Elementary Particles II

1 – QCD

Color, Gauge Fields, Gluons,
Asymptotic Freedom, Confinement,
Perturbative QCD, Quarkonium

Re-examining the Evidence

Experiments probing the EM structure, like DIS:

Scaling of the structure functions

Evidence for point-like constituents, funny behavior:

Like free particles when interacting with EM currents at high Q^2

Never observed outside hadrons $\rightarrow$ Tightly bound?

Experiments probing the strong interaction:

Large particle zoo

Evidence for highly symmetrical grouping and ordering

Strong suggestion of a substructure: Quarks

Funny, ad-hoc rules driving the observed symmetry

Constituents?

Besides the general, puzzling behavior observed, finally identify a number of serious problems, cast into a few issues:

Baryons and the Pauli Principle
R Ratio
π^0 *Decay Rate*
τ *Lepton Branching Ratios*

From all these questions a common conclusion:

Our picture of the quark model is not complete

Pauli Principle

Quark model:

Besides its many, remarkable successes, a central point is at issue:

The baryon wave function (space $\times$ spin $\times$ flavor) is symmetric

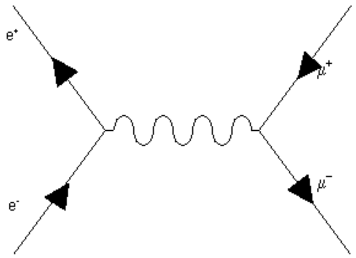
Pauli Principle seems to be lost, which is very bad news:

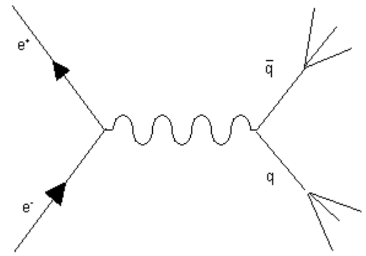
The Spin-Statistics Theorem is a consequence of very general principles of relativistic QFT, not easily dismissed

R Ratio - I

Assume the process $e^+e^- \to hadrons$ to proceed at the lowest order through

$e^+e^- \to q\,\bar{q} \to hadrons$





As for DIS:
Don't care about quark *hadronization*, assume the time scales for hard and soft sub-processes to be wildly different

$$\sigma\left(e^+e^- \to \mu^+\mu^-\right) = \frac{4\pi\alpha^2}{3s}$$

$$\sigma\left(e^+e^- \to q\,\bar{q}\right) = \frac{4\pi\alpha^2 Q_q^2}{3s}, \quad Q_q = \text{quark charge in } e \text{ units}$$

$$R\left(E_{CM}\right) = \frac{\sigma\left(e^+e^- \to adroni\right)}{\sigma\left(e^+e^- \to \mu^+\mu^-\right)} = \frac{\sum_q \sigma\left(e^+e^- \to q\bar{q}\right)}{\sigma\left(e^+e^- \to \mu^+\mu^-\right)} = \sum_q Q_q^2$$

Sum extended to all accessible quark flavors $\to 2m_q < E_{CM}$

R Ratio - II

R counts the number of different quark species created at any given E_{CM} . Expect:

$$u,d \rightarrow R = \left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 = \frac{5}{9} \qquad \textit{Low energy}$$

$$u,d,s \rightarrow R = \left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 = \frac{6}{9} \qquad E > 1\text{-}1.5\ GeV$$

$$u,d,s,c \rightarrow R = \left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 = \frac{10}{9} \qquad E > 3\ GeV$$

By taking 3 quark species
of any flavor:

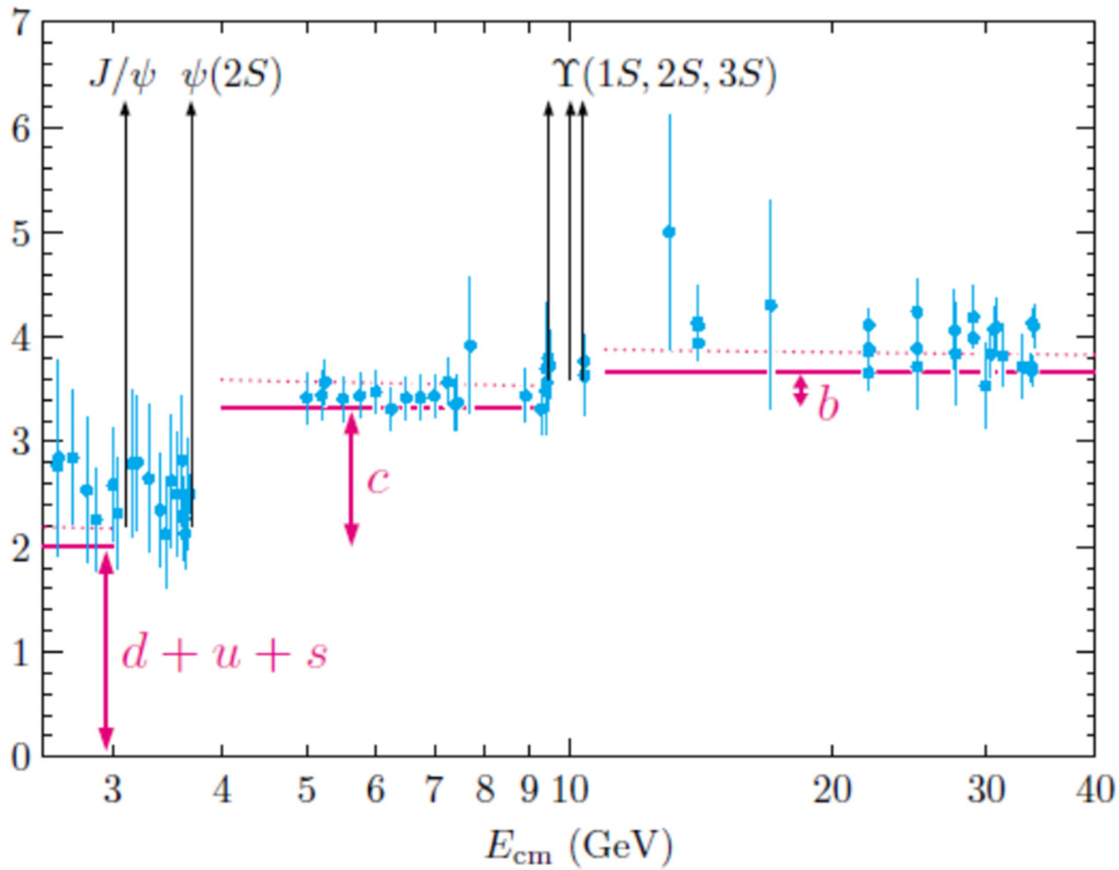
$$u,d \rightarrow R = \frac{15}{9}$$

$$u,d,s \rightarrow R = \frac{18}{9}$$

$$u,d,s,c \rightarrow R = \frac{30}{9}$$

R Ratio - III

$R = \sigma(\text{hadrons})/\sigma(\mu^+\mu^-)$



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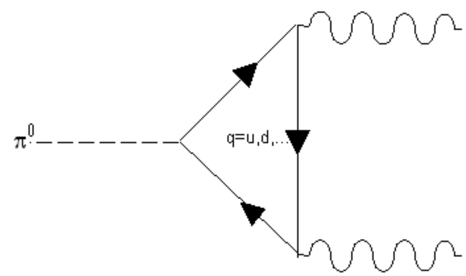
π^0 Decay Rate - I

Originally calculated by taking $p, \bar{p}$ in the triangle loop (Steinberger 1949)



Steinberger's calculation:
 Yukawa model to account for πpp vertex
 Point-like nucleons → QED couplings to photons

Nucleon current in the loop: 4-vector J^μ
[Actually *axial* vector, to match pion –ve parity]
π^0 spinless: Only 4-vector available p_μ

→Decay amplitude ~ $p_\mu J^\mu$

π^0 Decay Rate - II

With a proton loop rate OK (!)

By naively replacing the proton loop by a quark loop:

$$J^{\mu} \approx \sum_{i=u}^{d} q_i \bar{\psi}_i \gamma^{\mu} \gamma^5 \psi_i = e\left(\frac{2}{3}\bar{u}\gamma^{\mu}\gamma^5 u - \frac{1}{3}\bar{d}\gamma^{\mu}\gamma^5 d\right)$$

Amplitude: 2 vertexes

Each vertex $\propto \sqrt{\alpha} = e \rightarrow$ Amplitude $\propto e^2$

Sum over light quarks u, d:

$$\sum_{i=u,d} a_i Q_i^2 = e^2 \left[1 \cdot \left(\frac{2}{3}\right)^2 - 1 \cdot \left(-\frac{1}{3}\right)^2\right] = e^2 \left[\frac{4}{9} - \frac{1}{9}\right] = e^2 \frac{1}{3}$$

$$\Gamma_{quark}\left(\pi^0 \rightarrow \gamma\gamma\right) = \frac{1}{9}\Gamma_{proton}\left(\pi^0 \rightarrow \gamma\gamma\right) \quad ???$$

$\rightarrow$ Wrong by a factor 9!

Bad news for the quark model

π^0 Decay Rate - III

Upon looking more carefully to the problem, things look actually even worse:

By taking seriously the quark model, one cannot escape consequences of approximate *chiral symmetry* of light quarks

Then simple guess on approximate symmetry of the initial state would lead to conclude *the neutral pion is stable!* Another quark model puzzle..

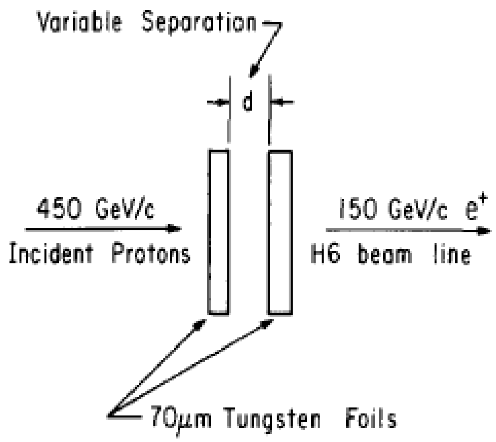
Explanation of this paradox led to discovery of the first *anomaly* in QFT
(Adler, Bell & Jackiw)

Advanced topic, quite relevant to the Standard Model:
Quantum field theories must be *anomaly free* in order to be renormalizable

Interesting conditions for SM to be anomaly free, including *charge quantization*

π^0 Decay Rate - IV

Direct method:



π^0 produced in a first thin foil, when not decayed do not contribute to e^+ yield from γ conversion in a second thin foil

$$Y(d) = N\{A + B[1 - \exp(-d/\lambda)]\}$$

$\lambda = \beta\gamma c\tau \simeq \gamma c\tau$ Energy dependent

Use known energy spectra for pions

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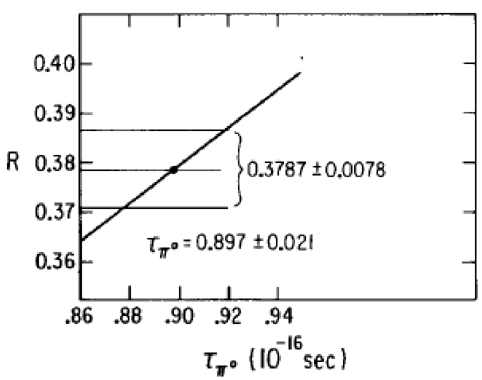
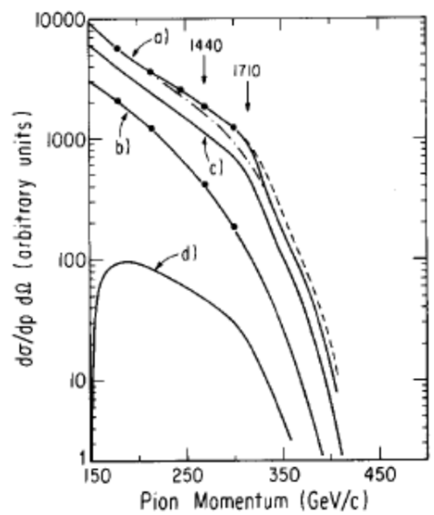
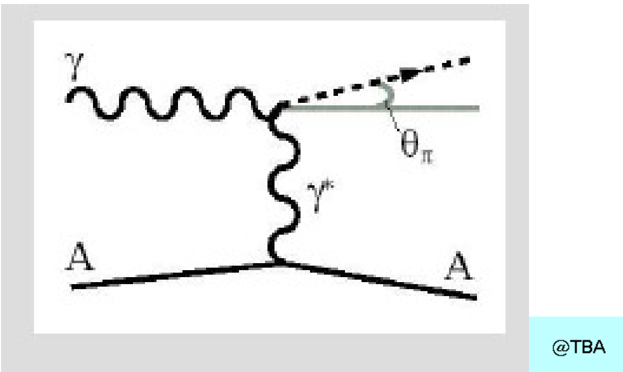


$$\tau = 0.897 \pm .021\ 10^{-16}\ s$$
$$\Gamma = 7.34 \pm 0.18 \pm 0.11\ eV$$



π^0 Decay Rate - V

Primakoff effect



Very simple idea:
Get a high energy photon beam + high Z target
Pick-up a virtual photon from the nuclear Coulomb field
2-photon coupling will (sometimes) create a π^0

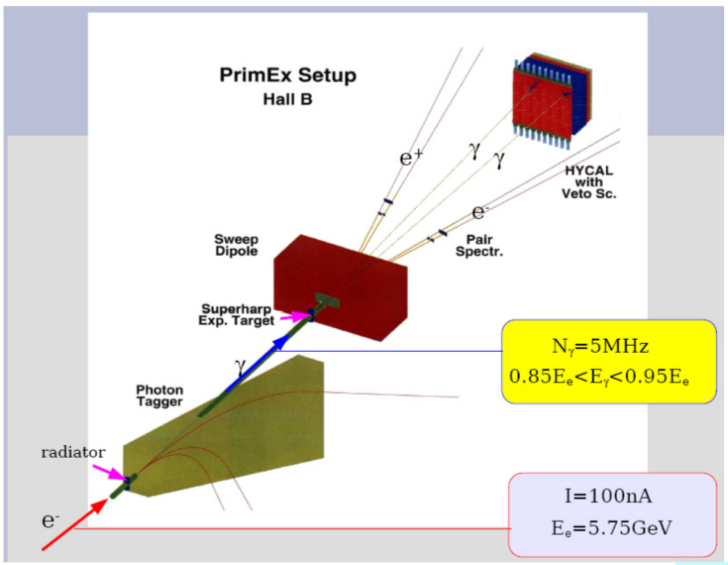
$$\frac{d\sigma_{\text{Prim}}}{d\Omega_{LAB}} \simeq \Gamma_{\pi^0 \to \gamma\gamma} Z^2 \frac{8\alpha E^4}{m_{\pi^0}^2} \frac{\left|F\left(q^2\right)\right|^2}{q^4} \sin^2 \theta_{\pi^0}$$

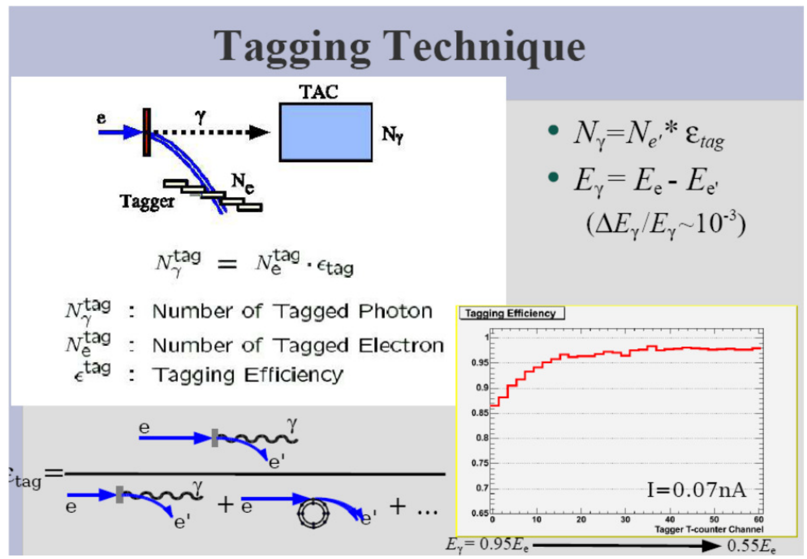
Strongly forward peaked
Quickly increasing with energy
Strongly *Z* dependent: Coherence

$\Gamma = 1/\tau$ extracted by measuring the differential cross-section
Nuclear form factor required

π^0 Decay Rate - VI

Recent experiment: PrimEx at Jefferson Lab (Virginia)





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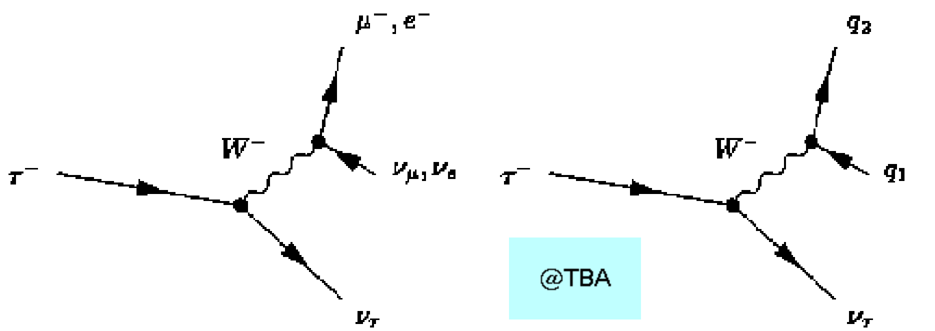
τ Lepton Decays

τ: Heavy brother of e and μ

m_τ= 1776 MeV

Weak decays:



$q, \bar{q} : u, d, s$

$BR(e) \quad \sim 18\%$

$BR(\mu) \quad \sim 17\%$

$BR(q\bar{q}) \quad \sim 65\%$

In the absence of color, weak interaction universality would lead to predict:

$BR(e) \sim BR(\mu) \sim BR(q\bar{q}) \sim 33\%$

With color:

$$\Gamma(q\bar{q}) \sim 3\,\Gamma(l\bar{l}) \rightarrow BR(q\bar{q}) \sim \frac{3\,\Gamma(l\bar{l})}{3\,\Gamma(l\bar{l}) + 2\,\Gamma(l\bar{l})} \sim 60\ \% \quad \text{OK}$$

Color - I

New hypothesis:

There is a new degree of freedom for quarks: Color

Each quark can be found in one of 3 different states

Internal space (mathematically identical to flavor):

States = 3-component complex vectors

Base states:

$$R(ed)=\begin{pmatrix}1\\0\\0\end{pmatrix},G(reen)=\begin{pmatrix}0\\1\\0\end{pmatrix},B(lue)=\begin{pmatrix}0\\0\\1\end{pmatrix}$$

Needless to say, nothing to do with our old, beloved color concept (nature, art, politics..):

Just a *name* for another, non-classical property of hadron constituents

Color - II

Provided one can build a color wave function for 3 fermions which is antisymmetric, the Fermi statistics problem is solved
Total wave function of a baryon

$$\psi = \psi_{color} \underbrace{\psi_{orbital}\psi_{spin}\psi_{flavor}}_{Symmetric} \to \psi_{color}: \text{ Antisymmetric}$$

To account for 3 different color states, the R ratio must be multiplied by *3* $\to$ OK with experimental data

Just the same conclusion for hadronic τ decays: Multiply rate by *3*

The correct π^0 rate is obtained by inserting a factor *9*

Color - III

Observe:

When computing *R*, τ decay rates we add the *rates* for different colors
$\rightarrow$Factor $\times$ *3*

We deal with quarks as with real, on-shell particles: Ignore fragmentation

When computing π^0 decay rate, we add the *amplitudes*
$\rightarrow$Factor $\times$ *9*

Quarks in the loop are virtual particles: *Amplitudes interfere*

Color - IV

Must be possible to build hadron states as color *singlets*

Do not expect hadrons to fill larger irr.reps.:
Would imply large degeneracies for hadron states, not observed
In other words:

Color is fine, but we do not observe any colored hadron

Therefore we assume the color charge is *confined*:
Never observed directly, as it is the electric charge

Why? For the moment, nobody really knows

Color - V

How colored hadrons would show up?
Just as an example:

Should the nucleon fill the ***3*** of $SU(3)_C$, there would be 3 different species of protons and neutrons.

Then each nucleon level in any nucleus could accommodate 3 particles instead of one:

The nuclear level scheme would be far different from the observed one

Color - VI

Guess *SU(3)* as the color group
Take the two fundamental decompositions:

$$\mathbf{3}\otimes\mathbf{3}\otimes\mathbf{3}=\mathbf{1}\oplus\mathbf{8}\oplus\mathbf{8}\oplus\mathbf{10}$$ Baryons

$$\mathbf{3}\otimes\mathbf{3}^{*}=\mathbf{1}\oplus\mathbf{8}$$ Mesons

Both feature a singlet in the direct sum: OK
No singlets in $\mathbf{3}\otimes\mathbf{3}$: OK

Can't say the same for other groups…
Take *SU(2)* as an example:
Say the quarks live in the adjoint *SU(2)* representation, ***3***
Then for qq

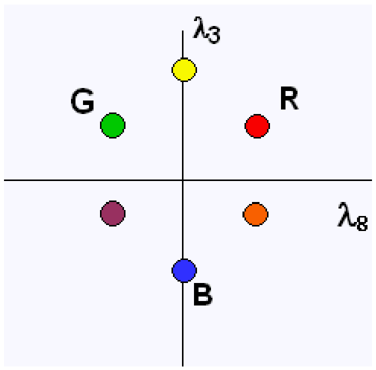
$$\boldsymbol{3}\otimes\boldsymbol{3}=\boldsymbol{1}\oplus\boldsymbol{3}\oplus\boldsymbol{5}$$

Observe: This is ***3*** of *SU(2),* which is quite different from ***3*** of *SU(3)*

Diquarks can be in color singlet
→Should find diquarks as commonly as baryons or mesons..

Colored Quarks

$$I_3^c = \frac{\lambda_3}{2}, \qquad Y^c = \frac{\lambda_8}{\sqrt{3}}$$



	I_3^c	Y^c
R	$+1/2$	$+1/3$
G	$-1/2$	$+1/3$
B	0	$-2/3$

	I_3^c	Y^c
$\bar{R}$	$-1/2$	$-1/3$
$\bar{G}$	$+1/2$	$-1/3$
$\bar{B}$	0	$+2/3$

SU(3)$_C$ is an *exact symmetry*:

$$m_u^{(R)} = m_u^{(G)} = m_u^{(B)}, \ldots$$

Beware: SU(3)$_C$ has nothing to do with *SU(3)*$_F$:
Quark quantum numbers are independent from their color state

They are left unchanged by QCD transitions

Uncolored Hadrons

According to our fundamental hypothesis:

Mesons: $\mathbf{3}\otimes\mathbf{3}^{*}=\mathbf{1}\oplus\mathbf{8}$

$$\rightarrow\psi_c=\frac{1}{\sqrt{3}}\left(R\bar{R}+G\bar{G}+B\bar{B}\right)$$

Baryons: $\mathbf{3}\otimes\mathbf{3}\otimes\mathbf{3}=\mathbf{1}\oplus\mathbf{8}\oplus\mathbf{8}\oplus\mathbf{10}$

$$\rightarrow\psi_c=\frac{1}{\sqrt{6}}\left(RGB-RBG+GBR-GRB+BRG-BGR\right)$$

In both cases, pick singlet

Mesons: *No particular exchange symmetry*
(2 non identical particles)

Baryons: *Fully antisymmetrical color wave function*
(3 identical particles)

Color Interaction: QCD

Color: A new degree of freedom for quarks
Compare to other quantum numbers:

Baryonic/Leptonic numbers
Conserved, *not originating interactions*

Electric charge
Conserved, *origin of the electromagnetic field*

A deep question:

What is the true origin of the electromagnetic interaction?

We have freely used the interaction term $j^{\mu} A_{\mu}$, only based on the classical analogy:
But supposedly quantum mechanics is more general than classical mechanics/electromagnetism..

Is there any deeper origin for it?

QED as a Gauge Theory - I

Symmetry:
Absolute phase not defined for a wave function.
Expect invariance as per our old acquaintance, Noether's Theorem

$L_0 = \bar{\psi}(x)\left(i\gamma^\mu \partial_\mu - m\right)\psi(x)$ Free Dirac Lagrangian

Global gauge (=Phase) transformation:

$G : \psi(x) \rightarrow \psi'(x) = U_\theta \psi(x) = e^{-iq\theta}\psi(x)$ $q\theta :$ New phase $\propto$ Charge

$\rightarrow L_0$ invariant wrt $G \rightarrow$ Charge conservation

Just meaning:

Take *all* particle states, re-phase each state proportionally to its charge

QED as a Gauge Theory - II

Generalize to local phase transformation:

$G_L : \psi(x) \to \psi'(x) = U_\theta \psi(x) = e^{-iq\theta(x)}\psi(x)$ Local gauge transformation

$\to L_0$ not invariant wrt G_L : Derivative term troublesome

$$L_0 = \bar{\psi}(x)\left(i\gamma^\mu \partial_\mu - m\right)\psi(x) \to L_0' = i\bar{\psi}(x)e^{+iq\theta(x)}\gamma^\mu \partial_\mu\left(e^{-iq\theta(x)}\psi(x)\right) - m\bar{\psi}(x)\psi(x)$$

$$L_0' = i\left[\bar{\psi}(x)\gamma^\mu \partial_\mu \psi(x) - iq\partial_\mu\left[\theta(x)\right]\psi(x)\right] - m\bar{\psi}(x)\psi(x)$$

$$L_0' = \left\{i\bar{\psi}(x)\gamma^\mu \partial_\mu \psi(x) + q\partial_\mu\left[\theta(x)\right]\psi(x) - m\bar{\psi}(x)\psi(x)\right\}$$

$$L_0' = \left[i\bar{\psi}(x)\gamma^\mu \partial_\mu \psi(x) - m\bar{\psi}(x)\psi(x)\right] + q\partial_\mu\left[\theta(x)\right]\psi(x) \neq L_0$$

$\to$ Local gauge invariance cannot hold in a world of free particles

Symmetry requires interaction

QED as a Gauge Theory - III

New transformation rule:

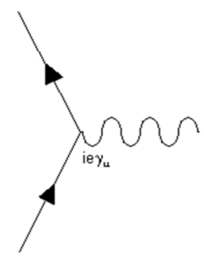
$$\begin{cases} \psi(x) \to \psi'(x) = U_\theta \psi(x) = e^{-iq\theta(x)}\psi(x) & \text{As before} \\ A_\mu(x) \to A_\mu(x) + q\ \partial_\mu \theta(x) & \text{New character in the comedy} \end{cases}$$

Equivalent to re-define derivative for ψ:

$\partial_\mu \to \partial_\mu + iqA_\mu$ Vector field

Add a new term to Lagrangian:

$L_i = -\underbrace{q\bar{\psi}(x)\gamma^\mu\psi(x)}_{j^\mu} A_\mu$ Interaction term



Same as classical electrodynamics

$$L_0 = \bar{\psi}(x)\left(i\gamma^\mu\partial_\mu - m\right)\psi(x) \to L_0 + L_i = \bar{\psi}(x)\left(i\gamma^\mu\partial_\mu - m\right)\psi(x) - q\bar{\psi}(x)\gamma^\mu\psi(x)A_\mu$$

Sum is invariant

QED as a Gauge Theory - IV

...And another one:

$$-\frac{1}{4}F^{\mu\nu}F_{\mu\nu},\quad F^{\mu\nu}=\partial^{\mu}A^{\nu}-\partial^{\nu}A^{\mu}\quad \text{Field energy}$$

Must be there because the field carries energy+momentum+angular momentum

Reminder:

$F^{\mu\nu}$ is the EM field

$$F^{\mu\nu}=\begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix}$$

Field must be massless to have L gauge invariant

$$\frac{1}{2}m^2A_{\mu}^2 \to \frac{1}{2}m^2\left(A_{\mu}(x)+q\,\partial_{\mu}\theta(x)\right)^2 \neq \frac{1}{2}m^2A_{\mu}^2 \quad \text{if } m\neq 0$$

QED as a Gauge Theory - V

Consider all the phase transformations as defined before:

$$\psi(x) \rightarrow \psi'(x) = U_\theta \psi(x) = e^{-iq\theta(x)}\psi(x)$$

The full set is a group: $U(1)$ Unitary, 1-dimensional

$$e^{-iq\theta_1(x)}e^{-iq\theta_2(x)}\psi(x) = e^{-iq[\theta_1(x)+\theta_2(x)]} \in U(1)$$

1 parameter : $\theta(x)$

Abelian : $e^{-iq\theta_1(x)}e^{-iq\theta_2(x)}\psi(x) = e^{-iq\theta_2(x)}e^{-iq\theta_1(x)}\psi(x)$

U(1) is the (Abelian) *gauge group* of QED

Equivalent to *SO(2),* group of 2D rotations

QCD as a Gauge Theory - I

Extend gauge transformations to a 3-component wave function:

$$\boldsymbol{\psi} \equiv \begin{pmatrix} \psi_R \\ \psi_G \\ \psi_B \end{pmatrix}$$

Global gauge transformation: Phase change for individual components

$\rightarrow$ Phase change will mix color components

$G_G^C : \boldsymbol{\psi}(x) \rightarrow \boldsymbol{\psi}'(x) = \mathbf{U}_G \cdot \boldsymbol{\psi}(x) = e^{-ig\mathbf{M}} \cdot \boldsymbol{\psi}(x) \qquad \mathbf{U}_G$ unitary $\rightarrow \mathbf{M}$ Hermitian

$e^{-ig\mathbf{M}} = \mathbf{1} - ig\mathbf{M} + \dfrac{(-ig\mathbf{M})^2}{2!} + \ldots \quad \mathbf{M}:$ 3×3 Hermitian matrix

$\mathbf{M}$ acting on the 3 color components of the quark state

Since the color symmetry group is $SU(3)_C$:

$$\mathbf{M} = \sum_{i=1}^{8} \boldsymbol{\lambda}_i \theta_i \equiv \vec{\boldsymbol{\lambda}} \cdot \vec{\theta}$$

$\vec{\boldsymbol{\lambda}}$: Vector of 8 3×3 Gell-Mann matrices; $\vec{\theta}$: Vector of 8 parameters

QCD as a Gauge Theory - II

As for QED, extend to *local* gauge transformations

As before, in order to guarantee invariance of L:

$\rightarrow$ Re-define derivative adding new vector fields:

$\partial_\mu \rightarrow \partial_\mu \mathbf{1} + ig\mathbf{C}_\mu$

$$\mathbf{C}_\mu : \begin{cases} \text{4-vector field} & \text{Lorentz structure} \\ \text{Matrix } \in SU(3)_C & \text{Color space} \end{cases}$$

We know how to express any Hermitian matrix $\in SU(3)_C$:

Use $SU(3)_C$ generators $\rightarrow$ Gell-Mann matrices

$\rightarrow \mathbf{C}_\mu = \frac{1}{2}\sum_{i=1}^{8} \mathrm{G}_\mu^a \boldsymbol{\lambda}_a \equiv \vec{G}_\mu \cdot \vec{\lambda}$ 8 fields required: Gluons

So gluons are a bit like 8 different "photons", exchanged between color charges

But: *They are non Abelian*

QCD as a Gauge Theory - III

Local gauge transformation for $SU(3)_C$:

$$\begin{cases} \psi(x) \to \psi'(x) = U_\theta \psi(x) = e^{-ig\vec{\lambda}\cdot\vec{\theta}(x)}\psi(x) \\ G^a_\mu(x) \to G^a_\mu{}'(x) = G^a_\mu(x) + \partial_\mu \theta^a + g\sum_{b,c=1}^{8} f^{abc} G^b_\mu(x)\,\theta^c(x) \quad a = 1,\ldots,8 \end{cases}$$

Very important: New term, coming from *SU(3)* being non Abelian

Reminder:

$$\left[\frac{\lambda_a}{2},\frac{\lambda_b}{2}\right] = if^{abc}\frac{\lambda_c}{2} \qquad f^{abc}: SU(3) \text{ structure constants}$$

$$L_0 = \bar{\psi}(x)\left(i\gamma^\mu\partial_\mu - m\right)\psi(x) \to L_0 + L_i$$

$$L_i = -g\underbrace{\left[\bar{\psi}(x)\gamma^\mu\left(\frac{\vec{\lambda}}{2}\right)\psi(x)\right]}_{\vec{j}_\mu}\cdot\vec{G}_\mu \quad \text{Interaction term}$$

$$-\frac{1}{4}\vec{G}_{\mu\nu}\cdot\vec{G}^{\mu\nu} = -\frac{1}{4}\sum_{a=1}^{8} G^a_{\mu\nu}\cdot G^{a\mu\nu} \quad \text{Field energy term}$$

QCD as a Gauge Theory - IV

Take the expression of fields in terms of potentials:

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu - g\sum_{b,c=1}^{8} f_{abc} G^b_\mu G^c_\nu$$

Very important: *Absent in QED (f=0)*
New term, coming from *SU(3)* being non Abelian

$$\rightarrow G^a_{\mu\nu} G^a_{\mu\nu} \quad \text{contains terms with} \quad \underbrace{\partial_\mu G^a_\nu \cdot G^b_\mu G^c_\upsilon}_{\text{3 gluons}}, \underbrace{G^b_\mu G^c_\upsilon \cdot G^b_\mu G^c_\upsilon}_{\text{4 gluons}}$$

When translated into Feynman rules/diagrams, these pieces of L correspond to 3 and 4 gluons vertices

So:

The form of QCD Lagrangian leads to predict the existence of a new kind of *gluon-gluon color interaction*

QCD as a Gauge Theory - V

Since color interaction is tied to color charge, we are saying *the gluons carry their own color charge*

Sounds unfamiliar? Well, that's all after playing with a non-Abelian gauge group.

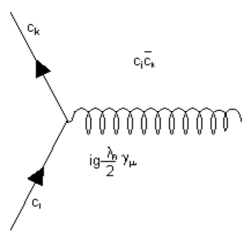
Unlike the electric charge, color charge can manifest itself in more than one way.

Indeed, gluons carry a type of color charge different from quarks/antiquarks:

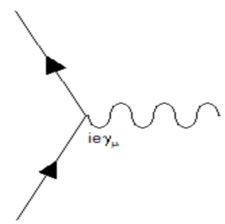
Color + Anticolor

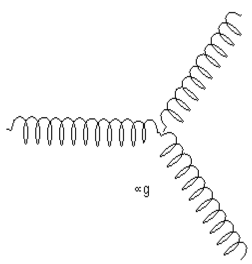
QCD as a Gauge Theory - VI

QCD Vertexes

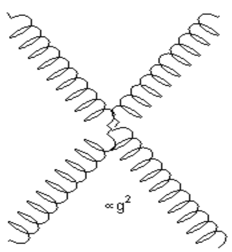


$ig\frac{\lambda_a}{2}\gamma_\mu$ Similar to QED:





$\propto g$ (Lorentz structure not shown)



$\propto g^2$ (Lorentz structure not shown)

Colored Gluons - I

Compare to mesons in $SU(3)_F$: *Flavor + Antiflavor*
But: *Gluons are not bound states of Color+Anticolor!*

Still, they share the same math:
Gluons live in the adjoint (***8***) irr.rep. of $SU(3)_C$

$$|1\rangle = \frac{1}{\sqrt{2}}\left(r\bar{b} + b\bar{r}\right), |2\rangle = -\frac{i}{\sqrt{2}}\left(r\bar{b} - b\bar{r}\right), |3\rangle = \frac{1}{\sqrt{2}}\left(r\bar{r} - b\bar{b}\right)$$

$$|4\rangle = \frac{1}{\sqrt{2}}\left(r\bar{g} + g\bar{r}\right), |5\rangle = -\frac{i}{\sqrt{2}}\left(r\bar{g} - g\bar{r}\right), |6\rangle = \frac{1}{\sqrt{2}}\left(b\bar{g} + g\bar{b}\right)$$

$$|7\rangle = -\frac{i}{\sqrt{2}}\left(b\bar{g} - g\bar{b}\right), |8\rangle = \frac{1}{\sqrt{6}}\left(r\bar{r} + b\bar{b} - 2g\bar{g}\right)$$

Colored Gluons - II

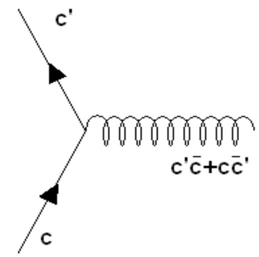
A very natural question: Gluons couple to $q\bar{q}$

Since one can decompose the total $q\bar{q}$ color state as:

$\mathbf{3}\otimes\mathbf{3}^{*}=\mathbf{1}\oplus\mathbf{8}$

Then: Where is the singlet gluon?

