Elementary Particles I

5 – QCD

Color, Gauge Fields, Gluons, Asymptotic Freedom, Perturbative QCD, Quarkonium

Hadrons:Re-Examining the Evidence

Experiments probing the EM structure, like DIS and similar:

Scaling of the structure functions

Evidence for point-like constituents, funny behavior:

Like free particles when interacting with EM currents at high Q^2

Never observed outside hadrons $\rightarrow$ *Tightly bound?*

Experiments probing the strong interaction:

Large particle zoo

Evidence for highly symmetrical grouping and ordering

Strong suggestion of a substructure: Quarks

Funny, ad-hoc rules driving the observed symmetry

Can We Believe in Constituents?

Besides the general, puzzling behavior observed, finally identify a number of serious problems, cast into a few, quantitative issues:

Baryons and the Pauli Principle
The R Ratio
The π^0 Decay Rate
The τ Lepton Branching Ratios

From all these questions, and others, a common conclusion:

Our picture of the quark model is not complete

The Pauli Principle

Quark model:

Besides its many, remarkable successes, a central point is at issue:

The baryon wave function

$(space \times spin \times flavor)$

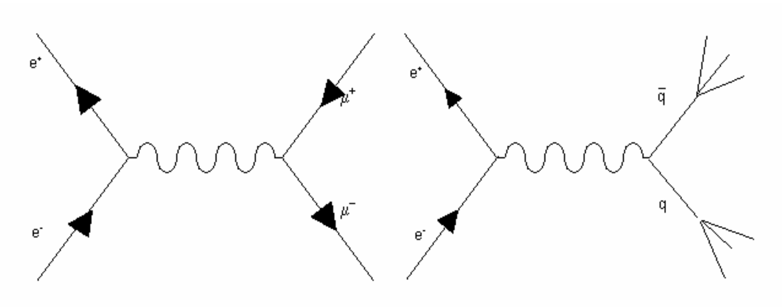
is symmetric

Pauli Principle seems to be lost, which is very bad news:

The Spin-Statistics Theorem is a consequence of very general principles of relativistic QFT, not easily dismissed

The R Ratio - I

Assume the process $e^+e^- \to hadrons$ to proceed at the lowest order through $e^+e^- \to q\ \overline{q} \to hadrons$



As for DIS:
Don't care about quark *hadronization*, assume the time scales for hard and soft sub-processes to be wildly different

$$\sigma\left(e^+e^- \to \mu^+\mu^-\right) = \frac{4\pi\alpha^2}{3s}$$

$$\sigma\left(e^+e^- \to q\ \overline{q}\right) = \frac{4\pi\alpha^2 Q_q^2}{3s}, \quad Q_q = \text{quark charge in } e \text{ units}$$

$$R\left(E_{CM}\right) = \frac{\sigma\left(e^+e^- \to adroni\right)}{\sigma\left(e^+e^- \to \mu^+\mu^-\right)} = \frac{\sum_q \sigma\left(e^+e^- \to q\overline{q}\right)}{\sigma\left(e^+e^- \to \mu^+\mu^-\right)} = \sum_q Q_q^2$$

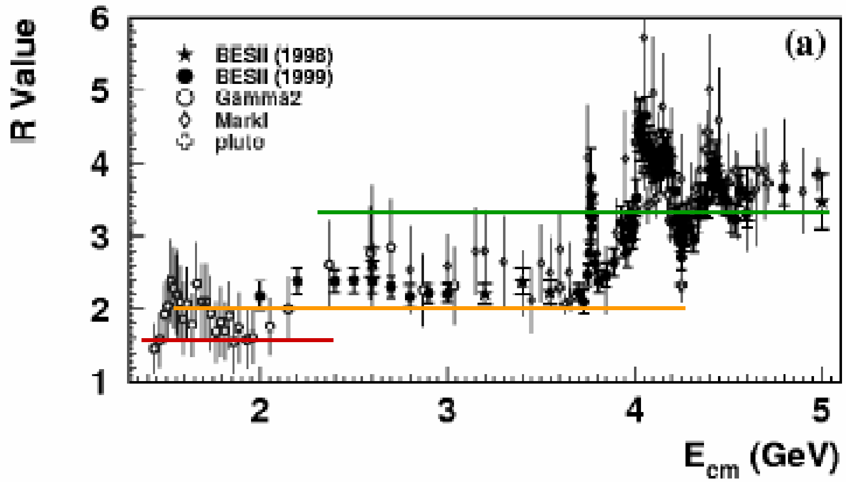
The R Ratio - II

R counts the number of different quark species created at any given E_{CM}.
Expect:

$$u,d \rightarrow R = \left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 = \frac{5}{9} \qquad Low\ energy$$

$$u,d,s \rightarrow R = \left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 = \frac{6}{9} \qquad E > 1\text{-}1.5\ GeV$$

$$u,d,s,c \rightarrow R = \left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 = \frac{10}{9} \qquad E > 3\ GeV$$



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By taking 3 quark species of any flavor:

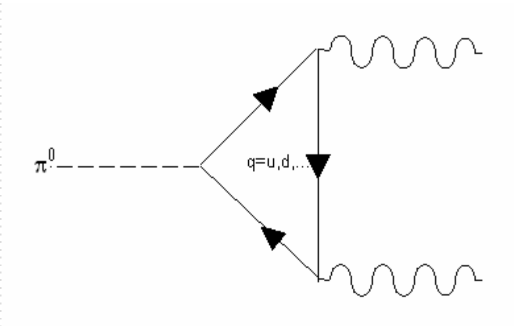
$$u,d \rightarrow R = \frac{15}{9}$$

$$u,d,s \rightarrow R = \frac{18}{9}$$

$$u,d,s,c \rightarrow R = \frac{30}{9}$$

The π^0 Decay Rate - I

Difficult subject: Strong interaction effects are *large*
Originally calculated by taking $p,\bar{p}$ in the triangle loop (Steinberger 1949)



As for similar cases: Initial state is *not* a plane wave

π^0 spinless: Only 4-vector available p_μ

$\rightarrow$Decay amplitude ~ $p_\mu J_\mu$

J_μ = Loop *axial* current, to match pion –ve parity

The π^0 Decay Rate - II

With a proton loop rate OK (!)
By replacing the proton loop by a quark loop:

$$J^{\mu}_{(A)} \approx \sum_{i=u}^{d} q_i \bar{\psi}_i \gamma^{\mu}\gamma^5 \tau_3^i \psi_i = e\left(\frac{2}{3}\bar{u}\gamma^{\mu}\gamma^5 u - \frac{1}{3}\bar{d}\gamma^{\mu}\gamma^5 d\right)$$

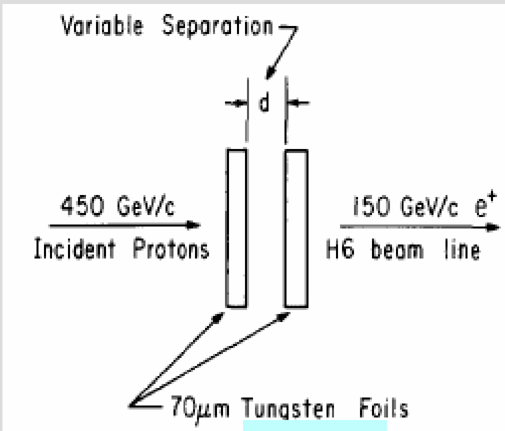
$$\sum_{i=u,d} \tau_3^i Q_i^2 = 1\cdot\left(\frac{2}{3}\right)^2 - 1\cdot\left(-\frac{1}{3}\right)^2 = \frac{4}{9} - \frac{1}{9} = \frac{1}{3}$$

$$\Gamma_{quark}\left(\pi^0 \to \gamma\gamma\right) = \frac{\alpha^2}{64\pi^3}\frac{m_\pi^3}{f_\pi^2}\sum_i g_A^{(i)} e_i^2 = \frac{1}{9}\Gamma_{proton}\left(\pi^0 \to \gamma\gamma\right) \to ???$$

NB: A whole *lot* of physics in this problem:
Simple guess on approximate symmetry of the initial state would lead to conclude *the neutral pion is stable!*

Explanation of this paradox led to discovery of the first *anomaly* in QFT (Adler, Bell, Jackiw)
Advanced topic, quite relevant to the Standard Model

The π^0 Lifetime: Direct Method



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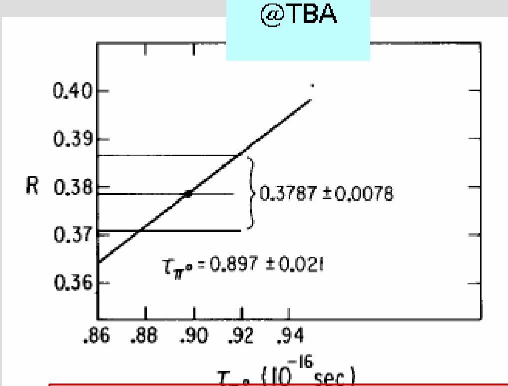


$$\tau = 0.897 \pm .021\ 10^{-16}\ s$$

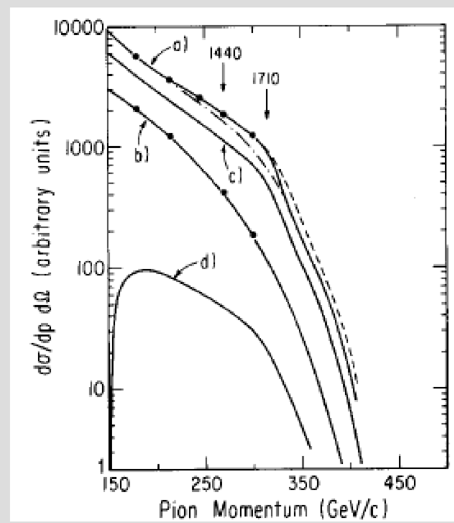
$$\Gamma = 7.34 \pm 0.18 \pm 0.11\ eV$$

π^0 produced in a first thin foil, when not decayed do not contribute to e^+ yield from γ conversion in a second thin foil

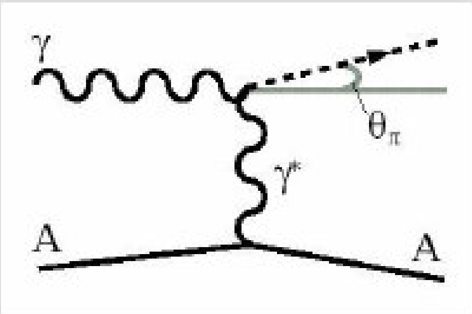
$$Y(d) = N\left\{A + B\left[1 - \exp\left(-d/\lambda\right)\right]\right\}$$

$\lambda = \beta\gamma c\tau \simeq \gamma c\tau$ Energy dependent

Use known energy spectra for pions



The π^0 Lifetime: Primakoff Effect



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Very simple idea:
Get a high energy photon beam + high Z target
Pick-up a virtual photon from the nuclear Coulomb field
2-photon coupling will (sometimes) create a π^0

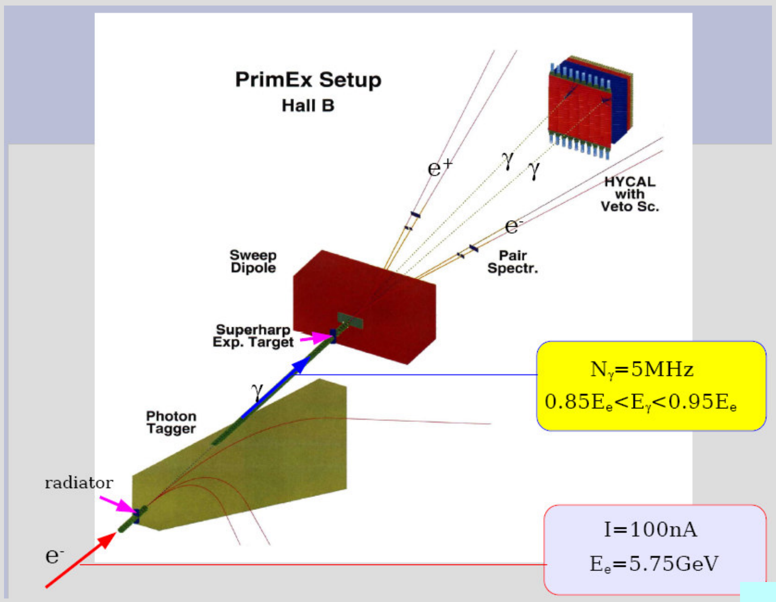
$$\frac{d\sigma_{\text{Prim}}}{d\Omega_{LAB}} \simeq \Gamma_{\pi^0\to\gamma\gamma} Z^2 \frac{8\alpha E^4}{m_{\pi^0}^2} \frac{\left|F\left(q^2\right)\right|^2}{q^4} \sin^2\theta_{\pi^0}$$

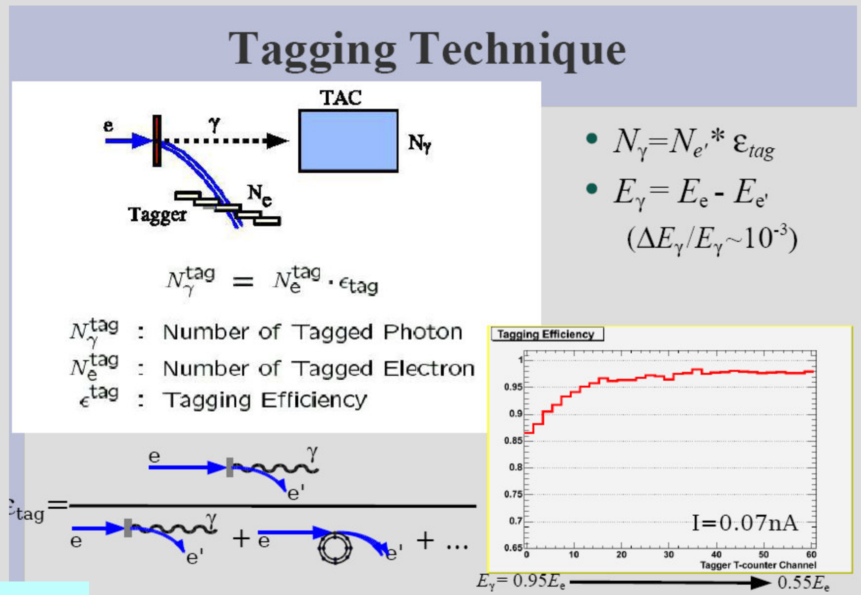
Strongly forward peaked
Quickly increasing with energy
Strongly Z dependent: Coherence

$\Gamma = 1/\tau$ extracted by measuring the differential cross-section
Nuclear form factor is required

A Recent Experiment

PrimEx at Jefferson Lab (Virginia)





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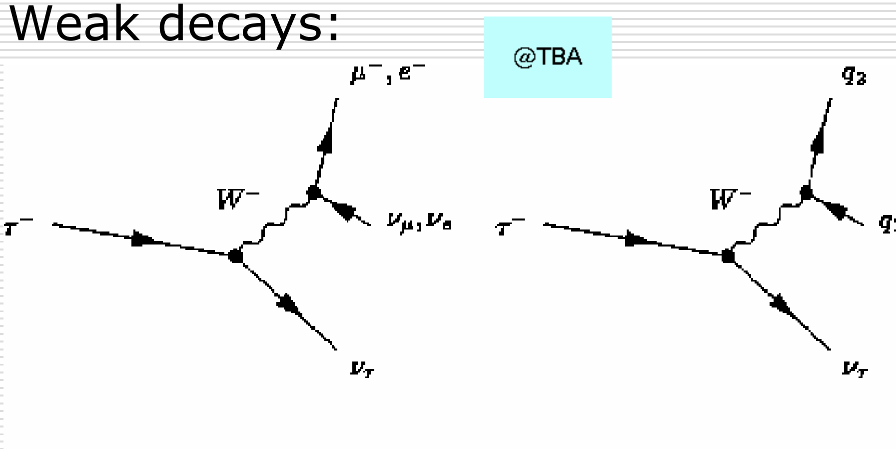
Results expected soon…

The τ Lepton Decays

τ: Heavy brother of e and μ

$m_\tau = 1776$ MeV

Weak decays:



$q, \bar{q} : u, d, s$

$BR(e) \sim 18\%$

$BR(\mu) \sim 17\%$

$BR(q\bar{q}) \sim 65\%$

In the absence of color, weak interaction universality would lead to predict:

$BR(e) \sim BR(\mu) \sim BR(q\bar{q}) \sim 33\%$

With color:

$$\Gamma(q\bar{q}) \sim 3\ \Gamma(l\bar{l}) \rightarrow BR(q\bar{q}) \sim \frac{3\ \Gamma(l\bar{l})}{3\ \Gamma(l\bar{l}) + 2\ \Gamma(l\bar{l})} \sim 60\ \%$$ OK

Color

New hypothesis:

There is a new degree of freedom for quarks: Color

Each quark can be found in one of 3 different states

Internal space (mathematically identical to flavor):

States = 3-component complex vectors

Base states:

$$R(ed) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, G(reen) = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, B(lue) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Needless to say, nothing to do with our old, beloved color concept (nature, art, politics..): Just a *name* for another, non-classical property of hadron constituents

Benefits from the Color Hypothesis

Provided one can build a color wave function for 3 fermions which is antisymmetric, the Fermi statistics problem is solved
Total wave function of a baryon

$$\psi = \psi_{color} \underbrace{\psi_{orbital}\psi_{spin}\psi_{flavor}}_{Symmetric} \rightarrow \psi_{color}: \text{ Antisymmetric}$$

To account for 3 different color states, the R ratio must be multiplied by *3* $\rightarrow$ OK with experimental data

Just the same conclusion for hadronic τ decays: Multiply rate by *3*

The correct π^0 rate is obtained by inserting a factor *9*

Real vs. Virtual Quarks

Observe:

When computing R, τ decay rates we add the *rates* for different colors
$\rightarrow$Factor $\times$ *3*

We deal with quarks as with real particles:
Ignore fragmentation

When computing π^0 decay rate, we add the *amplitudes*
$\rightarrow$Factor $\times$ *9*

Quarks in the loop are virtual particles:
Amplitudes interfere

Color as a Quantum Number

Must be possible to build hadron states as color *singlets*

Do not expect hadrons to fill larger irr.reps.:
Would imply large degeneracies for hadron states, not observed
In other words:

Color is fine, but we do not observe any colored hadron

Therefore we assume the color charge is *confined*: Never observed directly, as it is the electric charge

Why? For the moment, nobody really knows

Non Existing Colored Hadrons

How colored hadrons would show up?
Just as an example:

Should the nucleon fill the ***3*** of $SU(3)_C$, there would be 3 different species of protons and neutrons.

Then each nucleon level in any nucleus could accommodate 3 particles instead of one:

The nuclear level scheme would be far different from the observed one

The Color Group: *SU(3)*$_C$

Guess *SU(3)* as the color group
Take the two fundamental decompositions:

$\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{10}$ Baryons

$\mathbf{3} \otimes \mathbf{3}^* = \mathbf{1} \oplus \mathbf{8}$ Mesons

Both feature a singlet in the direct sum: OK
No singlets in **3**⊗**3**: OK

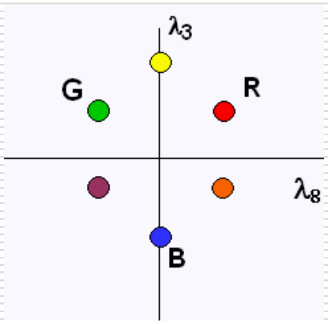
Can't say the same for other groups…
Take *SU(2)* as an example:
Say the quarks live in the adjoint *SU(2)* representation, ***3***
Then for qq :

$\boldsymbol{3} \otimes \boldsymbol{3} = \boldsymbol{1} \oplus \boldsymbol{3} \oplus \boldsymbol{5}$

Observe: This is ***3*** of *SU(2)*, which is quite different from ***3*** of *SU(3)*

Diquarks can be in color singlet
→Should find diquarks as commonly as baryons or mesons..

The Color of Quarks



$$I_3^c = \frac{\lambda_3}{2}, \qquad Y^c = \frac{\lambda_8}{\sqrt{3}}$$

	I_3^c	Y^c
R	$+1/2$	$+1/3$
G	$-1/2$	$+1/3$
B	0	$-2/3$

	I_3^c	Y^c
$\bar{R}$	$-1/2$	$-1/3$
$\bar{G}$	$+1/2$	$-1/3$
$\bar{B}$	0	$+2/3$

SU(3)$_C$ is an *exact symmetry*:

$$m_u^{(R)} = m_u^{(G)} = m_u^{(B)}, \ldots$$

Beware: SU(3)$_C$ has nothing to do with *SU(3)$_F$*:
Quark quantum numbers are independent from their color state

They are left unchanged by QCD transitions

The Color of Hadrons

According to our fundamental hypothesis:

Mesons: $\mathbf{3} \otimes \mathbf{3}^* = \mathbf{1} \oplus \mathbf{8}$

$$\rightarrow \psi_c = \frac{1}{\sqrt{3}}\left(R\overline{R} + G\overline{G} + B\overline{B}\right)$$

Baryons: $\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{10}$

$$\rightarrow \psi_c = \frac{1}{\sqrt{6}}\left(RGB - RBG + GBR - GRB + BRG - BGR\right)$$

In both cases, pick singlet
Mesons: *No particular exchange symmetry*
(2 non identical particles) [Some funny algebra on this point…]

Baryons: *Fully antisymmetrical color wave function*
(3 identical particles)

Extending the Color Hypothesis: *QCD*

Color: A new degree of freedom for quarks
Compare to other quantum numbers:

Baryonic/Leptonic numbers
Conserved, *not originating interactions*

Electric charge
Conserved, *origin of the electromagnetic field*

A deep question:

What is the true origin of the electromagnetic interaction?

We have used freely the interaction term $j^{\mu}A_{\mu}$, only based on the classical analogy: Is there a deeper origin for it?

