Elementary Particles I

6 – Weak Interaction

Beta Decay, *P & C* Violations, Current-Current Interaction, Charged Currents of Leptons and Quarks, Cabibbo Angle, GIM, Neutral Currents

The Electroweak Interaction

Standard Model:
Electromagnetic and Weak Interaction unified into Electroweak

Energy scale where unification is evident:

$E \sim M_W, M_Z \sim 100$ GeV

At lower energies:

Can still find tiny traces of the unification (Electroweak interference, Parity violation in atomic processes,...)

Electroweak interaction split into two, almost not interfering, *effective* interactions:

Electromagnetic Non fundamental, useful low energy approximations

Weak

The Weak Interaction

Compare:

Strong interaction – *All quarks*
Electromagnetic interaction – *All quarks + Charged Leptons*

Weak interaction – *All quarks + All leptons*

Large variety of phenomena

Classify weak processes into 3 types:

Leptonic $\mu^+ \to e^+ + \overline{\nu}_\mu + \nu_e, \quad \nu_e + e \to \nu_e + e$

Semileptonic $\pi^+ \to \mu^+ + \nu_\mu, \ \tau^+ \to \rho^+ + \nu_\tau$

Nonleptonic $K^0 \to \pi^+ + \pi^-, \ \Lambda^0 \to n + \pi^0$

Lost Symmetries

Many violations in weak processes:

Space Parity (large)
Charge Parity (large)
CP (very small)
T (very small)
Flavor conservation (Isospin, S,C,B,T) (larger + smaller)
Lepton numbers (?) (neutrino oscillations)

Fall of Parity - I

Discovery of parity non conservation:
Originated by the so-called "τ–θ puzzle"

Take K decays:
Weak process (S violation)
Observed decay modes (among many):

$$K^{\pm} \to \pi^{\pm}\pi^{0} \qquad BR = 21.2\ \%$$

$$K^{\pm} \to \pi^{\pm}\pi^{\pm}\pi^{\mp} \qquad BR = 5.6\ \%$$

Observe:

P_K=-1, as measured in strong processes
P_K= ?, as measured by its decays

Fall of Parity - II

Consider parity of the final states:

$$P\left|\pi\pi\right\rangle = (-1)(-1)(-1)^{l} = +1 \quad l = 0 \text{ because } J_K = 0,\ J_\pi = 0$$

$$P\left|\pi\pi\pi\right\rangle = (-1)(-1)(-1)P_{orb} = (-1)P_{orb}$$

$$J_K = 0 = L_{\pi_1\pi_2} \oplus L_{\pi_3} \to L_{\pi_1\pi_2} = L_{\pi_3}$$

$$\to P_{orb} = (-1)^{L_{\pi_1\pi_2}}(-1)^{L_{\pi_3}} = +1$$

$$\to P\left|\pi\pi\pi\right\rangle = -1$$

???
2 different particles, same mass, opposite parity?

Lee & Yang suggestion: *Parity is violated in weak processes*

Fall of Parity - III

Parity violation discovered almost simultaneously in 3 experiments

First : *Beta decay* (Wu et al.)
Others: π–μ decay (Garwin et al., Friedman et al.)

(see before)

Interesting question:
How does parity violation manifest itself?

Breaking of parity selection rules

Interference between even/odd amplitudes → Asymmetries

Non-zero value of parity-odd observables

Fall of Parity – IV

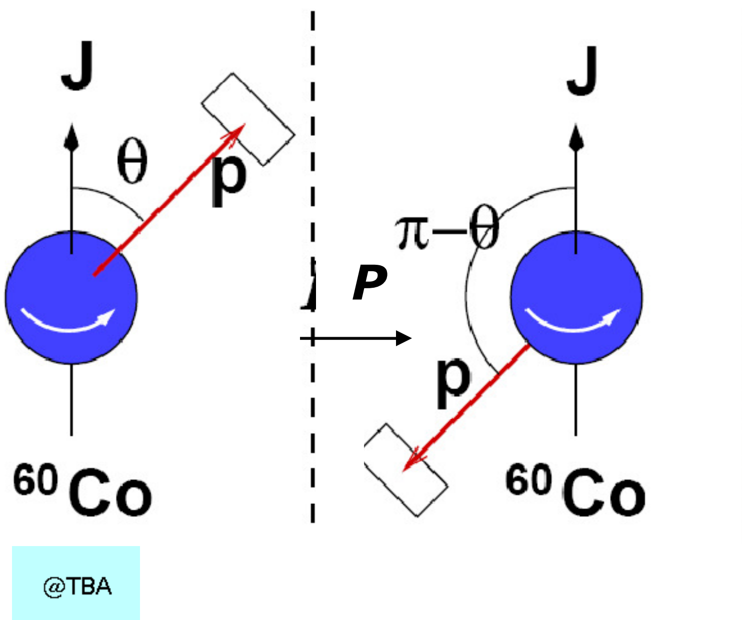
$^{60}Co \rightarrow {}^{60}Ni^{*} + e^{-} + \bar{\nu}_e$ β^- decay: Weak process

If weak interaction is parity invariant,

$\rightarrow Ampl(\theta) = Ampl(\pi - \theta)$

Otherwise:

Expect β^- direction *anisotropy*



Require nuclear polarization:
For an unpolarized sample,
by averaging over ***J*** z-projections
any possible anisotropy is washed out

Fall of Parity – V

The ^{60}Co Experiment: Polarization

Zeeman:

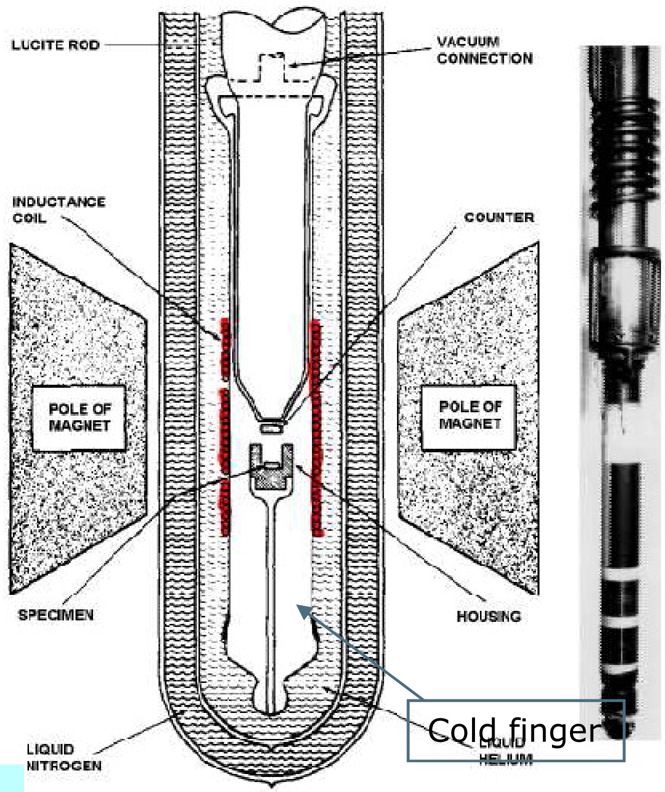
$$\mathcal{E}(M) = E_0 - \vec{\mu}\cdot\vec{B} = -g\mu_N BM$$

Boltzmann:

$$\frac{n(M')}{n(M)} = \frac{e^{\frac{\mathcal{E}(M)}{kT}}}{e^{\frac{\mathcal{E}(M')}{kT}}} = e^{\frac{(M-M')g\mu_N B}{kT}}$$

Magnetic field amplification in cerium-magnesium-nitrate crystal $0.05\,\mathrm{T} \to 10\text{–}100\,\mathrm{T}$

The ^{60}Co polarizes at a temperature of about 10 mK.



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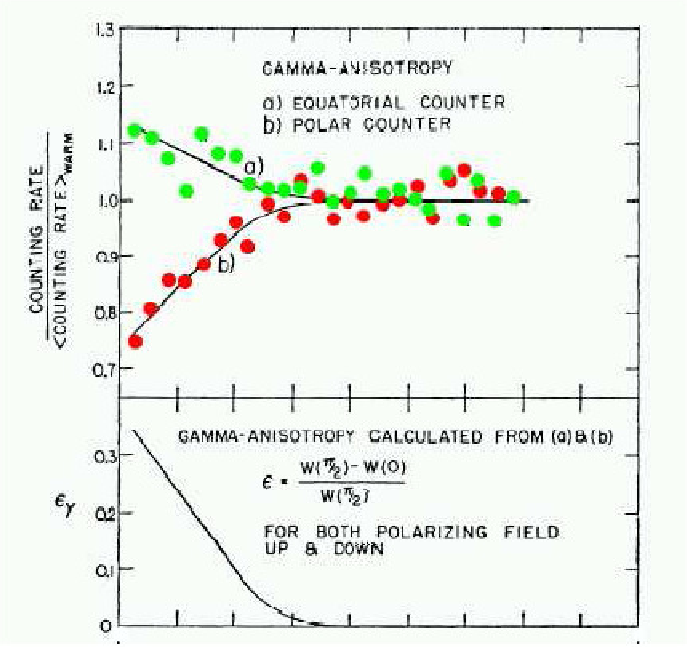
Fall of Parity – VI

To measure nuclear polarization:

γ anisotropy in $^{60}\text{Ni}^* \rightarrow ^{60}\text{Ni} + \gamma(E2)$

Electric quadrupole transition

$$\epsilon_\gamma = \frac{W(\pi/2) - W(0)}{W(\pi/2)}$$



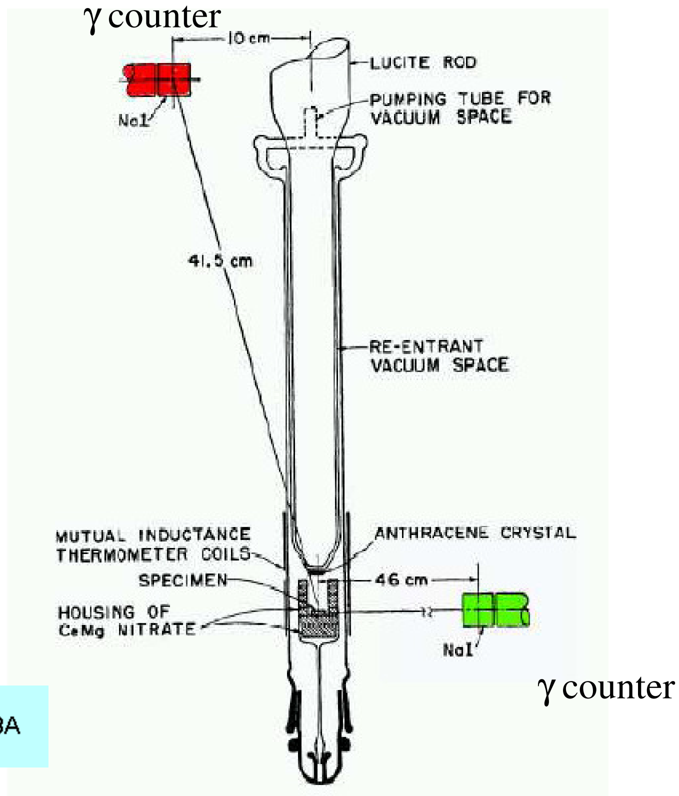
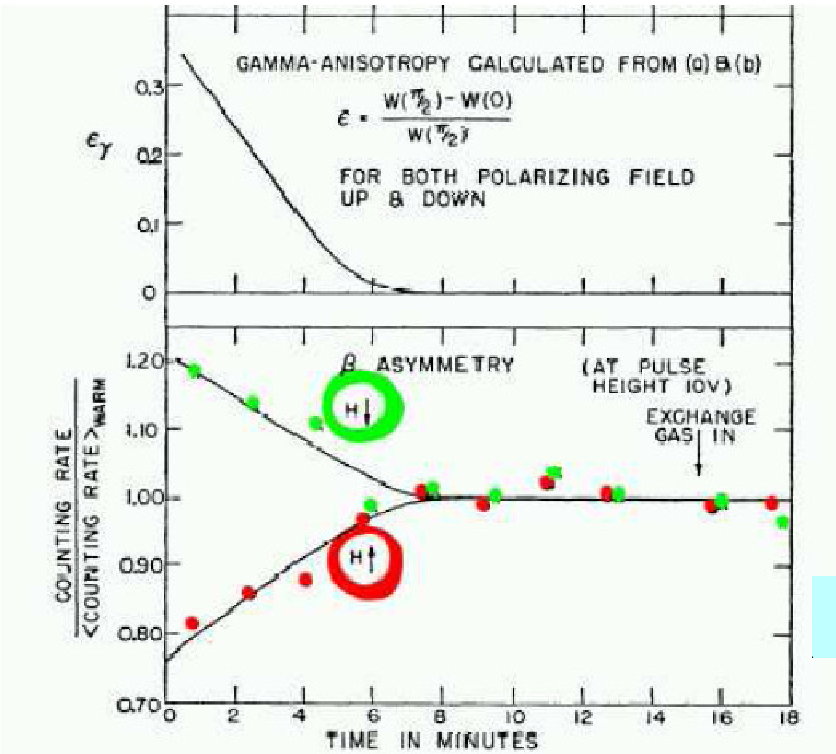


FIG. 1. Schematic drawing of the lower part of the cryostat.

@TBA

Fall of Parity – VII

$$^{60}\mathrm{Co} \rightarrow ^{60}\mathrm{Ni}^* + e^- + \bar{\nu}_e$$
$$\tau = 5.27\,\mathrm{year} \quad E_{e^-}^{max} = 2.824\,\mathrm{MeV}$$



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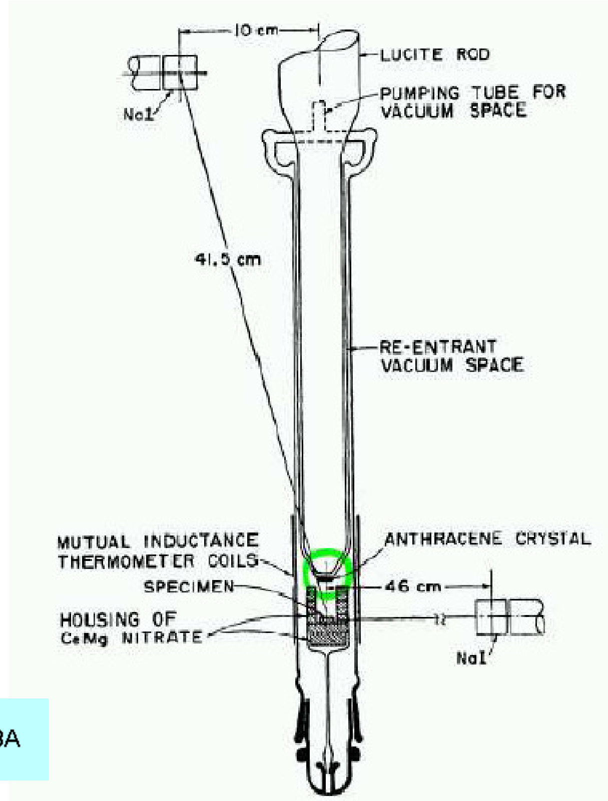


Fig. 1. Schematic drawing of the lower part of the cryostat

Fall of Parity - VIII

$$\pi^+ \to \mu^+ + \nu_\mu$$

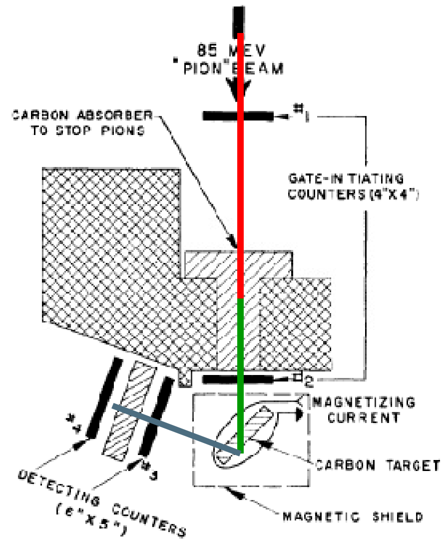
$$\boldsymbol{\sigma}_\mu \cdot \mathbf{p}_\mu \underset{U_P}{\to} \boldsymbol{\sigma}_\mu \cdot (-\mathbf{p}_\mu) = -\boldsymbol{\sigma}_\mu \cdot \mathbf{p}_\mu \qquad \text{Pseudoscalar observable}$$

$$\to P_\mu^{long} = \left\langle \frac{\boldsymbol{\sigma}_\mu \cdot \mathbf{p}_\mu}{|\mathbf{p}_\mu|} \right\rangle = 0 \ \text{ if parity is a good symmetry}$$

$$\mu^+ \to e^+ + \bar{\nu}_\mu + \nu_e$$

$$\boldsymbol{\sigma}_\mu \cdot \mathbf{p}_e \underset{U_P}{\to} \boldsymbol{\sigma}_\mu \cdot (-\mathbf{p}_e) = -\boldsymbol{\sigma}_\mu \cdot \mathbf{p}_e \qquad \text{Pseudoscalar observable}$$

$$\to \left\langle \frac{\boldsymbol{\sigma}_\mu \cdot \mathbf{p}_e}{|\mathbf{p}e|} \right\rangle = 0 \ \text{ if parity is a good symmetry}$$