Does not exist: There are only 8 gluons, not 9



Should the singlet gluon actually exist, it would behave more or less like a “photon”:

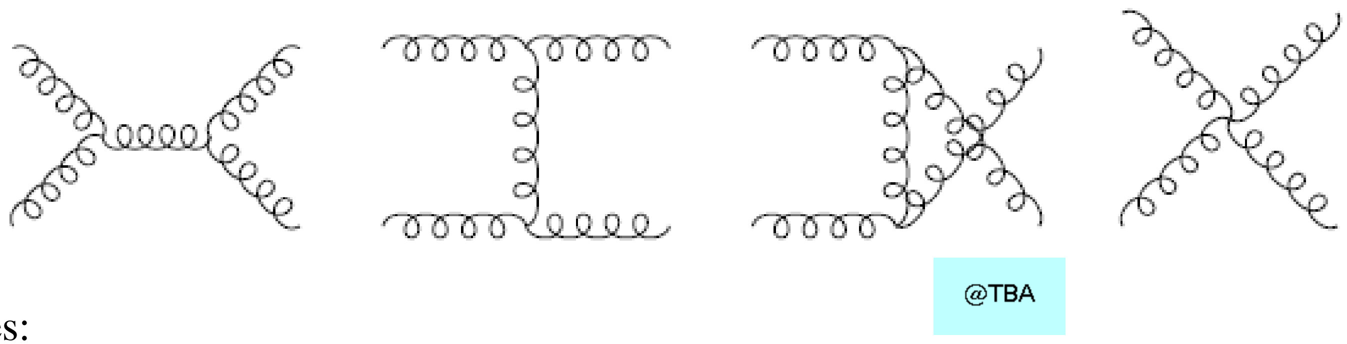
Would be ‘white’ (= Singlet)

Would couple to color charges in the same way as photon couples to electric charges

Would give rise to a sort of “QED-like”, long range color interaction, not observed

Colored Gluons - III

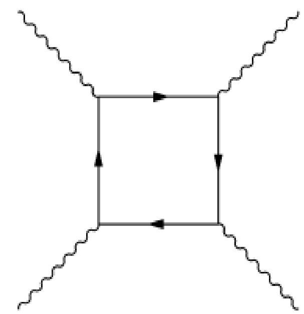
Non Abelian vertices: Gluon-Gluon scattering *at tree level* (no loops)



Vertexes:

$3-gluons: A \propto g$

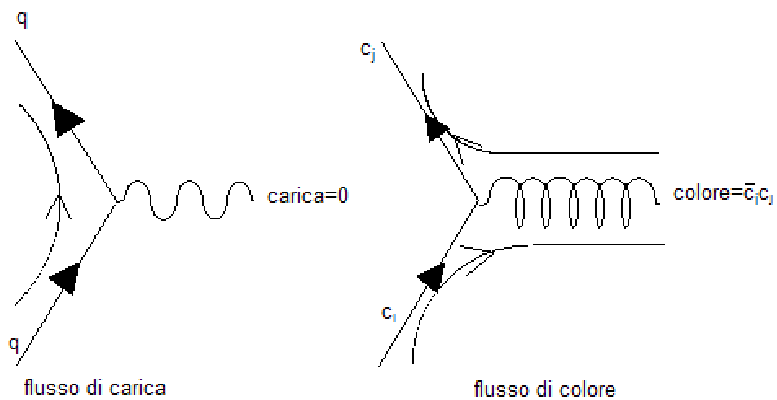
$4-gluons: A \propto g^2$



Compare:
In QED, photon-photon scattering amplitude occurs at order α^2 through the 1-loop diagram

Comparing QED and QCD - I

Compare the different situations:



QED
Photon is *neutral*

Neither sourcing,
nor sinking charge

QCD
Gluon is *colored*

Sourcing color,
sinking anti-color

Comparing QED and QCD - II

Comparison of coupling constants:

α vs. α_s Adimensional constants (*Interaction strength)*

Can define elementary charge in terms of α, α_s

Measure particle charge by its ratio to elementary charge:

Number

What are the allowed values for these numbers?

Comparing QED and QCD - III

QED: Gauge group is *Abelian*

Electric charge can be *any* number:
No reason for charge quantization → So electric charge quantization is a bit of a mistery

[Tricky business: Sticking to perturbation theory, one must have the SM *anomaly-free* in order to be renormalizable → This in turn *requires* charge quantization.
But: Is the SM just perturbation theory?
At a fundamental level, Grand Unified Theories explain charge quantization based on larger symmetry groups like *SU(5)*.
But: They fail to explain proton stability]

Photon charge is strictly *0*

QCD: Gauge group is *non Abelian*

"Color charge" value is *fixed* for every representation

Quarks: ***3***,***3**** → $Q = 4/3$
Gluons: ***8*** → $Q = 3$

Similar to $I(I+1)$ for any isospin ($SU(2)$) multiplet

Color Factors - I

Consider the static interaction between 2 charges:

QED For fixed $|q|$, the 'charge factor' can be defined as:

$$f_{12} = \frac{q_1 q_2}{|q|^2} = \begin{cases} +1 & q_1 q_2 > 0 \\ -1 & q_1 q_2 < 0 \end{cases}$$

Very simple for an Abelian interaction

QCD The ‘color factor’ depends on the irr.rep. of the color state

Representation dependent
Identical for any transition in a given representation
→*Color Conservation*

Less simple in this non-Abelian interaction

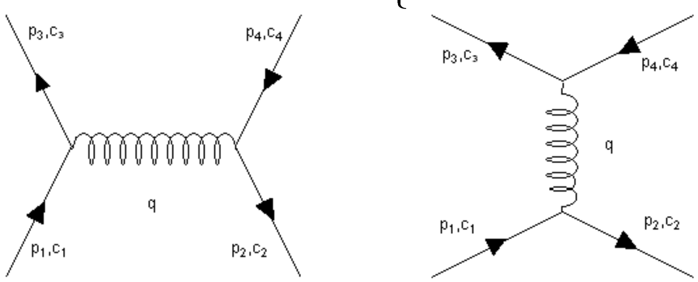
Color Factors - II

$q\bar{q} \to q\bar{q}$

$\mathbf{3} \otimes \mathbf{3}^* = \mathbf{1} \oplus \mathbf{8}$

Total color conservation: $\begin{cases} \mathbf{1} \to \mathbf{1} \\ \mathbf{8} \to \mathbf{8} \end{cases}$

Observe:
Similar to conservation of total I-spin



$$T_{fi} = i \sum_{\alpha,\beta=1}^{8} \underbrace{\left[\bar{u}(3) c_3^\dagger\right]\left[-i\frac{g_s}{2}\lambda^\alpha\gamma^\mu\right]\left[u(1)c_1\right]}_{\text{color current}} \underbrace{\left[-i\frac{g_{\mu\nu}\delta^{\alpha\beta}}{q^2}\right]}_{\text{propagator}} \underbrace{\left[\bar{v}(2)c_2^\dagger\right]\left[-i\frac{g_s}{2}\lambda^\beta\gamma^\nu\right]\left[v(4)c_4\right]}_{\text{color current}}$$

Sum is over all 8 color matrices

c_i are the color states of initial, final $q\bar{q}$

$$T_{fi} = \frac{-g_s^2}{q^2}\left[\bar{u}(3)\gamma^\mu u(1)\right]\left[\bar{v}(2)\gamma_\mu v(4)\right] \underbrace{\frac{1}{4}\sum_\alpha \left[c_3^\dagger\lambda^\alpha c_1\right]\left[c_2^\dagger\lambda^\alpha c_4\right]}_{\text{color factor}}$$

Color Factors - III

Octet

$r\bar{b}$ Just as an example: Result is the same for all octet states

$$\left.\begin{matrix} c_1 = c_3 = \begin{pmatrix}1\\0\\0\end{pmatrix} \\ c_2 = c_4 = \begin{pmatrix}0\\1\\0\end{pmatrix}\end{matrix}\right\} \rightarrow f = \frac{1}{4}\sum_{\alpha=1}^{8}\begin{pmatrix}1 & 0 & 0\end{pmatrix}\lambda^\alpha\begin{pmatrix}1\\0\\0\end{pmatrix}\begin{pmatrix}0 & 1 & 0\end{pmatrix}\lambda^\alpha\begin{pmatrix}0\\1\\0\end{pmatrix}$$

$$\rightarrow f = \frac{1}{4}\sum_{\alpha=1}^{8}\lambda_{11}^\alpha\lambda_{22}^\alpha = \frac{1}{4}\left(\lambda_{11}^3\lambda_{22}^3 + \lambda_{11}^8\lambda_{22}^8\right) = -\frac{1}{6}$$

Color Factors - IV

Singlet

$$\frac{1}{\sqrt{3}}\left(r\overline{r}+b\overline{b}+g\overline{g}\right)$$ Only this state in the singlet

But: *Any* component can go into *any other*..

$$f_i=\frac{1}{4}\frac{1}{\sqrt{3}}\frac{1}{\sqrt{3}}\sum_{j=1}^{3}\sum_{\alpha=1}^{8}\left[c_i^{\dagger}\lambda^{\alpha}c_j\right]\left[c_j^{\dagger}\lambda^{\alpha}c_i\right],\quad i=1,2,3$$

$$f=\sum_{i=1}^{3}f_i=\frac{1}{4}\frac{1}{\sqrt{3}}\frac{1}{\sqrt{3}}\sum_{\alpha=1}^{8}\sum_{i,j=1}^{3}\lambda_{ij}^{\alpha}\lambda_{ji}^{\alpha}=\frac{1}{12}\sum_{\alpha=1}^{8}Tr\left(\lambda^{\alpha}\lambda^{\alpha}\right)$$

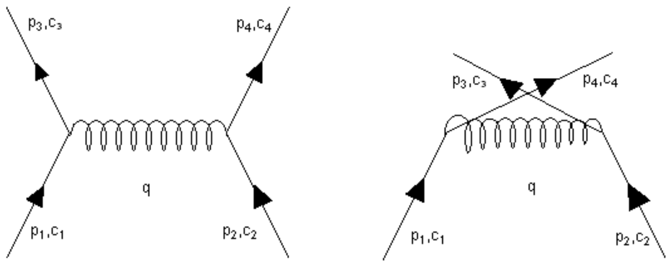
$$Tr\left(\lambda^{\alpha}\lambda^{\beta}\right)=2\delta^{\alpha\beta}\rightarrow\sum_{\alpha=1}^{8}Tr\left(\lambda^{\alpha}\lambda^{\alpha}\right)=16$$

$$\rightarrow f=\frac{4}{3}$$

Color Factors - V

$qq \to qq$

$\mathbf{3} \otimes \mathbf{3} = \mathbf{3}^* \oplus \mathbf{6}$



$$T_{fi} = \frac{-g_s^2}{q^2}\left[\overline{u}(3)\gamma^\mu u(1)\right]\left[\overline{u}(4)\gamma_\mu u(2)\right]\underbrace{\frac{1}{4}\sum_{\alpha=1}^{8}\left[c_3^\dagger \lambda^\alpha c_1\right]\left[c_4^\dagger \lambda^\alpha c_2\right]}_{\text{color factor}}$$

$$f_{qq} = \frac{1}{4}\sum_{\alpha=1}^{8}\left(c_3^\dagger \lambda^\alpha c_1\right)\left(c_4^\dagger \lambda^\alpha c_2\right)$$

Color Factors - VI

Color states: Triplet and sextet:

$$\mathbf{3}^*: \qquad \frac{1}{\sqrt{2}}(rb-br), \frac{1}{\sqrt{2}}(bg-gb), \frac{1}{\sqrt{2}}(gr-rg)$$

Antisymmetric

$$\mathbf{6}: \qquad rr, bb, gg, \frac{1}{\sqrt{2}}(rb+br), \frac{1}{\sqrt{2}}(bg+gb), \frac{1}{\sqrt{2}}(gr+rg)$$

Symmetric

Color Factors - VII

Sextet

rr Just as an example: Result is the same for all sextet states

$$c_1 = c_2 = c_3 = c_4 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$f = \frac{1}{4}\sum_{\alpha=1}^{8}\left[\begin{pmatrix} 1 & 0 & 0 \end{pmatrix}\lambda^{\alpha}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}\right]\left[\begin{pmatrix} 1 & 0 & 0 \end{pmatrix}\lambda^{\alpha}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}\right] = \frac{1}{4}\sum_{\alpha=1}^{8}\left(\lambda_{11}^{\alpha}\lambda_{11}^{\alpha}\right)$$

$$= \frac{1}{4}\left(\lambda_{11}^{3}\lambda_{11}^{3} + \lambda_{11}^{8}\lambda_{11}^{8}\right) = \frac{1}{3}$$

Color Factors - VIII

Triplet

$$\frac{1}{\sqrt{2}}(rb-br)$$

Just as an example as before

$$f=\frac{1}{4}\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\sum_{\alpha=1}^{8}$$

$$\left\{\left[\begin{pmatrix}1&0&0\end{pmatrix}\lambda^\alpha\begin{pmatrix}1\\0\\0\end{pmatrix}\right]\left[\begin{pmatrix}0&1&0\end{pmatrix}\lambda^\alpha\begin{pmatrix}0\\1\\0\end{pmatrix}\right]-\left[\begin{pmatrix}0&1&0\end{pmatrix}\lambda^\alpha\begin{pmatrix}1\\0\\0\end{pmatrix}\right]\left[\begin{pmatrix}1&0&0\end{pmatrix}\lambda^\alpha\begin{pmatrix}0\\1\\0\end{pmatrix}\right]\right.$$

$$\left.-\left[\begin{pmatrix}1&0&0\end{pmatrix}\lambda^\alpha\begin{pmatrix}0\\1\\0\end{pmatrix}\right]\left[\begin{pmatrix}0&1&0\end{pmatrix}\lambda^\alpha\begin{pmatrix}1\\0\\0\end{pmatrix}\right]-\left[\begin{pmatrix}0&1&0\end{pmatrix}\lambda^\alpha\begin{pmatrix}0\\1\\0\end{pmatrix}\right]\left[\begin{pmatrix}1&0&0\end{pmatrix}\lambda^\alpha\begin{pmatrix}1\\0\\0\end{pmatrix}\right]\right\}$$

$$f=\frac{1}{8}\sum_{\alpha=1}^{8}\left\{\lambda_{11}^\alpha\lambda_{22}^\alpha-\lambda_{21}^\alpha\lambda_{12}^\alpha-\lambda_{12}^\alpha\lambda_{21}^\alpha+\lambda_{22}^\alpha\lambda_{11}^\alpha\right\}$$

$$\to f=\frac{1}{4}\sum_{\alpha=1}^{8}\left\{\lambda_{11}^\alpha\lambda_{22}^\alpha-\lambda_{12}^\alpha\lambda_{21}^\alpha\right\}=\frac{1}{4}\left\{\lambda_{11}^3\lambda_{22}^3+\lambda_{11}^8\lambda_{22}^8-\lambda_{12}^1\lambda_{21}^1-\lambda_{12}^2\lambda_{21}^2\right\}=-\frac{2}{3}$$

Color Factors - IX

Matrix elements just calculated:
Very similar to the corresponding tree-level amplitudes in QED

→Expect similar, Coulomb-like, effective potential in the static limit

Constant depends on the color representation for the quark pair:

$$V_{q\bar{q}} = \begin{cases} -\frac{4}{3}\frac{\alpha_s}{r} & \text{singlet} \\ \frac{1}{6}\frac{\alpha_s}{r} & \text{octet} \end{cases}$$

Attractive

$$V_{qq} = \begin{cases} -\frac{2}{3}\frac{\alpha_s}{r} & \text{triplet} \\ \frac{1}{3}\frac{\alpha_s}{r} & \text{sextet} \end{cases}$$

Attractive

Expect maximal attraction in singlet

Color Factors - X

Baryons could be in any one of the ***1,8,10*** representations:
Why only the singlet is observed?
A hint of an explaination:

$$\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = (\mathbf{3} \otimes \mathbf{3}) \otimes \mathbf{3}$$

$$\mathbf{3} \otimes \mathbf{3} = \mathbf{6} \oplus \mathbf{3}^{*} \rightarrow (\mathbf{3} \otimes \mathbf{3}) \otimes \mathbf{3} = (\mathbf{6} \oplus \mathbf{3}^{*}) \otimes \mathbf{3}$$

$$\mathbf{6} \otimes \mathbf{3} = \mathbf{10} \oplus \mathbf{8}$$

$$\mathbf{3}^{*} \otimes \mathbf{3} = \mathbf{1} \oplus \mathbf{8}$$

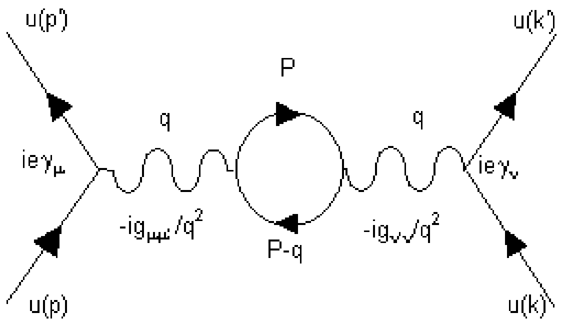
1: *each qq pair is a triplet* $\rightarrow$ *attractive*
8: *qq pair can be triplets, or sextet* $\rightarrow$ *attractive + repulsive*
10: *each qq pair is a sextet* $\rightarrow$ *repulsive*

So singlet is the state maximally attractive for 3 quarks

Does this explain the singlet-only mystery of bound states?

Running Coupling: QED - I

Consider the *one loop* modification to the photon propagator:



Includes a sum over P, the momentum circulating in the virtual loop. No obvious bounds on P..

$$M \propto \left[e\bar{u}(k')\gamma^{\mu}u(k)\right]\frac{g_{\mu\mu'}}{q^2}\frac{1}{(2\pi)^4}\int d^4P\frac{\left[e\bar{u}(P)\gamma^{\mu'}u(P-q)\right]}{P^2-m^2}\frac{\left[e\bar{u}(P-q)\gamma^{\nu'}u(P)\right]}{(P-q)^2-m^2}\frac{g_{\nu\nu'}}{q^2}\left[e\bar{u}(p')\gamma^{\nu}u(p)\right]$$

Modified propagator:

$$\frac{g_{\mu\nu}}{q^2}\to\frac{g_{\mu\nu}}{q^2}\left(1-I\left(q^2\right)\right),\quad I\left(q^2\right)=\frac{\alpha}{3\pi}\int_{m^2}^{\infty}\frac{dp^2}{p^2}-\frac{2\alpha}{\pi}\int_0^1 dx\,x(1-x)\ln\left[1-\frac{q^2x(1-x)}{m^2}\right]$$

Running Coupling: QED - II

Take the high q^2 approximation

$$-q^2 \gg m^2 \rightarrow \ln\left[1-\frac{q^2x(1-x)}{m^2}\right] \approx \ln\left[-\frac{q^2}{m^2}\right]$$

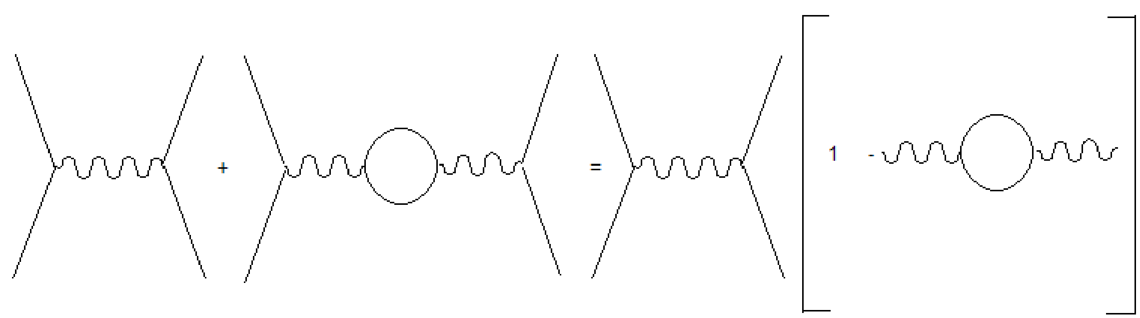
Provisional upper bound (cutoff) to make integral converging

$$I\left(q^2\right) \approx \frac{\alpha}{3\pi}\int_{m^2}^{M^2}\frac{dp^2}{p^2} - \frac{2\alpha}{\pi}\int_0^1 dx\,x(1-x)\ln\left[\frac{-q^2}{m^2}\right]$$

$$I\left(q^2\right) \approx \frac{\alpha}{3\pi}\ln\left(\frac{M^2}{m^2}\right) - \frac{2\alpha}{\pi}\frac{1}{6}\ln\left[\frac{-q^2}{m^2}\right] = \frac{\alpha}{3\pi}\left[\ln\left(\frac{M^2}{m^2}\right) - \ln\left[\frac{-q^2}{m^2}\right]\right] = \frac{\alpha}{3\pi}\ln\left(\frac{M^2}{-q^2}\right)$$

$$M \propto \alpha\left[\bar{u}(k')\gamma^\mu u(k)\right]\frac{g_{\mu\nu}}{q^2}\left[1-\frac{\alpha}{3\pi}\ln\left(\frac{M^2}{-q^2}\right)\right]\left[\bar{u}(p')\gamma^\nu u(p)\right]$$

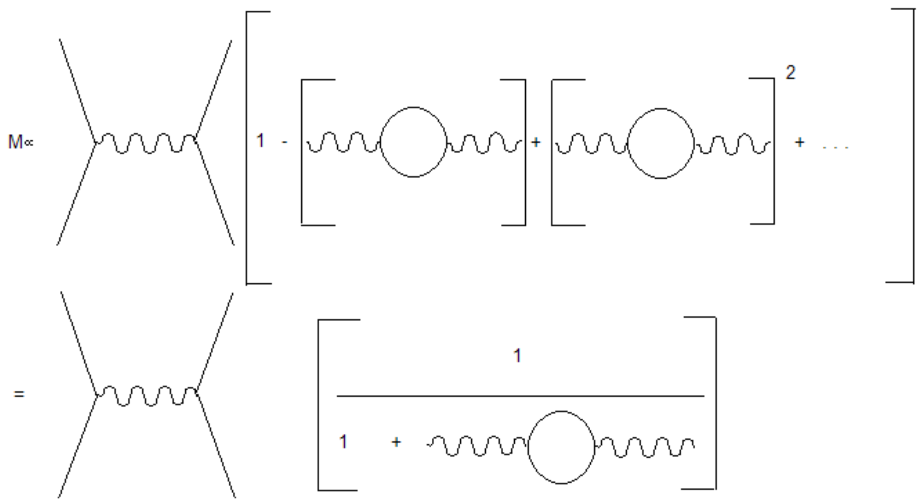
Cartoon translation:



Running Coupling: QED - III

Extend to diagrams with *2,3,…,n,…* loops: Add up all contributes

Sum of a 'geometrical series': Converging ??



Experts say this is the only contribution to running α to the 'leading logs' approximation, which means neglecting the next levels of iteration

Running Coupling: QED - IV

$$M \propto \left[\bar{u}(k')\gamma^{\mu}u(k)\right]\frac{g_{\mu\nu}}{q^2}\left[\frac{\alpha}{1+\alpha/3\pi\ln\left(M^2/-q^2\right)}\right]\left[\bar{u}(p')\gamma^{\nu}u(p)\right]$$

What is α?

Coupling 'constant' we would get should we turn off all loops

Call it α_0 = 'Bare' coupling constant, not physical:

Loops cannot be turned off

Then obtain an effective coupling, not constant but running:

$$\alpha\left(q^2\right)=\frac{\alpha_0}{1+\alpha_0/3\pi\ln\left(M^2/-q^2\right)}$$

α is q^2, or distance, dependent!

Running Coupling: QED - V

Running α is still cutoff dependent, which of course is uncomfortable
But: Not a real problem.
Indeed:

$$Q^2 = -q^2 \rightarrow \alpha\left(Q^2\right) = \frac{\alpha_0}{1 + \left(\alpha_0/3\pi\right)\ln\left(M^2/Q^2\right)}$$

Take a particular energy scale : $Q^2 = \mu^2$

$$\rightarrow \alpha\left(\mu^2\right) = \frac{\alpha_0}{1 + \left(\alpha_0/3\pi\right)\ln\left(M^2/\mu^2\right)}$$

Usually choose $\mu^2=0$, i.e. take α at distance $\rightarrow \infty$

Quite natural in QED (but not compulsory)

Running Coupling: QED - VI

$$\ln\left(\frac{M^2}{Q^2}\right) = \ln\left(\frac{M^2}{Q^2}\frac{\mu^2}{\mu^2}\right) = \ln\left(\frac{M^2}{\mu^2}\right) + \ln\left(\frac{\mu^2}{Q^2}\right)$$

$$\to \alpha\left(Q^2\right) = \frac{\alpha_0}{1 + (\alpha_0/3\pi)\left[\ln\left(M^2/\mu^2\right) + \ln\left(\mu^2/Q^2\right)\right]}$$

$$\to \frac{\alpha_0}{\alpha\left(\mu^2\right)} = 1 + (\alpha_0/3\pi)\ln\left(M^2/\mu^2\right)$$

$$\to \alpha\left(Q^2\right) = \frac{\alpha_0}{\alpha_0/\alpha\left(\mu^2\right) + (\alpha_0/3\pi)\ln\left(\mu^2/Q^2\right)} = \frac{\alpha\left(\mu^2\right)}{1 - \left[\alpha\left(\mu^2\right)/3\pi\right]\ln\left(Q^2/\mu^2\right)}$$

Very interesting result: Running α depends on q^2, through its own *measured* value at any chosen energy scale μ^2.

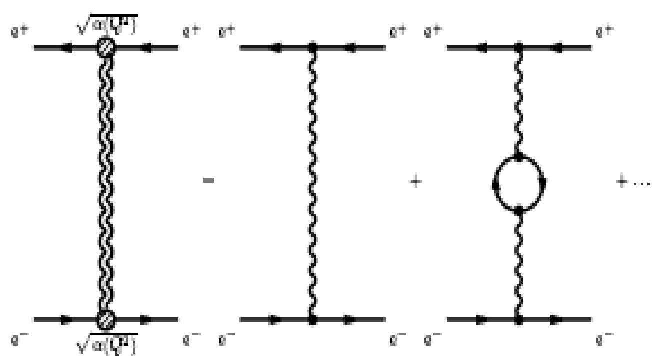
Cutoff has disappeared.

Running Coupling: QED - VII

Deep physics involved:

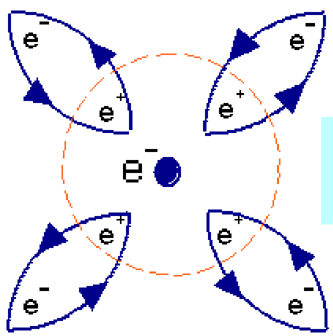
A ∞ number of diagrams can be formally replaced by a single, 1-photon diagram where the coupling 'constant' is running with q^2

Result valid to the 'leading log' approximation



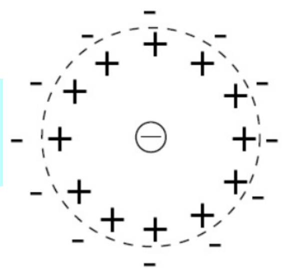
Running Coupling: QED - VIII

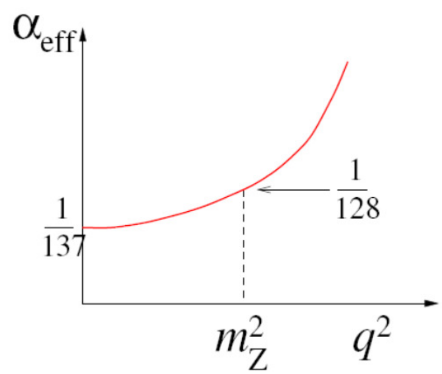
Virtual (loops) e^+e^- pairs



@TBA

Effective shielding





Picture the QED perturbative vacuum as a sort of (virtual) dielectric medium: Virtual photons and e^+e^- pairs continuously created/annihilated

Bare charge is shielded at large distance by the virtual pairs coming from loops.

The standard e charge is smaller than the bare charge

By probing the electron at smaller and smaller distance, observe an *increasing* effective charge

Running α at LEP - I

Experimental method: Bhabha scattering

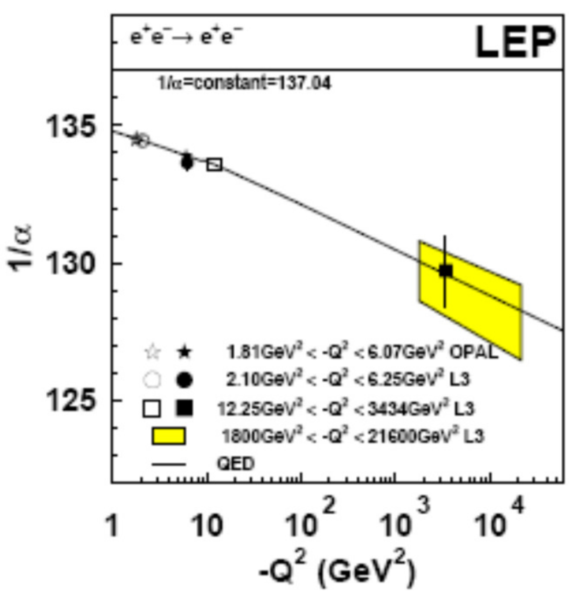
$$\frac{d\sigma}{dt} = \frac{d\sigma^0}{dt}\left(\frac{\alpha(t)}{\alpha_0}\right)^2 (1+\varepsilon)(1+\delta_\gamma)+\delta_Z,$$

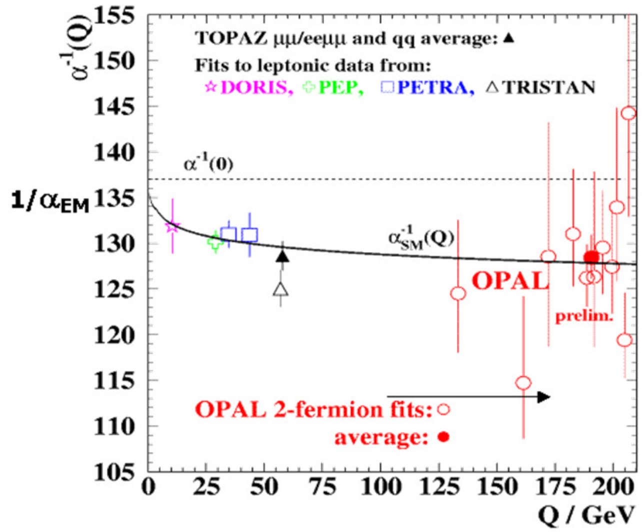
δ_γ, δ_Z s - channel contributions (small)

ε radiative corrections (known)

Use accurate, differential cross-section measurement to unfold $\alpha(t)$

[Total cross-section measurement would require a luminosity]



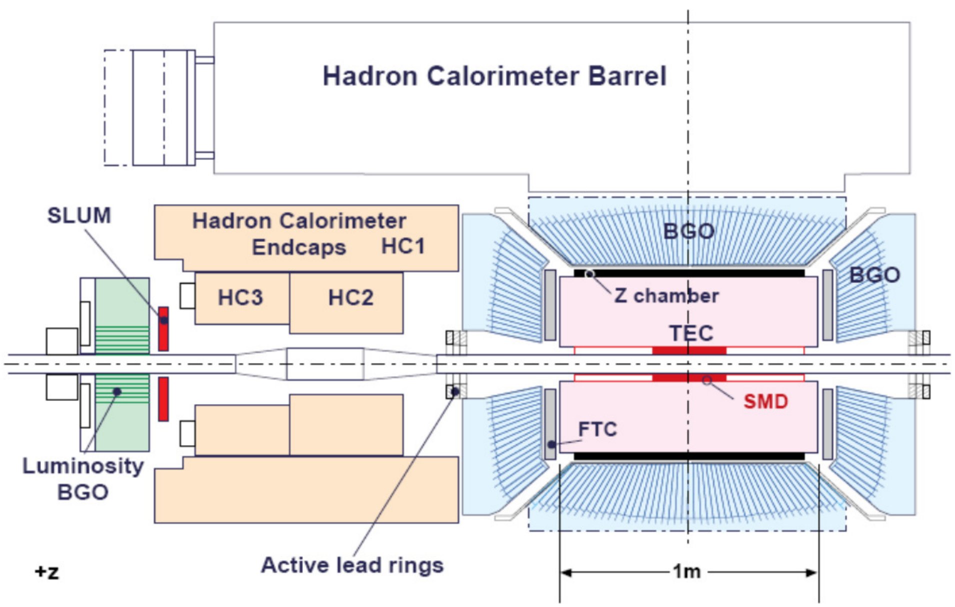


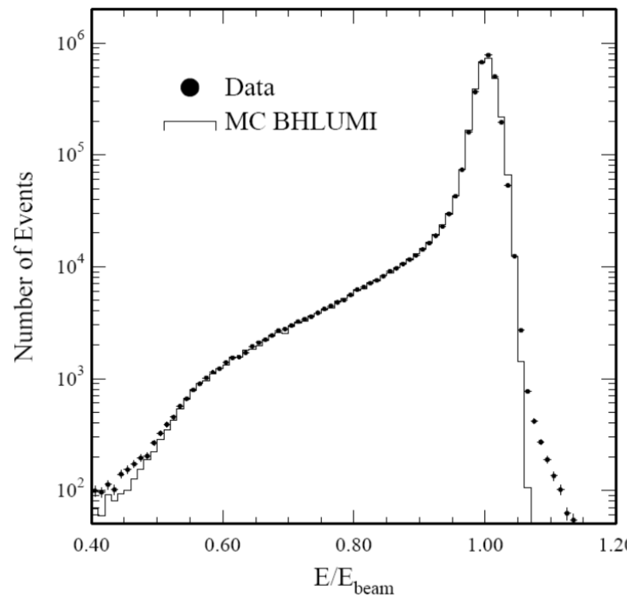
Running α at LEP - II

Just as an example, take L3 at LEP:
Relying on Bhabha scattering at small angle

$$\sigma = \frac{16\pi\alpha^2}{s}\left(\frac{1}{\theta^2_{min}} - \frac{1}{\theta^2_{max}}\right)$$

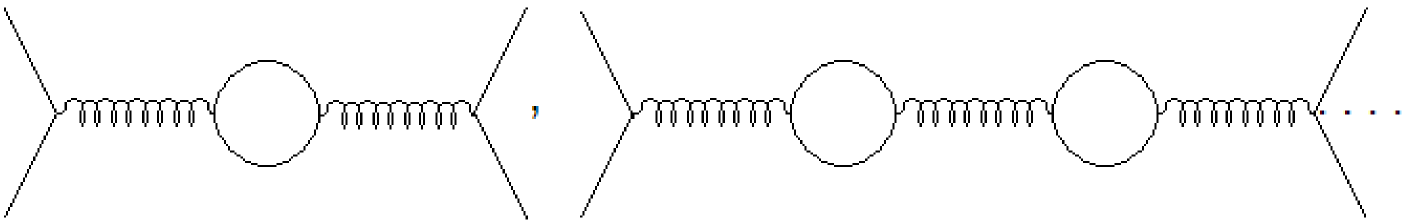
Calorimeter (Energy) + Silicon Tracker (Angle + Acceptance)



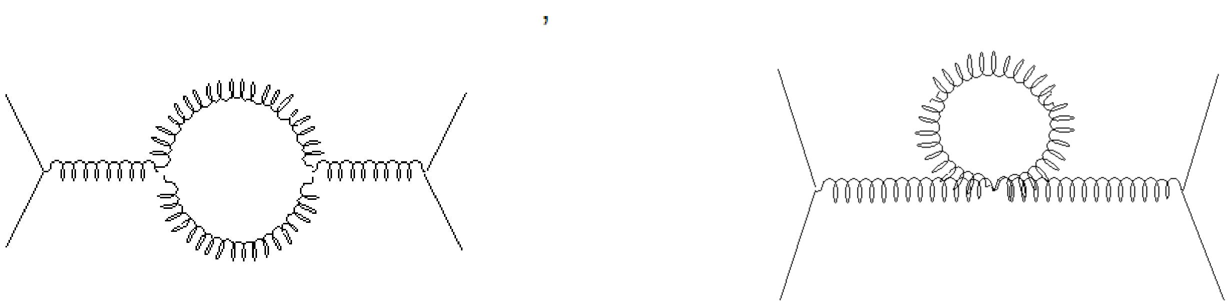


Running Coupling: QCD - I

Repeat all the steps: Loops etc



Except this time one has more loops: Gluons



Running Coupling: QCD - II

Turns out gluon loops yield *anti*-shielding effect
With 8 gluons and 6 quark flavors, gluons win

$$\alpha_s\left(\left|q^2\right|\right)=\frac{\alpha_s\left(\mu^2\right)}{1+\left(\alpha_s\left(\mu^2\right)/12\pi\right)\left(33-2n_{flavor}\right)\ln\left(\left|q^2\right|/\mu^2\right)}$$

Running coupling *decreases* with increasing q^2 (or at smaller distance)
This is known as *asymptotic freedom*:

Large q^2 processes feature small coupling $\rightarrow$ Perturbative!

