Elementary Particles II

1 – QCD

Color, Gauge Fields, Gluons, Asymptotic Freedom, Confinement, Perturbative QCD, Quarkonium

Re-Examining the Evidence

Experiments probing the EM structure, like DIS and similar:

Scaling of the structure functions

Evidence for point-like constituents, funny behavior:

Like free particles when interacting with EM currents at high Q^2

Never observed outside hadrons $\rightarrow$ *Tightly bound?*

Experiments probing the strong interaction:

Large particle zoo

Evidence for highly symmetrical grouping and ordering

Strong suggestion of a substructure: Quarks

Funny, ad-hoc rules driving the observed symmetry

Constituents?

Besides the general, puzzling behavior observed, finally identify a number of serious problems, cast into a few, quantitative issues:

Baryons and the Pauli Principle
The R Ratio
The π^0 *Decay Rate*
The τ *Lepton Branching Ratios*

From all these questions, and others, a common conclusion:

Our picture of the quark model is not complete

Pauli Principle

Quark model:

Besides its many, remarkable successes, a central point is at issue:

The baryon wave function (space $\times$ spin $\times$ flavor) is symmetric

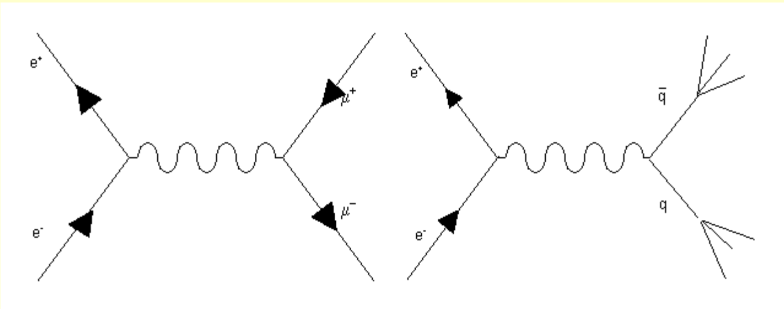
Pauli Principle seems to be lost, which is very bad news:

The Spin-Statistics Theorem is a consequence of very general principles of relativistic QFT, not easily dismissed

R Ratio - I

Assume the process $e^+e^- \to hadrons$ to proceed at the lowest order through

$e^+e^- \to q\,\bar{q} \to hadrons$



As for DIS:
Don't care about quark *hadronization*, assume the time scales for hard and soft sub-processes to be wildly different

$$\sigma\left(e^+e^- \to \mu^+\mu^-\right) = \frac{4\pi\alpha^2}{3s}$$

$$\sigma\left(e^+e^- \to q\,\bar{q}\right) = \frac{4\pi\alpha^2 Q_q^2}{3s}, \quad Q_q = \text{quark charge in } e \text{ units}$$

$$R\left(E_{CM}\right) = \frac{\sigma\left(e^+e^- \to adroni\right)}{\sigma\left(e^+e^- \to \mu^+\mu^-\right)} = \frac{\sum_q \sigma\left(e^+e^- \to q\bar{q}\right)}{\sigma\left(e^+e^- \to \mu^+\mu^-\right)} = \sum_q Q_q^2$$

Sum extended to all accessible quark flavors $\to 2m_q < E_{CM}$

R Ratio - II

R counts the number of different quark species created at any given E_{CM} . Expect:

$$u,d \rightarrow R = \left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 = \frac{5}{9} \qquad Low\ energy$$

$$u,d,s \rightarrow R = \left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 = \frac{6}{9} \qquad E > 1\text{-}1.5\ GeV$$

$$u,d,s,c \rightarrow R = \left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 = \frac{10}{9} \qquad E > 3\ GeV$$

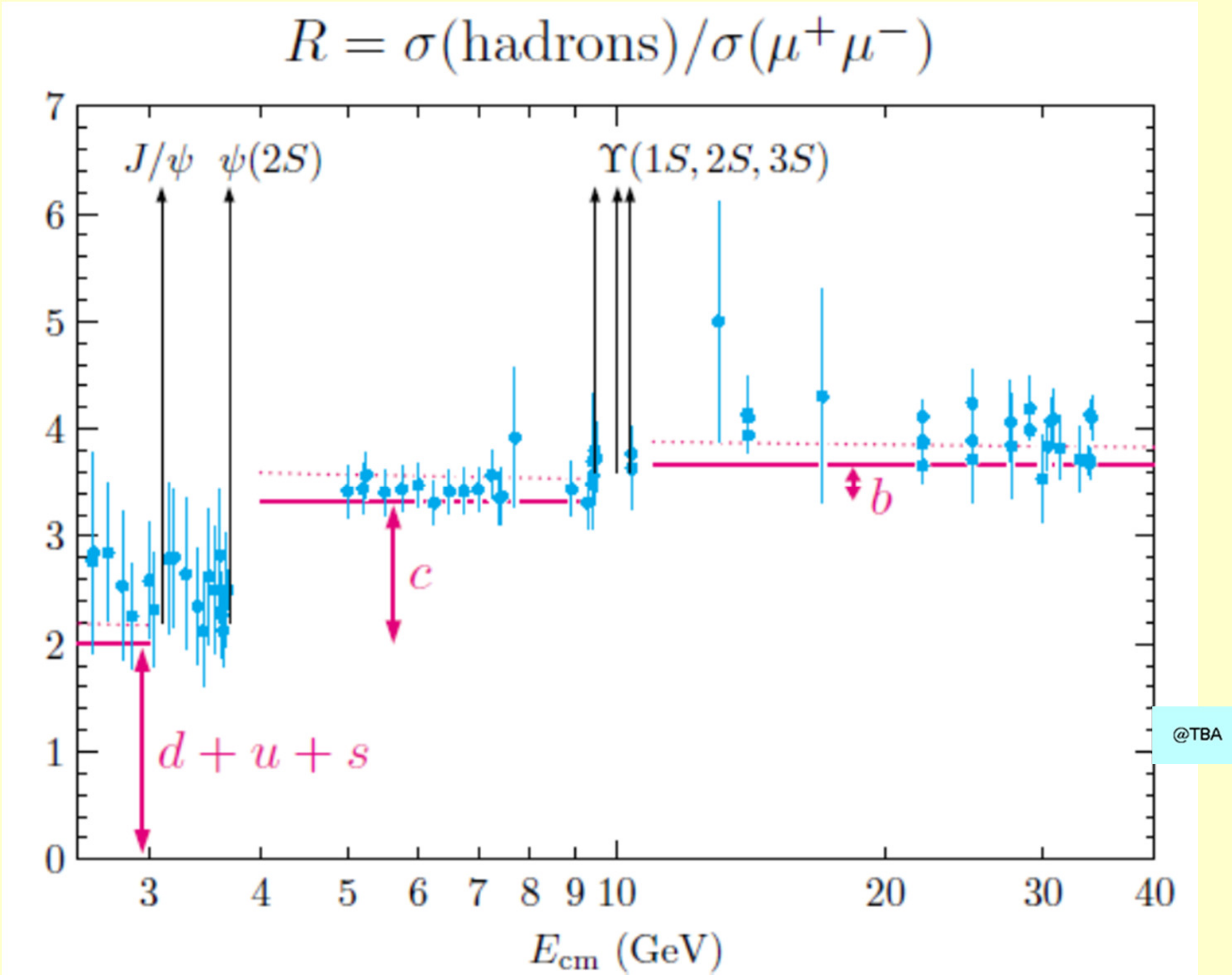
By taking 3 quark species
of any flavor:

$$u,d \rightarrow R = \frac{15}{9}$$

$$u,d,s \rightarrow R = \frac{18}{9}$$

$$u,d,s,c \rightarrow R = \frac{30}{9}$$

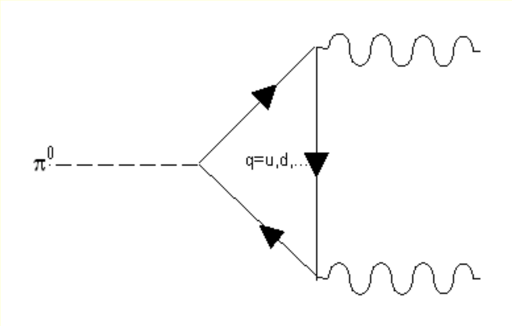
R Ratio - III



$R = \sigma(\text{hadrons})/\sigma(\mu^+\mu^-)$
J/ψ
$\psi(2S)$
$\Upsilon(1S, 2S, 3S)$
$d + u + s$
c
b
E_{cm} (GeV)
@TBA

π^0 Decay Rate - I

Difficult subject: Strong interaction effects are *large*
Originally calculated by taking $p, \bar{p}$ in the triangle loop (Steinberger 1949)



As for similar cases: Initial state is *not* a plane wave

π^0 spinless: Only 4-vector available p_μ

$\rightarrow$Decay amplitude ~ $p_\mu J_\mu$

J_μ = Loop *axial* current, to match pion –ve parity

π^0 Decay Rate - II

With a proton loop rate OK (!)
By replacing the proton loop by a quark loop:

$$J^{\mu}_{(A)} \approx \sum_{i=u}^{d} q_i \overline{\psi}_i \gamma^{\mu} \gamma^5 \tau_3^i \psi_i = e\left(\frac{2}{3}\overline{u}\gamma^{\mu}\gamma^5 u - \frac{1}{3}\overline{d}\gamma^{\mu}\gamma^5 d\right)$$

$$\sum_{i=u,d} \tau_3^i Q_i^2 = 1\cdot\left(\frac{2}{3}\right)^2 - 1\cdot\left(-\frac{1}{3}\right)^2 = \frac{4}{9} - \frac{1}{9} = \frac{1}{3}$$

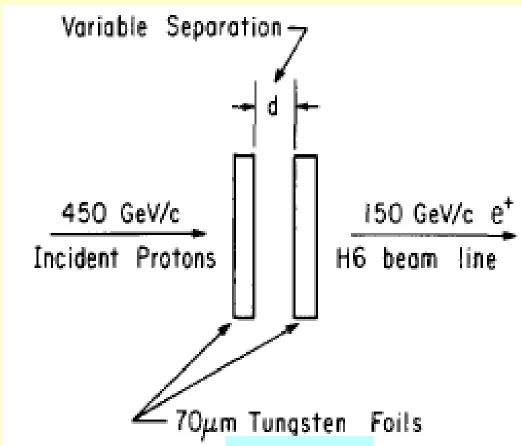
$$\Gamma_{quark}\left(\pi^0 \to \gamma\gamma\right) = \frac{\alpha^2}{64\pi^3}\frac{m_\pi^3}{f_\pi^2}\sum_i g_A^{(i)} e_i^2 = \frac{1}{9}\Gamma_{proton}\left(\pi^0 \to \gamma\gamma\right) \to ???$$

NB: A whole *lot* of physics in this problem:
Simple guess on approximate symmetry of the initial state would lead to conclude *the neutral pion is stable!*

Explanation of this paradox led to discovery of the first *anomaly* in QFT (Adler, Bell, Jackiw)
Advanced topic, quite relevant to the Standard Model

π^0 Decay rate - III

Direct method:



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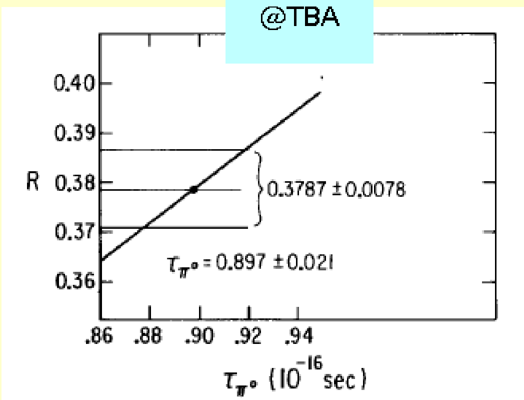


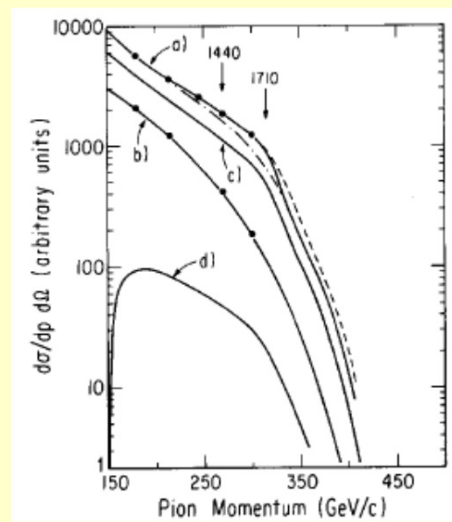
π^0 produced in a first thin foil, when not decayed do not contribute to e^+ yield from γ conversion in a second thin foil

$$Y(d) = N\left\{A + B\left[1 - \exp(-d/\lambda)\right]\right\}$$

$\lambda = \beta\gamma c\tau \simeq \gamma c\tau$ Energy dependent

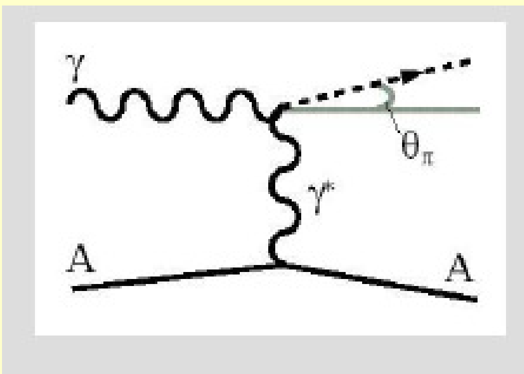
Use known energy spectra for pions

$$\tau = 0.897 \pm .021\ 10^{-16}\ s$$
$$\Gamma = 7.34 \pm 0.18 \pm 0.11\ eV$$



π^0 Decay rate - IV

Primakoff effect



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Very simple idea:
Get a high energy photon beam + high Z target
Pick-up a virtual photon from the nuclear Coulomb field
2-photon coupling will (sometimes) create a π^0

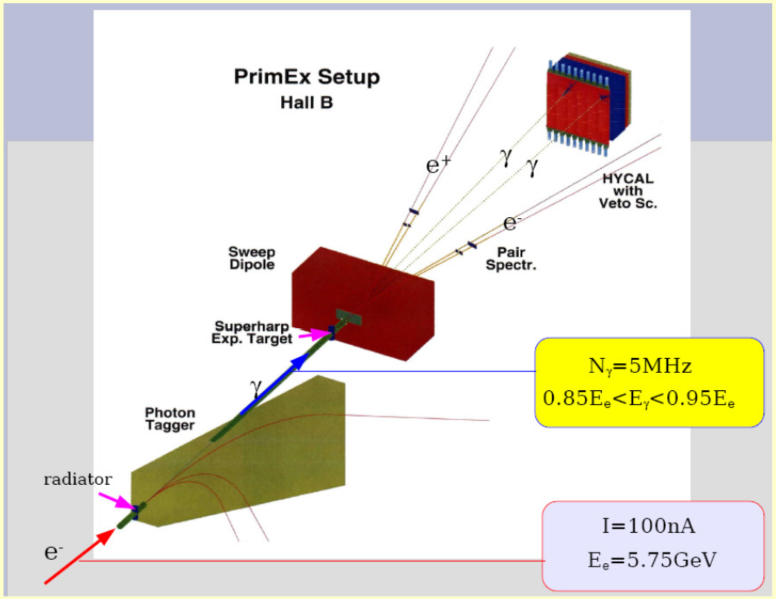
$$\frac{d\sigma_{\text{Prim}}}{d\Omega_{LAB}} \simeq \Gamma_{\pi^0\to\gamma\gamma} Z^2 \frac{8\alpha E^4}{m_{\pi^0}^2} \frac{\left|F\left(q^2\right)\right|^2}{q^4} \sin^2\theta_{\pi^0}$$

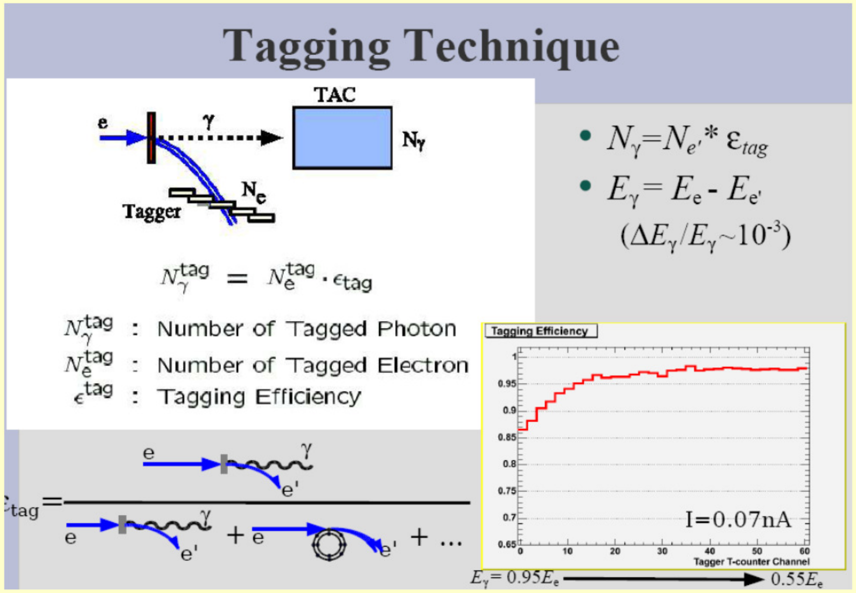
Strongly forward peaked
Quickly increasing with energy
Strongly *Z* dependent: Coherence

$\Gamma = 1/\tau$ extracted by measuring the differential cross-section
Nuclear form factor required

π^0 Decay rate - V

Recent experiment: PrimEx at Jefferson Lab (Virginia)





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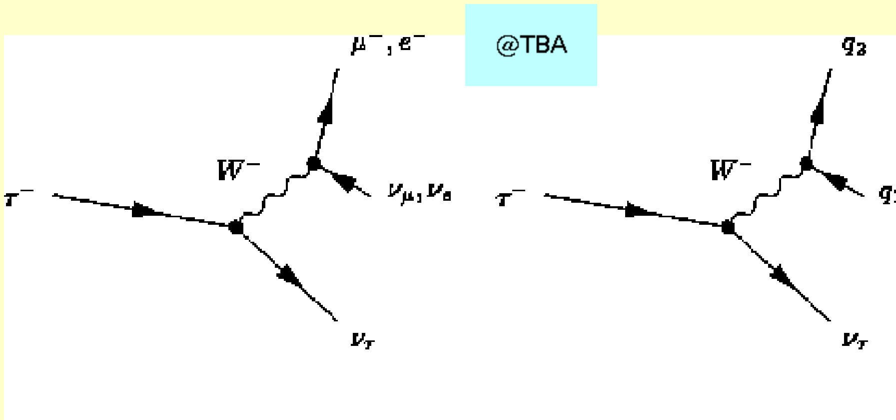
τ Lepton Decays

τ: Heavy brother of e and μ
m_τ= 1776 MeV
Weak decays:



$q, \bar{q} : u, d, s$
$BR(e) \sim 18\%$
$BR(\mu) \sim 17\%$
$BR(q\bar{q}) \sim 65\%$

In the absence of color, weak interaction universality would lead to predict:

$BR(e) \sim BR(\mu) \sim BR(q\bar{q}) \sim 33\%$

With color:

$$\Gamma(q\bar{q}) \sim 3\,\Gamma(l\bar{l}) \rightarrow BR(q\bar{q}) \sim \frac{3\,\Gamma(l\bar{l})}{3\,\Gamma(l\bar{l}) + 2\,\Gamma(l\bar{l})} \sim 60\ \% \quad \text{OK}$$

Color - I

New hypothesis:

There is a new degree of freedom for quarks: Color

Each quark can be found in one of 3 different states

Internal space (mathematically identical to flavor):

States = 3-component complex vectors

Base states:

$$R(ed) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, G(reen) = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, B(lue) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Needless to say, nothing to do with our old, beloved color concept (nature, art, politics..):

Just a *name* for another, non-classical property of hadron constituents

Color - II

Provided one can build a color wave function for 3 fermions which is antisymmetric, the Fermi statistics problem is solved

Total wave function of a baryon

$$\psi = \psi_{color} \underbrace{\psi_{orbital}\psi_{spin}\psi_{flavor}}_{Symmetric} \rightarrow \psi_{color}: \text{ Antisymmetric}$$

To account for 3 different color states, the *R* ratio must be multiplied by *3* $\rightarrow$ OK with experimental data

Just the same conclusion for hadronic τ decays: Multiply rate by *3*

The correct π^0 rate is obtained by inserting a factor *9*

Color - III

Observe:

When computing R, τ decay rates we add the *rates* for different colors
$\rightarrow$Factor $\times$ *3*

We deal with quarks as with real particles: Ignore fragmentation

When computing π^0 decay rate, we add the *amplitudes*
$\rightarrow$Factor $\times$ *9*

Quarks in the loop are virtual particles: Amplitudes interfere

Color - IV

Must be possible to build hadron states as color *singlets*

Do not expect hadrons to fill larger irr.reps.:
Would imply large degeneracies for hadron states, not observed
In other words:

Color is fine, but we do not observe any colored hadron

Therefore we assume the color charge is *confined*:
Never observed directly, as it is the electric charge

Why? For the moment, nobody really knows

Color - V

How colored hadrons would show up?
Just as an example:

Should the nucleon fill the ***3*** of $SU(3)_C$, there would be 3 different species of protons and neutrons.

Then each nucleon level in any nucleus could accommodate 3 particles instead of one:

The nuclear level scheme would be far different from the observed one

Color - VI

Guess *SU(3)* as the color group
Take the two fundamental decompositions:

$\mathbf{3}\otimes\mathbf{3}\otimes\mathbf{3}=\mathbf{1}\oplus\mathbf{8}\oplus\mathbf{8}\oplus\mathbf{10}$ Baryons

$\mathbf{3}\otimes\mathbf{3}^{*}=\mathbf{1}\oplus\mathbf{8}$ Mesons

Both feature a singlet in the direct sum: OK
No singlets in $\mathbf{3}\otimes\mathbf{3}$: OK

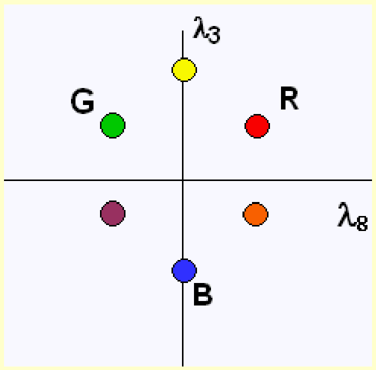
Can't say the same for other groups…
Take *SU(2)* as an example:
Say the quarks live in the adjoint *SU(2)* representation, ***3***
Then for qq

$\boldsymbol{3}\otimes\boldsymbol{3}=\boldsymbol{1}\oplus\boldsymbol{3}\oplus\boldsymbol{5}$

Observe: This is ***3*** of *SU(2),* which is quite different from ***3*** of *SU(3)*

Diquarks can be in color singlet
→Should find diquarks as commonly as baryons or mesons..

Colored Quarks



$$I_3^c = \frac{\lambda_3}{2}, \qquad Y^c = \frac{\lambda_8}{\sqrt{3}}$$

	I_3^c	Y^c
R	$+1/2$	$+1/3$
G	$-1/2$	$+1/3$
B	0	$-2/3$

	I_3^c	Y^c
$\bar{R}$	$-1/2$	$-1/3$
$\bar{G}$	$+1/2$	$-1/3$
$\bar{B}$	0	$+2/3$

$SU(3)_C$ is an *exact symmetry*:

$$m_u^{(R)} = m_u^{(G)} = m_u^{(B)}, \ldots$$

Beware: $SU(3)_C$ has nothing to do with $SU(3)_F$:
Quark quantum numbers are independent from their color state

They are left unchanged by QCD transitions

Uncolored Hadrons

According to our fundamental hypothesis:

Mesons: $\mathbf{3}\otimes\mathbf{3}^{*}=\mathbf{1}\oplus\mathbf{8}$

$$\rightarrow \psi_c = \frac{1}{\sqrt{3}}\left(R\bar{R}+G\bar{G}+B\bar{B}\right)$$

Baryons: $\mathbf{3}\otimes\mathbf{3}\otimes\mathbf{3}=\mathbf{1}\oplus\mathbf{8}\oplus\mathbf{8}\oplus\mathbf{10}$

$$\rightarrow \psi_c = \frac{1}{\sqrt{6}}\left(RGB-RBG+GBR-GRB+BRG-BGR\right)$$

In both cases, pick singlet

Mesons: *No particular exchange symmetry*
(2 non identical particles)

Baryons: *Fully antisymmetrical color wave function*
(3 identical particles)

Color Interaction: QCD

Color: A new degree of freedom for quarks
Compare to other quantum numbers:

Baryonic/Leptonic numbers
Conserved, *not originating interactions*

Electric charge
Conserved, *origin of the electromagnetic field*

A deep question:

What is the true origin of the electromagnetic interaction?

We have used freely the interaction term $j^{\mu} A_{\mu}$, only based on the classical analogy:
But supposedly quantum mechanics is more general of classical mechanics/electromagnetism..

Is there a deeper origin for it?

QED as a Gauge Theory - I

Symmetry:
Absolute phase not defined for a wave function.
Expect invariance as per our old acquaintance, Noether's Theorem

$L_0 = \bar{\psi}(x)\left(i\gamma^{\mu}\partial_{\mu} - m\right)\psi(x)$ Free Dirac Lagrangian

Global gauge (=Phase) transformation:

$G : \psi(x) \rightarrow \psi'(x) = U_{\theta}\psi(x) = e^{-iq\theta}\psi(x)$ $q\theta :$ New phase $\propto$ Charge

$\rightarrow L_0$ invariant wrt $G \rightarrow$ Charge conservation

Just meaning:

Take *all* particle states, re-phase each state proportionally to its charge

QED as a Gauge Theory - II

Generalize to local phase transformation:

$G_L : \psi(x) \to \psi'(x) = U_\theta \psi(x) = e^{-iq\theta(x)}\psi(x)$ Local gauge transformation

$\to L_0$ not invariant wrt G_L : Derivative term troublesome

$$L_0 = \bar{\psi}(x)\left(i\gamma^\mu \partial_\mu - m\right)\psi(x) \to L_0' = i\bar{\psi}(x)e^{+iq\theta(x)}\gamma^\mu \partial_\mu\left(e^{-iq\theta(x)}\psi(x)\right) - m\bar{\psi}(x)\psi(x)$$

$$L_0' = i\left[\bar{\psi}(x)\gamma^\mu \partial_\mu \psi(x) - iq\partial_\mu\left[\theta(x)\right]\psi(x)\right] - m\bar{\psi}(x)\psi(x)$$

$$L_0' = \left\{i\bar{\psi}(x)\gamma^\mu \partial_\mu \psi(x) + q\partial_\mu\left[\theta(x)\right]\psi(x) - m\bar{\psi}(x)\psi(x)\right\}$$

$$L_0' = \left[i\bar{\psi}(x)\gamma^\mu \partial_\mu \psi(x) - m\bar{\psi}(x)\psi(x)\right] + q\partial_\mu\left[\theta(x)\right]\psi(x) \neq L_0$$

$\to$ Local gauge invariance cannot hold in a world of free particles

Symmetry requires interaction

QED as a Gauge Theory - III

New transformation rule:

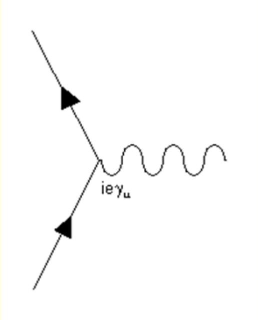
$$\begin{cases} \psi(x) \rightarrow \psi'(x) = U_\theta \psi(x) = e^{-iq\theta(x)}\psi(x) & \text{As before} \\ A_\mu(x) \rightarrow A_\mu(x) + q\,\partial_\mu \theta(x) & \text{New character in the comedy} \end{cases}$$

Equivalent to re-define derivative for ψ:

$\partial_\mu \rightarrow \partial_\mu + iqA_\mu$ Vector field

Add a new term to Lagrangian:

$L_i = -\underbrace{q\bar{\psi}(x)\gamma^\mu\psi(x)}_{j^\mu}A_\mu$ Interaction term



Same as classical electrodynamics

$$L_0 = \bar{\psi}(x)\left(i\gamma^\mu\partial_\mu - m\right)\psi(x) \rightarrow L_0 + L_i = \bar{\psi}(x)\left(i\gamma^\mu\partial_\mu - m\right)\psi(x) - q\bar{\psi}(x)\gamma^\mu\psi(x)A_\mu$$

Sum is invariant

QED as a Gauge Theory - IV

...And another one:

$$-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}, \quad F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu \quad \text{Field energy}$$

Must be there because the field carries energy+momentum+angular momentum

Reminder:

$F^{\mu\nu}$ is the EM field

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix}$$

Field must be massless to have *L* gauge invariant

$$\frac{1}{2}m^2A_\mu^2 \rightarrow \frac{1}{2}m^2\left(A_\mu(x) + q\ \partial_\mu\theta(x)\right)^2 \neq \frac{1}{2}m^2A_\mu^2 \quad \text{if } m \neq 0$$

QED as a Gauge Theory - V

Consider all the phase transformations as defined before:

$$\psi(x) \to \psi'(x) = U_\theta \psi(x) = e^{-iq\theta(x)}\psi(x)$$

The full set is a group: $U(1)$ Unitary, 1-dimensional

$$e^{-iq\theta_1(x)} e^{-iq\theta_2(x)}\psi(x) = e^{-iq[\theta_1(x)+\theta_2(x)]} \in U(1)$$

1 parameter: $\theta(x)$

Abelian: $e^{-iq\theta_1(x)} e^{-iq\theta_2(x)}\psi(x) = e^{-iq\theta_2(x)} e^{-iq\theta_1(x)}\psi(x)$

U(1) is the (Abelian) *gauge group* of QED
Equivalent to *SO(2),* group of 2D rotations

QCD as a Gauge Theory - I

Extend gauge transformations to a 3-component wave function:

$$\boldsymbol{\psi} \equiv \begin{pmatrix} \psi_R \\ \psi_G \\ \psi_B \end{pmatrix}$$

Global gauge transformation: Phase change for individual components

$\rightarrow$ Phase change will mix color components

$G_G^C : \boldsymbol{\psi}(x) \rightarrow \boldsymbol{\psi}'(x) = \mathbf{U}_G \cdot \boldsymbol{\psi}(x) = e^{-ig\mathbf{M}} \cdot \boldsymbol{\psi}(x)$ $\quad \mathbf{U}_G$ unitary $\rightarrow \mathbf{M}$ Hermitian

$e^{-ig\mathbf{M}} = \mathbf{1} - ig\mathbf{M} + \frac{(-ig\mathbf{M})^2}{2!} + \ldots$ $\quad \mathbf{M}$: 3×3 Hermitian matrix

$\mathbf{M}$ acting on the 3 color components of the quark state

Since the color symmetry group is $SU(3)_C$:

$$\mathbf{M} = \sum_{i=1}^{8} \boldsymbol{\lambda}_i \theta_i \equiv \vec{\boldsymbol{\lambda}} \cdot \vec{\theta}$$

$\vec{\boldsymbol{\lambda}}$: Vector of 8 3×3 Gell-Mann matrices; $\vec{\theta}$: Vector of 8 parameters

QCD as a Gauge Theory - II

As for QED, extend to *local* gauge transformations

As before, in order to guarantee invariance of L:

$\rightarrow$ Re-define derivative adding new vector fields:

$$\partial_\mu \rightarrow \partial_\mu \mathbf{1} + ig\mathbf{C}_\mu$$

$$\mathbf{C}_\mu : \begin{cases} \text{4-vector field} & \text{Lorentz structure} \\ \text{Matrix } \in SU(3)_C & \text{Color space} \end{cases}$$

We know how to express any Hermitian matrix $\in SU(3)_C$:

Use $SU(3)_C$ generators $\rightarrow$ Gell-Mann matrices

$$\rightarrow \mathbf{C}_\mu = \frac{1}{2}\sum_{i=1}^{8} \mathrm{G}_\mu^a \boldsymbol{\lambda}_a \equiv \vec{G}_\mu \cdot \vec{\boldsymbol{\lambda}}$$ 8 fields required: Gluons

So gluons are a bit like 8 different "photons", exchanged between color charges

But: *They are non Abelian*

QCD as a Gauge Theory - III

Local gauge transformation for $SU(3)_C$:

Very important: New term, coming from *SU(3)* being non Abelian

$$\begin{cases} \boldsymbol{\psi}(x) \to \boldsymbol{\psi}'(x) = U_\theta \boldsymbol{\psi}(x) = e^{-ig\vec{\boldsymbol{\lambda}}\cdot\vec{\theta}(x)}\boldsymbol{\psi}(x) \\ G^a_\mu(x) \to G^a_\mu{}'(x) = G^a_\mu(x) + \partial_\mu \theta^a + g\sum_{b,c=1}^{8} f^{abc} G^b_\mu(x)\,\theta^c(x) \quad a = 1,\ldots,8 \end{cases}$$

Reminder:

$$\left[\frac{\lambda_a}{2}, \frac{\lambda_b}{2}\right] = if^{abc}\frac{\lambda_c}{2}$$

f^{abc}: $SU(3)$ structure constants

$$L_0 = \overline{\boldsymbol{\psi}}(x)\left(i\gamma^\mu \partial_\mu - m\right)\boldsymbol{\psi}(x) \to L_0 + L_i$$

$$L_i = -g\underbrace{\left[\overline{\boldsymbol{\psi}}(x)\gamma^\mu\left(\frac{\vec{\boldsymbol{\lambda}}}{2}\right)\boldsymbol{\psi}(x)\right]}_{\vec{j}_\mu}\cdot\vec{G}_\mu$$

Interaction term

$$-\frac{1}{4}\vec{G}_{\mu\nu}\cdot\vec{G}^{\mu\nu} = -\frac{1}{4}\sum_{a=1}^{8} G^a_{\mu\nu}\cdot G^{a\mu\nu}$$

Field energy term

QCD as a Gauge Theory - IV

Take the expression of fields in terms of potentials:

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu - g\sum_{b,c=1}^{8} f_{abc} G^b_\mu G^c_\upsilon$$

Very important: *Absent in QED (f=0)*
New term, coming from *SU(3)* being non Abelian

$$\rightarrow G^a_{\mu\nu} G^a_{\mu\nu} \quad \text{contains terms with } \underbrace{\partial_\mu G^a_\nu \cdot G^b_\mu G^c_\upsilon}_{\text{3 gluons}}, \underbrace{G^b_\mu G^c_\upsilon \cdot G^b_\mu G^c_\upsilon}_{\text{4 gluons}}$$

These pieces of *L* correspond to 3 and 4 gluons vertices

The form of QCD Lagrangian leads to predict the existence of a new kind of gluon-gluon color interaction

QCD as a Gauge Theory - V

Since color interaction is tied to color charge, we are saying *the gluons carry their own color charge*

Sounds unfamiliar? Well, that's all after playing with a non-Abelian gauge group.

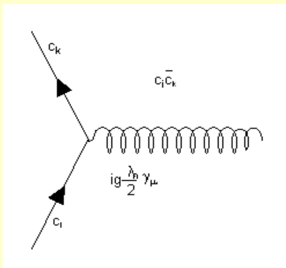
Unlike the electric charge, color charge can manifest itself in more than one way.

Indeed, gluons carry a type of color charge different from quarks/antiquarks:

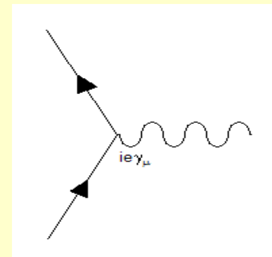
Color + Anticolor

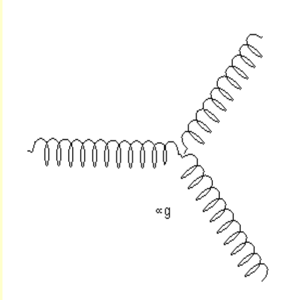
QCD as a Gauge Theory - VI

QCD Vertices

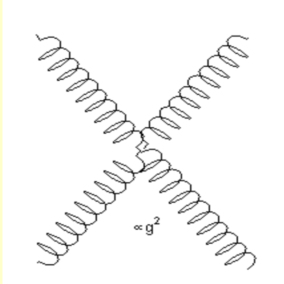


$ig\dfrac{\lambda_a}{2}\gamma_\mu$ Similar to QED:





$\propto g$ (Lorentz structure not shown)



$\propto g^2$ (Lorentz structure not shown)

Colored Gluons - I

Compare to mesons in $SU(3)_F$: *Flavor + Antiflavor*
But: *Gluons are not bound states of Color+Anticolor!*

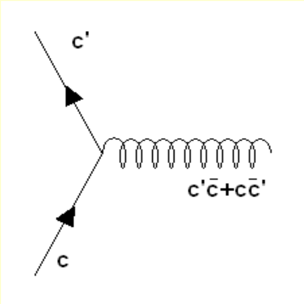
Still, they share the same math:
Gluons live in the adjoint (***8***) irr.rep. of $SU(3)_C$

$$|1\rangle = \frac{1}{\sqrt{2}}\left(r\bar{b} + b\bar{r}\right), |2\rangle = -\frac{i}{\sqrt{2}}\left(r\bar{b} - b\bar{r}\right), |3\rangle = \frac{1}{\sqrt{2}}\left(r\bar{r} - b\bar{b}\right)$$

$$|4\rangle = \frac{1}{\sqrt{2}}\left(r\bar{g} + g\bar{r}\right), |5\rangle = -\frac{i}{\sqrt{2}}\left(r\bar{g} - g\bar{r}\right), |6\rangle = \frac{1}{\sqrt{2}}\left(b\bar{g} + g\bar{b}\right)$$

$$|7\rangle = -\frac{i}{\sqrt{2}}\left(b\bar{g} - g\bar{b}\right), |8\rangle = \frac{1}{\sqrt{6}}\left(r\bar{r} + b\bar{b} - 2g\bar{g}\right)$$

Colored Gluons - II



A very natural question: Gluons couple to $q\overline{q}$

Since one can decompose the total $q\overline{q}$ color state as:

$$\mathbf{3} \otimes \mathbf{3}^* = \mathbf{1} \oplus \mathbf{8}$$

Then: Where is the singlet gluon?