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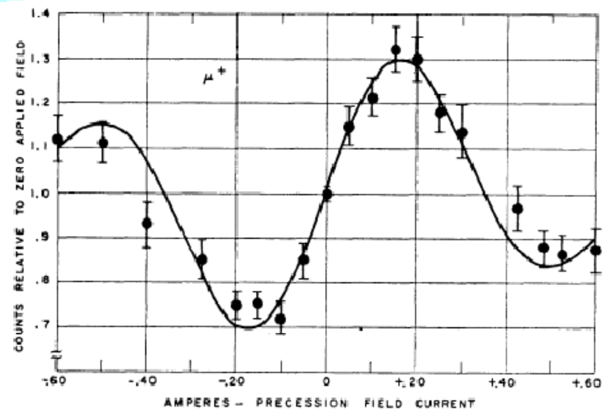
If parity is violated:

$$\begin{cases} \pi^+ \to \mu^+ + \nu_\mu & \text{Expect } \mu \text{ polarization along } \mathbf{p}_\mu \\ \mu^+ \to e^+ + \bar{\nu}_\mu + \nu_e & \text{Expect } e^+ \text{ direction correlated with } \mathbf{s}_\mu \end{cases}$$

In order to detect e^+ correlation:

μ spin precession in $\mathbf{B}$ →



Fall of Charge Parity

Immediate conclusion: *C*-parity is also violated

Indeed, it can be shown:

$$A = Scalar + Pseudoscalar \rightarrow |A|^2 = |S+P|^2 = |S|^2 + |P|^2 + 2\,\mathrm{Re}\left(SP^*\right)$$

$$\text{If } \begin{cases} CPT \text{ OK} \\ C \quad \text{OK} \end{cases}, \text{ by taking } \begin{cases} S \equiv |S| e^{i\alpha} \\ P \equiv |P| e^{i\beta} \end{cases} \rightarrow e^{i\alpha} = e^{i(\beta+\pi/2)}$$

$$\rightarrow SP^* = |S| e^{i\alpha} |P| e^{-i\beta} = |S||P| e^{i\alpha} e^{-i\beta} = |S||P| e^{i(\beta+\pi/2)} e^{-i\beta} = |S||P| e^{i\pi/2}$$

$$\rightarrow \mathrm{Re}\left(SP^*\right) = 0 \rightarrow \text{Interference term = 0} \rightarrow \text{Asymmetry=0}$$

Since asymmetries *are* observed, *C* must be KO

Beta Decay - I

Most common weak process in ordinary matter

3 nucleon 'decays':

$n \to p + e^- + \overline{\nu}_e$ Allowed for free neutrons

$p \to n + e^+ + \nu_e$ Energetically forbidden for free protons

$e^- + p \to n + \nu_e$ Atomic electron capture, usually from a K-shell (K-capture)

Both β^- and β^+ are observed for nucleons bound in a nucleus

$$(A,Z) \to (A,Z+1) + e^- + \overline{\nu}_e \qquad \beta^-$$
$$(A,Z) \to (A,Z-1) + e^+ + \nu_e \qquad \beta^+$$
$$e^- + (A,Z) \to (A,Z-1) + \nu_e \qquad K\text{-}Capture$$

Reminder: When found in a bound state, particles are *off-mass shell*

Beta Decay - II

Energy scale ~ few MeV

Small energy released to the pair $e\nu$

$e\nu$ orbital angular momentum

= *0* ‘*Allowed*’ Most frequent

= *1,2,..* ‘*Forbidden*’ Rare (long lifetime)

Allowed transitions:

$$J_{e\nu} = 1/2 \oplus 1/2 = \begin{cases} 0 & \text{singlet} \\ 1 & \text{triplet} \end{cases}$$

$$\rightarrow \Delta J_{nucleus} = \begin{cases} 0, & \Delta J_3 = 0 \quad \text{Fermi} \\ 1, & \Delta J_3 = 0, \pm 1 \quad \text{Gamow-Teller} \end{cases}$$

Beta Decay - III

Examples:

$$\underbrace{{}^{14}O}_{J=0} \rightarrow \underbrace{{}^{14}N}_{J=0} + \underbrace{e^+ + \nu_e}_{\rightarrow J=0} \quad \text{pure Fermi}$$

$$\underbrace{{}^{6}He}_{J=0} \rightarrow \underbrace{{}^{6}Li}_{J=1} + \underbrace{e^- + \overline{\nu}_e}_{\rightarrow J=1} \quad \text{pure Gamow-Teller}$$

$$\underbrace{{}^{1}n}_{J=1/2} \rightarrow \underbrace{{}^{1}p}_{J=1/2} + \underbrace{e^- + \overline{\nu}_e}_{\rightarrow J=0,1} \quad \text{mixed Fermi/Gamow-Teller}$$

Beta Decay - IV

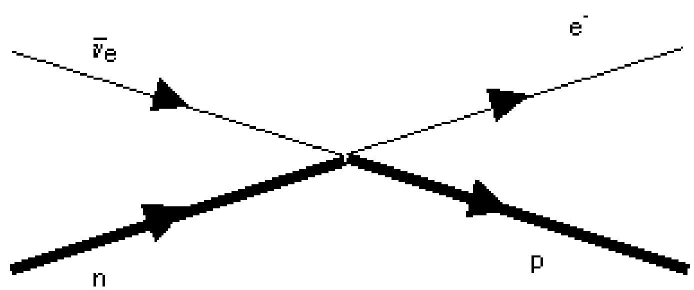
Take EM interaction as a model:

$$H_{\rm int}^{EM} = j^\mu A_\mu \to j_{(a)}^\mu \frac{1}{q^2} j_{(b)\mu} \quad \text{for 2 interacting currents}$$

Fermi guess:

Without introducing any intermediate particle, current-current interaction:

$$H_{\rm int}^{W} \propto j_{(a)} j_{(b)} \qquad j_{(a)}, j_{(b)} \text{ transition currents for leptons, nucleons}$$



Beta Decay - V

Observe:

Sticking for a moment to parity conservation, any *current*current* product which is a *Lorentz scalar* is acceptable for H_I.

So we are free to guess different forms for the weak current

$j_{(a)} \propto \overline{\psi}_{fin} \Gamma \psi_{in}$ Operator Γ fixes the Lorentz structure of the current

Transitions involve charge variation

$\rightarrow$ *Charged currents*

Beta Decay - VI

Fermi's original model: $\Gamma = \gamma_\mu$ Pure vector current

In general, Γ can be any of the set:

$$\begin{array}{ccccc} 1 & \gamma^\mu & \sigma^{\mu\nu} & \gamma^5\gamma^\mu & \gamma^5 \\ \Gamma_1 & \Gamma_2 & \Gamma_3 & \Gamma_4 & \Gamma_5 \end{array}$$

$\bar{\psi}\psi$	scalar S
$\bar{\psi}\gamma^\mu\psi$	vector V
$\bar{\psi}\sigma^{\mu\nu}\psi$	tensor T
$\bar{\psi}\gamma^5\gamma^\mu\psi$	axial vector A
$\bar{\psi}\gamma^5\psi$	pseudoscalar P

Most general form:

$$H_{\text{int}} = \sum_{i=S,V,T,A,P} C_i \left[\left(\bar{\psi}_p \Gamma_i \psi_n\right)\left(\bar{\psi}_e \Gamma^i \psi_\nu\right) + \underbrace{\left(\bar{\psi}_n \Gamma_i \psi_p\right)\left(\bar{\psi}_\nu \Gamma^i \psi_e\right)}_{=\text{herm. conj.}} \right]$$

Hermitian conjugate required in order to account for processes involving antiparticles

Rely on experiment to investigate the Lorentz structure

Beta Decay - VII

Non-relativistic limit of the different nucleon currents

S 1 $\rightarrow \chi_p^\dagger \chi_n$

V γ_μ $\begin{cases} \mu=0 \rightarrow \chi_p^\dagger \chi_n \\ \mu = 1,2,3 \rightarrow 0 \end{cases}$

T $\sigma_{\mu\nu}$ $\begin{cases} \mu=0,\nu \rightarrow 0 \\ \mu,\nu = 0 \rightarrow 0 \\ \mu,\nu = 1,2,3 \rightarrow \chi_p^\dagger \boldsymbol{\sigma} \chi_n \end{cases}$

A $\gamma_\mu \gamma_5$ $\begin{cases} \mu=0 \rightarrow 0 \\ \mu = 1,2,3 \rightarrow \chi_p^\dagger \boldsymbol{\sigma} \chi_n \end{cases}$

P γ_5 $\rightarrow 0$

Conclude:

P not relevant for β-decay
S,V do not change nucleon spin $\rightarrow$ OK for Fermi
T,A do change nucleon spin $\rightarrow$ OK for Gamow-Teller

Beta Decay - VIII

Attempts to understand which terms are present in the interaction

Pure Fermi : S and/or V
Pure Gamow-Teller: T and/or A

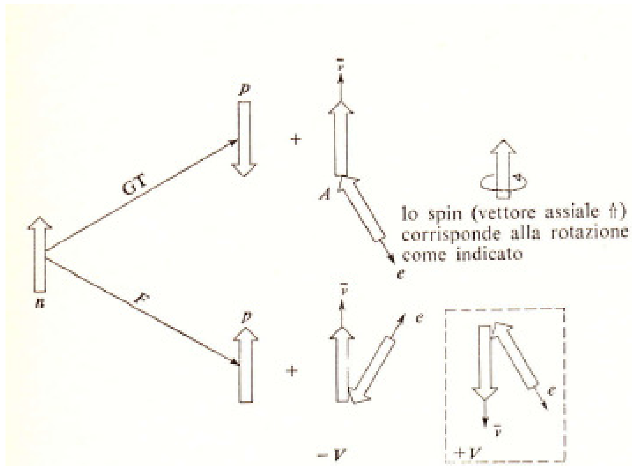
In both cases:

If both terms are present, expect a distortion of the electron energy spectrum
(*Fierz interference*)

Not observed → Fermi: Either S or V, Gamow-Teller: Either T or A

→Look for more indicators

Beta Decay – IX



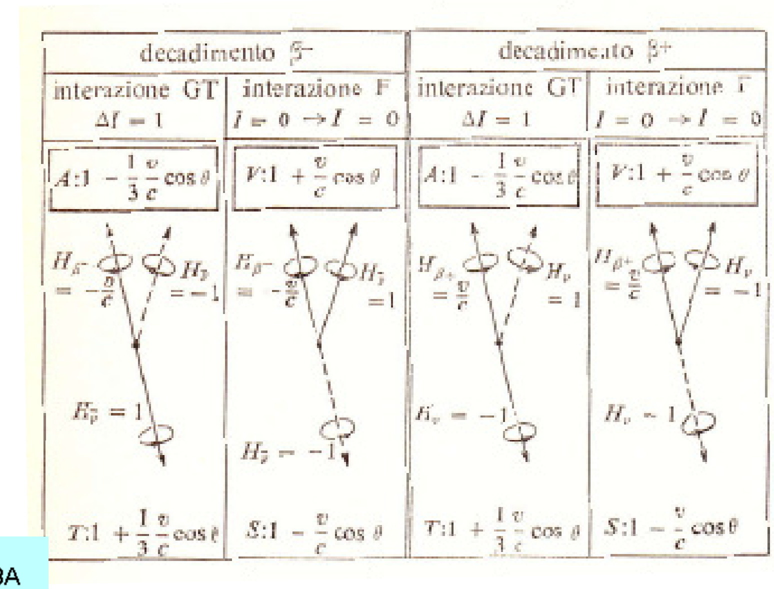


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Tavola 9-3 Correlazione tra impulsi ed elicità

Impulsi di $e^{\pm}$ e ν o $\bar{\nu}$	Paralleli	Opposti
GT Fermi	Tensore Vettore	Vettore assiale Scalare

Beta Decay - X

Angular correlation electron-neutrino. Expect:

$$\frac{dN}{d\cos\theta} = const\left(1+\lambda\beta\cos\theta\right), \qquad \lambda = \begin{cases} -1 & S \\ +1 & V \\ +1/3 & T \\ -1/3 & A \end{cases}$$

Cannot observe neutrino→Observe recoiling nucleus instead
Many experiments made :

Difficult, inconclusive, sometimes wrong, leading to mistakenly guess *S* & *T*

Solution finally found after the discovery of parity non conservation,
by ignoring (wrong) experimental data

Beta Decay - XI

To yield parity violation, H must include both *scalar* and *pseudo-scalar* terms.
Indeed, for any matrix element between initial and final states:

$$\left|\langle f|S+P|i\rangle\right|^2=\left|\langle f|S|i\rangle\right|^2+\left|\langle f|P|i\rangle\right|^2+2\langle f|S|i\rangle\langle f|P|i\rangle^*$$

$$\left|\langle f|S+P|i\rangle\right|^2\underset{Parity}{\rightarrow}\left|\langle f|S|i\rangle\right|^2+\left|\langle f|(-P)|i\rangle\right|^2+2\langle f|S|i\rangle\langle f|(-P)|i\rangle^*$$

$$=\left|\langle f|S|i\rangle\right|^2+\left|\langle f|P|i\rangle\right|^2-2\langle f|S|i\rangle\langle f|P|i\rangle^*$$

By allowing for parity non conservation, can write down:

$$H_{\text{int}}=\sum_{i=S,V,T,A}\left[C_i\underbrace{\left(\overline{\psi}_p\Gamma_i\psi_n\right)\left(\overline{\psi}_e\Gamma^i\psi_\nu\right)}_{S}+C_i'\underbrace{\left(\overline{\psi}_p\Gamma_i\psi_n\right)\left(\overline{\psi}_e\Gamma^i\gamma^5\psi_\nu\right)}_{P}\right]\quad C_i, C_i'\ \text{'constants'}$$

+ Hermitian conjugate always understood

Beta Decay - XII

Actually $C_i = C_i\left(q^2\right),\ C_i' = C_i'\left(q^2\right)$ Weak form factors

But: $q^2 \sim 0$ for β decay $\rightarrow$ Constants

γ^5 equivalently inserted in the nucleon current

No difference in full operator: Just a re-labeling of *C,C'*

Taking as an example V,A terms, either:

$$C_V\left(\bar{p}\gamma^\mu n\right)\left(\bar{e}\gamma_\mu \nu\right) + C_A\left(\bar{p}\gamma^\mu\gamma^5 n\right)\left(\bar{e}\gamma_\mu\gamma_5 \nu\right) +$$

$$+C_V'\left(\bar{p}\gamma^\mu n\right)\left(\bar{e}\gamma_\mu\gamma_5 \nu\right) + C_A'\left(\bar{p}\gamma^\mu\gamma^5 n\right)\left(\bar{e}\gamma_\mu \underbrace{\gamma_5\gamma_5}_{=1} \nu\right)$$

Or:

$$= C_V\left(\bar{p}\gamma^\mu n\right)\left(\bar{e}\gamma_\mu \nu\right) + C_A\left(\bar{p}\gamma^\mu\gamma^5 n\right)\left(\bar{e}\gamma_\mu\gamma_5 \nu\right) +$$

$$+C_V'\left(\bar{p}\gamma^\mu\gamma_5 n\right)\left(\bar{e}\gamma_\mu \nu\right) + C_A'\left(\bar{p}\gamma^\mu \underbrace{\gamma^5\gamma^5}_{=1} n\right)\left(\bar{e}\gamma_\mu\gamma_5 \nu\right)$$