QED as a Gauge Theory - I

Symmetry:
Absolute phase not defined for a wave function.
Expect invariance as per our old acquaintance, Noether's Theorem

$L_0 = \bar{\psi}(x)\left(i\gamma^\mu \partial_\mu - m\right)\psi(x)$ Free Dirac Lagrangian

Global gauge (=Phase) transformation:

$G : \psi(x) \rightarrow \psi'(x) = U_\theta \psi(x) = e^{-iq\theta}\psi(x)$ $q\theta :$ New phase $\propto$ Charge

$\rightarrow L_0$ invariant wrt $G \rightarrow$ Charge conservation

Just meaning:

Take *all* particle states,
Re-phase each state proportionally to its charge

QED as a Gauge Theory - II

Generalize to local phase transformation:

$G_L : \psi(x) \to \psi'(x) = U_\theta \psi(x) = e^{-iq\theta(x)}\psi(x)$ Local gauge transformation

$\to L_0$ not invariant wrt G_L: Derivative term troublesome

$$L_0 = \bar{\psi}(x)\left(i\gamma^\mu \partial_\mu - m\right)\psi(x) \to L_0' = i\bar{\psi}(x)e^{+iq\theta(x)}\gamma^\mu\partial_\mu\left(e^{-iq\theta(x)}\psi(x)\right) - m\bar{\psi}(x)\psi(x)$$

$$L_0' = i\left[\bar{\psi}(x)\gamma^\mu\partial_\mu\psi(x) - iq\partial_\mu\left[\theta(x)\right]\psi(x)\right] - m\bar{\psi}(x)\psi(x)$$

$$L_0' = \left\{i\bar{\psi}(x)\gamma^\mu\partial_\mu\psi(x) + q\partial_\mu\left[\theta(x)\right]\psi(x) - m\bar{\psi}(x)\psi(x)\right\}$$

$$L_0' = \left[i\bar{\psi}(x)\gamma^\mu\partial_\mu\psi(x) - m\bar{\psi}(x)\psi(x)\right] + q\partial_\mu\left[\theta(x)\right]\psi(x) \neq L_0$$

$\to$ Local gauge invariance cannot hold in a world of free particles

Symmetry requires interaction

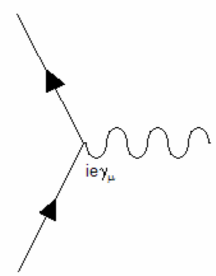
QED as a Gauge Theory - III

New transformation rule:

$$\begin{cases} \psi(x) \to \psi'(x) = U_\theta \psi(x) = e^{-iq\theta(x)}\psi(x) & \text{As before} \\ A_\mu(x) \to A_\mu(x) + q\ \partial_\mu \theta(x) & \text{New character in the comedy} \end{cases}$$

Equivalent to re-define derivative for ψ:

$\partial_\mu \to \partial_\mu + iqA_\mu$ Vector field



Add a new term to Lagrangian:

$L_i = -\underbrace{q\bar{\psi}(x)\gamma^\mu\psi(x)}_{j^\mu} A_\mu$ Interaction term

Same as classical electrodynamics

$$L_0 = \bar{\psi}(x)\left(i\gamma^\mu\partial_\mu - m\right)\psi(x) \to L_0 + L_i = \bar{\psi}(x)\left(i\gamma^\mu\partial_\mu - m\right)\psi(x) - q\bar{\psi}(x)\gamma^\mu\psi(x)A_\mu$$

Sum is invariant

QED as a Gauge Theory - IV

...And another one:

$$-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}, \quad F^{\mu\nu} = \partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu}$$ Field energy

Must be there because the field carries energy+momentum

Reminder:

$F^{\mu\nu}$ is the EM field

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix}$$

QED as a Gauge Theory - V

Field must be massless to have L gauge invariant

$$\frac{1}{2}m^2A_\mu^2 \to \frac{1}{2}m^2\left(A_\mu(x)+q\ \partial_\mu\theta(x)\right)^2 \neq \frac{1}{2}m^2A_\mu^2 \quad \text{if } m\neq 0$$

Consider all the phase transformations as defined before:

$$\psi(x)\to\psi'(x)=U_\theta\psi(x)=e^{-iq\theta(x)}\psi(x)$$

The full set is a group: $U(1)$ Unitary, 1-dimensional

$$e^{-iq\theta_1(x)}e^{-iq\theta_2(x)}\psi(x)=e^{-iq[\theta_1(x)+\theta_2(x)]}\in U(1)$$

1 parameter: $\theta(x)$

Abelian: $e^{-iq\theta_1(x)}e^{-iq\theta_2(x)}\psi(x)=e^{-iq\theta_2(x)}e^{-iq\theta_1(x)}\psi(x)$

U(1) is the (Abelian) *gauge group* of QED
Equivalent to *SO(2),* group of 2D rotations

QCD as a Gauge Theory - I

Extend gauge transformations to a 3-component wave function:

$$\boldsymbol{\psi} \equiv \begin{pmatrix} \psi_R \\ \psi_G \\ \psi_B \end{pmatrix}$$

Global gauge transformation: Phase change for individual components

$\rightarrow$Phase change will mix color components

$G_L^C : \boldsymbol{\psi}(x) \rightarrow \boldsymbol{\psi}'(x) = \mathbf{U}_G \cdot \boldsymbol{\psi}(x) = e^{-ig\mathbf{M}} \cdot \boldsymbol{\psi}(x)$ $\quad \mathbf{U}_G$ unitary $\rightarrow \mathbf{M}$ Hermitian

$e^{-ig\mathbf{M}} = \mathbf{1} - ig\mathbf{M} + \frac{(-ig\mathbf{M})^2}{2!} + \ldots$ $\quad \mathbf{M}:$ 3$\times$3 Hermitian matrix

$\mathbf{M}$ acting on the 3 color components of the quark state

Since the color symmetry group is $SU(3)_C$:

$$\mathbf{M} = \sum_{i=1}^{8} \boldsymbol{\lambda}_i \theta_i \equiv \vec{\boldsymbol{\lambda}} \cdot \vec{\theta}$$

$\vec{\boldsymbol{\lambda}}$: Vector of 8 3$\times$3 Gell-Mann matrices; $\vec{\theta}$: Vector of 8 parameters

QCD as a Gauge Theory - II

As for QED, extend to $local$ gauge transformations

As before, in order to guarantee invariance of L:

$\rightarrow$ Re-define derivative adding new vector fields:

$$\partial_\mu \rightarrow \partial_\mu \mathbf{1} + ig\mathbf{C}_\mu$$

$$\mathbf{C}_\mu : \begin{cases} \text{4-vector field} & \text{Lorentz structure} \\ \text{Matrix } \in SU(3)_C & \text{Color space} \end{cases}$$

We know how to express any Hermitian matrix $\in SU(3)_C$:

Use $SU(3)_C$ generators $\rightarrow$ Gell-Mann matrices

$\rightarrow \mathbf{C}_\mu = \frac{1}{2}\sum_{i=1}^{8} \mathrm{G}^a_\mu \boldsymbol{\lambda}_a \equiv \vec{G}_\mu \cdot \vec{\boldsymbol{\lambda}}$ 8 fields required: Gluons

So gluons are a bit like 8 different "photons", exchanged between color charges

But: *They are non Abelian*

QCD as a Gauge Theory - III

Local gauge transformation for $SU(3)_C$:

$$\begin{cases} \psi(x) \to \psi'(x) = U_\theta \psi(x) = e^{-ig\vec{\lambda}\cdot\vec{\theta}(x)}\psi(x) \\ G_\mu^a(x) \to G_\mu^a{}'(x) = G_\mu^a(x) + \partial_\mu \theta^a + g\sum_{b,c=1}^{8} f^{abc} G_\mu^b(x)\ \theta^c(x) \qquad a = 1,\ldots,8 \end{cases}$$

Very important: New term, coming from $SU(3)$ being non Abelian

Reminder:

$$\left[\frac{\lambda_a}{2}, \frac{\lambda_b}{2}\right] = if^{abc}\frac{\lambda_c}{2}$$

f^{abc}: $SU(3)$ structure constants

$$L_0 = \bar{\psi}(x)\left(i\gamma^\mu \partial_\mu - m\right)\psi(x) \to L_0 + L_i$$

$$L_i = -g\underbrace{\left[\bar{\psi}(x)\gamma^\mu\left(\frac{\vec{\lambda}}{2}\right)\psi(x)\right]}_{\vec{j}_\mu}\cdot\vec{G}_\mu$$ Interaction term

$$-\frac{1}{4}\vec{G}_{\mu\nu}\cdot\vec{G}^{\mu\nu} = -\frac{1}{4}\sum_{a=1}^{8} G_{\mu\nu}^a \cdot G^{a\mu\nu}$$ Field energy term

QCD as a Gauge Theory - IV

Take the expression of fields in terms of potentials:

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu - g \sum_{b,c=1}^{8} f_{abc} G^b_\mu G^c_\upsilon$$

Very important: *Absent in QED* $(f=0)$
New term, coming from $SU(3)$ being non Abelian

$$\rightarrow G^a_{\mu\nu} G^a_{\mu\nu} \quad \text{contains terms with} \quad \underbrace{\partial_\mu G^a_\nu \cdot G^b_\mu G^c_\upsilon}_{\text{3 gluons}}, \underbrace{G^b_\mu G^c_\upsilon \cdot G^b_\mu G^c_\upsilon}_{\text{4 gluons}}$$

These pieces of L correspond to 3 and 4 gluons vertexes

The form of QCD Lagrangian leads to predict the existence of a new kind of gluon-gluon color interaction

QCD as a Gauge Theory - V

Since color interaction is tied to color charge, we are saying *the gluons carry their own color charge*

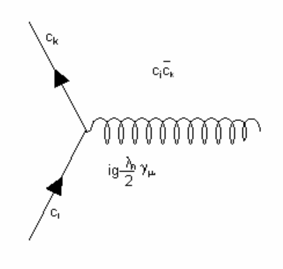
Sounds unfamiliar? Well, that’s all after playing with a non-Abelian gauge group.

Unlike the electric charge, color charge can manifest itself in more than one way.

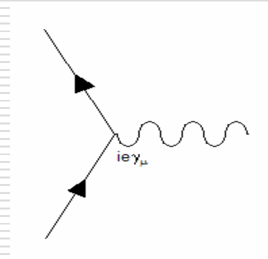
Indeed, gluons carry a type of color charge different from quarks/antiquarks:
Color + Anticolor

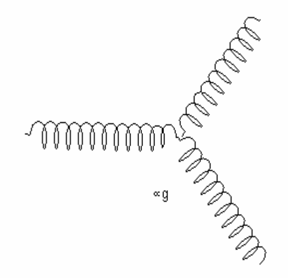
QCD as a Gauge Theory - VI

QCD Vertexes

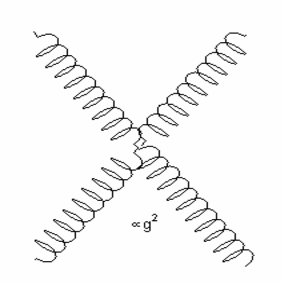


$ig\frac{\lambda_a}{2}\gamma_\mu$ Similar to QED:





$\propto g$ (Lorentz structure not shown)



$\propto g^2$ (Lorentz structure not shown)

The Color of Gluons - I

Compare to mesons in $SU(3)_F$: *Flavor + Antiflavor*
But: *Gluons are not bound states of Color+Anticolor!*

Still, they share the same math:
Gluons live in the adjoint (***8***) irr.rep. of $SU(3)_C$

$$|1\rangle = \frac{1}{\sqrt{2}}\left(r\bar{b} + b\bar{r}\right), |2\rangle = -\frac{i}{\sqrt{2}}\left(r\bar{b} - b\bar{r}\right), |3\rangle = \frac{1}{\sqrt{2}}\left(r\bar{r} - b\bar{b}\right)$$

$$|4\rangle = \frac{1}{\sqrt{2}}\left(r\bar{g} + g\bar{r}\right), |5\rangle = -\frac{i}{\sqrt{2}}\left(r\bar{g} - g\bar{r}\right), |6\rangle = \frac{1}{\sqrt{2}}\left(b\bar{g} + g\bar{b}\right)$$

$$|7\rangle = -\frac{i}{\sqrt{2}}\left(b\bar{g} - g\bar{b}\right), |8\rangle = \frac{1}{\sqrt{6}}\left(r\bar{r} + b\bar{b} - 2g\bar{g}\right)$$

The Color of Gluons - II

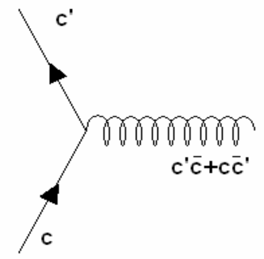
A very natural question: Gluons couple to $q\bar{q}$

Since one can decompose the total $q\bar{q}$ color state as:

$$\mathbf{3} \otimes \mathbf{3}^* = \mathbf{1} \oplus \mathbf{8}$$

Then: Where is the singlet gluon?

Does not exist: There are only 8 gluons, not 9

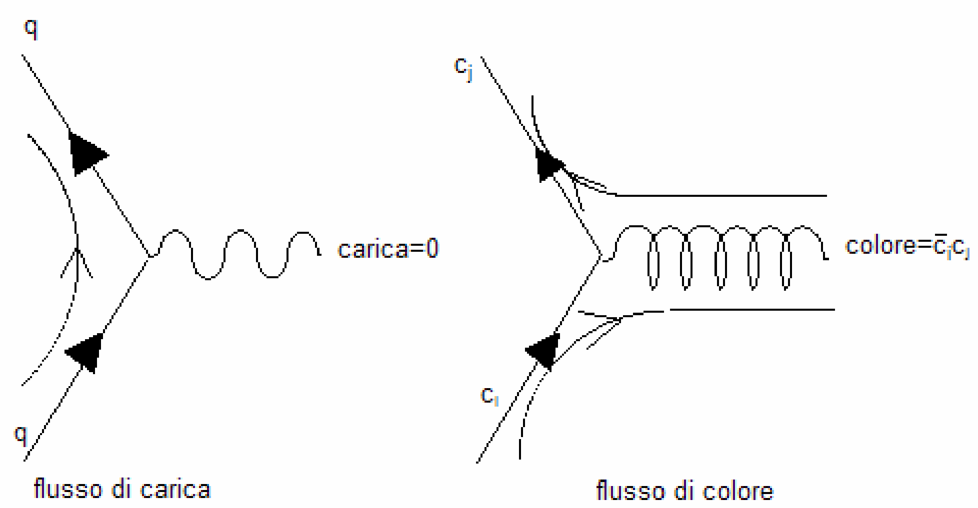


Should the singlet gluon actually exist, it would behave more or less like a "photon":

Would be 'white' (= Singlet)
Would couple to color charges in the same way as photon couples to electric charges
Would give rise to a sort of "QED-like" color interaction, not observed

Color vs. Charge Flow

Compare the different situations:



QED	QCD
Photon is *neutral*	Gluon is *colored*
Neither sourcing, nor sinking charge	Sourcing color, sinking anti-color

Comparing *QED* and *QCD* - I

Comparison of coupling constants:

α vs. α_s Adimensional constants (*Interaction strength)*

Can define elementary charge in terms of α, α_s

Measure particle charge by its ratio to elementary charge:

Number

What are the allowed values for these numbers?

Comparing *QED* and *QCD* - II

QED: Gauge group is *Abelian*

Electric charge can be *any* number:
No reason for charge quantization

Photon charge is strictly *0*

QCD: Gauge group is *non Abelian*

"Color charge" value is *fixed* for every representation

Quarks: $\mathbf{3},\mathbf{3}^*$ $\rightarrow Q = 4/3$
Gluons: $\mathbf{8}$ $\rightarrow Q = 3$

Similar to $I(I+1)$ for any isospin ($SU(2)$) multiplet

The Color Factor

Consider the static interaction between 2 charges:

QED For fixed $|q|$, the 'charge factor' can be defined as:

$$f_{12} = \frac{q_1 q_2}{|q|^2} = \begin{cases} +1 & q_1 q_2 > 0 \\ -1 & q_1 q_2 < 0 \end{cases}$$

Very simple for an Abelian interaction

QCD The ‘color factor’ depends on the irr.rep. of the color state

Representation dependent
Identical for any transition in a given representation
aka Color Conservation

Less simple in this non-Abelian interaction

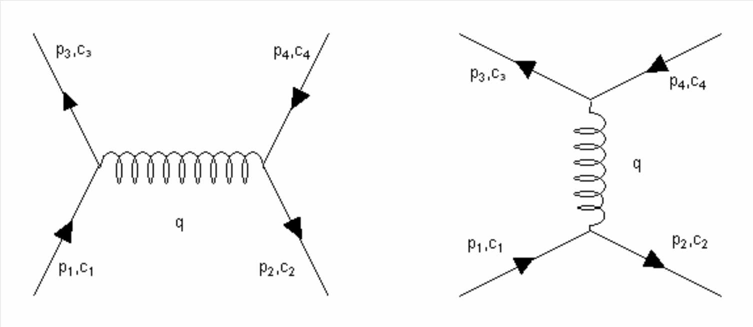
Color Interaction - I

$q\bar{q} \to q\bar{q}$

$\mathbf{3} \otimes \mathbf{3}^* = \mathbf{1} \oplus \mathbf{8}$

Total color conservation: $\begin{cases} \mathbf{1} \to \mathbf{1} \\ \mathbf{8} \to \mathbf{8} \end{cases}$

Observe:
Similar to conservation of total I-spin



$$T_{fi} = i \sum_{\alpha,\beta=1}^{8} \underbrace{\left[\bar{u}(3) c_3^\dagger\right]\left[-i \frac{g_s}{2} \lambda^\alpha \gamma^\mu\right]\left[u(1) c_1\right]}_{\text{color current}} \underbrace{\left[-i \frac{g_{\mu\nu} \delta^{\alpha\beta}}{q^2}\right]}_{\text{propagator}} \underbrace{\left[\bar{v}(2) c_2^\dagger\right]\left[-i \frac{g_s}{2} \lambda^\beta \gamma^\nu\right]\left[v(4) c_4\right]}_{\text{color current}}$$

Sum is over all 8 color matrices

c_i are the color states of initial, final $q\bar{q}$

$$T_{fi} = \frac{-g_s^2}{q^2}\left[\bar{u}(3)\gamma^\mu u(1)\right]\left[\bar{v}(2)\gamma_\mu v(4)\right] \underbrace{\frac{1}{4}\sum_\alpha \left[c_3^\dagger \lambda^\alpha c_1\right]\left[c_2^\dagger \lambda^\alpha c_4\right]}_{\text{color factor}}$$

Color Interaction - II

Octet

$r\bar{b}$ Just as an example: Result is the same for all octet states

$$\left.\begin{aligned} c_1 = c_3 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \\ c_2 = c_4 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \end{aligned}\right\} \rightarrow f = \frac{1}{4}\sum_{\alpha=1}^{8} \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \lambda^{\alpha} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \end{pmatrix} \lambda^{\alpha} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\rightarrow f = \frac{1}{4}\sum_{\alpha=1}^{8} \lambda_{11}^{\alpha}\lambda_{22}^{\alpha} = \frac{1}{4}\left(\lambda_{11}^{3}\lambda_{22}^{3} + \lambda_{11}^{8}\lambda_{22}^{8}\right) = -\frac{1}{6}$$

Color Interaction - III

Singlet

$$\frac{1}{\sqrt{3}}\left(r\bar{r}+b\bar{b}+g\bar{g}\right)$$ Only this state in the singlet

But: *Any* component can go into *any other*..

$$f_i=\frac{1}{4}\frac{1}{\sqrt{3}}\frac{1}{\sqrt{3}}\sum_{j=1}^{3}\sum_{\alpha=1}^{8}\left[c_i^\dagger\lambda^\alpha c_j\right]\left[c_j^\dagger\lambda^\alpha c_i\right],\quad i=1,2,3$$

$$f=\sum_{i=1}^{3}f_i=\frac{1}{4}\frac{1}{\sqrt{3}}\frac{1}{\sqrt{3}}\sum_{\alpha=1}^{8}\sum_{i,j=1}^{3}\lambda_{ij}^\alpha\lambda_{ji}^\alpha=\frac{1}{12}\sum_{\alpha=1}^{8}Tr\left(\lambda^\alpha\lambda^\alpha\right)$$

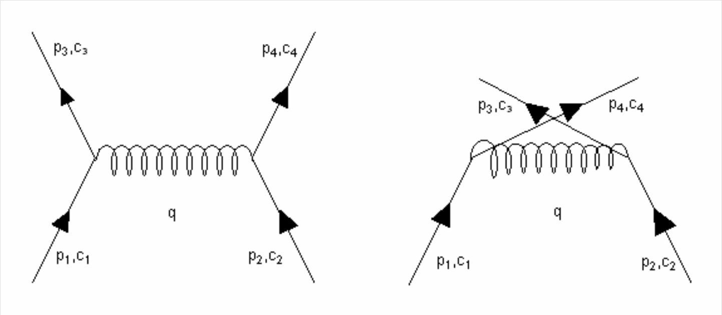
$$Tr\left(\lambda^\alpha\lambda^\beta\right)=2\delta^{\alpha\beta}\rightarrow\sum_{\alpha=1}^{8}Tr\left(\lambda^\alpha\lambda^\alpha\right)=16$$

$$\rightarrow f=\frac{4}{3}$$

Color Interaction - IV

qq

$\mathbf{3} \otimes \mathbf{3} = \mathbf{3}^* \oplus \mathbf{6}$



$$T_{fi} = \frac{-g_s^2}{q^2}\left[\bar{u}(3)\gamma^\mu u(1)\right]\left[\bar{u}(4)\gamma_\mu u(2)\right]\underbrace{\frac{1}{4}\sum_{\alpha=1}^{8}\left[c_3^\dagger \lambda^\alpha c_1\right]\left[c_4^\dagger \lambda^\alpha c_2\right]}_{\text{color factor}}$$

$$f_{qq} = \frac{1}{4}\sum_{\alpha=1}^{8}\left(c_3^\dagger \lambda^\alpha c_1\right)\left(c_4^\dagger \lambda^\alpha c_2\right)$$

