Elementary Particles II

2 – Electroweak Interaction

Universal Current-Current Interaction, Intermediate Vector Bosons, Gauge Symmetry, Spontaneous Symmetry Breaking, Electroweak Unification, Neutral Currents, Discovery of W & Z, Precision Measurements, Higgs

Electroweak Interaction

Standard Model:
Electromagnetic and Weak Interaction unified into Electroweak

Energy scale where unification is evident:

$E \sim M_W, M_Z \sim 100$ GeV

At lower energies:

Can still find tiny traces of the unification (Electroweak interference, Parity violation in atomic processes,...)

Electroweak interaction split into two, almost not interfering, *effective* interactions:

↑

Electromagnetic

Non fundamental, useful low energy approximations

Weak

$V - A$ - I

After a long history of beta decay experiments: *Current-Current (Fermi) Interaction* including *Vector* & *Axial Vector* terms in order to account for P & C violation

$$H_{\text{int}} = \frac{G_F}{\sqrt{2}} \sum_{i=V,A} C_i \left(\overline{\psi}_p \Gamma_i \psi_n\right) \left(\overline{\psi}_e \Gamma^i \left(1 + \frac{C_i'}{C_i}\gamma^5\right)\psi_\nu\right)$$

Neutrino helicity = -1 yields lepton current = $V - A$

$$C_i' = -C_i \rightarrow \left(1 + \frac{C_i'}{C_i}\gamma^5\right)\psi_\nu = \left(1-\gamma^5\right)\psi_\nu \qquad = -\gamma^\mu\left(1-\gamma^5\right)$$

$$\rightarrow H_{\text{int}} = \frac{G_F}{\sqrt{2}}\left[C_V\left(\overline{\psi}_p \gamma_\mu \psi_n\right)\left(\overline{\psi}_e \gamma^\mu\left(1-\gamma^5\right)\psi_\nu\right) + C_A\left(\overline{\psi}_p \gamma_\mu \gamma_5 \psi_n\right)\left(\overline{\psi}_e \gamma^\mu \gamma^5\left(1-\gamma^5\right)\psi_\nu\right)\right]$$

$$= \frac{G_F}{\sqrt{2}}\left[C_V\left(\overline{\psi}_p \gamma_\mu \psi_n\right)\left(\overline{\psi}_e \gamma^\mu\left(1-\gamma^5\right)\psi_\nu\right) - C_A\left(\overline{\psi}_p \gamma_\mu \gamma_5 \psi_n\right)\left(\overline{\psi}_e \gamma^\mu\left(1-\gamma^5\right)\psi_\nu\right)\right]$$

$$= \frac{G_F}{\sqrt{2}}\left[C_V\left(\overline{\psi}_p \gamma_\mu \psi_n\right) - C_A\left(\overline{\psi}_p \gamma_\mu \gamma_5 \psi_n\right)\right]\left(\overline{\psi}_e \gamma^\mu\left(1-\gamma^5\right)\psi_\nu\right)$$

$V - A$ - II

Therefore:

$$H_{\text{int}} = \frac{G_F}{\sqrt{2}} C_V \left(\bar{\psi}_p \gamma_\mu \left(1 - \frac{C_A}{C_V} \gamma_5 \right) \psi_n \right) \left(\bar{\psi}_e \gamma^\mu \left(1 - \gamma^5 \right) \psi_\nu \right)$$

Many violations in weak processes :

Space Parity (large)

Charge Parity (large)

CP (very small)

T (very small)

Flavor conservation (Isospin, S, C, B, T) (larger + smaller)

Lepton numbers (Neutrino oscillations)

$V - A$ - III

Observe:

$$H_{\text{int}} = \frac{2G_F}{\sqrt{2}}\left(\bar{\psi}_p\gamma_\mu\left(1-\frac{C_A}{C_V}\gamma_5\right)\psi_n\right)\left(\bar{\psi}_e\gamma^\mu\frac{\left(1-\gamma^5\right)}{2}\psi_\nu\right)$$

$$\frac{1-\gamma^5}{2}\ \text{Projection operator} \rightarrow \left[\frac{\left(1-\gamma^5\right)}{2}\right]^2 = \frac{1-\gamma^5}{2}$$

$$\rightarrow H_{\text{int}} = \frac{2G_F}{\sqrt{2}}\left(\bar{\psi}_p\gamma_\mu\left(1-\frac{C_A}{C_V}\gamma_5\right)\psi_n\right)\left(\bar{\psi}_e\gamma^\mu\left[\frac{\left(1-\gamma^5\right)}{2}\right]^2\psi_\nu\right)$$

$$\rightarrow H_{\text{int}} = \sqrt{2}G_F\left(\bar{\psi}_p\gamma_\mu\left(1-\frac{C_A}{C_V}\gamma_5\right)\psi_n\right)\left(\bar{\psi}_e\left(\frac{1+\gamma^5}{2}\right)\gamma^\mu\left(\frac{1-\gamma^5}{2}\right)\psi_\nu\right)$$

Lepton current written as *pure vector* between *chiral parts* of ν,e states

$\rightarrow$ The weak charged current is just the same as the e.m. current, except it operates between chiral projections with different charge $\Delta Q = \pm 1$

Universality: Leptons

Extend *V-A* to muon weak interactions:

$\mu^+ \to e^+ + \overline{\nu}_\mu + \nu_e, \;\; \mu^- \to e^- + \nu_\mu + \overline{\nu}_e \quad \mu$ decay

$\mu^- + p \to n + \nu_\mu \quad \mu$ capture, involves nucleon current

μ decay purely leptonic:

Guess: *Current-Current*, *V-A* for both electron and muon charged currents

Lagrangian density:

$$L_{V-A} = -\frac{G_F}{\sqrt{2}}\left[\overline{\nu}\gamma_\mu\left(1-\gamma_5\right)e\right]\left[\overline{e}\gamma^\mu\left(1-\gamma_5\right)\nu\right] + H.C.$$

Compute:

μ *Lifetime*
Electron energy spectrum
Electron longitudinal polarization

Universal Weak Coupling

Measure μ lifetime, get Fermi constant:

$$G_F^{(\mu)} = \sqrt{\frac{192\pi^3}{\tau_\mu m_\mu^5}} = 1.1638\ 10^{-5}\quad GeV^{-2}$$

After careful (radiative) corrections:

$$G_F^{(\mu)} = 1.16637\ 10^{-5}\quad GeV^{-2}$$

Measure τ lifetime and BR to electron, get Fermi constant:

$$G_F^{(\tau)} = \sqrt{\frac{\Gamma\left(\tau \to e\overline{v}_e v_\tau\right)}{\Gamma_{tot}} \frac{192\pi^3}{\tau_\tau m_\tau^5}} = 1.1642\ 10^{-5}\quad GeV^{-2}$$

Compare to Fermi constant from β decay:

$$G_F^{(\beta)} = 1.1639\ 10^{-5}\quad GeV^{-2}$$

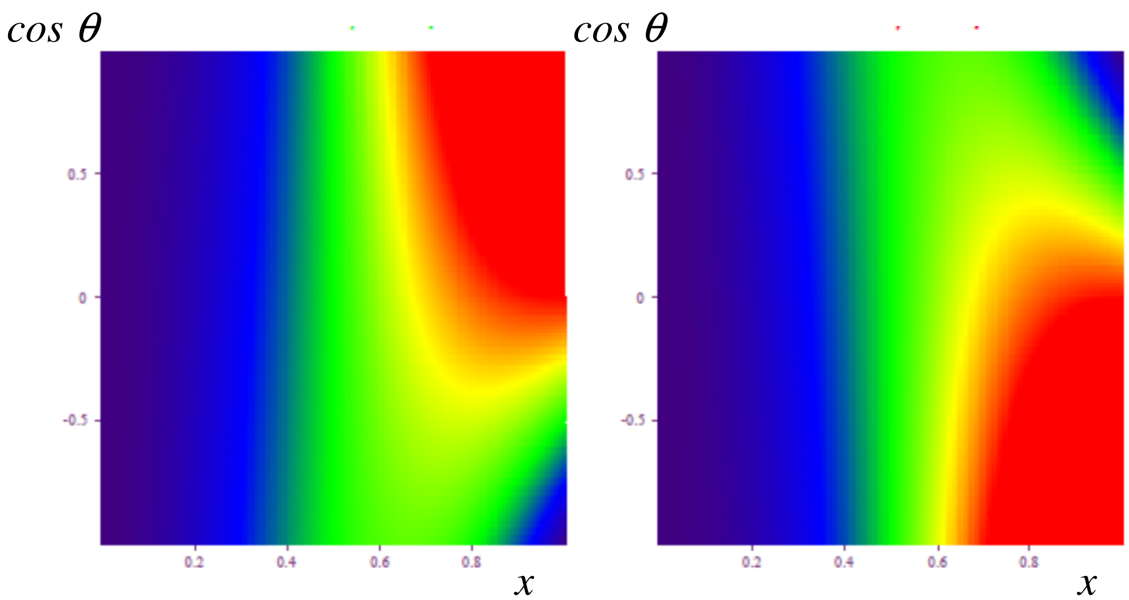
$\rightarrow$ Universal leptonic charged current!

μ Decay: C & P Violations

Polarized $\mu^{\pm}$ decay: μ rest frame

$$\frac{dN^{\pm}}{dx d\cos\theta} = x^2(3-2x)\left[1 \pm \cos\theta \frac{2x-1}{3-2x}\right]$$

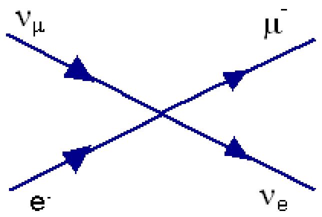
$$x = \frac{p_e}{p_e^{\max}}, \; \theta \sphericalangle \left(\mathbf{s}_\mu, \mathbf{p}_e\right)$$



Inverse Muon Decay - I

Charged current:

$$\nu_\mu + e^- \to \mu^- + \nu_e$$



$$\sum_{spin} T_{fi} T_{fi}^* = \sum_{spin} \frac{G_F}{\sqrt{2}} \left[\overline{u}(3) \gamma^\mu \left(1-\gamma^5\right) u(1) \right] \left[\overline{u}(3) \gamma_\nu \left(1-\gamma_5\right) u(1) \right]^* \cdot$$

$$\left[\overline{u}(4) \gamma_\mu \left(1-\gamma_5\right) u(2) \right] \left[\overline{u}(4) \gamma^\nu \left(1-\gamma^5\right) u(2) \right]^*$$

$$\sum_{spin} \left[\overline{u}(a) \Gamma_1 u(b) \right] \left[\overline{u}(a) \Gamma_2 u(b) \right]^* = Tr \left[\Gamma_1 \left(\not{p}_b + m_b \right) \Gamma_2 \left(\not{p}_a + m_a \right) \right]$$

$$\sum_{spin} \left| T_{fi} \right|^2 = 64 G_F^2 \left(p_1 \cdot p_2 \right) \left(p_3 \cdot p_4 \right)$$

$$\sum_{spin} \left| T_{fi} \right|^2 = 256 G_F^2 E^4 \left[1 - \left(\frac{m_\mu}{2E} \right)^2 \right]$$

Inverse Muon Decay - II

$$\frac{d\sigma}{d\Omega} = \frac{G_F^2}{\pi^2} E^{*2} \left[1 - \left(\frac{m_\mu}{2E^*} \right)^2 \right]^2$$

$$\rightarrow \sigma = \frac{4G_F^2}{\pi} E^{*2} \left[1 - \left(\frac{m_\mu}{2E^*} \right)^2 \right]^2, \quad E^* \text{ CM energy of } e, \nu$$

$$E^* \simeq \sqrt{2mE_\nu}$$

$$\rightarrow \sigma = \frac{8G_F^2 m_\mu}{\pi} E_\nu \propto E_\nu \quad \text{at high energy}$$

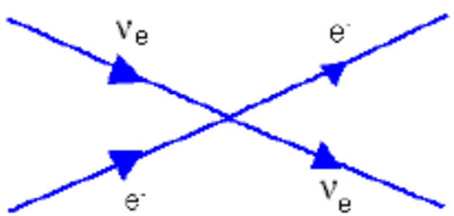
σ badly divergent

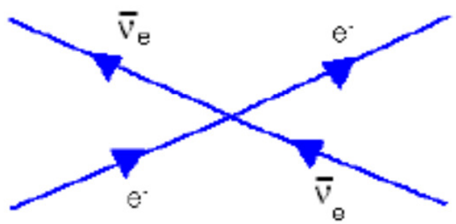
$\rightarrow$ Unphysical

Neutrino – Lepton Scattering - I

Charged current $\nu_e/\overline{\nu}_e - e$ scattering:

$$\nu_e + e^- \rightarrow e^- + \nu_e$$
$$\overline{\nu}_e + e^- \rightarrow \overline{\nu}_e + e^-$$



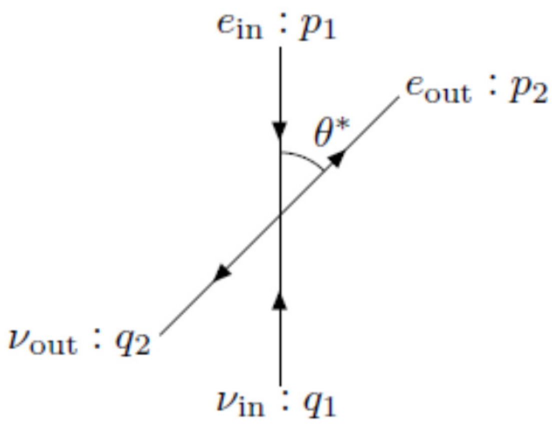


NB Actually incomplete:

Missing neutral current amplitude leading to the same final states

Cross sections must be avaluated by adding *all* the relevant amplitudes

Neutrino – Lepton Scattering - II



$$T_{fi} = -i\frac{G_F}{\sqrt{2}}\left[\overline{v}(\nu, q_1)\gamma_\mu(1-\gamma_5)u(e, p_1)\right]\cdot\left[\overline{u}(e, p_2)\gamma^\mu(1-\gamma_5)v(\nu, q_2)\right]$$

Neutrino – Lepton Scattering - III

$\bar{\nu} e \rightarrow \bar{\nu} e$

$$\frac{d\sigma_{\bar{\nu}e}}{d\Omega^*} = \frac{\left\langle \left| T_{fi} \right|^2 \right\rangle}{64\pi^2 s} = \frac{G_F^2\, 2mE_\nu \left(1 - \cos\theta^*\right)^2}{16\pi^2}$$

Total cross section:

$$\sigma_{\bar{\nu}e} = \frac{G_F^2\, 2mE_\nu}{3\pi} \approx 0.574\ 10^{-41} E_\nu \left(GeV\right)\ cm^2$$

$\nu e \rightarrow \nu e$

$$\frac{d\sigma_{\nu e}}{d\Omega^*} = \frac{\left\langle \left| T_{fi} \right|^2 \right\rangle}{64\pi^2 s} = \frac{G_F^2\, 2mE_\nu}{4\pi^2}$$

Total cross section:

$$\sigma_{\nu e} = \frac{G_F^2\, 2mE_\nu}{\pi} \approx 1.72\ 10^{-41} E_\nu \left(GeV\right)\ cm^2$$

Neutrino – Lepton Scattering - IV

Total cross sections:

$$\sigma_{\overline{\nu}e} = \frac{G_F^2 2mE_\nu}{3\pi} \approx 0.574\ 10^{-41} E_\nu \left(GeV\right)\ cm^2$$

$$\sigma_{\nu e} = \frac{G_F^2 2mE_\nu}{\pi} \approx 1.72\ 10^{-41} E_\nu \left(GeV\right)\ cm^2$$

as divergent at high energy as the inverse muon decay

NB Cross sections only crude approximations:

Neutral current contribute not included

Interesting factor $\times 3$ between νe and $\overline{\nu} e$

Neutrino – Lepton Scattering - V

Origin of factor $\times 3$:

allowed at all angles

forbidden (angular momentum) at $z = +1$

Unitarity Troubles - I

Divergence at high energy : Unitarity bound violated around $E_\nu^* \sim\ 300\ GeV$

$\sigma \leq \frac{4\pi}{k^2} = \frac{4\pi}{E^{*2}}$ Unitarity bound for S-Wave scattering

Reminder: (Simpler) Spinless potential scattering

Expand incident (plane) wave into angular momentum eigenstates

$$\Psi_i = e^{ikz} = \frac{i}{2kr}\sum_{l=0}^{\infty}(2l+1)\left[(-1)^l e^{-ikr} - e^{ikr}\right]P_l(\cos\theta)$$

Outgoing spherical wave phase shifted by potential:

$$\Psi_{total} = \Psi_{scattered} + \Psi_i = \frac{i}{2kr}\sum_{l=0}^{\infty}(2l+1)\left[(-1)^l e^{-ikr} - \eta_l e^{2i\delta_l} e^{ikr}\right]P_l(\cos\theta)$$

$$\Psi_{scattered} = \Psi_{total} - \Psi_i = \frac{e^{ikr}}{kr}\sum_{l=0}^{\infty}(2l+1)\frac{\eta_l e^{2i\delta_l} - 1}{2i}P_l(\cos\theta) = \frac{e^{ikr}}{r}f(\theta)$$

Unitarity Troubles - II

Outgoing elementary flux:

$$d\Phi_{out} = v_{out}\Psi_{scat}\Psi^*_{scat}r^2 d\Omega = v_{out}\left|F(\theta)\right|^2 d\Omega$$

Incident flux:

$$\Phi_{in} = \Psi_{in}\Psi^*_{in}v_{in} = v_{in}$$

$$\rightarrow d\sigma = \frac{\Phi_{out}}{\Phi_{in}} = \left|F(\theta)\right|^2 d\Omega$$

$$\sigma = \int \left|F(\theta)\right|^2 d\Omega$$

$$\sigma = \frac{1}{k^2}\sum_{l,m}(2l+1)\left[\frac{\eta_l e^{2i\delta_l}-1}{2i}\right](2m+1)\left[\frac{\eta_m e^{2i\delta_m}-1}{2i}\right]^*$$

$$\times\int P_l(\cos\theta)P_m(\cos\theta)d\Omega$$

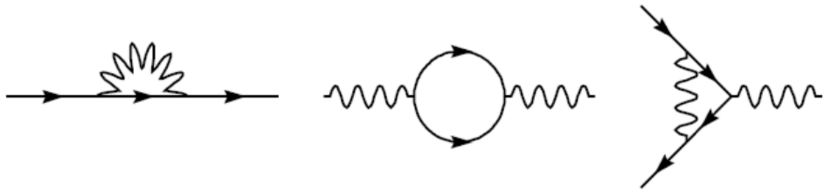
$$\sigma = \frac{4\pi}{k^2}\sum_l(2l+1)\left\|\frac{\eta_l e^{2i\delta_l}-1}{2i}\right\|^2 = \frac{4\pi}{k^2}\sum_l(2l+1)\sin^2\delta_l$$

$$\rightarrow \sigma_{l=0} = \frac{4\pi}{k^2}\sin^2\delta_0 \le \frac{4\pi}{k^2} = \frac{4\pi}{E^{*2}}$$ Unitarity bound for S-Wave scattering

Renormalization Troubles - I

Suppose Fermi's theory can be saved by radiative corrections:
Assume divergent cross-section as due to our limited, tree-level approximation
Maybe higher orders could fix it

Take QED as an example

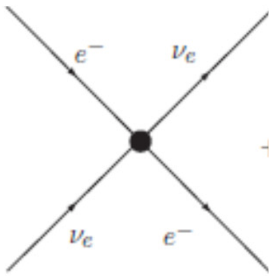


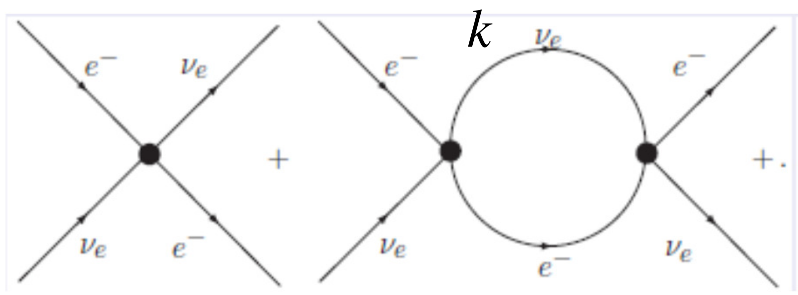
These diagrams (and higher orders) divergent:
However, nice fix available by renormalization procedure

Very successful program, leading to extraordinary accuracy & agreement between theory and experiment

Renormalization Troubles - II

Higher order diagrams in Fermi's theory:





Cannot be fixed by renormalization: Fermi's theory non-renormalizable

Indeed: Each vertex ~ G_F

Renormalization Troubles - III

Lagrangian density (μ decay etc)

$$L_{Fermi} = -\frac{G_F}{\sqrt{2}}\bar{\nu}_\mu \gamma^\alpha \left(1-\gamma^5\right)\mu \bar{e}\gamma_\alpha \left(1-\gamma^5\right)\nu_e$$

Action dimension:

$$S = \int L_{Fermi} d^4x \rightarrow [S] = ET = EE^{-1} = 0$$

$$\rightarrow [L_{Fermi}] = E^4$$

$$[L_{Fermi}] = [G_F]\left[\psi^4\right]$$

Field dimension: $[\psi] = E^{\frac{3}{2}}$

$$\rightarrow [L_{Fermi}] = [G_F] E^6 = E^4$$

$$\rightarrow [G_F] = E^{-2}$$

Amplitude dimension: $[A] = 0$

$\rightarrow$ Loop diagrams of higher orders include integrals of higher powers of k

