Elementary Particles I

5 – QCD

Color, Gauge Fields, Gluons, Asymptotic Freedom, Quarkonium

Hadrons:Re-Examining the Evidence

Experiments probing the EM structure, like DIS and similar:

Scaling of the structure functions

Evidence for point-like constituents, funny behavior:

Like free particles when interacting with EM currents at high Q^2

Never observed outside hadrons $\rightarrow$ Tightly bound?

Experiments probing the strong interaction:

Large particle zoo

Evidence for highly symmetrical grouping and ordering

Strong suggestion of a substructure: Quarks

Funny, ad-hoc rules driving the observed symmetry

Can We Believe in Constituents?

Besides the general, puzzling behavior observed, finally identify a number of serious problems, cast into a few, quantitative issues:

Baryons and the Pauli Principle
The R Ratio
The π^0 Decay Rate
The τ Lepton Branching Ratios

From all these questions, and others, a common conclusion:

Our picture of the quark model is not complete

The Pauli Principle

Quark model:

Besides its many, remarkable successes, a central point is at issue:

The baryon wave function

$$(space \times spin \times flavor)$$

is symmetric

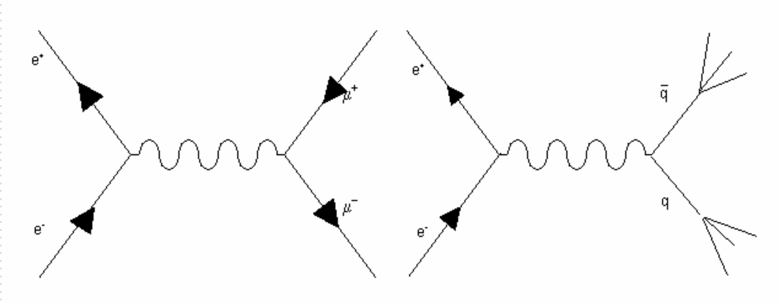
Pauli Principle seems to be lost, which is very bad news:

The Spin-Statistics Theorem is a consequence of very general principles of relativistic QFT, not easily dismissed

The R Ratio - I

Assume the process $e^+e^- \to hadrons$ to proceed at the lowest order through

$e^+e^- \to q\ \overline{q} \to hadrons$



As for DIS:
Don't care about quark *hadronization*, assume the time scales for hard and soft sub-processes to be wildly different

$$\sigma\left(e^+e^- \to \mu^+\mu^-\right) = \frac{4\pi\alpha^2}{3s}$$

$$\sigma\left(e^+e^- \to q\ \overline{q}\right) = \frac{4\pi\alpha^2 Q_q^2}{3s}, \quad Q_q = \text{quark charge in } e \text{ units}$$

$$R\left(E_{CM}\right) = \frac{\sigma\left(e^+e^- \to adroni\right)}{\sigma\left(e^+e^- \to \mu^+\mu^-\right)} = \frac{\sum_q \sigma\left(e^+e^- \to q\overline{q}\right)}{\sigma\left(e^+e^- \to \mu^+\mu^-\right)} = \sum_q Q_q^2$$

R counts the number of different quark species created at any given E_{CM}

The R Ratio - II

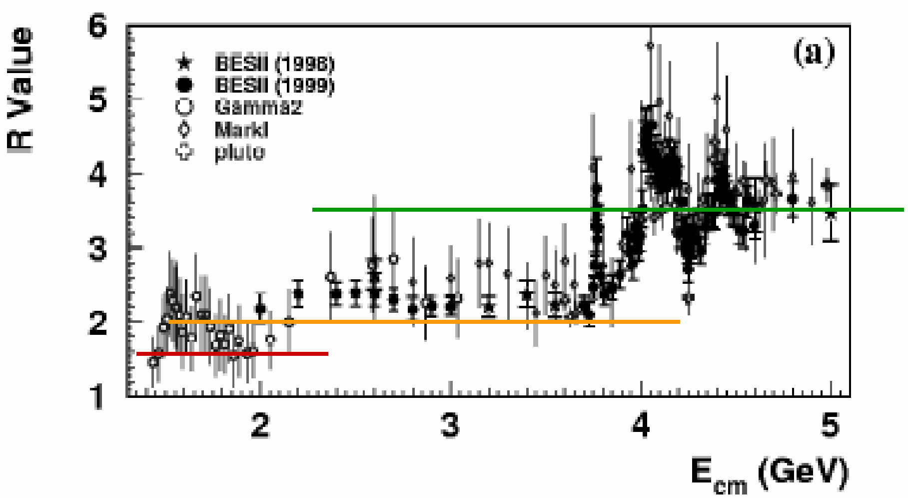
Expect:

$$u,d \to R = \left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 = \frac{5}{9} \qquad \textit{Low energy}$$

$$u,d,s \to R = \left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 = \frac{6}{9} \qquad E > 1\text{-}1.5\ GeV$$

$$u,d,s,c \to R = \left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 = \frac{10}{9} \qquad E > 3\ GeV$$

@TBA



By taking 3 quark species of any flavor:

$$u,d \to R = \frac{15}{9}$$

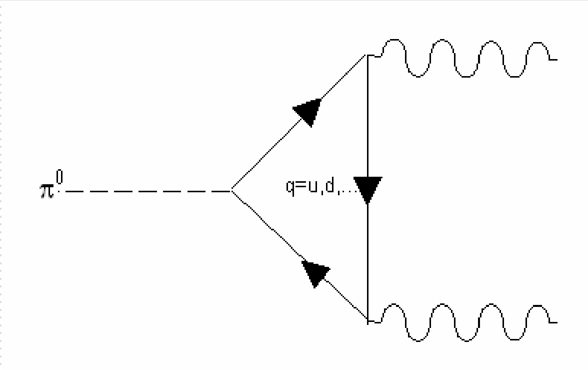
$$u,d,s \to R = \frac{18}{9}$$

$$u,d,s,c \to R = \frac{30}{9}$$

The π^0 Decay Rate

Difficult subject: Strong interaction effects are *large*

Basic diagram



Originally calculated by taking $p,\bar{p}$ in the triangle loop (Steinberger 1949)

$$J^{\mu}_{(A)} \approx e \sum_{i=u,d} \bar{\psi}_i \gamma^{\mu} \gamma^5 \tau_3^i \psi_i$$ Axial loop current matching π^0 - parity

$$\sum_{i=u,d} \tau_3^i Q_i^2 = 1 \cdot \left(\frac{2}{3}\right)^2 - 1 \cdot \left(-\frac{1}{3}\right)^2 = \frac{4}{9} - \frac{1}{9} = \frac{1}{3}$$

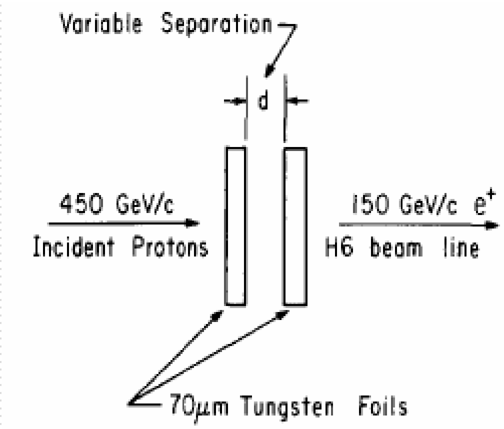
$$\Gamma_{quark}\left(\pi^0 \to \gamma\gamma\right) = \frac{\alpha^2}{64\pi^3} \frac{m_\pi^3}{f_\pi^2} \sum_i g_A^{(i)} e_i^2 = \frac{1}{9} \Gamma_{proton}\left(\pi^0 \to \gamma\gamma\right)$$

A whole *lot* of physics in this problem:

Simple guess on approximate symmetry of the initial state would lead to conclude that *the neutral pion is stable!*

Explaination of this paradox led to discovery of the first *anomaly* in QFT (Adler, Bell, Jackiw): Advanced topic, quite relevant to the Standard Model

The π^0 Lifetime: Direct Method

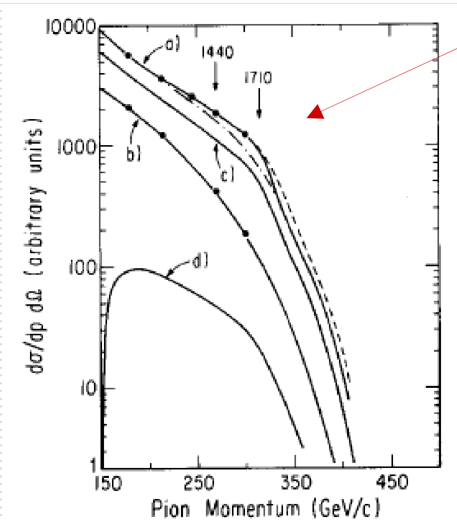


π^0 produced in a first thin foil, when not decayed do not contribute to e^+ yield from γ conversion in a second thin foil

$$Y(d) = N\left\{A + B\left[1 - \exp(-d/\lambda)\right]\right\}$$

$\lambda = \beta\gamma c\tau \simeq \gamma c\tau$ Energy dependent

Use known energy spectra for pions



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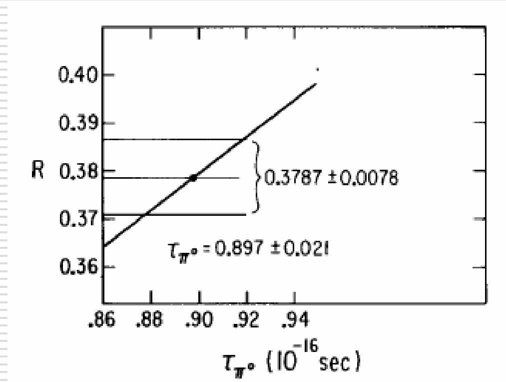
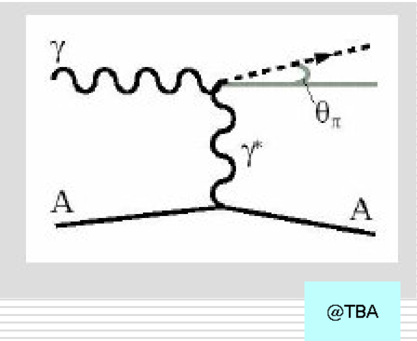


$$\tau = 0.897 \pm .021\ 10^{-16}\ s$$

$$\Gamma = 7.34 \pm 0.18 \pm 0.11\ eV$$

The π^0 Lifetime: Primakoff Effect



Very simple idea:
Get a high energy photon beam + high Z target
Pick-up a virtual photon from the nuclear Coulomb field
2-photon coupling will (sometimes) create a π^0

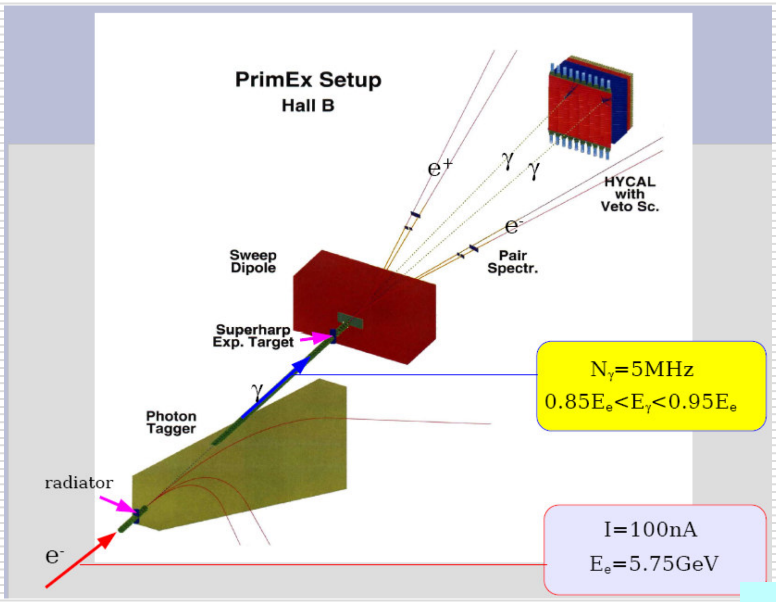
$$\frac{d\sigma_{\text{Prim}}}{d\Omega_{LAB}} \simeq \Gamma_{\pi^0\to\gamma\gamma} Z^2 \frac{8\alpha E^4}{m_{\pi^0}^2} \frac{\left|F\left(q^2\right)\right|^2}{q^4} \sin^2\theta_{\pi^0}$$

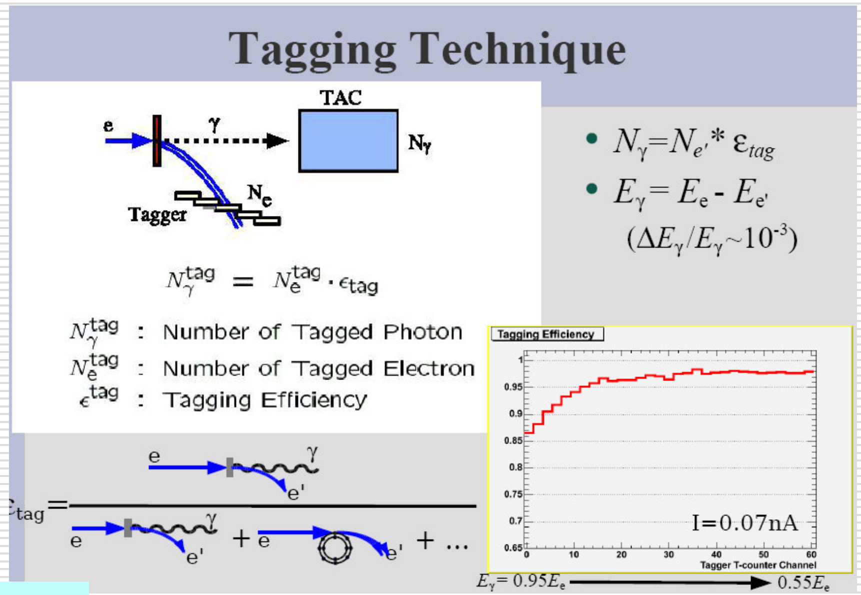
$\Gamma = 1/\tau$ can be extracted by measuring the differential cross-section
Nuclear form factor is required

Strongly forward peaked
Quickly increasing with energy
Strongly Z dependent: Coherence

A Recent Experiment

PrimEx at Jefferson Lab (Virginia)





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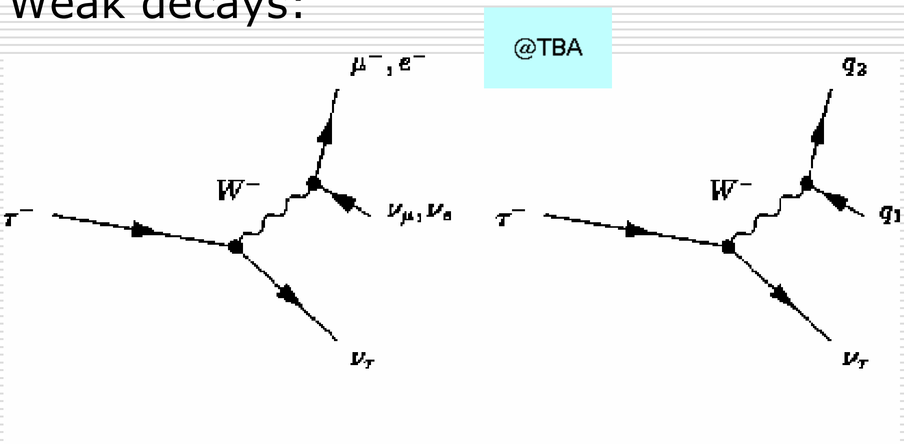
Results expected soon…

The τ Lepton Decays

τ: Heavy brother of e and μ

m_τ= 1776 MeV

Weak decays:



$q, \bar{q} : u, d, s$

$BR(e) \sim 18\%$

$BR(\mu) \sim 17\%$

$BR(q\bar{q}) \sim 65\%$

In the absence of color, weak interaction universality would lead to predict:

$BR(e) \sim BR(\mu) \sim BR(q\bar{q}) \sim 33\%$

With color:

$$\Gamma(q\bar{q}) \sim 3\ \Gamma(l\bar{l}) \to BR(q\bar{q}) \sim \frac{3\ \Gamma(l\bar{l})}{3\ \Gamma(l\bar{l}) + 2\ \Gamma(l\bar{l})} \sim 60\ \%$$ OK

Color

New hypothesis:

There is a new degree of freedom for quarks
Call it color

Each quark can be found in one of 3 different states

Internal space (mathematically identical to flavor):

States = 3-component complex vectors

Base states:

$$R(ed)=\begin{pmatrix}1\\0\\0\end{pmatrix},G(reen)=\begin{pmatrix}0\\1\\0\end{pmatrix},B(lue)=\begin{pmatrix}1\\0\\0\end{pmatrix}$$

Needless to say, nothing to do with our old, beloved color concept (nature, art, politics..):
Just a *name* for another, non-classical property of hadron constituents

Benefits from the Color Hypothesis

Provided one can build a color wave function for 3 fermions which is antisymmetric, the Fermi statistics problem is solved
Total wave function of a baryon

$$\psi = \psi_{color} \underbrace{\psi_{orbital} \psi_{spin} \psi_{flavor}}_{Symmetric} \rightarrow \psi_{color} : \ Antisymmetric$$

To account for 3 different color states, the R ratio must be multiplied by *3* $\rightarrow$ OK with experimental data

Just the same conclusion for hadronic τ decays: Multiply rate by *3*
The correct π^0 rate is obtained by inserting a factor *9*

Observe:
When computing R, τ decay rates we add the *rates* for different colors
$\rightarrow$ Factor $\times 3$
We deal with quarks as with real particles: Ignore fragmentation

When computing π^0 decay rate, we add the *amplitudes*
$\rightarrow$ Factor $\times 9$
Quarks are virtual particles: Amplitudes interfere

Color as a Quantum Number

Must be possible to build hadron states as color *singlets*
Do not expect hadrons to fill larger irr.reps.: Would imply large degeneracies for hadron states, not observed
In other words:

Color is fine, but we do not observe any colored hadron

How colored hadrons would show up? Just as an example:
Should the nucleon fill the ***3*** of $SU(3)_C$ there would be 3 different species of protons and neutrons. Then each nucleon level in any nucleus could accommodate 3 particles instead of one: The nuclear level scheme would be far different from the observed one

Therefore we assume the color charge is *confined*: Never observed directly, as it is the electric charge

Why? For the moment, nobody really knows

The Color Group: *SU(3)*$_C$

Guess *SU(3)* as the color group
Take the two fundamental decompositions:

$\mathbf{3}\otimes\mathbf{3}\otimes\mathbf{3}=\mathbf{1}\oplus\mathbf{8}\oplus\mathbf{8}\oplus\mathbf{10}$ Baryons

$\mathbf{3}\otimes\mathbf{3}^{*}=\mathbf{1}\oplus\mathbf{8}$ Mesons

Both feature a singlet in the direct sum: OK
No singlets in **3**⊗**3**: OK

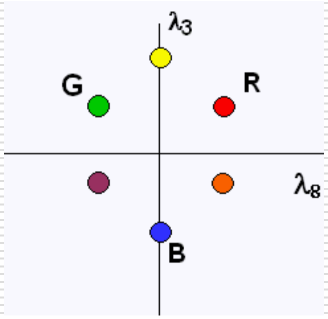
Can't say the same for other groups…
Take *SU(2)* as an example:
Say the quarks live in the adjoint *SU(2)* representation, ***3***
Then for qq :

$\boldsymbol{3}\otimes\boldsymbol{3}=\boldsymbol{1}\oplus\boldsymbol{3}\oplus\boldsymbol{5}$

Observe: This is ***3*** of *SU(2)*, which is quite different from ***3*** of *SU(3)*

Diquarks can be in color singlet
→Should find diquarks as commonly as baryons or mesons..

