Elementary Particles II

2 – Electroweak Interaction

Universal Current-Current Interaction, High Energy Problems, Intermediate Vector Bosons, Gauge Symmetry, Spontaneous Symmetry Breaking, Electroweak Unification, Neutral Currents, Discovery of W & Z, Precision Measurements, Higgs

Electroweak Interaction

Standard Model:
Electromagnetic and Weak Interaction unified into Electroweak

Energy scale where unification is evident:

$E \sim M_W, M_Z \sim 100$ GeV

At lower energies:

Can still find tiny traces of the unification (Electroweak interference, Parity violation in atomic processes,...)

Electroweak interaction split into two, almost not interfering, *effective* interactions:

↑ Non fundamental, useful low energy approximations

Electromagnetic

Weak

$V-A$ - I

After a long history of beta decay experiments: *Current-Current (Fermi) Interaction* including *Vector* & *Axial Vector* terms in order to account for P & C violation

$$H_{\text{int}} = \frac{G_F}{\sqrt{2}} \sum_{i=V,A} C_i \left(\bar{\psi}_p \Gamma_i \psi_n\right)\left(\bar{\psi}_e \Gamma^i \left(1+\frac{C_i\,'}{C_i}\gamma^5\right)\psi_\nu\right)$$

Neutrino helicity = -1 yields lepton current = $V-A$

$$C_i\,' = -C_i \rightarrow \left(1+\frac{C_i\,'}{C_i}\gamma^5\right)\psi_\nu = \left(1-\gamma^5\right)\psi_\nu \qquad\qquad = -\gamma^\mu\left(1-\gamma^5\right)$$

$$\rightarrow H_{\text{int}} = \frac{G_F}{\sqrt{2}}\left[C_V\left(\bar{\psi}_p\gamma_\mu\psi_n\right)\left(\bar{\psi}_e\gamma^\mu\left(1-\gamma^5\right)\psi_\nu\right) + C_A\left(\bar{\psi}_p\gamma_\mu\gamma_5\psi_n\right)\left(\bar{\psi}_e\gamma^\mu\gamma^5\left(1-\gamma^5\right)\psi_\nu\right)\right]$$

$$= \frac{G_F}{\sqrt{2}}\left[C_V\left(\bar{\psi}_p\gamma_\mu\psi_n\right)\left(\bar{\psi}_e\gamma^\mu\left(1-\gamma^5\right)\psi_\nu\right) - C_A\left(\bar{\psi}_p\gamma_\mu\gamma_5\psi_n\right)\left(\bar{\psi}_e\gamma^\mu\left(1-\gamma^5\right)\psi_\nu\right)\right]$$

$$= \frac{G_F}{\sqrt{2}}\left[C_V\left(\bar{\psi}_p\gamma_\mu\psi_n\right) - C_A\left(\bar{\psi}_p\gamma_\mu\gamma_5\psi_n\right)\right]\left(\bar{\psi}_e\gamma^\mu\left(1-\gamma^5\right)\psi_\nu\right)$$

$V - A$ - II

Therefore:

$$H_{\text{int}} = \frac{G_F}{\sqrt{2}} C_V \left(\bar{\psi}_p \gamma_\mu \left(1 - \frac{C_A}{C_V} \gamma_5 \right) \psi_n \right) \left(\bar{\psi}_e \gamma^\mu \left(1 - \gamma^5 \right) \psi_\nu \right)$$

Many violations in weak processes :

Space Parity (large)

Charge Parity (large)

CP (very small)

T(very small)

Flavor conservation(Isospin, S, C, B, T)(larger + smaller)

Lepton numbers (Neutrino oscillations)

V – A - III

Observe:

$$H_{\text{int}} = \frac{2G_F}{\sqrt{2}}\left(\bar{\psi}_p\gamma_\mu\left(1-\frac{C_A}{C_V}\gamma_5\right)\psi_n\right)\left(\bar{\psi}_e\gamma^\mu\frac{\left(1-\gamma^5\right)}{2}\psi_\nu\right)$$

$$\frac{1-\gamma^5}{2}\ \text{Projection operator} \rightarrow \left[\frac{\left(1-\gamma^5\right)}{2}\right]^2 = \frac{1-\gamma^5}{2}$$

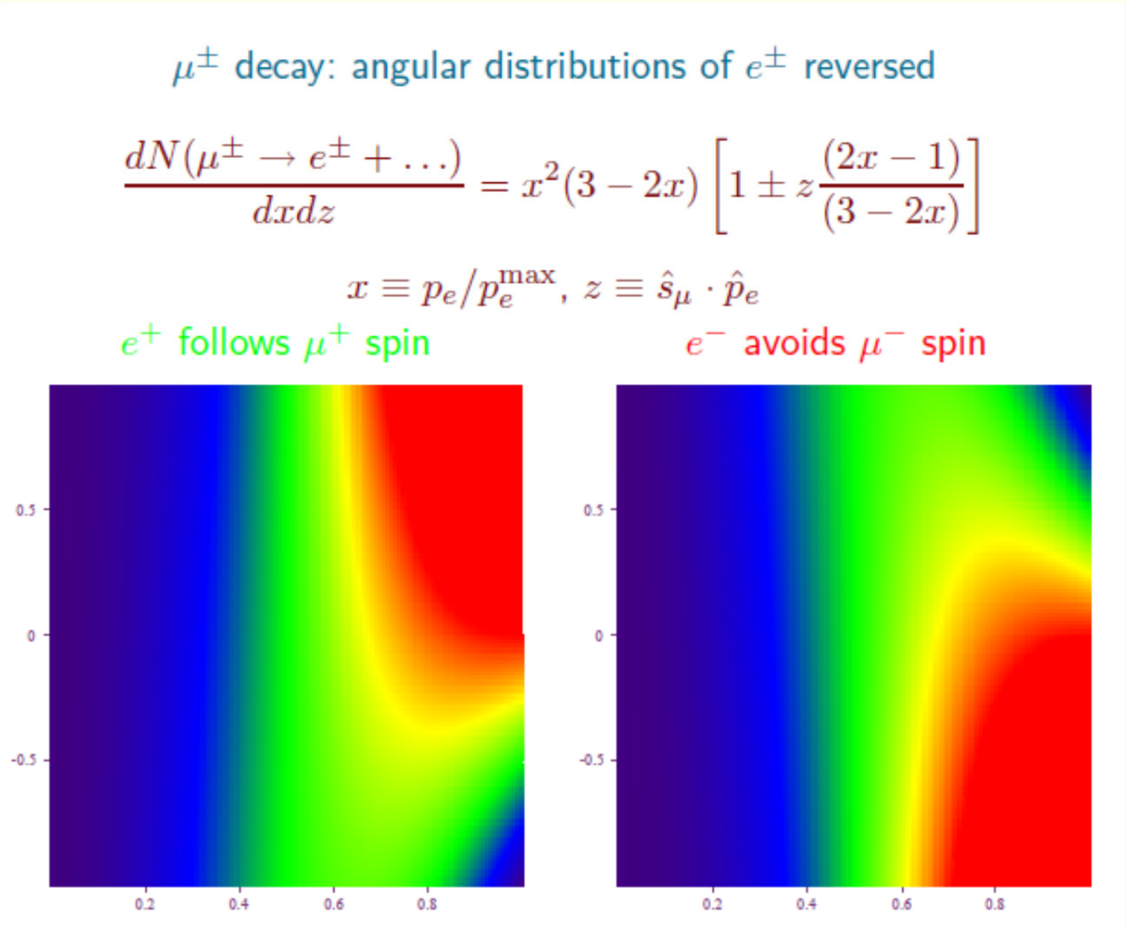
$$\rightarrow H_{\text{int}} = \frac{2G_F}{\sqrt{2}}\left(\bar{\psi}_p\gamma_\mu\left(1-\frac{C_A}{C_V}\gamma_5\right)\psi_n\right)\left(\bar{\psi}_e\gamma^\mu\left[\frac{\left(1-\gamma^5\right)}{2}\right]^2\psi_\nu\right)$$

$$\rightarrow H_{\text{int}} = \sqrt{2}G_F\left(\bar{\psi}_p\gamma_\mu\left(1-\frac{C_A}{C_V}\gamma_5\right)\psi_n\right)\left(\bar{\psi}_e\left(\frac{1+\gamma^5}{2}\right)\gamma^\mu\left(\frac{1-\gamma^5}{2}\right)\psi_\nu\right)$$

Lepton current written as *pure vector* between *chiral parts* of ν,*e* states

→ The weak charged current is just the same as the e.m. current, except it operates between chiral projections with different charge $\Delta Q = \pm 1$

μ Decay: C & P Violations



$\mu^\pm$ decay: angular distributions of $e^\pm$ reversed

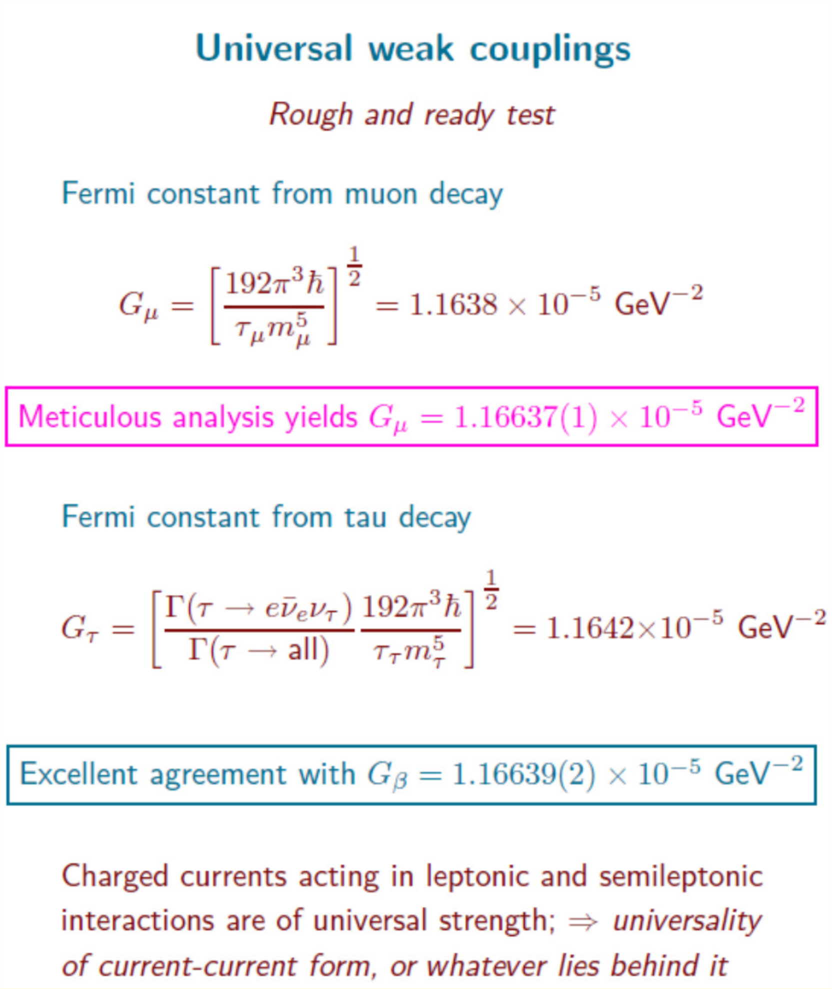
$$\frac{dN(\mu^\pm \to e^\pm + \ldots)}{dxdz} = x^2(3-2x)\left[1 \pm z\frac{(2x-1)}{(3-2x)}\right]$$

$$x \equiv p_e/p_e^{\max},\; z \equiv \hat{s}_\mu \cdot \hat{p}_e$$

e^+ follows μ^+ spin

e^- avoids μ^- spin

Universal Weak Coupling



Universal weak couplings

Rough and ready test

Fermi constant from muon decay

$$G_\mu = \left[\frac{192\pi^3\hbar}{\tau_\mu m_\mu^5}\right]^{\frac{1}{2}} = 1.1638 \times 10^{-5}\ \mathsf{GeV}^{-2}$$

Meticulous analysis yields $G_\mu = 1.16637(1) \times 10^{-5}\ \mathsf{GeV}^{-2}$

Fermi constant from tau decay

$$G_\tau = \left[\frac{\Gamma(\tau \to e\bar{\nu}_e\nu_\tau)}{\Gamma(\tau \to \mathsf{all})}\frac{192\pi^3\hbar}{\tau_\tau m_\tau^5}\right]^{\frac{1}{2}} = 1.1642\times10^{-5}\ \mathsf{GeV}^{-2}$$

Excellent agreement with $G_\beta = 1.16639(2) \times 10^{-5}\ \mathsf{GeV}^{-2}$

Charged currents acting in leptonic and semileptonic interactions are of universal strength; ⇒ *universality of current-current form, or whatever lies behind it*

Universality: Leptons

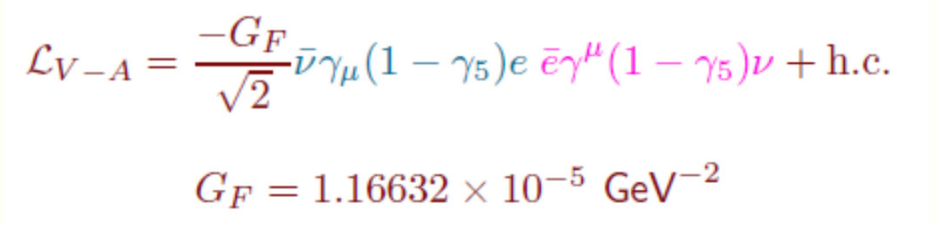
Extend *V-A* to muon weak interactions:

$\mu^+ \to e^+ + \overline{\nu}_\mu + \nu_e, \;\; \mu^- \to e^- + \nu_\mu + \overline{\nu}_e$ μ decay

$\mu^- + p \to n + \nu_\mu$ μ capture, involves nucleon current

μ decay purely leptonic:

Guess: *Current-Current*, *V-A* for both electron and muon charged currents



$$\mathcal{L}_{V-A} = \frac{-G_F}{\sqrt{2}} \bar{\nu}\gamma_\mu(1-\gamma_5)e \; \bar{e}\gamma^\mu(1-\gamma_5)\nu + \text{h.c.}$$

$$G_F = 1.16632 \times 10^{-5} \ \text{GeV}^{-2}$$

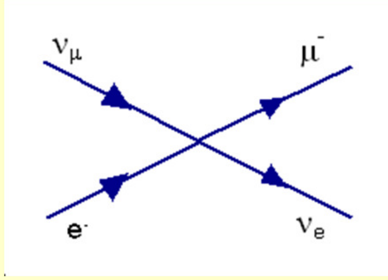
Compute:

μ *Lifetime*
Electron energy spectrum
Electron longitudinal polarization

Inverse Muon Decay - I

Charged current:

$\nu_\mu + e^- \to \mu^- + \nu_e$



$$\sum_{spin} T_{fi}T_{fi}^* = \sum_{spin} \frac{G_F}{\sqrt{2}} \left[\bar{u}(3)\gamma^\mu\left(1-\gamma^5\right)u(1)\right]\left[\bar{u}(3)\gamma_\nu\left(1-\gamma_5\right)u(1)\right]^* \cdot$$

$$\left[\bar{u}(4)\gamma_\mu\left(1-\gamma_5\right)u(2)\right]\left[\bar{u}(4)\gamma^\nu\left(1-\gamma^5\right)u(2)\right]^*$$

$$\sum_{spin}\left[\bar{u}(a)\Gamma_1 u(b)\right]\left[\bar{u}(a)\Gamma_2 u(b)\right]^* = Tr\left[\Gamma_1\left(\not{p}_b + m_b\right)\Gamma_2\left(\not{p}_a + m_a\right)\right]$$

$$\sum_{spin}\left|T_{fi}\right|^2 = 64G_F^2\left(p_1\cdot p_2\right)\left(p_3\cdot p_4\right)$$

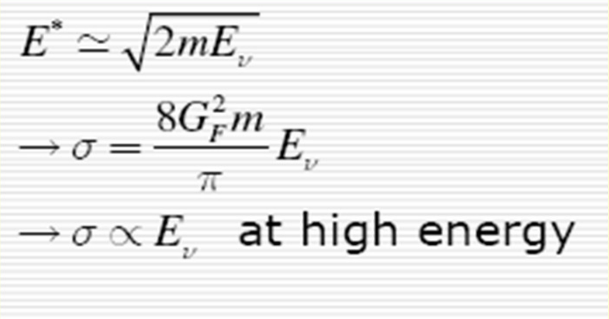
$$\sum_{spin}\left|T_{fi}\right|^2 = 256G_F^2E^4\left[1-\left(\frac{m_\mu}{2E}\right)^2\right]$$

Inverse Muon Decay - II

$$\frac{d\sigma}{d\Omega} = \frac{G_F^2}{\pi^2} E^{*2} \left[1 - \left(\frac{m_\mu}{2E^*} \right)^2 \right]^2$$

$$\rightarrow \sigma = \frac{4G_F^2}{\pi} E^{*2} \left[1 - \left(\frac{m_\mu}{2E^*} \right)^2 \right]^2 ,$$ E^* CM energy of e, v

$E^* \simeq \sqrt{2mE_\nu} \rightarrow \sigma \propto E_\nu$ at high energy

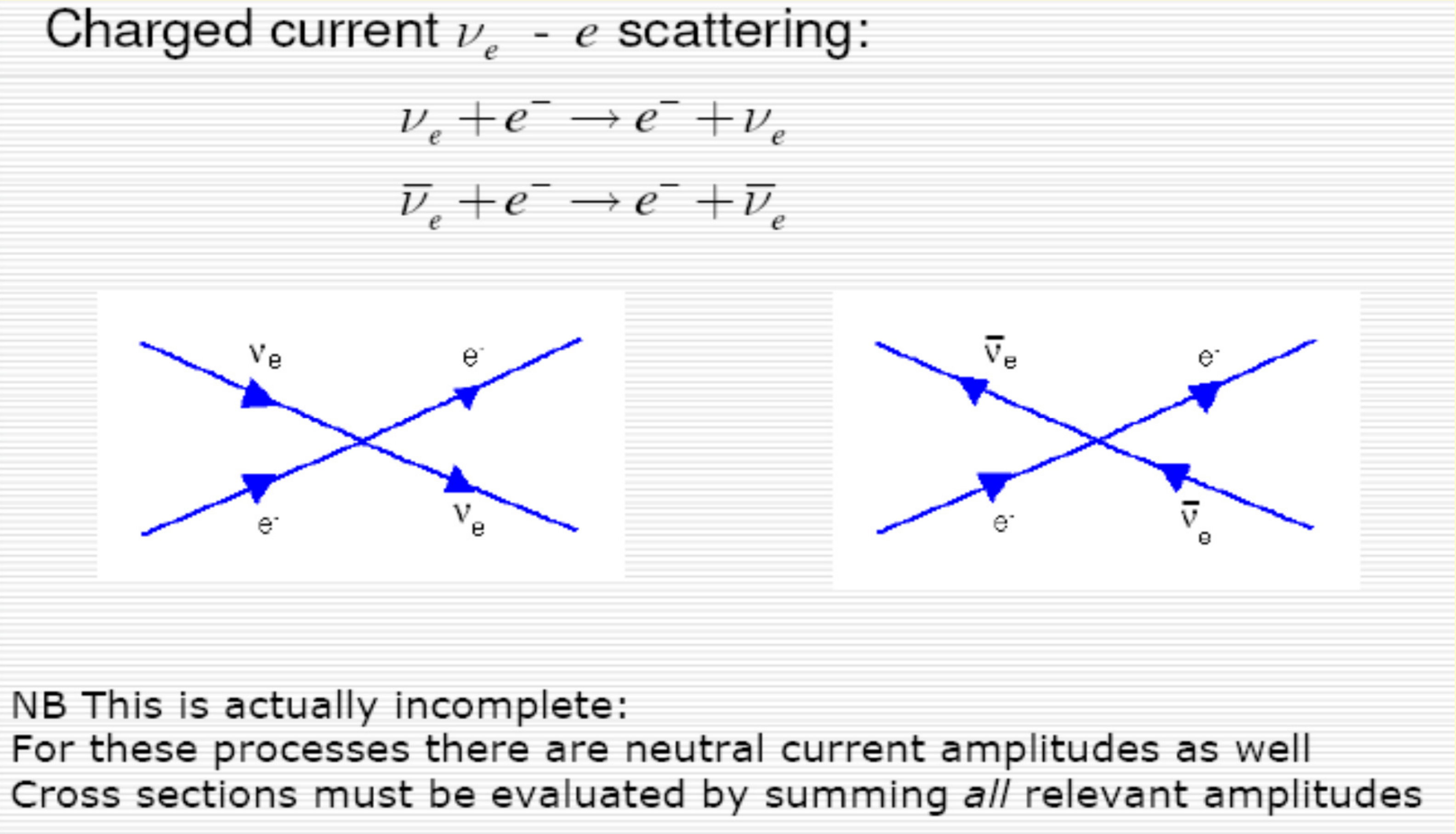


$$E^* \simeq \sqrt{2mE_\nu}$$

$$\rightarrow \sigma = \frac{8G_F^2 m}{\pi} E_\nu$$

$\rightarrow \sigma \propto E_\nu$ at high energy

Neutrino – Lepton Scattering - I



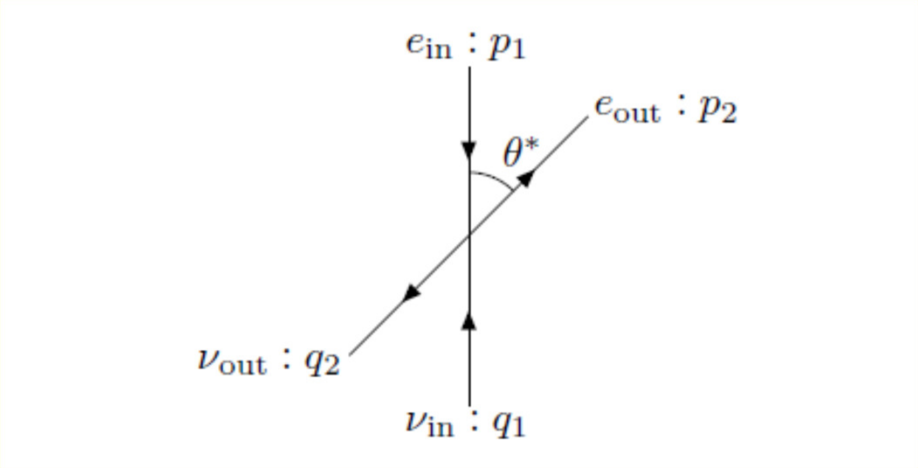
Charged current ν_e - e scattering:

$$\nu_e + e^- \rightarrow e^- + \nu_e$$

$$\overline{\nu}_e + e^- \rightarrow e^- + \overline{\nu}_e$$

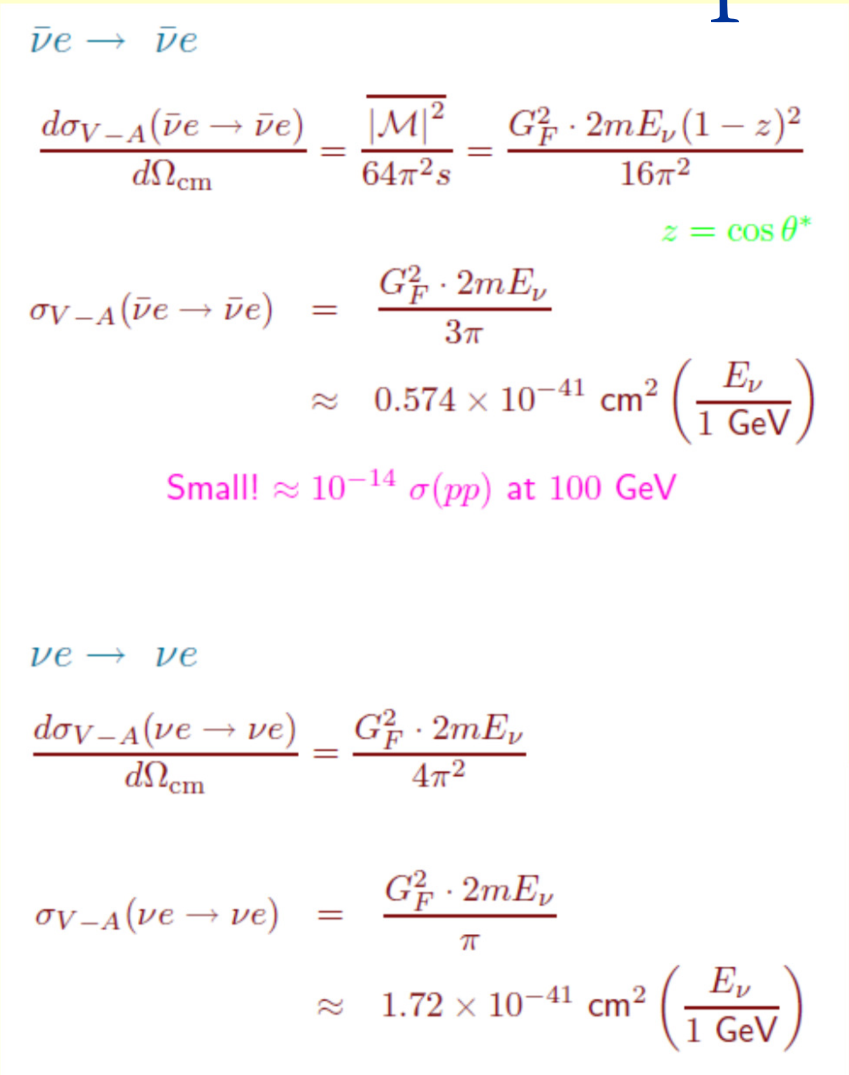
NB This is actually incomplete:
For these processes there are neutral current amplitudes as well
Cross sections must be evaluated by summing *all* relevant amplitudes

Neutrino – Lepton Scattering - II



$$\begin{aligned}\mathcal{M} &= -\frac{iG_F}{\sqrt{2}}\bar{v}(\nu,q_1)\gamma_\mu(1-\gamma_5)u(e,p_1) \\ &\quad \cdot \bar{u}(e,p_2)\gamma^\mu(1-\gamma_5)v(\nu,q_2)\end{aligned}$$

Neutrino – Lepton Scattering - III



$\bar{\nu}e \longrightarrow \bar{\nu}e$

$$\frac{d\sigma_{V-A}(\bar{\nu}e \to \bar{\nu}e)}{d\Omega_{\rm cm}} = \frac{\overline{|\mathcal{M}|^2}}{64\pi^2 s} = \frac{G_F^2 \cdot 2mE_\nu(1-z)^2}{16\pi^2}$$

$z = \cos\theta^*$

$$\sigma_{V-A}(\bar{\nu}e \to \bar{\nu}e) = \frac{G_F^2 \cdot 2mE_\nu}{3\pi}$$

$$\approx 0.574 \times 10^{-41}\ \text{cm}^2 \left(\frac{E_\nu}{1\ \text{GeV}}\right)$$

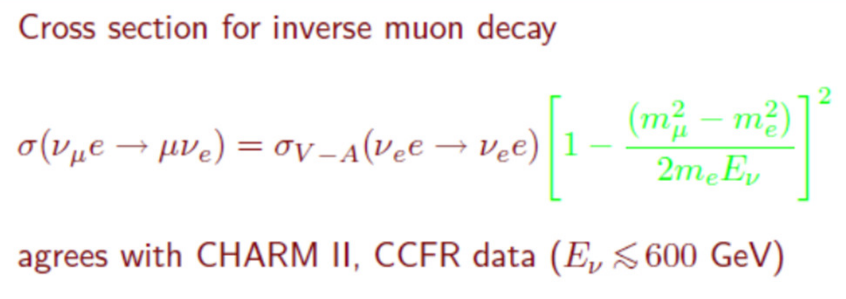
Small! $\approx 10^{-14}$ $\sigma(pp)$ at 100 GeV

$\nu e \longrightarrow \nu e$

$$\frac{d\sigma_{V-A}(\nu e \to \nu e)}{d\Omega_{\rm cm}} = \frac{G_F^2 \cdot 2mE_\nu}{4\pi^2}$$

$$\sigma_{V-A}(\nu e \to \nu e) = \frac{G_F^2 \cdot 2mE_\nu}{\pi}$$

$$\approx 1.72 \times 10^{-41}\ \text{cm}^2 \left(\frac{E_\nu}{1\ \text{GeV}}\right)$$

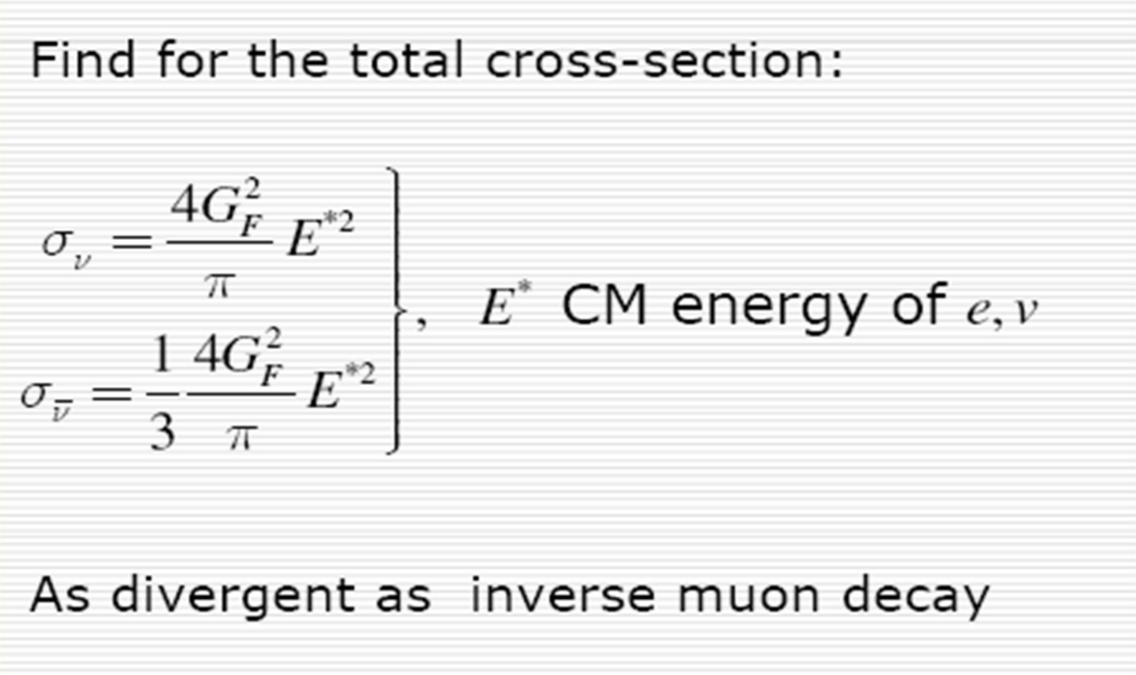


Cross section for inverse muon decay

$$\sigma(\nu_\mu e \to \mu\nu_e) = \sigma_{V-A}(\nu_e e \to \nu_e e)\left[1 - \frac{(m_\mu^2 - m_e^2)}{2m_e E_\nu}\right]^2$$

agrees with CHARM II, CCFR data ($E_\nu \lesssim 600$ GeV)

Neutrino – Lepton Scattering - IV



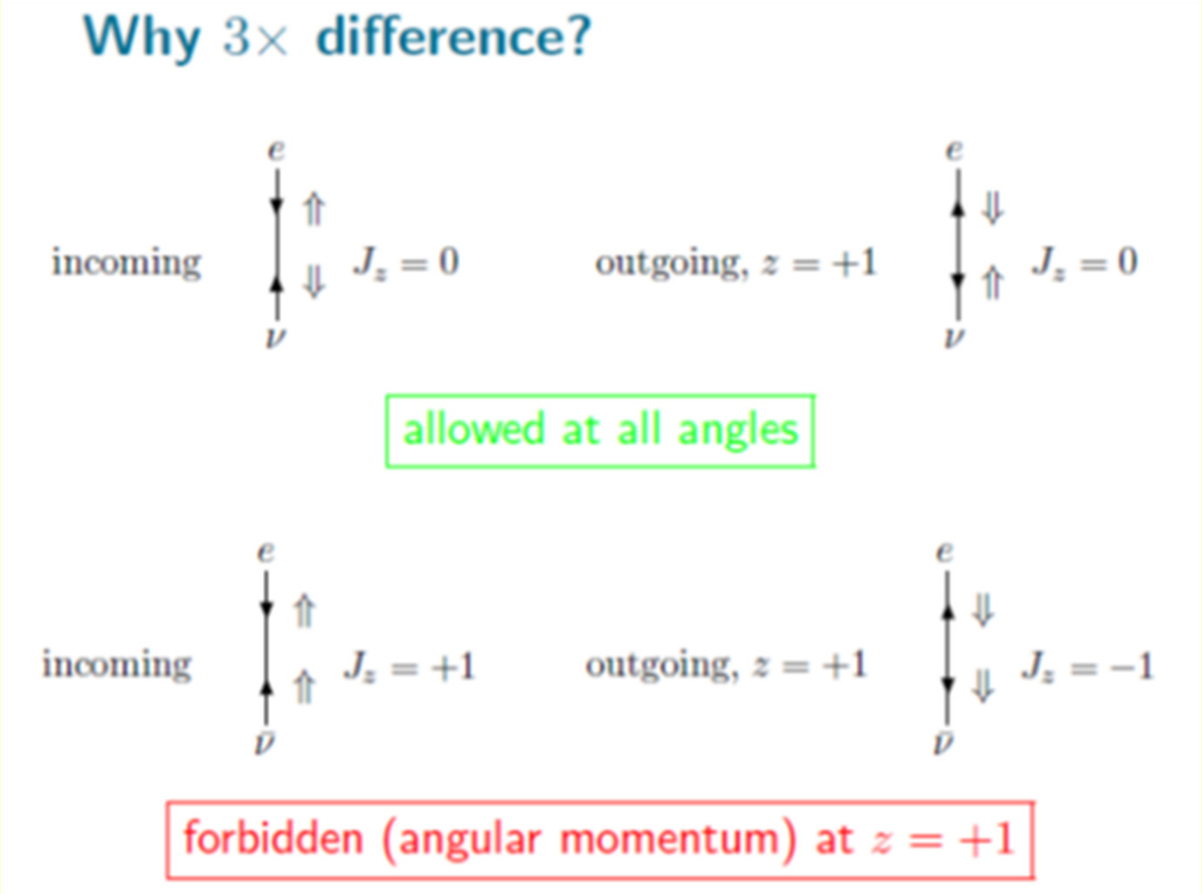
Find for the total cross-section:

$$\left.\begin{aligned}\sigma_\nu &= \frac{4G_F^2}{\pi}E^{*2}\\ \sigma_{\bar{\nu}} &= \frac{1}{3}\frac{4G_F^2}{\pi}E^{*2}\end{aligned}\right\},\quad E^* \text{ CM energy of } e,\nu$$

As divergent as inverse muon decay

NB These cross sections are only approximate, in that neutral current contribution is neglected

Neutrino – Lepton Scattering - V



Why $3\times$ difference?

incoming $J_z = 0$ outgoing, $z = +1$ $J_z = 0$

allowed at all angles

incoming $J_z = +1$ outgoing, $z = +1$ $J_z = -1$

forbidden (angular momentum) at $z = +1$

Unitarity Troubles - I

Cross section cannot be strictly proportional to E_n,, of course:

Divergence at high energy!

