Elementary Particles I

6 – Weak Interaction

Beta Decay, *P* & *C* Violations, Current-Current Interaction, Charged Currents for Leptons and Quarks, Cabibbo Angle, GIM, Neutral Currents

The Electroweak Interaction

Standard Model:
Electromagnetic and Weak Interaction unified into Electroweak

Energy scale where unification is evident:

$E \sim M_W, M_Z \sim 100 \text{ GeV}$

At lower energies:

Can still find tiny traces of the unification (Electroweak interference, Parity violation in atomic processes,...)

Electroweak interaction split into two, almost not interfering, *effective* interactions:

Non fundamental, useful low energy approximations

Electromagnetic
Weak

The Weak Interaction

Compare:

Strong interaction – *All quarks*
Electromagnetic interaction – *All quarks + Charged Leptons*

Weak interaction – *All quarks + All leptons*

Large variety of phenomena

Classify weak processes into 3 types:

Leptonic $\mu^+ \to e^+ + \overline{\nu}_\mu + \nu_e, \quad \nu_e + e \to \nu_e + e$

Semileptonic $\pi^+ \to \mu^+ + \nu_\mu, \; \tau^+ \to \rho^+ + \nu_\tau$

Nonleptonic $K^0 \to \pi^+ + \pi^-, \; \Lambda^0 \to n + \pi^0$

Lost Symmetries

Many violations in weak processes:

Space Parity (large)
Charge Parity (large)
CP (very small)
T (very small)
Flavor conservation (S,C,B,T) (larger + smaller)
Lepton numbers (?) (neutrino oscillations)

Fall of Parity

Discovery of parity non conservation: Originated by the so-called "τ–θ puzzle"

Take *K* decays:

Weak process (*S* violation)

Observed decay modes (among many):

$$K^{\pm} \to \pi^{\pm}\pi^{0} \qquad BR = 21.2\ \%$$

$$K^{\pm} \to \pi^{\pm}\pi^{\pm}\pi^{\mp} \qquad BR = 5.6\ \%$$

Observe:
P_K=-1, *as measured in strong processes*
P_K= ?, *as measured by its decays*

Consider parity of the final states:

$$P\left|\pi\pi\right\rangle = (-1)(-1)(-1)^{l} = +1 \qquad l = 0 \text{ because } J_K = 0,\ J_\pi = 0$$

$$P\left|\pi\pi\pi\right\rangle = (-1)(-1)(-1)P_{orb} = (-1)P_{orb}$$

$$J_K = 0 = L_{\pi_1\pi_2} \oplus L_{\pi_3} \to L_{\pi_1\pi_2} = L_{\pi_3}$$

$$\to P_{orb} = (-1)^{L_{\pi 1\pi 2}}(-1)^{L_{\pi 3}} = +1$$

$$\to P\left|\pi\pi\pi\right\rangle = -1$$

???
2 different particles, same mass, opposite parity?

Lee & Yang suggestion: *Parity is violated in weak processes*

Violation of Parity

Parity violation discovered almost simultaneously in 3 experiments
First : *Beta decay* (Wu et al.)
Others: π–μ decay (Garwin et al., Friedman et al.)
(see before)

Interesting question:
How does parity violation manifest itself?

Breaking of parity selection rules
Interference between even/odd amplitudes $\rightarrow$ Asymmetries
Non-zero value of parity-odd observables

Violation of Charge Parity

Immediate conclusion: C-parity is also violated
Indeed, it can be shown:

$$A = Scalar + Pseudoscalar \rightarrow |A|^2 = |S+P|^2 = |S|^2 + |P|^2 + 2\,\mathrm{Re}\left(SP^*\right)$$

If $\begin{cases} CPT \text{ OK} \\ C \quad \text{OK} \end{cases}$, by taking $\begin{cases} S \equiv |S| e^{i\alpha} \\ P \equiv |P| e^{i\beta} \end{cases} \rightarrow e^{i\alpha} = e^{i(\beta+\pi/2)}$

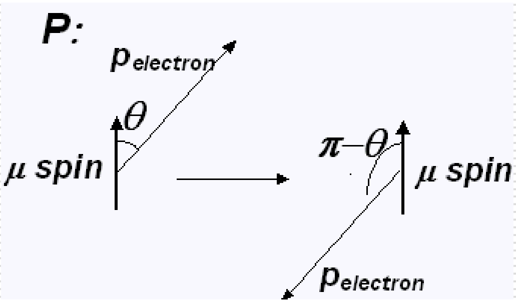
$$\rightarrow SP^* = |S| e^{i\alpha} |P| e^{-i\beta} = |S||P| e^{i\alpha} e^{-i\beta} = |S||P| e^{i(\beta+\pi/2)} e^{-i\beta} = |S||P| e^{i\pi/2}$$

$\rightarrow \mathrm{Re}\left(SP^*\right) = 0 \rightarrow$ Interference term = 0 $\rightarrow$ Asymmetry=0

Since asymmetries *are* observed, C must be KO

The Meaning of *P* and *C* Violations

Consider decay of a polarized muon: $\mu^+\uparrow \to e^+ + \bar{\nu}_\mu + \nu_e, \mu^-\uparrow \to e^- + \nu_\mu + \bar{\nu}_e$



P conservation: Predict

$$\left.\frac{d\Gamma_\pm}{d(\cos\theta)}\right|_\theta = \left.\frac{d\Gamma_\pm}{d(\cos\theta)}\right|_{\pi-\theta}$$

$$\to \frac{1}{2}\Gamma_\pm\left[1-\frac{\xi_\pm}{3}\cos\theta\right] = \frac{1}{2}\Gamma_\pm\left[1+\frac{\xi_\pm}{3}\cos\theta\right]$$

$\to \xi_\pm = 0$

Experiment:

$\xi_+ = -\xi_- = -1$

$\to P$ is violated

$C: \mu^\pm etc \to \mu^\mp etc$

$$\frac{d\Gamma_\pm}{d(\cos\theta)} = \frac{1}{2}\Gamma_\pm\left[1-\frac{\xi_\pm}{3}\cos\theta\right]$$

$$\Gamma_\pm = \frac{1}{\tau_\pm},\ \Gamma_+ = \Gamma_-$$

C conservation: Predict $\xi_+ = \xi_-$

Experiment:

$\xi_+ = -\xi_- = -1$

$\to C$ is violated

$$CP: \left.\frac{d\Gamma_+}{d(\cos\theta)}\right|_\theta = \left.\frac{d\Gamma_-}{d(\cos\theta)}\right|_{\pi-\theta}$$

$\to \xi_+ = -\xi_- \to$ Experiment OK

$\to CP$ is conserved

Beta Decay - I

Most common weak process in ordinary matter

3 nucleon 'decays':

$n \to p + e^- + \overline{\nu}_e$ Allowed for free neutrons

$p \to n + e^+ + \nu_e$ Energetically forbidden for free protons

$e^- + p \to n + \nu_e$ Atomic electron capture, usually from a K-shell (K-capture)

Both β^- and β^+ are observed for nucleons bound in a nucleus

$$(A,Z) \to (A,Z+1) + e^- + \overline{\nu}_e \qquad \beta^-$$

$$(A,Z) \to (A,Z-1) + e^+ + \nu_e \qquad \beta^+$$

$$e^- + (A,Z) \to (A,Z-1) + \nu_e \qquad K\text{-}Capture$$

Reminder: When found in a bound state, particles are *off-mass shell*

Beta Decay - II

Energy scale ~ few MeV

Small energy released to the pair $e\nu$

$\rightarrow e\nu$ orbital angular momentum = *0* *'Allowed'* Most frequent

= *1,2,..* *'Forbidden'* Rare (long lifetime)

Allowed transitions:

$$J_{e\nu} = 1/2 \oplus 1/2 = \begin{cases} 0 & \text{singlet} \\ 1 & \text{triplet} \end{cases}$$

$$\rightarrow \Delta J_{nucleus} = \begin{cases} 0, & \Delta J_3 = 0 \quad \text{Fermi} \\ 1, & \Delta J_3 = 0, \pm 1 \quad \text{Gamow-Teller} \end{cases}$$

Examples:

$$\underbrace{{}^{14}O}_{J=0} \rightarrow \underbrace{{}^{14}N}_{J=0} + \underbrace{e^+ + \nu_e}_{\rightarrow J=0}$$ pure Fermi

$$\underbrace{{}^{6}He}_{J=0} \rightarrow \underbrace{{}^{6}Li}_{J=1} + \underbrace{e^- + \overline{\nu}_e}_{\rightarrow J=1}$$ pure Gamow-Teller

$$\underbrace{{}^{1}n}_{J=1/2} \rightarrow \underbrace{{}^{1}p}_{J=1/2} + \underbrace{e^- + \overline{\nu}_e}_{\rightarrow J=0,1}$$ mixed Fermi/Gamow-Teller

Beta Decay - III

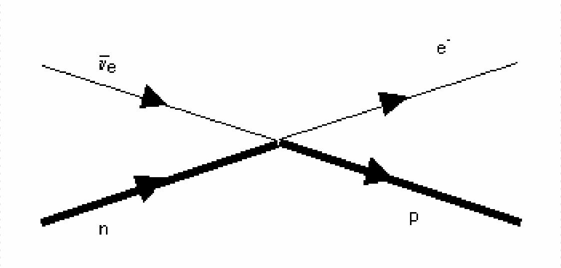
Take EM interaction as a model:

$$H_{\text{int}}^{EM} = j^{\mu} A_{\mu} \rightarrow j^{\mu}_{(a)} \frac{1}{q^2} j_{(b)\mu}$$ for 2 interacting currents

Fermi guess:

Without introducing any intermediate particle, current-current interaction:

$$H_{\text{int}}^{W} = j_{(a)} j_{(b)}$$ $j_{(a)}, j_{(b)}$ transition currents for leptons, nucleons



Transitions involve charge variation
$\rightarrow$ *Charged currents*

$$j_{(a)} \propto \bar{\psi}_{fin} \Gamma \psi_{in}$$ Operator Γ fixes the Lorentz structure of the current

Observe: Sticking for a moment to parity conservation, any *current*current* product which is a *Lorentz scalar* is acceptable for H_I. So we are free to guess different forms for the weak current

Beta Decay - IV

Fermi's original model had $\Gamma = \gamma_\mu$ Pure vector current

In general, Γ can be anyone of the following set:

$$1 \quad \gamma^\mu \quad \sigma^{\mu\nu} \quad \gamma^5\gamma^\mu \quad \gamma^5$$

$$\Gamma_1 \quad \Gamma_2 \quad \Gamma_3 \quad \Gamma_4 \quad \Gamma_5$$

$\bar{\psi}\psi$	scalar S
$\bar{\psi}\gamma^\mu\psi$	vector V
$\bar{\psi}\sigma^{\mu\nu}\psi$	tensor T
$\bar{\psi}\gamma^5\gamma^\mu\psi$	vector axial A
$\bar{\psi}\gamma^5\psi$	pseudoscalar P

Most general form:

$$H_{\text{int}} = \sum_{i=S,V,T,A,P} C_i \left[\left(\bar{\psi}_p \Gamma_i \psi_n\right)\left(\bar{\psi}_e \Gamma^i \psi_\nu\right) + \left(\bar{\psi}_n \Gamma_i \psi_p\right)\left(\bar{\psi}_\nu \Gamma^i \psi_e\right) \right]$$

Hermitian conjugate required in order to account for processes involving antileptons

Rely on experiment to investigate the Lorentz structure

Beta Decay - V

Non-relativistic limit of the different nucleon currents

S $\quad 1 \qquad \rightarrow \chi_p^\dagger \chi_n$

V $\quad \gamma_\mu \qquad \begin{cases} \mu=0 \rightarrow \chi_p^\dagger \chi_n \\ \mu = 1,2,3 \rightarrow 0 \end{cases}$

T $\quad \sigma_{\mu\nu} \qquad \begin{cases} \mu=0,\nu \rightarrow 0 \\ \mu,\nu = 0 \rightarrow 0 \\ \mu,\nu = 1,2,3 \rightarrow \chi_p^\dagger \boldsymbol{\sigma} \chi_n \end{cases}$