Beta Decay - XIII

Redefine constants by extracting $\frac{G_F}{\sqrt{2}}$ (G_F: Fermi constant)

$$H_{\text{int}} = \frac{G_F}{\sqrt{2}} \sum_{i=S,V,T,A} C_i \left(\bar{\psi}_p \Gamma_i \psi_n\right) \left(\bar{\psi}_e \Gamma^i \left(1 + \frac{C_i{}'}{C_i} \gamma^5\right) \psi_\nu\right)$$

Time reversal invariance: C_i, C'_i real

In order to investigate the form of lepton current:

Measurement of the lepton longitudinal polarization

Reminder:

$P_L(lept) = \langle lept | \boldsymbol{\sigma}_l \cdot \hat{\boldsymbol{p}}_l | lept \rangle$ Long. Polarization = Average Helicity

$P_T(lept) = 0,\ T$ non-invariant:

$$\boldsymbol{\sigma}_e \cdot (\boldsymbol{p}_e \times \boldsymbol{p}_\nu) \xrightarrow[T]{} -\boldsymbol{\sigma}_e \cdot (-\boldsymbol{p}_e \times -\boldsymbol{p}_\nu) = -\boldsymbol{\sigma}_e \cdot (\boldsymbol{p}_e \times \boldsymbol{p}_\nu)$$

Helicity/Chirality - I

With reference to Dirac equation:

$$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \boldsymbol{\gamma} = \begin{pmatrix} 0 & \boldsymbol{\sigma} \\ -\boldsymbol{\sigma} & 0 \end{pmatrix}, \gamma^5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$ Dirac representation

$$\mathbf{S} = \frac{\boldsymbol{\Sigma}}{2}, \boldsymbol{\Sigma} = \begin{pmatrix} \boldsymbol{\sigma} & 0 \\ 0 & \boldsymbol{\sigma} \end{pmatrix} = \underbrace{\gamma^0 \boldsymbol{\gamma}}_{\boldsymbol{\alpha}} \gamma^5 = \boldsymbol{\alpha}\gamma^5$$ Spin operator

$$\Lambda = \frac{\boldsymbol{\Sigma} \cdot \mathbf{p}}{|\mathbf{p}|}$$ Helicity operator

$$\left.\begin{array}{l} \Lambda u^{(+)} = +u^{(+)} \\ \Lambda u^{(-)} = -u^{(-)} \end{array}\right\}$$ Helicity eigenstates

$$P_{\pm} = \frac{1 \pm \Lambda}{2}$$ Projection operators onto helicity eigenstates

Helicity/Chirality - II

Projectors, indeed:

$$P_+P_+ = \left(\frac{1+\Lambda}{2}\right)\left(\frac{1+\Lambda}{2}\right) = \frac{1}{4}\left(1+\Lambda+\Lambda+\Lambda^2\right)$$

$$\Lambda^2 = \frac{\left(\mathbf{\Sigma}\cdot\mathbf{p}\right)^2}{|\mathbf{p}|^2} = 1 \rightarrow P_+P_+ = \frac{1}{4}(1+2\Lambda+1) = \left(\frac{1+\Lambda}{2}\right) = P_+, \quad P_-P_- = P_-$$

$$P_+P_- = \left(\frac{1+\Lambda}{2}\right)\left(\frac{1-\Lambda}{2}\right) = \frac{1}{4}\left(1+\Lambda-\Lambda-\Lambda^2\right) = 0 = P_-P_+$$

$$1 = \frac{1-\Lambda}{2} + \frac{1+\Lambda}{2} = P_- + P_+ \rightarrow 1u = \left(P_+ + P_-\right)u = u_+ + u_-$$

$$\Lambda = \frac{\mathbf{\Sigma}\cdot\mathbf{p}}{|\mathbf{p}|} = \underbrace{\gamma^0\boldsymbol{\gamma}}_{\boldsymbol{\alpha}}\gamma^5 \cdot \frac{\mathbf{p}}{|\mathbf{p}|} = \frac{\boldsymbol{\alpha}\cdot\mathbf{p}}{|\mathbf{p}|}\gamma^5 \rightarrow P_\pm = \frac{1 \pm \boldsymbol{\alpha}\cdot\hat{\mathbf{p}}\gamma^5}{2}$$

Helicity/Chirality - III

γ^5 Chirality operator

$P_L = \frac{1-\gamma^5}{2}, P_R = \frac{1+\gamma^5}{2}$ Projectors onto chirality eigenstates

$$\begin{cases} P_L u = u_L \\ P_R u = u_R \end{cases} \rightarrow 1u = \left(P_L + P_R\right)u = u_L + u_R$$

A very important limit:

$$\boldsymbol{\alpha}\cdot\mathbf{p} = E - \beta m$$

$$\Lambda = \frac{E\cdot 1 - m\cdot\beta}{p}\gamma^5 \underset{E\gg m}{\longrightarrow} \gamma^5$$

$$P_{\pm} = \frac{1\pm\Lambda}{2} \underset{E\gg m}{\longrightarrow} \frac{1\pm\gamma^5}{2} = P_{R,L}$$

For high energy, or massless, particles:
Helicity projectors $\rightarrow$ *Chirality* projectors

Helicity/Chirality - IV

$$Eu = (\boldsymbol{\alpha}\cdot\mathbf{p} + \beta m)u$$

$$u = \begin{pmatrix}\phi \\ \chi\end{pmatrix},\quad \phi,\chi$$ 2 components spinors

$$\boldsymbol{\alpha} = \begin{pmatrix}\boldsymbol{\sigma} & 0 \\ 0 & -\boldsymbol{\sigma}\end{pmatrix},\quad \beta = \begin{pmatrix}0 & 1 \\ 1 & 0\end{pmatrix}$$ Dirac matrices in chiral representation

$$\rightarrow \begin{cases} E\phi = (\boldsymbol{\sigma}\cdot\mathbf{p})\phi + m\chi \\ E\chi = -(\boldsymbol{\sigma}\cdot\mathbf{p})\chi + m\phi \end{cases}$$

$$\rightarrow \begin{cases} E\phi = (\boldsymbol{\sigma}\cdot\mathbf{p})\phi \\ E\chi = -(\boldsymbol{\sigma}\cdot\mathbf{p})\chi \end{cases}, m = 0 \rightarrow \begin{cases} \dfrac{(\boldsymbol{\sigma}\cdot\mathbf{p})}{E = |\mathbf{p}|}\phi = \phi \\ \dfrac{(\boldsymbol{\sigma}\cdot\mathbf{p})}{E = |\mathbf{p}|}\chi = -\chi \end{cases} \rightarrow \phi,\chi$$ Helicity eigenstates

Helicity/Chirality - V

States with definite value of chirality, massive or massless particles

Particle | *Antiparticle*

$$u_L = \frac{1}{2}\left(1-\gamma^5\right)u \qquad v_L = \frac{1}{2}\left(1+\gamma^5\right)v$$

$$u_R = \frac{1}{2}\left(1+\gamma^5\right)u \qquad v_R = \frac{1}{2}\left(1-\gamma^5\right)v$$

$$\overline{u}_L = \overline{u}\frac{1}{2}\left(1+\gamma^5\right) \qquad \overline{v}_L = \overline{v}\frac{1}{2}\left(1-\gamma^5\right)$$

$$\overline{u}_R = \overline{u}\frac{1}{2}\left(1-\gamma^5\right) \qquad \overline{v}_R = \overline{v}\frac{1}{2}\left(1+\gamma^5\right)$$

Is it true? Try one example:

$$\gamma^5 u_L = \gamma^5 \frac{1}{2}\left(1-\gamma^5\right)u = \frac{1}{2}\left(\gamma^5-1\right)u = -\frac{1}{2}\left(1-\gamma^5\right)u = -u_L \quad \text{OK}$$

Helicity/Chirality - VI

For chiral states:

Massless particle: $\begin{cases} u_L & \langle H \rangle = -1 \\ u_R & \langle H \rangle = +1 \end{cases}$

$\rightarrow$ Helicity defined $\equiv$ Full longitudinal polarization

Massive particle: $\begin{cases} u_L & \langle H \rangle = -\beta \\ u_R & \langle H \rangle = +\beta \end{cases}$

$\rightarrow$ Helicity undefined, find $\begin{array}{l} -1 \text{ prob. } \dfrac{1+\beta}{2}, \; +1 \text{ prob. } \dfrac{1-\beta}{2} \\ +1 \text{ prob. } \dfrac{1+\beta}{2}, \; -1 \text{ prob. } \dfrac{1-\beta}{2} \end{array}$

Massless particles: *Helicity is Lorentz invariant*

Lepton Helicity - I

Neutrino can be be *either R or L:* Must rely on experiment to decide

Goldhaber experiment: 2-steps decay

$$\underbrace{{}^{152}Eu}_{J=0} + \underbrace{e^-}_{J=1/2} \to \underbrace{{}^{152}Sm^*}_{J=1} + \underbrace{\nu_e}_{J=1/2} \quad K \text{ capture}$$

$$\underbrace{{}^{152}Sm^*}_{J=1} \to \underbrace{{}^{152}Sm}_{J=0} + \gamma$$

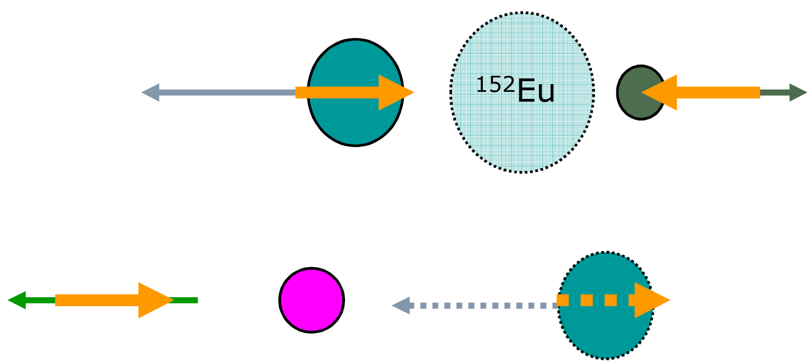
${}^{152}Sm^*$

ν_e

γ

Spin

${}^{152}Sm$



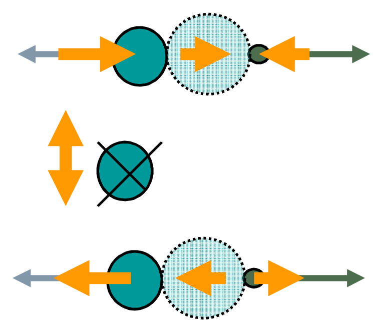
$$\to H\left({}^{152}Sm^*\right) = H\left(\nu_e\right)$$

Lepton Helicity - II

γ emitted along momentum do not contribute orbital angular momentum:

$^{152}Sm^*$ with $s_z = 0$ cannot yield a collinear γ:

Would yield a non existing γ with $H(\gamma) = 0$



$\rightarrow$ Collinear γ only from $^{152}Sm^*$ with $s_z = \pm 1$

$\rightarrow$ For collinear photons:

$$H(\gamma) = H\left({}^{152}Sm^*\right) = H(\nu_e)$$

Lepton Helicity - III

How to select collinear photons?

Choose a ^{152}Sm target:

$\gamma + {}^{152}Sm \to {}^{152}Sm^{*} \to {}^{152}Sm + \gamma$

Resonant scattering:

γ energy must be just larger than ΔE

in order to account for nuclear recoil

Only available from collinear photons

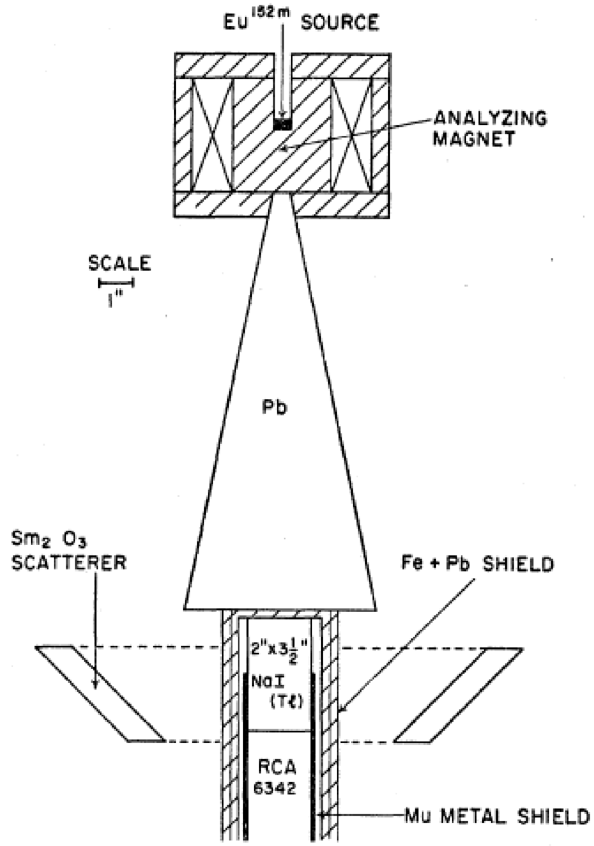
because includes a fraction of the decaying

nucleus kinetic energy

Measure $H(\gamma)$ by absorbing γ

in magnetized iron

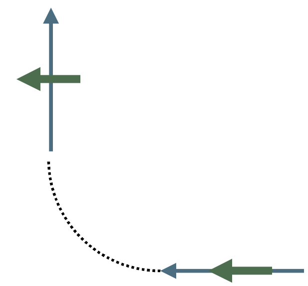
Result : $H(\nu_e) = -1$



Lepton Helicity - IV

Electron helicity:

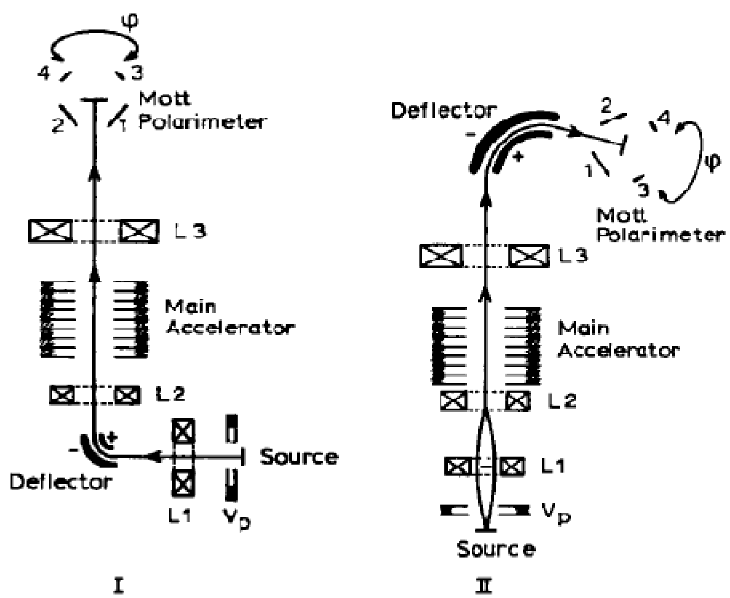
Method: Rotate *e* momentum by *90⁰* by means of an *electrostatic* field

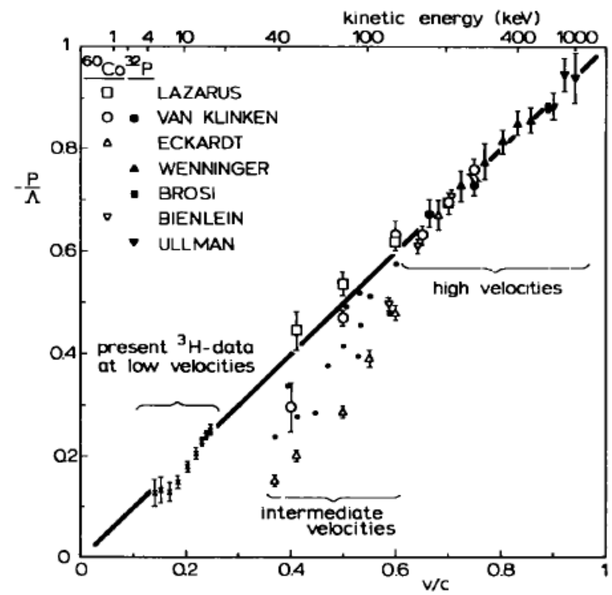


→Spin undeflected for non-relativistic e

→Longitudinal polarization becomes transverse

→Mott scattering sensitive to $P_\perp$





$\langle H \rangle = -\beta$ Average helicity $\equiv$ Longitudinal polarization

Mystery Solved: $V - A$ - I

Now closing on the analysis:

$$H_{\text{int}} = \frac{G_F}{\sqrt{2}} \sum_{i=V,A} C_i \left(\bar{\psi}_p \Gamma_i \psi_n\right)\left(\bar{\psi}_e \Gamma^i \left(1 + \frac{C_i\,'}{C_i}\gamma^5\right)\psi_\nu\right)$$