Indeed, unitarity bound is violated around E_ν *~ *300 GeV*

$\sigma \le \frac{4\pi}{k^2} = \frac{4\pi}{E^{*2}}$ Unitarity bound for S-Wave scattering

Unitarity Troubles - II

Reminder: (Simpler) Spinless potential scattering

Expand incident (plane) wave into angular momentum eigenstates

$$\Psi_i = e^{ikz} = \frac{i}{2kr}\sum_{l=0}^{\infty}(2l+1)\left[(-1)^l e^{-ikr} - e^{ikr}\right]P_l(\cos\theta)$$

Outgoing spherical wave phase shifted by potential:

$$\Psi_{total} = \Psi_{scattered} + \Psi_i = \frac{i}{2kr}\sum_{l=0}^{\infty}(2l+1)\left[(-1)^l e^{-ikr} - \eta_l e^{2i\delta_l} e^{ikr}\right]P_l(\cos\theta)$$

$$\Psi_{scattered} = \Psi_{total} - \Psi_i = \frac{e^{ikr}}{kr}\sum_{l=0}^{\infty}(2l+1)\frac{\eta_l e^{2i\delta_l} - 1}{2i}P_l(\cos\theta) = \frac{e^{ikr}}{r} f\left(\theta\right)$$

Outgoing elementary flux:

$$d\Phi_{out} = v_{out}\Psi_{scat}\Psi^*_{scat} r^2 d\Omega = v_{out}\left|F(\theta)\right|^2 d\Omega$$

Incident flux:

$$\Phi_{in} = \Psi_{in}\Psi^*_{in} v_{in} = v_{in}$$

Unitarity Troubles - III

$$d\sigma = \frac{F_{out}}{F_{in}} = \left|F(\theta)\right|^2 d\Omega$$

$$\sigma = \int \left|F(\theta)\right|^2 d\Omega$$

$$\sigma = \frac{1}{k^2} \sum_{l,m} (2l+1) \left[\frac{\eta_l e^{2i\delta_l} - 1}{2i}\right] (2m+1) \left[\frac{\eta_m e^{2i\delta_m} - 1}{2i}\right]^*$$

$$\times \int P_l(\cos\theta) P_m(\cos\theta) d\Omega$$

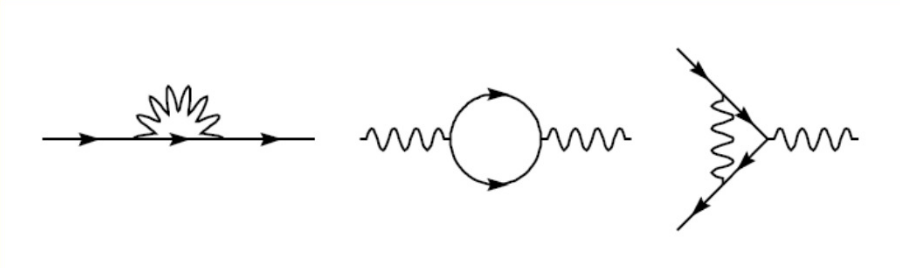
$$\sigma = \frac{4\pi}{k^2} \sum_l (2l+1) \left\| \left[\frac{\eta_l e^{2i\delta_l} - 1}{2i}\right] \right\|^2 = \frac{4\pi}{k^2} \sum_l (2l+1) \sin^2 \delta_l$$

$$\rightarrow \sigma_{l=0} = \frac{4\pi}{k^2} \sin^2 \delta_0 \leq \frac{4\pi}{k^2} = \frac{4\pi}{E^{*2}}$$ Unitarity bound for S-Wave scattering

Renormalization Troubles - I

Suppose Fermi's theory can be saved by radiative corrections:
Assume divergent cross-section as due to our limited, tree-level approximation
Maybe higher orders could fix it

Take QED as an example

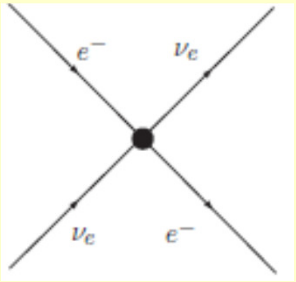


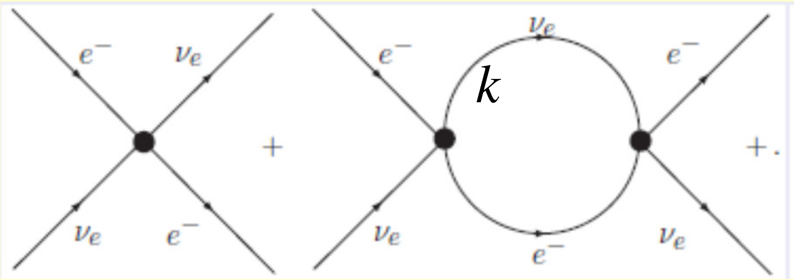
These diagrams (and higher orders) divergent:
However, nice fix available by renormalization procedure

Very successful program, leading to extraordinary accuracy & agreement between theory and experiment

Renormalization Troubles - II

Higher order diagrams in Fermi's theory:





Cannot fix them by renormalization: Fermi's theory non-renormalizable

Indeed: Each vertex ~ G_F

Renormalization Troubles - III

Lagrangian density (μ decay etc)

$$L_{Fermi} = -\frac{G_F}{\sqrt{2}} \bar{\nu}_\mu \gamma^\alpha \left(1-\gamma^5\right) \mu \bar{e} \gamma_\alpha \left(1-\gamma^5\right) \nu_e$$

Action dimension:

$$S = \int L_{Fermi} d^4 x \rightarrow [S] = ET = EE^{-1} = 0$$

$$\rightarrow [L_{Fermi}] = E^4$$

$$[L_{Fermi}] = [G_F][\psi^4]$$

Field dimension: $[\psi] = E^{\frac{3}{2}}$

$$\rightarrow [L_{Fermi}] = [G_F] E^6 = E^4$$

$$\rightarrow [G_F] = E^{-2}$$

Amplitude dimension: $[A] = 0$

→ Loop diagrams of higher orders include integrals of higher powers of k

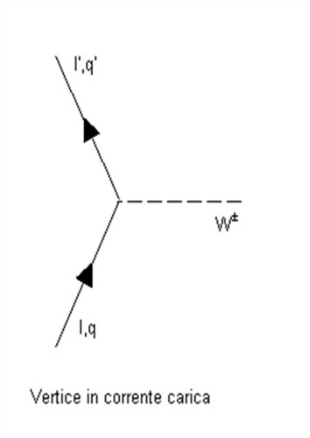
→ More and more divergent

Beyond Fermi's Theory

As anticipated:

Current-Current must be a *low energy effective theory:*
Low energy approximation of a more general theory

Replace *contact interaction* (current-current) by *boson exchange*:



Vertice in corrente carica

Modeled after the electromagnetic interaction

Exchanged particle must be

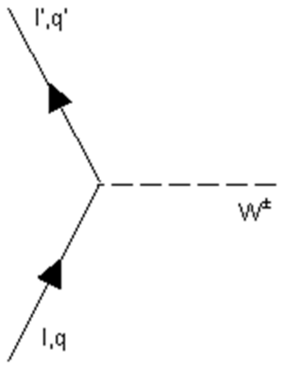
- *Charged* (Charged current ±)
- *Chiral* (Only coupled to left chiral parts: Parity violation)
- *Heavy* (Fermi's point-like interaction OK at low energy)

Intermediate Vector Boson - I

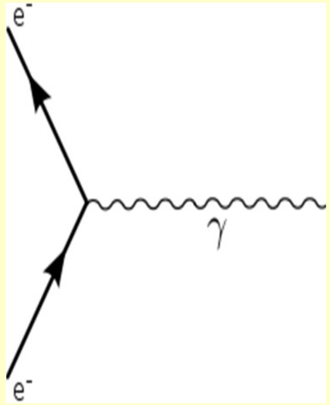
Some key points

A) (Quarks and) Leptons both interact through the exchange of *vector particles*

$\Delta Q = \pm 1$



Vertice in corrente carica



Vertice elettromagnetico

$\Delta Q = 0$

Intermediate Vector Boson - II

B) Exchanged vector bosons are *(very)* massive

Range of weak interaction quite small:

Compare β-decay of nuclei, $R<R_{nucleus}$

Cannot tell how large is boson mass, just raw estimate $M \geq 1\ GeV$

C) Exchanged vector bosons have undefined parity

Parity violation

Intermediate Vector Boson - III

W propagator: $-i\dfrac{g_{\mu\nu}-q_\mu q_\nu / M_W^2}{q^2-M_W^2}$

$q^2 \ll M_W^2 \rightarrow -i\dfrac{g_{\mu\nu}-q_\mu q_\nu / M_W^2}{q^2-M_W^2} \approx i\dfrac{g_{\mu\nu}}{M_W^2}$ q^2-independent

$$T_{fi} \cong \left(\frac{1}{2\sqrt{2}}\right)^2 g_W^2 \left(\overline{u}_f^{(1)}\frac{1}{2}\gamma^\mu\left(1-\gamma^5\right)u_i^{(1)}\right) i\frac{g_{\mu\nu}}{M_W^2}\left(\overline{u}_f^{(2)}\frac{1}{2}\gamma_\nu\left(1-\gamma_5\right)u_i^{(2)}\right)$$

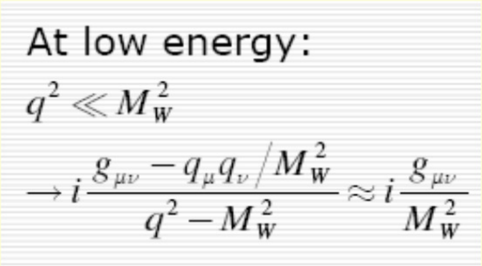
$$\rightarrow T_{fi} \cong i\frac{1}{8}4\sqrt{2}G_F \;\; j^{\mu(1)} j_\mu^{(2)}$$

$g_W^2 \equiv \alpha_W$ Charged current coupling constant

$G_F = \dfrac{\sqrt{2}}{8}\left(\dfrac{g_W}{M_W}\right)^2$ Fermi constant

Intermediate Vector Boson - IV

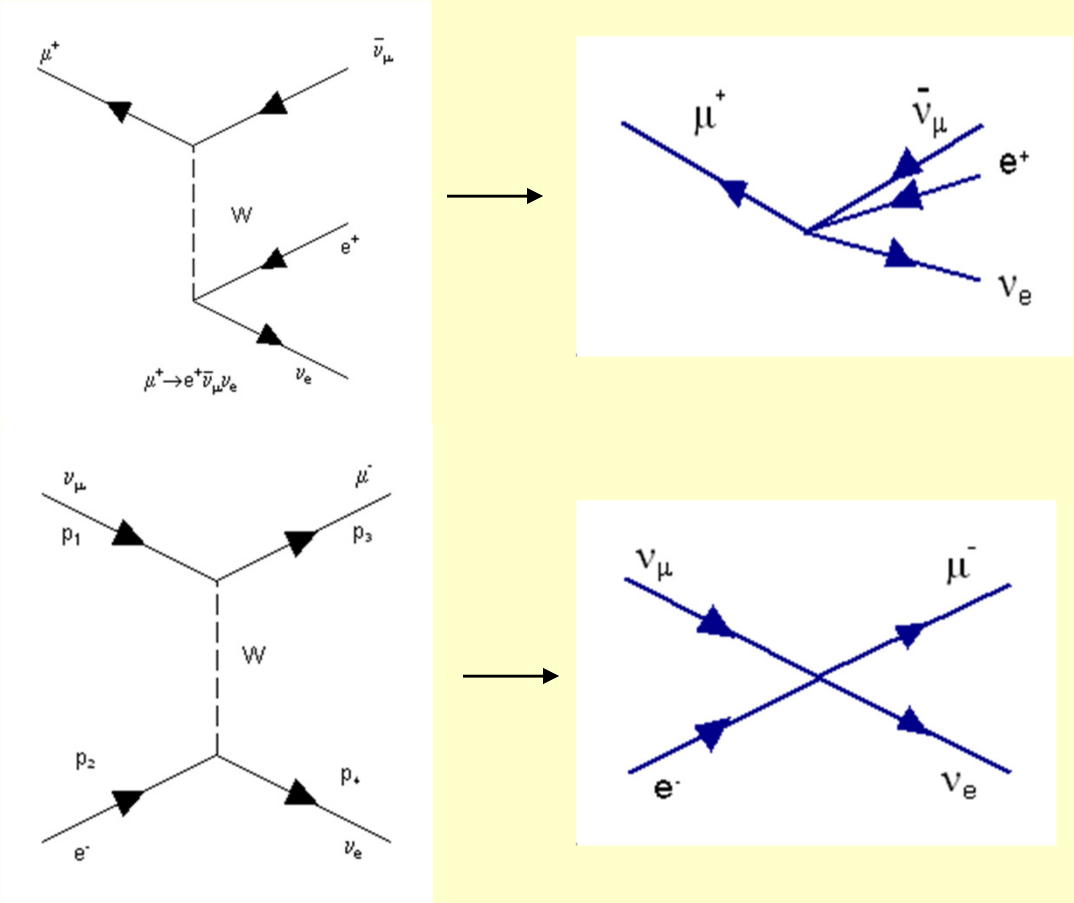
Showing how SM diagrams collapse into current-current:



At low energy:

$$q^2 \ll M_W^2$$

$$\to i\frac{g_{\mu\nu} - q_\mu q_\nu / M_W^2}{q^2 - M_W^2} \approx i\frac{g_{\mu\nu}}{M_W^2}$$

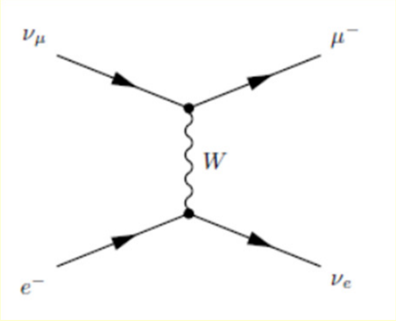


Intermediate Vector Boson - V

Good fix for some problems:

Cross sections of several neutrino reactions

Inverse Muon Decay:



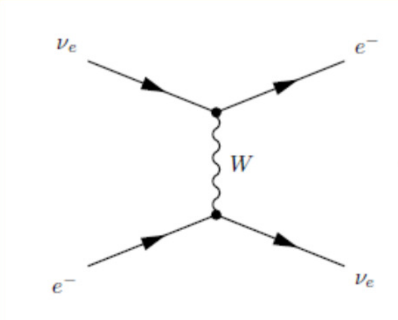
$$\frac{d\sigma}{d\Omega_{CM}} = \frac{G_F^2 M_W^4}{16\pi^2 k^2}\left(\frac{4k^2}{4k^2 - M_W^2}\right)^2 \rightarrow \frac{d\sigma}{d\Omega_{CM}} \approx \begin{cases} \dfrac{G_F^2 k^2}{\pi^2}, & 4k^2 \ll M_W^2 \\ \dfrac{G_F^2 M_W^4}{16\pi^2 k^2}, & 4k^2 \gg M_W^2 \end{cases}$$

$$\rightarrow \sigma \sim \begin{matrix} \dfrac{4G_F^2 k^2}{\pi}, & 4k^2 \ll M_W^2 \\ \dfrac{G_F^2 M_W^4}{4\pi k^2}, & 4k^2 \gg M_W^2 \end{matrix}$$

no divergence!

Intermediate Vector Boson - VI

Charged current (only), tree level elastic (anti) neutrino-electron cross sections:



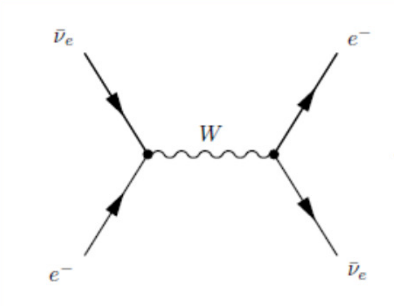
$$\nu_e + e \to \nu_e + e$$

$$\frac{d\sigma}{d\Omega_{CM}} \simeq \frac{16\mathrm{G}_F^2 M_W^2}{\pi^2} \frac{4k^2}{\left(q^2 - M_W^2\right)^2}, \quad k^2 \gg m_e^2$$

$$q^2 \simeq -2k^2\left(1-\cos\theta\right)$$

$$s \simeq 4k^2$$

$$\to \sigma \simeq \frac{G_F^2}{\pi} \frac{4k^2}{1+\frac{4k^2}{M_W^2}} \underset{k^2 \gg M_W^2}{\to} \frac{G_F^2 M_W^2}{\pi}, \quad \text{no divergence!}$$



$$\overline{\nu}_e + e \to \overline{\nu}_e + e$$

$$\sigma \simeq \frac{G_F^2 M_W^4}{3\pi} \frac{4k^2}{16k^4\left(1-\frac{M_W^2}{4k^2}\right)^2} = \frac{G_F^2 M_W^4}{3\pi} \frac{1}{4k^2\left(1-\frac{M_W^2}{4k^2}\right)^2}$$

$$\sigma \sim \frac{G_F^2 M_W^2}{3\pi 4k^2}, \quad k^2 \gg M_W^2 \quad \text{no divergence!}$$

Intermediate Vector Boson - VII

Still unitarity problems:
S-wave scattering amplitude for $\nu_e \, e$ (charged current)

$$a_0 = \frac{GM_W^2}{\sqrt{2}\pi} \log\left(1 + \frac{4k^2}{M_W^2}\right)$$

Unitarity violation, indeed, albeit at ridicolously huge energy

$$k \simeq \frac{M_W}{2} \exp\left(\frac{\pi}{\sqrt{2}GM_W^2}\right)$$

[Observe:
Constant cross-section does not imply constant scattering amplitude
Phase space + Flux factors enter to modify matrix element *k*-dependence]

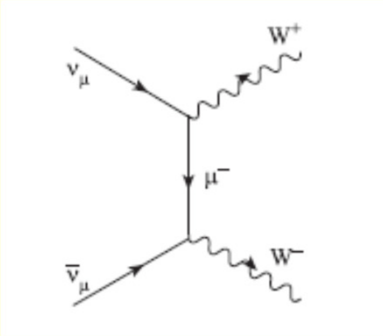
Similar situation for other neutrino processes

Intermediate Vector Boson - VIII

Another dark side of naive IVB model:

Take hypothetical reaction

$\nu_\mu + \overline{\nu}_\mu \rightarrow W^+ + W^-$



No question, not realistic to realize in the lab...

Nevertheless, it should be possible to compute cross section

Anyway, similar issues for the (realistic) reaction

$e^+ + e^- \rightarrow W^+ + W^-$

Intermediate Vector Boson - IX

Central issue:

Massive $W^{\pm}$ bosons in the final state

$\rightarrow$ 3 polarization states for a massive vector particle

Rest frame:

$$\left.\begin{array}{l}\varepsilon_x = (0,1,0,0) \\ \varepsilon_y = (0,0,1,0)\end{array}\right\} \varepsilon_T \quad \text{Transverse polarizatiom}$$

$$\varepsilon_z = (0,0,0,1) \quad \varepsilon_L \quad \text{Longitudinal polarization}$$

After a z-boost, carrying the W to 4-momentum $k^{\mu} = (k^0, 0, 0, k)$

$$\varepsilon_T(k) = \varepsilon_T(0)$$

$$\varepsilon_L(k) = \left(\frac{k}{M_W}, 0, 0, \frac{k_0}{M_W}\right) = \frac{k^{\mu}}{M_W} + O\left(\frac{M_W}{k_0}\right)$$

Intermediate Vector Boson - X

Not too difficult to show that:

$$\sigma\left(\nu\bar{\nu} \to W_{\mathrm{T}}^{+} W_{\mathrm{T}}^{-}\right) \xrightarrow{k^2 \to \infty} \text{constant}$$

$$\sigma\left(\nu\bar{\nu} \to W_{\mathrm{L}}^{+} W_{\mathrm{L}}^{-}\right) \xrightarrow{k^2 \to \infty} \left(g_W/M_W\right)^4 k^2$$

Matrix element:

$$\mathcal{M}_{\lambda_1\lambda_2} = g^2 \epsilon_{\mu}^{-*}(k_2, \lambda_2)\epsilon_{\nu}^{+*}(k_1, \lambda_1)\bar{v}(p_2)\gamma^{\mu}(1-\gamma_5) \times \frac{(\not{p}_1 - \not{k}_1 + m_{\mu})}{(p_1 - k_1)^2 - m_{\mu}^2}\gamma^{\nu}(1-\gamma_5)u(p_1)$$

since in the limit of high energy:

$$\epsilon(k, \lambda = 0) = \frac{k^{\mu}}{M_{\mathrm{W}}} + \frac{M_{\mathrm{W}}}{(k^0 + |\boldsymbol{k}|)}(-1, \hat{\boldsymbol{k}})$$

Intermediate Vector Boson - XI

By neglecting μ mass, limiting to longitudinally polarized *W* ($\lambda=0$), taking the high energy (>> M_W) limit for the polarization 4-vectors, commuting γ_5:

$$\frac{g^4}{M_W^4(p_1-k_1)^4}\mathrm{Tr}[\not{k}_2(1-\gamma_5)(\not{p}_1-\not{k}_1)\not{k}_1\not{p}_1\not{k}_1(\not{p}_1-\not{k}_1)\not{k}_2\not{p}_2]$$

$$\sum_{\text{spins}}|\mathcal{M}_{00}|^2 \sim (g^4/M_W^4)(p_1\cdot k_2)(p_2\cdot k_2) = (g^4/M_W^4)E^4(1-\cos^2\theta)$$

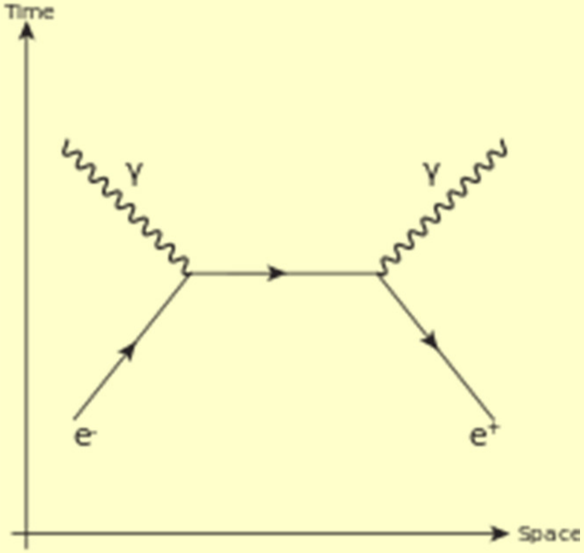
$$\frac{d\sigma}{d\Omega} = G_F^2\frac{E^2\sin^2\theta}{8\pi^2}$$

No simple solution for this problem:

Massive vector particles cannot make without 3 polarization states

Intermediate Vector Boson - XII

Compare to well known QED process



No contribution from longitudinal photons:

Real photons always transverse, as a consequence of *gauge invariance* of QED

Hope the gauge invariance benefits can be extended to weak interactions..

Intermediate Vector Boson - XIII

Is that a single trouble, unrelated to the full IVB scheme?

Have a look at diagrams including *virtual W*:

Discover that a new divergence hits hard our naive IVB model..

Looking at virtual *W* propagator:

$$\frac{-g_{\mu\nu} + k_\mu k_\nu / M_W^2}{k^2 - M_W^2} \xrightarrow[k^2 \to \infty]{} const$$

Will make loop diagrams divergent at high k^2

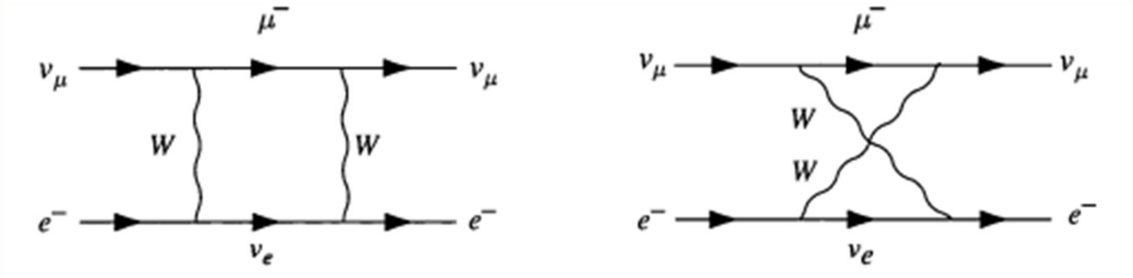
Intermediate Vector Boson - XIV

Neutral current reactions like:

$$\nu_\mu + e^- \rightarrow \nu_\mu + e^-$$

$$\bar{\nu}_\mu + e^- \rightarrow \bar{\nu}_\mu + e^-$$

Not allowed at tree level by our IVB model, only by loop diagrams:



But we can’t compute loop diagrams including virtual *W*:
IVB is not renormalizable

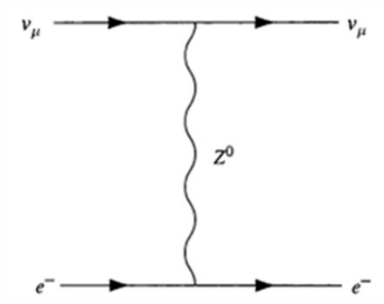
Intermediate Vector Boson - XV

Expect strong suppression, not observed: Cross sections ≈ to allowed processes

$$\nu_e + e^- \rightarrow \nu_e + e^-$$

$$\nu_\mu + e^- \rightarrow \nu_e + \mu^-$$

Suggestion:
Maybe neutral currents do exist *at tree level*, e.g.



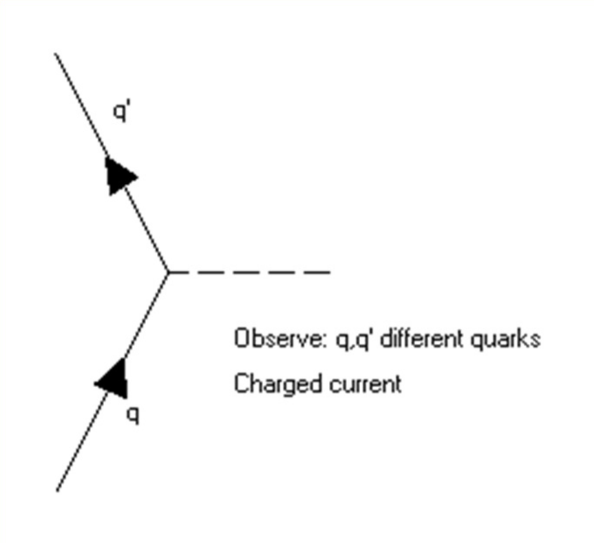
Indeed, neutral currents are *required* in standard electroweak theory

Basic requirement: Theory *must be* renormalizable, as is QED

Universality: Quarks

Semileptonic and non leptonic processes understood in terms of quarks

Basically similar coupling to leptonic charged currents:



Observe: q,q' different quarks

Charged current

Picture is slightly more complicated, however
Fundamental question:

Is the quark coupling identical to the lepton one?

$\nu, \bar{\nu}$ -Nucleon Cross Section - I

Extend V-A to neutrino-nucleon scattering

$$\nu_\mu + N \to \mu^- + X$$

$$\bar{\nu}_\mu + N \to \mu^+ + X$$

Somewhat similar to *e-N*, μ-*N* deep inelastic scattering
Modeling similar to DIS: Parton elastic scattering

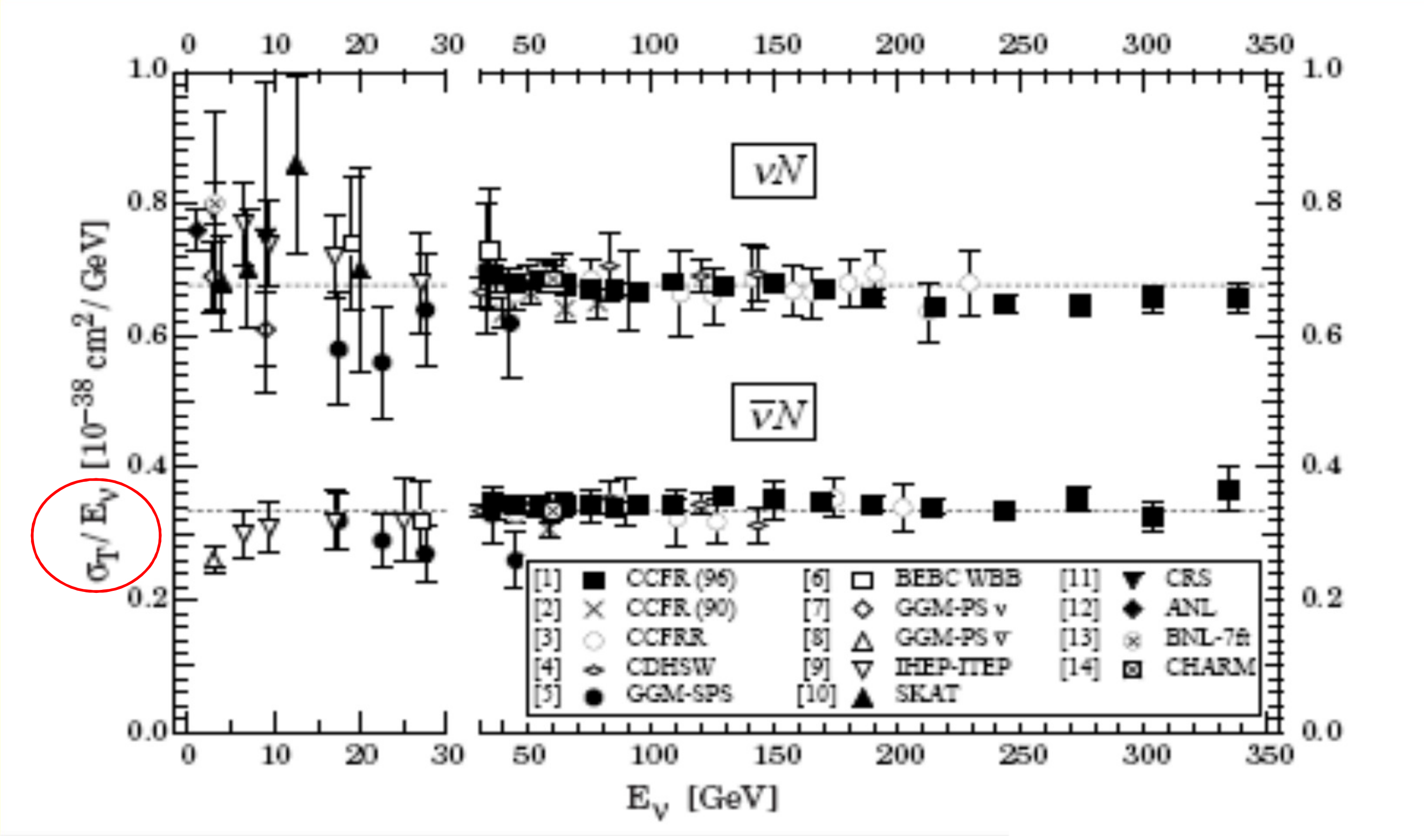
Deep ineastic neutrino scattering reveals the same structure as charged lepton DIS

More information: Charged current sensitive to parton charge sign

→ Can separate quark/antiquark contribution

And: Yes, by looking at (anti)neutrino-nucleon DIS structure functions (probing the parton structure by charged – and neutral – currents) one concludes that quarks couple to weak currents exactly as leptons

$\nu, \overline{\nu}$ -Nucleon Cross Section - II



Linearly rising cross section confirmed...

Families - I

Consider charged current of leptons:
Very natural to group charged and neutral leptons into *doublets*, or *families*

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix} \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix} \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}$$

Within each doublet, charged current transitions can be thought as due to emission/absorption of $W^\pm$ bosons , similar to (neutral) e.m. current transitions

$$\begin{tikzcd} W^- \arrow[r] & \nu_e \arrow[r] \arrow[d, shift left] & W^+ \\ W^- & e^- \arrow[l] \arrow[u, shift left] & W^+ \arrow[l] \end{tikzcd}$$

Similar for 2nd, 3rd family

Families - II

Natural to extend this scheme to quarks

$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ s \end{pmatrix} \quad \begin{pmatrix} t \\ b \end{pmatrix}$$

$$\begin{tikzcd}
W^- \arrow[r] & u \arrow[r] \arrow[d, shift left] & W^+ \\
W^- & d \arrow[l] \arrow[u, shift left] & W^+ \arrow[l]
\end{tikzcd}$$

Similar for 2nd, 3rd family

Almost correct, but incomplete:

Does not account for strangeness → flavour violating processes

Cabibbo’s very ingenious idea:

Quark flavor eigenstates (i.e., quark model eigenstates) are not to be identified with quark weak currents

→*Weak currents are mixtures of different flavors*

By universal convention, mixing is assumed between *d* , *s, b* quarks

Families - III

In terms of mixed "*d*-like" quarks, with just 2 families:

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix} \quad \theta_\text{c} \text{ Cabibbo's angle}$$

This explain *many* things….

How to extract θ_c?

Just one example: Get the angle from β decay

$$G_F^{(\beta)} = 0.975 G_F^{(\mu)} \text{(Remember that 2\% difference ?)}$$

$$\rightarrow G_F^{(\beta)} = \cos\theta_C G_F^{(\mu)}$$

$$\rightarrow \theta_c \simeq 13^0$$

Families - IV

Extend the idea to 3 families:
From Cabibbo’s angle to *Cabibbo-Kobayashi-Maskawa* matrix

$$\begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} |d\rangle \\ |s\rangle \\ |b\rangle \end{bmatrix} = \begin{bmatrix} |d'\rangle \\ |s'\rangle \\ |b'\rangle \end{bmatrix}$$

From unitarity:
3 mixing angles
1 complex phase This can account for CP violation

Experimental values:

$$\begin{bmatrix} 0.9753 & 0.221 & 0.003 \\ 0.221 & 0.9747 & 0.040 \\ 0.009 & 0.039 & 0.9991 \end{bmatrix}$$

Almost diagonal
Heavy quarks even more diagonal

Gauge Symmetry - I

What makes QED so successful?

Renormalization program allows for computing observables with high accuracy, comparable to experimental resolution

QED is a renormalizable field theory

Fermi's theory is a non-renormalizable theory

And:

Naive IVB theory of weak interactions is a non-renormalizable theory

Try to discover what makes the difference

Gauge Symmetry - II

Back to QED for a while: Reconsider global and local gauge invariance

Free Dirac Lagrangian:

$$L_0 = \bar{\psi}(x)\left(i\gamma^\mu \partial_\mu - m\right)\psi(x)$$

Invariant upon global gauge transformation:

$$\begin{cases} \psi(x) \to \psi'(x) = e^{-i\alpha}\psi(x) \\ \bar{\psi}(x) \to \bar{\psi}'(x) = e^{+i\alpha}\bar{\psi}(x) \end{cases}, \quad \alpha \text{ constant}$$

Noether's theorem → Conserved current:

$$\partial_\mu s^\mu(x) = 0, \quad s^\mu(x) = q\bar{\psi}(x)\gamma^\mu\psi(x)$$

→ Conserved charge:

$$Q = \int s^0(x)\, d^3\mathbf{r} = const$$

Gauge Symmetry - III

Non invariant under local gauge transformation:

$$\begin{cases} \psi(x) \to \psi'(x) = e^{-iqf(x)}\psi(x) \\ \overline{\psi}(x) \to \overline{\psi}'(x) = e^{+iqf(x)}\overline{\psi}(x) \end{cases}$$

$$L_0 \to L_0' = L_0 + q\overline{\psi}(x)\gamma^\mu\psi(x)\partial_\mu f(x)$$

Define then a *covariant derivative* as:

$$D_\mu\psi(x) = \left[\partial_\mu + iqA_\mu(x)\right]\psi(x)$$

where, upon the previous local gauge transformation:

$$A_\mu(x) \to A_\mu(x) = A_\mu(x) + \partial_\mu f(x)$$

Then the Lagrangian:

$$L = \overline{\psi}(x)\left(i\gamma^\mu D_\mu - m\right)\psi(x) = L_0 - q\overline{\psi}(x)\gamma^\mu\psi(x)A_\mu(x)$$

is invariant

L contains an *interaction* term $\left(\leftarrow j^\mu A_\mu\right)$

Gauge Symmetry - III

Consider a single family of massless leptons:

$$L_0 = i\bar{\psi}_l(x)\gamma^\mu\partial_\mu\psi_l(x) + i\bar{\psi}_{\nu_l}(x)\gamma^\mu\partial_\mu\psi_{\nu_l}(x)$$

Chiral spinors:

$$\psi^L(x) = P_L\psi(x) = \frac{1}{2}(1-\gamma_5)\psi(x)$$

$$\psi^R(x) = P_R\psi(x) = \frac{1}{2}(1+\gamma_5)\psi(x)$$

$$\rightarrow L_0 = i\bar{\psi}_l{}^L(x)\gamma^\mu\partial_\mu\psi_l{}^L(x) + i\bar{\psi}_{\nu_l}{}^L(x)\gamma^\mu\partial_\mu\psi_{\nu_l}{}^L(x) + i\bar{\psi}_l{}^R(x)\gamma^\mu\partial_\mu\psi_l{}^R(x) + i\bar{\psi}_{\nu_l}{}^R(x)\gamma^\mu\partial_\mu\psi_{\nu_l}{}^R(x)$$

Charged current: Connecting two leptons with $\Delta Q = \pm 1$

Attempting to encode this into a symmetry scheme, define the doublet:

$$\Psi_l{}^L(x) = \begin{pmatrix} \psi_{\nu_l}{}^L(x) \\ \psi_l{}^L(x) \end{pmatrix}, \bar{\Psi}_l{}^L(x) = \begin{pmatrix} \bar{\psi}_{\nu_l}{}^L(x) & \bar{\psi}_l{}^L(x) \end{pmatrix}$$

$$\rightarrow L_0 = i\bar{\Psi}_l{}^L(x)\gamma^\mu\partial_\mu\Psi_l{}^L(x) + i\bar{\psi}_l{}^R(x)\gamma^\mu\partial_\mu\psi_l{}^R(x) + i\bar{\psi}_{\nu_l}{}^R(x)\gamma^\mu\partial_\mu\psi_{\nu_l}{}^R(x)$$

Gauge Symmetry - IV

Suppose the *L*-doublet realizes the fundamental representation of a $SU(2)$ (gauge) symmetry of the weak interaction , exactly as $U(1)$ is is the (gauge) symmetry of QED

$$\left.\begin{aligned}&\Psi_l^{L}(x)\to\Psi_l^{L'}(x)=U(\alpha)\Psi_l^{L}(x)\equiv\exp\left(i\alpha_j\tau_j/2\right)\Psi_l^{L}(x)\\&\overline{\Psi}_l^{L}(x)\to\overline{\Psi}_l^{L'}(x)=\overline{\Psi}_l^{L}(x)U^{\dagger}(\alpha)\equiv\overline{\Psi}_l^{L}(x)\exp\left(-i\alpha_j\tau_j/2\right)\end{aligned}\right\}\qquad U(\alpha)\equiv\exp\left(i\alpha_j\tau_j/2\right)$$

α_i 3 parameters

τ_i Pauli matrices

$$\tau_1=\begin{pmatrix}0&1\\1&0\end{pmatrix},\quad\tau_2=\begin{pmatrix}0&-i\\i&0\end{pmatrix},\quad\tau_3=\begin{pmatrix}1&0\\0&-1\end{pmatrix}\qquad\left[\tau_i,\ \tau_j\right]=2i\varepsilon_{ijk}\tau_k$$

Also take *R*-spinors as *singlets*:

$$\left.\begin{aligned}&\psi_l^{R}(x)\to\psi_l^{R'}(x)=\psi_l^{R}(x),\quad\psi_{\nu_l}^{R}(x)\to\psi_{\nu_l}^{R'}(x)=\psi_{\nu_l}^{R}(x)\\&\overline{\psi}_l^{R}(x)\to\overline{\psi}_l^{R'}(x)=\overline{\psi}_l^{R}(x),\quad\overline{\psi}_{\nu_l}^{R}(x)\to\overline{\psi}_{\nu_l}^{R'}(x)=\overline{\psi}_{\nu_l}^{R}(x)\end{aligned}\right\}$$

Gauge Symmetry - V

According to Noether's theorem:
Expect conserved current after *L* invariance
Under infinitesimal *SU(2)* transformations:

$$\left.\begin{array}{l}\Psi_l^{\mathrm{L}}(x) \to \Psi_l^{\mathrm{L}'}(x) = \left(1 + \mathrm{i}\alpha_j \tau_j/2\right)\Psi_l^{\mathrm{L}}(x) \\ \overline{\Psi}_l^{\mathrm{L}}(x) \to \overline{\Psi}_l^{\mathrm{L}'}(x) = \overline{\Psi}_l^{\mathrm{L}}(x)\left(1 - \mathrm{i}\alpha_j \tau_j/2\right)\end{array}\right\}$$

Identify 3 weak isospin, conserved currents/charges

$$J_i^\alpha(x) = \frac{1}{2}\overline{\Psi}_l^{\mathrm{L}}(x)\gamma^\alpha \tau_i \Psi_l^{\mathrm{L}}(x)$$

$$I_i^W = \int \mathrm{d}^3\mathbf{x} J_i^0(x) = \frac{1}{2}\int \mathrm{d}^3\mathbf{x} \Psi_l^{\mathrm{L}\dagger}(x)\tau_i \Psi_l^{\mathrm{L}}(x)$$

Make 2 non-Hermitian, linear combinations:

$$\left.\begin{array}{l}J^\alpha(x) = 2\left[J_1^\alpha(x) - \mathrm{i}J_2^\alpha(x)\right] = \overline{\psi}_l(x)\gamma^\alpha(1-\gamma_5)\psi_{\nu_l}(x) \\ J^{\alpha\dagger}(x) = 2\left[J_1^\alpha(x) + \mathrm{i}J_2^\alpha(x)\right] = \overline{\psi}_{\nu_l}(x)\gamma^\alpha(1-\gamma_5)\psi_l(x)\end{array}\right\}$$

→ Just our weak charged currents!

Gauge Symmetry - VI

3rd weak isospin (conserved) current: Neutral!

$$J_3^\alpha(x) = \frac{1}{2}\overline{\Psi}_l^L(x)\gamma^\alpha\tau_3\Psi_l^L(x) = -\frac{1}{2}\left[\overline{\psi}_{\nu_l}^L(x)\gamma^\alpha\psi_{\nu_l}^L(x) - \overline{\psi}_l^L(x)\gamma^\alpha\psi_l^L(x)\right]$$

$$s^\alpha(x) = -e\overline{\psi}_l(x)\gamma^\alpha\psi_l(x),$$

Similar structure to electromagnetic current:
2° term is actually *part* of electromagnetic current! (Up to a factor..)

Strong indication that in this model EM and weak interactions may be unified:

3 weak isospin + 1 electromagnetic = 4 currents = 2 charged + 2 neutral

So the unifying symmetry group must be larger than *SU(2)*, which has only 3 parameters

Gauge Symmetry - VII

Early models (between ’50s and ’60s...):

Neutral current ≡ 3rd weak isospin current

Symmetry group is $SU(2)_L \times U(1)_Q$
$SU(2)_L$ (Non Abelian) symmetry group of weak interactions of *L*-fermions
$U(1)_Q$ (Abelian) symmetry group of QED

Then:

Neutral current has same V-A structure of charged current
(When finally observed, neutral current was found ≠ V-A)

Weak and Electromagnetic interactions stay independent
(At high energy, proofs of unification easily found)

Gauge Symmetry - VIII

Rather assume the symmetry group of (unified) Electroweak interaction is

$SU(2)_L \times U(1)_Y$

Where Y is a new observable called *weak hypercharge*

$$\begin{aligned} J_Y^\alpha(x) &= s^\alpha(x)/e - J_3^\alpha(x) \\ &= -\frac{1}{2}\overline{\Psi}_l^{\mathrm{L}}(x)\gamma^\alpha\Psi_l^{\mathrm{L}}(x) - \overline{\psi}_l^{\mathrm{R}}(x)\gamma^\alpha\psi_l^{\mathrm{R}}(x). \end{aligned}$$

Weak hypercharge current

$$Y = \int \mathrm{d}^3\mathbf{x} J_Y^0(x) \quad \rightarrow \quad Y = Q/e - I_3^W$$

Weak hypercharge, Electric charge

Fermion EW quantum numbers: Different for different chiralities!