A $\quad \gamma_\mu \gamma_5 \qquad \begin{cases} \mu=0 \rightarrow 0 \\ \mu = 1,2,3 \rightarrow \chi_p^\dagger \boldsymbol{\sigma} \chi_n \end{cases}$

P $\quad \gamma_5 \qquad \rightarrow 0$

Conclude:

P not much relevant for β-decay
S,V do not change nucleon spin $\rightarrow$ OK for Fermi
T,A do change nucleon spin $\rightarrow$ OK for Gamow-Teller

Beta Decay - VI

Attempts to understand which terms are present in the interaction

Pure Fermi : S and/or V

Pure Gamow-Teller: T and/or A

In both cases:

If both terms are there, get as a result a distortion of the electron energy spectrum (*Fierz interference*)

Not observed $\rightarrow$ Fermi: S or V, Gamow-Teller: T or A

Look for more indicators

Angular correlation electron-neutrino. Expect:

$$\frac{dN}{d\cos\theta} = const\left(1+\lambda\beta\cos\theta\right)$$

$$\lambda = \begin{cases} -1 & S \\ +1 & V \\ +1/3 & T \\ -1/3 & A \end{cases}$$

Cannot observe neutrino$\rightarrow$Observe recoiling nucleus instead
Many experiments made : Difficult, erratic, inconclusive, sometimes wrong, leading to mistakenly guess S & T
Solution finally found after the discovery of parity non conservation, *by ignoring (wrong) experimental data*

Beta Decay - VII

To yield parity violation, H must include both *scalar* and *pseudo-scalar* terms
Indeed, for any matrix element between initial and final states:

$$\left|\langle f|S+P|i\rangle\right|^2=\left|\langle f|S|i\rangle\right|^2+\left|\langle f|P|i\rangle\right|^2+2\langle f|S|i\rangle\langle f|P|i\rangle^*$$

$$\left|\langle f|S+P|i\rangle\right|^2\underset{Parity}{\longrightarrow}\left|\langle f|S|i\rangle\right|^2+\left|\langle f|(-P)|i\rangle\right|^2+2\langle f|S|i\rangle\langle f|(-P)|i\rangle^*$$

$$=\left|\langle f|S|i\rangle\right|^2+\left|\langle f|P|i\rangle\right|^2-2\langle f|S|i\rangle\langle f|P|i\rangle^*$$

By allowing for parity non conservation, can write down:

+Hermitian conjugate always understood

$$H_{\text{int}}=\sum_{i=S,V,T,A}\left[C_i\underbrace{\left(\bar{\psi}_p\Gamma_i\psi_n\right)\left(\bar{\psi}_e\Gamma^i\psi_\nu\right)}_{S}+C_i'\underbrace{\left(\bar{\psi}_p\Gamma_i\psi_n\right)\left(\bar{\psi}_e\Gamma^i\gamma^5\psi_\nu\right)}_{P}\right]$$

C_i, C_i' 'constants'

γ^5 equivalently inserted in the nucleon current, no difference in full operator: Just a re-labeling of C,C'

Actually $C_i=C_i\left(q^2\right),\ C_i'=C_i'\left(q^2\right)$ Weak form factors

But: $q^2\sim 0$ for β decay $\rightarrow$ Constants

Beta Decay - VIII

Redefine constants by extracting $\frac{G_F}{\sqrt{2}}$ (G_F: Fermi constant)

$$H_{\text{int}} = \frac{G_F}{\sqrt{2}} \sum_{i=S,V,T,A} C_i \left(\overline{\psi}_p \Gamma_i \psi_n\right) \left(\overline{\psi}_e \Gamma^i \left(1 + \frac{C_i'}{C_i} \gamma^5\right) \psi_\nu\right)$$

Time reversal invariance: C_i, C'_i real

In order to investigate the form of lepton current:

Measurement of the lepton longitudinal polarization

Reminder:

$P_L\left(lept\right) = \left\langle lept \middle| \boldsymbol{\sigma}_l \cdot \hat{\boldsymbol{p}}_l \middle| lept \right\rangle$ Long. Polarization = Average Helicity

$P_T\left(lept\right) = 0$ $\quad T$ non-invariant $\quad \boldsymbol{\sigma}_e \cdot \left(\boldsymbol{p}_e \times \boldsymbol{p}_\nu\right) \underset{T}{\rightarrow} -\boldsymbol{\sigma}_e \cdot \left(-\boldsymbol{p}_e \times -\boldsymbol{p}_\nu\right) = -\boldsymbol{\sigma}_e \cdot \left(\boldsymbol{p}_e \times \boldsymbol{p}_\nu\right)$

Helicity/Chirality Refresher - I

With reference to Dirac equation:

$$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \boldsymbol{\gamma} = \begin{pmatrix} 0 & \boldsymbol{\sigma} \\ -\boldsymbol{\sigma} & 0 \end{pmatrix}, \gamma^5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$ Dirac representation

$$\mathbf{S} = \frac{\boldsymbol{\Sigma}}{2}, \boldsymbol{\Sigma} = \begin{pmatrix} \boldsymbol{\sigma} & 0 \\ 0 & \boldsymbol{\sigma} \end{pmatrix} = \underbrace{\gamma^0 \boldsymbol{\gamma}}_{\boldsymbol{\alpha}} \gamma^5 = \boldsymbol{\alpha}\gamma^5$$ Spin operator

$$\Lambda = \frac{\boldsymbol{\Sigma}\cdot\mathbf{p}}{|\mathbf{p}|}$$ Helicity operator $$\left.\begin{aligned} \Lambda u^{(+)} &= +u^{(+)} \\ \Lambda u^{(-)} &= -u^{(-)} \end{aligned}\right\}$$ Helicity eigenstates

$$P_\pm = \frac{1 \pm \Lambda}{2}$$ Projection operators onto helicity eigenstates

Projectors, indeed:

$$P_+P_+ = \left(\frac{1+\Lambda}{2}\right)\left(\frac{1+\Lambda}{2}\right) = \frac{1}{4}\left(1+\Lambda+\Lambda+\Lambda^2\right),\ \Lambda^2 = \frac{\left(\boldsymbol{\Sigma}\cdot\mathbf{p}\right)^2}{|\mathbf{p}|^2} = 1 \to P_+P_+ = \frac{1}{4}\left(1+2\Lambda+1\right) = \left(\frac{1+\Lambda}{2}\right) = P_+,\quad P_-P_- = P_-$$

$$P_+P_- = \left(\frac{1+\Lambda}{2}\right)\left(\frac{1-\Lambda}{2}\right) = \frac{1}{4}\left(1+\Lambda-\Lambda-\Lambda^2\right) = 0 = P_-P_+ \quad 1 = \frac{1-\Lambda}{2} + \frac{1+\Lambda}{2} = P_- + P_+ \to 1u = \left(P_+ + P_-\right)u = u_+ + u_-$$

$$\Lambda = \frac{\boldsymbol{\Sigma}\cdot\mathbf{p}}{|\mathbf{p}|} = \underbrace{\gamma^0\boldsymbol{\gamma}}_{\boldsymbol{\alpha}}\gamma^5 \cdot \frac{\mathbf{p}}{|\mathbf{p}|} = \frac{\boldsymbol{\alpha}\cdot\mathbf{p}}{|\mathbf{p}|}\gamma^5 \to P_\pm = \frac{1 \pm \boldsymbol{\alpha}\cdot\hat{\mathbf{p}}\gamma^5}{2}$$

Helicity/Chirality Refresher - II

γ^5 Chirality operator

$P_L = \frac{1-\gamma^5}{2}, P_R = \frac{1+\gamma^5}{2}$ Projectors onto chirality eigenstates

$$\begin{cases} P_L u = u_L \\ P_R u = u_R \end{cases} \rightarrow 1u = \left(P_L + P_R\right)u = u_L + u_R$$

A very important limiting case:

$$\boldsymbol{\alpha}\cdot\mathbf{p} = E - \beta m$$
$$\Lambda = \frac{E\cdot 1 - m\cdot\beta}{p}\gamma^5 \underset{E\gg m}{\longrightarrow} \gamma^5$$
$$P_\pm = \frac{1\pm\Lambda}{2} \underset{E\gg m}{\longrightarrow} \frac{1\pm\gamma^5}{2} = P_{R,L}$$

Helicity projectors go into *chirality* projectors for high energy, or massless, particles

Helicity, Chirality Refresher - III

$$Eu = (\boldsymbol{\alpha}\cdot\mathbf{p} + \beta m)u$$

$u = \begin{pmatrix}\phi \\ \chi\end{pmatrix}$, ϕ, χ 2 components spinors

$\boldsymbol{\alpha} = \begin{pmatrix}\boldsymbol{\sigma} & 0 \\ 0 & -\boldsymbol{\sigma}\end{pmatrix}, \quad \beta = \begin{pmatrix}0 & 1 \\ 1 & 0\end{pmatrix}$ Dirac matrices in chiral representation

$$\rightarrow \begin{cases} E\phi = (\boldsymbol{\sigma}\cdot\mathbf{p})\phi + m\chi \\ E\chi = -(\boldsymbol{\sigma}\cdot\mathbf{p})\chi + m\phi \end{cases}$$

$\rightarrow \begin{cases} E\phi = (\boldsymbol{\sigma}\cdot\mathbf{p})\phi \\ E\chi = -(\boldsymbol{\sigma}\cdot\mathbf{p})\chi \end{cases}, m = 0 \rightarrow \begin{cases} \dfrac{(\boldsymbol{\sigma}\cdot\mathbf{p})}{E = |\mathbf{p}|}\phi = \phi \\ \dfrac{(\boldsymbol{\sigma}\cdot\mathbf{p})}{E = |\mathbf{p}|}\chi = -\chi \end{cases} \rightarrow \phi, \chi$ Helicity eigenstates

Chiral States

States with definite value of chirality, massive or massless particles

Particle	*Antiparticle*
$u_L = \frac{1}{2}\left(1-\gamma^5\right)u$	$v_L = \frac{1}{2}\left(1+\gamma^5\right)v$
$u_R = \frac{1}{2}\left(1+\gamma^5\right)u$	$v_R = \frac{1}{2}\left(1-\gamma^5\right)v$
$\overline{u}_L = \overline{u}\,\frac{1}{2}\left(1+\gamma^5\right)$	$\overline{v}_L = \overline{v}\,\frac{1}{2}\left(1-\gamma^5\right)$
$\overline{u}_R = \overline{u}\,\frac{1}{2}\left(1-\gamma^5\right)$	$\overline{v}_R = \overline{v}\,\frac{1}{2}\left(1+\gamma^5\right)$

Is it true? Try one example:

$$\gamma^5 u_L = \gamma^5\,\frac{1}{2}\left(1-\gamma^5\right)u = \frac{1}{2}\left(\gamma^5-1\right)u$$

$$= -\frac{1}{2}\left(1-\gamma^5\right)u = -u_L \quad \text{OK}$$

For chiral states:

Massless particle: $\begin{cases} u_L & \langle H\rangle = -1 \\ u_R & \langle H\rangle = +1 \end{cases}$ $\rightarrow$ Helicity defined $\equiv$ Full longitudinal polarization

Massive particle: $\begin{cases} u_L & \langle H\rangle = -\beta \\ u_R & \langle H\rangle = +\beta \end{cases}$ $\rightarrow$ Helicity undefined, find

-1 prob. $\frac{1+\beta}{2}$, $+1$ prob. $\frac{1-\beta}{2}$

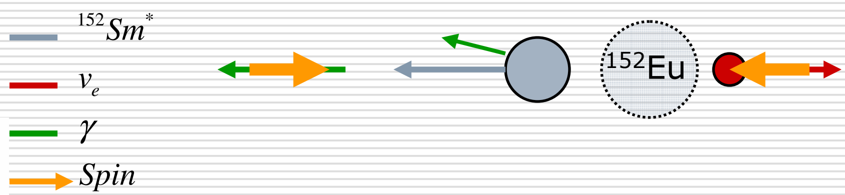
$+1$ prob. $\frac{1+\beta}{2}$, -1 prob. $\frac{1-\beta}{2}$