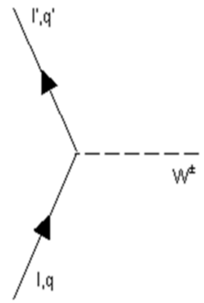
$\rightarrow$ More and more divergent

Beyond Fermi's Theory

As anticipated:

Current-Current must be a *low energy effective theory:*
Low energy approximation of a more general theory

Replace *contact interaction* (current-current) by *boson exchange*:



Vertice in corrente carica

Modeled after the electromagnetic interaction

Exchanged particle must be

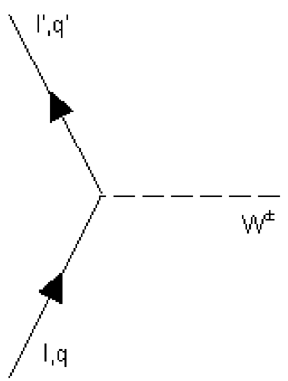
- *Charged* (Charged current ±)
- *Chiral* (Only coupled to left chiral parts: Parity violation)
- *Heavy* (Fermi's point-like interaction OK at low energy)

Intermediate Vector Boson - I

Some key points

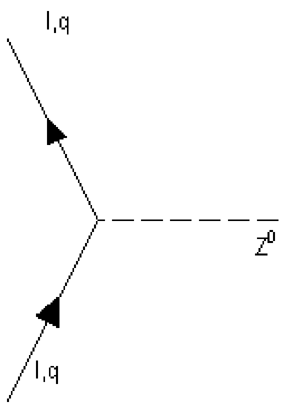
A) (Quarks and) Leptons both interact through the exchange of *vector particles*

$\Delta Q = \pm 1$

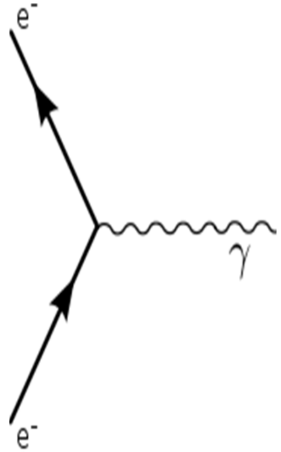


Vertice in corrente carica

$\Delta Q = 0$



Vertice in corrente neutra



Vertice elettromagnetico

Intermediate Vector Boson - II

B) Exchanged vector bosons are *(very)* massive

Range of weak interaction quite small:

Compare β-decay of nuclei, $R<R_{nucleus}$

Cannot tell how large is boson mass, just raw estimate $M \geq 1\ GeV$

C) Exchanged vector bosons have undefined parity

Parity violation

Intermediate Vector Boson - III

$$W \text{ propagator: } -i\frac{g_{\mu\nu} - q_\mu q_\nu / M_W^2}{q^2 - M_W^2} \quad \text{(Gauge dependent)}$$

$$q^2 \ll M_W^2 \rightarrow -i\frac{g_{\mu\nu} - q_\mu q_\nu / M_W^2}{q^2 - M_W^2} \approx i\frac{g_{\mu\nu}}{M_W^2} \quad q^2\text{-independent}$$

$$T_{fi} \cong \left(\frac{1}{2\sqrt{2}}\right)^2 g_W^2 \left(\overline{u}_f^{(1)} \frac{1}{2}\gamma^\mu \left(1-\gamma^5\right) u_i^{(1)}\right) i\frac{g_{\mu\nu}}{M_W^2}\left(\overline{u}_f^{(2)} \frac{1}{2}\gamma_\nu \left(1-\gamma_5\right) u_i^{(2)}\right)$$

$$\rightarrow T_{fi} \cong i\frac{1}{8} 4\sqrt{2} G_F \;\; j^{\mu(1)} j_\mu^{(2)}$$

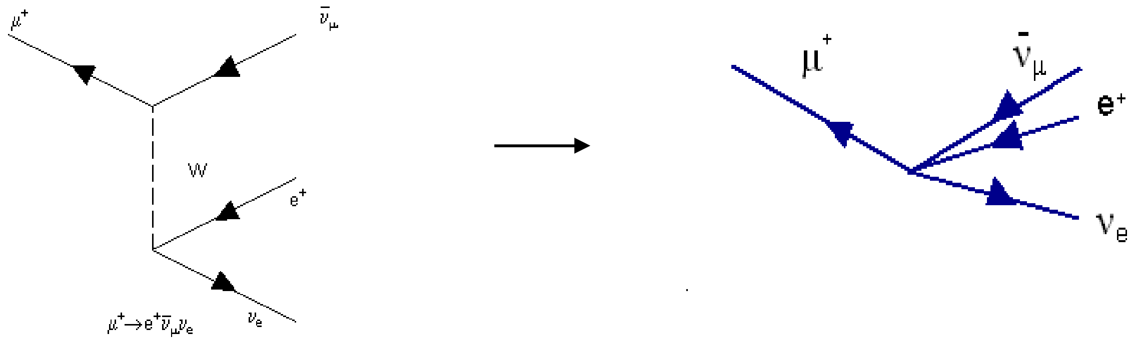
$$g_W^2 \equiv \alpha_W \quad \text{Charged current coupling constant}$$

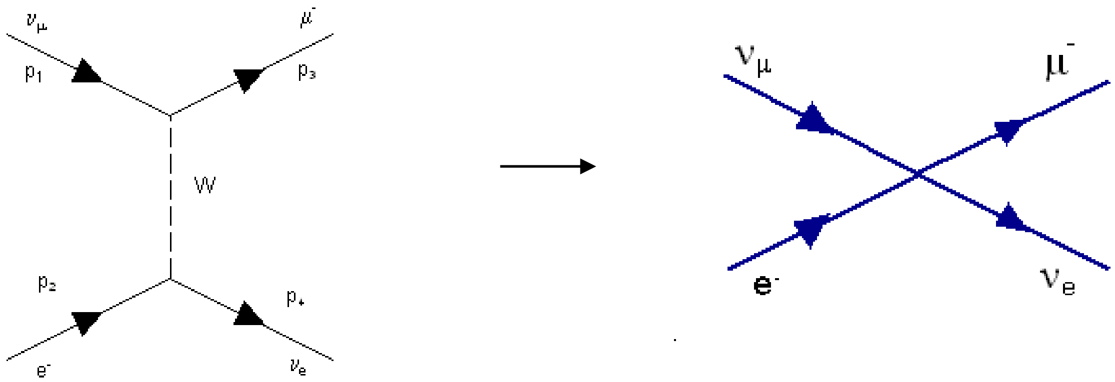
$$G_F = \frac{\sqrt{2}}{8}\left(\frac{g_W}{M_W}\right)^2 \quad \text{Fermi constant}$$

Intermediate Vector Boson - IV

Showing how Standard Model diagrams collapse into current-current at low energy:

$$q^2 \ll M_W^2 \to -i\frac{g_{\mu\nu} - q_\mu q_\nu / M_W^2}{q^2 - M_W^2} \approx i\frac{g_{\mu\nu}}{M_W^2}$$



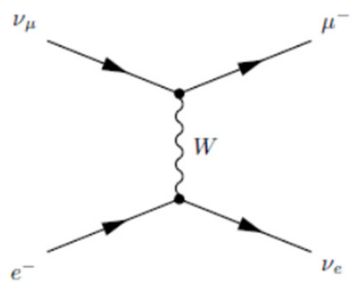


Intermediate Vector Boson - V

Good fix for some problems:

Cross sections of several neutrino reactions

Inverse Muon Decay:

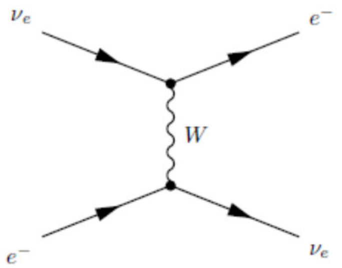


$$\frac{d\sigma}{d\Omega_{CM}} = \frac{G_F^2 M_W^4}{16\pi^2 k^2}\left(\frac{4k^2}{4k^2 - M_W^2}\right)^2 \rightarrow \frac{d\sigma}{d\Omega_{CM}} \approx \begin{cases} \dfrac{G_F^2 k^2}{\pi^2}, & 4k^2 \ll M_W^2 \\ \dfrac{G_F^2 M_W^4}{16\pi^2 k^2}, & 4k^2 \gg M_W^2 \end{cases} \rightarrow \sigma \sim \begin{cases} \dfrac{4G_F^2 k^2}{\pi}, & 4k^2 \ll M_W^2 \\ \dfrac{G_F^2 M_W^4}{4\pi k^2}, & 4k^2 \gg M_W^2 \end{cases}$$

No divergence!

Intermediate Vector Boson - VI

Charged current (only), tree level elastic (anti) electronic neutrino-electron cross sections:



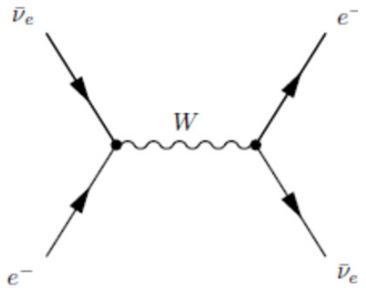
$$\nu_e + e \to e + \nu_e$$

$$\frac{d\sigma}{d\Omega_{CM}} \simeq \frac{16 G_F^2 M_W^2}{\pi^2} \frac{4k^2}{\left(q^2 - M_W^2\right)^2}, \quad k^2 \gg m_e^2$$

$$q^2 \simeq -2k^2\left(1-\cos\theta\right)$$

$$s \simeq 4k^2$$

$$\to \sigma \simeq \frac{G_F^2}{\pi} \frac{4k^2}{1+\frac{4k^2}{M_W^2}} \underset{k^2 \gg M_W^2}{\to} \frac{G_F^2 M_W^2}{\pi}, \quad \text{no divergence!}$$



$$\bar{\nu}_e + e \to \bar{\nu}_e + e$$

$$\sigma \simeq \frac{G_F^2 M_W^4}{3\pi} \frac{4k^2}{16k^4\left(1-\frac{M_W^2}{4k^2}\right)^2} = \frac{G_F^2 M_W^4}{3\pi} \frac{1}{4k^2\left(1-\frac{M_W^2}{4k^2}\right)^2}$$

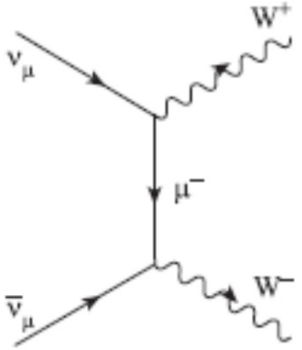
$$\sigma \underset{k^2 \gg M_W^2}{\to} \frac{G_F^2 M_W^4}{3\pi s} \quad \text{no divergence!}$$

Intermediate Vector Boson - VII

Another dark side of naive IVB model:

Take hypothetical reaction

$\nu_\mu + \overline{\nu}_\mu \rightarrow W^+ + W^-$



No question, not easy to realize in the lab...

Nevertheless, it should be possible to compute cross section

Anyway, similar issues for the (realistic) reaction

$e^+ + e^- \rightarrow W^+ + W^-$

Intermediate Vector Boson - VIII

Central issue:

Massive $W^{\pm}$ bosons in the final state

$\rightarrow$ 3 polarization states for a massive vector particle

Rest frame:

$$\left.\begin{aligned}\varepsilon_x &= (0,1,0,0)\\ \varepsilon_y &= (0,0,1,0)\end{aligned}\right\}\varepsilon_T \quad \text{Transverse polarizatiom}$$

$$\varepsilon_z = (0,0,0,1) \quad \varepsilon_L \quad \text{Longitudinal polarization}$$

After a z-boost, carrying the W to 4-momentum $k^{\mu} = \left(k^0, 0, 0, k\right)$

$$\varepsilon_T(k) = \varepsilon_T(0)$$

$$\varepsilon_L(k) = \left(\frac{k}{M_W}, 0, 0, \frac{k_0}{M_W}\right) = \frac{k^{\mu}}{M_W} + O\left(\frac{M_W}{k_0}\right)$$

Intermediate Vector Boson - IX

Matrix element $(1 = \nu, W^+, 2 = \overline{\nu}, W^-; p = \nu, k = W)$:

$$T_{fi}^{\lambda_1\lambda_2} = g^2 \varepsilon_\mu^{-*}(k_2,\lambda_2)\varepsilon_\mu^{+*}(k_1,\lambda_1)\overline{v}(p_2)\gamma^\mu(1-\gamma_5)\frac{(\not p_1 - \not k_1 + m_\mu)}{(p_1-k_1)^2 - m_\mu^2}\gamma^\nu(1-\gamma_5)u(p_1)$$

By:

Neglecting μ mass,

restricting to longitudinally polarized $W's$ $(\lambda = 0)$,

taking the high energy$(\gg M_W)$ limit for the polarization $4-$vectors,

commuting γ_5:

$$\left|T_{fi}^{00}\right|^2 = \frac{g^4}{M_W^4 (p_1-k_1)^4} Tr\left[k_2(1-\gamma_5)(\not p_1 - \not k_1)\not k_1 \not p_1 \not k_1 (\not p_1 - \not k_1)\not k_2 \not p_2\right]$$

Summing over initial spins:

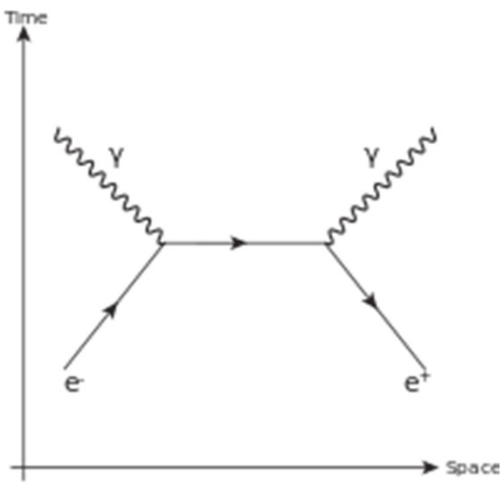
$$\sum_{spin}\left|T_{fi}^{00}\right|^2 = \frac{g^4}{M_W^4}(p_1\cdot k_1)(p_2\cdot k_2) = \frac{g^4}{M_W^4}E^4(1-\cos^2\theta) \rightarrow \frac{d\sigma}{d\Omega^*} = \frac{G_F^2}{8\pi^2}E^2\sin^2\theta$$

No simple solution for this problem:

Massive vector particles cannot make it without 3 polarization states

Intermediate Vector Boson - X

Compare to well known QED process



No contribution from longitudinal photons:

Real photons always transverse, as a consequence of *gauge invariance* of QED

Hope the gauge invariance benefits can be extended to weak interactions..

Intermediate Vector Boson - XI

Is that a single trouble, unrelated to the full IVB scheme?

Have a look at diagrams including *virtual W*:

Discover that a new divergence hits hard our naive IVB model..

Looking at virtual *W* propagator:

$$\frac{-g_{\mu\nu} + k_\mu k_\nu / M_W^2}{k^2 - M_W^2} \underset{k^2 \to \infty}{\longrightarrow} const$$

→ Will make diagrams with virtual *Ws* divergent at high energy

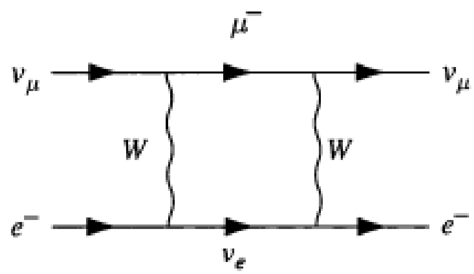
Serious illness of IVB model, particularly relevant for *neutral current* processes

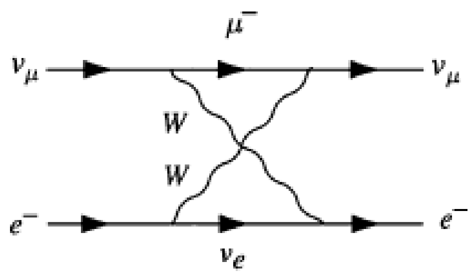
Intermediate Vector Boson - XII

Neutral current reactions like:

$$\nu_\mu + e^- \to \nu_\mu + e^-$$
$$\bar{\nu}_\mu + e^- \to \bar{\nu}_\mu + e^-$$

Not allowed at tree level by our IVB model, only by loop diagrams:





But we can’t compute loop diagrams including virtual *W*:

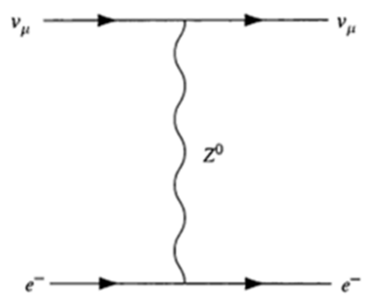
Divergent, IVB theory *not renormalizable*

Intermediate Vector Boson - XIII

Then: Expect strong suppression
After first observations: Cross sections ≈ to allowed processes

$$\nu_e + e^- \rightarrow e^- + \nu_e$$
$$\nu_\mu + e^- \rightarrow \mu^- + \nu_e$$

Suggestion:
Maybe neutral currents do exist *at tree level*, e.g.



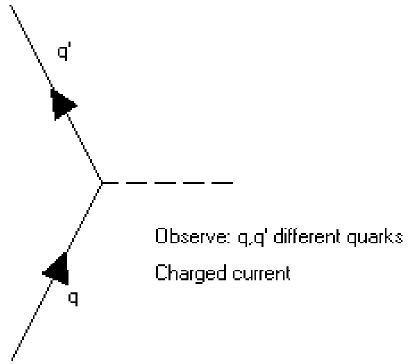
[Indeed, neutral currents are *required* in standard electroweak theory]

Basic requirement: Theory *must be* renormalizable, as is QED

Universality: Quarks

Semileptonic and non leptonic processes understood in terms of quarks

Basically similar coupling to leptonic charged currents:



Picture is slightly more complicated, however
Fundamental question:

Is the quark coupling identical to the lepton one?

Families - I

Consider charged current of leptons:
Very natural to group charged and neutral leptons into *doublets*, or *families*

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix} \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix} \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}$$

Within each doublet, charged current transitions can be thought as due to emission/absorption of $W^\pm$ bosons , similar to (neutral) e.m. current transitions

$$\begin{matrix} W^- \rightarrow & & \nu_e & & \rightarrow W^+ \\ & \uparrow & & \downarrow & \\ W^- \leftarrow & & e^- & & \leftarrow W^+ \end{matrix}$$

Similar for 2nd, 3rd family

Families - II

Natural to extend this scheme to quarks

$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ s \end{pmatrix} \quad \begin{pmatrix} t \\ b \end{pmatrix}$$

$$\begin{array}{ccccc} W^- \rightarrow & & u & & \rightarrow W^+ \\ & \uparrow & & \downarrow & \\ W^- \leftarrow & & d & & \leftarrow W^+ \end{array}$$

Similar for 2nd, 3rd family

Almost correct, but incomplete:
Does not account for strangeness (more generally, → flavour) violating processes

Cabibbo's very ingenious idea:

Quark flavor eigenstates (i.e., quark model eigenstates) are not to be identified with quark weak currents
→Weak currents are mixtures of different flavors

By universal convention, mixing is assumed between *d* , *s, b* quarks

Families - III

In terms of mixed "*d*-like" quarks, with just 2 families:

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$ θ_c Cabibbo's angle

This explain *many* things….

How to extract θ_c?

Just one example: Get the angle from β decay

$G_F^{(\beta)} = 0.975 G_F^{(\mu)}$ (Remember that 2% difference ?)