Defined by I, I_3, Y

Gauge Symmetry - IX

Now find the quantum numbers of (chiral) fermions:
Observe

$$\tau_3 \Psi_l^{\mathrm{L}}(x) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \psi_{\nu_l}^{\mathrm{L}}(x) \\ \psi_l^{\mathrm{L}}(x) \end{pmatrix} = \begin{pmatrix} \psi_{\nu_l}^{\mathrm{L}}(x) \\ -\psi_l^{\mathrm{L}}(x) \end{pmatrix}$$

Then find for different chiral fermions:

$$I_3^W |l^-, \mathrm{L}\rangle = -\frac{1}{2} |l^-, \mathrm{L}\rangle, \quad I_3^W |\nu_l, \mathrm{L}\rangle = +\frac{1}{2} |\nu_l, \mathrm{L}\rangle$$

$$I_3^W |l^-, \mathrm{R}\rangle = 0, \quad I_3^W |\nu_l, \mathrm{R}\rangle = 0.$$

$$\left.\begin{array}{c} Y|l^-, \mathrm{L}\rangle = -\frac{1}{2} |l^-, \mathrm{L}\rangle, \quad Y|\nu_l, \mathrm{L}\rangle = -\frac{1}{2} |\nu_l, \mathrm{L}\rangle \\ \\ Y|l^-, \mathrm{R}\rangle = -|l^-, \mathrm{R}\rangle \\ Y|\nu_l, \mathrm{R}\rangle = 0 \end{array}\right\}$$

Gauge Symmetry - X

Now consider *local* *SU(2)* gauge transformations for *L*- doublet:

$$\left.\begin{aligned} &\Psi_l^L(x) \to \Psi_l^{L'}(x) = \exp\left[ig\tau_j\omega_j(x)/2\right]\Psi_l^L(x) \\ &\overline{\Psi}_l^L(x) \to \overline{\Psi}_l^{L'}(x) = \overline{\Psi}_l^L(x)\exp\left[-ig\tau_j\omega_j(x)/2\right] \\ &\psi_l^R(x) \to \psi_l^{R'}(x) = \psi_l^R(x), \quad \overline{\psi}_{\nu_l}{}^R(x) \to \psi_{\nu_l}^{R'}(x) = \psi_{\nu_l}^R(x) \\ &\overline{\psi}_l^R(x) \to \overline{\psi}_l^{R'}(x) = \overline{\psi}_l^R(x), \quad \overline{\psi}_{\nu_l}^R(x) \to \overline{\psi}_{\nu_l}^{R'}(x) = \overline{\psi}_{\nu_l}^R(x). \end{aligned}\right\}$$

ω_j: 3 real parameters, functions of *(x,t)*

Same path as QED: L_0 not invariant

$$\mathscr{L}_0 \to \mathscr{L}'_0 = \mathscr{L}_0 + \delta\mathscr{L}_0 \equiv \mathscr{L}_0 - \frac{1}{2}g\overline{\Psi}_l^L(x)\tau_j\partial\!\!\!/\omega_j(x)\Psi_l^L(x).$$

Define a *covariant* derivative for the doublet:

$$\partial^\mu\Psi_l^L(x) \to D^\mu\Psi_l^L(x) = \left[\partial^\mu + ig\tau_j W_j^\mu(x)/2\right]\Psi_l^L(x)$$

W_j^μ : triplet of (charged, massless) , photon-like vector fields

Gauge Symmetry - XI

Requiring suitable transformation rules for the vector fields:

$$\begin{aligned} W_i^\mu(x) &\rightarrow W_i^{\mu'}(x) = W_i^\mu(x) + \delta W_i^\mu(x) \\ &\equiv W_i^\mu(x) - \partial^\mu \omega_i(x) - g\varepsilon_{ijk}\omega_j(x) W_k^\mu(x) \qquad [\text{small } \omega_j(x)] \end{aligned}$$

L invariance recovered

Now consider weak hypercharge *U(1)* gauge transformations:

$$\left.\begin{aligned} \psi(x) &\rightarrow \psi'(x) = \exp\left[ig' Yf(x)\right]\psi(x) \\ \overline{\psi}(x) &\rightarrow \overline{\psi}'(x) = \overline{\psi}(x)\exp\left[-ig' Yf(x)\right] \end{aligned}\right\}$$

Covariant derivative + (Single) Vector field transformation: Similar to EM case
g' new coupling constant

$$\partial^\mu \psi(x) \rightarrow D^\mu \psi(x) = \left[\partial^\mu + ig' YB^\mu(x)\right]\psi(x)$$

$$B^\mu(x) \rightarrow B^{\mu'}(x) = B^\mu(x) - \partial^\mu f(x)$$

L invariance recovered

Gauge Symmetry - XII

Collecting all pieces together, the full leptonic Lagrangian:

$$\mathcal{L}^{\mathrm{L}} = \mathrm{i}[\overline{\Psi}_l^{\mathrm{L}}(x)\not{D}\Psi_l^{\mathrm{L}}(x) + \overline{\psi}_l^{\mathrm{R}}(x)\not{D}\psi_l^{\mathrm{R}}(x) + \overline{\psi}_{\nu_l}^{\mathrm{R}}(x)\not{D}\psi_{\nu_l}^{\mathrm{R}}(x)]$$

where:

$$D^{\mu}\Psi_l^{\mathrm{L}}(x) = \left[\partial^{\mu} + \mathrm{i}g\tau_j W_j^{\mu}(x)/2 - \mathrm{i}g' B^{\mu}(x)/2\right]\Psi_l^{\mathrm{L}}(x)$$

$$D^{\mu}\psi_l^{\mathrm{R}}(x) = \left[\partial^{\mu} - \mathrm{i}g' B^{\mu}(x)\right]\psi_l^{\mathrm{R}}(x)$$

$$D^{\mu}\psi_{\nu_l}^{\mathrm{R}}(x) = \partial^{\mu}\psi_{\nu_l}^{\mathrm{R}}(x).$$

L can be written as:

$$\mathcal{L}^{\mathrm{L}} = \mathcal{L}_0 + \mathcal{L}_{\mathrm{I}},$$

Interaction part:

$$\mathcal{L}_{\mathrm{I}} = -gJ_i^{\mu}(x)W_{i\mu}(x) - g' J_Y^{\mu}(x)B_{\mu}(x)$$

Gauge Symmetry - XIII

Attempting to understand the meaning of the interaction terms

Interesting way of re-writing the interaction part:
Define

$$W_\mu(x) = \frac{1}{\sqrt{2}}\left[W_{1\mu}(x) - \mathrm{i}W_{2\mu}(x)\right]$$

And get for the first 2 terms:

$$-g\sum_{\mathrm{i}=1}^{2} J_i^\mu(x) W_{i\mu}(x) = \frac{-g}{2\sqrt{2}}\left[J^{\mu\dagger}(x)W_\mu(x) + J^\mu(x)W_\mu^\dagger(x)\right]$$

Looking like charged current interactions for *L*-fermions

Gauge Symmetry - XIV

Then define:

$$\left.\begin{aligned} W_{3\mu}(x) &= \cos\theta_{\mathrm{W}} Z_\mu(x) + \sin\theta_{\mathrm{W}} A_\mu(x) \\ B_\mu(x) &= -\sin\theta_{\mathrm{W}} Z_\mu(x) + \cos\theta_{\mathrm{W}} A_\mu(x) \end{aligned}\right\}$$

Remembering

$$J_Y^\mu(x) = s^\mu(x)/e - J_3^\mu(x)$$

Get for the remaining terms:

$$\begin{aligned} -gJ_3^\mu(x)W_{3\mu}(x) - g' J_Y^\mu(x) B_\mu(x) \\ = -\frac{g'}{e} s^\mu(x)\left[-\sin\theta_{\mathrm{W}} Z_\mu(x) + \cos\theta_{\mathrm{W}} A_\mu(x)\right] \\ - J_3^\mu(x)\left\{g\left[\cos\theta_{\mathrm{W}} Z_\mu(x) + \sin\theta_{\mathrm{W}} A_\mu(x)\right]\right. \\ \left. -g'\left[-\sin\theta_{\mathrm{W}} Z_\mu(x) + \cos\theta_{\mathrm{W}} A_\mu(x)\right]\right\}. \end{aligned}$$

Gauge Symmetry - XV

Most simple way of unifying the EM and weak interaction:
Require this condition on the g', θ_W constants

$$g\sin\theta_W = g'\cos\theta_W = e.$$

And contemplate the miracle:

$$\mathscr{L}_I = -s^\mu(x)A_\mu(x) - \frac{g}{2\sqrt{2}}\left[J^{\mu\dagger}(x)W_\mu(x) + J^\mu(x)W^\dagger_\mu(x)\right] - \frac{g}{\cos\theta_W}\left[J^\mu_3(x) - \sin^2\theta_W s^\mu(x)/e\right]Z_\mu(x).$$

Electromagnetic interaction
Charged current weak interaction
Neutral current weak interaction

$$g_W = \frac{g}{2\sqrt{2}}.$$

$$\sin^2\theta_W = 0.23122 \pm 0.00015,$$

Neutral Current - I

As a result of the weak-electromagnetic unification, neutral currents are *different* from charged

Lorentz structure not $V - A$

$$-i\frac{g_W}{\sqrt{2}}\gamma^\mu\frac{\left(1-\gamma^5\right)}{2} \quad \text{Charged}$$

$$-ig_Z\gamma^\mu\frac{\left(C_V^f-C_A^f\gamma^5\right)}{2} \quad \text{Neutral}$$

Coupling

Fermion	C_V	C_A
ν_e,ν_μ,ν_τ	$+1/2$	$+1/2$
e,μ,τ	$-1/2+2\sin\theta_W$	$-1/2$
u,c,t	$+1/2-4/3\sin^2\theta_W$	$+1/2$
d,s,b	$-1/2+2/3\sin^2\theta_W$	$-1/2$

θ_W new fundamental constant

What about interaction strength?

Neutral Currents - II

Tight relationship between weak and electromagnetic interactions

Coupling constants:

$$g_{\mathrm{w}} = \frac{e}{\sin\theta_W}$$

$$g_{\mathrm{z}} = \frac{e}{\sin\theta_W \cos\theta_W}$$

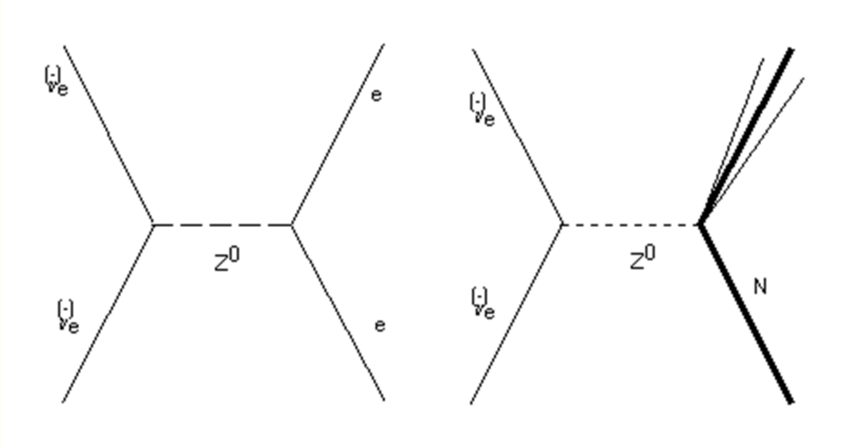
Charged, neutral and electromagnetic couplings fixed by 2 universal constants:

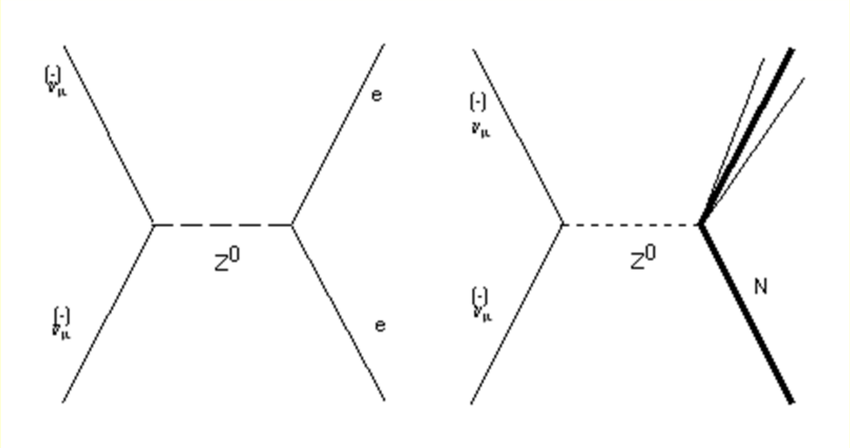
e : Elementary charge

θ_W: Weinberg angle, new fundamental constant

Neutral Currents - III

Expect to observe typical processes like:





$(\nu_e, \overline{\nu}_e) + e \rightarrow (\nu_e, \overline{\nu}_e) + e$ Contributing to elastic scattering

$$\left.\begin{array}{l}(\nu_\mu, \overline{\nu}_\mu) + e \rightarrow (\nu_\mu, \overline{\nu}_\mu) + e \\ (\nu_e, \overline{\nu}_e) + N \rightarrow (\nu_e, \overline{\nu}_e) + hadron \;\; shower \\ (\nu_\mu, \overline{\nu}_\mu) + N \rightarrow (\nu_\mu, \overline{\nu}_\mu) + hadron \;\; shower\end{array}\right\} \quad \text{New}$$

Gauge Boson Terms - I

As for *QED*:

Additional terms required in order to account for:

Energy, *Momentum*, *Angular Momentum*

carried over by the fields

Weak Hypercharge field:

$$-\frac{1}{4}B_{\mu\nu}(x)B^{\mu\nu}(x)$$

$$B^{\mu\nu}(x)=\partial^{\nu}B^{\mu}(x)-\partial^{\mu}B^{\nu}(x)$$

Similar to *QED*: Abelian symmetry $U(1)$

Weak Isospin fields:

$$-\frac{1}{4}\sum_{i=1}^{3}G^{(i)}{}_{\mu\nu}(x)G^{(i)\mu\nu}(x)$$

$$G^{(i)\mu\nu}(x)=\underbrace{\partial^{\nu}W^{(i)\mu}(x)-\partial^{\mu}W^{(i)\nu}(x)}_{F^{(i)\mu\nu}(x)}+g\sum_{i,j=1}^{3}\varepsilon_{ijk}W^{(j)\mu}(x)W^{(k)\nu}(x)$$

Similar to *QCD*: Non-Abelian symmetry $SU(2)_L$

Gauge Boson Terms - II

Gauge Boson Lagrangian:

$$L^B = -\frac{1}{4} B_{\mu\nu}(x) B^{\mu\nu}(x) - \frac{1}{4}\sum_{i=1}^{3} G^{(i)}{}_{\mu\nu}(x) G^{(i)\mu\nu}(x)$$

$$\rightarrow L^B = \underbrace{-\frac{1}{4} B_{\mu\nu}(x) B^{\mu\nu}(x) - \frac{1}{4}\sum_{i=1}^{3} F^{(i)}{}_{\mu\nu}(x) F^{(i)\mu\nu}(x)}_{L_0^B}$$

$$\underbrace{+ g \sum_{i,j,k=1}^{3} \varepsilon_{ijk} W^{(j)\mu}(x) W^{(k)\nu}(x) \partial^\mu W^{(k)\nu}(x) - \frac{1}{4} \sum_{i,j,k,l,m=1}^{3} \varepsilon_{ijk} \varepsilon_{ilm} g^2 W^{(j)\mu}(x) W^{(k)\nu}(x) W^{(l)}{}_\mu(x) W^{(m)}{}_\nu}_{L_{SI}^B}$$

$L_0^B =$ Free term

$L_{SI}^B =$ Self-Interaction term

Gauge Boson Terms - III

Free term: Rewrite using $A^{\mu}, W^{\mu}, W^{\dagger\mu}, Z^{\mu}$:

$$L_0^B = -\frac{1}{4} F_{\mu\nu}(x) F^{\mu\nu}(x) - \frac{1}{2} F^{W}{}_{\mu\nu}(x) F^{W\dagger\mu\nu}(x) - \frac{1}{4} Z_{\mu\nu}(x) Z^{\mu\nu}(x)$$

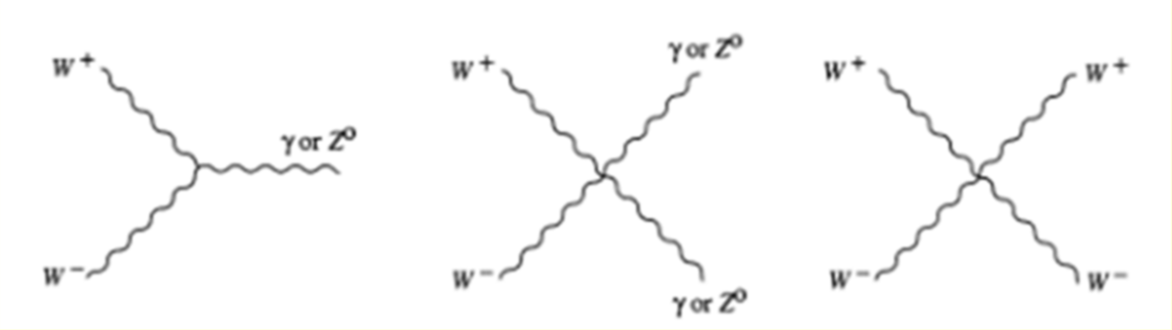
Field tensors:

$F_{\mu\nu}(x) = \partial_{\mu} A_{\nu}(x) - \partial_{\nu} A_{\mu}(x)$ Coupled to EM current

$$\left.\begin{array}{l} F^{W}{}_{\mu\nu}(x) = \partial_{\mu} W_{\nu}(x) - \partial_{\nu} W_{\mu}(x) \\ F^{W\dagger\mu\nu}(x) = \partial^{\mu} W^{\dagger\nu}(x) - \partial^{\nu} W^{\dagger\mu}(x) \end{array}\right\} \text{Coupled to Charged current}$$

$Z^{\mu\nu}(x) = \partial^{\mu} Z^{\nu}(x) - \partial^{\nu} Z^{\mu}(x)$ Coupled to Neutral current

Self-Interaction term: Similar to 3- and 4-gluons terms of QCD



Mass - I

Massless leptons & gauge bosons not physical: Mass must be there

But: Putting 'by hand' a mass term in *L* would spoil gauge invariance

Gauge bosons:

$$m_W^2 W_\mu^\dagger W^\mu + \frac{1}{2} m_Z^2 Z_\mu Z^\mu$$

Take W as an example:

$W_i^\mu \rightarrow W_i^\mu - \partial^\mu \omega_i(x) - g\varepsilon_{ijk}\omega_j(x) W_k^\mu$ infinitesimal parameters

Then:

$$m_W^2 W_\mu^\dagger W^\mu \rightarrow m_W^2 \left(W_i^{\dagger\mu} - \partial^\mu \omega_i(x) - g\varepsilon_{ijk}\omega_j(x) W_k^{\dagger\mu}\right)\left(W_i^\mu - \partial^\mu \omega_i(x) - g\varepsilon_{ijk}\omega_j(x) W_k^\mu\right)$$

$$\rightarrow \neq m_W^2 W_\mu^\dagger W^\mu$$

Mass - II

Leptons :

$$-m\overline{\psi}(x)\psi(x)$$

Write in terms of chiral parts :

$$-m\overline{\psi}(x)\psi(x)=-m\overline{\psi}(x)\left(\underbrace{P_R+P_L}_{=1}\right)\psi(x)$$

$$P_R=\frac{1+\gamma_5}{2},\quad P_L=\frac{1-\gamma_5}{2}$$

$$\rightarrow -m\overline{\psi}(x)\left(\frac{1+\gamma_5}{2}+\frac{1-\gamma_5}{2}\right)\psi(x)=-m\overline{\psi}(x)\left(\left(\frac{1+\gamma_5}{2}\right)^2+\left(\frac{1-\gamma_5}{2}\right)^2\right)\psi(x)$$

$$\rightarrow -m\overline{\psi}(x)\left(\frac{1+\gamma_5}{2}+\frac{1-\gamma_5}{2}\right)\psi(x)=-m\overline{\psi}_L(x)\psi_R(x)-m\overline{\psi}_R(x)\psi_L(x)$$

Mass - III

Not invariant wrt *SU(2)*:

L, R chiral parts live in different *SU(2)* representations

Bottom line: *Any* mass term not invariant

Glashow model (1961): Put mass by hand
→ Gauge invariance lost, back to naive IVB

Finally, discover a subtle mechanism to give mass to physical states,
without spoiling gauge invariance:

Spontaneous Symmetry Breaking

Broad phenomenology, also remotely rooted in classical physics

SSB - I

Symmetries: Frequently approximate $\rightarrow$ Broken

Breaking modes:

(a) Explicit breaking

$H = H_0 + H_b$

H_0 invariant

H_b non-invariant

Ex: Hydrogen atom in a magnetic field **B**

$H_0 = \frac{p^2}{2m} - \frac{Ze^2}{r}$ rotationally invariant

$H_b = -\boldsymbol{\mu} \cdot \mathbf{B}$ invariant wrt rotations around **B**

$\rightarrow H_0$ degeneracies removed by H_b

(b) Spontaneous breaking

H symmetric, ground state non symmetric

Ex: Ferromagnetism

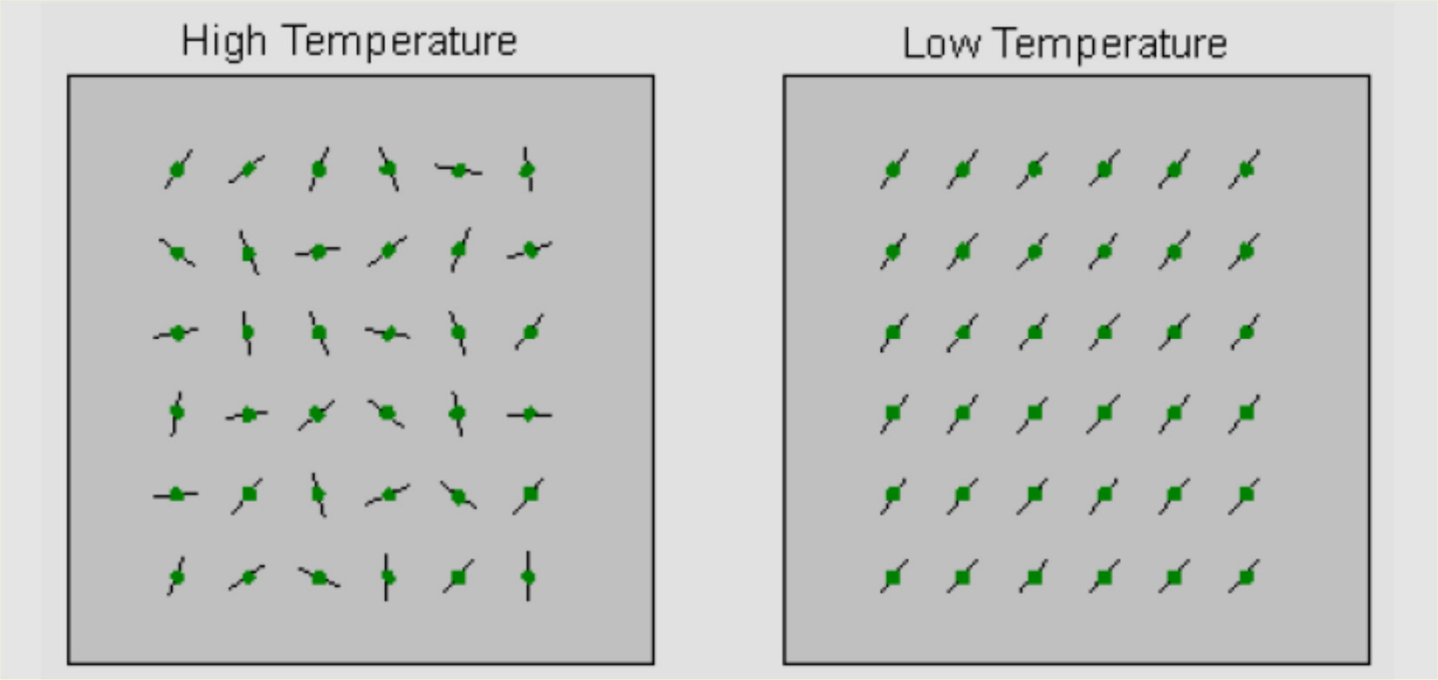
$T > T_c$: $\mathbf{M} = 0 \rightarrow$ Dipoles randomly oriented $\rightarrow$ Rotational symmetry

$T < T_c$: $\mathbf{M} \neq 0 \rightarrow$ Dipoles pick some direction

$\rightarrow H$ degeneracies not removed

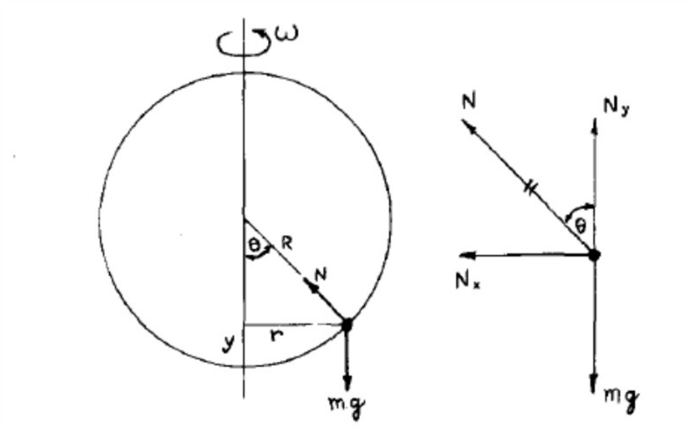
Ground state *degenerate*

SSB - II



SSB - III

Take a first year classical mechanics exercise:
Bead sliding frictionless along a spinning hoop
Find equilibrium angle



$$N = m\omega^2 R,$$
$$N_y = N\cos\theta = m\omega^2 R\cos\theta = mg,$$
$$\cos\theta_0 = g/\omega^2 R.$$

Funny observation:

Critical frequency

$$\cos\theta_0 = 1 = \frac{g}{\omega_0^2 R}$$

For $\omega < \omega_0$: Different solution

$$\theta_1 = 0$$

SSB - IV

Investigate by using Lagrangian formalism:

$$T = (1/2)mv^2 = (1/2)m(R^2\dot{\theta}^2 + \omega^2 R^2 \sin^2\theta)$$
$$V = mgy = mgR(1 - \cos\theta),$$

$$L = T - V = (1/2)mR^2\dot{\theta}^2 + (1/2)m\omega^2 R^2 \sin^2\theta$$
$$- mgR(1 - \cos\theta)$$

Define effective potential, including centrifugal term:

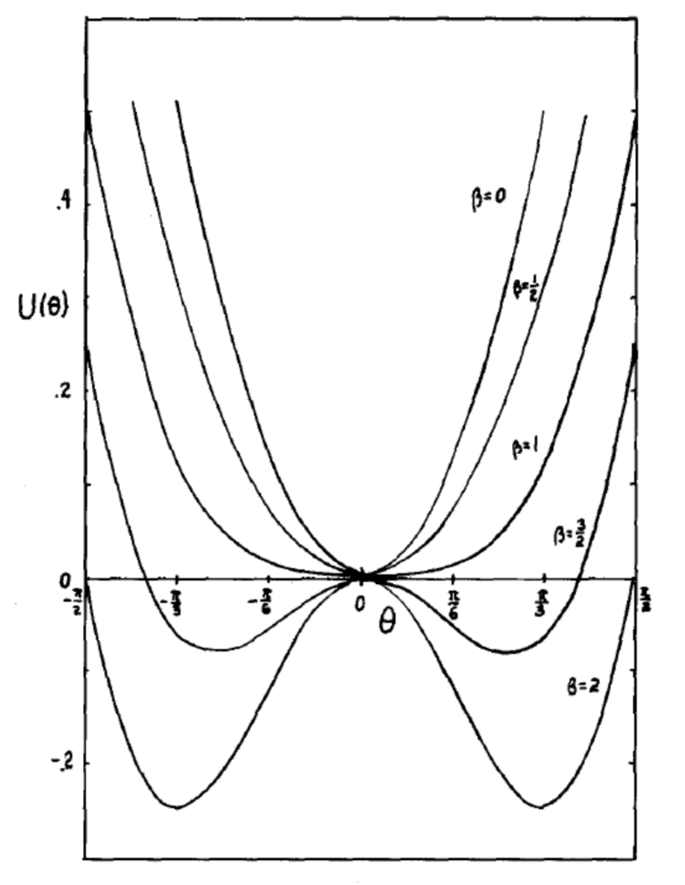
$$L = (1/2)mR^2\dot{\theta}^2 - V_e,$$
$$V_e \equiv mgR[(1 - \cos\theta) - (1/2)(\omega^2 R/g)\sin^2\theta]$$

Define reduced effective potential, β parameter:

$$\begin{aligned} U &\equiv V_e/mgR \\ &= (1 - \cos\theta) - (1/2)\beta\sin^2\theta \\ &= 2\sin^2(\theta/2)[1 - \beta\cos^2(\theta/2)], \qquad \beta = \omega^2 R/g \end{aligned}$$

SSB - V

Find equilibrium angles by zeroes of potential, identify *stable* and *unstable*:



$$\partial U/\partial\theta = \sin\theta(1 - \beta\cos\theta) = 0.$$

$$\cos\theta_0 = 1/\beta, \qquad \theta_1 = 0.$$

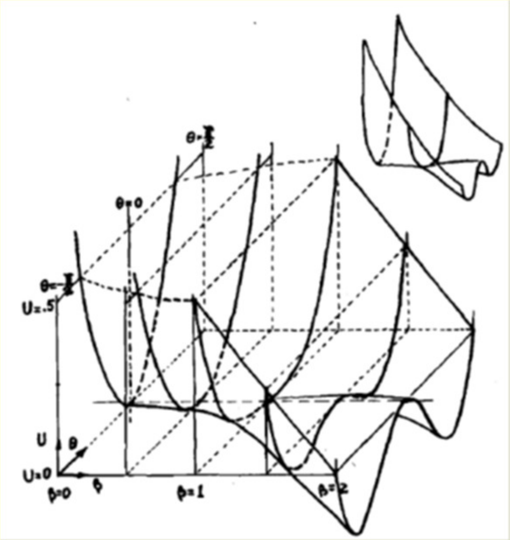
$$\partial^2 U/\partial\theta^2 \equiv U'' = \cos\theta - \beta\cos 2\theta.$$

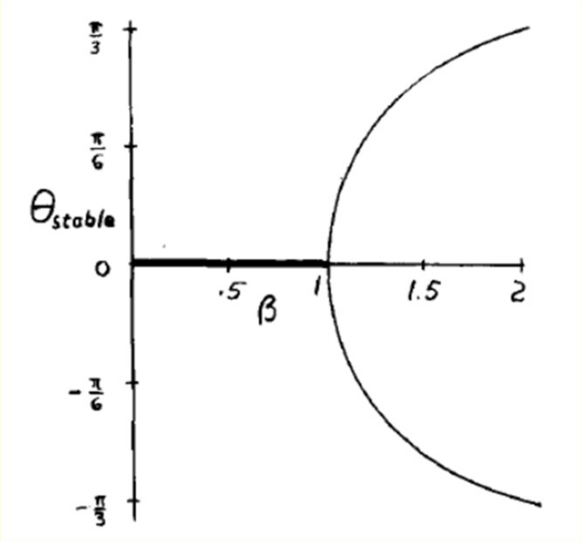
$$U''(\theta_1) = 1 - \beta, \qquad \text{stable for } \beta < 1$$

$$U''(\theta_0) = \beta - 1/\beta, \qquad \text{stable for } \beta > 1$$

SSB - VI

Showing how shape of potential curve, equilibrium angle change with β





a) $\beta<1 \rightarrow 1$ eq. angle
$\beta>1 \rightarrow 2$ eq. angles: Cannot tell which one will be found
Reflection symmetry of V lost (← spontaneously broken) in the solution of eq. of motion

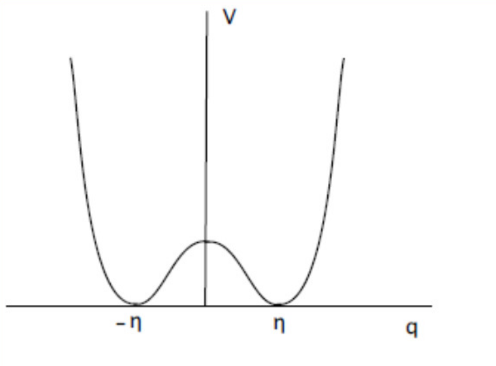
b) Small oscillations around equilibrium angle:
$\beta<1 \rightarrow OK$ Symmetrical wrt origin
$\beta>1 \rightarrow KO$ Non symmetrical wrt origin

SSB - VII

Quantum Mechanics: Simple system with 1 degree of freedom:

$$L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}\lambda^2\left(x^2 - \eta^2\right)^2$$

$$V\left(x\right) = \frac{1}{2}\lambda^2\left(x^2 - \eta^2\right)^2$$



Potential: Parity symmetric

$$V\left(x\right) = V\left(-x\right)$$

Expand around $\pm\eta$ to quadratic terms only: Harmonic oscillator

$\left|+\right\rangle$ solution, centered on $+\eta$

$\left|-\right\rangle$ solution centered on $-\eta$

Naively:

Expect 2, degenerate ground states

SSB - VIII

But:

H not diagonal in this basis

$\langle + | H | + \rangle = \langle - | H | - \rangle = a, \langle + | H | - \rangle = \langle - | H | + \rangle = b$

Physical reason : Tunneling through central barrier

$\rightarrow$ Diagonalize, find:

Eigenstates	Energies
$\lvert S \rangle = \lvert + \rangle + \lvert - \rangle$	$a + b$
$\lvert A \rangle = \lvert + \rangle - \lvert - \rangle$	$a - b$

$|S\rangle, |A\rangle$: Parity eigenstates

$\rightarrow$ Degeneracy removed: Just 1 ground state

$E_{ground} = a - |b|$

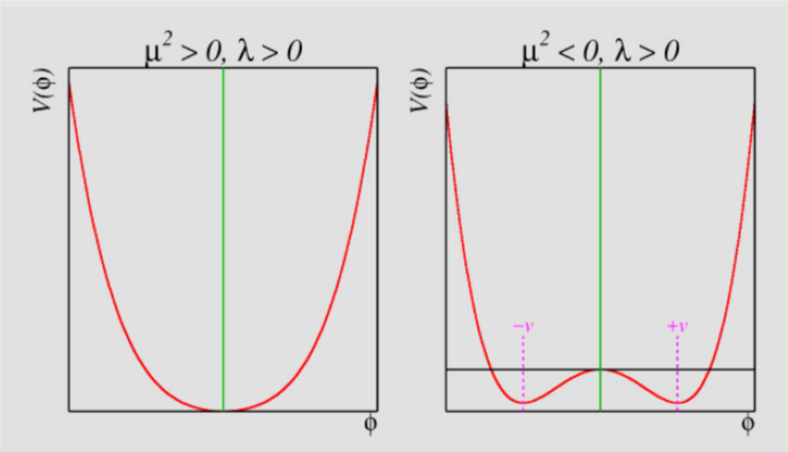
SSB - IX

Field theory: Real scalar field

$$L = \frac{1}{2}\left(\partial_\mu \phi\right)^2 - V\left(\varphi\right)$$

$$V\left(\varphi\right) = \frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda^2\phi^4, \lambda > 0$$

Reflection symmetric: $V\left(\varphi\right) = V\left(-\varphi\right)$



V Minima:

$\mu^2 > 0 : \phi = 0$

$\mu^2 < 0 : \phi = \nu = \pm\sqrt{-\frac{\mu^2}{\lambda}}$

V Minima: Defining *vacuum* state $\left(\leftarrow \text{Cannot have less energy}\right)$

$\mu^2 > 0$: Vacuum (non degenerate) ≡ Zero field

$\mu^2 < 0$: Vacuum (degenerate!) = $\nu \neq$ Zero field !!

ν = Vacuum Expectation Value (VEV) of ϕ

SSB - X

Choose vacuum state:

$\phi(x) = v$

Define: $\phi(x) = v + \eta(x)$

$$\rightarrow L = \frac{1}{2}(\partial_\mu \eta)^2 - \lambda\left(v^2\eta^2 - v\eta^3 - \frac{1}{4}\eta^4\right) = \left[\frac{1}{2}(\partial_\mu \eta)^2 - \lambda v^2\eta^2\right] - \lambda v\eta^3 - \frac{1}{4}\lambda\eta^4$$

$$L = \frac{1}{2}(\partial_\mu \eta)^2 - \lambda v^2\eta^2 \xcancel{+\text{higher powers of } \eta}$$

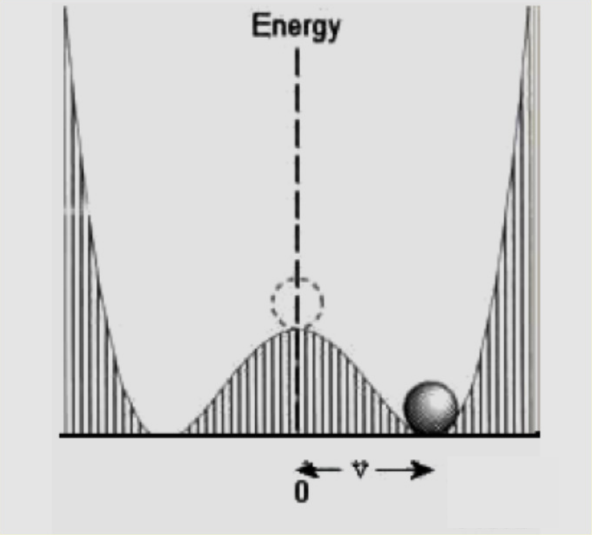
$\rightarrow$ Free Klein-Gordon equation $\rightarrow$ Scalar quantum field

$m^2 = 2\lambda v^2 \rightarrow m = \sqrt{-2\mu^2} > 0$

$\left[\text{Observe:} \mu^2 < 0 \rightarrow \text{Imaginary mass in original } L!\right]$

KO: $L(\eta) \neq L(-\eta)$

Reflection symmetry *spontaneously broken*



SSB - XI

What makes the difference between a single degree of freedom system and a field?