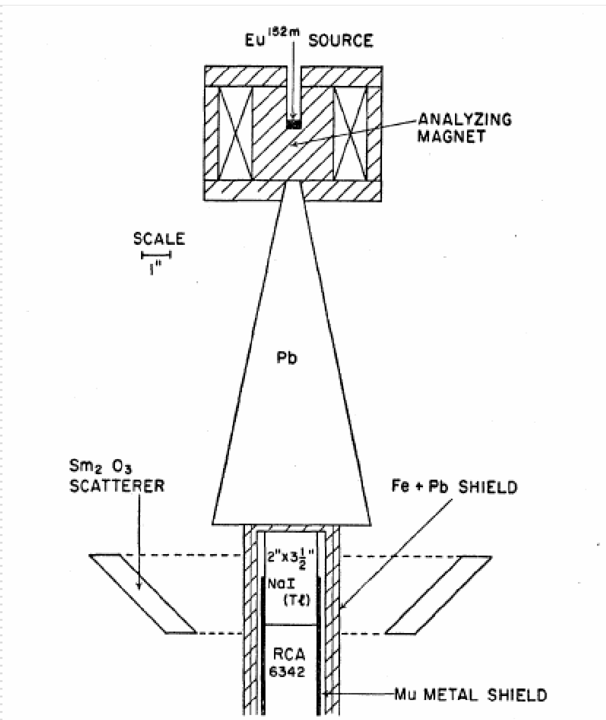
Massless particles: *Helicity is Lorentz invariant*

Neutrino Helicity

Neutrino can be be *either R or L:* Must rely on experiment to decide

Goldhaber experiment





$\boxed{\text{Result}: H(\nu_e) = -1}$

$\underbrace{^{152}Eu}_{J=0} + e^- \rightarrow \underbrace{^{152}Sm^*}_{J=1} + \nu_e \quad K$ capture

$\rightarrow H\left(^{152}Sm^*\right) = H(\nu_e)$

$\underbrace{^{152}Sm^*}_{J=1} \rightarrow \underbrace{^{152}Sm}_{J=0} + \gamma$

When γ is emitted along the nucleus momentum:

No orbital angular momentum

$\rightarrow H(\gamma) = H\left(^{152}Sm^*\right) = H(\nu_e)$ for collinear photons

How to select them?

Choose a ^{152}Sm target: $\gamma + {}^{152}Sm \rightarrow {}^{152}Sm^* \rightarrow {}^{152}Sm + \gamma$

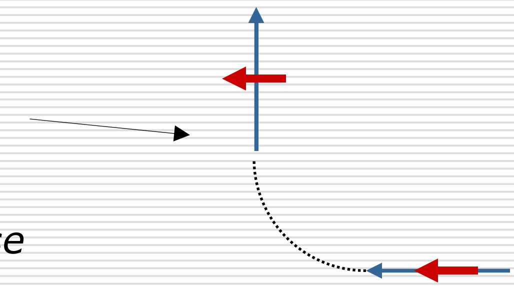
Resonant scattering:

Only possible for collinear photons (Energy)

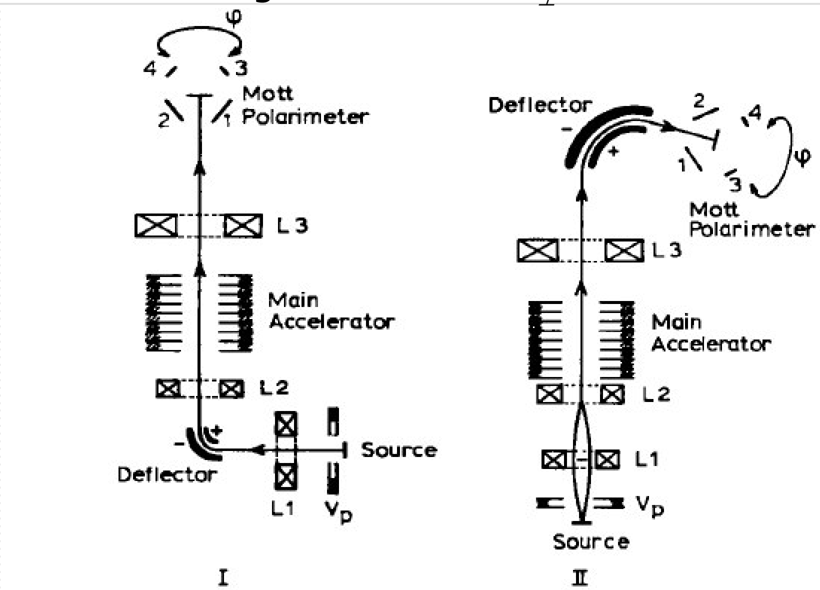
Measure γ polarization by absorbing in magnetized iron

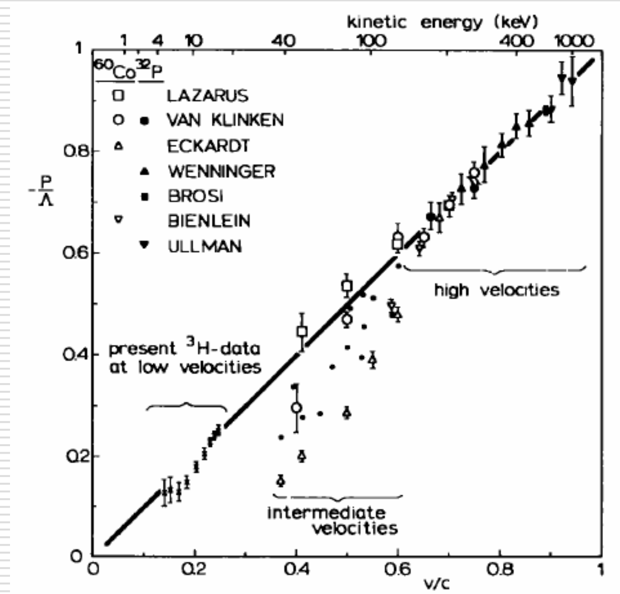
Electron Longitudinal Polarization

Method: Rotate e momentum by 90^0 by means of an *electrostatic* field



$\rightarrow$Spin undeflected for non relativistic e
$\rightarrow$Longitudinal polarization becomes *transverse*
$\rightarrow$Mott scattering sensitive to $P_\perp$





$\langle H \rangle = -\beta$ Average helicity $\equiv$ Long. polarization

Mystery Solved: V - A

Now closing on the analysis:

$$H_{\text{int}} = \frac{G_F}{\sqrt{2}} \sum_{i=V,A} C_i \left(\bar{\psi}_p \Gamma_i \psi_n\right)\left(\bar{\psi}_e \Gamma^i \left(1 + \frac{C_i\,'}{C_i}\gamma^5\right)\psi_\nu\right)$$

Neutrino helicity = -1 yields

Lepton current : V - A

$$\left(1 + \frac{C_i\,'}{C_i}\gamma^5\right)\psi_\nu = \left(1-\gamma^5\right)\psi_\nu \rightarrow C_i\,' = -C_i$$

$$= -\gamma^\mu\left(1-\gamma^5\right)$$

$$\rightarrow H_{\text{int}} = \frac{G_F}{\sqrt{2}}\left[C_V\left(\bar{\psi}_p\gamma_\mu\psi_n\right)\left(\bar{\psi}_e\gamma^\mu\left(1-\gamma^5\right)\psi_\nu\right) + C_A\left(\bar{\psi}_p\gamma_\mu\gamma_5\psi_n\right)\left(\bar{\psi}_e\gamma^\mu\gamma^5\left(1-\gamma^5\right)\psi_\nu\right)\right]$$

$$= \frac{G_F}{\sqrt{2}}\left[C_V\left(\bar{\psi}_p\gamma_\mu\psi_n\right)\left(\bar{\psi}_e\gamma^\mu\frac{\left(1-\gamma^5\right)^2}{2}\psi_\nu\right) - C_A\left(\bar{\psi}_p\gamma_\mu\gamma_5\psi_n\right)\left(\bar{\psi}_e\gamma^\mu\frac{\left(1-\gamma^5\right)^2}{2}\psi_\nu\right)\right]$$

$$= \frac{G_F}{\sqrt{2}}\left[C_V\left(\bar{\psi}_p\gamma_\mu\psi_n\right)\left(\frac{1}{2}\bar{\psi}_e\left(1+\gamma^5\right)\gamma^\mu\left(1-\gamma^5\right)\psi_\nu\right) + C_A\left(\bar{\psi}_p\gamma_\mu\gamma_5\psi_n\right)\left(\frac{1}{2}\bar{\psi}_e\left(1+\gamma^5\right)\gamma^\mu\left(1-\gamma^5\right)\psi_\nu\right)\right]$$

$$= \frac{G_F}{\sqrt{2}}\left[C_V\left(\bar{\psi}_p\gamma_\mu\psi_n\right) - C_A\left(\bar{\psi}_p\gamma_\mu\gamma_5\psi_n\right)\right]\left(\frac{1}{2}\bar{\psi}_e\left(1+\gamma^5\right)\gamma^\mu\left(1-\gamma^5\right)\psi_\nu\right)$$

$$= \frac{G_F}{\sqrt{2}}C_V\left[\left(\bar{\psi}_p\gamma_\mu\left(1-\frac{C_A}{C_V}\gamma_5\right)\psi_n\right)\left(\frac{1}{2}\bar{\psi}_e\left(1+\gamma^5\right)\gamma^\mu\left(1-\gamma^5\right)\psi_\nu\right)\right]$$

Nucleon current : V - ~ A

V-A and the Nucleon Current

Found ~ the same structure for lepton and nucleon current

$\bar{\psi}_e \gamma^\mu \left(1-\gamma^5\right)\psi_\nu$ Pure V-A

$\bar{\psi}_p \gamma_\mu \left(1-\frac{C_A}{C_V}\gamma_5\right)\psi_n$ V: identical coupling, A: modified by strong interaction

Consider β decay of O^{14} – pure Fermi $G_F = 1\cdot 10^{-5} M_p^{-2}$

Call this the β-decay Fermi constant

Measured value of $\frac{C_A}{C_V}$ for β decay of various baryons:

n	-1.267
Λ^0	-0.718
Σ^-	$+0.340$
Ξ^-	-0.25

Universality: Leptons

Consider muon weak interactions:

$\mu^+ \rightarrow e^+ + \overline{\nu}_\mu + \nu_e, \quad \mu^- \rightarrow e^- + \nu_\mu + \overline{\nu}_e \qquad \mu$ decay

$\mu^- + p \rightarrow n + \nu_\mu \qquad \mu$ capture

μ decay is purely leptonic:
Guess *current-current* + *V-A* for both electron and muon charged currents

Compute:
μ lifetime
Electron energy spectrum

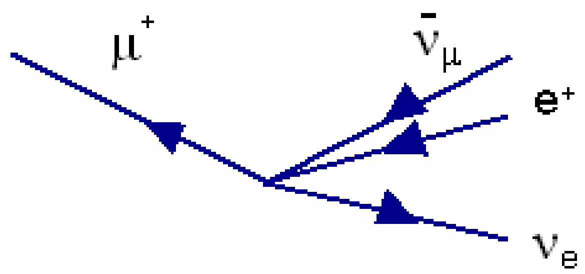
Extend to τ *leptonic* decays:

$\tau^+ \rightarrow e^+ + \overline{\nu}_\tau + \nu_e, \quad \tau^- \rightarrow e^- + \nu_\tau + \overline{\nu}_e$

$\tau^+ \rightarrow \mu^+ + \overline{\nu}_\tau + \nu_\mu, \quad \tau^- \rightarrow \mu^- + \nu_\tau + \overline{\nu}_\mu$