Most important consequence:

The fundamental hypothesis behind the successful parton model is finally understood and justified

Running Coupling: QCD - III

Rather than making reference to a specific value of α_s

$$\alpha_s\left(\left|q^2\right|\right)=\frac{\alpha_s\left(\mu^2\right)}{1+\left(\alpha_s\left(\mu^2\right)/12\pi\right)\left(33-2n_{flavor}\right)\ln\left(\left|q^2\right|/\mu^2\right)}$$

define a new constant

$$\ln\Lambda^2=\ln\mu^2-\frac{12\pi}{\left(33-2n_{flavor}\right)\alpha_s\left(\mu^2\right)}\rightarrow\Lambda^2=\mu^2 e^{-\frac{12\pi}{\left(33-2n_{flavor}\right)\alpha_s\left(\mu^2\right)}}$$

$$\rightarrow\alpha_s\left(\left|q^2\right|\right)\simeq\frac{12\pi}{\left(33-2n_{flavor}\right)\ln\left(\left|q^2\right|/\Lambda^2\right)}=\frac{12\pi}{21\ln\left(\left|q^2\right|/\Lambda^2\right)},\quad\left|q^2\right|\gg\Lambda^2$$

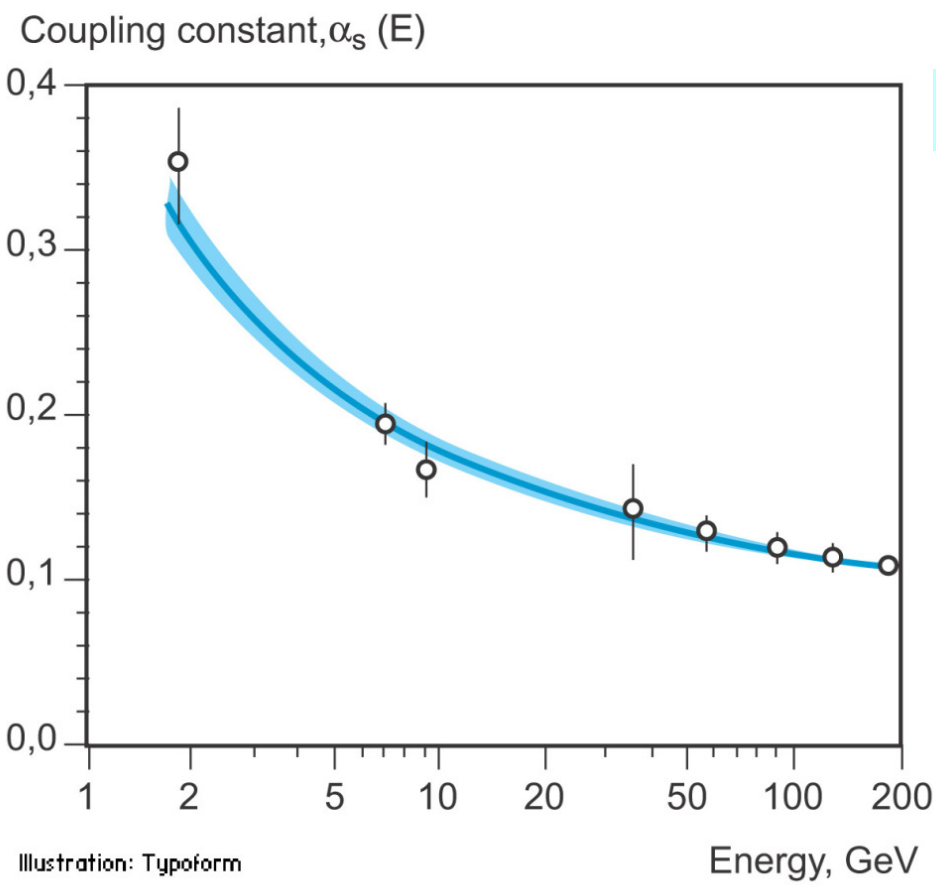
Λ = Renormalization scale $\rightarrow$ Fixes α_s at all q^2

$\Lambda\approx 200$ MeV yields the correct α_s at $\mu^2=M_{Z^0}^2$

Funny behavior, known as 'Dimensional Transmutation':

From an adimensional constant to a dimensional one $\alpha_s\rightarrow\Lambda$

Running Coupling: QCD - IV



@TBA

Sources:

Jets

DIS

Quarkonium

Annihilation: Muons vs Quarks

Electron-Positron annihilation:

Electroweak process

Low energy ($E_{CM} \ll M_{Z^0}$): Mostly electromagnetic

High energy ($E_{CM} \sim M_{Z^0}$): Mostly neutral current

Final state: *Fermion / Antifermion* pair

Muon vs quark pairs

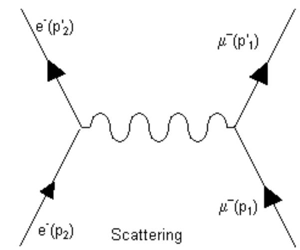
Best observed at e^+e^- colliders

Annihilation Cross-Section - I

Apply crossing symmetry to electron-muon scattering, take pure e.m. amplitude at tree level

$$e^- + \mu^- \to e^- + \mu^-$$ A: Scattering

$$e^- + \underset{crossed}{\left[e^-\right]} \to \underset{crossed}{\left[\mu^-\right]} + \mu^- \equiv e^- + e^+ \to \mu^+ + \mu^-$$ B: Annihilation



Amplitude for scattering:

$$T_{fi}(s,s',r,r') = (-e)\bar{u}_{(\mu)}(p_2',s')\gamma^\mu u_{(\mu)}(p_2,s)\frac{-ig_{\mu\nu}}{q^2}(-e)\bar{u}_{(e)}(p_1',r')\gamma^\nu u_{(e)}(p_1,r)$$

$$q = p_1 - p_1' \to q^2 = \left(p_1 - p_1'\right)^2 = p_1^2 + p_1'^2 - 2p_1 \cdot p_1'$$

$$q^2 = 2m_e^2 - 2\left(E_1E_1' - \mathbf{p}_1 \cdot \mathbf{p}_1'\right) \underset{E \gg m}{\simeq} -2\left(E_1E_1' - \mathbf{p}_1 \cdot \mathbf{p}_1'\right) < 0$$

q=4-momentum transfer

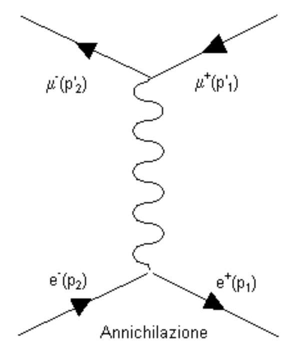
Amplitude for annihilation:

$$T_{fi}(s,s',r,r') = (-e)\bar{u}_{(\mu)}(p_2',s')\gamma^\mu v_{(\mu)}(p_1',r')\frac{-ig_{\mu\nu}}{q^2}(-e)\bar{v}_{(e)}(p_1,s)\gamma^\nu u_{(e)}(p_2,r)$$

$$q = p_1 + p_2 \to q^2 = \left(p_1 + p_2\right)^2 = p_1^2 + p_2^2 + 2p_1 \cdot p_2$$

$$q^2 = 2m_e^2 + 2\left(E_1E_2 - \mathbf{p}_1 \cdot \mathbf{p}_2\right) \underset{E \gg m}{\simeq} 2\left(E_1E_2 - \mathbf{p}_1 \cdot \mathbf{p}_2\right) > 0$$

q=total 4-momentum



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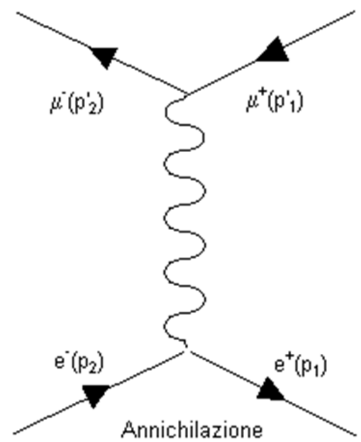
Annihilation Cross-Section - II



$$T_{fi} = \overline{v}(p_1, s')\gamma^{\mu} u(p_2, s)\frac{e^2}{q^2}\overline{u}(p_2', r)\gamma_{\mu} v(p_1', r')$$

$$\left|T_{fi}\right|^2 = \frac{e^4}{q^4}\left[\overline{v}(p_1, s')\gamma^{\mu} u(p_2, s)\overline{u}(p_2', s)\gamma^{\nu} v(p_1', s')\right]\left[\overline{v}(p_1, r)\gamma_{\mu} u(p_2, r')\overline{u}(p_2', r')\gamma_{\nu} v(p_1', r)\right]$$

Annihilation Cross-Section - III

$$\left|T_{fi}\right|^2 = \frac{e^4}{q^4}\left[\bar{v}(p_1,s')\gamma^\mu u(p_2,s)\bar{u}(p_2',s)\gamma^\nu v(p_1',s')\right]\left[\bar{v}(p_1,r)\gamma_\mu u(p_2,r')\bar{u}(p_2',r')\gamma_\nu v(p_1',r)\right]$$

$$\left\langle\left|T_{fi}\right|^2\right\rangle = \frac{1}{4}\sum_{s,s',r,r'}\left|T_{fi}\right|^2 = \frac{e^4}{4q^4}Tr\left[(\not{p}_1 - m)\gamma^\mu(\not{p}_2 + m)\gamma^\nu\right]Tr\left[(\not{p}_2' + M)\gamma_\mu(\not{p}_1' - M)\gamma_\nu\right]$$

$$Tr\left[(\not{p}_1 - m)\gamma^\mu(\not{p}_2 + m)\gamma^\nu\right] = 4\left[p_1^\mu p_2^\nu + p_1^\nu p_2^\mu - g^{\mu\nu}\left(p_2\cdot p_1 + m^2\right)\right]$$

$$Tr\left[(\not{p}_2' + M)\gamma_\mu(\not{p}_1' - M)\gamma_\nu\right] = 4\left[p_1^\mu{}' p_2^\nu{}' + p_1^\nu{}' p_2^\mu{}' - g^{\mu\nu}\left(p_2'\cdot p_1' + M^2\right)\right]$$

$$\rightarrow \left\langle\left|T_{fi}\right|^2\right\rangle = \frac{8e^4}{q^4}\left[(p_1\cdot p_1')(p_2\cdot p_2') + (p_1\cdot p_2')(p_2\cdot p_1') + M^2(p_1\cdot p_2)\right] \quad m \approx 0$$

$$CM: \begin{cases} p_1 = (E \quad 0 \quad 0 \quad E),\ p_2 = (E \quad 0 \quad 0 \quad -E) \\ |\mathbf{p}'| = |\mathbf{p}_1'| = |\mathbf{p}_2'| = \sqrt{E^2 - M^2} \end{cases}$$

$$p_1' = \left(E \quad |\mathbf{p}'|\sin\theta \quad 0 \quad |\mathbf{p}'|\cos\theta\right),\ p_2' = \left(E \quad -|\mathbf{p}'|\sin\theta \quad 0 \quad -|\mathbf{p}'|\cos\theta\right)$$

$$\rightarrow \left\langle\left|T_{fi}\right|^2\right\rangle = e^4\left[1 + \frac{M^2}{E^2} + \left(1 - \frac{M^2}{E^2}\right)\cos^2\theta\right]$$

Annihilation Cross-Section - IV

$$\left.\frac{d\sigma}{d\Omega}\right|_{CM} = \frac{\alpha^2}{4s}\sqrt{1-\frac{M^2}{E^2}}\left[1+\frac{M^2}{E^2}+\left(1-\frac{M^2}{E^2}\right)\cos^2\theta\right]$$

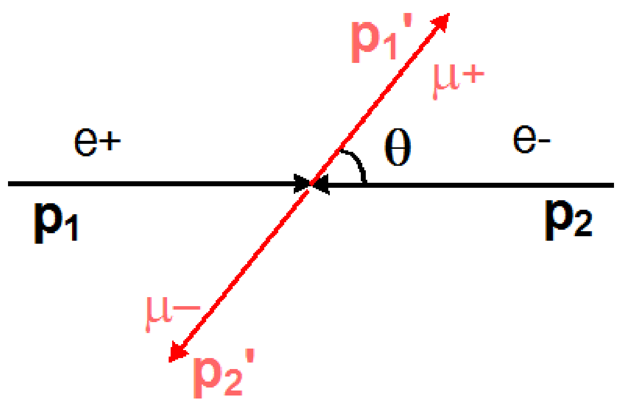
$$\rightarrow \left.\frac{d\sigma}{d\Omega}\right|_{CM} \approx \frac{\alpha^2}{4s}\left(1+\cos^2\theta\right), \qquad E \gg M$$

$$\sigma = \int_\Omega \frac{d\sigma}{d\Omega}d\Omega = \frac{4\pi\alpha^2}{3s}\sqrt{1-\frac{M^2}{E^2}}\left(1+\frac{1}{2}\frac{M^2}{E^2}\right)$$

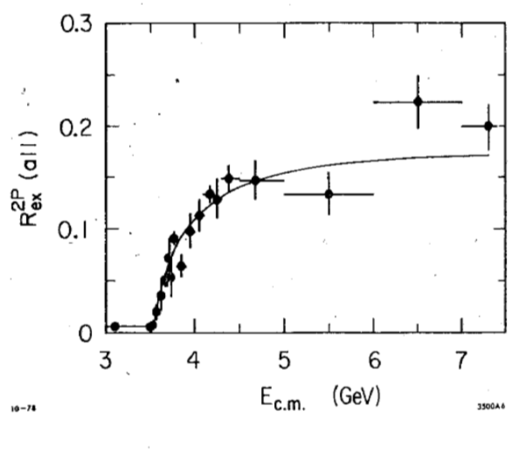
$$\gamma = \frac{E}{M} = \frac{1}{\sqrt{1-\beta^2}}$$

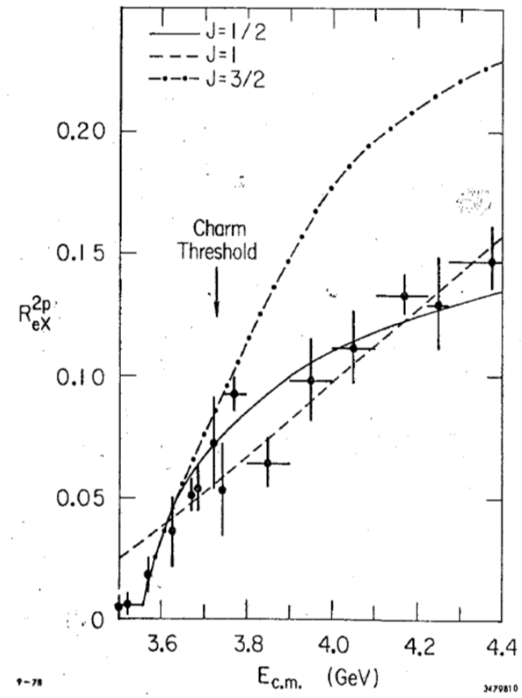
$$\rightarrow \sigma = \frac{4\pi\alpha^2}{3s}\underbrace{\sqrt{\frac{\gamma^2-1}{\gamma^2}}}_{\beta}\left(\frac{1+2\gamma^2}{2\gamma^2}\right) = \frac{4\pi\alpha^2}{3s}\beta\left(\frac{1}{2\gamma^2}+1\right) = \frac{4\pi\alpha^2}{3s}\beta\left(\frac{3-\beta^2}{2}\right)$$

$$\rightarrow \sigma \approx \frac{4\pi\alpha^2}{3s} = \frac{87}{s\left[GeV^2\right]}nb, \qquad E \gg M$$



Annihilation Cross-Section - V

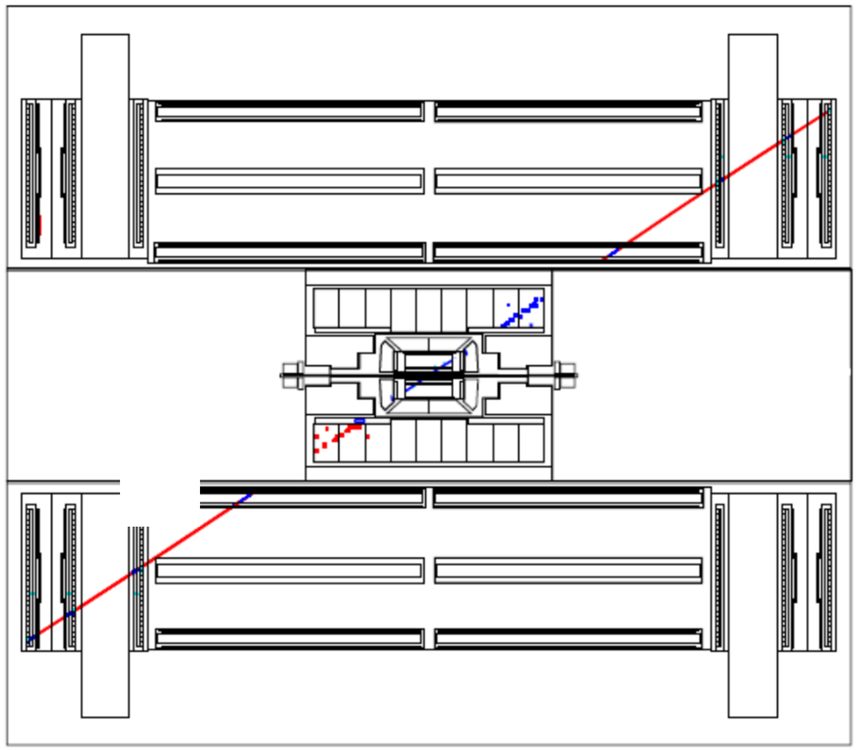




τ lepton discovery, mass & spin determination:

$$R_{eX}^{2p} \simeq \frac{\sigma\left(\tau^+\tau^-\right)}{\sigma\left(\mu^+\mu^-\right)} \simeq \sqrt{1-\frac{M_\tau^{\ 2}}{E^2}}\left(1+\frac{1}{2}\frac{M_\tau^{\ 2}}{E^2}\right), M_\mu \approx 0$$

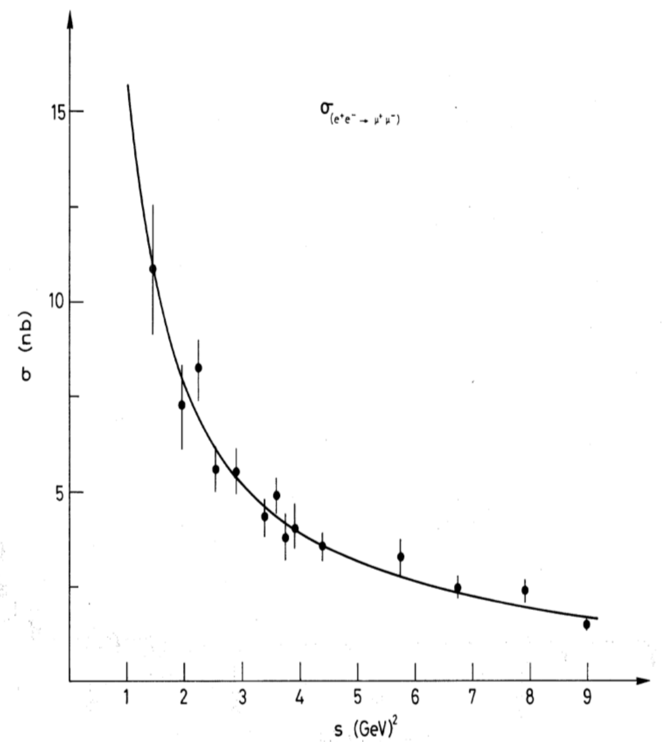
Annihilation Cross-Section - VI



$\mu^+ \mu^-$ event: L3 detector at LEP

Annihilation Cross-Section - VII

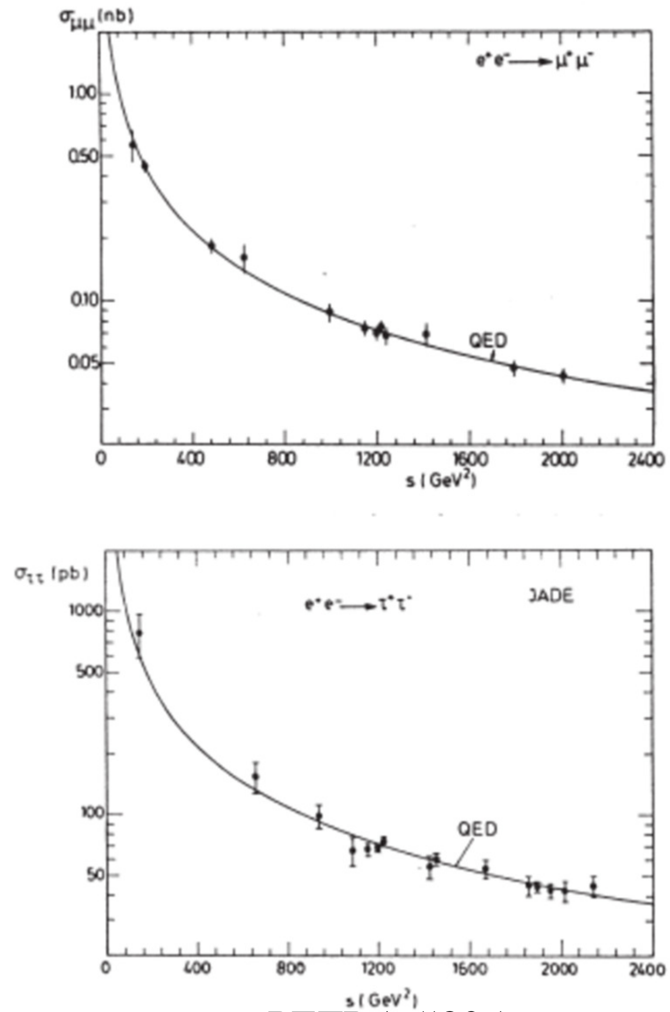
Total cross-section vs *s:*
Low energy



ADONE ('70s)

Annihilation Cross-Section - VIII

Total cross-section vs *s:*
Higher energy



PETRA (‘80s)

Annihilation Cross-Section - IX

Angular distribution: Low energy
1-photon, forward/backward symmetric

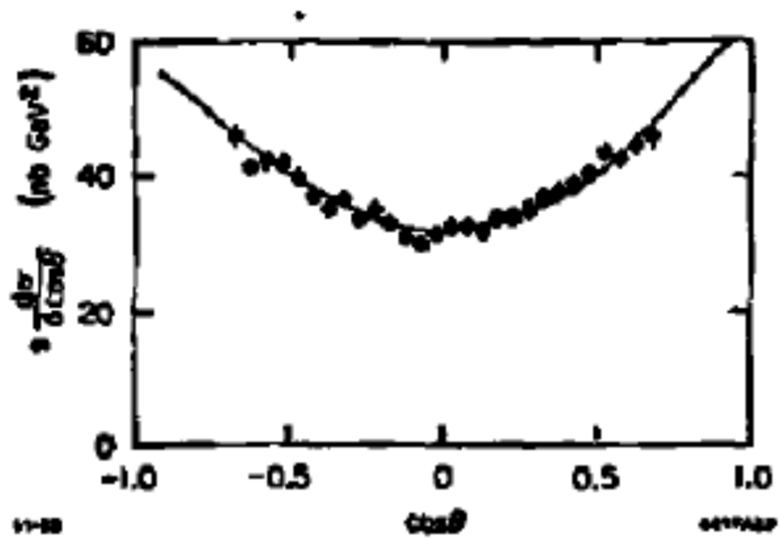
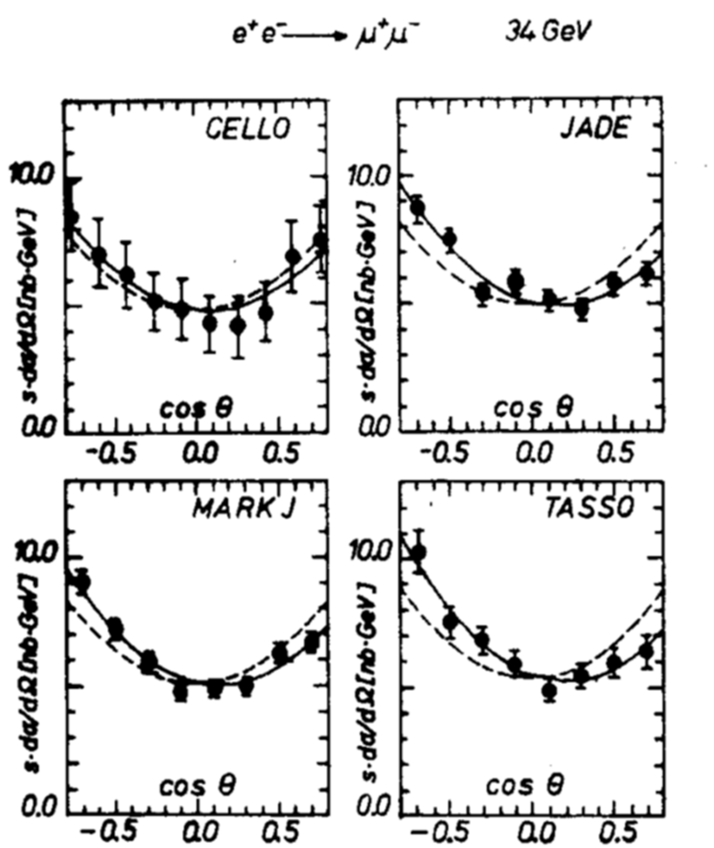


Fig. 15. MARK II $e^+e^- \to \mu^+\mu^-$ at $\langle E_{c.m.}\rangle = 5.847$ compared to $1+\cos^2\theta$.

SPEAR (‘70s)

Annihilation Cross-Section - X

Angular distribution: Higher energy:
Some contribution from Z^0 , forward/backward asymmetric



PETRA (‘80s)

Annihilation Cross-Section - XI

Forward/Backward asymmetry: Important subject

- Effective tool for precision tests of SM
- May probe physics BSM

Interesting point:

Some tiny asymmetry expected from pure QED

Coming from diagrams with >1 photon (Radiative correctio

Dominated by interference terms

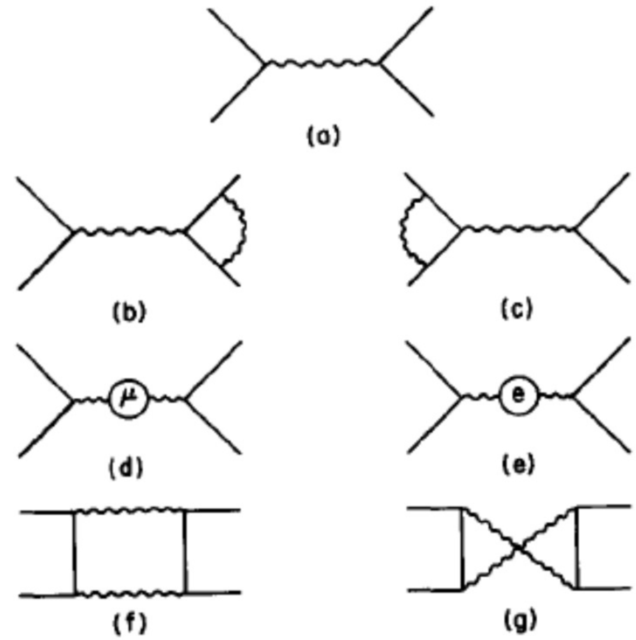
between amplitudes (a) and (b)-(g):

Opposite charge parity

Recent surge of interest from large asymmetry

found at Tevatron in $t\,\bar{t}$ production by 1 and 2 gluons:

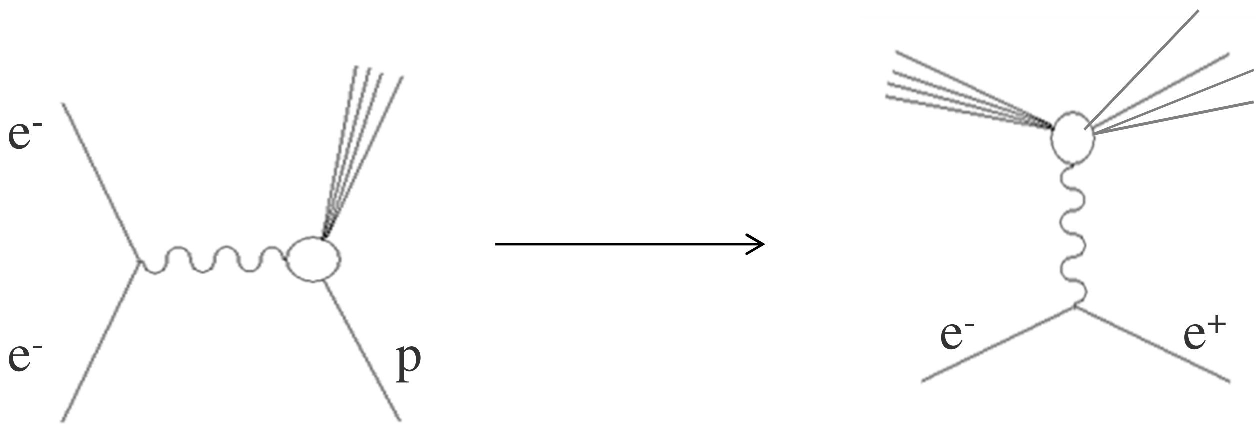
Similar physics, not fully understood



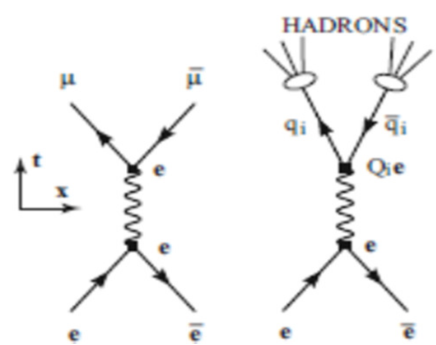
PQCD: Jets in e^+e^- Collisions - I

e^+e^- annihilation into hadrons:

At the parton level = Crossed Deep Inelastic Scattering

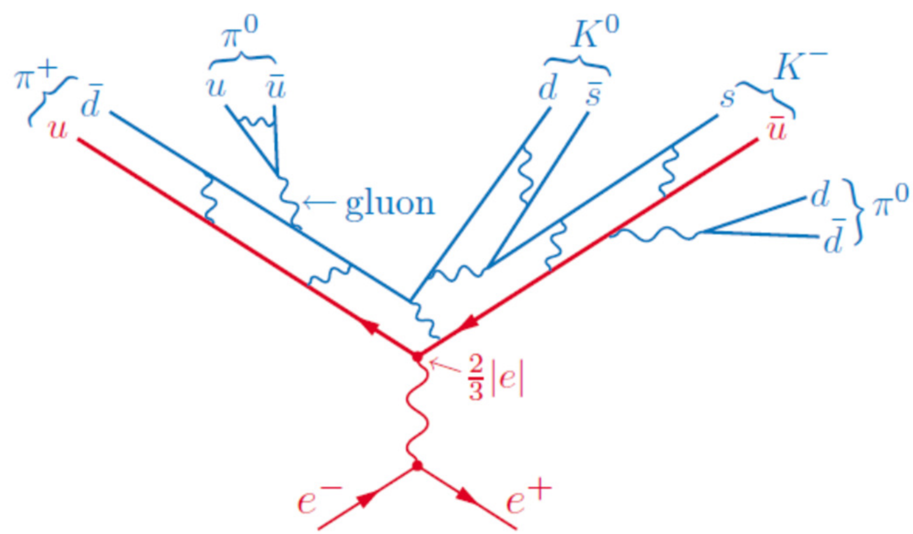


Understood as annihilation into a $q\overline{q}$ pair,
followed by quark fragmentation into hadrons