Neutrino helicity = - 1 yields lepton current = V - A

$$\left(1 + \frac{C_i\,'}{C_i}\gamma^5\right)\psi_\nu = \left(1-\gamma^5\right)\psi_\nu \rightarrow C_i\,' = -C_i$$

$$= -\gamma^\mu\left(1-\gamma^5\right)$$

$$\rightarrow H_{\text{int}} = \frac{G_F}{\sqrt{2}}\left[C_V\left(\bar{\psi}_p \gamma_\mu \psi_n\right)\left(\bar{\psi}_e \gamma^\mu\left(1-\gamma^5\right)\psi_\nu\right) + C_A\left(\bar{\psi}_p \gamma_\mu \gamma_5 \psi_n\right)\left(\bar{\psi}_e \gamma^\mu \gamma^5\left(1-\gamma^5\right)\psi_\nu\right)\right]$$

$$= \frac{G_F}{\sqrt{2}}\left[C_V\left(\bar{\psi}_p \gamma_\mu \psi_n\right)\left(\bar{\psi}_e \gamma^\mu\left(1-\gamma^5\right)\psi_\nu\right) - C_A\left(\bar{\psi}_p \gamma_\mu \gamma_5 \psi_n\right)\left(\bar{\psi}_e \gamma^\mu\left(1-\gamma^5\right)\psi_\nu\right)\right]$$

$$= \frac{G_F}{\sqrt{2}}\left[C_V\left(\bar{\psi}_p \gamma_\mu \psi_n\right) - C_A\left(\bar{\psi}_p \gamma_\mu \gamma_5 \psi_n\right)\right]\left(\bar{\psi}_e \gamma^\mu\left(1-\gamma^5\right)\psi_\nu\right)$$

Mystery Solved: $V-A$ - II

Therefore:

$$H_{\text{int}} = \frac{G_F}{\sqrt{2}}\left[C_V\left(\bar{\psi}_p\gamma_\mu\psi_n\right) - C_A\left(\bar{\psi}_p\gamma_\mu\gamma_5\psi_n\right)\right]\left(\bar{\psi}_e\gamma^\mu\left(1-\gamma^5\right)\psi_\nu\right)$$

$$H_{\text{int}} = \frac{G_F}{\sqrt{2}}C_V\left(\bar{\psi}_p\gamma_\mu\left(1-\frac{C_A}{C_V}\gamma_5\right)\psi_n\right)\left(\bar{\psi}_e\gamma^\mu\left(1-\gamma^5\right)\psi_\nu\right)$$

Fermi's theory confirmed, only adding parity violation

Parity violation is maximal:

Vector = Axial vector

Parity violation originating from V/A *interference*

(← Strictly quantum effect: No classical counterpart!)

Mystery Solved: *V* – *A* - III

Observe:

$$H_{\text{int}} = \frac{2G_F}{\sqrt{2}}\left(\bar{\psi}_p \gamma_\mu \left(1 - \frac{C_A}{C_V}\gamma_5\right)\psi_n\right)\left(\bar{\psi}_e \gamma^\mu \frac{\left(1-\gamma^5\right)}{2}\psi_\nu\right)$$

$$\frac{1-\gamma^5}{2}\ \text{Projection operator} \rightarrow \left[\frac{\left(1-\gamma^5\right)}{2}\right]^2 = \frac{1-\gamma^5}{2}$$

$$\rightarrow H_{\text{int}} = \frac{2G_F}{\sqrt{2}}\left(\bar{\psi}_p \gamma_\mu \left(1 - \frac{C_A}{C_V}\gamma_5\right)\psi_n\right)\left(\bar{\psi}_e \gamma^\mu \left[\frac{\left(1-\gamma^5\right)}{2}\right]^2 \psi_\nu\right)$$

$$\rightarrow H_{\text{int}} = \sqrt{2}G_F\left(\bar{\psi}_p \gamma_\mu \left(1 - \frac{C_A}{C_V}\gamma_5\right)\psi_n\right)\left(\bar{\psi}_e \left(\frac{1+\gamma^5}{2}\right)\gamma^\mu\left(\frac{1-\gamma^5}{2}\right)\psi_\nu\right)$$

Lepton current written as *pure vector* between *chiral parts* of ν,e states

→ The (charged current) weak interaction is just the same as the e.m. current, except operates between chiral states with different charge

Neutrinos - I

V-A Theory: Neutrino has very peculiar properties

1) Only left-handed neutrinos (and right- handed antineutrinos) exist
→ 4 component Dirac spinor not required

«Two component» neutrino theory

2) Consider P and C operations on neutrino states:

$$U_P|\nu_L\rangle = \eta_P|\nu_R\rangle$$

But: ν_R do not exist

$$U_C|\nu_L\rangle = \eta_C|\overline{\nu}_L\rangle$$

But: $\overline{\nu}_L$ do not exist

$$U_P U_C|\nu_L\rangle = \eta_P\eta_C|\overline{\nu}_R\rangle \quad \text{OK}$$

→CP symmetry apparently good for weak interactions (??)

Neutrinos - II

Recent, indirect yet convincing evidence that neutrinos have mass
(More and more direct, indeed…)

What about V-A?

Not so many changes in the Standard Model:

If neutrino have mass, they are no longer pure left-handed particles

Right-handed neutrinos exist

Separation into *+1* and *-1* *helicity* states is frame dependent

Separation into *+1* and *-1* *chirality* states is frame independent

Only the L-chirality states (R- for antineutrinos) contribute to charged current

Universality: Leptons

Consider muon weak interactions:

$\mu^+ \to e^+ + \bar{\nu}_\mu + \nu_e, \quad \mu^- \to e^- + \nu_\mu + \bar{\nu}_e \quad \mu$ decay

$\mu^- + p \to n + \nu_\mu \quad \mu$ capture, involves nucleon current

μ decay purely leptonic:

Guess

current-current, *V-A*

for both electron and muon charged currents

Compute:

μ *Lifetime*
Electron energy spectrum
Electron longitudinal polarization

Muon Decay - I

Consider decay of a polarized muon: $\mu^+ \uparrow \rightarrow e^+ + \bar{\nu}_\mu + \nu_e, \mu^- \uparrow \rightarrow e^- + \nu_\mu + \bar{\nu}_e$

P conservation: Predict

$$\left.\frac{d\Gamma_\pm}{d(\cos\theta)}\right|_\theta = \left.\frac{d\Gamma_\pm}{d(\cos\theta)}\right|_{\pi-\theta}$$

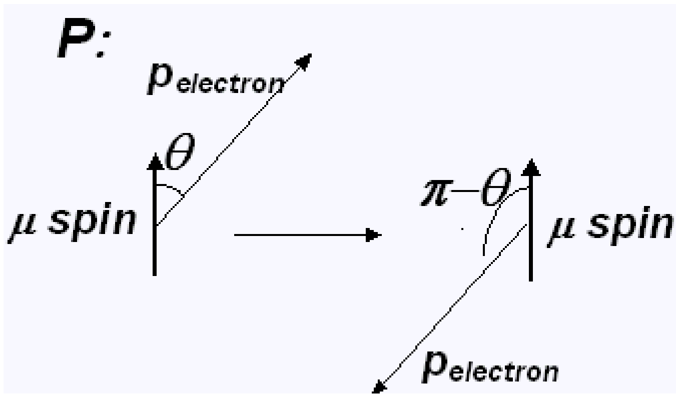
$$\rightarrow \frac{1}{2}\Gamma_\pm\left[1 - \frac{\xi_\pm}{3}\cos\theta\right] = \frac{1}{2}\Gamma_\pm\left[1 + \frac{\xi_\pm}{3}\cos\theta\right]$$

$$\rightarrow \xi_\pm = 0$$

Experiment:

$$\xi_+ = -\xi_- = -1$$

$\rightarrow P$ is violated



Muon Decay - II

$C : \mu^{\pm} etc \rightarrow \mu^{\mp} etc$

$$\frac{d\Gamma_{\pm}}{d(\cos\theta)} = \frac{1}{2}\Gamma_{\pm}\left[1 - \frac{\xi_{\pm}}{3}\cos\theta\right]$$

$$\Gamma_{\pm} = \frac{1}{\tau_{\pm}},\ \Gamma_{+} = \Gamma_{-}$$

C conservation: Predict $\xi_{+} = \xi_{-}$

Experiment:

$\xi_{+} = -\xi_{-} = -1$

$\rightarrow C$ is violated

CP :

$$\left.\frac{d\Gamma_{+}}{d(\cos\theta)}\right|_{\theta} = \left.\frac{d\Gamma_{-}}{d(\cos\theta)}\right|_{\pi-\theta}$$

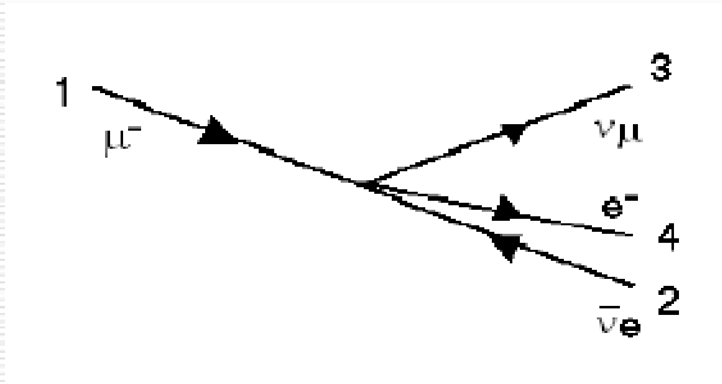
$\rightarrow \xi_{+} = -\xi_{-}$

Experiment:

OK

$\rightarrow CP$ is conserved

Muon Decay - III



Matrix element:

$$T_{fi} = \frac{G_F}{\sqrt{2}}\left[\bar{u}(3)\gamma^{\mu}\left(1-\gamma^{5}\right)u(1)\right]\left[\bar{u}(4)\gamma_{\mu}\left(1-\gamma_{5}\right)v(2)\right]$$

Squared m.e.:

$$T_{fi}T_{fi}^{*} = \frac{G_F^2}{2}\left[\bar{u}(3)\gamma^{\mu}\left(1-\gamma^{5}\right)u(1)\right]\left[\bar{u}(3)\gamma_{\nu}\left(1-\gamma_{5}\right)u(1)\right]^{*}$$

$$\cdot\left[\bar{u}(4)\gamma_{\mu}\left(1-\gamma_{5}\right)v(2)\right]\left[\bar{u}(4)\gamma^{\nu}\left(1-\gamma^{5}\right)v(2)\right]^{*}$$

Muon Decay - IV

Sum & Average over spin projections:

$$\left\langle \left|T_{fi}\right|^2 \right\rangle = 64G_F^2 \left(p_1 \cdot p_2\right)\left(p_3 \cdot p_4\right)$$

Muon rest frame:

$$p_1 = \left(m, 0, 0, 0\right)$$

$$\rightarrow p_1 \cdot p_2 = mE_2$$

$$p_1 = p_2 + p_3 + p_4$$

$$\rightarrow p_1 - p_2 = p_3 + p_4$$

Take electron as massless:

$$\rightarrow \left(p_3 + p_4\right)^2 = 2p_3 \cdot p_4$$

$$\rightarrow 2p_3 \cdot p_4 = p_1^2 - 2p_1 \cdot p_2$$

$$\rightarrow 2p_3 \cdot p_4 = m^2 - 2mE_2$$

Muon Decay - V

Therefore:

$$\left\langle \left| T_{fi} \right|^2 \right\rangle = 64 G_F^2 \left(p_1 \cdot p_2 \right) \left(p_3 \cdot p_4 \right)$$

$$\rightarrow \left\langle \left| T_{fi} \right|^2 \right\rangle = 32 G_F^2 m^2 E_2 \left(m - 2E_2 \right)$$

Differential decay rate:

$$d\Gamma = \left(2\pi\right)^4 \frac{\left\langle \left| T_{fi} \right|^2 \right\rangle}{2m} \delta\left(p_1 - p_2 - p_4 - p_4 \right) \frac{d^3\mathbf{p}_2}{\left(2\pi\right)^2 2\left|\mathbf{p}_2\right|} \frac{d^3\mathbf{p}_3}{\left(2\pi\right)^2 2\left|\mathbf{p}_3\right|} \frac{d^3\mathbf{p}_4}{\left(2\pi\right)^2 2\left|\mathbf{p}_4\right|}$$

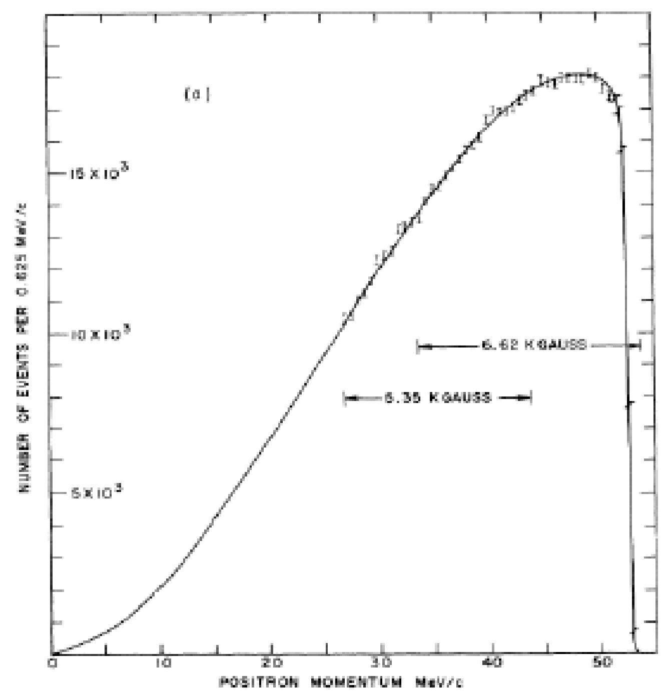
$$d\Gamma = 16 \left(2\pi\right)^4 G_F^2 m E_2 \left(m - 2E_2 \right) \delta\left(p_1 - p_2 - p_4 - p_4 \right) \frac{d^3\mathbf{p}_2}{\left(2\pi\right)^2 2\left|\mathbf{p}_2\right|} \frac{d^3\mathbf{p}_3}{\left(2\pi\right)^2 2\left|\mathbf{p}_3\right|} \frac{d^3\mathbf{p}_4}{\left(2\pi\right)^2 2\left|\mathbf{p}_4\right|}$$

Muon Decay - VI

If we are interested in the electron energy spectrum:
Integrate over all variables except $|\mathbf{p}_4|$

Spectrum shape:

$$\frac{d\Gamma}{dE} = 32 G_F^2 \frac{m_\mu^2 E^2}{2(4\pi)^3}\left(1 - \frac{4E}{3m_\mu}\right), E_{\max} = \frac{m}{2}$$



Muon Decay - VII

$\tau = \frac{1}{\Gamma}$ Lifetime and total rate

$$\Gamma = \int_{E_{\min}}^{E_{\max}} \frac{d\Gamma}{dE} dE = \frac{32 G_F^2 m_\mu^5}{12 (8\pi)^3} \rightarrow \tau = \frac{192\pi^3}{G_F^2 m_\mu^5}$$

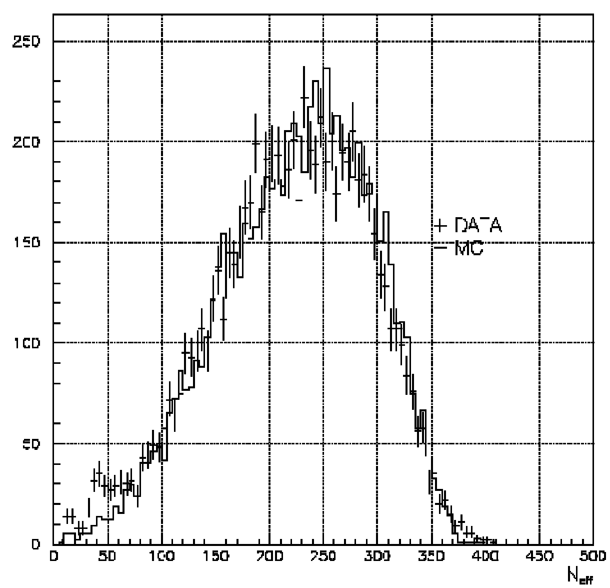
Extract G_F from measured lifetime:

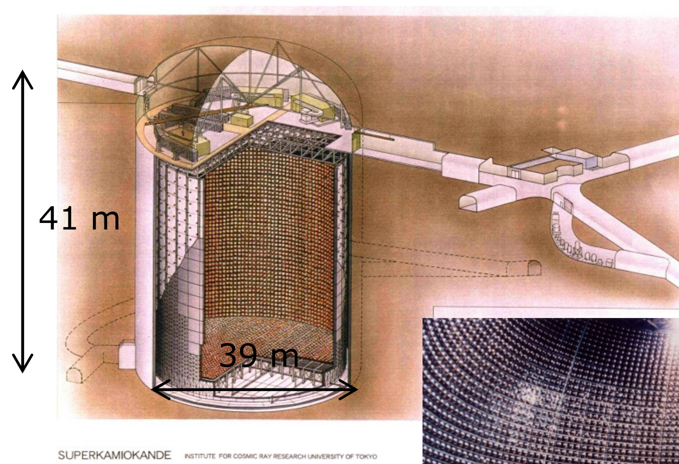
$G_F^{(\beta)} = 0.98 \quad G_F^{(\mu)}$ Almost identical!