τ has many more decay channels open into quark-antiquark pairs

Muon Decay - I



$$\mathcal{M} = -i2\sqrt{2}G_F(\overline{u}_3\gamma^\rho P_L u_a)(\overline{u}_1\gamma_\rho P_L v_2).$$

$$|\mathcal{M}|^2 = 8G_F^2\,(\overline{u}_3\gamma^\rho P_L u_a)(\overline{u}_a P_R\gamma^\sigma u_3)\,(\overline{u}_1\gamma_\rho P_L v_2)(\overline{v}_2 P_R\gamma_\sigma u_1).$$

$$\frac{1}{2}\sum_{\text{spins}}|\mathcal{M}|^2 = 32G_F^2(m_\mu^3 E_{\overline{\nu}_e} - 2m_\mu^2 E_{\overline{\nu}_e}^2).$$

$$d\Gamma = dE_e dE_{\overline{\nu}_e}\,\frac{G_F^2}{2\pi^3}(m_\mu^2 E_{\overline{\nu}_e} - 2m_\mu E_{\overline{\nu}_e}^2).$$

$$d\Gamma = dE_e\int_{\frac{m_\mu}{2}-E_e}^{\frac{m_\mu}{2}} dE_{\overline{\nu}_e}\,\frac{G_F^2}{2\pi^3}(m_\mu^2 E_{\overline{\nu}_e} - 2m_\mu E_{\overline{\nu}_e}^2) = dE_e\frac{G_F^2}{\pi^3}\left(\frac{m_\mu^2 E_e^2}{4} - \frac{m_\mu E_e^3}{3}\right)$$

Muon Decay - II

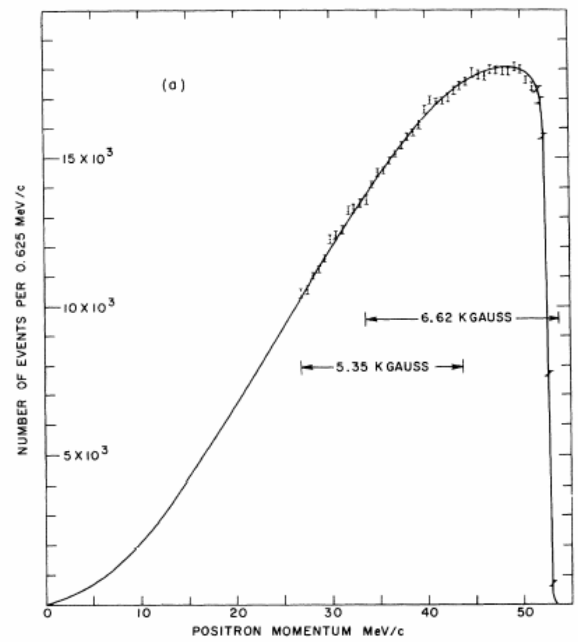
$$\Gamma = \int_{E_{\min}}^{E_{\max}} \frac{d\Gamma}{dE} dE = \frac{32 G_F^2 m_\mu^5}{12(8\pi)^3} \rightarrow \tau = \frac{192\pi^3}{G_F^2 m_\mu^5}$$

Extract G_F from measured lifetime: $G_F^{(\beta)} = 0.98 \quad G_F^{(\mu)}$ Almost identical!

2% difference ??

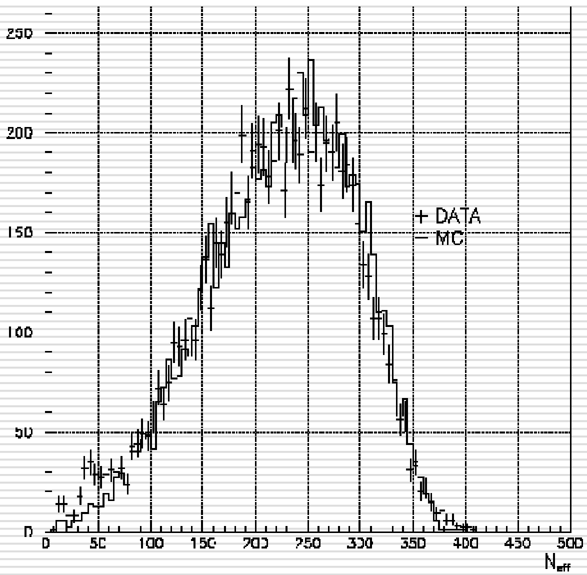
Spectrum shape:

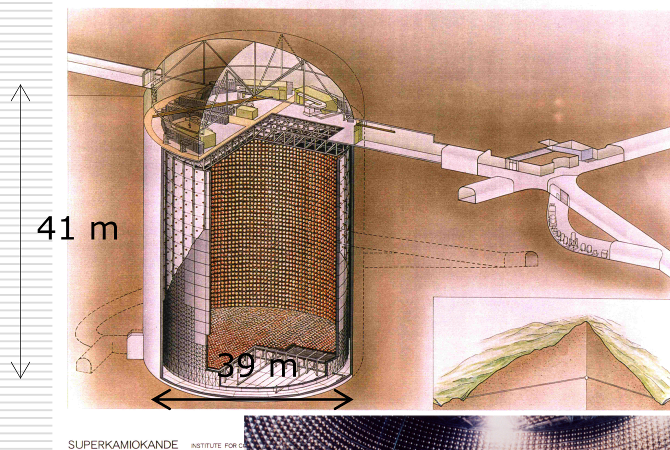
$$\frac{d\Gamma}{dE} = 32 G_F^2 \frac{m_\mu^2 E^2}{2(4\pi)^3} \left(1 - \frac{4E}{3m_\mu}\right)$$

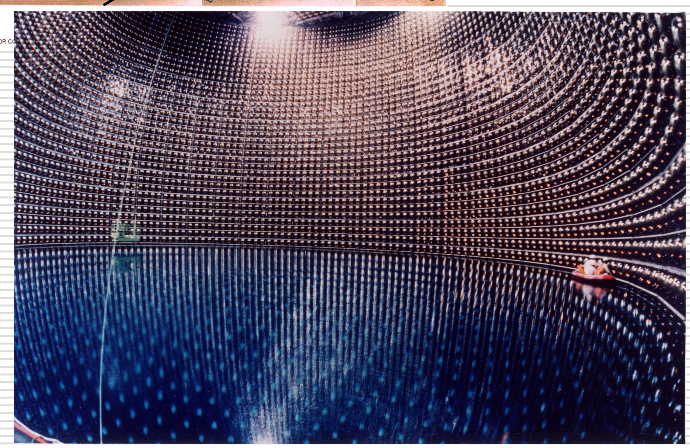


Super K

Electron spectrum from μ decay used as calibration at SuperKamiokande
Allows for absolute calibration of Cerenkov light signal vs. energy







Water Cerenkov

50000 t pure water

11200 photomultipliers (50 cm ∅!)

The μ Michel Parameter

Detailed analysis:

Differential decay rate depending on several parameters
Alternative structures of charged current cast into different parameter sets
Most important: *Michel parameter ρ*
Electron differential spectrum determined by ρ for unpolarized muon

$$\frac{dP}{d\Omega dx} = \frac{G^2 m_\mu^5}{192\pi^4} x^2 \left[3(1-x) + \frac{2}{3}\rho(4x-3)\right], \quad x = \frac{2E_e}{m_\mu}$$

V-A theory predicts $\rho = ¾$
Experimental result: 0.7517 ± 0.0026

Electron angular distribution for polarized muon decay:

$$\frac{dP}{d\Omega} = \frac{1}{4\pi}\left[1 - \frac{1}{3}\xi P \cos\theta\right], \ P$$ muon polarization

V-A: $\xi = +1$ Experiment OK

The τ Michel Parameter

Consider τ decays:

Many modes, including

Leptonic

$\tau^{\pm} \to \mu^{\pm} + \nu_{\mu} / \overline{\nu}_{\mu} + \overline{\nu}_{\tau} / \nu_{\tau}$

$\tau^{\pm} \to e^{\pm} + \nu_{e} / \overline{\nu}_{e} + \overline{\nu}_{\tau} / \nu_{\tau}$

Semileptonic

$\tau^{\pm} \to Hadrons + \overline{\nu}_{\tau} / \nu_{\tau}$

A simple question:
What is the τ charged current?
Investigations at LEP:

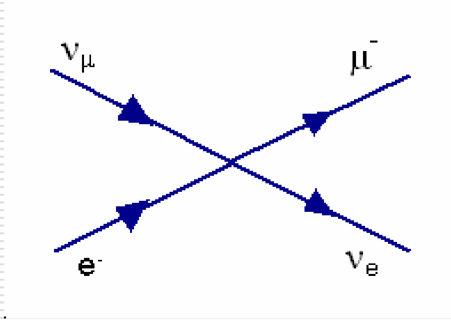
$e^{+} + e^{-} \to Z^{0} \to \tau^{+} + \tau^{-}$

Conclusion:

Current is $V - A$

$$\rho_{\tau} = \begin{cases} 0.747 \pm 0.024 \;\; e & \text{mode} \\ 0.776 \pm 0.049 \;\; \mu & \text{mode} \end{cases}$$

Neutrino Scattering



$$\sum_{spin} T_{fi} T_{fi}^* = \sum_{spin} \frac{G_F}{\sqrt{2}} \left[\bar{u}(3)\gamma^\mu\left(1-\gamma^5\right)u(1)\right]\left[\bar{u}(3)\gamma_\nu\left(1-\gamma_5\right)u(1)\right]^* \left[\bar{u}(4)\gamma_\mu\left(1-\gamma_5\right)u(2)\right]\left[\bar{u}(4)\gamma^\nu\left(1-\gamma^5\right)u(2)\right]^*$$

$$\sum_{spin} \left[\bar{u}(a)\Gamma_1 u(b)\right]\left[\bar{u}(a)\Gamma_2 u(b)\right]^* = Tr\left[\Gamma_1\left(\not{p}_b + m_b\right)\bar{\Gamma}_2\left(\not{p}_a + m_a\right)\right]$$

$$\sum_{spin} \left|T_{fi}\right|^2 = 64 G_F^2 \left(p_1 \cdot p_2\right)\left(p_3 \cdot p_4\right)$$

$$\sum_{spin} \left|T_{fi}\right|^2 = 256 G_F^2 E^4 \left[1 - \left(\frac{m_\mu}{2E}\right)^2\right]$$

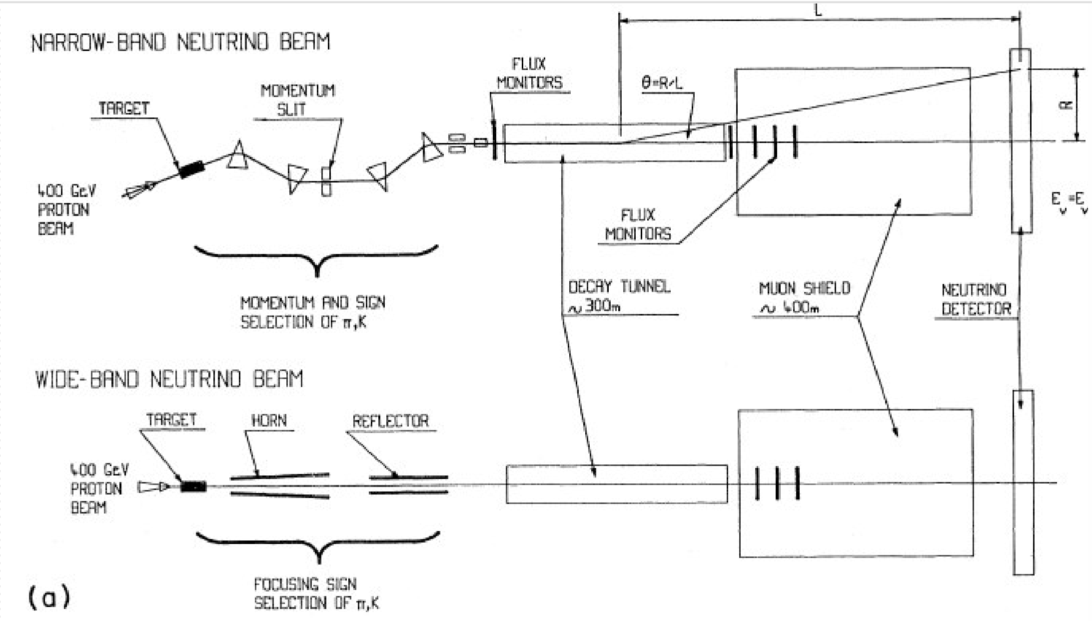
$$\frac{d\sigma}{d\Omega} = \frac{G_F^2}{\pi^2} E^4 \left[1 - \left(\frac{m_\mu}{2E}\right)^2\right]^2 \rightarrow \sigma = \frac{4G_F^2}{\pi} E^2 \left[1 - \left(\frac{m_\mu}{2E}\right)^2\right]^2$$