PQCD: Jets in e^+e^- Collisions - II

Picture of quark fragmentation



PQCD: Jets in e^+e^- Collisions - III

By ignoring *quark fragmentation* details

$$e^+ + e^- \to q + \overline{q} \to hadrons$$

$$\frac{d\sigma}{d\Omega} = \frac{3\alpha^2}{4s}\left(1+\cos^2\theta\right)\sum_{flavor} e_{flavor}^2$$

$$\to \sigma(s) = \frac{4\pi\alpha^2}{s}\sum_{flavor} e_{flavor}^2$$

Due to color confinement, quark/antiquark unobservable as free particles

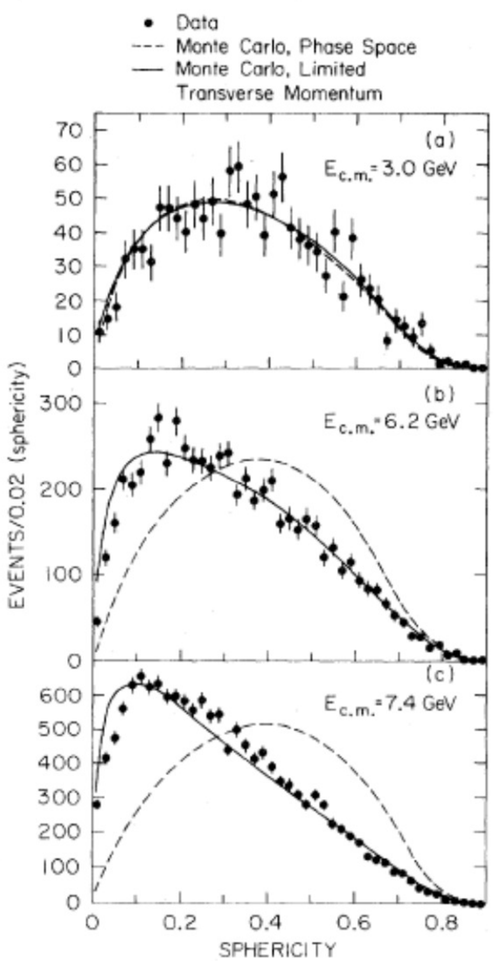
Expect hadron debris from quark fragmentation

$\to$ *Jets*
Hadrons: Trend towards grouping into narrow pencils of particles

Naive prediction: 2 jets

At high energy, expect hadronic annihilation events to be *non-spherical*

PQCD: Jets in e^+e^- Collisions - IV



Define *sphericity* of events:

$$S = \min \frac{3}{2} \frac{\sum_i p_{\perp i}^2}{\sum_i p_i^2} \rightarrow \begin{cases} 0 & \text{2-jets} \\ 1 & \text{spherical} \end{cases}$$

$\min :$ Choose axes which minimize $S\,(\leftarrow \textit{Iterative})$

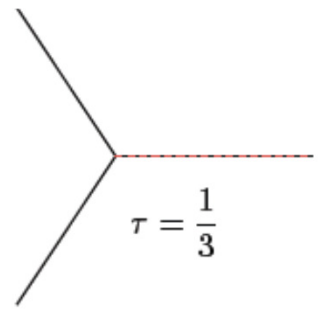
PQCD: Jets in e^+e^- Collisions - V

Interesting observable: *Thrust*

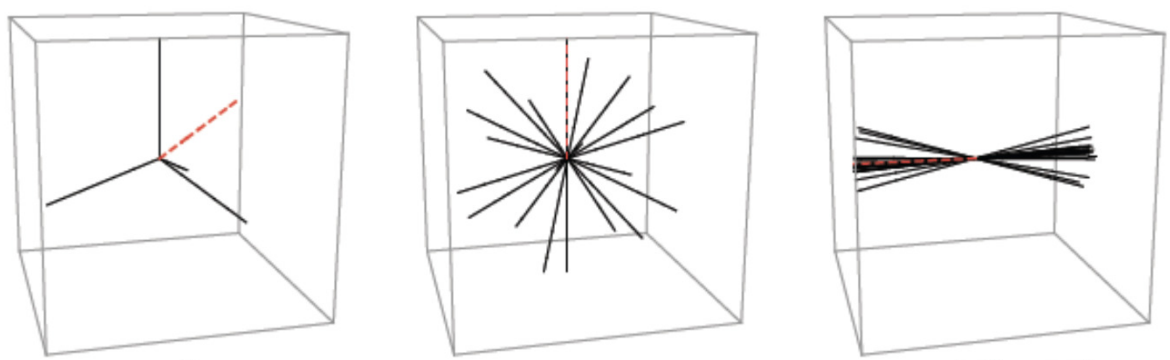
$$\tau = 0$$

$$T = \frac{1}{Q}\max_{\vec{n}} \sum_i |\vec{n}\cdot\vec{p}_i|$$

$$\tau = 1 - T$$

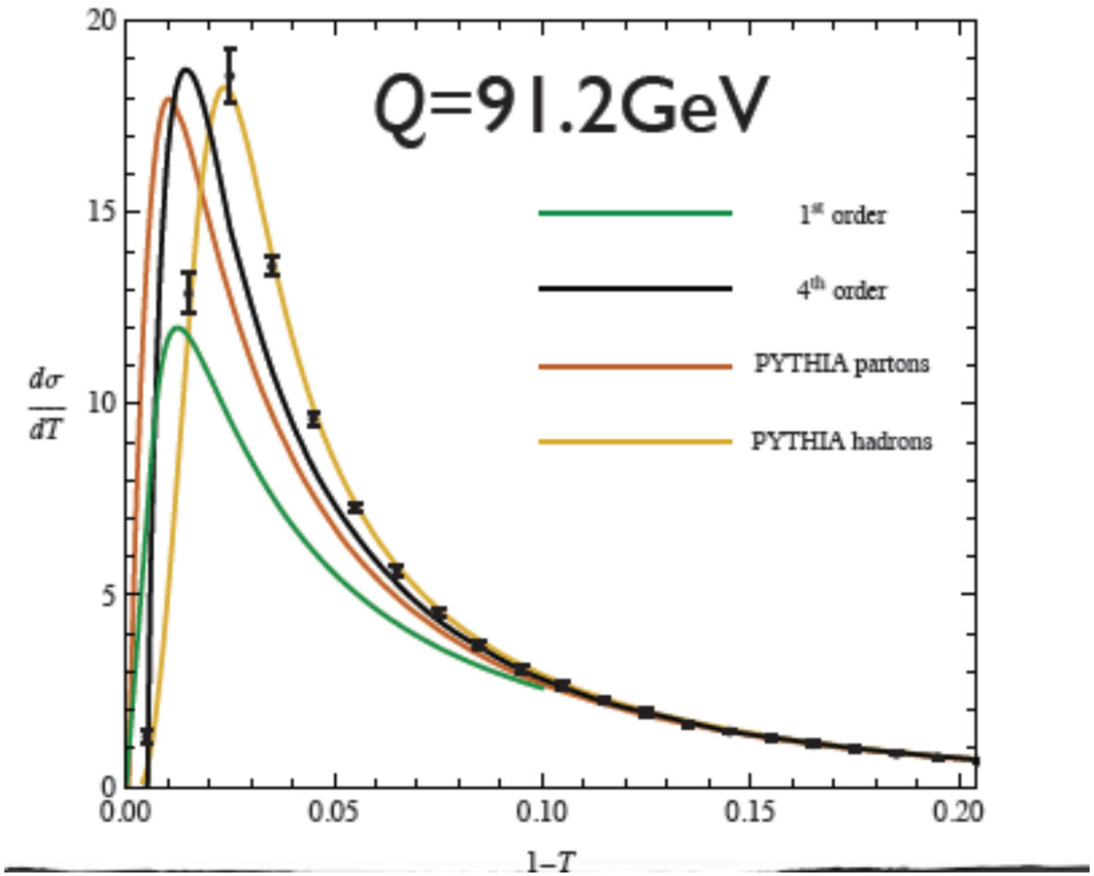


$$\tau = \frac{1}{3}$$



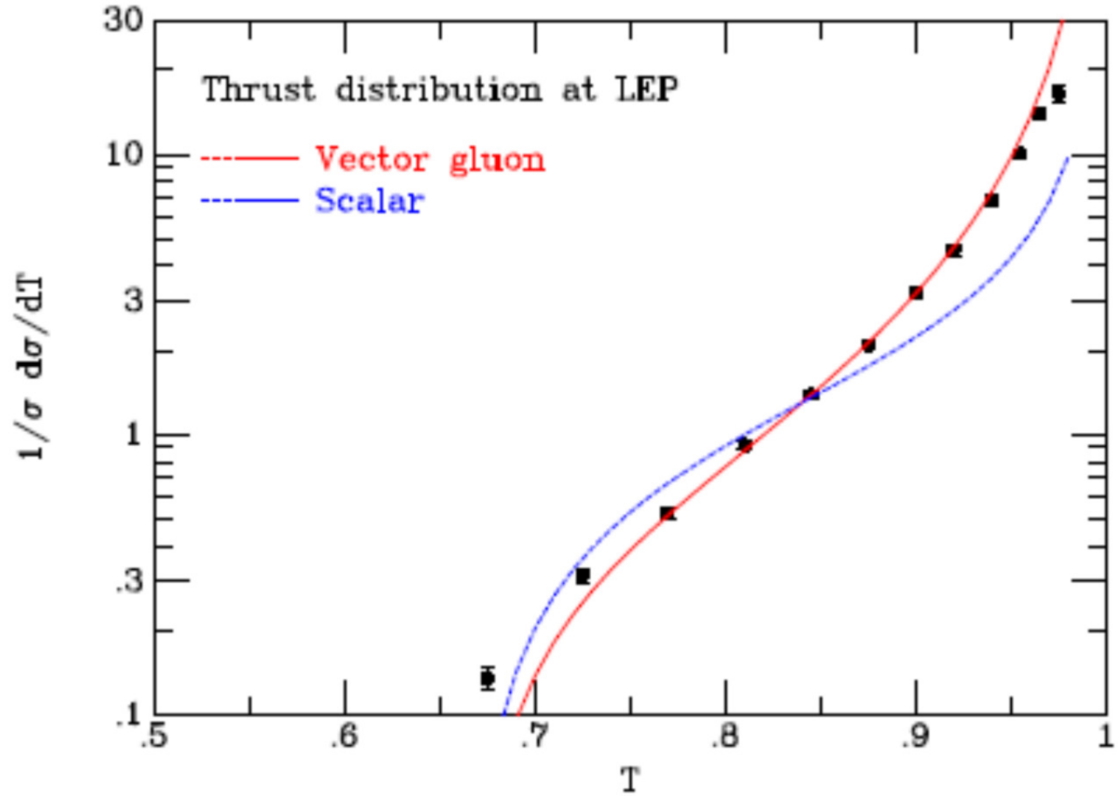
$$\tau = 1 - \frac{1}{\sqrt{3}} = 0.42 \qquad \tau = 0.48 \qquad \tau = \frac{M_1^2 + M_2^2}{Q^2}$$

PQCD: Jets in e^+e^- Collisions - VI



ALEPH

PQCD: Jets in $e^+ e^-$ Collisions - VII

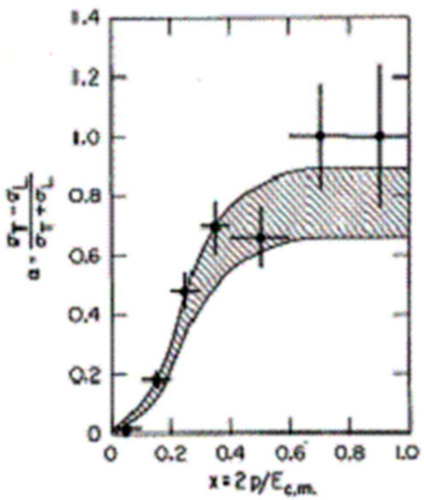


DELPHI

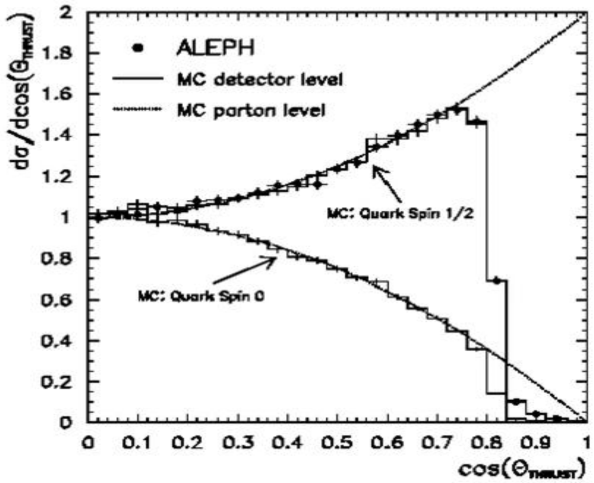
PQCD: Jets in $e^+ e^-$ Collisions - VIII

For 2 jets events

$$\left.\begin{array}{l}\dfrac{d\sigma}{d\Omega}\propto 1+\cos^2\theta \quad \text{quark spin} = 1/2 \\ \dfrac{d\sigma}{d\Omega}\propto 1-\cos^2\theta \quad \text{quark spin} = 0\end{array}\right\} \equiv 1+\alpha\cos^2\theta$$



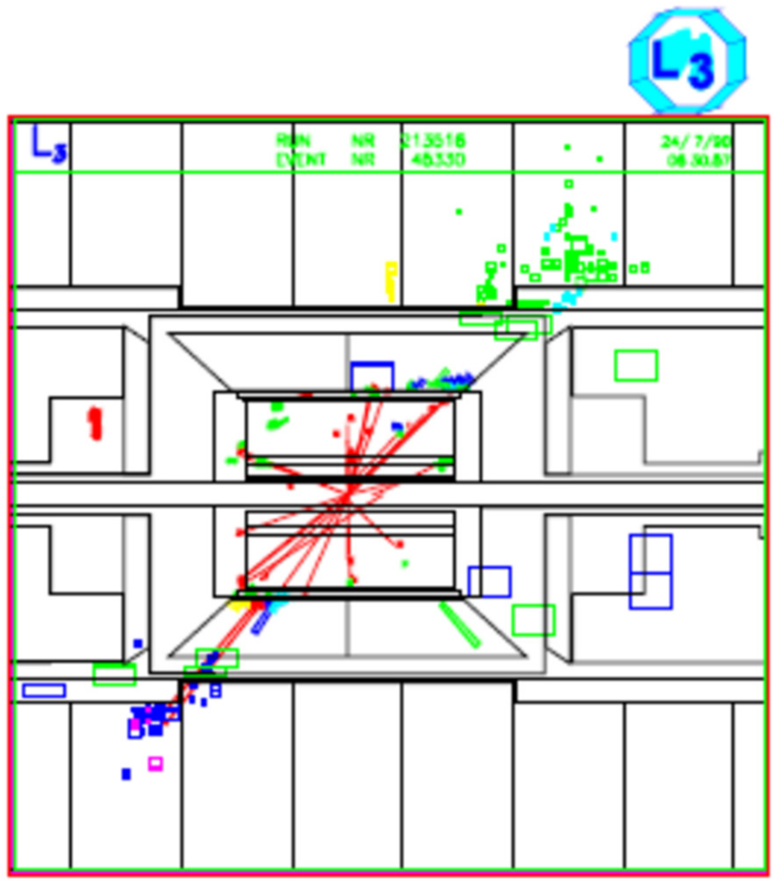
@TBA



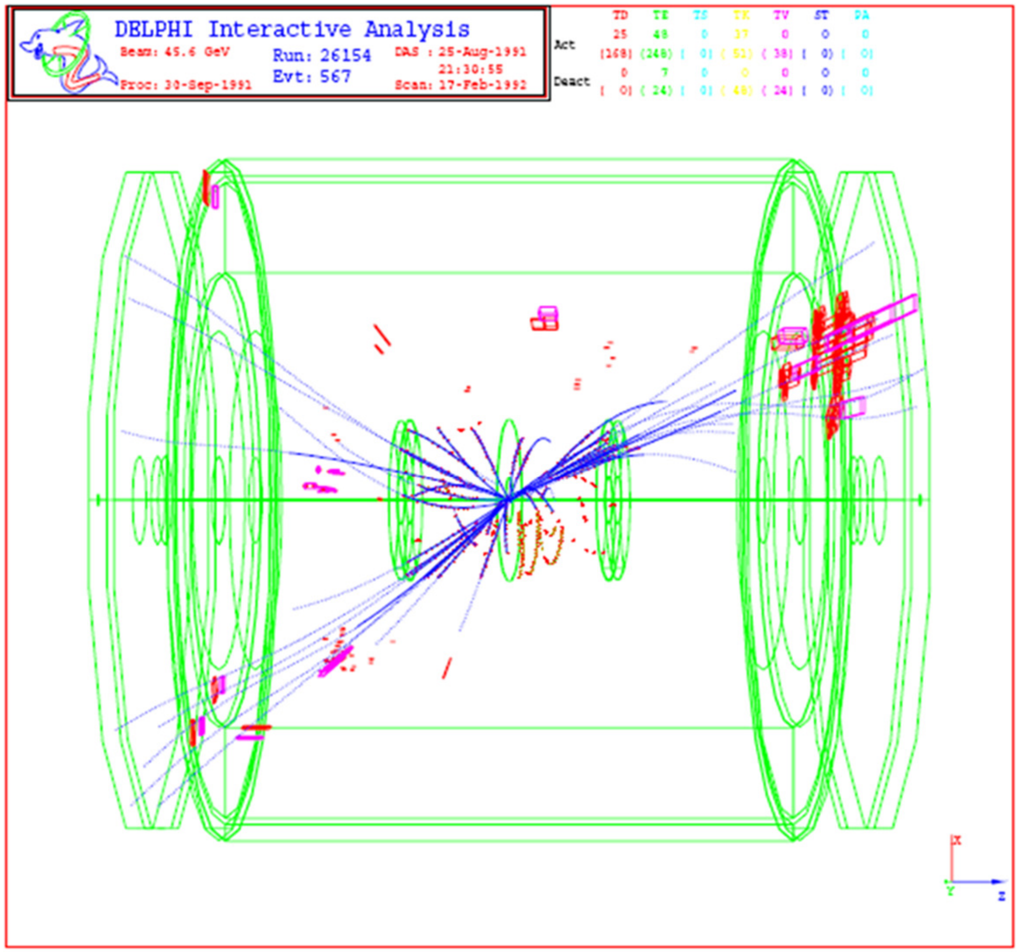
Mark I (SPEAR)
E = few GeV

ALEPH (LEP)
E = 90 GeV

PQCD: Jets in e^+e^- Collisions - IX



PQCD: Jets in e^+e^- Collisions - X



PQCD: Jets in $e^+ e^-$ Collisions - XI

Total hadronic cross section $\leftrightarrow$ R Ratio

Reminder:

Time scale of hard interaction

$$T \sim \frac{1}{Q} = \frac{1}{\sqrt{s}} \sim \frac{1}{\text{Many } GeV} \to \text{Very small}$$

Time scale of soft hadronization

$$\tau \sim \frac{1}{\Lambda} \sim \frac{1}{1\ GeV} \to \text{Large}$$

$\to$ Perturbative cross section OK

Expect at 30-40 GeV:

$$R = \frac{\sigma_{q\bar{q}}}{\sigma_{\mu\mu}} = 3 \sum_{i=u,d,s,c,b} Q_i^2 = \frac{11}{3} \approx 3.67 \left(+\ 0.05 \text{ coming from } Z^0\right)$$

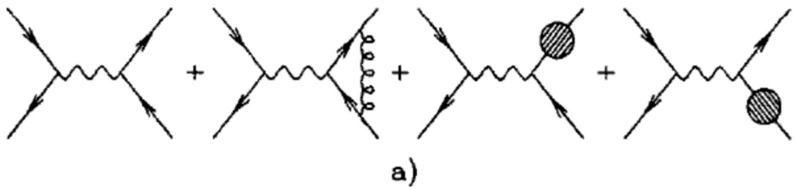
Measure :

$R \approx 3.9$

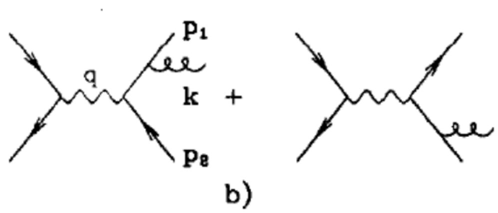
$\to$ QCD Correction required

PQCD: Jets in $e^+ e^-$ Collisions - XII

QCD corrections Next to Leading Order (NLO):



Virtual gluons



Real gluons

Real gluons: 3 particles in the final state

Some kinematics:

$$x_1 = \frac{2E_1}{\sqrt{s}}, x_2 = \frac{2E_2}{\sqrt{s}}$$

$$\rightarrow 0 \le x_1, x_2 \le 1, x_1 + x_2 \ge 1$$

$$x_3 = \frac{2E_g}{\sqrt{s}} = 2 - x_1 - x_2$$

PQCD: Jets in $e^+ e^-$ Collisions - XIII

Observe:

Plane (2D) event

Within the event plane: 2 degrees of freedom

Differential cross section:

$$\frac{1}{\sigma}\frac{d^2\sigma}{dx_1 dx_2} = \frac{2\alpha_s}{3\pi}\frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

Basic remark:

$x_1, x_2 \to 1 \quad \to \quad \sigma \to \infty$

Also true to higher perturbative orders

$\to$ 2 jets dominant over everything else

PQCD: Jets in $e^+ e^-$ Collisions - XIV

Total hadronic cross section:

$$\sigma^{q\bar{q}g} = \sigma_0 3\sum_q Q_q^2 \int \frac{d^2\sigma}{dx_1 dx_2} dx_1 dx_2 = \sigma_0 3\sum_q Q_q^2 \int \frac{2\alpha_s}{3\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)} dx_1 dx_2$$

In order to regularize diverging integrals: Funny and smart idea
Shift to $4\text{-}2\varepsilon$ space-time dimensions, make them nicely converging..

Diagrams with real gluons:

$$\sigma^{q\bar{q}g} = \sigma_0 3\sum_q Q_q^2 \frac{2\alpha_s}{3\pi} H(\varepsilon)\left[\frac{2}{\varepsilon^2} - \frac{3}{\varepsilon} + \frac{19}{2} + O(\varepsilon)\right]$$

Diagrams with virtual gluons:

$$\sigma^{q\bar{q}g} = \sigma_0 3\sum_q Q_q^2 \frac{2\alpha_s}{3\pi} H(\varepsilon)\left[-\frac{2}{\varepsilon^2} + \frac{3}{\varepsilon} - 8 + O(\varepsilon)\right]$$

Adding everything up, and reverting to 4D:

$$R \underset{\varepsilon\to 0}{\rightarrow} 3\sum_q Q_q^2 \left\{1 + \frac{\alpha_s}{\pi} + O(\alpha_s^2)\right\} \simeq 3\cdot\frac{11}{9}\left(1 + \frac{0.14}{3.14}\right) = 3.83$$

$$R + R_{Z^0} \simeq 3.83 + 0.05 = 3.88 \quad !!!$$

PQCD: Jets in $e^+ e^-$ Collisions - XV

Absolute number of jets ill defined, both in theory and experiment

Can define number of jets as a function of some *resolution parameter*, according to a *clustering algorithm*

Example: *Durham algorithm*

Take $q\bar{q}g$ final state

By fixing a y parameter as

$$m_{thresh}^2 = ys$$

compare the $(\text{invariant mass})^2$ of each parton pair to m_{thresh}^2

$$(p_i + p_j)^2 > ys \quad i, j = q, \bar{q}, g \quad \text{3 comb/event}$$

Then:

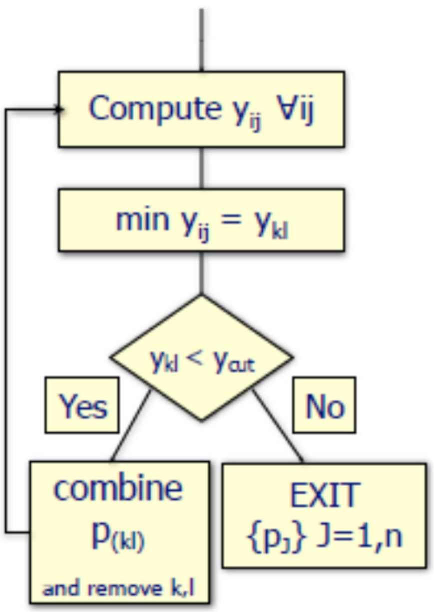
$$N_{hit} = N_{jet}$$

Of course,
$$R_{2\,jet} = R_{2\,jet}(y)$$
$$R_{3\,jet} = R_{3\,jet}(y)$$

Extend to n partons $\rightarrow$ QCD predicts $R_{k-jet}(y)$!

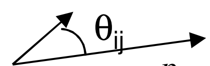
PQCD: Jets in $e^+ e^-$ Collisions - XVI

Jet algorithm for data: (modified) *Durham*
Define y_{cut}
Loop over all particle pairs



$$y_{ij} = \frac{2E_iE_j\left(1-\cos\theta_{ij}\right)}{s} \simeq \frac{m_{ij}^2}{s}$$

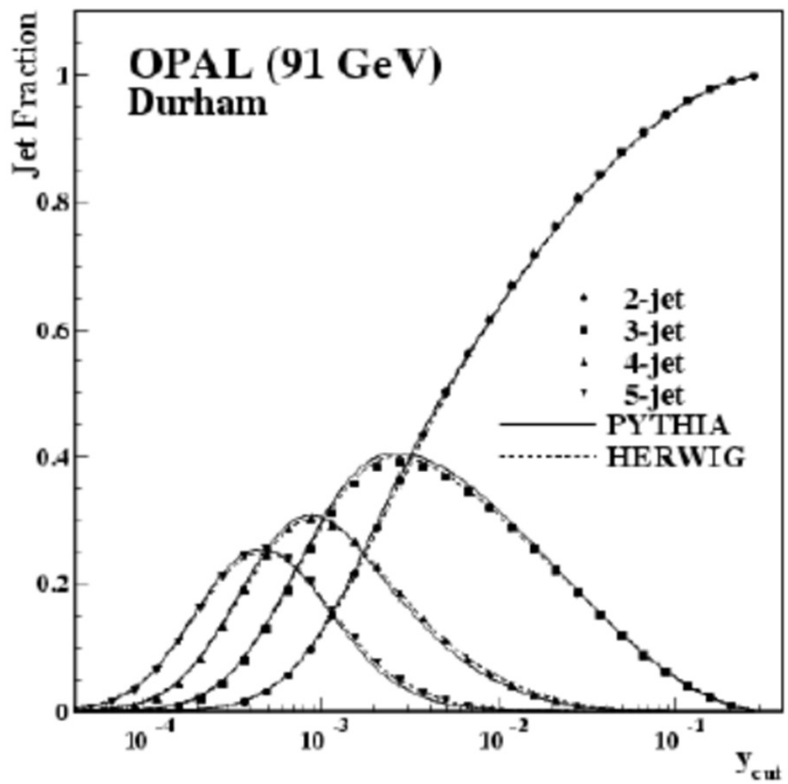
$$p_i = (E_i, \vec{p}_i)$$



$$p_j = (E_j, \vec{p}_j)$$

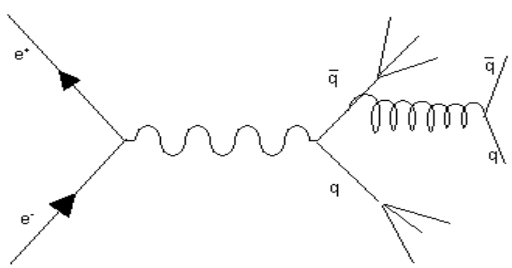
Exit with 4-momenta of *n* jets

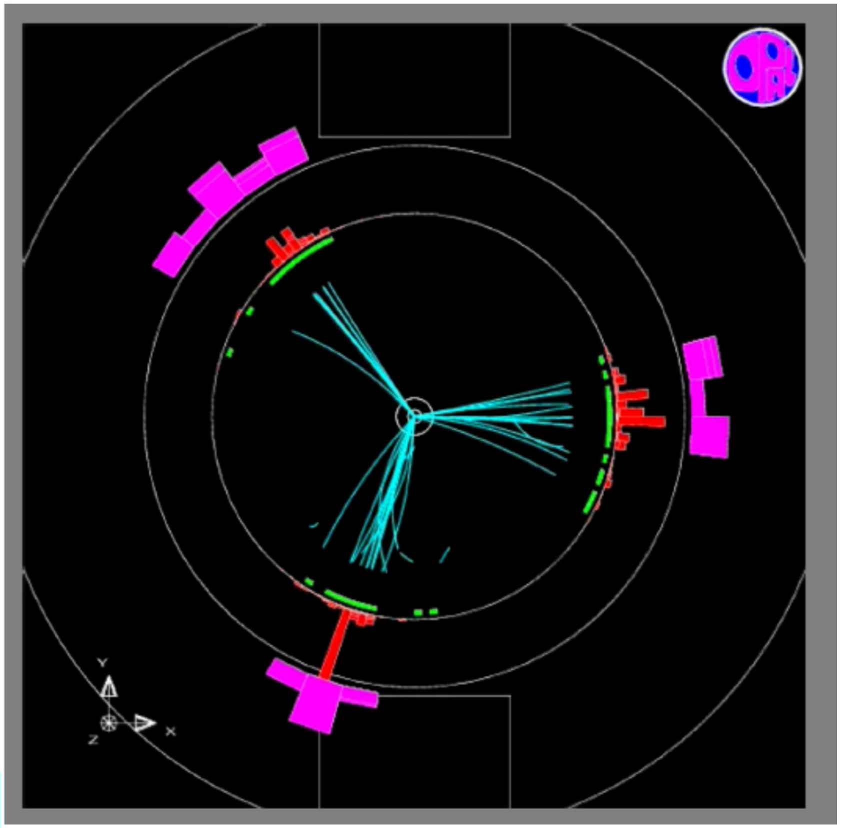
PQCD: Jets in $e^+ e^-$ Collisions - XVII



PQCD: Jets in $e^+ e^-$ Collisions - XVIII

Exceptional 3-jet event from OPAL





@TBA

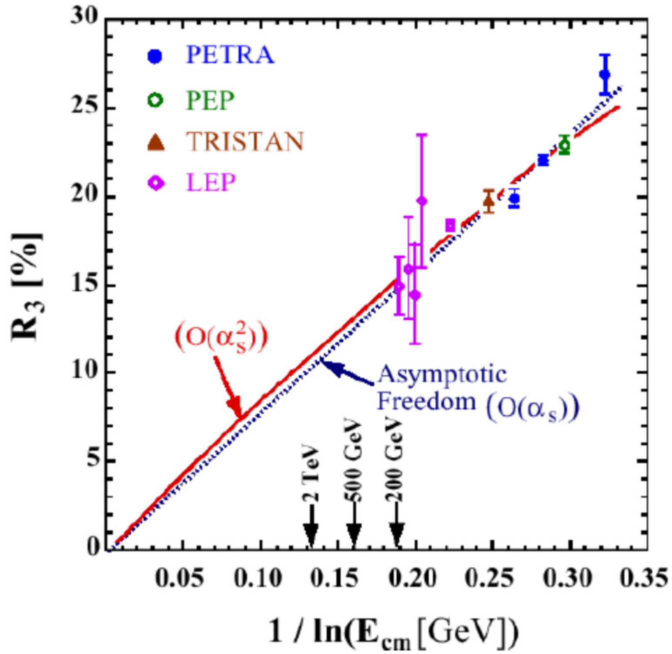
PQCD: Jets in e^+e^- Collisions - XIX

Get a measurement of α_s :

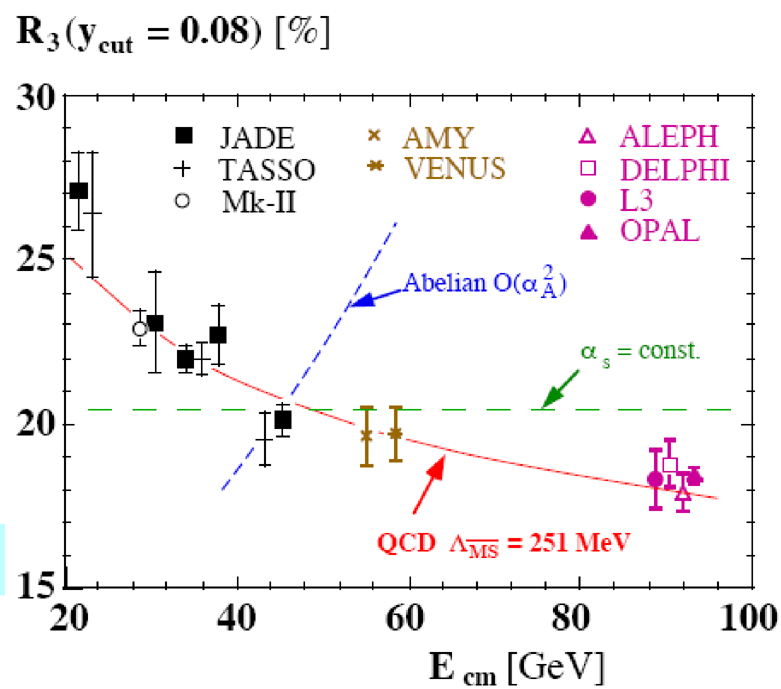
$$\alpha_S(35GeV) = 0.146 \pm 0.03$$

$$\alpha_S(M_{Z^0}) = 0.124 \pm 0.0043$$

$$R_3 \equiv \frac{\sigma_{3-\text{jet}}}{\sigma_{\text{tot}}} \propto \alpha_s(E_{\text{cm}}) \propto \frac{1}{\ln E_{\text{cm}}}$$



@TBA

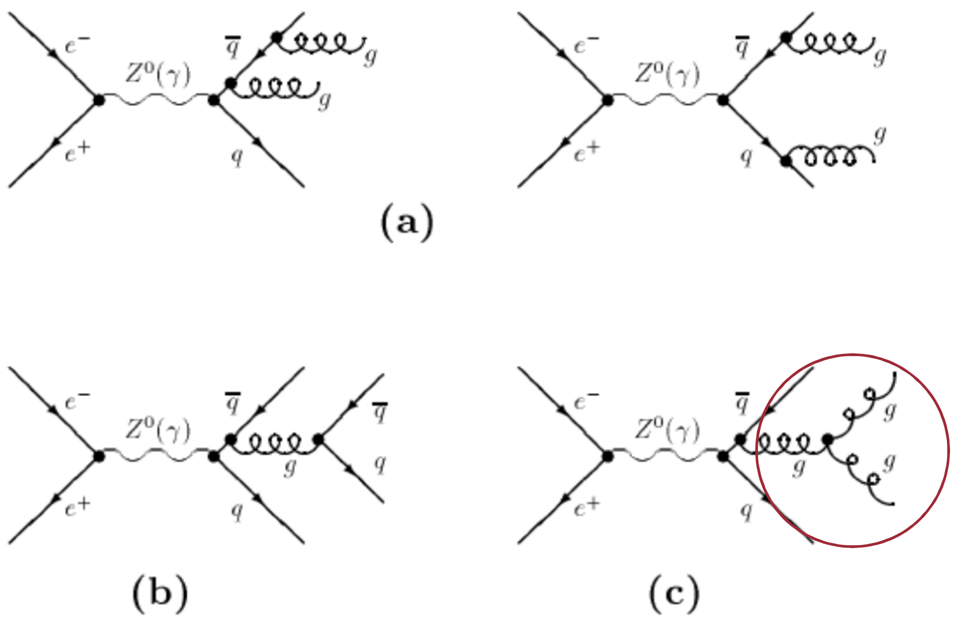


PQCD: Jets in e^+e^- Collisions- XX

Is QCD really *SU(3)* ?

