begin{array}{l} \sigma(\pi^+) \propto Q^2_{\bar{d}} = \dfrac{1}{9} \\ \sigma(\pi^-) \propto Q^2_{\bar{u}} = \dfrac{4}{9} \end{array}\right\} \rightarrow \frac{\sigma(\pi^-)}{\sigma(\pi^+)} = 4$$

More Quarks

Flavor	*Mass*	*Q*	*I*	I_3	*S*	*C*	*B*	*T*
Up	5.6 MeV	2/3	½	+1/2	0	0	0	0
Down	9.9 MeV	-1/3	½	-1/2	0	0	0	0
strange	199 MeV	-1/3	0	0	-1	0	0	0
charm	1350 MeV	2/3	0	0	0	1	0	0
bottom	4400 MeV	-1/3	0	0	0	0	-1	0
Top	174 GeV	2/3	0	0	0	0	0	1

Charm

Predicted to exist in order to solve a major puzzle in weak interactions via the *GIM Mechanism* (see later)

Expected mass much higher than *u,d,s*

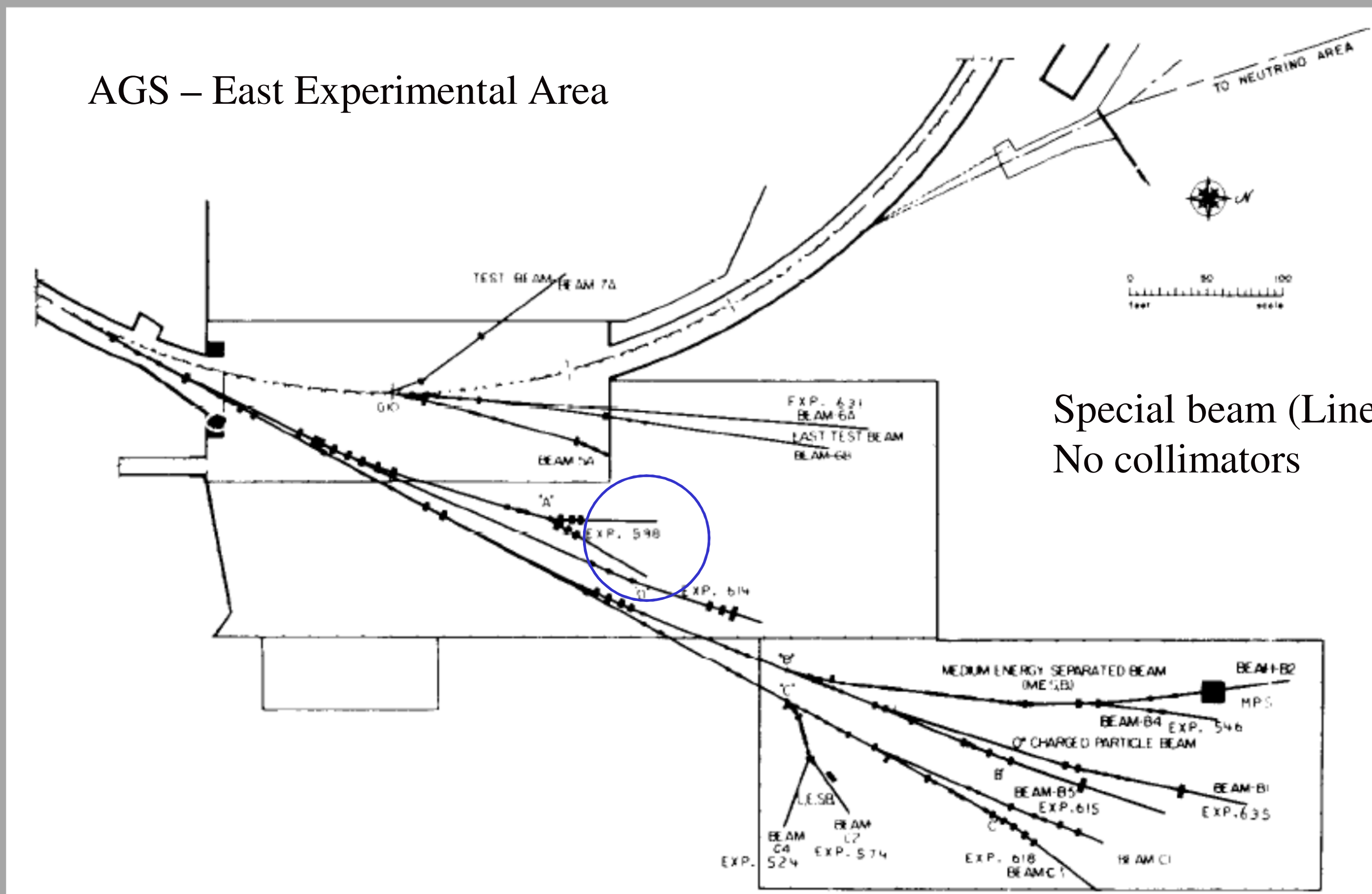
Phenomenology similar to strange quark *s*:
New breed of *charmed particles*, both mesons and baryons
Difference: Much larger mass

$\rightarrow$Many channels open to weak decays $\rightarrow$ *Shorter lifetime* $\sim 10^{-13}$ *s*
$\rightarrow$Extended symmetry severely broken $\rightarrow$ *SU(4) not useful*

Based on *asymptotic freedom* of QCD (see later), guess an exciting physics case for a *heavy, hidden charm bound state*

Discovered simultaneosly at SLAC (Mark I) and BNL (E598)

The J/ψ Particle at Brookhaven - I



The J/ψ Particle at Brookhaven - II

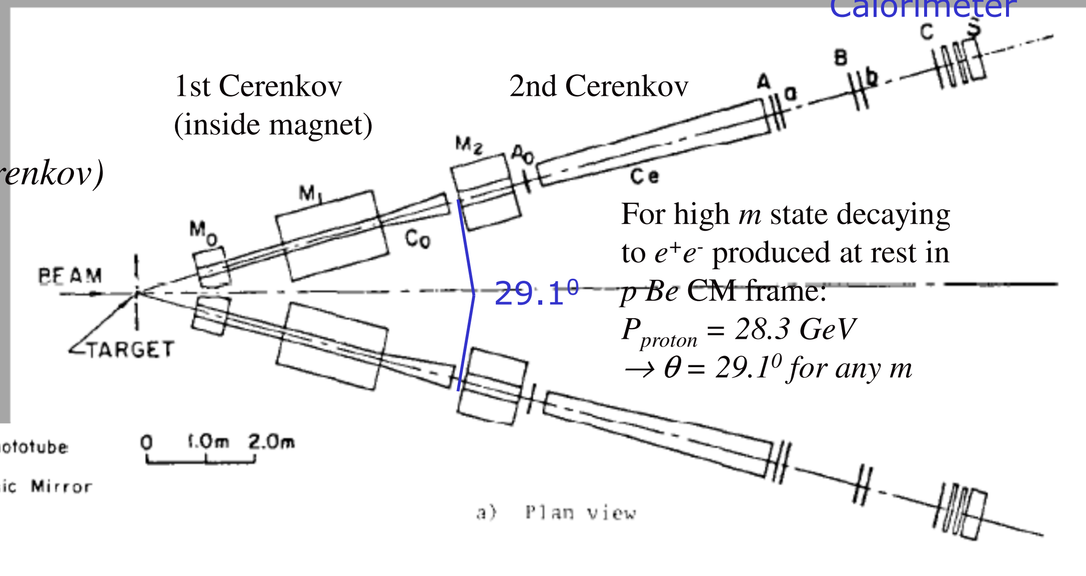
2-Arm Spectrometer

Magnetic analysis (3 dipoles)
Excellent electron identification (2 Cerenkov)
Very high intensity (2 10^{12} ppp)
Small spot size (3x6 mm^2)

Calorimeter

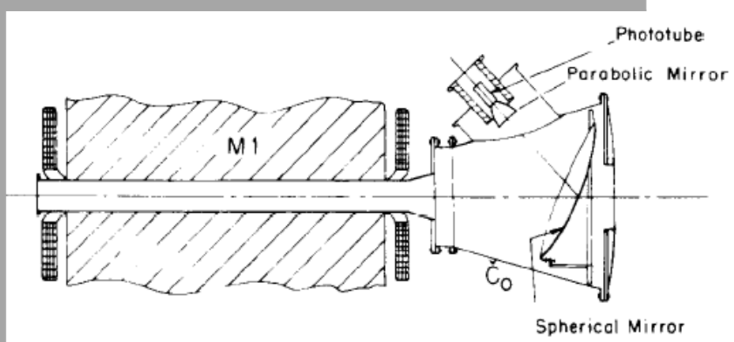
1st Cerenkov
(inside magnet)

2nd Cerenkov

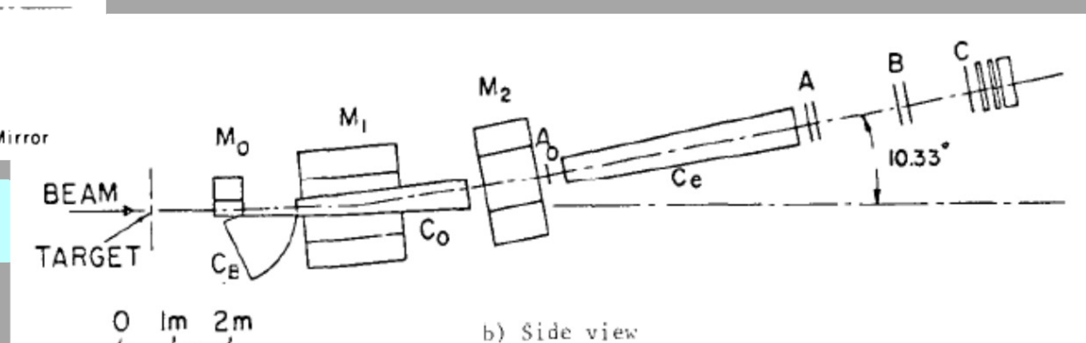


For high m state decaying to e^+e^- produced at rest in p Be CM frame:
P_{proton} = 28.3 GeV
$\rightarrow \theta = 29.1^0$ for any m