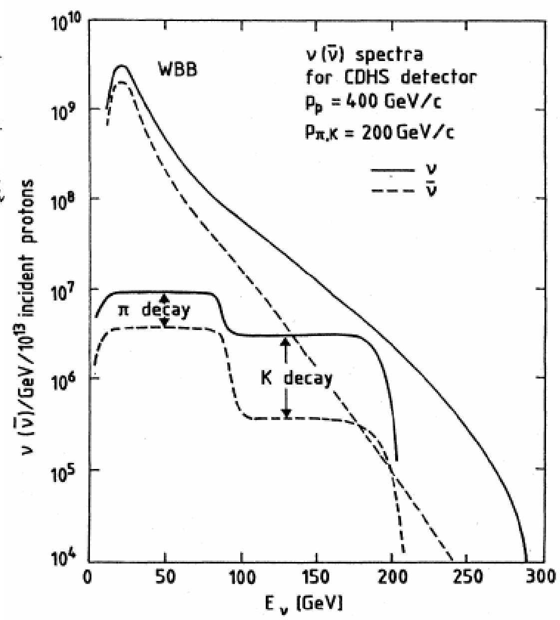
Neutrino Beams

Derived from 2-body π,K decay

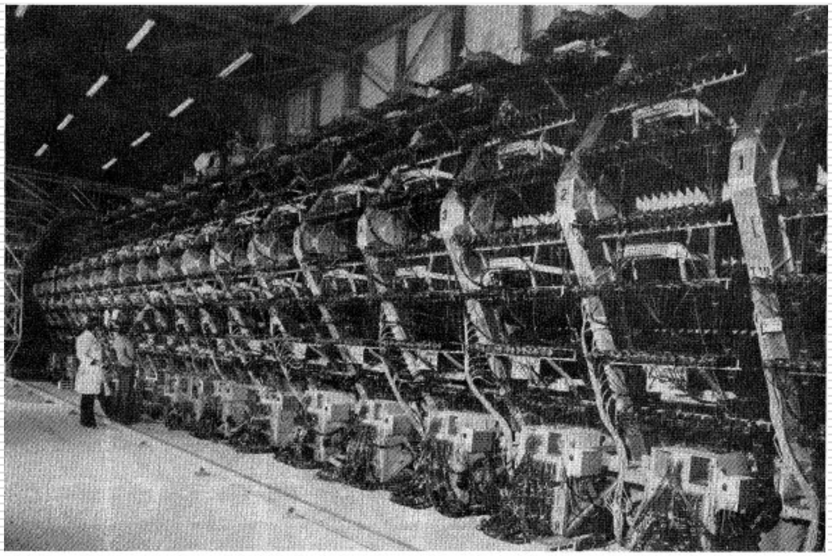
$$\pi^{\pm} \to \mu^{\pm} + \nu_{\mu}, \overline{\nu}_{\mu}$$