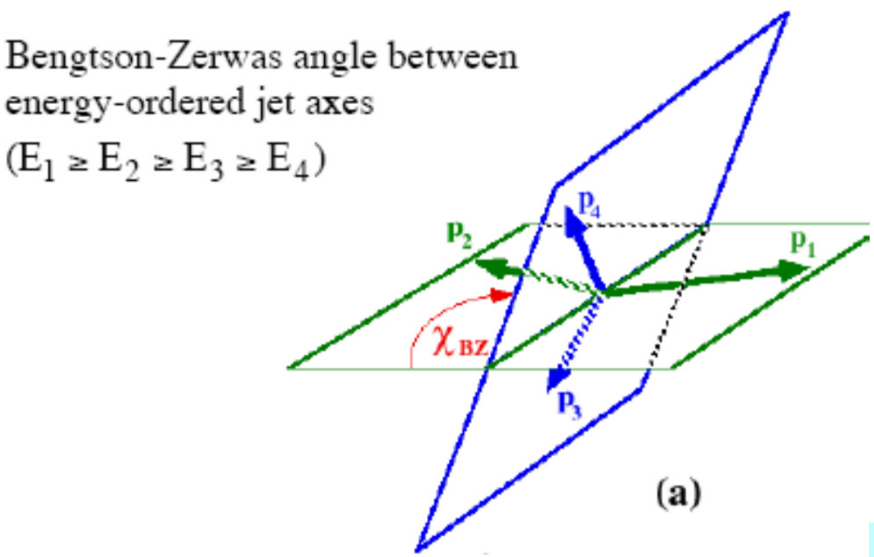
Test for non-Abelian couplings at LEP: 4 jets events

Special angular correlation from 3-gluon vertex amplitude

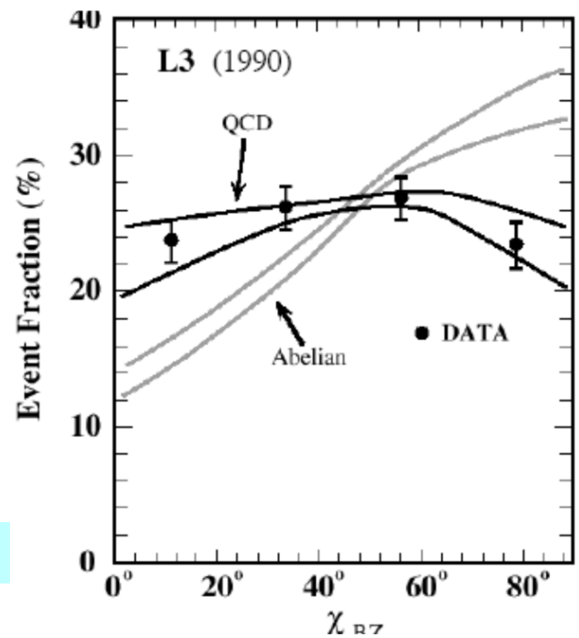


PQCD: Jets in $e^+ e^-$ Collisions - XXI

Look at distribution of a special angle, sensitive to non-Abelian couplings:



@TBA



Quark Parton Model - I

Write down F_2 in terms of PDFs

$$\frac{F_2(x)}{x} = \sum_i z_i^2 \underbrace{n_i \delta\left(x - \frac{m_i}{M}\right)} \rightarrow \frac{F_2(x)}{x} = \sum_i z_i^2 \underbrace{q_i(x)}$$

$$p = uud$$

$$F_2^p(x) = x\left[\left(\frac{2}{3}\right)^2 u_p(x) + \left(-\frac{1}{3}\right)^2 d_p(x)\right]$$

$$\rightarrow F_2^p(x) = x\left[\frac{4}{9}u_p(x) + \frac{1}{9}d_p(x)\right]$$

$$n = ddu$$

$$F_2^n(x) = x\left[\left(-\frac{1}{3}\right)^2 d_n(x) + \left(\frac{2}{3}\right)^2 u_n(x)\right]$$

From isospin symmetry:

$$F_2^n(x) = x\left[\left(-\frac{1}{3}\right)^2 u_p(x) + \left(\frac{2}{3}\right)^2 d_p(x)\right]$$

$$\rightarrow F_2^n(x) = x\left[\frac{1}{9}u_p(x) + \frac{4}{9}d_p(x)\right]$$

Quark Parton Model - II

Consider the deuteron structure function:

$$F_2^d(x) = \frac{1}{2}\left(F_2^p + F_2^n\right) = \frac{5}{9}\frac{x}{2}\left[u_p(x) + d_p(x)\right]$$

$$\rightarrow F_2^n(x) = F_2^d(x) - F_2^p(x)$$

$$= \frac{5}{18}x\left[u_p(x) + d_p(x)\right] - \frac{1}{9}x\left[u_p(x) - 4d_p(x)\right]$$

$$= \frac{3}{18}x\left[u_p(x) - d_p(x)\right]$$

Finally extract PDFs from measured F_2

$$xu_p(x) = xd_n(x) = 3F_2^p(x) - \frac{6}{5}F_2^d(x)$$

$$xu_n(x) = xd_p(x) = 3F_2^p(x) + \frac{24}{5}F_2^d(x)$$

Quark Parton Model - III

Take a Hydrogen atom:

Common wisdom: "A bound state of proton + electron"
But: Consider the effect of radiative corrections (e.g. loops)
Then we should be more precise:

Hydrogen = $(\text{Proton+Electron})_{\text{Valence}}$ + $(\text{Positrons+Electrons+Photons})_{\text{Sea}}$

Can we say valence and sea particles are fundamentally different? Well,…

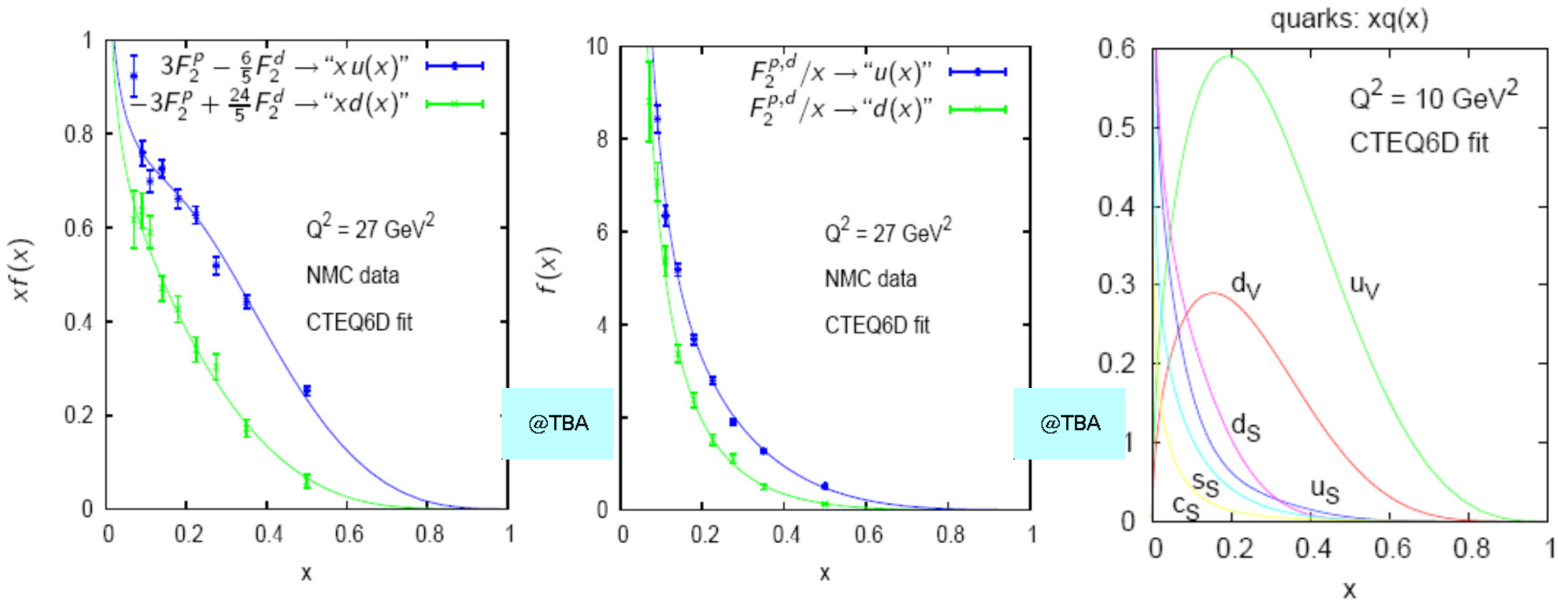
In a bound state, both are off mass shell
Both are active in yielding measurable effects (Coulomb levels vs. Lamb shift,..)
Sea particles yield small corrections to levels determined by valence e+p

Take a hadron:

Hadron = $(\text{Quarks/Antiquarks})_{\text{Valence}}$ + $(\text{Quarks/Antiquarks+Gluons})_{\text{Sea}}$

Since $a_s >> a$, ***sea effects are much larger in QCD***

Quark Parton Model - IV



Among parton model predictions: *Sum Rules* (= Integral relations) for PDFs
Examples: Proton quark content is *uud*

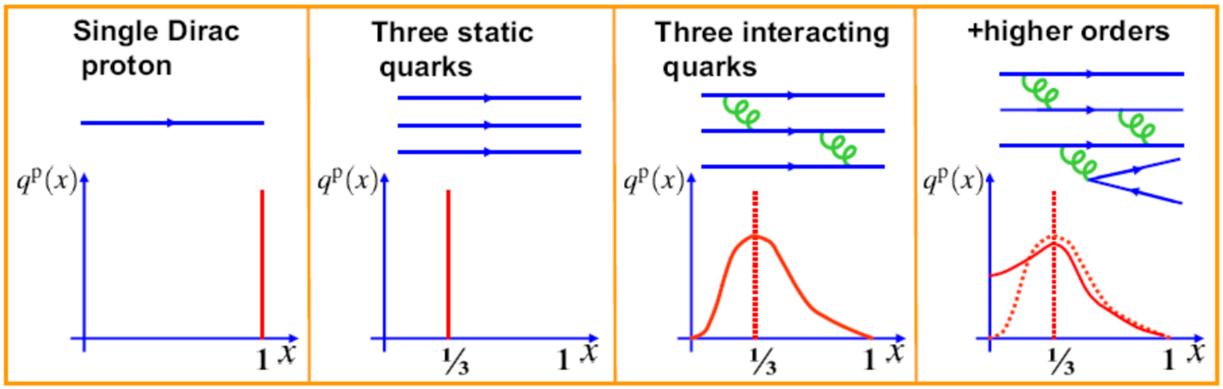
$$\int \left[u_p(x) - \bar{u}_p(x) \right] dx = 2$$

$$\int \left[d_p(x) - \bar{d}_p(x) \right] dx = 1$$

$$\int \left[s_p(x) - \bar{s}_p(x) \right] dx = 0$$

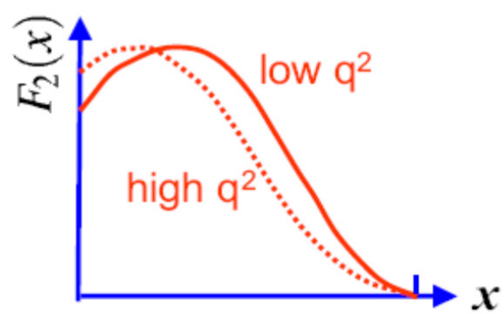
PQCD: DIS Scaling Violations - I

Our picture of structure functions



@TBA

Observe small deviations from scaling: $F_2(x) \rightarrow F_2(x, Q^2)$



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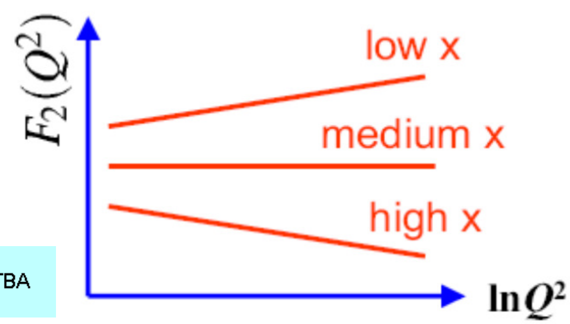


PQCD: DIS Scaling Violations - II

QCD on $F_2\left(x,Q^2\right)$:

$x-$dependence $\rightarrow$ Non perturbative $\rightarrow$ Not predicted

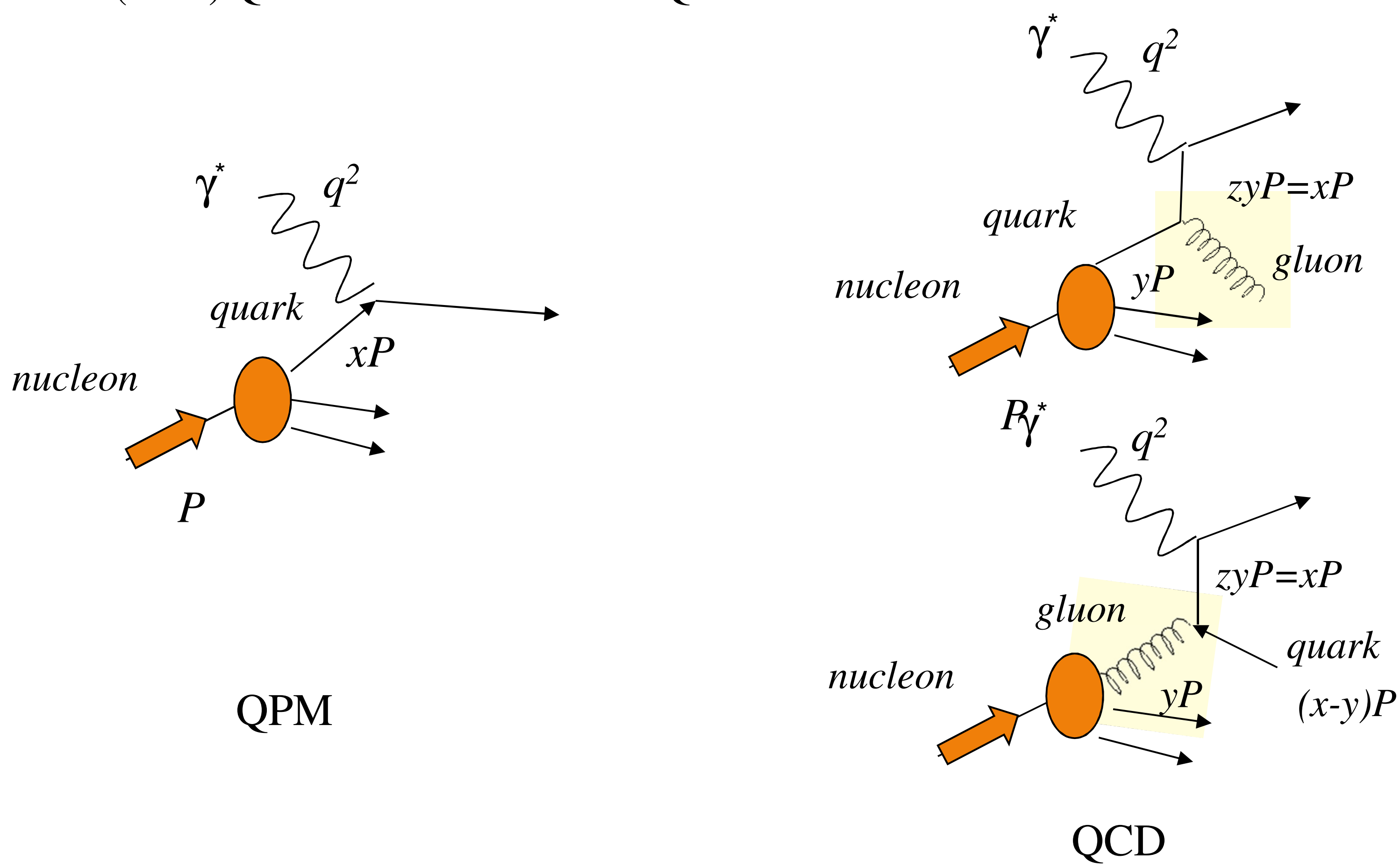
Q^2-dependence $\rightarrow$ Perturbative $\rightarrow$ Predicted !

Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) Equations:

Successful prediction of Q^2 evolution of structure functions

PQCD: DIS Scaling Violations - III

First order (NLO) QCD corrections to naive Quark Parton Model:



PQCD: DIS Scaling Violations - IV

The bottom line:
Measured structure functions at any given Bjorken *x* depend on quark & gluon densities taken at higher fractional momentum *y*>*x*
This originates a slow Q^2 dependence
Core physics: Probabilities of QCD radiative/scattering processes $P_{qq}(x/y)$, $P_{gq}(x/y)$
Usually called *splitting functions*

$$\frac{F_2(x)}{x} = 2F_1(x) = \sum_i e_i^2 q_i(x)$$

Quark-Parton Model

$$\rightarrow \frac{F_2(x)}{x} = \sum_i e_i^2 \int_0^1 q_i(y)\delta(x-y)dy$$

$$\rightarrow \frac{F_2(x)}{x} = \sum_i e_i^2 \int_0^1 q_i(y)\delta\left(1-\frac{x}{y}\right)\frac{dy}{y}$$

$$z = \frac{x}{y}$$

$$\rightarrow \frac{F_2(x)}{x} = \sum_i e_i^2 \int_0^1 q_i(y)\left[\delta\left(1-\frac{x}{y}\right)+\sigma_{qq}(z)\right]\frac{dy}{y}$$

QCD corrections

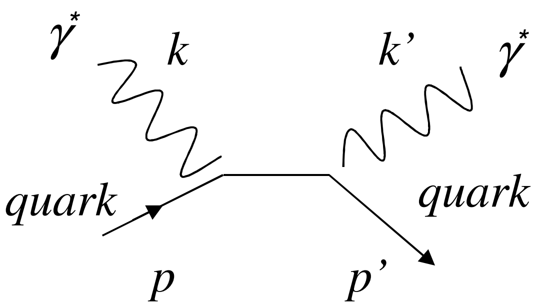
Reminder:

$$\delta(-x) = \delta(x), \quad \delta[a(1-x)] = \frac{1}{a}\delta(1-x)$$

$$\rightarrow \delta(x-y) = \delta\left[y\left(1-\frac{x}{y}\right)\right] = \frac{1}{y}\delta\left(1-\frac{x}{y}\right)$$

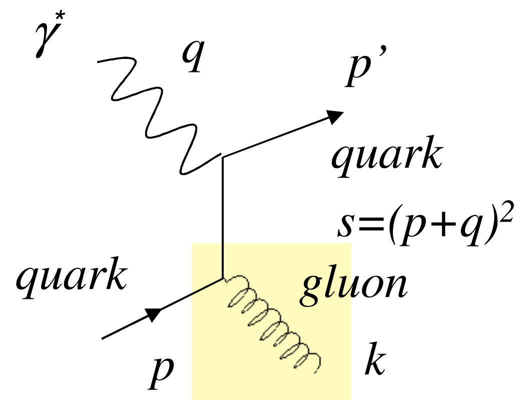
PQCD: DIS Scaling Violations - V

Just as an example: Gluon radiation splitting function at leading order (LO)
Almost carbon-copy of Compton effect



$k \leftrightarrow q$
$k' \leftrightarrow p'$

$u=(k-p')^2$ $t=(q-p')^2$



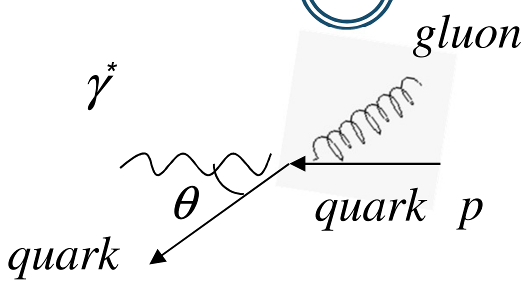
$\gamma^*(k)\, q(p) \to \gamma^*(k')\, q(p')$

$\gamma^*(q)\, q(p) \to q(p')g(k)$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\gamma q\to\gamma q} = \frac{\alpha^2 e_q^2}{2s}\left(\frac{-u}{s} - \frac{s}{u} - \frac{2tq^2}{su}\right)$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\gamma q\to gq} = \frac{C_F \alpha\alpha_S e_q^2}{2s}\left(\frac{-t}{s} - \frac{s}{t} - \frac{2uq^2}{st}\right)$$

PQCD: DIS Scaling Violations - VI



$$dp_T^2 = d(p^2 \sin^2\theta) = 2p^2 \sin\theta \underbrace{\cos\theta}_{\approx 1} d\theta = 2p^2 d(\cos\theta)$$

$$\rightarrow dp_T^2 = \frac{s}{2} d\left(\cos\theta\right) = \frac{s}{2}\frac{d\Omega}{4\pi} \rightarrow \frac{d\sigma}{d\Omega} = \frac{s}{8\pi}\frac{d\sigma}{dp_T^2} \rightarrow \frac{d\sigma}{dp_T^2} = \frac{8\pi}{s}\frac{d\sigma}{d\Omega}$$

$$\text{Reminder (Bjorken)}: \quad x = -\frac{q^2}{2P\cdot q}$$

$$\text{Define: } z = -\frac{q^2}{2p\cdot q} = -\frac{q^2}{s-q^2}$$

$$\rightarrow \left(\frac{d\sigma}{dp_T^2}\right)_{\gamma q\rightarrow gq} \propto \frac{C_F \alpha \alpha_S e_q^2}{p_T^2} P_{qq}(z), \quad P_{qq}(z) \equiv \frac{1+z^2}{1-z^2}$$

PQCD: DIS Scaling Violations - VII

Integrate 'Compton-like' differential cross-section between:

$$\begin{cases} \lambda \;\; \text{lower cutoff } (\leftarrow \text{no divergences}) \\ \dfrac{\hat{s}}{4} \;\; \text{upper cutoff } (\leftarrow \text{kinematical}),\ \hat{s} \text{ partonic CM energy squared} \end{cases}$$

$$\sigma_{qq}(z) = \int_{\lambda}^{p_{T\,\max}^2 = \frac{\hat{s}}{4}} \frac{d\sigma}{dp_T^2} dp_T^2 \propto \frac{C_F \alpha \alpha_S e_q^2}{s} P_{qq}(z) \ln\left(-\frac{q^2}{\lambda}\right)$$

$$\text{Redefine}: P_{qq}(z) \equiv \frac{\alpha e_q^2 C_F}{2\pi s} \frac{1+z^2}{1-z^2}$$

$$\rightarrow \frac{F_2(x)}{x} = \sum_i e_i^2 \int_x^1 q_i(y) \left[\delta(1-\frac{x}{y}) + \frac{\alpha_S}{2\pi} P_{qq}(z) \ln\left(\frac{Q^2}{\lambda}\right)\right] \frac{dy}{y}$$

$$\rightarrow \frac{F_2(x)}{x} = \sum_i e_i^2 \underbrace{\left[q_i(x) + \int_x^1 q_i(y) \frac{\alpha_S}{2\pi} \ln\left(\frac{Q^2}{\lambda}\right) P_{qq}\left(\frac{x}{y}\right) \frac{dy}{y}\right]}_{q_i(x,Q^2)}$$

$$q_i(x,Q^2) = q_i(x) + \int_x^1 q_i(y) \frac{\alpha_S}{2\pi} \ln\left(\frac{Q^2}{\lambda}\right) P_{qq}\left(\frac{x}{y}\right) \frac{dy}{y}$$

PQCD: DIS Scaling Violations -VIII

Evolution equation for each quark flavor:

$$\frac{dq_i(x,Q^2)}{d\ln(Q^2)} = \int_x^1 q_i(y,Q^2)\frac{\alpha_S}{2\pi}P_{qq}\left(\frac{x}{y}\right)\frac{dy}{y}$$

Observe: Since $q_i(x) \to q_i(x,Q^2)$, the evolution equation

should involve $q_i(x,Q^2)$, rather than $q_i(x)$, under the integral symbol

This is actually incomplete:

We forgot to add the contribution from the 2nd diagram..

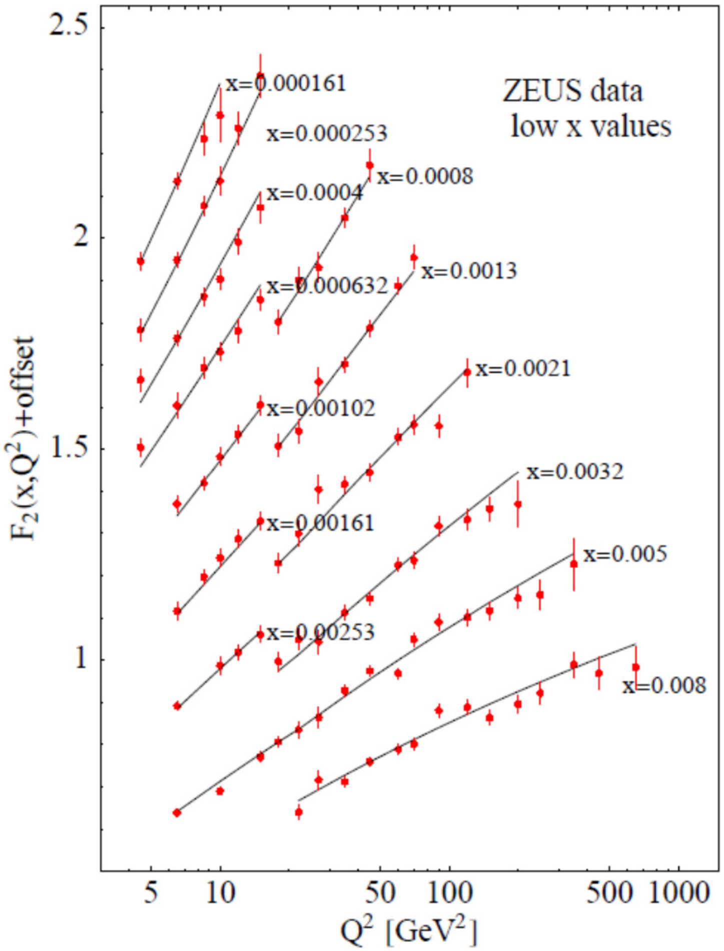
$$\to \frac{dq(x,Q^2)}{d\ln(Q^2)} = \frac{\alpha_S(Q^2)}{2\pi}\int_x^1\left[q(y,Q^2)P_{qq}\left(\frac{x}{y}\right) + g(y,Q^2)P_{gq}\left(\frac{x}{y}\right)\right]\frac{dy}{y}$$

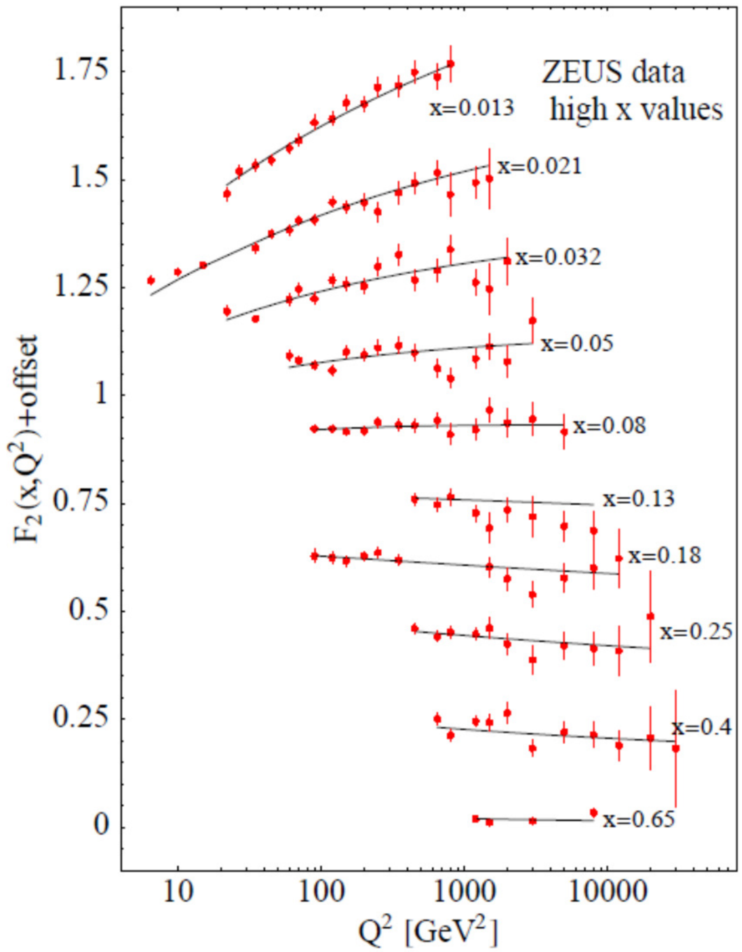
And there is another equation for the evolution of the *gluon* density:

$$\frac{dg(x,Q^2)}{d\ln(Q^2)} = \frac{\alpha_S(Q^2)}{2\pi}\int_x^1\left[\sum_i q_i(y,Q^2)P_{qg}\left(\frac{x}{y}\right) + g(y,Q^2)P_{gg}\left(\frac{x}{y}\right)\right]\frac{dy}{y}$$

Altarelli - Parisi, or *DGLAP*, equations for the parton densities

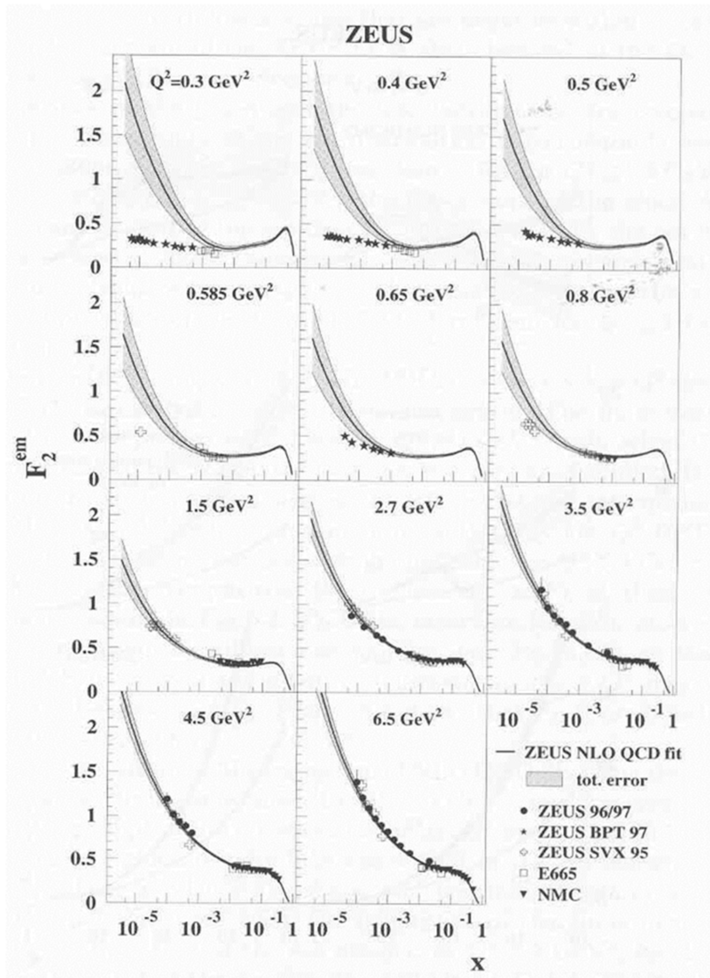
PQCD: DIS Scaling Violations -IX



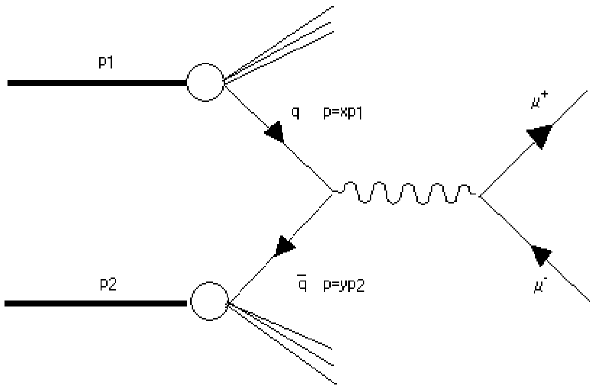


PQCD: DIS Scaling Violations -X

PDF Evolution with Q^2



PQCD: Drell-Yan - I



Angular distribution in the pair rest frame

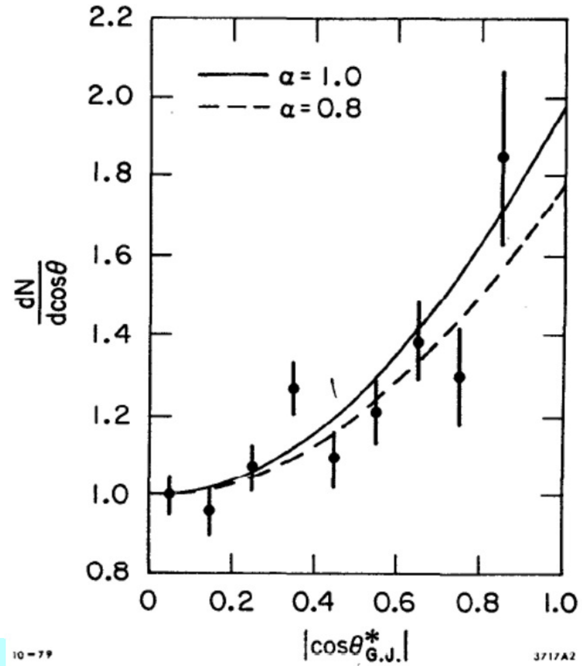
Expect $\propto 1+\cos^2\theta^*$ as usual for Fermion-Antifermion

$$\frac{d\sigma\left(q\bar{q}\to l^+l^-\right)}{dq^2}=\frac{4\pi\alpha^2}{3q^2}e_q^2\delta\left(q^2-s_{q\bar{q}}\right)$$

x_1, x_2 Bjorken x for $q, \bar{q}$

$$s_{q\bar{q}}=\left(p_q+p_{\bar{q}}\right)^2=\left(x_1p_1+x_2p_2\right)^2\simeq x_1x_2s$$

$$\sigma\left(q\bar{q}\to l^+l^-\right)=\frac{4\pi\alpha^2}{3q^2}e_q^2,\quad e_q = \text{ quark charge in } e \text{ units}$$



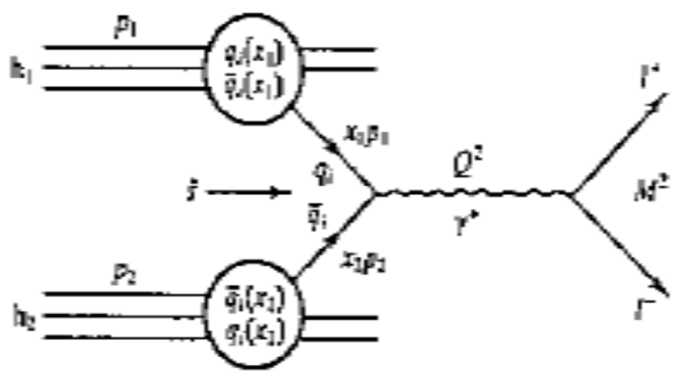
@TBA

PQCD: Drell – Yan - II

Reverse $e^+e^- \to q\overline{q}$ process: $q\overline{q} \to e^+e^-$

Obtained by any reaction

hadron + hadron $\to l^+ \; l^- \; + X$



Ignore parton transverse momentum

Ignore non-annihilating partons ($\to$ "spectators")

Ignore parton fragmentation

PQCD: Drell – Yan - III

$e^+e^- \to \mu^+\mu^-:$

$$\frac{d\sigma}{d\Omega^*} = \frac{\alpha^2}{4s}\left(1+\cos^2\theta^*\right)$$

$$\sigma = \frac{4\pi\alpha^2}{3s}$$

$q\bar{q} \to \mu^+\mu^-:$

$$\frac{d\sigma_q}{d\Omega^*} = \frac{Q_q^2\alpha^2}{4M^2}\left(1+\cos^2\theta^*\right)\cdot\frac{1}{3}$$