1 degree of freedom: Vacuum not degenerate

$\leftarrow$ Tunneling

∞ degrees of freedom: Vacuum degenerate

$\leftarrow$ Tunneling not effective

Indeed, it can be shown that:

$A_{tunnel} \propto e^{-aV}$, V system volume

$\rightarrow A_{tunnel} \sim 0$ for a $($infinite$)$ field

SSB - XII

Field theory: Complex scalar field

$$\phi=\frac{1}{\sqrt{2}}(\phi_1+i\phi_2),\phi^*=\frac{1}{\sqrt{2}}(\phi_1-i\phi_2)$$

$$L=(\partial_\mu\phi)(\partial^\mu\phi)^*-V(\phi)$$

$$V(\phi)=\mu^2\phi^*\phi+\lambda(\phi^*\phi)^2,\lambda>0,\mu^2<0$$

$U(1)$ symmetric: $\phi\rightarrow\phi'=e^{i\alpha}\phi$

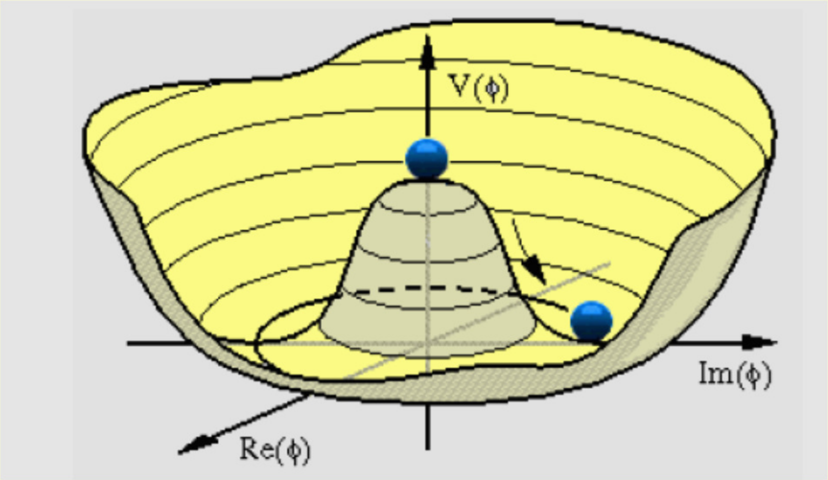
V Minima:

$$\mu^2<0:\phi_1^2+\phi_2^2=-\frac{\mu^2}{\lambda}$$

→ Vacuum infinitely degenerate

Choose vacuum = $(v,0)$

$\rightarrow U(1)$ symmetry *spontaneously broken*



SSB - XIII

Define:

$$\phi(x)=\frac{1}{\sqrt{2}}\left[v+\xi(x)+i\eta(x)\right]$$

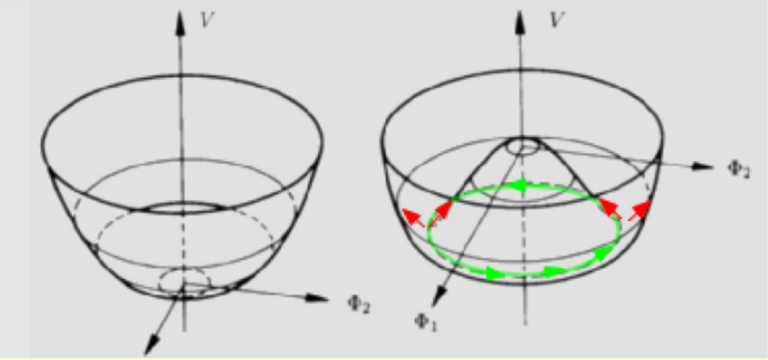
$$L=\frac{1}{2}\left[\left(\partial_\mu\xi\right)^2+\left(\partial_\mu\eta\right)^2\right]+\mu^2\eta^2\ \xcancel{+\text{higher powers of }\eta}$$

Free Klein-Gordon equations for (ξ,η)

But: "Kinetic energy" terms for both ξ,η; Mass term only for η

$\rightarrow$ η field excitations: *massive* scalar particles

ξ field excitations: *massless* scalar particles, aka *Goldstone Bosons*



Higgs Mechanism - I

Local gauge invariance + SSB: Higgs mechanism

Simple, yet subtle way of giving mass to gauge bosons

without spoiling gauge invariance (and renormalizability)

Higgs example:

$U(1)$ gauge group, require $local$ symmetry:

Gauge vector boson A_μ to be introduced, coupling to some current

Add "sombrero" potential for a complex, scalar field $\phi = \phi_1 + i\phi_2$

$$L = \left[\left(\partial_\mu - ieA_\mu\right)\phi^*\right]\left[\left(\partial^\mu + ieA^\mu\right)\phi\right] - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \mu^2\phi^*\phi - \lambda\left(\phi^*\phi\right)^2 + \underbrace{\ldots\ldots}_{\text{Current-field interaction etc}}$$

As found before:

Degenerate vacuum state, SSB by picking as vacuum state: $(v, 0)$

Higgs Mechanism - II

$\rightarrow \phi = v + \eta_1 + i\eta_2$

L written in terms of η_1, η_2 :

Upon quantization, 2 scalar particles $\rightarrow m_1 = \sqrt{2\lambda v^2}, m_2 = 0$

Plugging $\phi = v + \eta_1 + i\eta_2$ into L:

$$L = \frac{1}{2}\left(\partial_\mu \eta_1\right)\left(\partial^\mu \eta_1\right) - \frac{1}{2}\overbrace{2\lambda v^2}^{m_1^2}\eta_1^2 + \frac{1}{2}\left(\partial_\mu \eta_2\right)\left(\partial^\mu \eta_2\right)$$

$$+\underbrace{\frac{1}{2}\overbrace{(ev)^2}^{m_V^2} A^\mu A_\mu}_{Massive\ vector!} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \underbrace{evA^\mu \partial_\mu \eta_2}_{??} + \ldots$$

Attempting to understand L:

Massive vector field A^μ + Massive scalar field η_1 + Massless scalar field η_2

Troubling term coupling A^μ and η_2

Higgs Mechanism - III

Use gauge invariance:

$$\begin{cases}\phi \to \phi' = e^{-ie\theta(x)}\phi \\ A^{\mu} \to A'^{\mu} = A^{\mu} + \partial_{\mu}\theta\end{cases}$$

Choose θ to make ϕ real: Then $\eta_2 \equiv 0$ $(\leftarrow$ Unitary gauge$)$

$$\to L = \frac{1}{2}\left(\partial_{\mu}\eta_1\right)\left(\partial^{\mu}\eta_1\right) - \frac{1}{2}2\lambda v^2\eta_1^2 + \underbrace{\frac{1}{2}\overbrace{(ev)^2}^{M_V^2} A^{\mu}A_{\mu}}_{Massive\ vector!} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \dots$$

Massive vector field A^{μ} + Massive scalar field η_1

Counting degrees of freedom:

$$\underbrace{2}_{A_{\mu}} + \underbrace{1+1}_{\phi} = \underbrace{3}_{A_{\mu}} + \underbrace{1}_{\eta_1} \quad \text{OK}$$

Standard picture:

By effect of a smart gauge transformation, the massless vector field A_{μ}

has eaten the Goldstone boson η_2 to become massive

SM - I

Higgs mechanism exploited to fix troublesome massless gauge bosons in the unified electroweak interaction

Boson counting:

Local gauge symmetry $SU(2)_L \otimes U(1)_Y \rightarrow$ 4 vector bosons

Will need 3 symmetries spontaneously broken to give mass to 3 weak bosons: Photon *is* massless

Extend Abelian Higgs model to non-Abelian gauge symmetry:

Introduce a doublet of complex, scalar fields:

$$\phi(x) = \begin{pmatrix} \phi^+(x) \\ \phi^0(x) \end{pmatrix}$$

Assuming $y = 1$

$$\rightarrow \begin{cases} Q\left[\phi^+(x)\right] = +1 \\ Q\left[\phi^0(x)\right] = 0 \end{cases}$$

SM - II

$SU(2)_L \otimes U(1)$ Gauge transformation of doublet:

$$\phi \to \phi' = \exp\left\{-i\left[\frac{g}{2}\boldsymbol{\alpha}(x)\cdot\boldsymbol{\tau} + \frac{g'}{2}y\theta(x)I\right]\right\}\phi$$

$SU(2)_L \otimes U(1)$ Covariant derivative:

$$D^\mu = \partial^\mu + i\left[\frac{g}{2}\boldsymbol{\tau}\cdot\mathbf{W}^\mu + \frac{g'}{2}yB^\mu\right]$$

→ Additional term to EW lagrangian:

$$L_H = D_\mu\phi^\dagger D^\mu\phi - \mu^2\phi^\dagger\phi - \lambda\left(\phi^\dagger\phi\right)^2$$

Take $\mu^2 < 0, \quad \lambda > 0:$

$$\left|\phi\right|^2_{\min} = -\frac{\mu^2}{2\lambda} = \frac{v^2}{2} \to v = \sqrt{-\frac{\mu^2}{\lambda}}$$

Pick ground state (= vacuum) as

$$\left\langle\phi\right\rangle_0 = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \to \text{SSB of Electroweak gauge symmetry}$$

SM - III

Goldstone boson:

Associated with every generator of the gauge group not leaving invariant the vacuum

$\langle\phi\rangle_0$ Invariant under $G \to e^{i\alpha G}\langle\phi\rangle_0 =\simeq (1+i\alpha G)\langle\phi\rangle_0 = \langle\phi\rangle_0 \leftrightarrow \mathrm{G}\langle\phi\rangle_0 = 0$

Take generators of $SU(2)_L \otimes U(1)_Y$:

$$\tau_1\langle\phi\rangle_0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} v \\ 0 \end{pmatrix} \neq 0$$

$$\tau_2\langle\phi\rangle_0 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix} = -\frac{i}{\sqrt{2}}\begin{pmatrix} v \\ 0 \end{pmatrix} \neq 0$$

$$\tau_3\langle\phi\rangle_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ -v \end{pmatrix} \neq 0$$

$$Y\langle\phi\rangle_0 = +1\langle\phi\rangle_0 \neq 0$$

But: $Q\langle\phi\rangle_0 = \frac{1}{2}(Y+\tau_3)\langle\phi\rangle_0 = 0$

$\to \langle\phi\rangle_0 : U(1)_Q$ Invariant $\to U(1)_Q$ symmetry unbroken → Photon stays massless

SM - IV

As before for the Higgs model, rewrite:

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} \sigma_1 + i\sigma_2 \\ v + \eta_1 + i\eta_2 \end{pmatrix}$$

$$\rightarrow V = -\frac{\mu^4}{4\lambda} + \lambda v^2 \eta_1^2 + \lambda v \eta_1 \left(\sigma_1^2 + \sigma_2^2 + \eta_1^2 + \eta_2^2\right) + \frac{\lambda}{4}\left(\sigma_1^2 + \sigma_2^2 + \eta_1^2 + \eta_2^2\right)^2$$

3 massless scalars: $\sigma_1, \sigma_2, \eta_2 \leftarrow$ The Goldstones

1 massive scalar: $\eta_1, m_{\eta_1} \equiv m_H = \sqrt{2\lambda v^2} \leftarrow$ The Higgs

Gauge transformation suitable to get rid of 3 Goldstones:

$$\phi \rightarrow \phi' = U\phi = U\,\frac{1}{\sqrt{2}}\begin{pmatrix} \sigma_1 + i\sigma_2 \\ v + \eta_1 + i\eta_2 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + \eta_1 \end{pmatrix}$$

$$\rightarrow \begin{cases} SU(2)_L \text{ rotation of doublet to make it 'down'} \\ U(1)_Y \text{ re-phasing of doublet to make it real} \end{cases} \left(\leftarrow \text{Unitary gauge}\right)$$

SM - V

Re-write gauge terms of L in the unitary gauge, in terms of the physical fields:

$$L_B + L_H$$

$$= -\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x)$$ Photon

$$-\frac{1}{2}F^{W}{}_{\mu\nu}(x)F^{W\dagger\mu\nu}(x)+\frac{1}{2}m_W^2 W_\mu^\dagger W^\mu$$ $W^\pm$ boson

$$-\frac{1}{4}Z_{\mu\nu}(x)Z^{\mu\nu}(x)+\frac{1}{2}m_Z^2 Z_\mu Z^\mu$$ Z^0 boson

$$+\left(\partial_\mu\sigma\right)\left(\partial^\mu\sigma\right)-\frac{1}{2}m_H^2\sigma^2$$ H Higgs boson

$$+L_{BB}^I + L_{HH}^I + L_{HB}^I$$ Gauge-Higgs, Higgs self-, Gauge self-interactions

SM - VI

Finding the acquired mass of gauge bosons in terms of couplings and VEV of the Higgs field:

$$\left.\begin{aligned} m_{W^\pm} &= \frac{g}{2}\sqrt{-\frac{\mu^2}{\lambda}} \\ m_{Z^0} &= \frac{\sqrt{\left(g^2+g'^2\right)}}{2}\sqrt{-\frac{\mu^2}{\lambda}} \end{aligned}\right\} \rightarrow m_{Z^0} = m_{W^\pm}\sqrt{1+\frac{g'^2}{g^2}}$$

$$m_\gamma = 0$$

$$m_H = \sqrt{-2\mu^2} = ???$$

Model parameters v, λ and θ_W :

$$v = \sqrt{-\frac{\mu^2}{\lambda}}$$

$$\lambda = ???$$

$$g \sin\theta_W = g' \cos\theta_W = e$$

SM - VII

Lepton masses: Different mechanism required

$\rightarrow$ Yukawa $\left(\text{scalar}\right)$ coupling

Describing interaction between Dirac and scalar fields:

$V \approx g\bar{\Psi}\phi\Psi$, Static limit:

$$V = -\frac{g}{4\pi}\frac{e^{-\mu r}}{r}$$

$$L_{HL} = -g_l\left[\bar{\Psi}_l^L\Phi\psi_l^R + \bar{\psi}_l^R\Phi^\dagger\Psi_l^L\right] - \underbrace{g_{\nu_l}}_{=0 \text{ for massless neutrino}}\left[\bar{\Psi}_l^L\tilde{\Phi}\psi_{\nu_l}^R + \bar{\psi}_{\nu_l}^R\tilde{\Phi}^\dagger\Psi_l^L\right], \quad \tilde{\Phi} = \begin{pmatrix}\phi_b^* \\ -\phi_a^*\end{pmatrix}$$

In the unitary gauge:

$$L_{HL} = -\frac{1}{v}m_l\bar{\psi}_l\,\psi_l\,\sigma - \frac{1}{v}m_{\nu_l}\bar{\psi}_{\nu_l}\psi_{\nu_l}\sigma$$

Lepton masses in terms of model parameters:

$$m_l = \frac{vg_l}{\sqrt{2}}, m_{\nu_l} = \frac{vg_{\nu_l}}{\sqrt{2}}$$

SM - VIII

Model parameters:

$g, g', -\mu^2, \lambda, g_l, g_{\nu_l}$

Quite remarkably, get m_W, m_Z by measured constants:

$$\begin{cases} \alpha = \dfrac{1}{137.04} \\ G_F = 1.166\ 10^{-5} GeV^{-2} \\ \sin^2\theta_W = 0.23122 \end{cases}$$

$\rightarrow m_W = 77.5\ \ GeV, m_Z = 88.4\ \ GeV$

Experimental values:

$m_W = 80.40\ GeV, m_Z = 90.19\ GeV$

Difference originating from radiative corrections

Higgs:

$m_H = \sqrt{-2\mu^2} = \sqrt{2\lambda v^2} \rightarrow ???$

SM - IX

Relating model parameters to measured constants $e, G_F, \sin\theta_W$:

$$g = \frac{e}{\sin\theta_W}, \quad g' = \frac{e}{\cos\theta_W}$$

$$G_F = \frac{\sqrt{2}}{8}\left(\frac{g}{M_W}\right)^2 \to \frac{8G_F}{\sqrt{2}} = \left(\frac{g}{M_W}\right)^2 \to \frac{M_W}{g} = \sqrt{\frac{\sqrt{2}}{8G_F}}$$

$$\to M_W = \sqrt{\frac{\sqrt{2}g^2}{8G_F}} = \sqrt{\frac{\sqrt{2}e^2}{8G_F \sin^2\theta_W}} = \frac{37.3}{\sin\theta_W} \quad GeV$$

$$M_Z = \frac{M_W}{\cos\theta_W} = \frac{37.3}{\sin\theta_W \cos\theta_W} \quad GeV$$

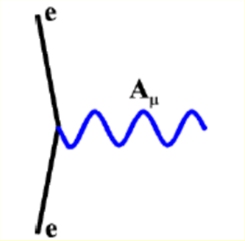
$$v = \frac{2M_W}{g} = 2\sqrt{\frac{\sqrt{2}}{8G_F}} = \frac{1}{\sqrt{\sqrt{2}G_F}}$$

$$\to \frac{v}{\sqrt{2}} = \frac{1}{\sqrt{\sqrt{2}G_F}}\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{\sqrt{8}G_F}} \approx 174 \text{ GeV} \to \langle\phi\rangle_0 = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \approx \begin{pmatrix} 0 \\ 174 \text{ GeV} \end{pmatrix} \quad \text{VEV of the Higgs field}$$

No clues on $\lambda \to$ No (direct) prediction of $M_H = \sqrt{2v^2\lambda}$

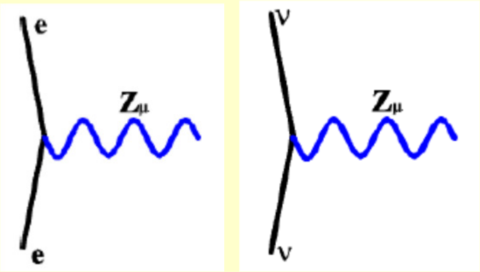
SM - XII

Lepton-Gauge Boson vertexes:



EM

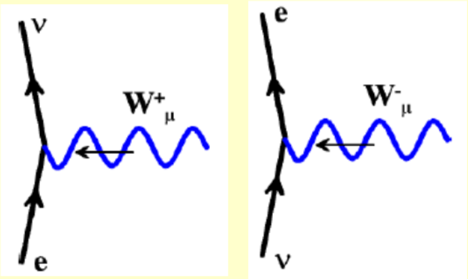
$$-iQ_e\bar{e}\gamma^\mu e A_\mu$$



Neutral Current - NC

$$-i\frac{g}{4\cos\theta_W}\bar{e}\gamma^\mu\left[2\sin^2\theta_W\left(1+\gamma_5\right)+\left(2\sin^2\theta_W-1\right)\left(1-\gamma_5\right)\right]eZ_\mu.$$

$$-i\frac{g}{4\cos\theta_W}\bar{\nu}\gamma^\mu\left(1-\gamma_5\right)\nu Z_\mu.$$



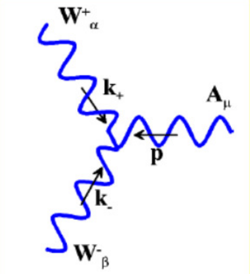
Charged Current - CC

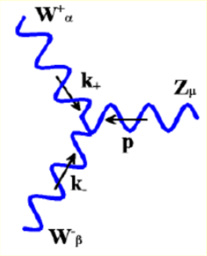
$$-i\frac{g}{2\sqrt{2}}\bar{\nu}\gamma^\mu\left(1-\gamma_5\right)eW^+_\mu$$

$$-i\frac{g}{2\sqrt{2}}\bar{e}\gamma^\mu\left(1-\gamma_5\right)\nu W^-_\mu$$

SM - XIII

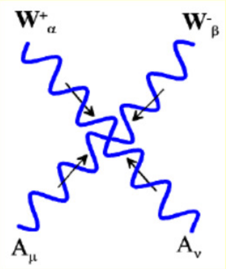
Gauge bosons self-interaction vertexes:

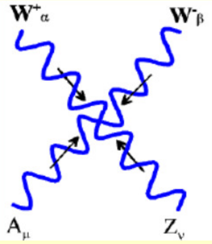




$$ig\sin\theta_W\left[g^{\alpha\beta}(k_+-k_-)^\mu+g^{\alpha\mu}(p-k_+)^\beta\right.$$
$$\left.+g^{\beta\mu}(k_--p)^\alpha\right]W^+_\alpha W^-_\beta A_\mu.$$

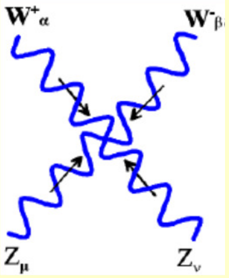
$$ig\cos\theta_W\left[g^{\alpha\beta}(k_+-k_-)^\mu+g^{\alpha\mu}(p-k_+)^\beta\right.$$
$$\left.+g^{\beta\mu}(k_--p)^\alpha\right]W^+_\alpha W^-_\beta Z_\mu.$$

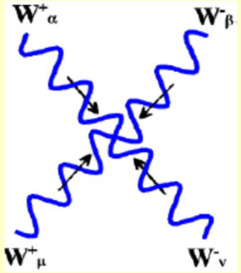




$$-ig^2\sin^2\theta_W\left[2g^{\mu\nu}g^{\alpha\beta}-g^{\mu\alpha}g^{\nu\beta}-g^{\mu\beta}g^{\alpha\nu}\right]$$
$$\times W^+_\alpha W^-_\beta A_\mu A_\nu,$$

$$-ig^2\sin\theta_W\cos\theta_W\left[2g^{\mu\nu}g^{\alpha\beta}-g^{\mu\alpha}g^{\nu\beta}\right.$$
$$\left.-g^{\mu\beta}g^{\alpha\nu}\right]W^+_\alpha W^-_\beta A_\mu Z_\nu.$$



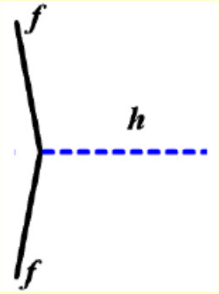


$$-ig^2\cos^2\theta_W\left[2g^{\mu\nu}g^{\alpha\beta}-g^{\mu\alpha}g^{\nu\beta}\right.$$
$$\left.-g^{\mu\beta}g^{\alpha\nu}\right]W^+_\alpha W^-_\beta Z_\mu Z_\nu,$$

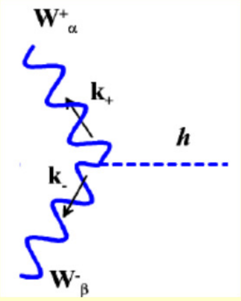
$$-ig^2\left[2g^{\mu\nu}g^{\alpha\beta}-g^{\mu\alpha}g^{\nu\beta}\right.$$
$$\left.-g^{\mu\beta}g^{\alpha\nu}\right]W^+_\alpha W^-_\beta W^+_\mu W^-_\nu$$

SM - XIV

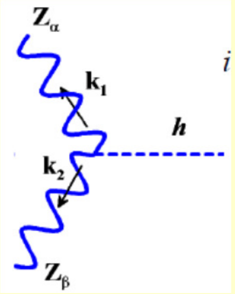
Higgs vertexes:



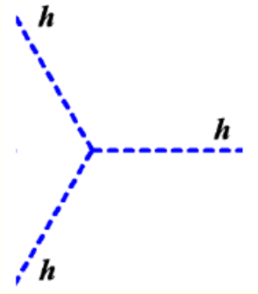
$-im_f\sqrt{\sqrt{2}G_F}\,\bar{f}\,f$



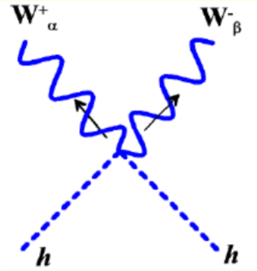
$2iM_W^2\sqrt{\sqrt{2}G_F}\,g^{\alpha\beta}W_\alpha^{+\dagger}W_\beta^{-\dagger}h$



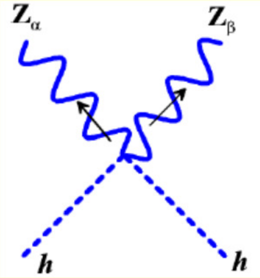
$2iM_Z^2\sqrt{\sqrt{2}G_F}\,g^{\alpha\beta}Z_\alpha^\dagger Z_\beta^\dagger h$



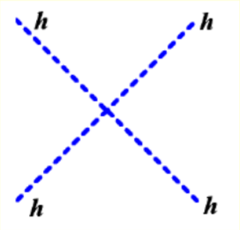
$-3im_h^2\sqrt{\sqrt{2}G_F}\,hhh$



$2iM_W^2\sqrt{2}G_F\,g^{\alpha\beta}hhW_\alpha^{+\dagger}W_\beta^{-\dagger}$



$2iM_Z^2\sqrt{2}G_F\,g^{\alpha\beta}hhZ_\alpha^\dagger Z_\beta^\dagger$



$-3im_h^2\sqrt{2}G_F\,hhhh$

SM - X

Extension to 2nd, 3rd lepton family: Straightforward

Will need 2+2 = 4 new parameters (Yukawa couplings)

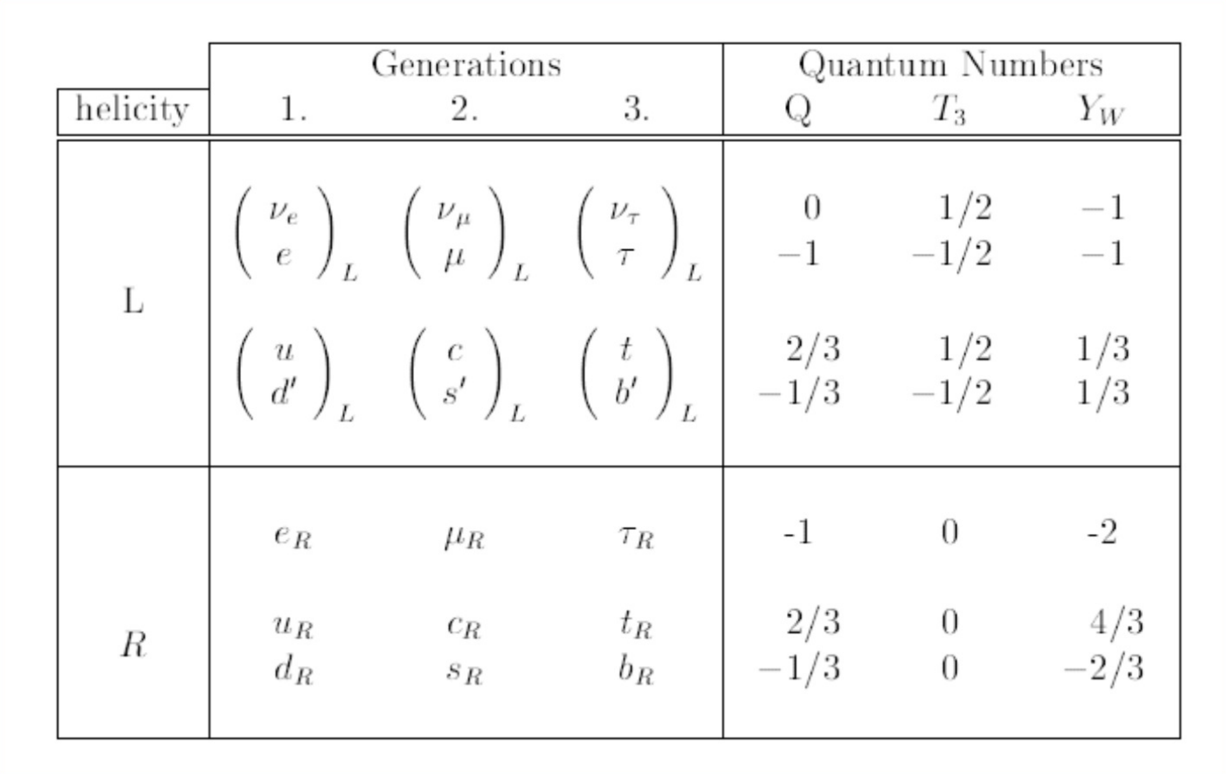
'Minimal' Standard Model:

Massless neutrinos $\rightarrow g_{\nu_l}^{(i)} = 0$

$$\left.\begin{array}{l}\text{'Non Minimal' Standard Model:}\\ \text{Neutrino mixing}\left(\leftarrow \text{Require massive neutrinos, mixing matrix}\right):\\ \text{Account for observed neutrino oscillations}\\ \text{May indicate physics beyond Standard Model}\\ \\ \text{Extension to 3 quark families: Similar to leptons}\\ \text{Will need 6 more parameters}\\ \text{Will require CKM 'flavor rotation' (see later)}\\ \text{Strong interaction effects}\end{array}\right\}\text{Flavor physics}$$

SM - XI

Fermion electroweak quantum numbers:



	Generations			Quantum Numbers		
helicity	1.	2.	3.	Q	T_3	Y_W
L	$\begin{pmatrix}\nu_e\\ e\end{pmatrix}_L$	$\begin{pmatrix}\nu_\mu\\ \mu\end{pmatrix}_L$	$\begin{pmatrix}\nu_\tau\\ \tau\end{pmatrix}_L$	0 -1	$1/2$ $-1/2$	-1 -1
	$\begin{pmatrix}u\\ d'\end{pmatrix}_L$	$\begin{pmatrix}c\\ s'\end{pmatrix}_L$	$\begin{pmatrix}t\\ b'\end{pmatrix}_L$	$2/3$ $-1/3$	$1/2$ $-1/2$	$1/3$ $1/3$
R	e_R	μ_R	τ_R	-1	0	-2
	u_R d_R	c_R s_R	t_R b_R	$2/3$ $-1/3$	0 0	$4/3$ $-2/3$

Neutral Currents Discovery - I

Predicted by Glashow-Salam-Weinberg model ('60s)

Not really accepted for a long time:

Mostly because of strong suppression of strangeness changing decays like:

$K^0 \to \mu^+\mu^- \quad BR \quad < 10^{-8}$

not accounted for. Compare:

$K^+ \to \mu^+\nu_\mu \quad BR \quad 63.4\ \%$

Two breakthroughs:

GIM prediction of charm to solve the $K^0 \to \mu^+\mu^-$ puzzle ('70)

GSW model shown to be renormalizable by 't Hooft ('71)

$\to$ Sudden wave of interest in gauge theories

Neutral Currents Discovery - II

Experimental point of view:

Main interest = Prediction of new phenomena

Most shocking prediction of GSW: neutral currents, never seen before

→ Try to find neutral currents to validate GSW

Best opportunity :

High energy neutrino interactions

No EM background

Larger cross sections

Drawback:

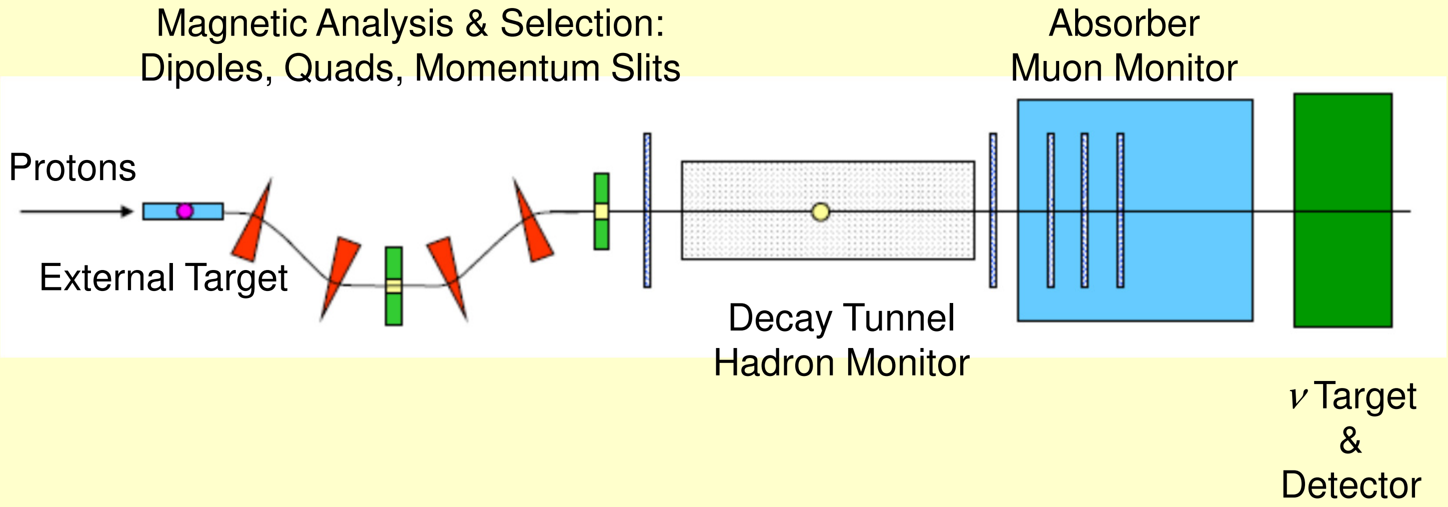
Neutrino experiments difficult

Neutral Currents Discovery - III

Neutrino beams

Take 2 body decays of π, K obtained from a high energy proton machine

a) Narrow Band Beam: ν energy known, low intensity



Neutral Currents Discovery - IV

2-body decay kinematics:

$$\pi^{\pm}, K^{\pm} \to \mu^{\pm} + \overset{(-)}{\nu}_{\mu}$$

$\beta, \gamma, M, |\mathbf{p}|$ K, π LAB

$p^*, E_\mu{}^*, \theta_\mu{}^*$ μ CM

p_μ, E_μ, θ_μ μ LAB

$$\left|\mathbf{p}_\nu{}^*\right| = E_\nu^* = \frac{M_{\pi,K}^2 - m_\mu^2}{2M_{\pi,K}} \simeq \begin{cases} 30 \\ 236 \end{cases} \text{MeV}$$

Isotropic decay:

$$\frac{dP}{d\left(\cos\theta^*\right)} = \frac{1}{2} \to \frac{dP}{dE} = \frac{dP}{d\left(\cos\theta^*\right)} \frac{d\left(\cos\theta^*\right)}{dE}$$

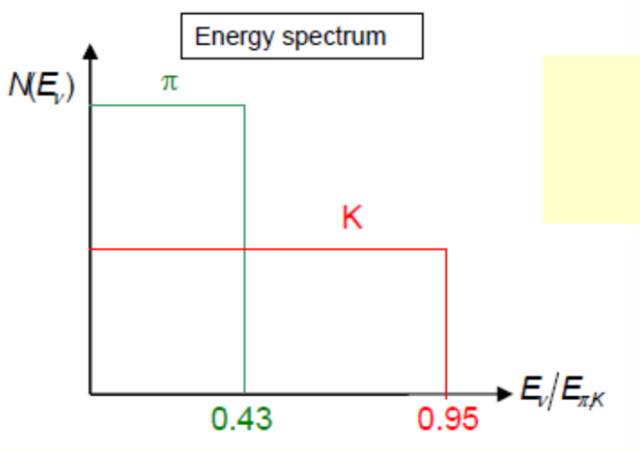
Neutral Currents Discovery - V

$$E=\gamma\left(E^{*}+\beta p^{*}\cos\theta^{*}\right)\rightarrow dE=\gamma\beta p^{*}d\left(\cos\theta^{*}\right)\rightarrow d\left(\cos\theta^{*}\right)=\frac{dE}{\gamma\beta p^{*}}$$

$$\rightarrow\frac{dP}{dE}=\frac{1}{2\gamma\beta p^{*}}\qquad\text{Flat distribution}\quad\begin{cases}\gamma(1+\beta)E^{*}=\gamma(1+\beta)\dfrac{M_{\pi,K}^{2}-m_{\mu}^{2}}{2M_{\pi,K}} & \text{max}\\ \gamma(1\text{-}\beta)E^{*}=\gamma(1\text{-}\beta)\dfrac{M_{\pi,K}^{2}-m_{\mu}^{2}}{2M_{\pi,K}} & \text{min}\end{cases}$$

$$\frac{dN}{dE}=\frac{M_{\pi,K}}{\gamma\beta\left(M_{\pi,K}^{2}-m_{\mu}^{2}\right)}=\frac{1}{\underbrace{\gamma\beta M_{\pi,K}}_{|\mathbf{p}|}\left(1-\dfrac{m_{\mu}^{2}}{M_{\pi,K}^{2}}\right)}=\frac{1}{|\mathbf{p}|\left(1-\dfrac{m_{\mu}^{2}}{M_{\pi,K}^{2}}\right)}$$

→ Broad LAB ν_{μ} energy distribution



Neutral Currents Discovery - VI

Measure ν energy by direction:

Exploit hadron beam $\sim$ monocromaticity

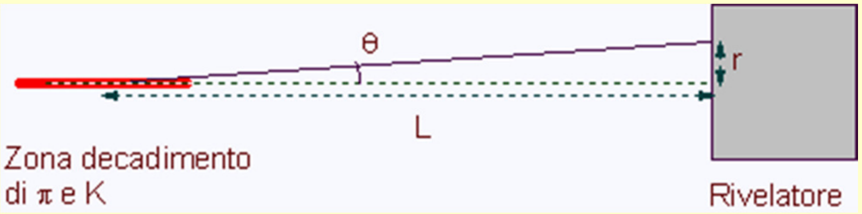
$$p_\pi = p_\mu + p_\nu \rightarrow p_\mu = p_\pi - p_\nu \rightarrow \left(p_\mu\right)^2 = \left(p_\pi - p_\nu\right)^2$$

$$m_\mu^2 = m_\pi^2 - 2p_\pi \cdot p_\nu \rightarrow m_\pi^2 - m_\mu^2 = 2\left(E_\pi E_\nu - \mathbf{p}_\pi \cdot \mathbf{p}_\nu\right)$$

$$E_\nu = \frac{m_{\pi,K}^2 - m_\mu^2}{2\left(E_{\pi,K} - p_{\pi,K}\cos\theta_\nu\right)} = \frac{m_\pi^2 - m_\mu^2}{2E_{\pi,K}\left(1-\beta\cos\theta_\nu\right)}$$

$$\tan\theta_\nu = \frac{\sin\theta_\nu^*}{\gamma\left(\cos\theta_\nu^* + \beta\right)} \rightarrow \tan\theta_{\max} = \frac{\sin\frac{\pi}{2}}{\gamma\left(\cos\frac{\pi}{2}+\beta\right)} = \frac{1}{\beta\gamma} \approx \frac{1}{\gamma} = \frac{m_{\pi,K}}{|\mathbf{p}|} \ll 1$$

$$E_\nu = \frac{m_{\pi,K}^2 - m_\mu^2}{2E_{\pi,K}\left(1-\beta\cos\theta\right)} \approx \frac{m_{\pi,K}^2 - m_\mu^2}{2E_{\pi,K}\left(1-\beta\left(1-\frac{\theta^2}{2}\right)\right)}$$



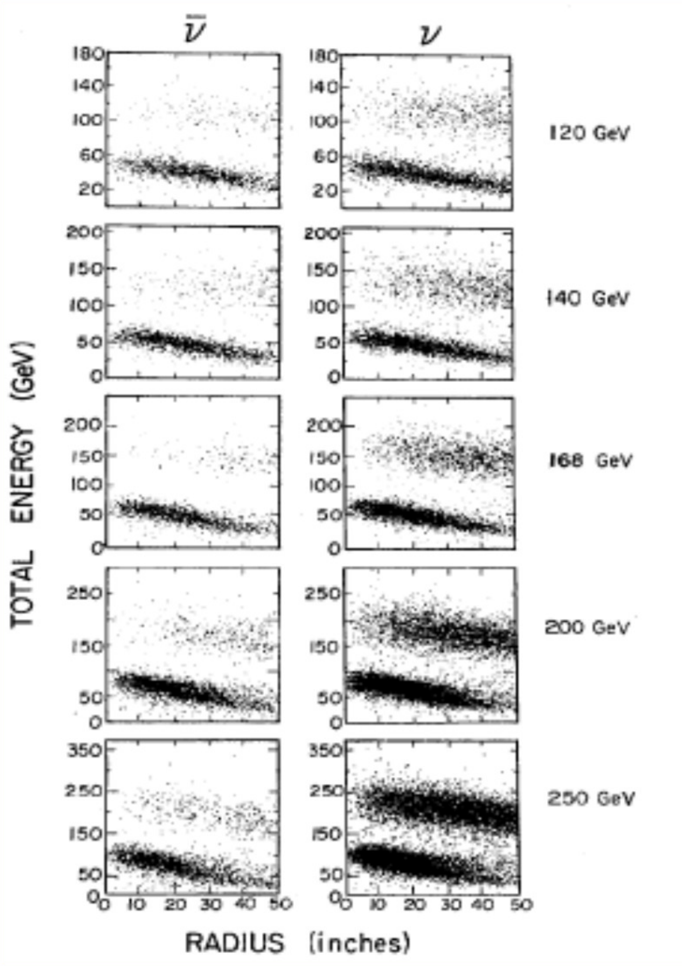
Neutral Currents Discovery - VII

$$E_\nu = \frac{m^2_{\pi,K} - m^2_\mu}{2E_{\pi,K}\left(1-\beta+\frac{\theta^2}{2}\right)} = \frac{m^2_{\pi,} - m^2_\mu}{2E_{\pi,K}\left(\frac{1-\beta^2}{1+\beta}+\frac{\theta^2}{2}\right)}$$

$$E_\nu \simeq \frac{m^2_{\pi,K} - m^2_\mu}{2E_{\pi,K}\left(\frac{1-\beta^2}{2}+\frac{\theta^2}{2}\right)}$$

$$E_\nu = \frac{m^2_{\pi,K} - m^2_\mu}{E_{\pi,K}\left(\frac{1}{\gamma^2}+\theta^2\right)} = \frac{m^2_{\pi,K}\left(1-\frac{m^2_\mu}{m^2_{\pi,K}}\right)E_{\pi,K}}{E^2_{\pi,K}\left(\frac{1}{\gamma^2}+\theta^2\right)}$$

$$E_\nu = \frac{m^2_{\pi,K}\left(1-\frac{m^2_\mu}{m^2_{\pi,K}}\right)E_{\pi,K}}{\gamma^2 m^2_{\pi,K}\left(\frac{1}{\gamma^2}+\theta^2\right)} = E_{\pi,K}\frac{\left(1-\frac{m^2_\mu}{m^2_{\pi,K}}\right)}{\left(1+\gamma^2\theta^2\right)}$$

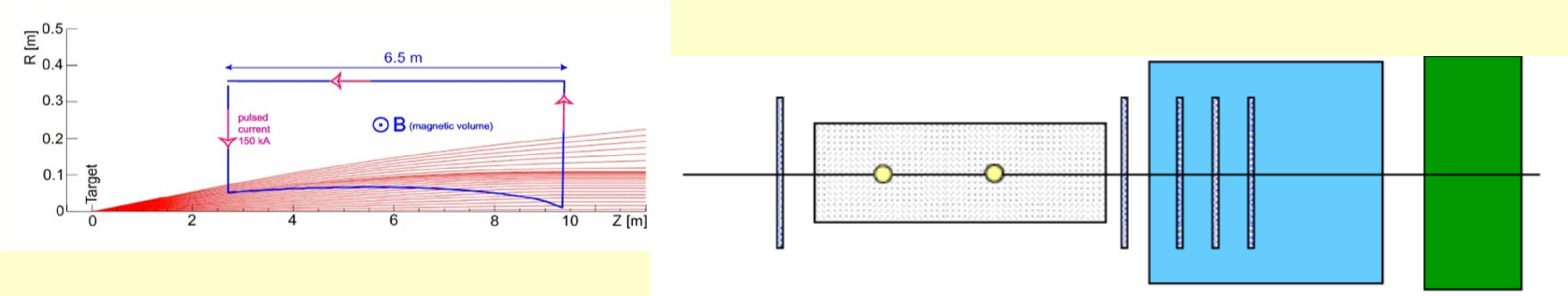


Neutral Currents Discovery - VIII

b) Wide Band Beam: ν energy unknown, high intensity

Replace magnetic selection by a special focussing device, suitable to make a low divergence hadron beam out of an uncollimated, divergent source:

Van der Meer Horn

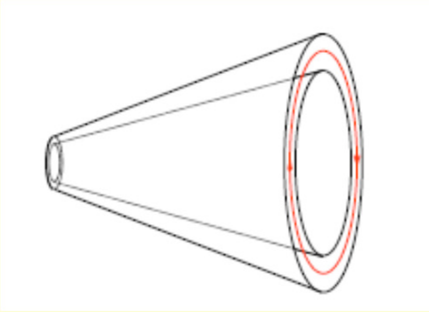


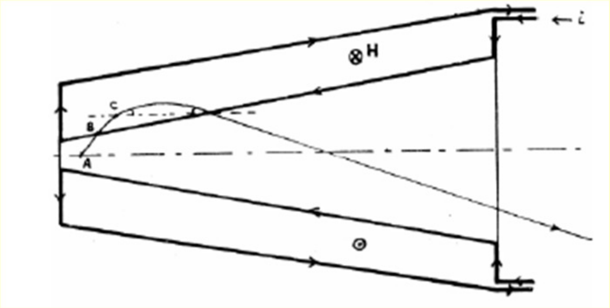
Neutral Currents Discovery - IX

2 conical (high) current sheets:

Equivalent to many trapezoidal current loops symmetrically placed around the axis

→ Circular magnetic field (red circumference)





Trajectory in the B field: $\sim$ Circular arc

$$|\mathbf{p}| = 0.3BR \quad \begin{cases} p & GeV \\ B & T \\ R & m \end{cases}$$

Deflection after a path length l in the field:

$$\Delta\theta = \frac{\Delta l}{R} = 0.3B\frac{\Delta l}{|\mathbf{p}|}$$

Deflection should compensate $\langle p_T \rangle$ of hadrons coming out of the target:

$\langle p_T \rangle \sim p\Delta\theta \sim 0.2\ GeV$ at PS energies

$$\rightarrow 0.2\ GeV \sim |\mathbf{p}|\Delta\theta = 0.3B\Delta l$$

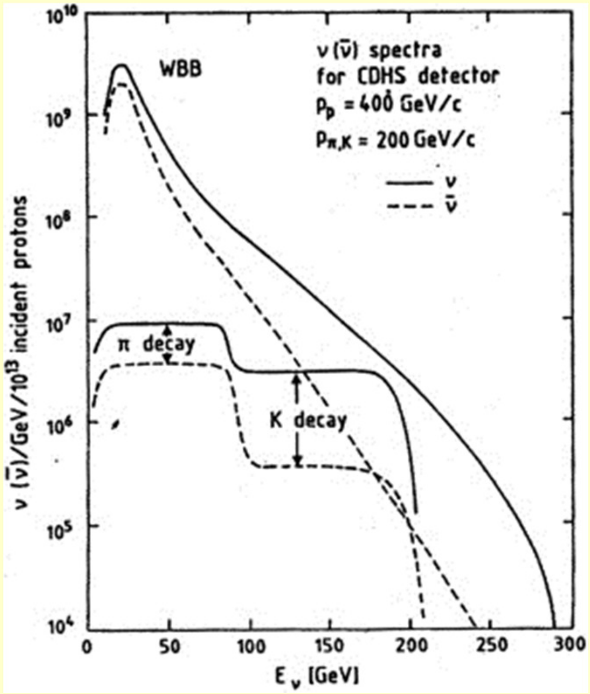
Simple guess:

$$B = \frac{\mu_0 I}{2\pi r} \rightarrow 0.2\ GeV \sim |\mathbf{p}|\Delta\theta \sim 0.3\frac{\mu_0 I}{2\pi r}\Delta l$$