$$K^{\pm} \to \mu^{\pm} + \nu_{\mu}, \overline{\nu}_{\mu}$$

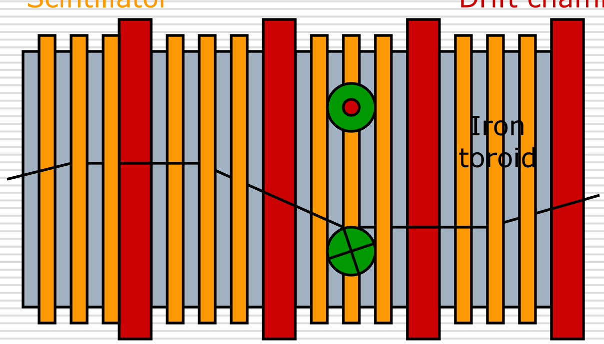


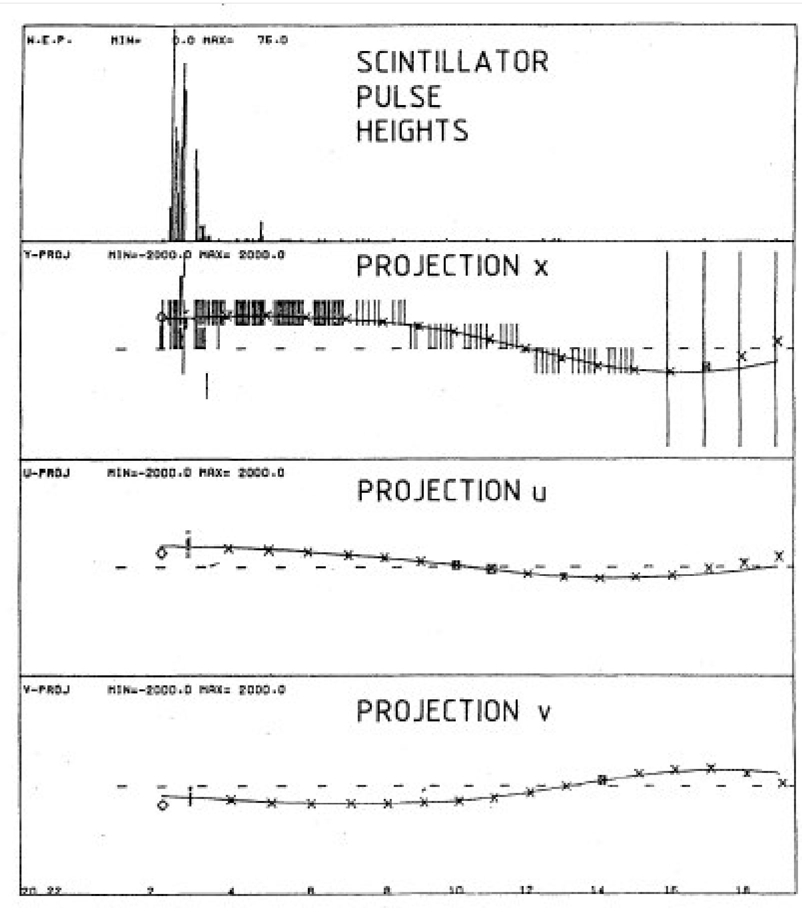


Neutrino Detectors - I

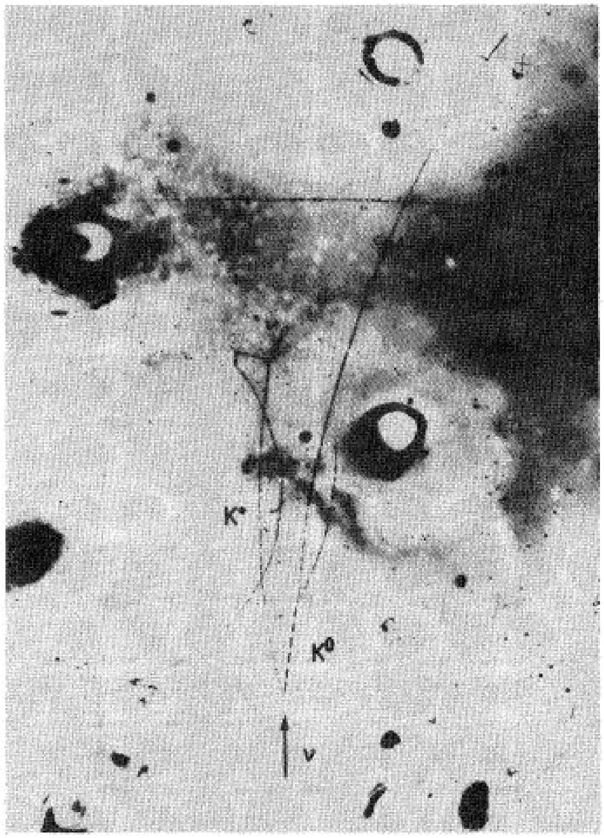


CDHS at CERN SPS

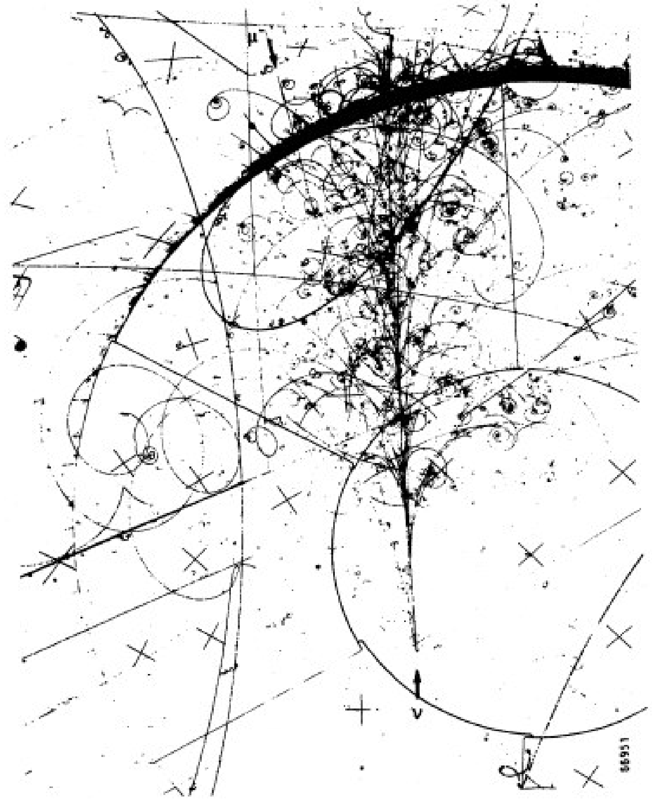




Neutrino Detectors - II

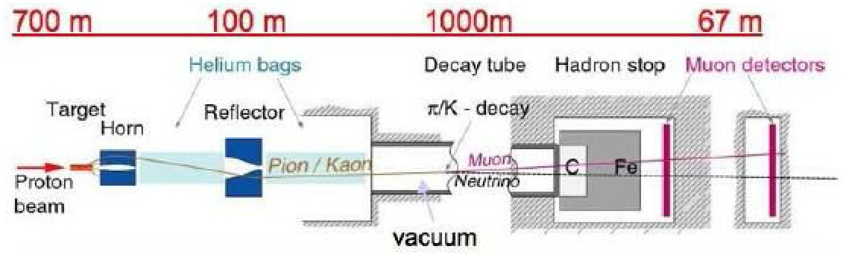


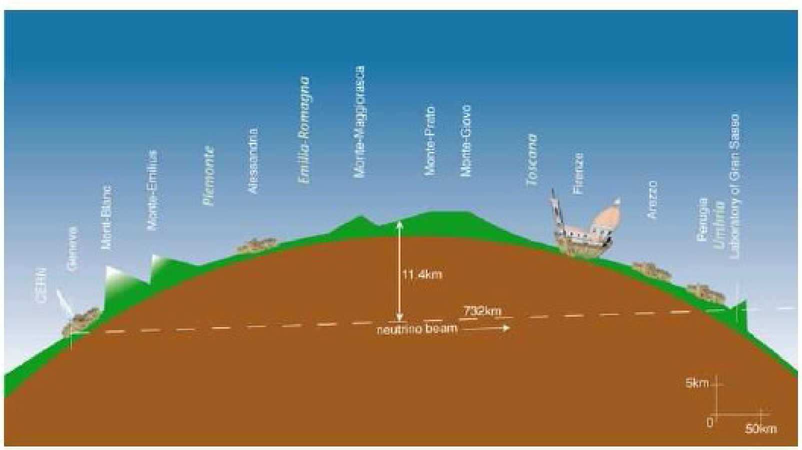
Gargamelle - Muonless

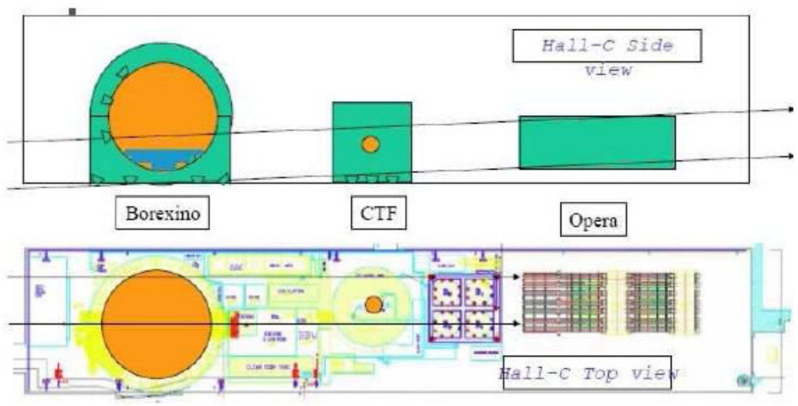


BEBC – Muon + Hadronic shower

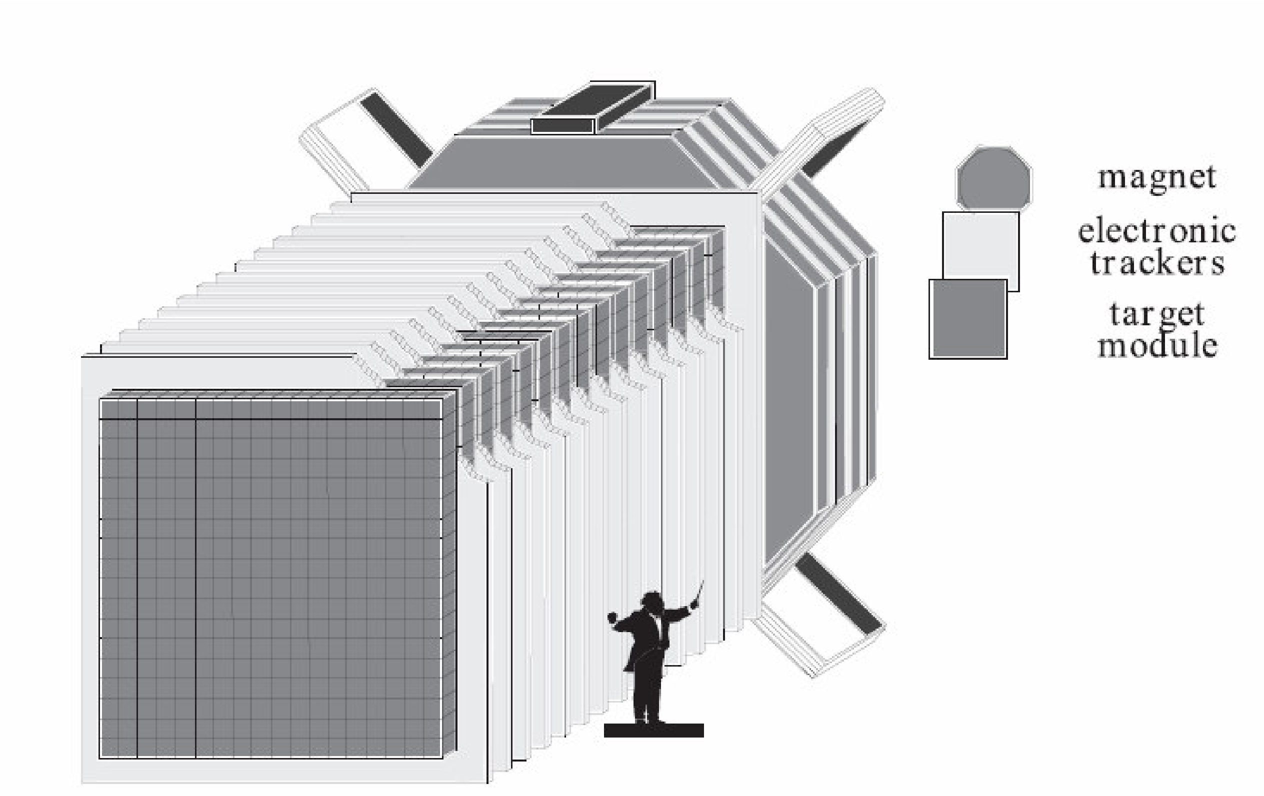
Neutrino Beam: CERN to Gran Sasso





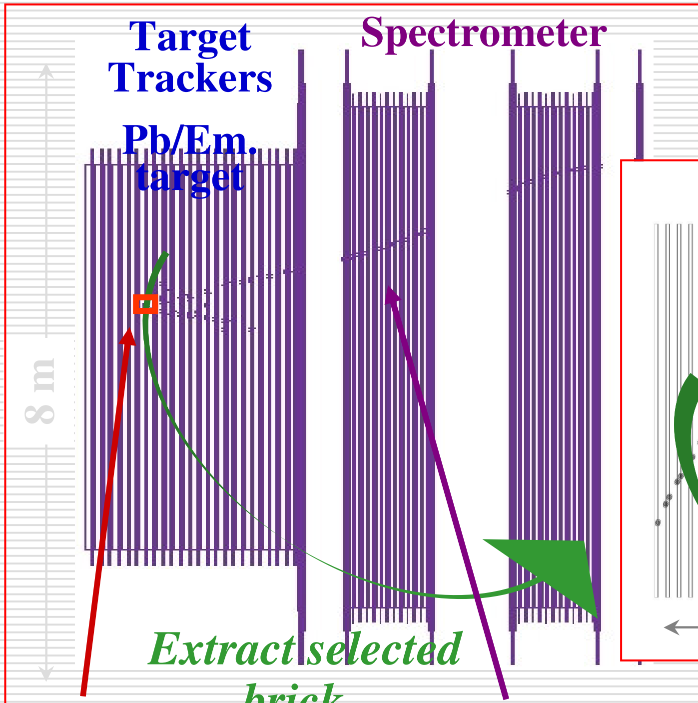


OPERA - I



OPERA - II

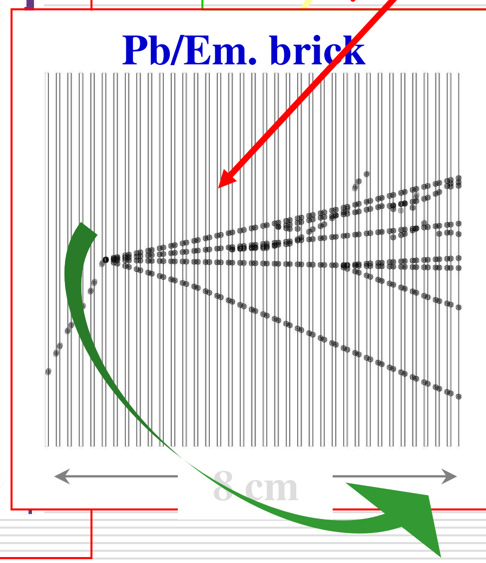
Electronic detectors:



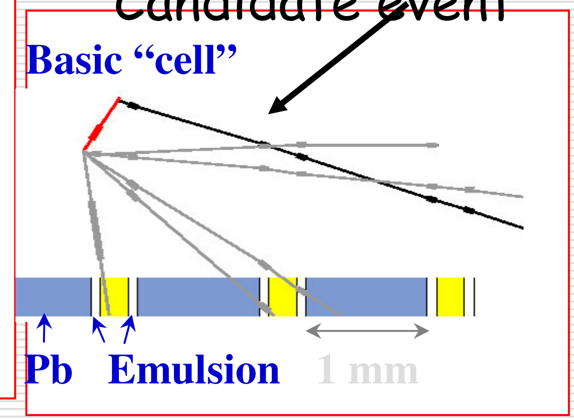
Brick finding   muon ID, charge and p

Emulsion analysis:

Vertex, decay kink e/γ ID, multiple scattering, kinematics



Link to mu ID, Candidate event



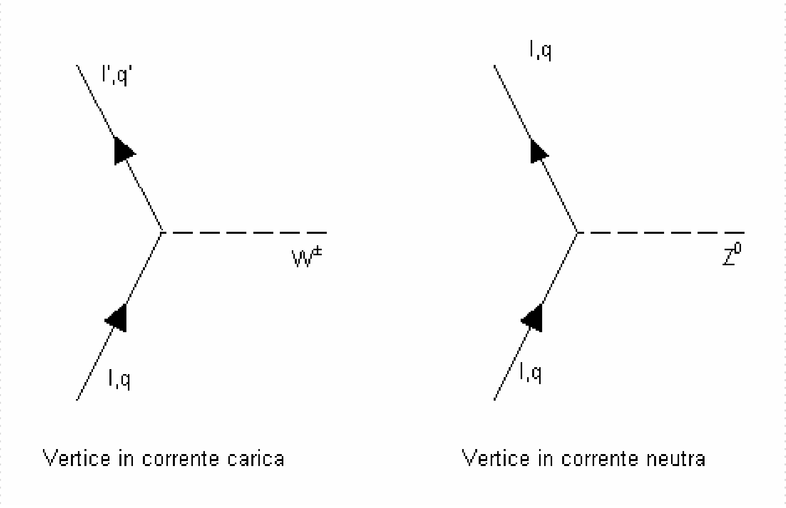
Weak Interaction - I

Full structure of the weak interaction: More advanced topic, at the heart of the Standard Model

Just listing some important conclusions

A) Quarks and leptons interact through the exchange of *vector particles*

Charged ($W^{\pm}$) Neutral (Z^0)

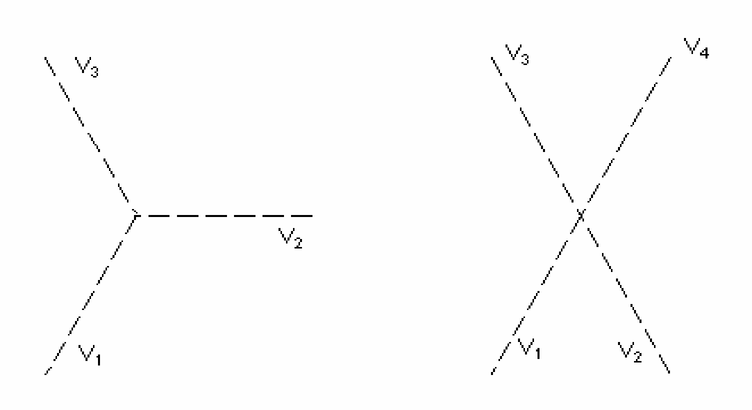


Weak Interaction - II

B) Exchanged vector bosons are *(very)* massive

C) Interaction form derived by a non-Abelian gauge symmetry
+
Special mechanism giving mass to some of the gauge fields

D) Non-Abelian vertexes



Weak Interaction - III

Weak charged current:

$$j^{\mu} = \bar{u}_f \gamma^{\mu} \frac{\left(1-\gamma^5\right)}{2} u_i$$

$$j_V^{\mu} = \frac{1}{2} \bar{u}_f \gamma^{\mu} u_i \qquad \text{Vector}$$

$$j_A^{\mu} = \frac{1}{2} \bar{u}_f \gamma^{\mu} \gamma^5 u_i \quad \text{Axial}$$

Electromagnetic current:

$$j^{\mu} = \bar{u} \gamma^{\mu} u$$

Apparently completely different. But:

$$\left(1-\gamma^5\right)^2 = \left(1-2\gamma^5+1\right) = \left(2-2\gamma^5\right) = 2\left(1-\gamma^5\right)$$

$$\rightarrow \left[\frac{\left(1-\gamma^5\right)}{2}\right]^2 = \frac{2\left(1-\gamma^5\right)}{4} = \frac{\left(1-\gamma^5\right)}{2}$$

$$\rightarrow \bar{u}_f \gamma^{\mu} \frac{\left(1-\gamma^5\right)}{2} u_i = \bar{u}_f \gamma^{\mu} \left[\frac{\left(1-\gamma^5\right)}{2}\right]^2 u_i = \underbrace{\bar{u}_f \frac{\left(1+\gamma^5\right)}{2}}_{\bar{u}_{fL}} \gamma^{\mu} \underbrace{\frac{\left(1-\gamma^5\right)}{2} u_i}_{u_{iL}} = \bar{u}_{fL} \gamma^{\mu} u_{iL}$$

Same form, but involving only the *LEFT* chiral states

Weak Interaction - IV

$$j_\mu^{em} = \overline{e}\gamma_\mu e$$

$$e \equiv \left(\frac{1-\gamma_5}{2}\right)e + \left(\frac{1+\gamma_5}{2}\right)e = e_L + e_R$$