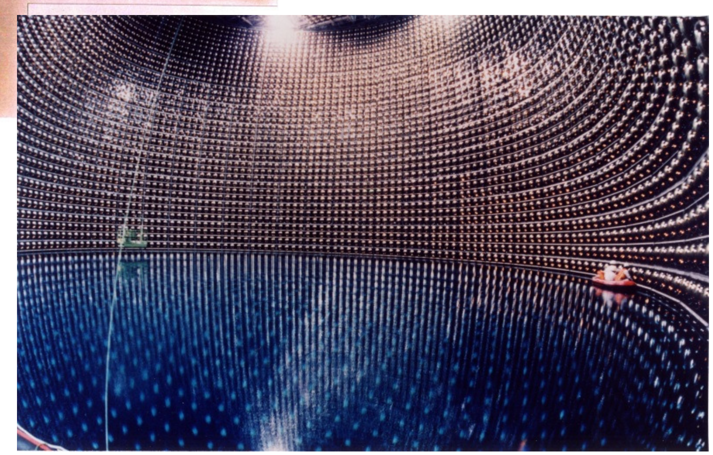
2% difference ??

Super K

Electron spectrum from *m* decay used as calibration at SuperKamiokande
Allows for absolute calibration of Cerenkov light signal vs. energy







Water Cerenkov

50000 *t* pure water

11200 photomultipliers (50 cm ∅!)

The μ Michel Parameter

Detailed analysis:
Differential decay rate depending on several parameters
Alternative structures of charged current cast into different parameter sets

Most important: *Michel parameter* ρ
Electron differential spectrum determined by ρ for unpolarized muon

$$\frac{dP}{d\Omega dx} = \frac{G^2 m_\mu^5}{192\pi^4} x^2 \left[3(1-x) + \frac{2}{3}\rho(4x-3) \right], \quad x = \frac{2E_e}{m_\mu}$$

V-A theory predicts $\rho = 3/4$
Experimental result: 0.7517 ± 0.0026

Electron angular distribution for polarized muon decay:

$$\frac{dP}{d\Omega} = \frac{1}{4\pi}\left[1 - \frac{1}{3}\xi P \cos\theta\right], \ P \text{ muon polarization}$$

V - A: $\xi = +1$ Experiment OK

Tau Decays

Extend to τ *leptonic* decays:

$$\tau^+ \rightarrow e^+ + \overline{\nu}_\tau + \nu_e, \;\; \tau^- \rightarrow e^- + \nu_\tau + \overline{\nu}_e$$

$$\tau^+ \rightarrow \mu^+ + \overline{\nu}_\tau + \nu_\mu, \; \tau^- \rightarrow \mu^- + \nu_\tau + \overline{\nu}_\mu$$

τ has many more decay channels open into quark-antiquark pairs

Leptonic decays: Similar to muon

Allow for checking of:

V – A Lorentz structure

Universality of lepton coupling

The τ Michel Parameters

Consider τ decays:

Many modes, including

Leptonic

$$\tau^\pm \to \mu^\pm + \nu_\mu / \overline{\nu}_\mu + \overline{\nu}_\tau / \nu_\tau$$

$$\tau^\pm \to e^\pm + \nu_e / \overline{\nu}_e + \overline{\nu}_\tau / \nu_\tau$$

Semileptonic

$$\tau^\pm \to Hadrons + \overline{\nu}_\tau / \nu_\tau$$

A simple question:
What is the τ charged current?
Investigations at LEP:

$$e^+ + e^- \to Z^0 \to \tau^+ + \tau^-$$

Conclusion: Current is $V - A$

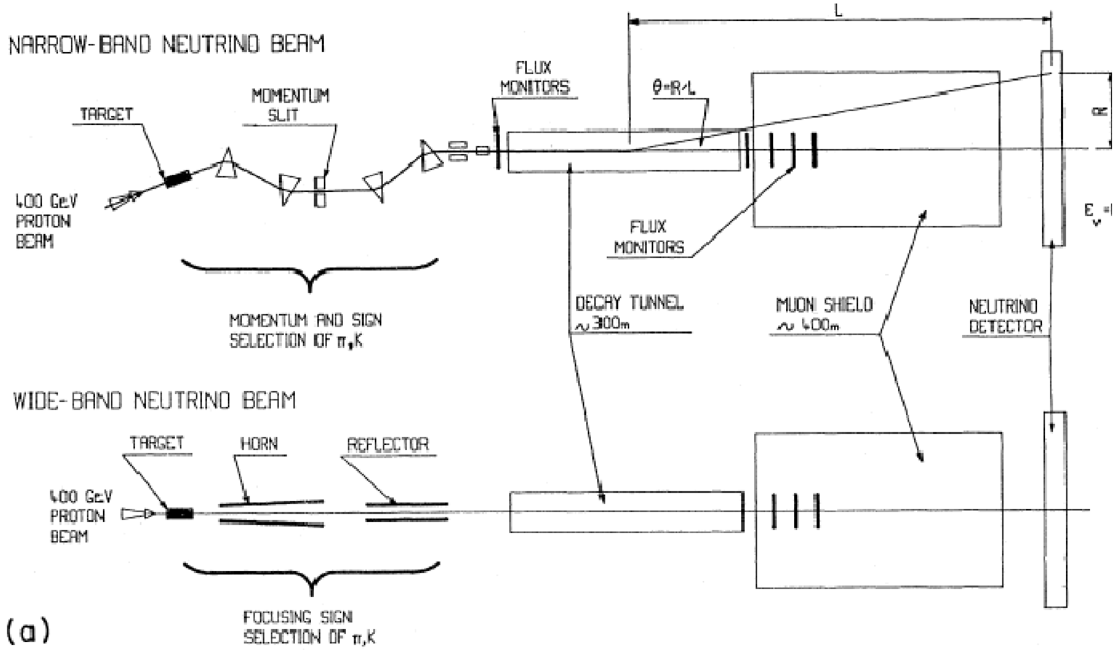
$$\rho_\tau = \begin{cases} 0.747 \pm 0.024 \ e \quad \text{mode} \\ 0.776 \pm 0.049 \ \mu \quad \text{mode} \end{cases}$$

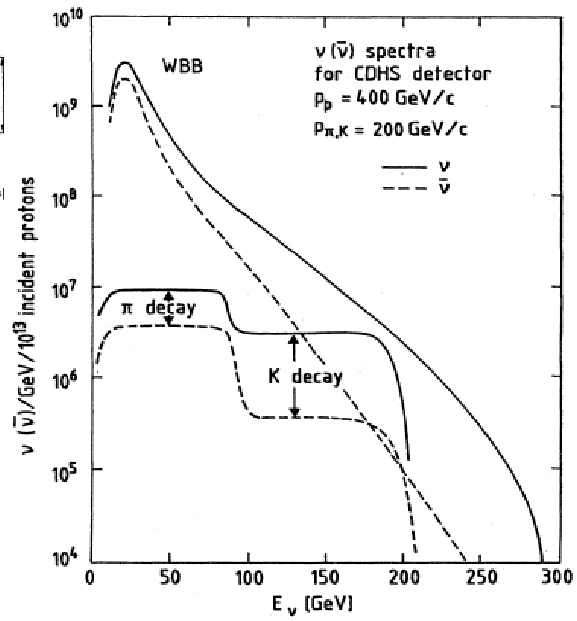
Neutrino Beams

Derived from 2-body π,K decay

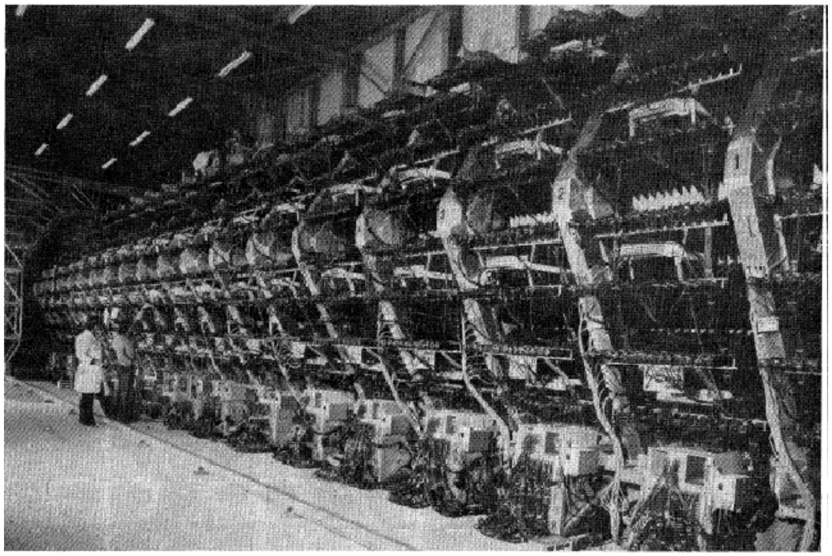
$$\pi^{\pm} \rightarrow \mu^{\pm} + \nu_{\mu}, \overline{\nu}_{\mu}$$