$\rightarrow G_F^{(\beta)} = \cos\theta_C G_F^{(\mu)}$

$\rightarrow \theta_c \simeq 13^0$

Families - IV

Extend the idea to 3 families:
From Cabibbo's angle to *Cabibbo-Kobayashi-Maskawa* matrix

$$\begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} |d\rangle \\ |s\rangle \\ |b\rangle \end{bmatrix} = \begin{bmatrix} |d'\rangle \\ |s'\rangle \\ |b'\rangle \end{bmatrix}$$

From unitarity:
3 mixing angles
1 complex phase   This can account for CP violation

Experimental values:

$$\begin{bmatrix} 0.9753 & 0.221 & 0.003 \\ 0.221 & 0.9747 & 0.040 \\ 0.009 & 0.039 & 0.9991 \end{bmatrix}$$

Almost diagonal
Heavy quarks even more diagonal

$\nu, \bar{\nu}$ - Nucleon Cross Section - I

Extend V-A to neutrino-nucleon scattering

$$\nu_\mu + N \to \mu^- + X$$

$$\bar{\nu}_\mu + N \to \mu^+ + X$$

Somewhat similar to *e-N*, μ*-N* deep inelastic scattering
Modeling similar to DIS: Parton elastic scattering

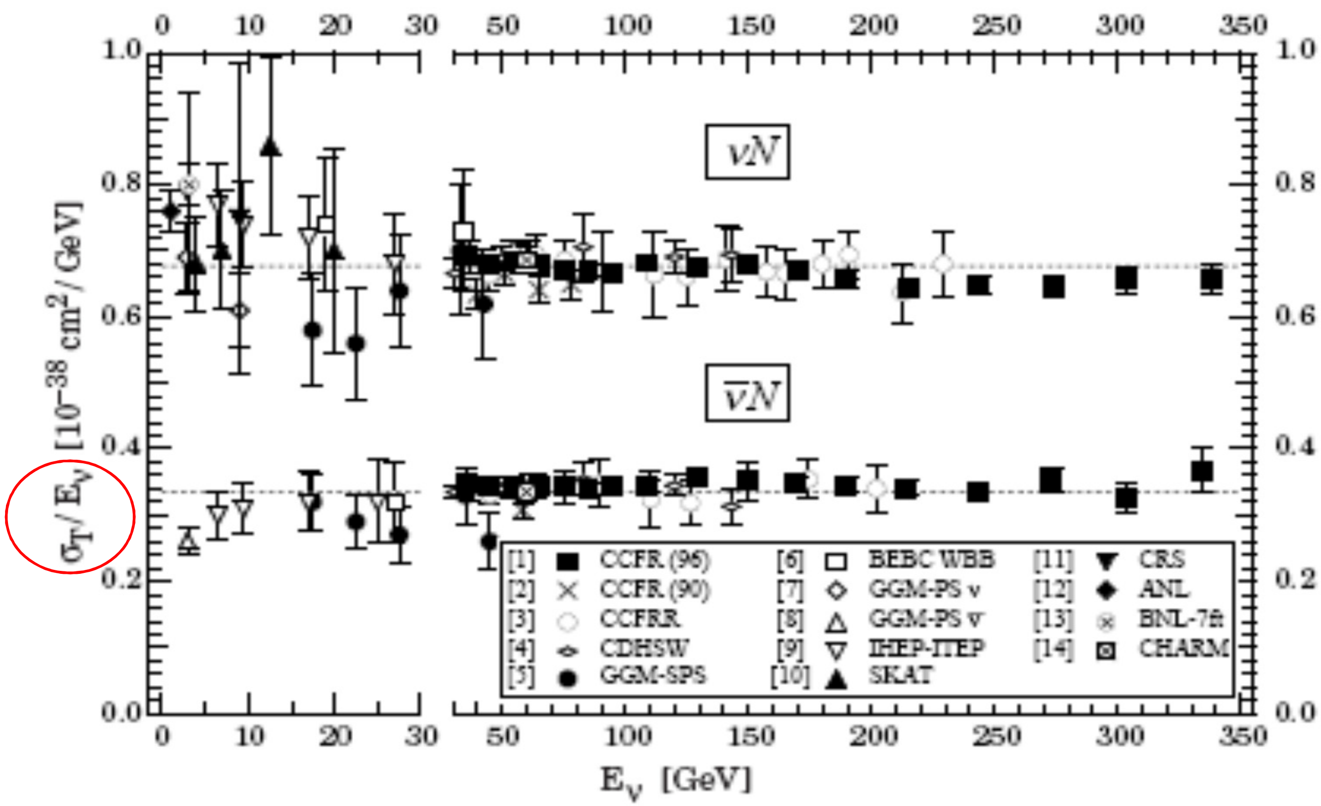
Deep ineastic neutrino scattering reveals the same structure as charged lepton DIS

More information: Charged current sensitive to parton charge sign

→ Can separate quark/antiquark contribution

And: Yes, by looking at (anti)neutrino-nucleon DIS structure functions (probing the parton structure by charged – and neutral – currents) one concludes that quarks couple to weak currents exactly as leptons

$\nu, \overline{\nu}$ - Nucleon Cross Section - II



Linearly rising cross section confirmed…

Gauge Symmetry - I

What makes QED so successful?

Renormalization program allows for computing observables with high accuracy, comparable to experimental resolution

QED is a renormalizable field theory

Fermi's theory is a non-renormalizable theory

And:

Naive IVB theory of weak interactions is a non-renormalizable theory

Try to discover what makes the difference

Gauge Symmetry - II

Back to QED for a while: Reconsider global and local gauge invariance

Free Dirac Lagrangian:

$$L_0 = \bar{\psi}(x)\left(i\gamma^\mu \partial_\mu - m\right)\psi(x)$$

Invariant upon global gauge transformation:

$$\begin{cases} \psi(x) \rightarrow \psi'(x) = e^{-i\alpha}\psi(x) \\ \bar{\psi}(x) \rightarrow \bar{\psi}'(x) = e^{+i\alpha}\bar{\psi}(x) \end{cases}, \quad \alpha \text{ constant}$$

Noether's theorem $\rightarrow$ Conserved current:

$$\partial_\mu s^\mu(x) = 0, \quad s^\mu(x) = q\bar{\psi}(x)\gamma^\mu\psi(x)$$

$\rightarrow$ Conserved charge:

$$Q = \int s^0(x)\, d^3\mathbf{r} = const$$

Gauge Symmetry - III

Non invariant under local gauge transformation:

$$\begin{cases} \psi(x) \to \psi'(x) = e^{-iqf(x)}\psi(x) \\ \overline{\psi}(x) \to \overline{\psi}'(x) = e^{+iqf(x)}\overline{\psi}(x) \end{cases}$$

$$L_0 \to L_0' = L_0 + q\overline{\psi}(x)\gamma^\mu\psi(x)\partial_\mu f(x)$$

Define then a *covariant derivative* as:

$$D_\mu\psi(x) = \left[\partial_\mu + iqA_\mu(x)\right]\psi(x)$$

where, upon the previous local gauge transformation:

$$A_\mu(x) \to A_\mu(x) = A_\mu(x) + \partial_\mu f(x)$$

Then the Lagrangian:

$$L = \overline{\psi}(x)\left(i\gamma^\mu D_\mu - m\right)\psi(x) = L_0 - q\overline{\psi}(x)\gamma^\mu\psi(x)A_\mu(x)$$

is invariant

L contains an *interaction* term $\left(\leftarrow j^\mu A_\mu\right)$

Gauge Symmetry - IV

Consider a single family of massless leptons:

$$L_0 = i\bar{\psi}(x)\gamma^\mu\partial_\mu\psi(x) + i\bar{\psi}_\nu(x)\gamma^\mu\partial_\mu\psi_\nu(x)$$

Chiral spinors:

$$\psi^L(x) = P_L\psi(x) = \frac{1}{2}(1-\gamma_5)\psi(x)$$

$$\psi^R(x) = P_R\psi(x) = \frac{1}{2}(1+\gamma_5)\psi(x)$$

$$\rightarrow L_0 = i\bar{\psi}^L(x)\gamma^\mu\partial_\mu\psi^L(x) + i\bar{\psi}_\nu{}^L(x)\gamma^\mu\partial_\mu\psi_\nu{}^L(x) + i\bar{\psi}^R(x)\gamma^\mu\partial_\mu\psi^R(x) + i\bar{\psi}_\nu{}^R(x)\gamma^\mu\partial_\mu\psi_\nu{}^R(x)$$

Observe: Do not require massless $(\leftarrow L-\text{only})$ neutrino at this stage

Charged current: Connecting two leptons with $\Delta Q = \pm 1$

To encode this into a symmetry scheme, define the doublet:

$$\Psi^L(x) = \begin{pmatrix} \psi_\nu{}^L(x) \\ \psi^L(x) \end{pmatrix}, \bar{\Psi}^L(x) = \begin{pmatrix} \bar{\psi}_\nu{}^L(x) & \bar{\psi}^L(x) \end{pmatrix}$$

$$\rightarrow L_0 = i\bar{\Psi}^L(x)\gamma^\mu\partial_\mu\Psi^L(x) + i\bar{\psi}^R(x)\gamma^\mu\partial_\mu\psi^R(x) + i\bar{\psi}_\nu{}^R(x)\gamma^\mu\partial_\mu\psi_\nu{}^R(x)$$

Gauge Symmetry - V

Suppose the L-doublet realizes the fundamental representation of a $SU(2)$ $(gauge)$ *symmetry* of the weak interaction , exactly as $U(1)$ is the (gauge) symmetry of QED

Then L-spinors will transform

$$\Psi^L(x) \to \Psi^L{}'(x) = U(\alpha)\Psi^L(x) = \exp\left[i\alpha_j\tau_j/2\right]\Psi^L(x)$$

$$\bar{\Psi}^L(x) \to \bar{\Psi}^L{}'(x) = \bar{\Psi}^L(x)U^\dagger(\alpha) = \bar{\Psi}^L(x)\exp\left[-i\alpha_j\tau_j/2\right]$$

$\alpha_1, \alpha_2, \alpha_3$ 3 continuous, real parameters

τ_1, τ_2, τ_3 Pauli matrices

$\left[\text{Reminder}: \left[\tau_i, \tau_j\right] = 2i\varepsilon_{ijk}\,\tau_k\right]$

Also take R-spinors as $SU(2)$ *singlets*:

$$\Psi^R(x) \to \Psi^R{}'(x) = \Psi^R(x), \Psi^R_\nu(x) \to \Psi^R_\nu{}'(x) = \Psi^R_\nu(x)$$

$$\bar{\Psi}^R(x) \to \bar{\Psi}^R{}'(x) = \bar{\Psi}^R(x), \bar{\Psi}^R_\nu(x) \to \bar{\Psi}^R_\nu{}'(x) = \bar{\Psi}^R_\nu(x)$$

Gauge Symmetry - VI

According to Noether's theorem:

Expect conserved current after L invariance under infinitesimal $SU(2)$ transformations:

$$\Psi^L(x) \to \Psi^L{}'(x) = \left(1 + i\alpha_j\tau_j/2\right)\Psi^L(x)$$

$$\bar{\Psi}^L(x) \to \bar{\Psi}^L{}'(x) = \bar{\Psi}^L(x)\left(1 - i\alpha_j\tau_j/2\right)$$

Identify 3 weak isospin, conserved currents / charges:

$$J_i^\mu(x) = \frac{1}{2}\bar{\Psi}^L(x)\gamma^\mu\tau_i\Psi^L(x)$$

$$I_i^W = \int d^3\mathbf{x} J_i^0(x) = \int \frac{1}{2}\Psi^{L\dagger}(x)\tau_i\Psi^L(x)$$

Make 2 non – Hermitian, linear combinations:

$$J^\mu(x) = 2\left[J_1^\mu(x) - iJ_2^\mu(x)\right] = \left[\bar{\Psi}^L(x)\gamma^\mu\tau_1\Psi^L(x) - i\bar{\Psi}^L(x)\gamma^\mu\tau_2\Psi^L(x)\right]$$

$$J^{\mu\dagger}(x) = 2\left[J_1^\mu(x) + iJ_2^\mu(x)\right] = \left[\bar{\Psi}^L(x)\gamma^\mu\tau_1\Psi^L(x) + i\bar{\Psi}^L(x)\gamma^\mu\tau_2\Psi^L(x)\right]$$

Gauge Symmetry - VII

Rememeber:

$$\Psi^L(x)=\begin{pmatrix}\psi_\nu{}^L(x)\\ \psi^L(x)\end{pmatrix},\bar{\Psi}^L(x)=\begin{pmatrix}\bar{\psi}_\nu{}^L(x) & \bar{\psi}^L(x)\end{pmatrix}$$

Then:

$$\bar{\Psi}^L(x)\gamma^\mu\tau_1\Psi^L(x)=\begin{pmatrix}\bar{\psi}_\nu{}^L(x) & \bar{\psi}^L(x)\end{pmatrix}\gamma^\mu\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}\psi_\nu{}^L(x)\\ \psi^L(x)\end{pmatrix}=\begin{pmatrix}\bar{\psi}_\nu{}^L(x) & \bar{\psi}^L(x)\end{pmatrix}\gamma^\mu\begin{pmatrix}\psi^L(x)\\ \psi_\nu{}^L(x)\end{pmatrix}$$

$$=\bar{\psi}_\nu{}^L(x)\gamma^\mu\psi^L(x)+\bar{\psi}^L(x)\gamma^\mu\psi_\nu{}^L(x)$$

$$i\bar{\Psi}^L(x)\gamma^\mu\tau_2\Psi^L(x)=i\begin{pmatrix}\bar{\psi}_\nu{}^L(x) & \bar{\psi}^L(x)\end{pmatrix}\gamma^\mu\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}\begin{pmatrix}\psi_\nu{}^L(x)\\ \psi^L(x)\end{pmatrix}=i\begin{pmatrix}\bar{\psi}_\nu{}^L(x) & \bar{\psi}^L(x)\end{pmatrix}\gamma^\mu\begin{pmatrix}-i\psi^L(x)\\ i\psi_\nu{}^L(x)\end{pmatrix}$$

$$=\bar{\psi}_\nu{}^L(x)\gamma^\mu\psi^L(x)-\bar{\psi}^L(x)\gamma^\mu\psi_\nu{}^L(x)$$

Therefore:

$$\begin{cases}J^\mu(x)=2\bar{\psi}^L(x)\gamma^\mu\psi_\nu{}^L(x)\\ J^{\mu\dagger}(x)=2\bar{\psi}_\nu{}^L(x)\gamma^\mu\psi^L(x)\end{cases}$$: Just our weak charged currents

Gauge Symmetry - VIII

$$J_3^\mu = \frac{1}{2}\bar{\Psi}^L(x)\gamma^\mu\tau_3\Psi^L(x)$$

$$\bar{\Psi}^L(x)\gamma^\mu\tau_3\Psi^L(x) = \begin{pmatrix}\bar{\psi}_\nu{}^L(x) & \bar{\psi}^L(x)\end{pmatrix}\gamma^\mu\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}\psi_\nu{}^L(x)\\ \psi^L(x)\end{pmatrix} = \begin{pmatrix}\bar{\psi}_\nu{}^L(x) & \bar{\psi}^L(x)\end{pmatrix}\gamma^\mu\begin{pmatrix}\psi_\nu{}^L(x)\\ -\psi^L(x)\end{pmatrix}$$

$$\rightarrow J_3^\mu = \frac{1}{2}\left[\bar{\psi}_\nu{}^L(x)\gamma^\mu\psi_\nu{}^L(x) - \bar{\psi}^L(x)\gamma^\mu\psi^L(x)\right]$$

Neutral (as opposed to charged) current

Observe:

2nd term is actually a part of the electromagnetic current, up to a constant factor

$$J_{EM}^\mu = -e\bar{\psi}(x)\gamma^\mu\psi(x)$$

Might be possible to unify EM and weak interactions.

But:

Count 3 weak isospin + 1 electromagnetic = 4 currents = 2 charged + 2 neutral

$\rightarrow$ Unified symmetry group must be larger that $SU(2)$, which has only 3 parameters

Gauge Symmetry - IX

Early models (between '50s and '60s...):

Neutral current ≡ 3rd weak isospin current

Symmetry group is $SU(2)_L \times U(1)_Q$
$SU(2)_L$ (Non Abelian) symmetry group of weak interactions of *L*-fermions
$U(1)_Q$ (Abelian) symmetry group of QED

Then:

Neutral current has same V-A structure of charged current
(Wrong: When finally observed, neutral current was found ≠ V-A)

Weak and Electromagnetic interactions stay independent
(Wrong: At high energy, proofs of unification easily found)

Gauge Symmetry - X

Rather assume the symmetry group of (unified) Electroweak interaction is

$$SU(2)_L \times U(1)_Y$$

where Y is a new observable called *weak hypercharge*

$$J_Y^\mu = \frac{1}{e} J_{EM}^\mu - J_3^\mu = -\bar{\psi}(x)\gamma^\mu\psi(x) - \frac{1}{2}\left[\bar{\psi}_\nu{}^L(x)\gamma^\mu\psi_\nu{}^L(x) - \bar{\psi}^L(x)\gamma^\mu\psi^L(x)\right]$$

$$\rightarrow J_Y^\mu = \frac{1}{2}\bar{\psi}^L(x)\gamma^\mu\psi^L(x) - \frac{1}{2}\bar{\psi}_\nu{}^L(x)\gamma^\mu\psi_\nu{}^L(x) - \bar{\psi}^R(x)\gamma^\mu\psi^R(x)$$

$\rightarrow$ Conserved charge:

$$Y_W = \int d^3\mathbf{x} J_Y^0(x) = \frac{Q}{e} - I_3^W$$

Fermion EW quantum numbers: Defined by I, I_3, Y

Different for different chiralities!

Gauge Symmetry - XI

Find the EW quantum numbers of (chiral) leptons :

$$I_3^W \left|l^-,L\right\rangle = -\frac{1}{2}\left|l^-,L\right\rangle \quad Y_W \left|l^-,L\right\rangle = -\frac{1}{2}\left|l^-,L\right\rangle$$

$$I_3^W \left|\nu_l,L\right\rangle = +\frac{1}{2}\left|\nu_l,L\right\rangle \quad Y_W \left|\nu_l,L\right\rangle = -\frac{1}{2}\left|\nu_l,L\right\rangle$$

$$I_3^W \left|l^-,R\right\rangle = 0 \qquad Y_W \left|l^-,R\right\rangle = -\left|l^-,L\right\rangle$$

$$I_3^W \left|\nu_l,R\right\rangle = 0 \qquad Y_W \left|\nu_l,R\right\rangle = 0$$

Gauge Symmetry - XII

Extend to *local* $SU(2)$ gauge transformations: Similar to QCD

$L -$ doublet

$$\Psi^L(x) \to \Psi^L{}'(x) = U(\alpha)\Psi^L(x) = \exp\left[i\,g\omega_j(x)\tau_j/2\right]\Psi^L(x)$$

$$\overline{\Psi}^L(x) \to \overline{\Psi}^L{}'(x) = \overline{\Psi}^L(x)U^\dagger(\alpha) = \overline{\Psi}^L(x)\exp\left[-i\,g\omega_j(x)\tau_j/2\right]$$

$R -$ singlet

$$\psi^R(x) \to \psi^R{}'(x) = \psi^R(x), \psi^R_\nu(x) \to \psi^R_\nu{}'(x) = \psi^R_\nu(x)$$

$$\overline{\psi}^R(x) \to \overline{\psi}^R{}'(x) = \overline{\psi}^R(x), \psi^R_\nu(x) \to \psi^R_\nu{}'(x) = \psi^R_\nu(x)$$

$\omega_j(x)$: 3 real parameters, functions of $(\mathbf{r},t)$

As for QED: L_0 not invariant

$$L_0 \to L_0{}' = L_0 + \delta L_0 \;\; = L_0 - \frac{1}{2} g\overline{\Psi}^L(x)\tau_j \not{\partial}\omega_j(x)\Psi^L(x)$$

Gauge Symmetry - XIII

$\rightarrow$ Define a covariant derivative for the doublet:

$$\partial^\mu \Psi^L(x) \rightarrow D^\mu \Psi^L(x) = \left[\partial^\mu + i\frac{g}{2}\tau_j W_j^\mu(x)\right]\Psi^L(x)$$

W_j^μ : triplet of (charged, massless) , photon / gluon – like vector fields

Requiring suitable transformation rules:

[Repeated indexes summed, ω_i infinitesimal]

$$W_j^\mu(x) \rightarrow W_j^\mu{}'(x) = W_j^\mu(x) - \partial^\mu \omega_j(x) - g\varepsilon_{jik}\omega_i(x)W_k^\mu(x)$$

$\rightarrow L$ invariant

Gauge Symmetry - XIV

Now consider weak hypercharge $U(1)$ gauge transformations: Similar to QED

$$\psi(x) \rightarrow \psi'(x) = e^{ig'Yf(x)}\psi(x)$$

$$\overline{\psi}(x) \rightarrow \overline{\psi}'(x) = \overline{\psi}(x)e^{ig'Yf(x)}$$

$\rightarrow$ Define a covariant derivative:

$$\partial^\mu \psi(x) \rightarrow D^\mu \psi(x) = \left[\partial^\mu + ig'B^\mu(x)\right]\psi(x)$$

B^μ : Neutral, massless photon / gluon – like vector field

g': New coupling constant

$$B^\mu(x) \rightarrow B^\mu{}'(x) = B^\mu(x) - \partial^\mu f(x)$$

$\rightarrow L$ invariant

Gauge Symmetry - XV

Collecting all pieces together:

$$L = i\left[\bar{\Psi}^L(x)\,\not{D}\,\Psi^L(x) + \bar{\psi}^R(x)\,\not{D}\,\psi^R(x) + \bar{\psi}_\nu^R(x)\,\not{D}\,\psi_\nu^R(x)\right]$$

$$D^\mu \Psi^L(x) = \left[\partial^\mu + i\frac{g}{2}\tau_j W_j^\mu(x) - i\frac{g'}{2}B^\mu(x)\right]\Psi^L(x)$$

$$D^\mu \psi^R(x) = \left[\partial^\mu - i\frac{g'}{2}B^\mu(x)\right]\psi^R(x)$$

$$D^\mu \psi_\nu^R(x) = \partial^\mu \psi_\nu^R(x)$$

Write it as:

$$L = L_0 + L_I$$

$$L_I = -gJ_1^\mu(x)W_{1\mu} - gJ_2^\mu(x)W_{2\mu} - gJ_3^\mu(x)W_{3\mu} - g'J_Y^\mu(x)B_\mu(x)$$

Gauge Symmetry - XVI

To understand the meaning of the interaction terms :

Re – write the interaction part

Define

$$W_\mu(x) = \frac{1}{\sqrt{2}}\left[W_{1\mu}(x) - iW_{2\mu}(x)\right]$$

$$W_\mu^\dagger(x) = \frac{1}{\sqrt{2}}\left[W_{1\mu}(x) + iW_{2\mu}(x)\right]$$

And get for the first 2 terms:

$$\rightarrow L_{I-ch} = -\frac{g}{2\sqrt{2}}\left[J^{\mu\dagger}(x)W_\mu(x) + J^\mu(x)W_\mu^\dagger(x)\right]$$

Charged current interaction of $L-$fermions

Gauge Symmetry - XVII

Define:

$$W_{3\mu}(x) = \cos\theta_W Z_\mu(x) + \sin\theta_W A_\mu(x)$$
$$B_\mu(x) = -\sin\theta_W Z_\mu(x) + \cos\theta_W A_\mu(x)$$

Reminder:

$$J_Y^\mu(x) = \frac{1}{e} J_{EM}^\mu(x) - J_3^\mu(x)$$

$$\rightarrow \begin{cases} -g' J_Y^\mu(x) B_\mu(x) = -g'\left[\frac{1}{e} J_{EM}^\mu(x) - J_3^\mu(x)\right]\left[-\sin\theta_W Z_\mu(x) + \cos\theta_W A_\mu(x)\right] \\ -g J_3^\mu(x) W_{3\mu} = -g J_3^\mu(x)\left[\cos\theta_W Z_\mu(x) + \sin\theta_W A_\mu(x)\right] \end{cases}$$

$\rightarrow$ Remaining terms:

$$-J_{EM}^\mu(x)\frac{g'}{e}\left[-\sin\theta_W Z_\mu(x) + \cos\theta_W A_\mu(x)\right] +$$
$$-J_3^\mu(x)\left\{g\left[\cos\theta_W Z_\mu(x) + \sin\theta_W A_\mu(x)\right] + g'\left[-\sin\theta_W Z_\mu(x) + \cos\theta_W A_\mu(x)\right]\right\}$$

Gauge Symmetry - XVIII

Most simple way of unifying the EM and weak interaction :
Require this condition on g', θ_W constants

$$g \sin\theta_W = g' \cos\theta_W = e$$

and contemplate the miracle:

$$L_I = -J^\mu_{EM}(x) A_\mu - \frac{g}{2\sqrt{2}} J^{\mu\dagger}(x) W_\mu + J^\mu(x) W_\mu^\dagger - \frac{g}{\cos\theta_W}\left[J^\mu_3(x) - \sin^2\theta_W \frac{J^\mu_{EM}}{e}(x) \right] Z_\mu(x)$$

Electromagnetic interaction
Charged current weak interaction
Neutral current weak interaction

Neutral Current - I

As a result of the weak-electromagnetic unification, neutral currents are *different* from charged

Lorentz structure not $V - A$

$$-i\frac{g_W}{\sqrt{2}}\gamma^\mu\frac{\left(1-\gamma^5\right)}{2} \quad \text{Charged}$$

$$-ig_Z\gamma^\mu\frac{\left(C_V^f - C_A^f\gamma^5\right)}{2} \quad \text{Neutral}$$

Coupling

Fermion	C_V	C_A
ν_e,ν_μ,ν_τ	$+1/2$	$+1/2$
e,μ,τ	$-1/2+2\sin\theta_W$	$-1/2$
u,c,t	$+1/2-4/3\sin^2\theta_W$	$+1/2$
d,s,b	$-1/2+2/3\sin^2\theta_W$	$-1/2$

θ_W new fundamental constant

What about interaction strength?