Electron identification



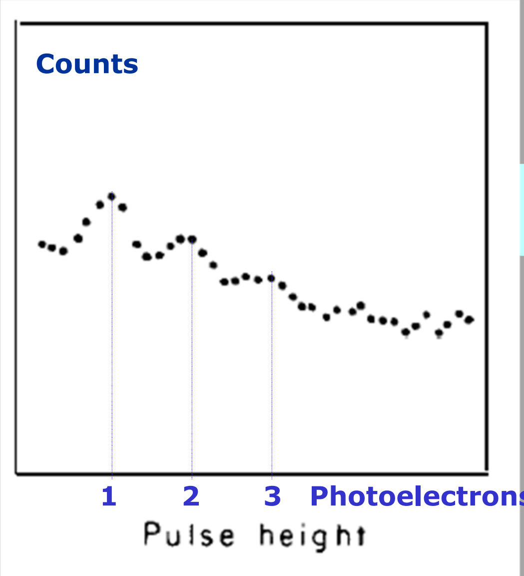
@TBA



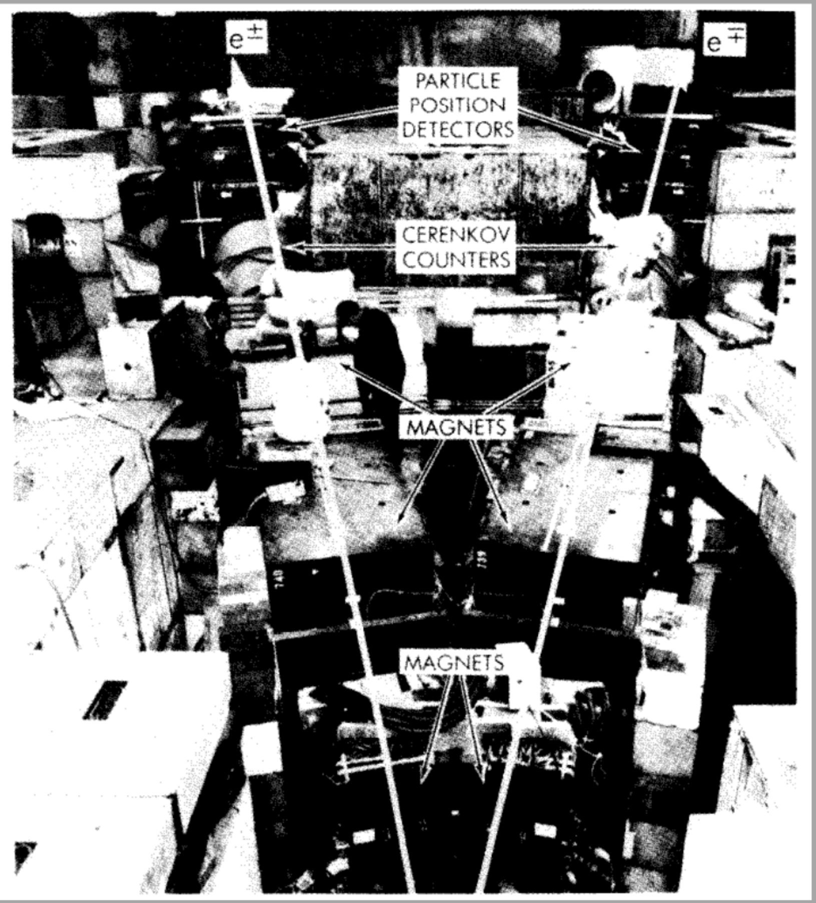
The J/ψ Particle at Brookhaven - III

Detectors in the experimental setup at BNL

Cherenkov photomultiplier spectrum
Excellent single electron resolution



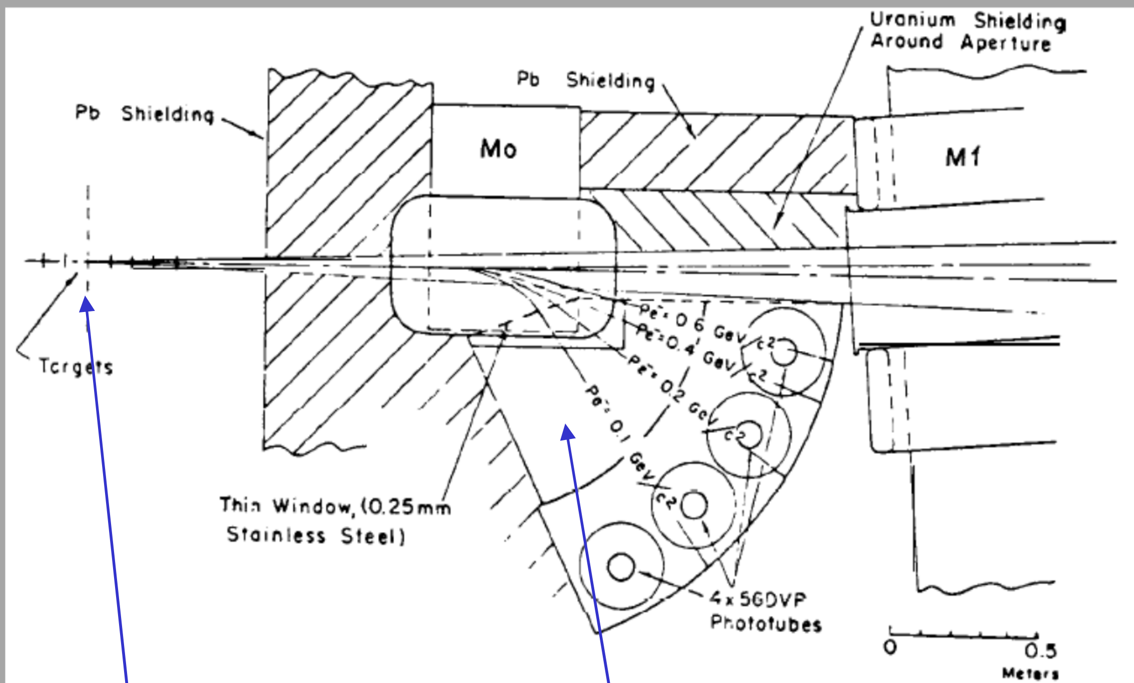
@TBA

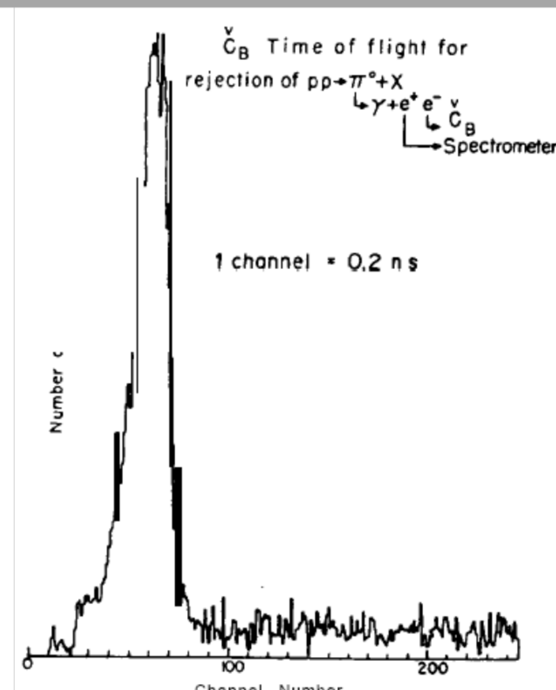


Good for signal efficiency, background rejection

The J/ψ Particle at Brookhaven - IV

Target area: Special Magnet (M_0) + Cerenkov (C_B) for calibration
Want to be sure not to trigger on e^+e^- pairs from γ conversion





$\pi^0 \to \gamma + e^+ + e^-$

Segmented target
Improved mass resolution

Beam Cerenkov
Detects electrons from Dalitz decays
Logic:Coincidence $C_B*(C_0+C_e)*Cal$

Time resolution ~ 1ns

The J/ψ Particle at Brookhaven - V

Large peak observed at *m = 3.1 GeV*

Still present at the same mass after reducing magnet currents by *10%*

Very large mass
Wrong by $6/3100 \sim 2\ 10^{-3}$
Excellent control of systematics!

Very small width
Consistent with experimental resolution

Totally inconsistent with standard picture of a high mass hadron:
In view of the many decay channels which are open, should be quite wide

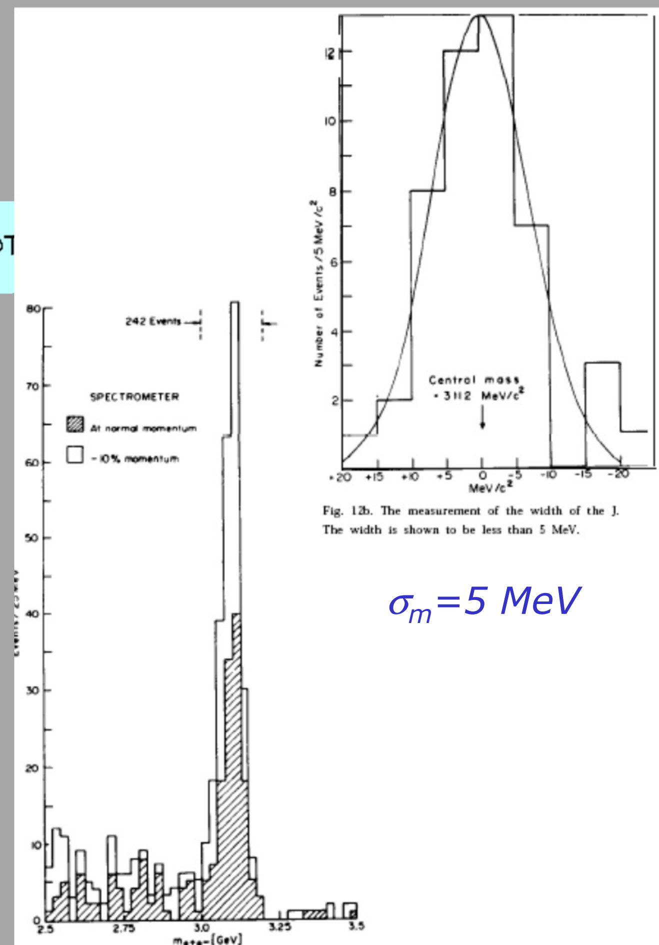
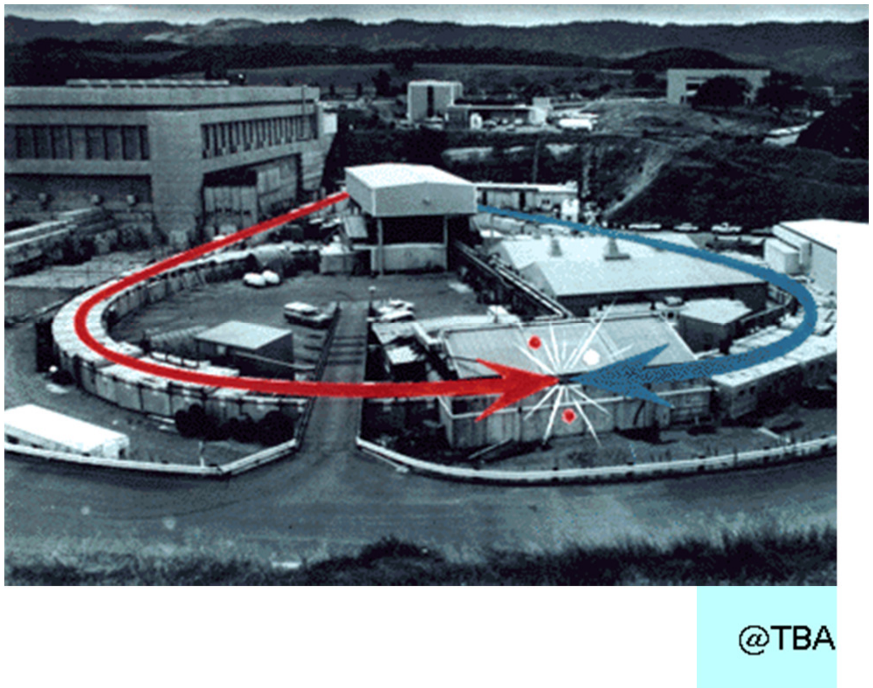


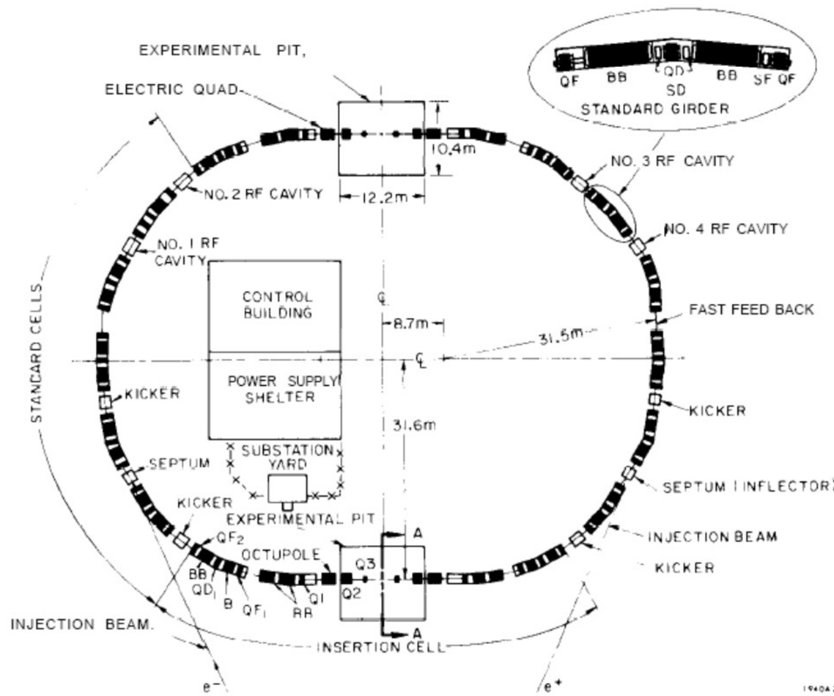
Fig. 12b. The measurement of the width of the J. The width is shown to be less than 5 MeV.