$$K^{\pm} \rightarrow \mu^{\pm} + \nu_{\mu}, \overline{\nu}_{\mu}$$

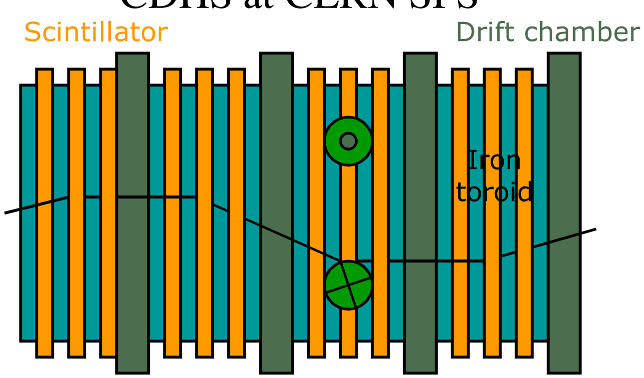


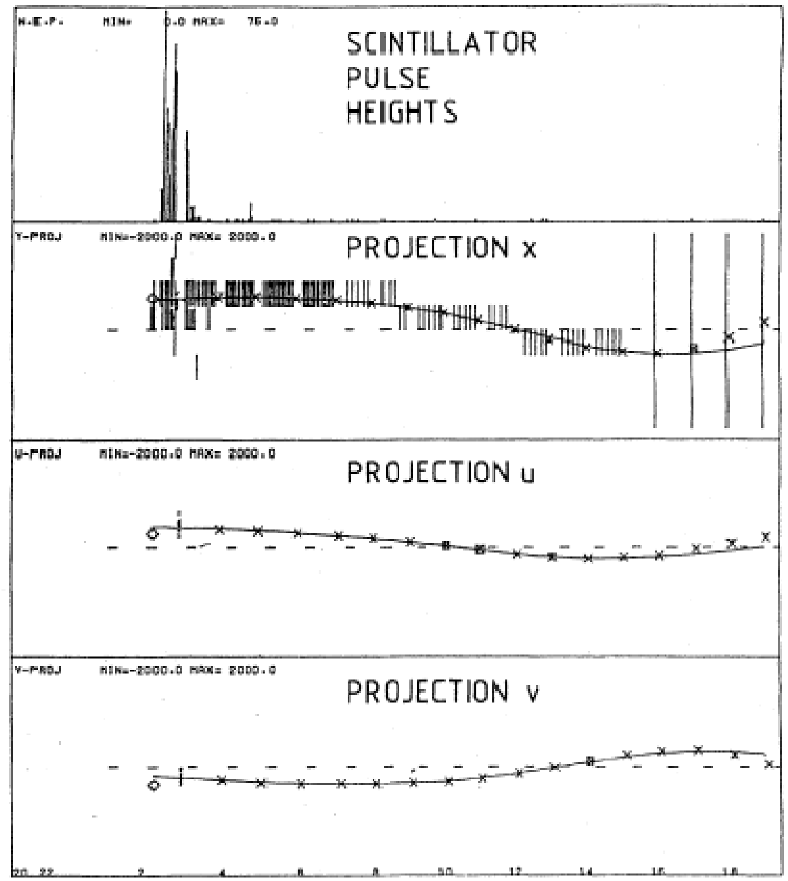


Neutrino Detectors - I

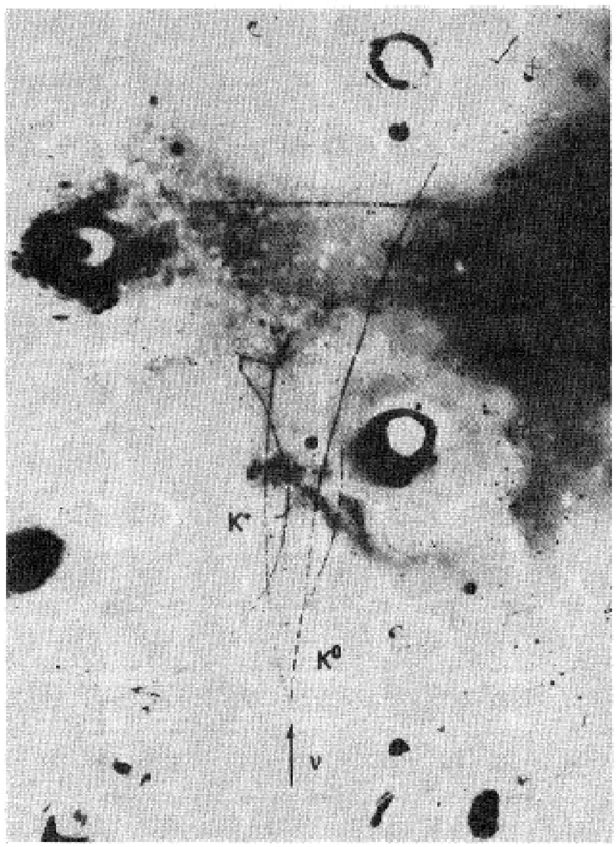


CDHS at CERN SPS

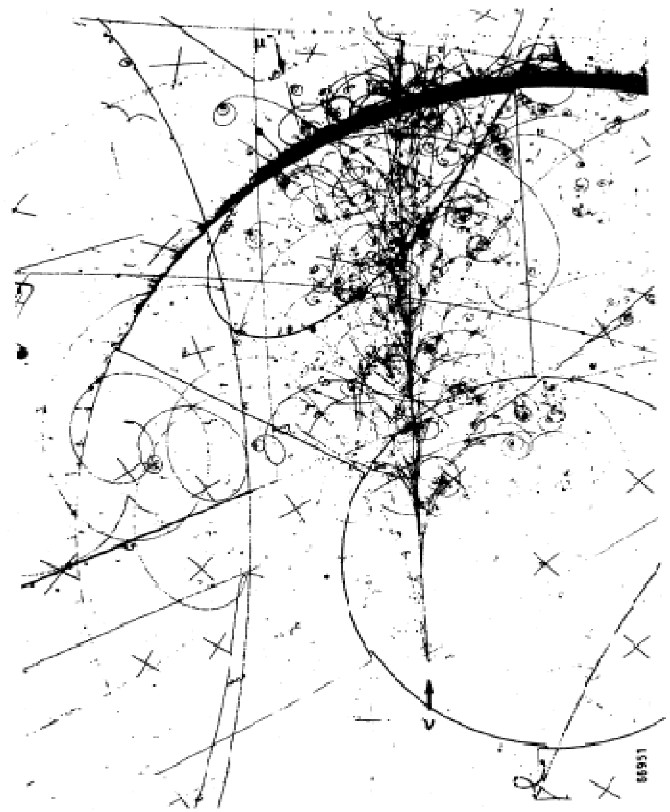




Neutrino Detectors - II

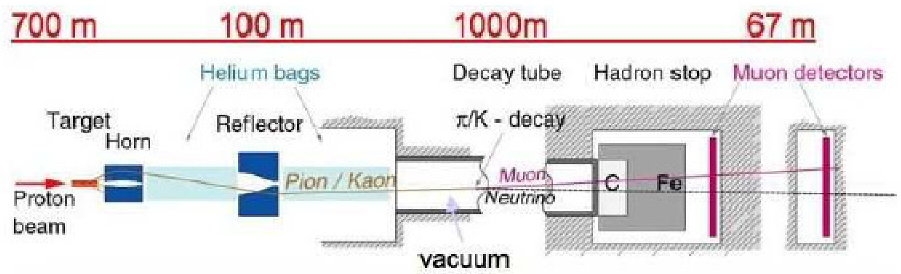


Gargamelle - Muonless

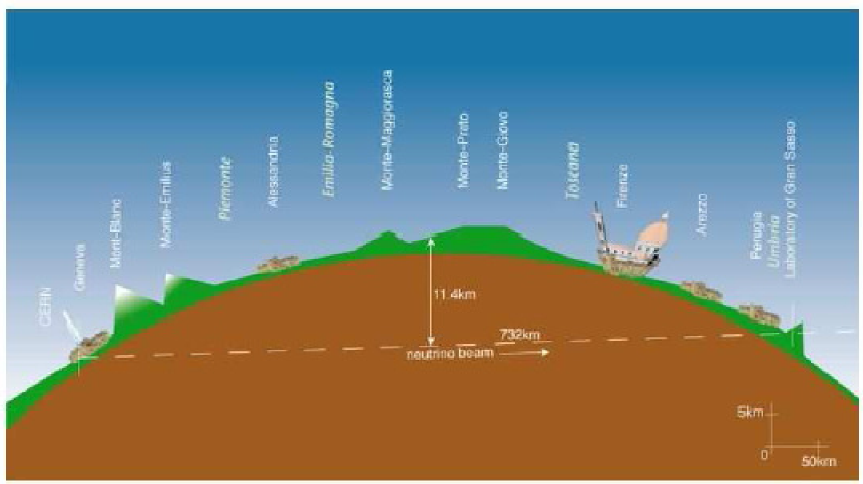


BEBC – Muon + Hadronic shower

OPERA - I

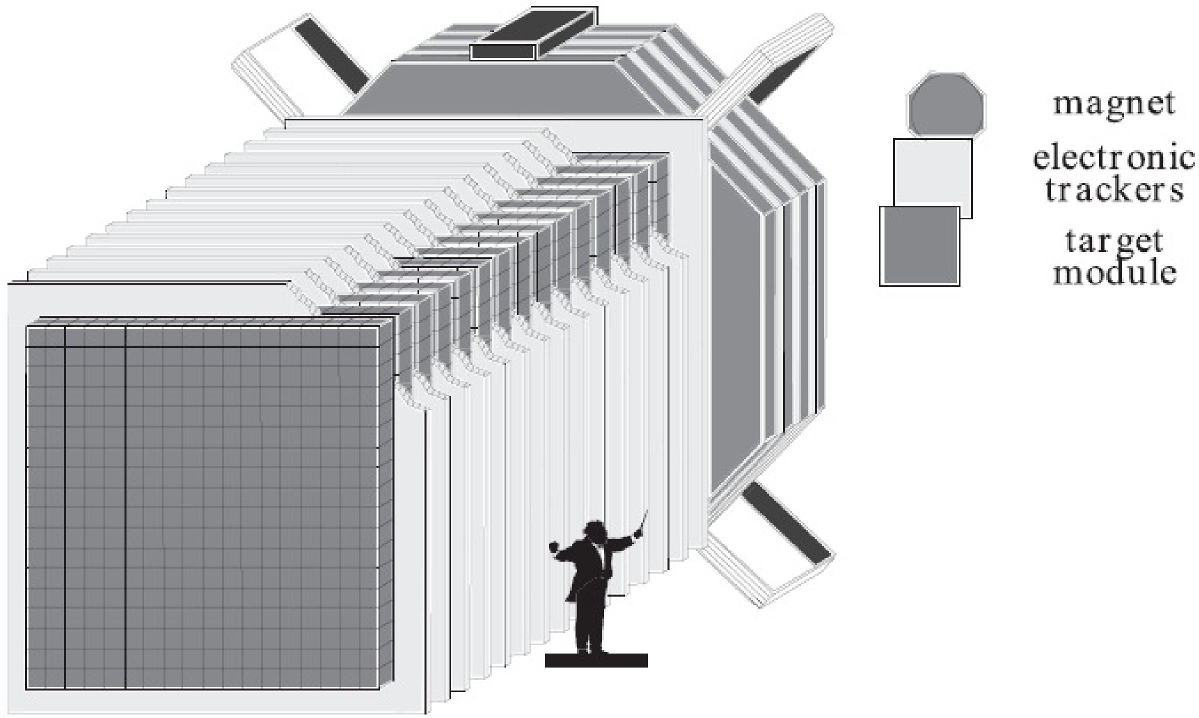


700 m
100 m
1000m
67 m
Helium bags
Decay tube
Hadron stop
Muon detectors
Target
Horn
Reflector
π/K - decay
Proton beam
Pion / Kaon
Muon
Neutrino
C
Fe
vacuum



CERN
Geneva
Mont-Blanc
Monte-Emilius
Piemonte
Alessandria
Emilia-Romagna
Monte-Maggiorasca
Monte-Prato
Monte-Giovo
Toscana
Firenze
Arezzo
Perugia
Umbria
Laboratory of Gran Sasso
11.4km
732km
neutrino beam
5km
0
50km

OPERA - II

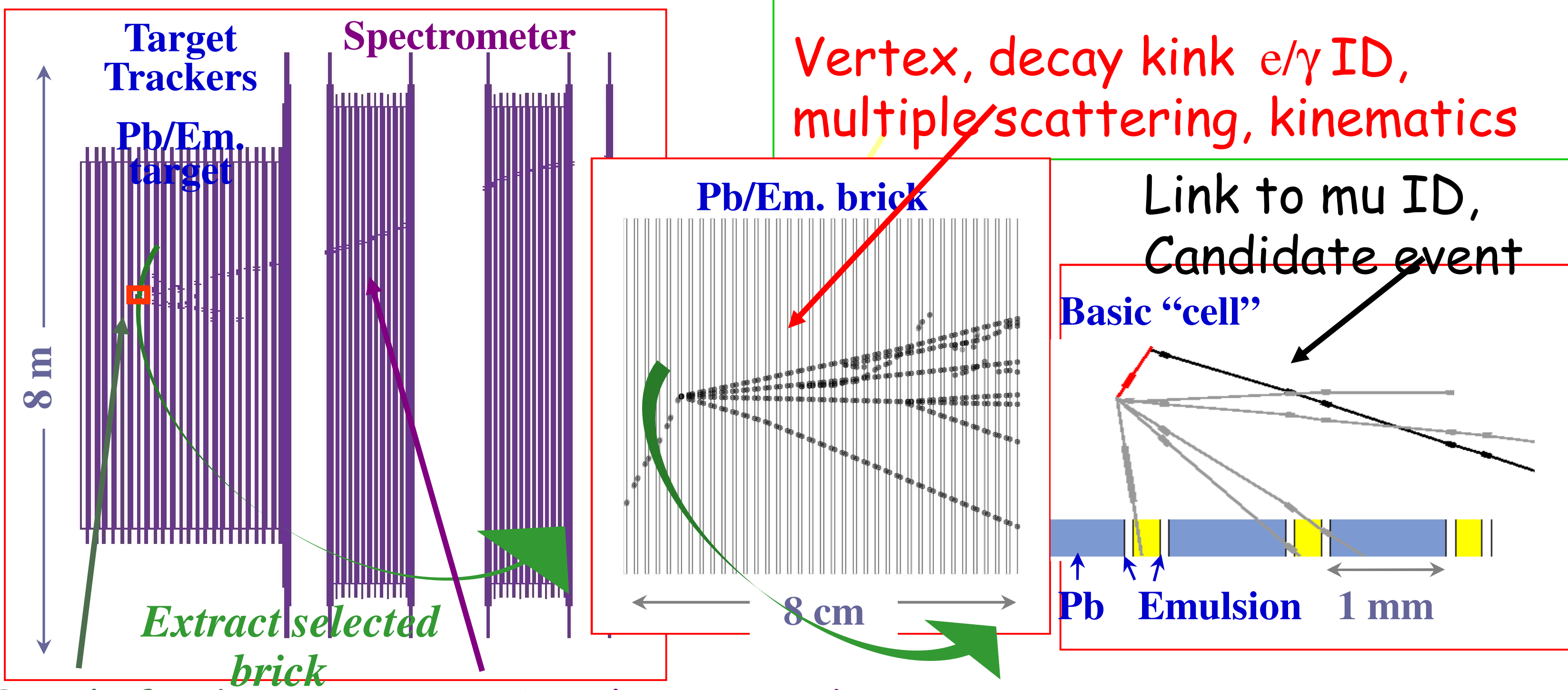


OPERA - III

Electronic detectors:

Emulsion analysis:

Vertex, decay kink e/γ ID, multiple scattering, kinematics



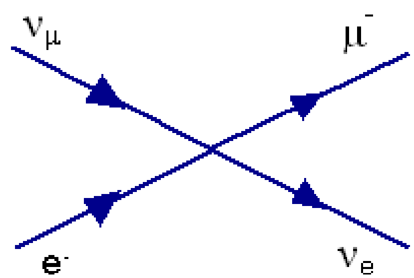
Brick finding

muon ID, charge and p

Inverse Muon Decay - I

Charged current:

$$\nu_\mu + e^- \to \mu^- + \nu_e$$



$$\sum_{spin} T_{fi} T_{fi}^* = \sum_{spin} \frac{G_F}{\sqrt{2}} \left[\bar{u}(3)\gamma^\mu\left(1-\gamma^5\right)u(1)\right]\left[\bar{u}(3)\gamma_\nu\left(1-\gamma_5\right)u(1)\right]$$

$$\left[\bar{u}(4)\gamma_\mu\left(1-\gamma_5\right)u(2)\right]\left[\bar{u}(4)\gamma^\nu\left(1-\gamma^5\right)u(2)\right]^*$$

$$\sum_{spin}\left[\bar{u}(a)\Gamma_1 u(b)\right]\left[\bar{u}(a)\Gamma_2 u(b)\right]^* = Tr\left[\Gamma_1\left(\not{p}_b + m_b\right)\Gamma_2\left(\not{p}_a + m_a\right)\right]$$

$$\sum_{spin}\left|T_{fi}\right|^2 = 64 G_F^2 \left(p_1 \cdot p_2\right)\left(p_3 \cdot p_4\right)$$

$$\sum_{spin}\left|T_{fi}\right|^2 = 256 G_F^2 E^4 \left[1 - \left(\frac{m_\mu}{2E}\right)^2\right]$$

Inverse Muon Decay - II

$$\frac{d\sigma}{d\Omega} = \frac{G_F^2}{\pi^2} E^{*2} \left[1 - \left(\frac{m_\mu}{2E^*} \right)^2 \right]^2$$

$$\rightarrow \sigma = \frac{4G_F^2}{\pi} E^{*2} \left[1 - \left(\frac{m_\mu}{2E^*} \right)^2 \right]^2, \quad E^* \text{ CM energy of } e, \nu$$

$E^* \simeq \sqrt{2mE_\nu} \rightarrow \sigma \propto E_\nu$ at high energy

$$E^* \simeq \sqrt{2mE_\nu}$$

$$\rightarrow \sigma = \frac{8G_F^2 m}{\pi} E_\nu$$

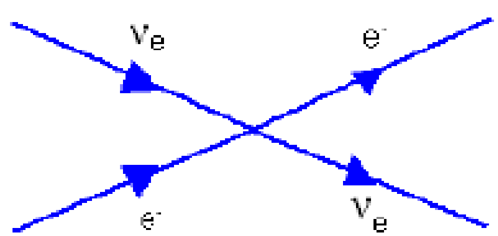
$\rightarrow \sigma \propto E_\nu$ at high energy

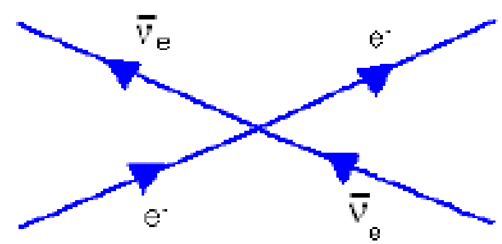
Neutrino – Lepton Scattering - I

Charged current ν_e - e scattering:

$$\nu_e + e^- \rightarrow e^- + \nu_e$$

$$\overline{\nu}_e + e^- \rightarrow e^- + \overline{\nu}_e$$





NB This is actually incomplete:
For these processes there are neutral current amplitudes as well
Cross sections must be evaluated by summing *all* relevant amplitudes

Neutrino – Lepton Scattering - II

Find for the total cross-section:

$$\left.\begin{aligned}\sigma_\nu &= \frac{4G_F^2}{\pi}E^{*2}\\ \sigma_{\bar\nu} &= \frac{1}{3}\frac{4G_F^2}{\pi}E^{*2}\end{aligned}\right\},\quad E^* \text{ CM energy of } e,\nu$$

As divergent as inverse muon decay

NB These cross sections are only approximate, in that neutral current contribution in neglected

V - A and the Nucleon Current

Nucleon current # $V - A$, rather $V - \alpha A$

Found ~ the same structure for lepton and nucleon current

$\bar{\psi}_e \gamma^\mu \left(1-\gamma^5\right)\psi_\nu$ Pure V-A

$$\bar{\psi}_p \gamma_\mu \left(1-\frac{C_A}{C_V}\gamma_5\right)\psi_n$$

V: identical coupling, A: modified by strong interaction

Consider β decay of O^{14} – pure Fermi $G_F = 1\cdot 10^{-5} M_p^{-2}$

Call this the β-decay Fermi constant

Measured value of $\frac{C_A}{C_V}$ for β decay of various baryons:

n	-1.267
Λ^0	-0.718
Σ^-	$+0.340$
Ξ^-	-0.25

$\nu, \bar{\nu}$ -Nucleon Cross Section - I

Extend V-A to neutrino-nucleon scattering

$$\nu_\mu + N \rightarrow \mu^- + X$$

$$\bar{\nu}_\mu + N \rightarrow \mu^+ + X$$

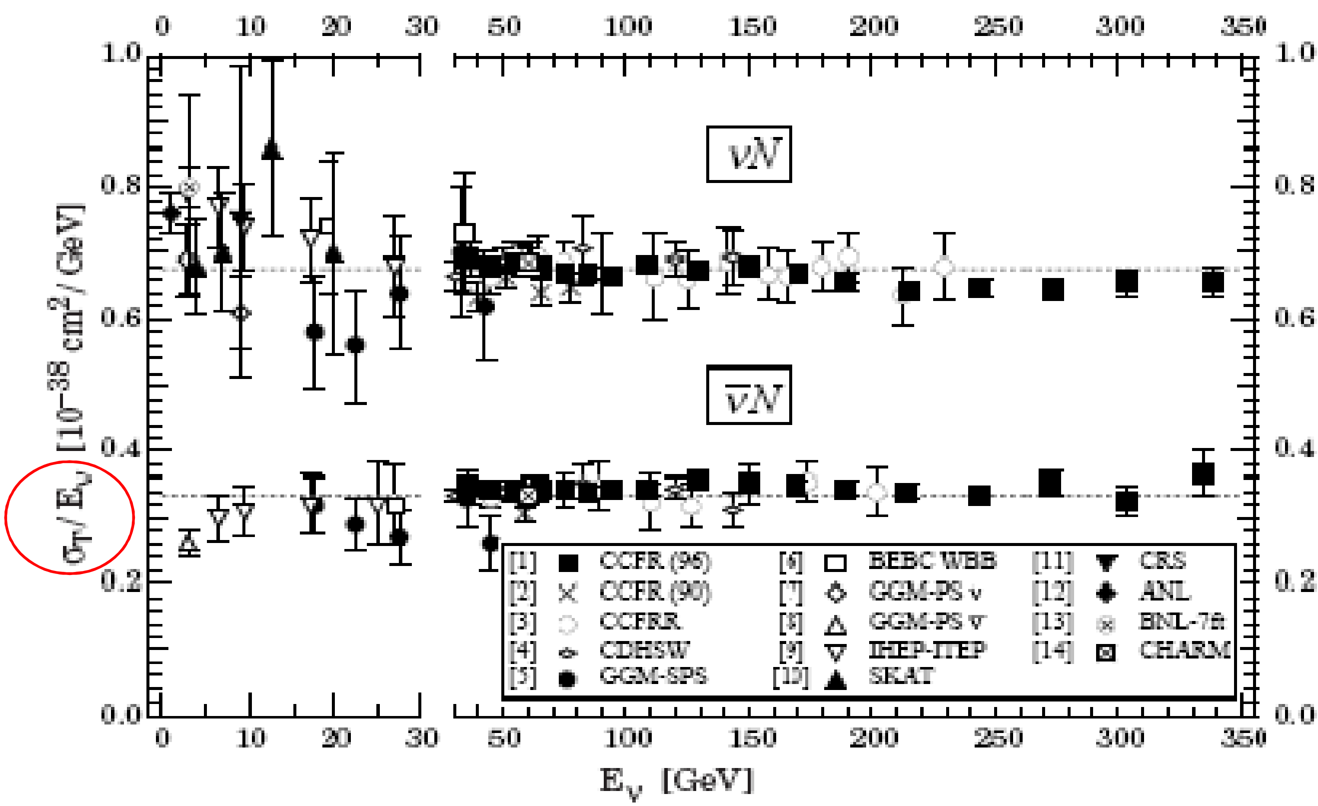
Somewhat similar to *e-N*, μ-*N* deep inelastic scattering
Modeling similar to DIS: Parton elastic scattering

