

Elementary Particles I

6 – Weak Interaction

Beta Decay, P & C Violations, Current-Current Interaction, Charged Currents of Leptons and Quarks, Cabibbo Angle, GIM, Neutral Currents

The Electroweak Interaction

Standard Model:

Electromagnetic and Weak Interaction unified into Electroweak

Energy scale where unification is evident:

$$E \sim M_W, M_Z \sim 100 \text{ GeV}$$

At lower energies:

Can still find tiny traces of the unification (Electroweak interference, Parity violation in atomic processes,...)

Electroweak interaction split into two, almost not interfering, *effective* interactions:

Non fundamental, useful low energy approximations

Electromagnetic
Weak

The Weak Interaction

Compare:

Strong interaction – *All quarks*

Electromagnetic interaction – *All quarks + Charged Leptons*

Weak interaction – *All quarks + All leptons*

Large variety of phenomena

Classify weak processes into 3 types:

Leptonic $\mu^+ \rightarrow e^+ + \bar{\nu}_\mu + \nu_e, \quad \nu_e + e \rightarrow \nu_e + e$

Semileptonic $\pi^+ \rightarrow \mu^+ + \nu_\mu, \quad \tau^+ \rightarrow \rho^+ + \nu_\tau$

Nonleptonic $K^0 \rightarrow \pi^+ + \pi^-, \quad \Lambda^0 \rightarrow n + \pi^0$

Lost Symmetries

Many violations in weak processes:

Space Parity (large)

Charge Parity (large)

CP (very small)

T (very small)

Flavor conservation (Isospin, S,C,B,T) (larger + smaller)

Lepton numbers (?) (neutrino oscillations)

Fall of Parity - I

Discovery of parity non conservation:
Originated by the so-called “ τ - θ puzzle”

Take K decays:
Weak process (S violation)
Observed decay modes (among many):

$$K^{\pm} \rightarrow \pi^{\pm} \pi^0 \qquad BR = 21.2 \%$$

$$K^{\pm} \rightarrow \pi^{\pm} \pi^{\pm} \pi^{\mp} \qquad BR = 5.6 \%$$

Observe:

$P_K = -1$, as measured in strong processes
 $P_K = ?$, as measured by its decays

Fall of Parity - II

Consider parity of the final states:

$$P|\pi\pi\rangle = (-1)(-1)(-1)^l = +1 \quad l=0 \quad \text{because } J_K = 0, J_\pi = 0$$

$$P|\pi\pi\pi\rangle = (-1)(-1)(-1)P_{orb} = (-1)P_{orb}$$

$$J_K = 0 = L_{\pi_1\pi_2} \oplus L_{\pi_3} \rightarrow L_{\pi_1\pi_2} = L_{\pi_3}$$

$$\rightarrow P_{orb} = (-1)^{L_{\pi_1\pi_2}} (-1)^{L_{\pi_3}} = +1$$

$$\rightarrow P|\pi\pi\pi\rangle = -1$$

???

2 different particles, same mass, opposite parity?

Lee & Yang suggestion: *Parity is violated in weak processes*

Violation of Parity

Parity violation discovered almost simultaneously in 3 experiments

First : *Beta decay* (Wu et al.)

Others: π - μ decay (Garwin et al., Friedman et al.)

(see before)

Interesting question:

How does parity violation manifest itself?

Breaking of parity selection rules

Interference between even/odd amplitudes $\rightarrow$ Asymmetries

Non-zero value of parity-odd observables

Violation of Charge Parity

Immediate conclusion: *C*-parity is also violated
Indeed, it can be shown:

$$A = \text{Scalar} + \text{Pseudoscalar} \rightarrow |A|^2 = |S + P|^2 = |S|^2 + |P|^2 + 2\text{Re}(SP^*)$$

$$\text{If } \begin{cases} CPT & \text{OK} \\ C & \text{OK} \end{cases}, \text{ by taking } \begin{cases} S \equiv |S|e^{i\alpha} \\ P \equiv |P|e^{i\beta} \end{cases} \rightarrow e^{i\alpha} = e^{i(\beta+\pi/2)}$$

$$\rightarrow SP^* = |S|e^{i\alpha} |P|e^{-i\beta} = |S||P|e^{i\alpha}e^{-i\beta} = |S||P|e^{i(\beta+\pi/2)}e^{-i\beta} = |S||P|e^{i\pi/2}$$

$$\rightarrow \text{Re}(SP^*) = 0 \rightarrow \text{Interference term} = 0 \rightarrow \text{Asymmetry} = 0$$

Since asymmetries *are* observed, *C* must be KO

P and *C* Violations - I

Consider decay of a polarized muon: $\mu^+ \uparrow \rightarrow e^+ + \bar{\nu}_\mu + \nu_e$, $\mu^- \uparrow \rightarrow e^- + \nu_\mu + \bar{\nu}_e$

P conservation: Predict

$$\left. \frac{d\Gamma_\pm}{d(\cos\theta)} \right|_\theta = \left. \frac{d\Gamma_\pm}{d(\cos\theta)} \right|_{\pi-\theta}$$

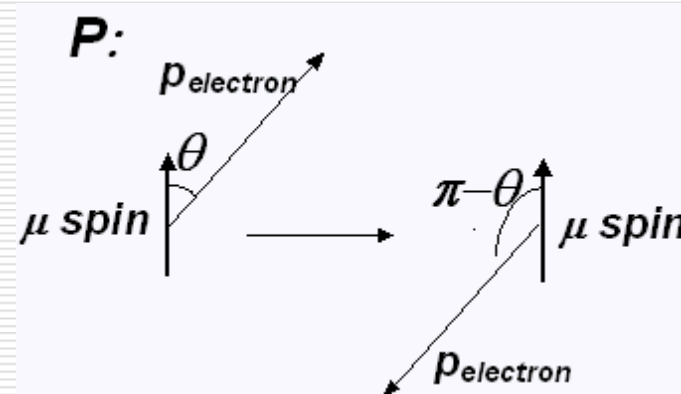
$$\rightarrow \frac{1}{2}\Gamma_\pm \left[1 - \frac{\xi_\pm}{3} \cos\theta \right] = \frac{1}{2}\Gamma_\pm \left[1 + \frac{\xi_\pm}{3} \cos\theta \right]$$

$$\rightarrow \xi_\pm = 0$$

Experiment:

$$\xi_+ = -\xi_- = -1$$

$\rightarrow P$ is violated



P and *C* Violations - II

C: $\mu^\pm \text{ etc} \rightarrow \mu^\mp \text{ etc}$

$$\frac{d\Gamma_\pm}{d(\cos\theta)} = \frac{1}{2}\Gamma_\pm \left[1 - \frac{\xi_\pm}{3} \cos\theta \right]$$

$$\Gamma_\pm = \frac{1}{\tau_\pm}, \quad \Gamma_+ = \Gamma_-$$

C conservation: Predict $\xi_+ = \xi_-$

Experiment:

$$\xi_+ = -\xi_- = -1$$

$\rightarrow$ *C* is violated

CP:

$$\left. \frac{d\Gamma_+}{d(\cos\theta)} \right|_\theta = \left. \frac{d\Gamma_-}{d(\cos\theta)} \right|_{\pi-\theta}$$

$$\rightarrow \xi_+ = -\xi_-$$

Experiment:

OK

$\rightarrow$ *CP* is conserved

Beta Decay - I

Most common weak process in ordinary matter

3 nucleon 'decays':

$n \rightarrow p + e^- + \bar{\nu}_e$ Allowed for free neutrons

$p \rightarrow n + e^+ + \nu_e$ Energetically forbidden for free protons

$e^- + p \rightarrow n + \nu_e$ Atomic electron capture, usually from a K-shell (K-capture)

Both β^- and β^+ are observed for nucleons bound in a nucleus

$(A, Z) \rightarrow (A, Z+1) + e^- + \bar{\nu}_e$ β^-

$(A, Z) \rightarrow (A, Z-1) + e^+ + \nu_e$ β^+

$e^- + (A, Z) \rightarrow (A, Z-1) + \nu_e$ *K - Capture*

Reminder: When found in a bound state, particles are *off-mass shell*

Beta Decay - II

Energy scale $\sim$ few MeV

Small energy released to the pair $e\nu$

$\rightarrow$ $e\nu$ orbital angular momentum

$= 0$ '*Allowed*' Most frequent

$= 1, 2, \dots$ '*Forbidden*' Rare (long lifetime)

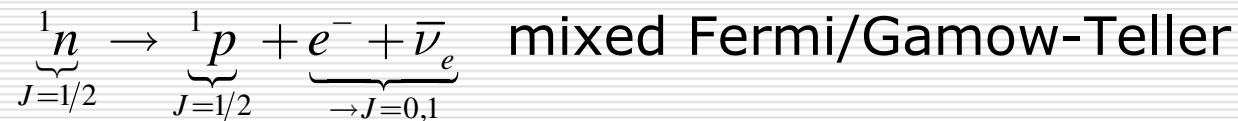
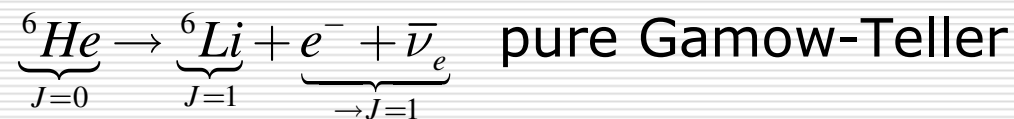
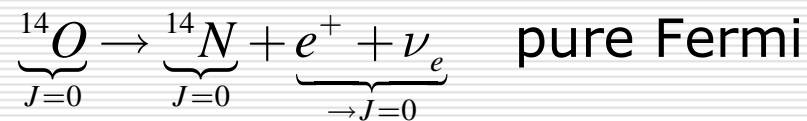
Allowed transitions:

$$J_{e\nu} = 1/2 \oplus 1/2 = \begin{cases} 0 & \text{singlet} \\ 1 & \text{triplet} \end{cases}$$

$$\rightarrow \Delta J_{nucleus} = \begin{cases} 0, & \Delta J_3 = 0 & \text{Fermi} \\ 1, & \Delta J_3 = 0, \pm 1 & \text{Gamow-Teller} \end{cases}$$

Beta Decay - III

Examples:



Beta Decay - IV

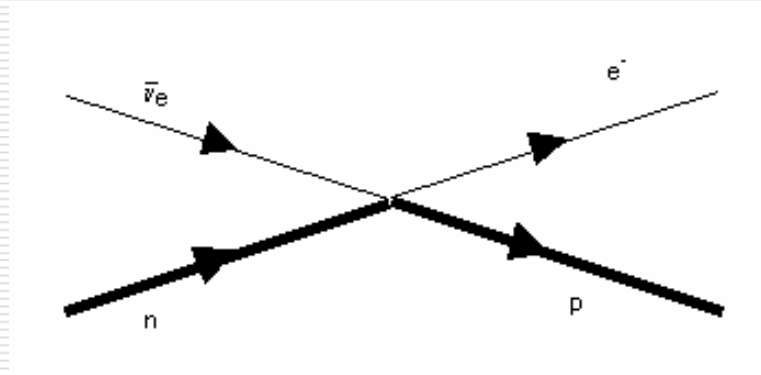
Take EM interaction as a model:

$$H_{\text{int}}^{EM} = j^\mu A_\mu \rightarrow j_{(a)}^\mu \frac{1}{q^2} j_{(b)\mu} \quad \text{for 2 interacting currents}$$

Fermi guess:

Without introducing any intermediate particle, current-current interaction:

$$H_{\text{int}}^W \propto j_{(a)} j_{(b)} \quad j_{(a)}, j_{(b)} \text{ transition currents for leptons, nucleons}$$



Beta Decay - V

Observe:

Sticking for a moment to parity conservation, any *current*current* product which is a *Lorentz scalar* is acceptable for H_I .

So we are free to guess different forms for the weak current

$j_{(a)} \propto \bar{\psi}_{fin} \Gamma \psi_{in}$ Operator Γ fixes the Lorentz structure of the current

Transitions involve charge variation

→ *Charged currents*

Beta Decay - VI

Fermi's original model: $\Gamma = \gamma_\mu$ Pure vector current

In general, Γ can be any of the set:

	1	γ^μ	$\sigma^{\mu\nu}$	$\gamma^5 \gamma^\mu$	γ^5
	Γ_1	Γ_2	Γ_3	Γ_4	Γ_5

$\bar{\psi}\psi$ scalar S

$\bar{\psi}\gamma^\mu\psi$ vector V

$\bar{\psi}\sigma^{\mu\nu}\psi$ tensor T

$\bar{\psi}\gamma^5\gamma^\mu\psi$ vector axial A

$\bar{\psi}\gamma^5\psi$ pseudoscalar P

Most general form: Hermitian conjugate required in order to account for processes involving antileptons

$$H_{\text{int}} = \sum_{i=S,V,T,A,P} C_i \left[(\bar{\psi}_p \Gamma_i \psi_n) (\bar{\psi}_e \Gamma^i \psi_\nu) + (\bar{\psi}_n \Gamma_i \psi_p) (\bar{\psi}_\nu \Gamma^i \psi_e) \right]$$

Rely on experiment to investigate the Lorentz structure

Beta Decay - VII

Non-relativistic limit of the different nucleon currents

$$S \quad 1 \quad \rightarrow \chi_p^\dagger \chi_n$$

$$V \quad \gamma_\mu \quad \left\{ \begin{array}{l} \mu=0 \rightarrow \chi_p^\dagger \chi_n \\ \mu=1,2,3 \rightarrow 0 \end{array} \right.$$

$$T \quad \sigma_{\mu\nu} \quad \left\{ \begin{array}{l} \mu=0, \nu \rightarrow 0 \\ \mu, \nu=0 \rightarrow 0 \\ \mu, \nu=1,2,3 \rightarrow \chi_p^\dagger \boldsymbol{\sigma} \chi_n \end{array} \right.$$

$$A \quad \gamma_\mu \gamma_5 \quad \left\{ \begin{array}{l} \mu=0 \rightarrow 0 \\ \mu=1,2,3 \rightarrow \chi_p^\dagger \boldsymbol{\sigma} \chi_n \end{array} \right.$$

$$P \quad \gamma_5 \quad \rightarrow 0$$

Conclude:

P not relevant for β -decay

S, V do not change nucleon spin $\rightarrow$ OK for Fermi

T, A do change nucleon spin $\rightarrow$ OK for Gamow-Teller

Beta Decay - VIII

Attempts to understand which terms are present in the interaction

Pure Fermi : S and/or V

Pure Gamow-Teller: T and/or A

In both cases:

If both terms are present, expect a distortion of the electron energy spectrum (*Fierz interference*)

Not observed $\rightarrow$ Fermi: S or V , Gamow-Teller: T or A

$\rightarrow$ Look for more indicators

Beta Decay - IX

Angular correlation electron-neutrino. Expect:

$$\frac{dN}{d\cos\theta} = \text{const} (1 + \lambda\beta\cos\theta), \quad \lambda = \begin{cases} -1 & S \\ +1 & V \\ +1/3 & T \\ -1/3 & A \end{cases}$$

Cannot observe neutrino → Observe recoiling nucleus instead
Many experiments made :

Difficult, inconclusive, sometimes wrong, leading to mistakenly guess S & T

Solution finally found after the discovery of parity non conservation,
by ignoring (wrong) experimental data

Beta Decay - X

To yield parity violation, H must include both *scalar* and *pseudo-scalar* terms.