σ_m=5 MeV

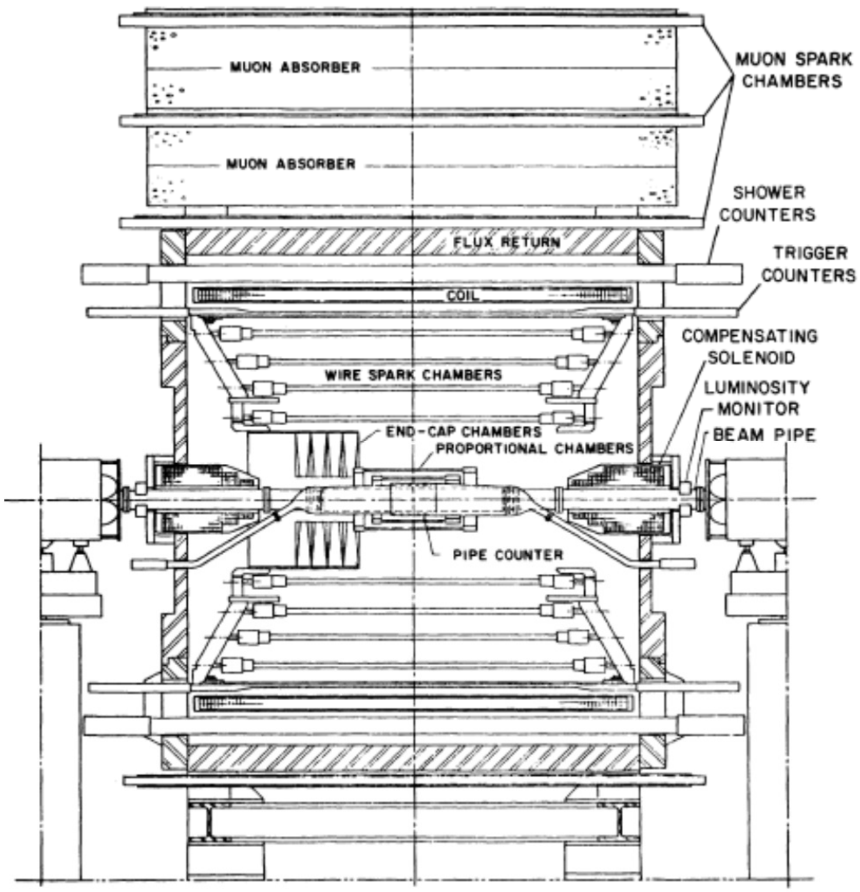
The J/ψ Particle at SLAC - I



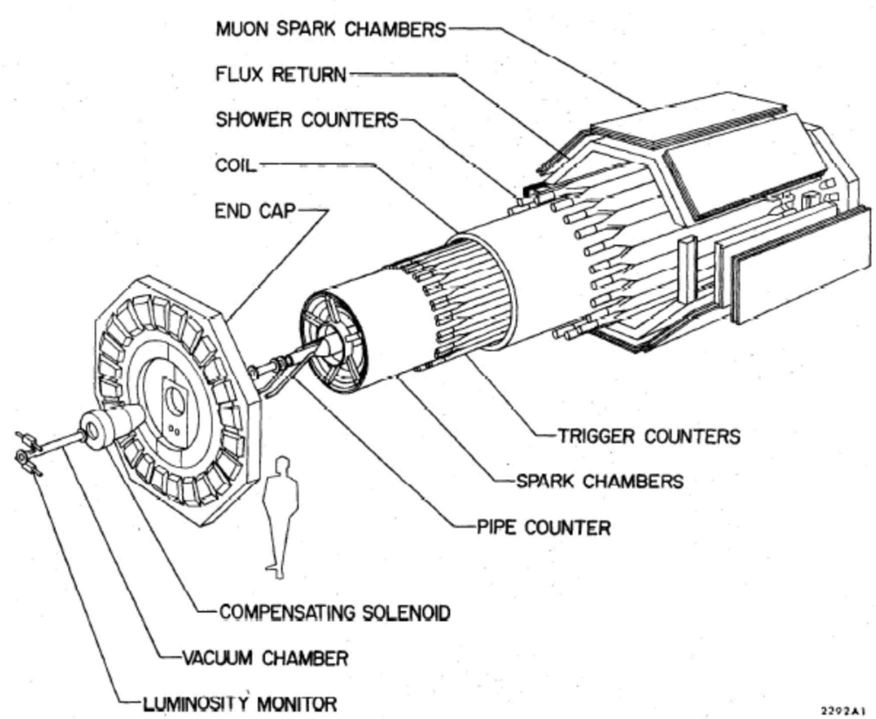
SPEAR



The J/ψ Particle at SLAC - II



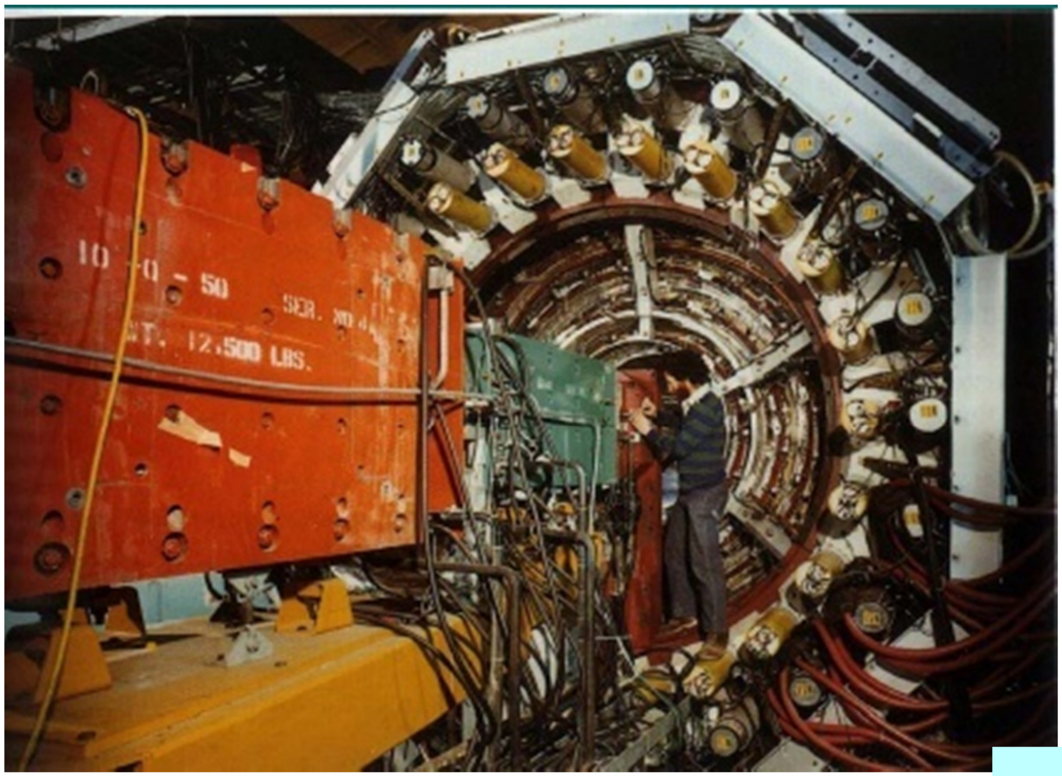
TBA

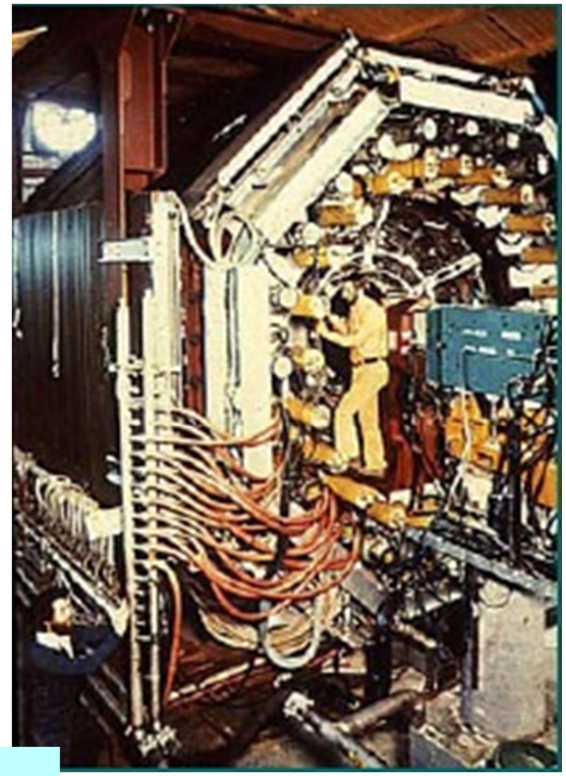


The Mark I Detector

The J/ψ Particle at SLAC - III

Mark I: First example of multi-purpose, collider detector





@TBA

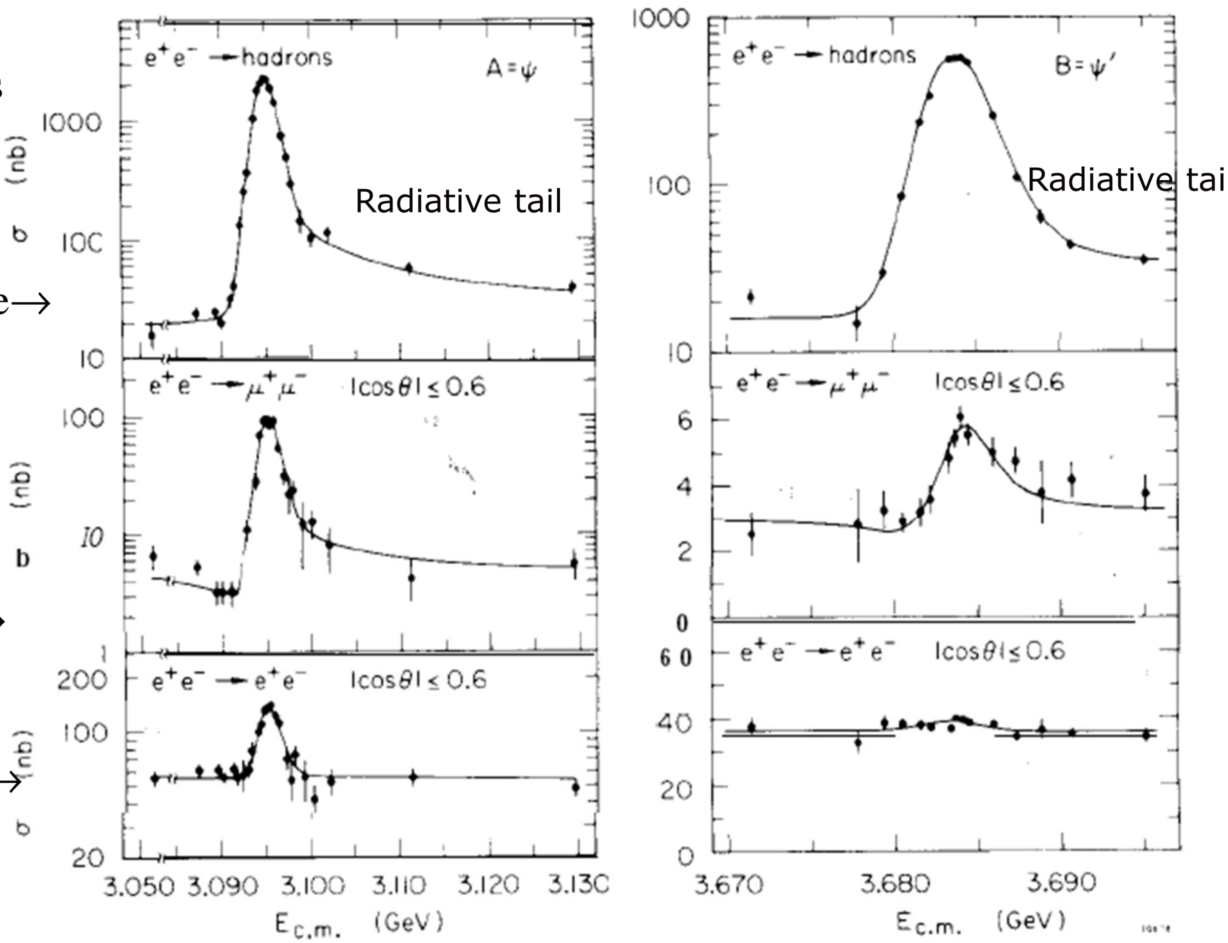
The J/ψ Particle at SLAC - IV

J/ψ and ψ' as seen
in different decay channels

No interference→

Interference →

No interference→



@TBA

What is the J/ψ?