Deep ineastic neutrino scattering reveals the same structure as charged lepton DIS

More information: Charged current sensitive to parton charge sign

→ Can separate quark/antiquark contribution

$\nu,\bar{\nu}$-Nucleon Cross Section - II



Linearly rising cross section confirmed…

$\nu, \bar{\nu}$ -Nucleon Cross Section - III

Cross section cannot be strictly proportional to E_n,, of course:

Divergence at high energy!

Indeed, unitarity bound is violated around E_n ~ *300 GeV*

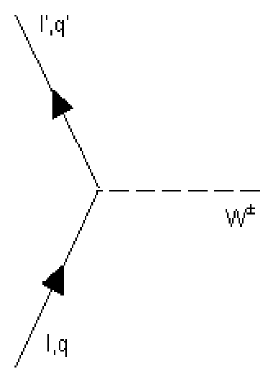
$\sigma \leq \frac{4\pi}{k^2} = \frac{4\pi}{E^{*2}}$ Unitarity bound for S-Wave scattering

Radiative corrections to Fermi's theory divergent:

No renormalization procedure available

Fermi theory is non renormalizable

Radical fix to the current*current model required:



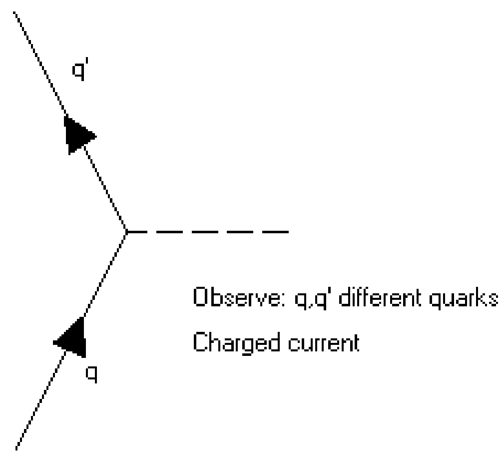
Vertice in corrente carica

For both lepton and nucleon currents, *exchange of a (heavy, charged) boson $W^{\pm}$*

Universality: Quarks

Semileptonic and non leptonic processes understood in terms of quarks

Basically similar coupling to leptonic charged currents:

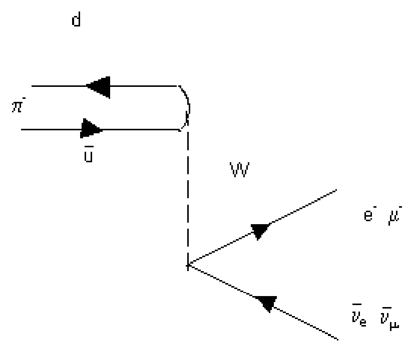


Picture is slightly more complicated, however
Fundamental question:

Is the quark coupling identical to the lepton one?

Semileptonic Decays: π, K - I

$$\pi \to \mu + \nu_\mu, \quad \pi \to e + \nu_e$$



$$T_{fi} = \frac{G_F}{\sqrt{2}} \left[\overline{u}(3) \gamma^\mu (1 - \gamma^5) v(2) \right] F_\mu$$

F_μ: equivalent of 'current' for the $q\overline{q}$ bound state

$F_\mu = f_\pi p_\mu$ 4-momentum is the only 4-vector available

Semileptonic Decays : π, K - II

Obtain:

$$\Gamma_l = \frac{f_\pi^2}{8m_\pi^3} G_F^2 m_l^2 \left(m_\pi^2 - m_l^2 \right)^2$$

$$\frac{\Gamma(e)}{\Gamma(\mu)} = \frac{m_e^2 \left(m_\pi^2 - m_e^2 \right)^2}{m_\mu^2 \left(m_\pi^2 - m_\mu^2 \right)^2} = 1.3\ 10^{-4}$$

Quite surprising: Phase space factor is much larger for *e*!
But:

Semileptonic Decays : π, K - III

Chirality rule:

ν is only L, $\bar{\nu}$ is only R

$$\rightarrow \begin{cases} \mu^-, e^- \text{ from } \pi,\ K \text{ decay forced to } R \\ \mu^+, e^+ \text{ from } \pi,\ K \text{ decay forced to } L \end{cases}$$

by angular momentum conservation:

$$\left.\begin{array}{l} H\left(\mu^-, e^-\right) = +1 \\ H\left(\mu^+, e^+\right) = -1 \end{array}\right\} \text{'Wrong' helicity}$$

e: Fully relativistic $\rightarrow P(wrong\ H) = \dfrac{1-\beta}{2} \sim 0$

μ : Not fully relativistic $\rightarrow P(wrong\ H) = \dfrac{1-\beta}{2} \sim$ substantial

Cabibbo Angle - I

Consider charged current of leptons:
Very natural to group charged and neutral leptons into *doublets*, or *families*

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix} \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix} \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}$$

Within each doublet, charged current transitions can be thought as due to emission/absorption of $W^\pm$ bosons , similar to (neutral) e.m. current transitions

$$\begin{matrix} W^- \rightarrow & & \nu_e & & \rightarrow W^+ \\ & \uparrow & & \downarrow & \\ W^- \leftarrow & & e^- & & \leftarrow W^+ \end{matrix}$$

Similar for 2nd, 3rd family

Charged intermediate bosons $W^\pm$ analog to photon

Cabibbo Angle - II

Would seem natural to extend this scheme to quarks

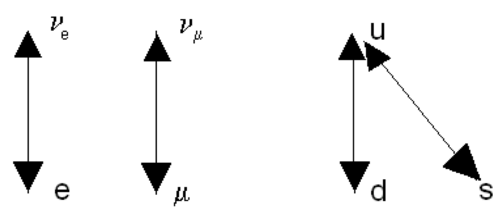
$$\begin{pmatrix} u \\ d \end{pmatrix} \qquad \begin{pmatrix} \\ s \end{pmatrix} \qquad\qquad \begin{array}{ccccc} W^- \rightarrow & & u & & \rightarrow W^+ \\ & \uparrow & & \downarrow & \\ W^- \leftarrow & & d & & \leftarrow W^+ \end{array}$$

Unfortunately, our scheme cannot work:

a) Parallelism quark-lepton is incomplete with 4 leptons and only 3 quarks

b) Does not account for strangeness violating processes

Cabibbo Angle - III



Cabibbo's very ingenious idea:

Quark flavor eigenstates (i.e., quark model eigenstates) are not to be identified with quark weak currents

$\rightarrow$*Weak currents are mixtures of different flavors*

By universal convention, mixing is assumed between *d* and *s* quarks:

$$\begin{cases} d' = \alpha d + \beta s \\ s' = \gamma d + \delta s \end{cases}$$

$\rightarrow$ By unitarity: $\begin{cases} d' = \cos\theta_c d + \sin\theta_c s \\ s' = -\sin\theta_c d + \cos\theta_c s \end{cases}$

Cabibbo Angle - IV

In terms of doublet:

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix} \quad \theta_\mathrm{c} \text{ Cabibbo's angle}$$

This explain *many* things….

Just one example:

Get the angle from β decay (Remember that *2%* difference..)

$$G_F^{(\beta)} = 0.975 G_F^{(\mu)}$$
$$\rightarrow \theta_c \simeq 13^0$$

Cabibbo Angle - V

Now assume for leptonic π, K decays (very simply!)

$$f_K = f \sin\theta_C$$
$$f_\pi = f \cos\theta_C$$
$$\rightarrow \frac{\Gamma(K \rightarrow l + \nu_l)}{\Gamma(\pi \rightarrow l + \nu_l)} = \tan^2\theta_C \frac{m_\pi^3 \left(m_K^2 - m_l^2\right)^2}{m_K^3 \left(m_\pi^2 - m_l^2\right)^2}$$
$$\rightarrow \frac{\Gamma(K \rightarrow l + \nu_l)}{\Gamma(\pi \rightarrow l + \nu_l)} = \begin{cases} (\text{teo.}) & 0.96 \; l = \mu \\ (\text{teo.}) & 0.19 \; l = e \end{cases}$$

Compare to experiment:

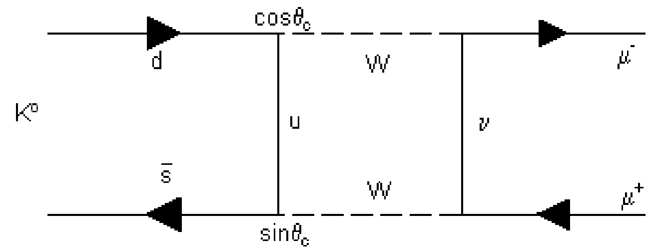
$$\begin{cases} (\text{exp.}) & 1.34 \; l = \mu \\ (\text{exp.}) & 0.19 \; l = e \end{cases}$$

Amazingly close!

GIM - I

Besides the many puzzles finally explained by Cabibbo, a few are left unexplained
Most relevant:

$K^0 \to \mu^+\mu^-$ strongly suppressed

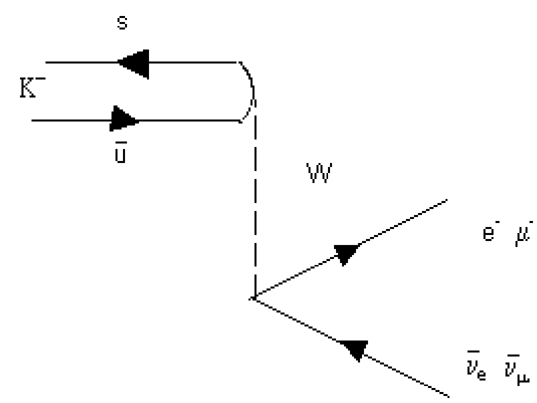


2nd order weak process, still quite easy to compute
Expect rate higher by orders of magnitude

$\mathrm{BR}_{\mathrm{meas}}(K^0 \to \mu\mu) = (6.87 \pm 0.11) 10^{-9}$

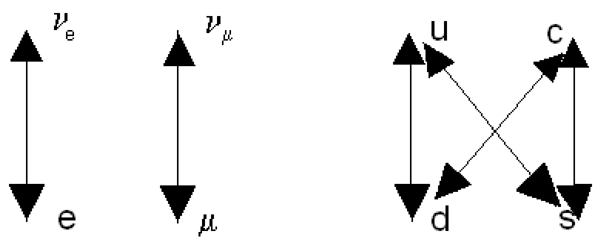
Compare charged decay:

$\mathrm{BR}_{\mathrm{meas}}(K^- \to \mu^- \bar{\nu}_\mu) = 63.5\ \%$



GIM - II

Glashow, Iliopoulos and Maiani:
There exists a fourth quark, call it c like charm
Full symmetry restored between lepton-quark families



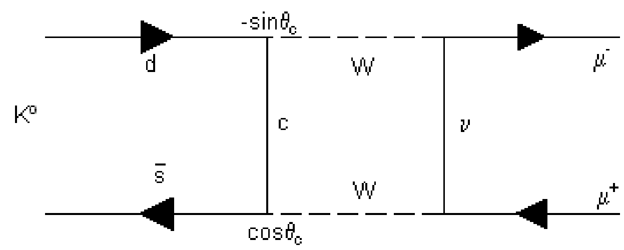
Two weak doublets:

$$\begin{pmatrix} u \\ d \end{pmatrix} \to \begin{pmatrix} u \\ d\cos\theta_C + s\sin\theta_C \end{pmatrix} \equiv \begin{pmatrix} u \\ d' \end{pmatrix}$$

$$\begin{pmatrix} c \\ s \end{pmatrix} \to \begin{pmatrix} c \\ -d\sin\theta_C + s\cos\theta_C \end{pmatrix} \equiv \begin{pmatrix} c \\ s' \end{pmatrix}$$

GIM - III

Then expect a second amplitude for K^0 decay



Total amplitude: Sum of two

$$\left.\begin{array}{l} A_1 \propto \sin\theta_c \cos\theta_c \\ A_2 \propto -\sin\theta_c \cos\theta_c \end{array}\right\} \rightarrow A_1 + A_2 \sim 0$$

Total amplitude not exactly *0* because $m_c \neq m_u$
From observed rate predict $m_c \sim 1-2 \quad GeV$

Leading to *November Revolution*: J/ψ discovery

CKM

Extend the idea to 3 families:
From Cabibbo's angle to *Cabibbo-Kobayashi-Maskawa* matrix

$$\begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} |d\rangle \\ |s\rangle \\ |b\rangle \end{bmatrix} = \begin{bmatrix} |d'\rangle \\ |s'\rangle \\ |b'\rangle \end{bmatrix}$$

From unitarity:
3 mixing angles
1 complex phase This can account for CP violation

Experimental values:

$$\begin{bmatrix} 0.9753 & 0.221 & 0.003 \\ 0.221 & 0.9747 & 0.040 \\ 0.009 & 0.039 & 0.9991 \end{bmatrix}$$

Almost diagonal
Heavy quarks even more diagonal

Beyond Fermi's Theory

As anticipated:

Current-Current must be a *low energy effective theory:*
Low energy approximation of a more general theory

Replace *contact interaction* (current-current) by *boson exchange*:

Modeled after the electromagnetic interaction

Exchanged particle must be

- *Charged* (Charged current ±)
- *Chiral* (Only coupled to left chiral parts: Parity violation)
- *Heavy* (Fermi's point-like interaction OK at low energy)

Remark

Topics to follow:

Just a short introduction to essential phenomenology

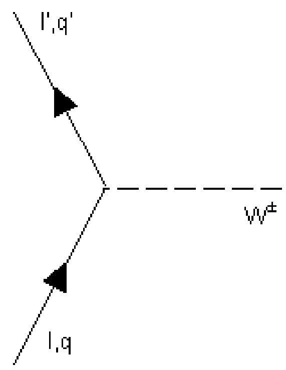
Core topics in the Standard Model

Fuller coverage of *electroweak interaction* in other courses

Weak Interaction - I

Just listing some important properties

A) Quarks and leptons both interact through the exchange of *vector particles*



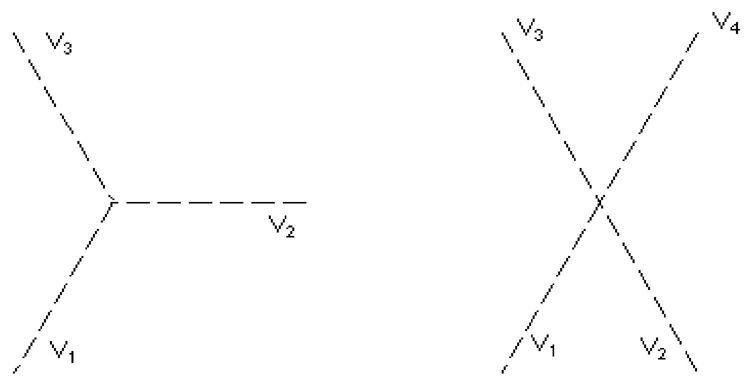
Vertice in corrente carica

Weak Interaction - II

B) Exchanged vector bosons are *(very)* massive

C) Interaction form derived by a non-Abelian gauge symmetry
+
Special mechanism giving mass to some of the gauge fields

D) Non-Abelian vertexes



Weak Interaction - III

Weak charged current:

$$j^{\mu} = \bar{u}_f \gamma^{\mu} \frac{\left(1-\gamma^5\right)}{2} u_i$$

$$j_V^{\mu} = \frac{1}{2} \bar{u}_f \gamma^{\mu} u_i \quad \text{Vector}$$

$$j_A^{\mu} = \frac{1}{2} \bar{u}_f \gamma^{\mu} \gamma^5 u_i \quad \text{Axial}$$

Compare to electromagnetic current:

$$j^{\mu} = \bar{u} \gamma^{\mu} u$$

Weak Interaction - IV

Apparently, totally different.