Does not exist: There are only 8 gluons, not 9

Should the singlet gluon actually exist, it would behave more or less like a "photon":

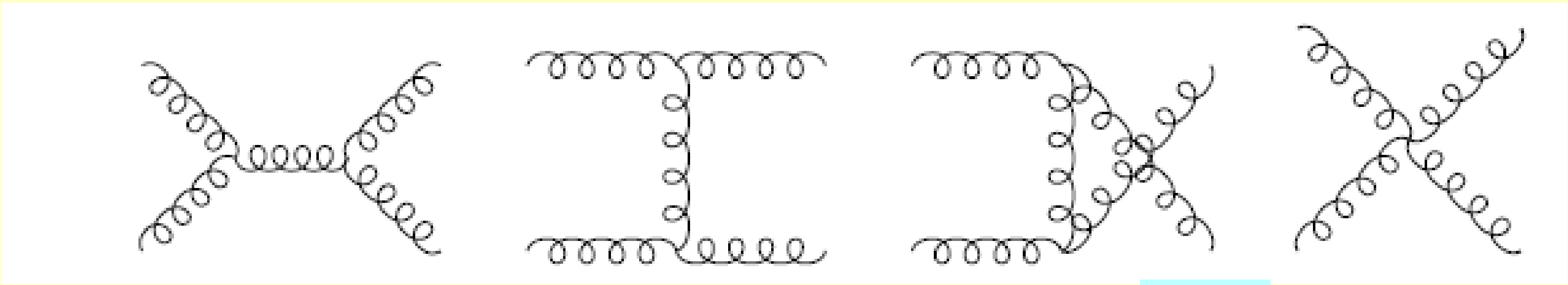
Would be 'white' (= Singlet)

Would couple to color charges in the same way as photon couples to electric charges

Would give rise to a sort of "QED-like", long range color interaction, not observed

Colored Gluons - III

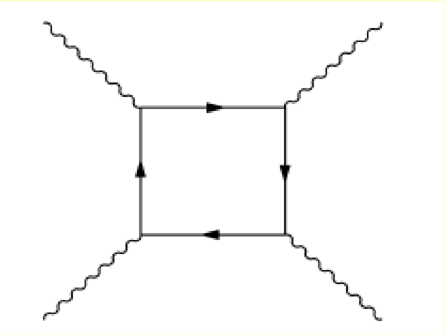
Non Abelian vertices: Gluon-Gluon scattering *at tree level*



@TBA

$3 - gluons: A \propto g$

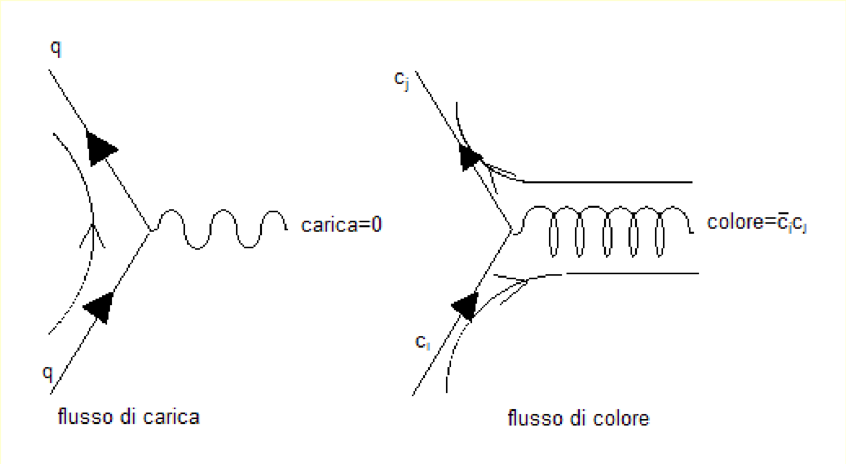
$4 - gluons: A \propto g^2$ Much harder to observe



Compare:
In QED, photon-photon scattering amplitude occurs at order α^2 through the 1-loop diagram

Comparing QED and QCD - I

Compare the different situations:



QED
Photon is *neutral*

Neither sourcing,
nor sinking charge

QCD
Gluon is *colored*

Sourcing color,
sinking anti-color

Comparing QED and QCD - II

Comparison of coupling constants:

α vs. α_s Adimensional constants (*Interaction strength)*

Can define elementary charge in terms of α, α_s

Measure particle charge by its ratio to elementary charge:

Number

What are the allowed values for these numbers?

Comparing QED and QCD - III

QED: Gauge group is *Abelian*

Electric charge can be *any* number:
No reason for charge quantization

Photon charge is strictly *0*

QCD: Gauge group is *non Abelian*

"Color charge" value is *fixed* for every representation

Quarks: ***3***,***3**** $\rightarrow Q = 4/3$
Gluons: ***8*** $\rightarrow Q = 3$

Similar to $I(I+1)$ for any isospin ($SU(2)$) multiplet

Color Factors - I

Consider the static interaction between 2 charges:

QED For fixed $|q|$, the 'charge factor' can be defined as:

$$f_{12} = \frac{q_1 q_2}{|q|^2} = \begin{cases} +1 & q_1 q_2 > 0 \\ -1 & q_1 q_2 < 0 \end{cases}$$

Very simple for an Abelian interaction

QCD The ‘color factor’ depends on the irr.rep. of the color state

Representation dependent
Identical for any transition in a given representation
→Color Conservation

Less simple in this non-Abelian interaction

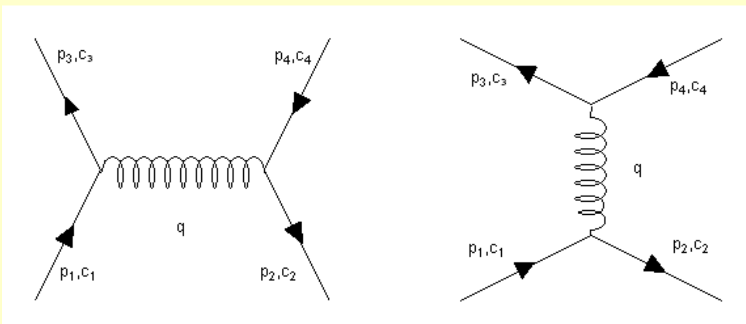
Color Factors - II

$q\bar{q} \to q\bar{q}$

$\mathbf{3} \otimes \mathbf{3}^* = \mathbf{1} \oplus \mathbf{8}$

Total color conservation: $\begin{cases} \mathbf{1} \to \mathbf{1} \\ \mathbf{8} \to \mathbf{8} \end{cases}$

Observe:
Similar to conservation of total I-spin



$$T_{fi} = i \sum_{\alpha,\beta=1}^{8} \underbrace{\left[\bar{u}(3) c_3^\dagger\right]\left[-i\frac{g_s}{2}\lambda^\alpha \gamma^\mu\right]\left[u(1) c_1\right]}_{\text{color current}} \underbrace{\left[-i\frac{g_{\mu\nu}\delta^{\alpha\beta}}{q^2}\right]}_{\text{propagator}} \underbrace{\left[\bar{v}(2) c_2^\dagger\right]\left[-i\frac{g_s}{2}\lambda^\beta \gamma^\nu\right]\left[v(4) c_4\right]}_{\text{color current}}$$

Sum is over all 8 color matrices

c_i are the color states of initial, final $q\bar{q}$

$$T_{fi} = \frac{-g_s^2}{q^2}\left[\bar{u}(3)\gamma^\mu u(1)\right]\left[\bar{v}(2)\gamma_\mu v(4)\right] \underbrace{\frac{1}{4}\sum_\alpha \left[c_3^\dagger \lambda^\alpha c_1\right]\left[c_2^\dagger \lambda^\alpha c_4\right]}_{\text{color factor}}$$

Color Factors - III

<u>Octet</u>

$r\bar{b}$ Just as an example: Result is the same for all octet states

$$\left.\begin{array}{l} c_1 = c_3 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \\ c_2 = c_4 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \end{array}\right\} \rightarrow f = \frac{1}{4}\sum_{\alpha=1}^{8} \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \lambda^{\alpha} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \end{pmatrix} \lambda^{\alpha} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\rightarrow f = \frac{1}{4}\sum_{\alpha=1}^{8} \lambda_{11}^{\alpha}\lambda_{22}^{\alpha} = \frac{1}{4}\left(\lambda_{11}^{3}\lambda_{22}^{3} + \lambda_{11}^{8}\lambda_{22}^{8}\right) = -\frac{1}{6}$$

Color Factors - IV

<u>Singlet</u>

$$\frac{1}{\sqrt{3}}\left(r\bar{r}+b\bar{b}+g\bar{g}\right)$$ Only this state in the singlet

But: *Any* component can go into *any other*..

$$f_i=\frac{1}{4}\frac{1}{\sqrt{3}}\frac{1}{\sqrt{3}}\sum_{j=1}^{3}\sum_{\alpha=1}^{8}\left[c_i^\dagger\lambda^\alpha c_j\right]\left[c_j^\dagger\lambda^\alpha c_i\right],\quad i=1,2,3$$

$$f=\sum_{i=1}^{3}f_i=\frac{1}{4}\frac{1}{\sqrt{3}}\frac{1}{\sqrt{3}}\sum_{\alpha=1}^{8}\sum_{i,j=1}^{3}\lambda_{ij}^\alpha\lambda_{ji}^\alpha=\frac{1}{12}\sum_{\alpha=1}^{8}Tr\left(\lambda^\alpha\lambda^\alpha\right)$$

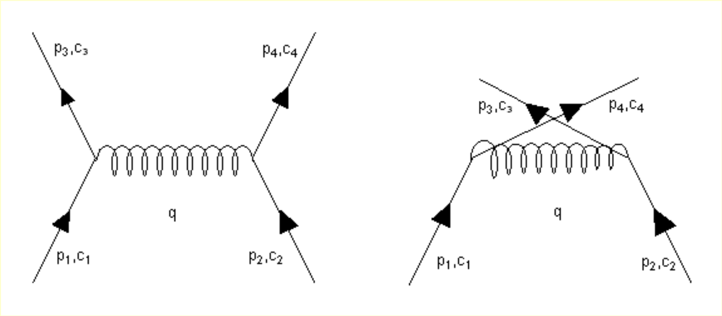
$$Tr\left(\lambda^\alpha\lambda^\beta\right)=2\delta^{\alpha\beta}\rightarrow\sum_{\alpha=1}^{8}Tr\left(\lambda^\alpha\lambda^\alpha\right)=16$$

$$\rightarrow f=\frac{4}{3}$$

Color Factors - V

qq

$\mathbf{3} \otimes \mathbf{3} = \mathbf{3}^* \oplus \mathbf{6}$



$$T_{fi} = \frac{-g_s^2}{q^2}\left[\overline{u}(3)\gamma^\mu u(1)\right]\left[\overline{u}(4)\gamma_\mu u(2)\right]\underbrace{\frac{1}{4}\sum_{\alpha=1}^{8}\left[c_3^\dagger\lambda^\alpha c_1\right]\left[c_4^\dagger\lambda^\alpha c_2\right]}_{\text{color factor}}$$

$$f_{qq} = \frac{1}{4}\sum_{\alpha=1}^{8}\left(c_3^\dagger\lambda^\alpha c_1\right)\left(c_4^\dagger\lambda^\alpha c_2\right)$$

Color Factors - VI

Color states of the triplet and sextet:

$$\mathbf{3}^*: \qquad \frac{1}{\sqrt{2}}(rb-br), \frac{1}{\sqrt{2}}(bg-gb), \frac{1}{\sqrt{2}}(gr-rg)$$

Antisymmetric

$$\mathbf{6}: \qquad rr, bb, gg, \frac{1}{\sqrt{2}}(rb+br), \frac{1}{\sqrt{2}}(bg+gb), \frac{1}{\sqrt{2}}(gr+rg)$$

Symmetric

Color Factors - VII

Sextet

rr Just as an example: Result is the same for all sextet states

$$c_1 = c_2 = c_3 = c_4 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$f = \frac{1}{4}\sum_{\alpha=1}^{8}\left[\begin{pmatrix} 1 & 0 & 0 \end{pmatrix}\lambda^{\alpha}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}\right]\left[\begin{pmatrix} 1 & 0 & 0 \end{pmatrix}\lambda^{\alpha}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}\right] = \frac{1}{4}\sum_{\alpha=1}^{8}\left(\lambda_{11}^{\alpha}\lambda_{11}^{\alpha}\right)$$

$$= \frac{1}{4}\left(\lambda_{11}^{3}\lambda_{11}^{3} + \lambda_{11}^{8}\lambda_{11}^{8}\right) = \frac{1}{3}$$

Color Factors - VIII

<u>Triplet</u>

$\frac{1}{\sqrt{2}}(rb-br)$ Just as an example as before

$$f=\frac{1}{4}\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\sum_{\alpha=1}^{8}$$

$$\left\{\left[\begin{pmatrix}1&0&0\end{pmatrix}\lambda^{\alpha}\begin{pmatrix}1\\0\\0\end{pmatrix}\right]\left[\begin{pmatrix}0&1&0\end{pmatrix}\lambda^{\alpha}\begin{pmatrix}0\\1\\0\end{pmatrix}\right]-\left[\begin{pmatrix}0&1&0\end{pmatrix}\lambda^{\alpha}\begin{pmatrix}1\\0\\0\end{pmatrix}\right]\left[\begin{pmatrix}1&0&0\end{pmatrix}\lambda^{\alpha}\begin{pmatrix}0\\1\\0\end{pmatrix}\right]\right.$$

$$\left.-\left[\begin{pmatrix}1&0&0\end{pmatrix}\lambda^{\alpha}\begin{pmatrix}0\\1\\0\end{pmatrix}\right]\left[\begin{pmatrix}0&1&0\end{pmatrix}\lambda^{\alpha}\begin{pmatrix}1\\0\\0\end{pmatrix}\right]-\left[\begin{pmatrix}0&1&0\end{pmatrix}\lambda^{\alpha}\begin{pmatrix}0\\1\\0\end{pmatrix}\right]\left[\begin{pmatrix}1&0&0\end{pmatrix}\lambda^{\alpha}\begin{pmatrix}1\\0\\0\end{pmatrix}\right]\right\}$$

$$f=\frac{1}{8}\sum_{\alpha=1}^{8}\left\{\lambda_{11}^{\alpha}\lambda_{22}^{\alpha}-\lambda_{21}^{\alpha}\lambda_{12}^{\alpha}-\lambda_{12}^{\alpha}\lambda_{21}^{\alpha}+\lambda_{22}^{\alpha}\lambda_{11}^{\alpha}\right\}$$

$$\rightarrow f=\frac{1}{4}\sum_{\alpha=1}^{8}\left\{\lambda_{11}^{\alpha}\lambda_{22}^{\alpha}-\lambda_{12}^{\alpha}\lambda_{21}^{\alpha}\right\}=\frac{1}{4}\left\{\lambda_{11}^{3}\lambda_{22}^{3}+\lambda_{11}^{8}\lambda_{22}^{8}-\lambda_{12}^{1}\lambda_{21}^{1}-\lambda_{12}^{2}\lambda_{21}^{2}\right\}=-\frac{2}{3}$$

Color Factors - IX

Matrix elements just calculated:
Very similar to the corresponding tree-level amplitudes in QED

→Expect similar, Coulomb-like, effective potential in the static limit

Constant depends on the color representation for the quark pair:

$$V_{q\bar{q}} = \begin{cases} -\frac{4}{3}\frac{\alpha_s}{r} & \text{singlet} \\ \frac{1}{6}\frac{\alpha_s}{r} & \text{octet} \end{cases}$$

Attractive

$$V_{qq} = \begin{cases} -\frac{2}{3}\frac{\alpha_s}{r} & \text{triplet} \\ \frac{1}{3}\frac{\alpha_s}{r} & \text{sextet} \end{cases}$$

Attractive

Expect maximal attraction in singlet

Color Factors - X

Baryons could be in any one of the ***1,8,10*** representations:
Why only the singlet is observed?
A hint of an explaination:

$$\mathbf{3}\otimes\mathbf{3}\otimes\mathbf{3}=(\mathbf{3}\otimes\mathbf{3})\otimes\mathbf{3}$$

$$\mathbf{3}\otimes\mathbf{3}=\mathbf{6}\oplus\mathbf{3}^{*}\rightarrow(\mathbf{3}\otimes\mathbf{3})\otimes\mathbf{3}=(\mathbf{6}\oplus\mathbf{3}^{*})\otimes\mathbf{3}$$

$$\mathbf{6}\otimes\mathbf{3}=\mathbf{10}\oplus\mathbf{8}$$

$$\mathbf{3}^{*}\otimes\mathbf{3}=\mathbf{1}\oplus\mathbf{8}$$

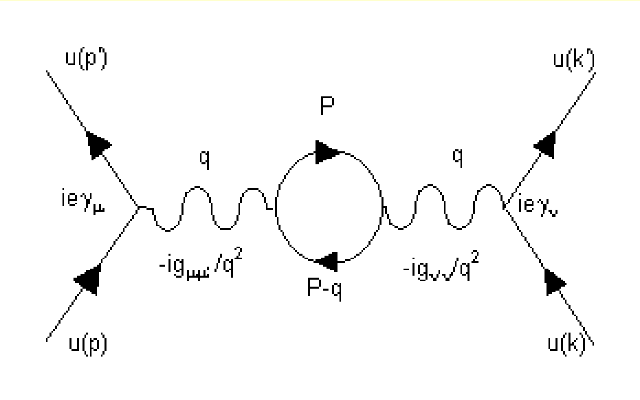
1: *each qq pair is a triplet* $\rightarrow$ *attractive*
8: *qq pair can be triplets, or sextet* $\rightarrow$ *attractive + repulsive*
10: *each qq pair is a sextet* $\rightarrow$ *repulsive*

So singlet is the state maximally attractive for 3 quarks

Does this explain the singlet-only mystery of bound states?

Running Coupling: QED - I

Consider the *one loop* modification to the photon propagator:



Includes a sum over *P*, the momentum circulating in the virtual loop. No obvious bounds on *P*..

$$M \propto \left[e\overline{u}(k')\gamma^{\mu}u(k)\right]\frac{g_{\mu\mu'}}{q^2}\frac{1}{(2\pi)^4}\int d^4P\frac{\left[e\overline{u}(P)\gamma^{\mu'}u(P-q)\right]}{P^2-m^2}\frac{\left[e\overline{u}(P-q)\gamma^{\nu'}u(P)\right]}{(P-q)^2-m^2}\frac{g_{\nu\nu'}}{q^2}\left[e\overline{u}(p')\gamma^{\nu}u(p)\right]$$

Modified propagator:

$$\frac{g_{\mu\nu}}{q^2}\to\frac{g_{\mu\nu}}{q^2}\left(1-I\left(q^2\right)\right),\ \ I\left(q^2\right)=\frac{\alpha}{3\pi}\int_{m^2}^{\infty}\frac{dp^2}{p^2}-\frac{2\alpha}{\pi}\int_0^1 dx\,x(1-x)\ln\left[1-\frac{q^2x(1-x)}{m^2}\right]$$

Running Coupling: QED - II

Take the high q^2 approximation

$$-q^2 \gg m^2 \rightarrow \ln\left[1-\frac{q^2 x(1-x)}{m^2}\right] \approx \ln\left[-\frac{q^2}{m^2}\right]$$

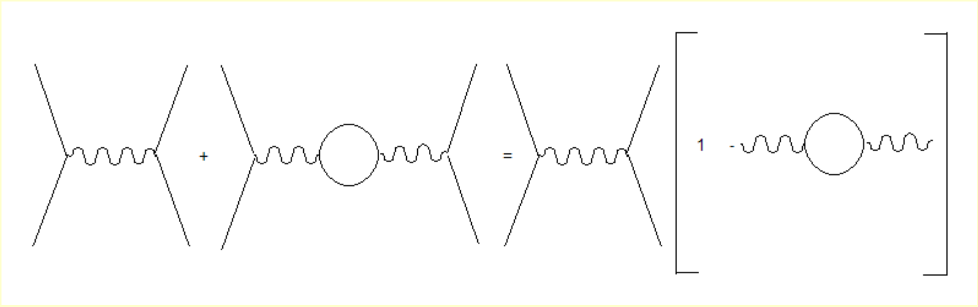
Provisional upper bound (cutoff) to make integral converging

$$I\left(q^2\right) \approx \frac{\alpha}{3\pi}\int_{m^2}^{M^2}\frac{dp^2}{p^2} - \frac{2\alpha}{\pi}\int_0^1 dx x(1-x)\ln\left[\frac{-q^2}{m^2}\right]$$

$$I\left(q^2\right) \approx \frac{\alpha}{3\pi}\ln\left(\frac{M^2}{m^2}\right) - \frac{2\alpha}{\pi}\frac{1}{6}\ln\left[\frac{-q^2}{m^2}\right] = \frac{\alpha}{3\pi}\left[\ln\left(\frac{M^2}{m^2}\right) - \ln\left[\frac{-q^2}{m^2}\right]\right] = \frac{\alpha}{3\pi}\ln\left(\frac{M^2}{-q^2}\right)$$

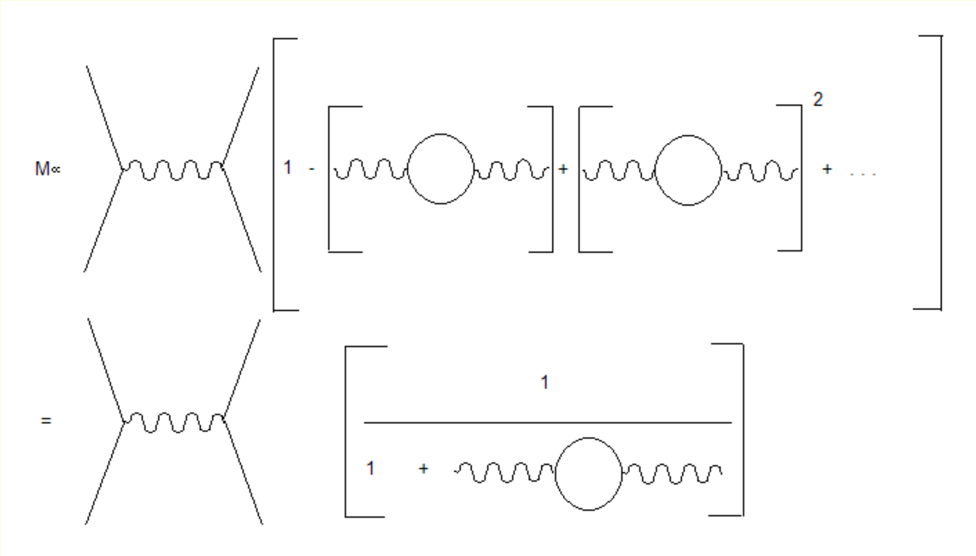
$$M \propto \alpha\left[\bar{u}(k')\gamma^\mu u(k)\right]\frac{g_{\mu\nu}}{q^2}\left[1-\frac{\alpha}{3\pi}\ln\left(\frac{M^2}{-q^2}\right)\right]\left[\bar{u}(p')\gamma^\nu u(p)\right]$$

Cartoon translation:



Running Coupling: QED - III

Extend to diagrams with 2,3,…,n,… loops: Add up all contributes
Sum of a 'geometrical series': Converging ??



Experts say this is the only contribution to running α to the 'leading logs' approximation, which means neglecting the next levels of iteration

Running Coupling: QED - IV

$$M \propto \left[\bar{u}(k')\gamma^{\mu}u(k)\right]\frac{g_{\mu\nu}}{q^2}\left[\frac{\alpha}{1+\alpha/3\pi\ln\left(M^2/-q^2\right)}\right]\left[\bar{u}(p')\gamma^{\nu}u(p)\right]$$

What is α?

Coupling 'constant' we would get should we turn off all loops

Call it α_0 = 'Bare' coupling constant, not physical:

Loops cannot be turned off

Then obtain an effective coupling, not constant but running:

$$\alpha\left(q^2\right)=\frac{\alpha_0}{1+\alpha_0/3\pi\ln\left(M^2/-q^2\right)}$$

α is q^2, or distance, dependent!

Running Coupling: QED - V

Running α is still cutoff dependent, which of course is uncomfortable
But: Not a real problem.
Indeed:

$$Q^2 = -q^2 \rightarrow \alpha\left(Q^2\right) = \frac{\alpha_0}{1 + \left(\alpha_0/3\pi\right)\ln\left(M^2/Q^2\right)}$$

Take a particular energy scale: $Q^2 = \mu^2$

$$\rightarrow \alpha\left(\mu^2\right) = \frac{\alpha_0}{1 + \left(\alpha_0/3\pi\right)\ln\left(M^2/\mu^2\right)}$$

Usually choose μ^2=0, i.e. take α at distance $\rightarrow \infty$

Quite natural in QED (but not compulsory)

Running Coupling: QED - VI

$$\ln\left(\frac{M^2}{Q^2}\right) = \ln\left(\frac{M^2}{Q^2}\frac{\mu^2}{\mu^2}\right) = \ln\left(\frac{M^2}{\mu^2}\right) + \ln\left(\frac{\mu^2}{Q^2}\right)$$

$$\rightarrow \alpha\left(Q^2\right) = \frac{\alpha_0}{1+\left(\alpha_0/3\pi\right)\left[\ln\left(M^2/\mu^2\right)+\ln\left(\mu^2/Q^2\right)\right]}$$

$$\rightarrow \frac{\alpha_0}{\alpha\left(\mu^2\right)} = 1+\left(\alpha_0/3\pi\right)\ln\left(M^2/\mu^2\right)$$

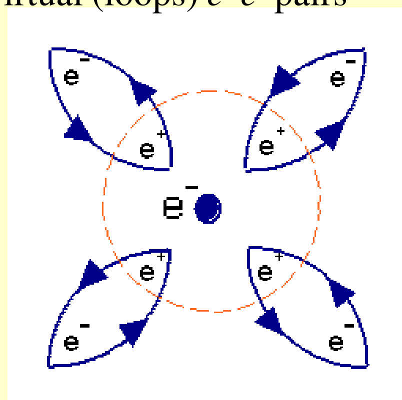
$$\rightarrow \alpha\left(Q^2\right) = \frac{\alpha_0}{\alpha_0/\alpha\left(\mu^2\right)+\left(\alpha_0/3\pi\right)\ln\left(\mu^2/Q^2\right)} = \frac{\alpha\left(\mu^2\right)}{1-\left[\alpha\left(\mu^2\right)/3\pi\right]\ln\left(Q^2/\mu^2\right)}$$

Very interesting result: Running α depends on q^2, through its own *measured* value at any chosen energy scale μ^2.

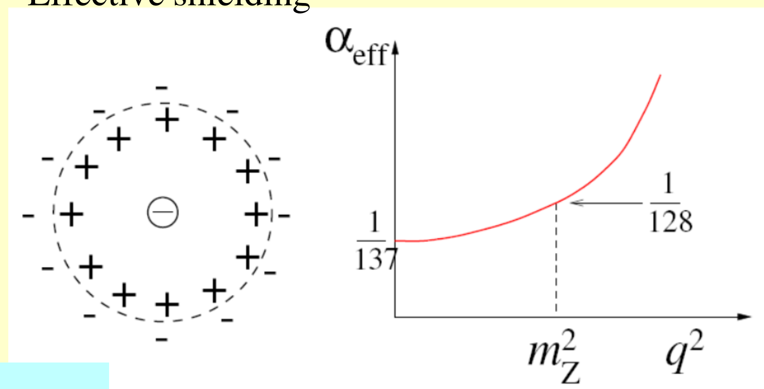
Cutoff has disappeared.

Running Coupling: QED - VII

Virtual (loops) e^+e^- pairs



Effective shielding



@TBA

Picture the QED perturbative vacuum as a sort of (virtual) dielectric medium: Virtual photons and e^+e^- pairs continuously created/annihilated

Bare charge is shielded at large distance by the virtual pairs coming from loops. The standard *e* charge is smaller than the bare charge

By probing the electron at smaller and smaller distance, observe an *increasing* effective charge

Running α at LEP - I

Experimental method: Bhabha scattering

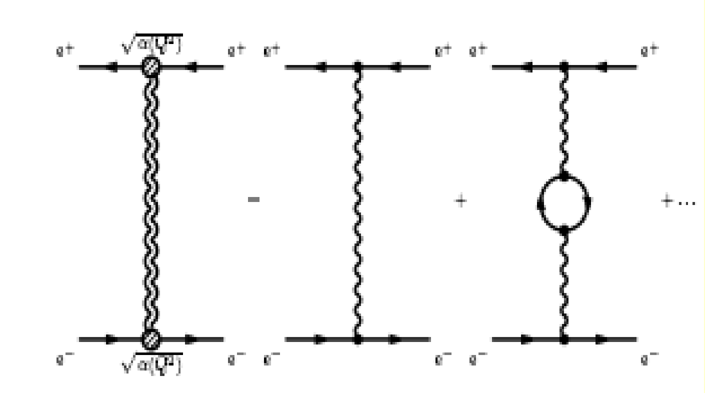
δ_γ, δ_Z s - channel contributions (small)

ε radiative corrections (known)

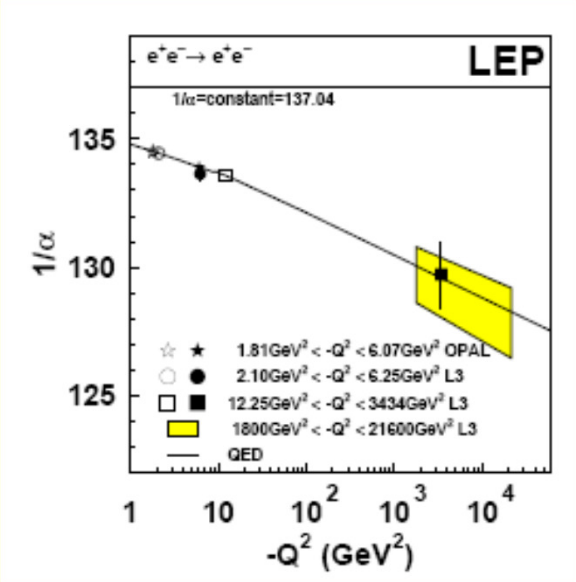
Use accurate, differential cross-section measurement to unfold $\alpha(t)$

Total cross-section measurement would require a luminosity..

$$\frac{d\sigma}{dt} = \frac{d\sigma^0}{dt}\left(\frac{\alpha(t)}{\alpha_0}\right)^2 (1+\varepsilon)(1+\delta_\gamma)+\delta_Z,$$



Results

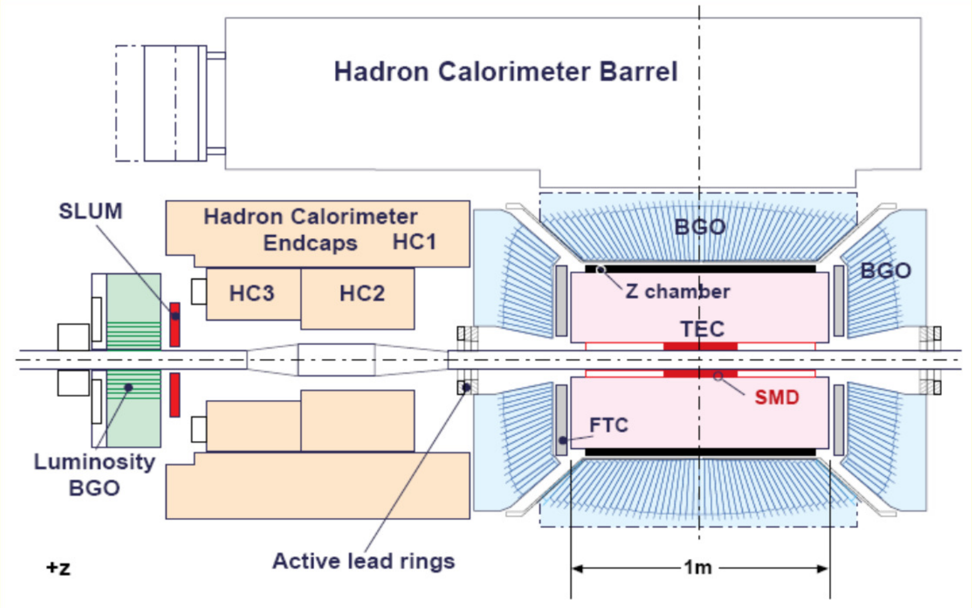


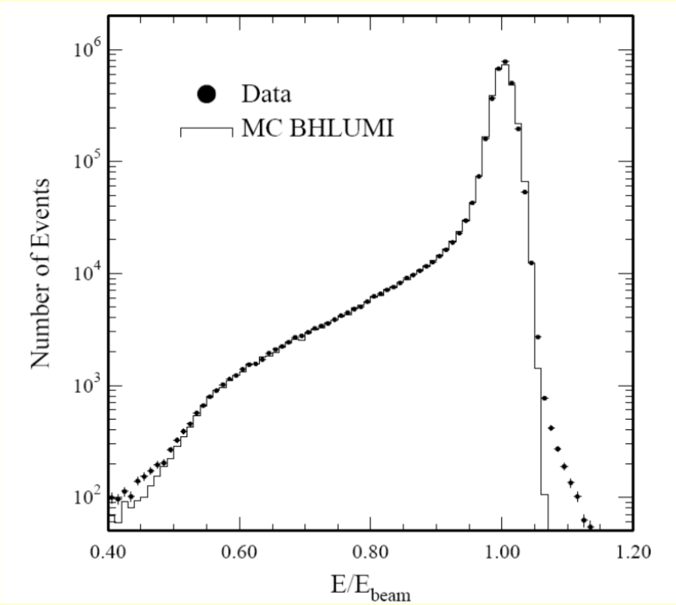
Running α at LEP - II

Just as an example, take L3 at LEP:
Relying on Bhabha scattering at small angle

$$\sigma = \frac{16\pi\alpha^2}{s}\left(\frac{1}{\theta^2_{min}} - \frac{1}{\theta^2_{max}}\right)$$

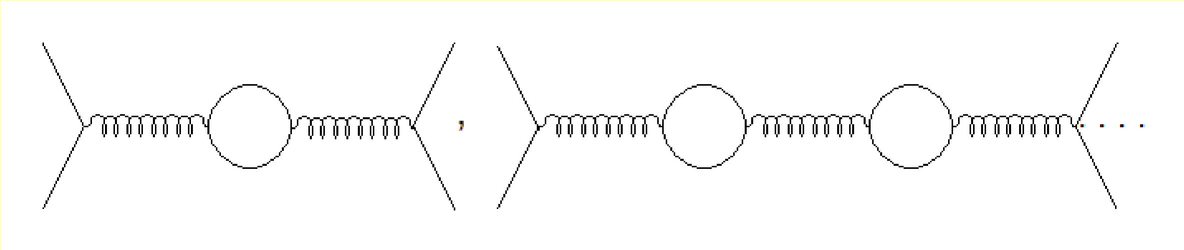
Calorimeter (Energy) + Silicon Tracker (Angle + Acceptance)



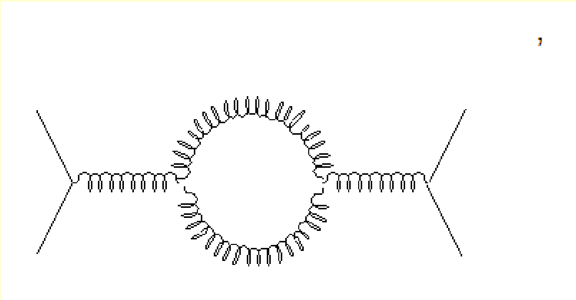


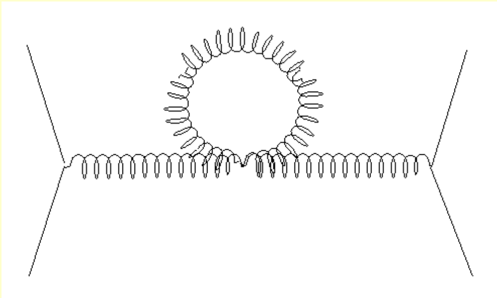
Running Coupling: QCD - I

Repeat all the steps: Loops etc



Except this time one has more loops: Gluons





Running Coupling: QCD - II

Turns out gluon loops yield *anti*-shielding effect
With 8 gluons and 6 quark flavors, gluons win

$$\alpha_s\left(\left|q^2\right|\right)=\frac{\alpha_s\left(\mu^2\right)}{1+\left(\alpha_s\left(\mu^2\right)/12\pi\right)\left(33-2n_{flavor}\right)\ln\left(\left|q^2\right|/\mu^2\right)}$$

Running coupling *decreases* with increasing q^2 (or at smaller distance)
This is known as *asymptotic freedom*:

Large q^2 processes feature small coupling $\rightarrow$ Perturbative!