Color Interaction - V

Color states of the triplet and sextet:

$$\mathbf{3}^*: \quad \frac{1}{\sqrt{2}}(rb-br), \frac{1}{\sqrt{2}}(bg-gb), \frac{1}{\sqrt{2}}(gr-rg)$$

Antisymmetric

$$\mathbf{6}: \quad rr, bb, gg, \frac{1}{\sqrt{2}}(rb+br), \frac{1}{\sqrt{2}}(bg+gb), \frac{1}{\sqrt{2}}(gr+rg)$$

Symmetric

Color Interaction - VI

<u>Sextet</u>

rr Just as an example: Result is the same for all sextet states

$$c_1 = c_2 = c_3 = c_4 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$f = \frac{1}{4}\sum_{\alpha=1}^{8}\left[\begin{pmatrix} 1 & 0 & 0 \end{pmatrix}\lambda^{\alpha}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}\right]\left[\begin{pmatrix} 1 & 0 & 0 \end{pmatrix}\lambda^{\alpha}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}\right] = \frac{1}{4}\sum_{\alpha=1}^{8}\left(\lambda_{11}^{\alpha}\lambda_{11}^{\alpha}\right)$$

$$= \frac{1}{4}\left(\lambda_{11}^{3}\lambda_{11}^{3} + \lambda_{11}^{8}\lambda_{11}^{8}\right) = \frac{1}{3}$$

Color Interaction - VII

<u>Triplet</u>

$$\frac{1}{\sqrt{2}}\left(rb-br\right)$$

Just as an example as before

$$f=\frac{1}{4}\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\sum_{\alpha=1}^{8}$$

$$\left\{\left[\begin{pmatrix}1&0&0\end{pmatrix}\lambda^{\alpha}\begin{pmatrix}1\\0\\0\end{pmatrix}\right]\left[\begin{pmatrix}0&1&0\end{pmatrix}\lambda^{\alpha}\begin{pmatrix}0\\1\\0\end{pmatrix}\right]-\left[\begin{pmatrix}0&1&0\end{pmatrix}\lambda^{\alpha}\begin{pmatrix}1\\0\\0\end{pmatrix}\right]\left[\begin{pmatrix}1&0&0\end{pmatrix}\lambda^{\alpha}\begin{pmatrix}0\\1\\0\end{pmatrix}\right]\right.$$

$$\left.-\left[\begin{pmatrix}1&0&0\end{pmatrix}\lambda^{\alpha}\begin{pmatrix}0\\1\\0\end{pmatrix}\right]\left[\begin{pmatrix}0&1&0\end{pmatrix}\lambda^{\alpha}\begin{pmatrix}1\\0\\0\end{pmatrix}\right]-\left[\begin{pmatrix}0&1&0\end{pmatrix}\lambda^{\alpha}\begin{pmatrix}0\\1\\0\end{pmatrix}\right]\left[\begin{pmatrix}1&0&0\end{pmatrix}\lambda^{\alpha}\begin{pmatrix}1\\0\\0\end{pmatrix}\right]\right\}$$

$$f=\frac{1}{8}\sum_{\alpha=1}^{8}\left\{\lambda_{11}^{\alpha}\lambda_{22}^{\alpha}-\lambda_{21}^{\alpha}\lambda_{12}^{\alpha}-\lambda_{12}^{\alpha}\lambda_{21}^{\alpha}+\lambda_{22}^{\alpha}\lambda_{11}^{\alpha}\right\}=\frac{1}{4}\sum_{\alpha=1}^{8}\left\{\lambda_{11}^{\alpha}\lambda_{22}^{\alpha}-\lambda_{12}^{\alpha}\lambda_{21}^{\alpha}\right\}=\frac{1}{4}\left\{\lambda_{11}^{3}\lambda_{22}^{3}+\lambda_{11}^{8}\lambda_{22}^{8}-\lambda_{12}^{1}\lambda_{21}^{1}-\lambda_{12}^{2}\lambda_{21}^{2}\right\}=-\frac{2}{3}$$

The Effective Potential

Matrix elements just calculated:
Very similar to the corresponding tree-level amplitudes in QED

→Expect similar, Coulomb-like, effective potential in the static limit

Constant depends on the color representation for the quark pair:

$$V_{q\bar{q}} = \begin{cases} -\frac{4}{3}\frac{\alpha_s}{r} & \text{singlet} \\ \frac{1}{6}\frac{\alpha_s}{r} & \text{octet} \end{cases}$$

Attractive

$$V_{qq} = \begin{cases} -\frac{2}{3}\frac{\alpha_s}{r} & \text{triplet} \\ \frac{1}{3}\frac{\alpha_s}{r} & \text{sextet} \end{cases}$$

Attractive

Expect maximal attraction in singlet

Baryons

Baryons could be in any one of the ***1,8,10*** representations:
Why only the singlet is observed?
A hint of an explaination:

$$\mathbf{3}\otimes\mathbf{3}\otimes\mathbf{3}=\left(\mathbf{3}\otimes\mathbf{3}\right)\otimes\mathbf{3}$$

$$\mathbf{3}\otimes\mathbf{3}=\mathbf{6}\oplus\mathbf{3}^{*}\rightarrow\left(\mathbf{3}\otimes\mathbf{3}\right)\otimes\mathbf{3}=\left(\mathbf{6}\oplus\mathbf{3}^{*}\right)\otimes\mathbf{3}$$

$$\mathbf{6}\otimes\mathbf{3}=\mathbf{10}\oplus\mathbf{8}$$

$$\mathbf{3}^{*}\otimes\mathbf{3}=\mathbf{1}\oplus\mathbf{8}$$

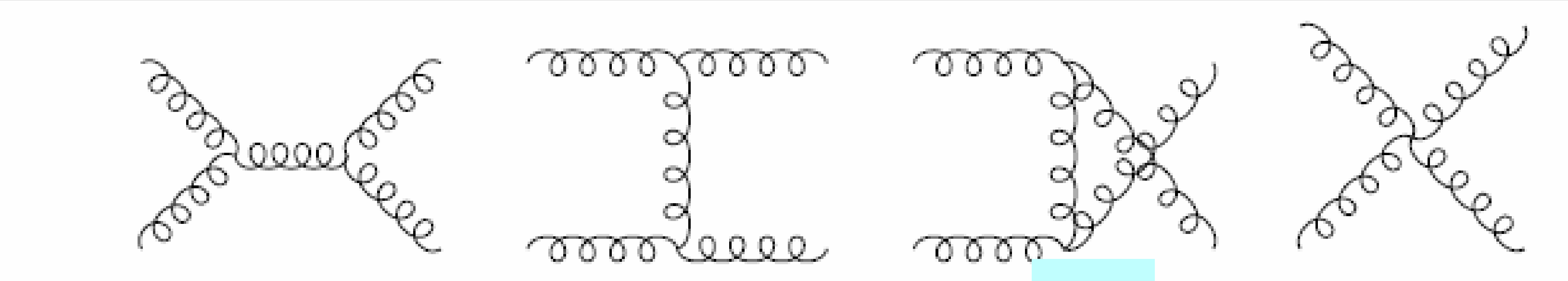
1: *each qq pair is a triplet $\rightarrow$ attractive*
8: *qq pair can be triplets, or sextet $\rightarrow$ attractive + repulsive*
10: *each qq pair is a sextet $\rightarrow$ repulsive*

So singlet is the state maximally attractive for 3 quarks

Does this explain the singlet-only mystery of bound states?

Another Color Interaction

Non Abelian vertices: Gluon-Gluon scattering *at tree level*

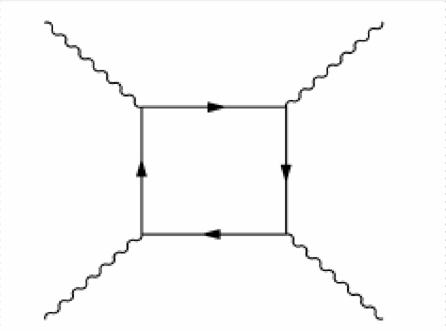


@TBA

$3 - gluons : A \propto g$

$4 - gluons : A \propto g^2$ Much harder to observe

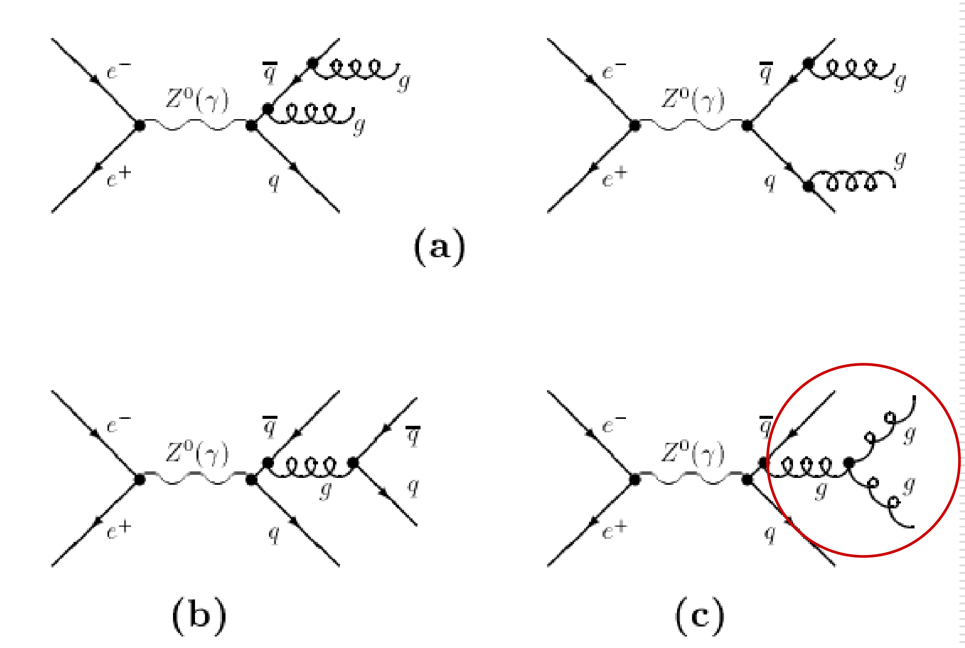
Compare:
In QED, photon-photon scattering amplitude occurs at order α^2 through the 1-loop diagram



Is QCD Really *SU(3)* ? - I

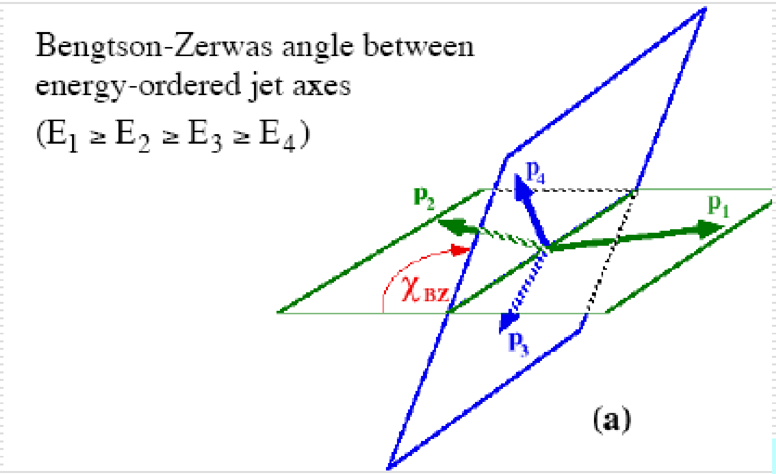
Test for non-Abelian couplings at LEP: 4 jets events

Special angular correlation from 3-gluon vertex amplitude



Is QCD Really *SU(3)* ? - II

Look at distribution of a special angle, sensitive to non-Abelian couplings:



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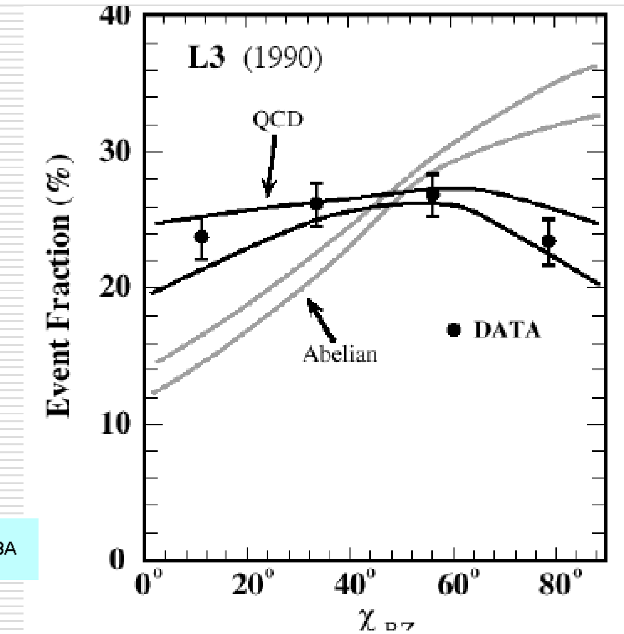
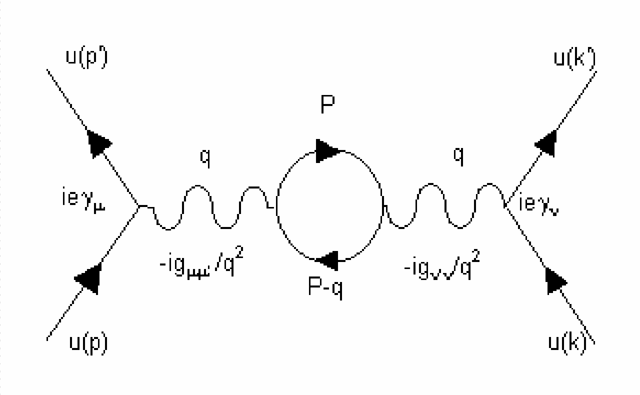


Running Coupling: *QED* - I

Consider the *one loop* modification to the photon propagator:



Includes a sum over P, the momentum circulating in the virtual loop.
No obvious bounds on P..

$$M \propto \left[e\bar{u}(k')\gamma^{\mu}u(k)\right]\frac{g_{\mu\mu'}}{q^2}\frac{1}{(2\pi)^4}\int d^4P\frac{\left[e\bar{u}(P)\gamma^{\mu'}u(P-q)\right]}{P^2-m^2}\frac{\left[e\bar{u}(P-q)\gamma^{\nu'}u(P)\right]}{(P-q)^2-m^2}\frac{g_{\nu\nu'}}{q^2}\left[e\bar{u}(p')\gamma^{\nu}u(p)\right]$$

Modified propagator:

$$\frac{g_{\mu\nu}}{q^2}\to\frac{g_{\mu\nu}}{q^2}\left(1-I(q^2)\right),\quad I(q^2)=\frac{\alpha}{3\pi}\int_{m^2}^{\infty}\frac{dp^2}{p^2}-\frac{2\alpha}{\pi}\int_0^1 dx\,x(1-x)\ln\left[1-\frac{q^2x(1-x)}{m^2}\right]$$

Running Coupling: *QED* - II

Take the high q^2 approximation

$$-q^2 \gg m^2 \to \ln\left[1-\frac{q^2x(1-x)}{m^2}\right] \approx \ln\left[-\frac{q^2}{m^2}\right]$$

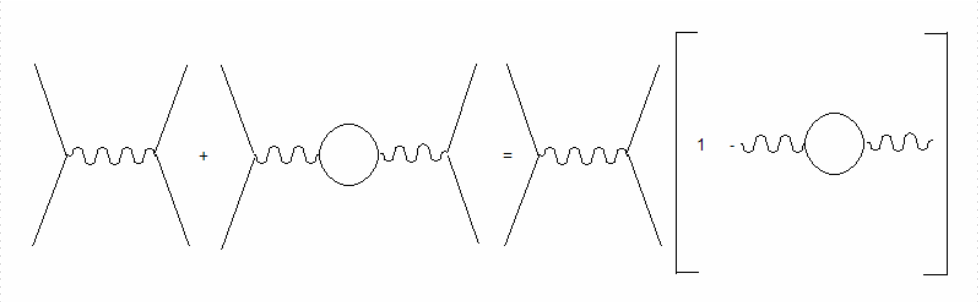
$$I(q^2) \approx \frac{\alpha}{3\pi}\int_{m^2}^{M^2}\frac{dp^2}{p^2} - \frac{2\alpha}{\pi}\int_0^1 dx\,x(1-x)\ln\left[\frac{-q^2}{m^2}\right]$$

Provisional upper bound (cutoff) to make integral to converge

$$I(q^2) \approx \frac{\alpha}{3\pi}\ln\left(\frac{M^2}{m^2}\right) - \frac{2\alpha}{\pi}\frac{1}{6}\ln\left[\frac{-q^2}{m^2}\right] = \frac{\alpha}{3\pi}\left[\ln\left(\frac{M^2}{m^2}\right) - \ln\left[\frac{-q^2}{m^2}\right]\right] = \frac{\alpha}{3\pi}\ln\left(\frac{M^2}{-q^2}\right)$$

$$M \propto \alpha\left[\bar{u}(k')\gamma^\mu u(k)\right]\frac{g_{\mu\nu}}{q^2}\left[1-\frac{\alpha}{3\pi}\ln\left(\frac{M^2}{-q^2}\right)\right]\left[\bar{u}(p')\gamma^\nu u(p)\right]$$

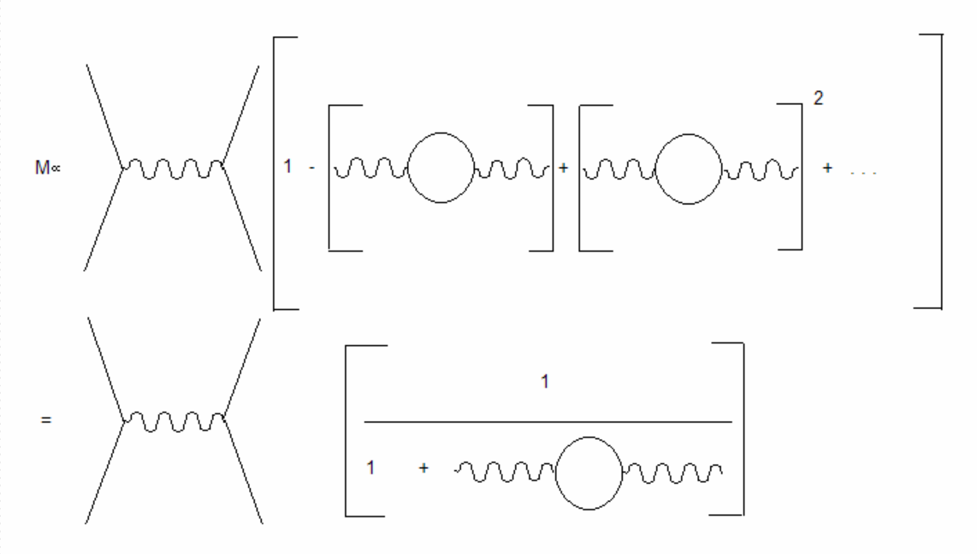
Cartoon translation:



Running Coupling: *QED* - III

Extend to diagrams with 2,3,…,n,… loops: Add up all contributes

Sum of a 'geometrical series': Converging ??



Experts say this is the only contribution to running α to the 'leading logs' approximation, which means neglecting the next levels of iteration

Running Coupling: *QED* - IV

$$M \propto \left[\bar{u}(k')\gamma^{\mu}u(k)\right]\frac{g_{\mu\nu}}{q^2}\left[\frac{\alpha}{1+\alpha/3\pi\ln\left(M^2/-q^2\right)}\right]\left[\bar{u}(p')\gamma^{\nu}u(p)\right]$$

What is α?

Coupling 'constant' we would get should we turn off all loops

Call it α_0 = 'Bare' coupling constant, not physical:

Loops cannot be turned off

Then obtain an effective coupling, not constant but running:

$$\alpha\left(q^2\right)=\frac{\alpha_0}{1+\alpha_0/3\pi\ln\left(M^2/-q^2\right)}$$

α is q^2, or distance, dependent!

Running Coupling: *QED* - V

Running α is still cutoff dependent, which of course is uncomfortable
But: Not a real problem.
Indeed:

$$Q^2 = -q^2 \rightarrow \alpha\left(Q^2\right) = \frac{\alpha_0}{1 + \left(\alpha_0 / 3\pi\right) \ln\left(M^2 / Q^2\right)}$$

Take a particular energy scale : $Q^2 = \mu^2$

$$\rightarrow \alpha\left(\mu^2\right) = \frac{\alpha_0}{1 + \left(\alpha_0 / 3\pi\right) \ln\left(M^2 / \mu^2\right)}$$

Usually choose $\mu^2 = 0$, i.e. take α at distance $\rightarrow \infty$

Quite natural in QED (but not compulsory)

Running Coupling: *QED* - VI

$$\ln\left(\frac{M^2}{Q^2}\right)=\ln\left(\frac{M^2}{Q^2}\frac{\mu^2}{\mu^2}\right)=\ln\left(\frac{M^2}{\mu^2}\right)+\ln\left(\frac{M^2}{Q^2}\right)$$

$$\rightarrow\alpha\left(Q^2\right)=\frac{\alpha_0}{1+(\alpha_0/3\pi)\left[\ln\left(M^2/\mu^2\right)+\ln\left(\mu^2/Q^2\right)\right]}$$

$$\rightarrow\frac{\alpha_0}{\alpha\left(\mu^2\right)}=1+(\alpha_0/3\pi)\ln\left(M^2/\mu^2\right)$$

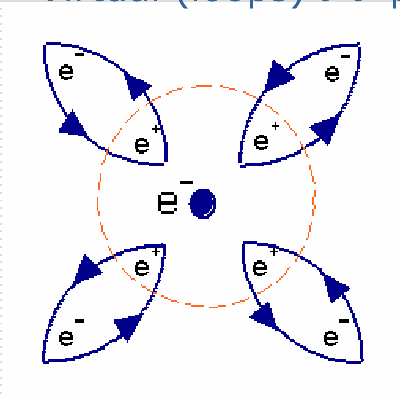
$$\rightarrow\alpha\left(Q^2\right)=\frac{\alpha_0}{\alpha_0/\alpha\left(\mu^2\right)+(\alpha_0/3\pi)\ln\left(\mu^2/Q^2\right)}=\frac{\alpha\left(\mu^2\right)}{1-\left[\alpha\left(\mu^2\right)/3\pi\right]\ln\left(Q^2/\mu^2\right)}$$

Very interesting result: Running α depends on q^2, through its own *measured* value at any chosen energy scale μ^2.