But, as anticipated:

$$\left(1-\gamma^5\right)^2=\left(1-2\gamma^5+1\right)=\left(2-2\gamma^5\right)=2\left(1-\gamma^5\right)$$

$$\rightarrow\left[\frac{\left(1-\gamma^5\right)}{2}\right]^2=\frac{2\left(1-\gamma^5\right)}{4}=\frac{\left(1-\gamma^5\right)}{2}$$

$$\rightarrow\bar{u}_f\gamma^\mu\frac{\left(1-\gamma^5\right)}{2}u_i=\bar{u}_f\gamma^\mu\left[\frac{\left(1-\gamma^5\right)}{2}\right]^2u_i=\underbrace{\bar{u}_f\frac{\left(1+\gamma^5\right)}{2}}_{\bar{u}_{fL}}\gamma^\mu\underbrace{\frac{\left(1-\gamma^5\right)}{2}u_i}_{u_{iL}}$$

$$\rightarrow\bar{u}_f\gamma^\mu\frac{\left(1-\gamma^5\right)}{2}u_i=\bar{u}_{fL}\gamma^\mu u_{iL}$$

Same form, but involving only *LEFT* chiral states

Weak Interaction - V

$$j_\mu^{em} = \bar{e}\gamma_\mu e$$

$$e \equiv \left(\frac{1-\gamma_5}{2}\right)e + \left(\frac{1+\gamma_5}{2}\right)e = e_L + e_R$$

Therefore:

$$\rightarrow j_\mu^{em} = \bar{e}\gamma_\mu e = \left(\bar{e}_L + \bar{e}_R\right)\gamma_\mu\left(e_L + e_R\right) = \bar{e}_L\gamma_\mu e_L + \bar{e}_R\gamma_\mu e_R$$

Because:

$$\bar{e}_L\gamma_\mu e_R = e\left(\frac{1+\gamma_5}{2}\right)\gamma_\mu\left(\frac{1+\gamma_5}{2}\right)e = e\gamma_\mu\left(\frac{1-\gamma_5}{2}\right)\left(\frac{1+\gamma_5}{2}\right)e = 0$$

$$\left(\frac{1-\gamma_5}{2}\right)\left(\frac{1+\gamma_5}{2}\right) = 0$$ Projectors 'orthogonal'

The bottom line:

Weak (charged) and Electromagnetic currents:

Same Lorentz structure (vector), but LL vs. LL+RR

Weak Interaction - VI

W propagator: $-i\dfrac{g_{\mu\nu}-q_\mu q_\nu/M_W^2}{q^2-M_W^2}$

$$q^2 \ll M_W^2 \rightarrow -i\frac{g_{\mu\nu}-q_\mu q_\nu/M_W^2}{q^2-M_W^2} \approx i\frac{g_{\mu\nu}}{M_W^2} \quad q^2\text{-independent}$$

$$T_{fi} \cong \left(\frac{1}{2\sqrt{2}}\right)^2 g_W^2 \left(\overline{u}_f^{(1)}\frac{1}{2}\gamma^\mu\left(1-\gamma^5\right)u_i^{(1)}\right) i\frac{g_{\mu\nu}}{M_W^2}\left(\overline{u}_f^{(2)}\frac{1}{2}\gamma_\nu\left(1-\gamma_5\right)u_i^{(2)}\right)$$

$$\rightarrow T_{fi} \cong i\frac{1}{8}4\sqrt{2}G_F \ \ j^{\mu(1)} j_\mu^{(2)}$$

$g_W^2 \equiv \alpha_W$ Charged current coupling constant

$G_F = \dfrac{\sqrt{2}}{8}\left(\dfrac{g_W}{M_W}\right)^2$ Fermi constant

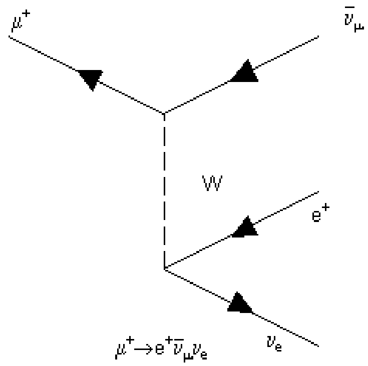
Weak Interaction - VII

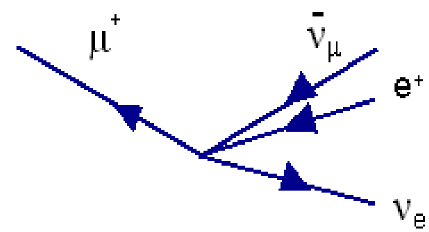
Showing how SM diagrams collapse into current-current:

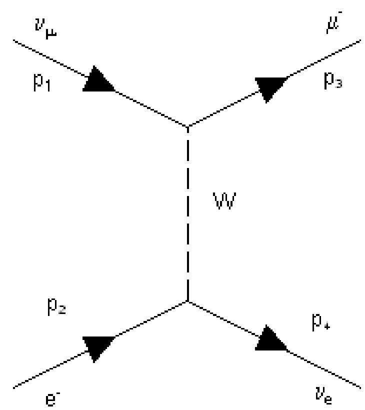
At low energy:

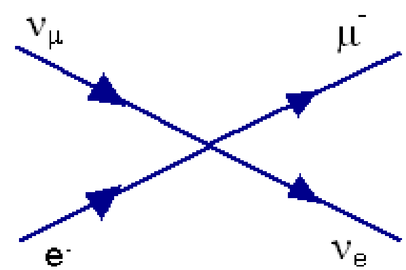
$$q^2 \ll M_W^2$$

$$\rightarrow i\frac{g_{\mu\nu} - q_\mu q_\nu / M_W^2}{q^2 - M_W^2} \approx i\frac{g_{\mu\nu}}{M_W^2}$$









Neutral Currents - I

Charged intermediate boson $W^{\pm}$ does not fix everything..
Typical issue left unsolved:

$$\nu + \overline{\nu} \to W^{+} + W^{-}$$

Amplitude still divergent due to *longitudinal W* contribution..
Similar to QED process

$$e^{+} + e^{-} \to \gamma + \gamma$$

In QED: Renormalization possible due to *gauge invariance*

Would like to upgrade charged current interaction model to a gauge theory to get finite, renormalized amplitudes

Charged current weak interaction to be understood as a gauge interaction
$\to$ $W^{\pm}$ as *gauge fields*

Neutral Currents - II

2 fields: Gauge group must be non-Abelian

Massive field: Gauge invariance apparently impossible

Nevertheless:

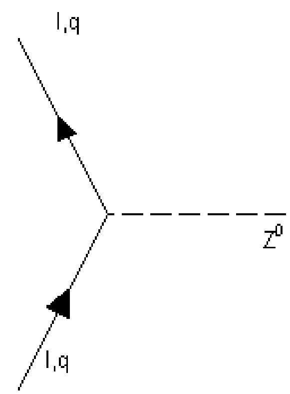
Assume some funny mechanism can preserve gauge invariance with a massive field (←Bold, but true)

Try *SU(2)* as gauge group:

Predict a triplet of gauge fields)

Neutral current should exist

Neutral gauge field must be massive



Vertice in corrente neutra

Neutral Currents - III

Exchange of *neutral intermediate boson* Z^0, rather than charged $W^{\pm}$

Expect processes similar to electromagnetic, with coupling to *all fermions*

Typical signature: *Parity violation*

However, do not expect large effects at low energy:

Large Z^0 mass quenching down amplitudes

Electromagnetic amplitudes dominating at low energy

Some hope to see them in neutrino interactions (no e.m. contributions)

Not observed for a long time: Neutrino experiments difficult

Finally observed at CERN in 1973 in a bubble chamber experiment

Neutral Currents - IV

As a result of the weak-electromagnetic unification, neutral currents are *different* from charged

Lorentz structure not $V - A$

$$-i\frac{g_W}{\sqrt{2}}\gamma^\mu\frac{\left(1-\gamma^5\right)}{2} \quad \text{Charged}$$

$$-ig_Z\gamma^\mu\frac{\left(C_V^f - C_A^f\gamma^5\right)}{2} \quad \text{Neutral}$$

Coupling

Fermion	C_V	C_A
ν_e, ν_μ, ν_τ	$+1/2$	$+1/2$
e, μ, τ	$-1/2 + 2\sin\theta_W$	$-1/2$
u, c, t	$+1/2 - 4/3\sin^2\theta_W$	$+1/2$
d, s, b	$-1/2 + 2/3\sin^2\theta_W$	$-1/2$

θ_W new fundamental constant

What about interaction strength?

Neutral Currents - V

Tight relationship between weak and electromagnetic interactions

Coupling constants:

$$g_{\mathrm{w}} = \frac{e}{\sin\theta_W}$$

$$g_{\mathrm{z}} = \frac{e}{\sin\theta_W \cos\theta_W}$$

$$\alpha = e^2$$

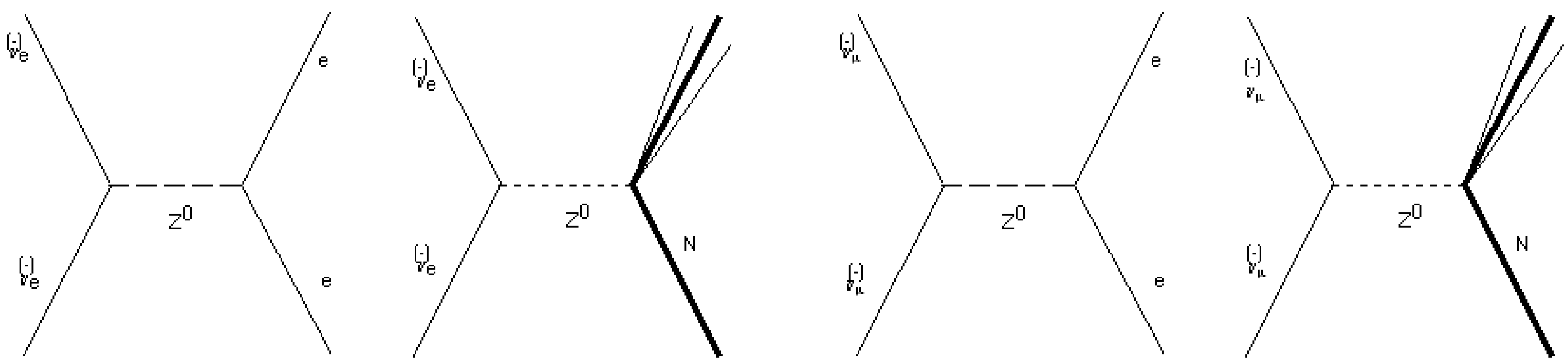
Charged, neutral and electromagnetic couplings fixed by 2 universal constants:

e : Elementary charge

θ_W: Weinberg angle, new fundamental constant

Neutral Currents - VI

Expect to observe typical processes like:



$(\nu_e, \overline{\nu}_e) + e \rightarrow (\nu_e, \overline{\nu}_e) + e$ Contributing to elastic scattering

$$\left.\begin{array}{l} (\nu_\mu, \overline{\nu}_\mu) + e \rightarrow (\nu_\mu, \overline{\nu}_\mu) + e \\ (\nu_e, \overline{\nu}_e) + N \rightarrow (\nu_e, \overline{\nu}_e) + hadron \;\; shower \\ (\nu_\mu, \overline{\nu}_\mu) + N \rightarrow (\nu_\mu, \overline{\nu}_\mu) + hadron \;\; shower \end{array}\right\} \text{New}$$

Discovery of NC: Gargamelle

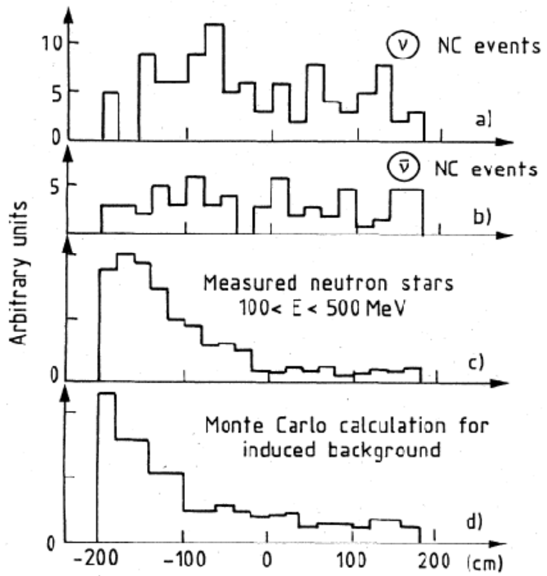
Main problem: Neutron background → Observe vertex position along the chamber

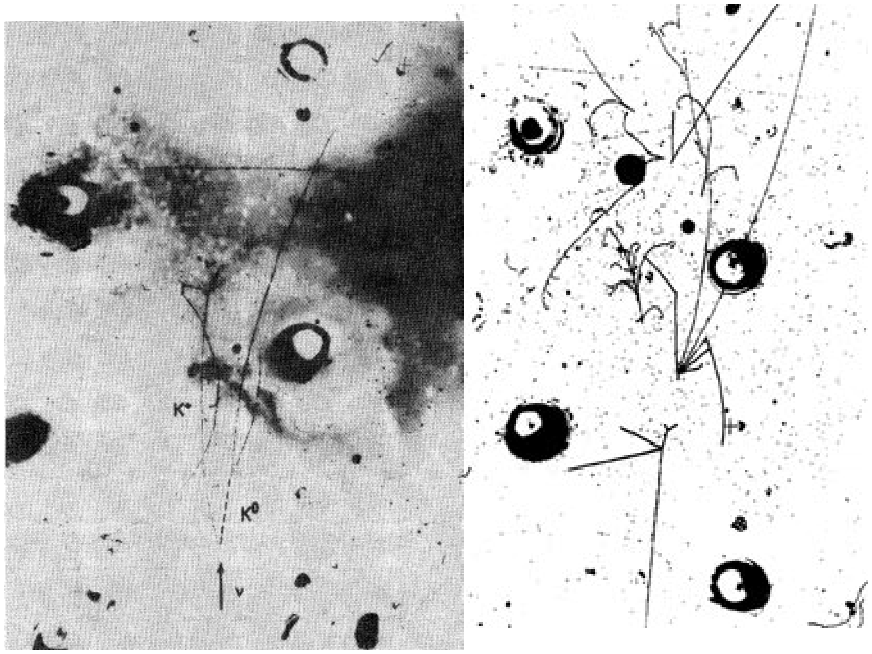
Exponential: Neutron background

$\lambda_{\text{int}}^{n} \sim 90 \ \ cm$

Flat: Neutrino signal

$\lambda_{\text{int}}^{\nu} \sim \infty$





Estimated n background: 9 6

Table 1

	ν-exposure	ν̄-exposure
No. of neutral-current candidates	102	64
No. of charged-current candidates	428	148