Indeed, for any matrix element between initial and final states:

$$\begin{aligned} |\langle f | S + P | i \rangle|^2 &= |\langle f | S | i \rangle|^2 + |\langle f | P | i \rangle|^2 + 2\langle f | S | i \rangle \langle f | P | i \rangle^* \\ \xrightarrow{\text{Parity}} |\langle f | S | i \rangle|^2 + |\langle f | (-P) | i \rangle|^2 + 2\langle f | S | i \rangle \langle f | (-P) | i \rangle^* \\ &= |\langle f | S | i \rangle|^2 + |\langle f | P | i \rangle|^2 - 2\langle f | S | i \rangle \langle f | P | i \rangle^* \end{aligned}$$

By allowing for parity non conservation, can write down:

$$H_{\text{int}} = \sum_{i=S,V,T,A} \left[C_i \underbrace{(\bar{\psi}_p \Gamma_i \psi_n)}_S (\bar{\psi}_e \Gamma^i \psi_\nu) + C_i' \underbrace{(\bar{\psi}_p \Gamma_i \psi_n)}_P (\bar{\psi}_e \Gamma^i \gamma^5 \psi_\nu) \right] \quad C_i, C_i' \text{ 'constants'}$$

+ Hermitian conjugate always understood

Beta Decay - XI

Actually $C_i = C_i(q^2)$, $C_i' = C_i'(q^2)$ Weak form factors

But: $q^2 \sim 0$ for β decay $\rightarrow$ Constants

γ^5 equivalently inserted in the nucleon current, no difference in full operator: Just a re-labeling of C, C'

Beta Decay - XII

Redefine constants by extracting $\frac{G_F}{\sqrt{2}}$ (G_F : Fermi constant)

$$H_{\text{int}} = \frac{G_F}{\sqrt{2}} \sum_{i=S,V,T,A} C_i (\bar{\psi}_p \Gamma_i \psi_n) \left(\bar{\psi}_e \Gamma^i \left(1 + \frac{C'_i}{C_i} \gamma^5 \right) \psi_\nu \right)$$

Time reversal invariance: C_i, C'_i real

In order to investigate the form of lepton current:

Measurement of the lepton longitudinal polarization

Reminder:

$P_L(\text{lept}) = \langle \text{lept} | \boldsymbol{\sigma}_l \cdot \hat{\mathbf{p}}_l | \text{lept} \rangle$ Long. Polarization = Average Helicity

$P_T(\text{lept}) = 0$, T non-invariant:

$$\boldsymbol{\sigma}_e \cdot (\mathbf{p}_e \times \mathbf{p}_\nu) \xrightarrow{T} -\boldsymbol{\sigma}_e \cdot (-\mathbf{p}_e \times -\mathbf{p}_\nu) = -\boldsymbol{\sigma}_e \cdot (\mathbf{p}_e \times \mathbf{p}_\nu)$$

Helicity/Chirality Refresher - I

With reference to Dirac equation:

$$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \boldsymbol{\gamma} = \begin{pmatrix} 0 & \boldsymbol{\sigma} \\ -\boldsymbol{\sigma} & 0 \end{pmatrix}, \gamma^5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \text{Dirac representation}$$

$$\mathbf{S} = \frac{\boldsymbol{\Sigma}}{2}, \boldsymbol{\Sigma} = \begin{pmatrix} \boldsymbol{\sigma} & 0 \\ 0 & \boldsymbol{\sigma} \end{pmatrix} = \underbrace{\gamma^0 \boldsymbol{\gamma}}_{\boldsymbol{\alpha}} \gamma^5 = \boldsymbol{\alpha} \gamma^5 \quad \text{Spin operator}$$

$$\Lambda = \frac{\boldsymbol{\Sigma} \cdot \mathbf{p}}{|\mathbf{p}|} \quad \text{Helicity operator}$$

$$\left. \begin{aligned} \Lambda u^{(+)} &= +u^{(+)} \\ \Lambda u^{(-)} &= -u^{(-)} \end{aligned} \right\} \quad \text{Helicity eigenstates}$$

$$P_{\pm} = \frac{1 \pm \Lambda}{2} \quad \text{Projection operators onto helicity eigenstates}$$

Helicity/Chirality Refresher - II

Projectors, indeed:

$$P_+ P_+ = \left(\frac{1+\Lambda}{2} \right) \left(\frac{1+\Lambda}{2} \right) = \frac{1}{4} (1 + \Lambda + \Lambda + \Lambda^2)$$

$$\Lambda^2 = \frac{(\boldsymbol{\Sigma} \cdot \mathbf{p})^2}{|\mathbf{p}|^2} = 1 \rightarrow P_+ P_+ = \frac{1}{4} (1 + 2\Lambda + 1) = \left(\frac{1+\Lambda}{2} \right) = P_+, \quad P_- P_- = P_-$$

$$P_+ P_- = \left(\frac{1+\Lambda}{2} \right) \left(\frac{1-\Lambda}{2} \right) = \frac{1}{4} (1 + \Lambda - \Lambda - \Lambda^2) = 0 = P_- P_+$$

$$1 = \frac{1-\Lambda}{2} + \frac{1+\Lambda}{2} = P_- + P_+ \rightarrow 1u = (P_+ + P_-)u = u_+ + u_-$$

$$\Lambda = \frac{\boldsymbol{\Sigma} \cdot \mathbf{p}}{|\mathbf{p}|} = \frac{\gamma^0 \boldsymbol{\gamma}}{\alpha} \gamma^5 \cdot \frac{\mathbf{p}}{|\mathbf{p}|} = \frac{\boldsymbol{\alpha} \cdot \mathbf{p}}{|\mathbf{p}|} \gamma^5 \rightarrow P_{\pm} = \frac{1 \pm \boldsymbol{\alpha} \cdot \hat{\mathbf{p}} \gamma^5}{2}$$

Helicity/Chirality Refresher - III

γ^5 Chirality operator

$P_L = \frac{1-\gamma^5}{2}, P_R = \frac{1+\gamma^5}{2}$ Projectors onto chirality eigenstates

$$\begin{cases} P_L u = u_L \\ P_R u = u_R \end{cases} \rightarrow 1u = (P_L + P_R)u = u_L + u_R$$

A very important limit:

$$\boldsymbol{\alpha} \cdot \mathbf{p} = E - \beta m$$

$$\Lambda = \frac{E - m \cdot \beta}{p} \gamma^5 \xrightarrow{E \gg m} \gamma^5$$

$$P_{\pm} = \frac{1 \pm \Lambda}{2} \xrightarrow{E \gg m} \frac{1 \pm \gamma^5}{2} = P_{R,L}$$

For high energy, or massless, particles:
Helicity projectors $\rightarrow$ *Chirality* projectors

Helicity, Chirality Refresher - IV

$$Eu = (\boldsymbol{\alpha} \cdot \mathbf{p} + \beta m)u$$

$$u = \begin{pmatrix} \phi \\ \chi \end{pmatrix}, \quad \phi, \chi \quad 2 \text{ components spinors}$$

$$\boldsymbol{\alpha} = \begin{pmatrix} \boldsymbol{\sigma} & 0 \\ 0 & -\boldsymbol{\sigma} \end{pmatrix}, \quad \beta = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \text{Dirac matrices in chiral representation}$$

$$\rightarrow \begin{cases} E\phi = (\boldsymbol{\sigma} \cdot \mathbf{p})\phi + m\chi \\ E\chi = -(\boldsymbol{\sigma} \cdot \mathbf{p})\chi + m\phi \end{cases}$$

$$\rightarrow \begin{cases} E\phi = (\boldsymbol{\sigma} \cdot \mathbf{p})\phi \\ E\chi = -(\boldsymbol{\sigma} \cdot \mathbf{p})\chi \end{cases}, m=0 \rightarrow \begin{cases} \frac{(\boldsymbol{\sigma} \cdot \mathbf{p})}{E=|\mathbf{p}|} \phi = \phi \\ \frac{(\boldsymbol{\sigma} \cdot \mathbf{p})}{E=|\mathbf{p}|} \chi = -\chi \end{cases} \rightarrow \phi, \chi \text{ Helicity eigenstates}$$

Chiral States - I

States with definite value of chirality, massive or massless particles

Particle

$$u_L = \frac{1}{2}(1 - \gamma^5)u$$

$$u_R = \frac{1}{2}(1 + \gamma^5)u$$

$$\bar{u}_L = \bar{u} \frac{1}{2}(1 + \gamma^5)$$

$$\bar{u}_R = \bar{u} \frac{1}{2}(1 - \gamma^5)$$

Antiparticle

$$v_L = \frac{1}{2}(1 + \gamma^5)v$$

$$v_R = \frac{1}{2}(1 - \gamma^5)v$$

$$\bar{v}_L = \bar{v} \frac{1}{2}(1 - \gamma^5)$$

$$\bar{v}_R = \bar{v} \frac{1}{2}(1 + \gamma^5)$$

Is it true? Try one example:

$$\gamma^5 u_L = \gamma^5 \frac{1}{2}(1 - \gamma^5)u = \frac{1}{2}(\gamma^5 - 1)u = -\frac{1}{2}(1 - \gamma^5)u = -u_L \quad \text{OK}$$

Chiral States - II

For chiral states:

$$\text{Massless particle: } \begin{cases} u_L & \langle H \rangle = -1 \\ u_R & \langle H \rangle = +1 \end{cases}$$

→ Helicity defined $\equiv$ Full longitudinal polarization

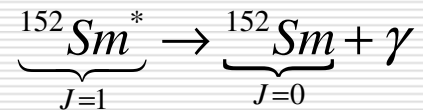
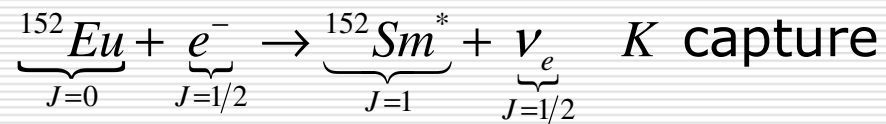
$$\text{Massive particle: } \begin{cases} u_L & \langle H \rangle = -\beta \\ u_R & \langle H \rangle = +\beta \end{cases}$$

$$\begin{aligned} \rightarrow \text{Helicity undefined, find} \quad & -1 \text{ prob. } \frac{1+\beta}{2}, \quad +1 \text{ prob. } \frac{1-\beta}{2} \\ & +1 \text{ prob. } \frac{1+\beta}{2}, \quad -1 \text{ prob. } \frac{1-\beta}{2} \end{aligned}$$

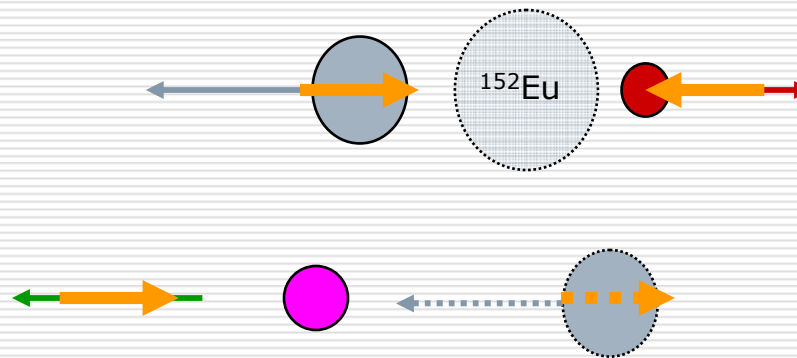
Massless particles: *Helicity is Lorentz invariant*

Neutrino Helicity - I

Neutrino can be *either R or L*: Must rely on experiment to decide
Goldhaber experiment: 2-steps decay



— ${}^{152}\text{Sm}^*$
— ν_e
— γ
→ *Spin*
— ${}^{152}\text{Sm}$



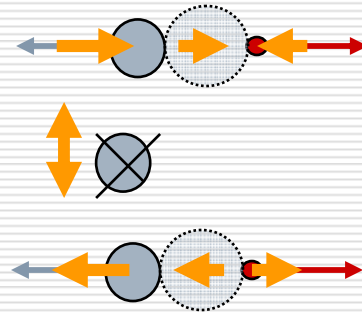
$$\rightarrow H({}^{152}\text{Sm}^*) = H(\nu_e)$$

Neutrino Helicity - II

γ emitted along momentum do not contribute orbital angular momentum:

$^{152}\text{Sm}^*$ with $s_z = 0$ cannot yield a collinear γ :

Would yield a non existing γ with $H(\gamma) = 0$



→ Collinear γ only from $^{152}\text{Sm}^*$ with $s_z = \pm 1$

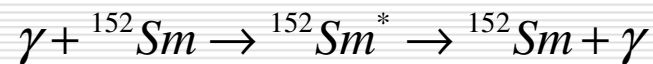
→ For collinear photons:

$$H(\gamma) = H(^{152}\text{Sm}^*) = H(\nu_e)$$

Neutrino Helicity - III

How to select collinear photons?