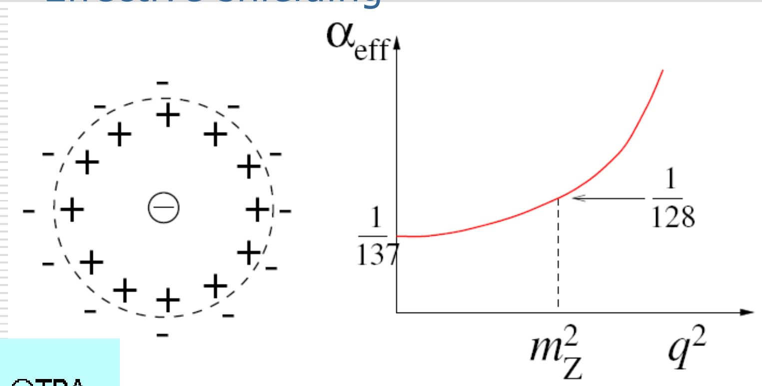
Cutoff has disappeared.

Cartooning Deep Physics

Virtual (loops) e^+e^- pairs



Effective shielding



@TBA

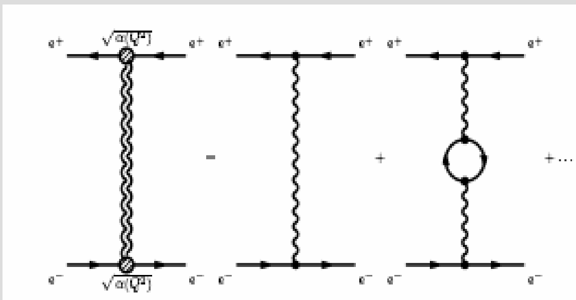
Picture the QED perturbative vacuum as a sort of (virtual) dielectric medium: Virtual photons and e^+e^- pairs continuously created/annihilated

Bare charge is shielded at large distance by the virtual pairs coming from loops. The standard e charge is smaller than the bare charge

By probing the electron at smaller and smaller distance, observe an *increasing* effective charge

Running α at LEP (and More)

Experimental method: Bhabha scattering



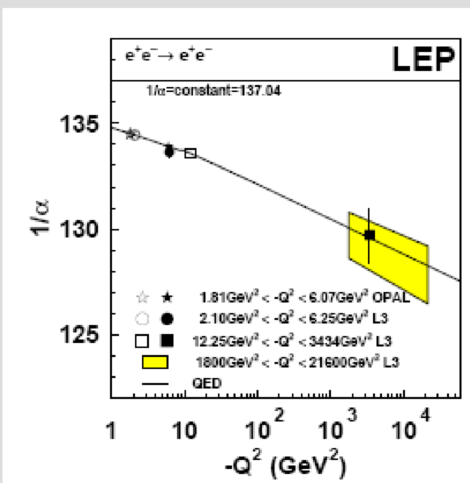
$$\frac{d\sigma}{dt} = \frac{d\sigma^0}{dt}\left(\frac{\alpha(t)}{\alpha_0}\right)^2 (1+\varepsilon)(1+\delta_\gamma)+\delta_Z,$$

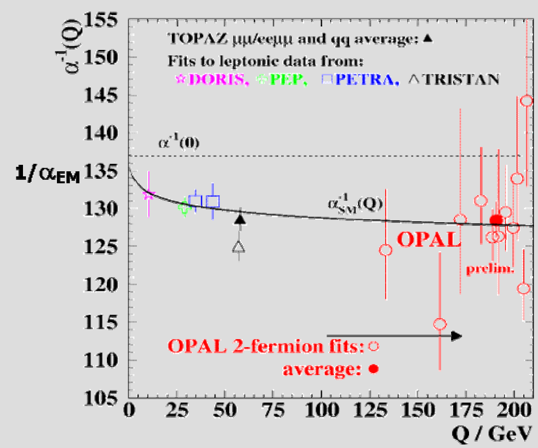
δ_γ, δ_Z s-channel contributions (small)

ε radiative corrections (known)

Use accurate, differential cross-section measurement to unfold $\alpha(t)$

Total cross-section measurement would require a luminosity..



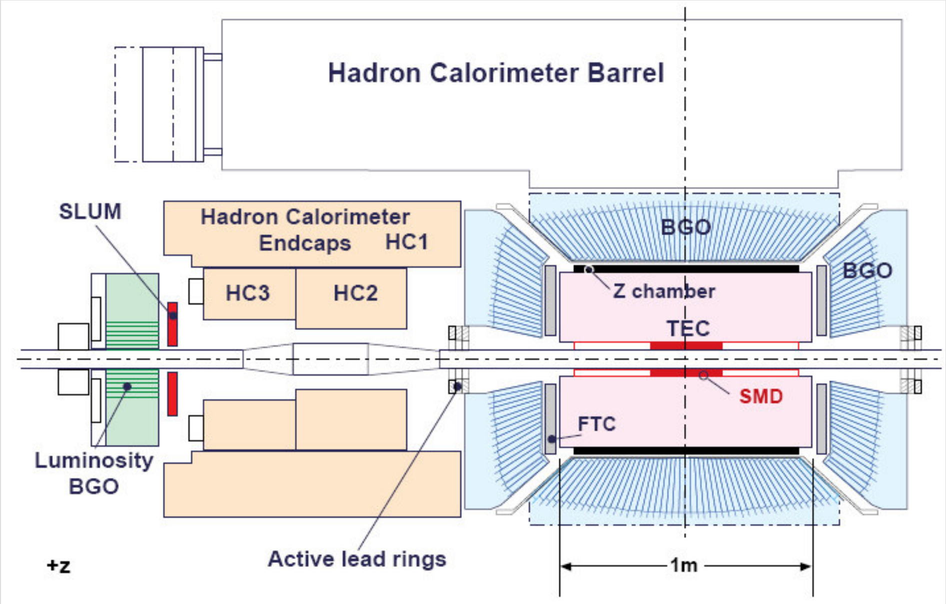


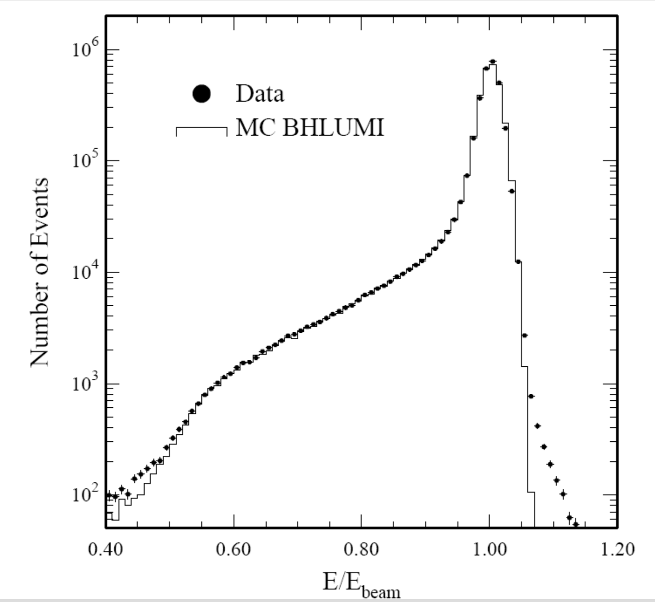
Luminosity Monitors

Just as an example, take L3 at LEP:
Relying on Bhabha scattering at small angle

$$\sigma = \frac{16\pi\alpha^2}{s}\left(\frac{1}{\theta^2_{min}} - \frac{1}{\theta^2_{max}}\right)$$

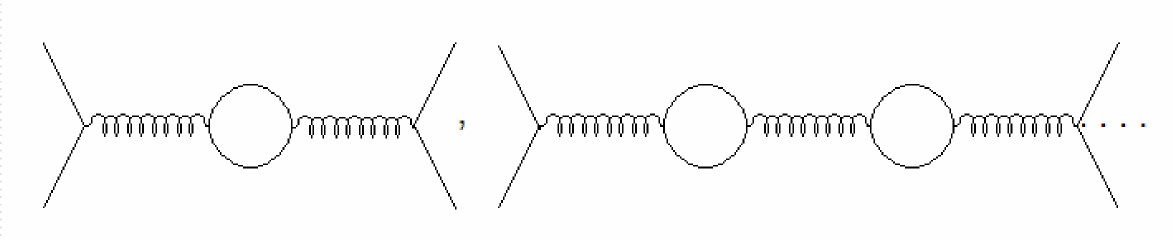
Calorimeter (Energy) + Silicon Tracker (Angle + Acceptance)



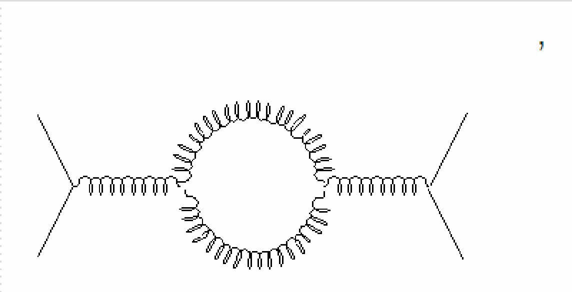


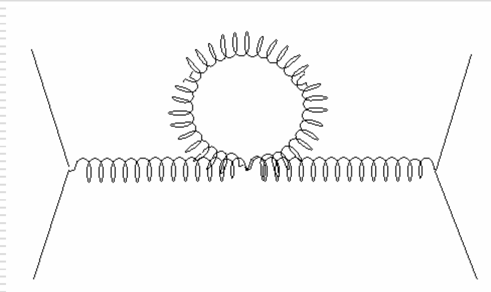
Running Coupling: *QCD* - I

Repeat all the steps: Loops etc



Except this time one has more loops: Gluons





Running Coupling: *QCD* - II

Turns out that gluon loops yield *anti*-shielding effect
With 8 gluons and 6 quark flavors, gluons win

$$\alpha_s\left(\left|q^2\right|\right)=\frac{\alpha_s\left(\mu^2\right)}{1+\left(\alpha_s\left(\mu^2\right)/12\pi\right)\left(33-2n_{flavor}\right)\ln\left(\left|q^2\right|/\mu^2\right)}$$

Running coupling *decreases* with increasing q^2 (or at smaller distance)
This is known as *asymptotic freedom*:

Large q^2 processes feature small coupling $\rightarrow$ Perturbative!

Most important consequence:

The fundamental hypothesis behind the successful parton model is finallyunderstood and justified

The Meaning of Λ

Rather than making reference to a specific value of α_s

$$\alpha_s\left(\left|q^2\right|\right)=\frac{\alpha_s\left(\mu^2\right)}{1+\left(\alpha_s\left(\mu^2\right)/12\pi\right)\left(33-2n_{flavor}\right)\ln\left(\left|q^2\right|/\mu^2\right)}$$

define a new constant

$$\ln\Lambda^2=\ln\mu^2-\frac{12\pi}{\left(33-2n_{flavor}\right)\alpha_s\left(\mu^2\right)}\rightarrow\Lambda^2=\mu^2 e^{-\frac{12\pi}{\left(33-2n_{flavor}\right)\alpha_s\left(\mu^2\right)}}$$

$$\rightarrow\alpha_s\left(\left|q^2\right|\right)\simeq\frac{12\pi}{\left(33-2n_{flavor}\right)\ln\left(\left|q^2\right|/\Lambda^2\right)}=\frac{12\pi}{21\ln\left(\left|q^2\right|/\Lambda^2\right)},\qquad\left|q^2\right|\gg\Lambda^2$$

Λ = Renormalization scale $\rightarrow$ Fixes α_s at all q^2

$\Lambda\approx 200$ MeV yields the correct α_s at $\mu^2=M_{Z^0}^2$

Funny behavior, known as 'Dimensional Transmutation':

From an adimensional constant to a dimensional one $\alpha_s\rightarrow\Lambda$

Confinement

$$\alpha_s\left(\left|q^2\right|\right) \simeq \frac{12\pi}{21\ln\left(\left|q^2\right|/\Lambda^2\right)}, \qquad \left|q^2\right| \gg \Lambda^2$$

When $\left|q^2\right| \sim \Lambda^2$, the previous expression does not apply

$\alpha_s\left(\Lambda^2\right)$ *is large*

Strong interaction is strong

Cannot rely on perturbative expansion

In a general sense, we expect Λ to mark the low energy range, corresponding to *soft* (low q^2) processes

Bound states: Non-perturbative, 'white', energy scale ≈ Λ

Does $\alpha_s(\Lambda^2)$ correspond to the *color confinement* range?

Very likely. But remember:

It is not yet convincingly shown that QCD is a confining theory

PQCD: Jets in $e^+ e^-$ Collisions - I

2 jets

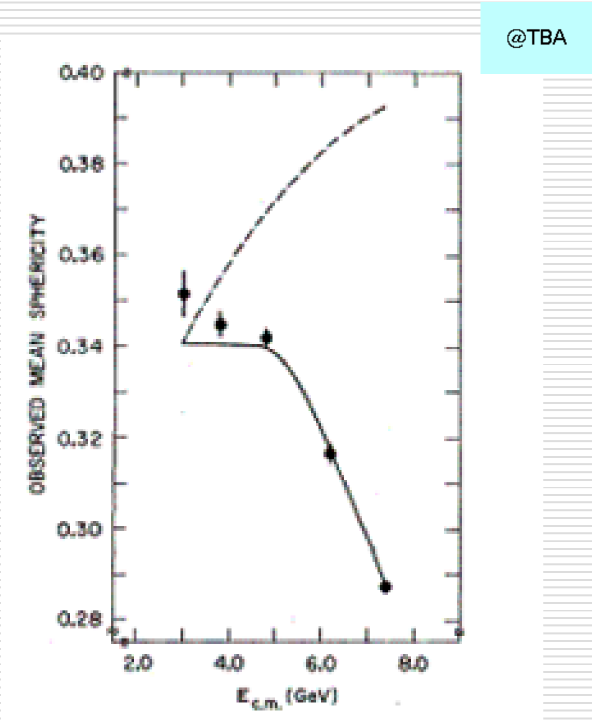
$$e^+ + e^- \to q + \bar{q} \to hadrons$$

$$\frac{d\sigma}{d\Omega} = \frac{3\alpha^2}{4s}\left(1+\cos^2\theta\right)\sum_{flavor} e^2_{flavor}$$

$$\to \sigma\left(s\right) = \frac{4\pi\alpha^2}{s}\sum_{flavor} e^2_{flavor}$$

Define sphericity of events:

$$S = \frac{3}{2}\frac{\sum_i p^2_{\perp i}}{\sum_i p^2_i} \to \begin{cases} 0 & \text{2-jets} \\ 1 & \text{spherical} \end{cases}$$

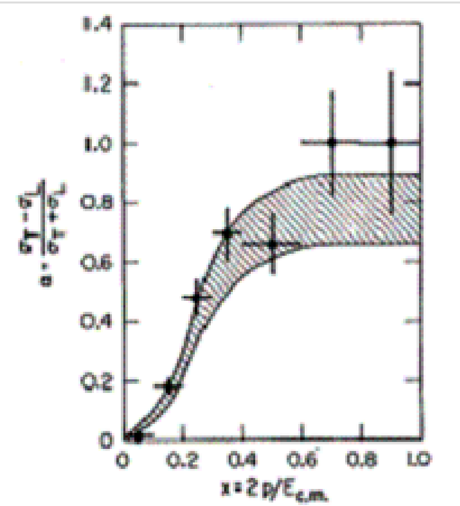


At high energy, events tend to be non-spherical

PQCD: Jets in $e^+ e^-$ Collisions - II

For 2 jets events

$$\left.\begin{array}{ll}\frac{d\sigma}{d\Omega} \propto 1+\cos^2\theta & \text{quark spin} = 1/2 \\ \frac{d\sigma}{d\Omega} \propto 1-\cos^2\theta & \text{quark spin} = 0\end{array}\right\} \equiv 1+\alpha\cos^2\theta$$



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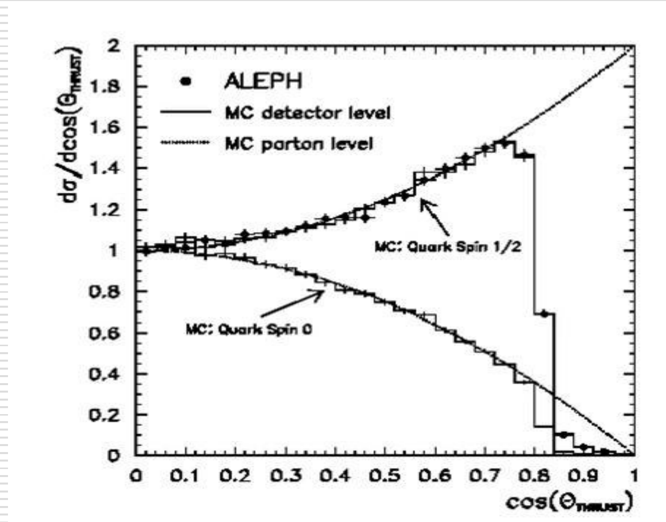
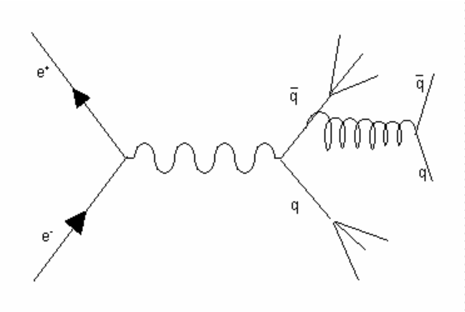


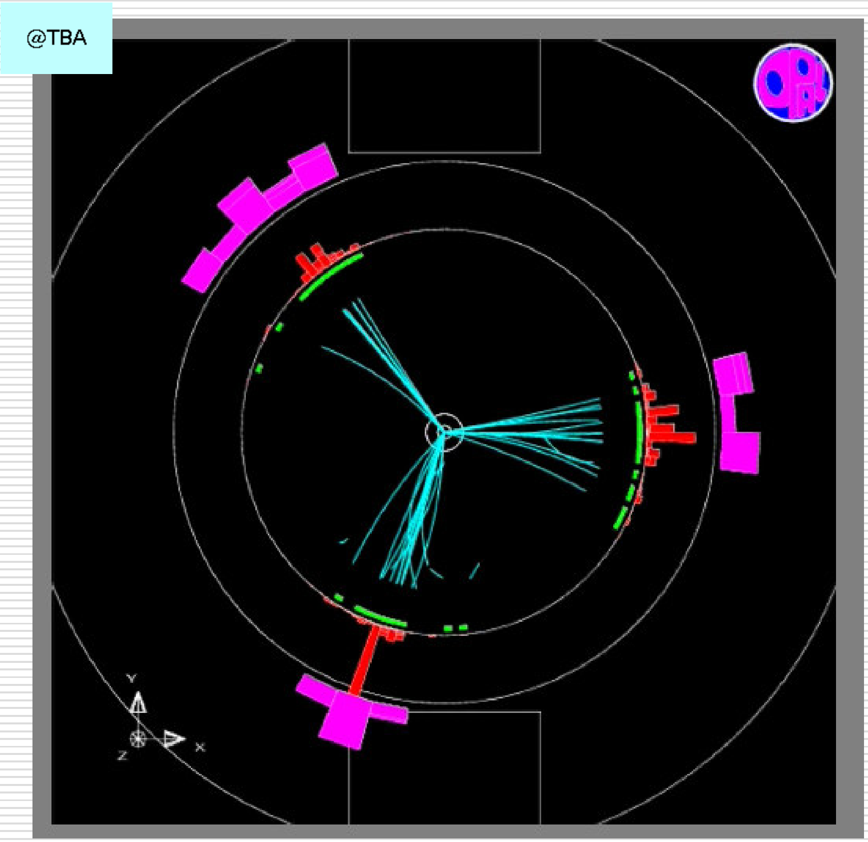
Mark I (SPEAR)
E = a few GeV

ALEPH (LEP)
E = 90 GeV

PQCD: Jets in $e^+ e^-$ Collisions - III

3 jets

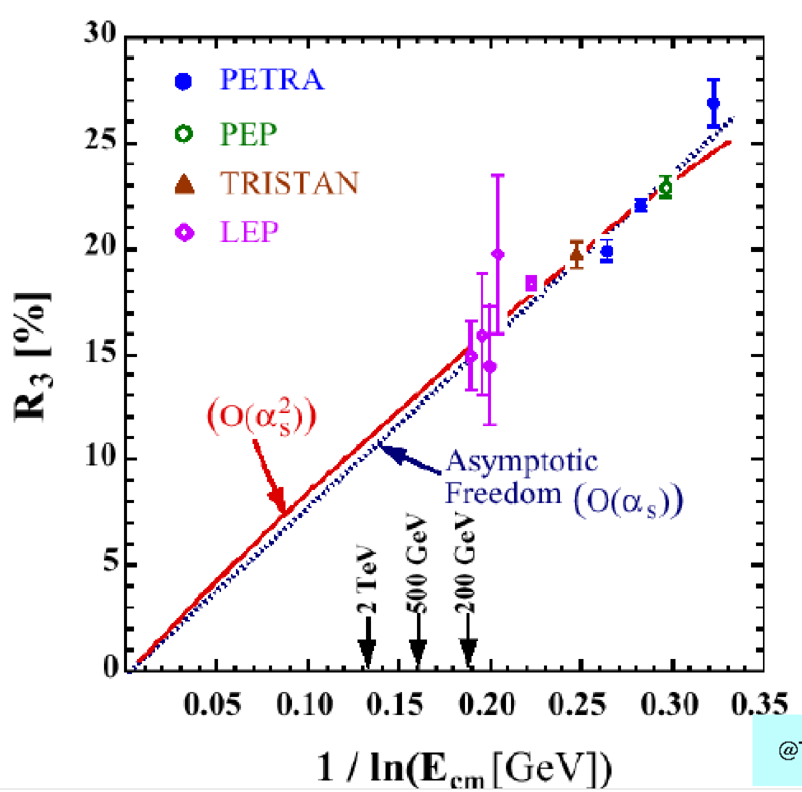




Left breathless by this exceptional 3-jet from OPAL? Relax, this is not exactly the rule…