Neutral Current - II

Tight relationship between weak and electromagnetic interactions

Coupling constants:

$$g_W = \frac{e}{\sin\theta_W}$$

$$g_Z = \frac{e}{\sin\theta_W \cos\theta_W}$$

Charged, neutral and electromagnetic couplings fixed by 2 universal constants:

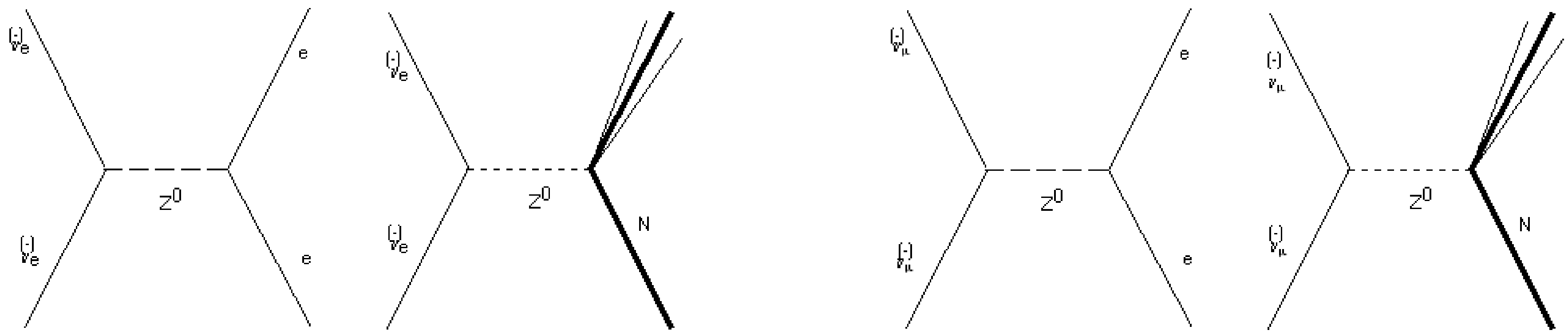
e : Elementary charge

θ_W: Weinberg angle, new fundamental constant

$\sin^2\theta_W = 0.23122 \pm 0.00015$

Neutral Current - III

Expect to observe typical processes like:



$$(\nu_e,\overline{\nu}_e)+e\rightarrow(\nu_e,\overline{\nu}_e)+e$$ Contributing to elastic scattering

$$\left.\begin{array}{l}(\nu_\mu,\overline{\nu}_\mu)+e\rightarrow(\nu_\mu,\overline{\nu}_\mu)+e\\(\nu_e,\overline{\nu}_e)+N\rightarrow(\nu_e,\overline{\nu}_e)+hadron\ \ shower\\(\nu_\mu,\overline{\nu}_\mu)+N\rightarrow(\nu_\mu,\overline{\nu}_\mu)+hadron\ \ shower\end{array}\right\}\quad \text{New}$$

Gauge Boson Terms - I

As for *QED*:

Additional terms required in order to account for:

Energy, *Momentum*, *Angular Momentum*

carried over by the fields

Weak Hypercharge field:

$$-\frac{1}{4}B_{\mu\nu}(x)B^{\mu\nu}(x)$$

$$B^{\mu\nu}(x)=\partial^{\nu}B^{\mu}(x)-\partial^{\mu}B^{\nu}(x)$$

Similar to *QED*: Abelian symmetry $U(1)$

Weak Isospin fields:

$$-\frac{1}{4}\sum_{i=1}^{3}G^{(i)}{}_{\mu\nu}(x)G^{(i)\mu\nu}(x)$$

$$G^{(i)\mu\nu}(x)=\underbrace{\partial^{\nu}W^{(i)\mu}(x)-\partial^{\mu}W^{(i)\nu}(x)}_{F^{(i)\mu\nu}(x)}+g\sum_{i,j=1}^{3}\varepsilon_{ijk}W^{(j)\mu}(x)W^{(k)\nu}(x)$$

Similar to *QCD*: Non-Abelian symmetry $SU(2)_L$

Gauge Boson Terms - II

Gauge Boson Lagrangian:

$$L^B = -\frac{1}{4} B_{\mu\nu}(x) B^{\mu\nu}(x) - \frac{1}{4}\sum_{i=1}^{3} G^{(i)}{}_{\mu\nu}(x) G^{(i)\mu\nu}(x)$$

$$\rightarrow L^B = \underbrace{-\frac{1}{4} B_{\mu\nu}(x) B^{\mu\nu}(x) - \frac{1}{4}\sum_{i=1}^{3} F^{(i)}{}_{\mu\nu}(x) F^{(i)\mu\nu}(x)}_{L_0^B}$$

$$\underbrace{+ g\sum_{i,j,k=1}^{3} \varepsilon_{ijk} W^{(j)\mu}(x) W^{(k)\nu}(x) \partial^\mu W^{(k)\nu}(x) - \frac{1}{4}\sum_{i,j,k,l,m=1}^{3} \varepsilon_{ijk}\varepsilon_{ilm} g^2 W^{(j)\mu}(x) W^{(k)\nu}(x) W^{(l)}{}_{\mu}(x) W^{(m)}{}_{\nu}}_{L_{SI}^B}$$

L_0^B = Free term

L_{SI}^B = Self-Interaction term

Gauge Boson Terms - III

Free term: Rewrite using $A^{\mu}, W^{\mu}, W^{\dagger\mu}, Z^{\mu}$:

$$L_0^B = -\frac{1}{4} F_{\mu\nu}(x) F^{\mu\nu}(x) - \frac{1}{2} F^{W}{}_{\mu\nu}(x) F^{W\dagger\mu\nu}(x) - \frac{1}{4} Z_{\mu\nu}(x) Z^{\mu\nu}(x)$$

Field tensors:

$$F_{\mu\nu}(x) = \partial_{\mu} A_{\nu}(x) - \partial_{\nu} A_{\mu}(x)$$ Coupled to EM current

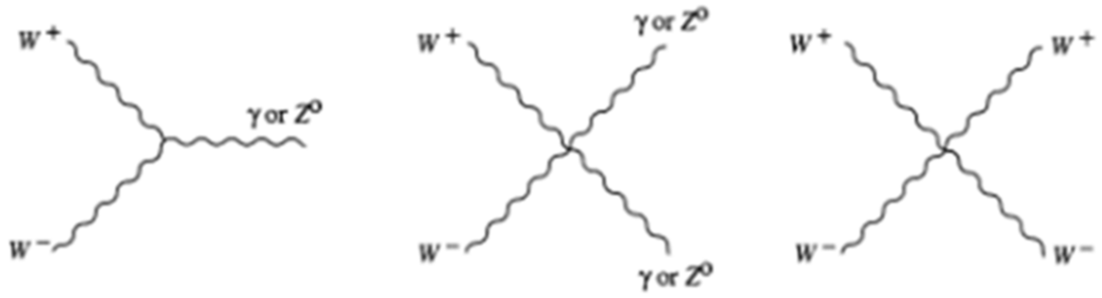
$$F^{W}{}_{\mu\nu}(x) = \partial_{\mu} W_{\nu}(x) - \partial_{\nu} W_{\mu}(x)$$

$$F^{W\dagger\mu\nu}(x) = \partial^{\mu} W^{\dagger\nu}(x) - \partial^{\nu} W^{\dagger\mu}(x)$$

Coupled to Charged current

$$Z^{\mu\nu}(x) = \partial^{\mu} Z^{\nu}(x) - \partial^{\nu} Z^{\mu}(x)$$ Coupled to Neutral current

Self-Interaction term: Similar to 3- and 4-gluons terms of QCD



Mass - I

Massless leptons & gauge bosons not physical: Mass must be there

But: Putting 'by hand' a mass term in L would spoil gauge invariance

Gauge bosons:

$$m_W^2 W_\mu^\dagger W^\mu + \frac{1}{2} m_Z^2 Z_\mu Z^\mu$$

Take W as an example:

$W_i^\mu \to W_i^\mu - \partial^\mu \omega_i(x) - g\varepsilon_{ijk}\omega_j(x)W_k^\mu$ infinitesimal parameters

Then:

$$m_W^2 W_\mu^\dagger W^\mu \to m_W^2 \left(W_i^{\dagger\mu} - \partial^\mu \omega_i(x) - g\varepsilon_{ijk}\omega_j(x)W_k^{\dagger\mu}\right)\left(W_i^\mu - \partial^\mu \omega_i(x) - g\varepsilon_{ijk}\omega_j(x)W_k^\mu\right)$$

$$\to \neq m_W^2 W_\mu^\dagger W^\mu$$

Mass - II

Leptons :

$$-m\overline{\psi}(x)\psi(x)$$

Write in terms of chiral parts :

$$-m\overline{\psi}(x)\psi(x) = -m\overline{\psi}(x)\left(\underbrace{P_R + P_L}_{=1}\right)\psi(x)$$

$$P_R = \frac{1+\gamma_5}{2}, \quad P_L = \frac{1-\gamma_5}{2}$$

$$\to -m\overline{\psi}(x)\left(\frac{1+\gamma_5}{2} + \frac{1-\gamma_5}{2}\right)\psi(x) = -m\overline{\psi}(x)\left(\left(\frac{1+\gamma_5}{2}\right)^2 + \left(\frac{1-\gamma_5}{2}\right)^2\right)\psi(x)$$

$$\to -m\overline{\psi}(x)\left(\frac{1+\gamma_5}{2} + \frac{1-\gamma_5}{2}\right)\psi(x) = -m\overline{\psi}_L(x)\psi_R(x) - m\overline{\psi}_R(x)\psi_L(x)$$

Not invariant under $SU(2)$:

L, R chiral parts live in different $SU(2)$ representations

Mass - III

Bottom line: *Any* mass term not invariant

Glashow model (1961): Put mass by hand
$\rightarrow$ Gauge invariance lost, back to naive IVB

Finally, discover a subtle mechanism to give mass to physical states,
without spoiling gauge invariance:

Spontaneous Symmetry Breaking

Broad phenomenology, also remotely rooted in classical physics

SSB - I

Symmetries: Frequently approximate $\rightarrow$ Broken

Breaking modes:

(a) Explicit breaking

$H = H_0 + H_b$

H_0 invariant

H_b non-invariant

Ex: Hydrogen atom in a magnetic field **B**

$H_0 = \frac{p^2}{2m} - \frac{Ze^2}{r}$ rotationally invariant

$H_b = -\boldsymbol{\mu} \cdot \mathbf{B}$ invariant wrt rotations around **B**

$\rightarrow H_0$ degeneracies removed by H_b

(b) Spontaneous breaking

H symmetric, ground state non symmetric

Ex: Ferromagnetism

$T > T_c$: $\mathbf{M} = 0 \rightarrow$ Dipoles randomly oriented

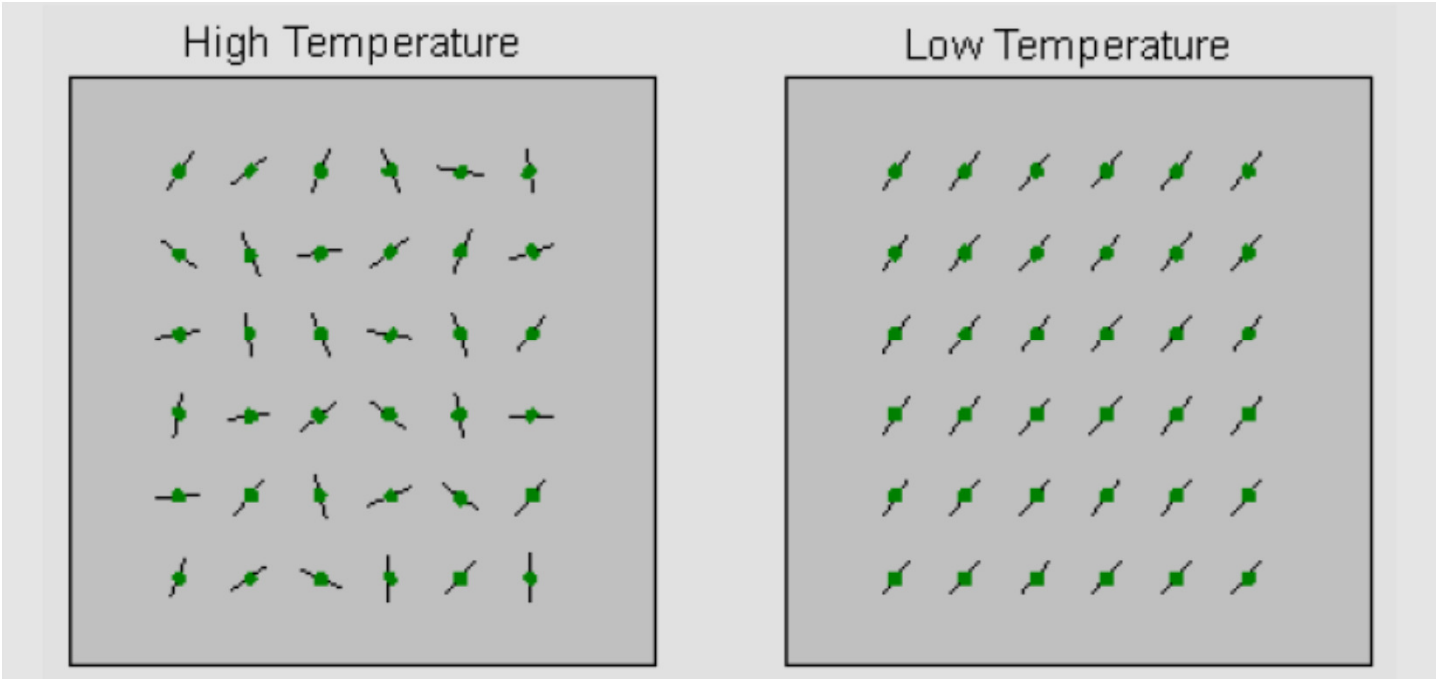
$\rightarrow$ Rotational symmetry

$T < T_c$: $\mathbf{M} \neq 0 \rightarrow$ Dipoles pick some direction

$\rightarrow H$ degeneracies not removed

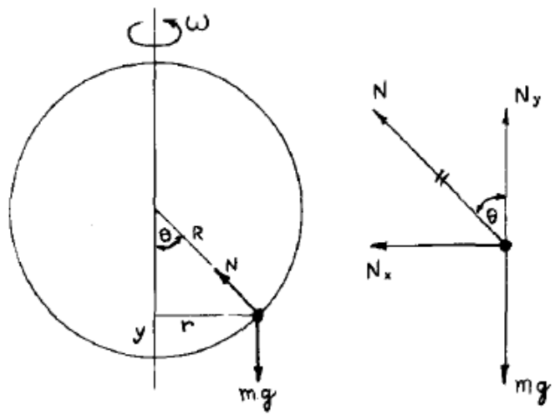
Ground state *degenerate*

SSB - II



SSB - III

Take a first year classical mechanics exercise:
Bead sliding frictionless along a spinning hoop
Find equilibrium angle



$$N = m\omega^2 R$$

$$N_y = N\cos\theta = m\omega^2 R\cos\theta = mg$$

$$\rightarrow \cos\theta_0 = \frac{g}{\omega^2 R}$$

Funny observation:

Critical frequency: $\cos\theta_0 = 1 = \frac{g}{\omega_0^2 R}$

For $\omega < \omega_0$: Different solution

$\theta_1 = 0$

SSB - IV

$$T=\frac{1}{2}mv^2=\frac{1}{2}m\left(R^2\dot{\theta}^2+\omega^2R^2\sin^2\theta\right)$$

$$V=mgy=mgR\left(1-\cos\theta\right)$$

$$L=T-V=\frac{1}{2}mR^2\dot{\theta}^2+\frac{1}{2}m\omega^2R^2\sin^2\theta-mgR\left(1-\cos\theta\right)$$

$$L=\frac{1}{2}mR^2\dot{\theta}^2-V_{eff}$$

Define effective potential, including centrifugal term:

$$V_{eff}\equiv-\frac{1}{2}m\omega^2R^2\sin^2\theta+mgR\left(1-\cos\theta\right)=mgR\left[\left(1-\cos\theta\right)-\frac{\omega^2R\sin^2\theta}{2g}\right]$$

Define reduced effective potential, β parameter:

$$U\equiv\frac{V_{eff}}{mgR}=\left(1-\cos\theta\right)-\frac{1}{2}\frac{\omega^2R}{g}\sin^2\theta$$

$$\beta\equiv\frac{\omega^2R}{g}$$

$$\rightarrow U=\left(1-\cos\theta\right)-\frac{1}{2}\beta\sin^2\theta=2\sin^2\frac{\theta}{2}-\frac{\beta}{2}\left(1-\cos^2\theta\right)=2\sin^2\frac{\theta}{2}\left(1-\beta\cos^2\frac{\theta}{2}\right)$$

SSB - V

Find equilibrium angles, identify stable and unstable:

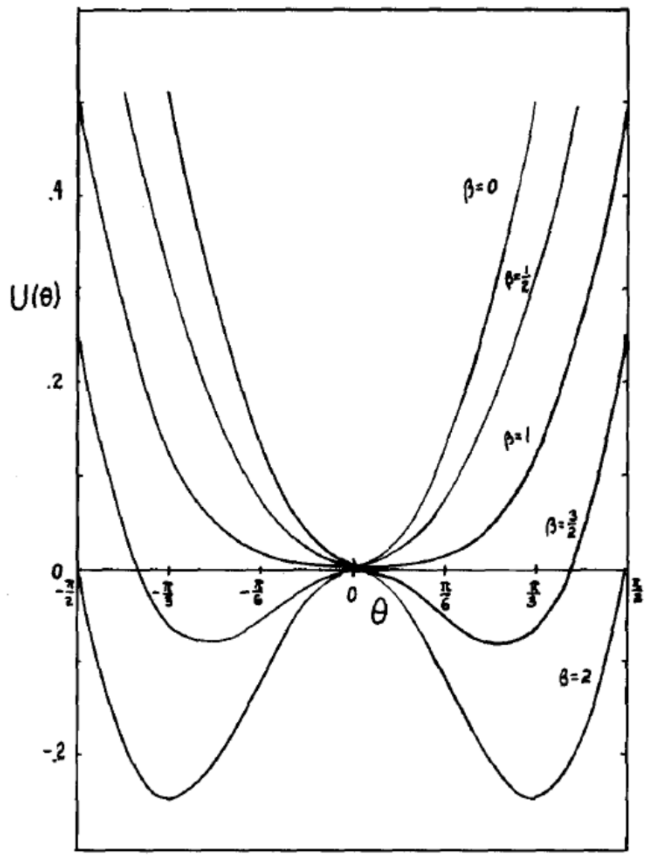
$$\frac{\partial U}{\partial \theta} = \sin\theta\left(1-\beta\cos\theta\right)=0$$

$$\rightarrow \begin{cases} \cos\theta_0 = \dfrac{1}{\beta} \\ \theta_1 = 0 \end{cases}$$

$$\frac{\partial^2 U}{\partial \theta^2} = \cos\theta - \beta\cos 2\theta$$

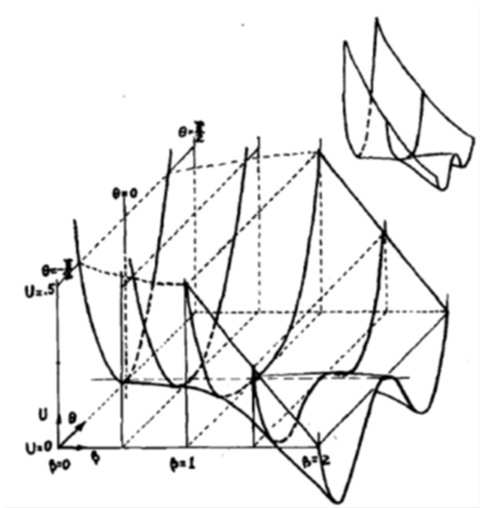
$$\left.\frac{\partial^2 U}{\partial \theta^2}\right|_{\theta=\theta_1} = 1-\beta \qquad \text{stable for } \beta<1$$