The Color of Quarks



$$I_3^c = \frac{\lambda_3}{2}, \qquad Y^c = \frac{\lambda_8}{\sqrt{3}}$$

	I_3^c	Y^c
R	$+1/2$	$+1/3$
G	$-1/2$	$+1/3$
B	0	$-2/3$

	I_3^c	Y^c
$\bar{R}$	$-1/2$	$-1/3$
$\bar{G}$	$+1/2$	$-1/3$
$\bar{B}$	0	$+2/3$

SU(3)$_C$ is an *exact symmetry*:

$$m_u^{(R)} = m_u^{(G)} = m_u^{(B)}, \ldots$$

Beware: SU(3)$_C$ has nothing to do with *SU(3)*$_F$:
Quark quantum numbers are independent from their color state
They are left unchanged by QCD transitions

The Color of Hadrons

According to our fundamental hypothesis:

Mesons: $\mathbf{3}\otimes\mathbf{3}^{*}=\mathbf{1}\oplus\mathbf{8}$

$$\rightarrow \psi_c = \frac{1}{\sqrt{3}}\left(R\bar{R}+G\bar{G}+B\bar{B}\right)$$

Pick singlet

Baryons: $\mathbf{3}\otimes\mathbf{3}\otimes\mathbf{3}=\mathbf{1}\oplus\mathbf{8}\oplus\mathbf{8}\oplus\mathbf{10}$

$$\rightarrow \psi_c = \frac{1}{\sqrt{6}}\left(RGB-RBG+GBR-GRB+BRG-BGR\right)$$

Mesons: *No particular exchange symmetry (2 non identical particles)*
However, by properly extending the Pauli principle to include particle-antiparticle pairs, singlet can be shown to be antisymmetrical (or so they say…)

Baryons: *Fully antisymmetrical color wave function (3 identical particles)*

Extending the Color Hypothesis: *QCD*

Color: A new degree of freedom for quarks
Compare to other quantum numbers:

Baryonic/Leptonic numbers, Flavor
Conserved, *not originating interactions*

Electric charge
Conserved, *origin of the electromagnetic field*

A deep question:

What is the true origin of the electromagnetic interaction?

We have used freely the interaction term $j^{\mu}A_{\mu}$, only based on the classical analogy: Is there a deeper origin for it?

QED as a Gauge Theory - I

Symmetry: Absolute phase not defined for a wave function.
Expect invariance as per our old acquaintance, Noether's Theorem

$L_0 = \bar{\psi}(x)\left(i\gamma^\mu \partial_\mu - m\right)\psi(x)$ Free Dirac Lagrangian

Global gauge (=Phase) transformation:

Just meaning:
Take *all* particle states; Re-phase each state proportionally to its charge

$G : \psi(x) \to \psi'(x) = U_\theta \psi(x) = e^{-iq\theta}\psi(x)$ $q\theta:$ New phase $\propto$ Charge

$\to L_0$ invariant wrt $G \to$ Charge conservation

Generalize to local phase transformation:

$G_L : \psi(x) \to \psi'(x) = U_\theta \psi(x) = e^{-iq\theta(x)}\psi(x)$ Local gauge transformation

$\to L_0$ not invariant wrt $G_L:$ Derivative term troublesome

$\to$ Local gauge invariance cannot hold in a world of free particles
Symmetry requires interaction

QED as a Gauge Theory - II

New transformation rule:

$$\begin{cases} \psi(x) \to \psi'(x) = U_\theta \psi(x) = e^{-iq\theta(x)}\psi(x) & \text{As before} \\ A_\mu(x) \to A_\mu(x) + q\ \partial_\mu \theta(x) & \text{New character in the comedy} \end{cases}$$

Equivalent to re-define derivative for ψ:

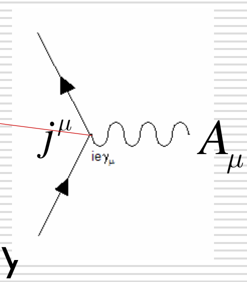
$\partial_\mu \to \partial_\mu + iqA_\mu$ Vector field

Experts say this has a *deep* geometrical meaning…

Add a new term to Lagrangian:

$L_0 = \bar{\psi}(x)\left(i\gamma^\mu \partial_\mu - m\right)\psi(x) \to L_0 + L_i$ Sum is invariant

$L_i = -q\underbrace{\bar{\psi}(x)\gamma^\mu \psi(x)}_{j^\mu} A_\mu$ Interaction term



…And another one:

$-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}, \quad F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ Field energy

Must be there because the field carries energy+momentum

Reminder:

$F^{\mu\nu}$ is the EM field

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix}$$

QED as a Gauge Theory - III

(One) Reason to insist on local transformations:

Global gauge changes would allow for non-local charge conservation:
Then one would happily violate our beloved Principle of Relativity…

Field must be massless to have L gauge invariant

Consider all the phase transformations as defined before:

$$\psi(x) \to \psi'(x) = U_\theta \psi(x) = e^{-iq\theta(x)}\psi(x)$$

The full set is a group: $U(1)$ Unitary, 1-dimensional

$$e^{-iq\theta_1(x)}e^{-iq\theta_2(x)}\psi(x) = e^{-iq[\theta_1(x)+\theta_2(x)]} \in U(1)$$

1 parameter: $\theta(x)$

Abelian: $e^{-iq\theta_1(x)}e^{-iq\theta_2(x)}\psi(x) = e^{-iq\theta_2(x)}e^{-iq\theta_1(x)}\psi(x)$

U(1) is the (Abelian) *gauge group* of QED
Equivalent to *SO(2),* group of 2D rotations

QCD as a Gauge Theory - I

Extend gauge transformations to a 3-component wave function:

$$\boldsymbol{\psi} \equiv \begin{pmatrix} \psi_R \\ \psi_G \\ \psi_B \end{pmatrix}$$

Global gauge transformation: Phase change for individual components

$\rightarrow$ Phase change will mix color components

$G_L^C : \boldsymbol{\psi}(x) \rightarrow \boldsymbol{\psi}'(x) = \mathbf{U}_G \cdot \boldsymbol{\psi}(x) = e^{-ig\mathbf{M}} \cdot \boldsymbol{\psi}(x)$ $\quad \mathbf{U}_G$ unitary $\rightarrow \mathbf{M}$ Hermitian

$g:$ Color charge

$e^{-ig\mathbf{M}} = \mathbf{1} - ig\mathbf{M} + \frac{(-ig\mathbf{M})^2}{2!} + \ldots$ $\quad \mathbf{M}:$ 3$\times$3 Hermitian matrix

$\mathbf{M}$ acting on the 3 color components of the quark state

Since the color symmetry group is $SU(3)_C$:

$$\mathbf{M} = \sum_{i=1}^{8} \boldsymbol{\lambda}_i \theta_i \equiv \vec{\boldsymbol{\lambda}} \cdot \vec{\theta}$$

$\vec{\boldsymbol{\lambda}}:$ Vector of 8 3$\times$3 Gell-Mann matrices; $\vec{\theta}:$ Vector of 8 parameters

QCD as a Gauge Theory - II

As for QED, extend to $local$ gauge transformations

As before, in order to guarantee invariance of L:

$\rightarrow$Re-define derivative adding new vector fields:

$\partial_\mu \rightarrow \partial_\mu \mathbf{1} + ig\mathbf{C}_\mu$

$$\mathbf{C}_\mu : \begin{cases} \text{4-vector field} & \text{Lorentz structure} \\ \text{Matrix } \in SU(3)_C & \text{Color space} \end{cases}$$

We know how to express any Hermitian matrix $\in SU(3)_C$:

Use $SU(3)_C$ generators $\rightarrow$ Gell-Mann matrices

$\rightarrow \mathbf{C}_\mu = \frac{1}{2}\sum_{i=1}^{8} \mathrm{G}_\mu^a \boldsymbol{\lambda}_a \equiv \vec{G}_\mu \cdot \vec{\lambda}$ 8 fields required: Gluons

So gluons are a bit like 8 different "photons", exchanged between color charges

But: *They are non Abelian*

QCD as a Gauge Theory - III

Local gauge transformation for $SU(3)_C$:

$$\begin{cases} \boldsymbol{\psi}(x) \to \boldsymbol{\psi}'(x) = U_\theta \boldsymbol{\psi}(x) = e^{-ig\vec{\boldsymbol{\lambda}}\cdot\vec{\theta}(x)}\boldsymbol{\psi}(x) \\ G_\mu^a(x) \to G_\mu^a{}'(x) = G_\mu^a(x) + \partial_\mu \theta^a + g\sum_{b,c=1}^{8} f^{abc} G_\mu^b(x)\ \theta^c(x) \qquad a = 1,\dots,8 \end{cases}$$

Very important:
New term, coming from $SU(3)$ being non Abelian

Reminder:

$$\left[\frac{\lambda_a}{2}, \frac{\lambda_b}{2}\right] = if^{abc}\frac{\lambda_c}{2}$$

f^{abc}: $SU(3)$ structure constants

A set of *integer numbers*,
kind of DNA of each group

$$L_0 = \bar{\boldsymbol{\psi}}(x)\left(i\gamma^\mu \partial_\mu - m\right)\boldsymbol{\psi}(x) \to L_0 + L_i$$

$$L_i = -g\underbrace{\left[\bar{\boldsymbol{\psi}}(x)\gamma^\mu\left(\frac{\vec{\boldsymbol{\lambda}}}{2}\right)\boldsymbol{\psi}(x)\right]}_{\vec{j}_\mu}\cdot\vec{G}_\mu$$ Interaction term

$$-\frac{1}{4}\vec{G}_{\mu\nu}\cdot\vec{G}^{\mu\nu} = -\frac{1}{4}\sum_{a=1}^{8} G_{\mu\nu}^a \cdot G^{a\mu\nu}$$ Field energy term

QCD as a Gauge Theory - IV

Take the expression of fields in terms of potentials:

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu - g\sum_{b,c=1}^{8} f_{abc} G^b_\mu G^c_\upsilon$$

Very important: *Absent in QED (f=0)*
New term, coming from *SU(3)* being non Abelian

$\rightarrow G^a_{\mu\nu} G^a_{\mu\nu}$ contains terms with $\underbrace{\partial_\mu G^a_\nu \cdot G^b_\mu G^c_\upsilon}_{\text{3 gluons}}, \underbrace{G^b_\mu G^c_\upsilon \cdot G^b_\mu G^c_\upsilon}_{\text{4 gluons}}$

These pieces of *L* correspond to 3 and 4 gluons vertexes

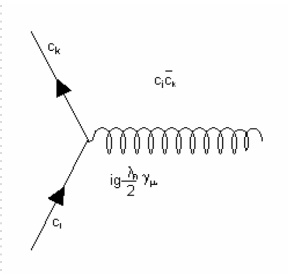
The form of QCD Lagrangian leads to predict the existence of a new kind of gluon-gluon color interaction

Since color interaction is tied to color charge, we are saying *the gluons carry their own color charge*

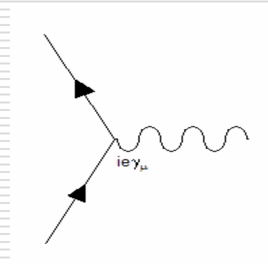
Sounds unfamiliar? Well, that's all after playing with a non-Abelian gauge group.
Unlike the electric charge, color charge can manifest itself in more than one way.
Indeed, gluons carry a type of color charge different from quarks/antiquarks:
Color + Anticolor

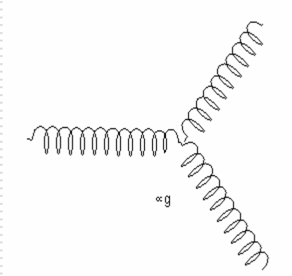
QCD as a Gauge Theory - V

QCD Vertexes

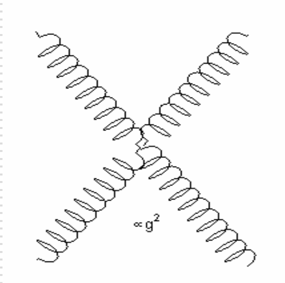


$ig\frac{\lambda_a}{2}\gamma_\mu$ Similar to QED:





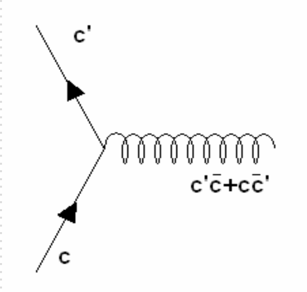
$\propto g$ (Lorentz structure not shown)



$\propto g^2$ (Lorentz structure not shown)

The Color of Gluons

Compare to mesons in *SU(3)*$_F$: *Flavor + Antiflavor*
But: *Gluons are not bound states of Color+Anticolor!*
Still, they share the same math:
Gluons live in the adjoint (***8***) irr.rep. of *SU(3)*$_C$



A very natural question: Gluons couple to $q\bar{q}$

Since one can decompose the total $q\bar{q}$ color state as:

$$\mathbf{3}\otimes\mathbf{3}^* = \mathbf{1}\oplus\mathbf{8}$$

Then: Where is the singlet gluon?

Does not exist: There are only 8 gluons, not 9

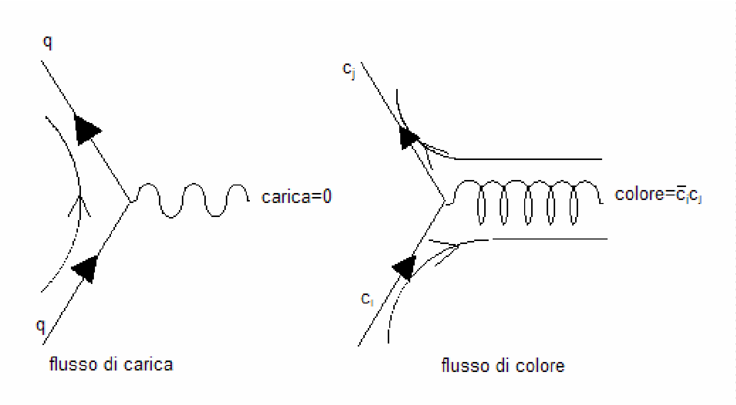
$$|1\rangle = \frac{1}{\sqrt{2}}\left(r\bar{b}+b\bar{r}\right), |2\rangle = -\frac{i}{\sqrt{2}}\left(r\bar{b}-b\bar{r}\right), |3\rangle = \frac{1}{\sqrt{2}}\left(r\bar{r}-b\bar{b}\right)$$

$$|4\rangle = \frac{1}{\sqrt{2}}\left(r\bar{g}+g\bar{r}\right), |5\rangle = -\frac{i}{\sqrt{2}}\left(r\bar{g}-g\bar{r}\right), |6\rangle = \frac{1}{\sqrt{2}}\left(b\bar{g}+g\bar{b}\right)$$

$$|7\rangle = -\frac{i}{\sqrt{2}}\left(b\bar{g}-g\bar{b}\right), |8\rangle = \frac{1}{\sqrt{6}}\left(r\bar{r}+b\bar{b}-2g\bar{g}\right)$$

Color vs. Charge Flow

Compare the different situations:



QED	QCD
Photon is neutral Neither sourcing, nor sinking charge	Gluon is colored Sourcing color, sinking anti-color

Should the singlet gluon actually exist, it would behave more or less like a "photon":
Would be 'white' (= Singlet)
Would couple to color charges in the same way as photon couples to electric charges
Would give rise to a sort of "QED-like" color interaction, not observed

Comparing *QED* and *QCD*

Comparison of coupling constants:

α vs. α_s Adimensional constants (*Interaction strength)*

Can define elementary charge in terms of α , α_s
Measure particle charge by its ratio to elementary charge: *Number*
What are the allowed values for these numbers?

QED: Gauge group is *Abelian*
Electric charge can be *any* number: No reason for charge quantization
Photon charge is strictly *0*

QCD: Gauge group is *non Abelian*
Color charge value is *fixed* for every representation

Quarks: ***3***,***3**** $\rightarrow Q = 4/3$
Gluons: ***8*** $\rightarrow Q = 8$

Similar to *I(I+1)* for any isospin (*SU(2)*) multiplet

The Color Factor

Consider the interaction between 2 charges:

QED

For fixed $|q|$, the 'charge factor' can be defined as:

$$f_{12} = \frac{q_1 q_2}{|q|^2} = \begin{cases} +1 & q_1 q_2 > 0 \\ -1 & q_1 q_2 < 0 \end{cases}$$

Very simple for an Abelian interaction

QCD

The ‘color factor’ depends on the irr.reps of the initial and final states
Since total color is conserved in all processes, expect a color factor:

Representation dependent
Identical for any transition in a given representation

Less simple in this non-Abelian interaction

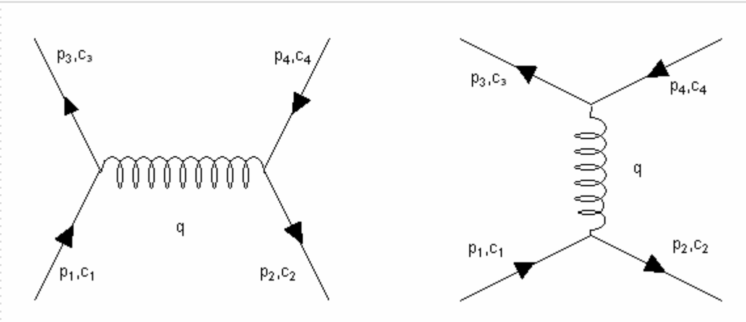
Color Interaction - I

$q\bar{q} \rightarrow q\bar{q}$

$\mathbf{3} \otimes \mathbf{3}^* = \mathbf{1} \oplus \mathbf{8}$

Total color conservation: $\begin{cases} \mathbf{1} \rightarrow \mathbf{1} \\ \mathbf{8} \rightarrow \mathbf{8} \end{cases}$

Observe:
Similar to conservation of total I-spin



$$T_{fi} = i \sum_{\alpha,\beta=1}^{8} \underbrace{\left[\bar{u}(3) c_3^\dagger\right]\left[-i\frac{g_s}{2}\lambda^\alpha \gamma^\mu\right]\left[u(1) c_1\right]}_{\text{color current}} \underbrace{\left[-i\frac{g_{\mu\nu}\delta^{\alpha\beta}}{q^2}\right]}_{\text{propagator}} \underbrace{\left[\bar{v}(2) c_2^\dagger\right]\left[-i\frac{g_s}{2}\lambda^\beta \gamma^\nu\right]\left[v(4) c_4\right]}_{\text{color current}}$$

Sum is over all 8 color matrices

c_i are the color states of initial, final $q\bar{q}$

$$T_{fi} = \frac{-g_s^2}{q^2}\left[\bar{u}(3)\gamma^\mu u(1)\right]\left[\bar{v}(2)\gamma_\mu v(4)\right] \underbrace{\frac{1}{4}\sum_\alpha \left[c_3^\dagger \lambda^\alpha c_1\right]\left[c_2^\dagger \lambda^\alpha c_4\right]}_{\text{color factor}}$$

Color Interaction - II

Octet

$r\bar{b}$ Just as an example: Result is the same for all octet states

$$\left.\begin{array}{l} c_1 = c_3 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \\ c_2 = c_4 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \end{array}\right\} \rightarrow f = \frac{1}{4}\sum_{\alpha=1}^{8} \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \lambda^{\alpha} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \end{pmatrix} \lambda^{\alpha} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\rightarrow f = \frac{1}{4}\sum_{\alpha=1}^{8} \lambda_{11}^{\alpha}\lambda_{22}^{\alpha} = \frac{1}{4}\left(\lambda_{11}^{3}\lambda_{22}^{3} + \lambda_{11}^{8}\lambda_{22}^{8}\right) = -\frac{1}{6}$$

Color Interaction - III

<u>Singlet</u>

$\frac{1}{\sqrt{3}}\left(r\bar{r}+b\bar{b}+g\bar{g}\right)$ Only this state in the singlet

But: *Any* component can go into *any other*..

$$f_i=\frac{1}{4}\frac{1}{\sqrt{3}}\sum_{\alpha=1}^{8}\left[c_3^\dagger\lambda^\alpha\begin{pmatrix}1\\0\\0\end{pmatrix}\right]\left[\begin{pmatrix}1&0&0\end{pmatrix}\lambda^\alpha c_4\right]+\left[c_3^\dagger\lambda^\alpha\begin{pmatrix}0\\1\\0\end{pmatrix}\right]\left[\begin{pmatrix}0&1&0\end{pmatrix}\lambda^\alpha c_4\right]+\left[c_3^\dagger\lambda^\alpha\begin{pmatrix}0\\0\\1\end{pmatrix}\right]\left[\begin{pmatrix}0&0&1\end{pmatrix}\lambda^\alpha c_4\right]$$