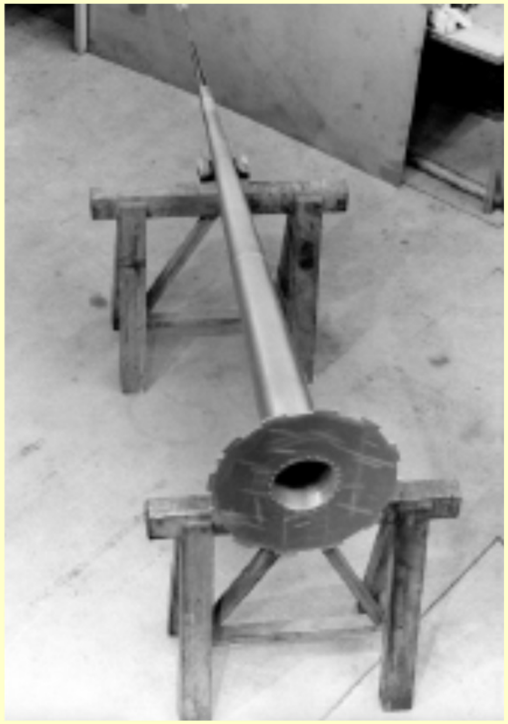
$$\rightarrow I \sim \frac{|\mathbf{p}|\Delta\theta 2\pi r}{0.3\mu_0\Delta l} \sim 10^5\, A!$$

Neutral Currents Discovery - X

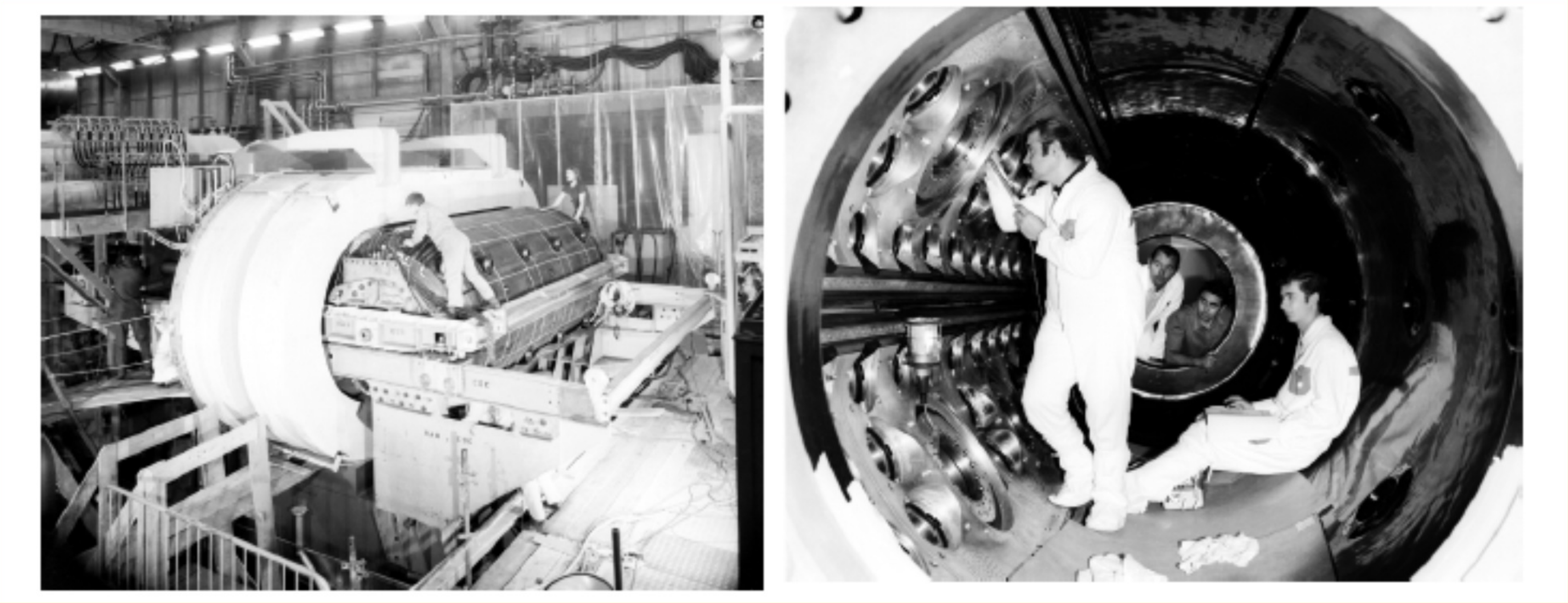
Wideband beam spectra
SPS beam



The Gargamelle horn



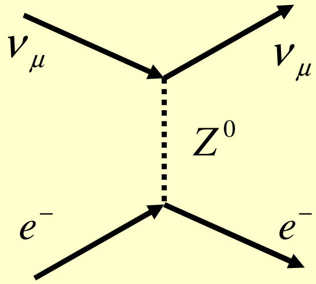
Neutral Currents Discovery - XII



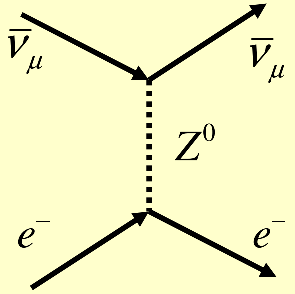
Length: 4.8 m
Diameter: 2 m
Liquid Freon: 12 m^3

Neutral Currents Discovery - XI

ν-e processes



Pure NC

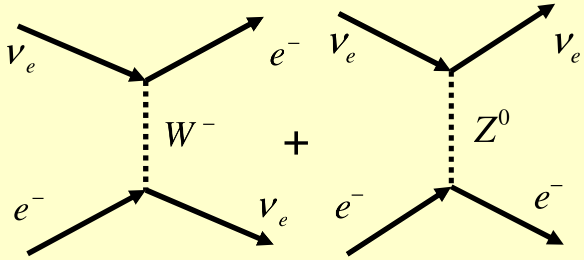


$$\sigma^{NC}_{\nu_\mu e^-}(E)=\frac{G_F{}^2 s}{\pi}\left[\left(-\frac{1}{2}+\sin^2\theta_W\right)^2+\frac{1}{3}\sin^4\theta_W\right]$$

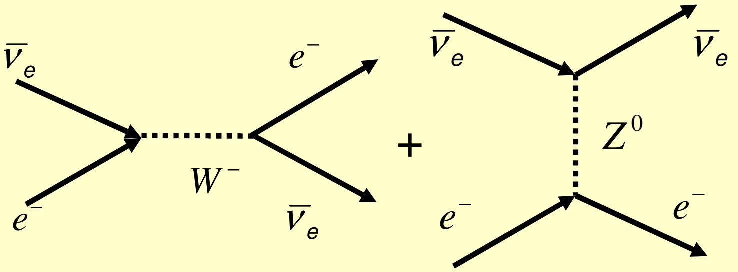
0.16x10^{-41} E(GeV) cm^2

$$\sigma^{NC}_{\bar{\nu}_\mu e^-}(E)=\frac{G_F{}^2 s}{\pi}\left[\frac{1}{3}\left(-\frac{1}{2}+\sin^2\theta_W\right)^2+\sin^4\theta_W\right]$$

0.13x10^{-41} E(GeV) cm^2



Interference
CC+NC



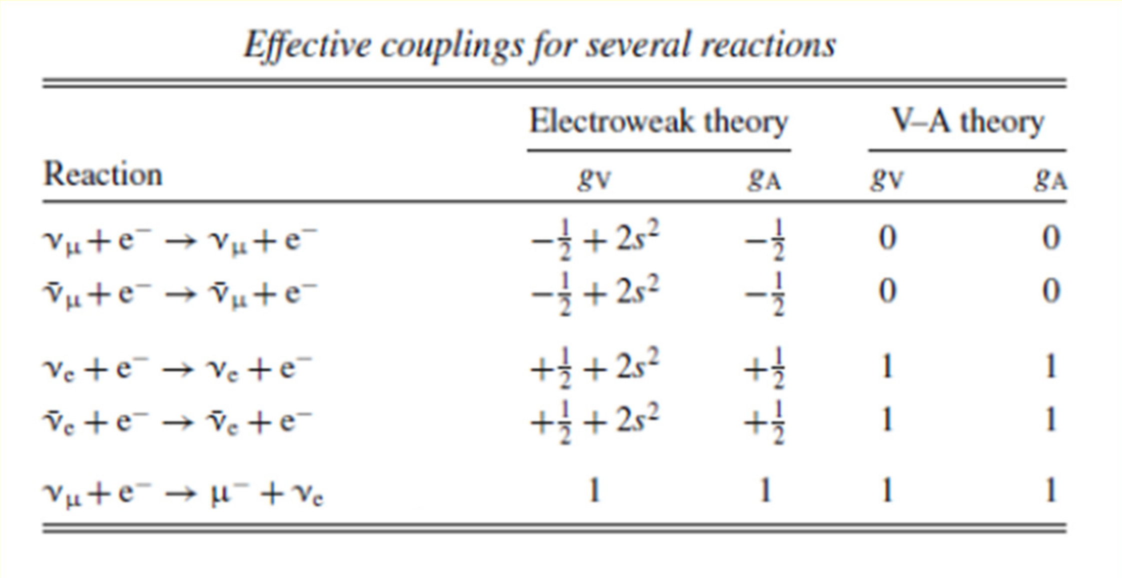
$$\sigma^{CC+NC}_{\nu_e e^-}(E)=\frac{G_F{}^2 s}{\pi}\left[\left(\frac{1}{2}+\sin^2\theta_W\right)^2+\frac{1}{3}\sin^4\theta_W\right]$$

0.96x10^{-41} E(GeV)

$$\sigma^{CC+NC}_{\bar{\nu}_e e^-}(E)=\frac{G_F{}^2 s}{\pi}\left[\frac{1}{3}\left(\frac{1}{2}+\sin^2\theta_W\right)^2+\sin^4\theta_W\right]$$

0.40x10^{-41} E(GeV) cm^2

Neutral Currents Discovery - XII



Effective couplings for several reactions

Reaction	Electroweak theory		V–A theory	
	g_V	g_A	g_V	g_A
$\nu_\mu + e^- \rightarrow \nu_\mu + e^-$	$-\frac{1}{2} + 2s^2$	$-\frac{1}{2}$	0	0
$\bar{\nu}_\mu + e^- \rightarrow \bar{\nu}_\mu + e^-$	$-\frac{1}{2} + 2s^2$	$-\frac{1}{2}$	0	0
$\nu_e + e^- \rightarrow \nu_e + e^-$	$+\frac{1}{2} + 2s^2$	$+\frac{1}{2}$	1	1
$\bar{\nu}_e + e^- \rightarrow \bar{\nu}_e + e^-$	$+\frac{1}{2} + 2s^2$	$+\frac{1}{2}$	1	1
$\nu_\mu + e^- \rightarrow \mu^- + \nu_e$	1	1	1	1

Neutral Currents Discovery - XIII

Differential cross sections:

$$y = 1 - \frac{E_\nu '}{E_\nu} \simeq \frac{E_e}{E_\nu} \quad \text{Bjorken } y$$

$$\frac{d\sigma_{\nu_\mu e}}{dy} = \frac{G_F^2 m_e}{2\pi} E_\nu \left[\left(g_V + g_A\right)^2 + \left(g_V - g_A\right)^2 \left(1 - y\right)^2 + \frac{m_e}{E_\nu}\left(g_A^2 - g_V^2\right) y \right]$$

$$\frac{d\sigma_{\bar{\nu}_\mu e}}{dy} = \frac{G_F^2 m_e}{2\pi} E_\nu \left[\left(g_V - g_A\right)^2 + \left(g_V + g_A\right)^2 \left(1 - y\right)^2 + \frac{m_e}{E_\nu}\left(g_A^2 - g_V^2\right) y \right]$$

$$\int_0^1 \left(1 - y\right)^2 dy = \frac{1}{3}, \quad \int_0^1 y dy = \frac{1}{2}$$

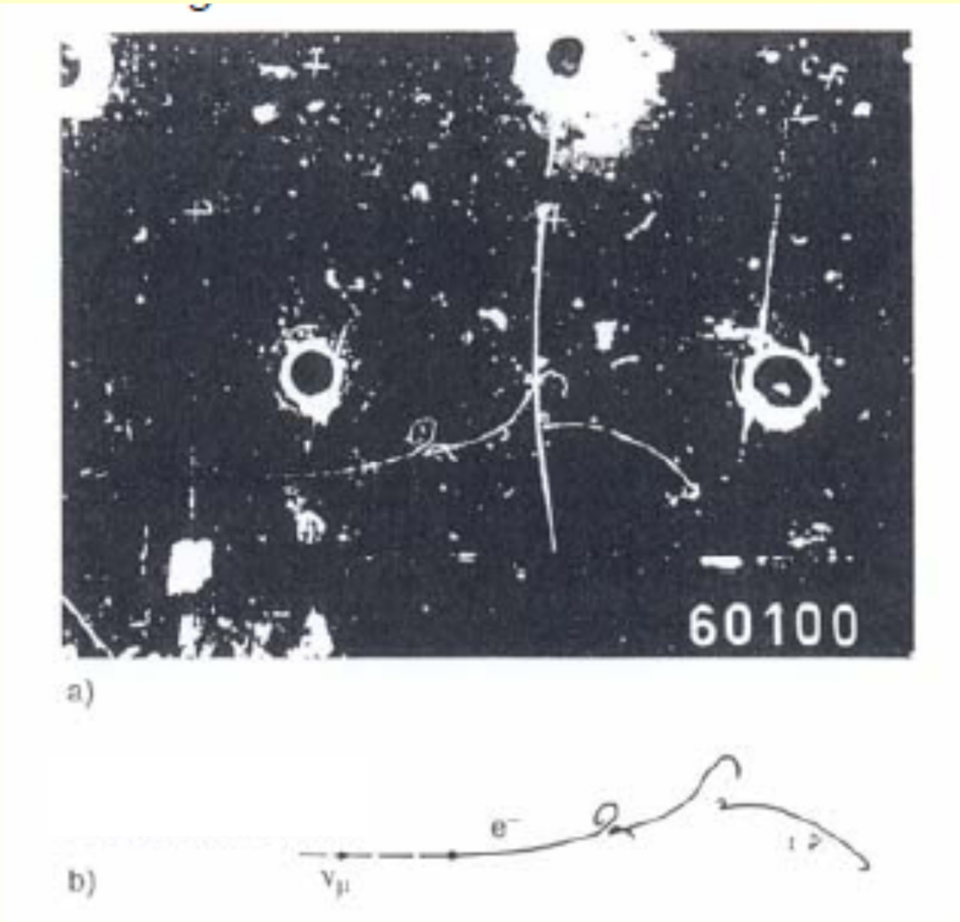
Total cross sections:

$$\rightarrow \sigma_{\nu_\mu e} = \frac{G_F^2 m_e}{2\pi} E_\nu \left[\left(g_V + g_A\right)^2 + \frac{1}{3}\left(g_V - g_A\right)^2 + \frac{m_e}{E_\nu}\frac{1}{2}\left(g_A^2 - g_V^2\right) \right]$$

$$\rightarrow \sigma_{\bar{\nu}_\mu e} = \frac{G_F^2 m_e}{2\pi} E_\nu \left[\left(g_V - g_A\right)^2 + \frac{1}{3}\left(g_V + g_A\right)^2 + \frac{m_e}{E_\nu}\frac{1}{2}\left(g_A^2 - g_V^2\right) \right]$$

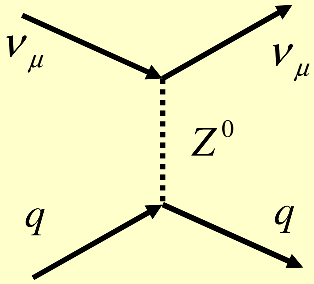
Neutral Currents Discovery - XIV

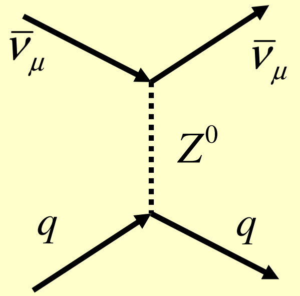
First Gargamelle
leptonic neutral current event

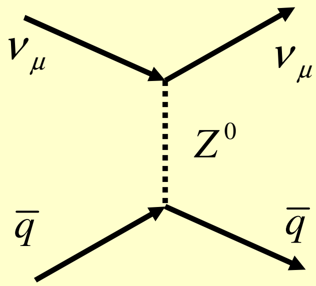


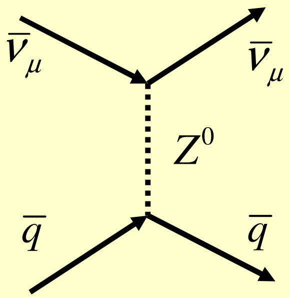
Neutral Currents Discovery - XV

$\overset{(-)}{\nu} - q, \bar{q}$ processes









Neutral Currents Discovery - XVI

$$g_V = \frac{1}{2} - \frac{4}{3}\sin^2\theta_W \qquad g_A = \frac{1}{2} \qquad u,c,t$$

$$g'_V = -\frac{1}{2} + \frac{4}{3}\sin^2\theta_W \qquad g'_A = \frac{1}{2} \qquad d,s,b$$

$$g_L = \frac{1}{2} - \frac{2}{3}\sin^2\theta_W \qquad g_R = -\frac{2}{3}\sin^2\theta_W \qquad u,c,t$$

$$g'_L = -\frac{1}{2} + \frac{1}{3}\sin^2\theta_W \qquad g_R = \frac{1}{3}\sin^2\theta_W \qquad d,s,b$$

$$\frac{d\sigma_{\nu_\mu q}}{dy} = \frac{d\sigma_{\bar{\nu}_\mu \bar{q}}}{dy} = \frac{G_F^2 m_q}{2\pi} E_\nu \left[\left(g_V + g_A\right)^2 + \left(g_V - g_A\right)^2 (1-y)^2 + \frac{m_q}{E}\left(g_A^2 - g_V^2\right) y \right]$$

$$\frac{d\sigma_{\bar{\nu}_\mu q}}{dy} = \frac{d\sigma_{\nu_\mu \bar{q}}}{dy} = \frac{G_F^2 m_q}{2\pi} E_\nu \left[\left(g_V - g_A\right)^2 + \left(g_V + g_A\right)^2 (1-y)^2 + \frac{m_q}{E_\nu}\left(g_A^2 - g_V^2\right) y \right]$$

Neutral Currents Discovery - XVII

$$\frac{d\sigma_{\overset{(-)}{\nu}_{\mu}}}{dxdy}=\sum_{q}q(x)\frac{d\sigma_{\overset{(-)}{\nu}_{\mu}q}}{dy}+\sum_{\bar{q}}\bar{q}(x)\frac{d\sigma_{\overset{(-)}{\nu}_{\mu}\bar{q}}}{dy}$$

$$\rightarrow\frac{d\sigma_{\nu_{\mu}N}}{dxdy}=\frac{G_F^2 m_N}{2\pi}xE_{\nu}\left[\left(g_L{}^2+g_L{}'^2\right)\left(q+\bar{q}\left(1-y\right)^2\right)+\left(g_R{}^2+g_R{}'^2\right)\left(\bar{q}+q\left(1-y\right)^2\right)\right]$$

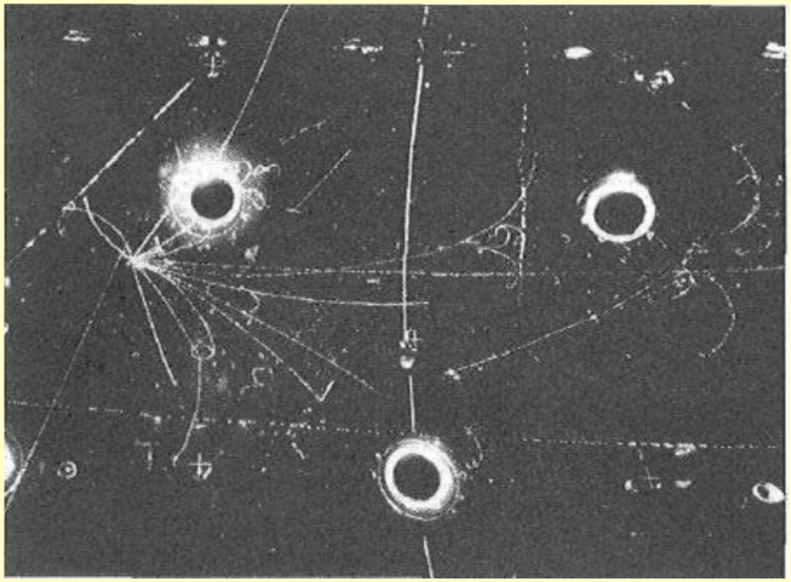
$$\rightarrow\frac{d\sigma_{\tilde{\nu}_{\mu}N}}{dxdy}=\frac{G_F^2 m_N}{2\pi}xE_{\nu}\left[\left(g_R{}^2+g_R{}'^2\right)\left(q+\bar{q}\left(1-y\right)^2\right)+\left(g_L{}^2+g_L{}'^2\right)\left(\bar{q}+q\left(1-y\right)^2\right)\right]$$

$$R_{\nu}^{N}=\frac{\sigma_{NC}(\nu)}{\sigma_{CC}(\nu)}\qquad R_{\bar{\nu}}^{N}=\frac{\sigma_{NC}(\bar{\nu})}{\sigma_{CC}(\bar{\nu})}\qquad r=\frac{\sigma_{CC}(\bar{\nu})}{\sigma_{CC}(\nu)}$$

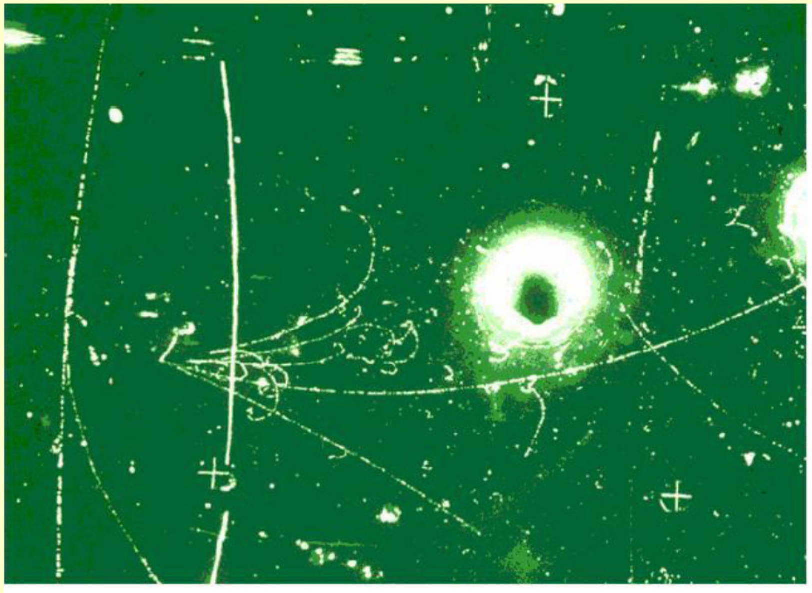
$$\rightarrow g_L{}^2+g_L{}'^2=\frac{R_{\nu}^{N}-r^2R_{\bar{\nu}}^{N}}{1-r^2}\qquad g_R{}^2+g_R{}'^2=\frac{r\left(R_{\nu}^{N}-R_{\bar{\nu}}^{N}\right)}{1-r^2}$$

Neutral Currents Discovery - XVIII

Gargamelle
charged current



Gargamelle
hadronic neutral current event

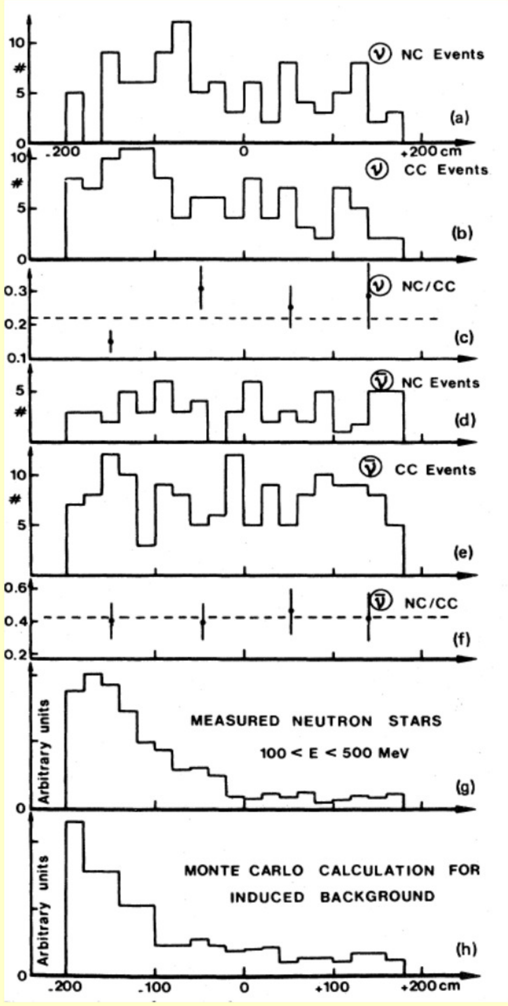


Neutral Currents Discovery - XIX

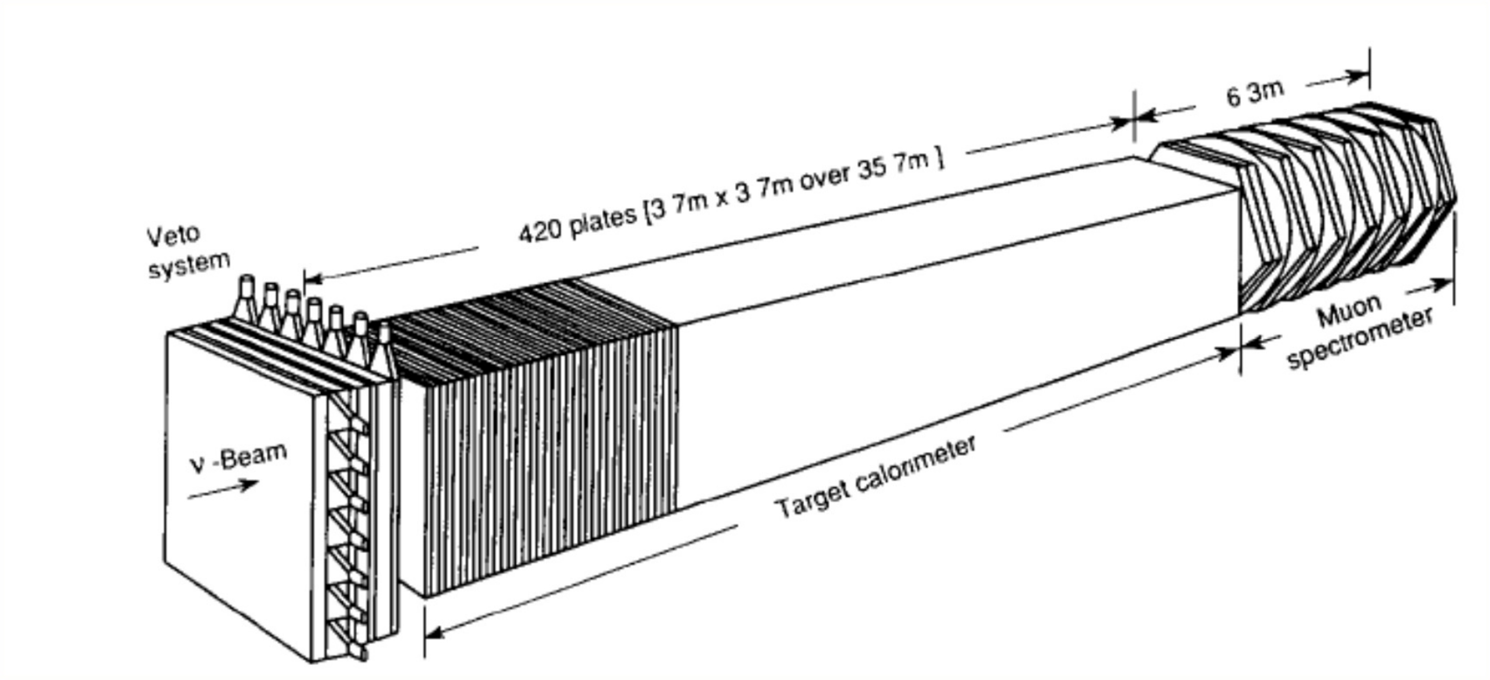
Background

Result

$$\boxed{\sin^2\theta_W = 0.3 \div 0.4}$$



Neutral Currents Discovery - XX

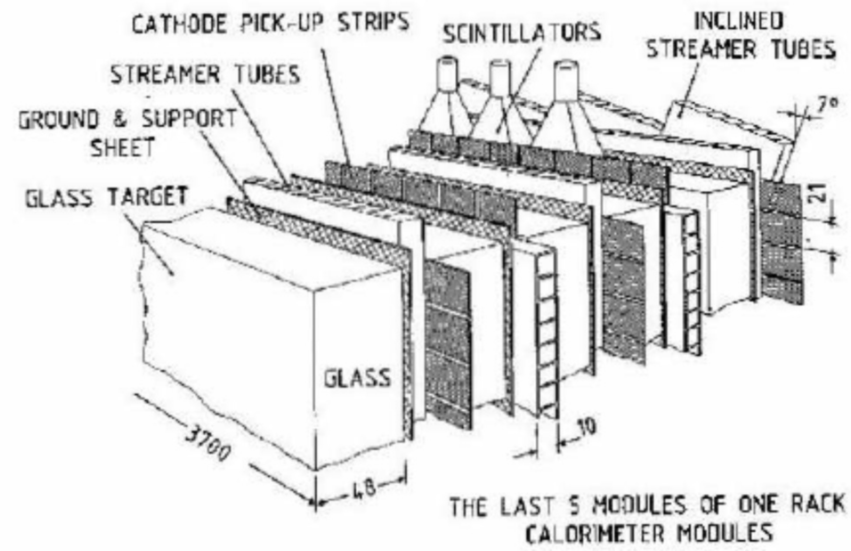


CHARM II

Neutral Currents Discovery - XXI

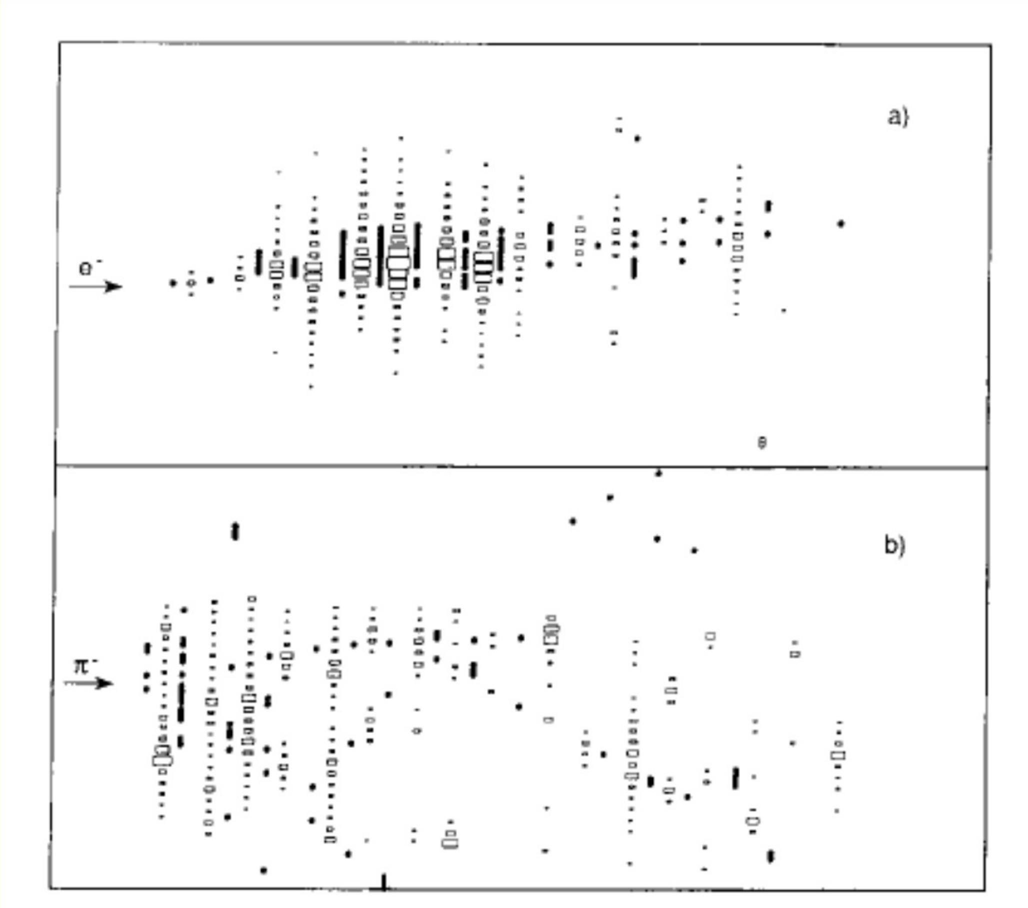
Experimental setup (CHARM II, 1987-1991)

electronic tracking detector



~ 700 t calorimeter, digital readout of energy and direction of produced particles

Neutral Currents Discovery - XXII



Electromagnetic shower

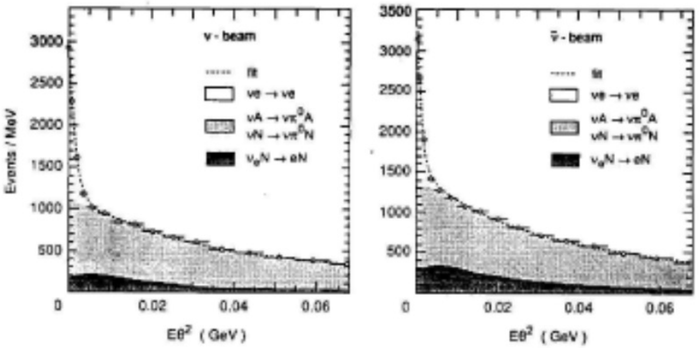
Hadronic shower

Neutral Currents Discovery- XXIII

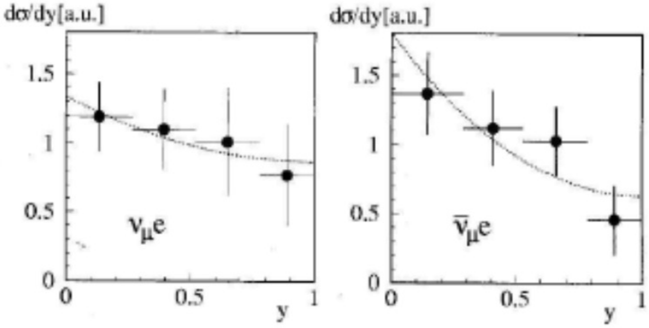
CHARM II data

Problem: discrimination of the NC events ($\sim$ 2500 for $\nu_\mu e^-$ and $\bar{\nu}_\mu e^-$ each) from the dominant background (CC scattering, inelastic scattering)

Solution: in processes of interest $\nu_\mu e \rightarrow \nu_\mu e$ and $\bar{\nu}_\mu e^- \rightarrow \bar{\nu}_\mu e^-$ the value $E_e\theta_e^2$ is kinematically restricted to small values



Phys.Lett. B335, 246 (1994)



Phys.Lett. B302, 351 (1993)

$$\sin^2\Theta_{\nu e} = 0.2324 \pm 0.0083$$

W & Z - I

Some reminescences about photons...

Free photons ($j^\mu = 0$) :

Lorentz condition

$\partial_\mu A^\mu = 0 \rightarrow \Box^2 A^\mu = 0$

$\rightarrow A^\mu = \varepsilon^\mu(q)e^{-iqx} \rightarrow q^2 = 0$ massless quanta

4 components ε^μ ??

$a)$ $\partial_\mu A^\mu = 0 \rightarrow \varepsilon^\mu(q)q_\mu = 0 \rightarrow 3$ components

$b)$ Gauge freedom:

$A^\mu \rightarrow A^\mu + \partial^\mu \Lambda,\ \Box^2 \Lambda = 0$

$\Lambda = iae^{-iqx}\left(\leftarrow\ \Box^2 \Lambda = q^2 \Lambda = 0 \text{ OK}\right)$

$\rightarrow \partial^\mu \Lambda = ia\partial^\mu e^{-iqx}$

$$A^\mu \rightarrow A^\mu + \partial^\mu \Lambda = \varepsilon^\mu(q)e^{-iqx} + ia\partial^\mu e^{-iqx} = \left[\varepsilon^\mu(q) + ia\left(-iq_\mu\right)\right]e^{-iqx} = \left[\varepsilon^\mu(q) + aq_\mu\right]e^{-iqx}$$

$\rightarrow$ EM field unchanged by $\varepsilon^\mu(q) \rightarrow \varepsilon^\mu(q) + aq^\mu$

Choose a to make $\varepsilon^0 = 0$

$\rightarrow \varepsilon^\mu(q)q_\mu = 0 \rightarrow \boldsymbol{\varepsilon}\cdot\mathbf{q} = 0 \rightarrow 2$ components

W & Z - II

2 components → 2 independent ε^{μ}

Take photon momentum along z

$\varepsilon_1^{\mu} = \begin{pmatrix} 0 & 1 & 0 & 0 \end{pmatrix}$ x - linear polarization

$\varepsilon_2^{\mu} = \begin{pmatrix} 0 & 0 & 1 & 0 \end{pmatrix}$ y - linear polarization

or

$\varepsilon_L^{\mu} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & -i & 0 \end{pmatrix}$ Left circular polarization: $S_z = -1$

$\varepsilon_R^{\mu} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & +i & 0 \end{pmatrix}$ Right circular polarization: $S_z = +1$

W & Z - III

Original wave equation:

$$\Box^2 A^\mu - \partial^\mu \left(\partial_\nu A^\nu \right) = j^\mu$$

For a massive vector boson:

$$\left(\Box^2 + m^2 \right) B^\mu - \partial^\mu \left(\partial_\nu B^\nu \right) = j^\mu$$

Free particle:

$$\left(\Box^2 + m^2 \right) B^\mu - \partial^\mu \left(\partial_\nu B^\nu \right) = 0$$

But:

$$\partial_\mu \left(\Box^2 + m^2 \right) B^\mu - \partial_\mu \partial^\mu \left(\partial_\nu B^\nu \right) = 0 \rightarrow \left(\Box^2 + m^2 \right) \partial_\mu B^\mu - \Box^2 \left(\partial_\nu B^\nu \right) = 0$$

$$\rightarrow m^2 \partial_\mu B^\mu = 0 \rightarrow \partial_\mu B^\mu = 0$$

Bottom line: Not an extra condition...

$$\rightarrow \left(\Box^2 + m^2 \right) B^\mu = 0$$

$B^\mu = \varepsilon^\mu \left(p \right) e^{-ipx} \rightarrow \varepsilon^\mu p_\mu = 0 \rightarrow 3$ independent components

No gauge freedom...