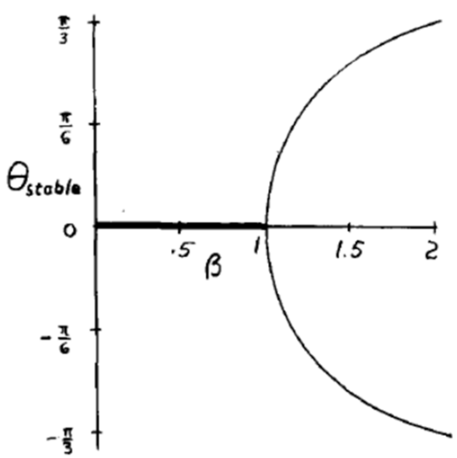
$$\left.\frac{\partial^2 U}{\partial \theta^2}\right|_{\theta=\theta_0} = \beta-\frac{1}{\beta} \qquad \text{stable for } \beta>1$$



SSB - VI

Showing how shape of potential curve, equilibrium angle change with β





a) $\beta<1 \rightarrow 1$ eq. angle

$\beta>1 \rightarrow 2$ eq. angles: Cannot tell which one will be found

Reflection symmetry of V lost (← spontaneously broken) in the solution of eq. of motion

b) Small oscillations around equilibrium angle:

$\beta<1 \rightarrow OK$ Symmetrical wrt origin

$\beta>1 \rightarrow KO$ Non symmetrical wrt origin

SSB - VII

Quantum Mechanics: Simple system with 1 degree of freedom:

$$L = \frac{1}{2} m\dot{x}^2 - \frac{1}{2}\lambda^2 \left(x^2 - \eta^2\right)^2$$

$$V(x) = \frac{1}{2}\lambda^2 \left(x^2 - \eta^2\right)^2$$

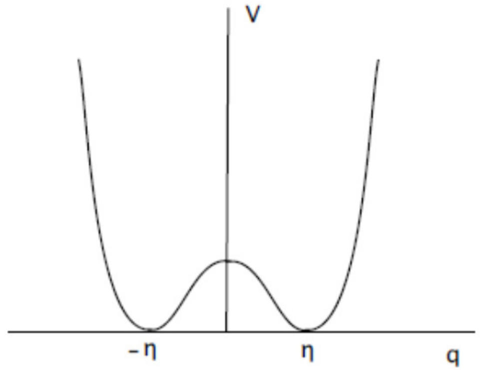
Potential: Parity symmetric

$$V(x) = V(-x)$$

Expand around $\pm\eta$ to quadratic terms only: Harmonic oscillator

$|+\rangle$ solution, centered on $+\eta$

$|-\rangle$ solution centered on $-\eta$



Naively:

Expect two degenerate ground states, both with undefined parity

SSB - VIII

But:

H not diagonal in this basis

$\langle + | H | + \rangle = \langle - | H | - \rangle = a, \langle + | H | - \rangle = \langle - | H | + \rangle = b$

Physical reason : Tunneling through central barrier

$\rightarrow$ Diagonalize, find:

Eigenstates	Energies
$\vert S \rangle = \vert + \rangle + \vert - \rangle$	$a + b$
$\vert A \rangle = \vert + \rangle - \vert - \rangle$	$a - b$

$|S\rangle, |A\rangle$: Parity eigenstates

$\rightarrow$ Degeneracy removed: Just 1 ground state

$E_{ground} = a - |b|$

SSB - IX

Field theory: Real scalar field

$$L = \frac{1}{2}\left(\partial_\mu \phi\right)^2 - V\left(\varphi\right)$$

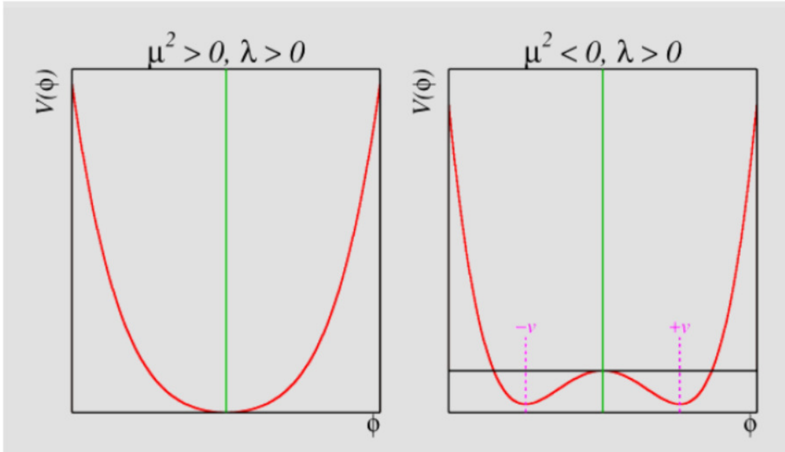
$$V\left(\varphi\right) = \frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda^2\phi^4, \lambda > 0$$

Reflection symmetric: $V\left(\varphi\right) = V\left(-\varphi\right)$

V Minima:

$\mu^2 > 0 : \phi = 0$

$\mu^2 < 0 : \phi = \nu = \pm\sqrt{-\frac{\mu^2}{\lambda}}$



V Minima: Defining *vacuum* state $\left(\leftarrow \text{Cannot have less energy}\right)$

$\mu^2 > 0 :$ Vacuum (non degenerate) $\equiv$ Zero field

$\mu^2 < 0 :$ Vacuum (degenerate!) = $\nu \neq$ Zero field !!

ν = Vacuum Expectation Value (VEV) of ϕ

SSB - X

Choose vacuum state:

$\langle \phi(x) \rangle_0 = v$ Spontaneous Symmetry Breakdown

Define: $\phi(x) = v + \eta(x)$

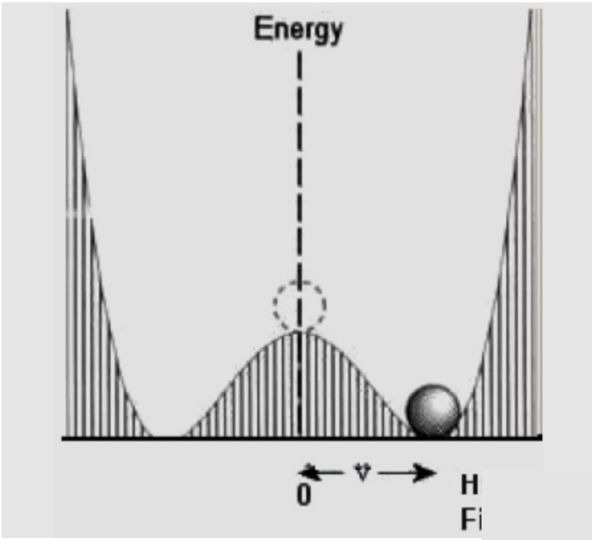
$$\rightarrow L = \frac{1}{2}(\partial_\mu \eta)^2 - \lambda\left(v^2\eta^2 - v\eta^3 - \frac{1}{4}\eta^4\right) = \left[\frac{1}{2}(\partial_\mu \eta)^2 - \lambda v^2 \eta^2\right] - \lambda v \eta^3 - \frac{1}{4}\lambda\eta^4$$

$$L = \frac{1}{2}(\partial_\mu \eta)^2 - \lambda v^2 \eta^2 \; \cancel{+\text{higher powers of } \eta}$$

$\rightarrow$ Free Klein-Gordon equation $\rightarrow$ Scalar quantum field

$m^2 = 2\lambda v^2 \rightarrow m = \sqrt{-2\mu^2} > 0$

$\left[\text{Observe:} \mu^2 < 0 \rightarrow \text{Imaginary mass in original } L!\right]$



KO: $L(\eta) \neq L(-\eta)$

Reflection symmetry *spontaneously broken*

SSB - XI

What makes the difference between a single degree of freedom system and a field?

1 degree of freedom: Vacuum not degenerate

$\leftarrow$ Tunneling

∞ degrees of freedom: Vacuum degenerate

$\leftarrow$ Tunneling not effective

Indeed, it can be shown that:

$A_{tunnel} \propto e^{-aV}, \quad V$ system volume

$\rightarrow A_{tunnel} \sim 0$ for a (infinite) field

SSB - XII

Field theory: Complex scalar field

$$\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2), \phi^* = \frac{1}{\sqrt{2}}(\phi_1 - i\phi_2)$$

$$L = (\partial_\mu \phi)(\partial^\mu \phi)^* - V(\phi)$$

$$V(\phi) = \mu^2 \phi^* \phi + \lambda(\phi^* \phi)^2, \lambda > 0, \mu^2 < 0$$

$U(1)$ symmetric: $\phi \to \phi' = e^{i\alpha}\phi$

Observe: $U(1)$ *continuous* symmetry

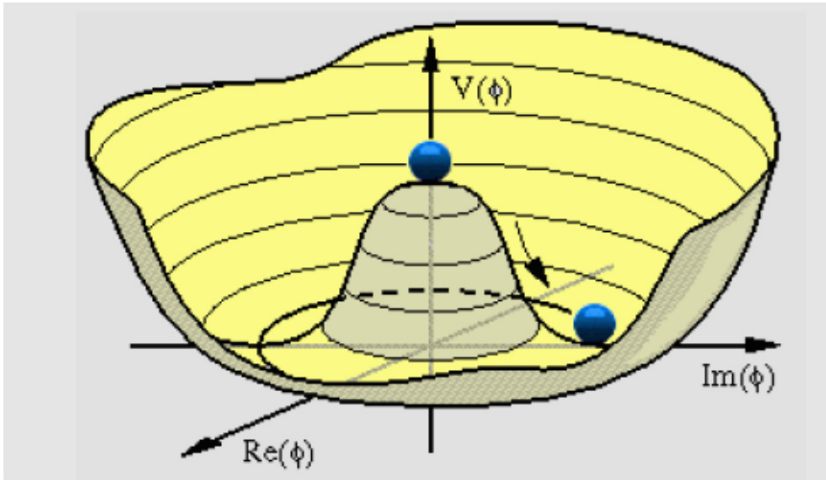
V Minima:

$$\mu^2 < 0 : \phi_1^2 + \phi_2^2 = -\frac{\mu^2}{\lambda}$$

$\rightarrow$ Vacuum infinitely degenerate

Choose vacuum = $(v, 0)$

$\rightarrow U(1)$ symmetry *spontaneously broken*



SSB - XIII

Define:

$$\phi(x) = \frac{1}{\sqrt{2}}\left[v + \xi(x) + i\eta(x)\right]$$

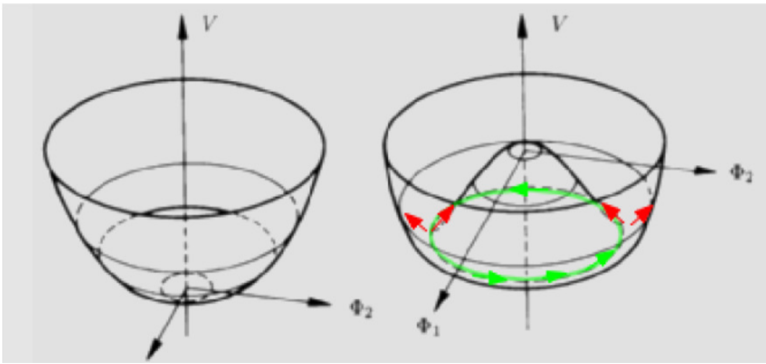
$$L = \frac{1}{2}\left[\left(\partial_\mu \xi\right)^2 + \left(\partial_\mu \eta\right)^2\right] + \mu^2\eta^2$$ ~~+higher powers of η~~

Free Klein-Gordon equations for (ξ, η)

But: "Kinetic energy" terms for both ξ, η; Mass term only for η

$\rightarrow$ η field excitations: *massive* scalar particles

ξ field excitations: *massless* scalar particles, aka *Goldstone Bosons*



SSB - XIV

Goldstone Theorem:

SSB of a continuous, global $(\leftarrow \text{non local})$ symmetry

Symmetry generators transform any vacuum state into another one
n generators not annihilating the vacuum $\rightarrow$ Appearance of n massless scalars = Goldstone bosons

Example form condensed matter physics:

Ferromagnet $\rightarrow$ Rotational symmetry lost $\rightarrow$ Spin waves = Goldstones

Would seem to definitely destroy our hint of a Standard Model:

$3+1$ massless gauge bosons, only one observed
4 massless scalar bosons, none observed

Higgs Mechanism - I

Local gauge invariance + SSB: Higgs mechanism, evading Goldstone's theorem

Simple, yet subtle way of giving mass to gauge bosons

without spoiling gauge invariance (and renormalizability)

Higgs example:

$U(1)$ gauge group, require *local* symmetry:

Gauge vector boson A_μ to be introduced, coupling to some current as usual

Now: Add "sombrero" potential for a complex, scalar field $\phi = \phi_1 + i\phi_2$

$$L = \left[\left(\partial_\mu - ieA_\mu\right)\phi^*\right]\left[\left(\partial^\mu + ieA^\mu\right)\phi\right] - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \mu^2\phi^*\phi - \lambda\left(\phi^*\phi\right)^2 + \underbrace{\ldots\ldots}_{\text{Current-field interaction etc}}$$

As found before:

Degenerate vacuum state $\rightarrow$ SSB picks as vacuum state $(v, 0)$

Higgs Mechanism - II

$\rightarrow \phi = v + \eta_1 + i\eta_2$

L written in terms of η_1, η_2 :

Upon quantization, 2 scalar particles $\rightarrow m_1 = \sqrt{2\lambda v^2}, m_2 = 0$

Plugging $\phi = v + \eta_1 + i\eta_2$ into L:

$$L = \frac{1}{2}(\partial_\mu \eta_1)(\partial^\mu \eta_1) - \frac{1}{2}\overbrace{2\lambda v^2}^{m_1^2}\eta_1^2 + \frac{1}{2}(\partial_\mu \eta_2)(\partial^\mu \eta_2)$$

$$+\underbrace{\frac{1}{2}\overbrace{(ev)^2}^{m_V^2} A^\mu A_\mu}_{Massive\ vector!} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \underbrace{evA^\mu \partial_\mu \eta_2}_{??} + \ldots$$

Attempting to understand L:

Massive vector field A^μ + Massive scalar field η_1 + Massless scalar field η_2

Troubling term coupling A^μ and η_2

Higgs Mechanism - III

Use gauge invariance:

$$\begin{cases} \phi \to \phi' = e^{-ie\theta(x)}\phi \\ A^\mu \to A'^\mu = A^\mu + \partial_\mu \theta \end{cases}$$

Choose θ to make ϕ real: Then $\eta_2 \equiv 0$ $(\leftarrow$ Unitary gauge$)$

$$\to L = \frac{1}{2}\left(\partial_\mu \eta_1\right)\left(\partial^\mu \eta_1\right) - \frac{1}{2} 2\lambda v^2 \eta_1^2 + \underbrace{\frac{1}{2}\overbrace{(ev)^2}^{M_V^2} A^\mu A_\mu}_{Massive\ vector!} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \ldots$$

Massive vector field A^μ + Massive scalar field η_1

Counting degrees of freedom:

$$\underbrace{2}_{A_\mu} + \underbrace{1 + 1}_{\phi} = \underbrace{3}_{A_\mu} + \underbrace{1}_{\eta_1} \quad \text{OK}$$

Standard picture:

By effect of a smart gauge transformation, the massless vector field A_μ has eaten the Goldstone boson η_2 to become massive

Higgs Mechanism - IV

Attempting to dissipate some misunderstandings likely to sneak in:
Mostly related to our naive perception of what is really a 'particle'

1) Where is the mass?
To identify mass terms in L: Not necessarily a trivial task
Key point: Particle content *only meaningful in perturbative expansion*

$$L = \left(\partial_\mu \phi\right)\left(\partial^\mu \phi\right)^* - \left(-\mu^2 \phi^* \phi\right) - \lambda\left(\phi^* \phi\right)^2, \lambda > 0, \mu^2 > 0$$

$-\mu^2 \rightarrow$ Imaginary mass $\rightarrow$ Nonsense $\rightarrow$???
But: To use this form of L to extract Feynman rules, should expand around $|\phi| = 0$
Unstable extremum $\rightarrow$ Can't make it

$$L = \frac{1}{2}\left(\partial_\mu \eta\right)^2 - \lambda \nu^2 \eta^2 + \cdots$$

Expand around $\eta = 0$
Stable extremum $\rightarrow$ OK $\rightarrow$ Particle content should be identified in this form

Higgs Mechanism - V

2)What's so special in unitary gauge?

Nothing: L invariant under local gauge transformations, including the one to unitary gauge:

$\rightarrow$ L describe the same physics before and after the gauge transofrmation

But: Particle content much easier to extract in the unitary gauge

3) Disappearing Goldstones !?

Indeed: And re-appearing as extra degrees of freedom for massive gauge bosons

See comment above on the tricky business of defining what is a particle...

4)What decides which vacuum is selected among the many?

Not really relevant

5) Could we make it with the SM without SSB and all that complicated swapping of degrees of freedom?

Actually no: SSB is an *intrinsic* feature of certain quantum systems

SM - I

Higgs mechanism exploited to fix troublesome massless gauge bosons in the unified electroweak interaction

Boson counting:

Local gauge symmetry $SU(2)_L \otimes U(1)_Y \rightarrow$ 4 vector bosons

Will need 3 symmetries spontaneously broken

to give mass to 3 weak bosons: Photon *is* massless

Extend Abelian Higgs model to non-Abelian gauge symmetry:

Introduce a doublet of complex, scalar fields:

$$\phi(x) = \begin{pmatrix} \phi^+(x) \\ \phi^0(x) \end{pmatrix}$$

Assuming $y = 1 \rightarrow \begin{cases} Q\left[\phi^+(x)\right] = +1 \\ Q\left[\phi^0(x)\right] = 0 \end{cases}$

SM - II

$SU(2)_L \otimes U(1)$ Gauge transformation of doublet:

$$\phi \to \phi' = \exp\left\{-i\left[\frac{g}{2}\boldsymbol{\alpha}(x)\cdot\boldsymbol{\tau} + \frac{g'}{2}y\theta(x)I\right]\right\}\phi$$

$SU(2)_L \otimes U(1)$ Covariant derivative:

$$D^\mu = \partial^\mu + i\left[\frac{g}{2}\boldsymbol{\tau}\cdot\mathbf{W}^\mu + \frac{g'}{2}yB^\mu\right]$$

$\rightarrow$ Additional term to EW lagrangian:

$$L_H = D_\mu\phi^\dagger D^\mu\phi - \mu^2\phi^\dagger\phi - \lambda\left(\phi^\dagger\phi\right)^2$$

Take $\mu^2 < 0, \quad \lambda > 0$:

$$|\phi|^2_{\min} = -\frac{\mu^2}{2\lambda} = \frac{v^2}{2} \to v = \sqrt{-\frac{\mu^2}{\lambda}}$$

Pick ground state (= vacuum) as

$$\langle\phi\rangle_0 = \begin{pmatrix} 0 \\ \dfrac{v}{\sqrt{2}} \end{pmatrix} \to \text{SSB of Electroweak gauge symmetry}$$

SM - III

Goldstone boson:

Associated with each generator of the gauge group not leaving invariant the vacuum

$\langle\phi\rangle_0$ Invariant under $G \to e^{i\alpha G}\langle\phi\rangle_0 = \simeq (1+i\alpha G)\langle\phi\rangle_0 = \langle\phi\rangle_0 \leftrightarrow \mathrm{G}\langle\phi\rangle_0 = 0$

Take generators of $SU(2)_L \otimes U(1)_Y$:

$$\tau_1\langle\phi\rangle_0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} v \\ 0 \end{pmatrix} \neq 0$$

$$\tau_2\langle\phi\rangle_0 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} = -\frac{i}{\sqrt{2}} \begin{pmatrix} v \\ 0 \end{pmatrix} \neq 0$$

$$\tau_3\langle\phi\rangle_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ -v \end{pmatrix} \neq 0$$

$$Y\langle\phi\rangle_0 = +1\langle\phi\rangle_0 \neq 0$$

But: $Q\langle\phi\rangle_0 = \frac{1}{2}(Y+\tau_3)\langle\phi\rangle_0 = 0$

$\to \langle\phi\rangle_0 : U(1)_Q$ Invariant $\to U(1)_Q$ symmetry unbroken $\to$ Photon stays massless

SM - IV

As before for the Higgs model, rewrite:

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} \sigma_1 + i\sigma_2 \\ v + \eta_1 + i\eta_2 \end{pmatrix}$$

$$\rightarrow V = -\frac{\mu^4}{4\lambda} + \lambda v^2 \eta_1^2 + \lambda v \eta_1 \left(\sigma_1^2 + \sigma_2^2 + \eta_1^2 + \eta_2^2\right) + \frac{\lambda}{4}\left(\sigma_1^2 + \sigma_2^2 + \eta_1^2 + \eta_2^2\right)^2$$