$Q_q e:$ Quark charge

$\frac{1}{3}:$ Color factor

$M^2: \ \mu^+\mu^-$ invariant mass = Total energy in partonic CM

PQCD: Drell – Yan - IV

$$p_1 = x_1 P_1 \simeq x_1 (E,0,0,P)$$
$$p_2 = x_2 P_2 \simeq x_2 (E,0,0,-P)$$
$$q = x_1 P_1 + x_2 P_2 \simeq \left[(x_1 + x_2) E, 0, 0, (x_1 - x_2) P\right]$$
$$M^2 = q^2 \simeq (x_1 + x_2)^2 E^2 - (x_1 - x_2)^2 P^2 \simeq 4 x_1 x_2 E^2 = x_1 x_2 s \equiv \tau s$$

Switch to more useful kinematical variables:

Either

$$\begin{cases} x_F \equiv x_1 - x_2 = \pm \dfrac{2|\mathbf{q}|}{\sqrt{s}} & \text{Feynman } x \text{ of parton pair} \\ M^2 = s x_1 x_2 \end{cases}$$

Or:

$$\begin{cases} y = \dfrac{1}{2} \ln \dfrac{E + p_3}{E - p_3} \simeq \dfrac{1}{2} \ln \dfrac{x_1}{x_2} & \text{Rapidity of parton pair (neglecting rest mass)} \\ M = \sqrt{s x_1 x_2} \end{cases}$$

PQCD: Drell – Yan - V

Inclusive cross-section:

Contribution by parton pair with (x_1, x_2) fractional momenta

$$d^2\sigma\left(pp \to \mu^+\mu^- + X\right) = \frac{4\pi\alpha^2}{9M^2}\sum_q Q_q^2\left[f_q\left(x_1\right)f_{\bar{q}}\left(x_2\right) + f_q\left(x_2\right)f_{\bar{q}}\left(x_1\right)\right]dx_1 dx_2$$

$$\begin{cases} y = \frac{1}{2}\ln\frac{E+p_3}{E-p_3} \simeq \frac{1}{2}\ln\frac{x_1}{x_2} \\ M = \sqrt{s x_1 x_2} \end{cases} \to \begin{matrix} x_1 = x_2 e^{2y} \\ M = \sqrt{s}\, x_2 e^{y} \end{matrix} \to x_2 = \frac{M}{\sqrt{s}}e^{-y}, x_1 = \frac{M}{\sqrt{s}}e^{+y}$$

$$dx_1 dx_2 = J dM dy$$

$$J = \frac{\partial\left(x_1, x_2\right)}{\partial\left(M, y\right)} = \begin{vmatrix} \frac{\partial x_1}{\partial M} & \frac{\partial x_1}{\partial y} \\ \frac{\partial x_2}{\partial M} & \frac{\partial x_2}{\partial y} \end{vmatrix} = \frac{\partial x_1}{\partial M}\frac{\partial x_2}{\partial y} - \frac{\partial x_1}{\partial y}\frac{\partial x_2}{\partial M}$$

$$\to J = \frac{1}{\sqrt{s}}e^{+y}\left(-\frac{M}{\sqrt{s}}e^{-y}\right) - \frac{M}{\sqrt{s}}e^{+y}\frac{1}{\sqrt{s}}e^{-y} = -\frac{2M}{s} = -2\sqrt{\frac{x_1 x_2}{s}} \to dx_1 dx_2 = \left(-2\sqrt{\frac{x_1 x_2}{s}}\right)dM dy$$

PQCD: Drell – Yan - VI

$$\rightarrow d^2\sigma = \frac{4\pi\alpha^2}{9M^2}\left|-2\sqrt{\frac{x_1 x_2}{s}}\right|\sum_q Q_q^2\left[f_q(x_1)f_{\bar{q}}(x_2)+f_q(x_2)f_{\bar{q}}(x_1)\right]dMdy$$

$$s\tau = M^2 \rightarrow M = \sqrt{s\tau}$$

$$\rightarrow \frac{d^2\sigma}{dMdy} = \frac{8\pi\alpha^2}{9M^2}\frac{\tau}{M}\sum_q Q_q^2\left[f_q(x_1)f_{\bar{q}}(x_2)+f_q(x_2)f_{\bar{q}}(x_1)\right]$$

$$\rightarrow \frac{d^2\sigma}{dMdy} = \frac{8\pi\alpha^2}{9Ms}\sum_q Q_q^2\left[f_q(x_1)f_{\bar{q}}(x_2)+f_q(x_2)f_{\bar{q}}(x_1)\right]$$

Central events:

$$y = 0, x_1 = x_2 = \sqrt{\tau}$$

$$\rightarrow \left.\frac{d^2\sigma}{dMdy}\right|_{y=0} = \frac{8\pi\alpha^2}{9Ms}\sum_q Q_q^2\left[f_q\left(\sqrt{\tau}\right)f_{\bar{q}}\left(\sqrt{\tau}\right)+f_q\left(\sqrt{\tau}\right)f_{\bar{q}}\left(\sqrt{\tau}\right)\right]$$

$$\rightarrow s^{3/2}\left.\frac{d^2\sigma}{dMdy}\right|_{y=0} = \frac{8\pi\alpha^2}{9}\frac{1}{\sqrt{\tau}}\sum_q Q_q^2\left[f_q\left(\sqrt{\tau}\right)f_{\bar{q}}\left(\sqrt{\tau}\right)+f_q\left(\sqrt{\tau}\right)f_{\bar{q}}\left(\sqrt{\tau}\right)\right]$$

PQCD: Drell – Yan - VII

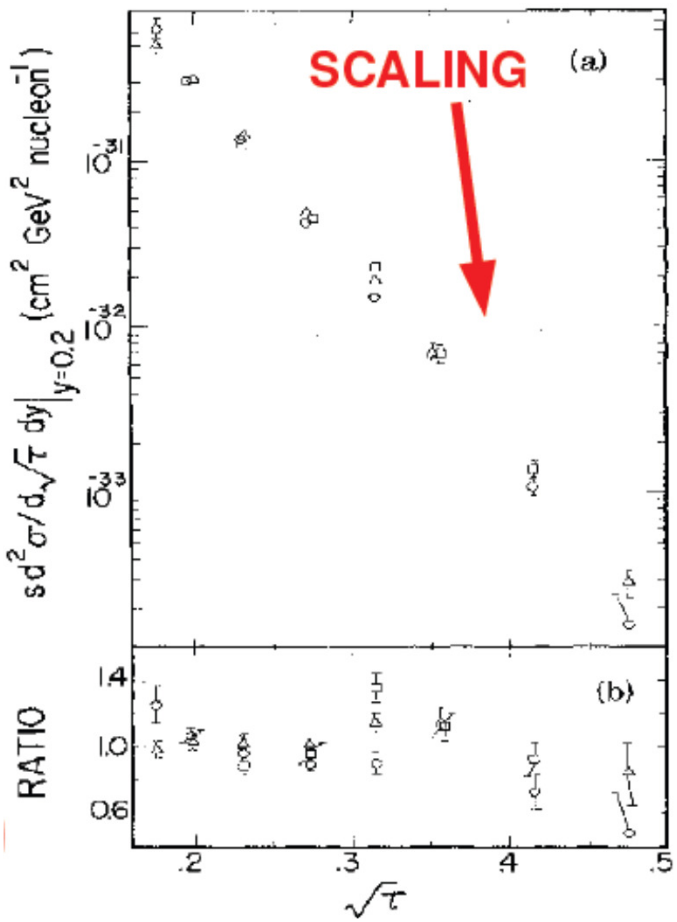
$$M=\sqrt{s\tau}\rightarrow dM=\sqrt{s}d\left(\sqrt{\tau}\right)$$

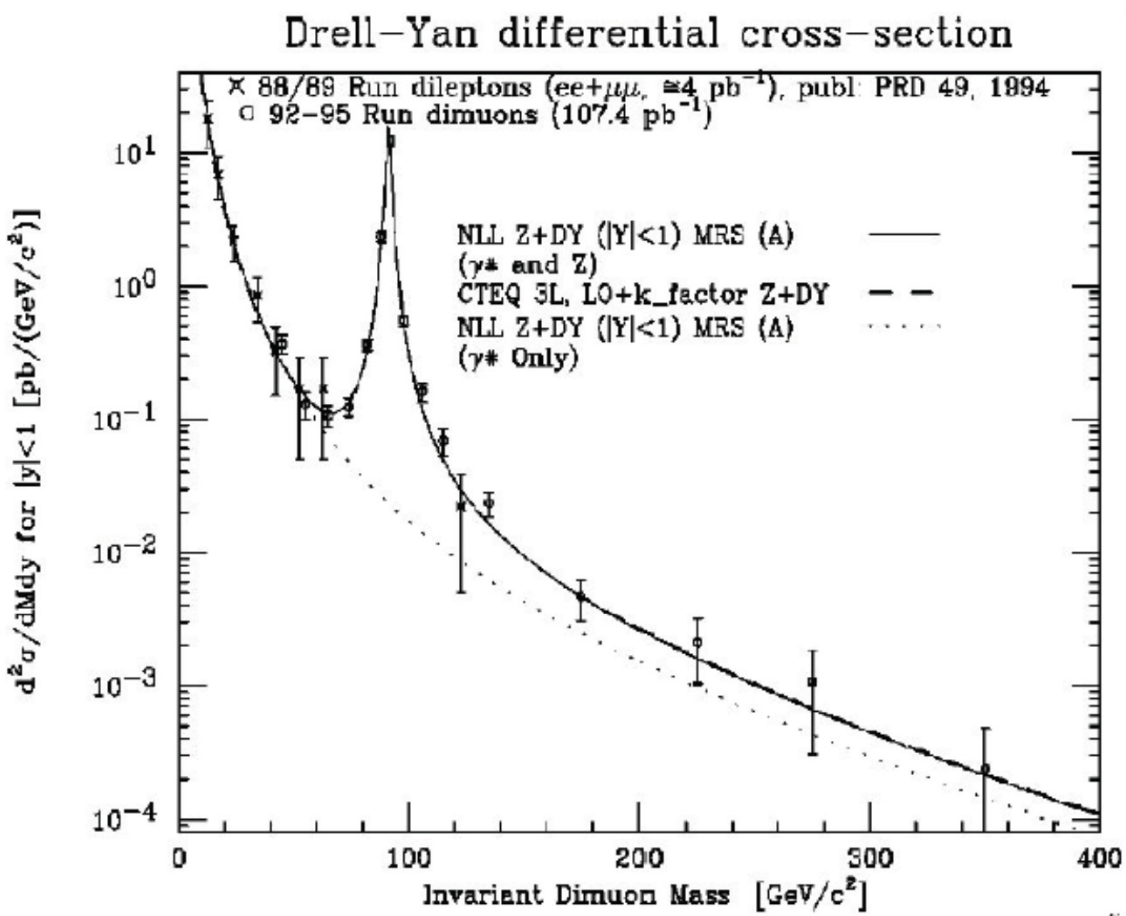
$$\rightarrow s^{3/2}\left.\frac{d^2\sigma}{dMdy}\right|_{y=0}=s^{3/2}\left.\frac{d^2\sigma}{\sqrt{s}d\left(\sqrt{\tau}\right)dy}\right|_{y=0}=s\left.\frac{d^2\sigma}{d\left(\sqrt{\tau}\right)dy}\right|_{y=0}$$

$$\rightarrow s\left.\frac{d^2\sigma}{d\left(\sqrt{\tau}\right)dy}\right|_{y=0}=\frac{8\pi\alpha^2}{9}\frac{1}{\sqrt{\tau}}\sum_q Q_q^2\left[f_q\left(\sqrt{\tau}\right)f_{\bar{q}}\left(\sqrt{\tau}\right)+f_q\left(\sqrt{\tau}\right)f_{\bar{q}}\left(\sqrt{\tau}\right)\right]$$

Scaling behavior: Compare to DIS

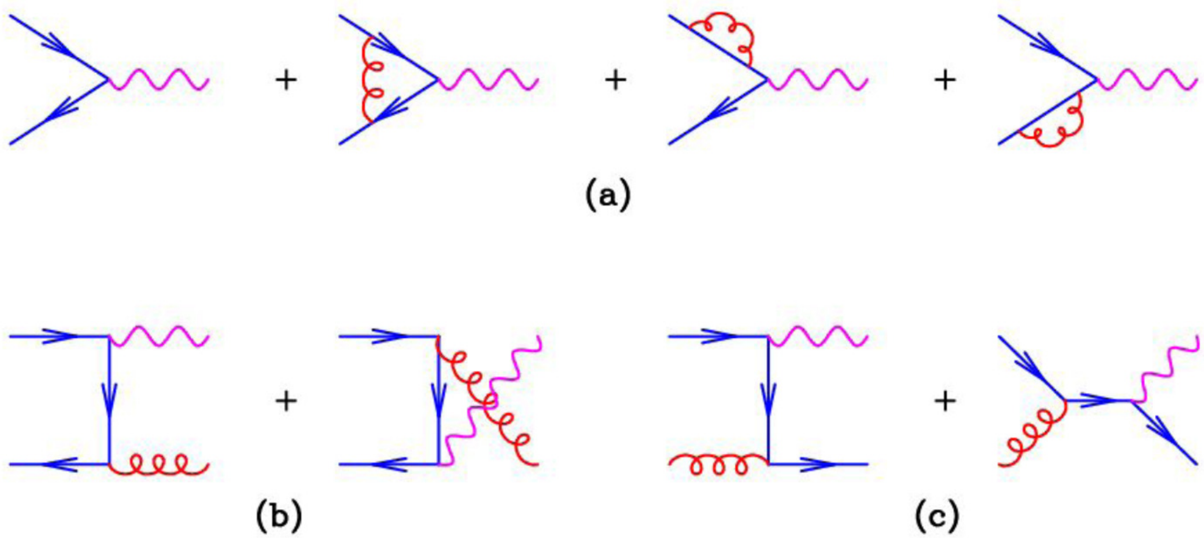
PQCD: Drell – Yan - VIII





PQCD: Drell – Yan - IX

NLO QCD corrections:



Quite similar to QCD corrections to:

$e^+e^- \rightarrow q\overline{q}$

PQCD: Drell – Yan - X

Total rate:

Same effect as for

$e^+e^- \to q\overline{q}$

Real gluons compensate virtual gluons

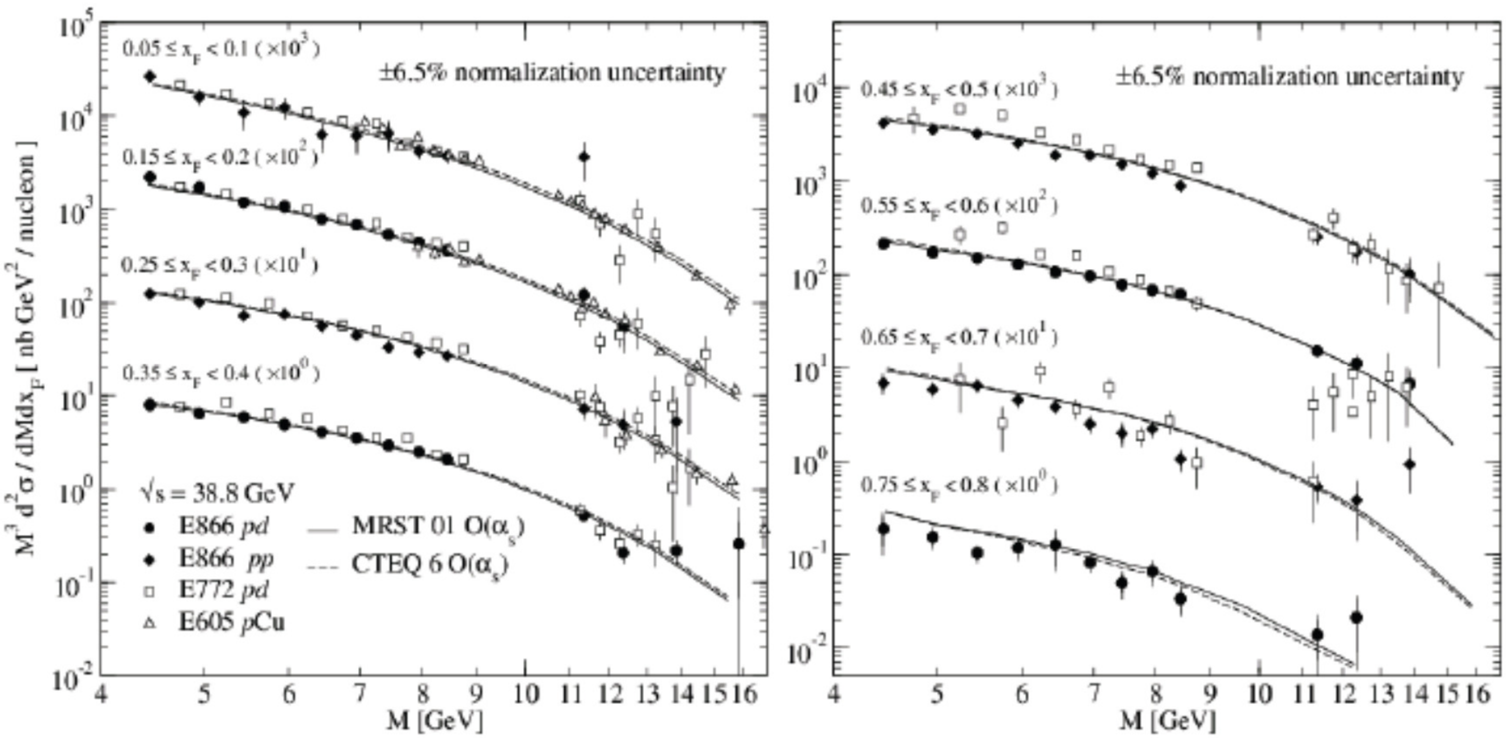
$$\sigma(real)+\sigma(virtual)=\frac{2\alpha_s}{3\pi}\sigma_0\left(\frac{4\pi^2}{3}-\frac{7}{2}\right)$$

$\to$ Overall effect lumped into a K-factor

$$K_{DY}^{(1)}=1+\frac{\alpha_s}{\pi}\left(\frac{8\pi^2}{9}-\frac{7}{3}\right)\approx 1+2.05\alpha_s\sim 2$$

PQCD: Drell – Yan - XI

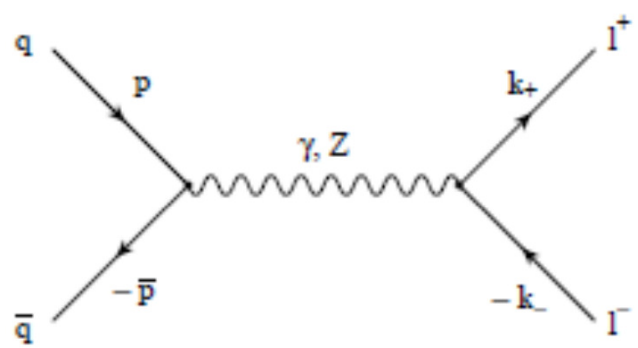
DY Scaling violation

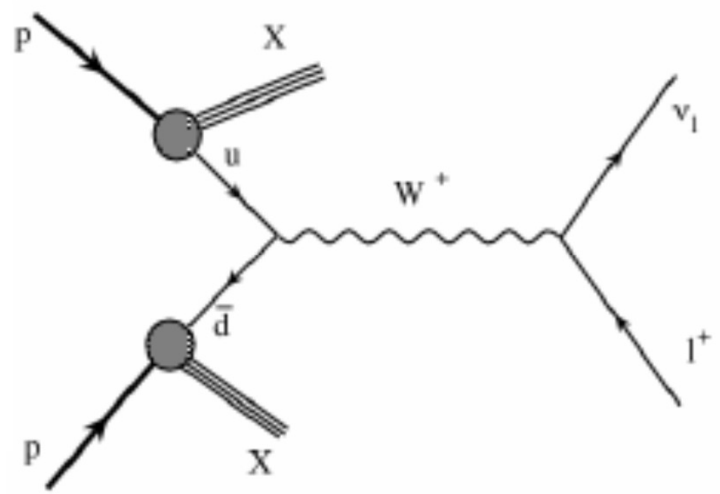


Feynman x: $\quad x_F = \frac{2}{\sqrt{s}}\left(p_+ + p_-\right) \approx x_1 - x_2$

PQCD: Drell – Yan - XII

Basic diagram generalized to include electroweak extensions:

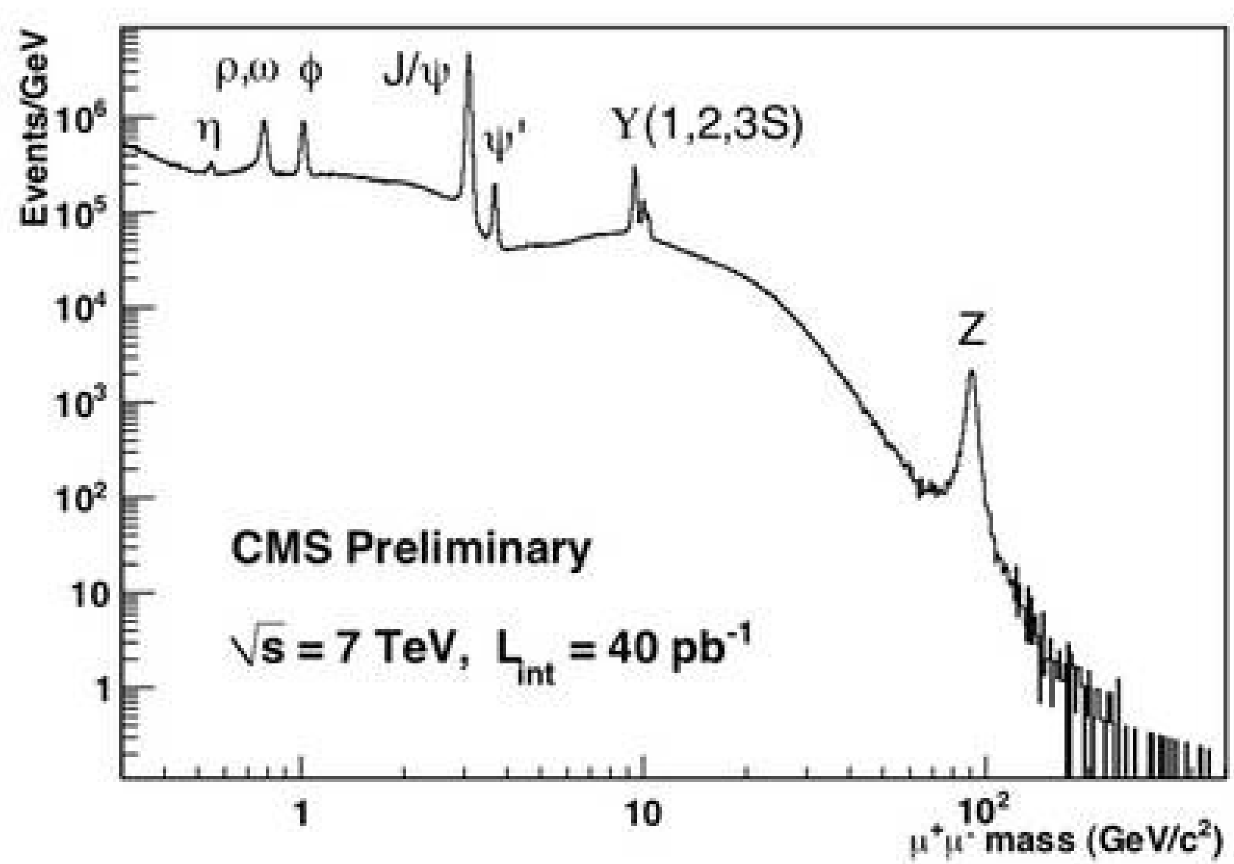




Quite important processes at hadron colliders

PQCD: Drell – Yan - XIII

Drell-Yan at large: LHC



PQCD: Hadron Collisions - I

Historically best observed and studied at hadron colliders

ISR = Intersecting Storage Ring (CERN '70s)

pp 31 GeV / beam

$Sp\bar{p}S$ = Super $p\bar{p}$ Synchrotron (CERN '80s)

$p\bar{p}$ 270 - 310 Gev / beam

Tevatron (Fermilab early '90s - 2011)

$p\bar{p}$ 1 TeV / beam

RHIC = Relativistic Heavy Ion Collider (BNL 3^{rd} Millennium)

ions 200 GeV / nucleon * beam

LHC = Large Hadron Collider (CERN 3rd Millennium)

pp 7 TeV / beam (presently 4 TeV)

ions 2.7 TeV / nucleon * beam

PQCD: Hadron Collisions - II

CM frame: usually identical to LAB

Important exception: ISR (collision angle 15^0)
Not relevant for LHC (collision angle 0.01^0)

But: Partonic collision CM $\neq$ Event CM
$\rightarrow E_{tot}, p$ of parton collision unknown

$\rightarrow$Initial state only partially known

$\rightarrow$ Separate collision kinematics into:

Transverse
Longitudinal

Introduce useful kinematical variables: *Rapidity, Transverse momentum*

PQCD: Hadron Collisions - III

Lorentz transformation $S \to S'(\beta)$:

$$\begin{pmatrix} z' \\ t' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} z \\ t \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{1-\beta^2}} & -\frac{\beta}{\sqrt{1-\beta^2}} \\ -\frac{\beta}{\sqrt{1-\beta^2}} & \frac{1}{\sqrt{1-\beta^2}} \end{pmatrix} \begin{pmatrix} z \\ t \end{pmatrix}$$

Compare:

$$\gamma^2 - \gamma^2\beta^2 = \frac{1}{1-\beta^2} - \frac{\beta^2}{1-\beta^2} = 1 \Leftrightarrow \cosh^2 y - \sinh^2 y = 1$$

$$\to \begin{cases} \gamma = \cosh y \\ \beta\gamma = \sinh y \end{cases} \to \beta = \tanh y, \quad y \ \textit{rapidity}$$

$$\to \begin{pmatrix} z' \\ t' \end{pmatrix} = \begin{pmatrix} \cosh y & -\sinh y \\ -\sinh y & \cosh y \end{pmatrix} \begin{pmatrix} z \\ t \end{pmatrix}$$

$\to$ Another way of writing a Lorentz transformation: y instead of β, γ

PQCD: Hadron Collisions - IV

Separate longitudinal/transverse momentum components:

$$E^2 = m^2 + |\boldsymbol{p}|^2$$

$$|\boldsymbol{p}|^2 = p_\parallel^2 + p_\perp^2$$

$$\rightarrow E^2 = m^2 + p_\parallel^2 + p_\perp^2 = m_\perp^2 + p_\parallel^2,\ m_\perp^2 = m^2 + p_\perp^2 \quad \text{transverse mass}$$

$$\rightarrow \left(\frac{E}{m_\perp}\right)^2 - \left(\frac{p_\parallel}{m_\perp}\right)^2 = 1$$

$$\rightarrow \begin{cases} E = m_\perp \cosh y \\ p_\parallel = m_\perp \sinh y \end{cases}$$

$$\rightarrow p_\parallel = E \tanh y$$

$$\rightarrow \begin{pmatrix} p_\parallel{}' \\ E' \end{pmatrix} = \begin{pmatrix} \cosh y & -\sinh y \\ -\sinh y & \cosh y \end{pmatrix} \begin{pmatrix} p_\parallel \\ E \end{pmatrix}$$

PQCD: Hadron Collisions - V

Most important properties:

Rapidity is *additive* under Lorentz boosts

Transverse momentum is *invariant* under Lorentz boosts

$$\begin{cases} E = m_\perp \cosh y \\ p_\parallel = m_\perp \sinh y \end{cases} \rightarrow \frac{p_\parallel}{E} = \tanh y = \frac{e^y - e^{-y}}{e^y + e^{-y}} = \frac{e^{2y} - 1}{e^{2y} + 1}$$

$$\rightarrow \frac{p_\parallel}{E}\left(e^{2y}+1\right) = e^{2y} - 1 \rightarrow e^{2y}\left(\frac{p_\parallel}{E} - 1\right) = -\left(1 + \frac{p_\parallel}{E}\right) \rightarrow e^{2y} = \frac{1 + \dfrac{p_\parallel}{E}}{1 - \dfrac{p_\parallel}{E}}$$

$$\rightarrow y = \frac{1}{2}\ln\frac{E + p_\parallel}{E - p_\parallel}$$

Boost along z by β, γ

$$\rightarrow y' = \frac{1}{2}\ln\frac{\gamma\left(E + \beta p_\parallel\right) + \gamma\left(p_\parallel + \beta E\right)}{\gamma\left(E + \beta p_\parallel\right) - \gamma\left(p_\parallel + \beta E\right)}$$

PQCD: Hadron Collisions - VI

$$\rightarrow y' = \frac{1}{2}\ln\frac{\left(E+p_{\parallel}\right)(1+\beta)}{\left(E-p_{\parallel}\right)(1-\beta)} = \underbrace{\frac{1}{2}\ln\frac{\left(E+p_{\parallel}\right)}{\left(E-p_{\parallel}\right)}}_{y} + \underbrace{\frac{1}{2}\ln\frac{(1+\beta)}{(1-\beta)}}_{y_{boost}}$$

Indeed:

$$y \rightarrow y' = y + y_{boost}$$

$$\rightarrow dy' = dy, \quad \Delta y' = \Delta y$$

Consistency check:

For momentum along z

$$y = \frac{1}{2}\ln\frac{E+p}{E-p} = \frac{1}{2}\ln\frac{\gamma m+\beta\gamma m}{\gamma m-\beta\gamma m} = \frac{1}{2}\ln\frac{1+\beta}{1-\beta}$$

$$\rightarrow \frac{1+\beta}{1-\beta} = e^{2y} \rightarrow 1+\beta = (1-\beta)e^{2y} \rightarrow \beta\left(1+e^{2y}\right) = e^{2y}-1$$

$$\rightarrow \beta = \frac{e^{2y}-1}{e^{2y}+1} = \tanh y$$

$$\rightarrow \gamma = \cosh y \rightarrow \text{OK}$$

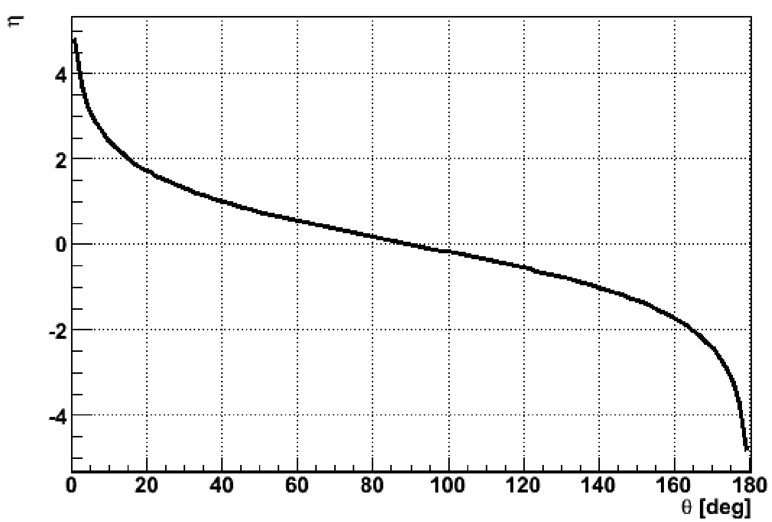
PQCD: Hadron Collisions - VII

Pseudo - rapidity η :

$$y = \frac{1}{2}\ln\frac{E+p_{\parallel}}{E-p_{\parallel}} \approx \frac{1}{2}\ln\frac{1+\cos\theta}{1-\cos\theta} = -\frac{1}{2}\ln\frac{1-\cos\theta}{1+\cos\theta} = -\frac{1}{2}\ln\frac{\sin^2\theta/2}{\cos^2\theta/2}$$

$$\rightarrow y \approx -\frac{1}{2}\ln\left(\tan^2\theta/2\right) = -\ln\left(\tan\theta/2\right) = \eta$$

$1-$to-1 correspondance to polar angle



PQCD: Hadron Collisions - VIII

Interesting processes: *Inclusive*

Ex: Inclusive production of c $\quad a+b \to c+X$

$\to$ Inclusive, invariant cross-sections

Reminder: $\dfrac{d^3\mathbf{P}}{E}$ Lorentz invariant quantity

Elementary volume (impulse space):

Same as cylindrical coordinate space

$$d^3\mathbf{r} = rdrdzd\varphi \to d^3\mathbf{P} = P_T dP_T dP_\parallel d\varphi \to \frac{d^3\mathbf{P}}{E} = \frac{P_T dP_T dP_\parallel d\varphi}{E}$$

$$dy = \frac{dP_\parallel}{E} \to \frac{d^3\mathbf{P}}{E} = dyP_T dP_T d\varphi$$

$\to$ Azimuthal integral:

$$\int_{\varphi=0}^{\varphi=2\pi} \frac{d^3\mathbf{P}}{E} = \int_{\varphi=0}^{\varphi=2\pi} \left(dyP_T dP_T\right) d\varphi = 2\pi dyP_T dP_T = 2\pi dy \frac{1}{2} d\left(P_T^2\right) = \pi dyd\left(P_T^2\right)$$

$\to$ Inclusive, invariant differential cross-section: $\dfrac{d\sigma}{\dfrac{d^3\mathbf{P}}{E}} = E\dfrac{d\sigma}{d^3\mathbf{P}} = \dfrac{1}{\pi}\dfrac{d\sigma}{dyd\left(P_T^2\right)} \equiv \dfrac{1}{2\pi P_T}\dfrac{d\sigma}{dydP_T}$

PQCD: Hadron Collisions - IX

Introducing pseudorapidity, transverse energy components of 4-momentum:

$$E_T = m_\perp \to \begin{matrix} E = E_T \cosh y \\ p_\parallel = E_T \sinh y \end{matrix} \to E_T = \frac{E}{\cosh y}, \quad y \approx \eta = -\ln\left(\tan\theta/2\right)$$

$$\frac{\sinh y}{\cosh y} = \tanh y = \frac{\sqrt{\cosh^2 y - 1}}{\cosh y} \to \tanh^2 y = 1 - \frac{1}{\cosh^2 y} \to \frac{1}{\cosh y} = \sqrt{1 - \tanh^2 y} \to E_T = E\sqrt{1 - \tanh^2 y}$$

$$\to E_T \approx E\sqrt{1 - \tanh^2 \eta} = E\left(1 - \frac{\dfrac{\cos\theta/2}{\sin\theta/2} - \dfrac{\sin\theta/2}{\cos\theta/2}}{\dfrac{\cos\theta/2}{\sin\theta/2} + \dfrac{\sin\theta/2}{\cos\theta/2}}\right)^{1/2} = E\left[1 - \left(\cos^2\theta/2 - \sin^2\theta/2\right)\right]^{1/2} = E\sin\theta$$

$$p = \left(E,\ \underbrace{P_x, P_y}_{P_T^2 = P_x^2 + P_y^2},\ \underbrace{P_z}_{P_\parallel = P_z}\right)$$

$$E \approx P, E_T \approx P_T$$

$$\to p \approx \left(E_T \cosh\eta, \underbrace{E_T \sin\phi, E_T \cos\phi,}_{E_T} E_T \sinh\eta\right)$$