W & Z - IV

3 independent components → 3 independent ε^{μ}

Take photon momentum along z

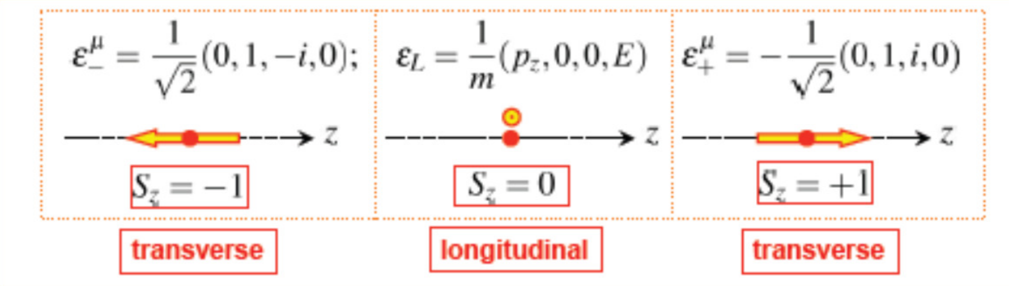
$$\varepsilon_L^{\mu} = \frac{1}{\sqrt{2}}\begin{pmatrix}0 & 1 & -i & 0\end{pmatrix}$$ Left circular polarization: $S_z = -1$

$$\varepsilon_R^{\mu} = -\frac{1}{\sqrt{2}}\begin{pmatrix}0 & 1 & +i & 0\end{pmatrix}$$ Right circular polarization: $S_z = +1$

To find 3rd polarization 4-vector:

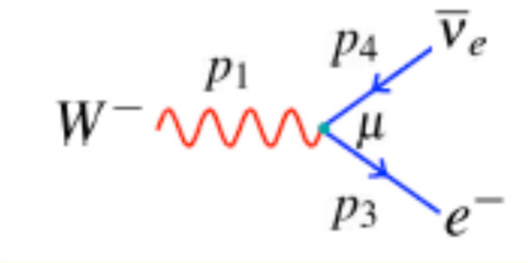
$$\varepsilon_0^{\mu} = \frac{1}{\sqrt{\alpha^2 - \beta^2}}\begin{pmatrix}\alpha & 0 & 0 & \beta\end{pmatrix} \rightarrow \varepsilon^{\mu} p_{\mu} = 0 \rightarrow \alpha E - \beta p_z = 0$$

$$\rightarrow \varepsilon_0^{\mu} = \frac{1}{m}\begin{pmatrix}p_z & 0 & 0 & E\end{pmatrix}$$ Longitudinal polarization: $S_z = 0$



W & Z - V

Decay: $W^- \to e^- + \bar{\nu}_e$



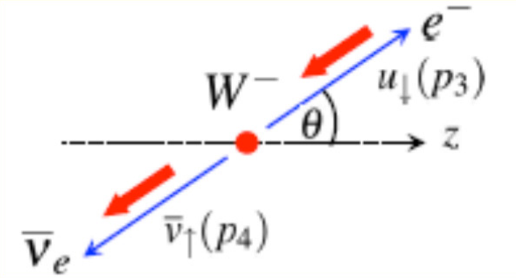
Matrix element:

$$M_{fi} = \frac{g}{\sqrt{2}}\varepsilon_\mu(p_1)\bar{u}(p_3)\frac{1}{2}\gamma^\mu(1-\gamma_5)v(p_4) = \frac{g}{\sqrt{2}}\varepsilon_\mu(p_1)\, j^\mu$$

$\bar{u}(p_3)$: outgoing fermion, $v(p_4)$: outgoing antifermion

$$\bar{u}(p_3)\gamma^\mu\frac{1}{2}(1-\gamma_5)v(p_4) = \bar{u}(p_3)\frac{1}{2}\gamma^\mu(1-\gamma_5)\frac{1}{2}\gamma^\mu(1-\gamma_5)v(p_4)$$

$$= \underbrace{\bar{u}(p_3)\frac{1}{2}(1+\gamma_5)}_{\bar{e}_L}\gamma^\mu\underbrace{\frac{1}{2}(1-\gamma_5)v(p_4)}_{v_R} = \bar{e}_L\gamma^\mu v_R$$



LR current :

Build from rotated *L*, *R* spinors

$$j^\mu = \bar{u}_\downarrow(p_3)\frac{1}{2}\gamma^\mu v_\uparrow(p_4) = 2E(0, -\cos\theta, -i, \sin\theta)$$

W & Z - VI

W polarization states in the rest system:

$$\varepsilon_L^\mu = \frac{1}{\sqrt{2}}\begin{pmatrix}0 & 1 & -i & 0\end{pmatrix}$$

$$\varepsilon_R^\mu = -\frac{1}{\sqrt{2}}\begin{pmatrix}0 & 1 & +i & 0\end{pmatrix}$$

$$\varepsilon_0^\mu = \frac{1}{m}\begin{pmatrix}0 & 0 & 0 & m\end{pmatrix} = \begin{pmatrix}0 & 0 & 0 & 1\end{pmatrix}$$

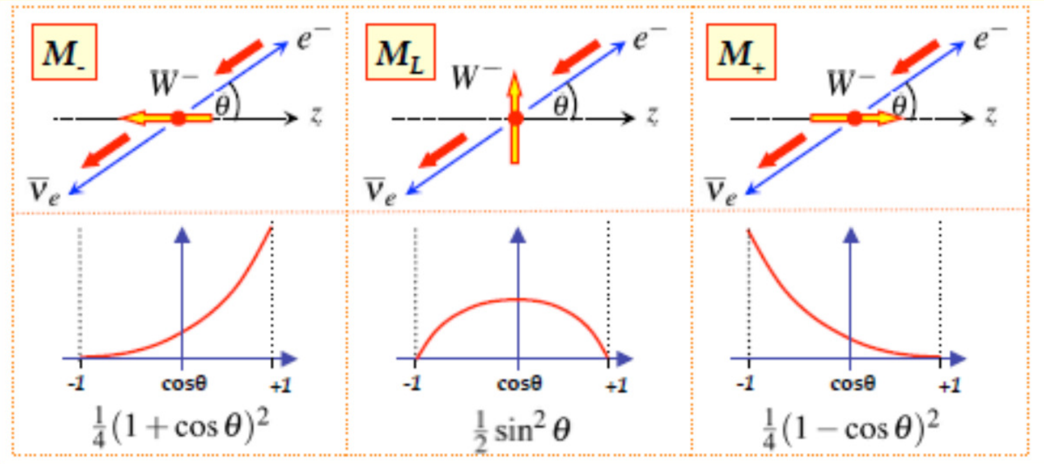
Matrix elements for different W polarization states in the rest system:

$$\varepsilon_L^\mu : \quad \frac{g}{\sqrt{2}}\frac{1}{\sqrt{2}}\begin{pmatrix}0 & 1 & -i & 0\end{pmatrix} 2\frac{M_W}{2}\left(0, -\cos\theta, -i, \sin\theta\right) = -\frac{gM_W}{2}\left(1+\cos\theta\right) \rightarrow \left|M_L\right|^2 = \frac{g^2M_W^2}{4}\left(1+\cos\theta\right)^2$$

$$\varepsilon_R^\mu : \quad \frac{g}{\sqrt{2}}\left[-\frac{1}{\sqrt{2}}\begin{pmatrix}0 & 1 & +i & 0\end{pmatrix}\right] 2\frac{M_W}{2}\left(0, -\cos\theta, -i, \sin\theta\right) = -\frac{gM_W}{2}\left(1-\cos\theta\right) \rightarrow \left|M_R\right|^2 = \frac{g^2M_W^2}{4}\left(1-\cos\theta\right)^2$$

$$\varepsilon_0^\mu : \quad \frac{g}{\sqrt{2}}\frac{1}{m}\begin{pmatrix}0 & 0 & 0 & m\end{pmatrix} 2\frac{M_W}{2}\left(0, -\cos\theta, -i, \sin\theta\right) = \frac{gM_W}{\sqrt{2}}\sin\theta \rightarrow \left|M_0\right|^2 = \frac{g^2M_W^2}{2}\sin^2\theta$$

W & Z - VII



$$|M_L|^2 = \frac{g^2 M_W^2}{4}(1+\cos\theta)^2$$

$$|M_R|^2 = \frac{g^2 M_W^2}{4}(1-\cos\theta)^2$$

$$|M_0|^2 = \frac{g^2 M_W^2}{2}\sin^2\theta$$

2-body differential decay rate:

$$\frac{d\Gamma_{L,R,0}}{d\Omega} = \frac{p}{32\pi^2 M_W^2}|M|^2 = \frac{1}{64\pi^2 M_W}|M|^2 = \frac{g^2 M_W}{64\pi^2}\begin{cases}\frac{1}{4}(1+\cos\theta)^2 \\ \frac{1}{2}\sin^2\theta \\ \frac{1}{4}(1-\cos\theta)^2\end{cases}$$

Total rates:

$$\int \frac{1}{4}(1\pm\cos\theta)^2\, d\cos\theta d\varphi = \int \frac{1}{2}\sin^2\theta d\cos\theta d\varphi = \frac{4\pi}{3}$$

$$\rightarrow \Gamma_L = \Gamma_R = \Gamma_0 = \frac{g^2 M_W}{48\pi}$$

W & Z - VIII

$$|M_L|^2 = \frac{g^2 M_W^2}{4}(1+\cos\theta)^2$$

$$|M_R|^2 = \frac{g^2 M_W^2}{4}(1-\cos\theta)^2$$

$$|M_0|^2 = \frac{g^2 M_W^2}{2}\sin^2\theta$$

Averaging over the initial spin states:

$$\left\langle |M|^2 \right\rangle = \frac{1}{3} g^2 M_W^2 \left[\frac{1}{4}(1+\cos\theta)^2 + \frac{1}{4}(1-\cos\theta)^2 + \frac{1}{2}\sin^2\theta\right]$$

$$\rightarrow \left\langle |M|^2 \right\rangle = \frac{1}{3} g^2 M_W^2$$

Isotropic: OK for an unpolarized mother particle

$$\rightarrow \Gamma\left(W^- \rightarrow e^- + \overline{\nu}_e\right) = \frac{g^2 M_W}{48\pi}$$

W & Z - IX

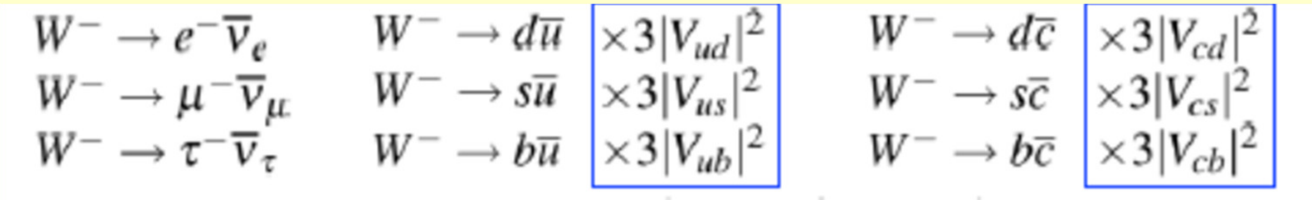
Considering all the others decay modes: Large W mass → All fermions ≈ massless

Do *not* count Top: Too heavy, decay energetically forbidden

Color factor = 3

Similar to $e^{+}e^{=} \to q\bar{q}$: Take quarks as free, on shell particles

Taking into account CKM mixing:



$W^- \to e^-\overline{\nu}_e$	$W^- \to d\overline{u}$	$\times 3\|V_{ud}\|^2$	$W^- \to d\overline{c}$	$\times 3\|V_{cd}\|^2$
$W^- \to \mu^-\overline{\nu}_\mu$	$W^- \to s\overline{u}$	$\times 3\|V_{us}\|^2$	$W^- \to s\overline{c}$	$\times 3\|V_{cs}\|^2$
$W^- \to \tau^-\overline{\nu}_\tau$	$W^- \to b\overline{u}$	$\times 3\|V_{ub}\|^2$	$W^- \to b\overline{c}$	$\times 3\|V_{cb}\|^2$

CKM Unitarity:

$$e.g.\quad |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 \quad \text{etc}$$

$$\to \Gamma_W = 9\Gamma_{W\to e\nu} = \frac{3g^2 M_W}{16\pi} = 2.07\ GeV$$

Experiment :

$2.14 \pm 0.04\ \ GeV$

QCD corrections..

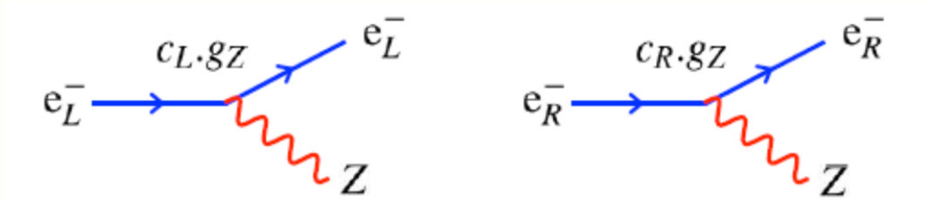
W & Z - X

Z couplings:

$$c_L = I_3 - Q\sin^2\theta_W$$

$$c_R = -Q\sin^2\theta_W$$

$$j_\mu^Z = g_Z \bar{u}\gamma_\mu \left[c_L \frac{1}{2}(1-\gamma_5) + c_R \frac{1}{2}(1+\gamma_5) \right] u$$

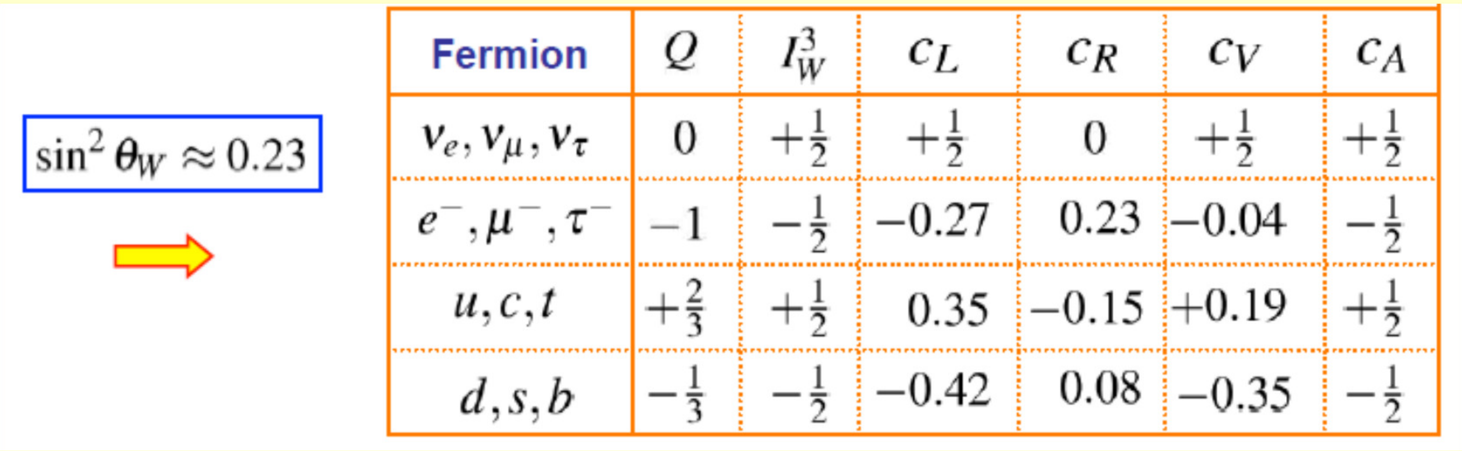


$$c_V = c_L + c_R = I_3 - 2Q\sin^2\theta_W$$

$$c_A = c_L - c_R = I_3$$

W & Z - XI

Therefore:



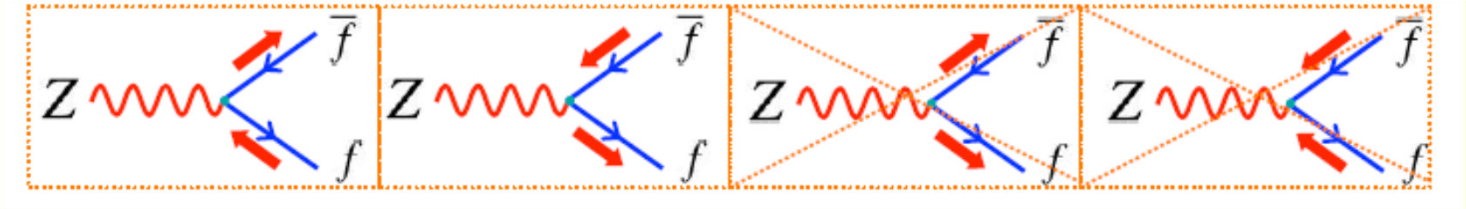
$\boxed{\sin^2\theta_W \approx 0.23}$ ⇨

Fermion	Q	I_W^3	c_L	c_R	c_V	c_A
ν_e, ν_μ, ν_τ	0	$+\frac{1}{2}$	$+\frac{1}{2}$	0	$+\frac{1}{2}$	$+\frac{1}{2}$
e^-, μ^-, τ^-	-1	$-\frac{1}{2}$	-0.27	0.23	-0.04	$-\frac{1}{2}$
u, c, t	$+\frac{2}{3}$	$+\frac{1}{2}$	0.35	-0.15	$+0.19$	$+\frac{1}{2}$
d, s, b	$-\frac{1}{3}$	$-\frac{1}{2}$	-0.42	0.08	-0.35	$-\frac{1}{2}$

W & Z - XII

Z couplings: Both to *L* and *R* fermions

Nevertheless:
Only 2 vertexes, remaining 2 = 0



To show that RR vertex is 0 (LL similar):

$$\bar{u}_R = u_R^\dagger \gamma^0 = u^\dagger \frac{1+\gamma^5}{2}\gamma^0, \quad v_R = \frac{1-\gamma^5}{2} v$$

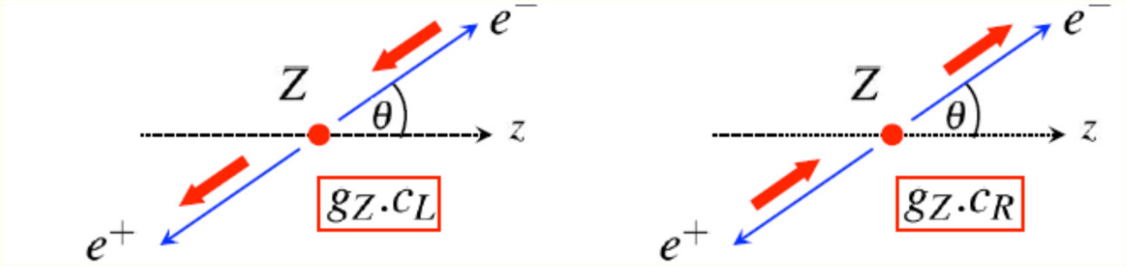
$$\bar{u}_R \gamma^\mu \left(c_V + c_A \gamma_5\right) v_R = u^\dagger \frac{1+\gamma^5}{2} \gamma^0 \gamma^\mu \left(c_V + c_A \gamma_5\right) \frac{1-\gamma^5}{2} v$$

$$= \frac{1}{4} u^\dagger \gamma^0 \left(1-\gamma^5\right)\left(1-\gamma^5\right)\left(c_V + c_A \gamma_5\right) v$$

$$= \frac{1}{4} \bar{u} \gamma^\mu \left(1+\gamma^5\right)\left(1-\gamma^5\right)\left(c_V + c_A \gamma_5\right) v = 0$$

W & Z - XIII

Decay: $Z^0 \to e^+ + e^-$



$$\left\langle |M|^2 \right\rangle = \frac{2}{3} g^2 \cos^2\theta_W M_Z^2 \left[c_L^2 + c_R^2\right]$$

$$2\left[c_L^2 + c_R^2\right] = \left[c_V^2 + c_A^2\right]$$

$$\to \Gamma\left(Z \to e^+e^-\right) = \frac{g^2 \cos^2\theta_W M_Z}{48\pi}\left[c_V^2 + c_A^2\right]$$

W & Z - XIV

$$Br(Z \to e^+e^-) = Br(Z \to \mu^+\mu^-) = Br(Z \to \tau^+\tau^-) \boxed{\approx 3.5\%}$$

$$Br(Z \to \nu_1\overline{\nu}_1) = Br(Z \to \nu_2\overline{\nu}_2) = Br(Z \to \nu_3\overline{\nu}_3) \boxed{\approx 6.9\%}$$

$$Br(Z \to d\overline{d}) = Br(Z \to s\overline{s}) = Br(Z \to b\overline{b}) \boxed{\approx 15\%}$$

$$Br(Z \to u\overline{u}) = Br(Z \to c\overline{c}) \boxed{\approx 12\%}$$

$$Br(Z \to \text{hadrons}) \approx 69\%$$

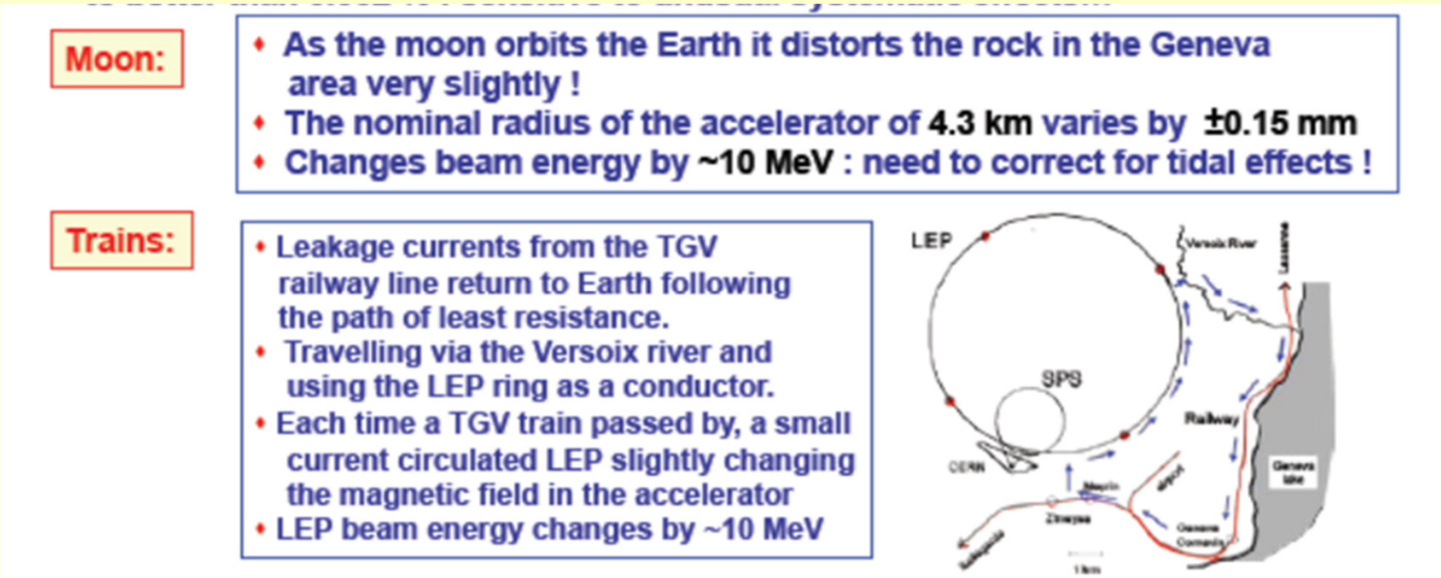
$$\to \Gamma_Z = \sum_i \Gamma_i = 2.5 \quad GeV$$

$$\text{Experiment: } \Gamma_Z = 2.4952 \pm 0.0023 \quad GeV$$

!!!

W & Z - XV

Ultimate systematics….



Moon:

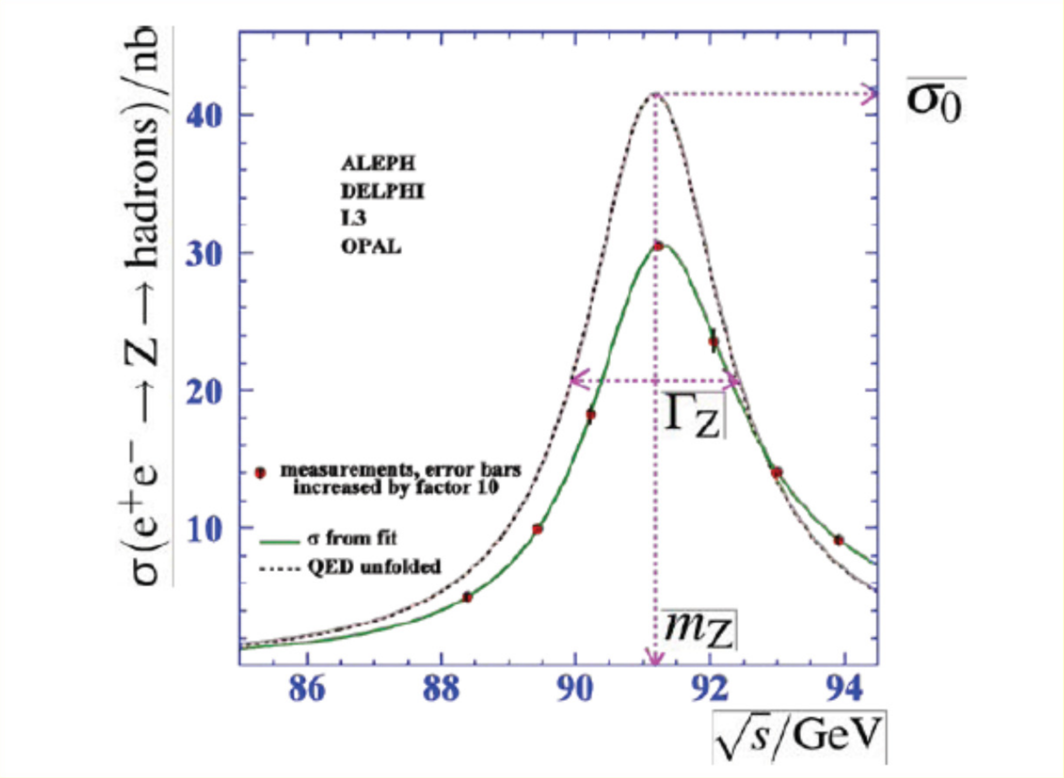
- As the moon orbits the Earth it distorts the rock in the Geneva area very slightly !
- The nominal radius of the accelerator of 4.3 km varies by ±0.15 mm
- Changes beam energy by ~10 MeV : need to correct for tidal effects !

Trains:

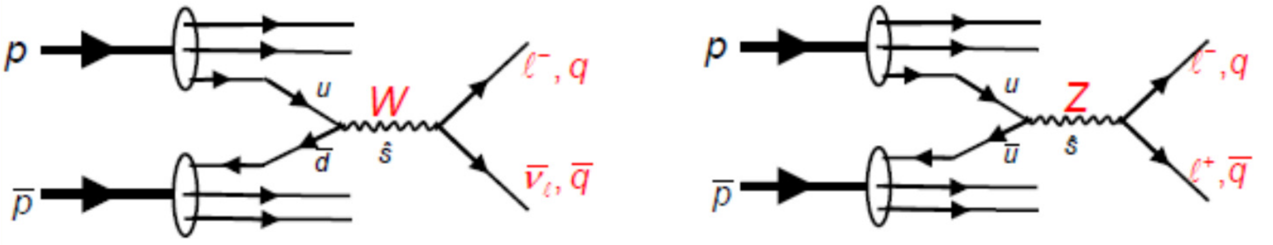
- Leakage currents from the TGV railway line return to Earth following the path of least resistance.
- Travelling via the Versoix river and using the LEP ring as a conductor.
- Each time a TGV train passed by, a small current circulated LEP slightly changing the magnetic field in the accelerator
- LEP beam energy changes by ~10 MeV

W & Z - XVI

Measure the number of SM neutrinos from the Z width: $N_\nu=2.9840\pm0.0082$

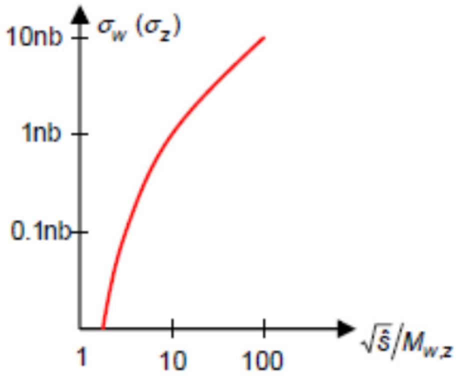


W & Z Discovery - I



$p\bar{p} \to W \to \ell \bar{\nu}_\ell + X$

$p\bar{p} \to Z \to f\bar{f} + X$



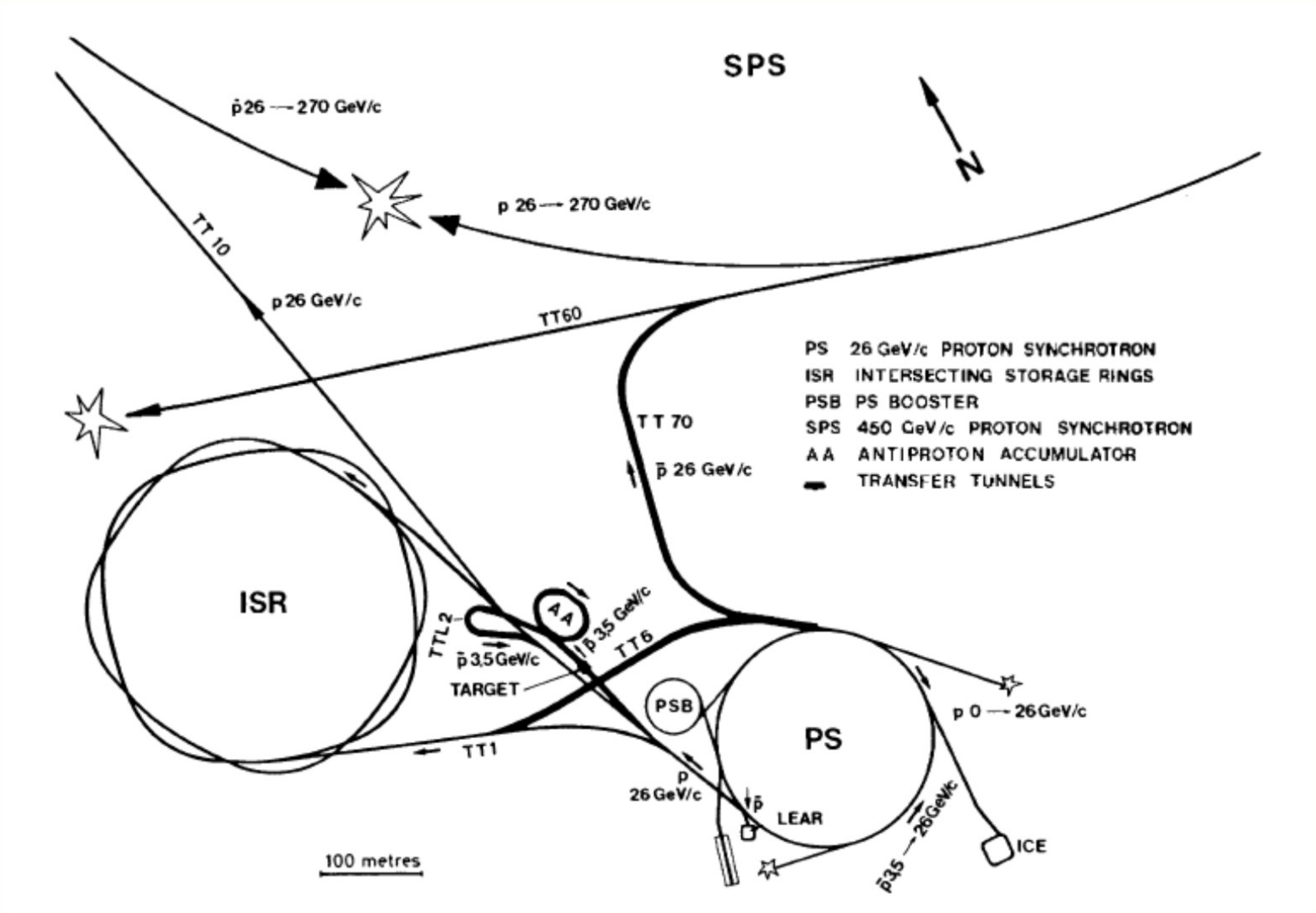
Similar to Drell-Yan: (photon instead of W)

$\hat{s} = x_q x_{\bar{q}} s \qquad \langle x_q \rangle \approx 0.12$

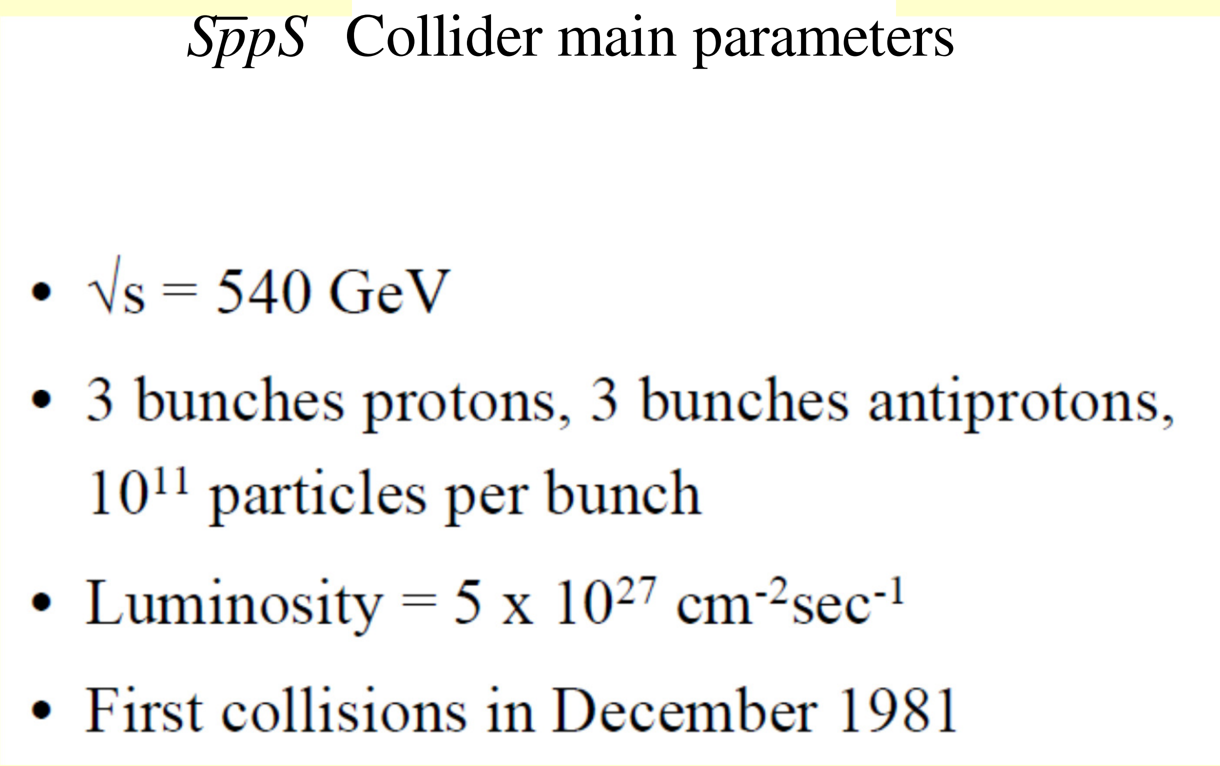
$\hat{s} = \langle x_q \rangle^2 s \approx 0.014 s = (65\ \text{GeV})^2$

→ Cross section is small !

W & Z Discovery - II



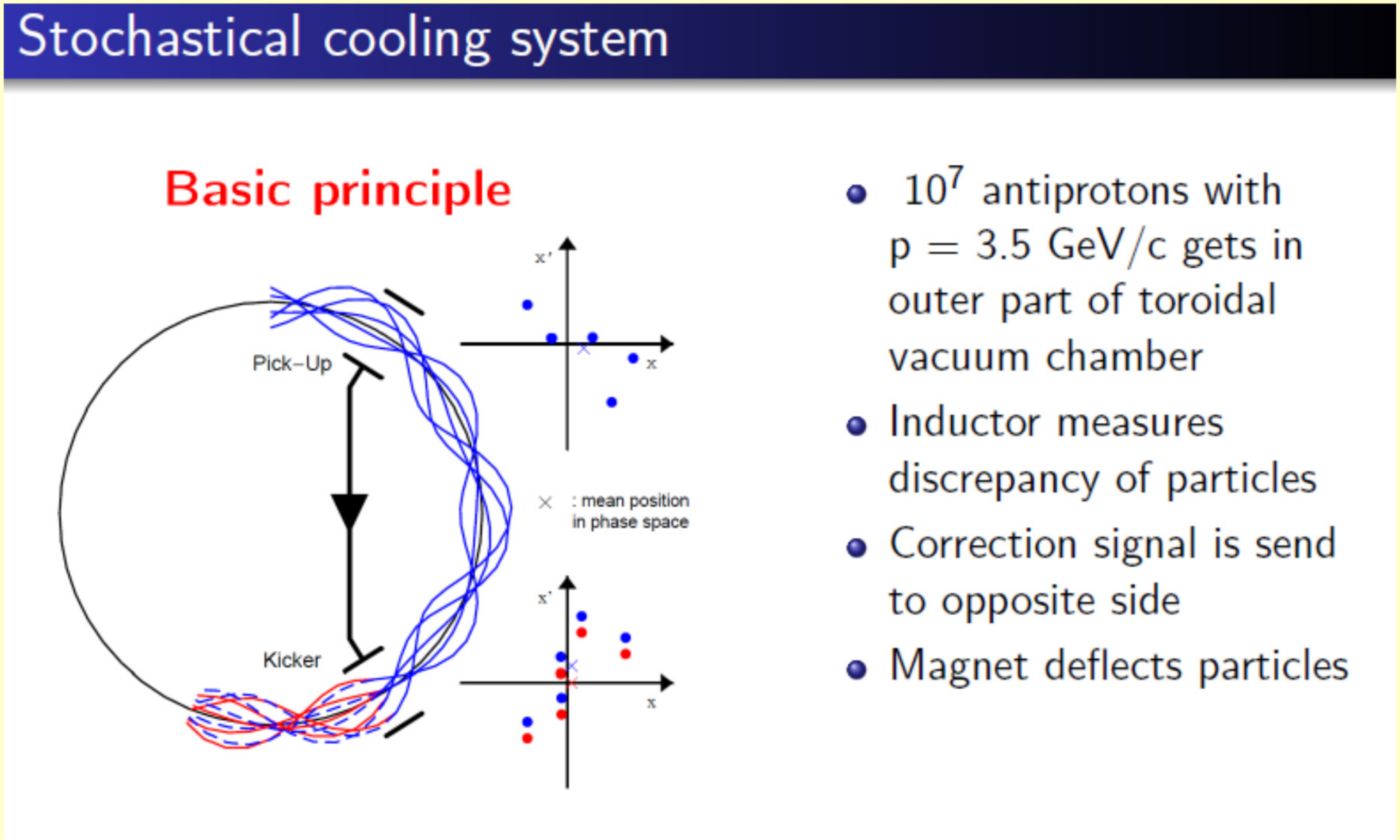
W & Z Discovery - III



$S\bar{p}pS$ Collider main parameters

- $\sqrt{s} = 540$ GeV
- 3 bunches protons, 3 bunches antiprotons, 10^{11} particles per bunch
- Luminosity = 5×10^{27} cm^{-2}sec^{-1}
- First collisions in December 1981

W & Z Discovery - IV



Stochastical cooling system

Basic principle

- 10^7 antiprotons with $p = 3.5$ GeV/c gets in outer part of toroidal vacuum chamber
- Inductor measures discrepancy of particles
- Correction signal is send to opposite side
- Magnet deflects particles