Choose a ^{152}Sm target:



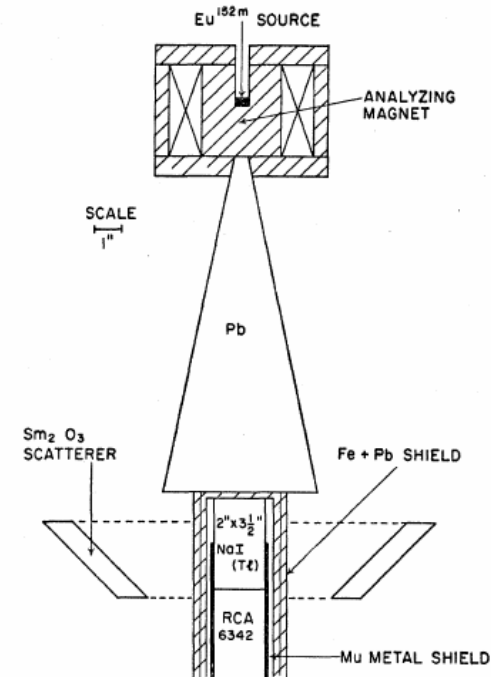
Resonant scattering:

γ energy must be just larger than ΔE

in order to account for nuclear recoil

Only available from collinear photons
because includes a fraction of the decaying
nucleus kinetic energy

Measure $H(\gamma)$ by absorbing γ
in magnetized iron



Result: $H(\nu_e) = -1$

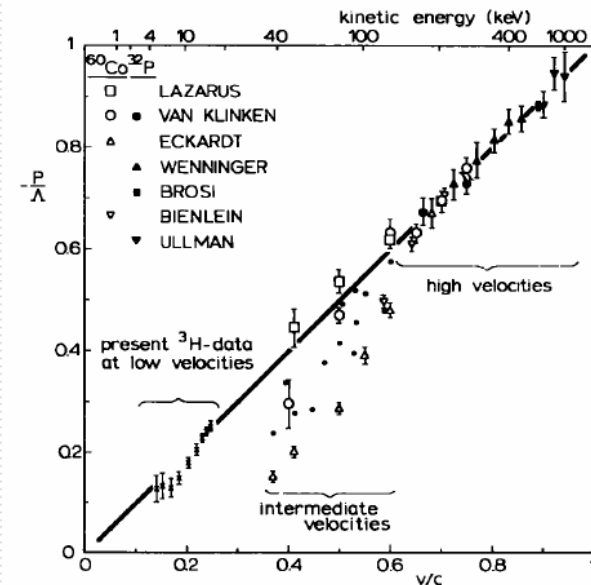
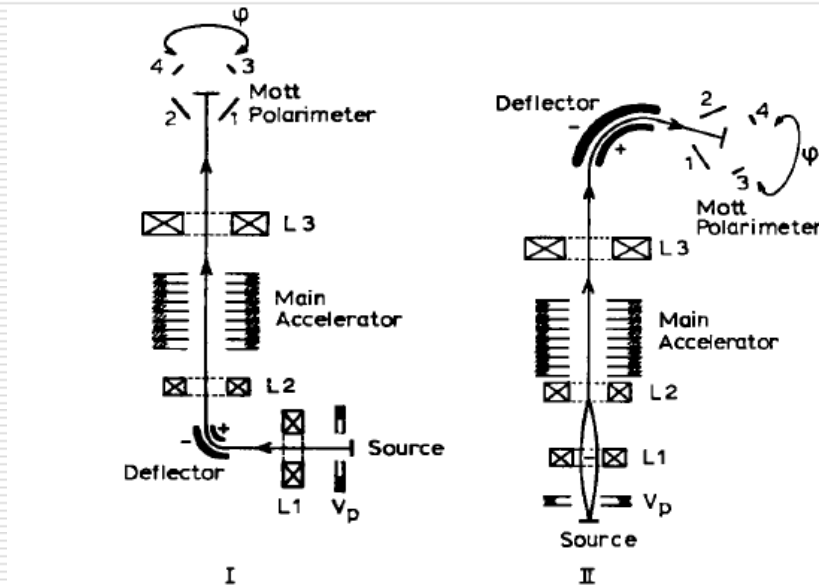
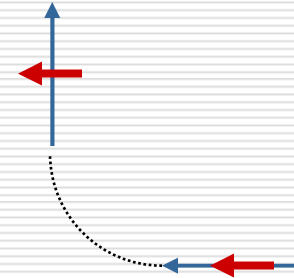
β Electron Helicity

Method: Rotate e momentum by 90° by means of an *electrostatic* field

→ Spin undeflected for non-relativistic e

→ Longitudinal polarization becomes transverse

→ Mott scattering sensitive to $P_\perp$



$$\langle H \rangle = -\beta \quad \text{Average helicity} \equiv \text{Long. polarization}$$

Mystery Solved: $V - A$ - I

Now closing on the analysis:

$$H_{\text{int}} = \frac{G_F}{\sqrt{2}} \sum_{i=V,A} C_i (\bar{\psi}_p \Gamma_i \psi_n) \left(\bar{\psi}_e \Gamma^i \left(1 + \frac{C_i'}{C_i} \gamma^5 \right) \psi_\nu \right)$$

Neutrino helicity = - 1 yields lepton current = $V - A$

$$\left(1 + \frac{C_i'}{C_i} \gamma^5 \right) \psi_\nu = (1 - \gamma^5) \psi_\nu \rightarrow C_i' = -C_i$$

$$= -\gamma^\mu (1 - \gamma^5)$$

$$\rightarrow H_{\text{int}} = \frac{G_F}{\sqrt{2}} \left[C_V (\bar{\psi}_p \gamma_\mu \psi_n) (\bar{\psi}_e \gamma^\mu (1 - \gamma^5) \psi_\nu) + C_A (\bar{\psi}_p \gamma_\mu \gamma_5 \psi_n) (\bar{\psi}_e \gamma^\mu \gamma^5 (1 - \gamma^5) \psi_\nu) \right]$$

$$= \frac{G_F}{\sqrt{2}} \left[C_V (\bar{\psi}_p \gamma_\mu \psi_n) (\bar{\psi}_e \gamma^\mu (1 - \gamma^5) \psi_\nu) - C_A (\bar{\psi}_p \gamma_\mu \gamma_5 \psi_n) (\bar{\psi}_e \gamma^\mu (1 - \gamma^5) \psi_\nu) \right]$$

$$= \frac{G_F}{\sqrt{2}} \left[C_V (\bar{\psi}_p \gamma_\mu \psi_n) - C_A (\bar{\psi}_p \gamma_\mu \gamma_5 \psi_n) \right] (\bar{\psi}_e \gamma^\mu (1 - \gamma^5) \psi_\nu)$$

Mystery Solved: $V - A$ - II

Therefore:

$$H_{\text{int}} = \frac{G_F}{\sqrt{2}} \left[C_V (\bar{\psi}_p \gamma_\mu \psi_n) - C_A (\bar{\psi}_p \gamma_\mu \gamma_5 \psi_n) \right] (\bar{\psi}_e \gamma^\mu (1 - \gamma^5) \psi_\nu)$$

$$H_{\text{int}} = \frac{G_F}{\sqrt{2}} C_V \left(\bar{\psi}_p \gamma_\mu \left(1 - \frac{C_A}{C_V} \gamma_5 \right) \psi_n \right) (\bar{\psi}_e \gamma^\mu (1 - \gamma^5) \psi_\nu)$$

Observe:

$$H_{\text{int}} = \frac{2G_F}{\sqrt{2}} \left(\bar{\psi}_p \gamma_\mu \left(1 - \frac{C_A}{C_V} \gamma_5 \right) \psi_n \right) \left(\bar{\psi}_e \gamma^\mu \frac{(1 - \gamma^5)}{2} \psi_\nu \right)$$

$$H_{\text{int}} = \frac{2G_F}{\sqrt{2}} \left(\bar{\psi}_p \gamma_\mu \left(1 - \frac{C_A}{C_V} \gamma_5 \right) \psi_n \right) \left(\bar{\psi}_e \gamma^\mu \left[\frac{(1 - \gamma^5)}{2} \right]^2 \psi_\nu \right)$$

$\frac{(1 - \gamma^5)}{2}$: Projection operator

$$\rightarrow H_{\text{int}} = \sqrt{2} G_F \left(\bar{\psi}_p \gamma_\mu \left(1 - \frac{C_A}{C_V} \gamma_5 \right) \psi_n \right) \left(\bar{\psi}_e \left(\frac{1 + \gamma^5}{2} \right) \gamma^\mu \left(\frac{1 - \gamma^5}{2} \right) \psi_\nu \right)$$

Lepton current written
as pure vector
between *chiral parts*
of ν, e states

Nucleon current $\#$ $V - A$, rather $V - \alpha A$

Neutrinos - I

$V-A$ theory: Neutrino has very peculiar properties

Remember: Helicity is a Lorentz invariant for a massless particle

Taking neutrinos as massless:

All neutrinos are left-handed

All anti-neutrinos are right-handed

Now consider P and C operations on neutrino states:

$$U_P |\nu_L\rangle = \eta_P |\nu_R\rangle$$

But: ν_R does not exist

$$U_C |\nu_L\rangle = \eta_C |\bar{\nu}_L\rangle$$

But: $\bar{\nu}_L$ does not exist

$$U_P U_C |\nu_L\rangle = \eta_P \eta_C |\bar{\nu}_R\rangle$$

$\rightarrow$ OK

Neutrinos - II

Recent, indirect yet convincing evidence that neutrinos have mass

What about $V-A$?

Not so many changes in the Standard Model:

If neutrinos have mass, they are no longer pure left-handed particles

Right-handed neutrinos exist

Separation into $+1$ and -1 *helicity* states is frame dependent

Separation into L and R *chirality* states is frame independent

Only the L chiral states (R for anti-neutrinos) contribute to charged current

Universality: Leptons

Consider muon weak interactions:

$$\mu^+ \rightarrow e^+ + \bar{\nu}_\mu + \nu_e, \quad \mu^- \rightarrow e^- + \nu_\mu + \bar{\nu}_e \quad \mu \text{ decay}$$

$$\mu^- + p \rightarrow n + \nu_\mu \quad \mu \text{ capture, involves nucleon current}$$

μ decay purely leptonic:

Guess

current-current, V-A

for both electron and muon charged currents

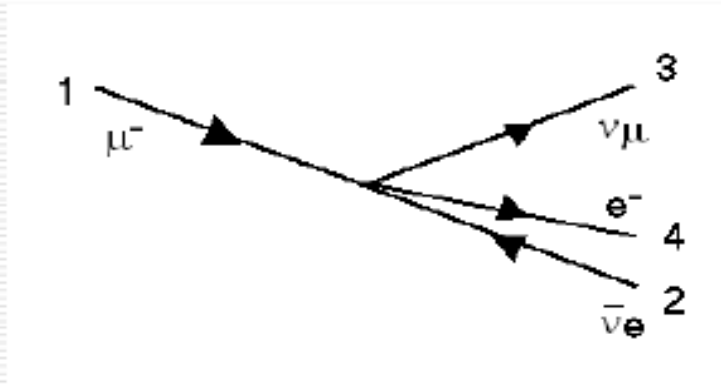
Compute:

μ Lifetime

Electron energy spectrum,

Electron longitudinal polarization

Muon Decay - I



Matrix element:

$$T_{fi} = \frac{G_F}{\sqrt{2}} [\bar{u}(3) \gamma^\mu (1 - \gamma^5) u(1)] [\bar{u}(4) \gamma_\mu (1 - \gamma_5) v(2)]$$

Squared m.e.:

$$T_{fi} T_{fi}^* = \frac{G_F^2}{2} [\bar{u}(3) \gamma^\mu (1 - \gamma^5) u(1)] [\bar{u}(3) \gamma_\nu (1 - \gamma_5) u(1)]^* \\ \cdot [\bar{u}(4) \gamma_\mu (1 - \gamma_5) v(2)] [\bar{u}(4) \gamma^\nu (1 - \gamma^5) v(2)]^*$$

Muon Decay - II

Sum & Average over spin projections:

$$\left\langle |T_{fi}|^2 \right\rangle = 64 G_F^2 (p_1 \cdot p_2)(p_3 \cdot p_4)$$

Muon rest frame:

$$p_1 = (m, 0, 0, 0)$$

$$\rightarrow p_1 \cdot p_2 = m E_2$$

$$p_1 = p_2 + p_3 + p_4$$

$$\rightarrow p_1 - p_2 = p_3 + p_4$$

Take electron as massless:

$$\rightarrow (p_3 + p_4)^2 = 2 p_3 \cdot p_4$$

$$\rightarrow 2 p_3 \cdot p_4 = p_1^2 - 2 p_1 \cdot p_2$$

$$\rightarrow 2 p_3 \cdot p_4 = m^2 - 2 m E_2$$

Muon Decay - III

Therefore:

$$\begin{aligned}\langle |T_{fi}|^2 \rangle &= 64 G_F^2 (p_1 \cdot p_2)(p_3 \cdot p_4) \\ \rightarrow \langle |T_{fi}|^2 \rangle &= 32 G_F^2 m^2 E_2 (m - 2E_2)\end{aligned}$$

Differential decay rate:

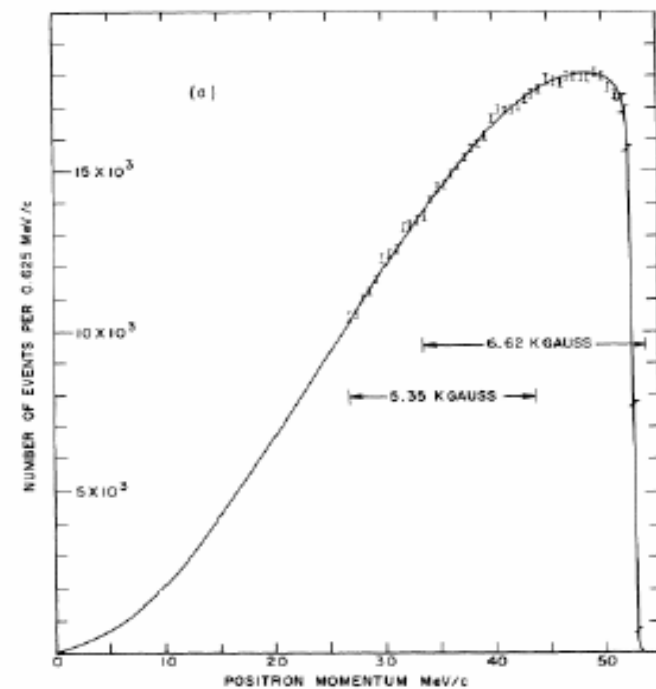
$$\begin{aligned}d\Gamma &= (2\pi)^4 \frac{\langle |T_{fi}|^2 \rangle}{2m} \delta(p_1 - p_2 - p_3 - p_4) \frac{d^3\mathbf{p}_2}{(2\pi)^2 2|\mathbf{p}_2|} \frac{d^3\mathbf{p}_3}{(2\pi)^2 2|\mathbf{p}_3|} \frac{d^3\mathbf{p}_4}{(2\pi)^2 2|\mathbf{p}_4|} \\ d\Gamma &= 16(2\pi)^4 G_F^2 m E_2 (m - 2E_2) \delta(p_1 - p_2 - p_3 - p_4) \frac{d^3\mathbf{p}_2}{(2\pi)^2 2|\mathbf{p}_2|} \frac{d^3\mathbf{p}_3}{(2\pi)^2 2|\mathbf{p}_3|} \frac{d^3\mathbf{p}_4}{(2\pi)^2 2|\mathbf{p}_4|}\end{aligned}$$

Muon Decay - IV

If we are interested in the electron energy spectrum:
Integrate over all variables except $|\mathbf{p}_4|$

Spectrum shape:

$$\frac{d\Gamma}{dE} = 32G_F^2 \frac{m_\mu^2 E^2}{2(4\pi)^3} \left(1 - \frac{4E}{3m_\mu}\right), E_{\max} = \frac{m}{2}$$



Muon Decay - V

$\tau = \frac{1}{\Gamma}$ Lifetime and total rate

$$\Gamma = \int_{E_{\min}}^{E_{\max}} \frac{d\Gamma}{dE} dE = \frac{32G_F^2 m_\mu^5}{12(8\pi)^3} \rightarrow \tau = \frac{192\pi^3}{G_F^2 m_\mu^5}$$

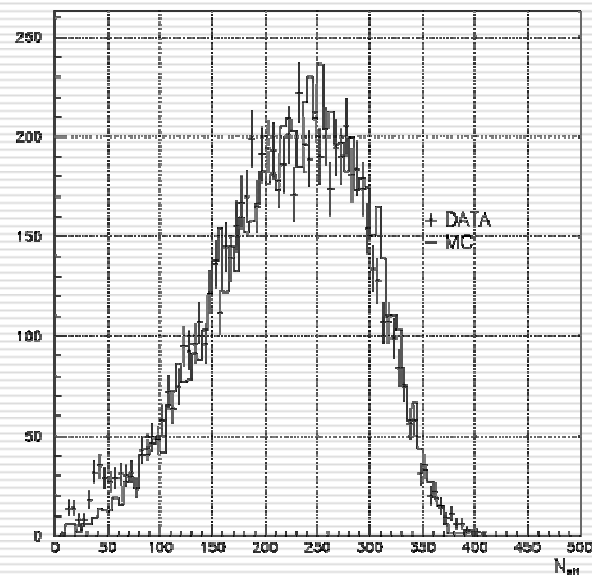
Extract G_F from measured lifetime:

$G_F^{(\beta)} = 0.98 \quad G_F^{(\mu)}$ Almost identical!

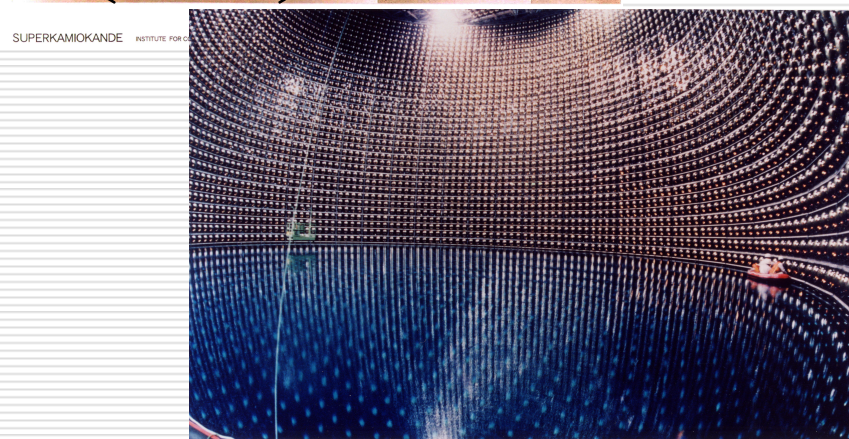
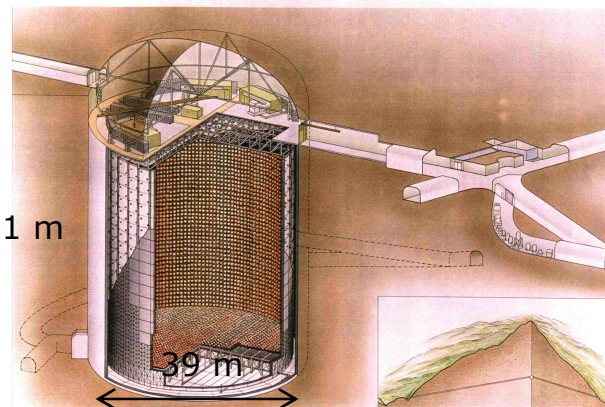
2% difference ??

Super K

Electron spectrum from μ decay used as calibration at SuperKamiokande
Allows for absolute calibration of Cerenkov light signal vs. energy



Water Cerenkov
50000 t pure water
11200 photomultipliers (50 cm Ø!)



The μ Michel Parameter

Detailed analysis:

Differential decay rate depending on several parameters

Alternative structures of charged current cast into different parameter sets

Most important: *Michel parameter* ρ

Electron differential spectrum determined by ρ for unpolarized muon

$$\frac{dP}{d\Omega dx} = \frac{G^2 m_\mu^5}{192\pi^4} x^2 \left[3(1-x) + \frac{2}{3}\rho(4x-3) \right], \quad x = \frac{2E_e}{m_\mu}$$

V-A theory predicts $\rho = 3/4$

Experimental result: 0.7517 ± 0.0026

Electron angular distribution for polarized muon decay:

$$\frac{dP}{d\Omega} = \frac{1}{4\pi} \left[1 - \frac{1}{3}\xi P \cos\theta \right], \quad P \text{ muon polarization}$$

V-A: $\xi = +1$ Experiment OK

Tau

Extend to τ *leptonic* decays:

$$\tau^+ \rightarrow e^+ + \bar{\nu}_\tau + \nu_e, \quad \tau^- \rightarrow e^- + \nu_\tau + \bar{\nu}_e$$

$$\tau^+ \rightarrow \mu^+ + \bar{\nu}_\tau + \nu_\mu, \quad \tau^- \rightarrow \mu^- + \nu_\tau + \bar{\nu}_\mu$$

τ has many more decay channels open into quark-antiquark pairs

Leptonic decays: Similar to muon

Allow for checking of:

V – A Lorentz structure

Universality of lepton coupling

The τ Michel Parameter

Consider τ decays:

Many modes, including

Leptonic

Semileptonic

$$\tau^\pm \rightarrow \mu^\pm + \nu_\mu / \bar{\nu}_\mu + \bar{\nu}_\tau / \nu_\tau \quad \tau^\pm \rightarrow \text{Hadrons} + \bar{\nu}_\tau / \nu_\tau$$

$$\tau^\pm \rightarrow e^\pm + \nu_e / \bar{\nu}_e + \bar{\nu}_\tau / \nu_\tau$$

A simple question:

What is the τ charged current?

Investigations at LEP:

$$e^+ + e^- \rightarrow Z^0 \rightarrow \tau^+ + \tau^-$$

Conclusion: Current is $V - A$

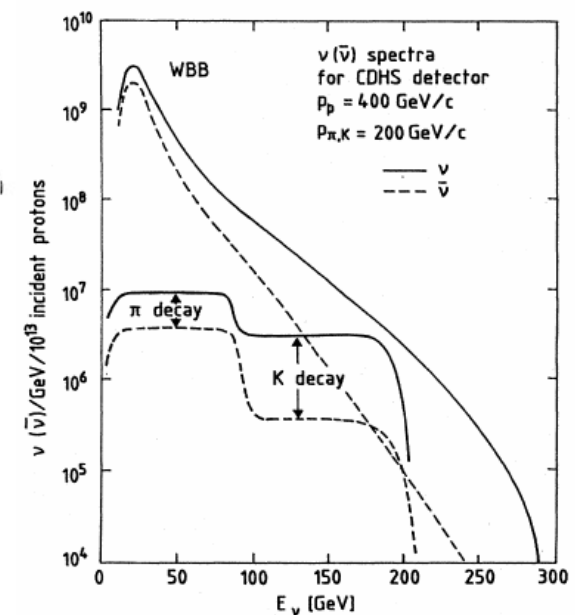
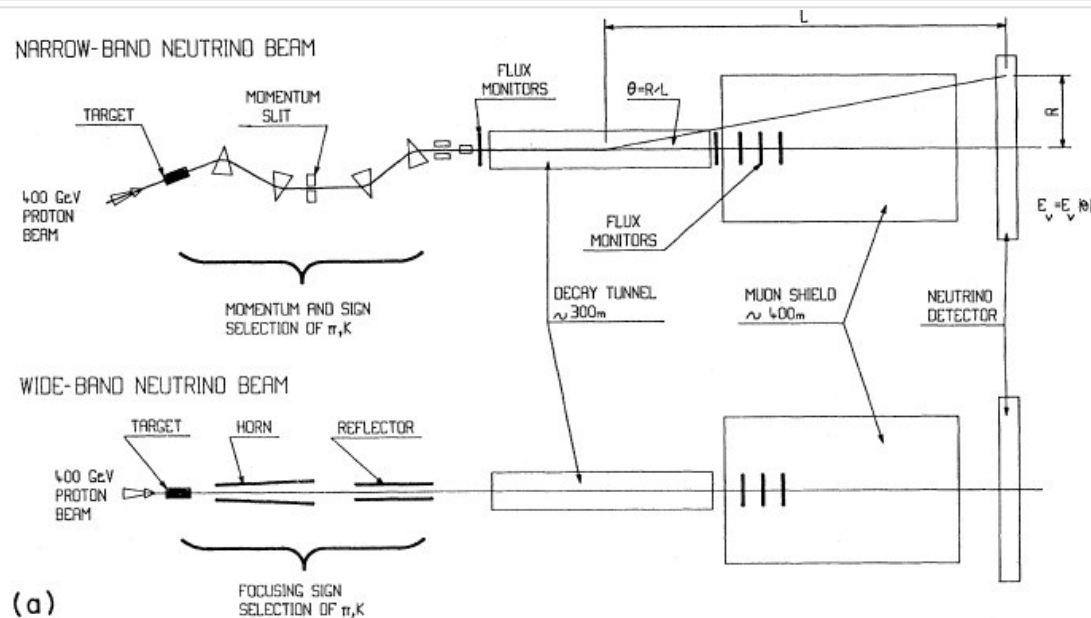
$$\rho_\tau = \begin{cases} 0.747 \pm 0.024 & e \text{ mode} \\ 0.776 \pm 0.049 & \mu \text{ mode} \end{cases}$$