Most important consequence:

The fundamental hypothesis behind the successful parton model is finally understood and justified

Running Coupling: QCD - III

Rather than making reference to a specific value of α_s

$$\alpha_s\left(\left|q^2\right|\right)=\frac{\alpha_s\left(\mu^2\right)}{1+\left(\alpha_s\left(\mu^2\right)/12\pi\right)\left(33-2n_{flavor}\right)\ln\left(\left|q^2\right|/\mu^2\right)}$$

define a new constant

$$\ln\Lambda^2=\ln\mu^2-\frac{12\pi}{\left(33-2n_{flavor}\right)\alpha_s\left(\mu^2\right)}\rightarrow\Lambda^2=\mu^2e^{-\frac{12\pi}{\left(33-2n_{flavor}\right)\alpha_s\left(\mu^2\right)}}$$

$$\rightarrow\alpha_s\left(\left|q^2\right|\right)\simeq\frac{12\pi}{\left(33-2n_{flavor}\right)\ln\left(\left|q^2\right|/\Lambda^2\right)}=\frac{12\pi}{21\ln\left(\left|q^2\right|/\Lambda^2\right)},\quad\left|q^2\right|\gg\Lambda^2$$

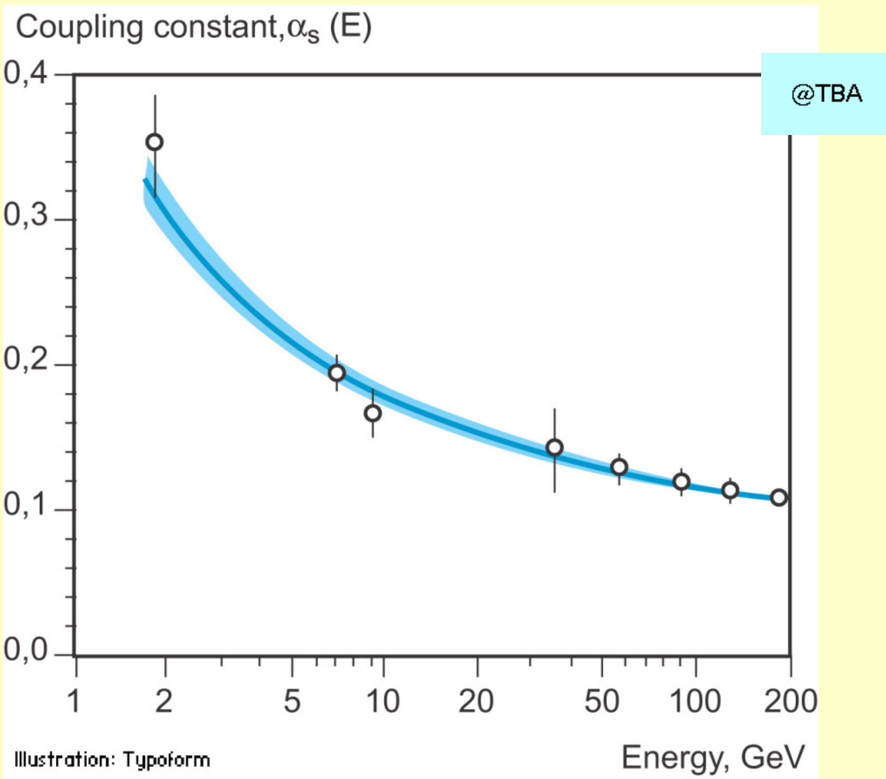
Λ = Renormalization scale $\rightarrow$ Fixes α_s at all q^2

$\Lambda\approx 200$ MeV yields the correct α_s at $\mu^2=M_{Z^0}^2$

Funny behavior, known as 'Dimensional Transmutation':

From an adimensional constant to a dimensional one $\alpha_s\rightarrow\Lambda$

Running Coupling: QCD - IV



Sources:

Jets

DIS

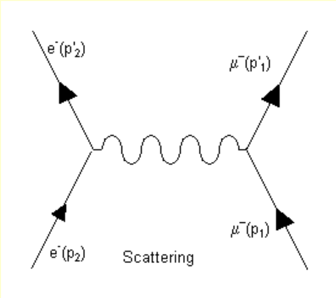
Quarkonium

Annihilation Cross-Section - I

Apply crossing symmetry to electron-muon scattering

$$e^- + \mu^- \to e^- + \mu^- \qquad \text{A: Scattering}$$

$$e^- + \underbrace{\left[e^-\right]}_{crossed} \to \underbrace{\left[\mu^-\right]}_{crossed} + \mu^- \equiv e^- + e^+ \to \mu^+ + \mu^- \qquad \text{B: Annihilation}$$



Amplitude for scattering:

$$T_{fi}(s,s',r,r') = (-e)\bar{u}_{(\mu)}(p_2',s')\gamma^\mu u_{(\mu)}(p_2,s)\frac{-ig_{\mu\nu}}{q^2}(-e)\bar{u}_{(e)}(p_1',r')\gamma^\nu u_{(e)}(p_1,r)$$

$$q = p_1 - p_1' \to q^2 = \left(p_1 - p_1'\right)^2 = p_1^2 + p_1'^2 - 2p_1 \cdot p_1'$$

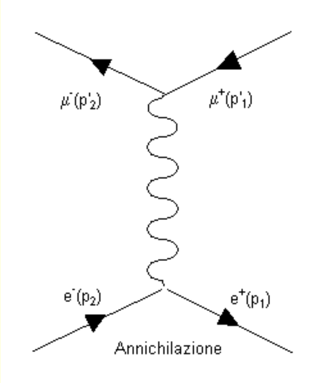
$$q^2 = 2m_e^2 - 2\left(E_1E_1' - \mathbf{p}_1 \cdot \mathbf{p}_1'\right) \underset{E \gg m}{\simeq} -2\left(E_1E_1' - \mathbf{p}_1 \cdot \mathbf{p}_1'\right) < 0 \qquad q\text{=4-momentum transfer}$$

Amplitude for annihilation:

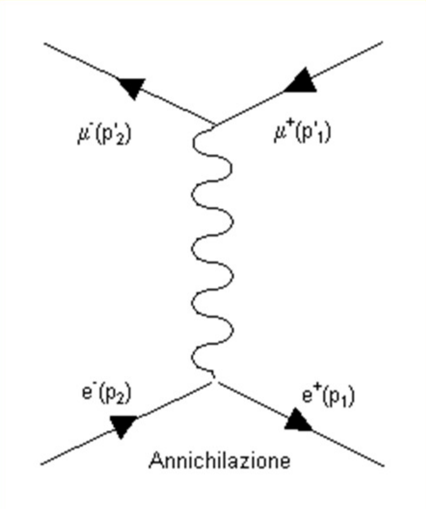
$$T_{fi}(s,s',r,r') = (-e)\bar{u}_{(\mu)}(p_2',s')\gamma^\mu v_{(\mu)}(p_1',r')\frac{-ig_{\mu\nu}}{q^2}(-e)\bar{v}_{(e)}(p_1,s)\gamma^\nu u_{(e)}(p_2,r)$$

$$q = p_1 + p_2 \to q^2 = \left(p_1 + p_2\right)^2 = p_1^2 + p_2^2 + 2p_1 \cdot p_2$$

$$q^2 = 2m_e^2 + 2\left(E_1E_2 - \mathbf{p}_1 \cdot \mathbf{p}_2\right) \underset{E \gg m}{\simeq} 2\left(E_1E_2 - \mathbf{p}_1 \cdot \mathbf{p}_2\right) > 0 \qquad q\text{=total 4-momentum}$$



Annihilation Cross-Section - II



$$T_{fi} = \overline{v}(p_1, s')\gamma^\mu u(p_2, s)\frac{e^2}{q^2}\overline{u}(p_2', r)\gamma_\mu v(p_1', r')$$

$$\left|T_{fi}\right|^2 = \frac{e^4}{q^4}\left[\overline{v}(p_1, s')\gamma^\mu u(p_2, s)\overline{u}(p_2', s)\gamma^\nu v\left(p_1^{'}, s'\right)\right]\left[\overline{v}(p_1, r)\gamma_\mu u(p_2, r')\overline{u}(p_2', r')\gamma_\nu v\left(p_1^{'}, r\right)\right]$$

Annihilation Cross-Section - III

$$\left|T_{fi}\right|^2 = \frac{e^4}{q^4}\left[\bar{v}(p_1,s')\gamma^\mu u(p_2,s)\bar{u}(p_2',s)\gamma^\nu v(p_1',s')\right]\left[\bar{v}(p_1,r)\gamma_\mu u(p_2,r')\bar{u}(p_2',r')\gamma_\nu v(p_1',r)\right]$$

$$\left\langle\left|T_{fi}\right|^2\right\rangle = \frac{1}{4}\sum_{s,s',r,r'}\left|T_{fi}\right|^2 = \frac{e^4}{4q^4}Tr\left[(\not{p}_1 - m)\gamma^\mu(\not{p}_2 + m)\gamma^\nu\right]Tr\left[(\not{p}_2' + M)\gamma_\mu(\not{p}_1' - M)\gamma_\nu\right]$$

$$Tr\left[(\not{p}_1 - m)\gamma^\mu(\not{p}_2 + m)\gamma^\nu\right] = 4\left[p_1^\mu p_2^\nu + p_1^\nu p_2^\mu - g^{\mu\nu}\left(p_2\cdot p_1 + m^2\right)\right]$$

$$Tr\left[(\not{p}_2' + M)\gamma_\mu(\not{p}_1' - M)\gamma_\nu\right] = 4\left[p_1^{\mu}{}' p_2^{\nu}{}' + p_1^{\nu}{}' p_2^{\mu}{}' - g^{\mu\nu}\left(p_2'\cdot p_1' + M^2\right)\right]$$

$$\rightarrow \left\langle\left|T_{fi}\right|^2\right\rangle = \frac{8e^4}{q^4}\left[(p_1\cdot p_1')(p_2\cdot p_2') + (p_1\cdot p_2')(p_2\cdot p_1') + M^2(p_1\cdot p_2)\right] \quad m \approx 0$$

$$CM: \begin{cases} p_1 = (E \quad 0 \quad 0 \quad E),\ p_2 = (E \quad 0 \quad 0 \quad -E) \\ \left|\mathbf{p}'\right| = \left|\mathbf{p}_1'\right| = \left|\mathbf{p}_2'\right| = \sqrt{E^2 - M^2} \end{cases}$$

$$p_1' = \left(E \quad \left|\mathbf{p}'\right|\sin\theta \quad 0 \quad \left|\mathbf{p}'\right|\cos\theta\right),\ p_2' = \left(E \quad -\left|\mathbf{p}'\right|\sin\theta \quad 0 \quad -\left|\mathbf{p}'\right|\cos\theta\right)$$

$$\rightarrow \left\langle\left|T_{fi}\right|^2\right\rangle = e^4\left[1 + \frac{M^2}{E^2} + \left(1 - \frac{M^2}{E^2}\right)\cos^2\theta\right]$$

Annihilation Cross-Section - IV

$$\left.\frac{d\sigma}{d\Omega}\right|_{CM} = \frac{\alpha^2}{4s}\sqrt{1-\frac{M^2}{E^2}}\left[1+\frac{M^2}{E^2}+\left(1-\frac{M^2}{E^2}\right)\cos^2\theta\right]$$

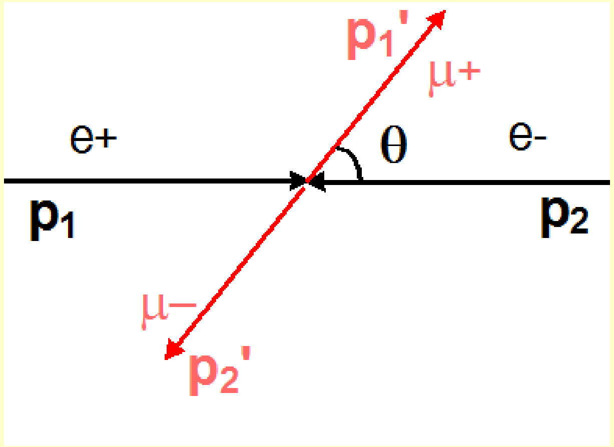
$$\rightarrow \left.\frac{d\sigma}{d\Omega}\right|_{CM} \approx \frac{\alpha^2}{4s}\left(1+\cos^2\theta\right), \qquad E \gg M$$

$$\sigma = \int_\Omega \frac{d\sigma}{d\Omega}d\Omega = \frac{4\pi\alpha^2}{3s}\sqrt{1-\frac{M^2}{E^2}}\left(1+\frac{1}{2}\frac{M^2}{E^2}\right)$$

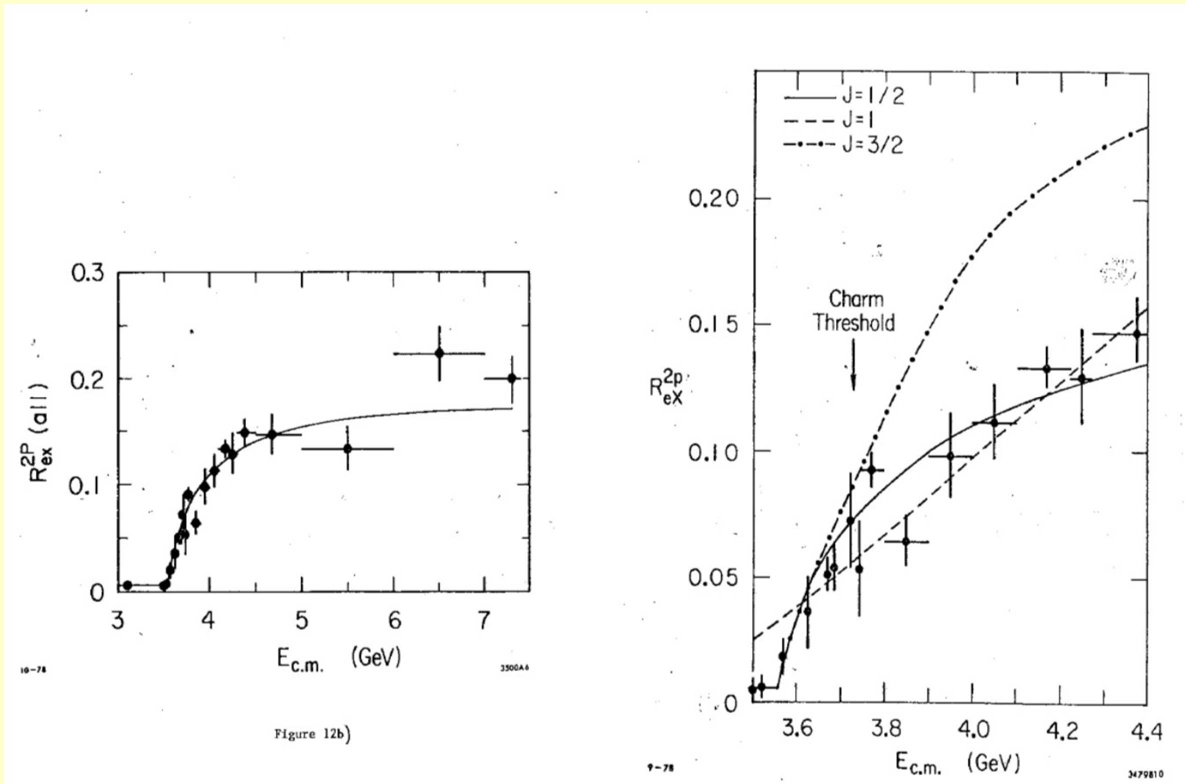
$$\gamma = \frac{E}{M} = \frac{1}{\sqrt{1-\beta^2}}$$

$$\rightarrow \sigma = \frac{4\pi\alpha^2}{3s}\underbrace{\sqrt{\frac{\gamma^2-1}{\gamma^2}}}_{\beta}\left(\frac{1+2\gamma^2}{2\gamma^2}\right) = \frac{4\pi\alpha^2}{3s}\beta\left(\frac{1}{2\gamma^2}+1\right) = \frac{4\pi\alpha^2}{3s}\beta\left(\frac{3-\beta^2}{2}\right)$$

$$\rightarrow \sigma \approx \frac{4\pi\alpha^2}{3s} = \frac{87}{s\left[GeV^2\right]}nb, \qquad E \gg M$$



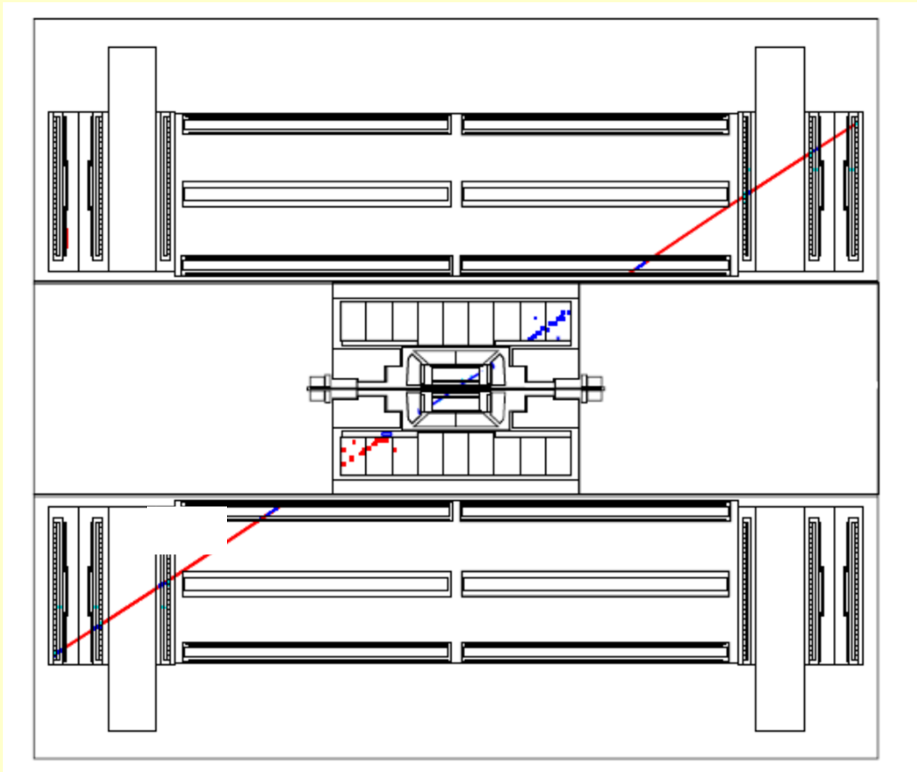
Annihilation Cross-Section - VII



τ lepton discovery, mass & spin determination:

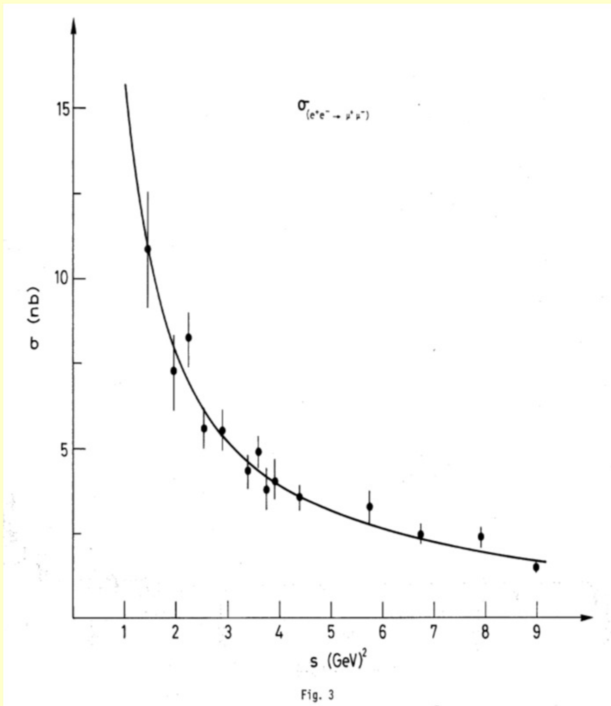
$$R_{eX}^{2p} \simeq \frac{\sigma\left(\tau^+\tau^-\right)}{\sigma\left(\mu^+\mu^-\right)} \simeq \sqrt{1-\frac{M_\tau^{\ 2}}{E^2}}\left(1+\frac{1}{2}\frac{M_\tau^{\ 2}}{E^2}\right), M_\mu \approx 0$$

Annihilation Cross-Section - VIII



$\mu^+ \mu^-$ event: L3 detector at LEP

Annihilation Cross-Section - IX



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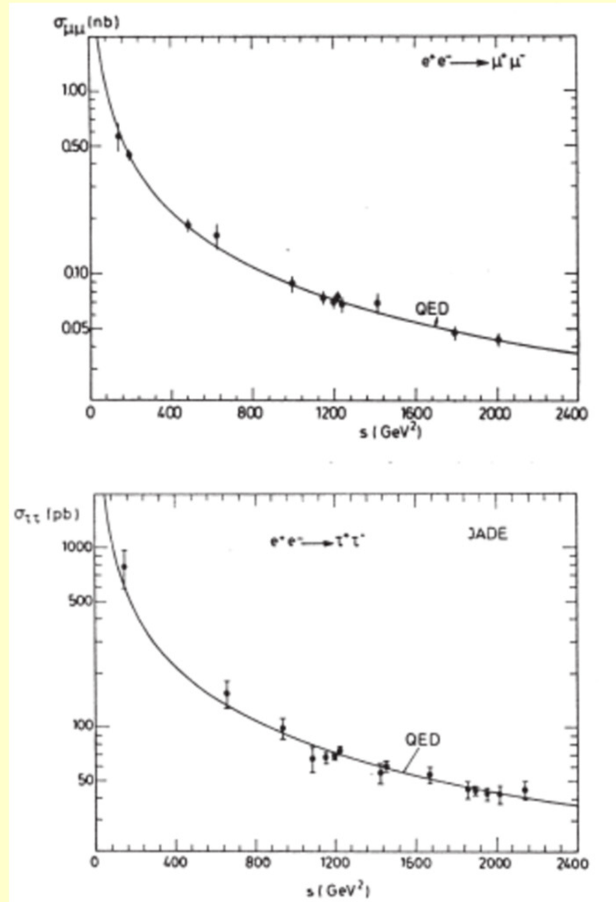


PETRA

Annihilation Cross-Section - X

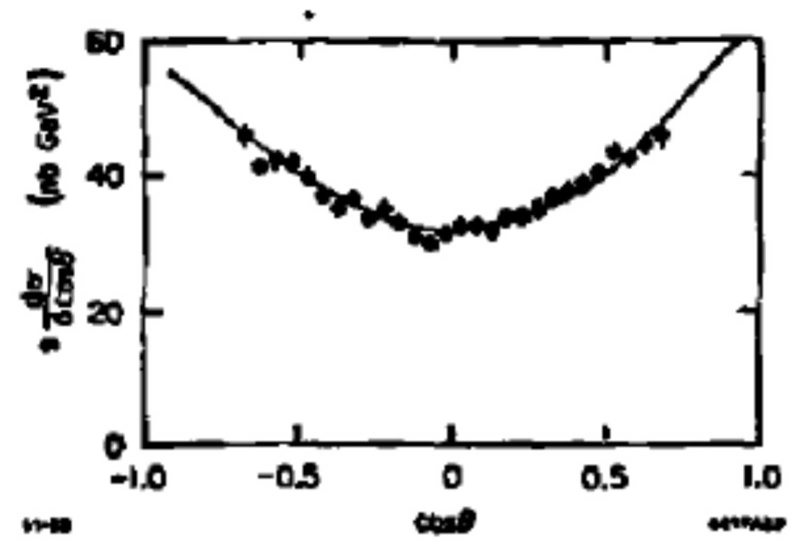
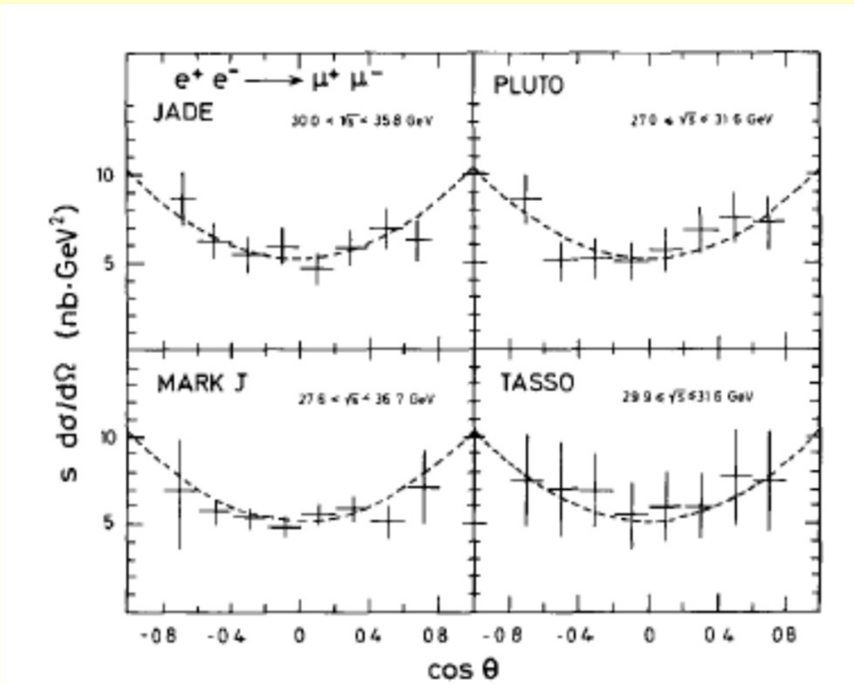
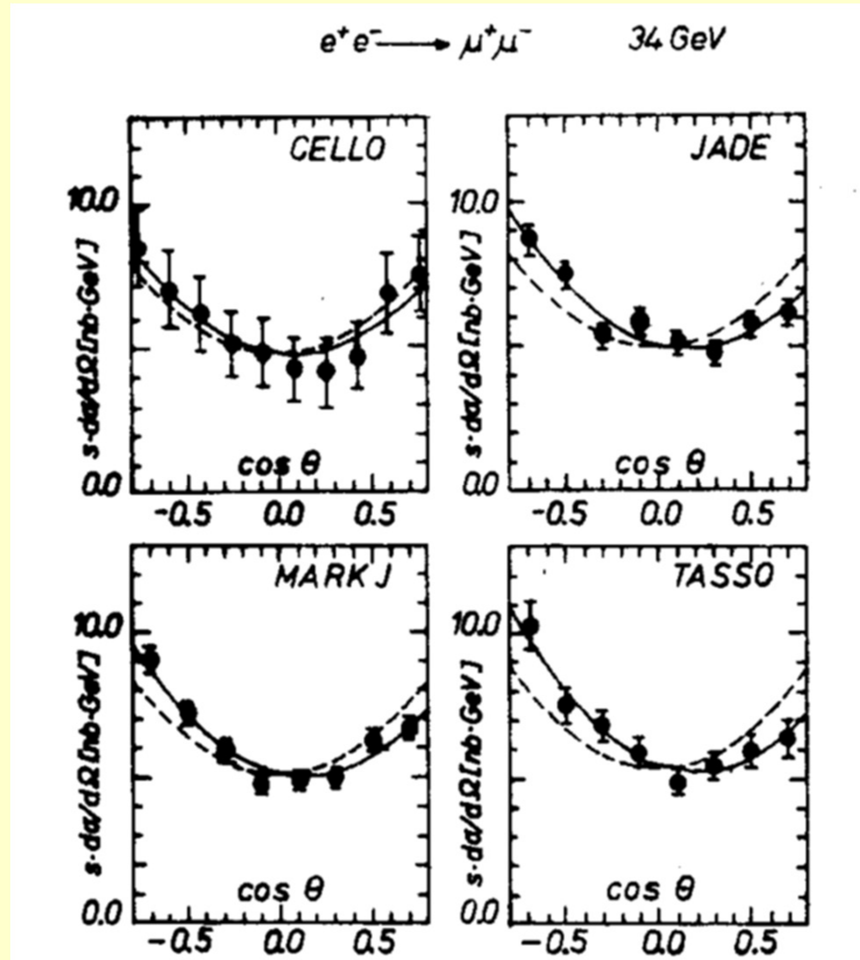


Fig. 15. MARK II $e^+e^- \to \mu^+\mu^-$ at $\langle E_{c.m.}\rangle^{4} = 5.847$ compared to $1+\cos^2\theta$.

PEP: Angular distribution

Annihilation Cross-Section - XI



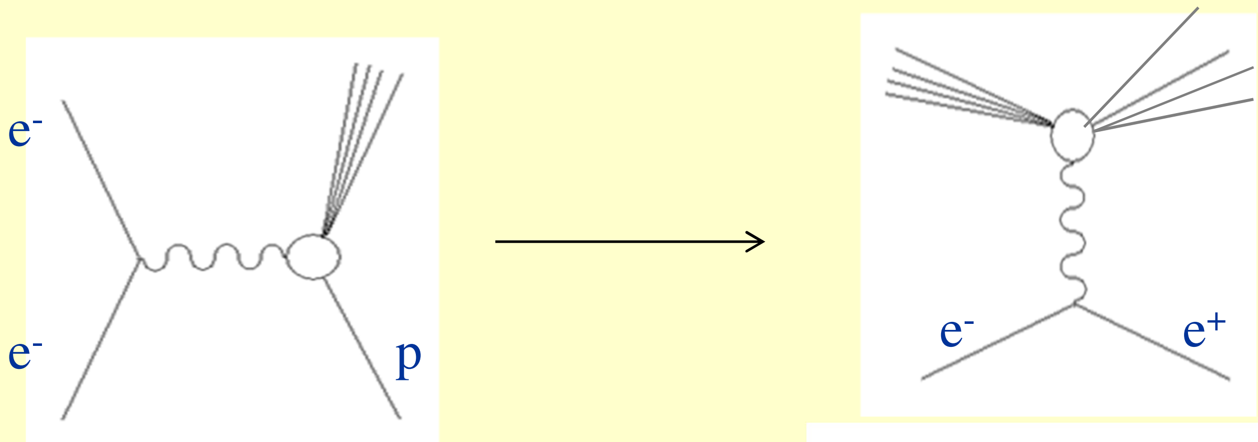


PETRA: Electroweak interference

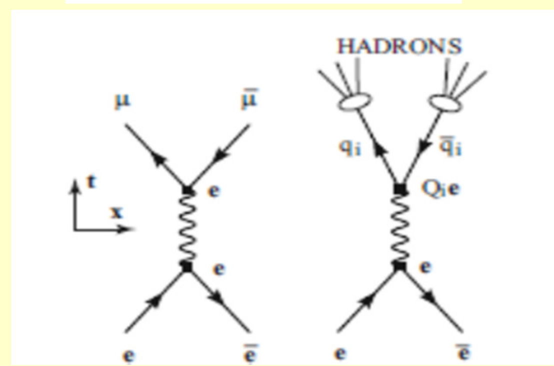
PQCD: Jets in e^+e^- Collisions - I

e^+e^-annihilation into hadrons:

At the parton level = Crossed Deep Inelastic Scattering

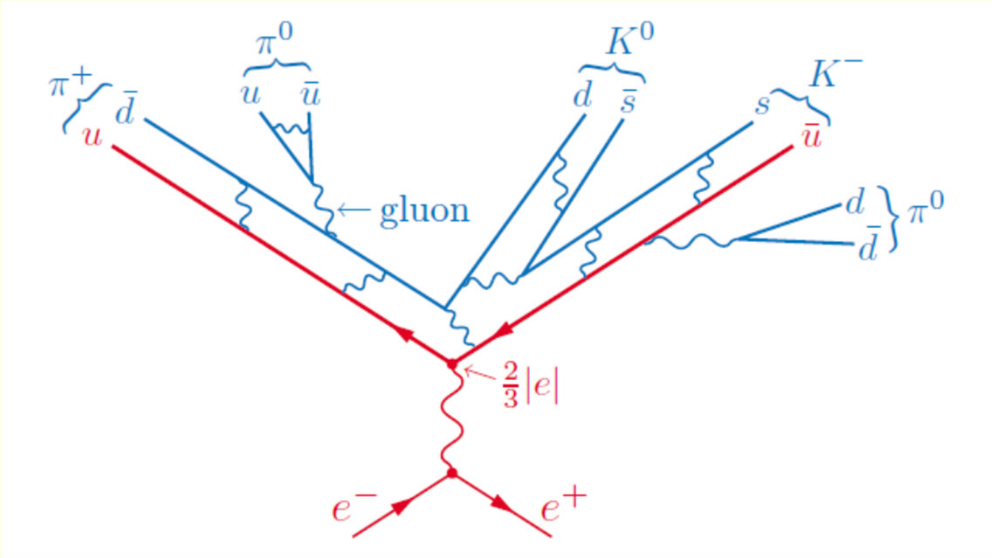


Interpreted as annihilation into a $q\bar{q}$ pair,
followed by quark fragmentation into hadrons



PQCD: Jets in e^+e^- Collisions - II

Picture of quark fragmentation



PQCD: Jets in e^+e^- Collisions - III

By ignoring *quark fragmentation* details

$$e^+ + e^- \to q + \overline{q} \to hadrons$$

$$\frac{d\sigma}{d\Omega} = \frac{3\alpha^2}{4s}\left(1+\cos^2\theta\right)\sum_{flavor} e^2_{flavor}$$

$$\to \sigma\left(s\right) = \frac{4\pi\alpha^2}{s}\sum_{flavor} e^2_{flavor}$$

Due to color confinement, quark/antiquark unobservable as free particles

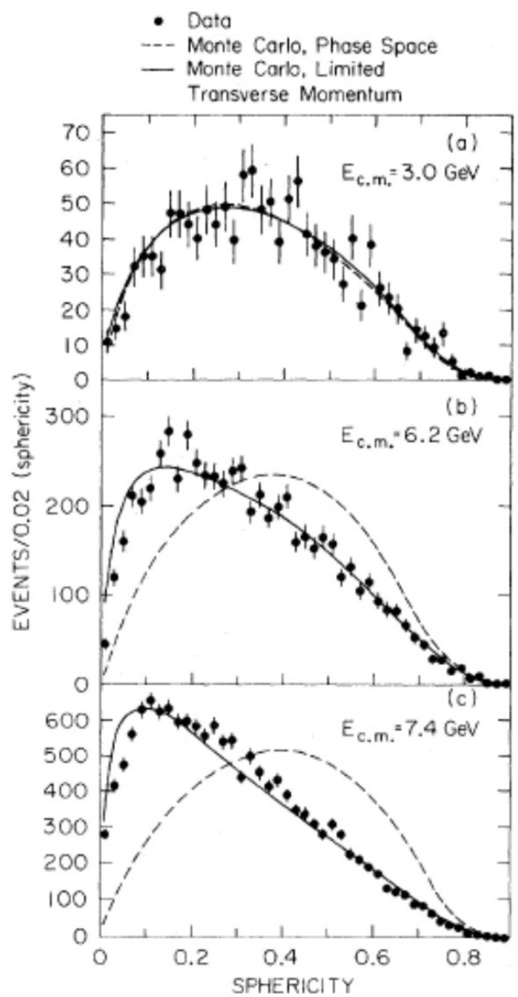
Expect hadron debris from quark fragmentation

→ *Jets*
Hadrons: Trend towards grouping into narrow pencils of particles

Naive prediction: 2 jets

At high energy, expect hadronic annihilation events to be *non-spherical*

PQCD: Jets in e^+e^- Collisions - IV



Define *sphericity* of events:

$$S = \min \frac{3}{2} \frac{\sum_i p_{\perp i}^2}{\sum_i p_i^2} \rightarrow \begin{cases} 0 & \text{2-jets} \\ 1 & \text{spherical} \end{cases}$$

$\min:$ Choose axes which minimize S ($\leftarrow$ *Iterative*)