Quickly understood as the first, indirect evidence for charm

Bound state of quark-antiquark pair $c, \overline{c}$

Another member of the vector mesons family

Main differences:

Charm quark has a large mass 1.5 GeV

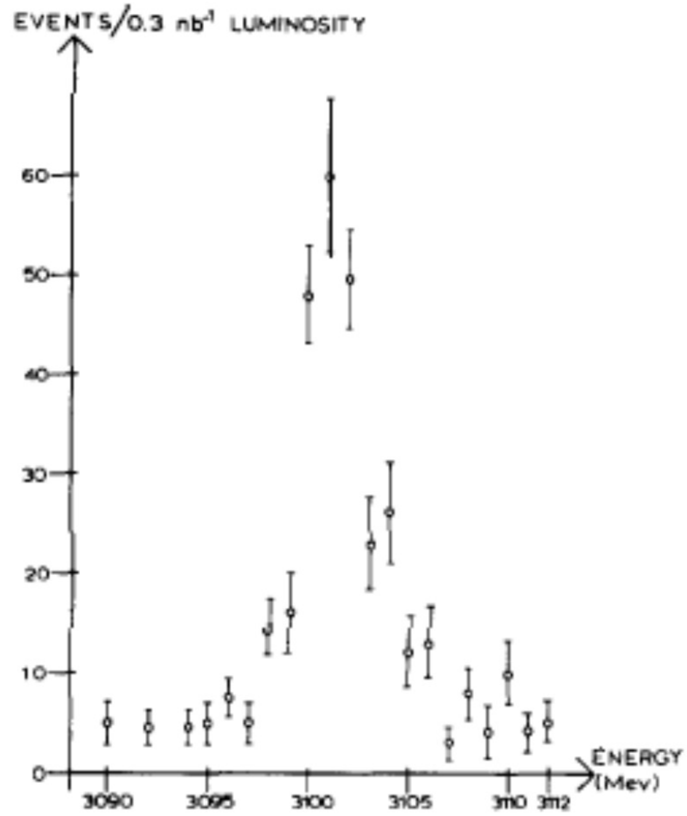
Lightest charmed particles are so heavy the J/ψ cannot decay into a pair of them$\rightarrow$Most decays channels are closed

The J/ψ Particle at Frascati

One day of November, back in 1974, Frascati got a phone call from Brookhaven..

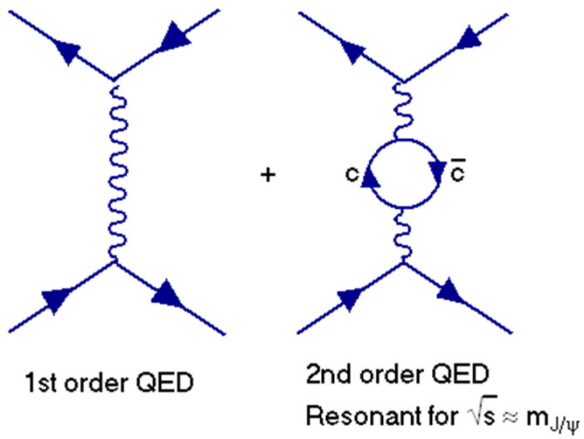
But: J/ψ mass was just beyond the energy range of ADONE

So what?
Push magnet currents up…



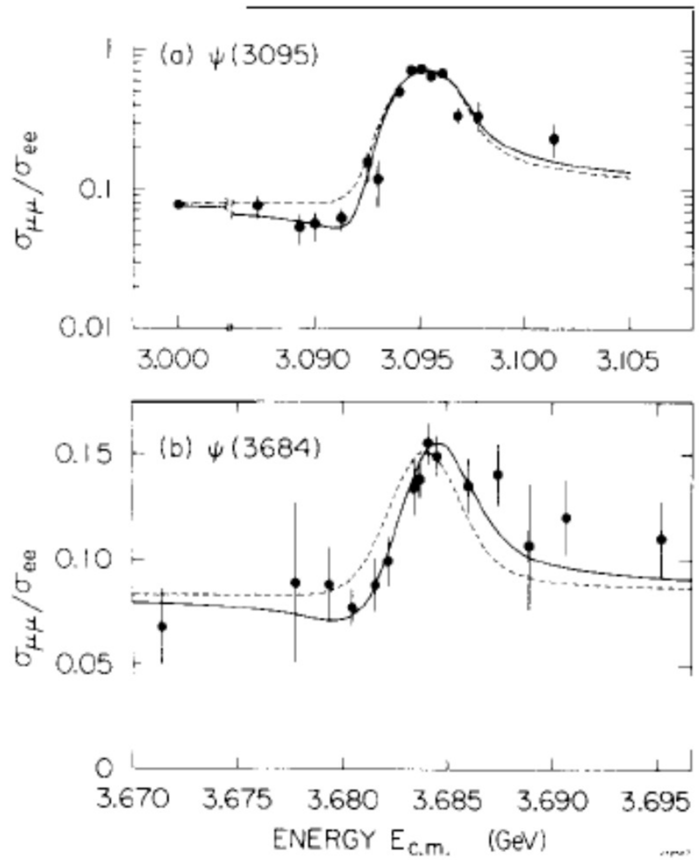
J/ψ Quantum Numbers

An interesting example of quantum interference
Take the 2 annihilation diagrams:



@TBA

Take the ratio to minimize
point-to-point luminosity systematics



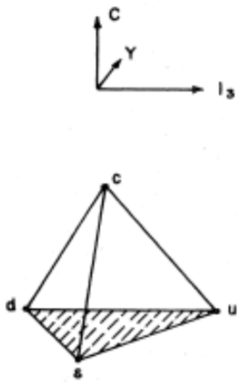
Interference between the 2 shows up in the total cross-section as a result of the resonant state being $J^{PC}=1^{--}$ like the photon

SU(4) Multiplets

Fundamental rep. ***4, 4*, 6***

$4 \cdot 4 - 1 = 15$ generators

3 fundamental, non equivalent irr. reps.



@TBA

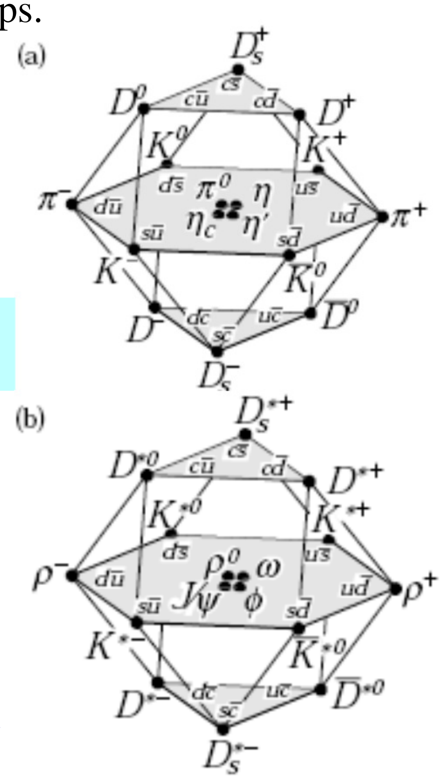
Spin 3/2 Spin 1/2

$$\mathbf{4} \otimes \mathbf{4}^* = \mathbf{1} \oplus \mathbf{15}$$

$$\mathbf{4} \otimes \mathbf{4} \otimes \mathbf{4} = \mathbf{20}_S \oplus \mathbf{20}_M \oplus \mathbf{20}_M \oplus \cancel{\mathbf{4}^*_A}$$

Mesons

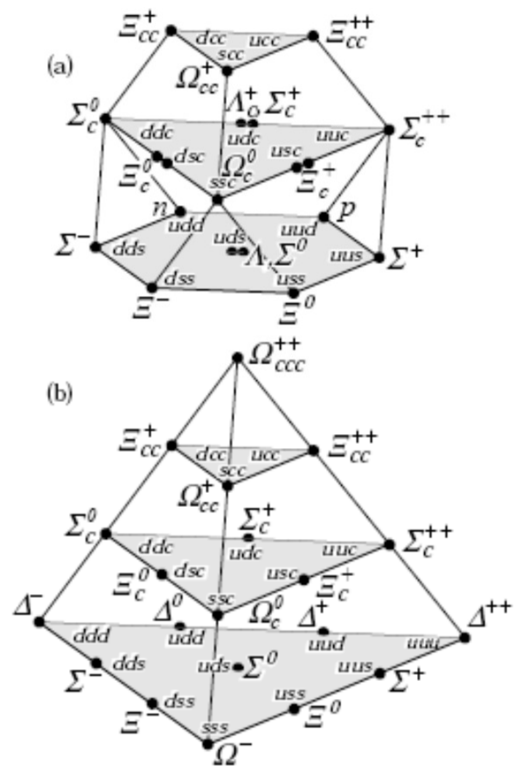
Pseudoscalars



Vectors

Baryons

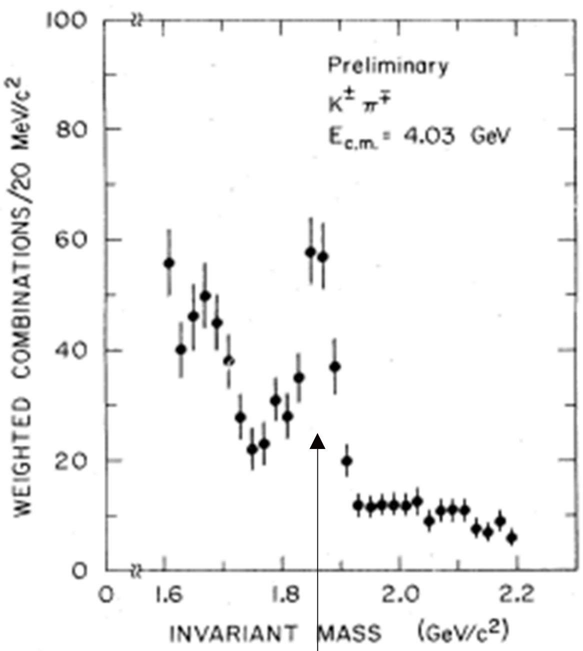
Spin 1/2



Spin 3/2

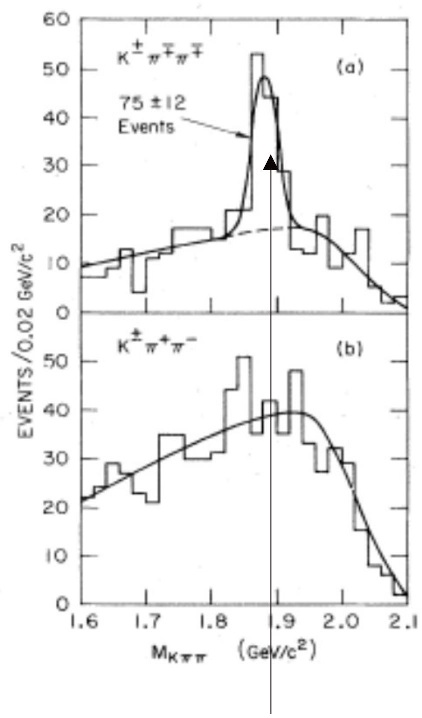
Open Charm

SLAC-LBL Collaboration – Mark I



D^0

@TBA



D^{0*}

The Charmed Zoo

Λ_c^+	****
$\Lambda_c(2593)^+$	***
$\Lambda_c(2625)^+$	***
$\Lambda_c(2765)^+$	*
$\Lambda_c(2880)^+$	**
$\Sigma_c(2455)$	****
$\Sigma_c(2520)$	***
$\Sigma_c(2800)$	***
Ξ_c^+	***
Ξ_c^0	***
$\Xi_c'^+$	***
$\Xi_c'^0$	***
$\Xi_c(2645)$	***
$\Xi_c(2790)$	***
$\Xi_c(2815)$	***
Ω_c^0	***
Ξ_{cc}^+	*