Useful in clustering algorithms

PQCD: Hadron Collisions - X

Partonic kinematics: Relevant for 'hard' collisions

Event CM frame:

$$\begin{cases} P_A = \left(E_A, 0, 0, p_A\right) \\ P_B = \left(E_A, 0, 0, -p_A\right) \end{cases}$$ 4-momenta incident particles

$$\begin{cases} p_1 = x_1 P_A \\ p_2 = x_2 P_B \end{cases}$$ 4-momenta incident partons

$$\beta_{CM} = \frac{x_1 - x_2}{x_1 + x_2}$$ Parton CM speed as seen by event CM $\left(= \text{LAB for most colliders}\right)$

$$x_F = x_1 - x_2$$ Parton Feynman x

$$y = \frac{1}{2}\ln\frac{1+\beta_{CM}}{1-\beta_{CM}} = \frac{1}{2}\ln\frac{x_1}{x_2}$$ Parton CM rapidity as seen by event CM

x_1, x_2 varying on event-by-event basis

$\rightarrow \beta_{CM},\ x_F,\ y$ not fixed, rather statistically distributed

Distribution depending on event type

PQCD: Hadron Collisions - XI

Hadronic, *inclusive* cross sections for hard processes

$A+B \to c+X$

result from a convolution of *partonic*, *exclusive* cross sections

$a+b \to c+X$

at any given partonic CM energy $\hat{s}$

$$\sigma_{A+B\to f+X} = \sum_{a,b} C_{ab} \int \sigma_{ab\to c+X} \left[f_{a/A}\left(x_a\right) f_{bB}\left(x_b\right) + \left(A \leftrightarrow B\right)_{b\neq a} \right] dx_a dx_b$$

with PDF over full range of $\left(x_1, x_2\right)$

$$\sigma_{ab\to c+X} \begin{cases} \text{total/differential in the final state } c \\ \text{summed over initial, final colors} \end{cases}$$

C_{ab} color-averaging factor, different for $qq,\ q\bar{q},\ qg,\ gg$

PQCD: Hadron Collisions - XII

Partonic CM energy :

$$\sqrt{\hat{s}} = \sqrt{s x_a x_b}$$

Introduce τ:

$$\tau = x_a x_b$$

Switching to x_a, τ independent variables:

$$x_b = \frac{\tau}{x_a} \rightarrow dx_b = \frac{d\tau}{x_a}$$

$\rightarrow$ Hadronic cross-section in terms of partonic subprocess contributions:

$$\sigma_{A+B\rightarrow f+X} = \sum_{a,b} C_{ab} \int_0^1 d\tau \int_\tau^1 \sigma_{ab\rightarrow c+X}\left(\hat{s}\right)\left[f_{a/A}\left(x_a\right) f_{b/B}\left(\tau / x_a\right) + \left(A \leftrightarrow B\right)_{b\neq a}\right] \frac{dx_a}{x_a}$$

PQCD: Hadron Collisions - XIII

$\rightarrow$ Re-write cross-section as *differential* in τ:

$$\frac{d\sigma_{A+B\to f+X}}{d\tau} = \sum_{a,b} \sigma_{ab\to c+X}\left(\hat{s}\right)\frac{dL_{ab}}{d\tau}$$

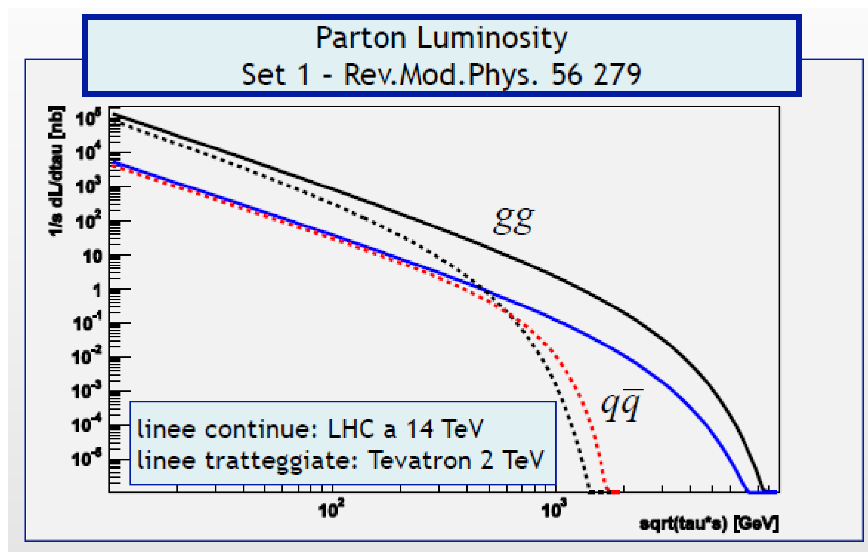
Introducing *differential luminosity* for parton collisions occurring within $\left(\tau, \tau + d\tau\right)$:

$$\frac{dL_{ab}}{d\tau} = C_{ab}\int_{\tau}^{1}\left[f_{a/A}\left(x_a\right)f_{b/B}\left(\tau / x_a\right) + \left(A \leftrightarrow B\right)_{b\neq a}\right]\frac{dx_a}{x_a}$$

$$\rightarrow \frac{\tau}{\hat{s}}\frac{dL_{ab}}{d\tau} = \frac{1}{s}\frac{dL_{ab}}{d\tau} = \frac{dL_{ab}}{d\hat{s}}$$

Parton luminosities quite relevant to assess production rates:

Ex. Higgs at LHC



PQCD: Hadron Collisions - XIV

τ alone does not fix kinematics of the initial state: Use y Rapidity

$$\begin{cases} \tau = x_a x_b \\ y = \frac{1}{2}\ln\frac{x_a}{x_b} \end{cases} \rightarrow \begin{matrix} x_a = \sqrt{\tau}e^{y} \\ x_b = \sqrt{\tau}e^{-y} \end{matrix} \rightarrow \sqrt{\hat{s}} \equiv \sqrt{\tau s}, \quad dx_1 dx_2 = d\tau dy$$

For any given τ: $|y| < -\frac{1}{2}\ln\tau$

Re-write hadronic cross-section as *doubly differential* in both y, τ :

$$\frac{d\sigma_{A+B\to c+X}}{dx_a dx_b} = \sum_{a,b} C_{ab}\sigma_{ab\to c+X}\left[f_{a/A}(x_a) f_{b/B}(x_b) + (A \leftrightarrow B)_{b\neq a}\right] = \frac{d\sigma}{d\tau dy}$$

Ex. : *Central production*:

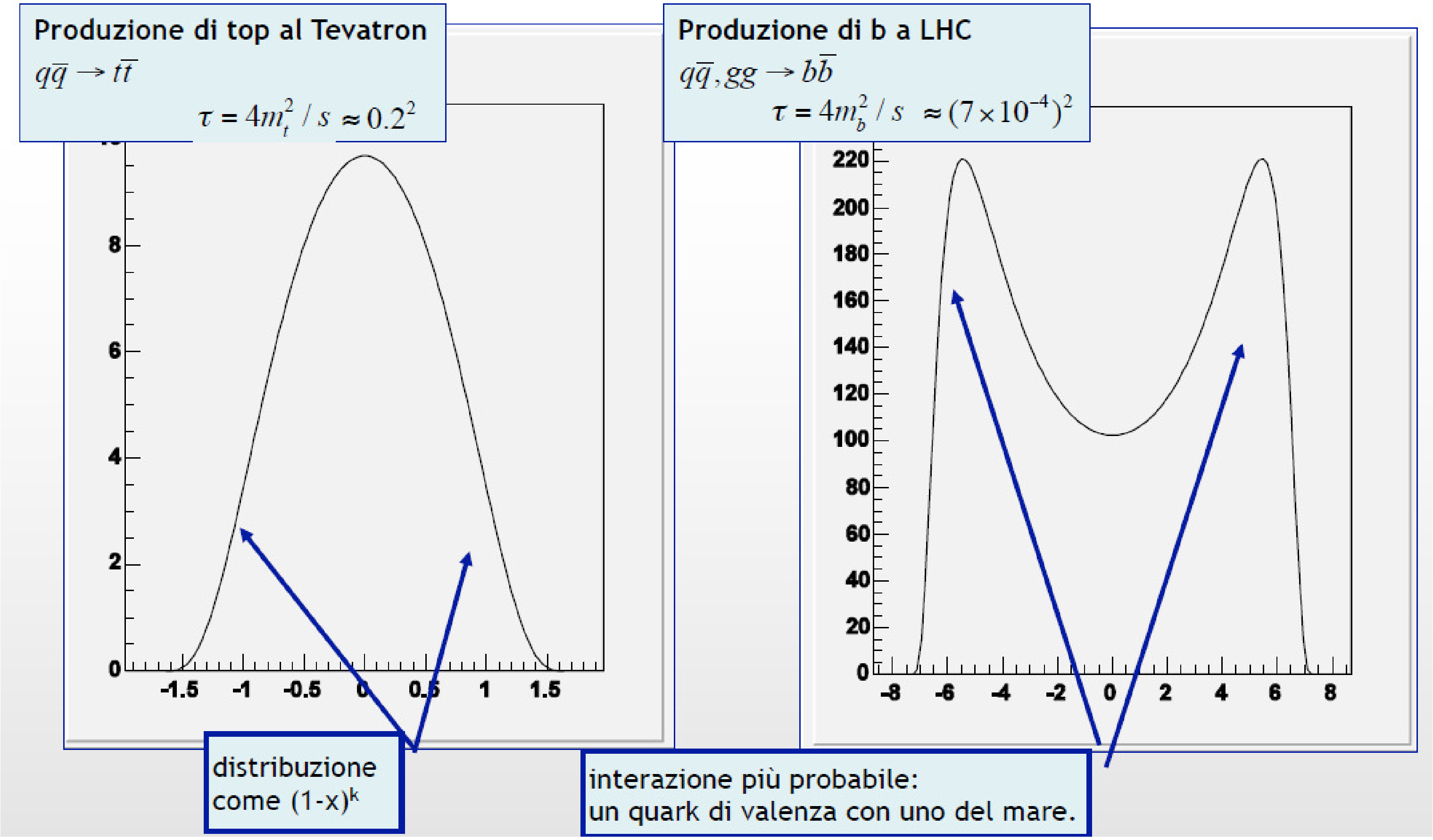
$$y = 0 \rightarrow x_a = x_b = \sqrt{\tau} \rightarrow \left.\frac{d\sigma}{d\tau dy}\right|_{y=0} = \sum_{a,b} C_{ab}\sigma_{ab\to c+X}\left[f_{a/A}(\sqrt{\tau}) f_{b/B}(\sqrt{\tau}) + (A \leftrightarrow B)_{b\neq a}\right]$$

Ex. : *Threshold production*

$$\sqrt{\tau} = \frac{m_c}{\sqrt{s}}$$

PQCD: Hadron Collisions - XV

Examples of rapidity distribution: Threshold $t\bar{t}$, $b\bar{b}$ production at Tevatron, LHC



PQCD: Hadron Collisions - XVI

Consider all the 2-body processes in QCD:

$qq \to qq, q\bar{q} \to q\bar{q}$ Quarks only

$qg \to qg, \bar{q}g \to \bar{q}g, gg \to gg, q\bar{q} \to gg, gg \to q\bar{q}$ Quarks and/or Gluons

All yielding 2 jets to LO

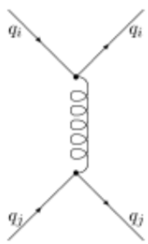


Figure 1: Feynman diagram for $q_i q_j \to q_i q_j$, $i \neq j$

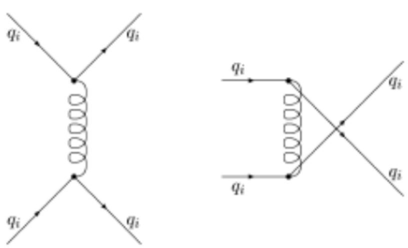


Figure 2: Feynman diagrams for $q_i q_i \to q_i q_i$

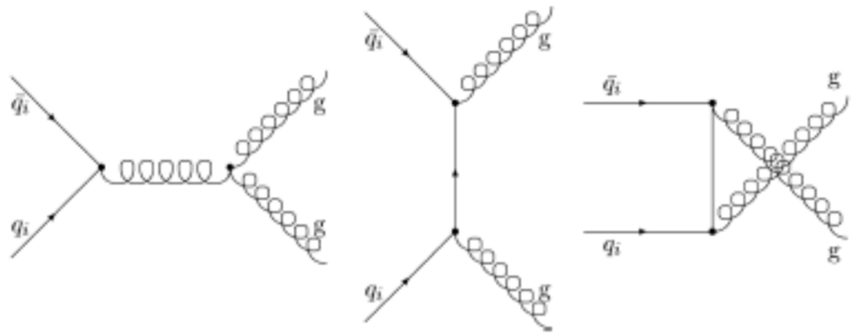


Figure 5: Feynman diagrams for $q_i \bar{q}_i \to gg$

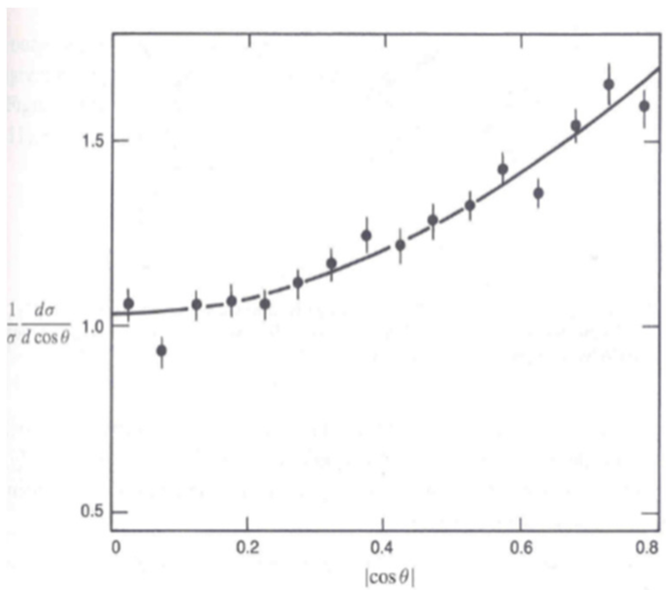
PQCD: Hadron Collisions - XVII

When quark only processes can be identified, expect

$$\frac{d\sigma}{d\left(\cos\theta^*\right)} = \frac{\pi\alpha_s^2}{2s_{ij}}\left|M\right|^2$$

$$s_{ij} = \left(x_i p_i + x_j p_j\right)^2 \approx x_i x_j s$$

$$\rightarrow \left|M\right|^2 \propto 1 + \cos^2\theta^* \quad \text{as usual}$$



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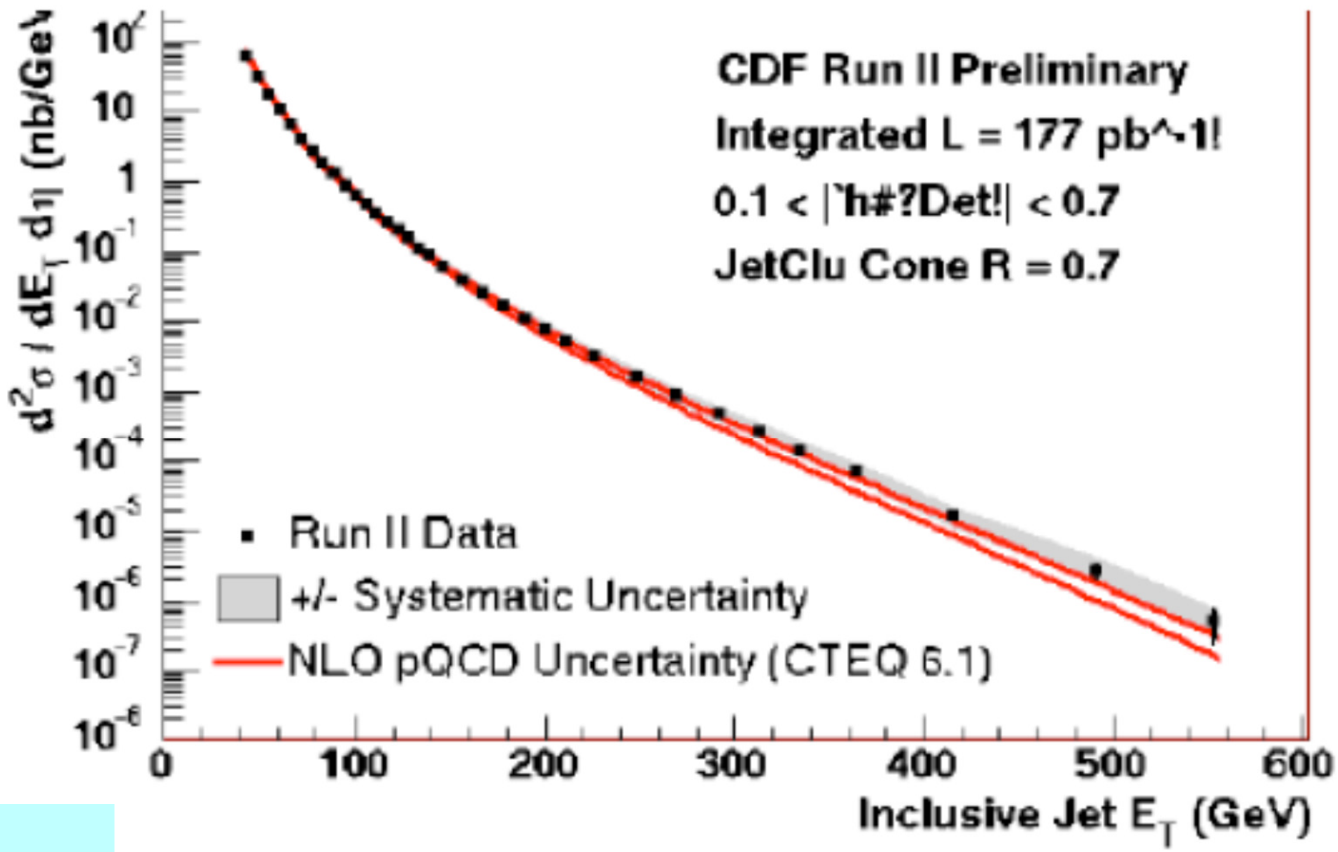
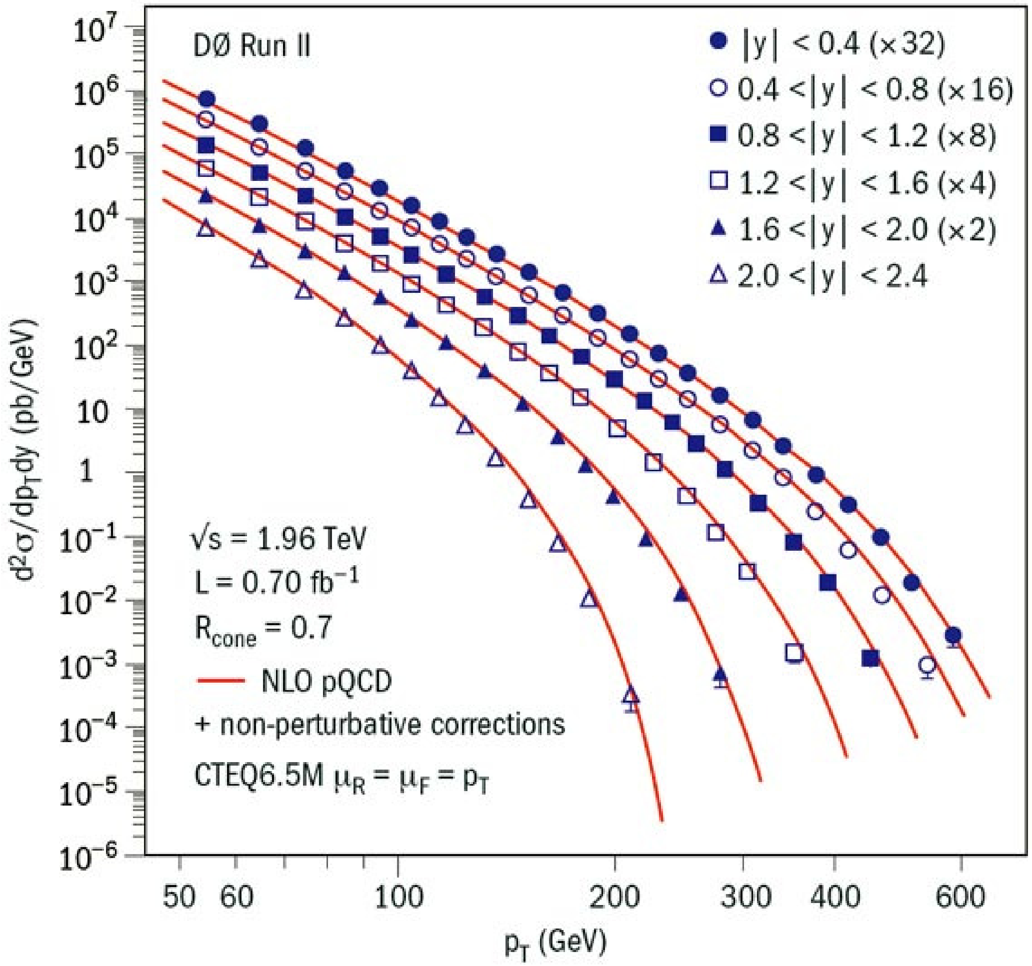
PQCD: Hadron Collisions - XVIII



@TBA

PQCD: Hadron Collisions - XIX



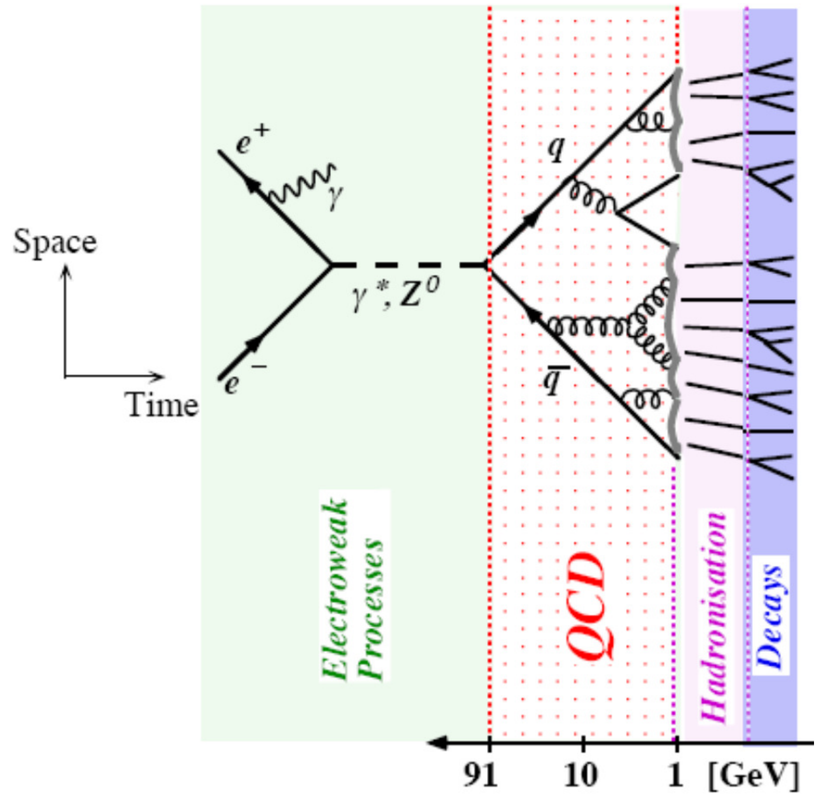
PQCD: Hadron Collisions - XX

Complete jet evolution cannot be computed in a full QCD framework:

As shown in the picture, *increasing* time scales correspond to *decreasing* Q^2 scale, down to a region where perturbative expansion cannot be granted

Conversion of quarks and gluons into hadrons (*Fragmentation* or *Hadronization*) must be evaluated by ad-hoc prescriptions (models).

QCD-inspired, successful models often based on string-like behavior of $q\bar{q}$ pairs



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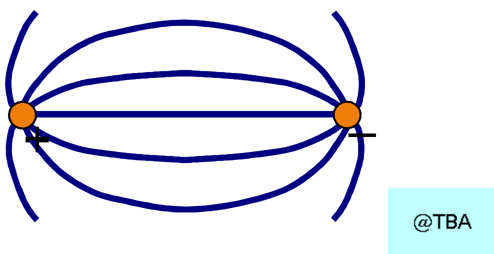
PQCD: Hadron Collisions - XXI

Typical model implemented in fragmentation Montecarlo programs

$q\bar{q}$ Interaction

QED-like at small distance



Gluon self-interaction yields *string* (flux tube) pattern at large distance: $F = const$

Picture baryons as ‘mesons’:

$$\boldsymbol{3} \otimes \boldsymbol{3} = \boldsymbol{3}^* \oplus \boldsymbol{6}$$

$$qqq = \underbrace{qq}_{\sim \bar{q}} + q$$

Confinement - I

$$\alpha_s\left(\left|q^2\right|\right) \simeq \frac{12\pi}{21\ln\left(\left|q^2\right|/\Lambda^2\right)}, \quad \left|q^2\right| \gg \Lambda^2$$

When $\left|q^2\right| \sim \Lambda^2$, the previous expression does not apply

$\alpha_s\left(\Lambda^2\right)$ *is large*
Strong interaction is strong
Cannot rely on perturbative expansion

In a general sense, expect Λ to mark the low energy range, corresponding to *soft* (low q^2) processes

Bound states: Non-perturbative, 'white', energy scale ≈ Λ
Does $a_s(\Lambda^2)$ correspond to the *color confinement* range?
Very likely. But remember:

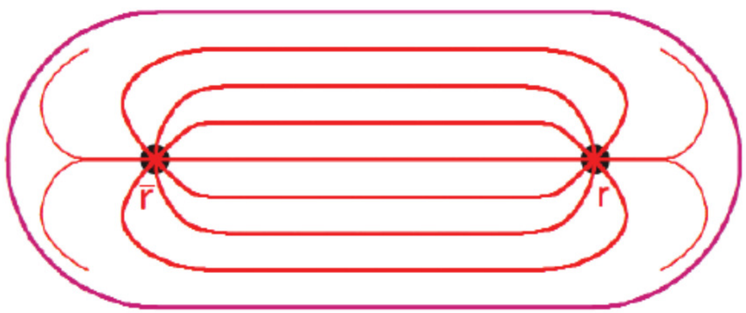
It is not yet convincingly shown that QCD is a confining theory

Confinement - II

QCD: At large color charges separation, field lines compressed to tube-like regions

Reason: Gluon-gluon interaction

$\rightarrow \quad \sim$ String

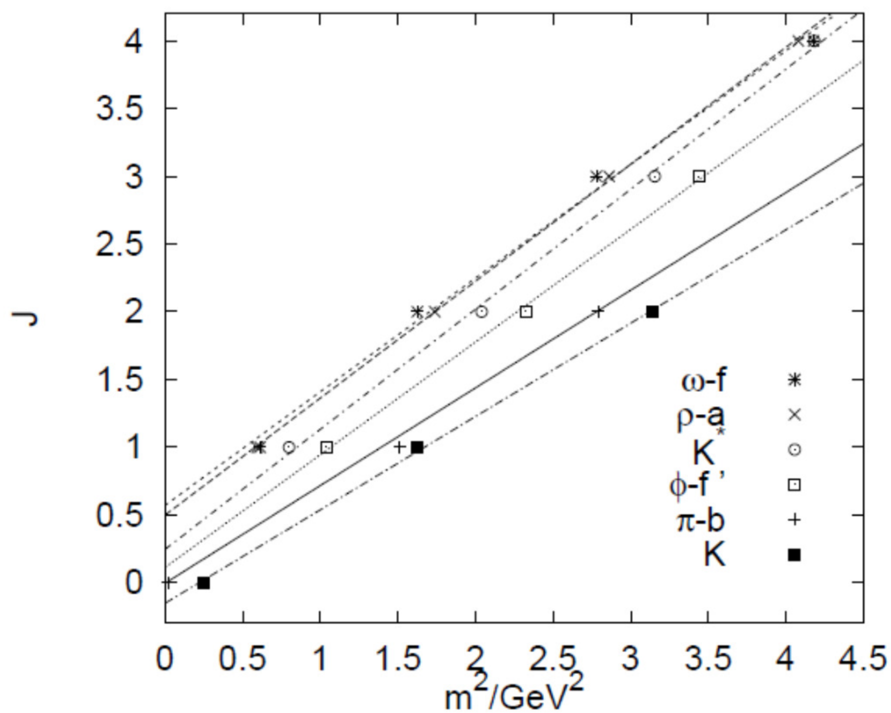


$\rightarrow F(r) \approx const \rightarrow V(r) = kr$ Linearly confining potential

$k \sim 1\ GeV/fm$

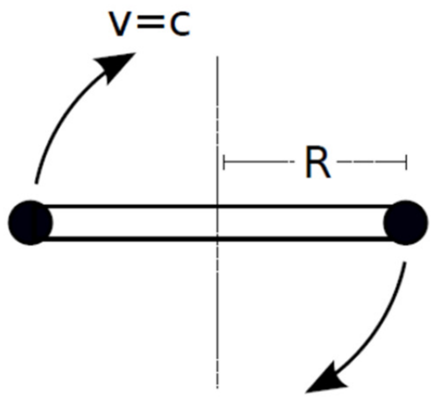
Confinement - III

Regge trajectories: Old concept, adapted by potential scattering theory
Very general property, not related to any constituent model:
Linear relationship between angular momentum and $(\text{mass})^2$ of resonances



Confinement - IV

String model of mesons: Simple 'explaination' of Regge trajectories



$$F^{\mu} = \frac{dP^{\mu}}{d\tau}, F^{\mu} = (\gamma \mathbf{F} \cdot \mathbf{u}, \gamma \mathbf{F}) \text{ relativistic 2nd law}$$

$$\rightarrow m = E = W = 2\int_0^R \gamma \mathbf{F} \cdot d\mathbf{r}$$

$$\mathbf{F} = k\hat{\mathbf{r}} \quad \leftrightarrow \quad \text{linear potential}$$

$$\rightarrow m = E = 2\int_0^R \gamma k\hat{\mathbf{r}} \cdot d\mathbf{r} = 2\int_0^R \frac{k}{\sqrt{1-\beta^2}} dr$$

$$\beta = \frac{r}{R} \rightarrow m = E = 2k\int_0^R \frac{dr}{\sqrt{1-\left(\frac{r}{R}\right)^2}} = \pi k R$$

$$J = 2k\int_0^R \frac{\frac{r}{R}}{\sqrt{1-\left(\frac{r}{R}\right)^2}} dr = \frac{1}{2}\pi k R^2 = \frac{m^2}{2\pi k}$$

Gluonia - I

QCD : Leading to predict new, 'exotic' (= non $q\bar{q}$) mesonic states

Quarkless mesons: no valence quarks

$\rightarrow$ *Gluonium*, aka *Glueball*

Expect, among others, exotic quantum numbers

Flavor: **1** Singlet ($\leftarrow$ no quark)

Color: Bound state $\rightarrow$ Must be **1** Singlet ($\leftarrow$ 'white')

$\rightarrow 2\ g$ at least

$\mathbf{8} \otimes \mathbf{8} = \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{10} \oplus \overline{\mathbf{10}} \oplus \mathbf{27}$

Pick singlet:

1 $\leftrightarrow$ Symmetric

Bose statistics $\rightarrow$ Spin$\times$Orbital: Symmetric

Observe:

1 of $SU(3)_C$ exchange-symmetric

when originated by

$\mathbf{8} \otimes \mathbf{8} = \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{10} \oplus \mathbf{10}^* \oplus \mathbf{27}$

Gluonia - II

J=L+S

By taking $S-$wave (space symmetric):

$L=0 \rightarrow J=S=1\oplus 1=0,1,2$

$S=0,2$ Symmetric $\rightarrow$ OK

$$\left.\begin{array}{l} P=(-1)^{L}=+1 \\ C=(-1)^{2}=+1 \end{array}\right\} \rightarrow 0^{++}, 2^{++}$$

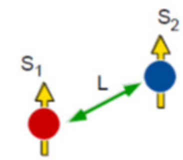
By taking $P-$wave (space antisymmetric):

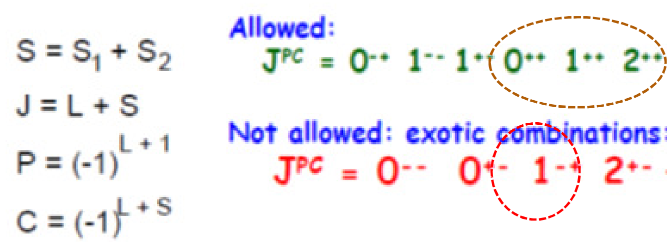
$L=1 \rightarrow J=1\oplus 1\oplus 1=0,1,2,3$

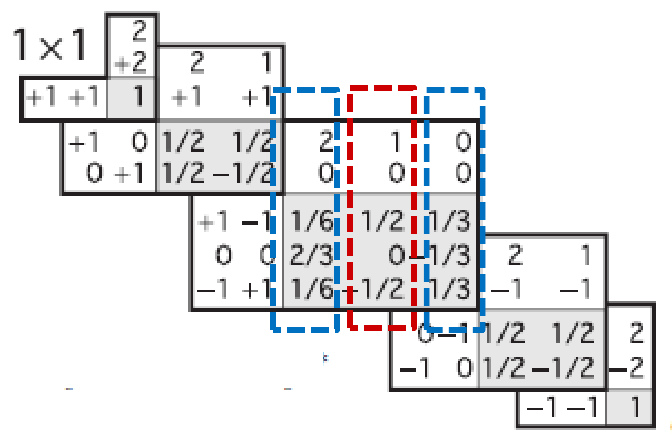
$S=1$ Antisymmetric $\rightarrow$ OK

$$\left.\begin{array}{l} P=(-1)^{L}=-1 \\ C=(-1)^{2}=+1 \end{array}\right\} \rightarrow 1^{-+}, \text{ Exotic!}$$

Compare to $q\bar{q}$, standard mesons:







Gluonia - III

Indeed, build $2g$ state out of single gluon states with defined helicity:

$$U_P\left|\mathbf{k},R;-\mathbf{k},R\right\rangle = \left|-\mathbf{k},L;\mathbf{k},L\right\rangle \quad \text{not a } U_P \text{ eigenstate}$$

$$U_P\left|\mathbf{k},L;-\mathbf{k},L\right\rangle = \left|-\mathbf{k},R;\mathbf{k},R\right\rangle \quad \text{not a } U_P \text{ eigenstate}$$

$$U_P\left|\mathbf{k},R;-\mathbf{k},L\right\rangle = \left|-\mathbf{k},L;\mathbf{k},R\right\rangle \equiv \left|\mathbf{k},R;-\mathbf{k},L\right\rangle$$

$$\rightarrow U_P \text{ eigenstate, } \eta_P = +1,\ J_3 = +2 \rightarrow J = 2$$

$$U_P\left|\mathbf{k},L;-\mathbf{k},R\right\rangle = \left|-\mathbf{k},R;\mathbf{k},L\right\rangle \equiv \left|\mathbf{k},L;-\mathbf{k},R\right\rangle$$

$$\rightarrow U_P \text{ eigenstate, } \eta_P = +1\ \ J_3 = -2 \rightarrow J = 2$$

$$\left|\mathbf{k},R;-\mathbf{k},R\right\rangle \pm \left|\mathbf{k},L;-\mathbf{k},L\right\rangle$$

$$\rightarrow U_P \text{ eigenstates,} \eta_P = \pm 1, J_3 = 0 \rightarrow J = 0,2$$

$$\rightarrow \text{Pick } \left|\mathbf{k},R;-\mathbf{k},R\right\rangle + \left|\mathbf{k},L;-\mathbf{k},L\right\rangle \text{ (symmetric)} \rightarrow \eta_P = +1$$

Quarkonium - I

Small distance: Perturbative!