3 massless scalars: $\sigma_1, \sigma_2, \eta_2 \leftarrow$ The Goldstones

1 massive scalar: $\eta_1, m_{\eta_1} \equiv m_H = \sqrt{2\lambda v^2} \leftarrow$ The Higgs

Gauge transformation suitable to get rid of 3 Goldstones:

$$\phi \rightarrow \phi' = U\phi = U\frac{1}{\sqrt{2}}\begin{pmatrix} \sigma_1 + i\sigma_2 \\ v + \eta_1 + i\eta_2 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + \eta_1 \end{pmatrix}$$

$$\rightarrow \begin{cases} SU(2)_L \text{ rotation of doublet to make it 'down'} \\ U(1)_Y \text{ re-phasing of doublet to make it real} \end{cases} \left(\leftarrow \text{Unitary gauge}\right)$$

SM - V

Re-write gauge terms of L in the unitary gauge, in terms of the physical fields:

$$L_B + L_H$$

$$= -\frac{1}{4} F_{\mu\nu}(x) F^{\mu\nu}(x) \qquad \text{Photon}$$

$$-\frac{1}{2} F^{W}{}_{\mu\nu}(x) F^{W\dagger\mu\nu}(x) + \frac{1}{2} m_W^2 W_\mu^\dagger W^\mu \qquad W^\pm \text{ boson}$$

$$-\frac{1}{4} Z_{\mu\nu}(x) Z^{\mu\nu}(x) + \frac{1}{2} m_Z^2 Z_\mu Z^\mu \qquad Z^0 \text{ boson}$$

$$+\left(\partial_\mu \sigma\right)\left(\partial^\mu \sigma\right) - \frac{1}{2} m_H^2 \sigma^2 \qquad H \text{ Higgs boson}$$

$$+L_{BB}^I + L_{HH}^I + L_{HB}^I \qquad \text{Gauge-Higgs, Higgs self-, Gauge self-interactions}$$

SM - VI

Finding the acquired mass of gauge bosons in terms of couplings and VEV of the Higgs field:

$$\left.\begin{aligned} m_{W^\pm} &= \frac{g}{2}\sqrt{-\frac{\mu^2}{\lambda}} \\ m_{Z^0} &= \frac{\sqrt{\left(g^2+g'^2\right)}}{2}\sqrt{-\frac{\mu^2}{\lambda}} \end{aligned}\right\} \rightarrow m_{Z^0} = m_{W^\pm}\sqrt{1+\frac{g'^2}{g^2}}$$

$$m_\gamma = 0$$

$$m_H = \sqrt{-2\mu^2} = ???$$

Model parameters v, λ and θ_W :

$$v = \sqrt{-\frac{\mu^2}{\lambda}}$$

$$\lambda = ???$$

$$g\sin\theta_W = g'\cos\theta_W = e$$

SM - VII

Lepton masses: Different mechanism required

$\rightarrow$ Yukawa (scalar) coupling

Describing interaction between Dirac and scalar fields:

$V \approx g\overline{\Psi}\phi\Psi$, Static limit:

$$V = -\frac{g}{4\pi}\frac{e^{-\mu r}}{r}$$

$$L_{HL} = -g_l\left[\overline{\Psi}_l^L \Phi \psi_l^R + \overline{\psi}_l^R \Phi^\dagger \Psi_l^L\right] - \underbrace{g_{\nu_l}}_{=0 \text{ for massless neutrino}} \left[\overline{\Psi}_l^L \tilde{\Phi} \psi_{\nu_l}^R + \overline{\psi}_{\nu_l}^R \tilde{\Phi}^\dagger \Psi_l^L\right], \quad \tilde{\Phi} = \begin{pmatrix} \phi_b^* \\ -\phi_a^* \end{pmatrix}$$

In the unitary gauge:

$$L_{HL} = -\frac{1}{v} m_l \overline{\psi}_l \psi_l \sigma - \frac{1}{v} m_{\nu_l} \overline{\psi}_{\nu_l} \psi_{\nu_l} \sigma$$

Lepton masses in terms of model parameters:

$$m_l = \frac{v g_l}{\sqrt{2}}, m_{\nu_l} = \frac{v g_{\nu_l}}{\sqrt{2}}$$

SM - VIII

Model parameters:

$g, g', -\mu^2, \lambda, g_l, g_{\nu_l}$

Quite remarkably, get m_W, m_Z by measured constants:

$$\begin{cases} \alpha = \dfrac{1}{137.04} \\ G_F = 1.166\ 10^{-5}\, GeV^{-2} \\ \sin^2\theta_W = 0.23122 \end{cases}$$

$$\rightarrow m_W = 77.5\ \ GeV, m_Z = 88.4\ \ GeV$$

Experimental values:

$$m_W = 80.40\ GeV, m_Z = 90.19\ GeV$$

Difference originating from radiative corrections

Higgs:

$$m_H = \sqrt{-2\mu^2} = \sqrt{2\lambda v^2} \rightarrow ???$$

SM - IX

Relating model parameters to measured constants $e, G_F, \sin\theta_W$:

$$g = \frac{e}{\sin\theta_W}, \quad g' = \frac{e}{\cos\theta_W}$$

$$G_F = \frac{\sqrt{2}}{8}\left(\frac{g}{M_W}\right)^2 \rightarrow \frac{8G_F}{\sqrt{2}} = \left(\frac{g}{M_W}\right)^2 \rightarrow \frac{M_W}{g} = \sqrt{\frac{\sqrt{2}}{8G_F}}$$

$$\rightarrow M_W = \sqrt{\frac{\sqrt{2}g^2}{8G_F}} = \sqrt{\frac{\sqrt{2}e^2}{8G_F\sin^2\theta_W}} = \frac{37.3}{\sin\theta_W} \quad GeV$$

$$M_Z = \frac{M_W}{\cos\theta_W} = \frac{37.3}{\sin\theta_W\cos\theta_W} \quad GeV$$

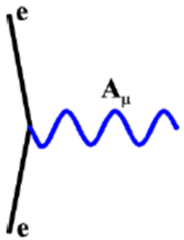
$$v = \frac{2M_W}{g} = 2\sqrt{\frac{\sqrt{2}}{8G_F}} = \frac{1}{\sqrt{\sqrt{2}G_F}}$$

$$\rightarrow \frac{v}{\sqrt{2}} = \frac{1}{\sqrt{\sqrt{2}G_F}}\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{\sqrt{8}G_F}} \approx 174 \text{ GeV} \rightarrow \langle\phi\rangle_0 = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \approx \begin{pmatrix} 0 \\ 174 \text{ GeV} \end{pmatrix} \quad \text{VEV of the Higgs field}$$

No clues on $\lambda \rightarrow$ No (direct) prediction of $M_H = \sqrt{2v^2\lambda}$

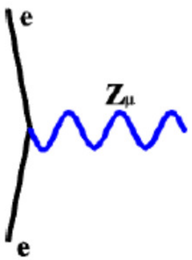
SM - X

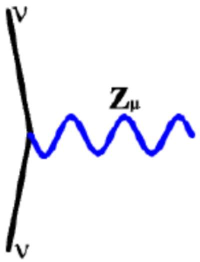
Lepton-Gauge Boson vertexes:



EM

$$-iQ_e\,\bar{e}\gamma^\mu e\,A_\mu$$

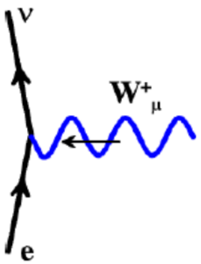


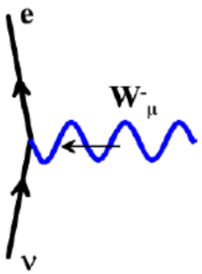


Neutral Current - NC

$$-i\frac{g}{4\cos\theta_W}\bar{e}\gamma^\mu\left[2\sin^2\theta_W(1+\gamma_5)+\left(2\sin^2\theta_W-1\right)(1-\gamma_5)\right]e\,Z_\mu.$$

$$-i\frac{g}{4\cos\theta_W}\bar{\nu}\gamma^\mu(1-\gamma_5)\nu\,Z_\mu.$$





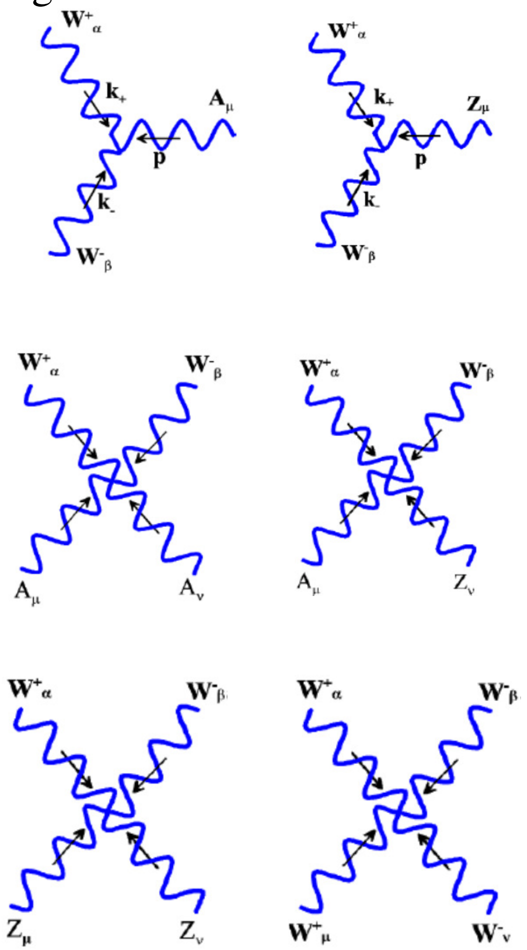
Charged Current - CC

$$-i\frac{g}{2\sqrt{2}}\bar{\nu}\gamma^\mu(1-\gamma_5)e\,W^+_\mu$$

$$-i\frac{g}{2\sqrt{2}}\bar{e}\gamma^\mu(1-\gamma_5)\nu\,W^-_\mu$$

SM - XI

Gauge bosons self-interaction vertexes:



$$ig\sin\theta_W\Big[g^{\alpha\beta}(k_+-k_-)^\mu+g^{\alpha\mu}(p-k_+)^\beta$$
$$+g^{\beta\mu}(k_--p)^\alpha\Big]W^+_\alpha W^-_\beta A_\mu .$$

$$ig\cos\theta_W\Big[g^{\alpha\beta}(k_+-k_-)^\mu+g^{\alpha\mu}(p-k_+)^\beta$$
$$+g^{\beta\mu}(k_--p)^\alpha\Big]W^+_\alpha W^-_\beta Z_\mu .$$

$$-ig^2\sin^2\theta_W\left[2g^{\mu\nu}g^{\alpha\beta}-g^{\mu\alpha}g^{\nu\beta}-g^{\mu\beta}g^{\alpha\nu}\right]$$
$$\times W^+_\alpha W^-_\beta A_\mu A_\nu ,$$

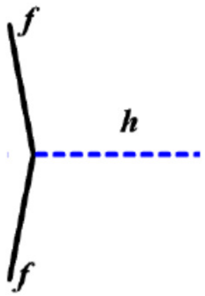
$$-ig^2\sin\theta_W\cos\theta_W\Big[2g^{\mu\nu}g^{\alpha\beta}-g^{\mu\alpha}g^{\nu\beta}$$
$$-g^{\mu\beta}g^{\alpha\nu}\Big]W^+_\alpha W^-_\beta A_\mu Z_\nu .$$

$$-ig^2\cos^2\theta_W\Big[2g^{\mu\nu}g^{\alpha\beta}-g^{\mu\alpha}g^{\nu\beta}$$
$$-g^{\mu\beta}g^{\alpha\nu}\Big]W^+_\alpha W^-_\beta Z_\mu Z_\nu ,$$

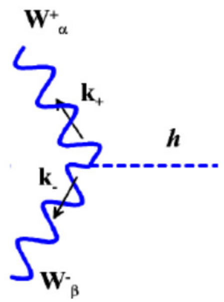
$$-ig^2\Big[2g^{\mu\nu}g^{\alpha\beta}-g^{\mu\alpha}g^{\nu\beta}$$
$$-g^{\mu\beta}g^{\alpha\nu}\Big]W^+_\alpha W^-_\beta W^+_\mu W^-_\nu$$

SM - XII

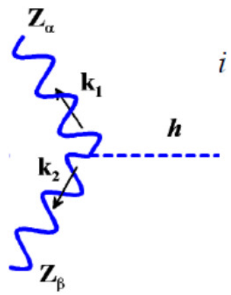
Higgs vertexes:



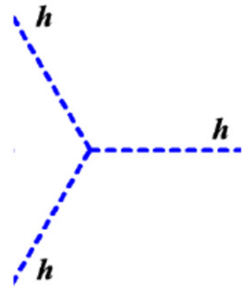
$$-im_f\sqrt{\sqrt{2}G_F}\,\bar{f}\,f$$



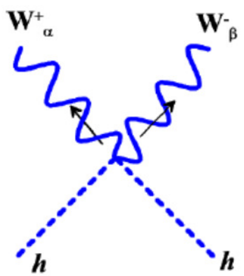
$$2iM_W^2\sqrt{\sqrt{2}G_F}\,g^{\alpha\beta}W_\alpha^{+\dagger}W_\beta^{-\dagger}h$$



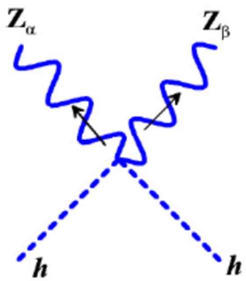
$$2iM_Z^2\sqrt{\sqrt{2}G_F}\,g^{\alpha\beta}Z_\alpha^{\dagger}Z_\beta^{\dagger}h$$



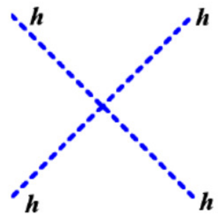
$$-3im_h^2\sqrt{\sqrt{2}G_F}\,hhh$$



$$2iM_W^2\sqrt{2}G_F\,g^{\alpha\beta}hhW_\alpha^{+\dagger}W_\beta^{-\dagger}$$



$$2iM_Z^2\sqrt{2}G_F\,g^{\alpha\beta}hhZ_\alpha^{\dagger}Z_\beta^{\dagger}$$



$$-3im_h^2\sqrt{2}G_F\,hhhh$$

SM - XIII

Extension to 2nd, 3rd lepton family: Straightforward

Will need 2+2 = 4 new parameters (Yukawa couplings)

'Minimal' Standard Model:

Massless neutrinos $\rightarrow g_{\nu_l}^{(i)} = 0$

'Non Minimal' Standard Model:

Neutrino mixing $\left(\leftarrow \text{Require massive neutrinos, mixing matrix}\right)$:

Account for observed neutrino oscillations

May indicate physics beyond Standard Model

Extension to 3 quark families: Similar to leptons

Will need 6 more parameters

Will require CKM 'flavor rotation' (see later)

Strong interaction effects

} Flavor physics

SM - XIV

Fermion electroweak quantum numbers:

	Generations			Quantum Numbers		
helicity	1.	2.	3.	Q	T_3	Y_W
L	$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$	$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$	$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$	0 -1	$1/2$ $-1/2$	-1 -1
	$\begin{pmatrix} u \\ d' \end{pmatrix}_L$	$\begin{pmatrix} c \\ s' \end{pmatrix}_L$	$\begin{pmatrix} t \\ b' \end{pmatrix}_L$	$2/3$ $-1/3$	$1/2$ $-1/2$	$1/3$ $1/3$
R	e_R	μ_R	τ_R	-1	0	-2
	u_R d_R	c_R s_R	t_R b_R	$2/3$ $-1/3$	0 0	$4/3$ $-2/3$

Neutral Currents Discovery - I

Predicted by Glashow-Salam-Weinberg model ('60s)

Not really accepted for a long time:

Mostly because of strong suppression of strangeness changing decays like:

$K^0 \to \mu^+\mu^- \quad BR \quad < 10^{-8}$

not accounted for. Compare:

$K^+ \to \mu^+\nu_\mu \quad BR \quad 63.4\ \%$

Also because it was not clearly demonstrated that GSW was renormalizable

Two breakthroughs:

GIM prediction of charm to solve the $K^0 \to \mu^+\mu^-$ puzzle ('70)

GSW model shown to be renormalizable by 't Hooft ('71)

$\to$ Sudden wave of interest in gauge theories

Neutral Currents Discovery - II

Main interest = Prediction of new phenomena

Most shocking prediction of GSW: neutral currents, never seen before

$\rightarrow$ Try to find neutral currents to validate GSW

Best opportunity :

High energy neutrino interactions

Larger cross sections

No EM background

Drawback:

Neutrino experiments difficult

Neutral Currents Discovery - III

Neutrino beams

Take 2 body decays of π, K obtained from a high energy proton machine

Kinematics:

$$\pi^{\pm}, K^{\pm} \rightarrow \mu^{\pm} + \overset{(-)}{\nu}_{\mu}$$

$\beta, \gamma, M, \lvert\mathbf{p}\rvert$	K, π	LAB
$p^*, E_\mu{}^*, \theta_\mu{}^*$	μ	CM
p_μ, E_μ, θ_μ	μ	LAB

$$\left|\mathbf{p}_\nu{}^*\right| = E_\nu^* = \frac{M_{\pi,K}^2 - m_\mu^2}{2M_{\pi,K}} \simeq \begin{cases} 30 \\ 236 \end{cases} \text{MeV}$$

CM : Isotropic decay

$$\frac{dP}{d\left(\cos\theta^*\right)} = \frac{1}{2} \rightarrow \frac{dP}{dE} = \frac{dP}{d\left(\cos\theta^*\right)} \frac{d\left(\cos\theta^*\right)}{dE}$$

Neutral Currents Discovery - IV

Lorentz transform to LAB:

$$E = \gamma\left(E^* + \beta p^* \cos\theta^*\right) \to dE = \gamma\beta p^* d\left(\cos\theta^*\right) \to d\left(\cos\theta^*\right) = \frac{dE}{\gamma\beta p^*}$$

$$\to \frac{dP}{dE} = \frac{1}{2\gamma\beta p^*}$$

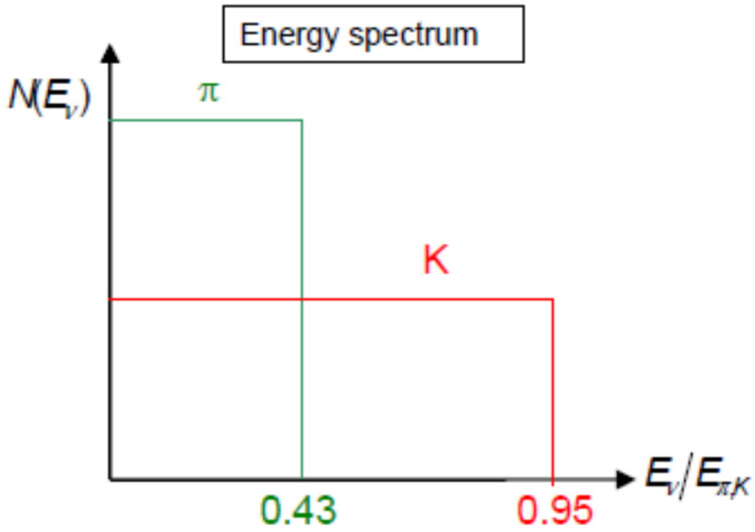
Flat distribution over wide interval:

$$\begin{cases} \gamma(1+\beta)E^* = \gamma(1+\beta)\dfrac{M_{\pi,K}^2 - m_\mu^2}{2M_{\pi,K}} & \text{max} \\ \gamma(1-\beta)E^* = \gamma(1-\beta)\dfrac{M_{\pi,K}^2 - m_\mu^2}{2M_{\pi,K}} & \text{min} \end{cases}$$

$$\frac{dN}{dE} = \frac{M_{\pi,K}}{\gamma\beta\left(M_{\pi,K}^2 - m_\mu^2\right)} = \frac{1}{\underbrace{\gamma\beta M_{\pi,K}}_{|\mathbf{p}|}\left(1 - \dfrac{m_\mu^2}{M_{\pi,K}^2}\right)} = \frac{1}{|\mathbf{p}|\left(1 - \dfrac{m_\mu^2}{M_{\pi,K}^2}\right)}$$