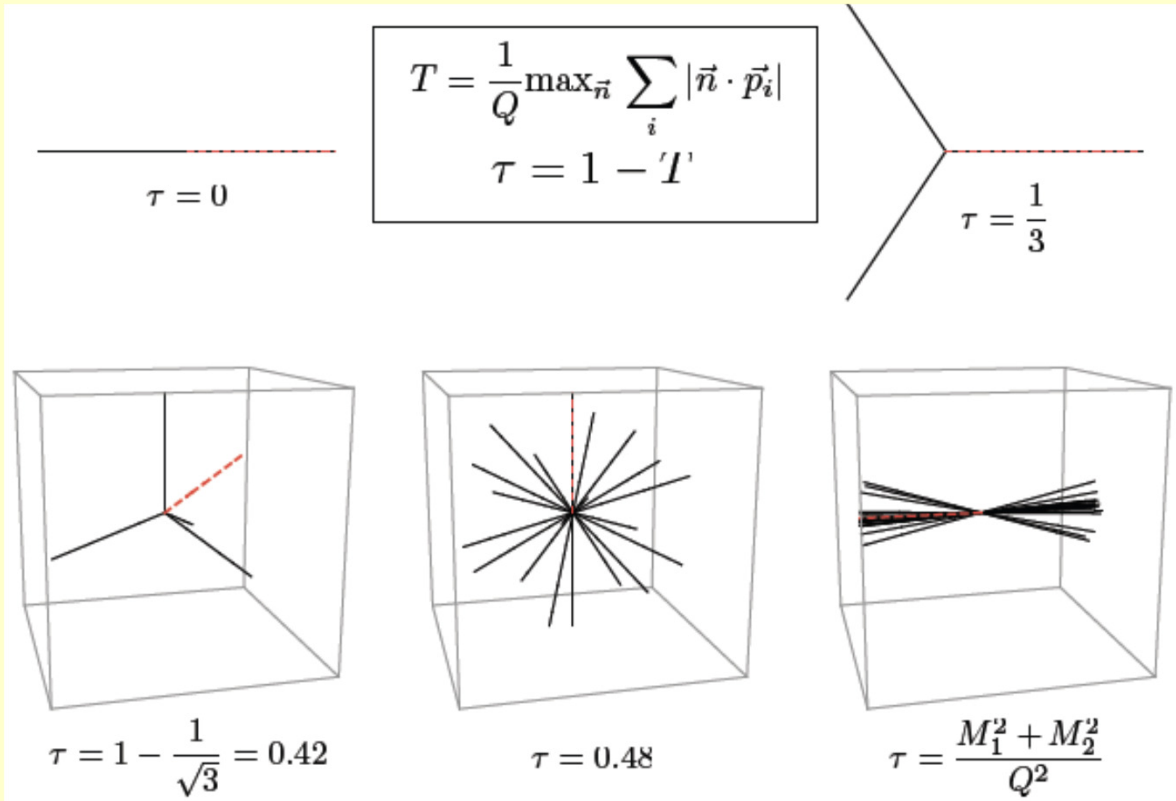
full line jet model, dashed line phase space model

sphericity distribution

for $E_{c.m.} = 3.0,\ 6.2,\ 7.4$ GeV

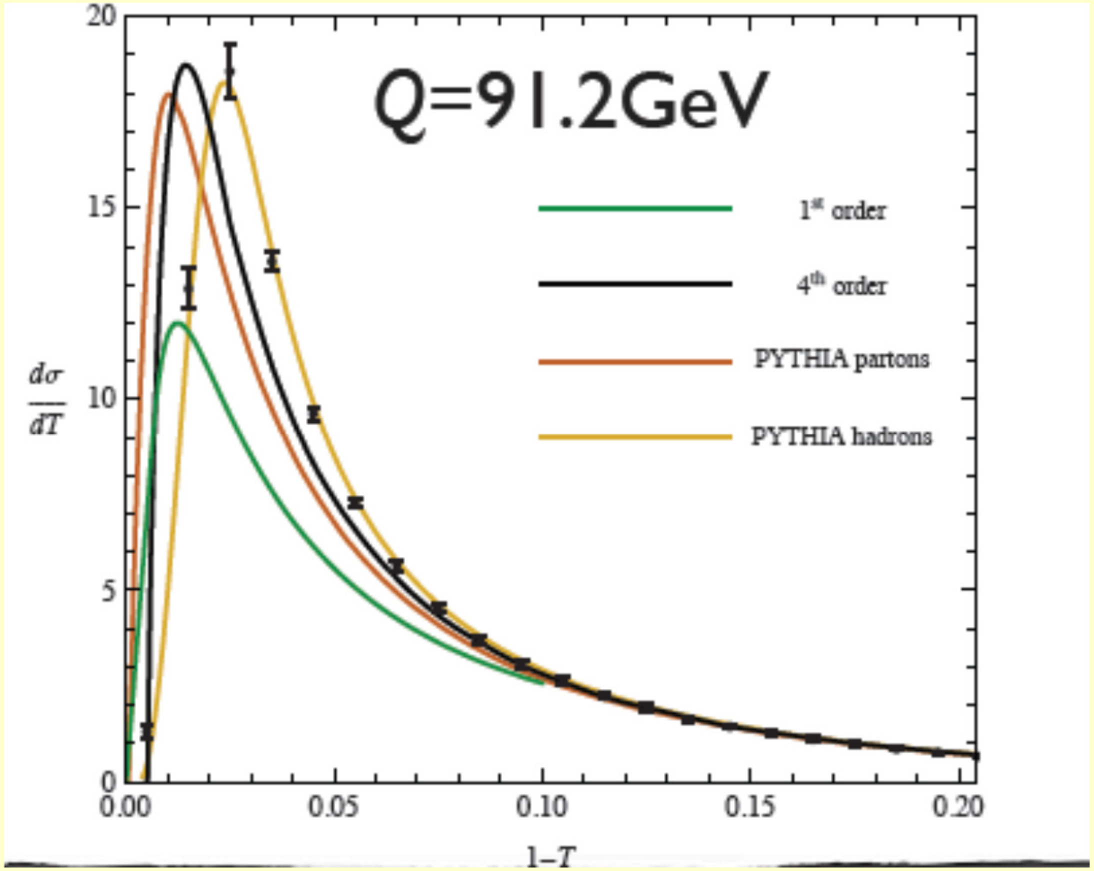
PQCD: Jets in e^+e^- Collisions - V

Interesting observable: *Thrust*



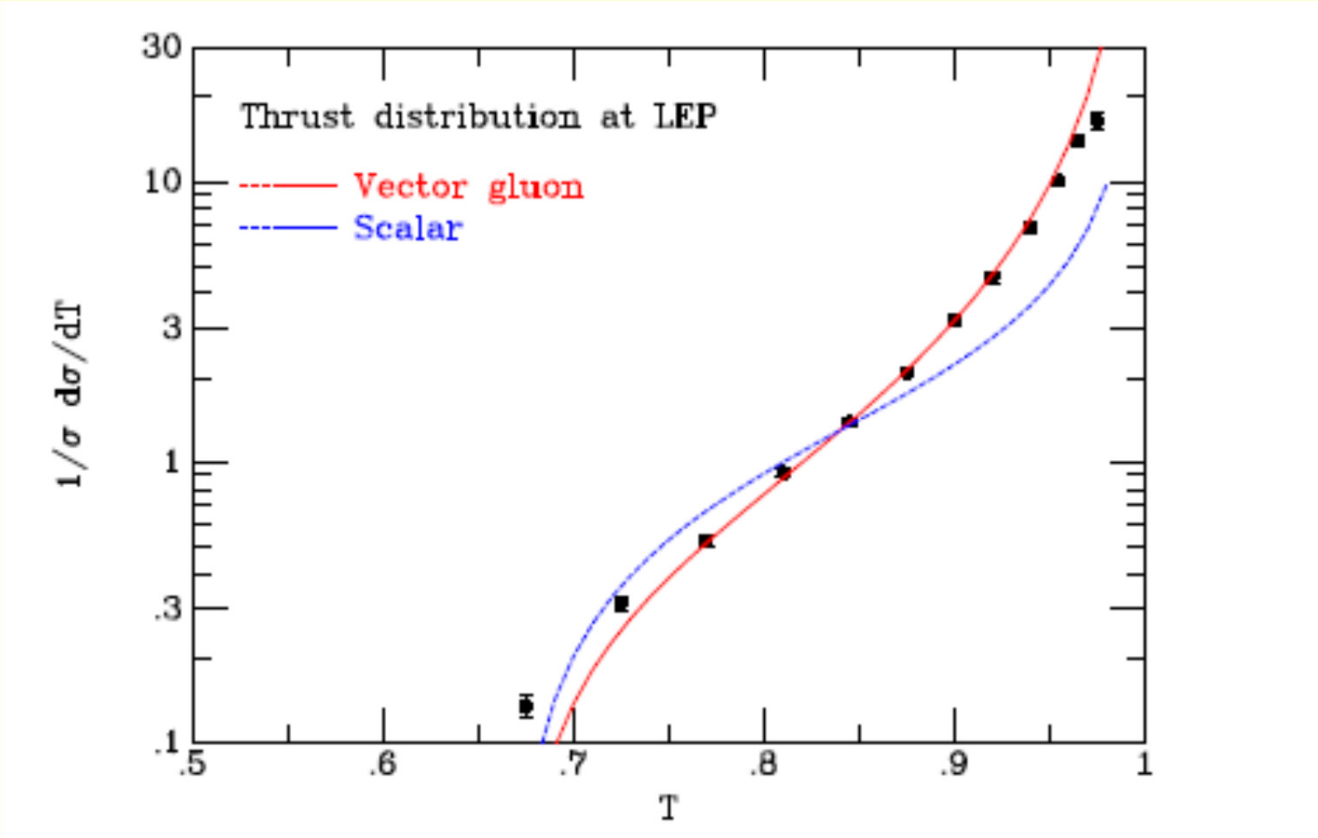
PQCD: Jets in e^+e^- Collisions - VI

ALEPH



PQCD: Jets in $e^+ e^-$ Collisions - VII

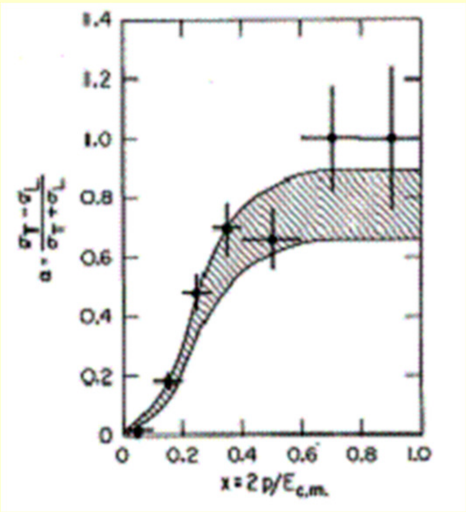
DELPHI



PQCD: Jets in $e^+ e^-$ Collisions - VIII

For 2 jets events

$$\left.\begin{array}{l} \frac{d\sigma}{d\Omega} \propto 1+\cos^2\theta \;\; \text{quark spin} = 1/2 \\ \frac{d\sigma}{d\Omega} \propto 1-\cos^2\theta \;\; \text{quark spin} = 0 \end{array}\right\} \equiv 1+\alpha\cos^2\theta$$



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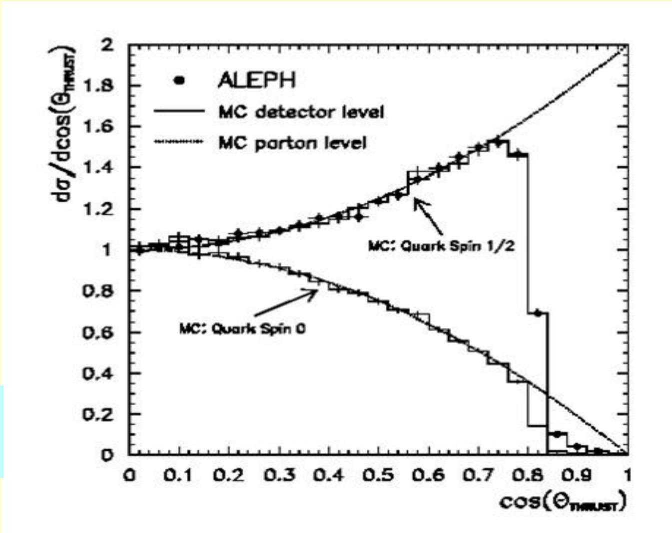
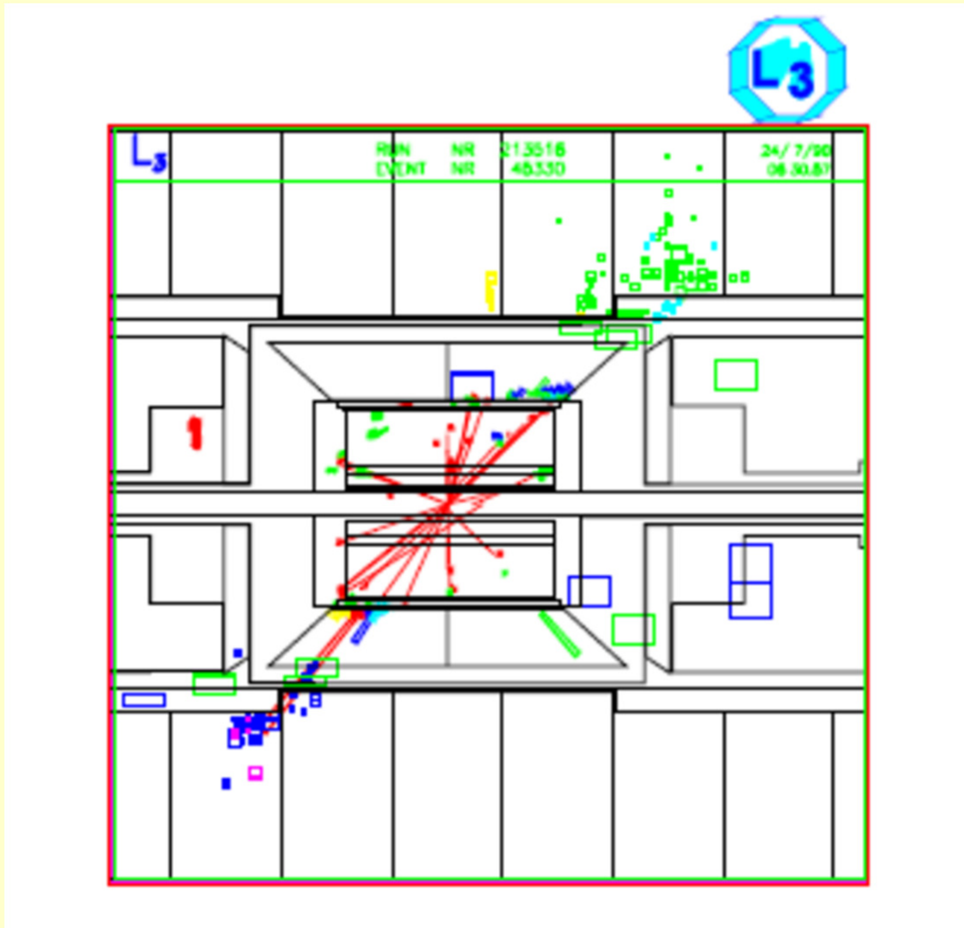


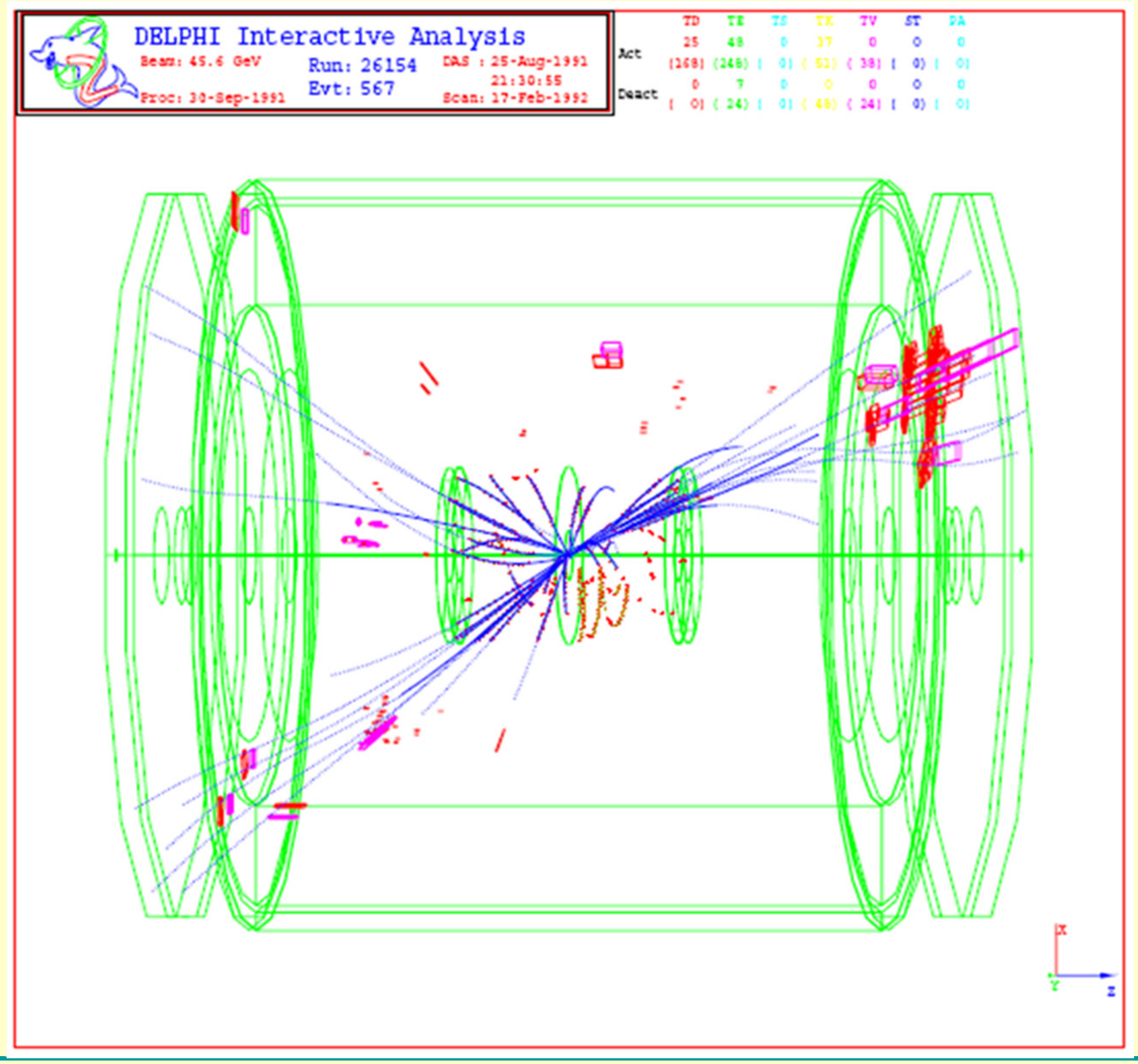
Mark I (SPEAR)
E = few GeV

ALEPH (LEP)
E = 90 GeV

PQCD: Jets in e^+e^- Collisions - IX



PQCD: Jets in e^+e^- Collisions - X



PQCD: Jets in $e^+ e^-$ Collisions - XI

Total hadronic cross section $\leftrightarrow$ R Ratio

Reminder:

Time scale of hard interaction

$$T \sim \frac{1}{Q} = \frac{1}{\sqrt{s}} \sim \frac{1}{\text{Many } GeV} \rightarrow \text{Very small}$$

Time scale of soft hadronization

$$\tau \sim \frac{1}{\Lambda} \sim \frac{1}{1\ GeV} \rightarrow \text{Large}$$

$\rightarrow$ Perturbative cross section OK

Expect at 30-40 GeV:

$$R = \frac{\sigma_{q\bar{q}}}{\sigma_{\mu\mu}} = 3 \sum_{i=u,d,s,c,b} Q_i^2 = \frac{11}{3} \approx 3.67 \left(+ \ 0.05 \text{ coming from } Z^0 \right)$$

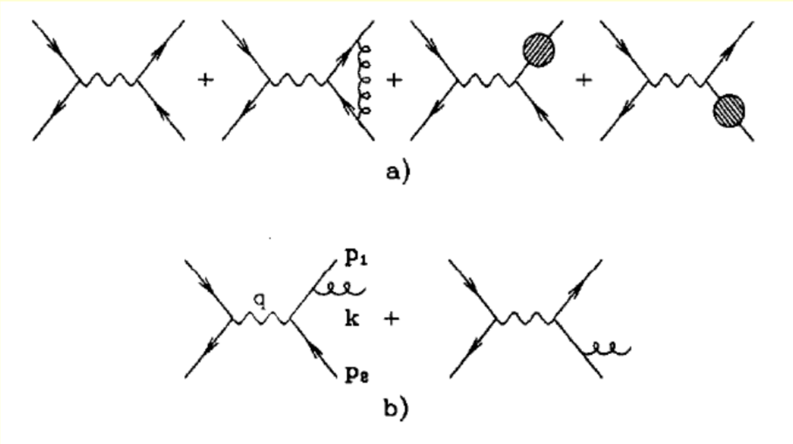
Measure :

$R \approx 3.9$

$\rightarrow$ QCD Correction required

PQCD: Jets in $e^+ e^-$ Collisions - XII

QCD corrections Next to Leading Order (NLO):



Virtual gluons

Real gluons

Real gluons: 3 particles in the final state

Some kinematics:

$x_1 = 2E_1/\sqrt{s}$ $\quad$ $x_2 = 2E_2/\sqrt{s}$

$0 \le x_1, x_2 \le 1,\ x_1 + x_2 \ge 1.$

$x_3 = 2k \cdot q/q^2 = 2E_g/\sqrt{s} = 2 - x_1 - x_2.$

PQCD: Jets in $e^+ e^-$ Collisions - XIII

Differential cross section

$$\frac{1}{\sigma}\frac{d^2\sigma}{dx_1 dx_2} = \frac{2\alpha_S}{3\pi}\frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

Basic remark:

$x_1, x_2 \rightarrow 1 \quad \rightarrow \quad \sigma \rightarrow \infty$

Also true to higher perturbative orders

$\rightarrow$ 2 jets dominant over everything else

PQCD: Jets in $e^+ e^-$ Collisions - XIV

Total hadronic cross section:

$$\sigma^{q\bar{q}g} = \sigma_0 \, 3 \sum_q Q_q^2 \int dx_1 dx_2 \, \frac{2\alpha_S}{3\pi} \, \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

In order to regularize diverging integrals:

Shift to *4-2ε* dimensions, make them converging..

$$\sigma^{q\bar{q}g} = \sigma_0 \, 3 \sum_q Q_q^2 \frac{2\alpha_S}{3\pi} H(\epsilon) \left[\frac{2}{\epsilon^2} - \frac{3}{\epsilon} + \frac{19}{2} + O(\epsilon)\right]$$

Real gluons

$$\sigma^{q\bar{q}(g)} = \sigma_0 \, 3 \sum_q Q_q^2 \frac{2\alpha_S}{3\pi} H(\epsilon) \left[-\frac{2}{\epsilon^2} + \frac{3}{\epsilon} - 8 + O(\epsilon)\right]$$

Virtual gluons

$$\underset{\varepsilon \to 0}{\longrightarrow} \quad R^{e^+e^-} = 3 \sum_q Q_q^2 \left\{1 + \frac{\alpha_S}{\pi} + O(\alpha_S^2)\right\}.$$

PQCD: Jets in $e^+ e^-$ Collisions - XV

Absolute number of jets ill defined, both in theory and experiment

Can define number of jets as a function of some *resolution parameter*, according to a *clustering algorithm*

Just as an example: *Durham algorithm*

Take $q\bar{q}g$ final state

By fixing a y parameter as

$m_{thresh}^2 = ys$

compare the $(\text{invariant mass})^2$ of each parton pair to m_{thresh}^2

$$(p_i + p_j)^2 > ys, \qquad i,j = q,\bar{q},g.$$

3 combinations/event

Then:

$N_{hit} = N_{jet}$

Of course, $\begin{matrix} R_{2\,jet} = R_{2\,jet}(y) \\ R_{3\,jet} = R_{3\,jet}(y) \end{matrix} \rightarrow$ QCD predicts $R_{k-jet}(y)$!

PQCD: Jets in $e^+ e^-$ Collisions - XVI

Typical jet algorithm: (modified) *Durham*

To define fraction f_n of n-jet final states ($n = 2, 3, \ldots$), must specify jet algorithm.

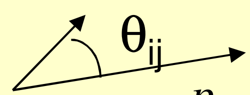
Most common is k_T or Durham algorithm:

- Define jet resolution y_{cut} (dimensionless).
- For each pair of final-state momenta p_i, p_j define

$$y_{ij} = 2\min\{E_i^2, E_j^2\}(1 - \cos\theta_{ij})/s$$

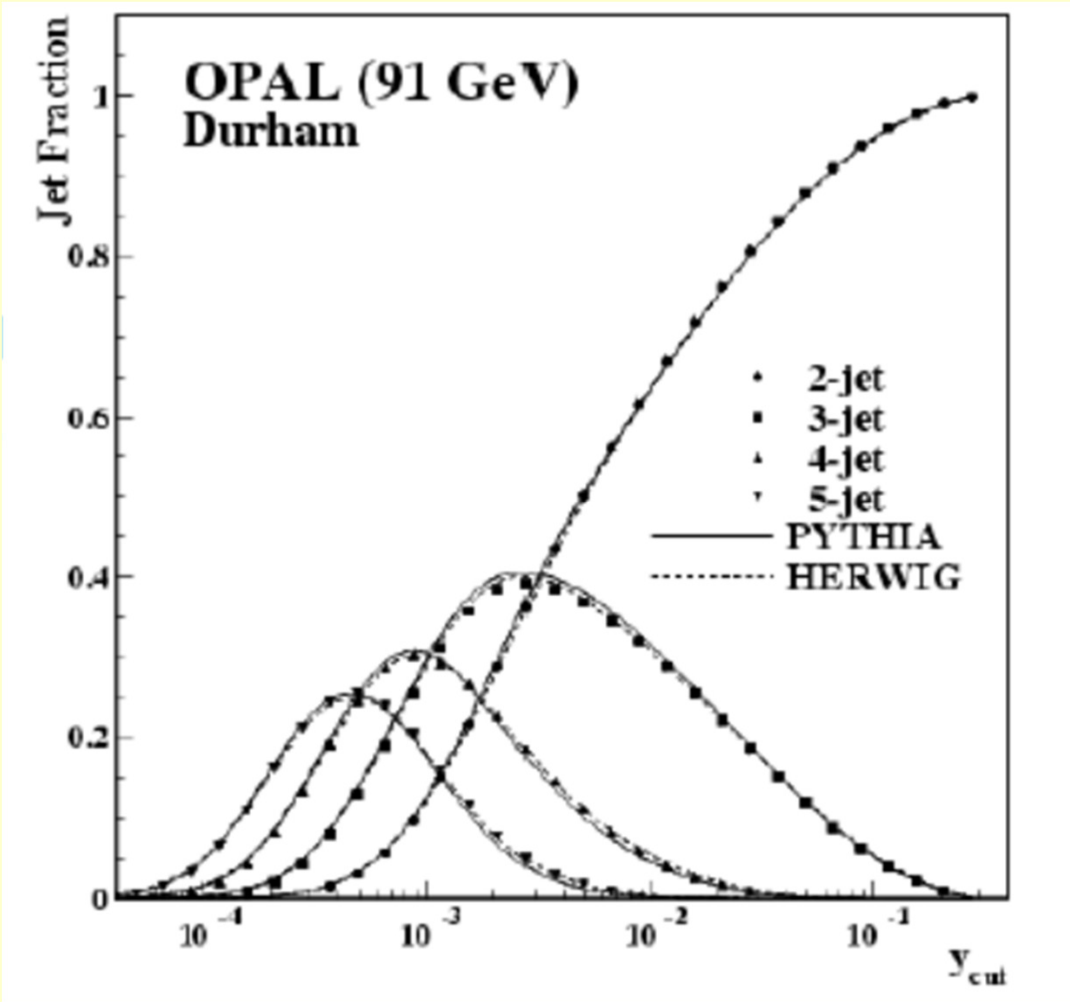
- If $y_{IJ} = \min\{y_{ij}\} < y_{\text{cut}}$, combine I, J into one object K with $p_K = p_I + p_J$.
- Repeat until $y_{IJ} > y_{\text{cut}}$. Then remaining objects are jets.

$p_i = (E_i, \vec{p}_i)$



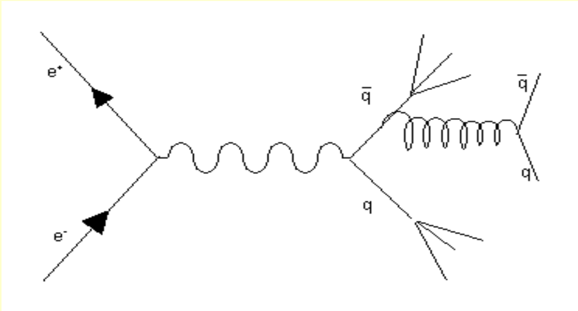
$p_j = (E_j, \vec{p}_j)$

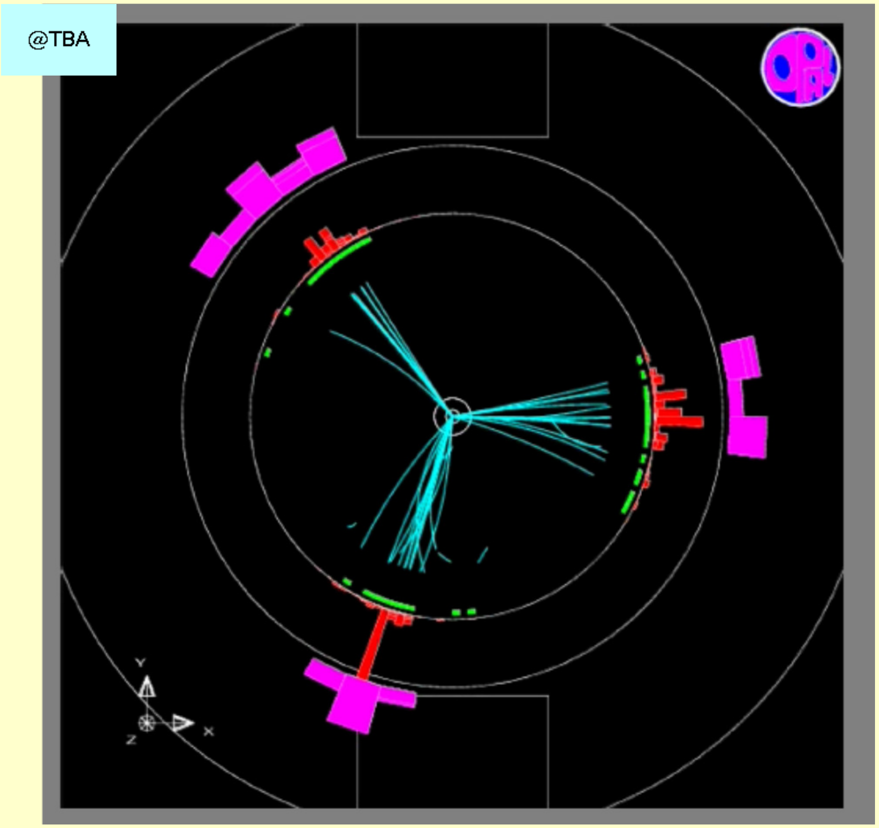
PQCD: Jets in $e^+ e^-$ Collisions - XVII



PQCD: Jets in $e^+ e^-$ Collisions - XVIII

Exceptional 3-jet event from OPAL



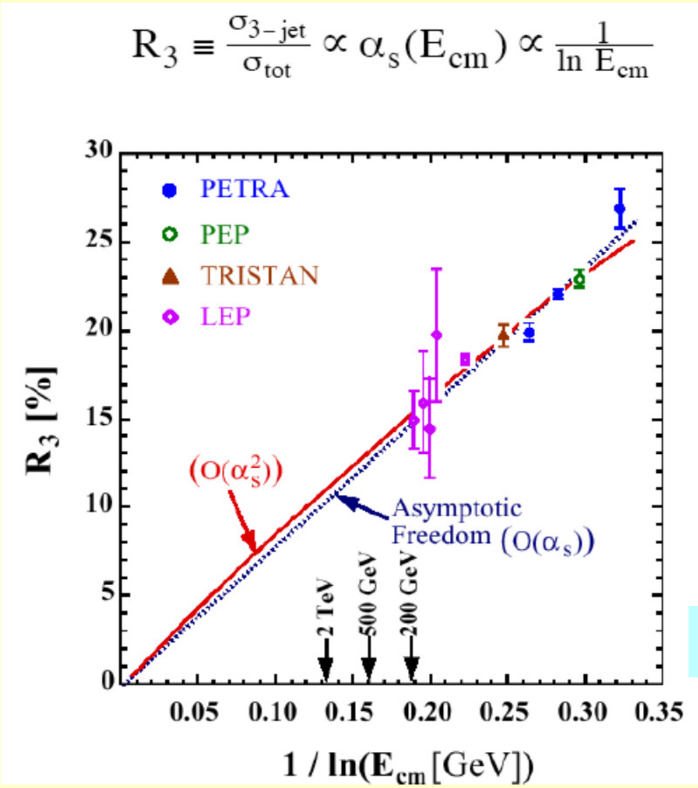


PQCD: Jets in e^+e^- Collisions - XIX

Get a measurement of α_s :

$$\alpha_S(35GeV) = 0.146 \pm 0.03$$

$$\alpha_S(M_{Z^0}) = 0.124 \pm 0.0043$$



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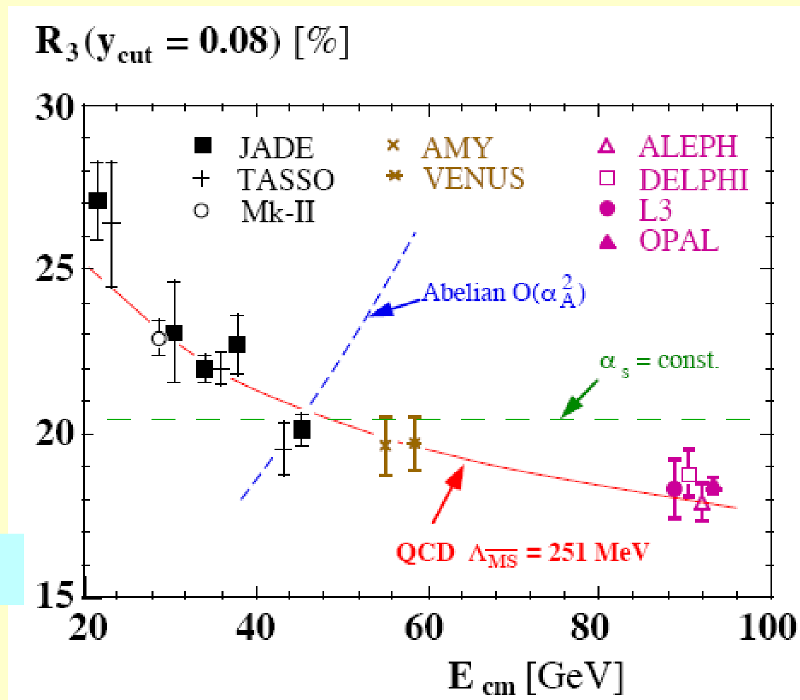
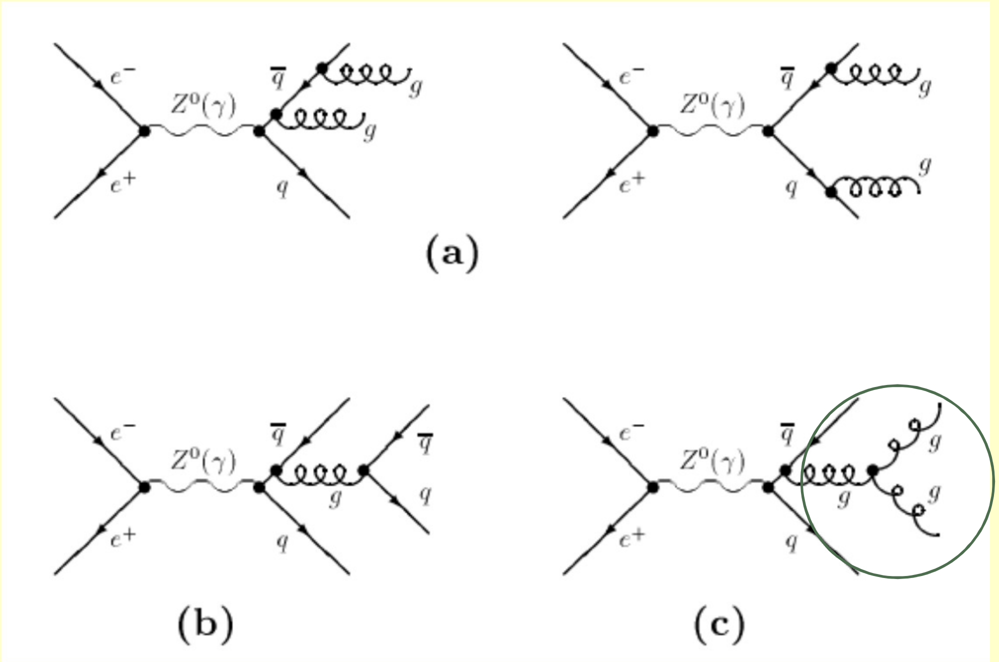


PQCD: Jets in e^+e^- Collisions- XX

Is QCD Really SU(3) ?