Baryons

@TBA

CHARMED ($C = \pm1$)	
• $D^\pm$	$1/2(0^-)$
• D^0	$1/2(0^-)$
• $D^*(2007)^0$	$1/2(1^-)$
• $D^*(2010)^\pm$	$1/2(1^-)$
$D_0^*(2400)^0$	$1/2(0^+)$
$D_0^*(2400)^\pm$	$1/2(0^+)$
• $D_1(2420)^0$	$1/2(1^+)$
$D_1(2420)^\pm$	$1/2(?^?)$
$D_1(2430)^0$	$1/2(1^+)$
• $D_2^*(2460)^0$	$1/2(2^+)$
• $D_2^*(2460)^\pm$	$1/2(2^+)$
$D^*(2640)^\pm$	$1/2(?^?)$
CHARMED, STRANGE ($C = S = \pm1$)	
• $D_s^\pm$	$0(0^-)$
• $D_s^{*\pm}$	$0(?^?)$
• $D_{s0}^*(2317)^\pm$	$0(0^+)$
• $D_{s1}(2460)^\pm$	$0(1^+)$
• $D_{s1}(2536)^\pm$	$0(1^+)$
• $D_{s2}(2573)^\pm$	$0(?^?)$

Mesons

Bottom

3rd family (*Bottom, Top*) predicted in order to ‘explain’ (Better: Account for) CP violation (see later) – Kobayashi & Maskawa, 1973

Bound $b\bar{b}$ states first observed at Fermilab in 1977

Discovery subsequently confirmed at e^+e^- machines (DESY, Cornell)

Several *b*-hadrons observed

Very large *b*-quark mass ~ 4-5 GeV

Situation somewhat similar to charm

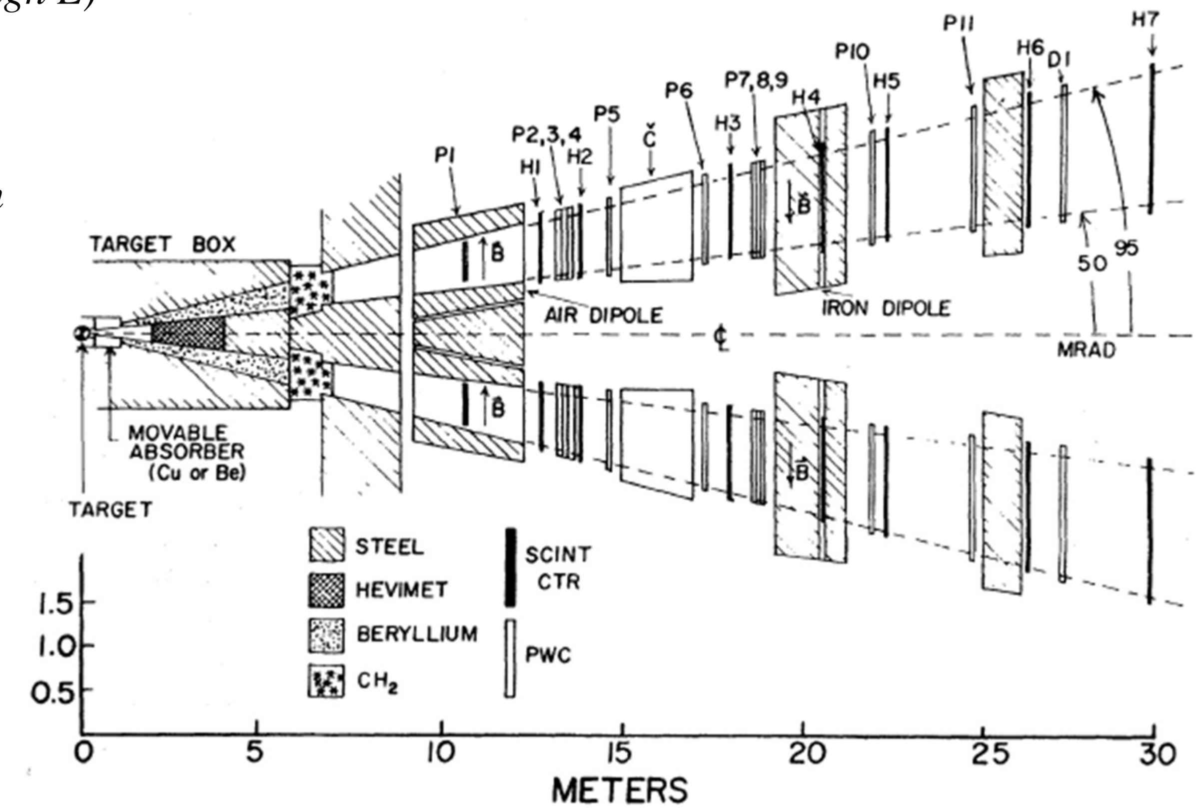
The Υ Discovery at FNAL - I

Design similar to J/ψ experiment:

Switch from electrons to muons
(Easier to handle at high E)

High intensity

~Good mass resolution



@TBA

The Υ Discovery at FNAL - II

Mass distribution for exclusive process:
$p + Be \rightarrow \mu^+ + \mu^- + X$

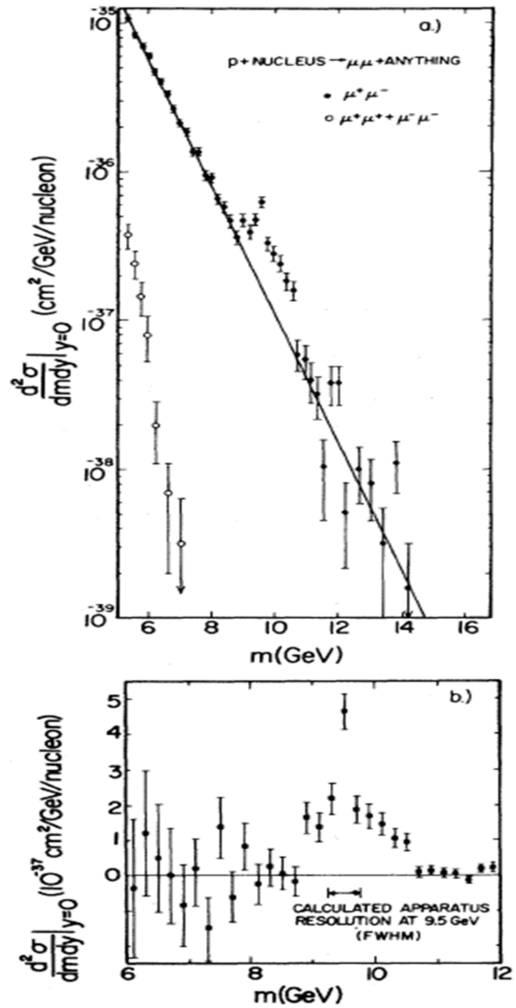
y: Pseudorapidity of the muon pair
(Related to CM angle)