$i=1,2,3$

$$f=\frac{1}{4}\frac{1}{\sqrt{3}}\frac{1}{\sqrt{3}}\sum_{\alpha=1}^{8}\sum_{i,j=1}^{3}\lambda_{ij}^\alpha\lambda_{ji}^\alpha=\frac{1}{12}Tr\left(\lambda^\alpha\lambda^\alpha\right)$$

$$Tr\left(\lambda^\alpha\lambda^\beta\right)=2\delta^{\alpha\beta}$$

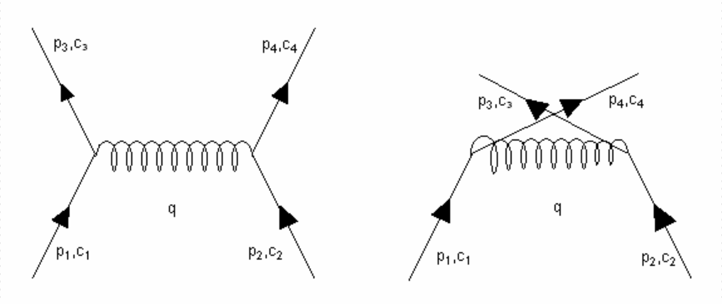
$$Tr\left(\lambda^\alpha\lambda^\alpha\right)=16$$

$$\rightarrow f=\frac{4}{3}$$

Color Interaction - IV

qq

$\mathbf{3} \otimes \mathbf{3} = \mathbf{3}^* \oplus \mathbf{6}$



$$T_{fi} = \frac{-g_s^2}{q^2}\left[\bar{u}(3)\gamma^\mu u(1)\right]\left[\bar{u}(4)\gamma_\mu u(2)\right]\underbrace{\frac{1}{4}\sum_{\alpha=1}^{8}\left[c_3^\dagger \lambda^\alpha c_1\right]\left[c_4^\dagger \lambda^\alpha c_2\right]}_{\text{color factor}}$$

$$f_{qq} = \frac{1}{4}\sum_{\alpha=1}^{8}\left(c_3^\dagger \lambda^\alpha c_1\right)\left(c_4^\dagger \lambda^\alpha c_2\right)$$

Color states of the triplet and sextet:

$\mathbf{3}^*$: $\frac{1}{\sqrt{2}}(rb - br), \frac{1}{\sqrt{2}}(bg - gb), \frac{1}{\sqrt{2}}(gr - rg)$ Antisymmetric

$\mathbf{6}$: $rr, bb, gg, \frac{1}{\sqrt{2}}(rb + br), \frac{1}{\sqrt{2}}(bg + gb), \frac{1}{\sqrt{2}}(gr + rg)$ Symmetric

Color Interaction - V

Sextet

rr Just as an example: Result is the same for all sextet states

$$c_1 = c_2 = c_3 = c_4 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$f = \frac{1}{4}\sum_{\alpha=1}^{8}\left[\begin{pmatrix} 1 & 0 & 0 \end{pmatrix}\lambda^{\alpha}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}\right]\left[\begin{pmatrix} 1 & 0 & 0 \end{pmatrix}\lambda^{\alpha}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}\right] = \frac{1}{4}\sum_{\alpha=1}^{8}\left(\lambda_{11}^{\alpha}\lambda_{11}^{\alpha}\right)$$

$$= \frac{1}{4}\left(\lambda_{11}^{3}\lambda_{11}^{3} + \lambda_{11}^{8}\lambda_{11}^{8}\right) = \frac{1}{3}$$

Color Interaction - VI

Triplet

$\frac{1}{\sqrt{2}}\left(rb-br\right)$ Just as an example as before

$$f=\frac{1}{4}\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\sum_{\alpha=1}^{8}$$

$$\left\{\left[\begin{pmatrix}1&0&0\end{pmatrix}\lambda^{\alpha}\begin{pmatrix}1\\0\\0\end{pmatrix}\right]\left[\begin{pmatrix}0&1&0\end{pmatrix}\lambda^{\alpha}\begin{pmatrix}0\\1\\0\end{pmatrix}\right]-\left[\begin{pmatrix}0&1&0\end{pmatrix}\lambda^{\alpha}\begin{pmatrix}1\\0\\0\end{pmatrix}\right]\left[\begin{pmatrix}1&0&0\end{pmatrix}\lambda^{\alpha}\begin{pmatrix}0\\1\\0\end{pmatrix}\right]\right.$$

$$\left.-\left[\begin{pmatrix}1&0&0\end{pmatrix}\lambda^{\alpha}\begin{pmatrix}0\\1\\0\end{pmatrix}\right]\left[\begin{pmatrix}0&1&0\end{pmatrix}\lambda^{\alpha}\begin{pmatrix}1\\0\\0\end{pmatrix}\right]-\left[\begin{pmatrix}0&1&0\end{pmatrix}\lambda^{\alpha}\begin{pmatrix}0\\1\\0\end{pmatrix}\right]\left[\begin{pmatrix}1&0&0\end{pmatrix}\lambda^{\alpha}\begin{pmatrix}1\\0\\0\end{pmatrix}\right]\right\}$$

$$f=\frac{1}{8}\sum_{\alpha=1}^{8}\left\{\lambda_{11}^{\alpha}\lambda_{22}^{\alpha}-\lambda_{21}^{\alpha}\lambda_{12}^{\alpha}-\lambda_{12}^{\alpha}\lambda_{21}^{\alpha}+\lambda_{22}^{\alpha}\lambda_{11}^{\alpha}\right\}=\frac{1}{4}\sum_{\alpha=1}^{8}\left\{\lambda_{11}^{\alpha}\lambda_{22}^{\alpha}-\lambda_{12}^{\alpha}\lambda_{21}^{\alpha}\right\}=\frac{1}{4}\left\{\lambda_{11}^{3}\lambda_{22}^{3}+\lambda_{11}^{8}\lambda_{22}^{8}-\lambda_{12}^{1}\lambda_{21}^{1}-\lambda_{12}^{2}\lambda_{21}^{2}\right\}=-\frac{2}{3}$$

The Effective Potential

Matrix elements just calculated: Very similar to the corresponding tree-level amplitudes in QED

→Expect similar, Coulomb-like, effective potential in the static limit

Constant depends on the color representation for the quark pair:

$$V_{q\bar{q}} = \begin{cases} -\frac{4}{3}\frac{\alpha_s}{r} & \text{singlet} \\ \frac{1}{6}\frac{\alpha_s}{r} & \text{octet} \end{cases}$$

$$V_{qq} = \begin{cases} -\frac{2}{3}\frac{\alpha_s}{r} & \text{triplet} \\ \frac{1}{3}\frac{\alpha_s}{r} & \text{sextet} \end{cases}$$

Attractive

Expect maximal attraction in singlets
Diquarks?

Baryons

Baryons could be in any one of the ***1,8,10*** representations:
Why only the singlet is observed?
A hint of an explaination:

$$\mathbf{3}\otimes\mathbf{3}\otimes\mathbf{3}=(\mathbf{3}\otimes\mathbf{3})\otimes\mathbf{3}$$

$$\mathbf{3}\otimes\mathbf{3}=\mathbf{6}\oplus\mathbf{3}^{*}\rightarrow(\mathbf{3}\otimes\mathbf{3})\otimes\mathbf{3}=(\mathbf{6}\oplus\mathbf{3}^{*})\otimes\mathbf{3}$$

$$\mathbf{6}\otimes\mathbf{3}=\mathbf{10}\oplus\mathbf{8}$$

$$\mathbf{3}^{*}\otimes\mathbf{3}=\mathbf{1}\oplus\mathbf{8}$$

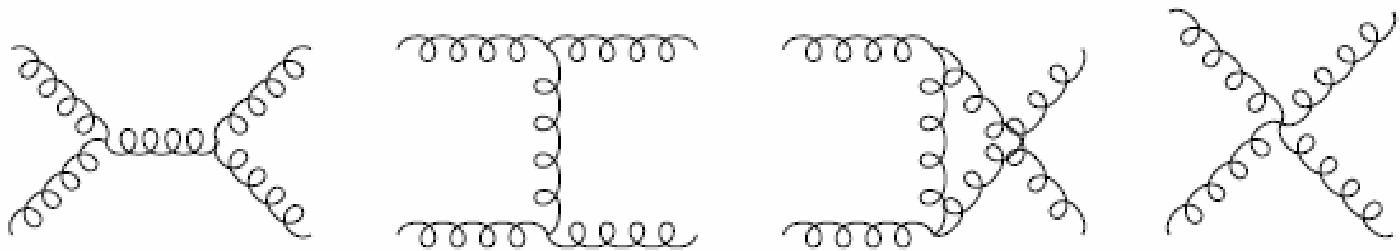
1: *each qq pair is a triplet $\rightarrow$ attractive*
8: *qq pair can be triplets, or sextet $\rightarrow$ attractive + repulsive*
10: *each qq pair is a sextet $\rightarrow$ repulsive*

So singlet is the state maximally attractive for 3 quarks

Does this explain the singlet-only mystery for bound states?

Another Color Interaction

Non Abelian vertices: Gluon-Gluon scattering *at tree level*



@TBA

$3-gluons: A \propto g$

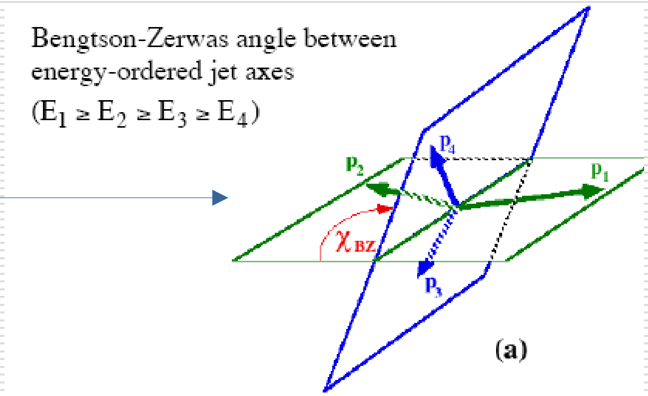
$4-gluons: A \propto g^2$ Much harder to observe

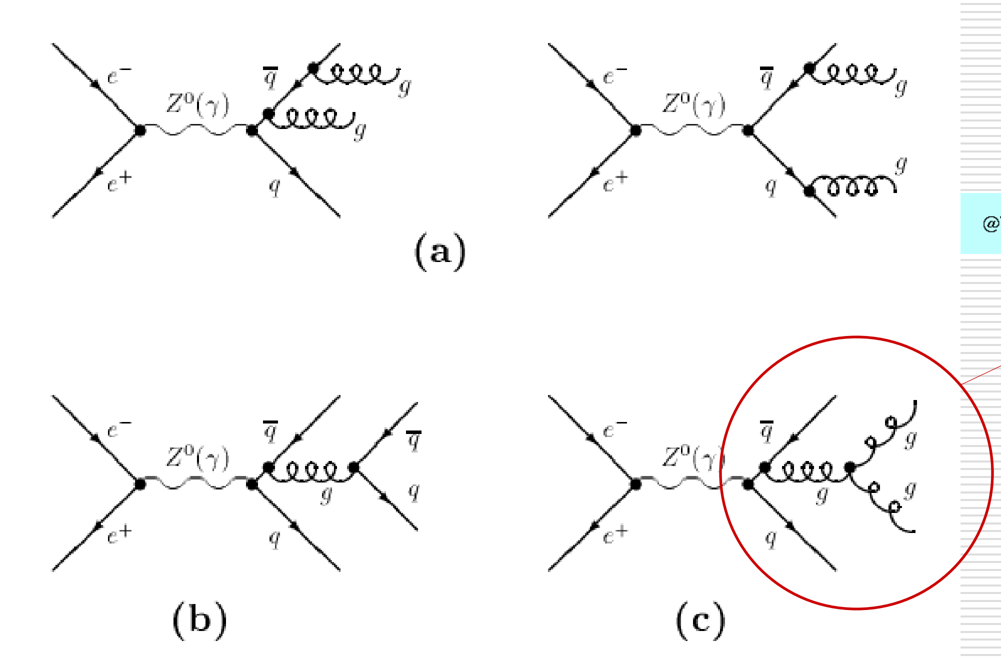
Compare: In QED, photon-photon scattering amplitude occurs at order α^2 through the 1-loop diagram

Color Interaction - VIII

Test for non-Abelian couplings at LEP: 4 jets events

Special angular correlation from 3-gluon vertex amplitude





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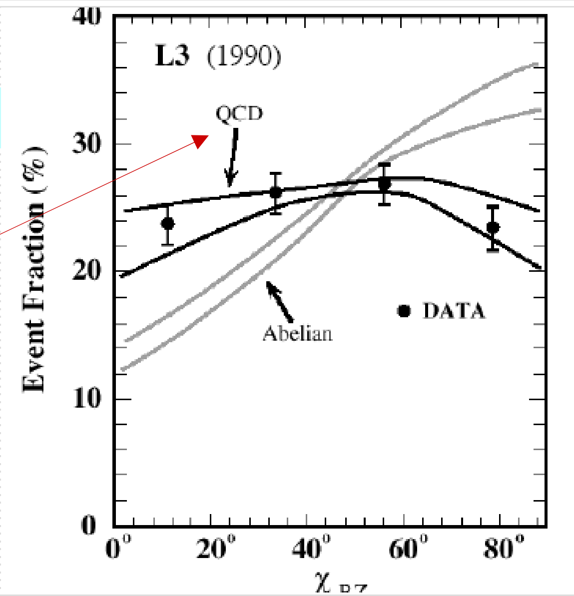
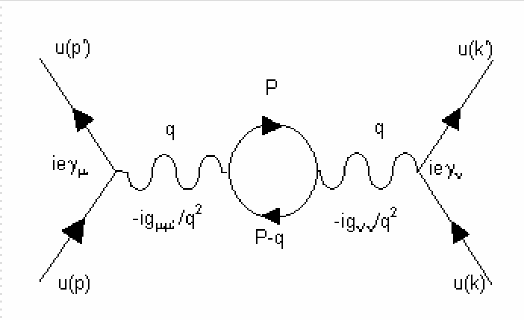


Running Coupling: *QED* - I

Consider the *one loop* modification to the photon propagator:



Includes a sum over P, the momentum circulating in the virtual loop. No obvious bounds on P..

$$M \propto \left[e\bar{u}(k')\gamma^{\mu}u(k)\right]\frac{g_{\mu\mu'}}{q^2}\frac{1}{(2\pi)^4}\int d^4P\frac{\left[e\bar{u}(P)\gamma^{\mu'}u(P-q)\right]\left[e\bar{u}(P-q)\gamma^{\nu'}u(P)\right]}{\left(P^2-m^2\right)\left((P-q)^2-m^2\right)}\frac{g_{\nu\nu'}}{q^2}\left[e\bar{u}(p')\gamma^{\nu}u(p)\right]$$

Modified propagator:

$$\frac{g_{\mu\nu}}{q^2}\rightarrow\frac{g_{\mu\nu}}{q^2}\left(1-I\left(q^2\right)\right),\quad I\left(q^2\right)=\frac{\alpha}{3\pi}\int_{m^2}^{\infty}\frac{dp^2}{p^2}-\frac{2\alpha}{\pi}\int_0^1 dx\,x(1-x)\ln\left[1-\frac{q^2x(1-x)}{m^2}\right]$$

Running Coupling: *QED* - II

Take the high q^2 approximation

$$-q^2 \gg m^2 \rightarrow \ln\left[1-\frac{q^2 x(1-x)}{m^2}\right] \approx \ln\left[-\frac{q^2}{m^2}\right]$$

$$I(q^2) \approx \frac{\alpha}{3\pi}\int_{m^2}^{M^2}\frac{dp^2}{p^2} - \frac{2\alpha}{\pi}\int_0^1 dx\,x(1-x)\ln\left[\frac{-q^2}{m^2}\right]$$

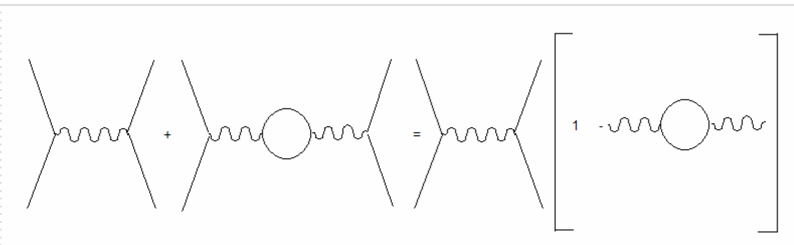
Provisional upper bound (cutoff) to make integral to converge

$$= \frac{\alpha}{3\pi}\ln\left(\frac{M^2}{m^2}\right) - \frac{2\alpha}{\pi}\frac{1}{6}\ln\left[\frac{-q^2}{m^2}\right]$$

$$= \frac{\alpha}{3\pi}\left[\ln\left(\frac{M^2}{m^2}\right) - \ln\left[\frac{-q^2}{m^2}\right]\right] = \frac{\alpha}{3\pi}\ln\left(\frac{M^2}{-q^2}\right)$$

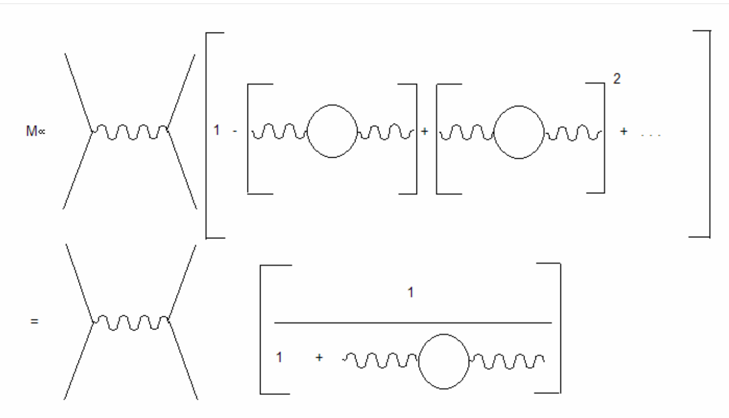
$$M \propto \alpha\left[\bar{u}(k')\gamma^\mu u(k)\right]\frac{g_{\mu\nu}}{q^2}\left[1-\frac{\alpha}{3\pi}\ln\left(\frac{M^2}{-q^2}\right)\right]\left[\bar{u}(p')\gamma^\nu u(p)\right]$$

Cartoon translation:



Running Coupling: *QED* - III

Extend to diagrams with 2,3,…,n,… loops: Add up all contributes



Experts say this is the only contribution to running α to the 'leading logs' approximation, which means neglecting the next levels of iteration

$$M \propto \left[\bar{u}(k')\gamma^{\mu}u(k)\right]\frac{g_{\mu\nu}}{q^2}\left[\frac{\alpha}{1+\alpha/3\pi\ln\left(M^2/-q^2\right)}\right]\left[\bar{u}(p')\gamma^{\nu}u(p)\right]$$

Sum of a 'geometrical series'
Converging??

What is α? Coupling 'constant' we would get should we turn off all loops

Call it α_0 = 'Bare' coupling constant, not physical: Loops cannot be turned off

Then obtain an effective coupling, not constant but *running*:

$$\alpha\left(q^2\right)=\frac{\alpha_0}{1+\alpha_0/3\pi\ln\left(M^2/-q^2\right)}$$

α is q^2, or distance, dependent!