Therefore:

$$\rightarrow j_\mu^{em} = \overline{e}\gamma_\mu e = \left(\overline{e}_L + \overline{e}_R\right)\gamma_\mu\left(e_L + e_R\right) = \overline{e}_L\gamma_\mu e_L + \overline{e}_R\gamma_\mu e_R$$

Because:

$$\overline{e}_L\gamma_\mu e_R = e\left(\frac{1+\gamma_5}{2}\right)\gamma_\mu\left(\frac{1+\gamma_5}{2}\right)e = e\gamma_\mu\left(\frac{1-\gamma_5}{2}\right)\left(\frac{1+\gamma_5}{2}\right)e = 0$$

$$\left(\frac{1-\gamma_5}{2}\right)\left(\frac{1+\gamma_5}{2}\right) = 0$$ Projectors 'orthogonal'

The bottom line:

Weak (charged) and Electromagnetic currents:
Same Lorentz structure (vector), but LL *vs.* $LL+RR$

Weak Interaction - V

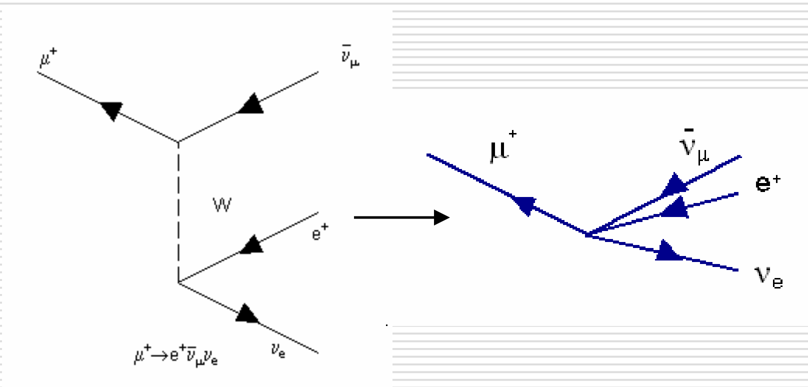
W propagator: $-i\dfrac{g_{\mu\nu}-q_\mu q_\nu / M_W^2}{q^2-M_W^2}$

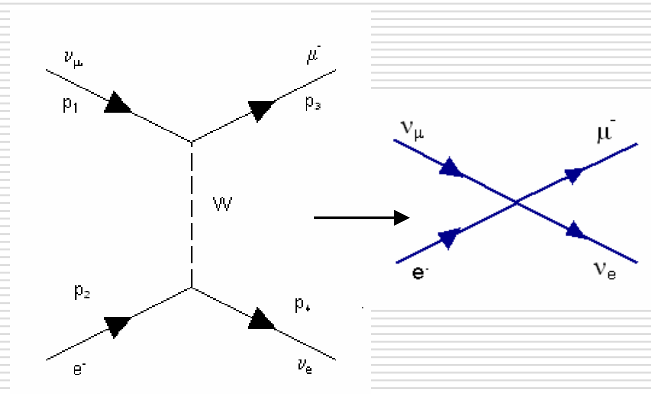
$q^2 \ll M_W^2 \rightarrow -i\dfrac{g_{\mu\nu}-q_\mu q_\nu / M_W^2}{q^2-M_W^2} \approx i\dfrac{g_{\mu\nu}}{M_W^2}$ $\quad q^2$-independent

$$T_{fi} \cong \left(\frac{1}{2\sqrt{2}}\right)^2 g_W^2 \left(\overline{u}_f^{(1)} \frac{1}{2}\gamma^\mu\left(1-\gamma^5\right)u_i^{(1)}\right) i\frac{g_{\mu\nu}}{M_W^2}\left(\overline{u}_f^{(2)} \frac{1}{2}\gamma_\nu\left(1-\gamma_5\right)u_i^{(2)}\right) = i\frac{1}{8}4\sqrt{2}G_F \quad j^{\mu(1)} j_\mu^{(2)}$$

$g_W^2 \equiv \alpha_W$ Charged current coupling constant

$G_F = \dfrac{\sqrt{2}}{8}\left(\dfrac{g_W}{M_W}\right)^2$ Fermi constant

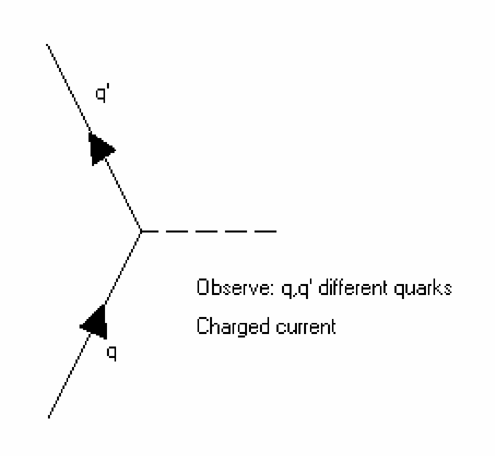




Universality: Quarks

Semileptonic and non leptonic processes understood in terms of quarks

Basically similar coupling to leptonic charged currents:

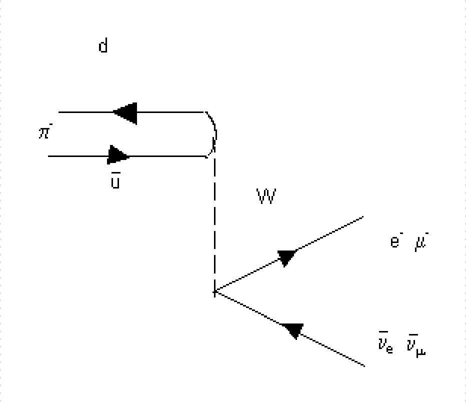


Picture is slightly more complicated, however
Fundamental question:

Is the quark coupling identical to the lepton one?

Semileptonic: π, *K* Decay - I

$\pi \to \mu + \nu_\mu, \;\; \pi \to e + \nu_e$



$$T_{fi} = \frac{G_F}{\sqrt{2}} \left[\bar{u}(3) \gamma^\mu \left(1 - \gamma^5\right) v(2) \right] F_\mu$$

F_μ: equivalent of 'current' for the $q\bar{q}$ bound state

$F_\mu = f_\pi p_\mu$ 4-momentum is the only 4-vector available

$$\Gamma_l = \frac{f_\pi^2}{8 m_\pi^3} G_F^2 m_l^2 \left(m_\pi^2 - m_l^2 \right)^2$$

Semileptonic: π, K Decay - II

$$\frac{\Gamma(e)}{\Gamma(\mu)} = \frac{m_e^2\left(m_\pi^2 - m_e^2\right)^2}{m_\mu^2\left(m_\pi^2 - m_\mu^2\right)^2} = 1.3\ 10^{-4}$$

Quite surprising: Phase space factor is much larger for e!
But:

Chirality rule:

ν is only L, $\bar{\nu}$ is only R

$$\rightarrow \begin{cases} \mu^-, e^- \text{ from } \pi,\ K \text{ decay forced to } R \\ \mu^+, e^+ \text{ from } \pi,\ K \text{ decay forced to } L \end{cases}$$

by angular momentum conservation:

For K decay:

$$\left.\begin{array}{l} H\left(\mu^-, e^-\right) = +1 \\ H\left(\mu^+, e^+\right) = -1 \end{array}\right\} \text{'Wrong' helicity}$$

e: Fully relativistic $\rightarrow P(wrong\ \ H) = \dfrac{1-\beta}{2} \sim 0$

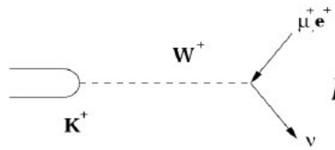
μ: Not fully relativistic $\rightarrow P(wrong\ \ H) = \dfrac{1-\beta}{2} \sim$ substantial

K Decays Ride Again: NA48/2

$$\frac{\Gamma(K \to l + \nu_l)}{\Gamma(\pi \to l + \nu_l)} = \frac{f_K^2 m_\pi^3 \left(m_K^2 - m_l^2\right)^2}{f_\pi^2 m_K^3 \left(m_\pi^2 - m_l^2\right)^2} = \begin{cases} (\text{exp.})\ 1.34 & l=\mu \\ (\text{exp.})0.19 & l=e \end{cases}$$

Within the Standard Model:

M. Finkemeier: Phys.Lett.B387:391-394,1996



$$R_M := \frac{\Gamma(M \to e\nu_e(\gamma))}{\Gamma(M \to \mu\nu_\mu(\gamma))} = \frac{m_e^2}{m_\mu^2}\left(\frac{m_M^2 - m_e^2}{m_M^2 - m_\mu^2}\right)^2 \left(1 + \delta R_M\right)$$

where δR_M arises from the radiative corrections, $M=\pi^\pm,K^\pm$

For $K^\pm$: $\delta R_K = -(3.78\pm0.04)\%$, leading to

$R_K = (2.472 \pm 0.001) * 10^{-5}$

The value of R_K could be different in case of SUSY and LFV models – the correction could be as high as 3% in both directions

Measurement of R_K tests the μ-e universality and provides a sensible test of the SM

Data taking starting now at the CERN SPS…

Cabibbo Angle - I

Consider charged current of leptons:
Very natural to group charged and neutral leptons into *doublets*, or *families*

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix} \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix} \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}$$

Within each doublet, charged current transitions are driven by the exchange of $W^\pm$ bosons

$$\begin{array}{ccccc} W^+ \longrightarrow & & \nu_e & & \longrightarrow W^+ \\ & \uparrow & & \downarrow & \\ W^- \longleftarrow & & e^- & & \longleftarrow W^- \end{array}$$

+ Similar for 2nd, 3rd family

Would seem natural to extend this scheme to quarks

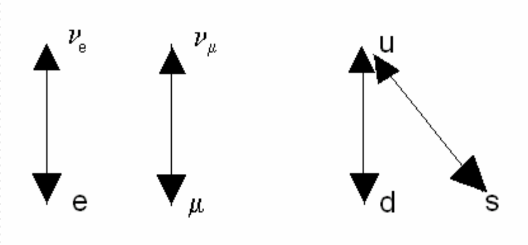
$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ s \end{pmatrix} \quad \begin{pmatrix} t \\ b \end{pmatrix} \qquad \begin{array}{ccccc} W^+ \longrightarrow & & u & & \longrightarrow W^+ \\ & \uparrow & & \downarrow & \\ W^- \longleftarrow & & d & & \longleftarrow W^- \end{array}$$

+ Similar for 2nd, 3rd family

Cabibbo Angle - II

Unfortunately, our scheme cannot work:

a) Parallelism quark-lepton is incomplete with 4 leptons and only 3 quarks
b) Does not account for strangeness violating processes



Cabibbo's very ingenious idea:

Quark flavor eigenstates (i.e., quark model eigenstates) are not to be identified with quark weak currents
$\rightarrow$*Weak currents are mixtures of different flavors*

By universal convention, mixing is assumed between d and s quarks:

$$\begin{cases} d' = \alpha d + \beta s \\ s' = \gamma d + \delta s \end{cases} \rightarrow \text{By unitarity: } \begin{cases} d' = \cos\theta_c d + \sin\theta_c s \\ s' = -\sin\theta_c d + \cos\theta_c s \end{cases}$$

Cabibbo Angle - III

In terms of doublet:

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix} \quad \theta_c \text{ Cabibbo's angle}$$

This explain *many* things:
Just one example
Get the angle from β decay

$$G_F^{(\beta)} = 0.975 G_F^{(\mu)}$$

$$\rightarrow \theta_c \simeq 13^0$$

Now assume for leptonic π, K decays (very simply!)

$$f_K = f \sin\theta_C$$

$$f_\pi = f \cos\theta_C$$

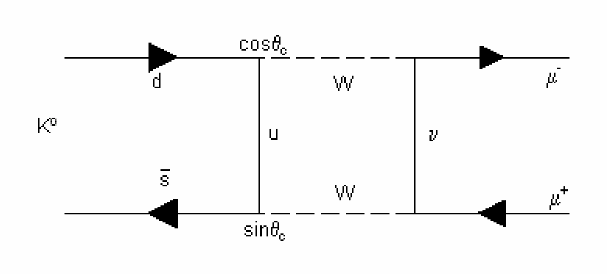
$$\rightarrow \frac{\Gamma(K \rightarrow l + \nu_l)}{\Gamma(\pi \rightarrow l + \nu_l)} = \tan^2\theta_C \frac{m_\pi^3 \left(m_K^2 - m_l^2\right)^2}{m_K^3 \left(m_\pi^2 - m_l^2\right)^2} = \begin{cases} (\text{teo.})\ 0.96\ l{=}\mu \\ (\text{teo.})\ 0.19\ l{=}e \end{cases} \begin{cases} (\text{exp.})\ 1.34\ l{=}\mu \\ (\text{exp.})\ 0.19\ l{=}e \end{cases}$$

Amazingly close!