Indeed: Large quark mass $\rightarrow$ Large Q^2

$\rightarrow$ One gluon exchange OK

Non relativistic effective potential $\sim$ Coulomb-like

$$\rightarrow V\left(r \ll \frac{1}{m_q}\right) \approx -\frac{4}{3}\frac{\alpha_s}{r}$$

Add phenomenological confining term: String inspired

$\rightarrow$ Full potential:

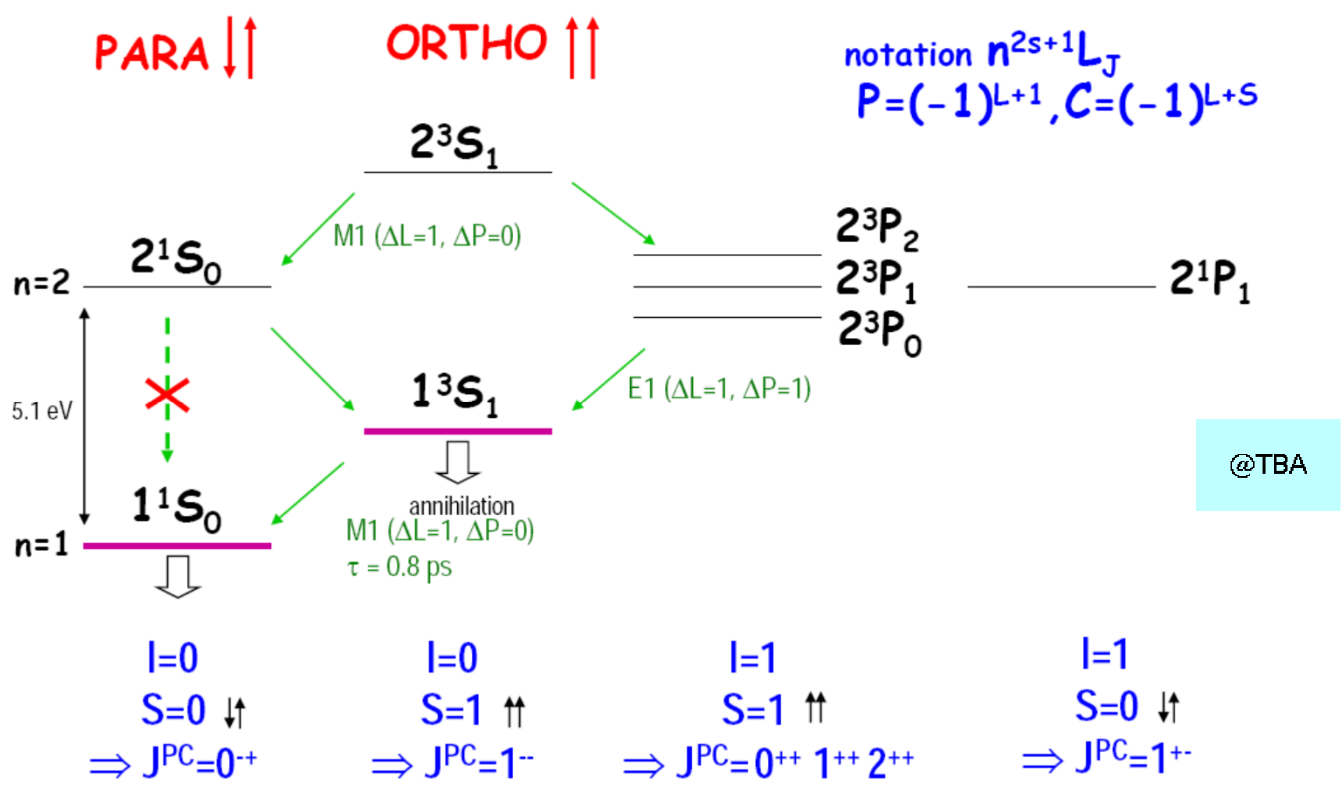
$$V(r) \approx -\frac{4}{3}\frac{\alpha_s}{r} + kr$$

Most interesting $q\bar{q}$ physics case: Heavy, neutral, flavorless mesons

In order to better understand it, revert for a while to simple QED bound state: *Positronium*

Quarkonium - II

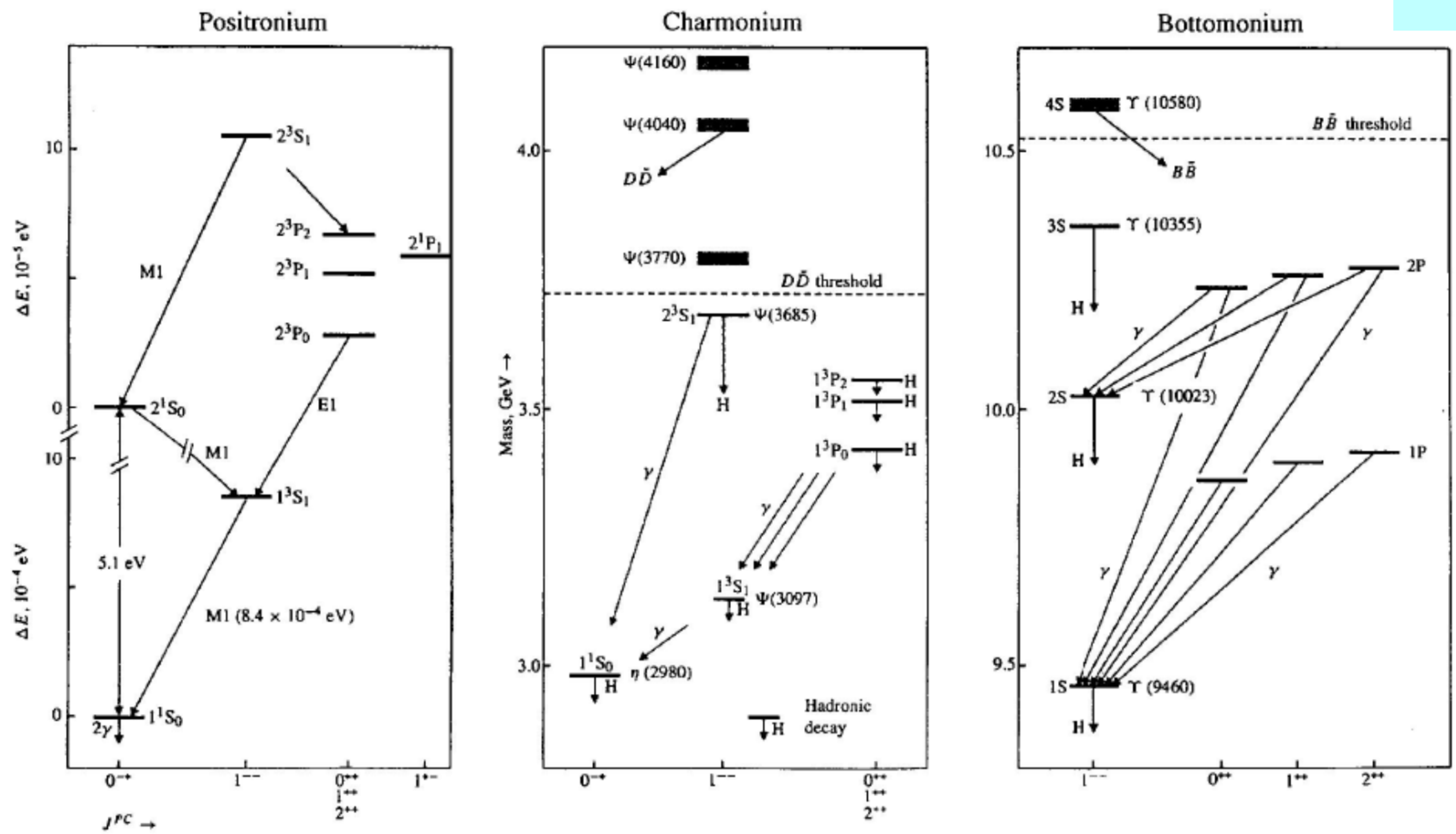
Bound state of electron - positron: Similar to Hydrogen atom



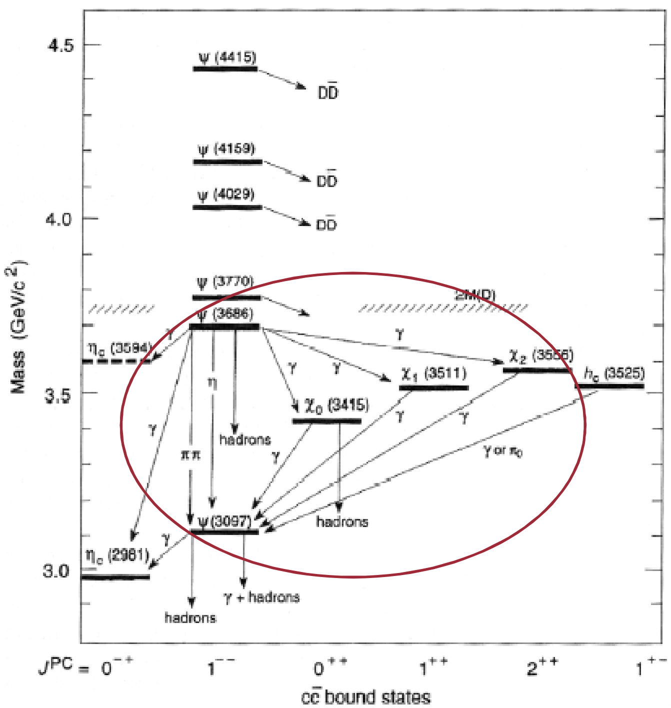
Quarkonium - III

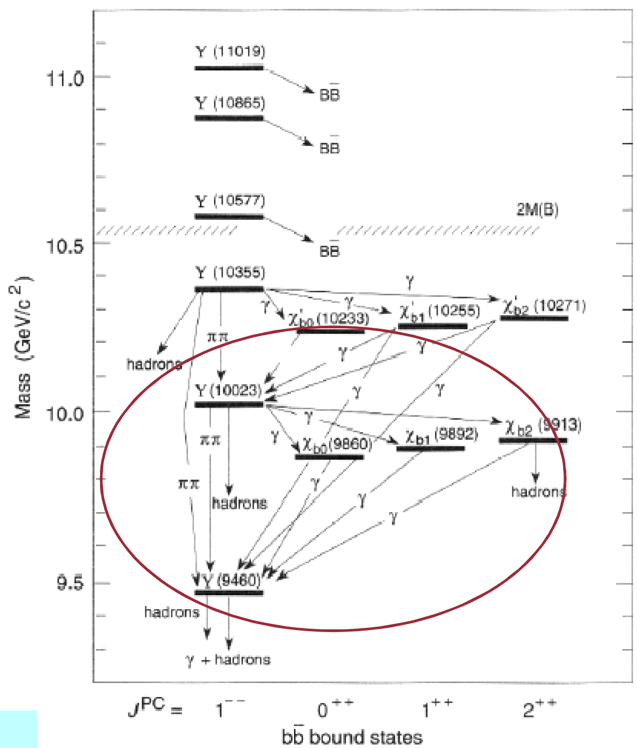
Family portrait of *–onia*:

@TBA



Quarkonium - IV





@TBA

Striking similarity, ≈ same energy scale *above ground state*

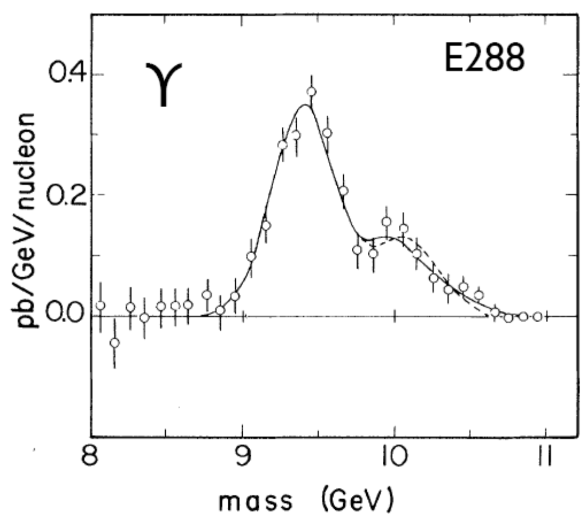
Quarkonium - V

Original interest for non-relativistic, Schrodinger equation approach:

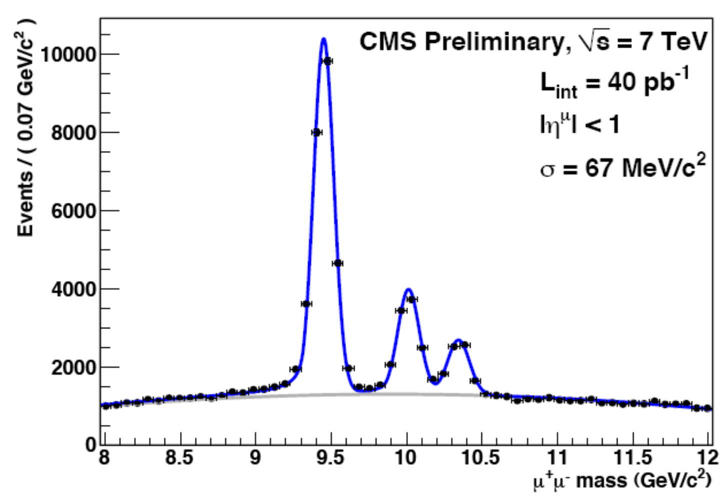
$\Delta E(charm) \simeq \Delta E(bottom)$ Amazingly close

E288	$M(\Upsilon') - M(\Upsilon)$	$M(\Upsilon'') - M(\Upsilon)$
Two-level fit	650 ± 30 MeV	
Three-level fit	610 ± 40 MeV	1000 ± 120 MeV
$M(\psi') - M(\psi)$	≈ 590 MeV	

Yesterday 1977



Today 2012



Quarkonium - VI

Reminder: Bohr radius

$$R = \frac{n^2}{\alpha m} \to R_0 = \frac{1}{\alpha m}$$

Consequence of Coulomb potential, static limit of 1 photon diagram

If 1 gluon exchange approximation can be granted, expect for quarkonium

$$R_{q\bar{q}} \sim \frac{1}{\alpha_s m}$$

Observe: m large $\to$ R small $\to$ α_s small$\to$ 1 gluon appr. OK: Self-consistent

Use phenomenological, $q\bar{q}$ confining term like this:

$$V(r) = -\frac{4}{3}\frac{\alpha_s}{r} + kr$$

Solve Schrodinger equation with these terms
(Add more terms to take into account relativistic & color-hyperfine effects)

Question: Which form of effective potential would yield m-independent ΔE ??

Quarkonium - VII

Scaling Schroedinger:

$$\psi(r) = R(r) Y_{lm}(\theta,\varphi), \quad u(r) = rR(r)$$

$$\mu = \frac{m}{2}$$ Reduced mass

$$V(r) = \lambda r^{\nu}$$ Power law potential

$$\frac{\hbar^2}{2\mu}\frac{d^2u}{dr^2} + \left[E - \lambda r^{\nu} - \frac{l(l+1)}{2\mu r^2}\right] u(r) = 0$$ Radial Schrodinger equation

$$r = \rho \left(\frac{\hbar}{2\mu|a|}\right)^{\frac{1}{2+\nu}}$$ Scale radial distance

$$E = \varepsilon \left(\frac{\hbar}{2\mu|\lambda|}\right)^{-\frac{2}{2+\nu}} \frac{\hbar^2}{2\mu}$$ Scale energy

$$w(\rho) = u(r)$$

$$\rightarrow \frac{d^2w}{d\rho^2} + \left[\varepsilon - \rho^{\nu} - \frac{l(l+1)}{\rho^2}\right] w(\rho) = 0$$ Adimensional radial equation

3 parameters:

μ reduced mass $\equiv \frac{m_q}{2}$ for $q\bar{q}$

λ strength

ν exponent

Quarkonium - VIII

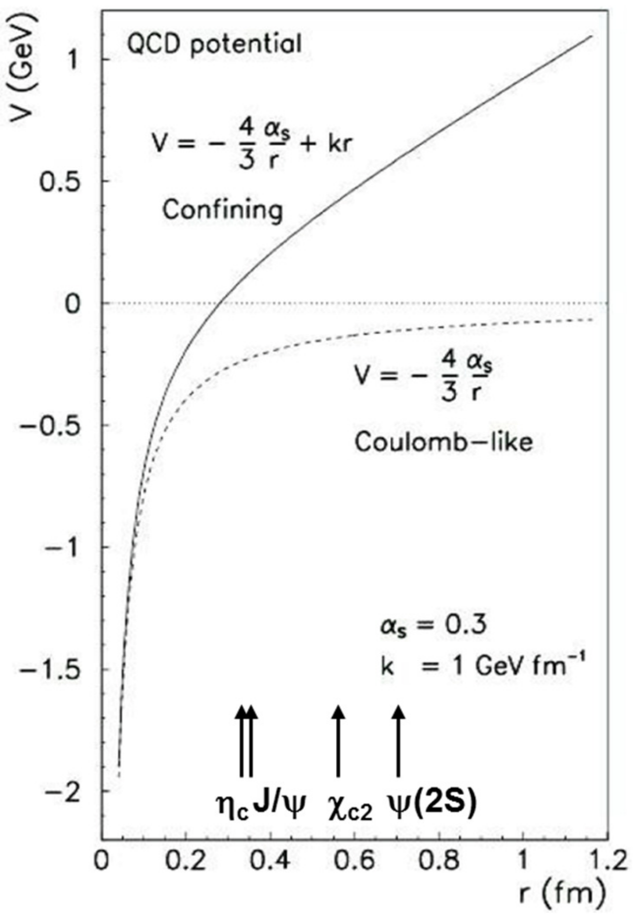
Scaling laws

Length: $L \propto \left(\mu|\lambda|\right)^{-\frac{1}{\nu+2}}$ Energy: $E \propto (\mu)^{-\frac{\nu}{\nu+2}} |\lambda|^{\frac{2}{\nu+2}}$

	Length	Energy
Coulomb	$\left(\mu\|\lambda\|\right)^{-1}$	$\mu\|\lambda\|^2$
Logarithmic	$\left(\mu\|\lambda\|\right)^{-\frac{1}{2}}$	$\mu^0 \|\lambda\|$
Linear	$\left(\mu\|\lambda\|\right)^{-\frac{1}{3}}$	$\mu^{-\frac{1}{3}} \|\lambda\|^{\frac{2}{3}}$
Harmonic	$\left(\mu\|\lambda\|\right)^{-\frac{1}{4}}$	$\mu^{-\frac{1}{2}} \|\lambda\|^{\frac{1}{2}}$
Well	$\left(\mu\|\lambda\|\right)^{0}$	μ^{-1}

Quarkonium - IX

Cornell potential



Quarkonium - X

Several interesting applications:

1) Logarithmic potential yielding ΔE mass independent

Also obtained by properly fitted Cornell potential

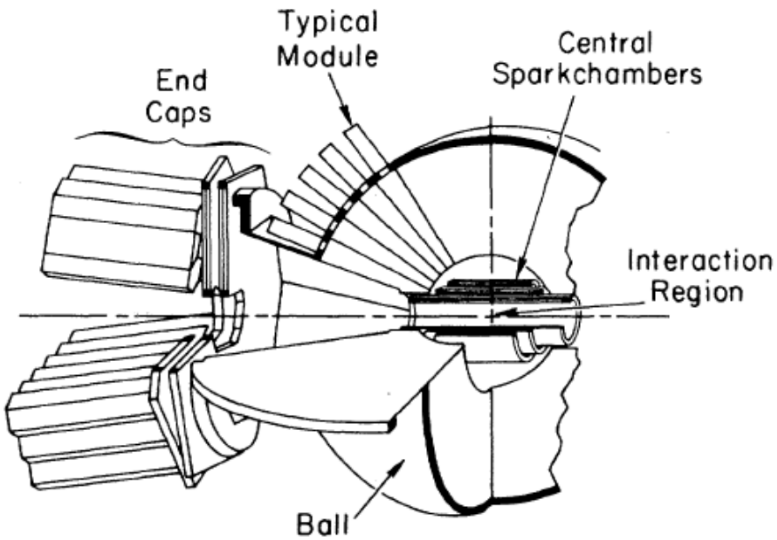
$$\rightarrow \text{Fit} \begin{cases} \alpha_s\left(q^2=\left(2m_q\right)^2\right)\sim 0.25-0.35 \\ k\sim 1 \quad GeV\,/\,fm \end{cases}$$

2)Extra bonus:

Probability density $\propto L^{-3} \rightarrow \left|\psi(0)\right|^2 \sim \left(\mu\left|\lambda\right|\right)^{\frac{3}{\nu+2}}$

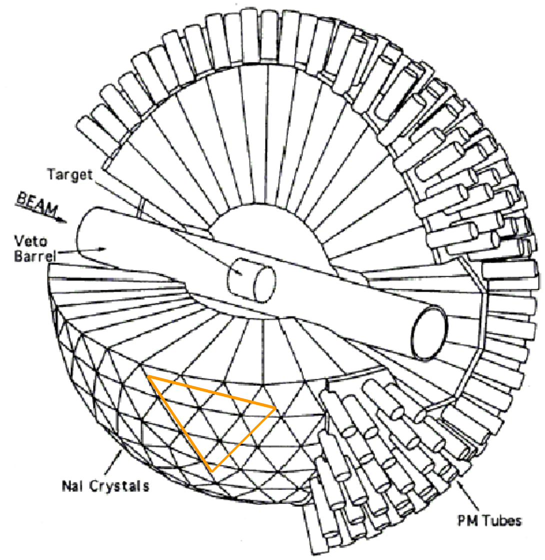
$\rightarrow$ Fix partial width to e^+e^- of vector mesons

Crystal Ball - I



@TBA

94% of solid angle covered



Sodium Iodide

$NaI(Tl)$: Inorganic scintillating crystal; Tl is an activator

Merits:

Can grow large crystals

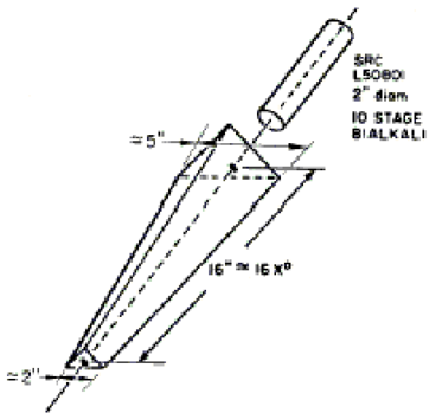
Lots of light

Crystal Ball - II

672 optically isolated NaI(Tl) crystals, 15.7 radiation lengths thick
Inner radius 25.3 cm; Outer radius 66.0 cm

CB geometry: Based on *icosahedron.*
Each of the 20 triangular faces (major triangles) is divided into four minor triangles, each consisting of nine separated crystals.

Each crystal: Truncated triangular pyramid, 40.6 cm high, pointing towards the center of the Ball.
Side on the inner end: 5.1 cm ; Side on the outer end: 12.7 cm

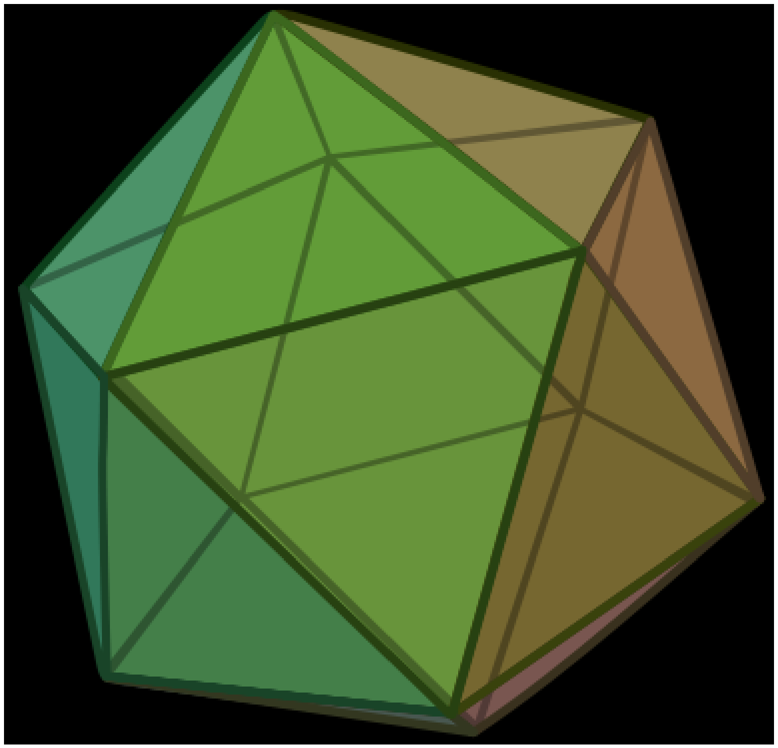


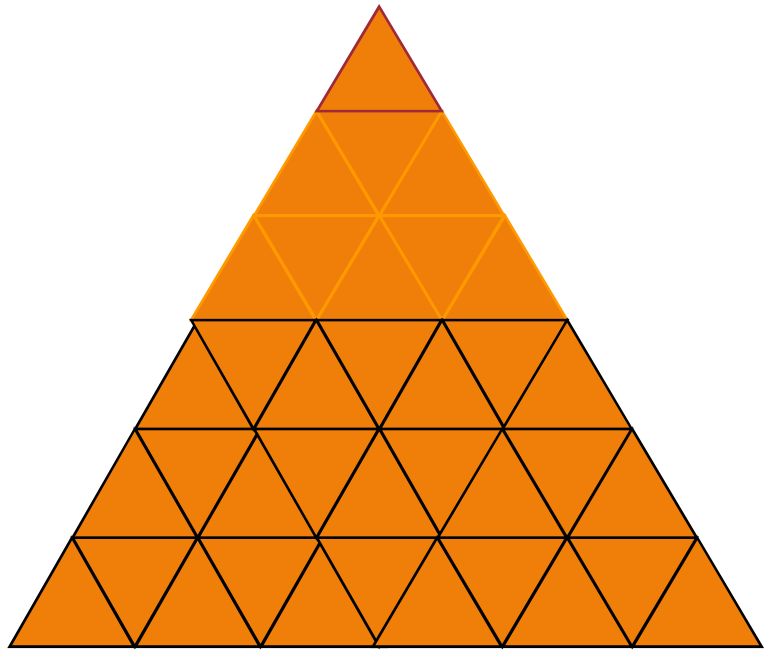
@TBA

Crystal & Photomutiplier

Crystal Ball - III

Icosahedron magic: Platonic solid (!) , 20 equilateral triangle faces





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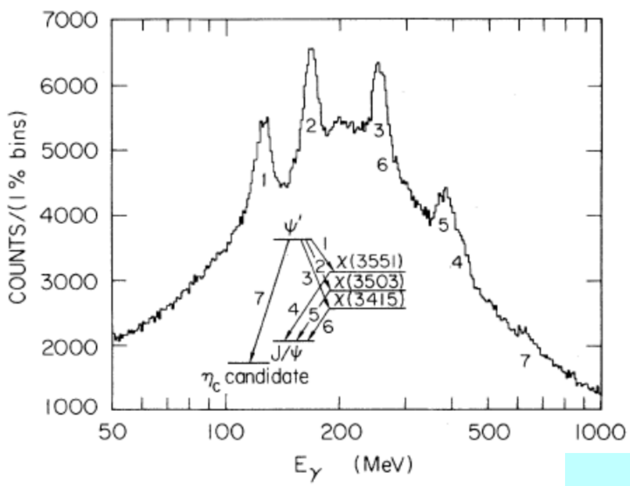
Triangle count:

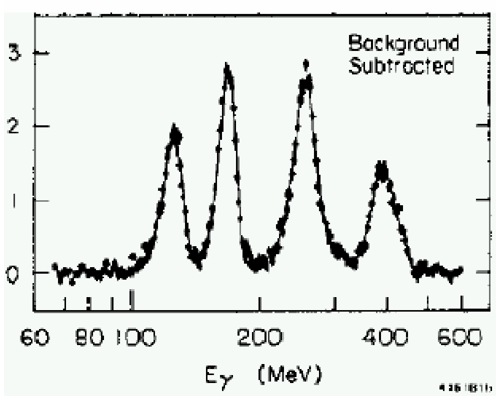
Large triangle 20 Small triangle 80 Crystal < 720 (edges)

Crystal Ball - IV



Inclusive photon spectrum

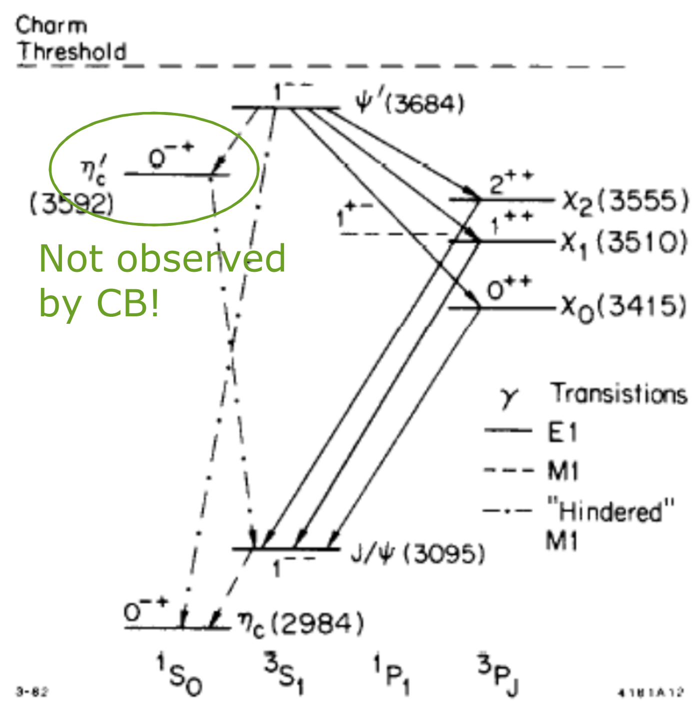
@TBA



Most important results, among many:

Tune beam energy as to form ψ'(3686)

Observe decays into photon + X



Crystal Ball - V

After moving the CB detector to DORIS (DESY, Hamburg): Bottomonium!
Observation of the P-wave triplets

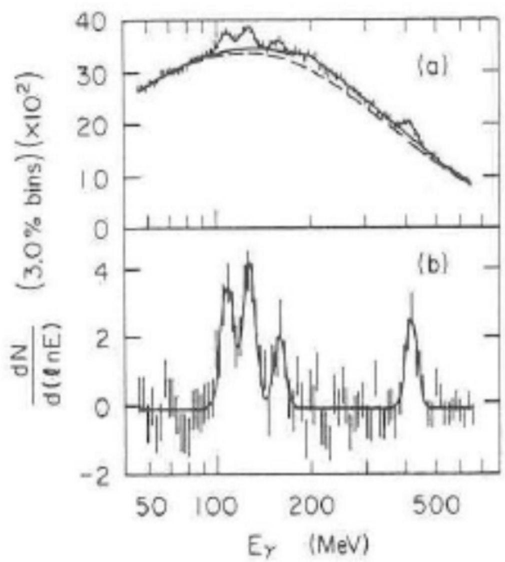


Figure 11.2: The photon spectrum from Υ' decays obtained by the Crystal Ball Collaboration at DORIS II. A triplet of lines corresponding to $\Upsilon' \to \gamma\chi_b(^3P_{2,1,0})$ is seen between 100 and 200 MeV. The decays $\chi_b \to \gamma\Upsilon$ produce the unresolved signal between 400 and 500 MeV [R. Nernst *et al.*, *Phys. Rev. Lett.* **54**, 2195 (1985)].

@TBA

Non-Perturbative QCD - I

Needed to deal with *bound states* and *soft interaction* regime

Very difficult problem

Different approaches available:

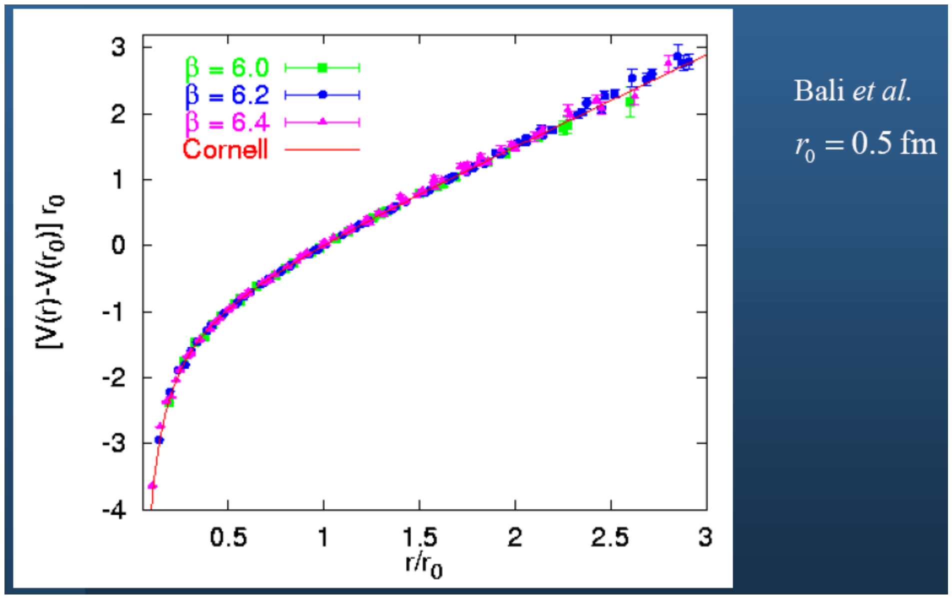
Lattice QCD
Chiral Pertubation Theory
Non-Relativistic QCD
Heavy Quark Effective Theory
…

Deep waters, not even surfed in this course

Non Perturbative QCD - II

Perform QCD calculations over a discretized space-time (lattice)

@TBA



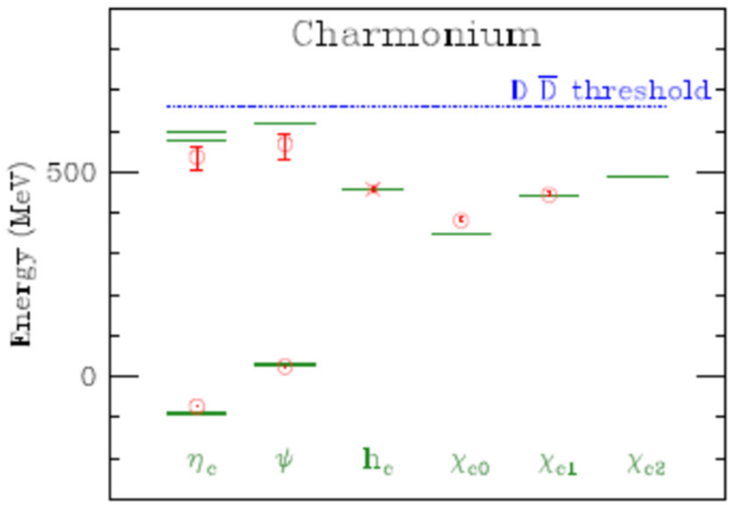
$q\bar{q}$ potential from lattice

$$V(r) = -\frac{4}{3}\frac{\alpha_s}{r} + ar$$: Not a bad idea after all…

Non Perturbative QCD - III

Examples:

Charmonium levels from lattice



@TBA

Predicted glueball spectrum from lattice