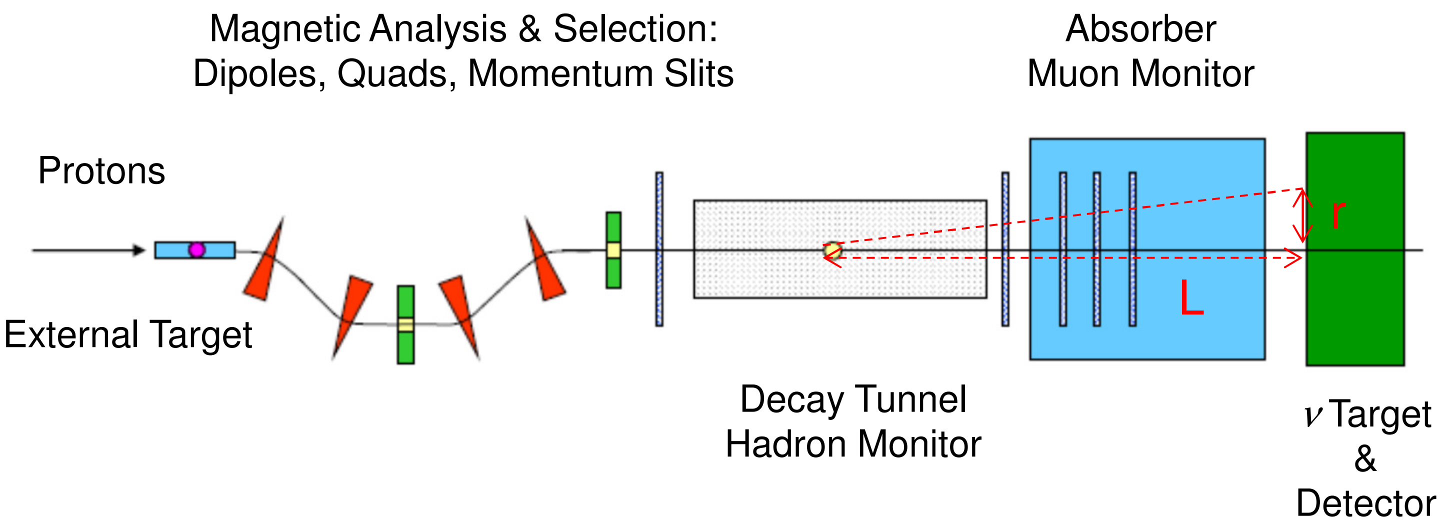
Neutral Currents Discovery - V

$\rightarrow$ Broad LAB ν_μ energy distribution



Neutral Currents Discovery - VI

a) Narrow Band Beam: ν energy known, low intensity
Magnetic selection of a narrow π/K momentum window



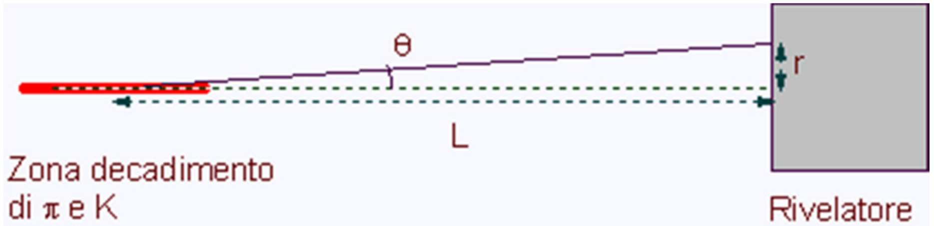
π/K momentum well defined

Beam/Detector geometry: $r \ll L$

Neutral Currents Discovery - VI

Measure ν energy by direction:

Exploit hadron beam $\sim$ monocromaticity



$$p_\pi = p_\mu + p_\nu \rightarrow p_\mu = p_\pi - p_\nu \rightarrow \left(p_\mu\right)^2 = \left(p_\pi - p_\nu\right)^2$$

$$m_\mu^2 = m_\pi^2 - 2p_\pi \cdot p_\nu \rightarrow m_\pi^2 - m_\mu^2 = 2\left(E_\pi E_\nu - \mathbf{p}_\pi \cdot \mathbf{p}_\nu\right)$$

$$E_\nu = \frac{m_{\pi,K}^2 - m_\mu^2}{2\left(E_{\pi,K} - p_{\pi,K}\cos\theta_\nu\right)} = \frac{m_\pi^2 - m_\mu^2}{2E_{\pi,K}\left(1 - \beta\cos\theta_\nu\right)}$$

$$\tan\theta_\nu = \frac{\sin\theta_\nu^{\;*}}{\gamma\left(\cos\theta_\nu^{\;*} + \beta\right)} \rightarrow \tan\theta_{\max} = \frac{\sin\frac{\pi}{2}}{\gamma\left(\cos\frac{\pi}{2} + \beta\right)} = \frac{1}{\beta\gamma} \approx \frac{1}{\gamma} = \frac{m_{\pi,K}}{|\mathbf{p}|} \ll 1$$

$$E_\nu = \frac{m_{\pi,K}^2 - m_\mu^2}{2E_{\pi,K}\left(1 - \beta\cos\theta\right)} \approx \frac{m_{\pi,K}^2 - m_\mu^2}{2E_{\pi,K}\left(1 - \beta\left(1 - \frac{\theta^2}{2}\right)\right)}$$

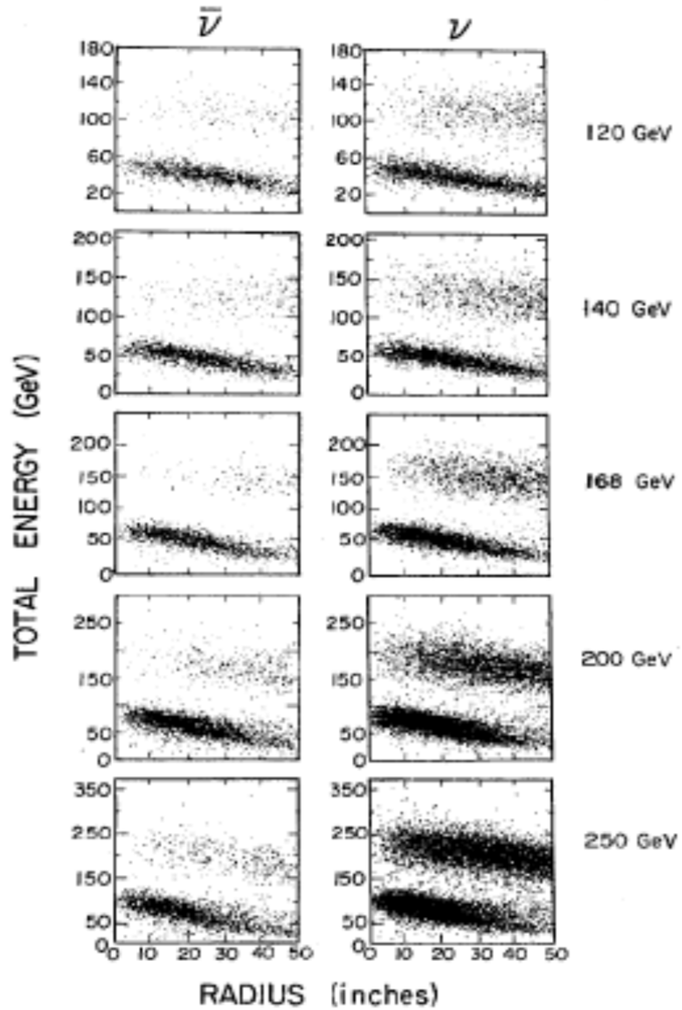
Neutral Currents Discovery - VII

$$E_\nu = \frac{m^2_{\pi,K} - m^2_\mu}{2E_{\pi,K}\left(1-\beta+\beta\dfrac{\theta^2}{2}\right)} = \frac{m^2_{\pi,} - m^2_\mu}{2E_{\pi,K}\left(\dfrac{1-\beta^2}{1+\beta}+\beta\dfrac{\theta^2}{2}\right)}$$

$$E_\nu \simeq \frac{m^2_{\pi,K} - m^2_\mu}{2E_{\pi,K}\left(\dfrac{1-\beta^2}{2}+\dfrac{\theta^2}{2}\right)}, \beta \approx 1$$

$$E_\nu = \frac{m^2_{\pi,K} - m^2_\mu}{E_{\pi,K}\left(\dfrac{1}{\gamma^2}+\theta^2\right)} = \frac{m^2_{\pi,K}\left(1-\dfrac{m^2_\mu}{m^2_{\pi,K}}\right)E_{\pi,K}}{E^2_{\pi,K}\left(\dfrac{1}{\gamma^2}+\theta^2\right)}$$

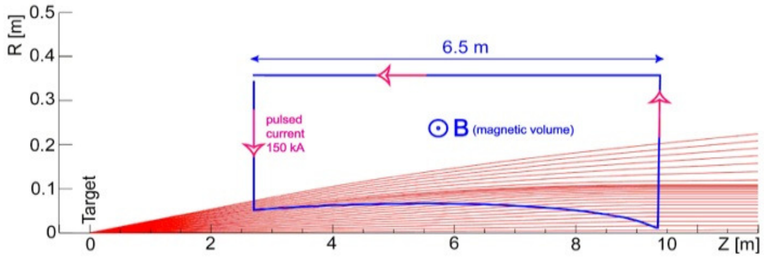
$$E_\nu = \frac{m^2_{\pi,K}\left(1-\dfrac{m^2_\mu}{m^2_{\pi,K}}\right)E_{\pi,K}}{\gamma^2 m^2_{\pi,K}\left(\dfrac{1}{\gamma^2}+\theta^2\right)} = E_{\pi,K}\frac{\left(1-\dfrac{m^2_\mu}{m^2_{\pi,K}}\right)}{\left(1+\gamma^2\theta^2\right)}$$

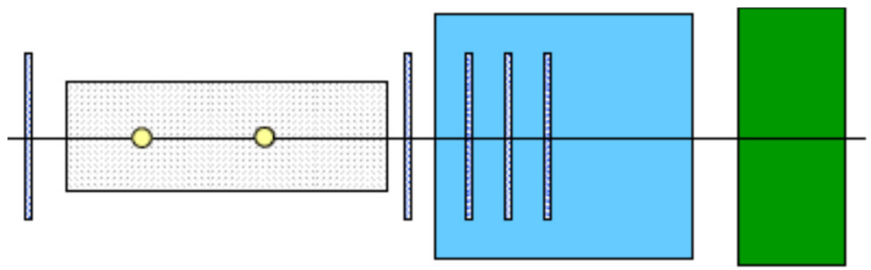


Neutral Currents Discovery - VIII

b) Wide Band Beam: ν energy unknown, high intensity

Replace magnetic selection by a special focussing device, suitable to make a low divergence hadron beam out of an uncollimated, divergent source: Van der Meer Horn

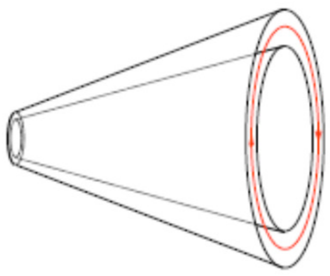


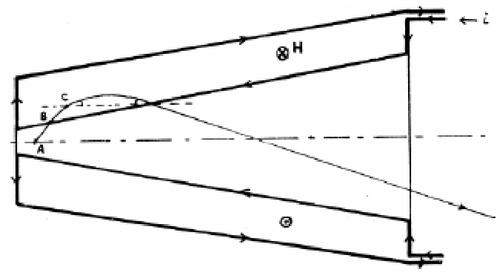


Collect a wide momentum window, focus into a narrow, intense beam

Neutral Currents Discovery - IX

2 conical (high) current sheets:
Equivalent to many trapezoidal current loops symmetrically placed around the axis
$\rightarrow$ Circular magnetic field (red circumference)





Trajectory in the B field: $\sim$ Circular arc

$$|\mathbf{p}| = 0.3BR \quad \begin{cases} p & GeV \\ B & T \\ R & m \end{cases}$$

Deflection after a path length l in the field:

$$\Delta\theta = \frac{\Delta l}{R} = 0.3B\frac{\Delta l}{|\mathbf{p}|}$$

Deflection should compensate $\langle p_T \rangle$ of hadrons coming out of the target:

$\langle p_T \rangle \sim p\Delta\theta \sim 0.2\ GeV$ at PS energies

$\rightarrow 0.2\ GeV \sim |\mathbf{p}|\Delta\theta = 0.3B\Delta l$

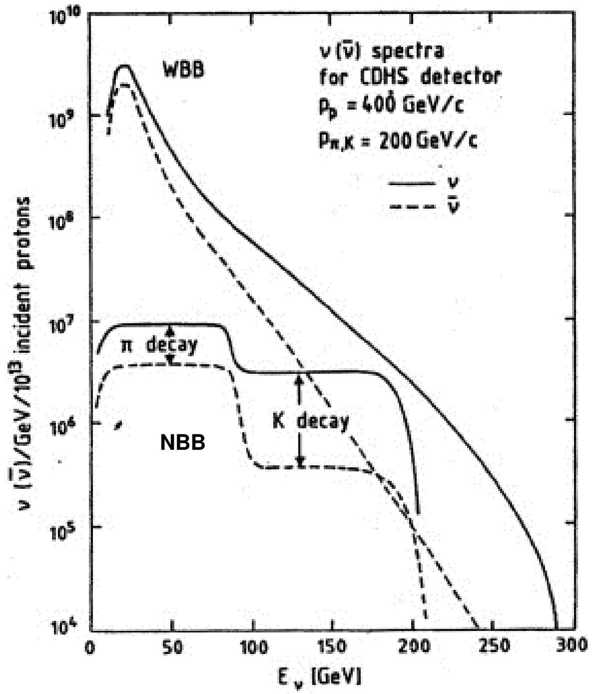
Simple guess:

$$B = \frac{\mu_0 I}{2\pi r} \rightarrow 0.2\ GeV \sim |\mathbf{p}|\Delta\theta \sim 0.3\frac{\mu_0 I}{2\pi r}\Delta l$$

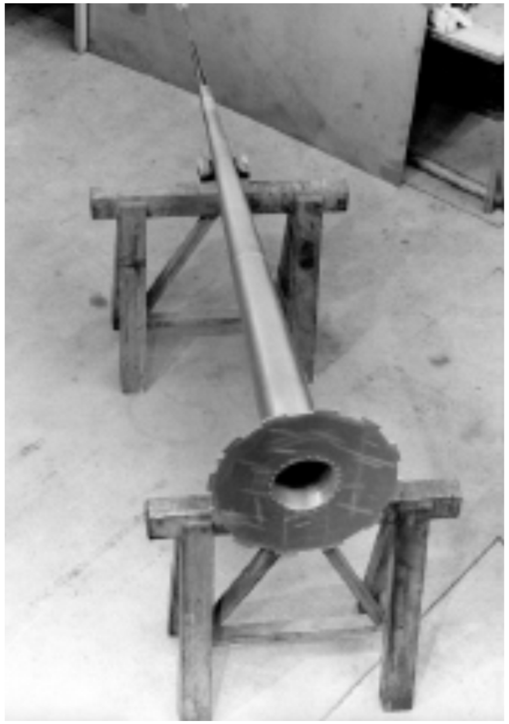
$$\rightarrow I \sim \frac{|\mathbf{p}|\Delta\theta 2\pi r}{0.3\mu_0 \Delta l} \sim 10^5\ A!$$

Neutral Currents Discovery - X

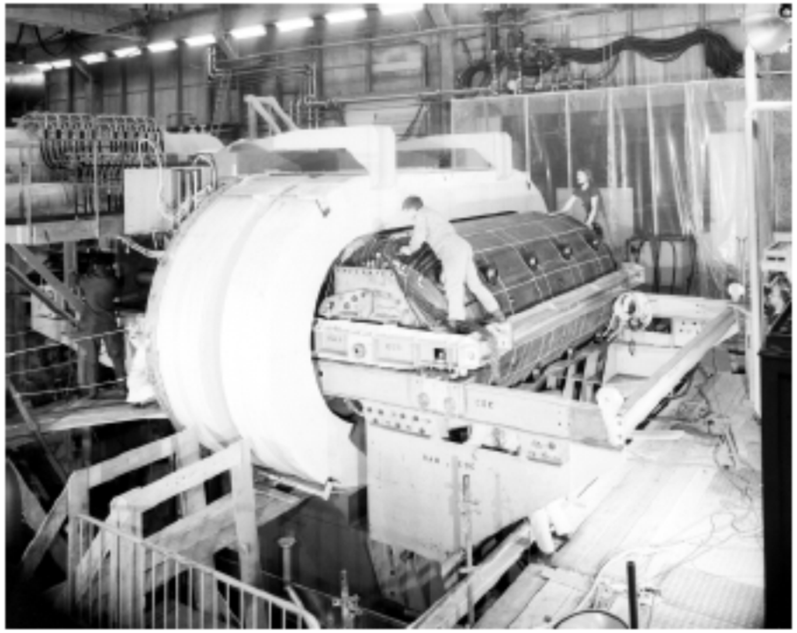
Narrow/Wideband beam spectra
SPS beam



The Gargamelle horn



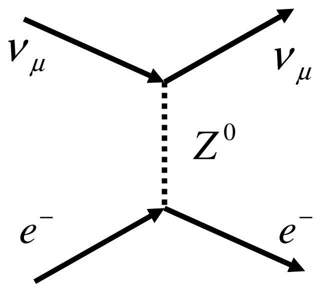
Neutral Currents Discovery - XI



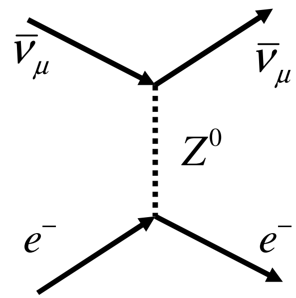
Length: 4.8 m
Diameter: 2 m
Liquid Freon: 12 m^3

Neutral Currents Discovery - XII

ν-e processes



Pure NC

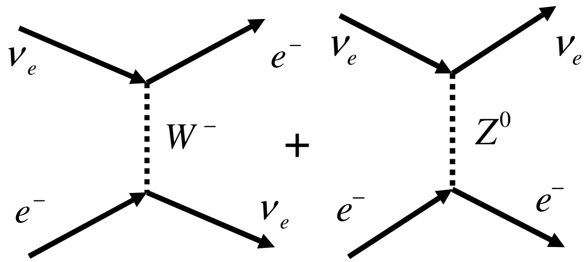


$$\sigma^{NC}_{\nu_\mu e^-}(E) = \frac{G_F{}^2 s}{\pi}\left[\left(-\frac{1}{2}+\sin^2\theta_W\right)^2+\frac{1}{3}\sin^4\theta_W\right]$$

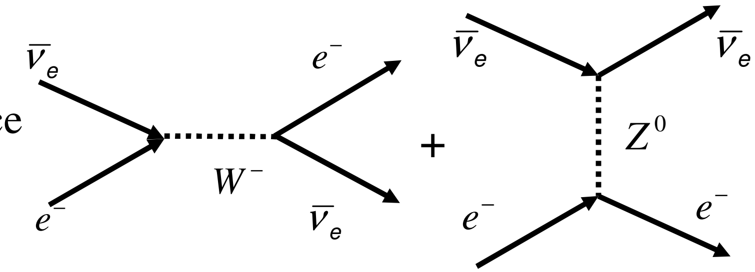
0.16x10^{-41} E(GeV) cm^2

$$\sigma^{NC}_{\bar{\nu}_\mu e^-}(E) = \frac{G_F{}^2 s}{\pi}\left[\frac{1}{3}\left(-\frac{1}{2}+\sin^2\theta_W\right)^2+\sin^4\theta_W\right]$$

0.13x10^{-41} E(GeV) cm^2



Interference
CC+NC



$$\sigma^{CC+NC}_{\nu_e e^-}(E) = \frac{G_F{}^2 s}{\pi}\left[\left(\frac{1}{2}+\sin^2\theta_W\right)^2+\frac{1}{3}\sin^4\theta_W\right]$$

0.96x10^{-41} E(GeV)

$$\sigma^{CC+NC}_{\bar{\nu}_e e^-}(E) = \frac{G_F{}^2 s}{\pi}\left[\frac{1}{3}\left(\frac{1}{2}+\sin^2\theta_W\right)^2+\sin^4\theta_W\right]$$

0.40x10^{-41} E(GeV) cm^2

Neutral Currents Discovery - XIII

Effective couplings for several reactions

		Electroweak theory		V–A theory	
Reaction	ε	g_V	g_A	g_V	g_A
$\nu_\mu + e^- \to \nu_\mu + e^-$	+1	$-\frac{1}{2} + 2s^2$	$-\frac{1}{2}$	0	0
$\bar{\nu}_\mu + e^- \to \bar{\nu}_\mu + e^-$	−1	$-\frac{1}{2} + 2s^2$	$-\frac{1}{2}$	0	0
$\nu_e + e^- \to \nu_e + e^-$	+1	$+\frac{1}{2} + 2s^2$	$+\frac{1}{2}$	1	1
$\bar{\nu}_e + e^- \to \bar{\nu}_e + e^-$	−1	$+\frac{1}{2} + 2s^2$	$+\frac{1}{2}$	1	1
$\nu_\mu + e^- \to \mu^- + \nu_e$	+1	1	1	1	1