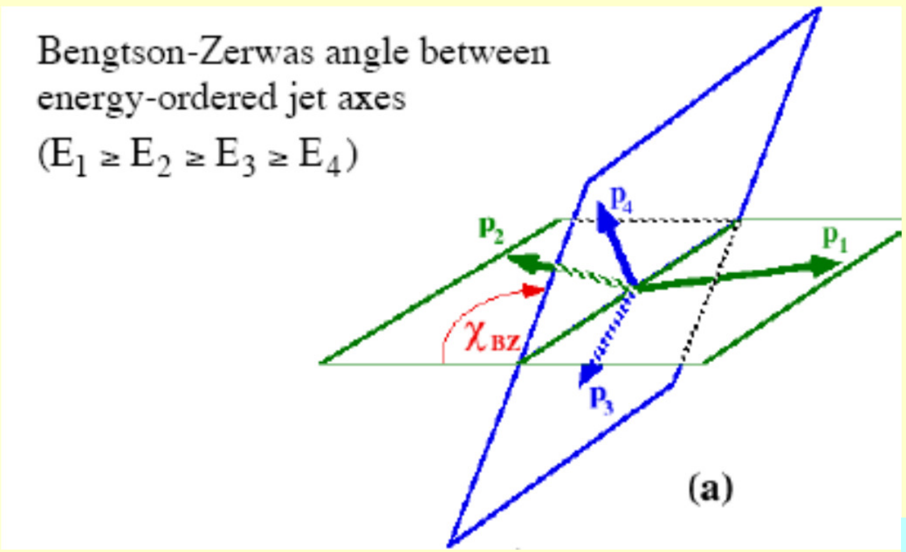
Test for non-Abelian couplings at LEP: 4 jets events

Special angular correlation from 3-gluon vertex amplitude



PQCD: Jets in $e^+ e^-$ Collisions - XXI

Look at distribution of a special angle, sensitive to non-Abelian couplings:



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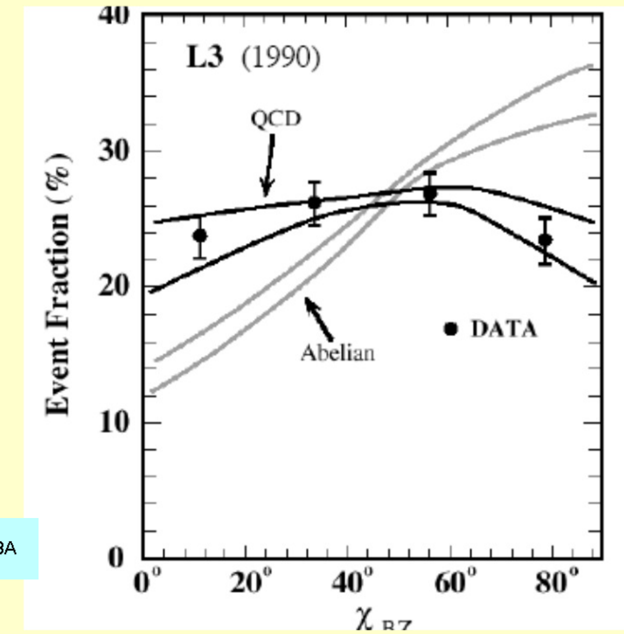


Quark Parton Model - I

Write down F_2 in terms of PDFs

$$\frac{F_2(x)}{x} = \sum_i z_i^2 \underbrace{n_i \delta\left(x - \frac{m_i}{M}\right)} \rightarrow \frac{F_2(x)}{x} = \sum_i z_i^2 \underbrace{q_i(x)}$$

$$p = uud$$

$$F_2^p(x) = x\left[\left(\frac{2}{3}\right)^2 u_p(x) + \left(-\frac{1}{3}\right)^2 d_p(x)\right]$$

$$\rightarrow F_2^p(x) = x\left[\frac{4}{9} u_p(x) + \frac{1}{9} d_p(x)\right]$$

$$n = ddu$$

$$F_2^n(x) = x\left[\left(-\frac{1}{3}\right)^2 d_n(x) + \left(\frac{2}{3}\right)^2 u_n(x)\right]$$

From isospin symmetry:

$$F_2^n(x) = x\left[\left(-\frac{1}{3}\right)^2 u_p(x) + \left(\frac{2}{3}\right)^2 d_p(x)\right]$$

$$\rightarrow F_2^n(x) = x\left[\frac{1}{9} u_p(x) + \frac{4}{9} d_p(x)\right]$$

Quark Parton Model - II

Consider the deuteron structure function:

$$F_2^d(x) = \frac{1}{2}\left(F_2^p + F_2^n\right) = \frac{5}{9}\frac{x}{2}\left[u_p(x) + d_p(x)\right]$$

$$\rightarrow F_2^n(x) = F_2^d(x) - F_2^p(x)$$

$$= \frac{5}{18}x\left[u_p(x) + d_p(x)\right] - \frac{1}{9}x\left[u_p(x) - 4d_p(x)\right]$$

$$= \frac{3}{18}x\left[u_p(x) - d_p(x)\right]$$

Finally extract PDFs from measured F_2

$$xu_p(x) = xd_n(x) = 3F_2^p(x) - \frac{6}{5}F_2^d(x)$$

$$xu_n(x) = xd_p(x) = 3F_2^p(x) + \frac{24}{5}F_2^d(x)$$

Quark Parton Model - III

Take a Hydrogen atom:

Common wisdom: “A bound state of proton + electron”
But: Consider the effect of radiative corrections (e.g. loops)
Then we should be more precise:

Hydrogen = $(\text{Proton+Electron})_{\text{Valence}}$ + $(\text{Positrons+Electrons+Photons})_{\text{Sea}}$

Can we say valence and sea particles are fundamentally different? Well,…

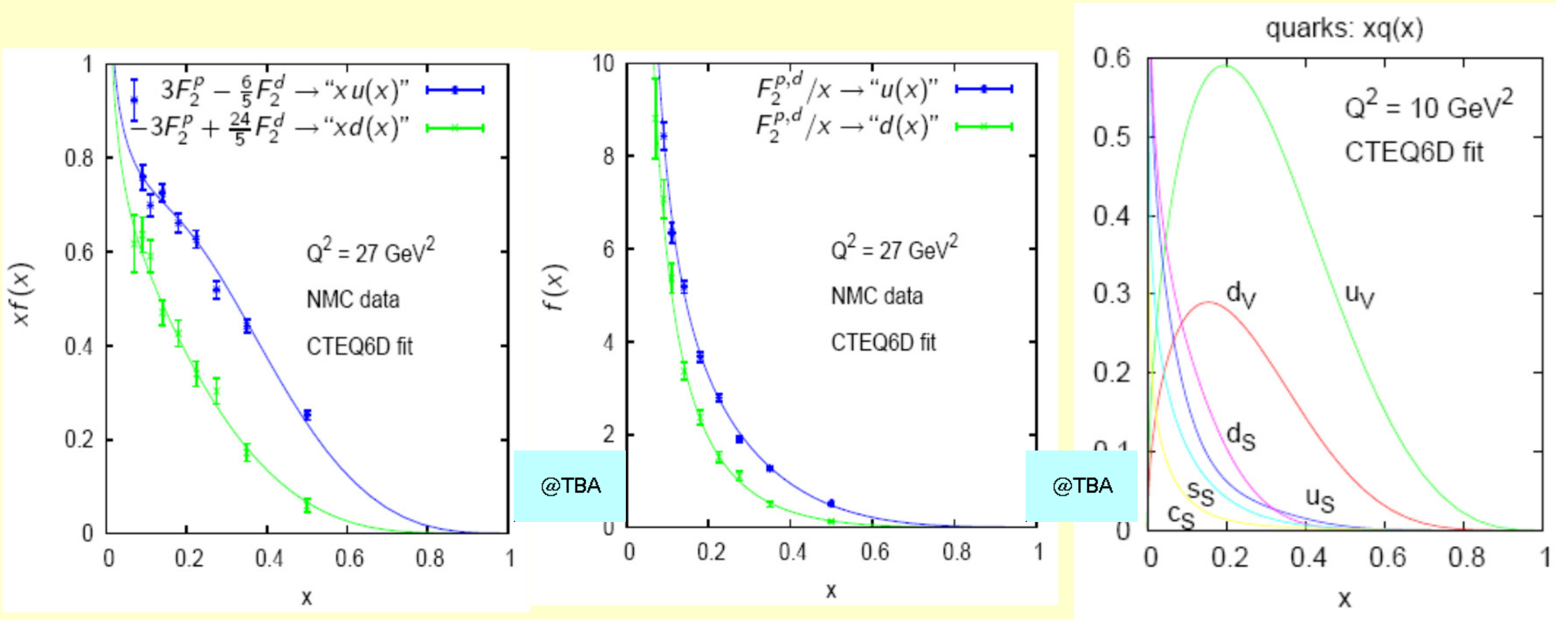
In a bound state, both are off mass shell
Both are active in yielding measurable effects (Coulomb levels vs. Lamb shift,..)
Sea particles yield small corrections to levels determined by valence e+p

Take a hadron:

Hadron = $(\text{Quarks/Antiquarks})_{\text{Valence}}$ + $(\text{Quarks/Antiquarks+Gluons})_{\text{Sea}}$

Since $a_s >> a$, ***sea effects are much larger in QCD***

Quark Parton Model - IV



Among parton model predictions: *Sum Rules* (= Integral relations) for PDFs
Examples: Proton quark content is *uud*

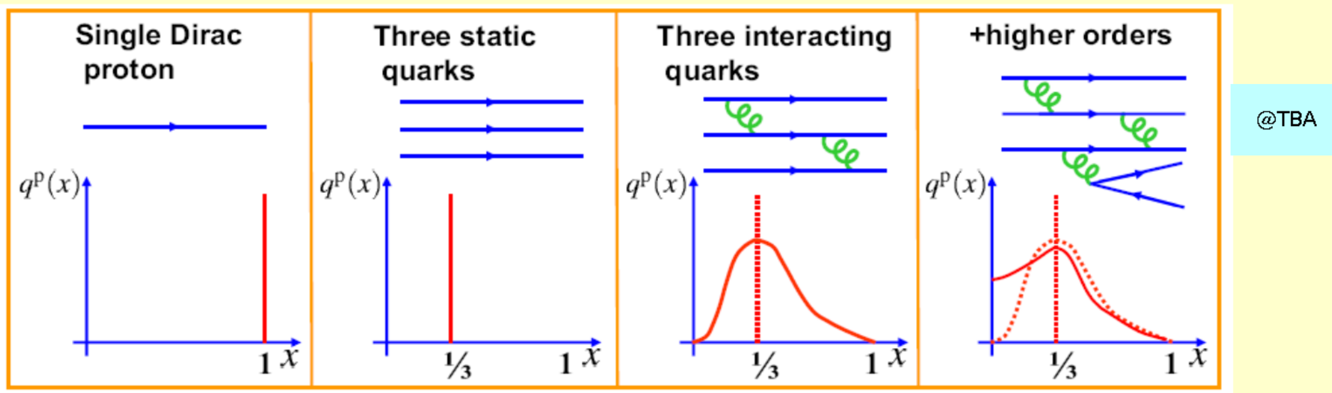
$$\int \left[u_p(x) - \bar{u}_p(x)\right] dx = 2$$

$$\int \left[d_p(x) - \bar{d}_p(x)\right] dx = 1$$

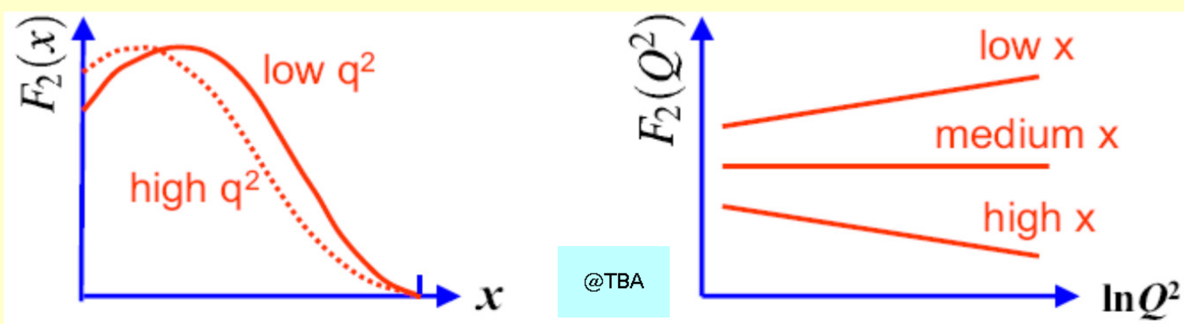
$$\int \left[s_p(x) - \bar{s}_p(x)\right] dx = 0$$

PQCD: DIS Scaling Violations - I

Our picture of structure functions



Observe small deviations from scaling: $F_2\left(x\right) \to F_2\left(x, Q^2\right)$

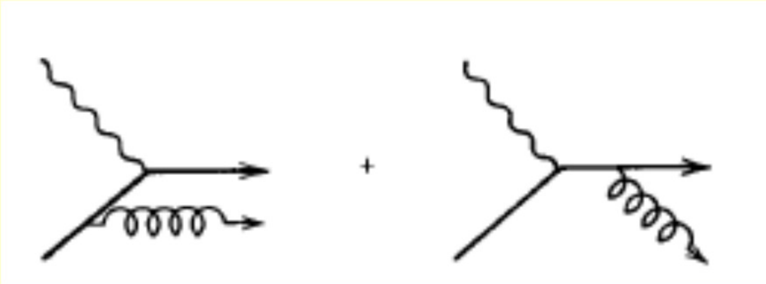


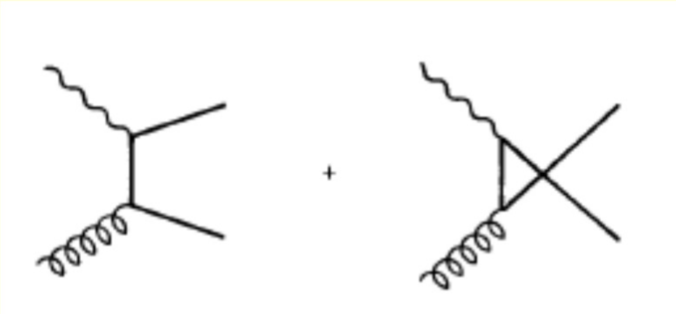
PQCD: DIS Scaling Violations - II

QCD on $F_2\left(x,Q^2\right)$:

$x-$dependence $\rightarrow$ Not predicted

Q^2-dependence $\rightarrow$ Predicted !



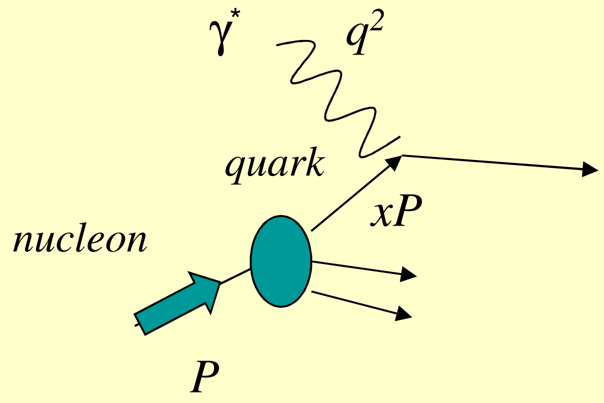


Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) Equations:

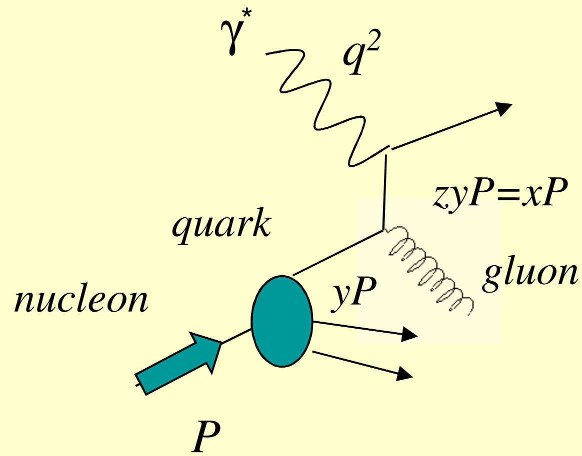
Successful prediction of Q^2 evolution of structure functions

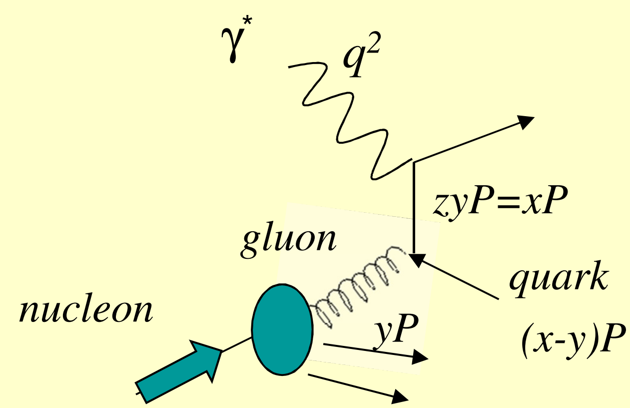
PQCD: DIS Scaling Violations - III

First order (NLO) QCD corrections to naive Quark Parton Model:



QPM





QCD

PQCD: DIS Scaling Violations - IV

The bottom line:
Parton Density Functions at any given Bjorken x , $q(x)$:
Depending on quark & gluon densities taken at higher fractional momentum $y>x$

Also depending on probabilities of radiative/scattering processes
$P_{qq}(x/y)$, $P_{gq}(x/y)$ usually called *splitting functions*

$$\frac{F_2(x)}{x} = 2F_1(x) = \sum_i e_i^2 q_i(x)$$

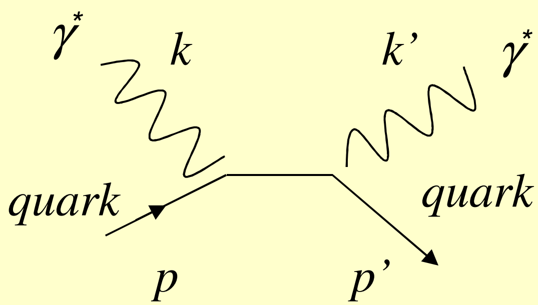
$$\rightarrow \frac{F_2(x)}{x} = \sum_i e_i^2 \int_0^1 q_i(y)\delta(x-y)dy = \sum_i e_i^2 \int_0^1 q_i(y)\delta\left(1-\frac{x}{y}\right)\frac{dy}{y}$$

$$z = \frac{x}{y}$$

$$\rightarrow \frac{F_2(x)}{x} = \sum_i e_i^2 \int_0^1 q_i(y)\left[\delta\left(1-\frac{x}{y}\right)+\sigma_{qq}(z)\right]\frac{dy}{y}$$

PQCD: DIS Scaling Violations - V

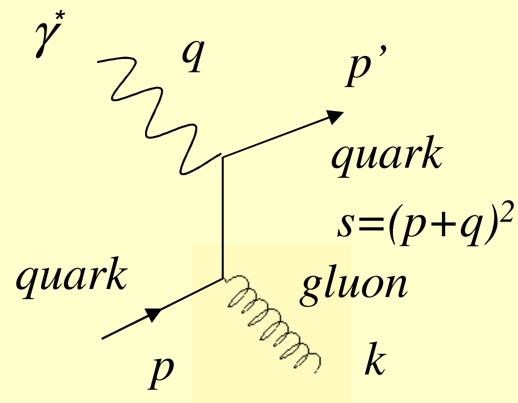
Just as an example: Gluon radiation splitting function at leading order (LO)
Almost carbon-copy of Compton effect in QED..



$k \leftrightarrow q$

$k' \leftrightarrow p'$

$u=(k-p')^2$ $t=(q-p')^2$



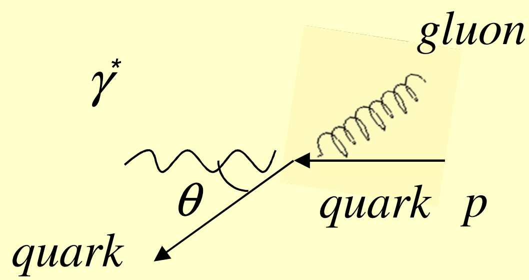
$\gamma^*(k)\ q(p) \rightarrow \gamma^*(k')\ q(p')$

$\gamma^*(q)\ q(p) \rightarrow q(p')g(k)$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\gamma q\rightarrow\gamma q} = \frac{\alpha^2 e_q^2}{2s}\left(\frac{-u}{s} - \frac{s}{u} - \frac{2tq^2}{su}\right)$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\gamma q\rightarrow gq} = \frac{C_F \alpha\alpha_S e_q^2}{2s}\left(\frac{-t}{s} - \frac{s}{t} - \frac{2uq^2}{st}\right)$$

PQCD: DIS Scaling Violations - VI



$$dp_T^2 = d(p^2 \sin^2\theta) = 2p^2 \sin\theta \underbrace{\cos\theta}_{\approx 1} d\theta = 2p^2 d(\cos\theta)$$

$$\rightarrow dp_T^2 = \frac{s}{2} d\left(\cos\theta\right) = \frac{s}{2}\frac{d\Omega}{4\pi} \rightarrow \frac{d\sigma}{d\Omega} = \frac{s}{8\pi}\frac{d\sigma}{dp_T^2} \rightarrow \frac{d\sigma}{dp_T^2} = \frac{8\pi}{s}\frac{d\sigma}{d\Omega}$$

$$\text{Reminder (Bjorken)}: \; x = -\frac{q^2}{2P\cdot q}$$

$$\text{Define: } z = -\frac{q^2}{2p\cdot q} = -\frac{q^2}{s-q^2}$$

$$\rightarrow \left(\frac{d\sigma}{dp_T^2}\right)_{\gamma q\rightarrow gq} \propto \frac{C_F \alpha \alpha_S e_q^2}{p_T^2} P_{qq}(z), \quad P_{qq}(z) \equiv \frac{1+z^2}{1-z^2}$$

PQCD: DIS Scaling Violations - VII

Integrate 'Compton-like' differential cross-section between:

$$\begin{cases} \lambda \text{ lower cutoff } (\leftarrow \text{no divergences}) \\ \dfrac{\hat{s}}{4} \text{ upper cutoff } (\leftarrow \text{kinematical}),\ \hat{s} \text{ partonic CM energy squared} \end{cases}$$

$$\sigma_{qq}(z) = \int_{\lambda}^{p_{T\,\max}^2 = \frac{\hat{s}}{4}} \frac{d\sigma}{dp_T^2} dp_T^2 \propto \frac{C_F \alpha \alpha_S e_q^2}{s} P_{qq}(z) \ln\left(-\frac{q^2}{\lambda}\right)$$

$$\rightarrow \frac{F_2(x)}{x} = \sum_i e_i^2 \int_x^1 q_i(y) \left[\delta(1-\frac{x}{y}) + \frac{\alpha_S}{2\pi} P_{qq}(z) \ln\left(\frac{Q^2}{\lambda}\right)\right] \frac{dy}{y}$$

$$P_{qq}(z) \equiv \frac{\alpha e_q^2 C_F}{2\pi s} \frac{1+z^2}{1-z^2}$$

$$\rightarrow \frac{F_2(x)}{x} = \sum_i e_i^2 \underbrace{\left[q_i(x) + \frac{\alpha_S}{2\pi} \ln\left(\frac{Q^2}{\lambda}\right) \int_x^1 q_i(y) P_{qq}\left(\frac{x}{y}\right) \frac{dy}{y}\right]}_{q_i(x,Q^2)}$$

PQCD: DIS Scaling Violations -VIII

Evolution equation for each quark flavor:

$$\frac{dq_i(x,Q^2)}{d\ln(Q^2)} = \int_x^1 q_i(y,Q^2)\frac{\alpha_S}{2\pi}P_{qq}\left(z=\frac{x}{y}\right)\frac{dy}{y}$$

This is actually incomplete:

We forgot to add the contribution from the 2nd diagram..

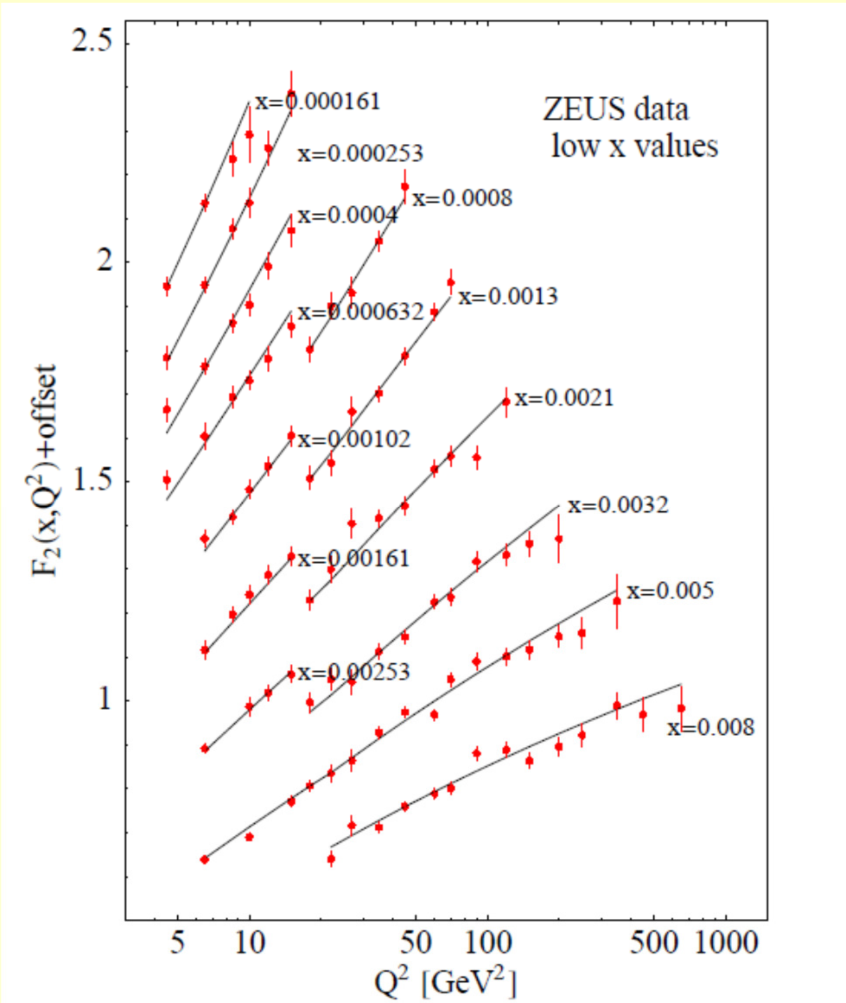
$$\rightarrow \frac{dq(x,Q^2)}{d\ln(Q^2)} = \frac{\alpha_S(Q^2)}{2\pi}\int_x^1 \left[q(y,Q^2)P_{qq}\left(z=\frac{x}{y}\right)+g(y,Q^2)P_{gq}\left(z=\frac{x}{y}\right)\right]\frac{dy}{y}$$

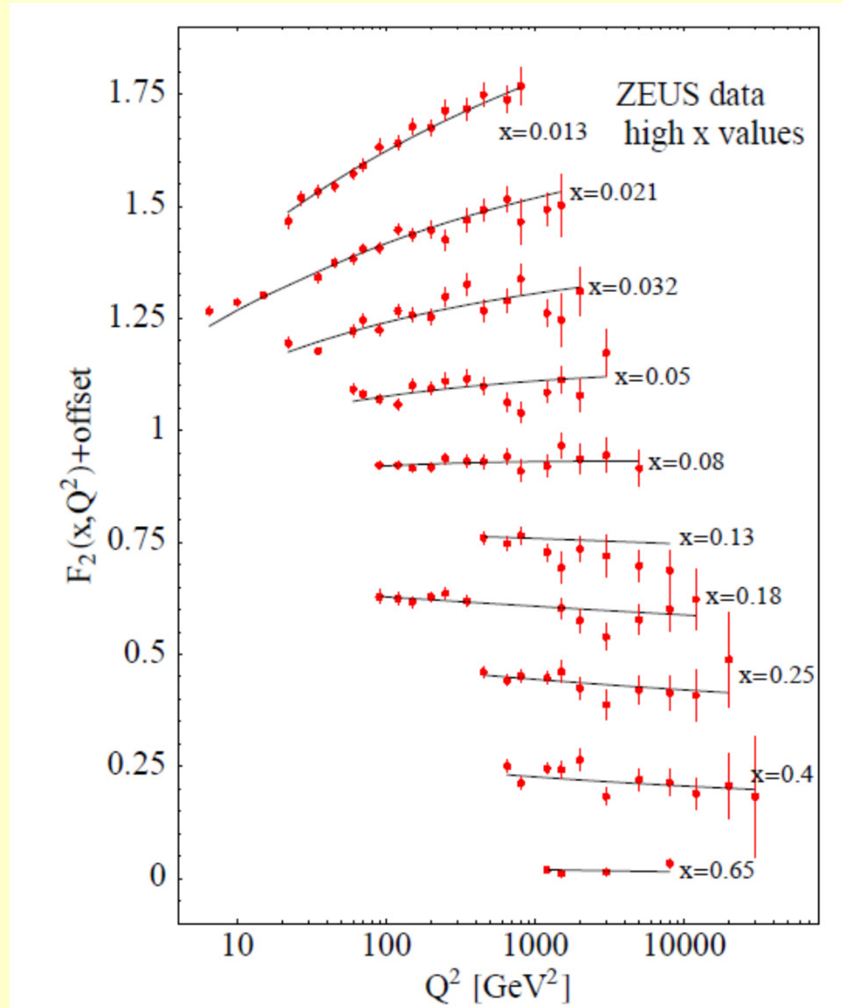
And there is another equation for the evolution of the *gluon* density:

$$\frac{dg(x,Q^2)}{d\ln(Q^2)} = \frac{\alpha_S(Q^2)}{2\pi}\int_x^1 \left[\sum_i q_i(y,Q^2)P_{qg}\left(z=\frac{x}{y}\right)+g(y,Q^2)P_{gg}\left(z=\frac{x}{y}\right)\right]\frac{dy}{y}$$

These are the *Altarelli - Parisi*, or *DGLAP*, equations for the parton densities

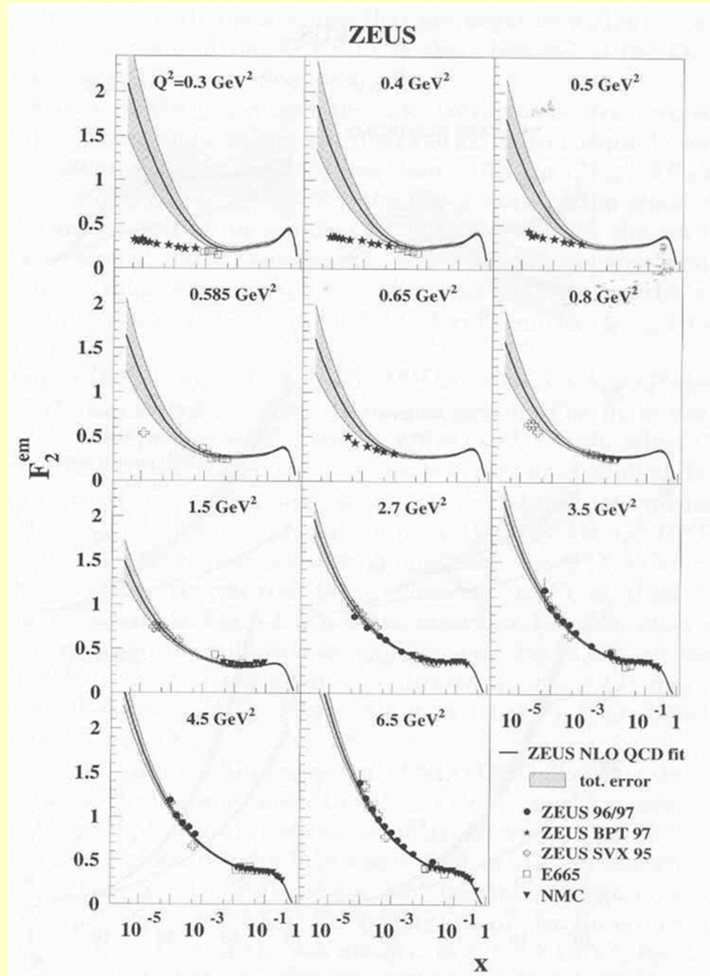
PQCD: DIS Scaling Violations -IX



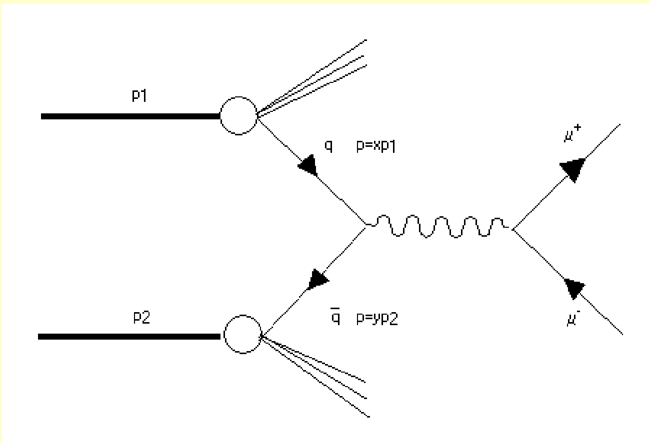


PQCD: DIS Scaling Violations -X

PDF Evolution with Q^2



PQCD: Drell-Yan - I



$$\frac{d\sigma\left(q\bar{q}\to l^+l^-\right)}{dq^2}=\frac{4\pi\alpha^2}{3q^2}e_q^2\delta\left(q^2-s_{q\bar{q}}\right)$$

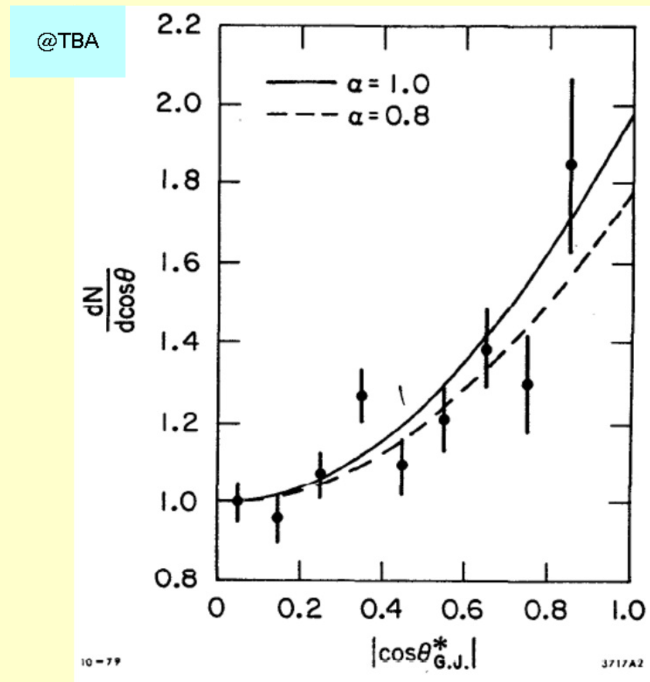
x_1, x_2 Bjorken x for $q, \bar{q}$

$$s_{q\bar{q}}=\left(p_q+p_{\bar{q}}\right)^2=\left(x_1p_1+x_2p_2\right)^2\simeq x_1x_2s$$

$$\sigma\left(q\bar{q}\to l^+l^-\right)=\frac{4\pi\alpha^2}{3q^2}e_q^2,\quad e_q=\text{ quark charge in } e \text{ units}$$

Angular distribution in the pair rest frame

Expect $\propto 1+\cos^2\theta^*$ as usual

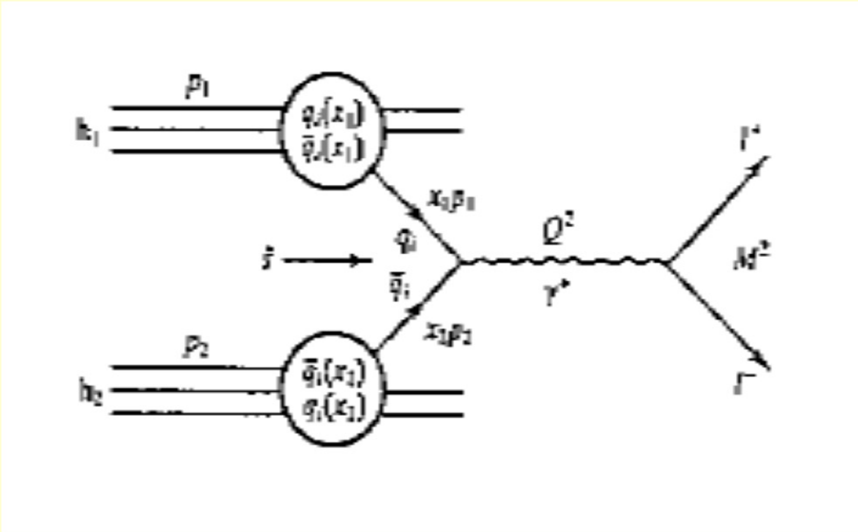


PQCD: Drell – Yan - II

Reverse $e^+e^- \to q\bar{q}$ process: $q\bar{q} \to e^+e^-$

Obtained by any reaction

hadron + hadron $\to l^+ \, l^- + X$



Ignore parton transverse momentum

Ignore non-annihilating partons ($\to$ "spectators")

Ignore parton fragmentation

PQCD: Drell – Yan - III

$$e^+e^- \to \mu^+\mu^- :$$

$$\frac{d\sigma}{d\Omega^*} = \frac{\alpha^2}{4s}\left(1+\cos^2\theta^*\right)$$

$$\sigma = \frac{4\pi\alpha^2}{3s}$$

$$q\overline{q} \to \mu^+\mu^- :$$

$$\frac{d\sigma_q}{d\Omega^*} = \frac{Q_q^2\alpha^2}{4M^2}\left(1+\cos^2\theta^*\right)\cdot\frac{1}{3}$$

$Q_q e$: Quark charge

$\frac{1}{3}$: Color factor

M^2: $\mu^+\mu^-$ invariant mass = Total energy in partonic CM

PQCD: Drell – Yan - IV

$$p_1 = x_1 P_1 \simeq x_1 \left(E, 0, 0, P\right)$$

$$p_2 = x_2 P_2 \simeq x_2 \left(E, 0, 0, -P\right)$$

$$q = x_1 P_1 + x_2 P_2 \simeq \left[\left(x_1 + x_2\right) E, 0, 0, \left(x_1 - x_2\right) P\right]$$

$$M^2 = q^2 \simeq \left(x_1 + x_2\right)^2 E^2 - \left(x_1 - x_2\right)^2 P^2 \simeq 4 x_1 x_2 E^2 = x_1 x_2 s \equiv \tau s$$

Shift to more useful kinematical variables:

Either

$$\begin{cases} x_F \equiv x_1 - x_2 = \pm \dfrac{2|\mathbf{q}|}{\sqrt{s}} & \text{Feynman } x \text{ of parton pair} \\ M^2 = s x_1 x_2 \end{cases}$$

Or:

$$\begin{cases} y = \dfrac{1}{2} \ln \dfrac{E + p_3}{E - p_3} \simeq \dfrac{1}{2} \ln \dfrac{x_1}{x_2} & \text{Rapidity of parton pair (neglecting rest mass)} \\ M = \sqrt{s x_1 x_2} \end{cases}$$

PQCD: Drell – Yan - V

Inclusive cross-section:

Contribution by parton pair with (x_1, x_2) fractional momenta

$$d^2\sigma\left(pp \to \mu^+\mu^- + X\right) = \frac{4\pi\alpha^2}{9M^2}\sum_q Q_q^2\left[f_q\left(x_1\right)f_{\bar{q}}\left(x_2\right) + f_q\left(x_2\right)f_{\bar{q}}\left(x_1\right)\right]dx_1dx_2$$

$$\begin{cases} y = \dfrac{1}{2}\ln\dfrac{E+p_3}{E-p_3} \simeq \dfrac{1}{2}\ln\dfrac{x_1}{x_2} \\ M = \sqrt{sx_1x_2} \end{cases} \to \begin{matrix} x_1 = x_2e^{2y} \\ M = \sqrt{s}x_2e^y \end{matrix} \to x_2 = \frac{M}{\sqrt{s}}e^{-y}, x_1 = \frac{M}{\sqrt{s}}e^{+y}$$

$$dx_1dx_2 = JdMdy$$

$$J = \frac{\partial\left(x_1, x_2\right)}{\partial\left(M, y\right)} = \begin{vmatrix} \dfrac{\partial x_1}{\partial M} & \dfrac{\partial x_1}{\partial y} \\ \dfrac{\partial x_2}{\partial M} & \dfrac{\partial x_2}{\partial y} \end{vmatrix} = \frac{\partial x_1}{\partial M}\frac{\partial x_2}{\partial y} - \frac{\partial x_1}{\partial y}\frac{\partial x_2}{\partial M}$$

$$\to J = \frac{1}{\sqrt{s}}e^{+y}\left(-\frac{M}{\sqrt{s}}e^{-y}\right) - \frac{M}{\sqrt{s}}e^{+y}\frac{1}{\sqrt{s}}e^{-y} = -\frac{2M}{s} = -2\sqrt{\frac{x_1x_2}{s}} \to dx_1dx_2 = \left(-2\sqrt{\frac{x_1x_2}{s}}\right)dMdy$$

PQCD: Drell – Yan - VI

$$\rightarrow d^2\sigma = \frac{4\pi\alpha^2}{9M^2}\left|-2\sqrt{\frac{x_1x_2}{s}}\right|\sum_q Q_q^2\left[f_q(x_1)f_{\bar{q}}(x_2)+f_q(x_2)f_{\bar{q}}(x_1)\right]dMdy$$

$$s\tau = M^2 \rightarrow M = \sqrt{s\tau}$$

$$\rightarrow \frac{d^2\sigma}{dMdy} = \frac{8\pi\alpha^2}{9M^2}\frac{\tau}{M}\sum_q Q_q^2\left[f_q(x_1)f_{\bar{q}}(x_2)+f_q(x_2)f_{\bar{q}}(x_1)\right]$$

$$\rightarrow \frac{d^2\sigma}{dMdy} = \frac{8\pi\alpha^2}{9Ms}\sum_q Q_q^2\left[f_q(x_1)f_{\bar{q}}(x_2)+f_q(x_2)f_{\bar{q}}(x_1)\right]$$