PQCD: Jets in $e^+ e^-$ Collisions - IV

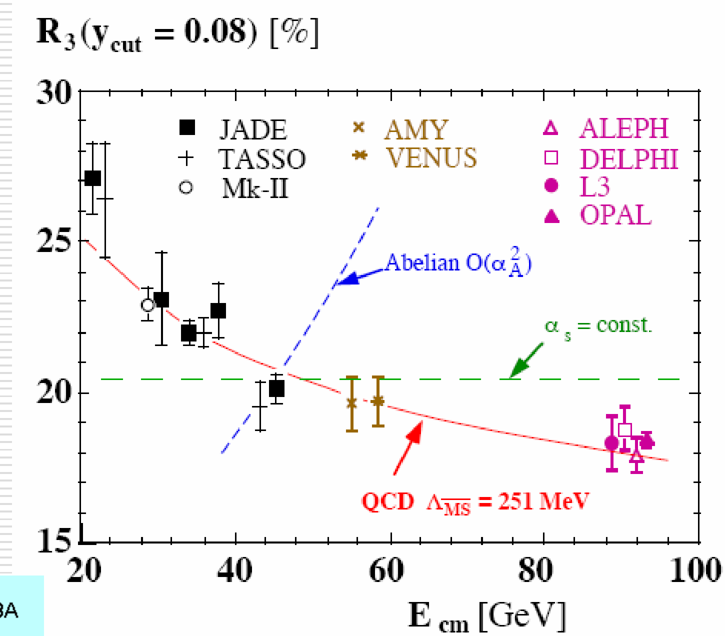
$$R_3 \equiv \frac{\sigma_{3-jet}}{\sigma_{tot}} \propto \alpha_s(E_{cm}) \propto \frac{1}{\ln E_{cm}}$$



Get a measurement of α_s :

$$\alpha_S(35GeV) = 0.146 \pm 0.03$$

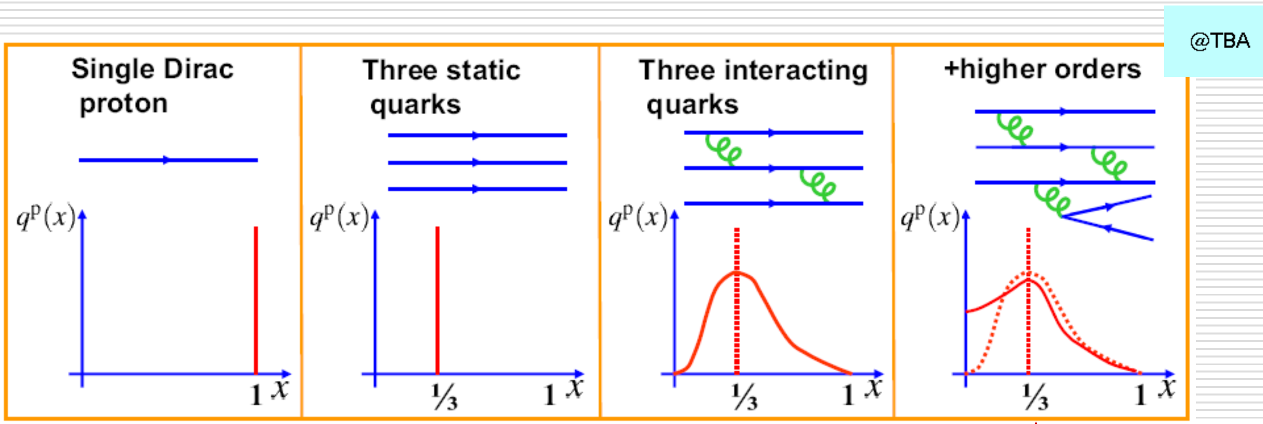
$$\alpha_S(M_{Z^0}) = 0.124 \pm 0.0043$$



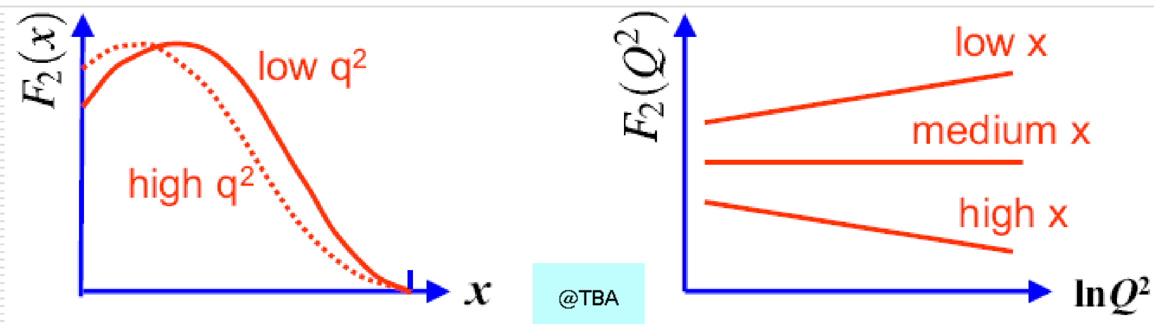
@TBA

PQCD: DIS Scaling Violations - I

Our picture of structure functions



Observe small deviations from scaling: $F_2(x) \rightarrow F_2(x, Q^2) \rightarrow$ QCD!

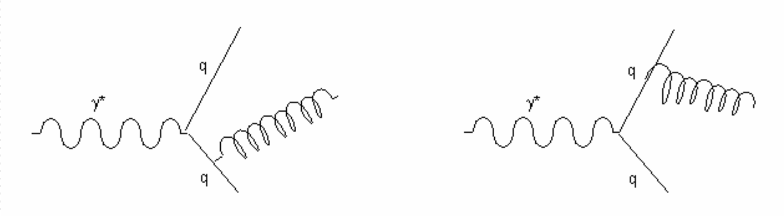


PQCD: DIS Scaling Violations - II

QCD on $F_2\left(x,Q^2\right)$:

$x-$dependence $\rightarrow$ Not predicted

Q^2-dependence $\rightarrow$ Predicted !



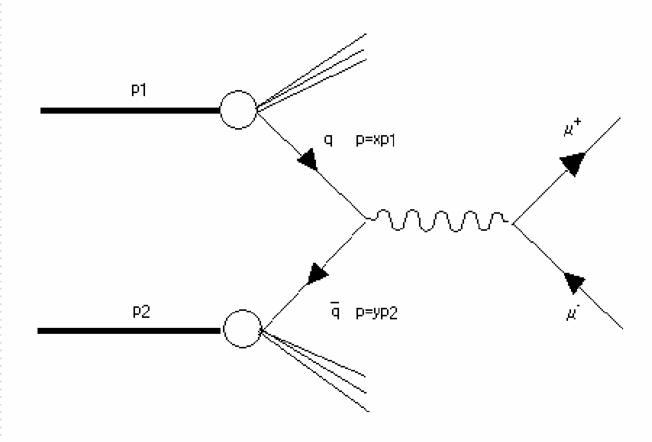
Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equation:
Successful prediction of Q^2 evolution of structure function

$$F_2\left(x,Q^2\right)=\sum_q xe^2\left[q\left(x\right)+\Delta q\left(x,Q^2\right)\right]$$

$$\Delta q\left(x,Q^2\right)=\frac{\alpha_s}{2\pi}\int_x^1 \frac{dx'}{x} q\left(x'\right) P_{qq}\left(\frac{x}{x'}\right)\ln\left(\frac{Q^2}{k^2}\right)+\ldots$$

Deep waters…

PQCD: Drell-Yan



$$\frac{d\sigma\left(q\bar{q}\to l^+l^-\right)}{dq^2}=\frac{4\pi\alpha^2}{3q^2}e_q^2\delta\left(q^2-s_{q\bar{q}}\right)$$

x_1, x_2 Bjorken x for $q, \bar{q}$

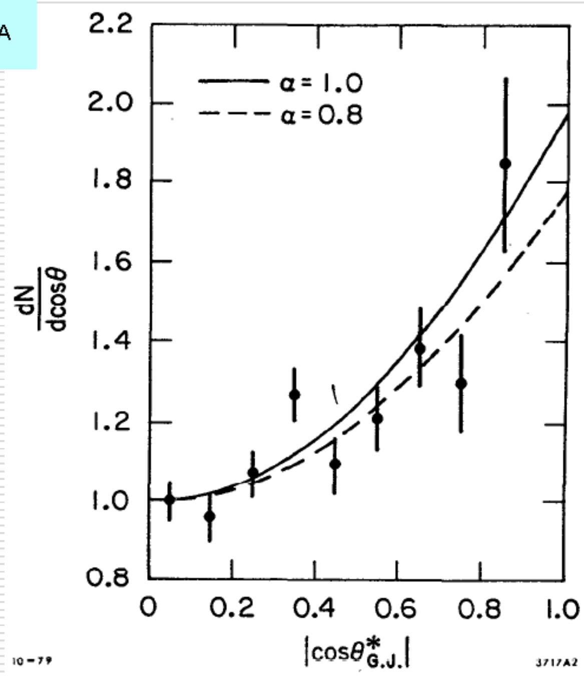
$$s_{q\bar{q}}=\left(p_q+p_{\bar{q}}\right)^2=\left(x_1p_1+x_2p_2\right)^2\simeq x_1x_2s$$

$$\sigma\left(q\bar{q}\to l^+l^-\right)=\frac{4\pi\alpha^2}{3q^2}e_q^2, \quad e_q = \text{ quark charge in } e \text{ units}$$

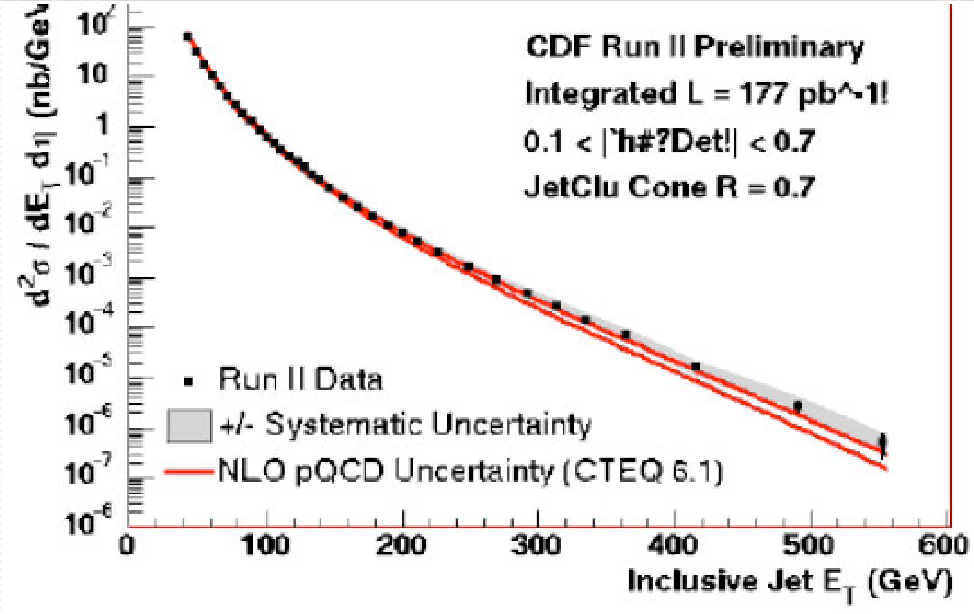
Angular distribution
in the pair rest frame

Expect $\propto 1+\cos^2\theta^*$ as usual

@TBA



PQCD: Jets in Hadron Collisions



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Cannot rely on triggering on a single, high $p_\perp$ particle
Devise a calorimeter trigger based on *total transverse energy* observed

$$\sum p_\perp^{(i)} = \sum p_i \sin\theta_i \sim \sum E_T^{(i)}$$

PQCD: 2-Body Partonic Processes

Consider all the 2-body processes in QCD:

$qq \to qq, q\bar{q} \to q\bar{q}$ Quarks only

$qg \to qg, \bar{q}g \to \bar{q}g, gg \to gg, q\bar{q} \to gg, gg \to q\bar{q}$ Quarks and/or Gluons

All will yield 2 jets to first approximation

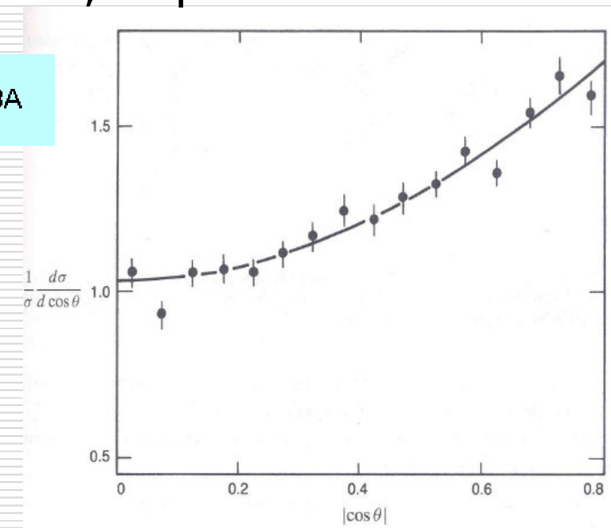
When quark only processes can be identified, expect:

$$\frac{d\sigma}{d\left(\cos\theta^*\right)} = \frac{\pi\alpha_s^2}{2s_{ij}}\left|M\right|^2$$

$$s_{ij} = \left(x_i p_i + x_j p_j\right)^2 \approx x_i x_j s$$

$$\to \left|M\right|^2 \propto 1 + \cos^2\theta^*$$ as usual

@TBA



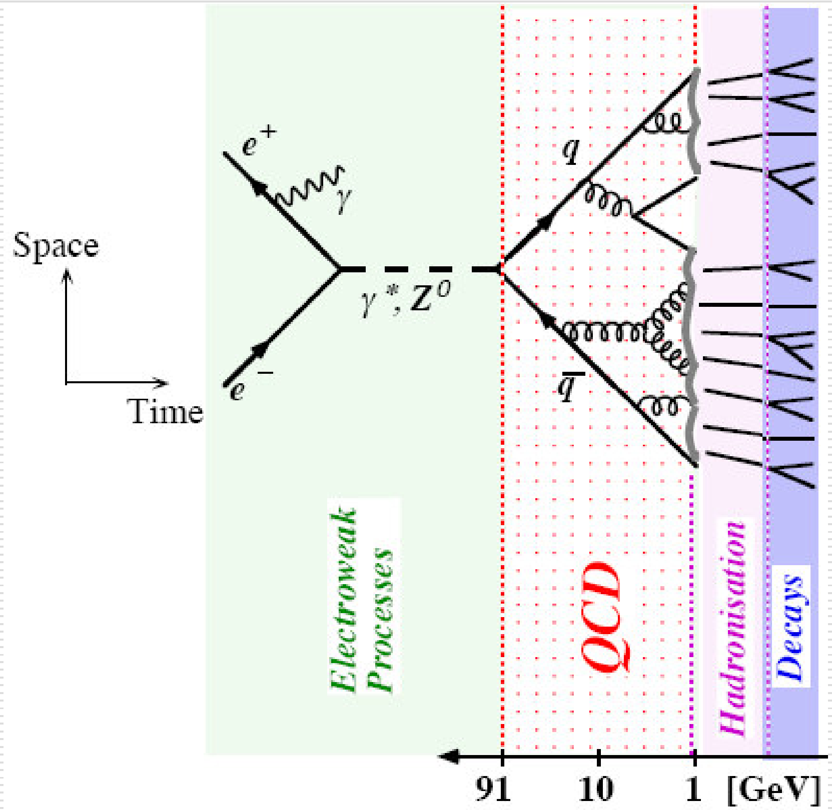
Jet Fragmentation

Complete jet evolution cannot be computed in a full QCD framework:

As shown in the picture, *increasing* time scales correspond to *decreasing* Q^2 scale, down to a region where perturbative expansion cannot be granted

Conversion of quarks and gluons into hadrons (*Fragmentation* or *Hadronization*) must be evaluated by ad-hoc prescriptions (models).

QCD-inspired, successful $q\overline{q}$ models often based on string-like behavior of pairs



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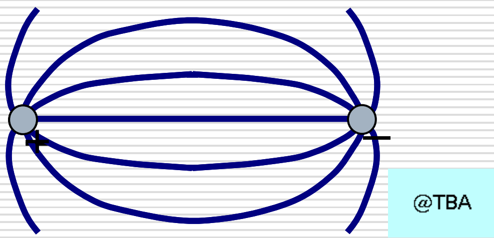
Stringy QCD

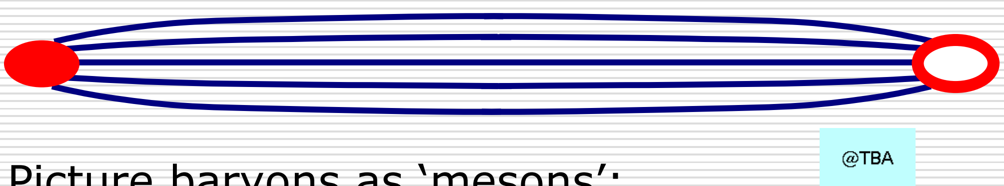
Typical model implemented in fragmentation Montecarlo programs

$q\bar{q}$ Interaction

QED-like at small distance



Gluon self-interaction yields *string* (flux tube) pattern at large distance



Picture baryons as ‘mesons’:

$$\boldsymbol{3}\otimes\boldsymbol{3}=\boldsymbol{3}^{*}\oplus\boldsymbol{6}$$

$$qqq=\underbrace{qq}_{\sim\bar{q}}+q$$

Valence and Sea

Take a Hydrogen atom:

= Chemistry!

Common wisdom: "A bound state of proton + electron"
But: Consider the effect of radiative corrections (e.g. loops)
Then we should be more precise:

Hydrogen = $(\text{Proton+Electron})_{\text{Valence}}$ + $(\text{Positrons+Electrons+Photons})_{\text{Sea}}$

Can we say valence and sea particles are fundamentally different? Well,…

In a bound state, both are off mass shell
Both are active in yielding measurable effects (Coulomb levels vs. Lamb shift,..)
Sea particles yield small corrections to levels determined by valence $e+p$

Take a hadron:

Hadron = $(\text{Quarks/Antiquarks})_{\text{Valence}}$ + $(\text{Quarks/Antiquarks+Gluons})_{\text{Sea}}$

Since $\alpha_s >> \alpha$, sea effects are much larger in QCD

The Quark Parton Model - I

Write down F_2 in terms of PDFs

$$F_2 = \left(\sum_i z_i^2 n_i\right) x\delta\left(x - \frac{m}{M}\right)$$

$$F_2(x) = x\left(\sum_i z_i^2 q_i(x)\right)$$

$$p = uud$$

$$F_2^p(x) = x\left[\left(\frac{2}{3}\right)^2 u_p(x) + \left(-\frac{1}{3}\right)^2 d_p(x)\right]$$

$$\rightarrow F_2^p(x) = x\left[\frac{4}{9}u_p(x) + \frac{1}{9}d_p(x)\right]$$

$$n = ddu$$

$$F_2^n(x) = x\left[\left(-\frac{1}{3}\right)^2 d_n(x) + \left(\frac{2}{3}\right)^2 u_n(x)\right]$$

From isospin symmetry:

$$F_2^n(x) = x\left[\left(-\frac{1}{3}\right)^2 u_p(x) + \left(\frac{2}{3}\right)^2 d_p(x)\right]$$

$$\rightarrow F_2^n(x) = x\left[\frac{1}{9}u_p(x) + \frac{4}{9}d_p(x)\right]$$

The Quark Parton Model - II

Consider the deuteron structure function:

$$F_2^d(x) = \frac{1}{2}\left(F_2^p + F_2^n\right) = \frac{5}{9}\frac{x}{2}\left[u_p(x) + d_p(x)\right]$$

$$\rightarrow F_2^n(x) = F_2^d(x) - F_2^p(x)$$

$$= \frac{5}{18}x\left[u_p(x) + d_p(x)\right] - \frac{1}{9}x\left[u_p(x) - 4d_p(x)\right]$$

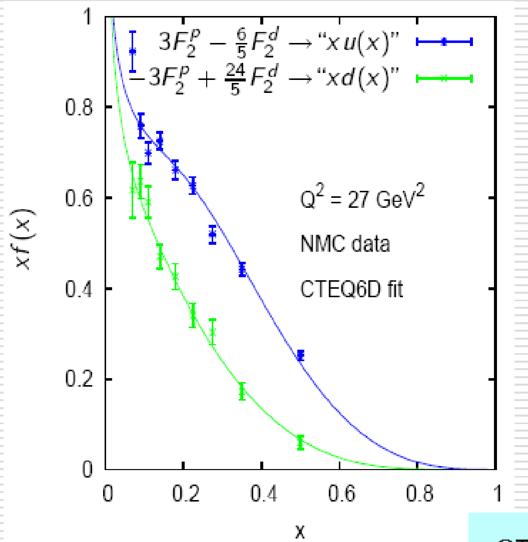
$$= \frac{3}{18}x\left[u_p(x) - d_p(x)\right]$$