Running Coupling: *QED* - IV

Running α is still cutoff dependent, which of course is uncomfortable
But: Not a real problem. Indeed:

$$Q^2 = -q^2 \to \alpha\left(Q^2\right) = \frac{\alpha_0}{1+\left(\alpha_0/3\pi\right)\ln\left(M^2/Q^2\right)}$$

Usually choose $\mu^2=0$, i.e. take α at distance$\to\infty$
Quite natural in QED, but not compulsory

Take a particular energy scale: $Q^2 = \mu^2 \to \alpha\left(\mu^2\right) = \frac{\alpha_0}{1+\left(\alpha_0/3\pi\right)\ln\left(\Lambda^2/\mu^2\right)}$

$$\ln\left(\frac{M^2}{Q^2}\right) = \ln\left(\frac{M^2}{Q^2}\frac{\mu^2}{\mu^2}\right) = \ln\left(\frac{M^2}{\mu^2}\right) + \ln\left(\frac{M^2}{Q^2}\right) \to \alpha\left(Q^2\right) = \frac{\alpha_0}{1+\left(\alpha_0/3\pi\right)\left[\ln\left(M^2/\mu^2\right)+\ln\left(\mu^2/Q^2\right)\right]}$$

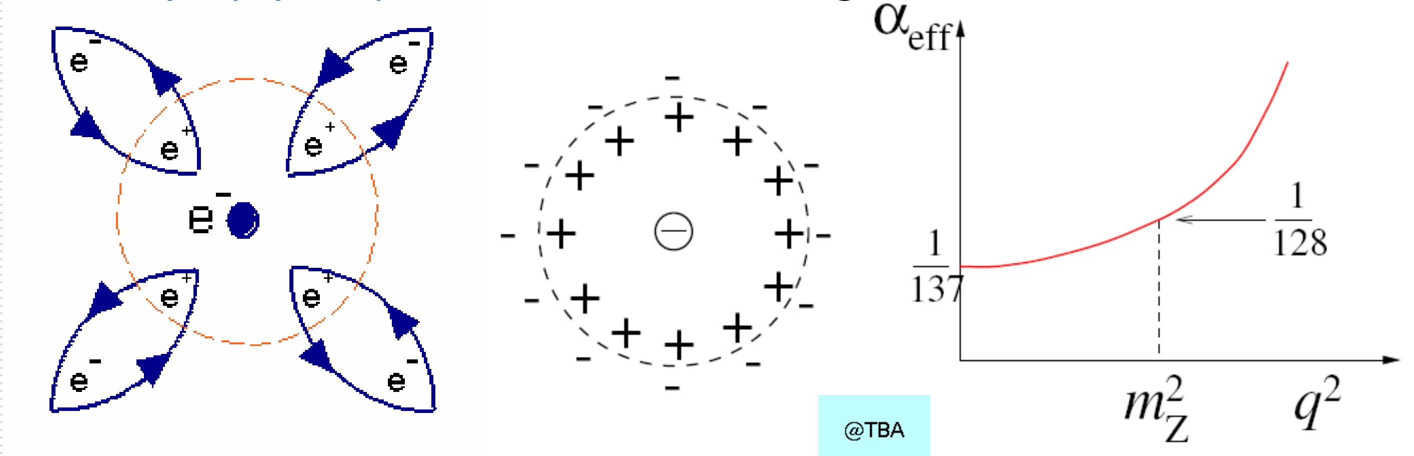
$$\to \frac{\alpha_0}{\alpha\left(\mu^2\right)} = 1+\left(\alpha_0/3\pi\right)\ln\left(M^2/\mu^2\right)$$

$$\to \alpha\left(Q^2\right) = \frac{\alpha_0}{\alpha_0/\alpha\left(\mu^2\right)+\left(\alpha_0/3\pi\right)\ln\left(\mu^2/Q^2\right)} = \frac{\alpha\left(\mu^2\right)}{1-\left[\alpha\left(\mu^2\right)/3\pi\right]\ln\left(Q^2/\mu^2\right)}$$

Very interesting result: Running α depends on q^2, through its own *measured* value at any chosen energy scale μ^2. Cutoff has disappeared.

Cartooning Deep Physics

Virtual (loops) e^+e^- pairs — Effective shielding



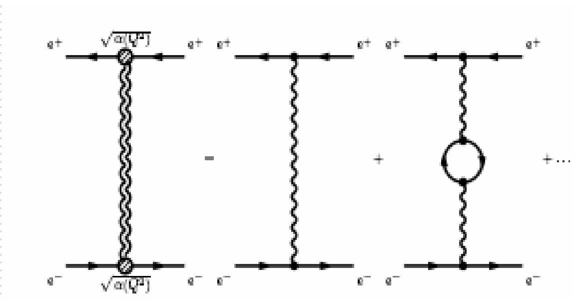
Picture the QED perturbative vacuum as a sort of (virtual) dielectric medium:
Virtual photons and e^+e^- pairs continuously created/annihilated

Bare charge is shielded at large distance by the virtual pairs coming from loops
The standard e charge is smaller than the bare charge

By probing the electron at smaller and smaller distance, one observe an *increasing* effective charge

Running α at LEP (and More)

Experimental method: Bhabha scattering



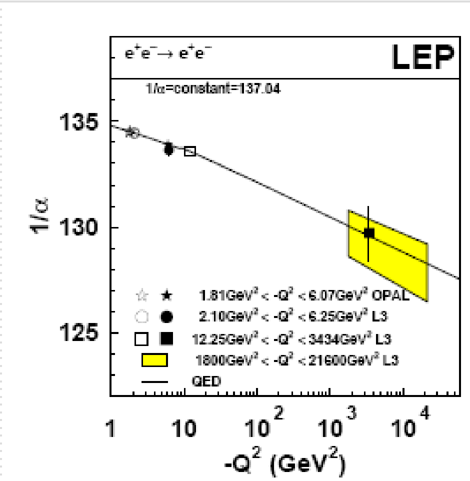
$$\frac{d\sigma}{dt} = \frac{d\sigma^0}{dt}\left(\frac{\alpha(t)}{\alpha_0}\right)^2 (1+\varepsilon)(1+\delta_\gamma)+\delta_Z,$$

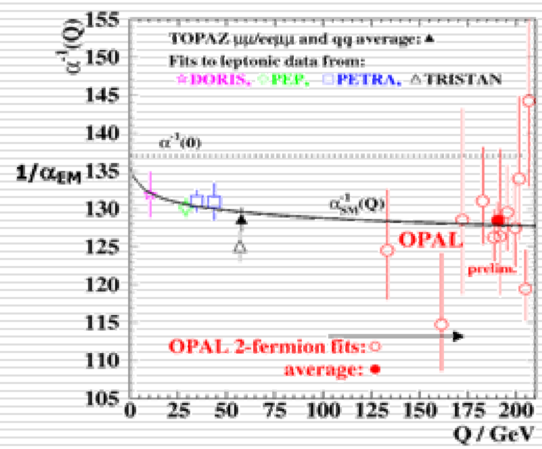
δ_γ, δ_Z s-channel contributions (small)

ε radiative corrections (known)

Use accurate, differential cross-section measurement to unfold $\alpha(t)$

Total cross-section measurement would require a luminosity..



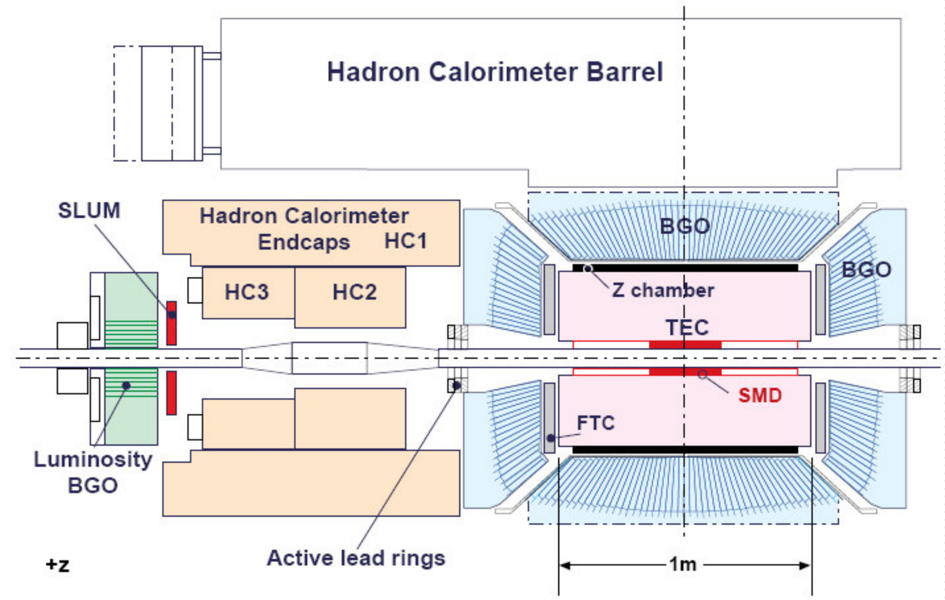


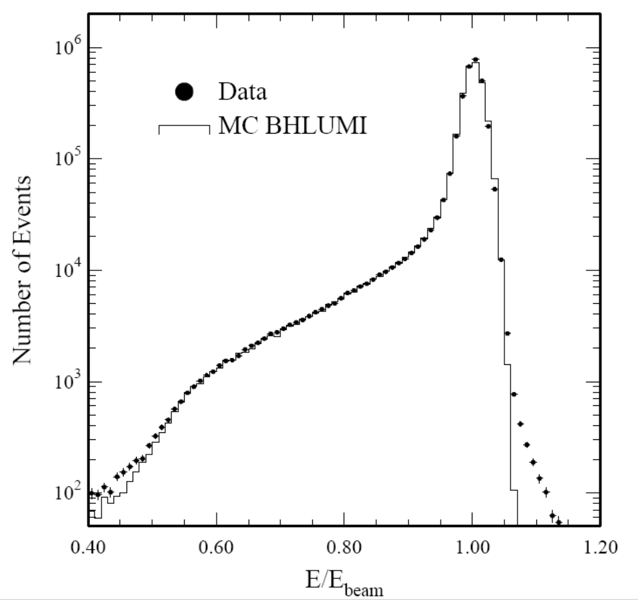
Luminosity Monitors

Just as an example, take L3 at LEP:
Relying on Bhabha scattering at small angle

$$\sigma = \frac{16\pi\alpha^2}{s}\left(\frac{1}{\theta^2_{min}} - \frac{1}{\theta^2_{max}}\right)$$

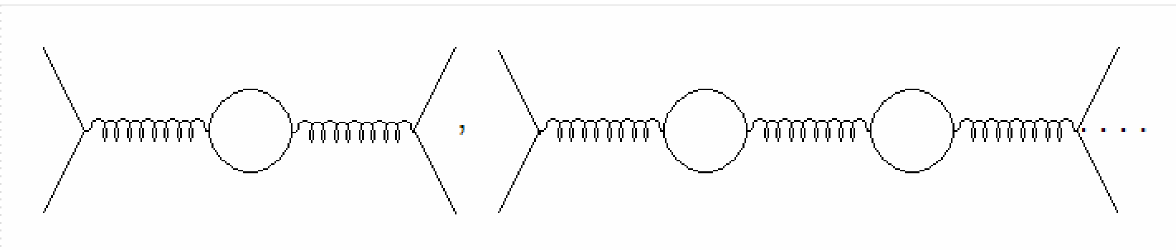
Calorimeter (Energy) + Silicon Tracker (Angle + Acceptance)



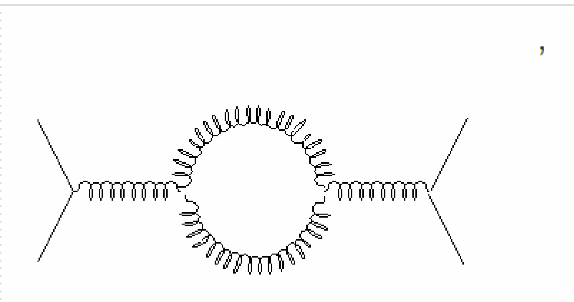


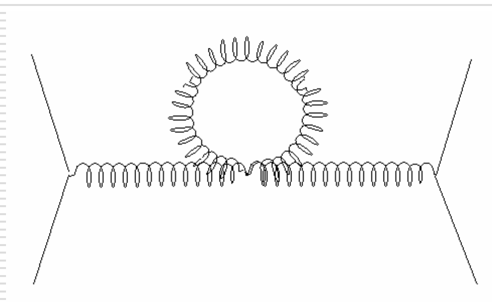
Running Coupling: *QCD* - I

Repeat all the steps: Loops etc



Except this time one has more loops: Gluons





Running Coupling: *QCD* - II

Turns out that gluon loops yield *anti*-shielding effect
With 8 gluons and 6 quark flavors, gluons win

$$\alpha_s\left(\left|q^2\right|\right)=\frac{\alpha_s\left(\mu^2\right)}{1+\left(\alpha_s\left(\mu^2\right)/12\pi\right)\left(33-2n_{flavor}\right)\ln\left(\left|q^2\right|/\mu^2\right)}$$

Running coupling *decreases* with increasing q^2 (or at smaller distance)
This is knwon as *asymptotic freedom*:

Large q^2 processes feature small coupling $\rightarrow$ Perturbative!

Most important consequence:

The fundamental hypothesis behind the successful parton model is finally understood and justified

The Meaning of Λ

Rather than making reference to a specific value of α_s

$$\alpha_s\left(\left|q^2\right|\right)=\frac{\alpha_s\left(\mu^2\right)}{1+\left(\alpha_s\left(\mu^2\right)/12\pi\right)\left(33-2n_{flavor}\right)\ln\left(\left|q^2\right|/\mu^2\right)}$$

define a new constant

$$\ln\Lambda^2=\ln\mu^2-\frac{12\pi}{\left(33-2n_{flavor}\right)\alpha_s\left(\mu^2\right)}\rightarrow\Lambda^2=\mu^2 e^{-\frac{12\pi}{\left(33-2n_{flavor}\right)\alpha_s\left(\mu^2\right)}}$$

$$\rightarrow\alpha_s\left(\left|q^2\right|\right)\simeq\frac{12\pi}{\left(33-2n_{flavor}\right)\ln\left(\left|q^2\right|/\Lambda^2\right)}=\frac{12\pi}{21\ln\left(\left|q^2\right|/\Lambda^2\right)},\qquad\left|q^2\right|\gg\Lambda^2$$

Λ = Renormalization scale $\rightarrow$ Fixes α_s at all q^2

$\Lambda\approx 200\ MeV$ yields the correct α_s at $\mu^2=M_{Z^0}^2$

Funny behavior, known as 'Dimensional Transmutation':

From an adimensional constant to a dimensional one $\alpha_s\rightarrow\Lambda$

Confinement

$$\alpha_s\left(\left|q^2\right|\right) \simeq \frac{12\pi}{21\ln\left(\left|q^2\right|/\Lambda^2\right)}, \qquad \left|q^2\right| \gg \Lambda^2$$

When $\left|q^2\right| \sim \Lambda^2$, the previous expression does not apply

$\alpha_s\left(\Lambda^2\right)$ *is large*
Strong interaction is strong
Cannot rely on perturbative expansion

In a general sense, we expect Λ to mark the low energy range, corresponding to *soft* (low q^2) processes

Bound states: Non-perturbative, 'white', energy scale ≈ Λ

Does $\alpha_s(\Lambda^2)$ correspond to the *color confinement* range?
Very likely. But remember:

It is not yet convincingly shown that QCD is a confining theory

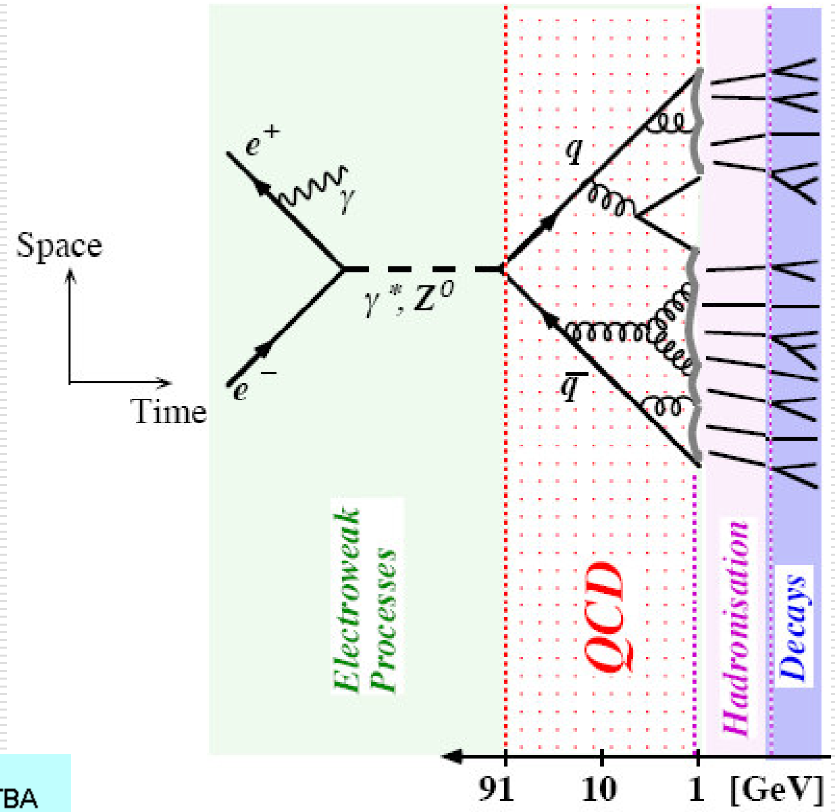
Jet Fragmentation

Complete jet evolution cannot be computed in a full QCD framework:

As shown in the picture, *increasing* time scales correspond to *decreasing* Q^2 scale, down to a region where perturbative expansion cannot be granted

Conversion of quarks and gluons into hadrons (*Fragmentation* or *Hadronization*) must be evaluated by ad-hoc prescriptions (models).

QCD-inspired, successful models often based on string-like behavior of pairs $q\overline{q}$



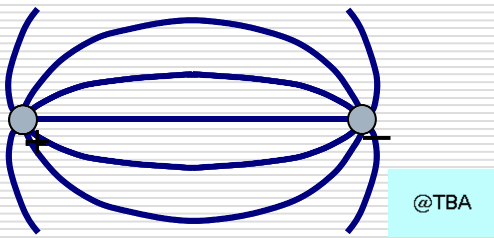
@TBA

Stringy QCD

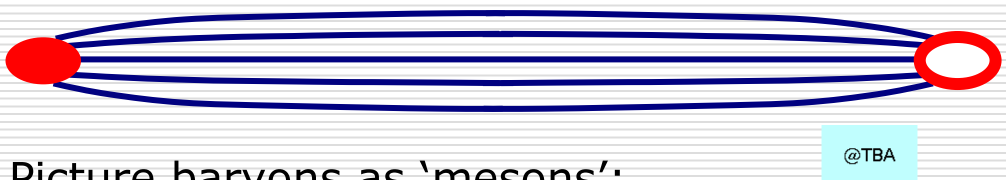
Typical model implemented in fragmentation Montecarlo programs

$q\bar{q}$ Interaction

QED-like at small distance



Gluon self-interaction yields *string* (flux tube) pattern at large distance



Picture baryons as ‘mesons’:

$$\boldsymbol{3} \otimes \boldsymbol{3} = \boldsymbol{3}^* \oplus \boldsymbol{6}$$

$$qqq = \underbrace{qq}_{\sim \bar{q}} + q$$

PQCD: Jets in $e^+ e^-$ Collisions - I

2 jets

$$e^+ + e^- \rightarrow q + \bar{q} \rightarrow hadrons$$

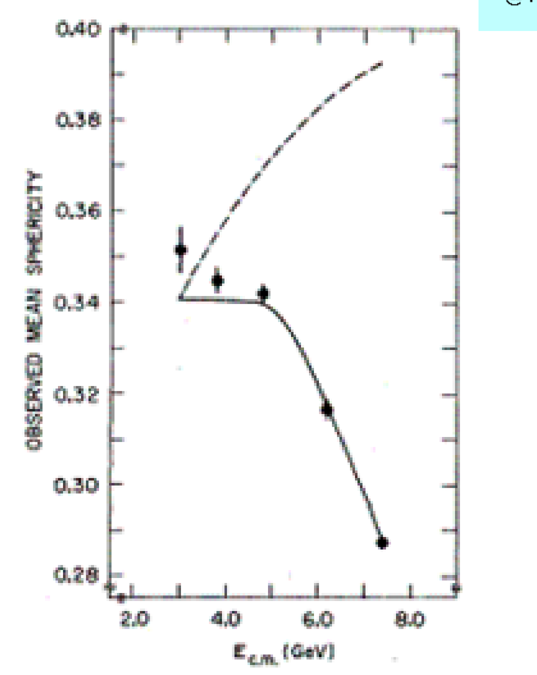
$$\frac{d\sigma}{d\Omega} = \frac{3\alpha^2}{4s}\left(1+\cos^2\theta\right)\sum_{flavor} e_{flavor}^2$$

$$\rightarrow \sigma(s) = \frac{4\pi\alpha^2}{s}\sum_{flavor} e_{flavor}^2$$

Define sphericity of events:

$$S = \frac{3}{2}\frac{\sum_i p_{\perp i}^2}{\sum_i p_i^2} \rightarrow \begin{cases} 0 & \text{2-jets} \\ 1 & \text{spherical} \end{cases}$$