Neutrino Beams

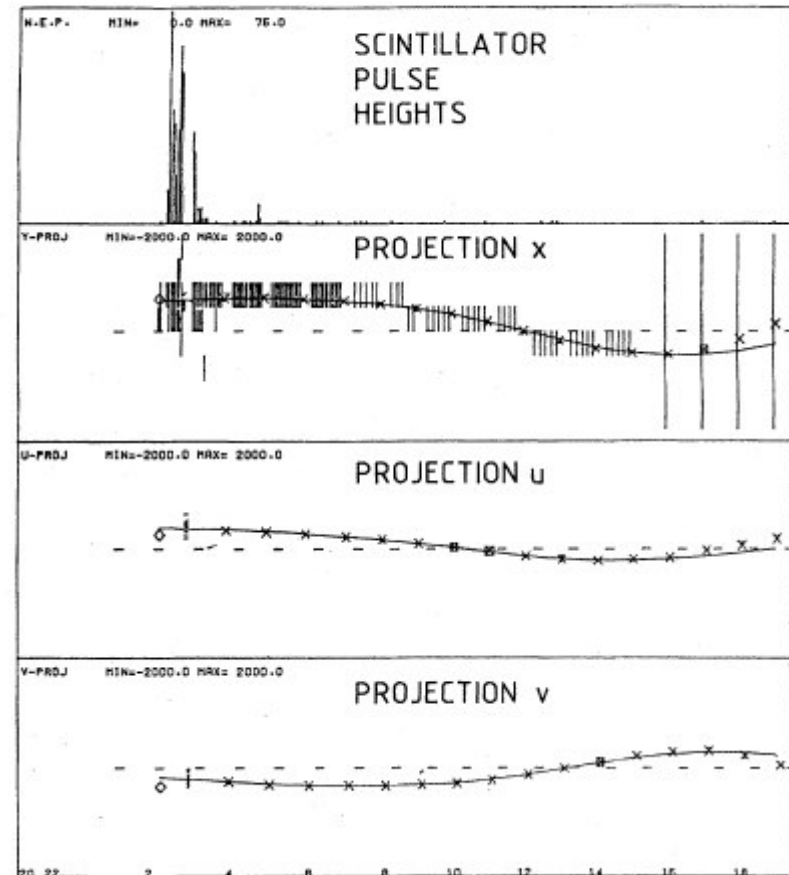
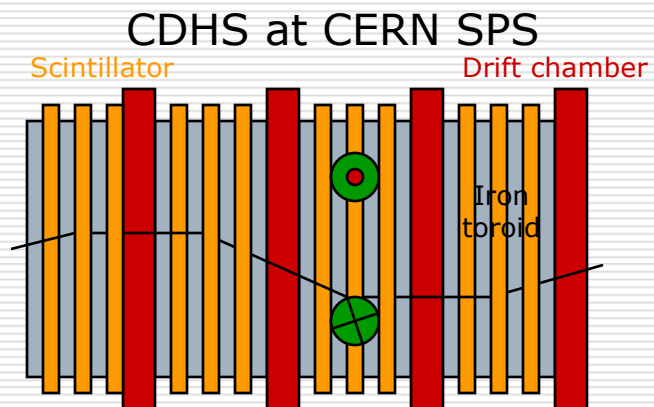
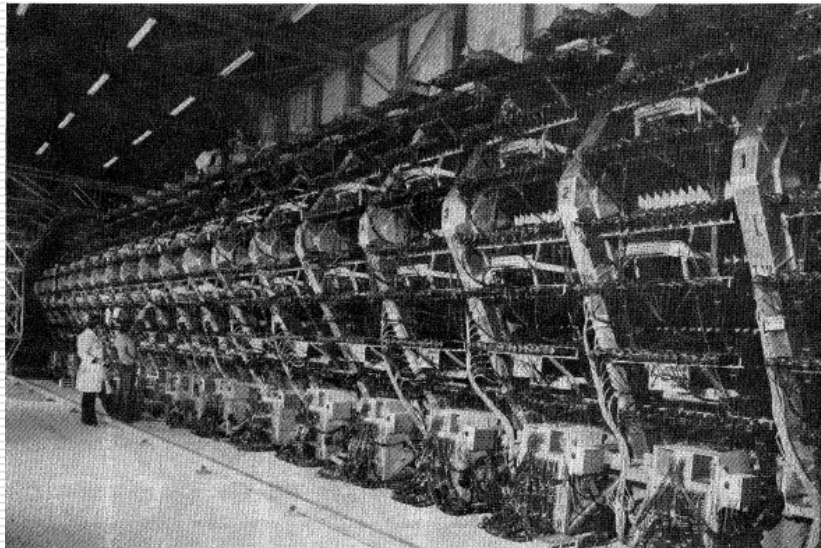
Derived from 2-body π, K decay

$$\pi^{\pm} \rightarrow \mu^{\pm} + \nu_{\mu}, \bar{\nu}_{\mu}$$

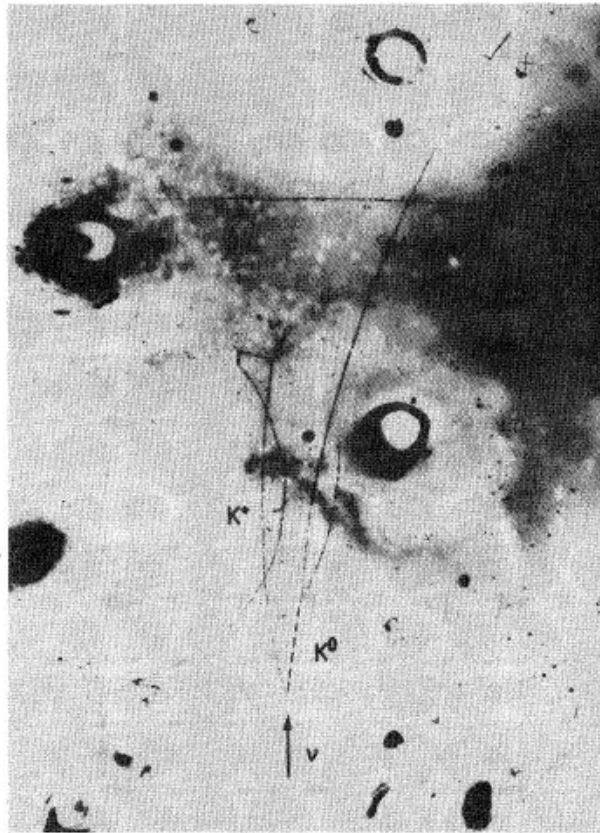
$$K^{\pm} \rightarrow \mu^{\pm} + \nu_{\mu}, \bar{\nu}_{\mu}$$



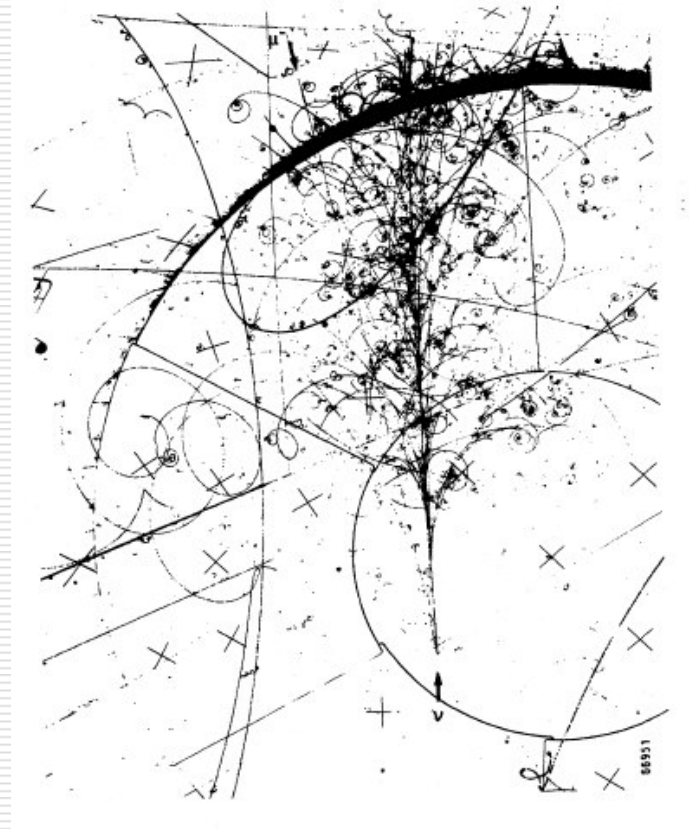
Neutrino Detectors - I



Neutrino Detectors - II

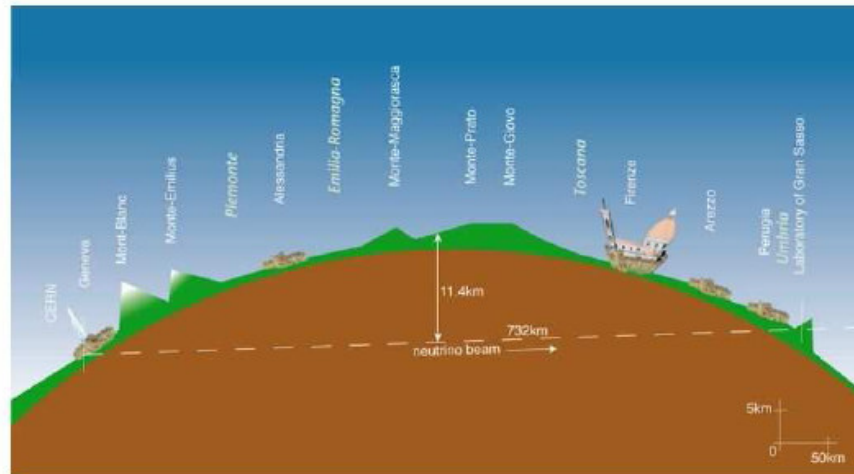
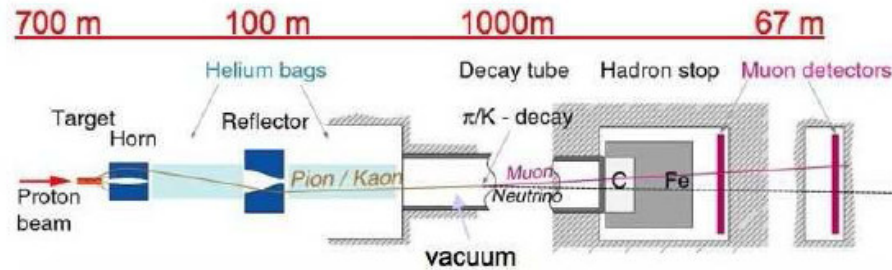


Gargamelle - Muonless

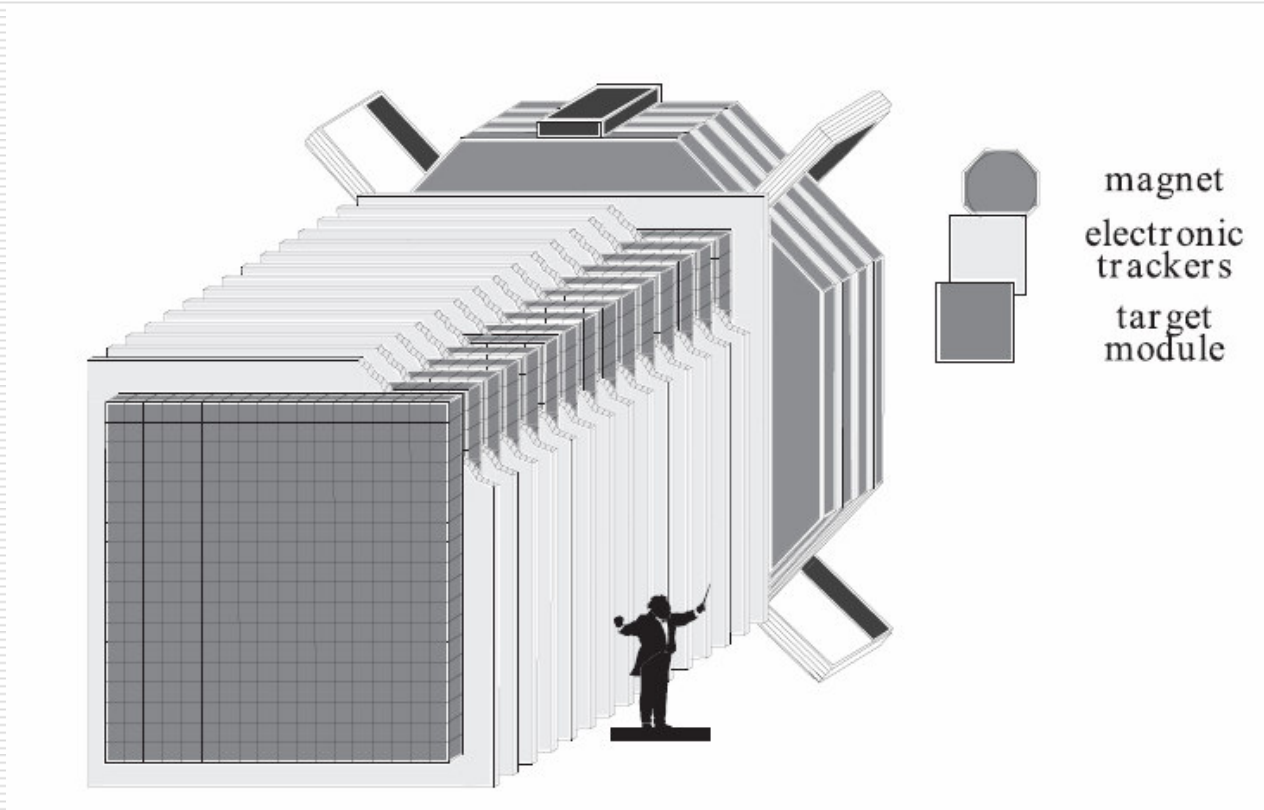


BEBC - Muon + Hadronic shower

Neutrino Beam: CERN to Gran Sasso

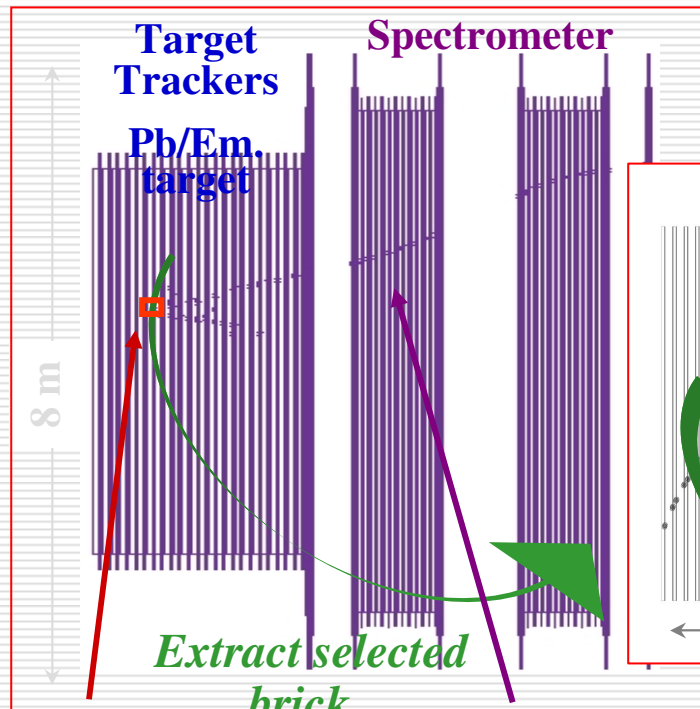


OPERA - I



OPERA - II

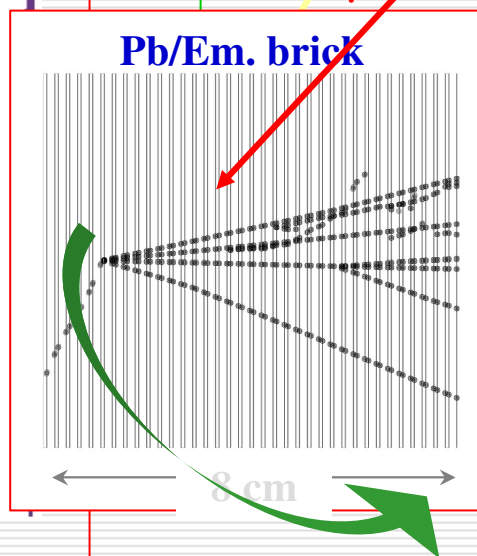
Electronic detectors:



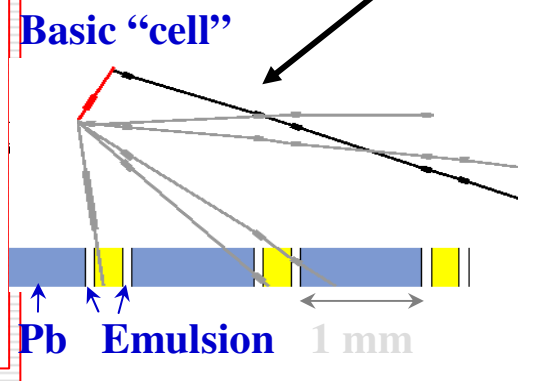
Brick finding muon ID, charge and p

Emulsion analysis:

Vertex, decay kink e/γ ID,
multiple scattering, kinematics



Link to mu ID,
Candidate event



Inverse Muon Decay - I

Charged current:

$$\nu_\mu + e^- \rightarrow \mu^- + \nu_e$$

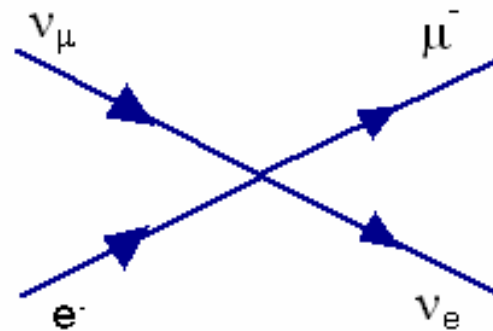
$$\sum_{spin} T_{fi} T_{fi}^* = \sum_{spin} \frac{G_F}{\sqrt{2}} [\bar{u}(3) \gamma^\mu (1 - \gamma^5) u(1)] [\bar{u}(3) \gamma_\nu (1 - \gamma^5) u(1)]^* \cdot$$

$$[\bar{u}(4) \gamma_\mu (1 - \gamma^5) u(2)] [\bar{u}(4) \gamma^\nu (1 - \gamma^5) u(2)]^*$$

$$\sum_{spin} [\bar{u}(a) \Gamma_1 u(b)] [\bar{u}(a) \Gamma_2 u(b)]^* = Tr[\Gamma_1 (\not{p}_b + m_b) \Gamma_2 (\not{p}_a + m_a)]$$

$$\sum_{spin} |T_{fi}|^2 = 64 G_F^2 (p_1 \cdot p_2) (p_3 \cdot p_4)$$

$$\sum_{spin} |T_{fi}|^2 = 256 G_F^2 E^4 \left[1 - \left(\frac{m_\mu}{2E} \right)^2 \right]$$



Inverse Muon Decay - II

$$\frac{d\sigma}{d\Omega} = \frac{G_F^2}{\pi^2} E^{*2} \left[1 - \left(\frac{m_\mu}{2E^*} \right)^2 \right]^2$$

$$\rightarrow \sigma = \frac{4G_F^2}{\pi} E^{*2} \left[1 - \left(\frac{m_\mu}{2E^*} \right)^2 \right]^2, \quad E^* \text{ CM energy of } e, \nu$$

$$E^* \simeq \sqrt{2mE_\nu}$$

$$\rightarrow \sigma = \frac{8G_F^2 m}{\pi} E_\nu$$

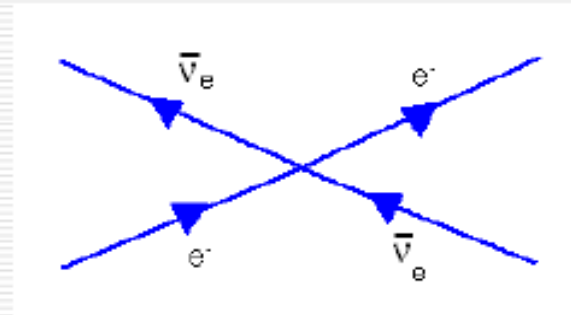
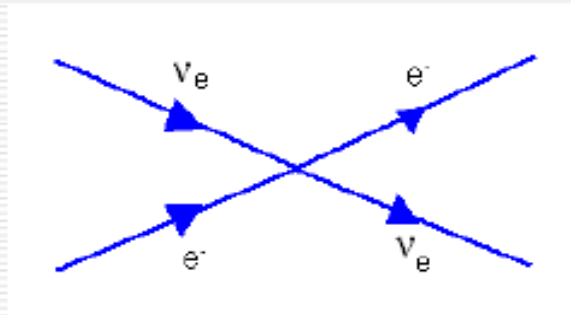
$$\rightarrow \sigma \propto E_\nu \text{ at high energy}$$

Neutrino – Lepton Scattering - I

Charged current ν_e - e scattering:

$$\nu_e + e^- \rightarrow e^- + \nu_e$$

$$\bar{\nu}_e + e^- \rightarrow e^- + \bar{\nu}_e$$



NB This is actually incomplete:

For these processes there are neutral current amplitudes as well

Cross sections must be evaluated by summing *all* relevant amplitudes

Neutrino – Lepton Scattering - II

Find for the total cross-section:

$$\left. \begin{aligned} \sigma_{\nu} &= \frac{4G_F^2}{\pi} E^{*2} \\ \sigma_{\bar{\nu}} &= \frac{1}{3} \frac{4G_F^2}{\pi} E^{*2} \end{aligned} \right\}, \quad E^* \text{ CM energy of } e, \nu$$

As divergent as inverse muon decay

NB These cross sections are only approximate, in that neutral current contribution is neglected

Beyond Fermi's Theory - I

Cross section cannot be strictly proportional to E_ν , of course:

Divergence at high energy!

Indeed, unitarity bound is violated around $E_\nu \sim 300 \text{ GeV}$

$$\sigma \leq \frac{4\pi}{k^2} = \frac{4\pi}{E^{*2}} \quad \text{Unitarity bound for S-Wave scattering}$$

Radiative corrections to Fermi's theory divergent:

No renormalization procedure available

Fermi theory is non renormalizable

Beyond Fermi's Theory - II

Current-Current must be a *low energy effective theory*:
Low energy approximation of a more general theory

Replace *contact interaction* (current-current) by *boson exchange*:
Modeled after the electromagnetic interaction

Exchanged particle must be

Charged (Charged current $\pm$)

Chiral (Parity violation)

Heavy (Fermi's point-like interaction OK at low energy)

Remark

Topics to follow:

Just a short introduction to essential phenomenology

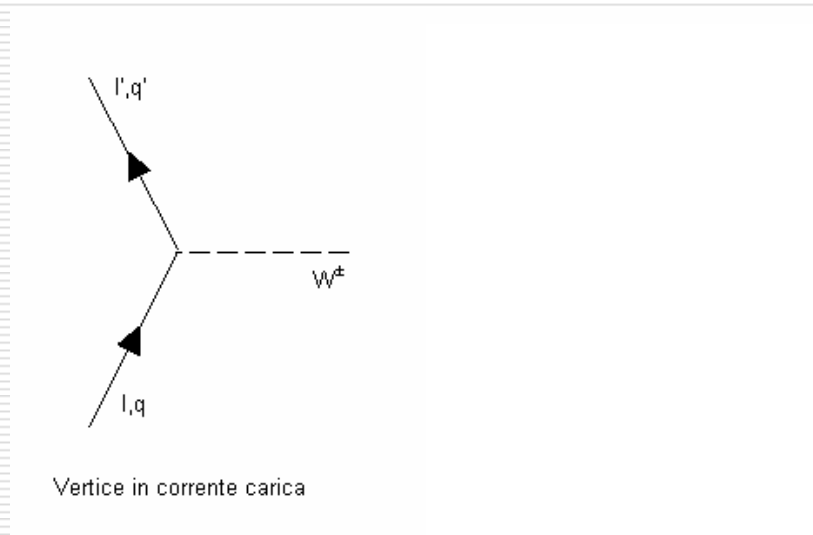
Core topics in the Standard Model

Fuller coverage of *electroweak interaction* in other courses

Weak Interaction - I

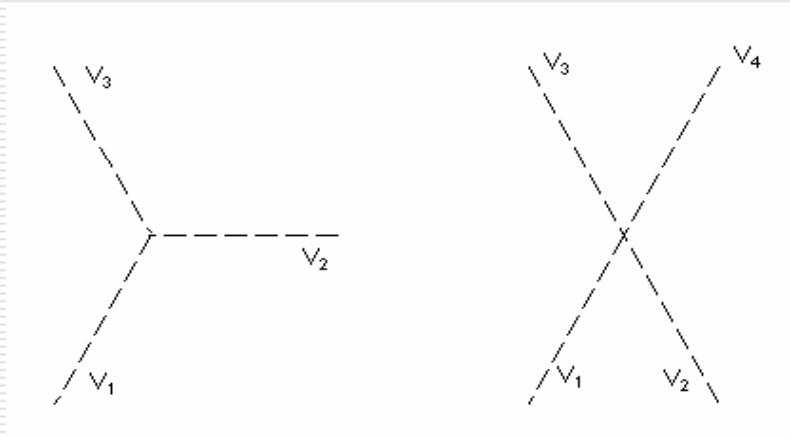
Just listing some important properties

A) Quarks and leptons both interact through the exchange of *vector particles*



Weak Interaction - II

- B) Exchanged vector bosons are (*very*) massive
- C) Interaction form derived by a non-Abelian gauge symmetry
+
Special mechanism giving mass to some of the gauge fields
- D) Non-Abelian vertexes



Weak Interaction - III

Weak charged current:

$$j^\mu = \bar{u}_f \gamma^\mu \frac{(1 - \gamma^5)}{2} u_i$$

$$j_V^\mu = \frac{1}{2} \bar{u}_f \gamma^\mu u_i \quad \text{Vector}$$

$$j_A^\mu = \frac{1}{2} \bar{u}_f \gamma^\mu \gamma^5 u_i \quad \text{Axial}$$

Compare to electromagnetic current:

$$j^\mu = \bar{u} \gamma^\mu u$$

Weak Interaction - IV

Apparently, totally different.

But:

$$(1 - \gamma^5)^2 = (1 - 2\gamma^5 + 1) = (2 - 2\gamma^5) = 2(1 - \gamma^5)$$

$$\rightarrow \left[\frac{(1 - \gamma^5)}{2} \right]^2 = \frac{2(1 - \gamma^5)}{4} = \frac{(1 - \gamma^5)}{2}$$

$$\rightarrow \bar{u}_f \gamma^\mu \frac{(1 - \gamma^5)}{2} u_i = \bar{u}_f \gamma^\mu \left[\frac{(1 - \gamma^5)}{2} \right]^2 u_i = \underbrace{\bar{u}_f \frac{(1 + \gamma^5)}{2}}_{\bar{u}_{fL}} \gamma^\mu \underbrace{\frac{(1 - \gamma^5)}{2} u_i}_{u_{iL}}$$

$$\rightarrow \bar{u}_f \gamma^\mu \frac{(1 - \gamma^5)}{2} u_i = \bar{u}_{fL} \gamma^\mu u_{iL}$$

Same form, but involving only *LEFT* chiral states

Weak Interaction - V

$$j_\mu^{em} = \bar{e} \gamma_\mu e$$

$$e \equiv \left(\frac{1-\gamma_5}{2} \right) e + \left(\frac{1+\gamma_5}{2} \right) e = e_L + e_R$$

Therefore:

$$\rightarrow j_\mu^{em} = \bar{e} \gamma_\mu e = (\bar{e}_L + \bar{e}_R) \gamma_\mu (e_L + e_R) = \bar{e}_L \gamma_\mu e_L + \bar{e}_R \gamma_\mu e_R$$

Because:

$$\bar{e}_L \gamma_\mu e_R = e \left(\frac{1+\gamma_5}{2} \right) \gamma_\mu \left(\frac{1+\gamma_5}{2} \right) e = e \gamma_\mu \left(\frac{1-\gamma_5}{2} \right) \left(\frac{1+\gamma_5}{2} \right) e = 0$$

$$\left(\frac{1-\gamma_5}{2} \right) \left(\frac{1+\gamma_5}{2} \right) = 0 \quad \text{Projectors 'orthogonal'}$$

The bottom line:

Weak (charged) and Electromagnetic currents:
Same Lorentz structure (vector), but LL vs. LL+RR

Weak Interaction - VI

$$W \text{ propagator: } -i \frac{g_{\mu\nu} - q_\mu q_\nu / M_W^2}{q^2 - M_W^2}$$

$$q^2 \ll M_W^2 \rightarrow -i \frac{g_{\mu\nu} - q_\mu q_\nu / M_W^2}{q^2 - M_W^2} \approx i \frac{g_{\mu\nu}}{M_W^2} \quad q^2\text{-independent}$$

$$T_{fi} \cong \left(\frac{1}{2\sqrt{2}} \right)^2 g_W^2 \left(\bar{u}_f^{(1)} \frac{1}{2} \gamma^\mu (1 - \gamma^5) u_i^{(1)} \right) i \frac{g_{\mu\nu}}{M_W^2} \left(\bar{u}_f^{(2)} \frac{1}{2} \gamma_\nu (1 - \gamma_5) u_i^{(2)} \right)$$

$$\rightarrow T_{fi} \cong i \frac{1}{8} 4\sqrt{2} G_F \quad j^{\mu(1)} j_\mu^{(2)}$$

$$g_W^2 \equiv \alpha_W \quad \text{Charged current coupling constant}$$

$$G_F = \frac{\sqrt{2}}{8} \left(\frac{g_W}{M_W} \right)^2 \quad \text{Fermi constant}$$