Finally extract PDFs from measured F_2

$$xu_p(x) = xd_n(x) = 3F_2^p(x) - \frac{6}{5}F_2^d(x)$$

$$xu_n(x) = xd_p(x) = 3F_2^p(x) + \frac{24}{5}F_2^d(x)$$

The Parton Distribution Functions



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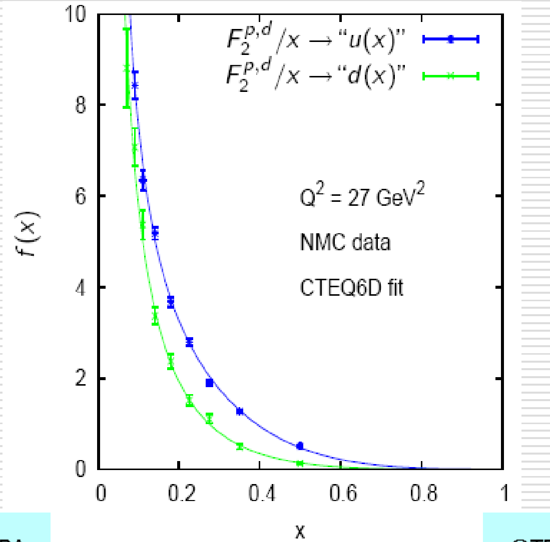

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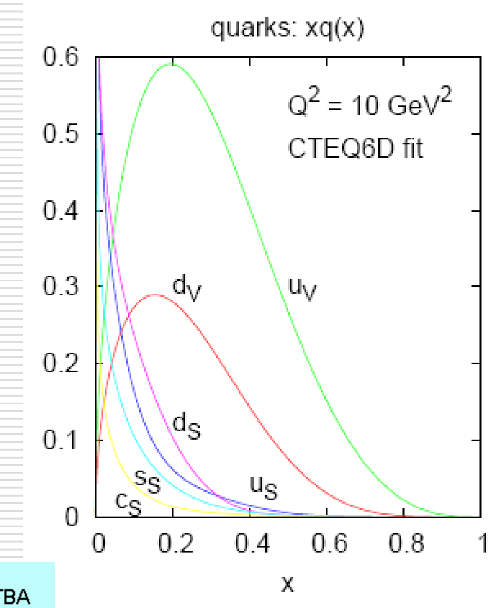


Among parton model predictions: *Sum Rules* (= Integral relations) for PDFs
Examples: Proton quark content is *uud*

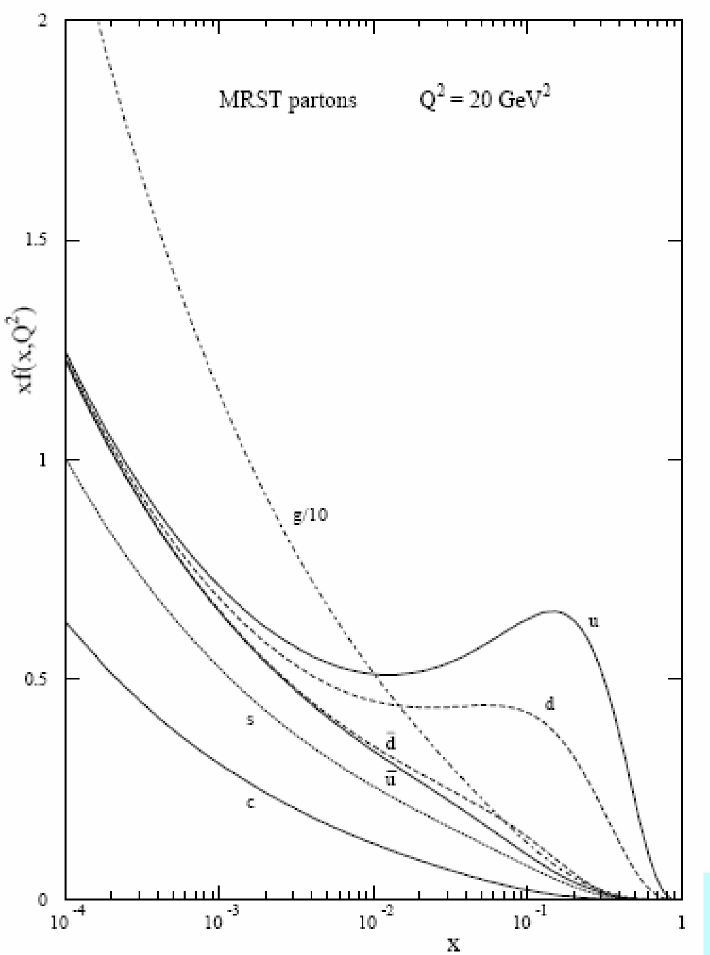
$$\int\left[u_p(x)-\bar{u}_p(x)\right]dx = 2$$

$$\int\left[d_p(x)-\bar{d}_p(x)\right]dx = 1$$

$$\int\left[s_p(x)-\bar{s}_p(x)\right]dx = 0$$

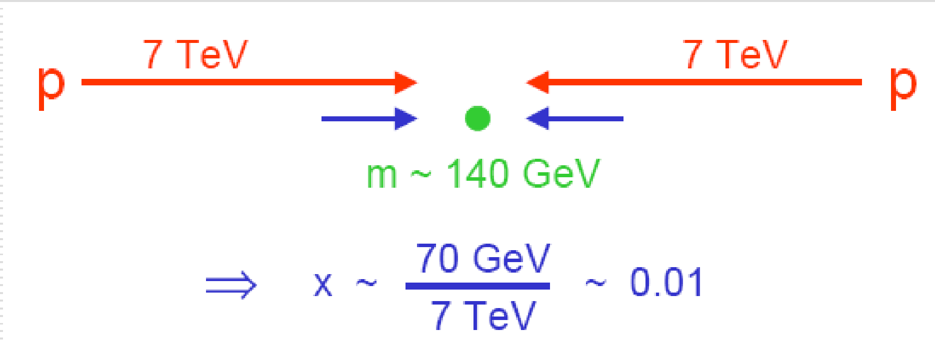
What's the origin of antiquarks in the nucleon?
QCD! See later..

The PDFs at Low *x*



Data-based calculation
Low-x region very important at LHC

Example:
Production of a Higgs with m_H = *140 GeV*



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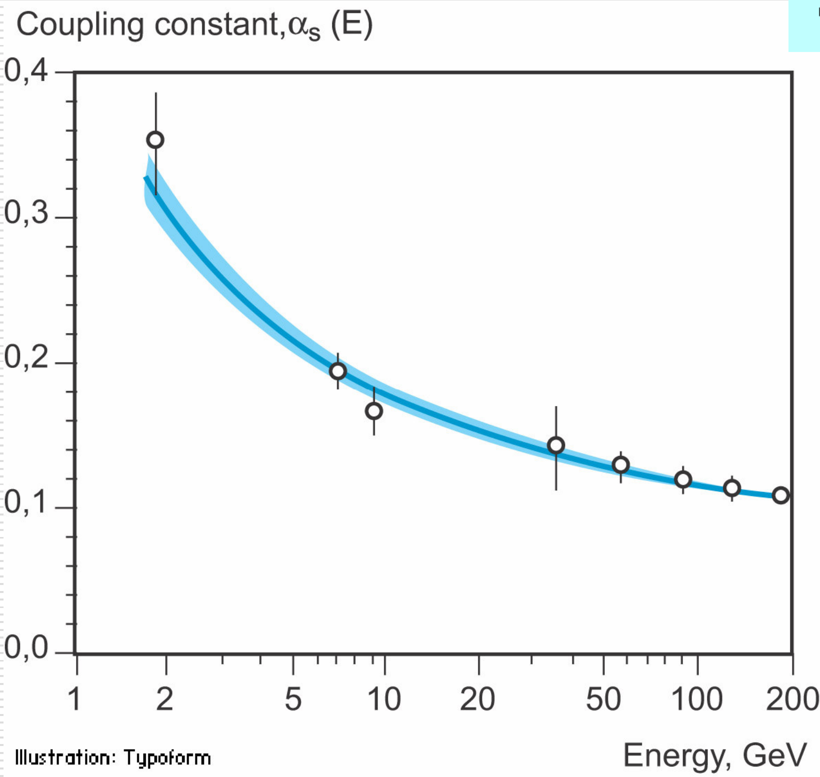
Running α_s



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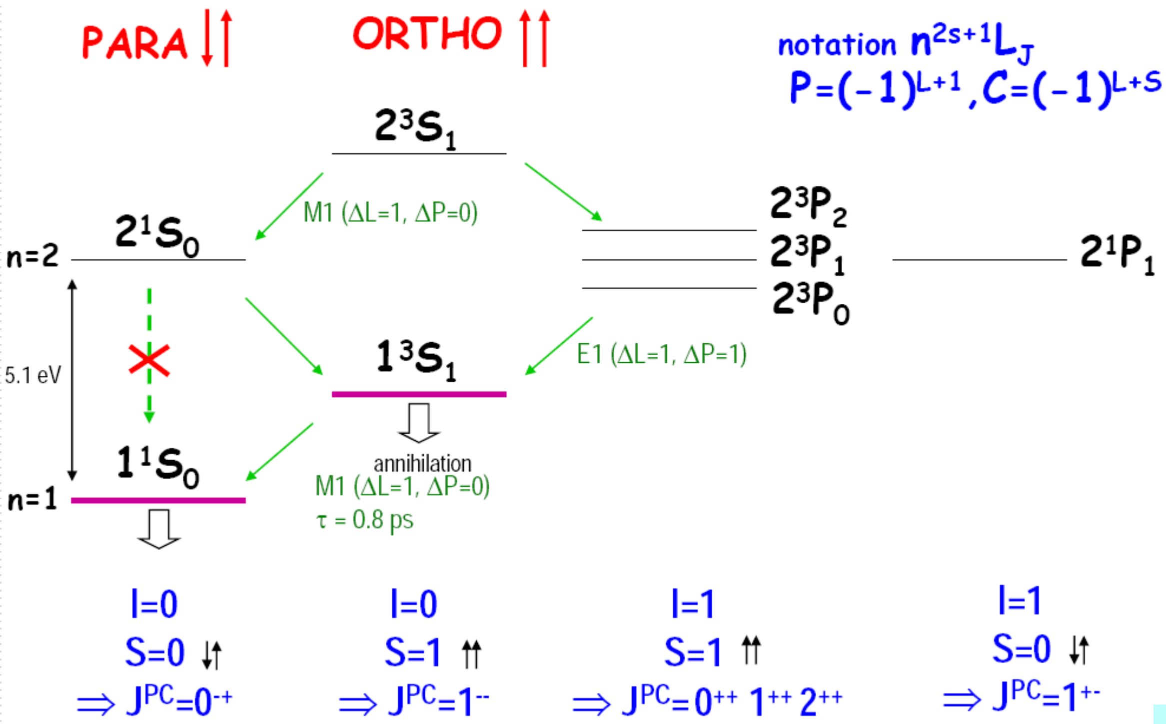
Sources:

DIS
Jets
Quarkonium

Positronium



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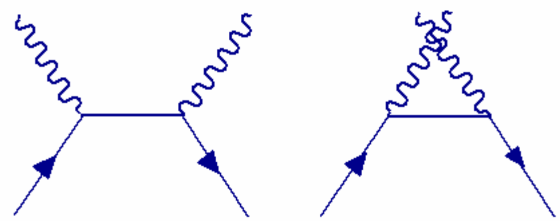
e^+- e^- : 2 Photons Annihilation - I

Transition amplitude in the small speed limit ($\beta \to 0$):



Permutations of 2 photons
$\to$2 diagrams altogether

$$T_{fi} = T_1 + T_2$$

$$T_1 = \frac{e^2}{(p_1 - p_3)^2 - m^2} \bar{v}(2) \not{\varepsilon}_4 \left(\not{p}_1 - \not{p}_3 + m \right) \not{\varepsilon}_3 u(1), \quad T_2 = \frac{e^2}{(p_1 - p_4)^2 - m^2} \bar{v}(2) \not{\varepsilon}_3 \left(\not{p}_1 - \not{p}_4 + m \right) \not{\varepsilon}_4 u(1)$$

$p_1 = m(1,0,0,0),\ p_2 = m(1,0,0,0),\ p_3 = m(1,0,0,1),\ p_4 = m(1,0,0,-1)$ γ rays emitted along z

$$(p_1 - p_3)^2 - m^2 = (p_1 - p_4)^2 - m^2 = -2m^2$$

$\to T = -4e^2$ Averaged over initial, summed over final spin projections

e^+- e^- : 2 Photons Annihilation - II

Cross-section from amplitude: 2-body reaction

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi s}\frac{|\mathbf{p}_f|}{|\mathbf{p}_i|}|T|^2$$

$$|\mathbf{p}_f| = m, \quad |\mathbf{p}_i| \simeq m\beta, \quad s = (2m)^2 = 4m^2$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi 4m^2}\frac{m}{m\beta}16\alpha^2 = \frac{\alpha^2}{16\pi m^2\beta} \rightarrow \sigma = 4\pi\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4m^2\beta}$$

Selection rule for bound state annihilation

$$U_C|2\gamma\rangle = (-1)^2|2\gamma\rangle \rightarrow \eta_c(2\gamma) = +1$$

$$\rightarrow (-1)^{L+S} = +1$$

$$\Rightarrow L = 0 \rightarrow S = 0$$

S-wave: Singlet only

Positronium: 2 γ Annihilation - I

Proceed as for Van-Royen - Weisskopf

$$A_{pos} = \sum_p \underbrace{\langle \gamma\gamma | T | p \rangle}_{A(\mathbf{p})} \underbrace{\langle p | \pi^0 \rangle}_{\psi(\mathbf{p})}$$

$A(\mathbf{p})$ plane wave amplitude, $\psi(\mathbf{p})$ momentum space wave function

$$\rightarrow A_{pos} = \int d^3\mathbf{p} A(\mathbf{p}) \psi(\mathbf{p}) = \frac{1}{(2\pi)^{3/2}} \int d^3\mathbf{p} A(\mathbf{p}) \int \psi(\mathbf{r}) e^{i\mathbf{p}\cdot\mathbf{r}} d^3\mathbf{r}$$

$$\rightarrow A_{pos} = \frac{1}{(2\pi)^{3/2}} \int d^3\mathbf{r}\, \psi(\mathbf{r}) \int A(\mathbf{p}) e^{i\mathbf{p}\cdot\mathbf{r}} d^3\mathbf{r}$$

Take $A(\mathbf{p}) \approx A = const$ (can be shown to be true)

$$\rightarrow A_{pos} \approx \frac{A}{(2\pi)^{3/2}} \int d^3\mathbf{r}\, \psi(\mathbf{r}) \underbrace{\int e^{i\mathbf{p}\cdot\mathbf{r}} d^3\mathbf{p}}_{(2\pi)^3 \delta^3(\mathbf{r})} = (2\pi)^{3/2} A\psi(0)$$

$$\rightarrow \Gamma_{pos} = |A_{pos}|^2 \approx (2\pi)^3 |A|^2 |\psi(0)|^2$$

Positronium: 2 γ Annihilation - II

$$\sigma = \frac{\alpha^2}{4m^2\beta} = |A|^2 \frac{(2\pi)^3}{\beta} \to |A|^2 = \frac{\alpha^2}{4m^2\beta}\frac{\beta}{(2\pi)^3} = \frac{\alpha^2}{(2\pi)^3 4m^2}$$

$$\to \Gamma_{pos} \approx (2\pi)^3 |A|^2 |\psi(0)|^2 = \frac{\alpha^2}{4m^2}|\psi(0)|^2$$

Since only singlet initial state contributes:

$$\Gamma_{pos} \to \frac{1}{1/4}\cdot\frac{\alpha^2}{4m^2}|\psi(0)|^2 = \frac{\alpha^2}{m^2}|\psi(0)|^2$$

Positronium: 2 γ Annihilation - III

Taking scaled Hydrogen wave function

$$\psi(r) = \frac{2}{(a_0)^{3/2}} e^{-\frac{2r}{a_0}}, a_0 = \frac{1}{\alpha m}, m$$ reduced mass

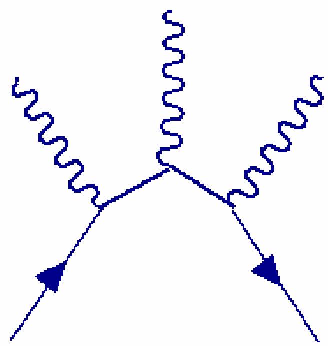
$$Hyd: \quad m \simeq m_e \rightarrow a_0 \approx \frac{1}{\alpha m_e}$$

$$Pos: \ m = \frac{m_e}{2} \rightarrow a_0 = \frac{2}{\alpha m_e}$$

$$\rightarrow \psi_{pos}(0) = \frac{2}{(a_0)^{3/2}} = \frac{2(\alpha m_e)^{3/2}}{(2)^{3/2}} \rightarrow |\psi_{pos}(0)|^2 = \frac{4\alpha^3 m_e^3}{8} = \frac{\alpha^3 m_e^3}{2}$$

$$\rightarrow \Gamma_{pos} \approx (2\pi)^3 |A|^2 |\psi(0)|^2 = \frac{\alpha^2}{m_e^2} \frac{\alpha^3 m_e^3}{2} = \frac{\alpha^5 m_e}{2}$$

Positronium: 3 γ Annihilation



Selection rule:

$$U_C\left|3\gamma\right\rangle=(-1)^3=-1$$

$$\rightarrow(-1)^{L+S}=-1$$

$$\rightarrow\begin{cases}L=0\\S=1\end{cases}$$ Triplet only

After *some* algebra…

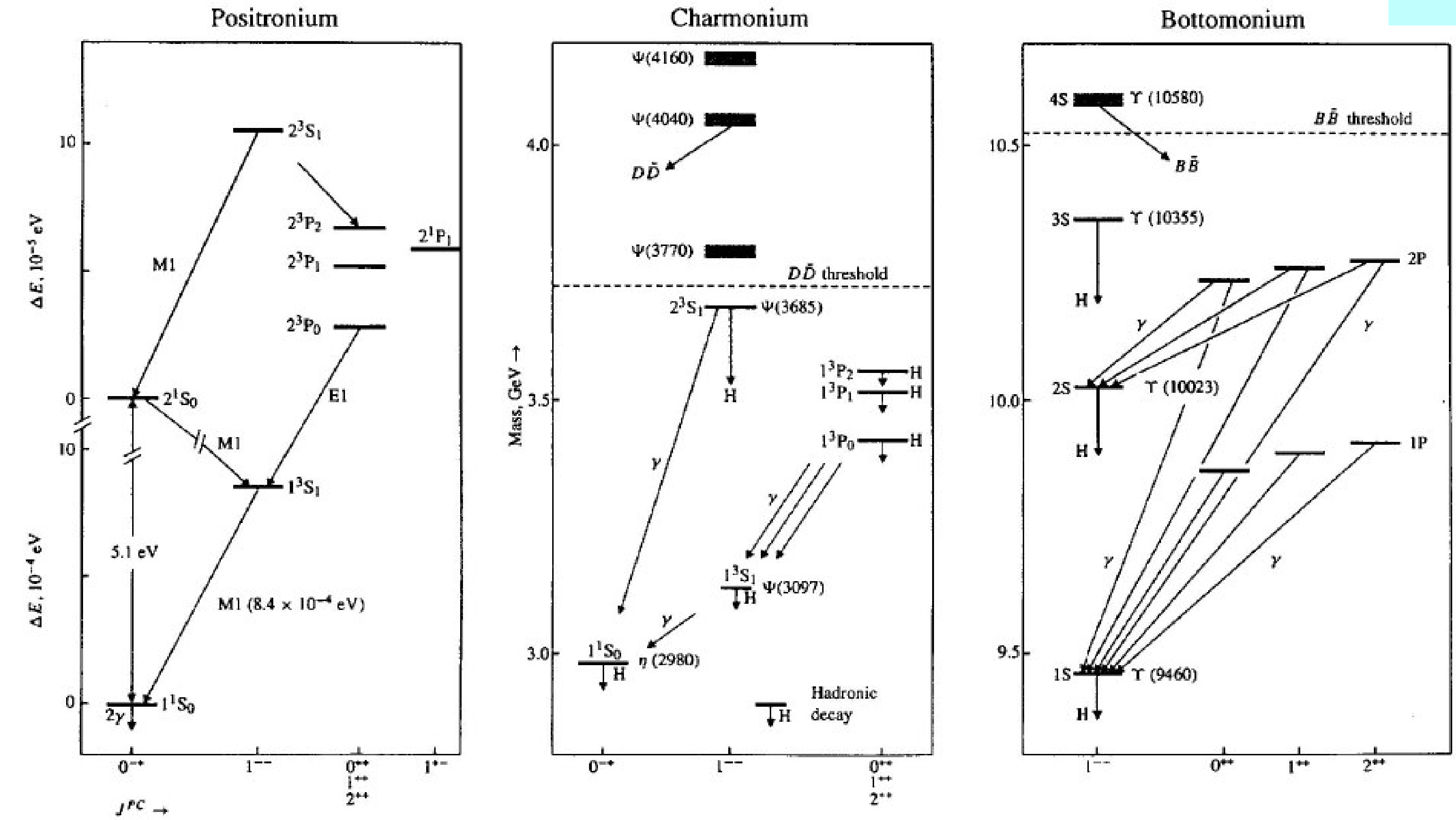
$$\Gamma_{pos}^{3\gamma}=\frac{4}{9\pi}\left(\pi^2-9\right)\frac{\alpha^3}{m_e^2}\left|\psi(0)\right|^2=\frac{2}{9\pi}\left(\pi^2-9\right)m_e\alpha^6$$

$$\rightarrow\frac{\Gamma_{pos}^{3\gamma}}{\Gamma_{pos}^{2\gamma}}=\frac{\frac{2}{9\pi}\left(\pi^2-9\right)m_e\alpha^6}{\frac{\alpha^5 m_e}{2}}=\frac{4\left(\pi^2-9\right)}{9\pi}\alpha\sim10^{-3}$$