Central events:

$$y = 0, x_1 = x_2 = \sqrt{\tau}$$

$$\rightarrow \left.\frac{d^2\sigma}{dMdy}\right|_{y=0} = \frac{8\pi\alpha^2}{9Ms}\sum_q Q_q^2\left[f_q(\sqrt{\tau})f_{\bar{q}}(\sqrt{\tau})+f_q(\sqrt{\tau})f_{\bar{q}}(\sqrt{\tau})\right]$$

$$\rightarrow s^{3/2}\left.\frac{d^2\sigma}{dMdy}\right|_{y=0} = \frac{8\pi\alpha^2}{9}\frac{1}{\sqrt{\tau}}\sum_q Q_q^2\left[f_q(\sqrt{\tau})f_{\bar{q}}(\sqrt{\tau})+f_q(\sqrt{\tau})f_{\bar{q}}(\sqrt{\tau})\right]$$

PQCD: Drell – Yan - VII

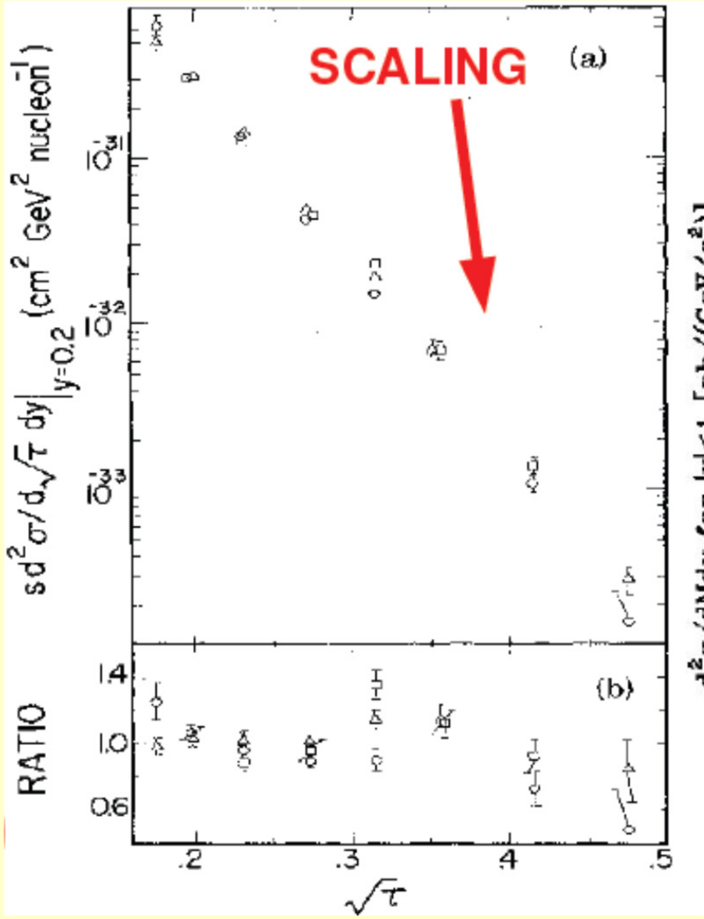
$$M = \sqrt{s\tau} \to dM = \sqrt{s}\, d\left(\sqrt{\tau}\right)$$

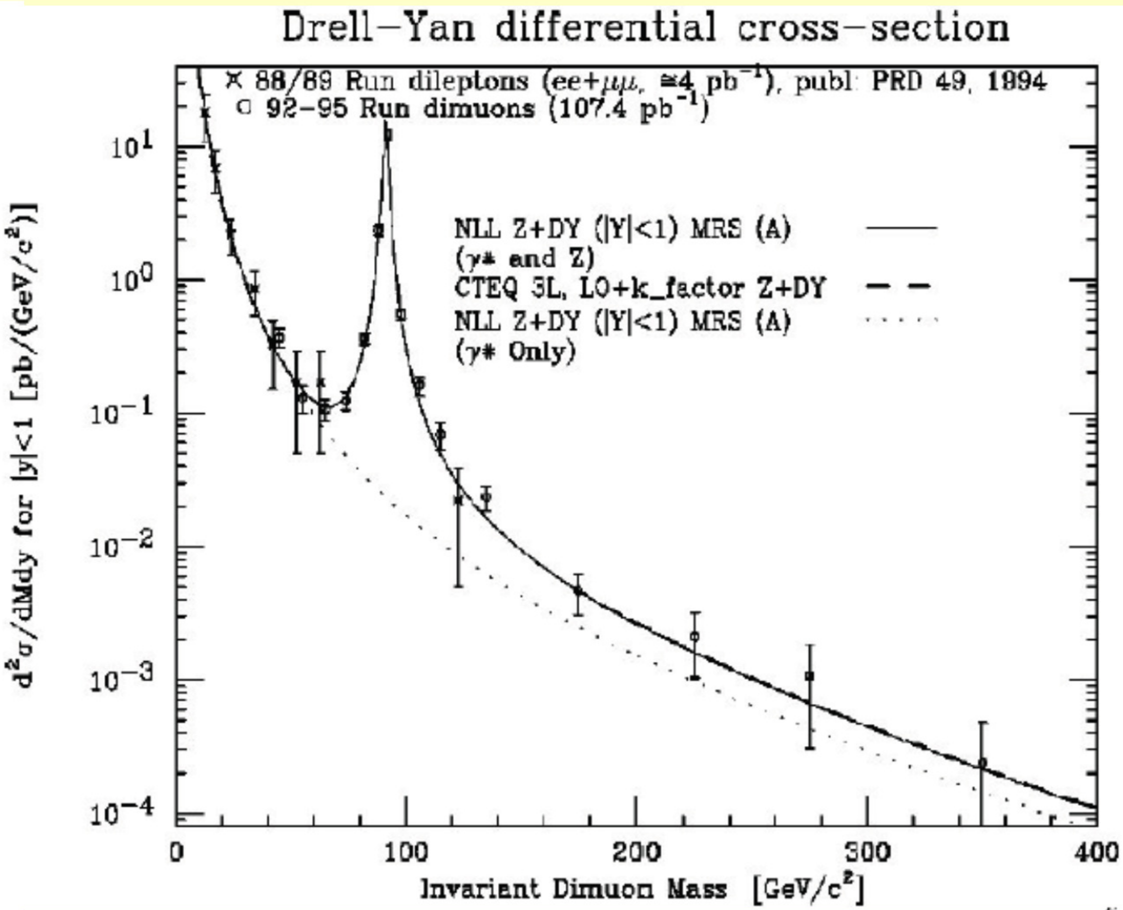
$$\to s^{3/2} \left.\frac{d^2\sigma}{dMdy}\right|_{y=0} = s^{3/2} \left.\frac{d^2\sigma}{\sqrt{s}\, d\left(\sqrt{\tau}\right) dy}\right|_{y=0} = s \left.\frac{d^2\sigma}{d\left(\sqrt{\tau}\right) dy}\right|_{y=0}$$

$$\to s \left.\frac{d^2\sigma}{d\left(\sqrt{\tau}\right) dy}\right|_{y=0} = \frac{8\pi\alpha^2}{9} \frac{1}{\sqrt{\tau}} \sum_q Q_q^2 \left[f_q\left(\sqrt{\tau}\right) f_{\overline{q}}\left(\sqrt{\tau}\right) + f_q\left(\sqrt{\tau}\right) f_{\overline{q}}\left(\sqrt{\tau}\right) \right]$$

Scaling behavior: Compare to DIS

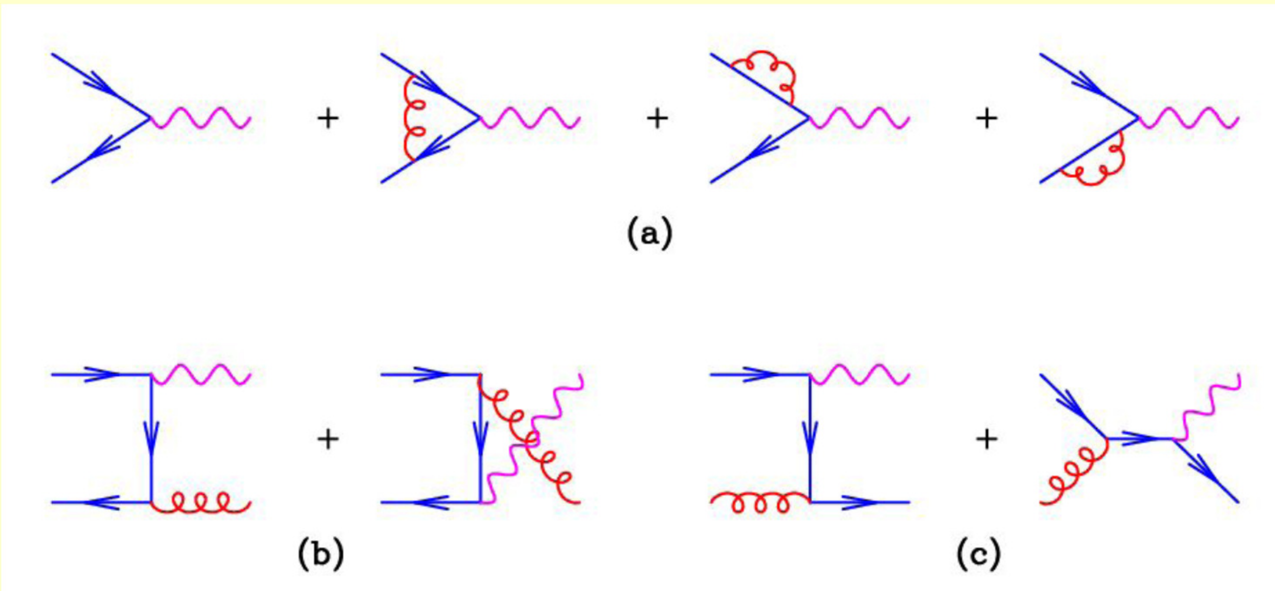
PQCD: Drell – Yan - VIII





PQCD: Drell – Yan - IX

NLO QCD corrections:



Quite similar to QCD corrections to:

$e^+e^- \to q\overline{q}$

PQCD: Drell – Yan - X

Total rate:

Same effect as for

$e^+e^- \to q\overline{q}$

Real gluons compensate virtual gluons

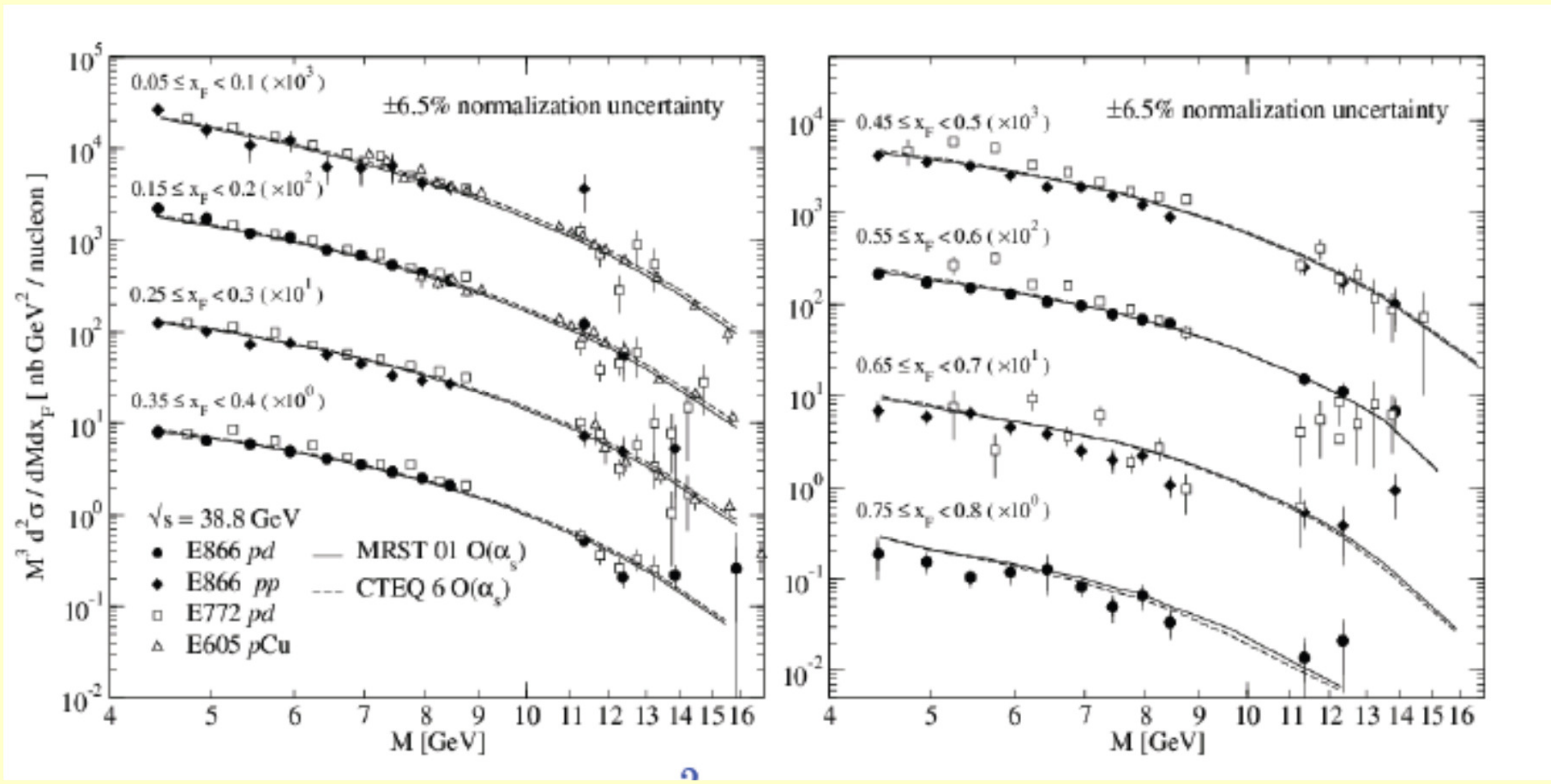
$$(\hat{\sigma}_{MG}(real) + \hat{\sigma}_{MG}(virtual))_{DY} = \frac{2\alpha_s}{3\pi}\hat{\sigma}_0\left[\frac{4\pi^2}{3} - \frac{7}{2}\right]$$

$\to$ Overall effect lumped into a K-factor

$$K^{DY}(\text{1st order}) = 1 + \frac{\alpha_s}{\pi}\left[\frac{8\pi^2}{9} - \frac{7}{3}\right] = 1 + 2.05\alpha_s \sim 2$$

PQCD: Drell – Yan - XI

DY Scaling violation



$$x_F = \frac{2}{\sqrt{s}}(p_{l+} + p_{l-}) \sim x_1 - x_2$$

PQCD: Hadron Collisions - I

Historically best observed and studied at hadron colliders

ISR = Intersecting Storage Ring (CERN '70s)

pp 31 GeV / beam

$Sp\bar{p}S$ = Super $p\bar{p}$ Synchrotron (CERN '80s)

$p\bar{p}$ 270 - 310 Gev / beam

Tevatron (Fermilab early '90s - 2011)

$p\bar{p}$ 1 TeV / beam

RHIC = Relativistic Heavy Ion Collider (BNL 3^{rd} Millennium)

ions 200 GeV / nucleon * beam

LHC = Large Hadron Collider (CERN 3^{0} Millennium)

pp 7 TeV / beam (presently 4 TeV)

ions 2.7 TeV / nucleon * beam

PQCD: Hadron Collisions - II

CM frame: usually identical to LAB

Important exception: ISR (collision angle *15^0*)
Not relevant for LHC (collision angle *0.01^0*)

But: Partonic collision CM $\neq$ Event CM
→ E_{tot}, p of parton collision unknown

→Initial state only partially known

→ Separate collision kinematics into:

Transverse
Longitudinal

PQCD: Hadron Collisions - III

Rapidity :

$$y = \frac{1}{2}\ln\frac{E+p_\parallel}{E-p_\parallel} \leftrightarrow p_\parallel = E\tanh y$$

Pseudo - rapidity :

$$\eta = -\ln\left(\tan\frac{\theta}{2}\right)$$

$$y = \frac{1}{2}\ln\frac{E+p_\parallel}{E-p_\parallel} \approx \frac{1}{2}\ln\frac{1+\cos\theta}{1-\cos\theta} = -\frac{1}{2}\ln\frac{1-\cos\theta}{1+\cos\theta} = -\frac{1}{2}\ln\frac{\sin^2\theta/2}{\cos^2\theta/2}$$

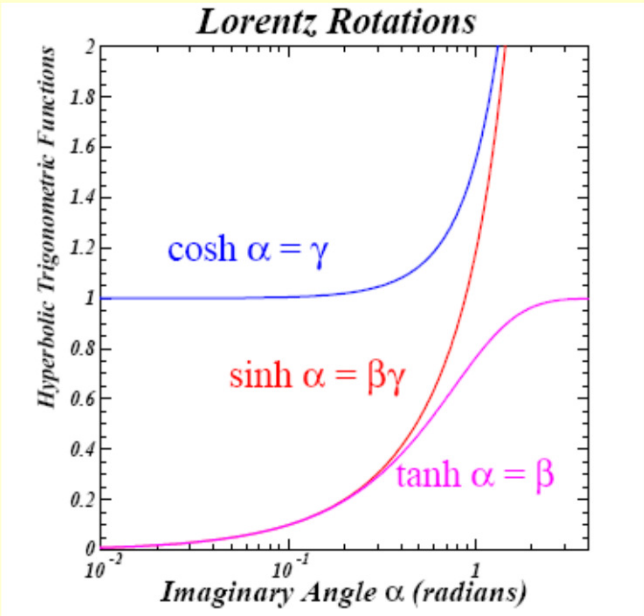
$$\rightarrow y \approx -\frac{1}{2}\ln\left(\tan^2\theta/2\right) = -\ln\left(\tan\theta/2\right)$$

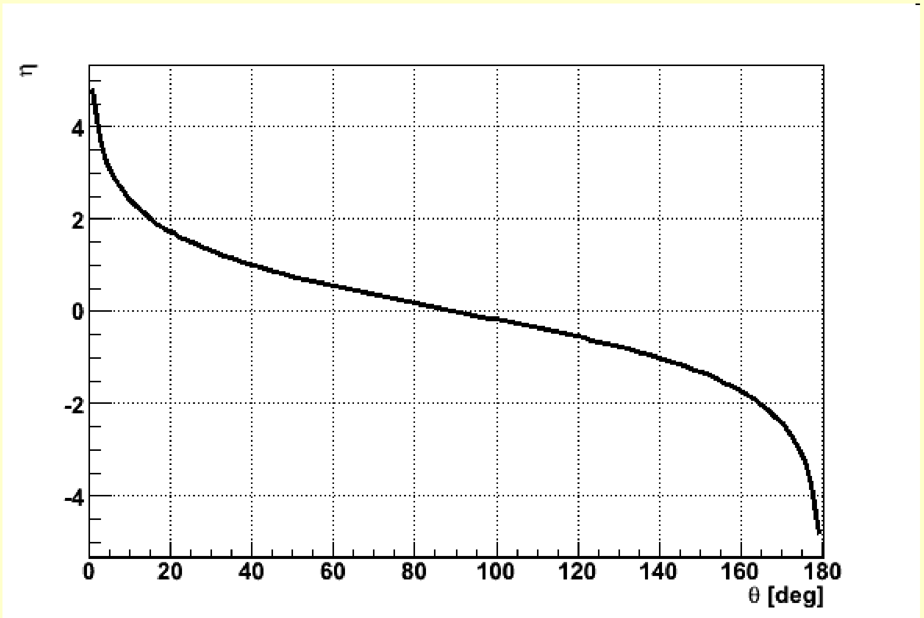
Examples: $P_T = 0 \rightarrow y_{\max}$

	$p\bar{p}$	pp
TeV	2	14
$y_{\max}$	7.7	9.6

PQCD: Hadron Collisions - IV

Pseudorapidity & Polar Angle





$$y = \frac{1}{2}\ln\frac{E+p_{\parallel}}{E-p_{\parallel}} \to \frac{1}{2}\ln\frac{\gamma\left(E+\beta p_{\parallel}\right)+\gamma\left(p_{\parallel}+\beta E\right)}{\gamma\left(E+\beta p_{\parallel}\right)-\gamma\left(p_{\parallel}+\beta E\right)} = \frac{1}{2}\ln\frac{\left(E+p_{\parallel}\right)(1+\beta)}{\left(E-p_{\parallel}\right)(1-\beta)} = \frac{1}{2}\ln\frac{\left(E+p_{\parallel}\right)}{\left(E-p_{\parallel}\right)} + \frac{1}{2}\ln\frac{(1+\beta)}{(1-\beta)}$$

Indeed:

$$y \to y + y_b$$

PQCD: Hadron Collisions - V

Elementary volume (impulse space):

$$d^3\mathbf{P} = P^2 dP d\Omega = dP_{\parallel} P_T dP_T d\varphi$$

$$\frac{d^3\mathbf{P}}{E} = \frac{dP_{\parallel} P_T dP_T d\varphi}{E}$$

$$dy = \frac{dP_{\parallel}}{E} \rightarrow \frac{d^3\mathbf{P}}{E} = dy P_T dP_T d\varphi$$

$$\int \left(dy P_T dP_T \right) d\varphi = 2\pi dy P_T dP_T = 2\pi dy \frac{1}{2} d\left(P_T^2\right) = \pi dy d\left(P_T^2\right)$$

Differential cross-section, invariant:

$$\rightarrow \frac{d\sigma}{\frac{d^3\mathbf{P}}{E}} = E \frac{d\sigma}{d^3\mathbf{P}} = \frac{1}{\pi} \frac{d\sigma}{dy d\left(P_T^2\right)} = \frac{1}{2\pi P_T} \frac{d\sigma}{dy dP_T}$$

PQCD: Hadron Collisions - VI

$$\begin{pmatrix} p_{\parallel}' \\ E' \end{pmatrix} = \begin{pmatrix} \cosh y & -\sinh y \\ -\sinh y & \cosh y \end{pmatrix} \begin{pmatrix} p_{\parallel} \\ E \end{pmatrix} \underset{\text{High Energy}}{\longrightarrow} \begin{pmatrix} p_{\parallel}' \\ E' \end{pmatrix} \simeq \begin{pmatrix} \cosh \eta & -\sinh \eta \\ -\sinh \eta & \cosh \eta \end{pmatrix} \begin{pmatrix} p_{\parallel} \\ E \end{pmatrix}$$

(Event) CM frame:

$$\begin{cases} P_A = \left(E_A, 0, 0, p_A\right) \\ P_B = \left(E_A, 0, 0, -p_A\right) \end{cases}$$ 4-momenta incident particles

$$\begin{cases} p_1 = x_1 P_A \\ p_2 = x_2 P_B \end{cases}$$ 4-momenta incident partons

$$\beta_{CM} = \frac{x_1 - x_2}{x_1 + x_2}$$ Parton CM speed as seen by CM = LAB

$$y_{CM} = \frac{1}{2} \ln \frac{1 + \beta_{CM}}{1 - \beta_{CM}} = \frac{1}{2} \ln \frac{x_1}{x_2}$$ Parton CM rapidity as seen by CM = LAB

PQCD: Hadron Collisions - VII

$$\begin{pmatrix} E' \\ p_{\parallel}' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta_{CM} \\ -\gamma\beta_{CM} & \gamma \end{pmatrix} \begin{pmatrix} E \\ p_{\parallel} \end{pmatrix} = \begin{pmatrix} \cosh y_{CM} & -\sinh y_{CM} \\ -\sinh y_{CM} & \cosh y_{CM} \end{pmatrix} \begin{pmatrix} E \\ p_{\parallel} \end{pmatrix}$$

$$p_T^2 = p_x^2 + p_y^2$$

$$p_T = p \sin\theta$$

$$E_T^2 = p_x^2 + p_y^2 + m^2 = p_T^2 + m^2 = E^2 - p_{\parallel}^2$$

$$p_{\parallel} = E \tanh y$$

$$E_T^2 = E^2 - p_{\parallel}^2 = E^2 - E^2 \tanh^2 y$$

$$\rightarrow E = E_T \cosh y$$

$$\rightarrow p_{\parallel} = E_T \sinh y$$

$$y \approx -\ln\left(\tan\theta/2\right) \rightarrow E_T = E\left(1 - \tanh^2 y\right)^{1/2} \approx E\left(1 - \frac{\dfrac{\cos\theta/2}{\sin\theta/2} - \dfrac{\sin\theta/2}{\cos\theta/2}}{\dfrac{\cos\theta/2}{\sin\theta/2} + \dfrac{\sin\theta/2}{\cos\theta/2}}\right)^{1/2}$$

$$E_T = E\left[1 - \left(\cos^2\theta/2 - \sin^2\theta/2\right)\right]^{1/2} \rightarrow E_T \approx E\sin\theta$$

PQCD: Hadron Collisions - VIII

Express 4-momentum in terms of longitudinal, transverse quantities

$$p = \left(E,\ \underbrace{P_x, P_y}_{P_T^2 = P_x^2 + P_y^2},\ \underbrace{P_z}_{P_\parallel = P_z} \right)$$

$$P_T = \sqrt{P^2 - P_\parallel^2}$$

$$\rightarrow \begin{cases} P = P_T \cosh\eta \\ P_\parallel \simeq P_T \sinh\eta \end{cases}$$

$$E \approx P, E_T \approx P_T$$

$$\rightarrow p \approx \left(E_T \cosh\eta, \underbrace{E_T \sin\phi, E_T \cos\phi,}_{E_T} E_T \sinh\eta \right)$$

Useful in clustering algorithms

PQCD: Hadron Collisions - IX

Consider all the 2-body processes in QCD:

$$qq \to qq, q\overline{q} \to q\overline{q}$$ Quarks only

$$qg \to qg, \overline{q}g \to \overline{q}g, gg \to gg, q\overline{q} \to gg, gg \to q\overline{q}$$ Quarks and/or Gluons

All will yield 2 jets to LO

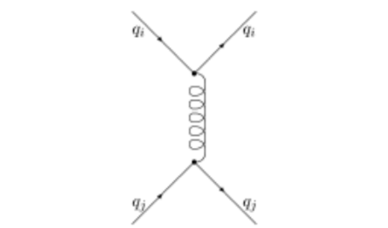


Figure 1: Feynman diagram for $q_i q_j \to q_i q_j,\ i \neq j$

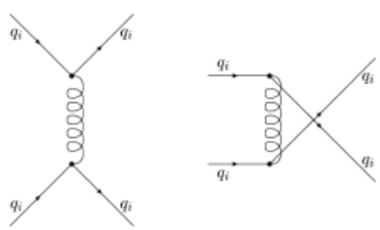


Figure 2: Feynman diagrams for $q_i q_i \to q_i q_i$

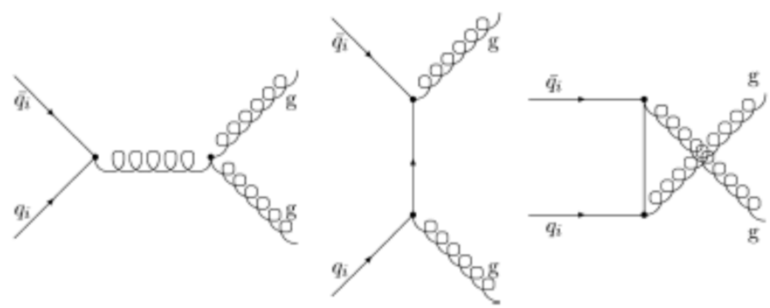


Figure 5: Feynman diagrams for $q_i \bar{q}_i \to gg$

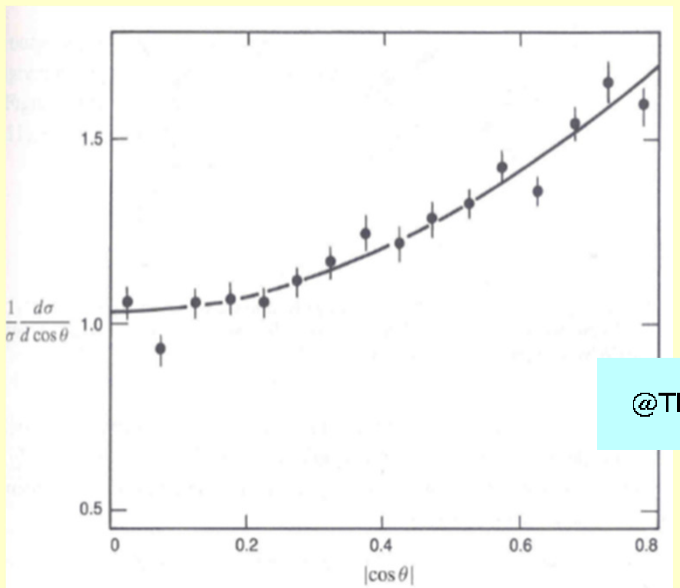
PQCD: Hadron Collisions - X

When quark only processes can be identified, expect

$$\frac{d\sigma}{d\left(\cos\theta^*\right)} = \frac{\pi\alpha_s^2}{2s_{ij}}\left|M\right|^2$$

$$s_{ij} = \left(x_i p_i + x_j p_j\right)^2 \approx x_i x_j s$$

$$\rightarrow \left|M\right|^2 \propto 1 + \cos^2\theta^* \quad \text{as usual}$$



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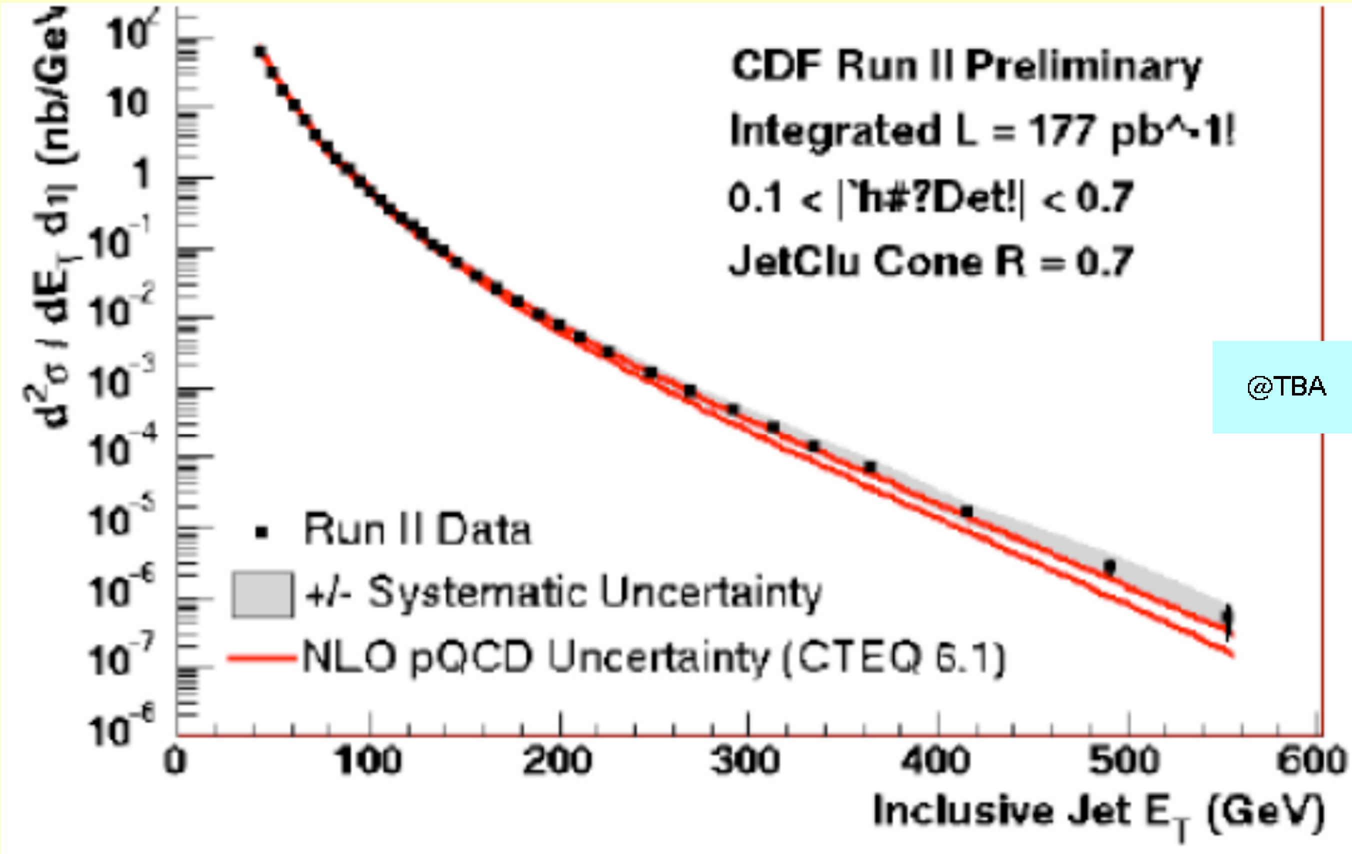
PQCD: Hadron Collisions - XI



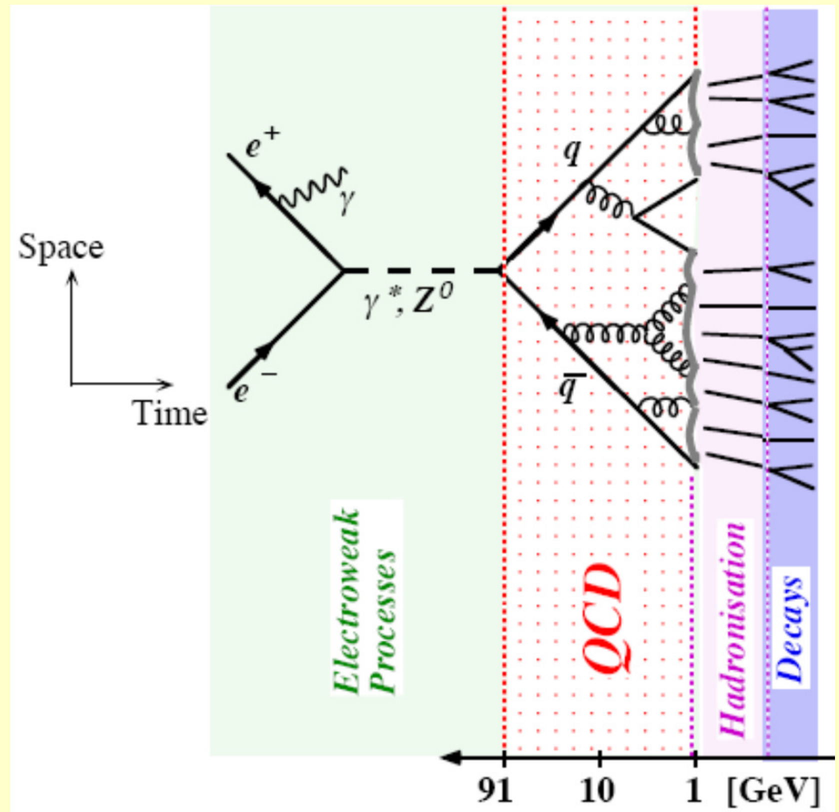
PQCD: Hadron Collisions - XII

Complete jet evolution cannot be computed in a full QCD framework:

As shown in the picture, *increasing* time scales correspond to *decreasing* Q^2 scale, down to a region where perturbative expansion cannot be granted

Conversion of quarks and gluons into hadrons (*Fragmentation* or *Hadronization*) must be evaluated by ad-hoc prescriptions (models).

QCD-inspired, successful models often based on string-like behavior of $q\bar{q}$ pairs



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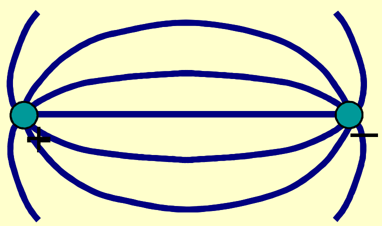
PQCD: Hadron Collisions - XIII

Typical model implemented in fragmentation Montecarlo programs

$q\bar{q}$ Interaction

QED-like at small distance



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Gluon self-interaction yields *string* (flux tube) pattern at large distance: F = *const*

@TBA

Picture baryons as 'mesons':

$$\boldsymbol{3} \otimes \boldsymbol{3} = \boldsymbol{3}^* \oplus \boldsymbol{6}$$

$$qqq = \underbrace{qq}_{\sim\bar{q}} + q$$

Confinement - I

$$\alpha_s\left(\left|q^2\right|\right) \simeq \frac{12\pi}{21\ln\left(\left|q^2\right|/\Lambda^2\right)}, \quad \left|q^2\right| \gg \Lambda^2$$

When $\left|q^2\right| \sim \Lambda^2$, the previous expression does not apply

$\alpha_s\left(\Lambda^2\right)$ *is large*
Strong interaction is strong
Cannot rely on perturbative expansion

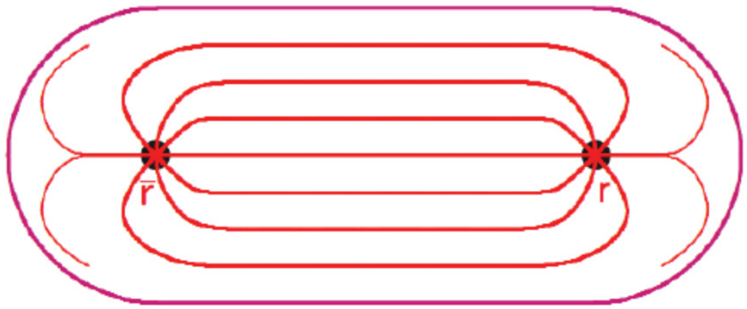
In a general sense, we expect Λ to mark the low energy range, corresponding to *soft* (low q^2) processes

Bound states: Non-perturbative, 'white', energy scale ≈ Λ
Does $a_s(\Lambda^2)$ correspond to the *color confinement* range?
Very likely. But remember:

It is not yet convincingly shown that QCD is a confining theory

Confinement - II

In QCD, for large charge separation, field lines seem to be compressed to tubelike region(s) $\Rightarrow$ **string(s)**



by self-interactions among soft gluons in the "vacuum".