@TBA

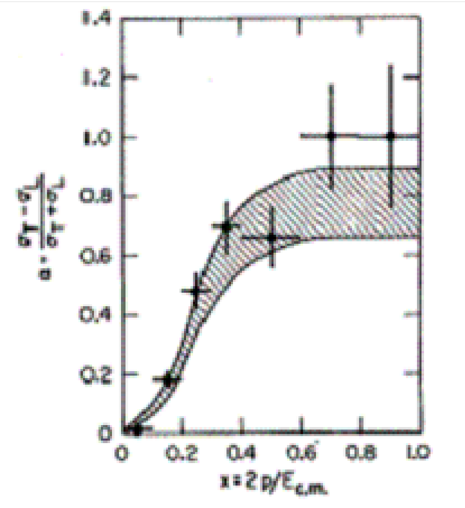


At high energy, events tend to be non-spherical

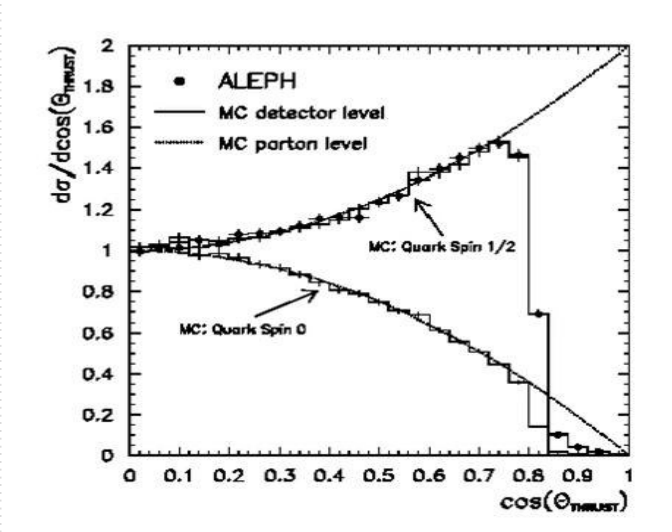
PQCD: Jets in $e^+ e^-$ Collisions - II

For 2 jets events

$$\left.\begin{array}{l} \frac{d\sigma}{d\Omega} \propto 1+\cos^2\theta \quad \text{quark spin} = 1/2 \\ \frac{d\sigma}{d\Omega} \propto 1-\cos^2\theta \quad \text{quark spin} = 0 \end{array}\right\} \equiv 1+\alpha\cos^2\theta$$



@TBA

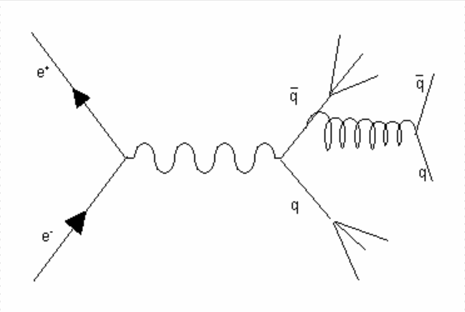


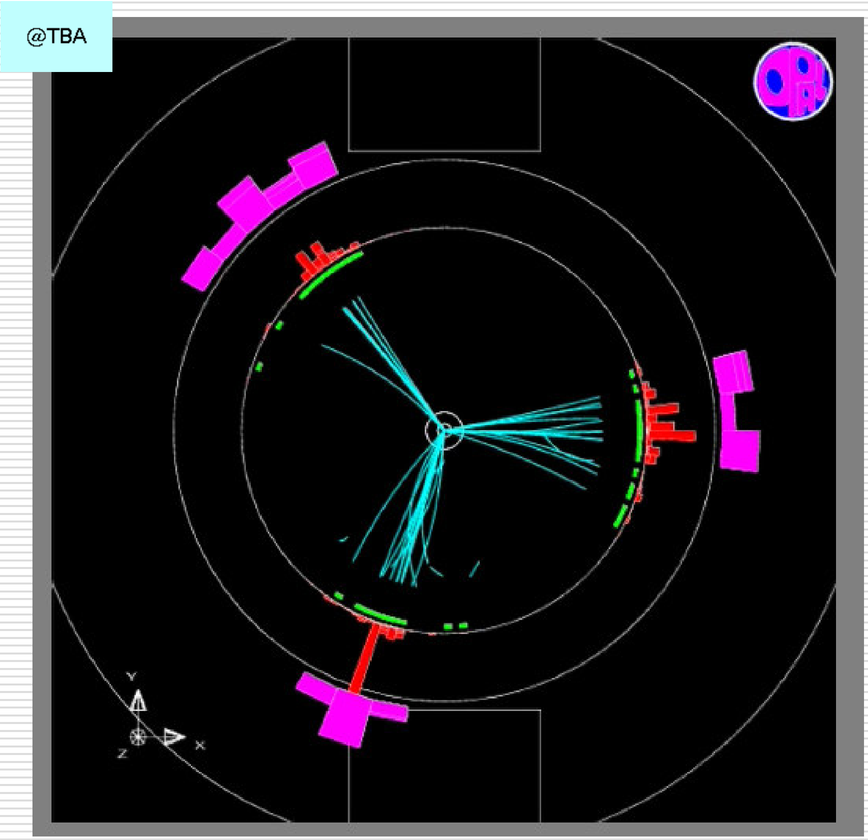
Mark I (SPEAR)
E = a few GeV

ALEPH (LEP)
E = 90 GeV

PQCD: Jets in $e^+ e^-$ Collisions - III

3 jets

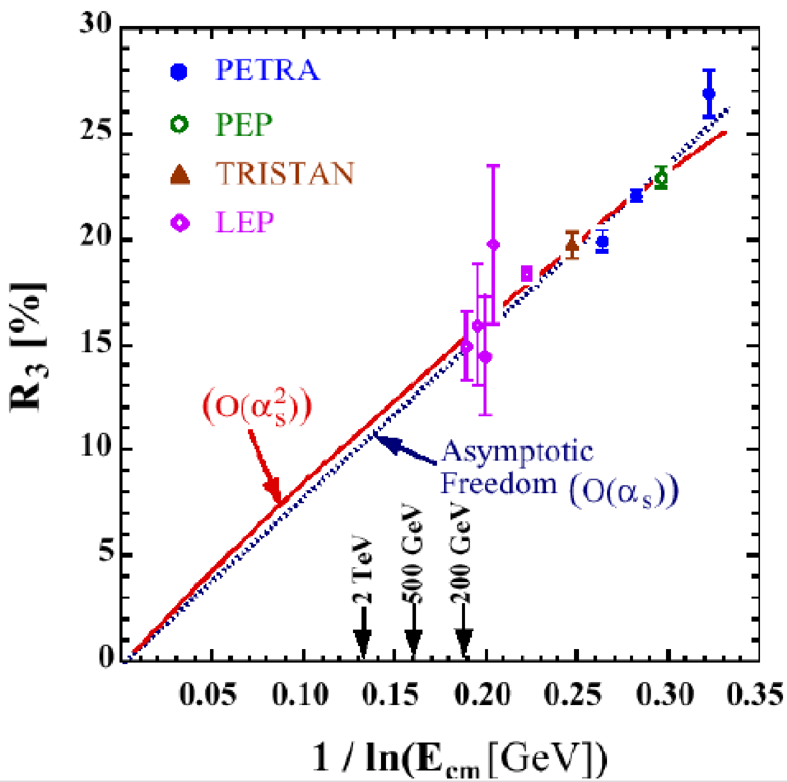




Left breathless by this exceptional 3-jet from OPAL? Relax, this is not exactly the rule…

PQCD: Jets in $e^+ e^-$ Collisions - IV

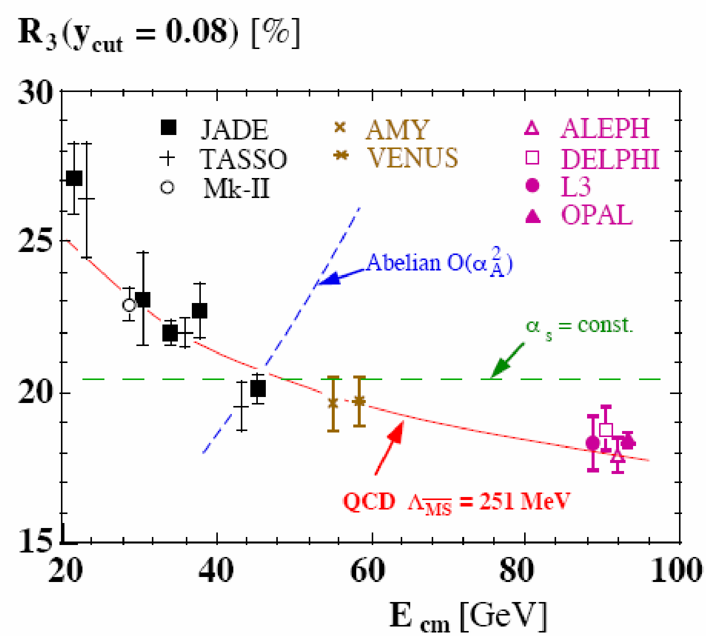
$$R_3 \equiv \frac{\sigma_{3-\text{jet}}}{\sigma_{\text{tot}}} \propto \alpha_s(E_{cm}) \propto \frac{1}{\ln E_{cm}}$$



Get a measurement of α_s :

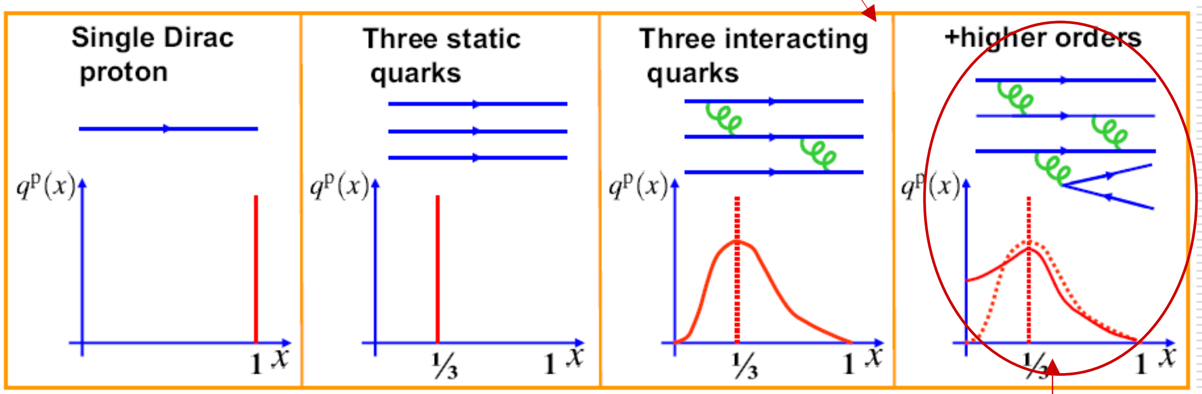
$$\alpha_S(35GeV) = 0.146 \pm 0.03$$

$$\alpha_S(M_{Z^0}) = 0.124 \pm 0.0043$$

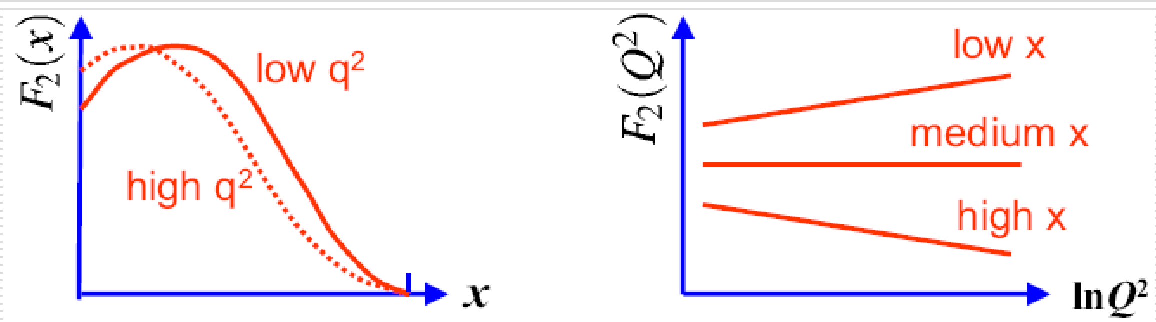


PQCD: DIS Scaling Violations - I

Our picture of structure functions



Observe small deviations from scaling: $F_2(x) \to F_2(x, Q^2) \to$ QCD!

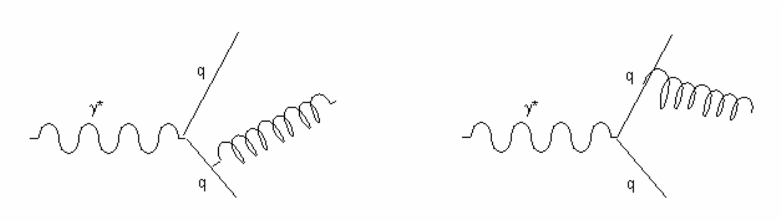


PQCD: DIS Scaling Violations - II

QCD on $F_2\left(x,Q^2\right)$:

$x-$dependence $\rightarrow$ Not predicted

Q^2-dependence $\rightarrow$ Predicted !



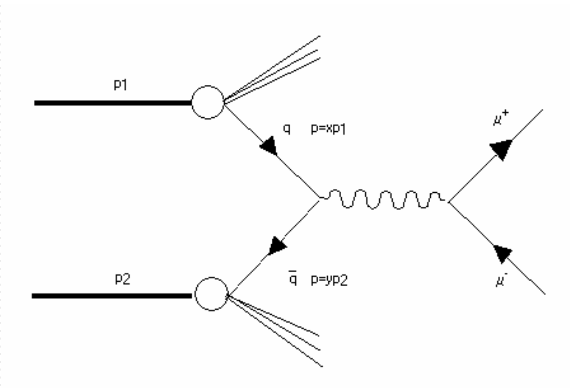
Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equation:
Successful prediction of Q^2 evolution of structure function

$$F_2\left(x,Q^2\right)=\sum_q xe^2\left[q\left(x\right)+\Delta q\left(x,Q^2\right)\right]$$

$$\Delta q\left(x,Q^2\right)=\frac{\alpha_s}{2\pi}\int_x^1\frac{dx'}{x}q\left(x'\right)P_{qq}\left(\frac{x}{x'}\right)\ln\left(\frac{Q^2}{k^2}\right)+\ldots$$

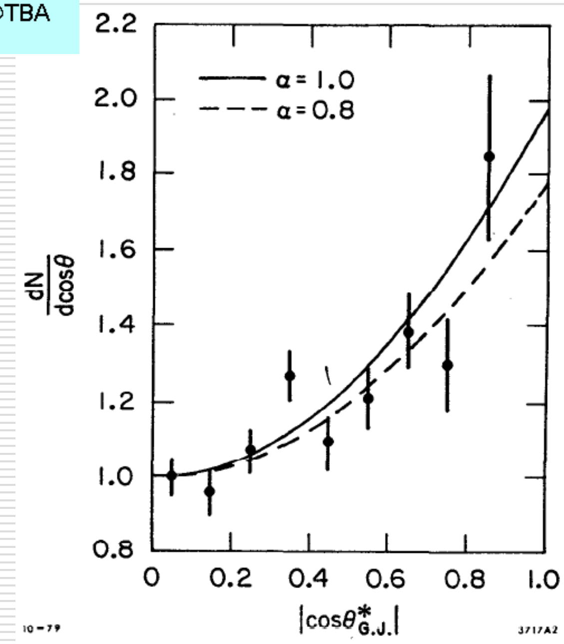
Deep waters…

PQCD: Drell-Yan - I



Angular distribution
in the pair rest frame

@TBA



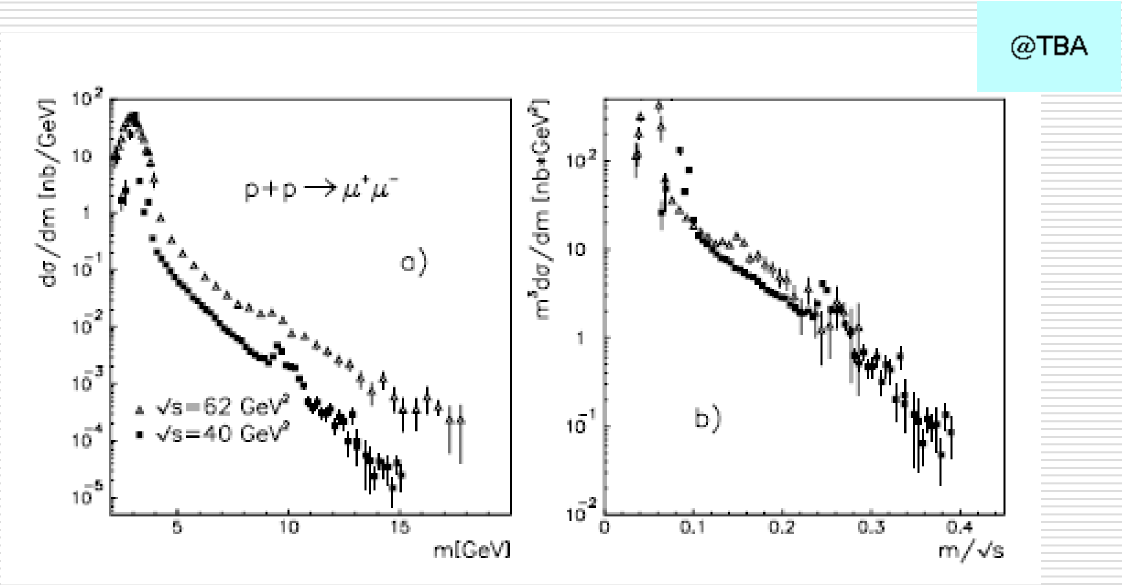
$$\frac{d\sigma\left(q\bar{q}\to l^+l^-\right)}{dq^2}=\frac{4\pi\alpha^2}{3q^2}e_q^2\delta\left(q^2-s_{q\bar{q}}\right)$$

x_1, x_2 Bjorken x for $q,\bar{q}$

$$s_{q\bar{q}}=\left(p_q+p_{\bar{q}}\right)^2=\left(x_1p_1+x_2p_2\right)^2\simeq x_1x_2s$$

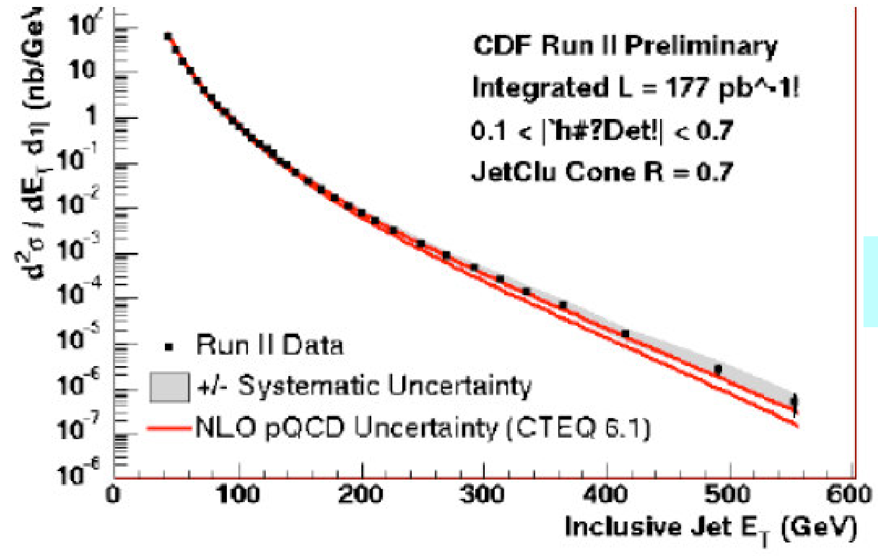
$\sigma\left(q\bar{q}\to l^+l^-\right)=\frac{4\pi\alpha^2}{3q^2}e_q^2,\quad e_q=$ quark charge in e units   Expect $\propto 1+\cos^2\theta^*$ as usual

PQCD: Drell-Yan - II



Scaling of the pair mass distribution (= differential cross-section)

PQCD: Jets in Hadron Collisions



@TBA

Cannot rely on triggering on a single, high $p_\perp$ particle
Devise a calorimeter trigger based on *total transverse energy* observed

$$\sum p_\perp^{(i)} = \sum p_i \sin\theta_i \sim \sum E_T^{(i)}$$

PQCD: 2-Body Partonic Processes

Consider all the 2-body processes in QCD:

$qq \to qq, q\overline{q} \to q\overline{q}$ Quarks only

$qg \to qg, \overline{q}g \to \overline{q}g, gg \to gg, q\overline{q} \to gg, gg \to q\overline{q}$ Quarks and/or Gluons

All will yield 2 jets to first approximation

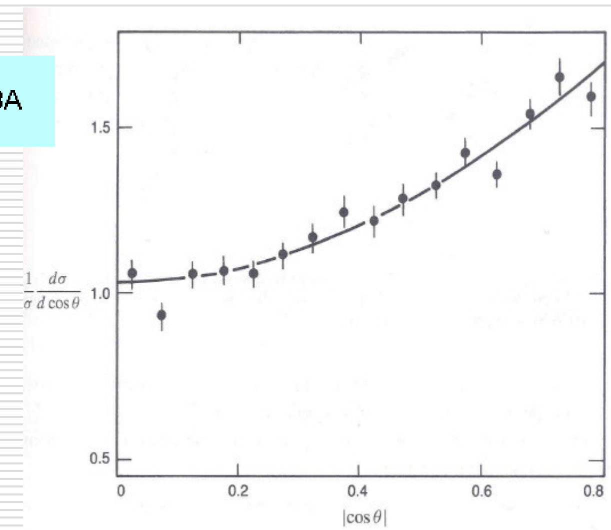
When quark only processes can be identified, expect:

$$\frac{d\sigma}{d\left(\cos\theta^*\right)} = \frac{\pi\alpha_s^2}{2s_{ij}}|M|^2$$

$$s_{ij} = \left(x_i p_i + x_j p_j\right)^2 \approx x_i x_j s$$

$\to |M|^2 \propto 1 + \cos^2\theta^*$ as usual

@TBA



Valence and Sea

Take a Hydrogen atom:

= Chemistry!