W & Z Discovery - V

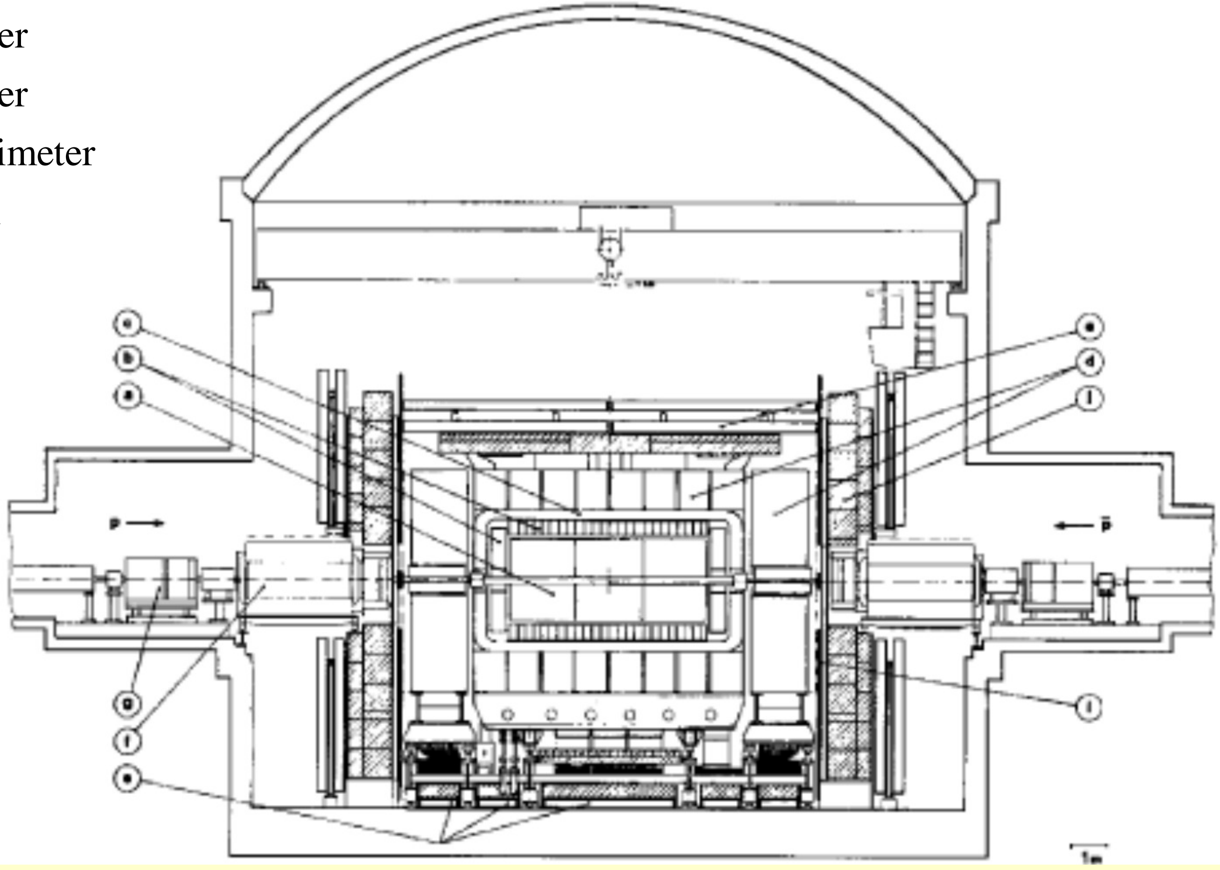
*UA*1 Detector

Central Tracker

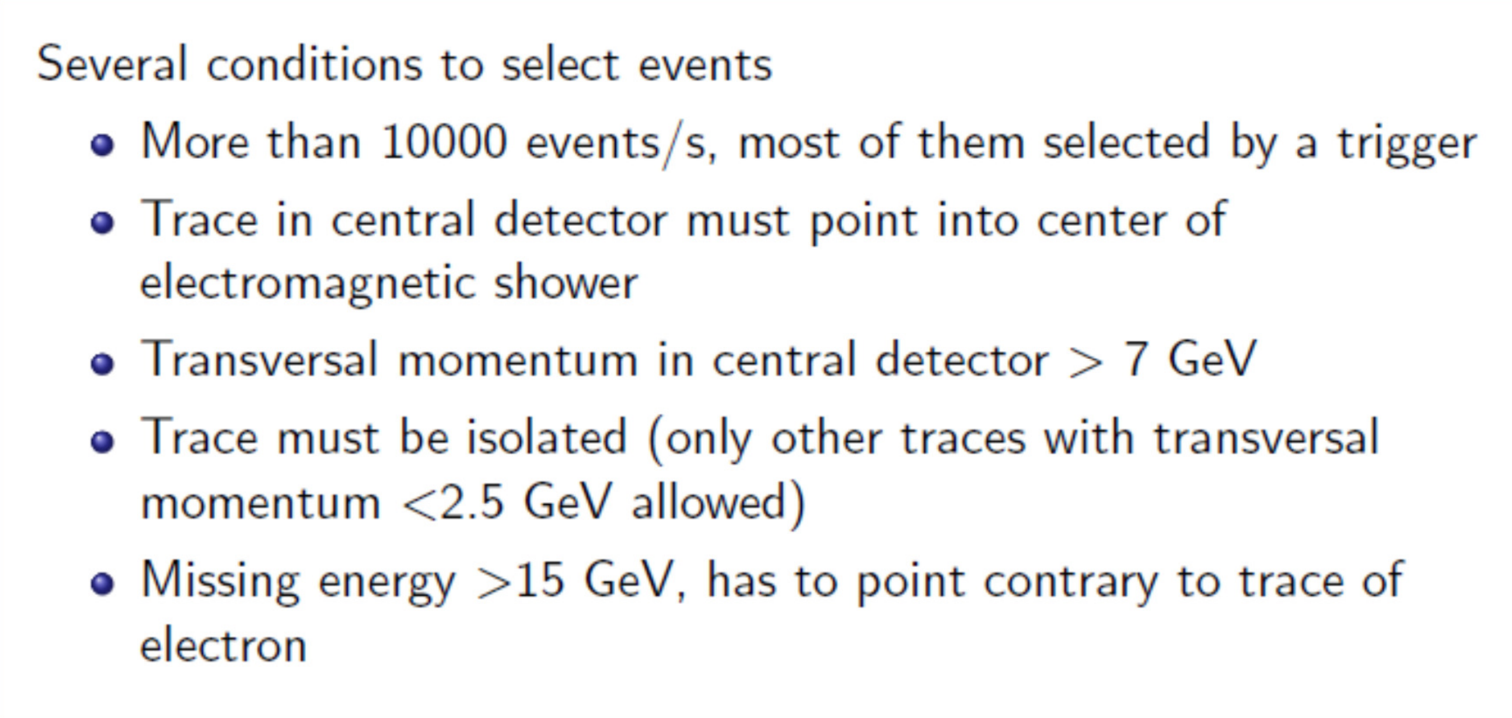
EM calorimeter

Hadron Calorimeter

Muon System



W & Z Discovery - VI

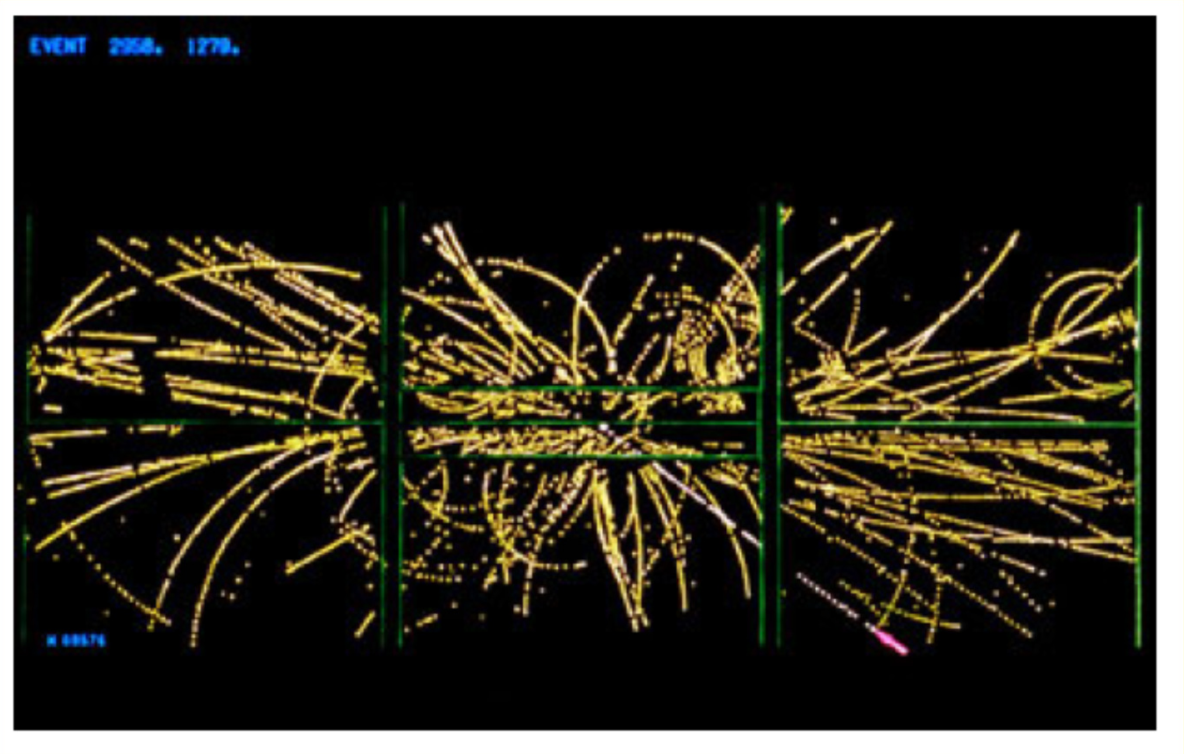


Several conditions to select events

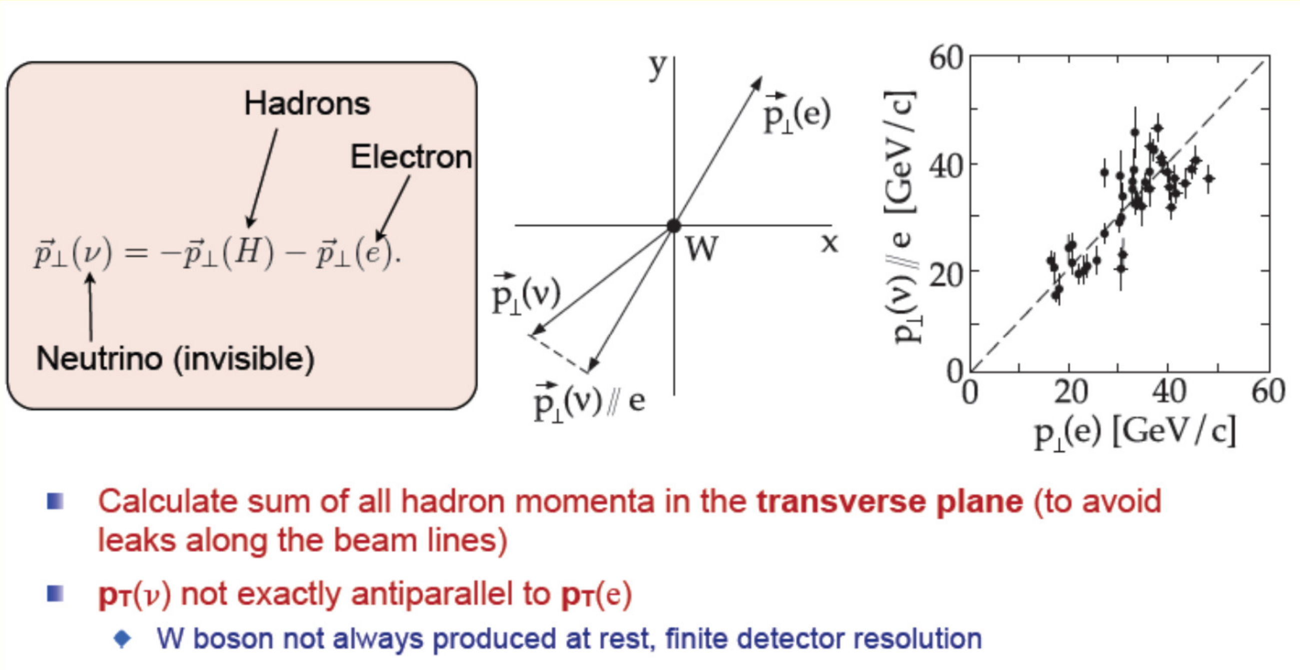
- More than 10000 events/s, most of them selected by a trigger
- Trace in central detector must point into center of electromagnetic shower
- Transversal momentum in central detector > 7 GeV
- Trace must be isolated (only other traces with transversal momentum <2.5 GeV allowed)
- Missing energy >15 GeV, has to point contrary to trace of electron

W & Z Discovery - VII

UA1 $W\rightarrow e\nu$ candidate event



W & Z Discovery - VIII

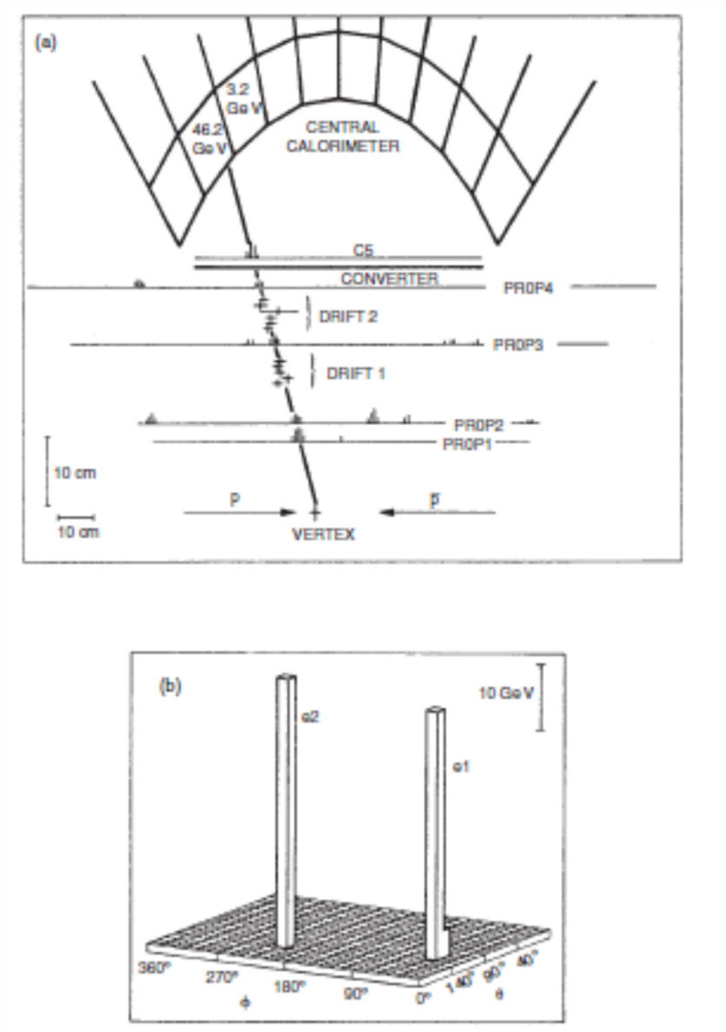


$$\vec{p}_\perp(\nu) = -\vec{p}_\perp(H) - \vec{p}_\perp(e).$$

- Calculate sum of all hadron momenta in the **transverse plane** (to avoid leaks along the beam lines)
- **p_T**(ν) not exactly antiparallel to **p_T**(e)
  - W boson not always produced at rest, finite detector resolution

W & Z Discovery - IX

UA2 Candidate Z event



W & Z Discovery - X

$$\frac{d\sigma}{d\cos\theta^*} = \text{const} \qquad \text{Just an approximation}$$

$$\frac{d\sigma}{dp_T} = \frac{d\sigma}{d\cos\theta^*}\frac{d\cos\theta^*}{dp_T}$$

$$p_T = p^*\sin\theta^* = \frac{M_W}{2}\sin\theta^*$$

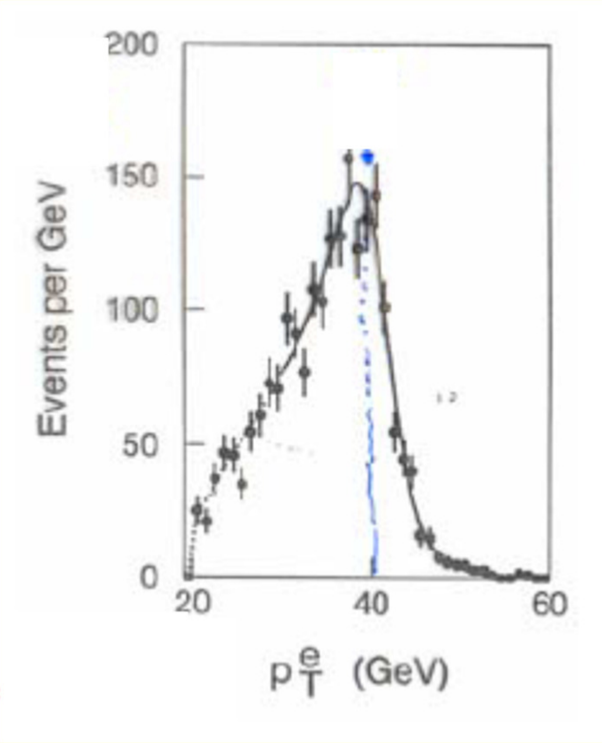
$$\to \sin\theta^* = \frac{2p_T}{M_W}$$

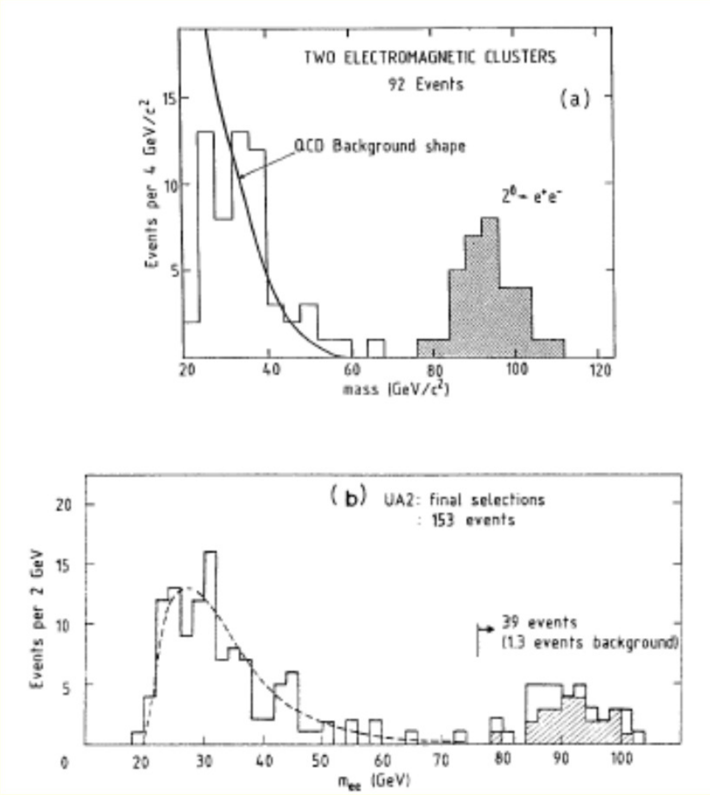
$$\to \cos\theta^* = \sqrt{1-\sin^2\theta^*} = \sqrt{1-\left(\frac{2p_T}{M_W}\right)^2}$$

$$\to \frac{d\cos\theta^*}{dp_T} = \frac{\frac{4p_T}{M_W}}{2\sqrt{1-\left(\frac{2p_T}{M_W}\right)^2}} = \frac{2p_T}{M_W\sqrt{1-\left(\frac{2p_T}{M_W}\right)^2}}$$

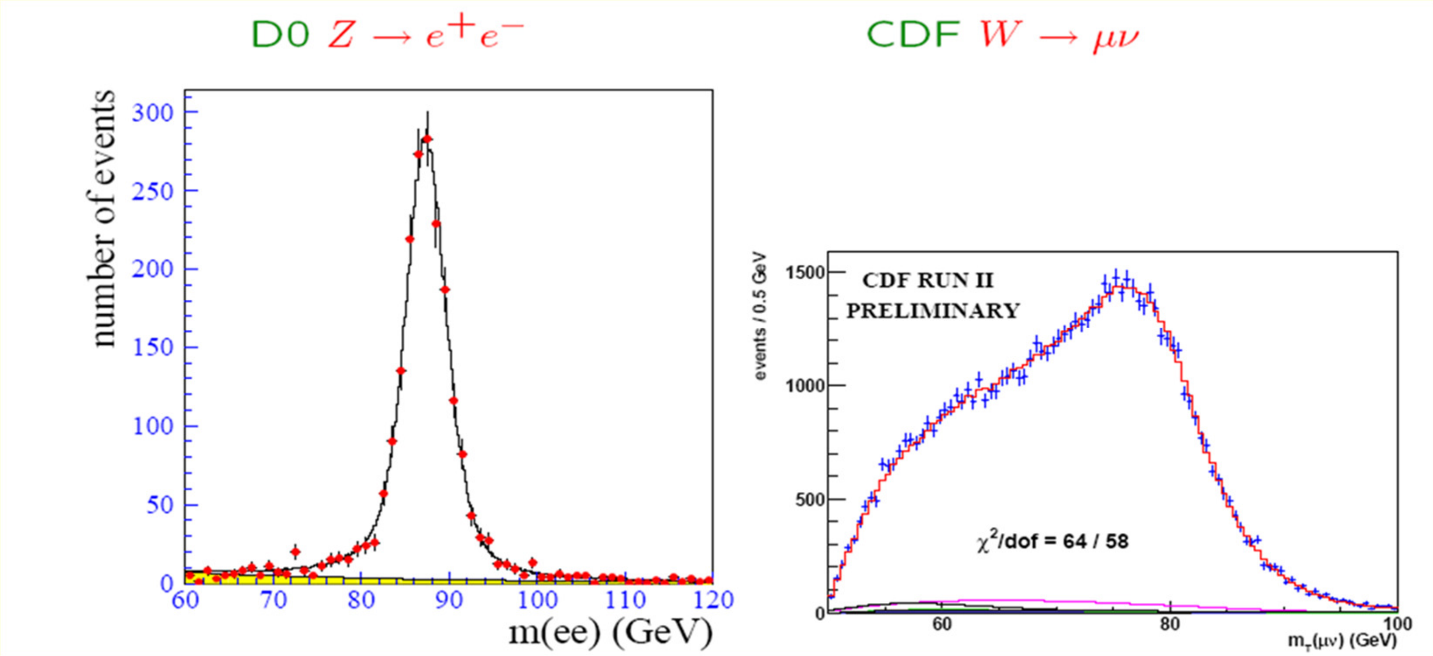
$$\to \frac{d\sigma}{dp_T} = A\left(\cos\theta^*\right)\frac{d\cos\theta^*}{dp_T} \approx K\frac{2p_T}{M_W\sqrt{1-\left(\frac{2p_T}{M_W}\right)^2}} \quad \text{Jacobian peak at } \frac{M_W}{2}$$

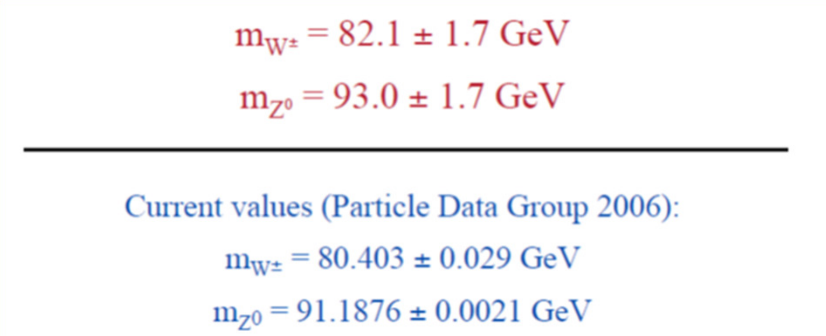
W & Z Discovery - XI





W & Z Discovery - XII





$m_{W^\pm} = 82.1 \pm 1.7$ GeV

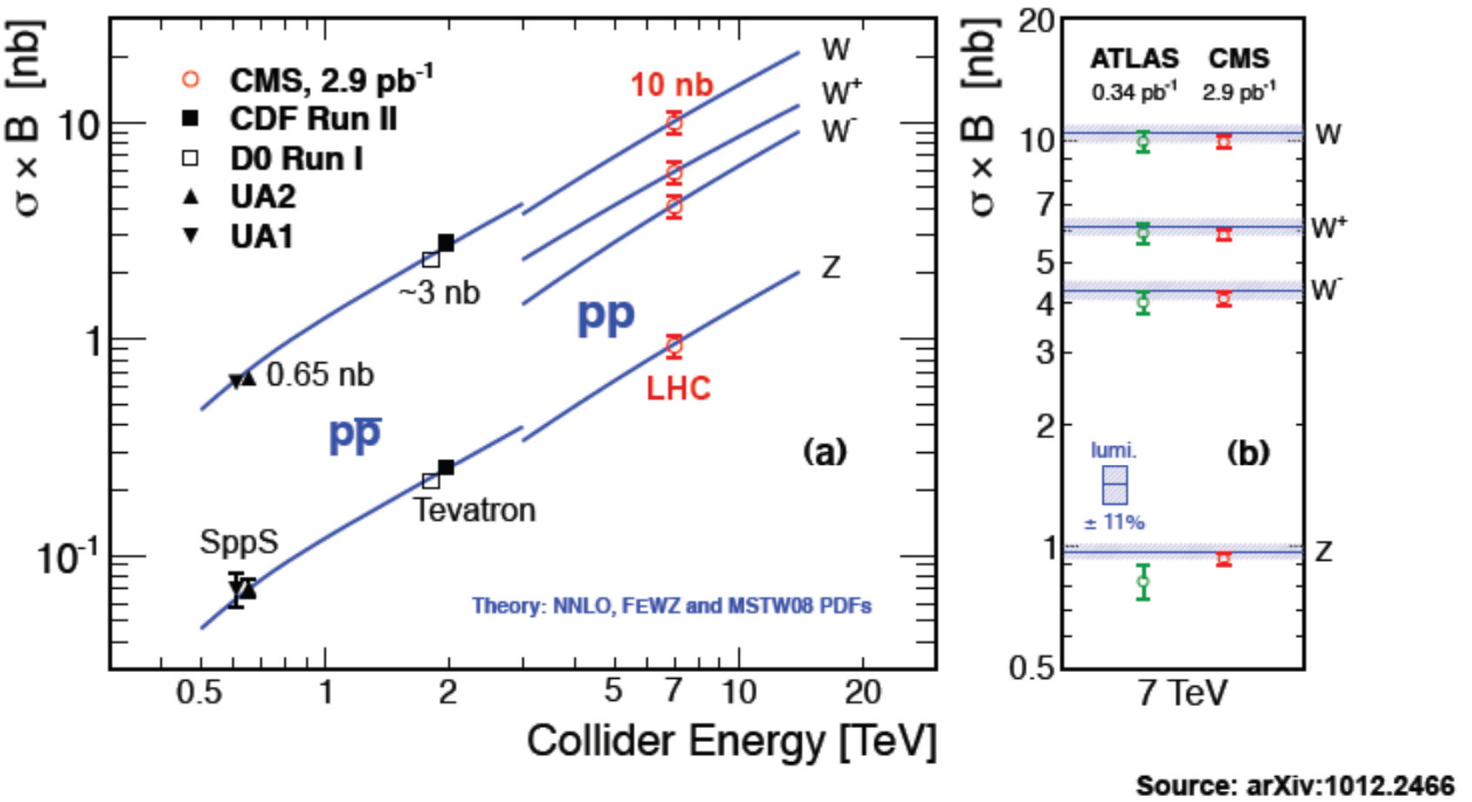
$m_{Z^0} = 93.0 \pm 1.7$ GeV

Current values (Particle Data Group 2006):

$m_{W^\pm} = 80.403 \pm 0.029$ GeV

$m_{Z0} = 91.1876 \pm 0.0021$ GeV

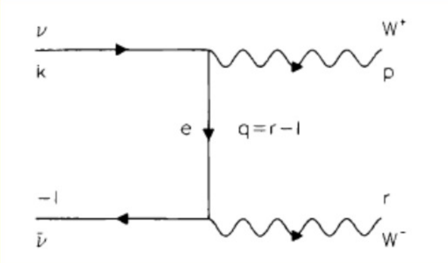
W & Z Discovery - XIII



Source: arXiv:1012.2466

- Z^0 cross section ~10 times smaller than $W^{\pm}$ boson production
- W^+ cross section ~43% larger than W^- at LHC (pp collider!)

SM Internal Consistency - I



Reconsidering hypothetical, troublesome reaction

$\nu\bar{\nu} \to W_L^+ W_L^-$

at very high energy

Polarization 4-vectors of longitudinally polarized *W*s:

$$\varepsilon_L^\mu(p) = \frac{p^\mu}{m_W} + O\left(\frac{m_W}{p^0}\right) \sim \frac{p^\mu}{m_W}$$

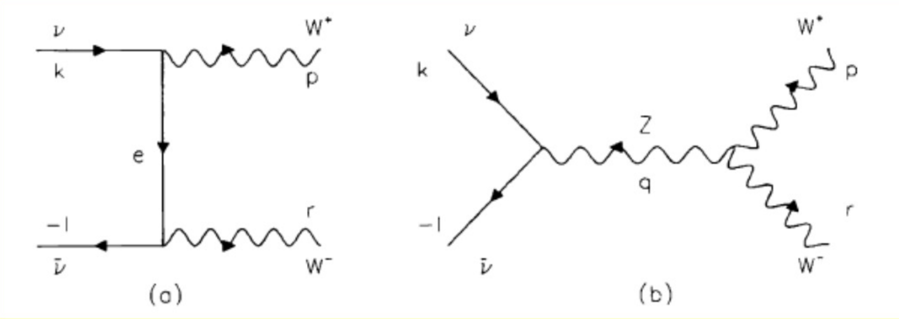
Divergent term of matrix element:

$$M_{fi} \approx -\frac{g^2}{8}\bar{v}(l)(1-\gamma_5)\frac{1}{\not{q}-m}(1-\gamma_5)u(k)\frac{r^\mu}{m_W}\frac{p^\nu}{m_W}$$

$$M_{fi} \approx -\frac{g^2}{4m_W^2}\bar{v}(l)\not{p}(1-\gamma_5)u(k) - \frac{g^2}{8m_W^2}m\bar{v}(l)(1+\gamma_5)\frac{\not{q}+m}{q^2-m^2}\not{p}(1-\gamma_5)u(k)$$

$$M_{fi}^a \approx -\frac{g^2}{4m_W^2}\bar{v}(l)\not{p}(1-\gamma_5)u(k)$$

SM Internal Consistency - II



Standard Model: Neutral Current → Two diagrams instead of one

$M_{fi}^{b} : Z^{0}$ matrix element:

$\nu\nu Z, WWZ$ vertexes, Z propagator

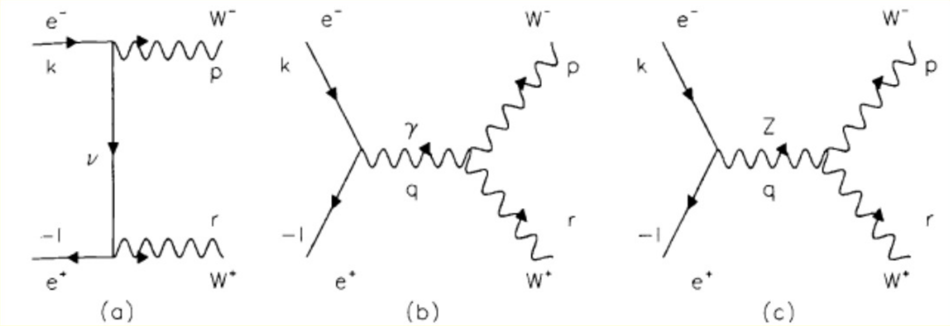
After quite intense calculations....

$$M_{fi}^{b} \approx \frac{g^{2}}{4m_{W}^{2}} \overline{v}(l) \not{p} (1-\gamma_{5}) u(k)$$

$$\rightarrow M_{fi}^{b} + M_{fi}^{b} = 0$$

Divergence fixed in a gauge theory!

SM Internal Consistency - III



Another, similar reaction

$e^+e^- \to W_L^+W_L^-$

Quite realistic!

$$M_{fi}^a \approx -\frac{g^2}{4m_W^2}\bar{v}(l)\not{p}(1-\gamma_5)u(k)$$

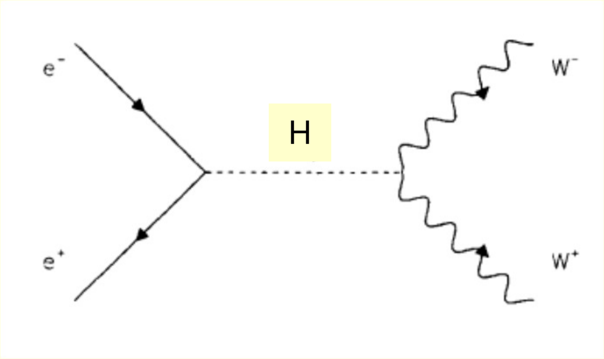
$$M_{fi}^b \approx \frac{e^2}{m_W^2}\bar{v}(l)\not{p}u(k)$$

$$M_{fi}^c \approx -\frac{g_{WWZ}}{2m_W^2}\bar{v}(l)\left[g_L\not{p}(1-\gamma_5)+g_R\not{p}(1+\gamma_5)\right]u(k)$$

$\to M_{fi}^a + M_{fi}^b + M_{fi}^c \approx --\frac{g^2}{4m_W^2}m\bar{v}(l)u(k)$ Still (weakly) divergent at high energy

SM Internal Consistency - IV

Reason of extra divergence: *R* chiral parts of massive fermions



Higgs diagram:

$$M_{fi}^{H} \approx -\frac{1}{2m_W^2} g_{eeH} g_{WWH} \overline{v}(l) u(k)$$

→ Correct compensation with gauge theory & SSB

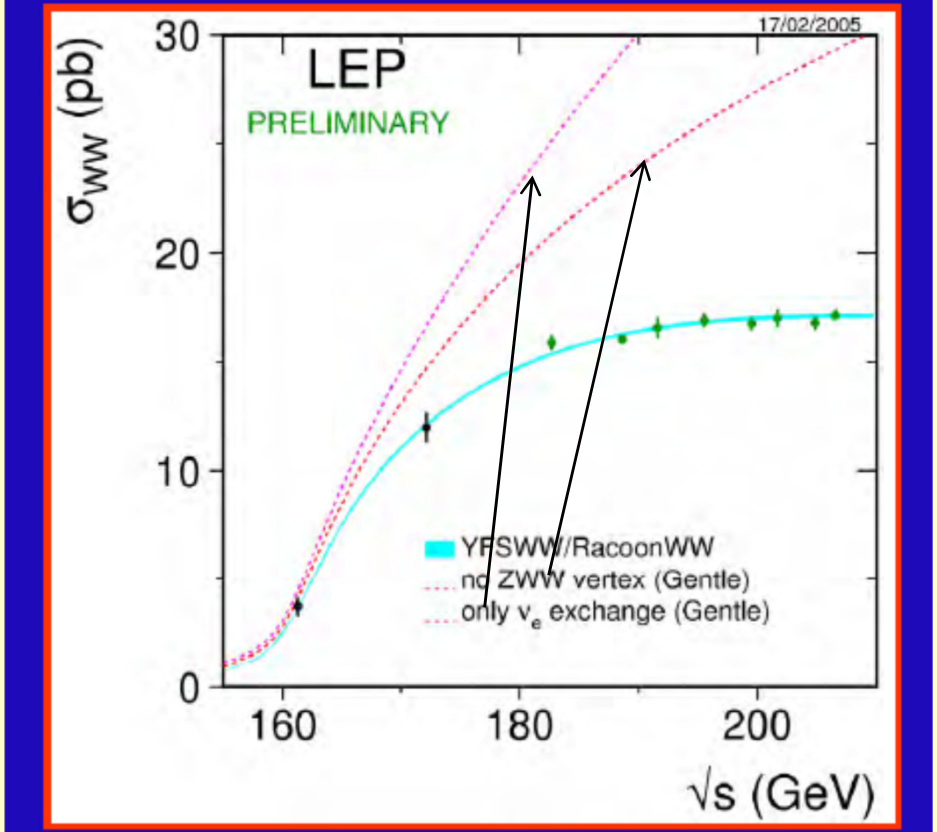
Strong support for the Standard Model:

Higgs *must* be there

(or something really new must happen at $\sim$ 1 TeV to save unitarity)

SM Internal Consistency - V

$$e^+ e^- \rightarrow W^+ W^-$$



Precision Tests - I

LEP – Precision tests of SM 1989-2000

$1989-1995$

$\sqrt{s} = 91.2\ GeV$

$17\ 10^6\ Z^0$ detected

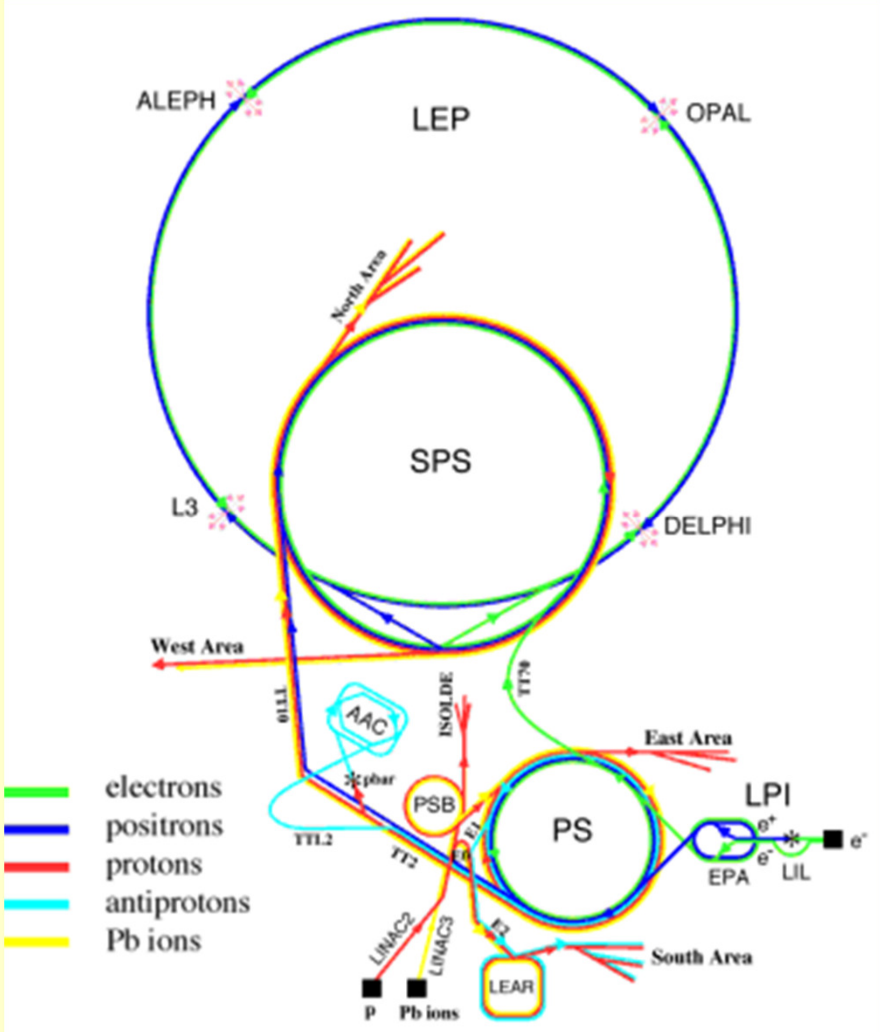
$1996-2000$

$\sqrt{s} = 161-208\ GeV$

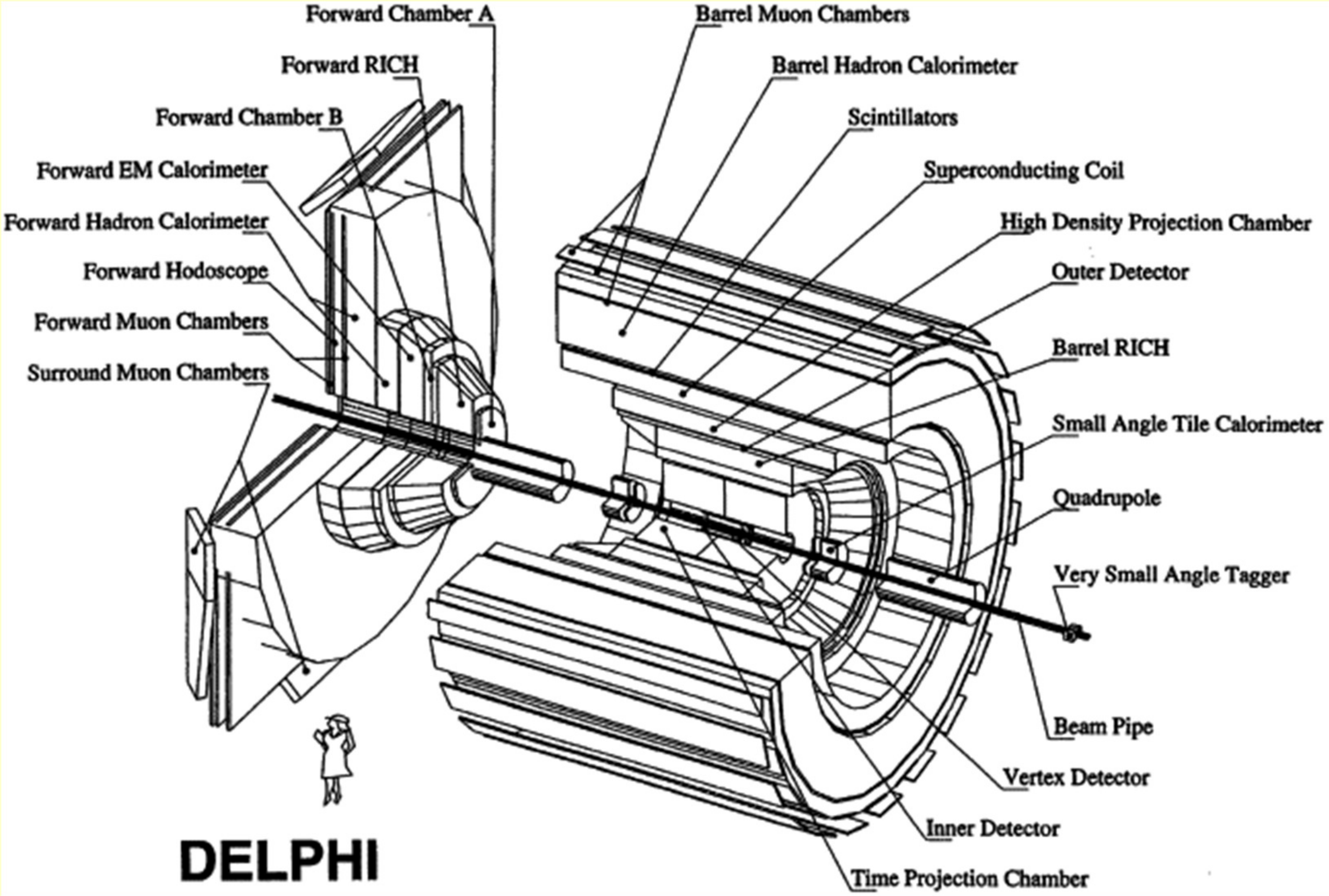
$30\ 10^3\ \ WW$ detected

26 km circumference
4 large experiments: ALEPH, DELPHI, L3, OPAL

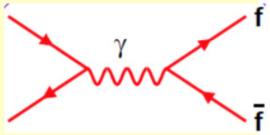
Precision Tests - II

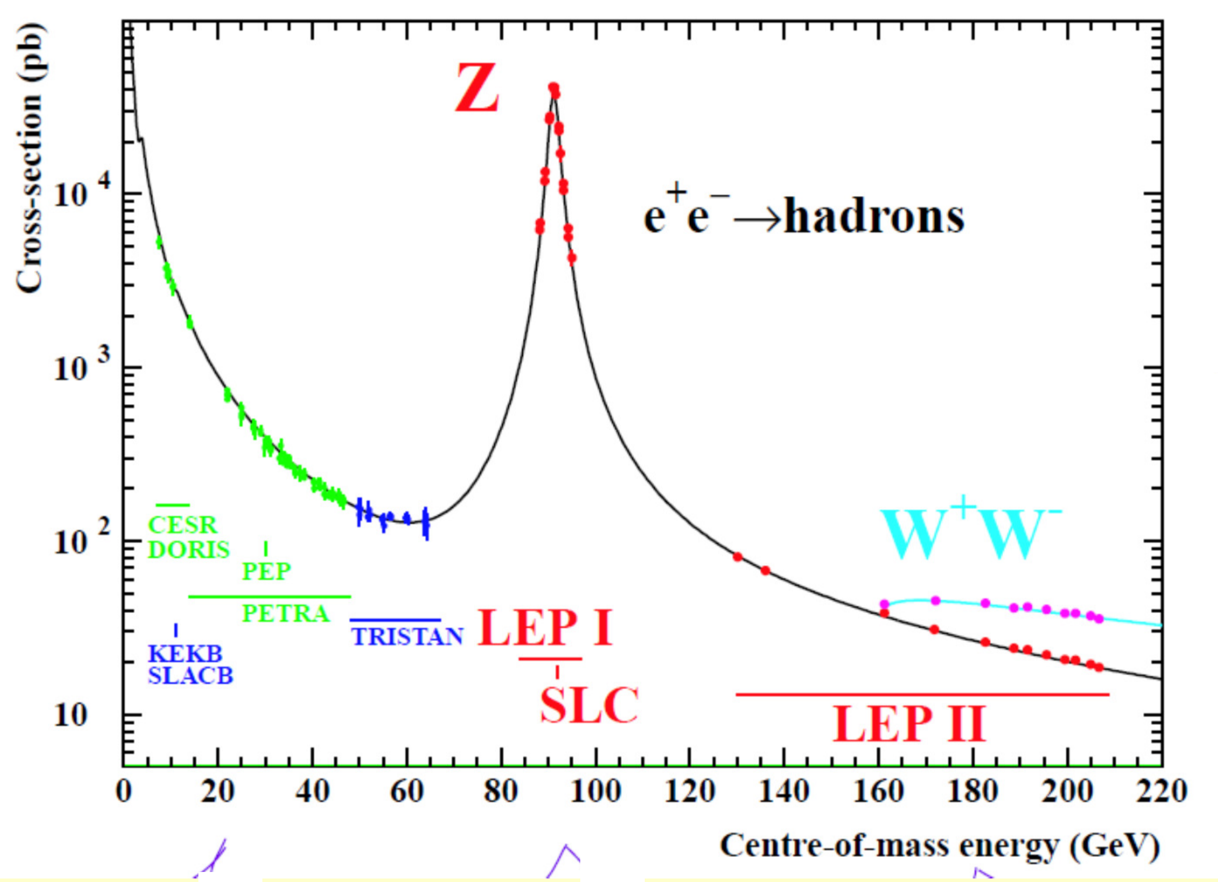


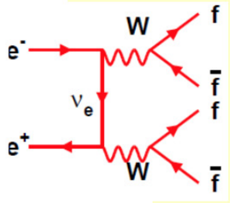
Precision Tests - III

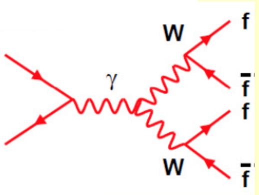


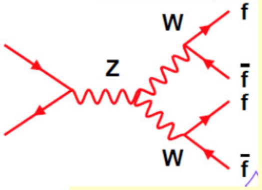
Precision Tests - IV

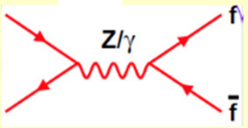


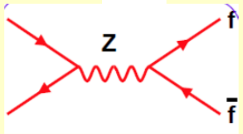


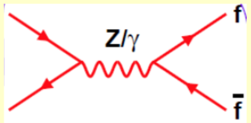








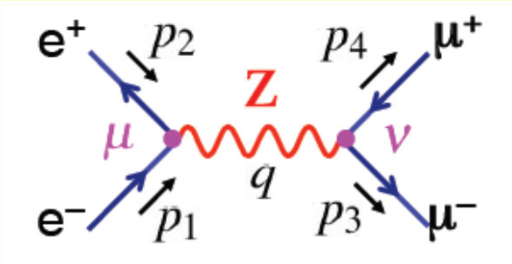




Precision Tests - V

$e^+e^- \to \mu^+\mu^-$ at the Z peak

Only consider (dominant) Z diagram



Electron vertex:

$$\bar{v}(p_2)\left(-ig_Z\gamma^\mu\right)\frac{1}{2}\left(c_V - c_A\gamma^5\right)u(p_1) \quad \text{Electron } c_V, c_A$$