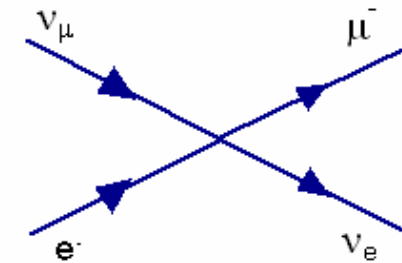
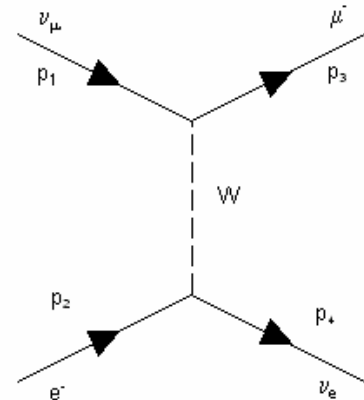
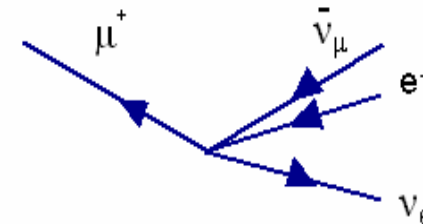
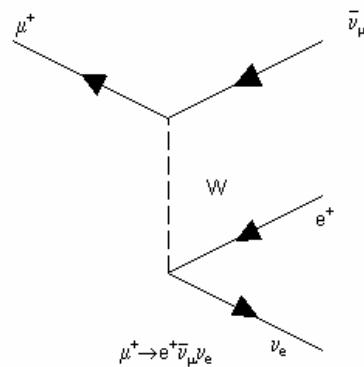
Weak Interaction - VII

Showing how SM diagrams collapse into current-current:

At low energy:

$$q^2 \ll M_W^2$$

$$\rightarrow i \frac{g_{\mu\nu} - q_\mu q_\nu / M_W^2}{q^2 - M_W^2} \approx i \frac{g_{\mu\nu}}{M_W^2}$$



V - A and the Nucleon Current

Found $\sim$ the same structure for lepton and nucleon current

$$\bar{\psi}_e \gamma^\mu (1 - \gamma^5) \psi_\nu \quad \text{Pure V-A}$$

$$\bar{\psi}_p \gamma_\mu \left(1 - \frac{C_A}{C_V} \gamma_5 \right) \psi_n$$

V: identical coupling, A: modified by strong interaction

Consider β decay of O^{14} – pure Fermi $G_F = 1 \cdot 10^{-5} M_p^{-2}$

Call this the β -decay Fermi constant

Measured value of $\frac{C_A}{C_V}$ for β decay of various baryons:

$$n \quad -1.267$$

$$\Lambda^0 \quad -0.718$$

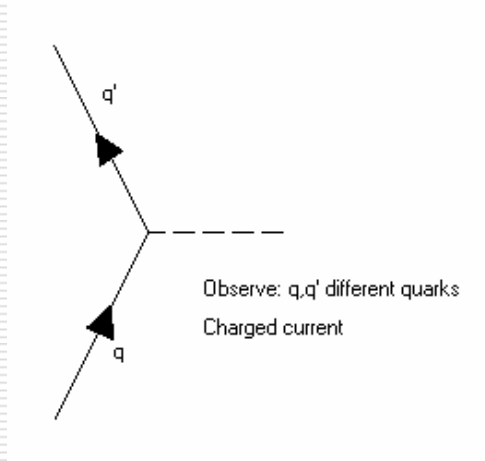
$$\Sigma^- \quad +0.340$$

$$\Xi^- \quad -0.25$$

Universality: Quarks

Semileptonic and non leptonic processes understood in terms of quarks

Basically similar coupling to leptonic charged currents:



Picture is slightly more complicated, however
Fundamental question:

Is the quark coupling identical to the lepton one?

Neutrino – Nucleon Scattering

Extend to ν_e - N scattering:

$$\nu_\mu + N \rightarrow \mu^- + X$$

$$\bar{\nu}_\mu + N \rightarrow \mu^+ + X$$

Somewhat similar to e - N and μ - N scattering

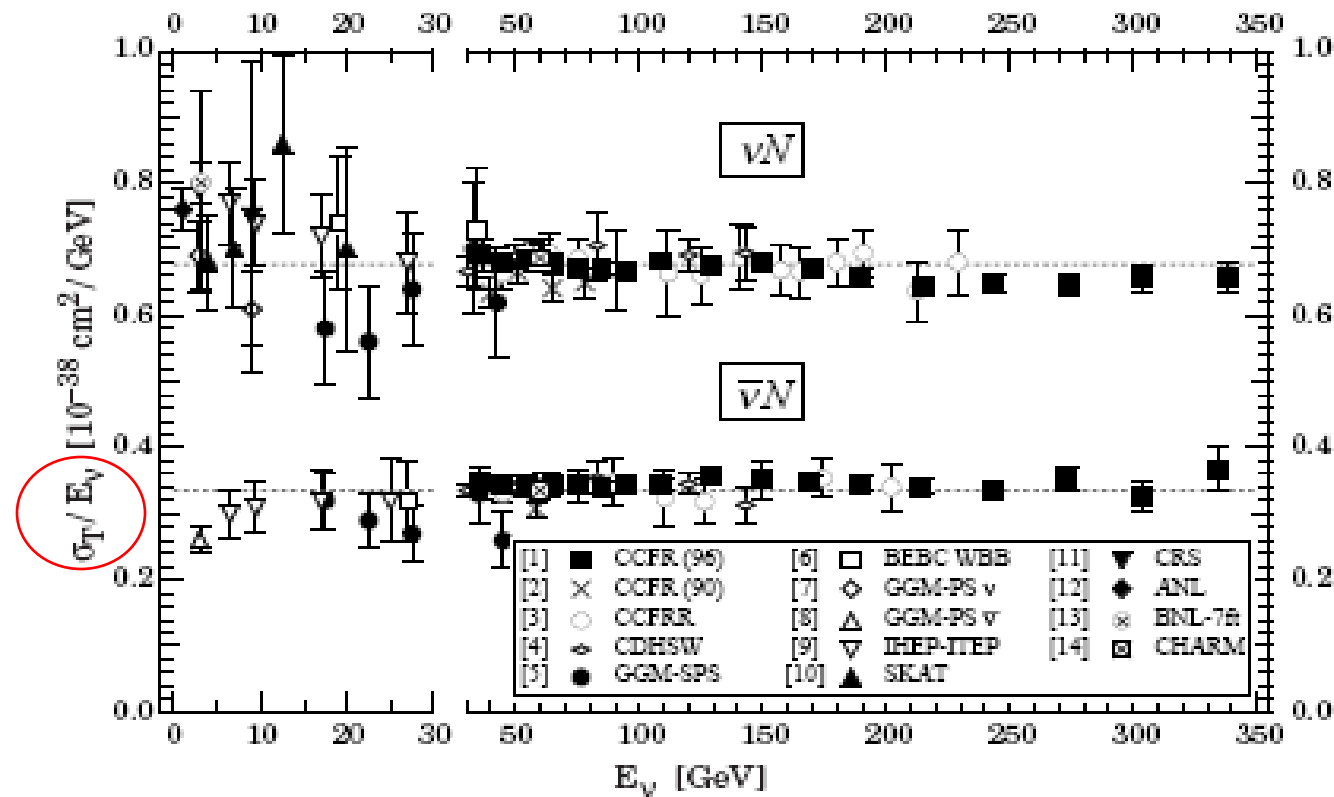
Modeling similar to DIS: Parton elastic scattering

Deep inelastic neutrino scattering reveals the same structure as DIS

More information available: Charged current sensitive to parton charge *sign*

→ *Can separate quark / antiquark contributions*

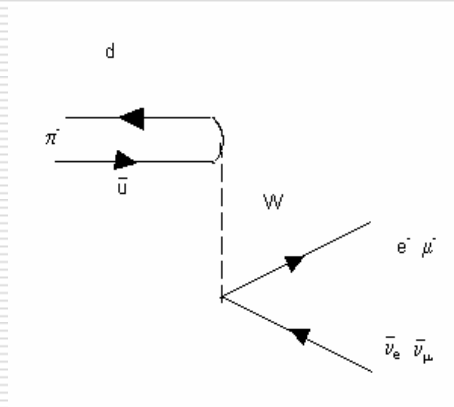
Total $\nu, \bar{\nu}$ - Nucleon Cross Section



Linearly rising cross section confirmed...

Semileptonic Decays: π, K - I

$$\pi \rightarrow \mu + \nu_\mu, \quad \pi \rightarrow e + \nu_e$$



$$T_{fi} = \frac{G_F}{\sqrt{2}} [\bar{u}(3) \gamma^\mu (1 - \gamma^5) v(2)] F_\mu$$

F_μ : equivalent of 'current' for the $q\bar{q}$ bound state

$F_\mu = f_\pi p_\mu$ 4-momentum is the only 4-vector available

Semileptonic Decays : π, K - II

Obtain:

$$\Gamma_l = \frac{f_\pi^2}{8m_\pi^3} G_F^2 m_l^2 (m_\pi^2 - m_l^2)^2$$

$$\frac{\Gamma(e)}{\Gamma(\mu)} = \frac{m_e^2 (m_\pi^2 - m_e^2)^2}{m_\mu^2 (m_\pi^2 - m_\mu^2)^2} = 1.3 \cdot 10^{-4}$$

Quite surprising: Phase space factor is much larger for e !
But:



Semileptonic Decays : π, K - III

Chirality rule:

ν is only L , $\bar{\nu}$ is only R

$$\rightarrow \begin{cases} \mu^-, e^- \text{ from } \pi, K \text{ decay forced to } R \\ \mu^+, e^+ \text{ from } \pi, K \text{ decay forced to } L \end{cases}$$

by angular momentum conservation:

$$\left. \begin{array}{l} H(\mu^-, e^-) = +1 \\ H(\mu^+, e^+) = -1 \end{array} \right\} \text{'Wrong' helicity}$$

$$e: \text{ Fully relativistic } \rightarrow P(\text{wrong } H) = \frac{1-\beta}{2} \sim 0$$

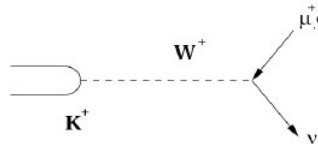
$$\mu: \text{ Not fully relativistic } \rightarrow P(\text{wrong } H) = \frac{1-\beta}{2} \sim \text{substantial}$$

K Decays Ride Again: NA48/2

$$\frac{\Gamma(K \rightarrow l + \nu_l)}{\Gamma(\pi \rightarrow l + \nu_l)} = \frac{f_K^2 m_\pi^3 (m_K^2 - m_l^2)^2}{f_\pi^2 m_K^3 (m_\pi^2 - m_l^2)^2} = \begin{cases} (\text{exp.}) 1.34 & l=\mu \\ (\text{exp.}) 0.19 & l=e \end{cases}$$

Within the Standard Model:

M. Finkemeier: Phys.Lett.B387:391-394,1996



$$R_M := \frac{\Gamma(M \rightarrow e \nu_e(\gamma))}{\Gamma(M \rightarrow \mu \nu_\mu(\gamma))} = \frac{m_e^2}{m_\mu^2} \left(\frac{m_M^2 - m_e^2}{m_M^2 - m_\mu^2} \right)^2 (1 + \delta R_M)$$

where δR_M arises from the radiative corrections, $M=\pi^\pm, K^\pm$

For $K^\pm$: $\delta R_K = -(3.78 \pm 0.04)\%$, leading to

$$R_K = (2.472 \pm 0.001) \times 10^{-5}$$

The value of R_K could be different in case of SUSY and LFV models – the correction could be as high as 3% in both directions
Measurement of R_K tests the μ -e universality and provides a sensible test of the SM

Data taking starting now at the CERN SPS...

Cabibbo Angle - I

Consider charged current of leptons:

Very natural to group charged and neutral leptons into *doublets*, or *families*

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix} \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix} \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}$$

Within each doublet, charged current transitions are driven by the emission/absorption of $W^\pm$ bosons

$$\begin{array}{ccccc} W^- & \rightarrow & \nu_e & \rightarrow & W^+ \\ & \uparrow & & \downarrow & \\ W^- & \leftarrow & e^- & \leftarrow & W^+ \end{array}$$

Similar for 2nd, 3rd family

Cabibbo Angle - II

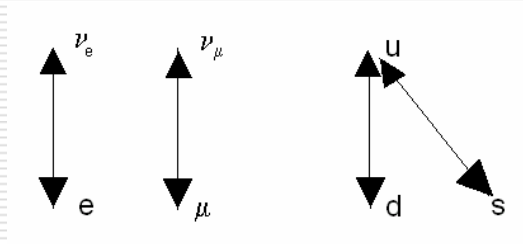
Would seem natural to extend this scheme to quarks

$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} \\ s \end{pmatrix} \quad \begin{array}{ccccc} W^- & \rightarrow & u & \rightarrow & W^+ \\ & \uparrow & & \downarrow & \\ W^- & \leftarrow & d & \leftarrow & W^+ \end{array}$$

Unfortunately, our scheme cannot work:

- a) Parallelism quark-lepton is incomplete with 4 leptons and only 3 quarks
- b) Does not account for strangeness violating processes

Cabibbo Angle - III



Cabibbo's very ingenious idea:

Quark flavor eigenstates (i.e., quark model eigenstates) are not to be identified with quark weak currents

→ Weak currents are mixtures of different flavors

By universal convention, mixing is assumed between d and s quarks:

$$\begin{cases} d' = \alpha d + \beta s \\ s' = \gamma d + \delta s \end{cases} \rightarrow \text{By unitarity: } \begin{cases} d' = \cos \theta_c d + \sin \theta_c s \\ s' = -\sin \theta_c d + \cos \theta_c s \end{cases}$$

Cabibbo Angle - IV

In terms of doublet:

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos \theta_c & \sin \theta_c \\ -\sin \theta_c & \cos \theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix} \quad \theta_c \text{ Cabibbo's angle}$$

This explain *many* things:
Just one example

Get the angle from β decay (Remember that 2% difference..)

$$G_F^{(\beta)} = 0.975 G_F^{(\mu)}$$

$$\rightarrow \theta_c \simeq 13^\circ$$

Cabibbo Angle - V

Now assume for leptonic π , K decays (very simply!)

$$f_K = f \sin \theta_C$$

$$f_\pi = f \cos \theta_C$$

$$\rightarrow \frac{\Gamma(K \rightarrow l + \nu_l)}{\Gamma(\pi \rightarrow l + \nu_l)} = \tan^2 \theta_C \frac{m_\pi^3 (m_K^2 - m_l^2)^2}{m_K^3 (m_\pi^2 - m_l^2)^2}$$

$$\rightarrow \frac{\Gamma(K \rightarrow l + \nu_l)}{\Gamma(\pi \rightarrow l + \nu_l)} = \begin{cases} (\text{teo.}) & 0.96 \quad l = \mu \\ (\text{teo.}) & 0.19 \quad l = e \end{cases}$$

Compare to experiment:

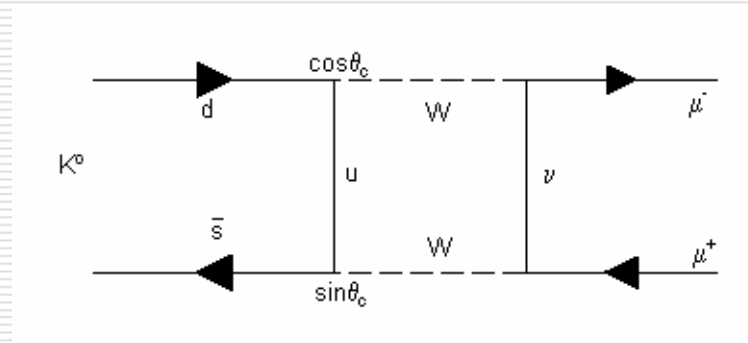
$$\begin{cases} (\text{exp.}) & 1.34 \quad l = \mu \\ (\text{exp.}) & 0.19 \quad l = e \end{cases}$$

Amazingly close!