Common wisdom: "A bound state of proton + electron"
But: Consider the effect of radiative corrections (e.g. loops)
Then we should be more precise:

Hydrogen = $(\text{Proton+Electron})_{\text{Valence}}$ + $(\text{Positrons+Electrons+Photons})_{\text{Sea}}$

Can we say valence and sea particles are fundamentally different? Well,…

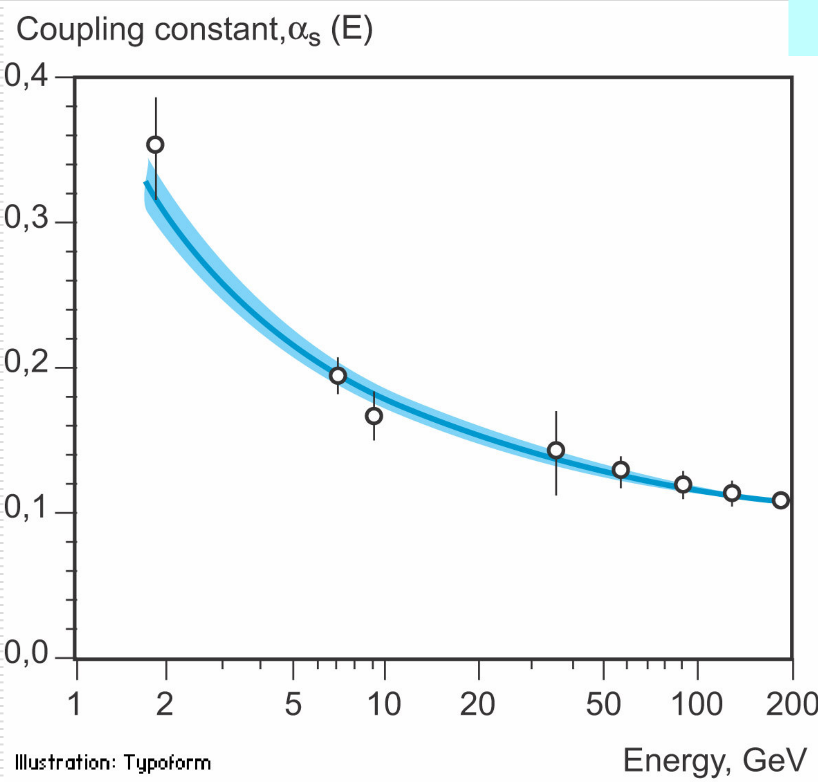
In a bound state, both are off mass shell
Both are active in yielding measurable effects (Coulomb levels vs. Lamb shift,..)
Sea particles yield small corrections to levels determined by valence $e+p$

Take a hadron:

Hadron = $(\text{Quarks/Antiquarks})_{\text{Valence}}$ + $(\text{Quarks/Antiquarks+Gluons})_{\text{Sea}}$

Since $\alpha_S >> \alpha$, sea effects are much larger in QCD

Running α_s

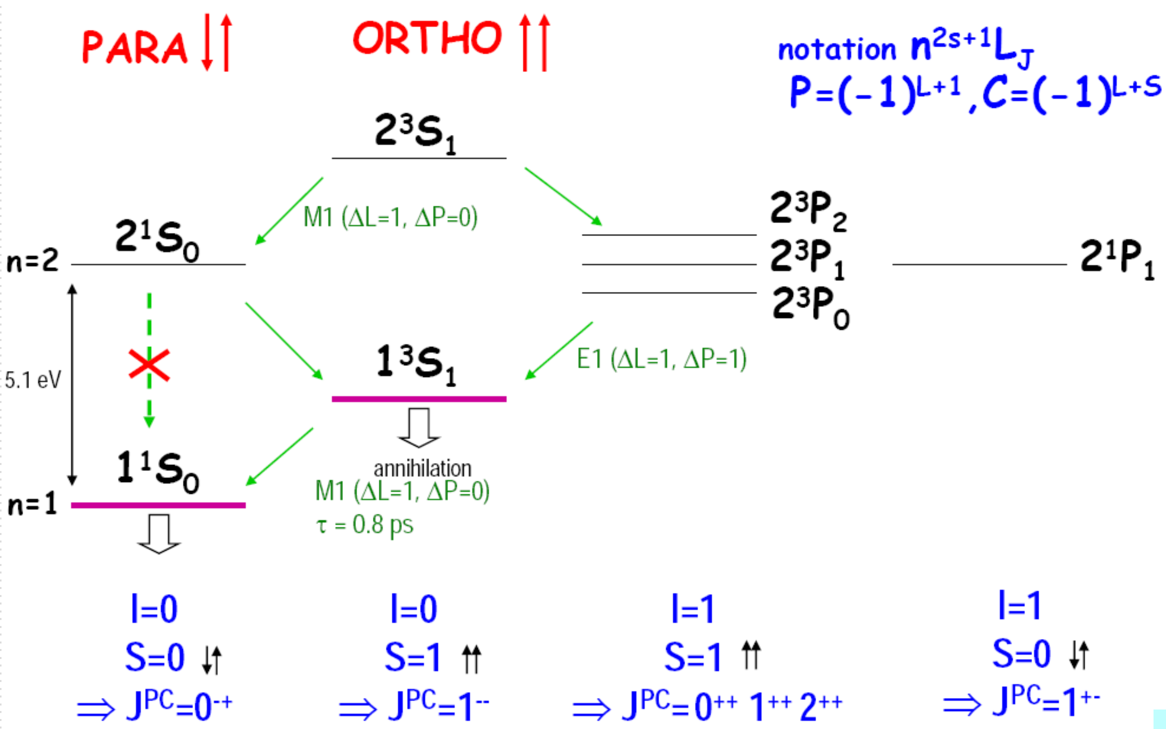


@TBA

Sources:

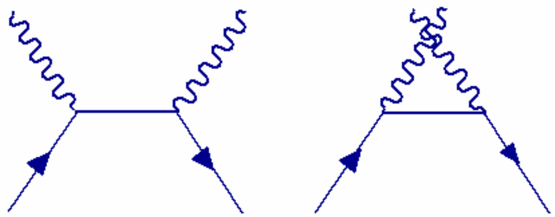
DIS
Jets
Quarkonium

Positronium



@TBA

e^+- e^- : 2 Photons Annihilation



$$T_{fi} = T_1 + T_2$$

$$T_1 = \frac{e^2}{(p_1 - p_3)^2 - m^2} \bar{v}(2) \not{\varepsilon}_4 (\not{p}_1 - \not{p}_3 + m) \not{\varepsilon}_3 u(1), \quad T_2 = \frac{e^2}{(p_1 - p_4)^2 - m^2} \bar{v}(2) \not{\varepsilon}_3 (\not{p}_1 - \not{p}_4 + m) \not{\varepsilon}_4 u(1)$$

$$p_1 = m(1,0,0,0),\ p_2 = m(1,0,0,0),\ p_3 = m(1,0,0,1),\ p_4 = m(1,0,0,-1)$$

$$(p_1 - p_3)^2 - m^2 = (p_1 - p_4)^2 - m^2 = -2m^2 \to T = -4e^2$$ Averaged over initial, summed over final spin projections

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi s} \frac{|\mathbf{p}_f|}{|\mathbf{p}_i|} |T|^2$$

$$|\mathbf{p}_f| = m, \quad |\mathbf{p}_i| \simeq m\beta, \quad s = (2m)^2 = 4m^2$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi 4m^2} \frac{m}{m\beta} 16\alpha^2 = \frac{\alpha^2}{16\pi m^2 \beta} \to \sigma = 4\pi \frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4m^2\beta}$$

Positronium: 2 Photons Annihilation

Proceed as for Van-Royen - Weisskopf

$$A_{pos} = \sum_p \underbrace{\langle \gamma\gamma | T | p \rangle}_{A(\mathbf{p})} \underbrace{\langle p | \pi^0 \rangle}_{\psi(\mathbf{p})}$$

$A(\mathbf{p})$ plane wave amplitude, $\psi(\mathbf{p})$ momentum space wave function

$$\rightarrow A_{pos} = \int d^3\mathbf{p} A(\mathbf{p})\psi(\mathbf{p}) = \frac{1}{(2\pi)^{3/2}} \int d^3\mathbf{p} A(\mathbf{p}) \int \psi(\mathbf{r}) e^{i\mathbf{p}\cdot\mathbf{r}} d^3\mathbf{r} = \frac{1}{(2\pi)^{3/2}} \int d^3\mathbf{r}\, \psi(\mathbf{r}) \int A(\mathbf{p}) e^{i\mathbf{p}\cdot\mathbf{r}} d^3\mathbf{r}$$

Take $A(\mathbf{p}) \approx A = const$

Can be shown to be true

$$\rightarrow A_{pos} \approx \frac{A}{(2\pi)^{3/2}} \int d^3\mathbf{r}\, \psi(\mathbf{r}) \underbrace{\int e^{i\mathbf{p}\cdot\mathbf{r}} d^3\mathbf{p}}_{(2\pi)^3 \delta^3(\mathbf{r})} = (2\pi)^{3/2} A\psi(0)$$

$$\rightarrow \Gamma_{pos} = |A_{pos}|^2 \approx (2\pi)^3 |A|^2 |\psi(0)|^2$$

$$\sigma = \frac{\alpha^2}{4m^2\beta} = |A|^2 \frac{(2\pi)^3}{\beta} \rightarrow |A|^2 = \frac{\alpha^2}{4m^2\beta} \frac{\beta}{(2\pi)^3} = \frac{\alpha^2}{(2\pi)^3 4m^2}$$

$$\rightarrow \Gamma_{pos} \approx (2\pi)^3 |A|^2 |\psi(0)|^2 = \frac{\alpha^2}{4m^2} |\psi(0)|^2$$

$U_C |2\gamma\rangle = (-1)^2 |2\gamma\rangle \rightarrow \eta_c(2\gamma) = +1$

$\rightarrow (-1)^{L+S} = +1$

$\Rightarrow L = 0 \rightarrow S = 0$

S-wave: Singlet only

Since only singlet initial state contributes: $\Gamma_{pos} \rightarrow \frac{1}{1/4} \cdot \frac{\alpha^2}{4m^2} |\psi(0)|^2 = \frac{\alpha^2}{m^2} |\psi(0)|^2$

Positronium: 2 Photons Annihilation

Taking scaled Hydrogen wave function

$$\psi(r) = \frac{2}{(a_0)^{3/2}} e^{-\frac{2r}{a_0}}, a_0 = \frac{1}{\alpha m}, m$$ reduced mass

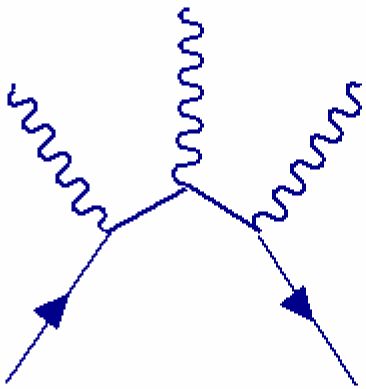
$$Hyd: \quad m \simeq m_e \to a_0 \approx \frac{1}{\alpha m_e}$$

$$Pos: \; m = \frac{m_e}{2} \to a_0 = \frac{2}{\alpha m_e}$$

$$\to \psi_{pos}(0) = \frac{2}{(a_0)^{3/2}} = \frac{2(\alpha m_e)^{3/2}}{(2)^{3/2}} \to |\psi_{pos}(0)|^2 = \frac{4\alpha^3 m_e^3}{8} = \frac{\alpha^3 m_e^3}{2}$$

$$\to \Gamma_{pos} \approx (2\pi)^3 |A|^2 |\psi(0)|^2 = \frac{\alpha^2}{m_e^2} \frac{\alpha^3 m_e^3}{2} = \frac{\alpha^5 m_e}{2}$$

Positronium: 3 Photons Annihilation



Permutations of 3 photons
$\rightarrow$6 diagrams altogether

$$U_C\left|3\gamma\right\rangle = (-1)^3 = -1 \rightarrow (-1)^{L+S} = -1 \rightarrow \begin{cases} L=0 \\ S=1 \end{cases}$$ Triplet only

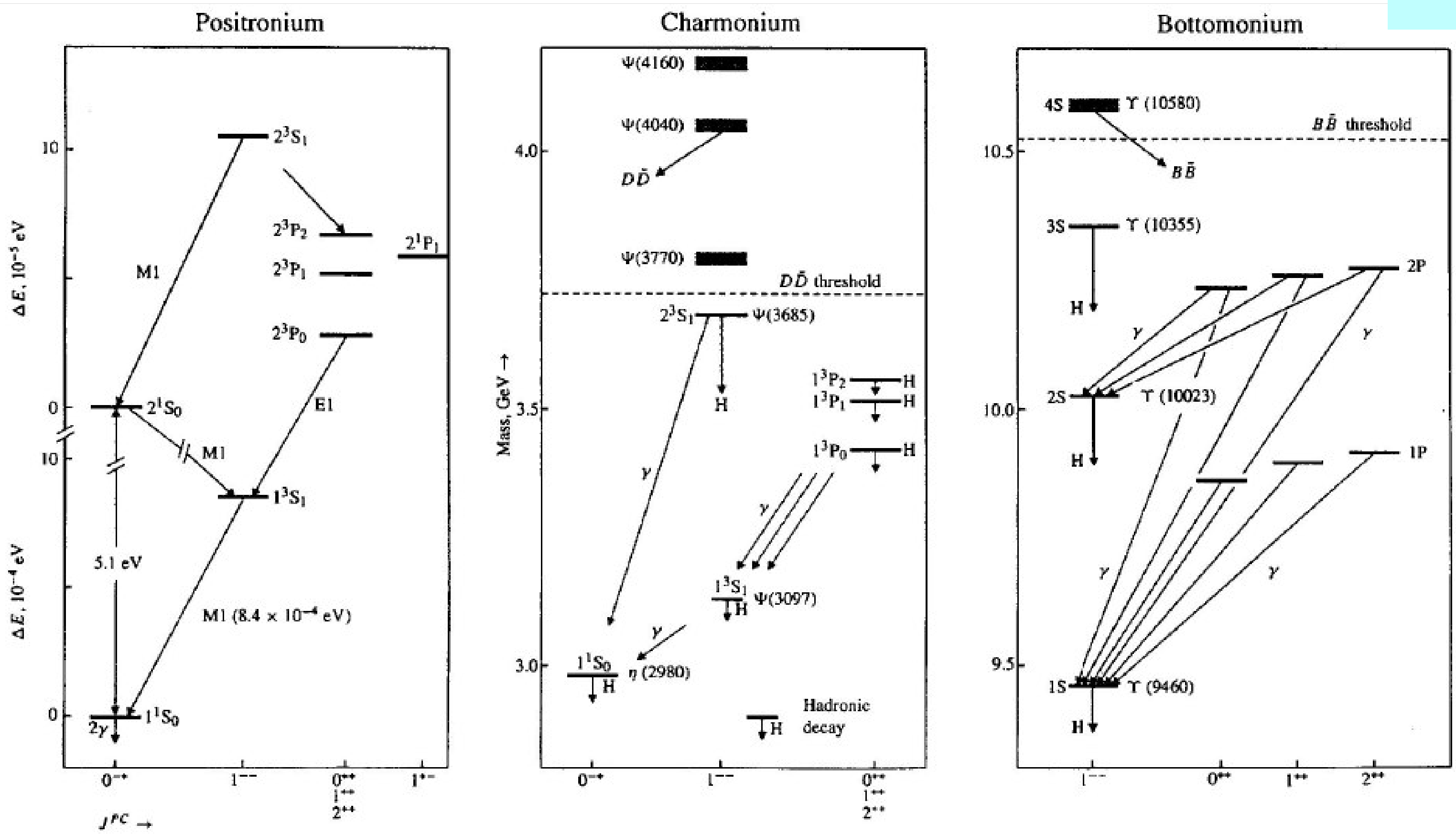
After *some* algebra…

$$\Gamma_{pos}^{3\gamma} = \frac{4}{9\pi}\left(\pi^2-9\right)\frac{\alpha^3}{m_e^2}\left|\psi(0)\right|^2 = \frac{2}{9\pi}\left(\pi^2-9\right)m_e\alpha^6$$

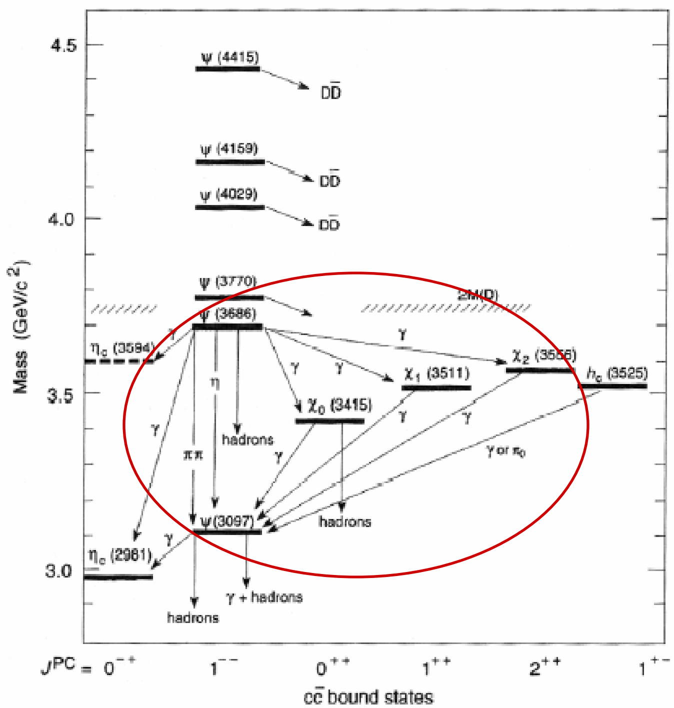
Quarkonium

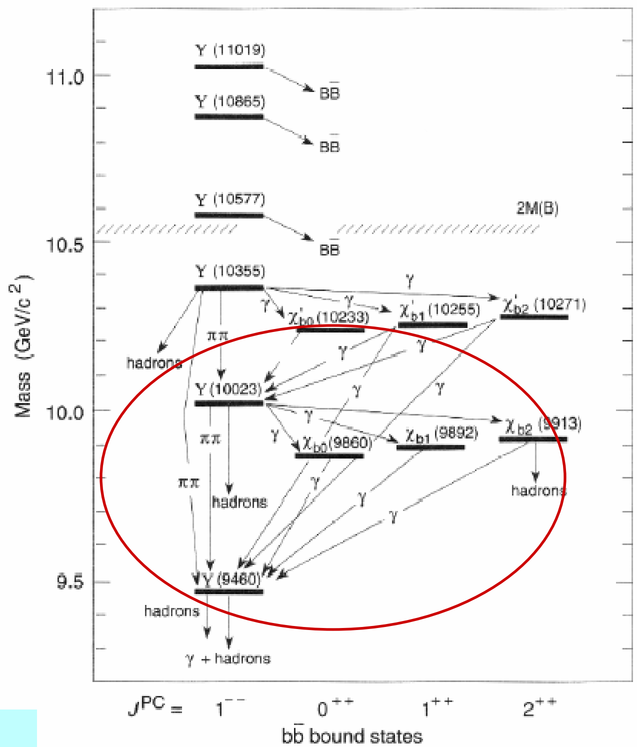
Family portrait of *–onia*:

@TBA



Real Life Quarkonia





@TBA

Striking similarity, same energy scale *above ground state*

Quarkonium: Schrodinger Equation

Reminder: Bohr radius

$$R = \frac{n^2}{\alpha m} \rightarrow R_0 = \frac{1}{\alpha m}$$

Consequence of Coulomb potential, static limit of 1 photon diagram

If 1 gluon exchange approximation can be granted, expect for quarkonium

$$R_{q\bar{q}} \sim \frac{1}{\alpha_s m}$$

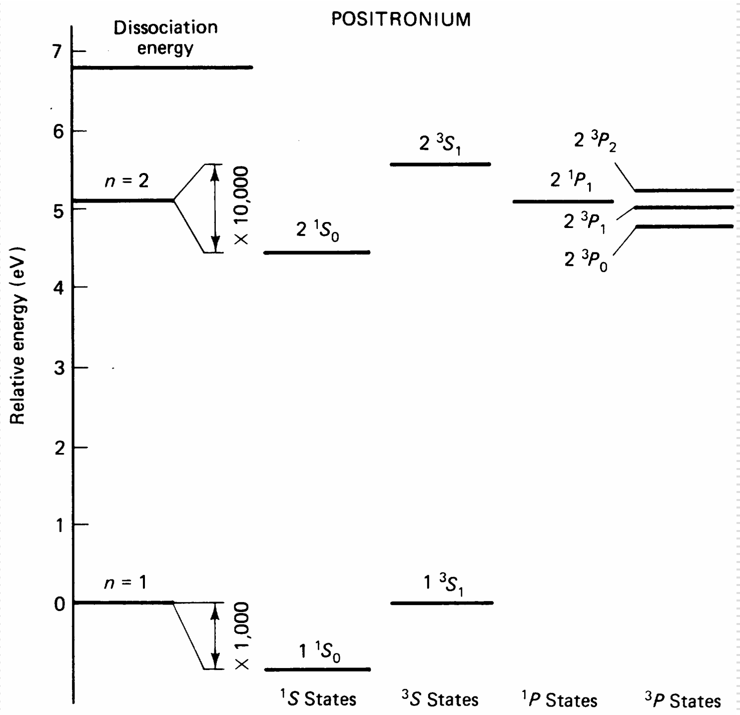
Observe: m large $\rightarrow$ R small $\rightarrow$ α_s small QCD OK

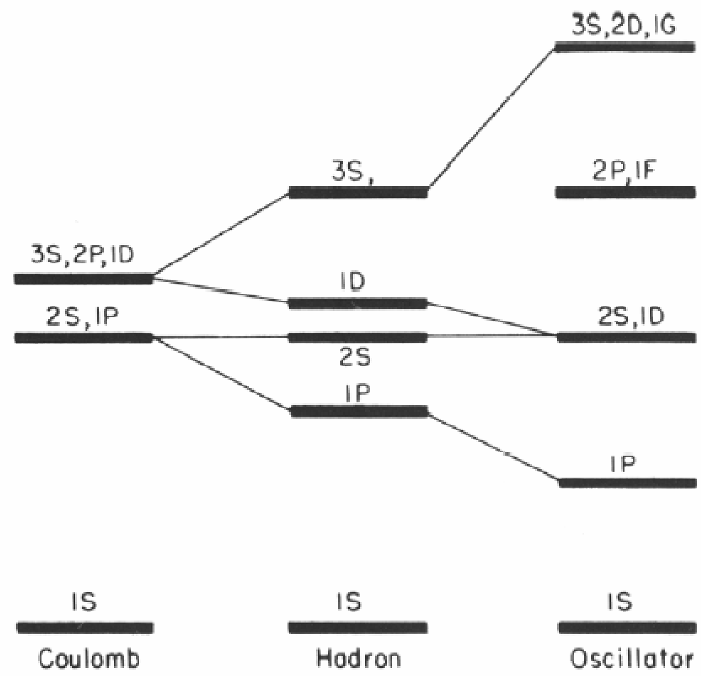
Must keep in mind the $q\bar{q}$ potential is confining
Add a phenomenological, confining term like this:

$$V(r) = -\frac{4}{3}\frac{\alpha_s}{r} + ar$$

Solve Schrodinger equation with these terms
Add more terms to take into account relativistic & color-hyperfine effects

The $q\bar{q}$ Effective Potential: Levels





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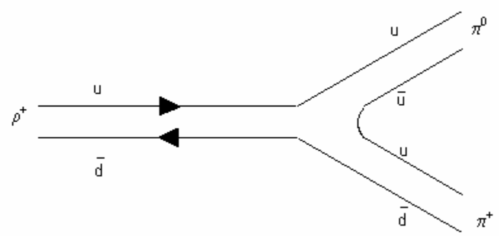
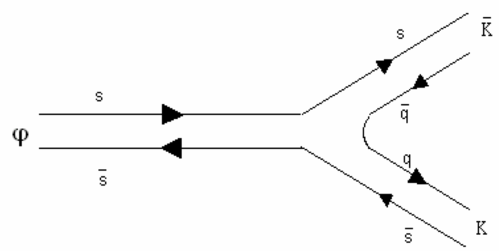
Quark Flow Diagrams: The OZI Rule

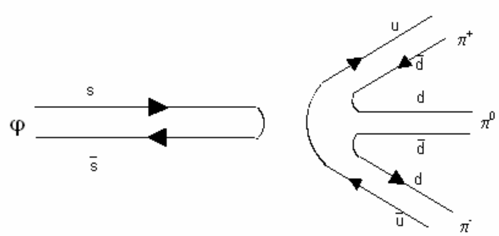
Okubo-Zweig-Iizuka Rule: *Disconnected diagrams are suppressed*



This diagram is connected

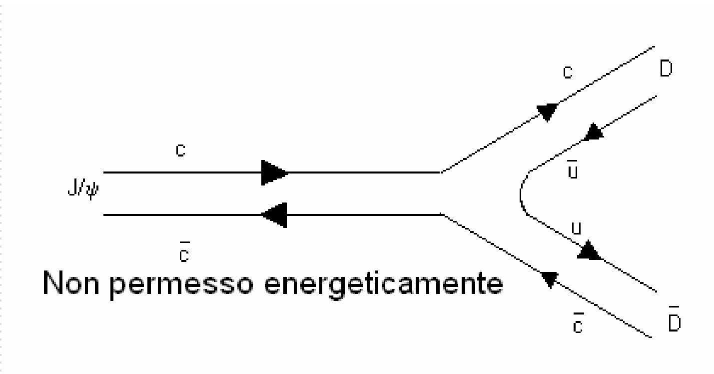


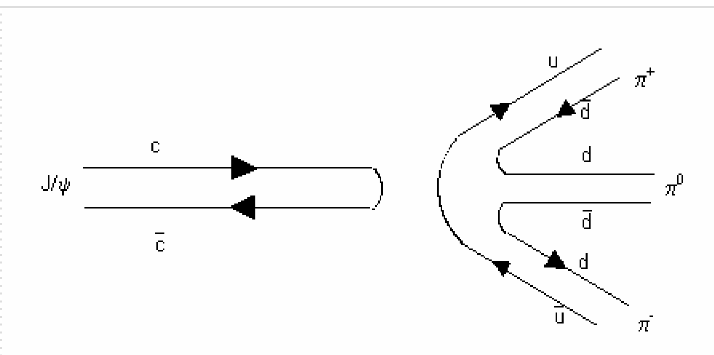
This diagram is connected: *BR 83 %*
(with smallish phase space)



This diagram is disconnected: *BR 15 %*
(with much larger phase space)

The OZI Rule and Charmonium





Compare mass and width

$$m_{J/\psi} = 3097\ \text{MeV},\ \Gamma_{J/\psi} = 80\ \text{keV} \qquad J^{PC} = 1^{--}$$
$$m_{\psi'} = 3686\ \text{MeV},\ \Gamma_{\psi'} = 250\ \text{keV} \qquad J^{PC} = 1^{--}$$

Explaining the small width:

$m_{D^0} = 1865\ MeV \rightarrow 2 \times m_{D^0} = 3730\ MeV > m_{J/\psi}, m_{\psi'}$

Therefore J/ψ, ψ' decay to open charm is energetically forbidden

$\rightarrow$ Decay diagrams are disconnected

$\rightarrow$ OZI rule: Decay is suppressed

$\rightarrow$ *States are very narrow*

The Origin of the OZI Rule

As a general rule

$\rightarrow A \propto \alpha_s^n \quad n =$ number of gluons

Connected diagrams: Small number of soft gluons $\rightarrow A$ *= large*
Disconnected diagrams: Large number of hard gluons $\rightarrow A$ *= small*

Indeed:

1) Single gluon annihilation is forbidden for mesons by color conservation (meson = ***1***, gluon = ***8***)

2) Annihilation of massive quarks yields hard gluons $\rightarrow \alpha_s$ is small

3) Connected diagrams involve softer gluons $\rightarrow \alpha_s$ is large

Quarkonium: 2,3 Gluons

Consider quarkonium annihilation into gluons:

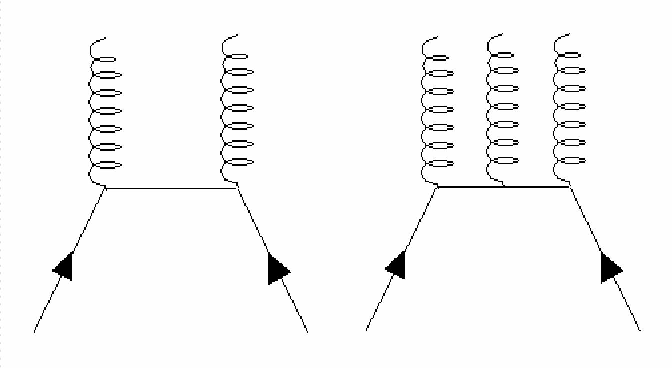
$q\bar{q} \to g$ Excluded: $(q\bar{q})_1 \not\to (1g)_8$

$q\bar{q} \to gg$ Allowed

$q\bar{q} \to ggg$ Allowed

Decompose the direct product of 2 octets:

$\boldsymbol{8 \otimes 8 = 1 \oplus 8 \oplus 8 \oplus 10 \oplus 10^* \oplus 27}$



Charge Parity:

$J^{PC} = 0^{-+} \to C = +1 \to 2g$ OK

$J^{PC} = 1^{--} \to C = -1 \to 3g$ OK

Perturbative regime: $A(2g)>A(3g)$

$\to$Pseudoscalars wider than vectors

Quarkonium Annihilations

By comparison with positronium:

$$\left(e^+e^-\right)_{positronium} \rightarrow \gamma\gamma$$

$$\Gamma\left[\left(e^+e^-\right) \rightarrow \gamma\gamma\right] = \frac{\alpha^2}{m^2}\left|\psi(0)\right|^2$$

$$\left(c\overline{c}\right)_{charmonium} \rightarrow \gamma\gamma$$

$$\begin{cases} e \rightarrow \frac{2}{3}e \rightarrow \alpha \rightarrow \frac{4}{9}\alpha & \text{Quark charge} \\ \times 9 & \text{Sum amplitude over colors} \end{cases}$$

$$\Gamma\left[\left(c\overline{c}\right) \rightarrow \gamma\gamma\right] = \frac{48\alpha^2}{27m_c^2}\left|\psi_{c\overline{c}}(0)\right|^2$$

$$\left(c\overline{c}\right)_{charmonium} \rightarrow gg$$

Color factor $= \frac{9}{8}$ From SU(3) algebra: 2 g in a color singlet state

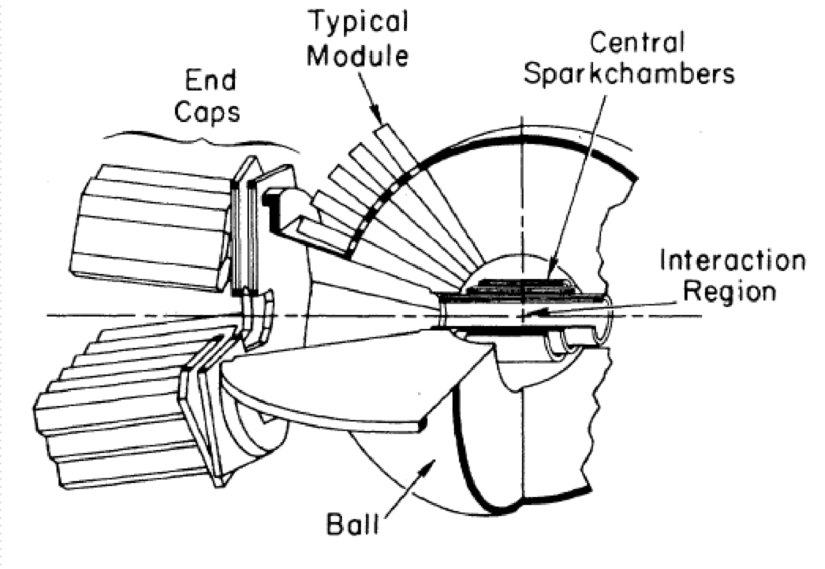
$$\Gamma\left[\left(c\overline{c}\right) \rightarrow gg\right] = \frac{2\alpha_s^2}{3m_c^2}\left|\psi_{c\overline{c}}(0)\right|^2$$

But remember:

Positronium rate was obtained by taking the low speed limit of scattering amplitude to 1-photon approx

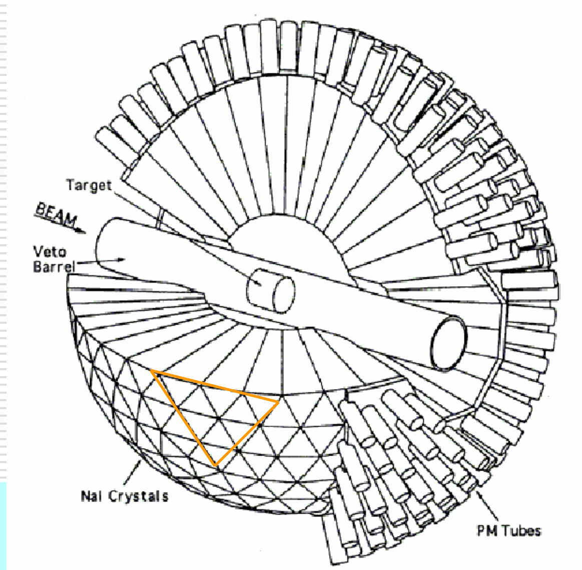
Is it granted for $c\overline{c}$?

Crystal Ball - I



@TBA

94% of solid angle covered



Sodium Iodide

$NaI(Tl)$: Inorganic scintillating crystal; Tl is an activator

Merits:

Can grow large crystals

Lots of light

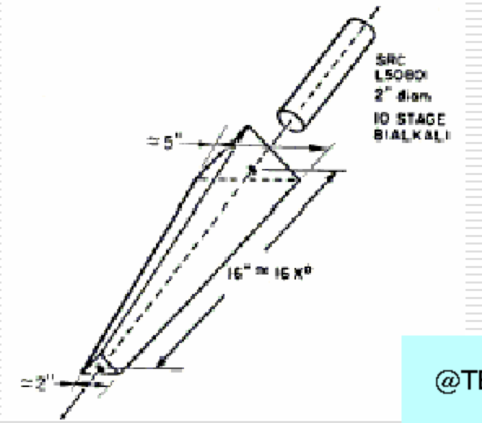
Crystal Ball - II

672 optically isolated NaI(Tl) crystals, 15.7 radiation lengths thick
Inner radius 25.3 cm; Outer radius 66.0 cm

CB geometry: Based on icosahedron.
Each of the 20 triangular faces (major triangles) is divided into four minor triangles, each consisting of nine separated crystals.

Each crystal: Truncated triangular pyramid, 40.6 cm high, pointing towards the center of the Ball.
Side on the inner end: 5.1 cm ; Side on the outer end: 12.7 cm

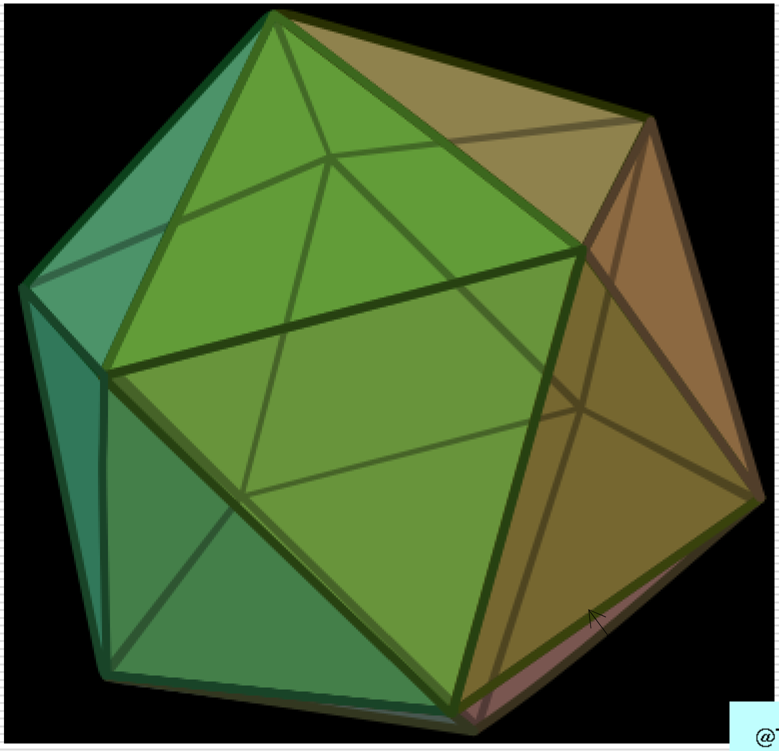
Crystal & Photomutiplier

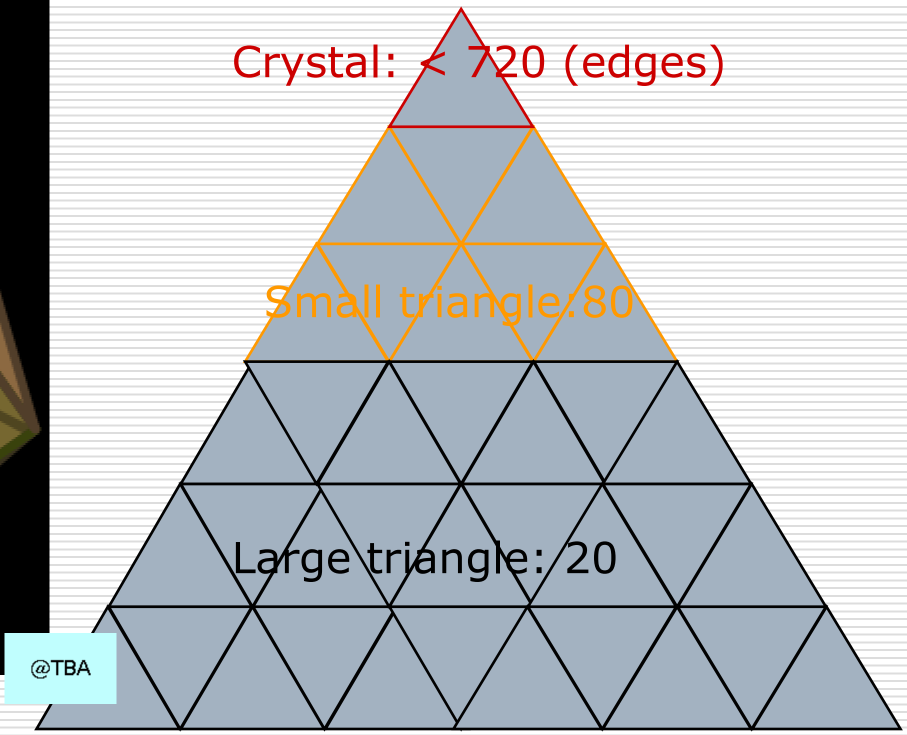


@TBA

Crystal Ball - III

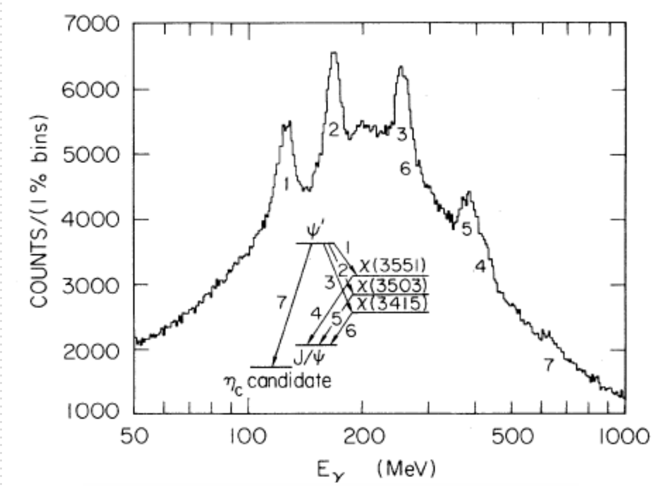
Icosahedron magic: Platonic solid, 20 equilateral triangle faces