GIM - I

Besides the many puzzles finally explained by Cabibbo, a few are left unexplained
Most relevant:

$K^0 \rightarrow \mu^+\mu^-$ strongly suppressed



2[nd] order weak process, still quite easy to compute
Expect rate higher by orders of magnitude

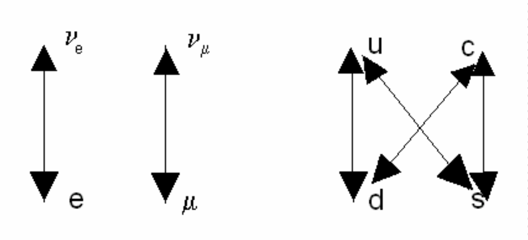
$BR_{meas}(K^0 \rightarrow \mu\mu) = (6.87 \pm 0.11)10^{-9}$

Glashow, Iliopoulos and Maiani:

There exists a fourth quark, call it c like charm

GIM - II

Full symmetry restored between lepton-quark families

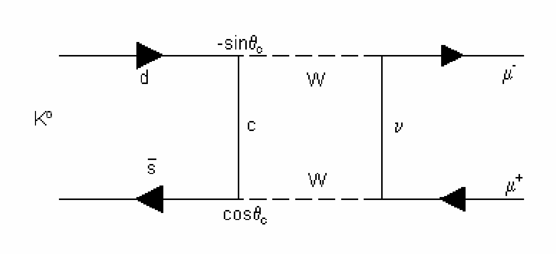


Weak doublets

$$\begin{pmatrix} u \\ d \end{pmatrix} \rightarrow \begin{pmatrix} u \\ d\cos\theta_C + s\sin\theta_C \end{pmatrix} \equiv \begin{pmatrix} u \\ d' \end{pmatrix}$$

$$\begin{pmatrix} c \\ s \end{pmatrix} \rightarrow \begin{pmatrix} c \\ -d\sin\theta_C + s\cos\theta_C \end{pmatrix} \equiv \begin{pmatrix} c \\ s' \end{pmatrix}$$

Then expect another amplitude for K^0 decay



$$\left.\begin{array}{l} A_1 \propto \sin\theta_c \cos\theta_c \\ A_2 \propto -\sin\theta_c \cos\theta_c \end{array}\right\} \rightarrow A_1 + A_2 \sim 0$$

Total amplitude not exactly *0* because $m_c \neq m_u$
From observed rate predict $m_c \sim 1-2 \quad GeV$
Leading to November Revolution: J/ψ discovery

CKM

Extend the idea to 3 families:
From Cabibbo's angle to Cabibbo-Kobayashi-Maskawa matrix

$$\begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} |d\rangle \\ |s\rangle \\ |b\rangle \end{bmatrix} = \begin{bmatrix} |d'\rangle \\ |s'\rangle \\ |b'\rangle \end{bmatrix}$$

From unitarity:
3 mixing angles
1 complex phase This can account for CP violation
(Complex phase not discussed)
Experimental values:

$$\begin{bmatrix} 0.9753 & 0.221 & 0.003 \\ 0.221 & 0.9747 & 0.040 \\ 0.009 & 0.039 & 0.9991 \end{bmatrix}$$

Observe:
Almost diagonal
Heavy quarks even more diagonal

Neutral Currents - I

General warning:

This subject is fully entangled with the building of the Standard Model core
Not covered in this course: Just a few remarks

Existence required by Standard Model consistence
Exchange of *neutral intermediate boson* Z^0, rather than charged $W^{\pm}$
Expect processes similar to electromagnetic, with coupling to *all fermions*
Typical signature: *Parity violation*

However, do not expect large effects at low energy, because:

Large Z^0 mass quenching down amplitudes
Electromagnetic amplitudes dominating at low energy

Some hope to see them in neutrino interactions (no e.m. contributions)
Not observed for a long time: Neutrino experiments difficult
Finally observed at CERN in 1973 in a bubble chamber experiment

Neutral Currents - II

As a result of the weak-electromagnetic unification, neutral currents are *different* from charged

Lorentz structure, not $V\text{-}A$
Coupling, not g_W

$$-i\frac{g_W}{\sqrt{2}}\gamma^\mu\frac{\left(1-\gamma^5\right)}{2}$$ Charged

$$-ig_Z\gamma^\mu\frac{\left(C_V^f-C_A^f\gamma^5\right)}{2}$$ Neutral

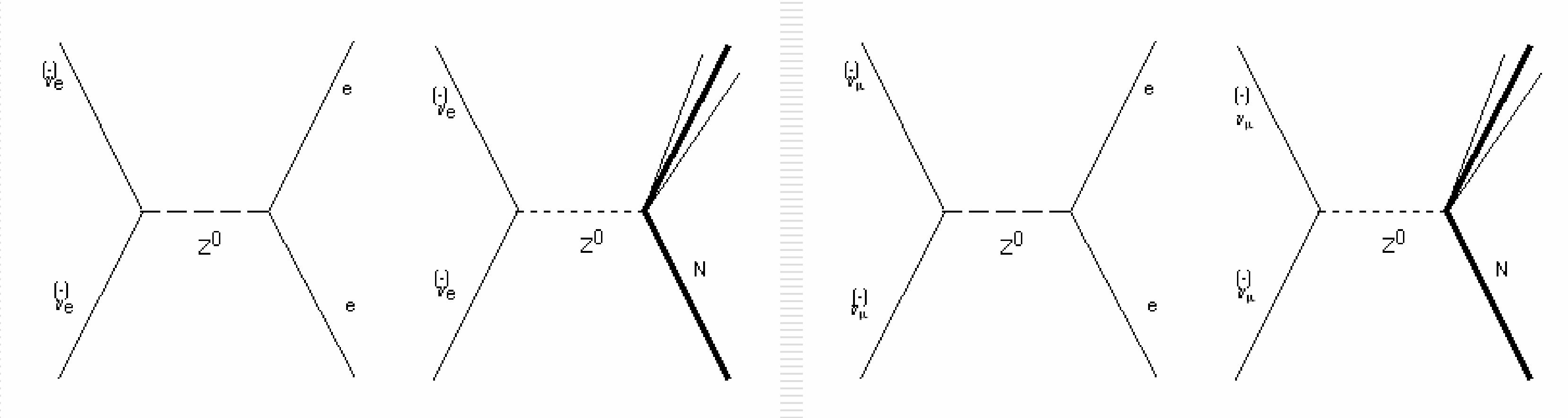
Particle	C_V	C_A
ν_e,ν_μ,ν_τ	$+1/2$	$+1/2$
e,μ,τ	$-1/2+2\sin\theta_W$	$-1/2$
u,c,t	$+1/2-4/3\sin^2\theta_W$	$+1/2$
d,s,b	$-1/2+2/3\sin^2\theta_W$	$-1/2$

Couplings:

$$\left.\begin{aligned} g_w &= \frac{e}{\sin\theta_W} \\ g_z &= \frac{e}{\sin\theta_W\cos\theta_W} \end{aligned}\right\}\theta_W\text{: Weinberg angle, new fundamental constant}$$

Neutral Currents - III

Expect to observe typical processes like:



$$\left(\nu_e,\bar{\nu}_e\right)+e\rightarrow\left(\nu_e,\bar{\nu}_e\right)+e$$

$$\left(\nu_\mu,\bar{\nu}_\mu\right)+e\rightarrow\left(\nu_\mu,\bar{\nu}_\mu\right)+e$$

$$\left(\nu_e,\bar{\nu}_e\right)+N\rightarrow\left(\nu_e,\bar{\nu}_e\right)+hadron\quad shower$$

$$\left(\nu_\mu,\bar{\nu}_\mu\right)+N\rightarrow\left(\nu_\mu,\bar{\nu}_\mu\right)+hadron\quad shower$$

Discovery of NC: Gargamelle

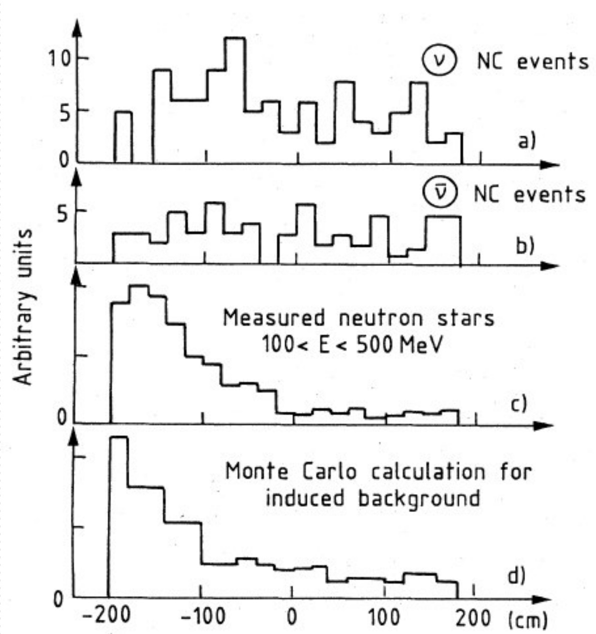
Main problem: Neutron background
Observe: Vertex position along the chamber

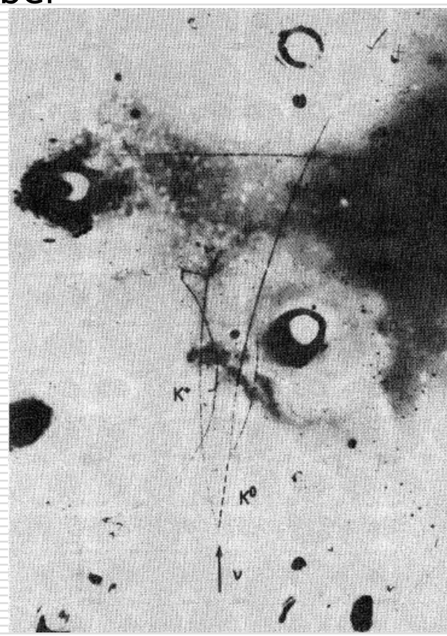
Exponential: Neutron background

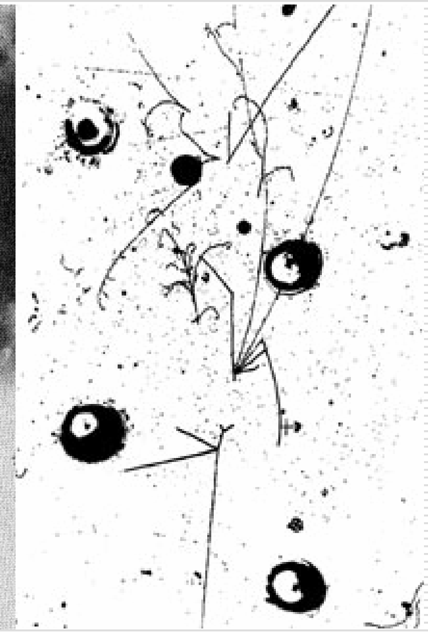
$\lambda_{\text{int}}^{n} \sim 90 \quad cm$

Flat: Neutrino signal

$\lambda_{\text{int}}^{\nu} \sim \infty$







Estimated n background: $9 \qquad 6$

Table 1

	ν-exposure	ν̄-exposure
No. of neutral-current candidates	102	64
No. of charged-current candidates	428	148