y=0 Central region

High mass region shown
Exponential trend + peak

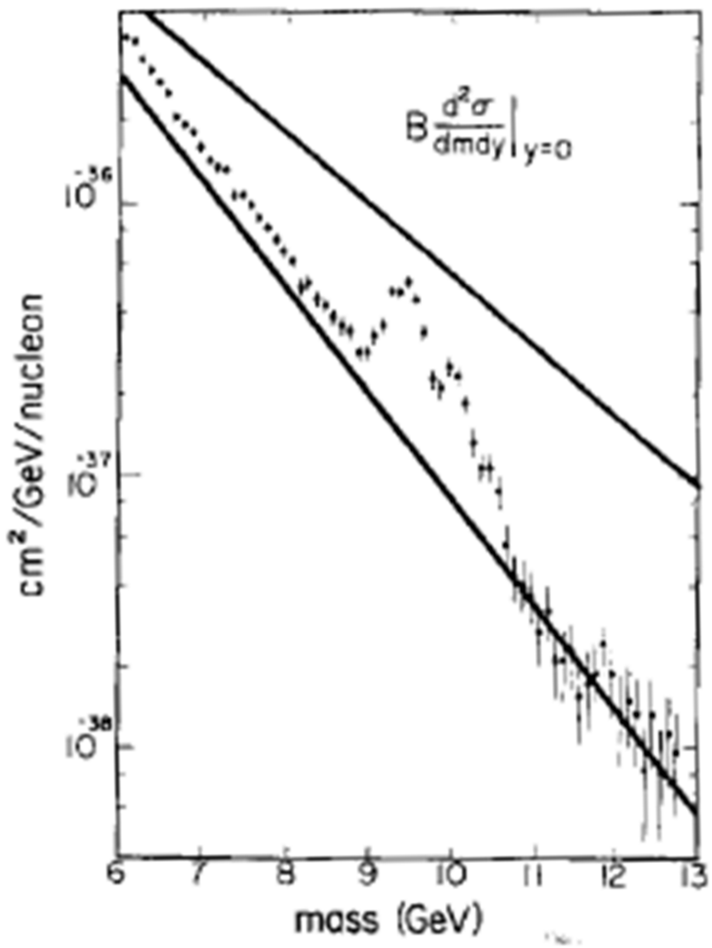
Mass resolution ~ 180 MeV

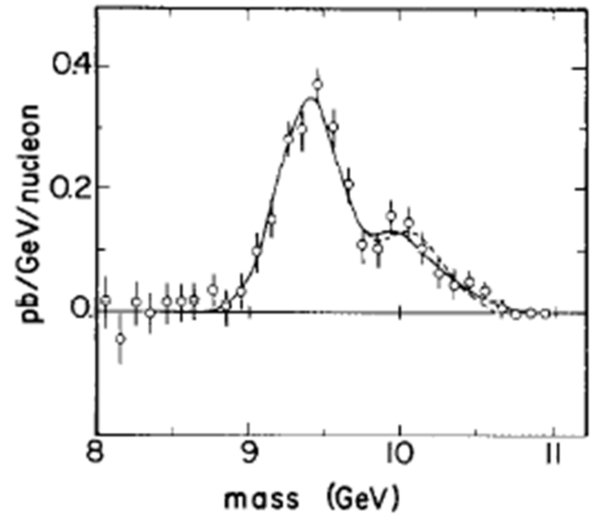
@TBA



The Υ Discovery at FNAL - III

With some more statistics

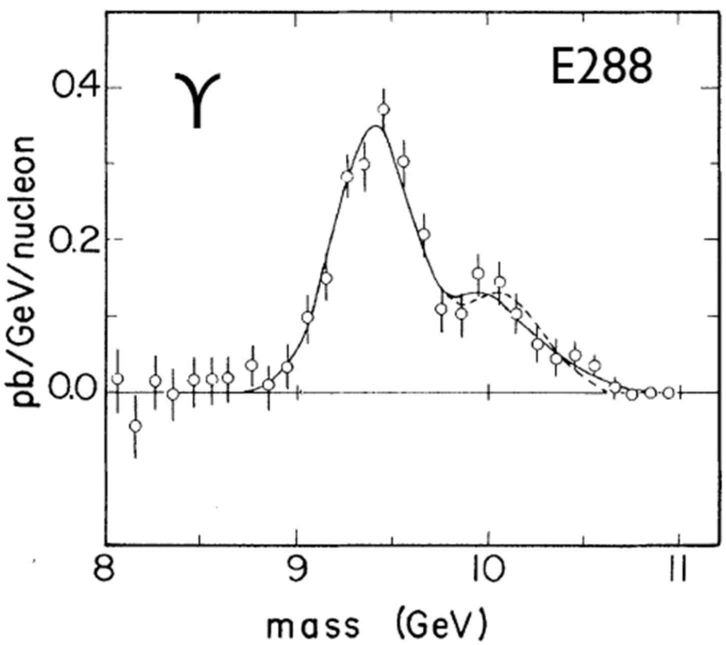




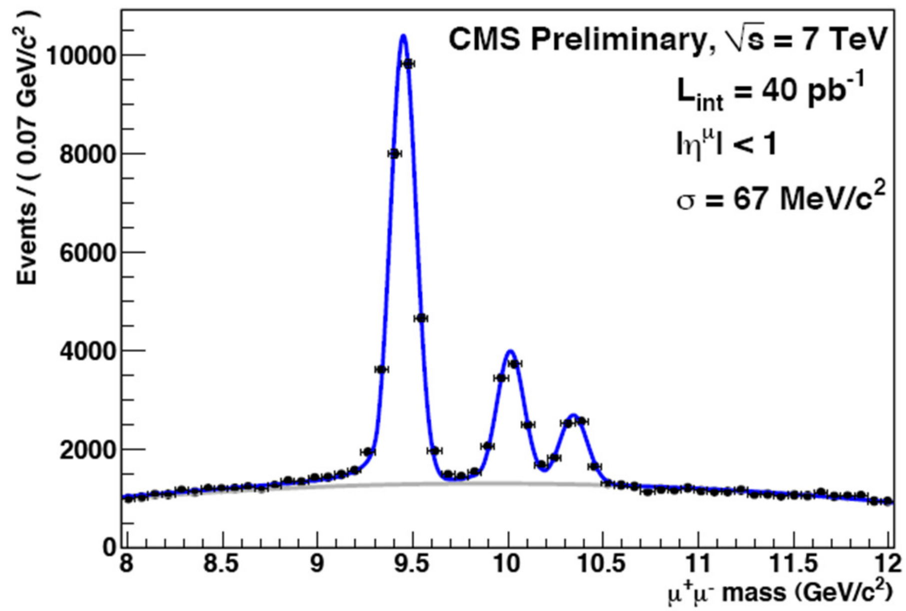
Background subtracted

The Υ Discovery at FNAL - IV

Yesterday



Today



The Last (?) Zoo

Λ_b^0	***
Ξ_b^0, Ξ_b^-	*

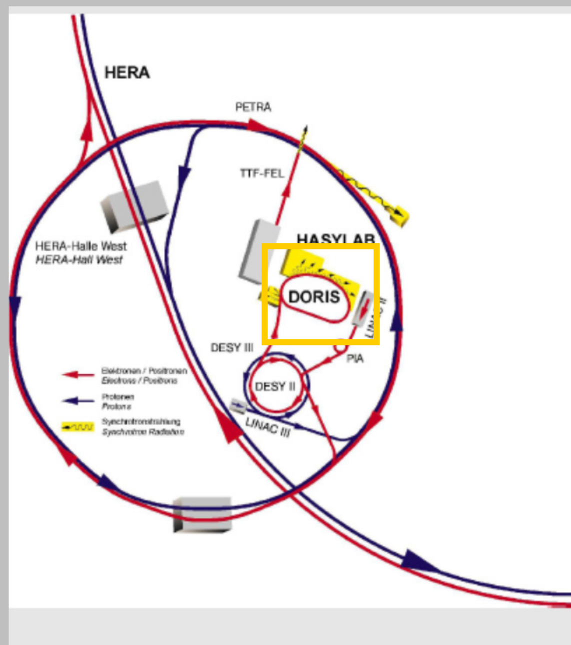
Baryons

BOTTOM ($B = \pm1$)	$I^G(J^{PC})$
• $B^\pm$	$1/2(0^-)$
• B^0	$1/2(0^-)$
• $B^\pm/B^0$ ADMIXTURE	
• $B^\pm/B^0/B_s^0/b$-baryon ADMIXTURE	
V_{cb} and V_{ub} CKM Matrix Elements	
• B^*	$1/2(1^-)$
$B_J^*(5732)$	$?(?^?)$
BOTTOM, STRANGE ($B = \pm1, S = \mp1$)	
• B_s^0	$0(0^-)$
B_s^*	$0(1^-)$
$B_{sJ}^*(5850)$	$?(?^?)$
BOTTOM, CHARMED ($B = C = \pm1$)	
• $B_c^\pm$	$0(0^-)$

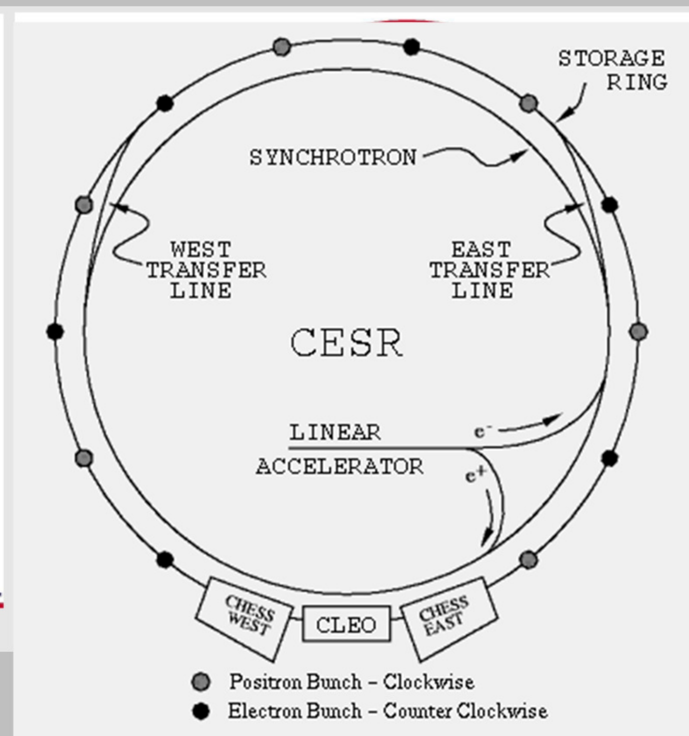
@TBA

Mesons

DORIS & CESR

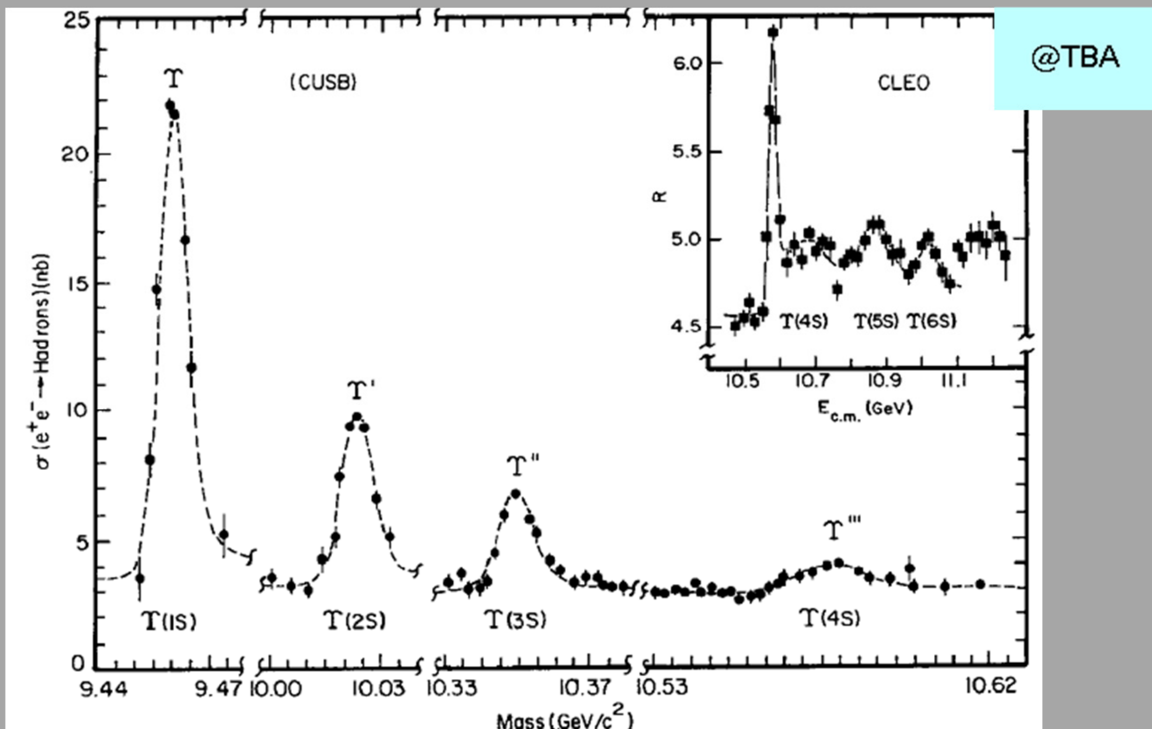


DESY, Hamburg



Cornell, Ithaca, NY

The Υ Family



3 *radial excitations* of the Υ
observed as narrow peaks

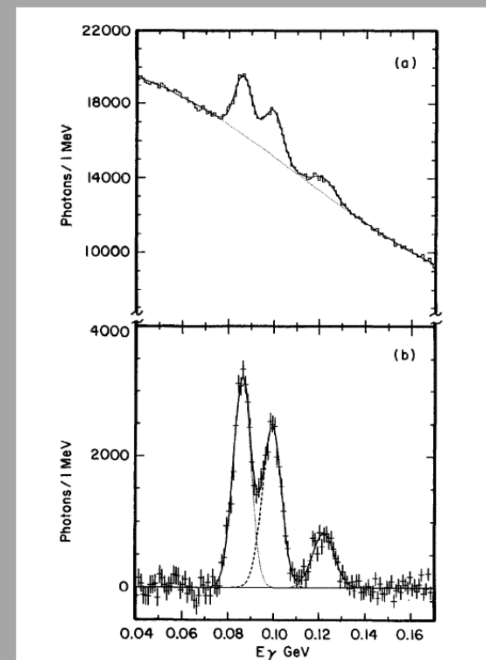


Figure 5 (a) Inclusive $\Upsilon(3S)$ photon energy spectrum from the CLEO-II collaboration. (b) Background subtracted spectrum.

Inclusive γ-ray spectrum
from $\Upsilon(3S)$

ARGUS

One of the first examples
of modern collider detector design

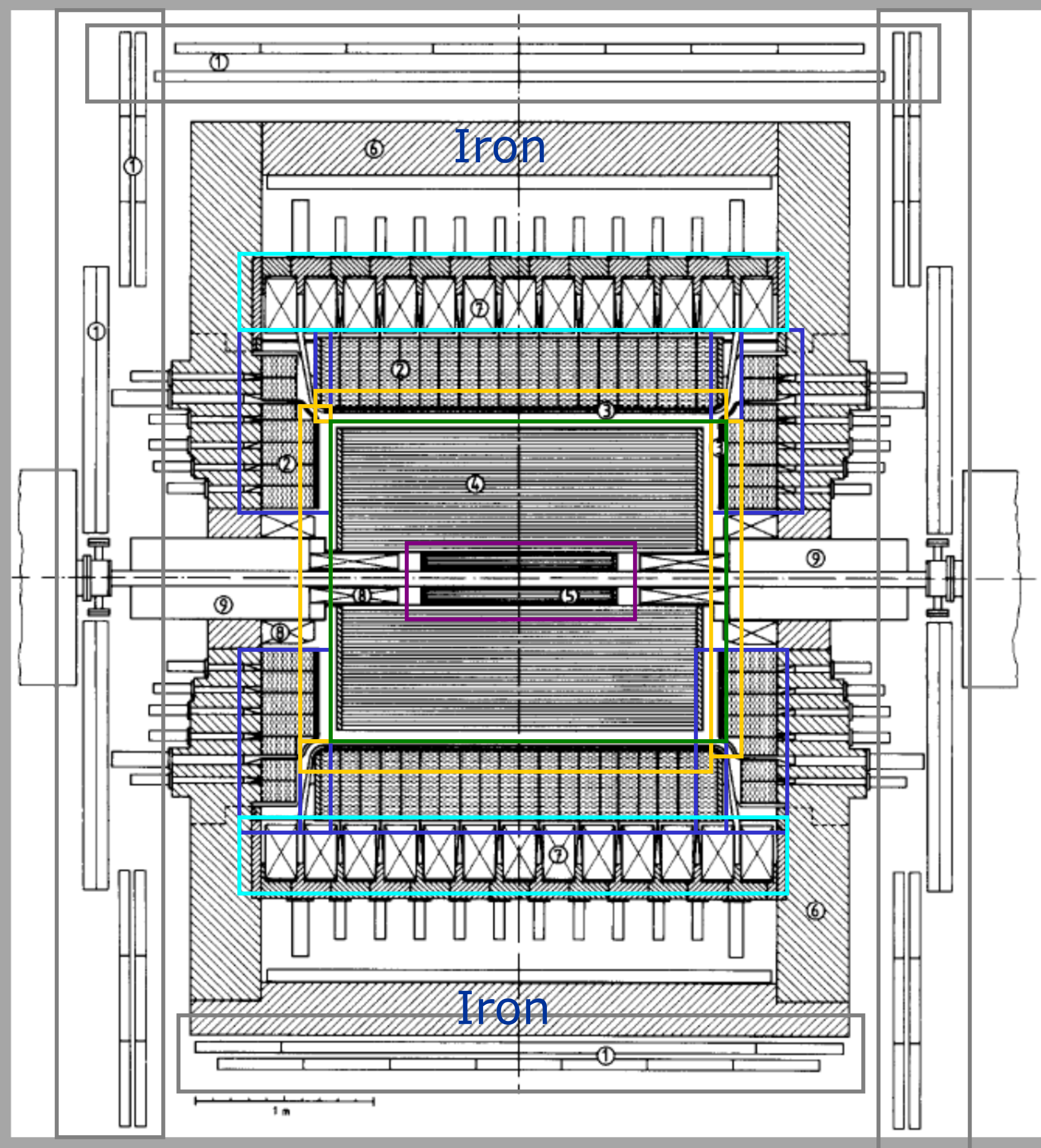
Large size
(6m ∅, 6m L: High ***p*** resol.)