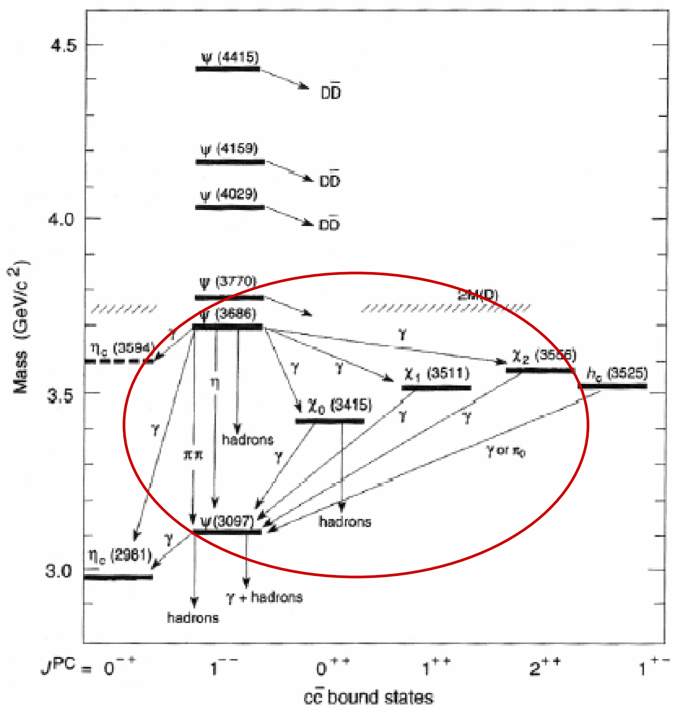
Quarkonium

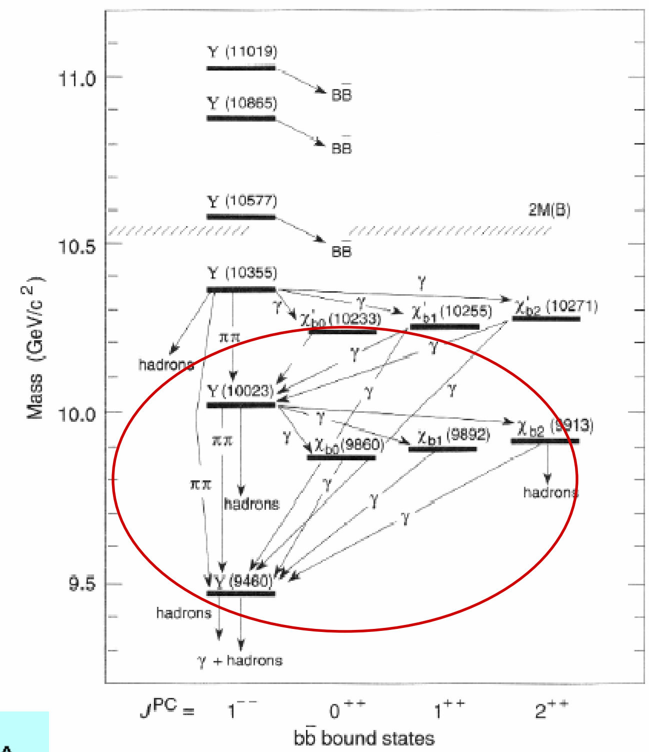
Family portrait of *–onia*:

@TBA



Real Life Quarkonia





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Striking similarity, same energy scale *above ground state*

Quarkonium: Schrodinger Equation

Reminder: Bohr radius

$$R = \frac{n^2}{\alpha m} \rightarrow R_0 = \frac{1}{\alpha m}$$

Consequence of Coulomb potential, static limit of 1 photon diagram

If 1 gluon exchange approximation can be granted, expect for quarkonium

$$R_{q\bar{q}} \sim \frac{1}{\alpha_s m}$$

Observe: m large $\rightarrow$ R small $\rightarrow$ α_s small

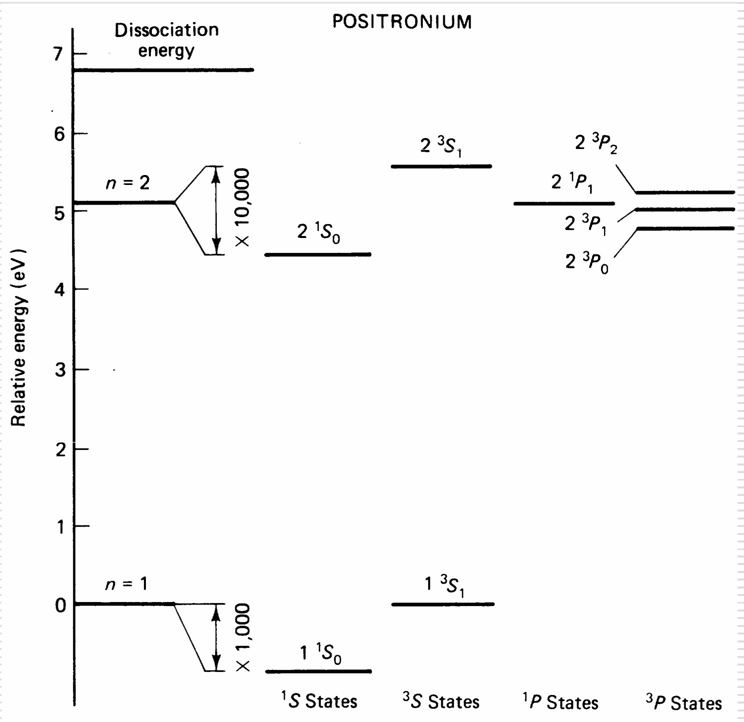
Must keep in mind the $q\bar{q}$ potential is confining
Add a phenomenological, confining term like this:

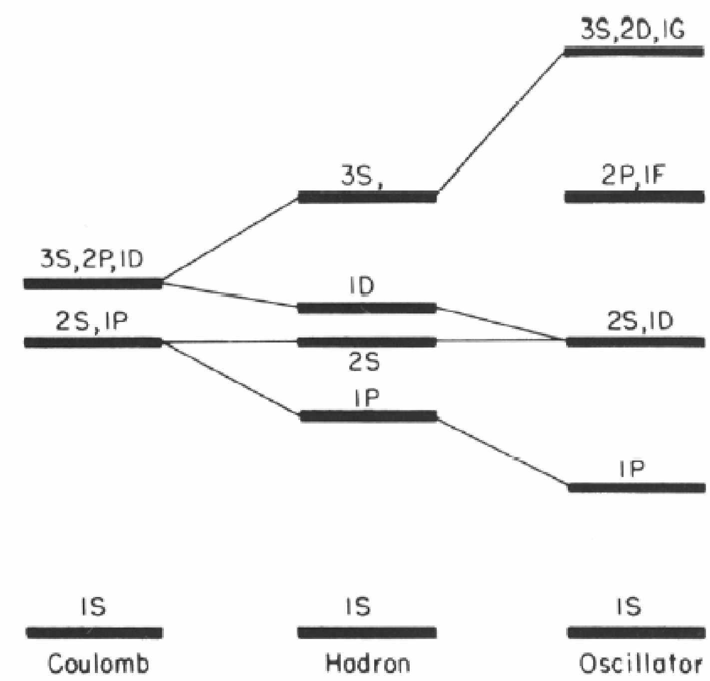
$$V(r) = -\frac{4}{3}\frac{\alpha_s}{r} + ar$$

Solve Schrodinger equation with these terms

Add more terms to take into account relativistic & color-hyperfine effects

The $q\bar{q}$ Effective Potential: Levels





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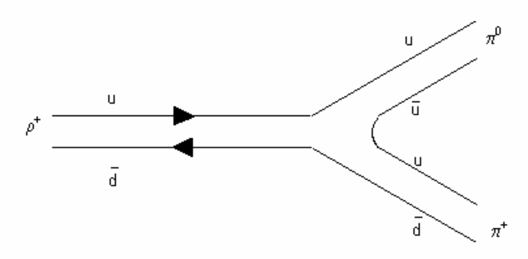
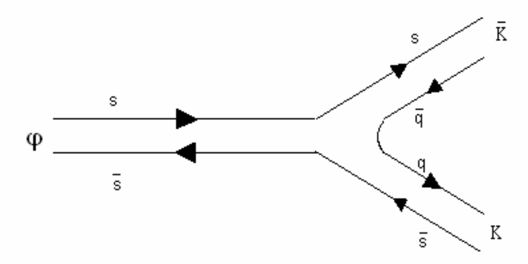
Quark Flow Diagrams: The OZI Rule

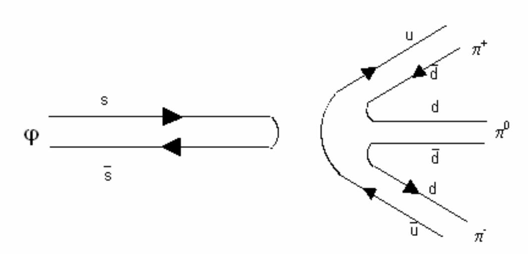
Okubo-Zweig-Iizuka Rule: *Disconnected diagrams are suppressed*



This diagram is connected

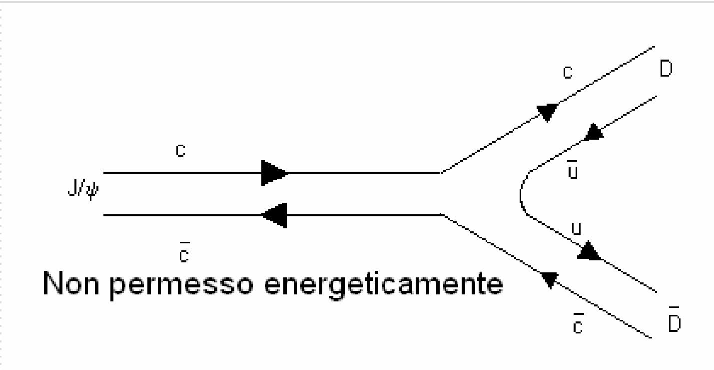


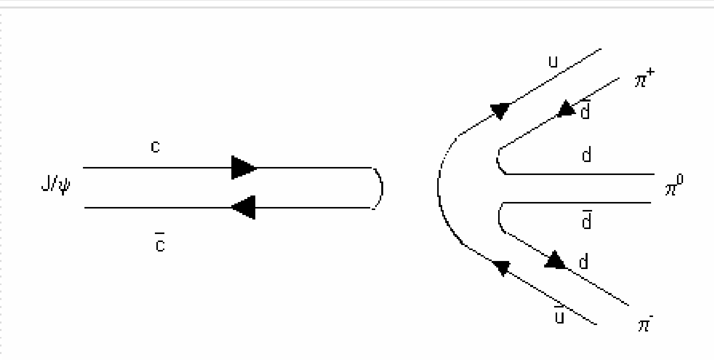
This diagram is connected: *BR 83 %*
(with smallish phase space)



This diagram is disconnected: *BR 15 %*
(with much larger phase space)

The OZI Rule and Charmonium





Compare mass and width

$$m_{J/\psi} = 3097 \ \text{MeV}, \ \Gamma_{J/\psi} = 80 \ \text{keV} \qquad J^{PC} = 1^{--}$$

$$m_{\psi'} = 3686 \ \text{MeV}, \ \Gamma_{\psi'} = 250 \ \text{keV} \qquad J^{PC} = 1^{--}$$

Explaining the small width:

$$m_{D^0} = 1865 \ MeV \to 2 \times m_{D^0} = 3730 \ MeV > m_{J/\psi}, m_{\psi'}$$

Therefore J/ψ, ψ' decay to open charm is energetically forbidden

$\to$ Decay diagrams are disconnected

$\to$ OZI rule: Decay is suppressed

$\to$ *States are very narrow*

The Origin of the OZI Rule

As a general rule

$\rightarrow A \propto \alpha_s^n \quad n =$ number of gluons

Connected diagrams: Small number of soft gluons $\rightarrow A$ *= large*
Disconnected diagrams: Large number of hard gluons $\rightarrow A$ *= small*

Indeed:

1) Single gluon annihilation is forbidden for mesons by color conservation (meson = ***1***, gluon = ***8***)

2) Annihilation of massive quarks yields hard gluons $\rightarrow \alpha_s$ is small

3) Connected diagrams involve softer gluons $\rightarrow \alpha_s$ is large

Quarkonium: 2,3 Gluons

Consider quarkonium annihilation into gluons:

$q\bar{q} \to g$ Excluded: $(q\bar{q})_1 \not\leftrightarrow (1g)_8$

$q\bar{q} \to gg$ Allowed

$q\bar{q} \to ggg$ Allowed

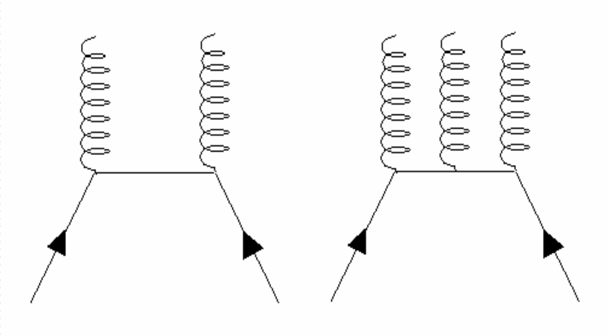
Decompose the direct product of 2 octets:

$$\boldsymbol{8 \otimes 8 = 1 \oplus 8 \oplus 8 \oplus 10 \oplus 10^* \oplus 27}$$

Charge Parity:

$J^{PC} = 0^{-+} \to C = +1 \to 2g$ OK

$J^{PC} = 1^{--} \to C = -1 \to 3g$ OK



Perturbative regime: $A(2g) > A(3g)$
$\to$Pseudoscalars wider than vectors

Quarkonium Annihilations

By comparison with positronium:

$$\left(e^+e^-\right)_{positronium} \to \gamma\gamma$$

$$\Gamma\left[\left(e^+e^-\right) \to \gamma\gamma\right] = \frac{\alpha^2}{m^2}\left|\psi(0)\right|^2$$

$$\left(c\bar{c}\right)_{charmonium} \to \gamma\gamma$$

$$\begin{cases} e \to \frac{2}{3}e \to \alpha \to \frac{4}{9}\alpha & \text{Quark charge} \\ \times 9 & \text{Sum amplitude over colors} \end{cases}$$

$$\Gamma\left[\left(c\bar{c}\right) \to \gamma\gamma\right] = \frac{48\alpha^2}{27m_c^2}\left|\psi_{c\bar{c}}(0)\right|^2$$

$$\left(c\bar{c}\right)_{charmonium} \to gg$$

Color factor $= \frac{9}{8}$ From SU(3) algebra: 2 g in a color singlet state

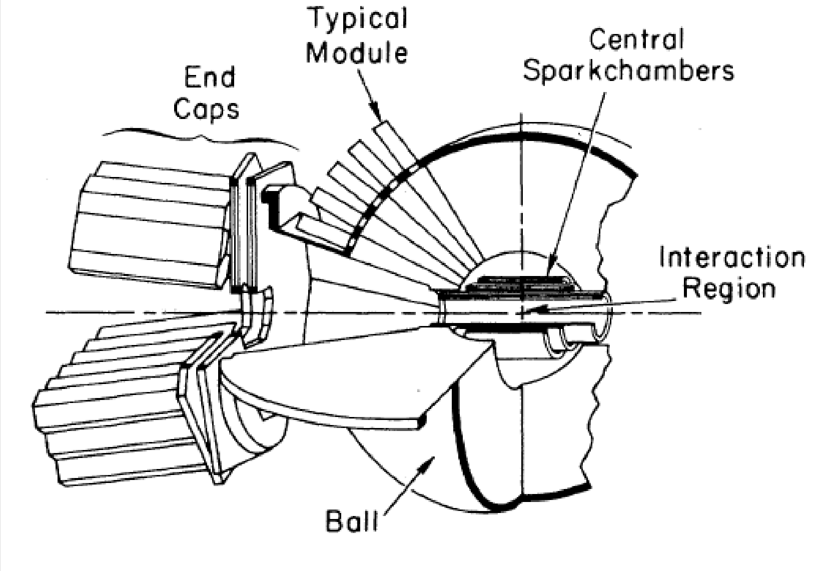
$$\Gamma\left[\left(c\bar{c}\right) \to gg\right] = \frac{2\alpha_s^2}{3m_c^2}\left|\psi_{c\bar{c}}(0)\right|^2$$

But:

Positronium rate was obtained by taking the low speed limit of scattering amplitude to 1-photon approx

Is it granted for $c\bar{c}$?

Crystal Ball - I



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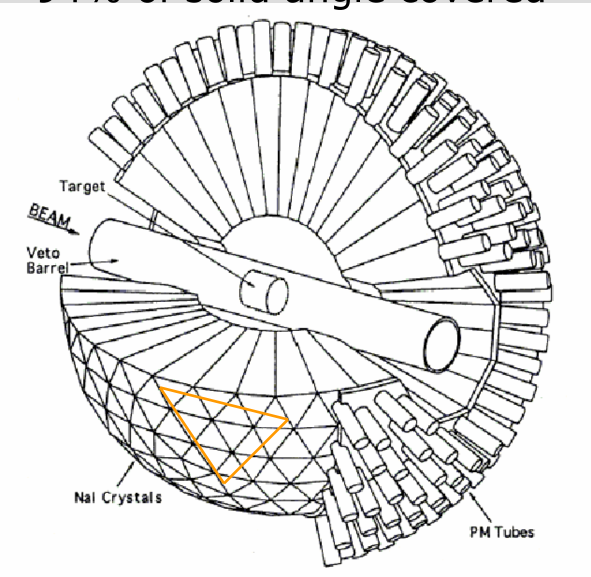
94% of solid angle covered



Sodium Iodide

$NaI(Tl)$: Inorganic scintillating crystal; Tl is an activator

Merits:

Can grow large crystals

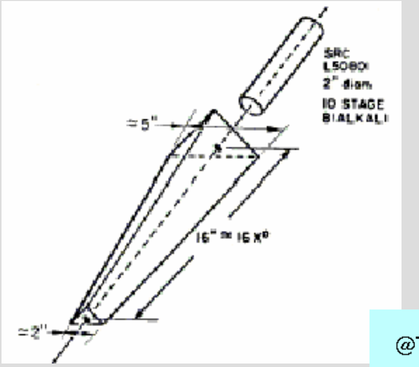
Lots of light

Crystal Ball - II

672 optically isolated NaI(Tl) crystals, 15.7 radiation lengths thick
Inner radius 25.3 cm; Outer radius 66.0 cm

CB geometry: Based on icosahedron.
Each of the 20 triangular faces (major triangles) is divided into four minor triangles, each consisting of nine separated crystals.

Each crystal: Truncated triangular pyramid, 40.6 cm high, pointing towards the center of the Ball.
Side on the inner end: 5.1 cm ; Side on the outer end: 12.7 cm



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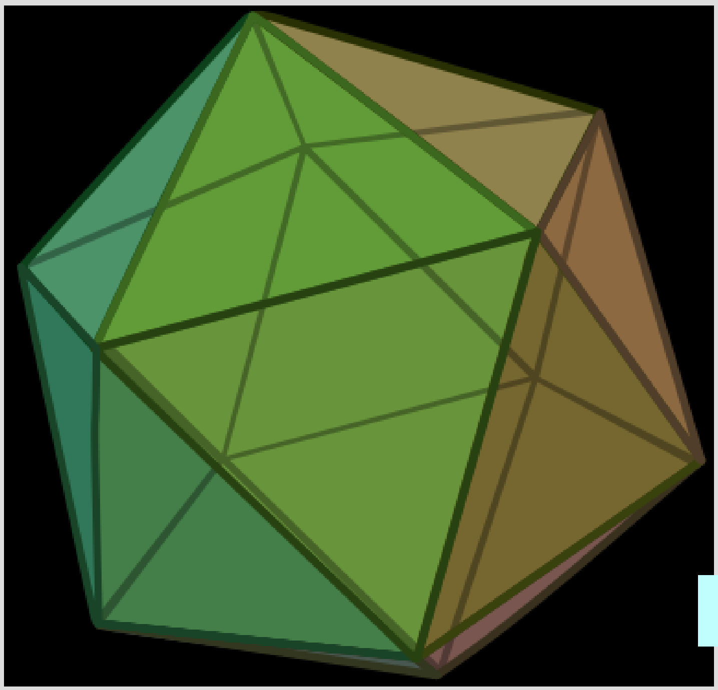
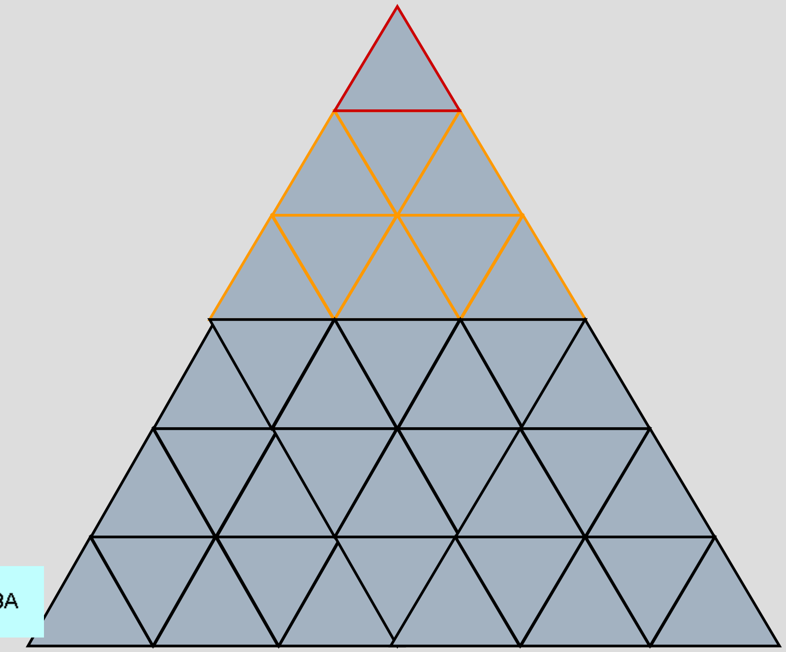
Crystal & Photomutiplier