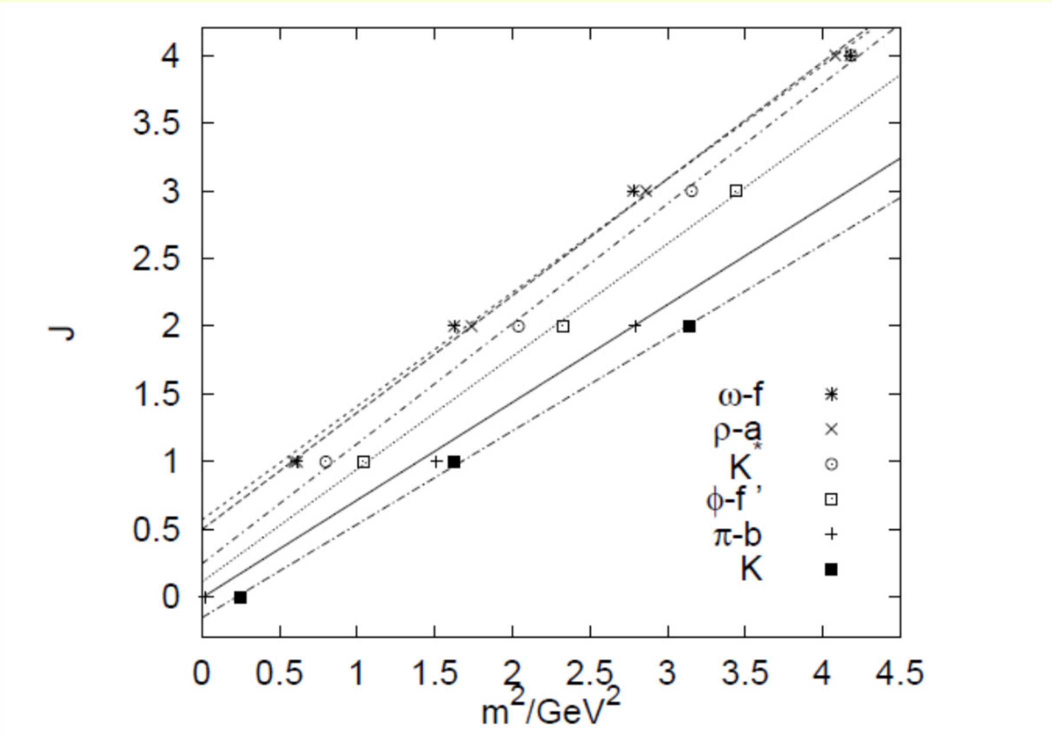
Gives linear confinement with string tension:

$F(r) \approx \text{const} = \kappa \approx 1\ \text{GeV/fm} \quad \Longleftrightarrow \quad V(r) \approx \kappa r$

Separation of transverse and longitudinal degrees of freedom $\Rightarrow$ simple description as 1+1-dimensional object – **string** – with Lorentz invariant formalism

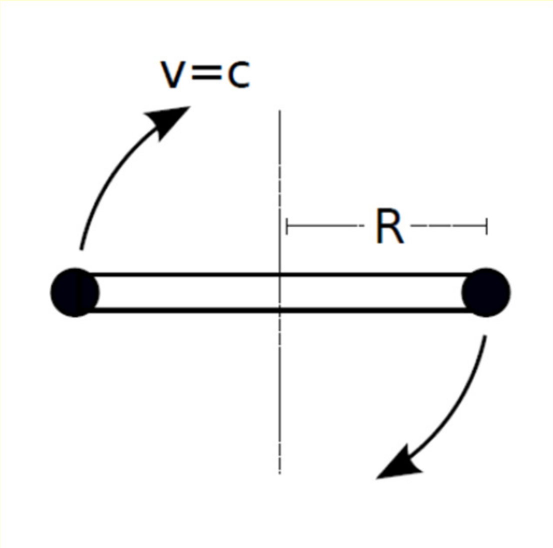
Confinement - III

Regge trajectories:



Confinement - IV

String model of mesons: Simple explaination of Regge trajectories



$$F^{\mu} = \frac{dP^{\mu}}{d\tau}, F^{\mu} = \left(\gamma \mathbf{F} \cdot \mathbf{u}, \gamma \mathbf{F}\right) \text{ relativistic 2nd law}$$

$$\rightarrow m = E = W = 2\int_0^R \gamma \mathbf{F} \cdot d\mathbf{r}$$

$$\mathbf{F} = k\hat{\mathbf{r}} \quad \leftrightarrow \quad \text{linear potential}$$

$$\rightarrow m = E = 2\int_0^R \gamma k\hat{\mathbf{r}} \cdot d\mathbf{r} = 2\int_0^R \frac{k}{\sqrt{1-\beta^2}} dr$$

$$\beta = \frac{r}{R}$$

$$\rightarrow m = E = 2k\int_0^R \frac{dr}{\sqrt{1-\left(\frac{r}{R}\right)^2}} = \pi k R$$

$$J = 2k\int_0^R \frac{\frac{r}{R}}{\sqrt{1-\left(\frac{r}{R}\right)^2}} dr = \frac{1}{2}\pi k R^2 = \frac{m^2}{2\pi k}$$

Confinement - V

Small distance:Perturbative!

Large quark mass $\rightarrow$ Large Q^2

$\rightarrow$ One gluon exchange OK

$$\rightarrow V\left(r \ll \frac{1}{m_q}\right) \approx -\frac{4}{3}\frac{\alpha_s}{r}$$

$\rightarrow$ Full potential:

$$V\left(r\right) \approx -\frac{4}{3}\frac{\alpha_s}{r} + kr$$

Confinement - VI

QCD : Leading to predict new, 'exotic' (= non $q\bar{q}$) mesonic states

Quarkless mesons: no valence quarks

$\rightarrow$ *Gluonium*, aka *Glueball*

Expect, among others, exotic quantum numbers

Flavor:

1 Singlet

Color:

Bound state $\rightarrow$ Must be color singlet ($\leftarrow$ 'white')

$\rightarrow 2\ g$ at least

$$\mathbf{8}\otimes\mathbf{8}=\mathbf{1}\oplus\mathbf{8}\oplus\mathbf{8}\oplus\mathbf{10}\oplus\overline{\mathbf{10}}\oplus\mathbf{27}$$

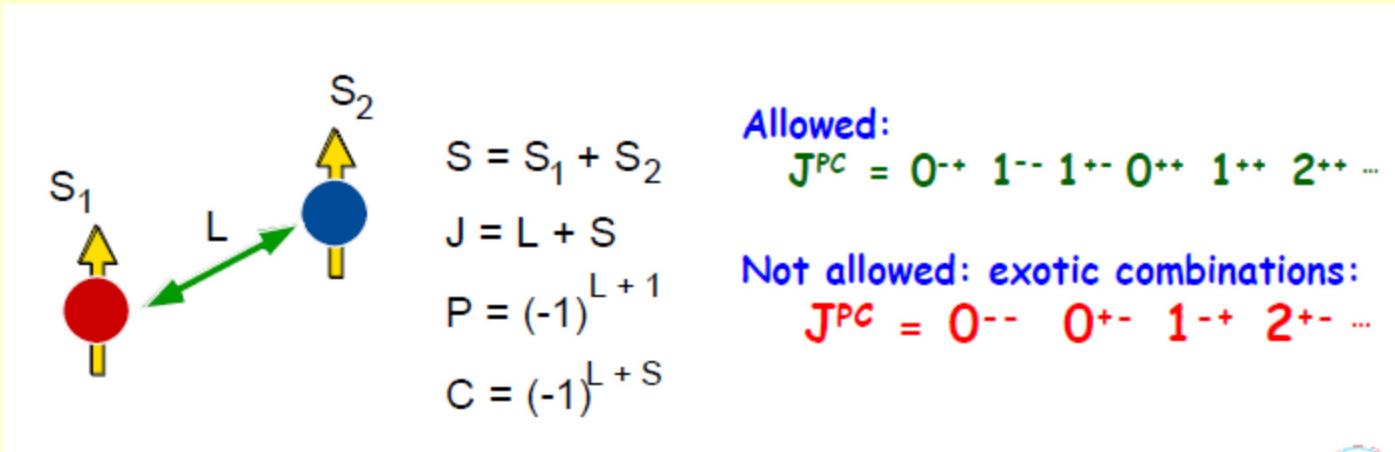
Pick singlet:

1 $\leftrightarrow$ Symmetric

$\rightarrow$ Spin+Orbital: Symmetric (Bose statistics)

Confinement - VII

Compare to $q\bar{q}$, standard mesons:



2 gluons:

J=L+S

By taking simplest case, $S-$wave:

$$\left.\begin{array}{l} L=0 \rightarrow J=1\oplus 1=0,1,2 \\ P=(-1)^L=+1 \\ C=(-1)^2=+1 \end{array}\right\} \rightarrow 0^{++}, 2^{++}$$

Confinement - VIII

$J=1$ excluded by symmetry argument:

$2g$ state must have defined parity

$2g$ state must have defined symmetry

Build $2g$ state by single gluon states with defined helicity:

$U_P\left|\mathbf{k},R;-\mathbf{k},R\right\rangle=\left|-\mathbf{k},L;\mathbf{k},L\right\rangle$ not a U_P eigenstate

$U_P\left|\mathbf{k},L;-\mathbf{k},L\right\rangle=\left|-\mathbf{k},R;\mathbf{k},R\right\rangle$ not a U_P eigenstate

$U_P\left|\mathbf{k},R;-\mathbf{k},L\right\rangle=\left|-\mathbf{k},L;\mathbf{k},R\right\rangle\equiv\left|\mathbf{k},R;-\mathbf{k},L\right\rangle$

$\rightarrow U_P$ eigenstate, $\eta_P=+1,\ J_3=+2$

$U_P\left|\mathbf{k},L;-\mathbf{k},R\right\rangle=\left|-\mathbf{k},R;\mathbf{k},L\right\rangle\equiv\left|\mathbf{k},L;-\mathbf{k},R\right\rangle$

$\rightarrow U_P$ eigenstate, $\eta_P=+1\ \ J_3=-2$

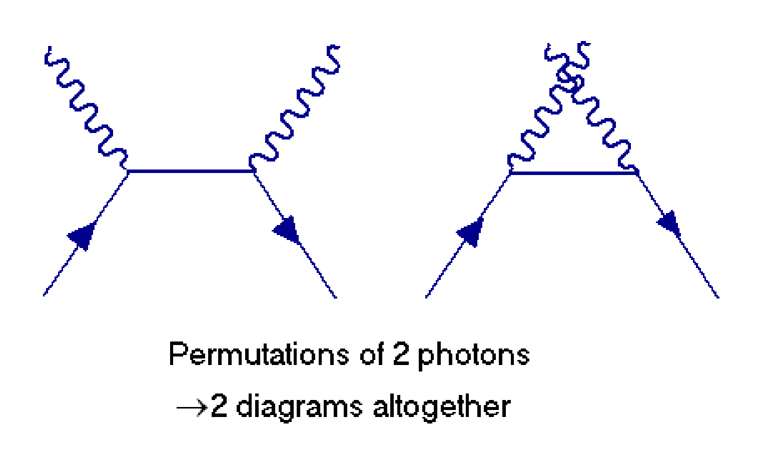
$\left|\mathbf{k},R;-\mathbf{k},R\right\rangle\pm\left|\mathbf{k},L;-\mathbf{k},L\right\rangle$

$\rightarrow U_P$ eigenstates, $\eta_P=\pm1, J_3=0$

$\rightarrow$ Pick $\left|\mathbf{k},R;-\mathbf{k},R\right\rangle+\left|\mathbf{k},L;-\mathbf{k},L\right\rangle$ (symmetric) $\rightarrow\eta_P=+1$

e^+- e^-: 2 Photons Annihilation - I

Transition amplitude in the small speed limit ($\beta \to 0$):
2 diagrams, similar to (rotated) Compton scattering



$$T_{fi} = T_1 + T_2$$

$$T_1 = \frac{e^2}{(p_1 - p_3)^2 - m^2} \overline{v}(2) \not{\varepsilon}_4 \left(\not{p}_1 - \not{p}_3 + m \right) \not{\varepsilon}_3 u(1), \quad T_2 = \frac{e^2}{(p_1 - p_4)^2 - m^2} \overline{v}(2) \not{\varepsilon}_3 \left(\not{p}_1 - \not{p}_4 + m \right) \not{\varepsilon}_4 u(1)$$

$p_1 = m(1,0,0,0), p_2 = m(1,0,0,0), p_3 = m(1,0,0,1), p_4 = m(1,0,0,-1)$ γ rays emitted along z

$$(p_1 - p_3)^2 - m^2 = (p_1 - p_4)^2 - m^2 = -2m^2$$

$\to T = -4e^2$ Averaged over initial, summed over final spin projections

e^+- e^-: 2 Photons Annihilation - II

Cross-section from amplitude: 2-body reaction

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi s}\frac{\left|\mathbf{p}_f\right|}{\left|\mathbf{p}_i\right|}\left|T\right|^2$$

$$\left|\mathbf{p}_f\right| = m, \quad \left|\mathbf{p}_i\right| \simeq m\beta, \quad s = \left(2m\right)^2 = 4m^2$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi 4m^2}\frac{m}{m\beta}16\alpha^2 = \frac{\alpha^2}{16\pi m^2\beta}$$

$$\to \sigma = 4\pi\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4m^2\beta}$$

e^+- e^-: 2 Photons Annihilation - III

Selection rule for bound state annihilation into 2,3 photons

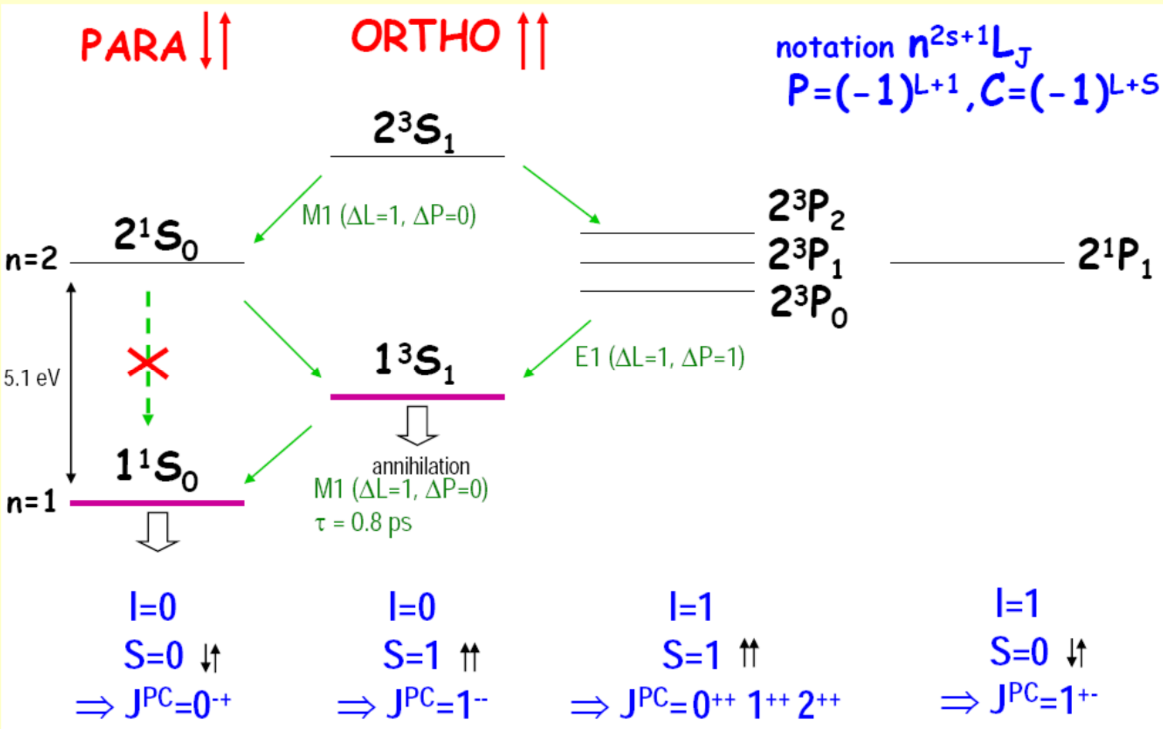
$$U_C\left|2\gamma\right\rangle=\left(-1\right)^2\left|2\gamma\right\rangle\rightarrow\eta_c\left(2\gamma\right)=+1$$
$$\rightarrow\left(-1\right)^{L+S}=+1$$
$$\rightarrow L=0\Rightarrow S=0$$

S-wave: Singlet only

$$U_C\left|3\gamma\right\rangle=\left(-1\right)^3=-1$$
$$\rightarrow\left(-1\right)^{L+S}=-1$$
$$\rightarrow L=0\Rightarrow S=1$$

S-wave: Triplet only

Positronium - I



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Positronium - II

2 γ Annihilation : Proceed as for Van-Royen - Weisskopf

$$A_{pos} = \sum_p \underbrace{\langle \gamma\gamma | T | p \rangle}_{A(\mathbf{p})} \underbrace{\langle p | \Pi \rangle}_{\psi(\mathbf{p})}$$

$A(\mathbf{p})$ plane wave amplitude, $\psi(\mathbf{p})$ momentum space wave function

$$\to A_{pos} = \int d^3\mathbf{p} A(\mathbf{p}) \psi(\mathbf{p}) = \frac{1}{(2\pi)^{3/2}} \int d^3\mathbf{p} A(\mathbf{p}) \int \psi(\mathbf{r}) e^{i\mathbf{p}\cdot\mathbf{r}} d^3\mathbf{r}$$

$$\to A_{pos} = \frac{1}{(2\pi)^{3/2}} \int d^3\mathbf{r} \psi(\mathbf{r}) \int A(\mathbf{p}) e^{i\mathbf{p}\cdot\mathbf{r}} d^3\mathbf{r}$$

Take $A(\mathbf{p}) \approx A = const$ (can be shown to be true)

$$\to A_{pos} \approx \frac{A}{(2\pi)^{3/2}} \int d^3\mathbf{r} \psi(\mathbf{r}) \underbrace{\int e^{i\mathbf{p}\cdot\mathbf{r}} d^3\mathbf{p}}_{(2\pi)^3 \delta^3(\mathbf{r})} = (2\pi)^{3/2} A\psi(0)$$

$$\to \Gamma_{pos} = \left| A_{pos} \right|^2 \approx (2\pi)^3 \left| A \right|^2 \left| \psi(0) \right|^2$$

Positronium - III

$$\sigma = \frac{\alpha^2}{4m^2\beta} = |A|^2 \frac{(2\pi)^3}{\beta} \to |A|^2 = \frac{\alpha^2}{4m^2\beta}\frac{\beta}{(2\pi)^3} = \frac{\alpha^2}{(2\pi)^3 4m^2}$$

$$\to \Gamma_{pos} \approx (2\pi)^3 |A|^2 |\psi(0)|^2 = \frac{\alpha^2}{4m^2}|\psi(0)|^2$$

Computed by using averaged matrix element: 3+1 = 4 spin states
Since only singlet initial state contributes:

$$\Gamma_{pos} \to \frac{1}{1/4}\cdot\frac{\alpha^2}{4m^2}|\psi(0)|^2 = \frac{\alpha^2}{m^2}|\psi(0)|^2$$

Ground state wave function required:
Use scaled Hydrogen..

Positronium - IV

Taking scaled Hydrogen wave function

$$\psi(r) = \frac{2}{(a_0)^{3/2}} e^{-\frac{2r}{a_0}}, a_0 = \frac{1}{\alpha m}, m \text{ reduced mass}$$

$$Hyd: \quad m \simeq m_e \to a_0 \approx \frac{1}{\alpha m_e}$$

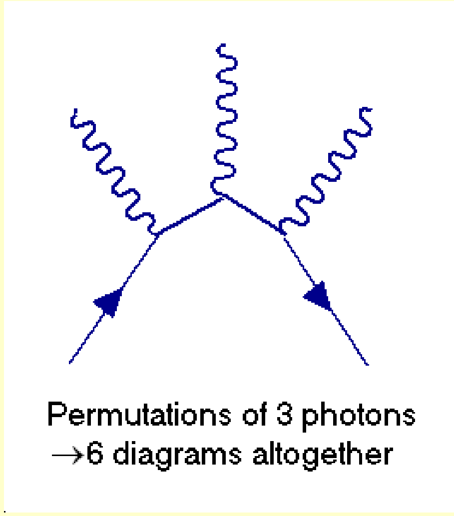
$$Pos: \ m = \frac{m_e}{2} \to a_0 = \frac{2}{\alpha m_e}$$

$$\to \psi_{pos}(0) = \frac{2}{(a_0)^{3/2}} = \frac{2(\alpha m_e)^{3/2}}{(2)^{3/2}} \to \left|\psi_{pos}(0)\right|^2 = \frac{4\alpha^3 m_e^3}{8} = \frac{\alpha^3 m_e^3}{2}$$

$$\to \Gamma_{pos} \approx (2\pi)^3 |A|^2 \left|\psi(0)\right|^2 = \frac{\alpha^2}{m_e^2} \frac{\alpha^3 m_e^3}{2} = \frac{\alpha^5 m_e}{2}$$

Positronium - V

3 γ Annihilation



After *some* algebra…

$$\Gamma_{pos}^{3\gamma} = \frac{4}{9\pi}\left(\pi^2 - 9\right)\frac{\alpha^3}{m_e^2}\left|\psi(0)\right|^2 = \frac{2}{9\pi}\left(\pi^2 - 9\right)m_e\alpha^6$$

$$\rightarrow \frac{\Gamma_{pos}^{3\gamma}}{\Gamma_{pos}^{2\gamma}} = \frac{\frac{2}{9\pi}\left(\pi^2 - 9\right)m_e\alpha^6}{\frac{\alpha^5 m_e}{2}} = \frac{4\left(\pi^2 - 9\right)}{9\pi}\alpha \sim 10^{-3}$$

Quarkonium - I

Family portrait of –*onia*:

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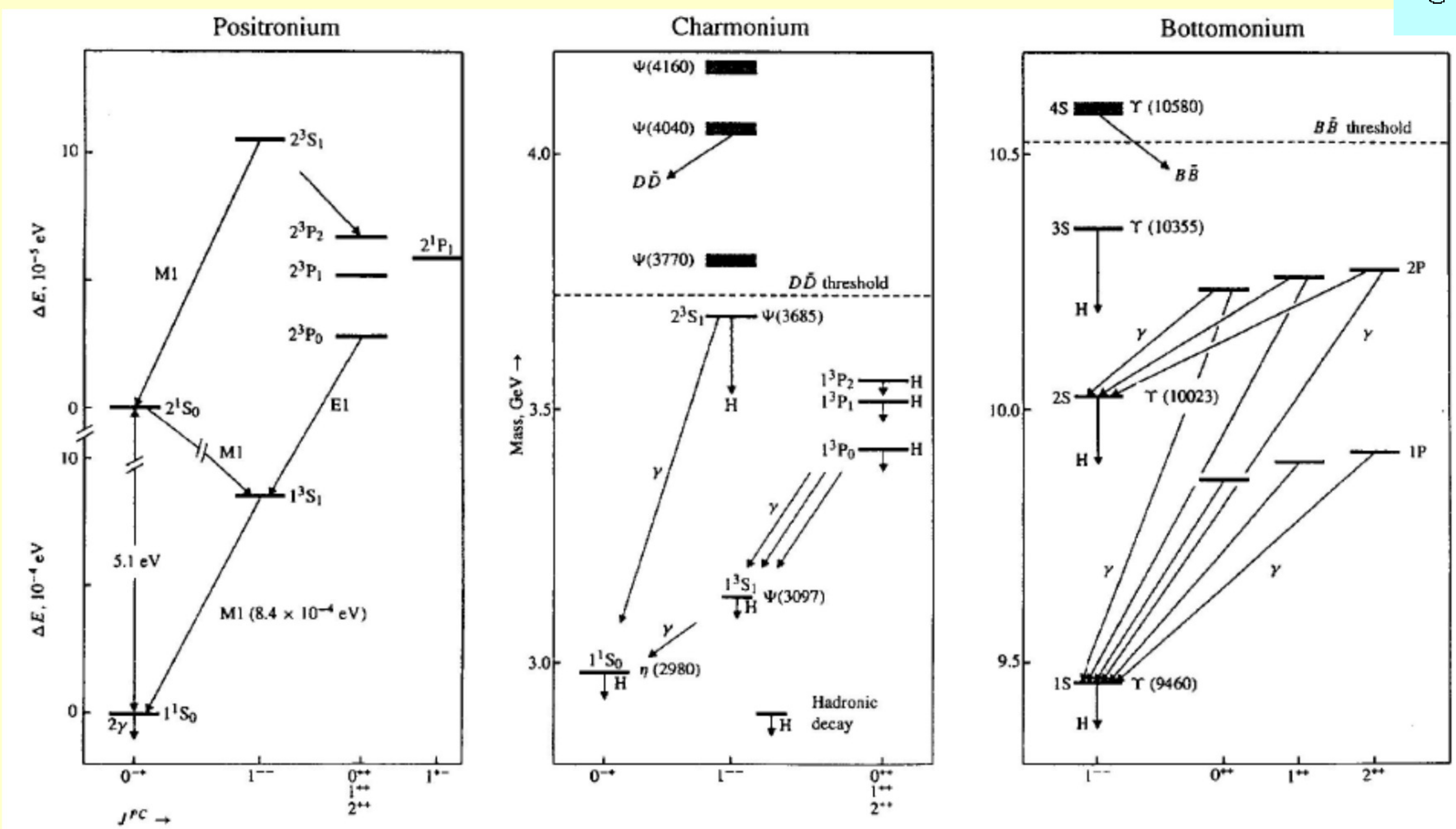
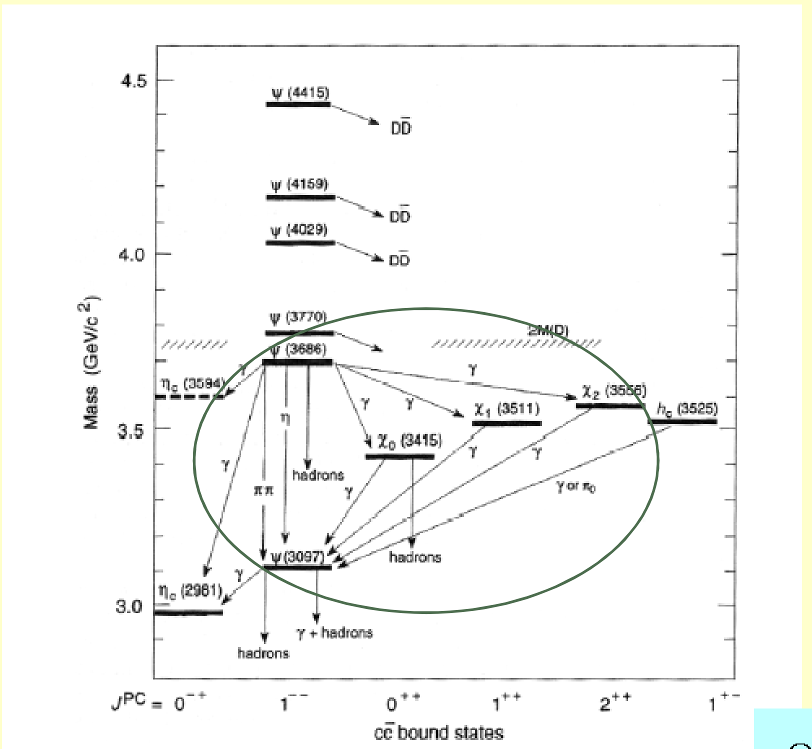
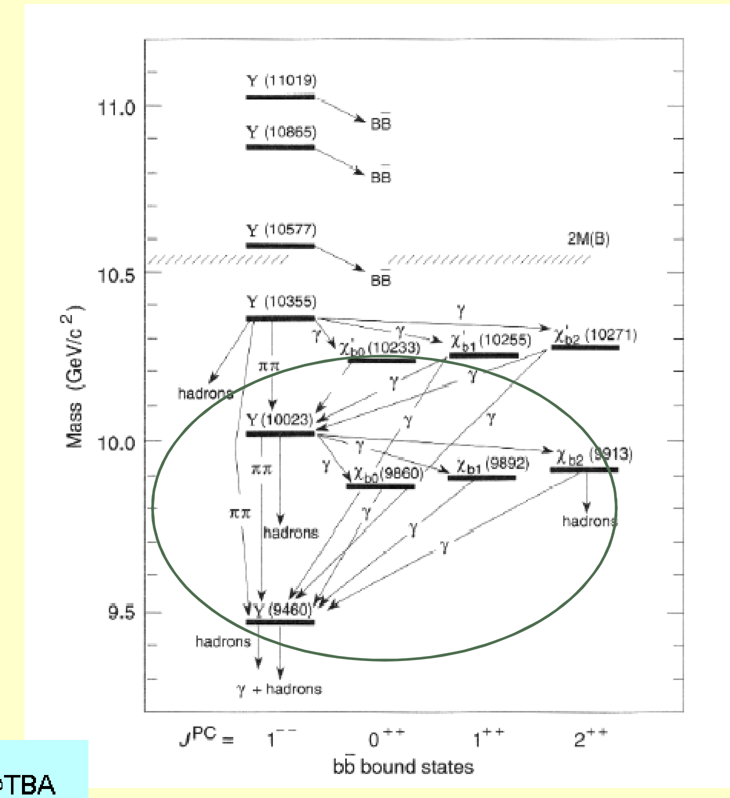


Quarkonium - II





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Striking similarity, ≈ same energy scale *above ground state*

Quarkonium - III

Reminder: Bohr radius

$$R = \frac{n^2}{\alpha m} \to R_0 = \frac{1}{\alpha m}$$

Consequence of Coulomb potential, static limit of 1 photon diagram

If 1 gluon exchange approximation can be granted, expect for quarkonium

$$R_{q\bar{q}} \sim \frac{1}{\alpha_s m}$$

Observe: m large $\to$ R small $\to$ α_s small

Must keep in mind the $q\bar{q}$ potential is confining
Add a phenomenological, confining term like this:

$$V(r) = -\frac{4}{3}\frac{\alpha_s}{r} + ar$$

Solve Schrodinger equation with these terms

Add more terms to take into account relativistic & color-hyperfine effects

Quarkonium - IV

$$V(r) = \lambda r^{\nu}$$

$$\mu = \frac{m}{2}$$

$$\frac{\hbar^2}{2\mu}\frac{d^2u}{dr^2} + \left[E - \lambda r^{\nu} - \frac{l(l+1)}{2\mu r^2}\right]u(r) = 0$$ Radial Schrodinger equation

$$r = \rho\left(\frac{\hbar}{2\mu|a|}\right)^{\frac{1}{2+\nu}}$$

$$E = \varepsilon\left(\frac{\hbar}{2\mu|\lambda|}\right)^{-\frac{2}{2+\nu}}\frac{\hbar^2}{2\mu}$$

$$\rightarrow \frac{d^2w}{d\rho^2} + \left[\varepsilon - \rho^{\nu} - \frac{l(l+1)}{\rho^2}\right]w(\rho) = 0$$ Adimensional radial equation

Quarkonium - V

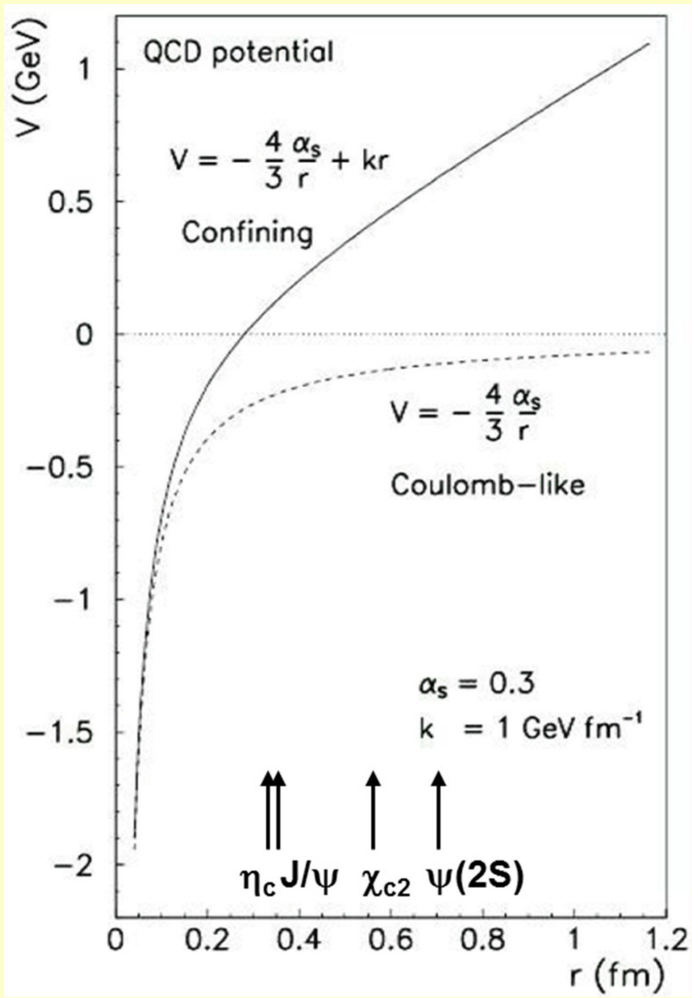
Length scale $[L] \propto (\mu|\lambda|)^{-1/(2+\nu)}$

Coulomb:	$(\mu\vert\lambda\vert)^{-1}$
Log: $V(r) = C \log r$	$(C\mu)^{-1/2}$
Linear:	$(\mu\vert\lambda\vert)^{-1/3}$
SHO:	$(\mu\vert\lambda\vert)^{-1/4}$
Square well:	$(\mu\vert\lambda\vert)^{0}$

Energies $[\Delta E] \propto (\mu)^{-\nu/(2+\nu)}(|\lambda|)^{2/(2+\nu)}$

Coulomb:	$\mu\vert\lambda\vert^2$
Log: $V(r) = C \log r$	$C\mu^0$
Linear:	$\mu^{-1/3}\vert\lambda\vert^{2/3}$
SHO:	$\mu^{-1/2}\vert\lambda\vert^{1/2}$
Square well:	μ^{-1}

Quarkonium - VI



Quarkonium - VII

$$\Gamma_V = \left|A_V\right|^2 \approx \left(2\pi\right)^3 \left|A\right|^2 \left|\psi\left(0\right)\right|^2$$

$$\left|A\left(p\right)\right|^2 = \frac{\pi\alpha^2 Q^2}{m_q^3}\frac{p_e}{v}\frac{v}{\left(2\pi\right)^3} = \frac{\pi\alpha^2 Q^2}{m_q^3}\frac{p_e}{\left(2\pi\right)^3}$$

Neglect quark momentum, electron mass: $p_e \approx m_q$

$$\rightarrow \left|A\left(p\right)\right|^2 \approx \frac{\pi\alpha^2 Q^2}{\left(2\pi\right)^3 m_q^2}$$

$$\rightarrow \Gamma_V \approx \left(2\pi\right)^3 \frac{\pi\alpha^2 Q^2}{\left(2\pi\right)^3 m_q^2}\left|\psi\left(0\right)\right|^2$$ Van Royen - Weisskopf formula (from the roaring '60s..)