Vertex chamber
(Aiming to short lifetimes..)

Good EM Calorimetry
(Electron/Photon detection)

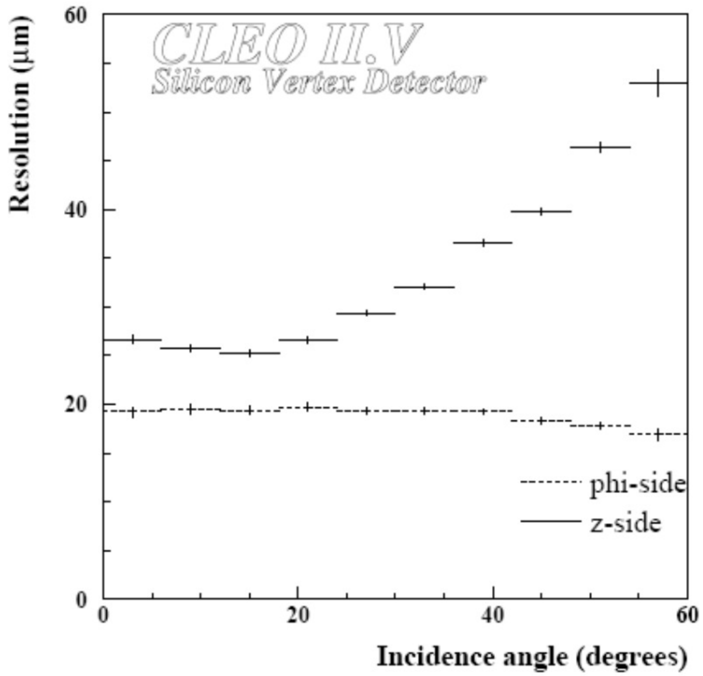
Machine improvements
(Low β quads for luminosity)

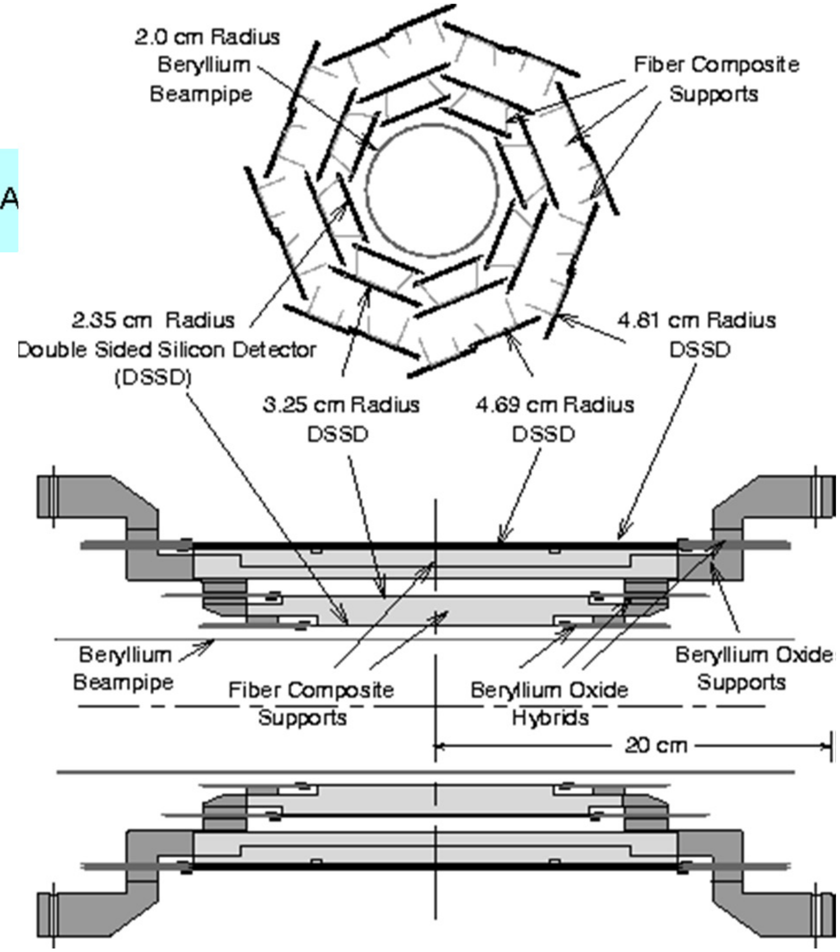
Muon Chambers
Electromagnetic Calorimeter
Time of Flight
Drift Chamber
Vertex Detector
Iron Yoke
Solenoid



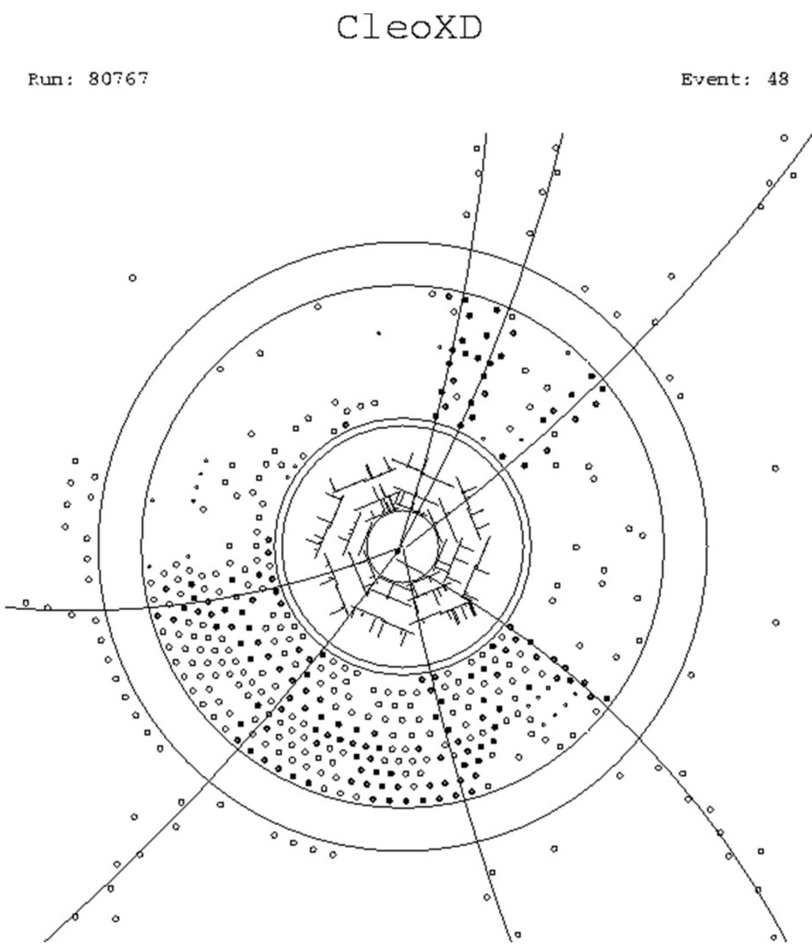
CLEO: The Vertex Detector

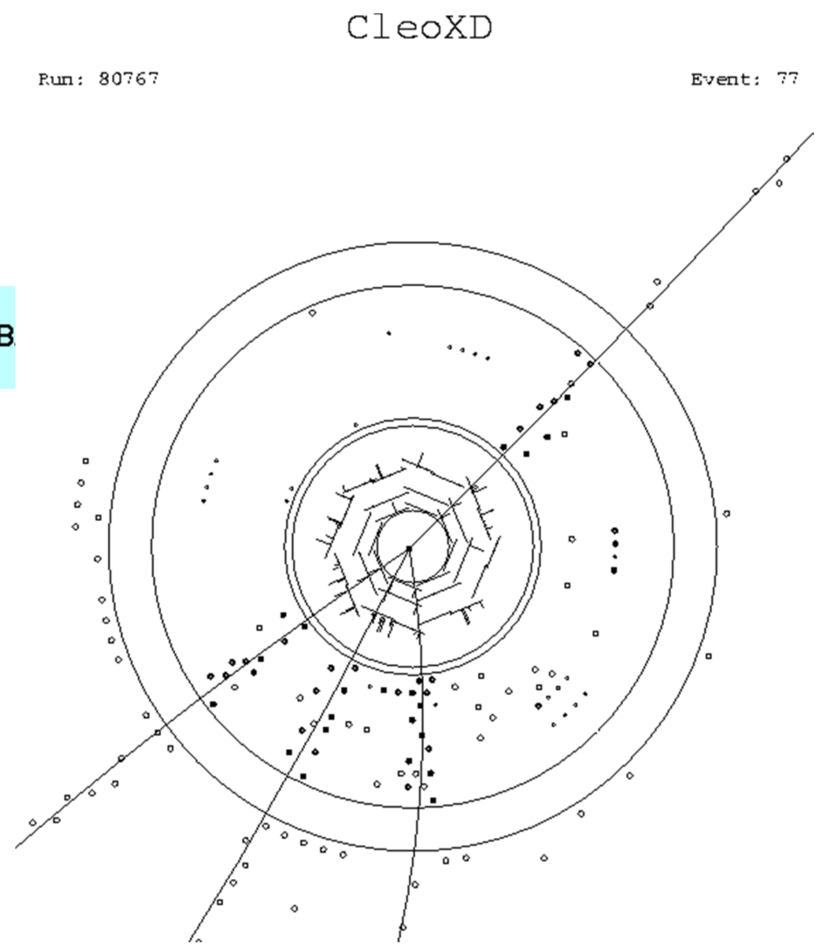
20208 strips



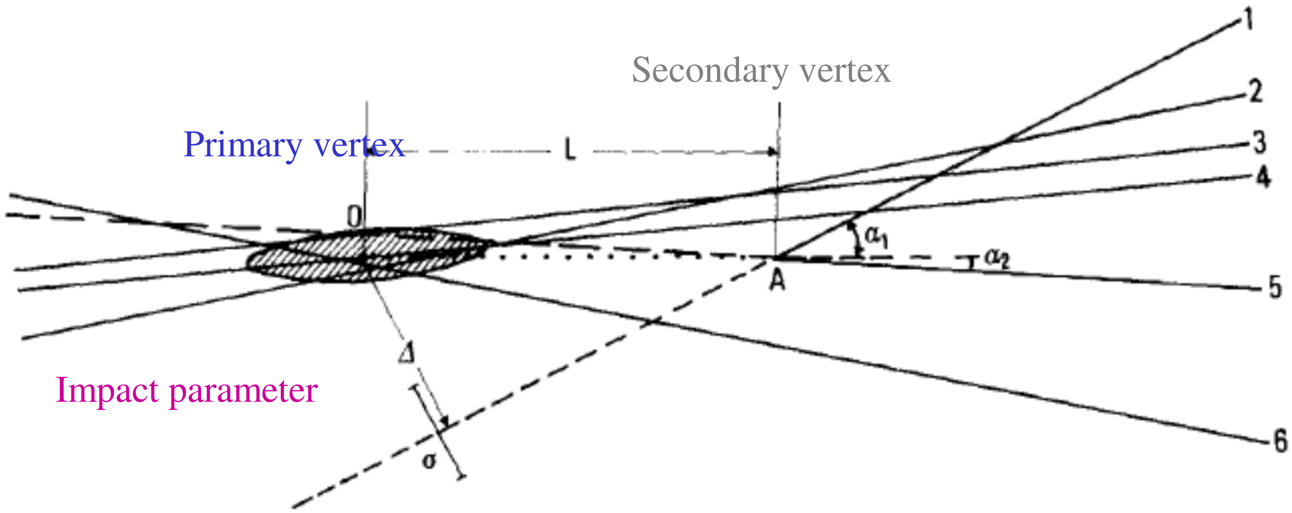


Effects on Tracking





Vertex Detection - I



$$\theta \equiv \alpha_1$$

Plane defined by primary vertex,track direction

Consider a particle produced at primary vertex with speed β

When it decays to another particle, call speed β^*, decay angle in CM θ^*

$$\tan\theta = \frac{\sin\theta^*}{\gamma\cos\theta^* + \beta/\beta^*}$$ Lorentz transformation to LAB