GIM - I

Besides the many puzzles finally explained by Cabibbo, a few are left unexplained

Most relevant: $K^0 \rightarrow \mu^+ \mu^-$ strongly suppressed



2nd order weak process, still quite easy to compute
Expect rate higher by orders of magnitude

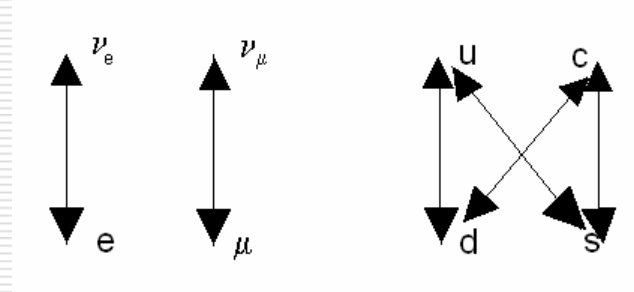
$$\text{BR}_{\text{meas}}(K^0 \rightarrow \mu\mu) = (6.87 \pm 0.11) 10^{-9}$$

GIM - II

Glashow, Iliopoulos and Maiani:

There exists a fourth quark, call it c like charm

Full symmetry restored between lepton-quark families



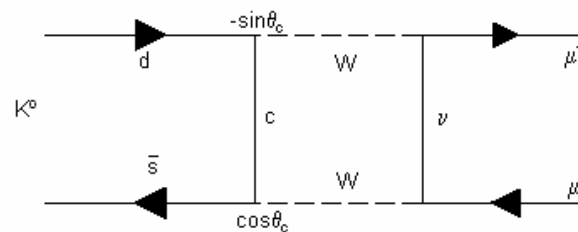
Two weak doublets:

$$\begin{pmatrix} u \\ d \end{pmatrix} \rightarrow \begin{pmatrix} u \\ d \cos \theta_c + s \sin \theta_c \end{pmatrix} \equiv \begin{pmatrix} u \\ d' \end{pmatrix}$$

$$\begin{pmatrix} c \\ s \end{pmatrix} \rightarrow \begin{pmatrix} c \\ -d \sin \theta_c + s \cos \theta_c \end{pmatrix} \equiv \begin{pmatrix} c \\ s' \end{pmatrix}$$

GIM - III

Then expect a second amplitude for K^0 decay



Total amplitude: Sum of two

$$\left. \begin{array}{l} A_1 \propto \sin\theta_c \cos\theta_c \\ A_2 \propto -\sin\theta_c \cos\theta_c \end{array} \right\} \rightarrow A_1 + A_2 \sim 0$$

Total amplitude not exactly 0 because $m_c \neq m_u$
 From observed rate predict $m_c \sim 1-2 \text{ GeV}$
 Leading to *November Revolution*: J/ψ discovery

CKM

Extend the idea to 3 families:

From Cabibbo's angle to *Cabibbo-Kobayashi-Maskawa* matrix

$$\begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} |d\rangle \\ |s\rangle \\ |b\rangle \end{bmatrix} = \begin{bmatrix} |d'\rangle \\ |s'\rangle \\ |b'\rangle \end{bmatrix}$$

From unitarity:

3 mixing angles

1 complex phase

This can account for CP violation

Experimental values:

$$\begin{bmatrix} 0.9753 & 0.221 & 0.003 \\ 0.221 & 0.9747 & 0.040 \\ 0.009 & 0.039 & 0.9991 \end{bmatrix}$$

Almost diagonal

Heavy quarks even more diagonal

Neutral Currents - I

Charged intermediate boson $W^\pm$ does not fix everything..
Typical issue left unsolved:

$$\nu + \bar{\nu} \rightarrow W^+ + W^-$$

Amplitude still divergent due to *longitudinal* W contribution..
Similar to QED process

$$e^+ + e^- \rightarrow \gamma + \gamma$$

In QED: Renormalization possible due to *gauge invariance*

Would like to upgrade charged current interaction model to a gauge theory to get finite, renormalized amplitudes

Charged current weak interaction to be understood as a gauge interaction $\rightarrow W^\pm$ as *gauge fields*

Neutral Currents - II

2 fields: Gauge group must be non-Abelian

Massive field: Gauge invariance apparently impossible

Nevertheless:

Assume some funny mechanism can preserve gauge invariance with a massive field (←Bold, but true)

Try $SU(2)$ as gauge group:

Predict a triplet of gauge fields (Adjoint representation)

Neutral current should exist

Neutral gauge field must be massive

Neutral Currents - III

Exchange of *neutral intermediate boson Z^0* , rather than charged $W^\pm$
Expect processes similar to electromagnetic, with coupling to *all fermions*

Typical signature: *Parity violation*

However, do not expect large effects at low energy:

Large Z^0 mass quenching down amplitudes

Electromagnetic amplitudes dominating at low energy

Some hope to see them in neutrino interactions (no e.m. contributions)

Not observed for a long time: Neutrino experiments difficult

Finally observed at CERN in 1973 in a bubble chamber experiment

Neutral Currents - IV

As a result of the weak-electromagnetic unification, neutral currents are *different* from charged

Lorentz structure not $V - A$	$-i \frac{g_W}{\sqrt{2}} \gamma^\mu \frac{(1 - \gamma^5)}{2}$	Charged
	$-i g_Z \gamma^\mu \frac{(C_V^f - C_A^f \gamma^5)}{2}$	Neutral
Coupling, not g_W	Fermion C_V	C_A
	ν_e, ν_μ, ν_τ	$+1/2$
	e, μ, τ	$-1/2 + 2 \sin \theta_W$
	u, c, t	$+1/2 - 4/3 \sin^2 \theta_W$
	d, s, b	$-1/2 + 2/3 \sin^2 \theta_W$

θ_W new fundamental constant

Neutral Currents - V

Tight relationship between weak and electromagnetic interactions

Couplings:

$$g_w = \frac{e}{\sin \theta_w}$$

$$g_z = \frac{e}{\sin \theta_w \cos \theta_w}$$

$$\alpha = e^2$$

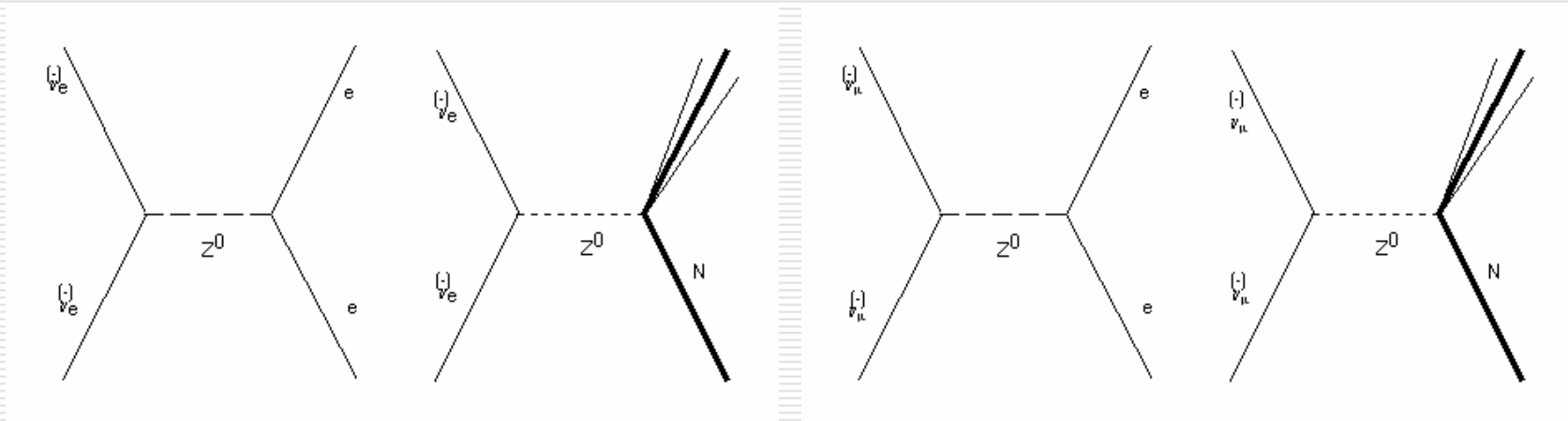
Charged, neutral and electromagnetic couplings
fixed by 2 universal constants:

e : Elementary charge

θ_w : Weinberg angle, new fundamental constant

Neutral Currents - VI

Expect to observe typical processes like:



$(\nu_e, \bar{\nu}_e) + e \rightarrow (\nu_e, \bar{\nu}_e) + e$ Contributing to elastic scattering

$(\nu_\mu, \bar{\nu}_\mu) + e \rightarrow (\nu_\mu, \bar{\nu}_\mu) + e$
 $(\nu_e, \bar{\nu}_e) + N \rightarrow (\nu_e, \bar{\nu}_e) + \text{hadron shower}$
 $(\nu_\mu, \bar{\nu}_\mu) + N \rightarrow (\nu_\mu, \bar{\nu}_\mu) + \text{hadron shower}$

New

Discovery of NC: Gargamelle

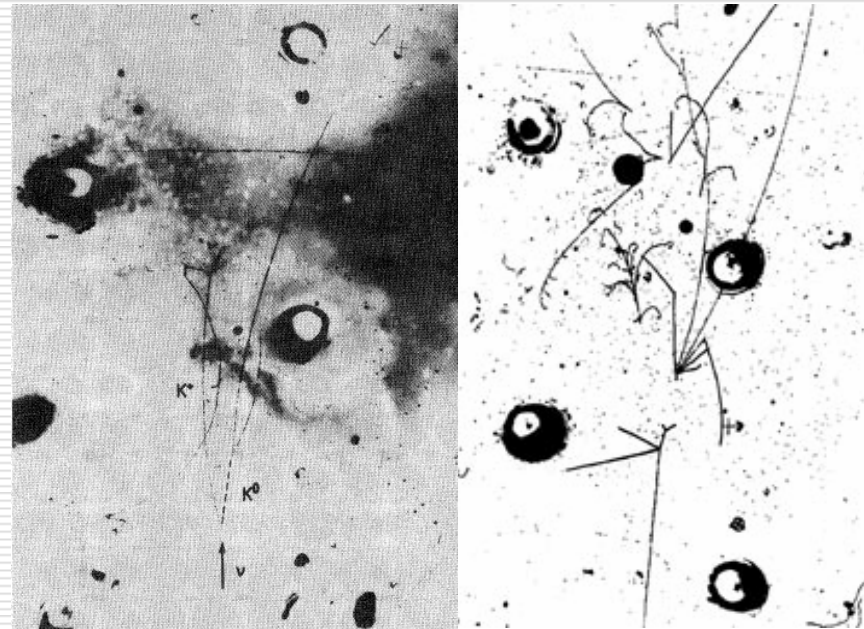
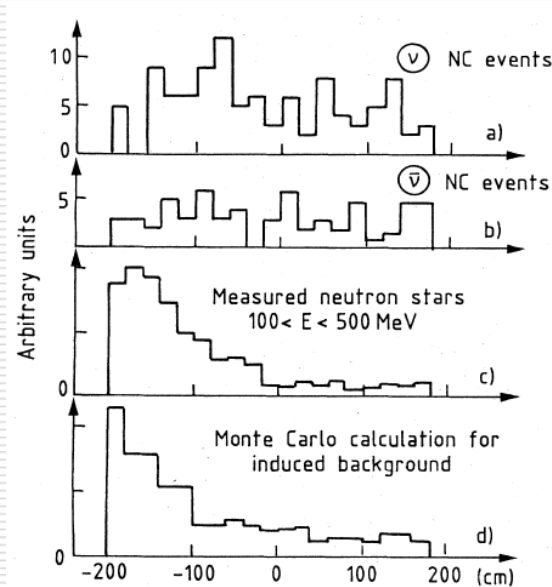
Main problem: Neutron background → Observe vertex position along the chamber

Exponential: Neutron background

$$\lambda_{\text{int}}^n \sim 90 \text{ cm}$$

Flat: Neutrino signal

$$\lambda_{\text{int}}^\nu \sim \infty$$



Estimated n background: 9 6

Table 1

	ν -exposure	$\bar{\nu}$ -exposure
No. of neutral-current candidates	102	64
No. of charged-current candidates	428	148