For Bottomonium and Charmonium:

$$\rightarrow \frac{\Gamma_{\Upsilon_n}}{\Gamma_{\psi_n}} \approx \frac{Q_b^2}{Q_c^2}\frac{m_c^2}{m_b^2}\frac{\left|\psi_{\Upsilon_n}\left(0\right)\right|^2}{\left|\psi_{\psi_n}\left(0\right)\right|^2}$$

Quarkonium - VIII

Measuring the b-quark's charge

For $\nu \leq 1$, scaling laws imply

$$|\Psi_b(0)|^2 \geq \frac{m_b}{m_c}|\Psi_c(0)|^2$$

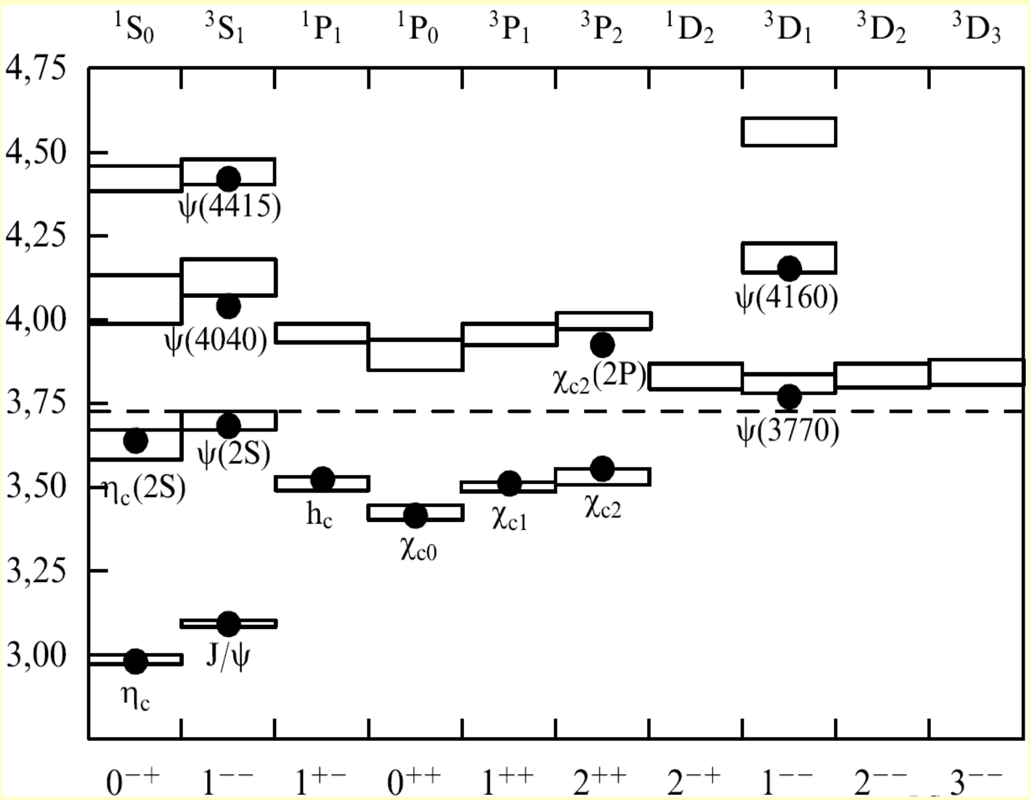
$$\rightsquigarrow \Gamma(\Upsilon_n \to \ell^+\ell^-) \geq \frac{e_b^2}{e_c^2} \cdot \frac{m_b}{m_c} \cdot \frac{M(\psi_n)^2}{M(\Upsilon_n)^2}\Gamma(\psi_n \to \ell^+\ell^-)$$

DORIS results presented at 1978 ICHEP (Tokyo)

$$\begin{array}{ll} \Gamma(\Upsilon \to \ell^+\ell^-) & 1.26 \pm 0.21 \text{ keV} \\ \Gamma(\Upsilon' \to \ell^+\ell^-) & 0.36 \pm 0.09 \text{ keV} \end{array}$$

established $e_b = -\frac{1}{3}$

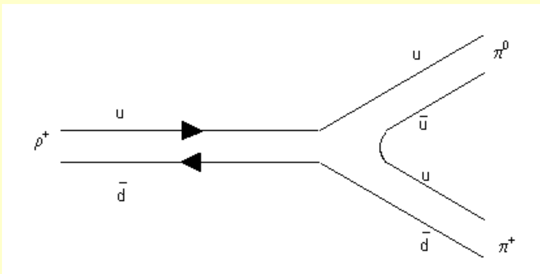
Quarkonium - IX



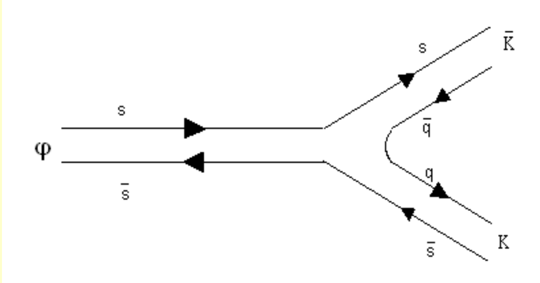
@TBA

OZI Rule - I

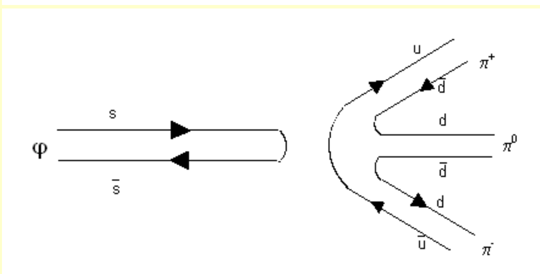
Okubo-Zweig-Iizuka Rule: *Disconnected diagrams are suppressed*



This diagram is connected

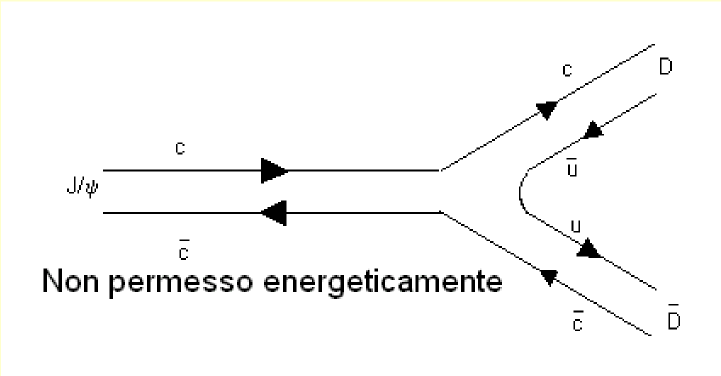


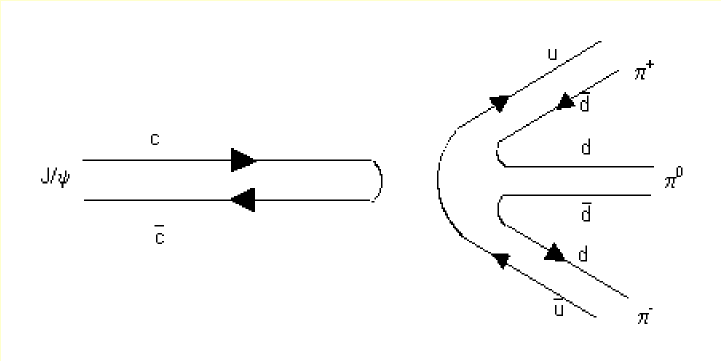
This diagram is connected: *BR 83 %*
(with smallish phase space)



This diagram is disconnected: *BR 15 %*
(with much larger phase space)

OZI Rule- II





Compare mass and width

$m_{J/\psi} = 3097\ \text{MeV},\ \Gamma_{J/\psi} = 80\ \text{keV}$ $\quad J^{PC}=1^{--}$

$m_{\psi'} = 3686\ \text{MeV},\ \Gamma_{\psi'} = 250\ \text{keV}$ $\quad J^{PC}=1^{--}$

Explaining the small width:

$m_{D^0} = 1865\ MeV \rightarrow 2 \times m_{D^0} = 3730\ MeV > m_{J/\psi}, m_{\psi'}$

Therefore $J/\psi,\ \psi'$ decay to open charm is energetically forbidden

$\rightarrow$ Decay diagrams are disconnected

$\rightarrow$ OZI rule: Decay is suppressed

$\rightarrow$ *States are very narrow*

OZI Rule - III

As a general rule

$\rightarrow A \propto \alpha_s^n \quad n =$ number of gluons

Connected diagrams: Small number of soft gluons $\rightarrow$ *A = large*
Disconnected diagrams: Large number of hard gluons $\rightarrow$ *A = small*

Indeed:

Single gluon annihilation is forbidden for mesons by color conservation (meson = ***1***, gluon = ***8***)

Annihilation of massive quarks yields hard gluons $\rightarrow \alpha_s$ is small

Connected diagrams involve softer gluons $\rightarrow \alpha_s$ is large

OZI Rule - IV

Consider quarkonium annihilation into gluons:

$q\bar{q} \to g$ Excluded: $(q\bar{q})_1 \not\to (1g)_8$

$q\bar{q} \to gg$ Allowed

$q\bar{q} \to ggg$ Allowed

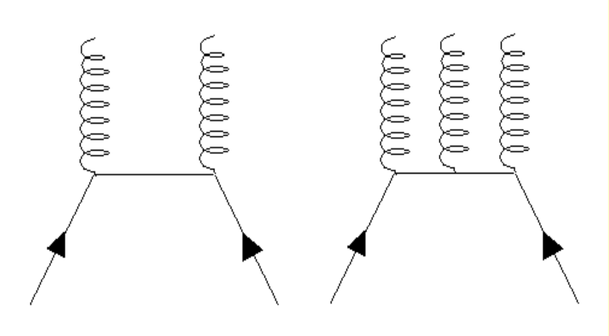
Decompose the direct product of 2 octets:

$$\boldsymbol{8 \otimes 8 = 1 \oplus 8 \oplus 8 \oplus 10 \oplus 10^* \oplus 27}$$

Charge Parity:

$J^{PC} = 0^{-+} \to C = +1 \to 2g$ OK

$J^{PC} = 1^{--} \to C = -1 \to 3g$ OK



Perturbative regime: *A(2g)>A(3g)*

→Pseudoscalars wider than vectors

OZI Rule - V

By comparison with positronium:

$$\left(e^+e^-\right)_{positronium} \to \gamma\gamma$$

$$\Gamma\left[\left(e^+e^-\right) \to \gamma\gamma\right] = \frac{\alpha^2}{m^2}\left|\psi(0)\right|^2$$

$$\left(c\overline{c}\right)_{charmonium} \to \gamma\gamma$$

$$\begin{cases} e \to \frac{2}{3}e \to \alpha \to \frac{4}{9}\alpha \text{ Quark charge} \\ \times 9 \quad \text{Sum amplitude over colors} \end{cases}$$

$$\Gamma\left[\left(c\overline{c}\right) \to \gamma\gamma\right] = \frac{48\alpha^2}{27m_c^2}\left|\psi_{c\overline{c}}(0)\right|^2$$

$$\left(c\overline{c}\right)_{charmonium} \to gg$$

From SU(3) algebra: 2 g in a color singlet state

Color factor $= \frac{9}{8}$

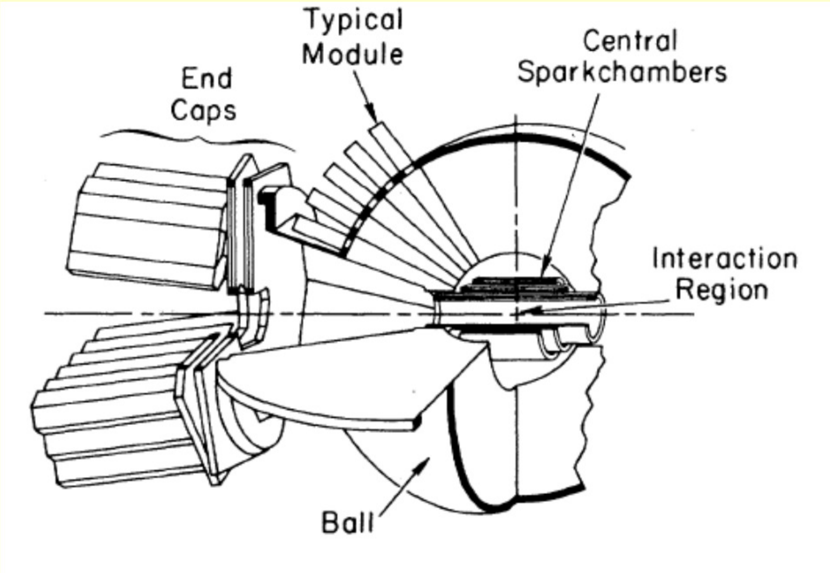
$$\Gamma\left[\left(c\overline{c}\right) \to gg\right] = \frac{2\alpha_s^2}{3m_c^2}\left|\psi_{c\overline{c}}(0)\right|^2$$

But:

Positronium rate was obtained by taking the low speed limit of scattering amplitude to 1-photon approx

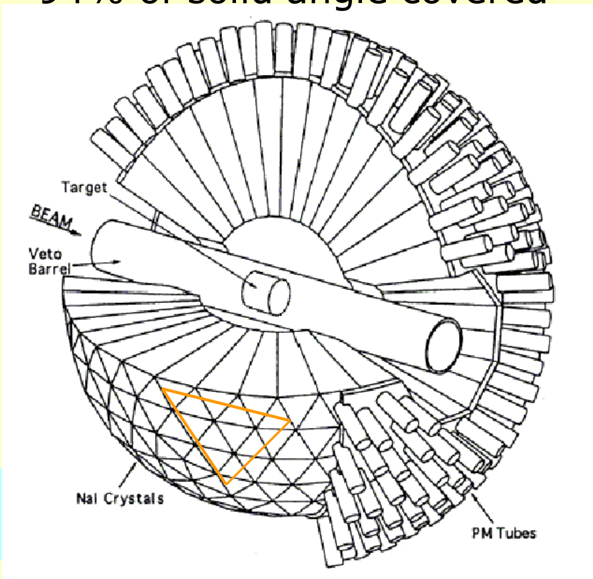
Is it granted for $c\overline{c}$?

Crystal Ball - I



@TBA

94% of solid angle covered



Sodium Iodide

$NaI\left(Tl\right)$: Inorganic scintillating crystal; Tl is an activator

Merits:

Can grow large crystals

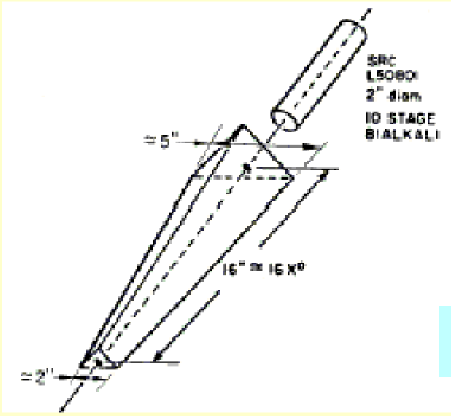
Lots of light

Crystal Ball - II

672 optically isolated NaI(Tl) crystals, 15.7 radiation lengths thick
Inner radius 25.3 cm; Outer radius 66.0 cm

CB geometry: Based on *icosahedron.*
Each of the 20 triangular faces (major triangles) is divided into four minor triangles, each consisting of nine separated crystals.

Each crystal: Truncated triangular pyramid, 40.6 cm high, pointing towards the center of the Ball.
Side on the inner end: 5.1 cm ; Side on the outer end: 12.7 cm

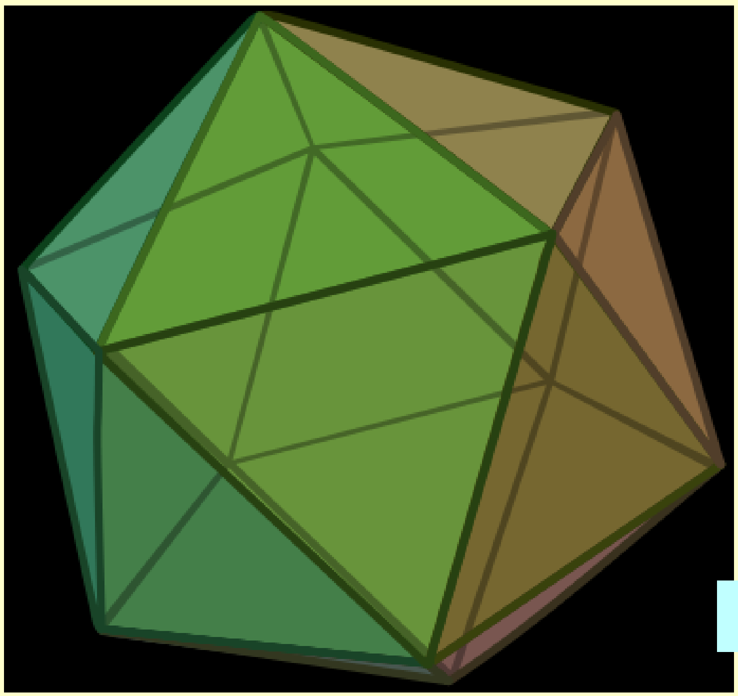


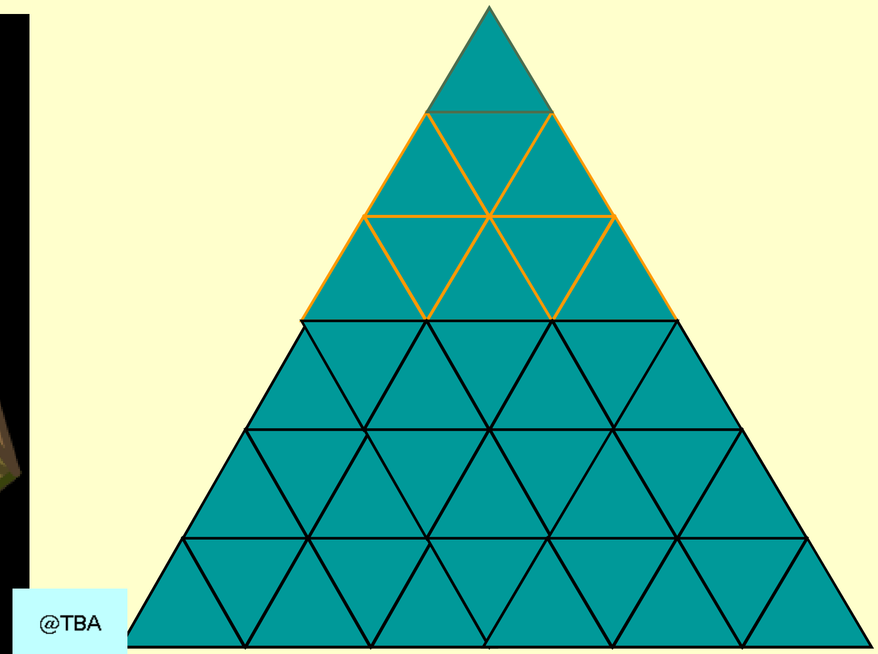
Crystal & Photomutiplier

@TBA

Crystal Ball - III

Icosahedron magic: Platonic solid, 20 equilateral triangle faces



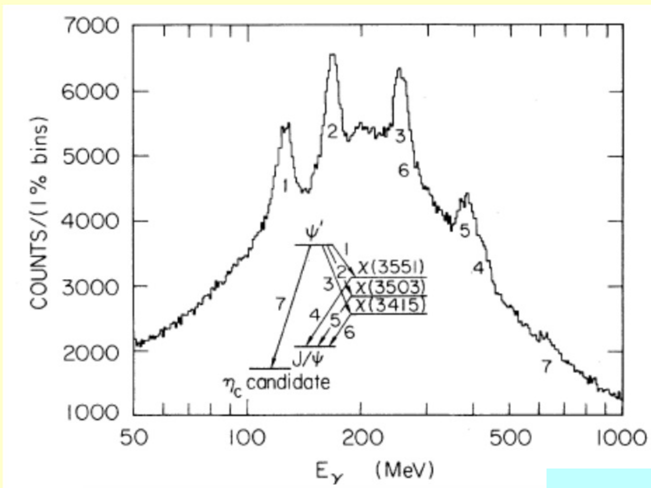


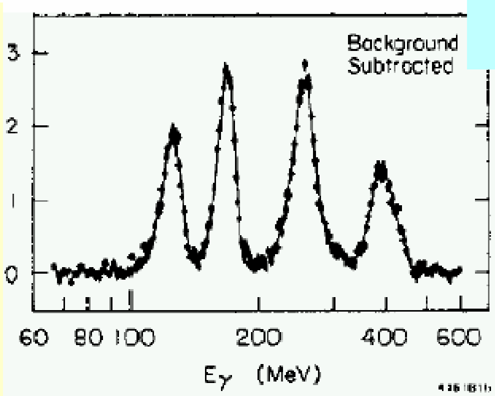
Triangle count:

Large triangle 20 Small triangle 80 Crystal < 720 (edges)

Crystal Ball - IV

Inclusive photon spectrum



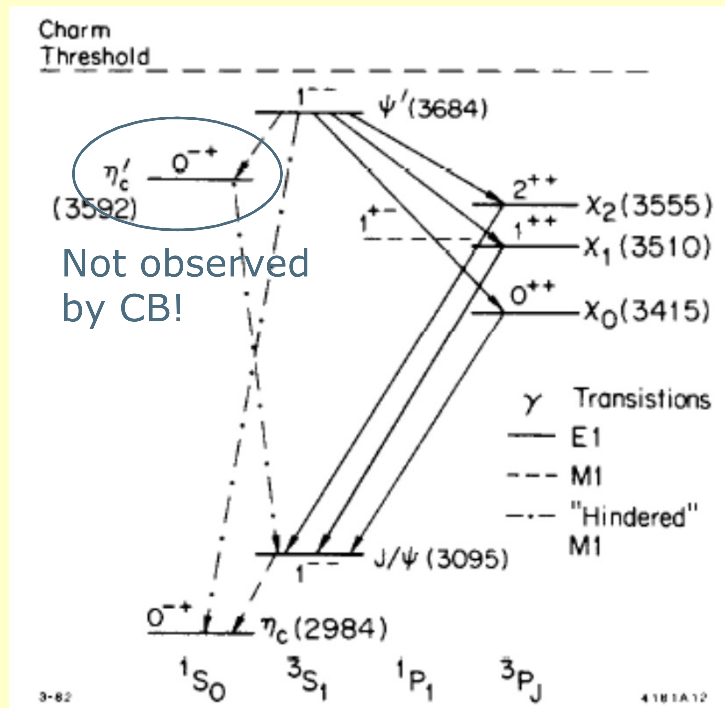


@TBA

Most important results, among many:

Tune beam energy as to form ψ'(3686)

Observe decays into photon + X



Crystal Ball - V

After moving the CB detector to DORIS (DESY, Hamburg): Bottomonium!
Observation of the P-wave triplets

@TBA

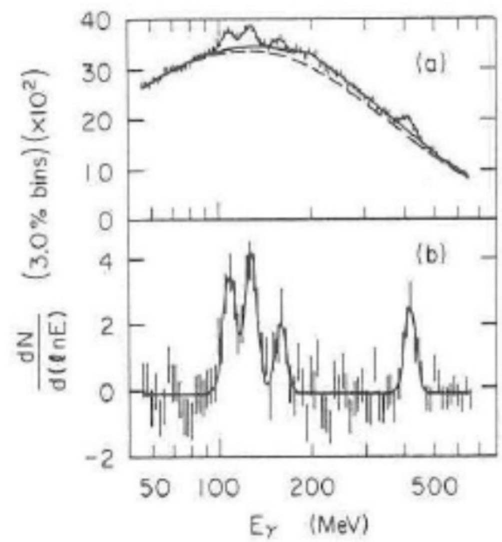


Figure 11.2: The photon spectrum from Υ' decays obtained by the Crystal Ball Collaboration at DORIS II. A triplet of lines corresponding to $\Upsilon' \to \gamma\chi_b(^3P_{2,1,0})$ is seen between 100 and 200 MeV. The decays $\chi_b \to \gamma\Upsilon$ produce the unresolved signal between 400 and 500 MeV [R. Nernst *et al.*, *Phys. Rev. Lett.* **54**, 2195 (1985)].

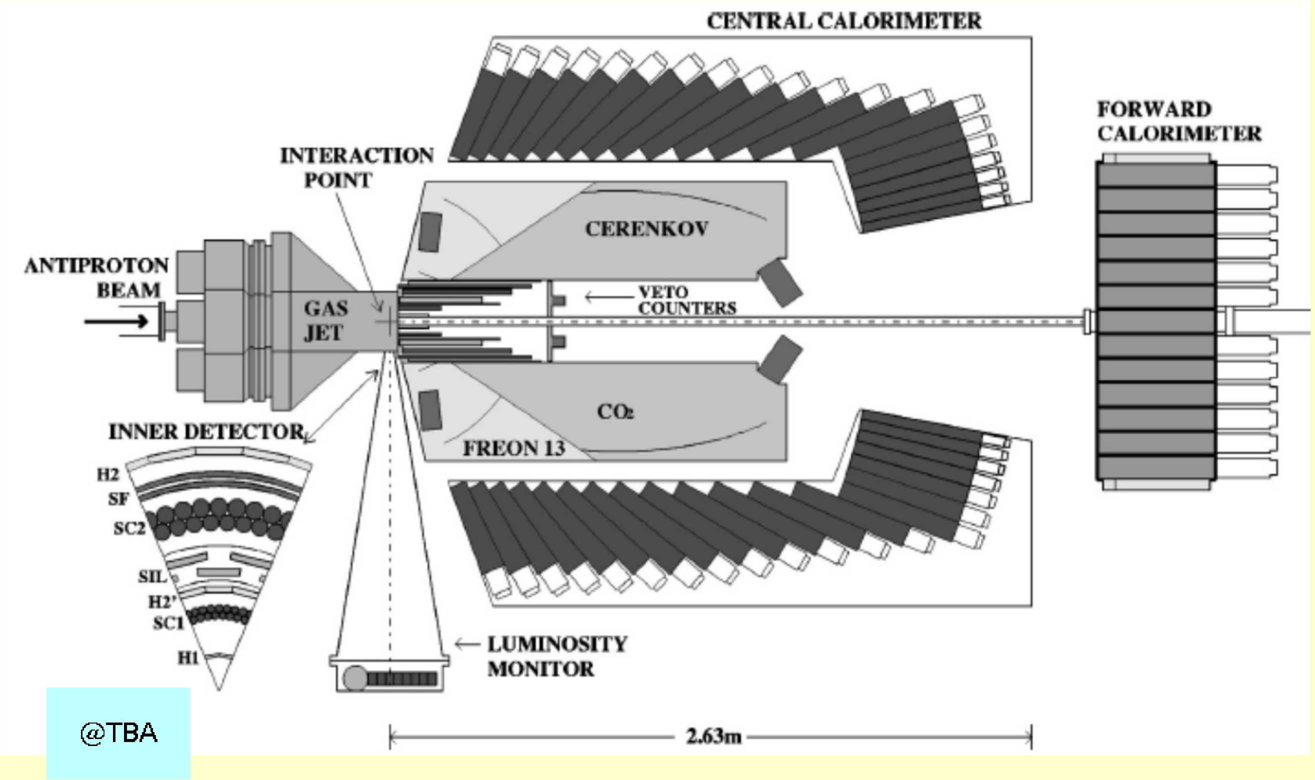
Another Side of Charmonium - I

$$p + \overline{p} \rightarrow \underbrace{c\overline{c}}_{Charmonium} \rightarrow Electromagnetic\ \ decay$$

Circulating $\overline{p}$Beam:
Excellent E resolution

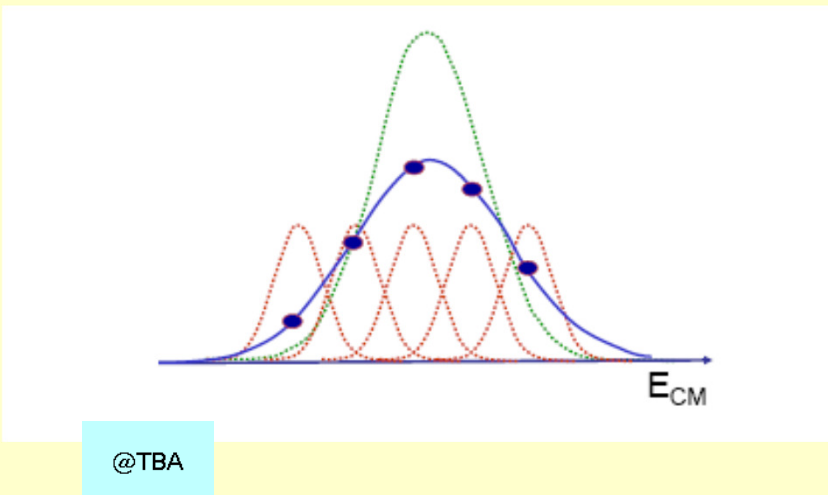
Gas jet target:
Reduced E loss

Non magnetic detector:
EM Calorimeter,
Tracking,
Cerenkov



Another Side of Charmonium - II

Concept of resonance scan: A fixed target, formation experiment
Move the beam energy in small steps across the energy range of any given resonant state
Measure the decay rate of the state at each step



Rate

Resonance profile
Typical width $\Gamma < 1$ MeV for $c\overline{c}$

Beam profile
Typical resolution $\sigma\left(E_{CM}\right) \sim 0.2\ MeV$

Get resonance mass, width, coupling by deconvolving the beam profile from the observed rate

Another Side of Charmonium - III

Electrons: *Cerenkov + Calorimeter + Tracking*
$\rightarrow$ Very low background to $e^+ e^-$

$$\psi' \rightarrow J/\psi + X$$
$$\searrow e^+ e^-$$

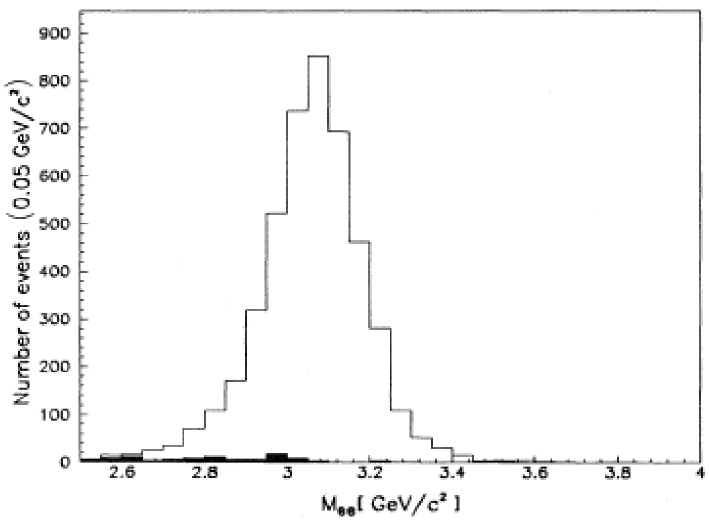


FIG. 5. Invariant mass distribution of electron pairs for the 1991 J/ψ scan (open area) and for the off-resonance background normalized to the same luminosity (shaded area). For this figure only the level of background has been multiplied by a factor of 10 to make it discernible.

@TBA

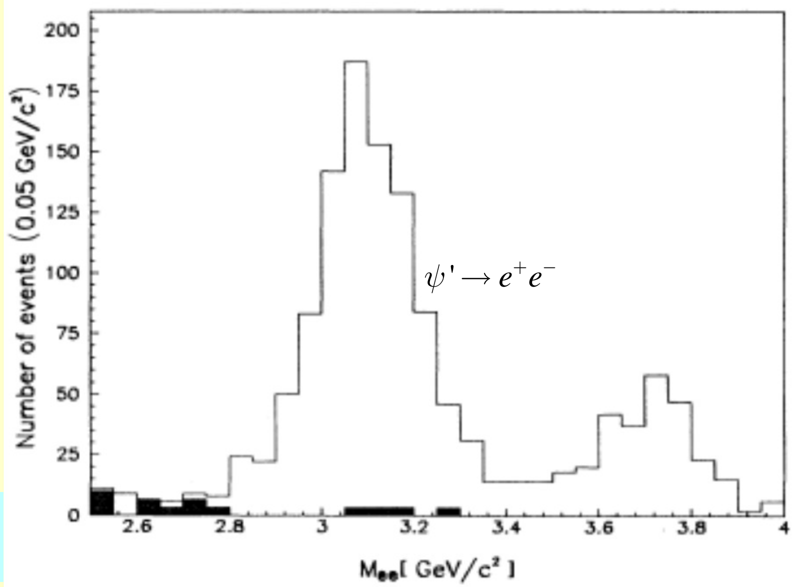


FIG. 6. Invariant mass distribution of electron pairs for the 1991 ψ' scan (open area) and for the off-resonance background normalized to the same luminosity (shaded area).

$M_{e^+e^-}$ from scan across J/ψ $\qquad$ $M_{e^+e^-}$ from scan across ψ'

Another Side of Charmonium - IV

A few results..

@TBA

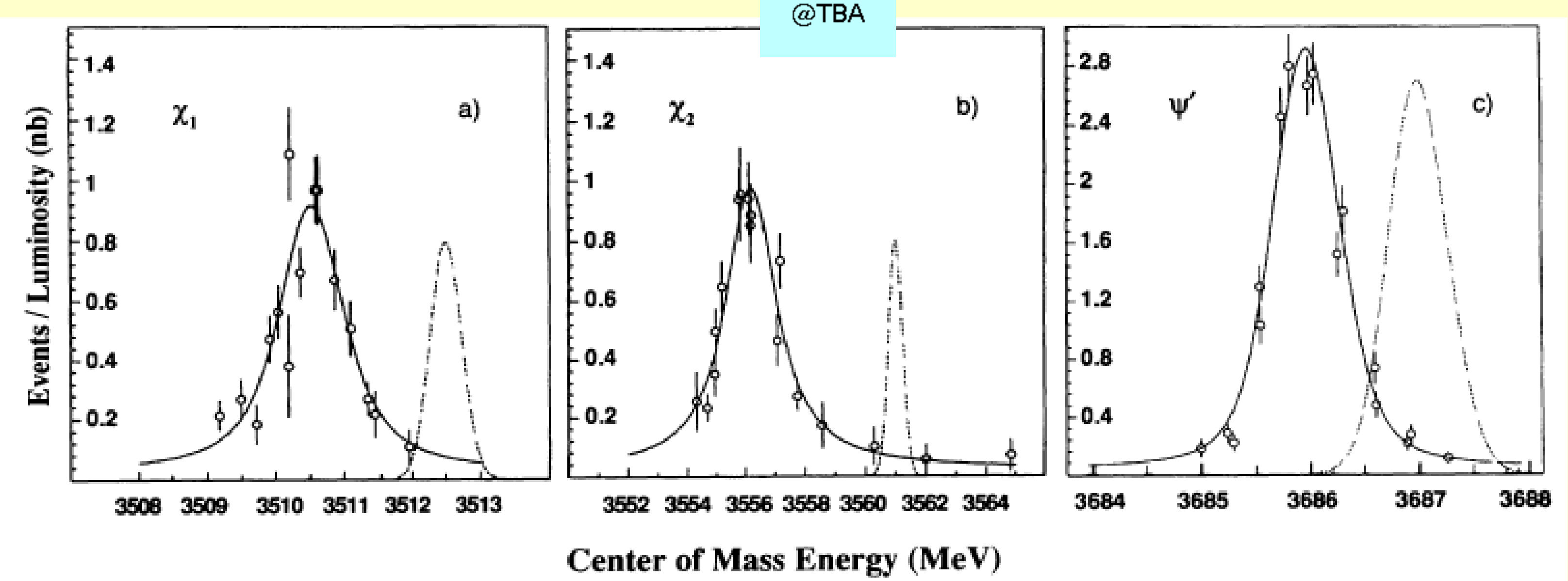


FIG. 3. Events per unit luminosity for the energy scan at (a) the χ_{c1}, (b) the χ_{c2}, and (c) the ψ'. The solid line represents the best fit with the data. The dashed line shows a typical resolution in the center-of-mass energy (arbitrary vertical units).

Quarkonia on PDG

Hidden Charm

$c\bar{c}$	
• $\eta_c(1S)$	$0^+(0^{-+})$
• $J/\psi(1S)$	$0^-(1^{--})$
• $\chi_{c0}(1P)$	$0^+(0^{++})$
• $\chi_{c1}(1P)$	$0^+(1^{++})$
$h_c(1P)$	$?^?(?^{??})$
• $\chi_{c2}(1P)$	$0^+(2^{++})$
• $\eta_c(2S)$	$0^+(0^{-+})$
• $\psi(2S)$	$0^-(1^{--})$
• $\psi(3770)$	$0^-(1^{--})$
• $X(3872)$	$0^?(?^{?+})$
• $\chi_{c2}(2P)$	$0^+(2^{++})$
$Y(3940)$	$?^?(?^{??})$
• $\psi(4040)$	$0^-(1^{--})$
• $\psi(4160)$	$0^-(1^{--})$
$Y(4260)$	$?^?(1^{--})$
• $\psi(4415)$	$0^-(1^{--})$

Hidden Bottom

$b\bar{b}$	
$\eta_b(1S)$	$0^+(0^{-+})$
• $\Upsilon(1S)$	$0^-(1^{--})$
• $\chi_{b0}(1P)$	$0^+(0^{++})$
• $\chi_{b1}(1P)$	$0^+(1^{++})$
• $\chi_{b2}(1P)$	$0^+(2^{++})$
• $\Upsilon(2S)$	$0^-(1^{--})$
$\Upsilon(1D)$	$0^-(2^{--})$
• $\chi_{b0}(2P)$	$0^+(0^{++})$
• $\chi_{b1}(2P)$	$0^+(1^{++})$
• $\chi_{b2}(2P)$	$0^+(2^{++})$
• $\Upsilon(3S)$	$0^-(1^{--})$
• $\Upsilon(4S)$	$0^-(1^{--})$
• $\Upsilon(10860)$	$0^-(1^{--})$
• $\Upsilon(11020)$	$0^-(1^{--})$

@TBA

Non-Perturbative QCD - I

Needed to deal with *bound states* and *soft interaction* regime

Very difficult problem

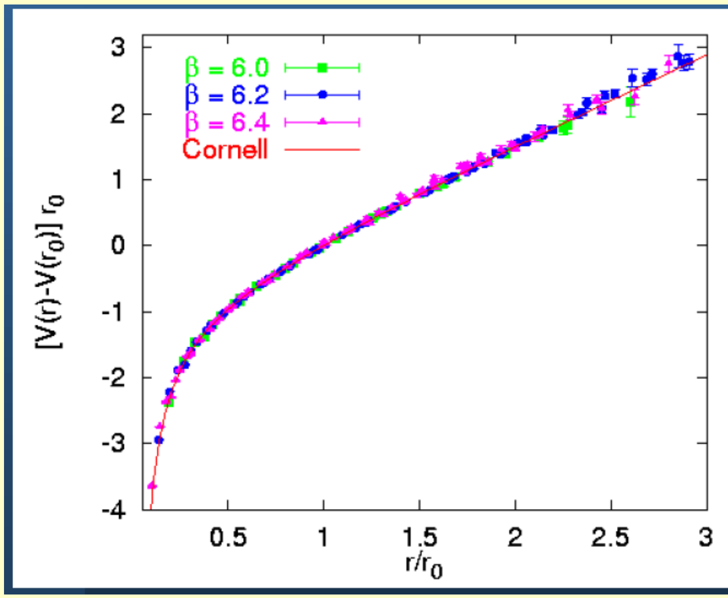
Different approaches available:

Lattice QCD
Chiral Pertubation Theory
Non-Relativistic QCD
Heavy Quark Effective Theory
…

Deep waters, not even surfed in this course

Non Perturbative QCD - II

Perform QCD calculations over a discretized space-time (lattice)



@TBA

Bali *et al.*

$r_0 = 0.5$ fm

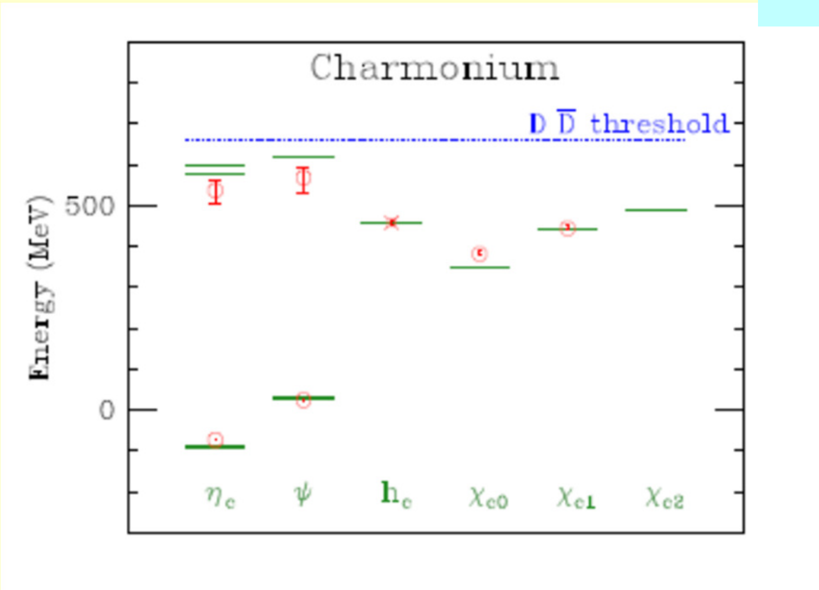
$q\bar{q}$ potential from lattice

$V(r) = -\frac{4}{3}\frac{\alpha_s}{r} + ar$: Not a bad idea after all...

Non Perturbative QCD - III

Examples:

Charmonium levels from lattice



Predicted glueball spectrum from lattice