Vertex Detection - II

$L = \beta\gamma\tau$ Decay length

Define impact parameter Δ in terms of decay length, L, and angle θ :

$$\Delta = L\sin\theta = L\frac{\tan\theta}{\sqrt{1+\tan^2\theta}} = L\frac{\dfrac{\sin\theta^*}{\gamma\left(\cos\theta^* + \beta/\beta^*\right)}}{\sqrt{1+\left(\dfrac{\sin\theta^*}{\gamma\left(\cos\theta^* + \beta/\beta^*\right)}\right)^2}} = L\frac{\sin\theta^*}{\sqrt{\left(\gamma\left(\cos\theta^* + \beta/\beta^*\right)\right)^2 + \sin^2\theta^*}}$$

$$\rightarrow \Delta = L\frac{1}{\gamma}\frac{\sin\theta^*}{\sqrt{\left(\cos\theta^* + \beta/\beta^*\right)^2 + \dfrac{1}{\gamma^2}\sin^2\theta^*}} = \beta\tau\frac{\sin\theta^*}{\sqrt{\left(\cos\theta^* + \beta/\beta^*\right)^2 + \dfrac{1}{\gamma^2}\sin^2\theta^*}}$$

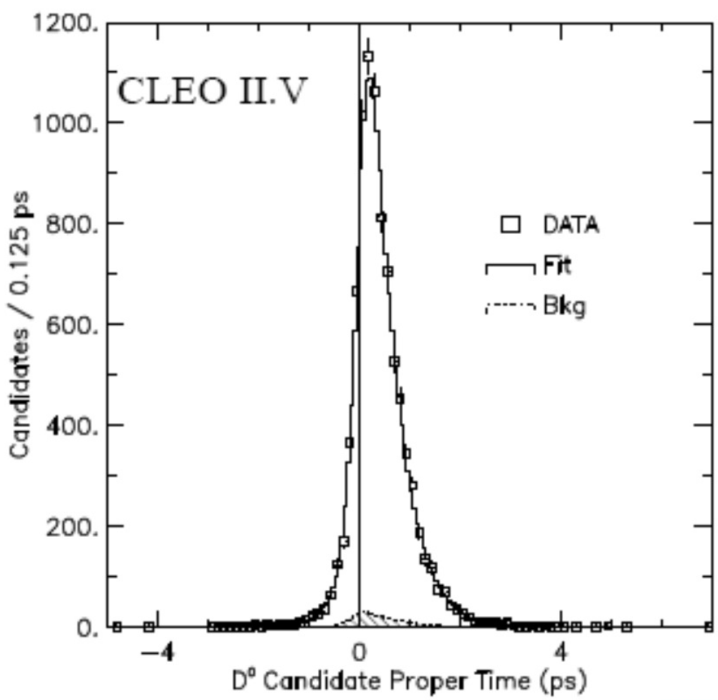
$$\Delta \underset{\beta,\beta^*\rightarrow 1}{\longrightarrow} \beta\tau\frac{\sin\theta^*}{1+\cos\theta^*} = \beta\tau\tan\frac{\theta^*}{2}$$

$y \equiv \dfrac{\Delta}{\tau} \rightarrow$ Find statistical distribution of y for isotropic θ^*, exponential τ

$\rightarrow \langle y \rangle = \dfrac{\pi}{2} \rightarrow \langle \Delta \rangle = \dfrac{\langle \tau \rangle \pi}{2}$ Get a measurement of the decay lifetime

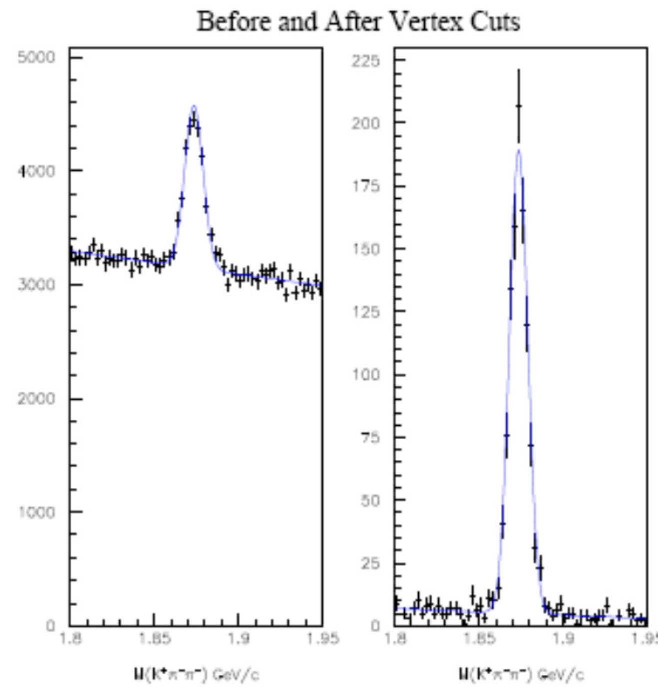
In the limit of relativistic speeds, only from impact parameter !
Full decay reconstruction not required

Vertex Detection: Charm



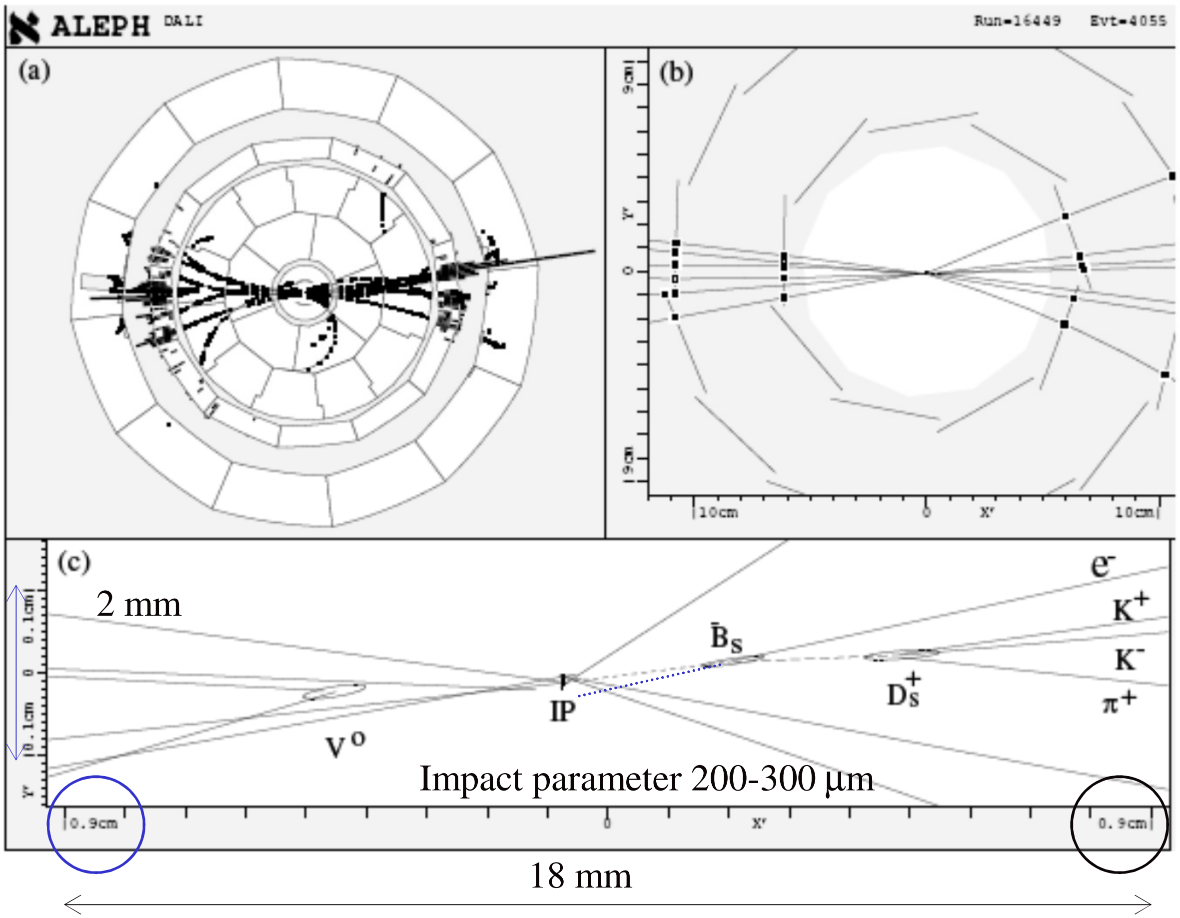
D^0 Lifetime

@TBA



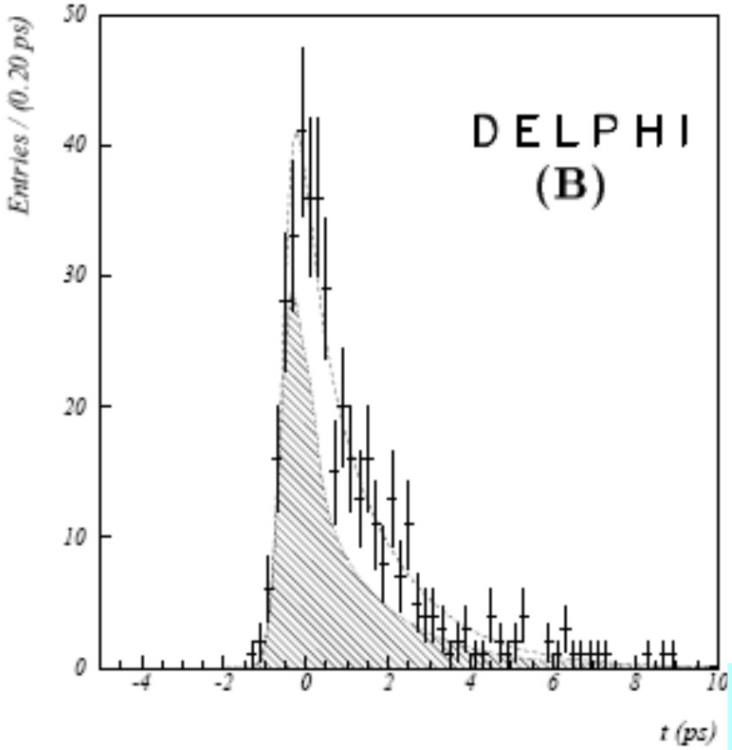
D^* selection,
with and without
secondary vertex

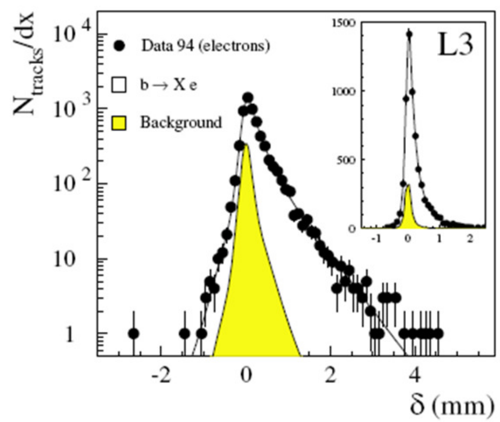
B Tagging: ALEPH



@TBA

DELPHI and L3: *B* Lifetime





Top

Heaviest quark, predicted together with *b* as a member of the 3rd family

Finally found at Fermilab in 1995, after more than 20 years of theoretical and experimental hunting

Very peculiar quark, whose mass of 171 GeV is so large as to allow for weak decays into *b* + *real* W,Z^0

$\rightarrow$ *Very large weak decay rate, short lifetime similar to strong interaction resonances*

$\rightarrow$ *Does not bind into mesons, baryons*

Best understood while discussing weak interactions (see later)