Z propagator :

$$-i\frac{g_{\mu\nu}}{q^2 - m_Z^2} \quad \text{Approximate, see later}$$

Muon vertex:

$$\bar{u}(p_3)\left(-ig_Z\gamma^\nu\right)\frac{1}{2}\left(c_V - c_A\gamma^5\right)v(p_4) \quad \text{Muon } c_V, c_A$$

Precision Tests- VI

Ultrarelativistic limit → Chirality ≃ Helicity

→ Use helicity eigenstates for electron, muon vertexes

$$c_L = c_V + c_A, c_R = c_V - c_A$$

$$\rightarrow c_V = \frac{1}{2}\left(c_L + c_R\right), c_A = \frac{1}{2}\left(c_L - c_R\right)$$

$$\frac{1}{2}\left(c_V - c_A\gamma^5\right) \rightarrow \frac{1}{2}c_L\left(1-\gamma^5\right) + \frac{1}{2}c_R\left(1+\gamma^5\right)$$

→ Matrix element:

$$-\frac{g_Z}{q^2 - m_Z^2} g_{\mu\nu}$$

$$\times\left[c_L\bar{v}\left(p_2\right)\gamma^\mu \frac{1}{2}\left(1-\gamma^5\right)u\left(p_1\right) + c_R\bar{v}\left(p_2\right)\gamma^\mu \frac{1}{2}\left(1+\gamma^5\right)u\left(p_1\right)\right]$$

$$\times\left[c_L\bar{u}\left(p_3\right)\gamma^\mu \frac{1}{2}\left(1-\gamma^5\right)v\left(p_4\right) + c_R\bar{u}\left(p_3\right)\gamma^\mu \frac{1}{2}\left(1+\gamma^5\right)v\left(p_4\right)\right]$$

Introduce chirality ≃ helicity projectors:

$$\frac{1}{2}\left(1-\gamma^5\right)u \simeq u_\downarrow, \frac{1}{2}\left(1+\gamma^5\right)u \simeq u_\uparrow, \frac{1}{2}\left(1-\gamma^5\right)v \simeq v_\uparrow, \frac{1}{2}\left(1+\gamma^5\right)v \simeq v_\downarrow$$

Precision Tests- VII

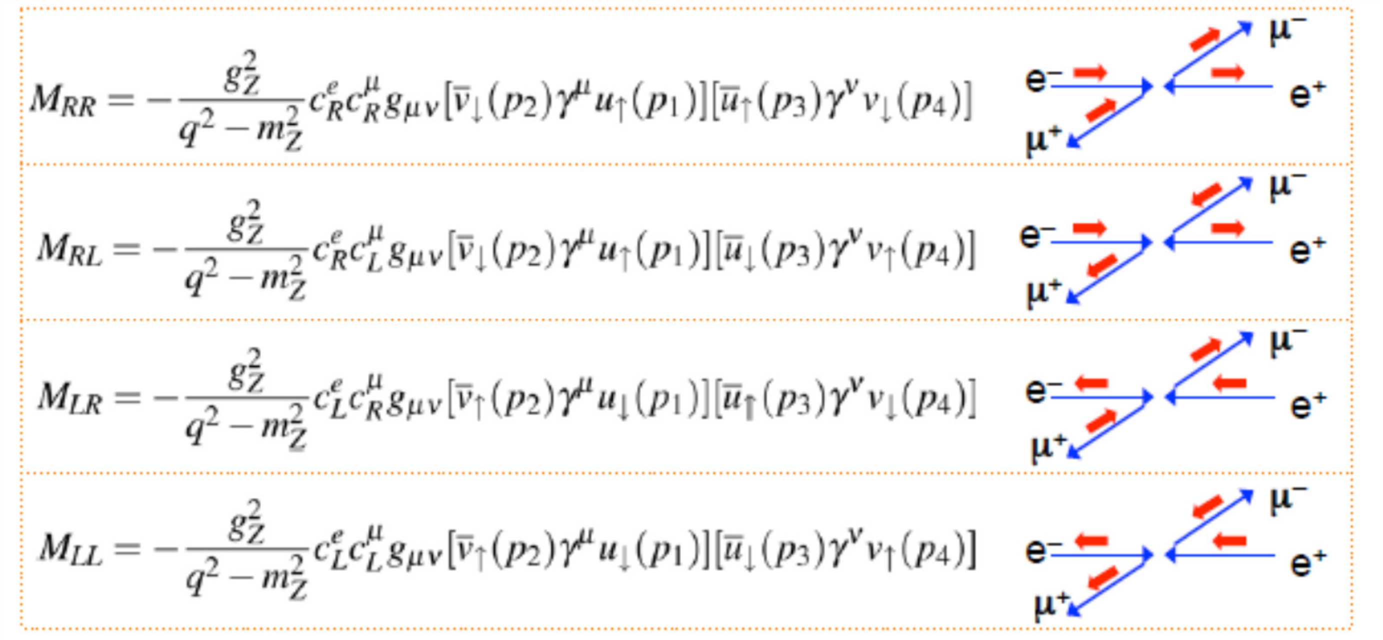
→ Matrix element:

$$-\frac{g_Z}{q^2-m_Z^2}g_{\mu\nu}\left[c_L\bar{v}(p_2)\gamma^\mu u_\downarrow(p_1)+c_R\bar{v}(p_2)\gamma^\mu u_\uparrow(p_1)\right]\left[c_L\bar{u}(p_3)\gamma^\nu v_\uparrow(p_4)+c_R\bar{u}(p_3)\gamma^\nu v_\downarrow(p_4)\right]$$

$$\bar{v}(p_2)=\bar{v}_\uparrow(p_2)+\bar{v}_\downarrow(p_2),\bar{u}(p_3)=\bar{u}_\uparrow(p_3)+\bar{u}_\downarrow(p_3)$$

Surviving terms in both e,μ currents: LR, RL only

$$\to -\frac{g_Z}{q^2-m_Z^2}g_{\mu\nu}\left[c_L\bar{v}_\uparrow(p_2)\gamma^\mu u_\downarrow(p_1)+c_R\bar{v}_\downarrow(p_2)\gamma^\mu u_\uparrow(p_1)\right]\left[c_L\bar{u}_\downarrow(p_3)\gamma^\nu v_\uparrow(p_4)+c_R\bar{u}_\uparrow(p_3)\gamma^\nu v_\downarrow(p_4)\right]$$



$$M_{RR}=-\frac{g_Z^2}{q^2-m_Z^2}c_R^e c_R^\mu g_{\mu\nu}[\bar{v}_\downarrow(p_2)\gamma^\mu u_\uparrow(p_1)][\bar{u}_\uparrow(p_3)\gamma^\nu v_\downarrow(p_4)]$$

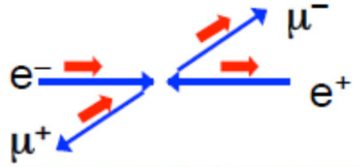
$$M_{RL}=-\frac{g_Z^2}{q^2-m_Z^2}c_R^e c_L^\mu g_{\mu\nu}[\bar{v}_\downarrow(p_2)\gamma^\mu u_\uparrow(p_1)][\bar{u}_\downarrow(p_3)\gamma^\nu v_\uparrow(p_4)]$$

$$M_{LR}=-\frac{g_Z^2}{q^2-m_Z^2}c_L^e c_R^\mu g_{\mu\nu}[\bar{v}_\uparrow(p_2)\gamma^\mu u_\downarrow(p_1)][\bar{u}_\uparrow(p_3)\gamma^\nu v_\downarrow(p_4)]$$

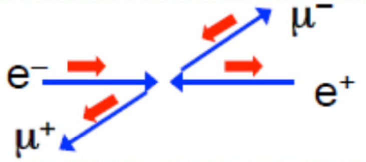
$$M_{LL}=-\frac{g_Z^2}{q^2-m_Z^2}c_L^e c_L^\mu g_{\mu\nu}[\bar{v}_\uparrow(p_2)\gamma^\mu u_\downarrow(p_1)][\bar{u}_\downarrow(p_3)\gamma^\nu v_\uparrow(p_4)]$$

Precision Tests- VIII

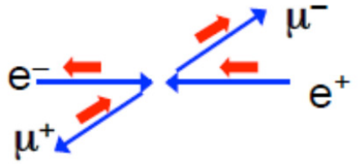
$$M_{RR} = -\frac{g_Z^2}{q^2 - m_Z^2} c_R^e c_R^\mu g_{\mu\nu} [\bar{v}_\downarrow(p_2)\gamma^\mu u_\uparrow(p_1)][\bar{u}_\uparrow(p_3)\gamma^\nu v_\downarrow(p_4)]$$



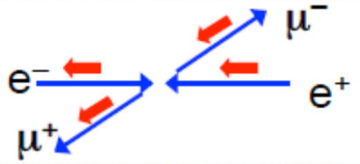
$$M_{RL} = -\frac{g_Z^2}{q^2 - m_Z^2} c_R^e c_L^\mu g_{\mu\nu} [\bar{v}_\downarrow(p_2)\gamma^\mu u_\uparrow(p_1)][\bar{u}_\downarrow(p_3)\gamma^\nu v_\uparrow(p_4)]$$



$$M_{LR} = -\frac{g_Z^2}{q^2 - m_Z^2} c_L^e c_R^\mu g_{\mu\nu} [\bar{v}_\uparrow(p_2)\gamma^\mu u_\downarrow(p_1)][\bar{u}_\uparrow(p_3)\gamma^\nu v_\downarrow(p_4)]$$



$$M_{LL} = -\frac{g_Z^2}{q^2 - m_Z^2} c_L^e c_L^\mu g_{\mu\nu} [\bar{v}_\uparrow(p_2)\gamma^\mu u_\downarrow(p_1)][\bar{u}_\downarrow(p_3)\gamma^\nu v_\uparrow(p_4)]$$



Precision Tests- IX

Almost ‘Cut & Paste’ from QED case:

$$|M_{RR}|^2 = s^2 \left|\frac{g_Z^2}{s-m_Z^2}\right|^2 (c_R^e)^2 (c_R^\mu)^2 (1+\cos\theta)^2$$

$$|M_{RL}|^2 = s^2 \left|\frac{g_Z^2}{s-m_Z^2}\right|^2 (c_R^e)^2 (c_L^\mu)^2 (1-\cos\theta)^2$$

$$|M_{LR}|^2 = s^2 \left|\frac{g_Z^2}{s-m_Z^2}\right|^2 (c_L^e)^2 (c_R^\mu)^2 (1-\cos\theta)^2$$

$$|M_{LR}|^2 = s^2 \left|\frac{g_Z^2}{s-m_Z^2}\right|^2 (c_L^e)^2 (c_L^\mu)^2 (1+\cos\theta)^2$$

Precision Tests- X

Now take correct Z propagator: Z *unstable*

$$-i\frac{g_{\mu\nu}}{q^2-m_Z^2}=-i\frac{g_{\mu\nu}}{s-m_Z^2}\rightarrow -i\frac{g_{\mu\nu}}{s-m_Z^2+im_Z\Gamma_Z}$$

$$\rightarrow\left|-i\frac{1}{s-m_Z^2+im_Z\Gamma_Z}\right|^2=\frac{1}{\left(s-m_Z^2\right)^2+m_Z^2\Gamma_Z^2}$$

$$\frac{d\sigma}{d\Omega}=\frac{\left|M_{fi}\right|^2}{64\pi^2 s}$$

$\rightarrow$ Differential cross-section for the 4 combinations:

$$\frac{\mathrm{d}\sigma_{RR}}{\mathrm{d}\Omega}=\frac{1}{64\pi^2}\frac{g_Z^4 s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}(c_R^e)^2(c_R^\mu)^2(1+\cos\theta)^2$$

$$\frac{\mathrm{d}\sigma_{LL}}{\mathrm{d}\Omega}=\frac{1}{64\pi^2}\frac{g_Z^4 s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}(c_L^e)^2(c_L^\mu)^2(1+\cos\theta)^2$$

$$\frac{\mathrm{d}\sigma_{LR}}{\mathrm{d}\Omega}=\frac{1}{64\pi^2}\frac{g_Z^4 s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}(c_L^e)^2(c_R^\mu)^2(1-\cos\theta)^2$$

$$\frac{\mathrm{d}\sigma_{RL}}{\mathrm{d}\Omega}=\frac{1}{64\pi^2}\frac{g_Z^4 s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}(c_L^e)^2(c_R^\mu)^2(1-\cos\theta)^2$$

Precision Tests- XI

Most interesting difference wrt *QED* case:

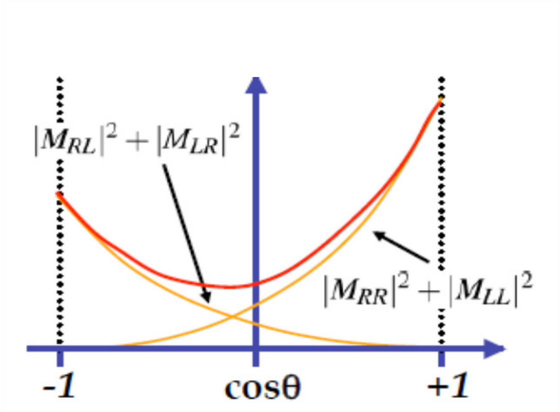
$$|M_{LL}|^2 + |M_{RR}|^2 \neq |M_{LR}|^2 + |M_{RL}|^2$$

Unpolarized cross section: Average & Sum over spins

$$\rightarrow \frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2}\frac{1}{4}\frac{g_Z^4 s}{\left(s-m_Z^2\right)^2 + m_Z^2\Gamma_Z^2}\frac{1}{4}\left(c_V^2 + c_A^2\right)_e \left(c_V^2 + c_A^2\right)_\mu \left(1+\cos^2\theta\right) + 2\left(c_V c_A\right)_e \left(c_V c_A\right)_\mu \cos\theta$$

Sizeable forward-backward asymmetry!

Neutral current parity violation



Precision Tests - XII

Integrate over solid angle, get total cross section:

$$\sigma_{e^+e^-\to Z\to\mu^+\mu^-} = \frac{1}{192\pi}\frac{g_Z^4 s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}[(c_V^e)^2+(c_A^e)^2][(c_V^\mu)^2+(c_A^\mu)^2]$$

Recall partial Z widths:

$$\Gamma(Z\to e^+e^-) = \frac{g_Z^2 m_Z}{48\pi}[(c_V^e)^2+(c_A^e)^2] \quad \text{ar} \quad \Gamma(Z\to\mu^+\mu^-) = \frac{g_Z^2 m_Z}{48\pi}[(c_V^\mu)^2+(c_A^\mu)^2]$$

$$\sigma = \frac{12\pi}{m_Z^2}\frac{s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}\Gamma(Z\to e^+e^-)\Gamma(Z\to\mu^+\mu^-)$$

$$\to \sigma_{peak} \simeq \frac{12\pi\left(BR\right)^2}{m_Z^2} \approx \frac{37.7\left(3.510^{-2}\right)^2}{\left(91.2\right)^2} \approx 55\ 10^{-7}GeV^{-2}$$

$$\left(\hbar c\right)^2 \simeq 0.389GeV^2mb$$

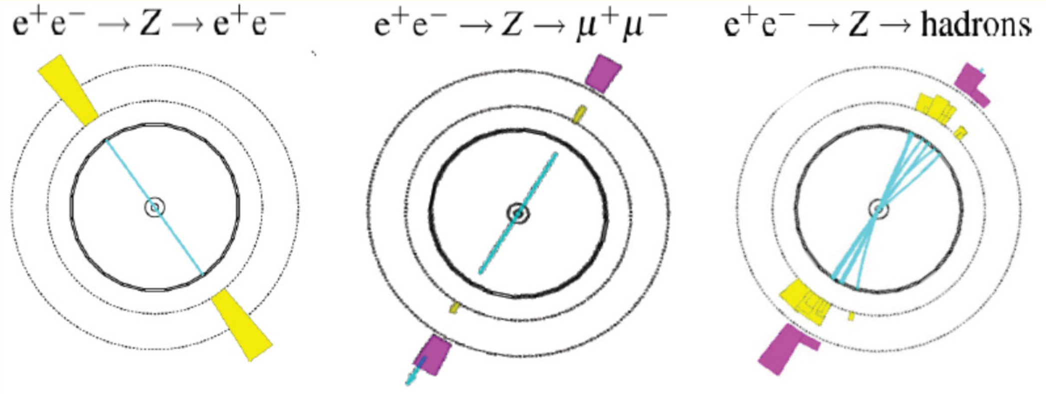
$$\to \sigma_{peak} \simeq 55\ 10^{-7}GeV^{-2}0.389GeV^2mb \approx 2.1410^{-6}mb = 2.14nb$$

Precision Tests - XIII

Z peak: Essentially 4 types of events

$e^+e^- \to e^+e^-, \mu^+\mu^-, \tau^+\tau^-, q\bar{q}\left(\to hadrons\right)$

Different topologies in the detectors:



Measuring cross sections:

Count events$\left(!\right)$

Subtract background

Correct for inefficiency

Get integrated luminosity $\left(\text{Most of the time from independent counting of Bhabha events}\right)$

$$\to \sigma = \frac{N - N_{bck}}{\varepsilon}\frac{1}{L_{\text{int}}}$$

Precision Tests - XIV

Among other results at the peak: Z lineshape

Meaning in practice:

m_Z	Z mass
Γ_Z	Z total width
Γ_f	Z partial width to fermion type f
N_ν	Number of (SM) neutrino species

Obtained by 'scanning' the Z^0 peak:

Move $E_{beam} = \frac{\sqrt{s}}{2}$ in steps through the peak

Measure relevant σ at each step

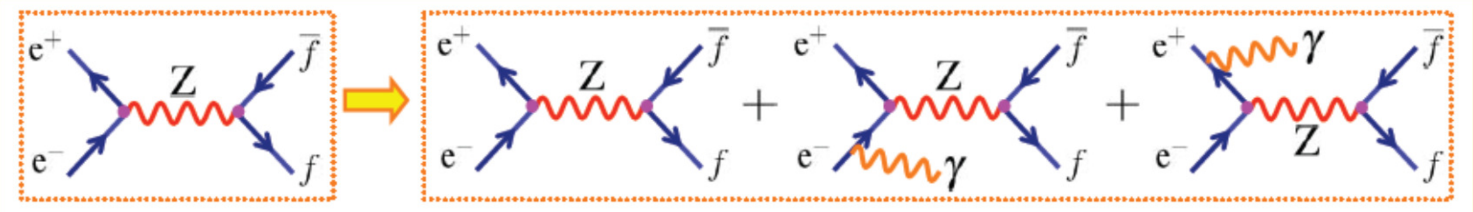
Fit profile:

$$\sigma\left(e^+e^- \to Z \to f\bar{f}\right) = \frac{12\pi}{m_Z^2} \frac{s\Gamma_{ee}\Gamma_{ff}}{\left(s - m_Z^2\right)^2 + m_Z^2\Gamma_Z^2}$$

Precision Tests - XV

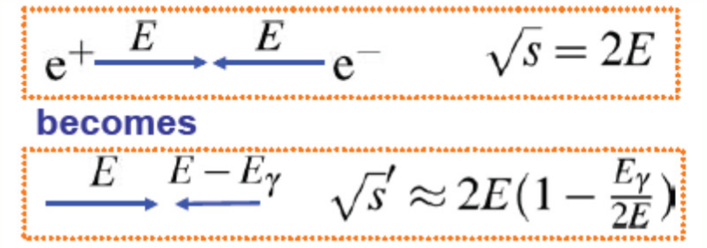
Lineshape quite distorted by several effects

Main effect: Initial State Radiation (*ISR*)



Result:

Collision *CM* energy $\neq 2E_{beam}$



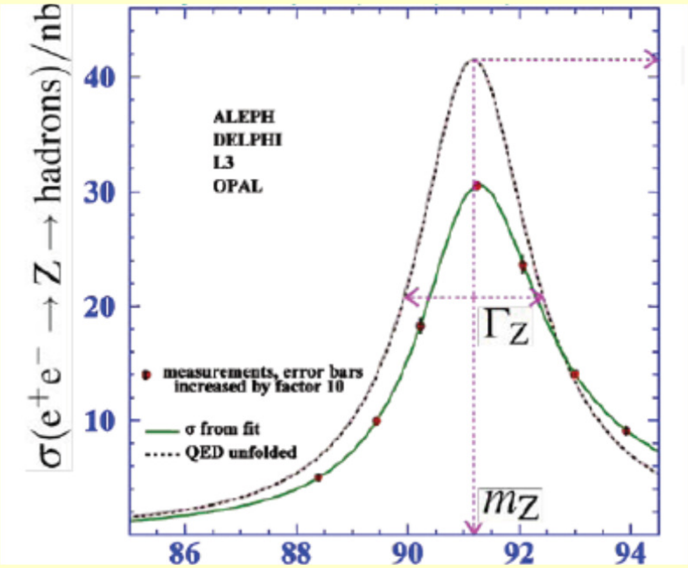
$$\sqrt{s} = 2E$$

becomes

$$\sqrt{s'} \approx 2E(1-\frac{E_\gamma}{2E})$$

$$\boxed{m_Z = 91.1875 \pm 0.0021\,\text{GeV}}$$

$$\boxed{\Gamma_Z = 2.4952 \pm 0.0023\,\text{GeV}}$$



Precision Tests - XVI

Finding the number of Standard Model neutrinos

(Meaning: With standard coupling to Z)

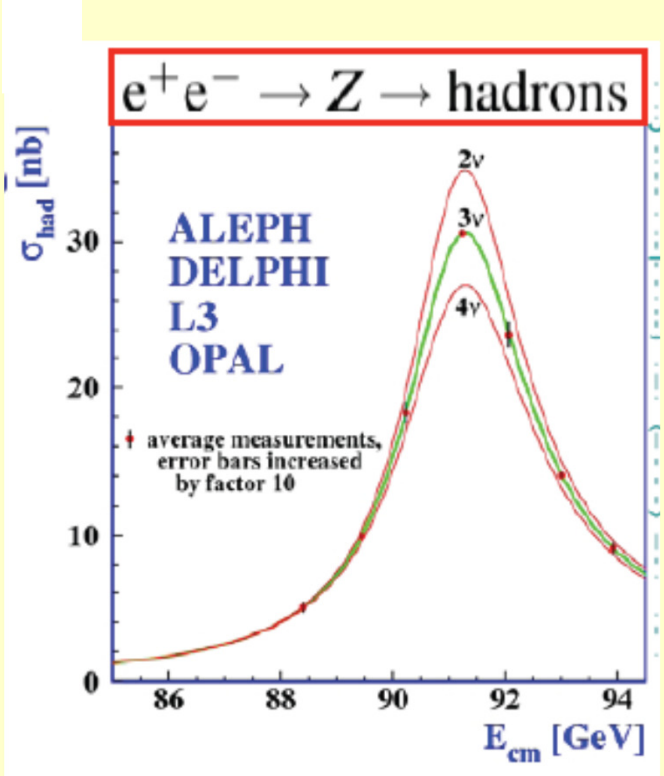
Total width:

$$\Gamma = \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + \Gamma_{had} + \Gamma_{\nu_1\nu_1} + \Gamma_{\nu_2\nu_2} + \Gamma_{\nu_3\nu_3} + ?$$

$$\Gamma = 3\Gamma_{ll} + \Gamma_{had} + N_\nu \Gamma_{\nu\nu}$$

Measure partial widths from peak cross sections:

$$\sigma_0^{ff} = \frac{12\pi}{m_Z^2}\frac{\Gamma_{ee}\Gamma_{ff}}{\Gamma_Z^2}$$



$$N_\nu = 2.9840 \pm 0.0082$$

Precision Tests - XVII

Write differential cross section $\left(\text{ e.g. for } e^+e^- \to \mu^+\mu^-\right)$ as:

$$\frac{d\sigma}{d\Omega} = k\left[A\left(1+\cos^2\theta\right)+B\cos\theta\right]$$

$$A = \left[\left(c_L^e\right)^2+\left(c_R^e\right)^2\right]\left[\left(c_L^\mu\right)^2+\left(c_R^\mu\right)^2\right], \quad B = \left[\left(c_L^e\right)^2-\left(c_R^e\right)^2\right]\left[\left(c_L^\mu\right)^2-\left(c_R^\mu\right)^2\right]$$

Forward/Backward cross sections:

$$\sigma_F = \int_0^1 \frac{d\sigma}{d\cos\theta} d\cos\theta, \quad \sigma_B = \int_{-1}^0 \frac{d\sigma}{d\cos\theta} d\cos\theta$$

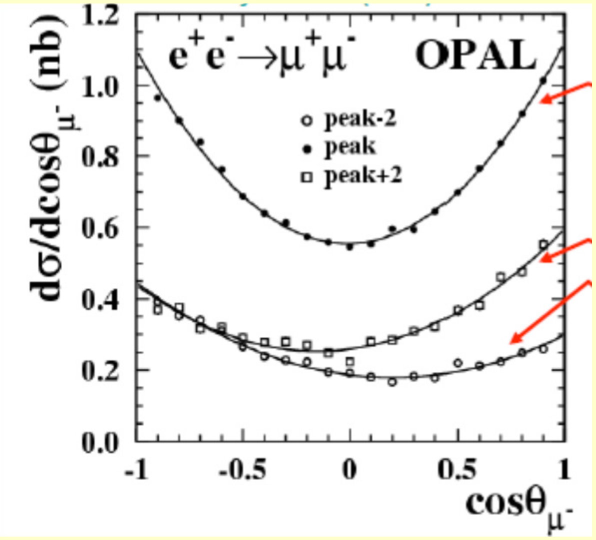
FB Asymmetry:

$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$

$$\sigma_F = k\left(\frac{4}{3}A+\frac{1}{2}B\right), \sigma_B = k\left(\frac{4}{3}A-\frac{1}{2}B\right)$$

$$\to A_{FB} = \frac{3}{8}\frac{B}{A} = \frac{3}{4}\frac{\left[\left(c_L^e\right)^2-\left(c_R^e\right)^2\right]}{\left[\left(c_L^e\right)^2+\left(c_R^e\right)^2\right]}\frac{\left[\left(c_L^\mu\right)^2-\left(c_R^\mu\right)^2\right]}{\left[\left(c_L^\mu\right)^2+\left(c_R^\mu\right)^2\right]} = \frac{3}{4}A_e A_\mu$$

Precision Tests - XVIII



$A_{FB}\left(peak\right) \sim 0$ for leptons ($\sin^2\theta_W \approx 0.25$)

$A_{FB}\left(peak \pm 2 \text{ GeV}\right) \neq 0$:

Interference with QED

$$A_{FB}^{0,e} = 0.0145 \pm 0.0025$$
$$A_{FB}^{0,\mu} = 0.0169 \pm 0.0013$$
$$A_{FB}^{0,\tau} = 0.0188 \pm 0.0017$$

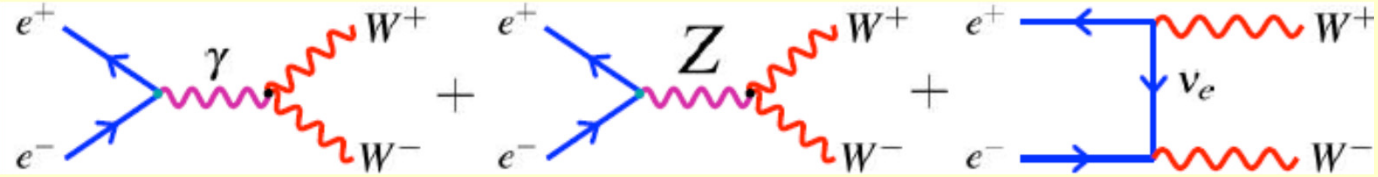
$$A_e = 0.1514 \pm 0.0019$$
$$A_\mu = 0.1456 \pm 0.0091$$
$$A_\tau = 0.1449 \pm 0.0040$$

$$= 2\frac{c_V/c_A}{1+(c_V/c_A)^2}$$

$$\sin^2\theta_W = 0.23154 \pm 0.00016$$

Precision Tests - XIX

LEP2: Study of *WW* production

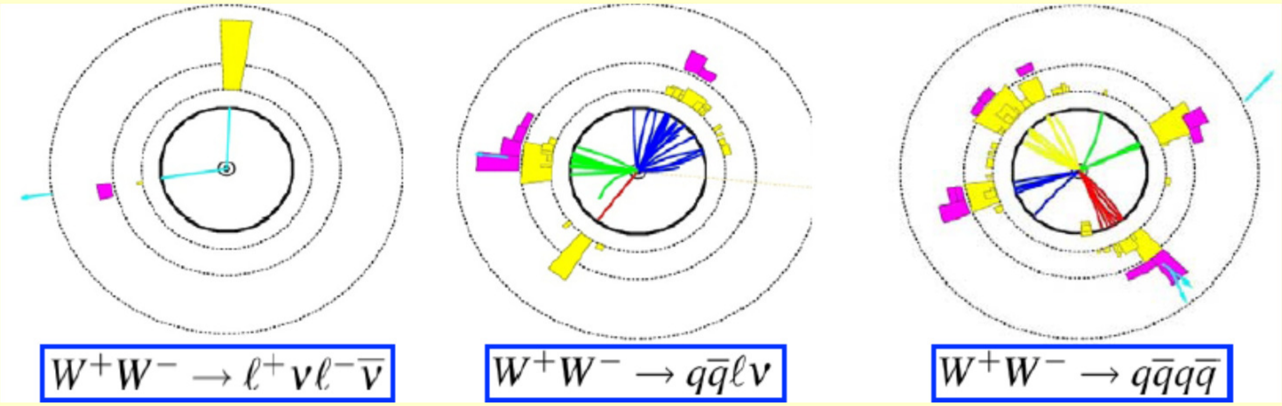


3 main cathegories of *WW* events:

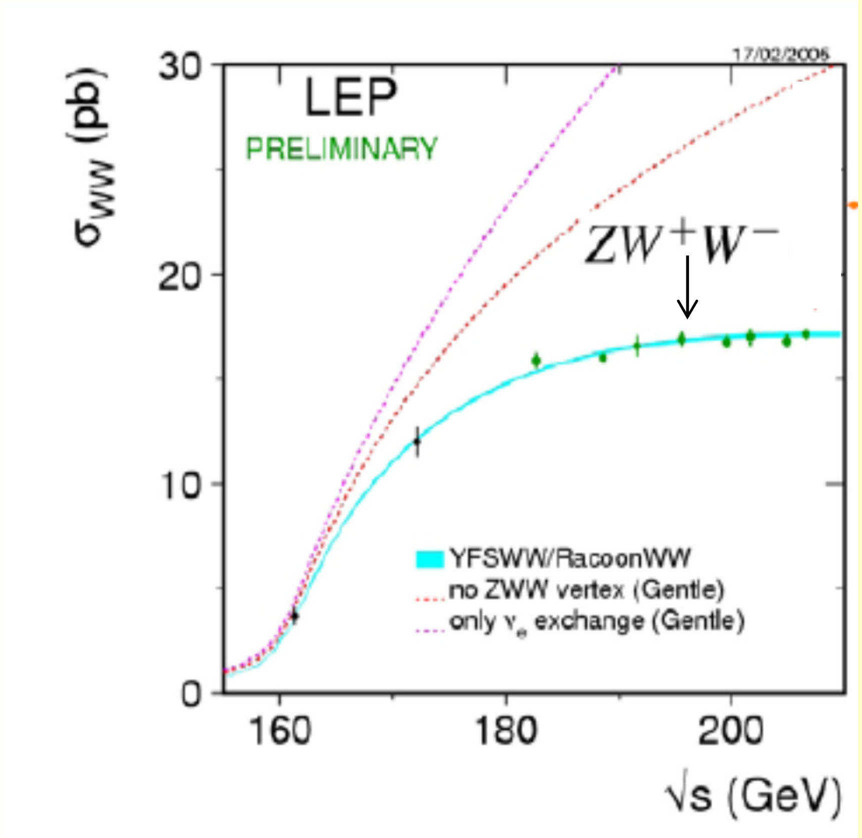
$W_1 \to l\nu, W_2 \to l\nu$

$W_1 \to l\nu, W_2 \to q\bar{q}$

$W_1 \to q\bar{q}, W_2 \to q\bar{q}$



Precision Tests - XX



Maybe one of the best results of the whole LEP saga

Precision Tests - XXI

Measurement of m_W : Kinematical fit

Example:

$$W^+W^- \to q\bar{q}e^-\bar{\nu}$$

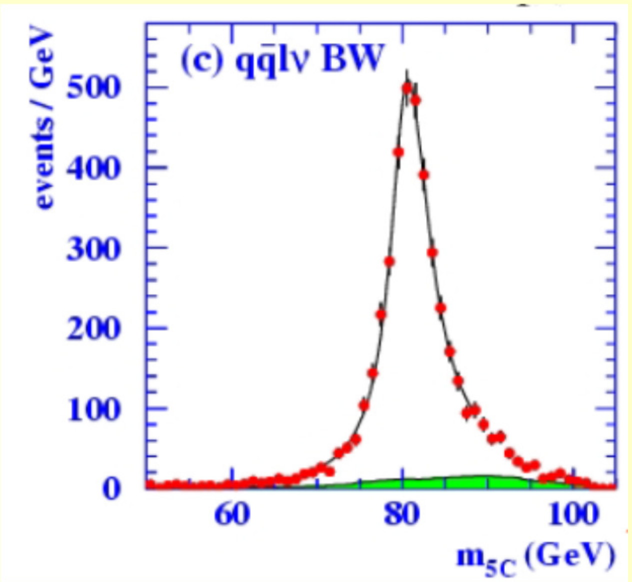
Get $\bar{\nu}$ 4-momentum from:

$$p_q + p_{\bar{q}} + p_{e^-} + p_{\bar{\nu}} = \left(\sqrt{s}, 0\right)$$

Make W bosons masses :

$$M_{W^+} = \left(p_q + p_{\bar{q}}\right)^2$$

$$M_{W^-} = \left(p_{e^-} + p_{\bar{\nu}}\right)^2$$



$$m_W = 80.376 \pm 0.033\,\mathrm{GeV}$$

$$\Gamma_W = 2.196 \pm 0.083\,\mathrm{GeV}$$

Precision Tests - XXII

Standard Model :

$$M_W = M_Z \cos\theta_W$$

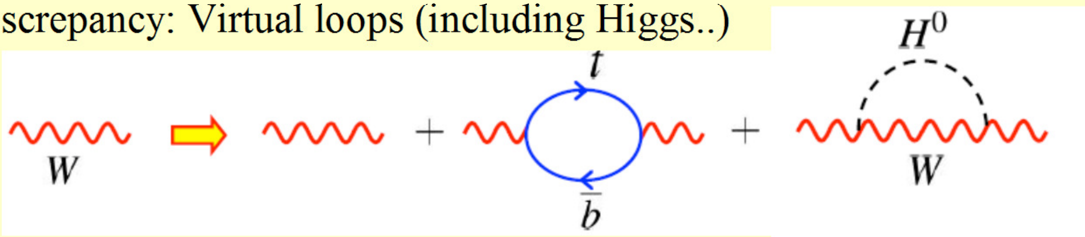
$$\text{Measure: } \begin{cases} M_Z = 91.1875 \pm 0.0021 \;\; GeV \\ \sin^2\theta_W = 0.23154 \pm 0.00016 \end{cases}$$

$\rightarrow$ Predict $M_W = 79.946 \pm 0.008 \;\; GeV$

Measure

$M_W = 80.376 \pm 0.033 \;\; GeV$

Discrepancy: Virtual loops (including Higgs..)



$$m_W = m_W^0 + a m_t^2 + b \ln\left(\frac{m_H}{m_W}\right)$$

Precision Tests - XXIII

Applied loopology:

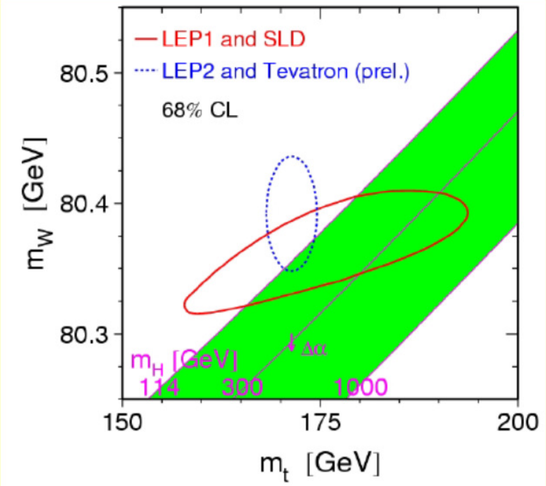
Predict: $m_t^{\text{loop}} = 173 \pm 11\,\text{GeV}$

!!!

Observe: $m_t^{\text{meas}} = 174.2 \pm 3.3\,\text{GeV}$

Can’t do the same for Higgs, loop effects *logarithmic* in m_H

Nevertheless:



$m_H < 200\,\text{GeV}$