Crystal Ball - III

Icosahedron magic: Platonic solid, 20 equilateral triangle faces





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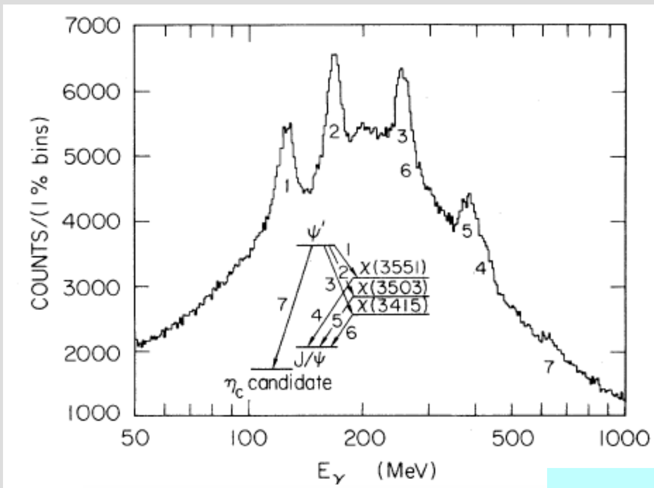
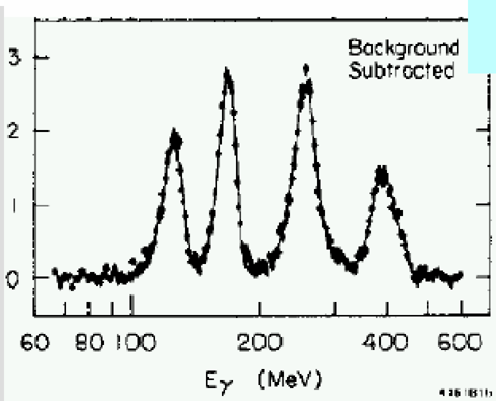
Triangle count:

Large triangle 20 Small triangle 80 Crystal < 720 (edges)

Crystal Ball - IV

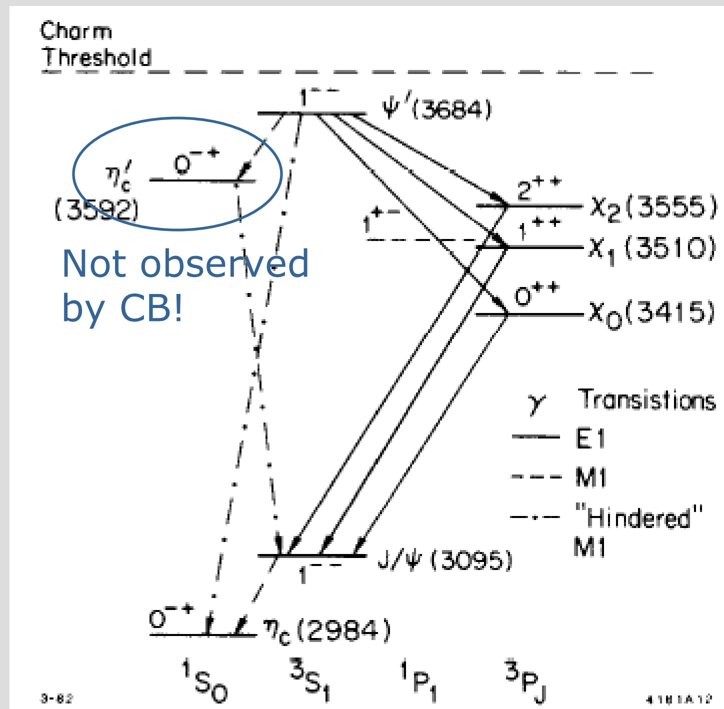
Inclusive photon spectrum





@TBA

Most important results, among many:
Tune beam energy as to form ψ′(3686)
Observe decays into photon + X



Crystal Ball - V

After moving the CB detector to DORIS (DESY, Hamburg):
Bottomonium!   Observation of the P-wave triplets

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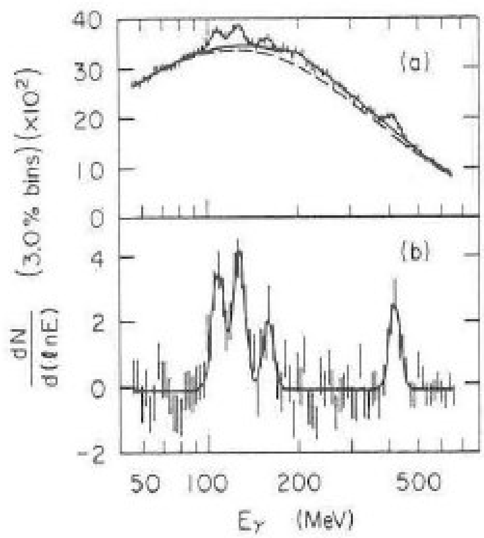


Figure 11.2: The photon spectrum from Υ' decays obtained by the Crystal Ball Collaboration at DORIS II. A triplet of lines corresponding to $\Upsilon' \to \gamma\chi_b(^3P_{2,1,0})$ is seen between 100 and 200 MeV. The decays $\chi_b \to \gamma\Upsilon$ produce the unresolved signal between 400 and 500 MeV [R. Nernst *et al.*, *Phys. Rev. Lett.* 54, 2195 (1985)].

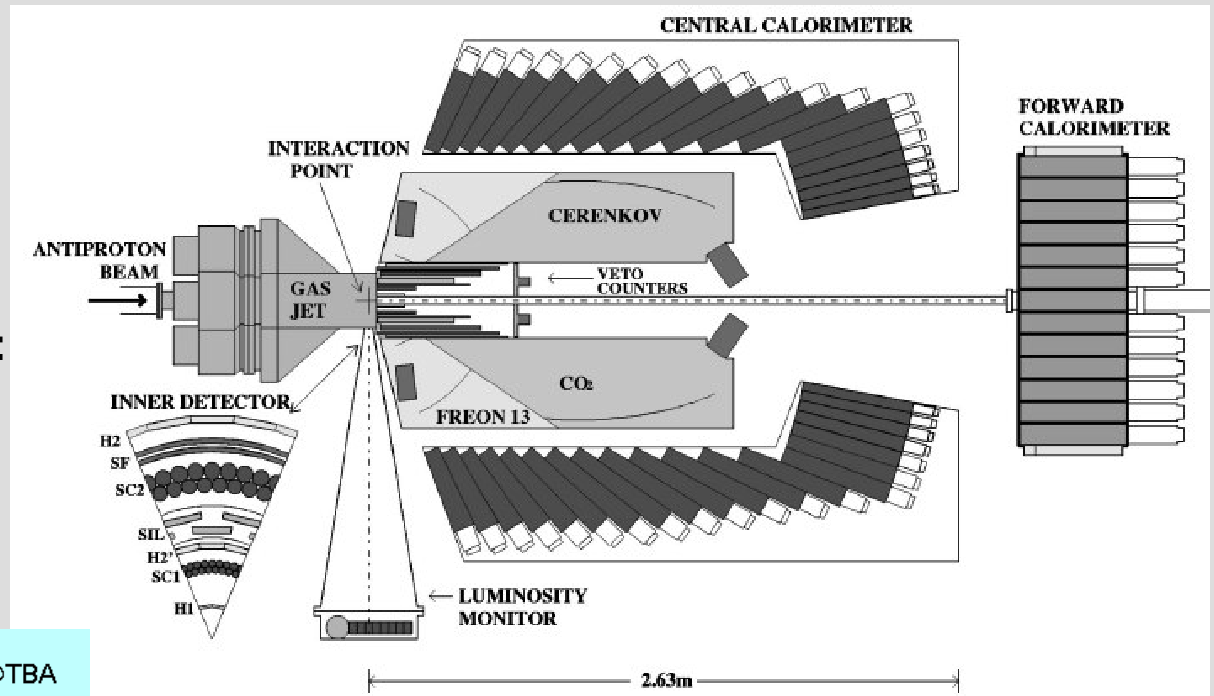
Another Side of Charmonium - I

$$p+\bar{p} \to \underbrace{c\bar{c}}_{Charmonium} \to Electromagnetic\ \ decay$$

Circulating $\bar{p}$ Beam:
Excellent E *resolution*

Gas jet target:
Reduced E *loss*

Non magnetic detector:
EM Calorimeter,
Tracking,
Cerenkov



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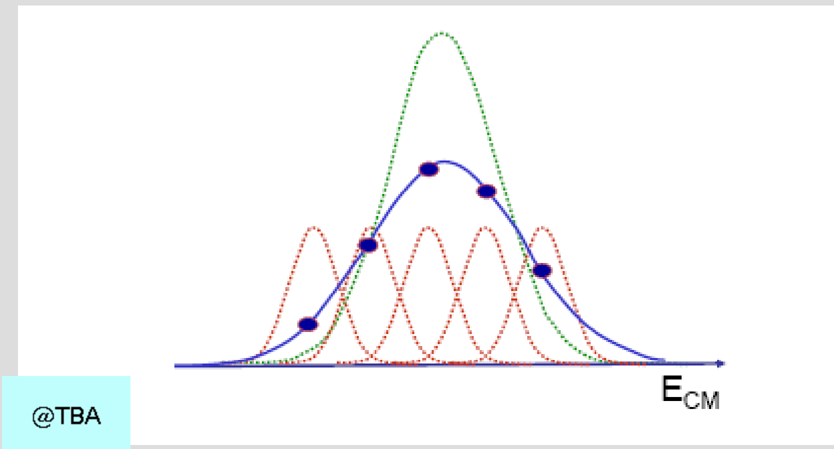
Another Side of Charmonium - II

Concept of resonance scan: A fixed target, formation experiment

Move the beam energy in small steps across the energy range of any given resonant state

Measure the decay rate of the state at each step



Rate

Resonance profile

Typical width $\Gamma < 1$ *MeV for* $c\bar{c}$

Beam profile

Typical resolution $\sigma(E_{CM}) \sim 0.2\ MeV$

Get resonance mass, width, coupling by deconvolving the beam profile from the observed rate

Another Side of Charmonium - III

Electrons: *Cerenkov* + *Calorimeter* + *Tracking*
$\rightarrow$ Very low background to $e^+ e^-$

$$\psi' \rightarrow J/\psi + X$$
$$\searrow e^+ e$$

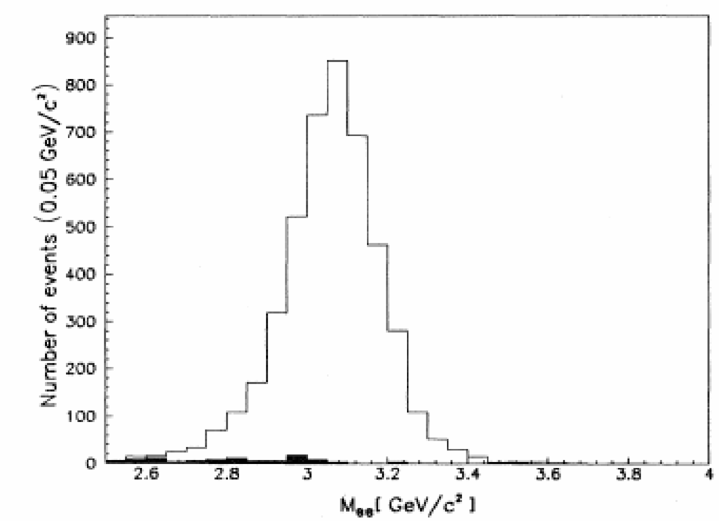


FIG. 5. Invariant mass distribution of electron pairs for the 1991 J/ψ scan (open area) and for the off-resonance background normalized to the same luminosity (shaded area). For this figure only the level of background has been multiplied by a factor of 10 to make it discernible.

$M_{e^+e^-}$ from scan across J/ψ

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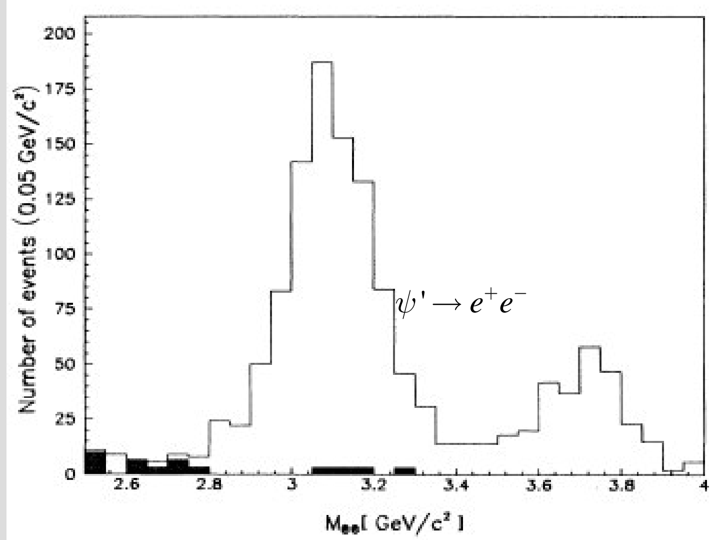


FIG. 6. Invariant mass distribution of electron pairs for the 1991 ψ' scan (open area) and for the off-resonance background normalized to the same luminosity (shaded area).

$M_{e^+e^-}$ from scan across ψ'

Another Side of Charmonium - IV

A few results..

@TBA

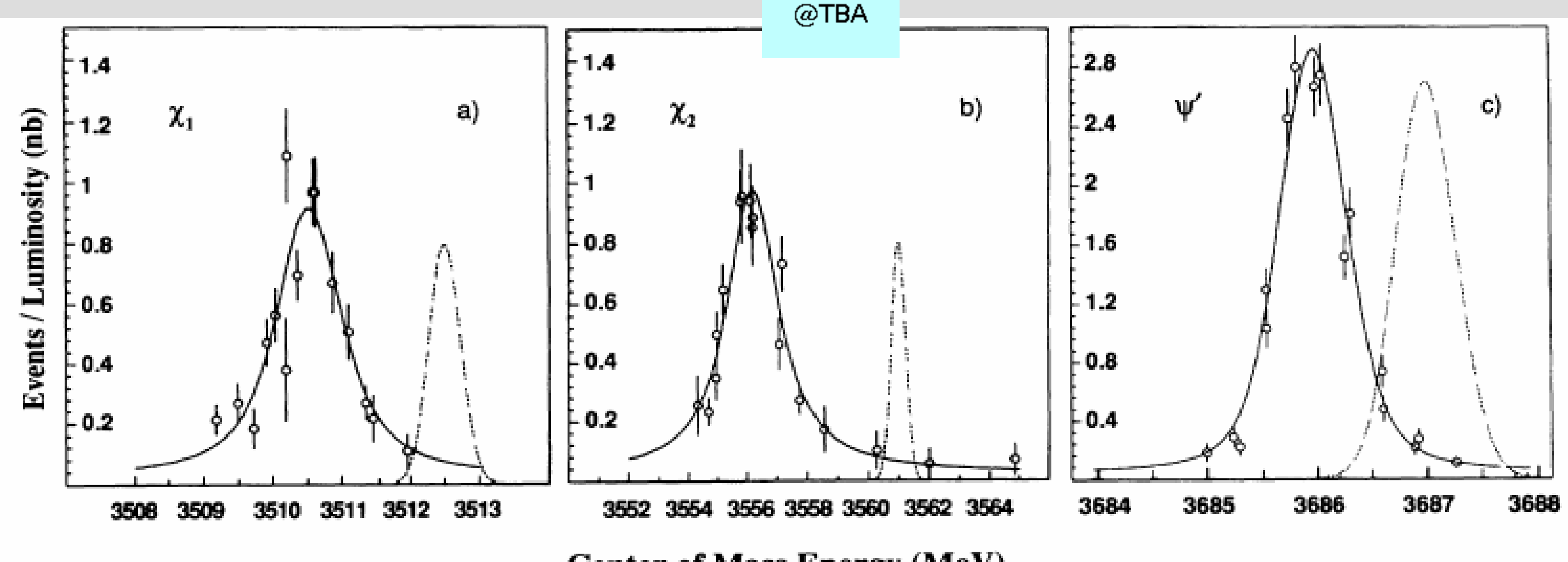


FIG. 3. Events per unit luminosity for the energy scan at (a) the χ_{c1}, (b) the χ_{c2}, and (c) the ψ'. The solid line represents the best fit with the data. The dashed line shows a typical resolution in the center-of-mass energy (arbitrary vertical units).

Quarkonia on PDG

Hidden Charm

$c\bar{c}$	
• $\eta_c(1S)$	$0^+(0^{-+})$
• $J/\psi(1S)$	$0^-(1^{--})$
• $\chi_{c0}(1P)$	$0^+(0^{++})$
• $\chi_{c1}(1P)$	$0^+(1^{++})$
$h_c(1P)$	$?^?(?^{??})$
• $\chi_{c2}(1P)$	$0^+(2^{++})$
• $\eta_c(2S)$	$0^+(0^{-+})$
• $\psi(2S)$	$0^-(1^{--})$
• $\psi(3770)$	$0^-(1^{--})$
• $X(3872)$	$0^?(?^{?+})$
• $\chi_{c2}(2P)$	$0^+(2^{++})$
$Y(3940)$	$?^?(?^{??})$
• $\psi(4040)$	$0^-(1^{--})$
• $\psi(4160)$	$0^-(1^{--})$
$Y(4260)$	$?^?(1^{--})$
• $\psi(4415)$	$0^-(1^{--})$

Hidden Bottom

$b\bar{b}$	
$\eta_b(1S)$	$0^+(0^{-+})$
• $\Upsilon(1S)$	$0^-(1^{--})$
• $\chi_{b0}(1P)$	$0^+(0^{++})$
• $\chi_{b1}(1P)$	$0^+(1^{++})$
• $\chi_{b2}(1P)$	$0^+(2^{++})$
• $\Upsilon(2S)$	$0^-(1^{--})$
$\Upsilon(1D)$	$0^-(2^{--})$
• $\chi_{b0}(2P)$	$0^+(0^{++})$
• $\chi_{b1}(2P)$	$0^+(1^{++})$
• $\chi_{b2}(2P)$	$0^+(2^{++})$
• $\Upsilon(3S)$	$0^-(1^{--})$
• $\Upsilon(4S)$	$0^-(1^{--})$
• $\Upsilon(10860)$	$0^-(1^{--})$
• $\Upsilon(11020)$	$0^-(1^{--})$

@TBA

Non-Perturbative QCD

Needed to deal with *bound states* and *soft interaction* regime

Very difficult problem

Different approaches available:

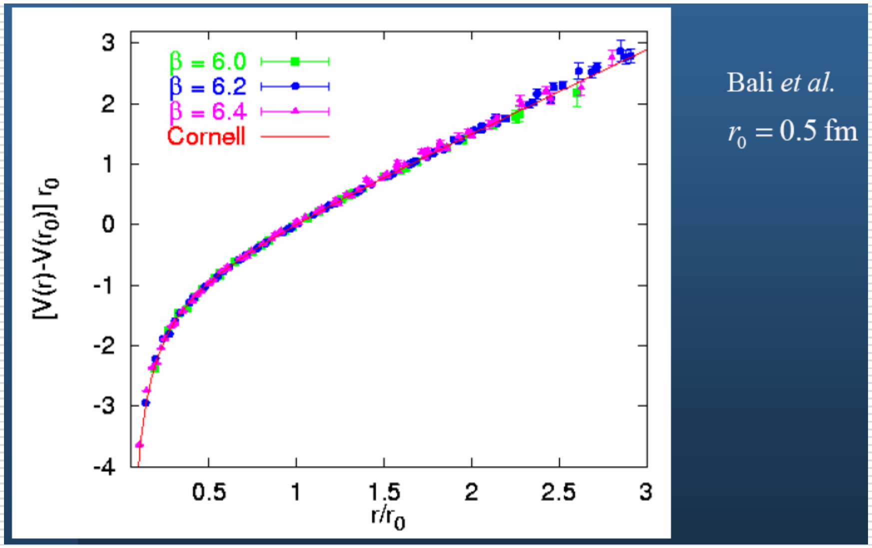
Lattice QCD
Chiral Pertubation Theory
Non-Relativistic QCD
Heavy Quark Effective Theory
…

Deep waters, not even surfed in this course

Lattice QCD

Perform QCD calculations over a discretized space-time (lattice)

$q\bar{q}$ potential from lattice



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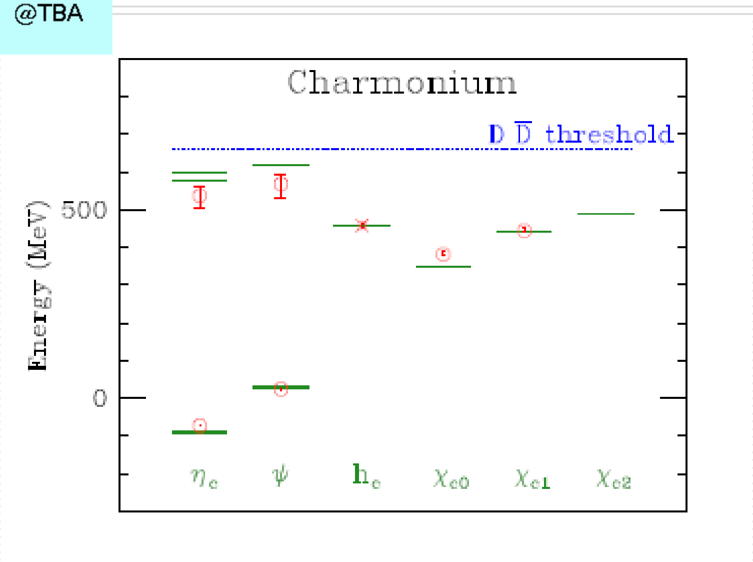

$V(r) = -\frac{4}{3}\frac{\alpha_s}{r} + ar$: Not a bad idea after all...

Example:
Charmonium levels from lattice