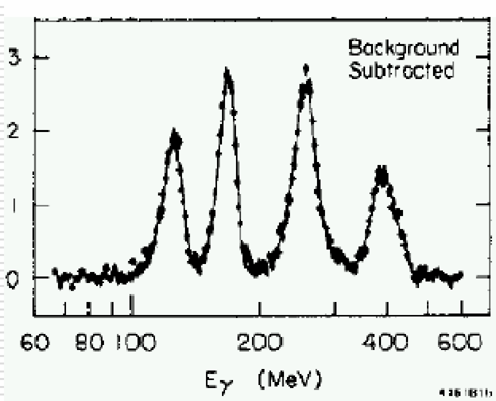




Crystal Ball - IV

Inclusive photon spectrum



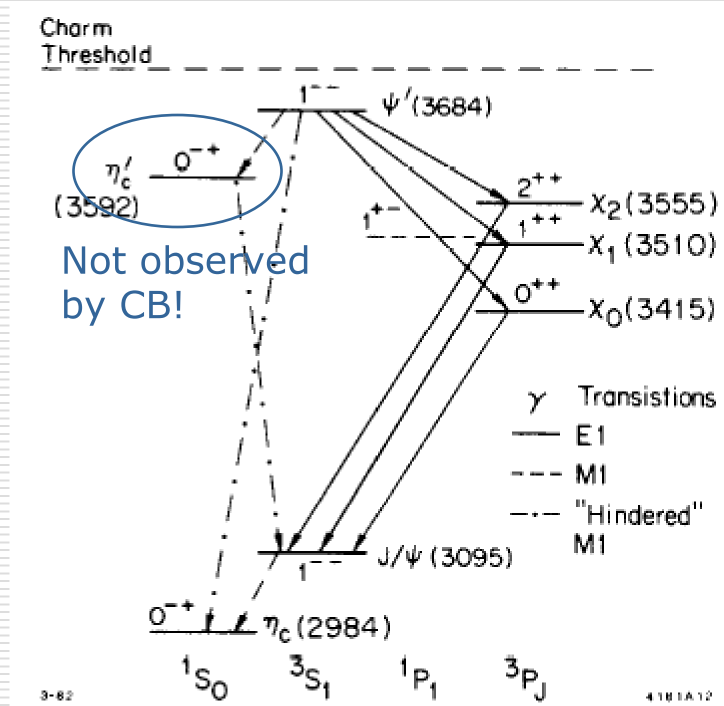


@TBA

Most important results, among many:

Tune beam energy as to form ψ′(3686)

Observe decays into photon + X



Crystal Ball - V

After moving the CB detector to DORIS (DESY, Hamburg): Bottomonium!
Observation of the P-wave triplets

@TBA

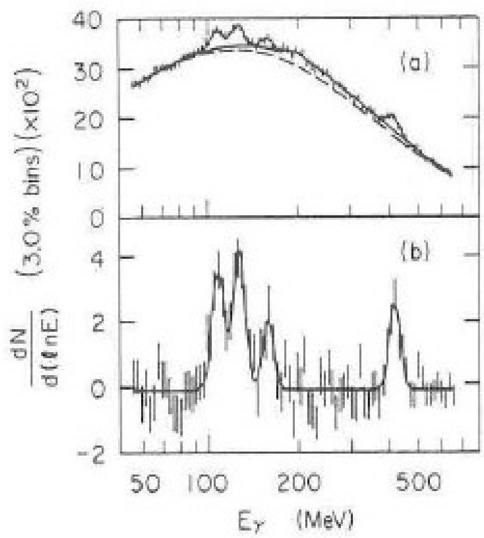


Figure 11.2: The photon spectrum from Υ' decays obtained by the Crystal Ball Collaboration at DORIS II. A triplet of lines corresponding to $\Upsilon' \to \gamma\chi_b(^3P_{2,1,0})$ is seen between 100 and 200 MeV. The decays $\chi_b \to \gamma\Upsilon$ produce the unresolved signal between 400 and 500 MeV [R. Nernst *et al.*, *Phys. Rev. Lett.* **54**, 2195 (1985)].

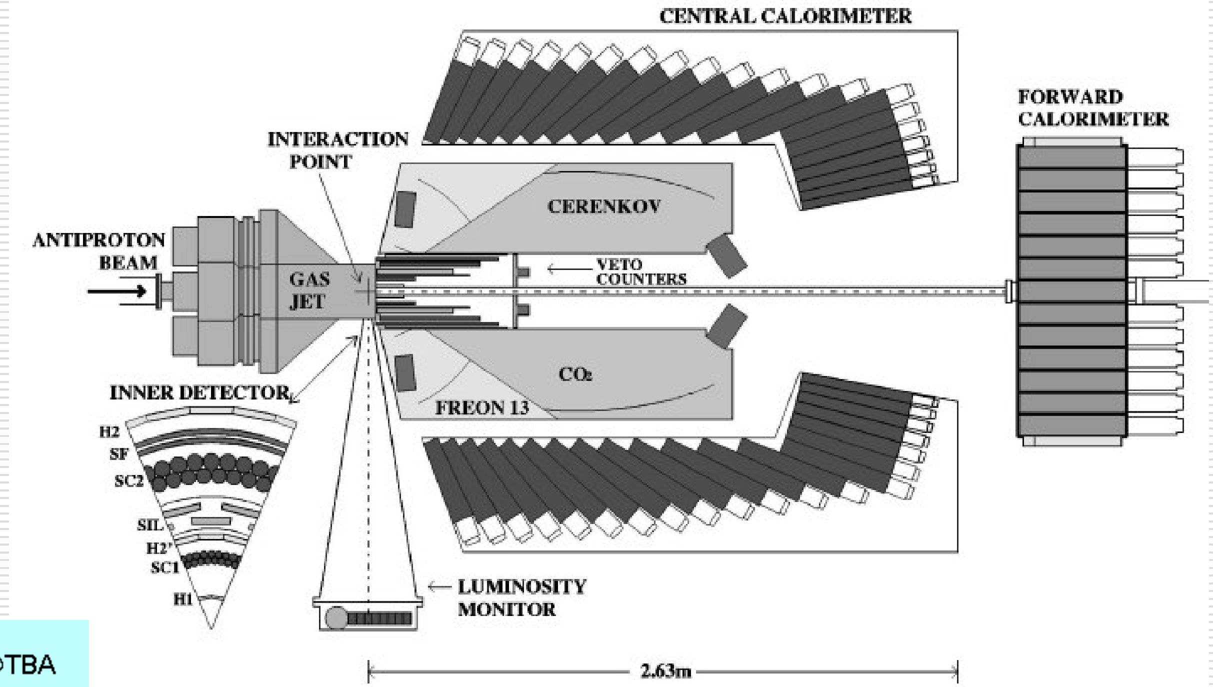
Another Side of Charmonium - I

$$p+\overline{p}\rightarrow \underbrace{c\overline{c}}_{Charmonium} \rightarrow Electromagnetic\ \ decay$$

Circulating $\overline{p}$ Beam:
Excellent E *resolution*

Gas jet target:
Reduced E *loss*

Non magnetic detector:
EM Calorimeter, Tracking, Cerenkov



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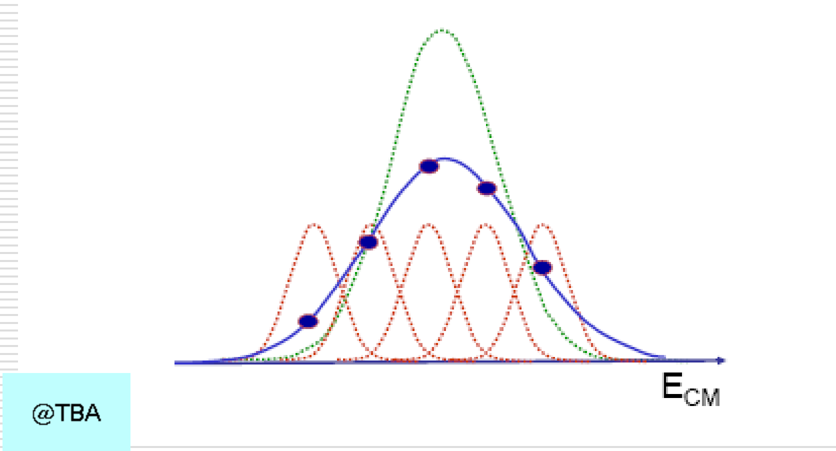
Another Side of Charmonium - II

Concept of resonance scan: A fixed target, formation experiment

Move the beam energy in small steps across the energy range of a given resonant state

Measure the decay rate of the state at each step



Rate

Resonance profile

Typical width Γ < 1 MeV for $c\overline{c}$

Beam profile

Typical resolution $\sigma(E_{CM}) \sim 0.2\ MeV$

Get resonance mass, width, coupling by deconvolving the beam profile from the observed rate

Another Side of Charmonium - III

Electrons: *Cerenkov* + *Calorimeter* + *Tracking*
→ Very low background to $e^+ e^-$

$$\psi' \to J/\psi + X$$
$$\searrow e^+ e$$

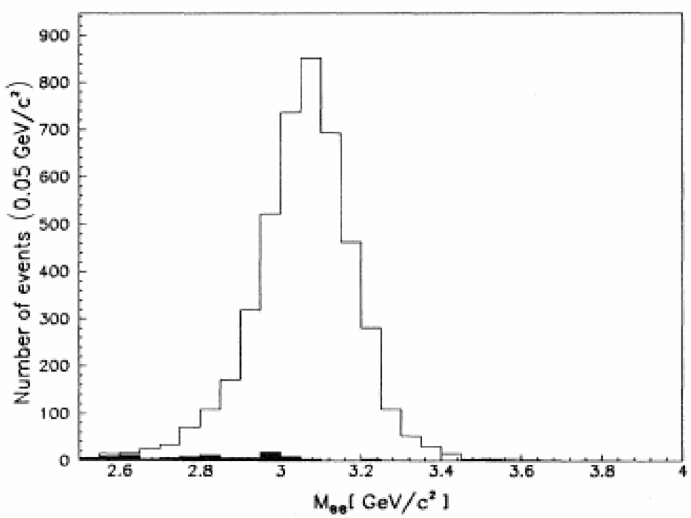


FIG. 5. Invariant mass distribution of electron pairs for the 1991 J/ψ scan (open area) and for the off-resonance background normalized to the same luminosity (shaded area). For this figure only the level of background has been multiplied by a factor of 10 to make it discernible.

@TBA

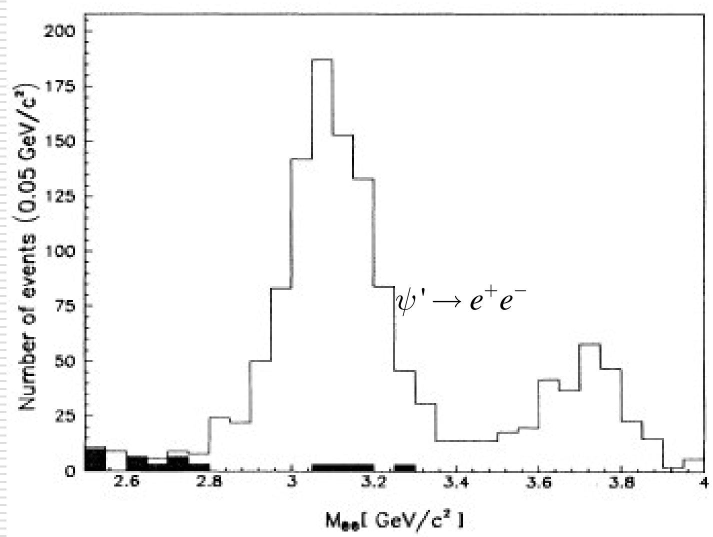


FIG. 6. Invariant mass distribution of electron pairs for the 1991 ψ' scan (open area) and for the off-resonance background normalized to the same luminosity (shaded area).

$M_{e^+e^-}$ from scan across J/ψ

$M_{e^+e^-}$ from scan across ψ'

Another Side of Charmonium - IV

A few results..

@TBA

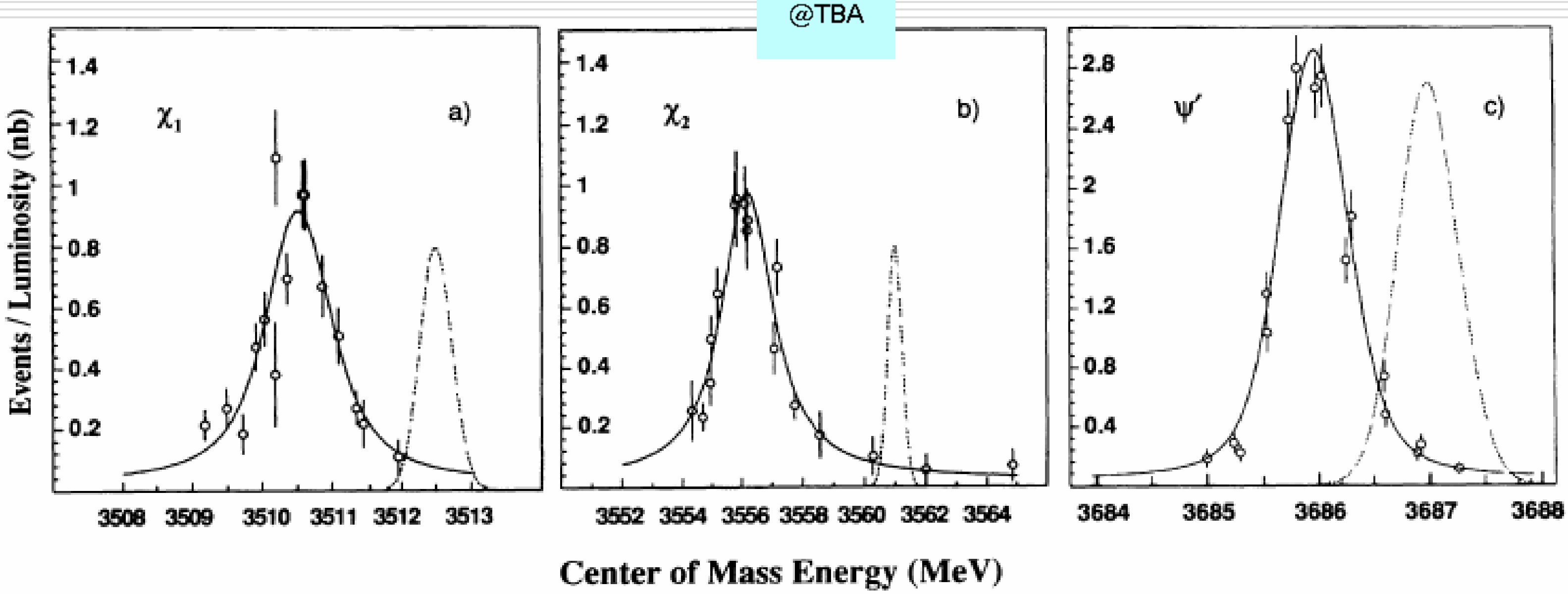


FIG. 3. Events per unit luminosity for the energy scan at (a) the χ_{c1}, (b) the χ_{c2}, and (c) the ψ'. The solid line represents the best fit with the data. The dashed line shows a typical resolution in the center-of-mass energy (arbitrary vertical units).

Quarkonia on PDG

Hidden Charm

$c\bar{c}$	
• $\eta_c(1S)$	$0^+(0^{-+})$
• $J/\psi(1S)$	$0^-(1^{--})$
• $\chi_{c0}(1P)$	$0^+(0^{++})$
• $\chi_{c1}(1P)$	$0^+(1^{++})$
$h_c(1P)$	$?^?(?^{??})$
• $\chi_{c2}(1P)$	$0^+(2^{++})$
• $\eta_c(2S)$	$0^+(0^{-+})$
• $\psi(2S)$	$0^-(1^{--})$
• $\psi(3770)$	$0^-(1^{--})$
• $X(3872)$	$0^?(?^{?+})$
• $\chi_{c2}(2P)$	$0^+(2^{++})$
$Y(3940)$	$?^?(?^{??})$
• $\psi(4040)$	$0^-(1^{--})$
• $\psi(4160)$	$0^-(1^{--})$
$Y(4260)$	$?^?(1^{--})$
• $\psi(4415)$	$0^-(1^{--})$

Hidden Bottom

$b\bar{b}$	
$\eta_b(1S)$	$0^+(0^{-+})$
• $\Upsilon(1S)$	$0^-(1^{--})$
• $\chi_{b0}(1P)$	$0^+(0^{++})$
• $\chi_{b1}(1P)$	$0^+(1^{++})$
• $\chi_{b2}(1P)$	$0^+(2^{++})$
• $\Upsilon(2S)$	$0^-(1^{--})$
$\Upsilon(1D)$	$0^-(2^{--})$
• $\chi_{b0}(2P)$	$0^+(0^{++})$
• $\chi_{b1}(2P)$	$0^+(1^{++})$
• $\chi_{b2}(2P)$	$0^+(2^{++})$
• $\Upsilon(3S)$	$0^-(1^{--})$
• $\Upsilon(4S)$	$0^-(1^{--})$
• $\Upsilon(10860)$	$0^-(1^{--})$
• $\Upsilon(11020)$	$0^-(1^{--})$

@TBA

Non perturbative QCD

Needed to deal with *bound states* and *soft interaction* regime

Very difficult problem

Different approaches available:

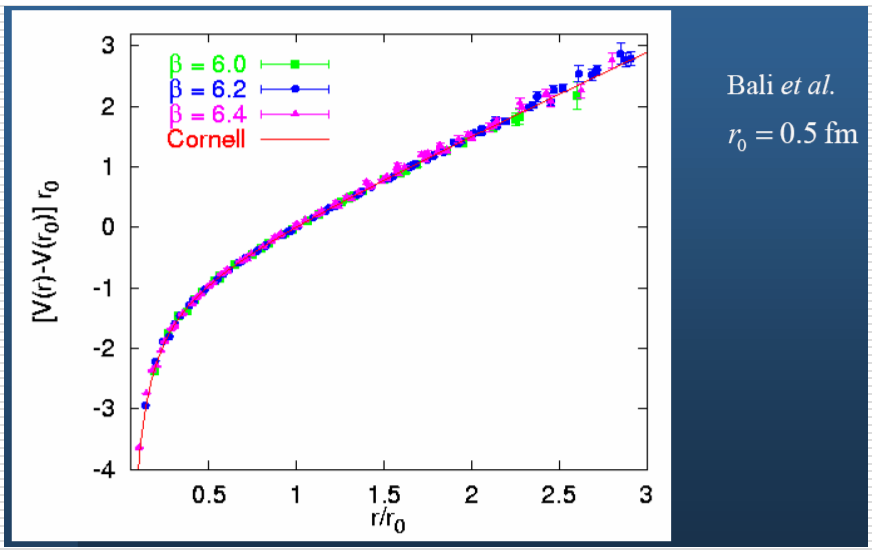
Lattice QCD
Chiral pertubation theory
NRQCD
Heavy quark effective theory
…

Deep waters, not even surfed in this course

Lattice QCD

Perform QCD calculations over a discretized space-time (lattice)

$q\bar{q}$ potential from lattice



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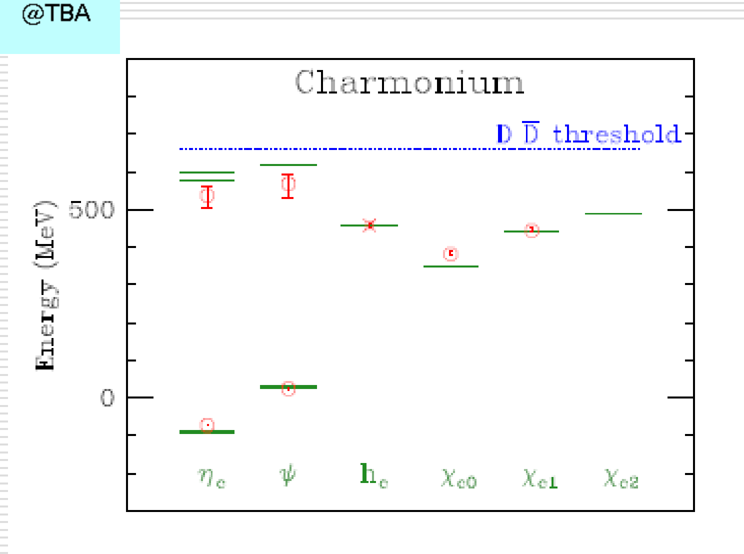

$V(r) = -\frac{4}{3}\frac{\alpha_s}{r} + ar$: Not a bad idea after all...

Example:
Charmonium levels from lattice