Neutral Currents Discovery - XIV

Differential cross sections:

$$y = 1 - \frac{E_\nu'}{E_\nu} \simeq \frac{E_e}{E_\nu} \quad \text{Bjorken } y$$

$$\frac{d\sigma_{\nu_\mu e}}{dy} = \frac{G_F^2 m_e}{2\pi} E_\nu \left[(g_V + g_A)^2 + (g_V - g_A)^2 (1-y)^2 + \frac{m_e}{E_\nu} (g_A^2 - g_V^2) y \right]$$

$$\frac{d\sigma_{\bar{\nu}_\mu e}}{dy} = \frac{G_F^2 m_e}{2\pi} E_\nu \left[(g_V - g_A)^2 + (g_V + g_A)^2 (1-y)^2 + \frac{m_e}{E_\nu} (g_A^2 - g_V^2) y \right]$$

$$\int_0^1 (1-y)^2 dy = \frac{1}{3}, \quad \int_0^1 y dy = \frac{1}{2}$$

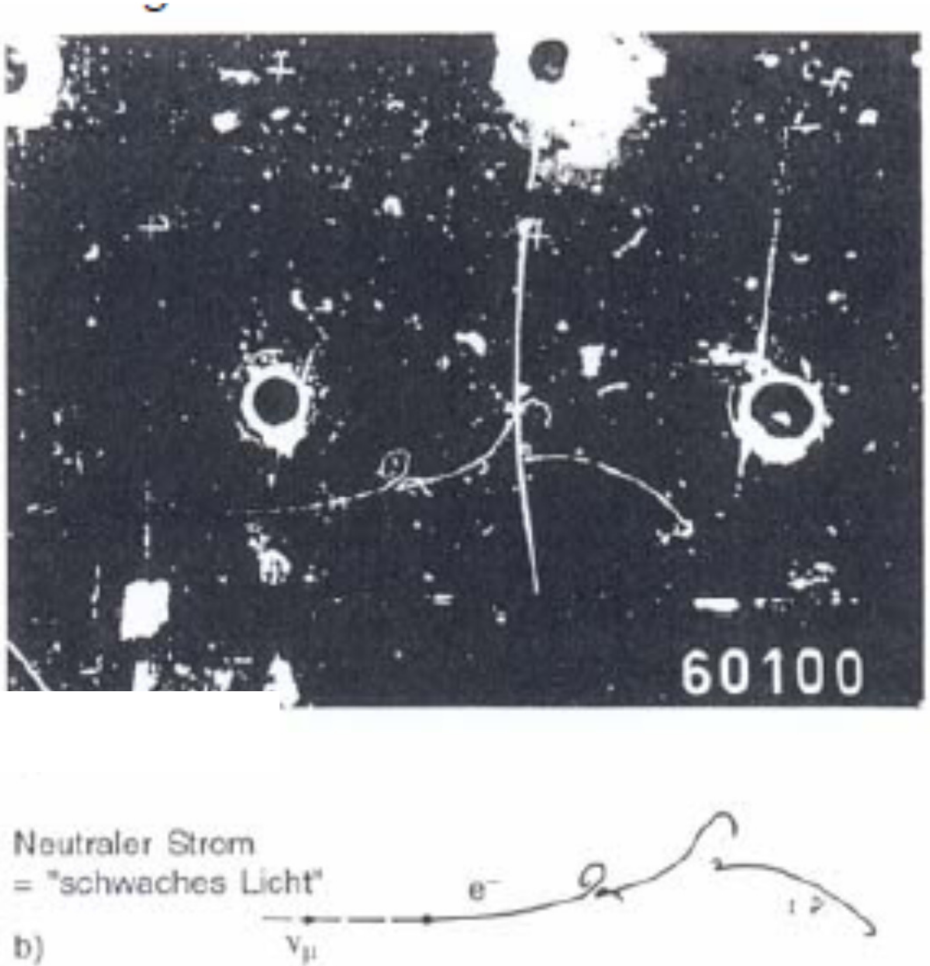
Total cross sections:

$$\rightarrow \sigma_{\nu_\mu e} = \frac{G_F^2 m_e}{2\pi} E_\nu \left[(g_V + g_A)^2 + \frac{1}{3}(g_V - g_A)^2 + \frac{m_e}{E_\nu} \frac{1}{2} (g_A^2 - g_V^2) \right]$$

$$\rightarrow \sigma_{\bar{\nu}_\mu e} = \frac{G_F^2 m_e}{2\pi} E_\nu \left[(g_V - g_A)^2 + \frac{1}{3}(g_V + g_A)^2 + \frac{m_e}{E_\nu} \frac{1}{2} (g_A^2 - g_V^2) \right]$$

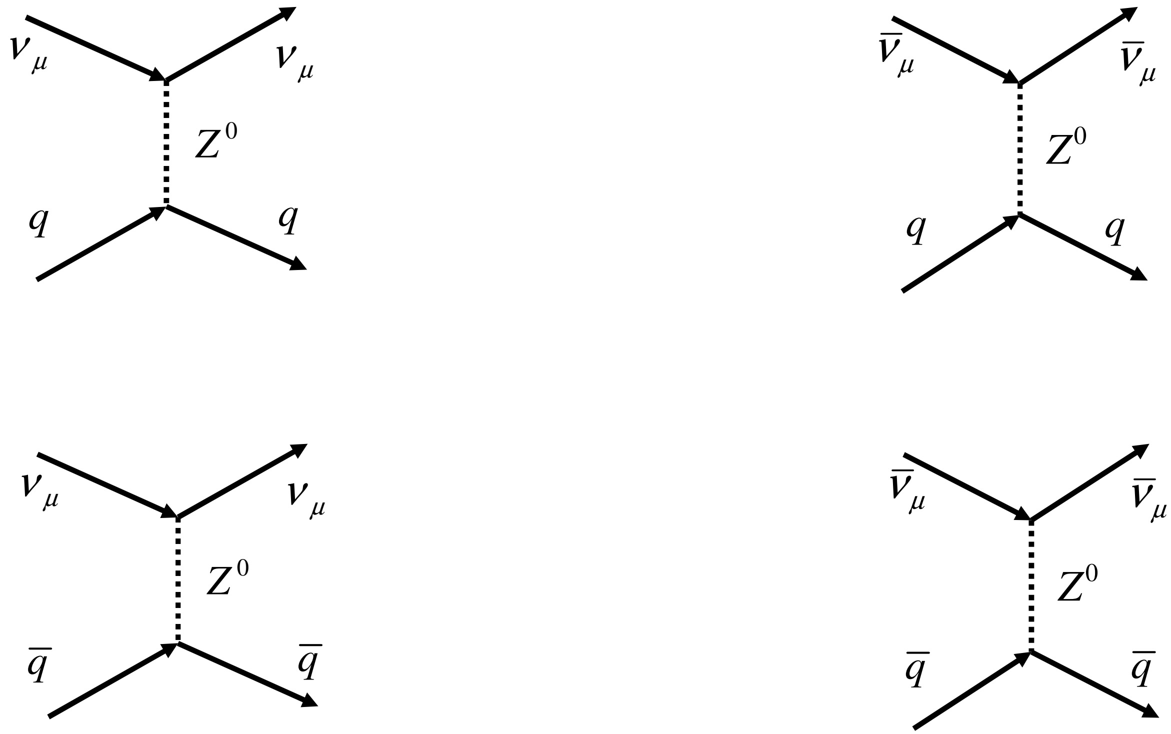
Neutral Currents Discovery - XV

First Gargamelle
leptonic neutral current event



Neutral Currents Discovery - XVI

$\overset{(-)}{\nu} - q, \bar{q}$ processes



Neutral Currents Discovery - XVII

$$g_V = \frac{1}{2} - \frac{4}{3}\sin^2\theta_W \qquad g_A = \frac{1}{2} \qquad u,c,t$$

$$g'_V = -\frac{1}{2} + \frac{4}{3}\sin^2\theta_W \qquad g'_A = \frac{1}{2} \qquad d,s,b$$

$$g_L = \frac{1}{2} - \frac{2}{3}\sin^2\theta_W \qquad g_R = -\frac{2}{3}\sin^2\theta_W \qquad u,c,t$$

$$g'_L = -\frac{1}{2} + \frac{1}{3}\sin^2\theta_W \qquad g_R = \frac{1}{3}\sin^2\theta_W \qquad d,s,b$$

$$\frac{d\sigma_{\nu_\mu q}}{dy} = \frac{d\sigma_{\bar{\nu}_\mu \bar{q}}}{dy} = \frac{G_F^2 m_q}{2\pi} E_\nu \left[\left(g_V + g_A\right)^2 + \left(g_V - g_A\right)^2 \left(1-y\right)^2 + \frac{m_q}{E}\left(g_A^2 - g_V^2\right) y \right]$$

$$\frac{d\sigma_{\bar{\nu}_\mu q}}{dy} = \frac{d\sigma_{\nu_\mu \bar{q}}}{dy} = \frac{G_F^2 m_q}{2\pi} E_\nu \left[\left(g_V - g_A\right)^2 + \left(g_V + g_A\right)^2 \left(1-y\right)^2 + \frac{m_q}{E_\nu}\left(g_A^2 - g_V^2\right) y \right]$$

Neutral Currents Discovery - XVIII

$$\frac{d\sigma_{\overset{(-)}{\nu}_\mu}}{dxdy} = \sum_q q(x)\frac{d\sigma_{\overset{(-)}{\nu}_\mu q}}{dy} + \sum_{\bar{q}} \bar{q}(x)\frac{d\sigma_{\overset{(-)}{\nu}_\mu \bar{q}}}{dy}$$

$$\rightarrow \frac{d\sigma_{\nu_\mu N}}{dxdy} = \frac{G_F^2 m_N}{2\pi} xE_\nu \left[\left(g_L{}^2 + g_L{}'^2\right)\left(q + \bar{q}(1-y)^2\right) + \left(g_R{}^2 + g_R{}'^2\right)\left(\bar{q} + q(1-y)^2\right)\right]$$

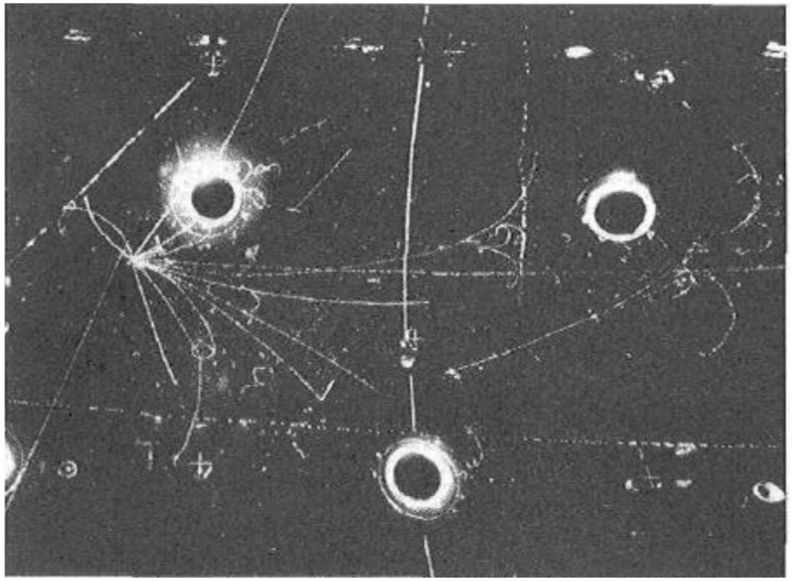
$$\rightarrow \frac{d\sigma_{\tilde{\nu}_\mu N}}{dxdy} = \frac{G_F^2 m_N}{2\pi} xE_\nu \left[\left(g_R{}^2 + g_R{}'^2\right)\left(q + \bar{q}(1-y)^2\right) + \left(g_L{}^2 + g_L{}'^2\right)\left(\bar{q} + q(1-y)^2\right)\right]$$

$$R_\nu^N = \frac{\sigma_{NC}(\nu)}{\sigma_{CC}(\nu)} \qquad R_{\bar{\nu}}^N = \frac{\sigma_{NC}(\bar{\nu})}{\sigma_{CC}(\bar{\nu})} \qquad r = \frac{\sigma_{CC}(\bar{\nu})}{\sigma_{CC}(\nu)}$$

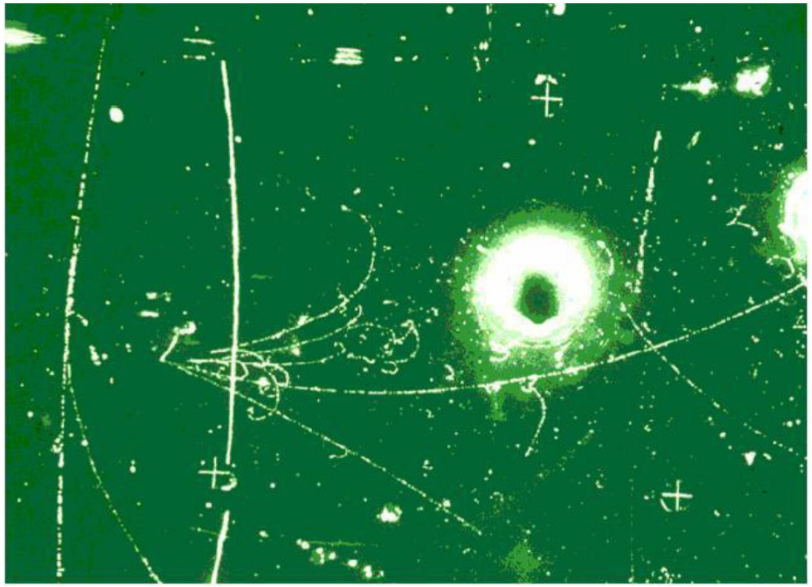
$$\rightarrow g_L{}^2 + g_L{}'^2 = \frac{R_\nu^N - r^2 R_{\bar{\nu}}^N}{1-r^2} \qquad g_R{}^2 + g_R{}'^2 = \frac{r\left(R_\nu^N - R_{\bar{\nu}}^N\right)}{1-r^2}$$

Neutral Currents Discovery - XIX

Gargamelle
charged current



Gargamelle
hadronic neutral current event

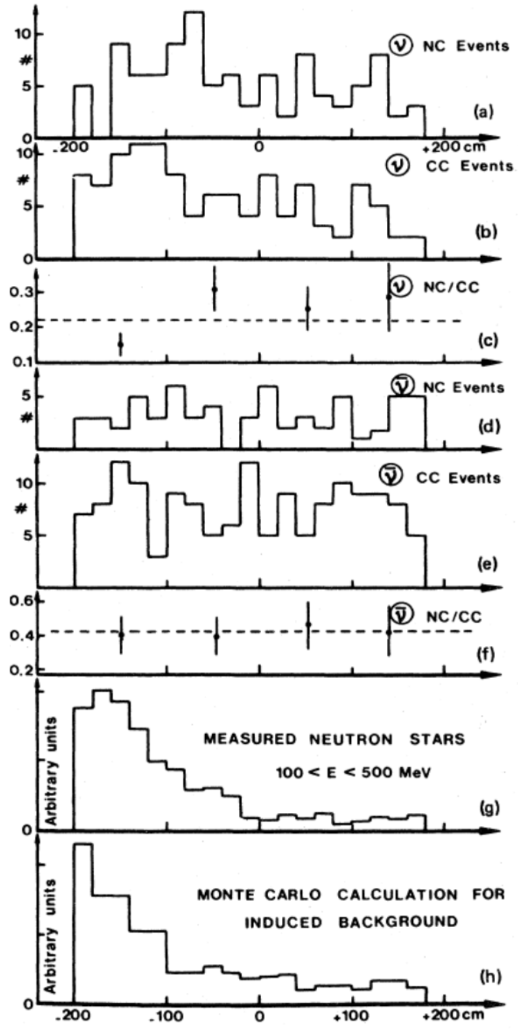


Neutral Currents Discovery - XX

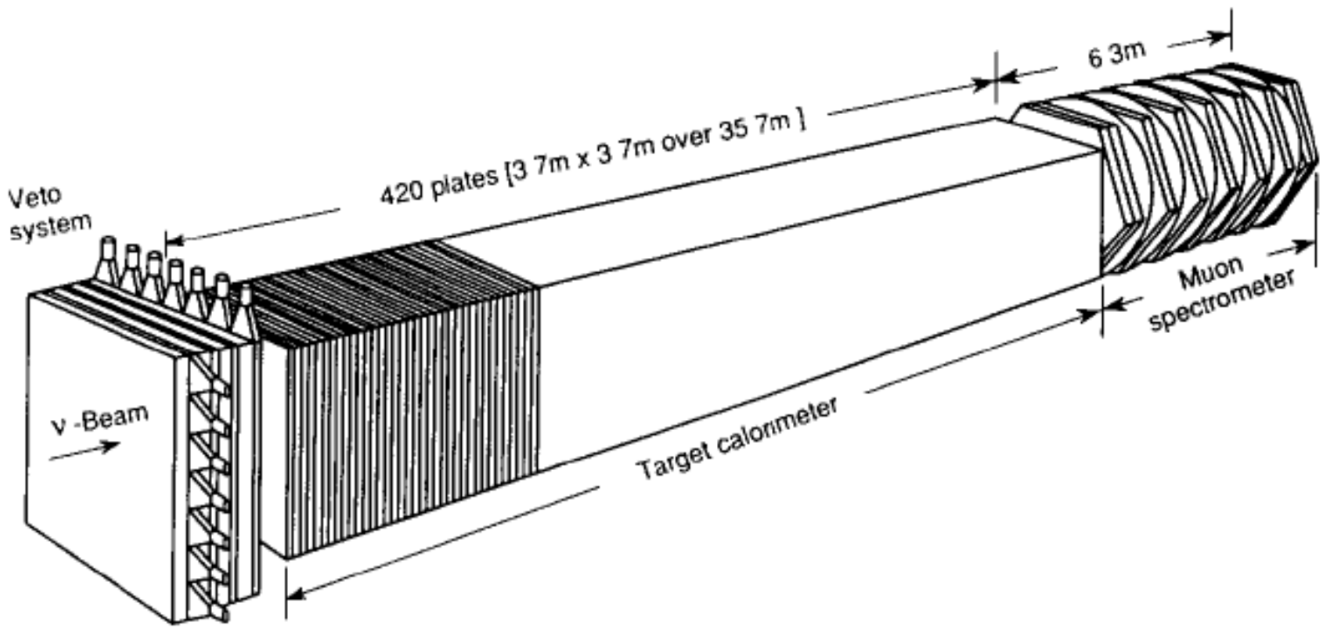
Background

Result

$$\sin^2\theta_W = 0.3 \div 0.4$$



Neutral Currents Discovery - XXI

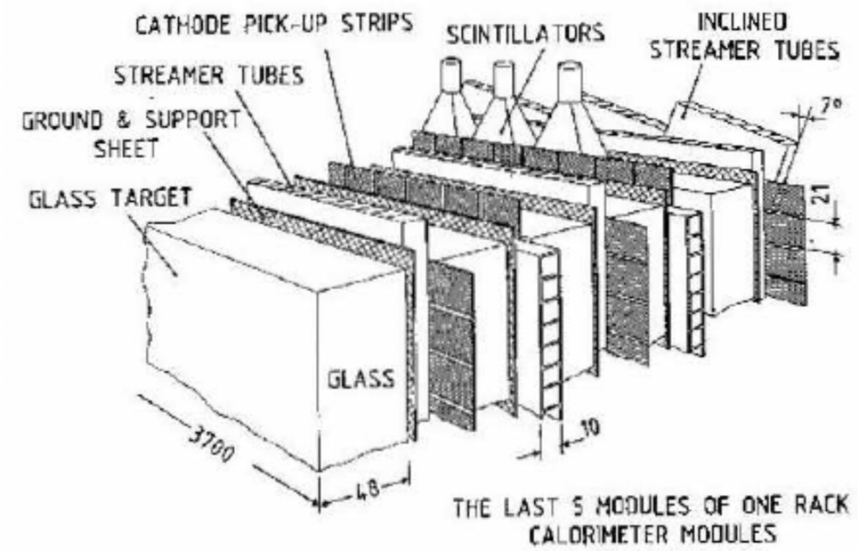


CHARM II

Neutral Currents Discovery - XXII

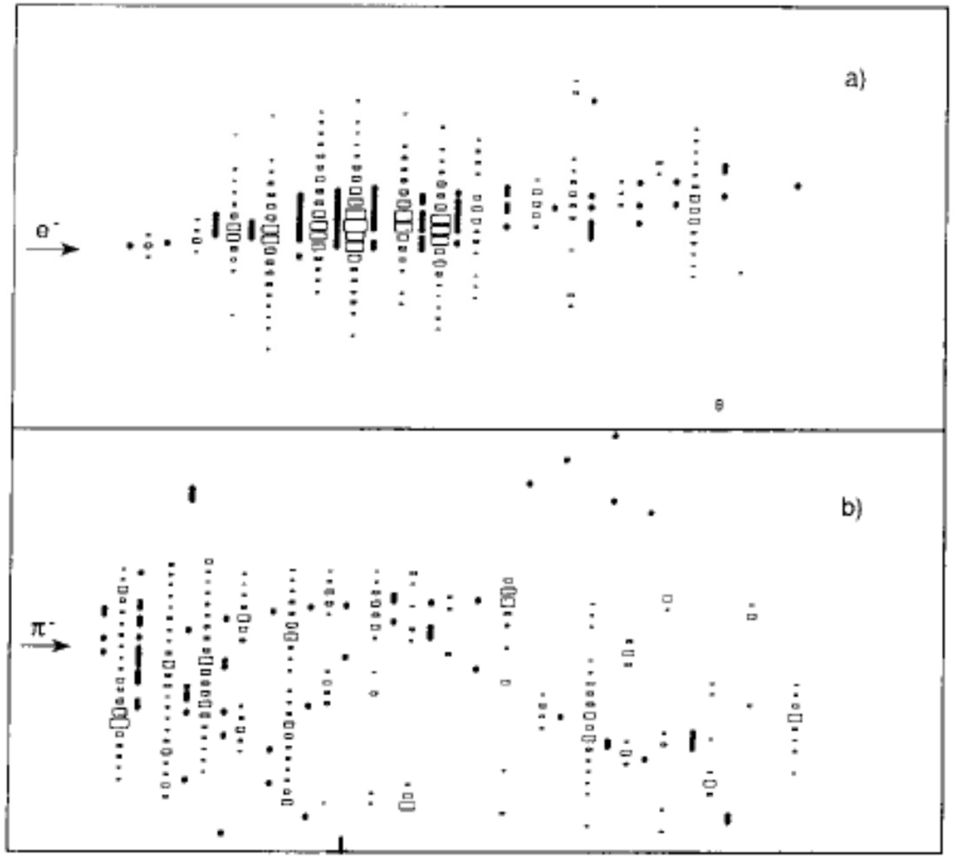
Experimental setup (CHARM II, 1987-1991)

electronic tracking detector



$\sim 700\,t$ calorimeter, digital readout of energy and direction of produced particles

Neutral Currents Discovery - XXIII



Electromagnetic shower

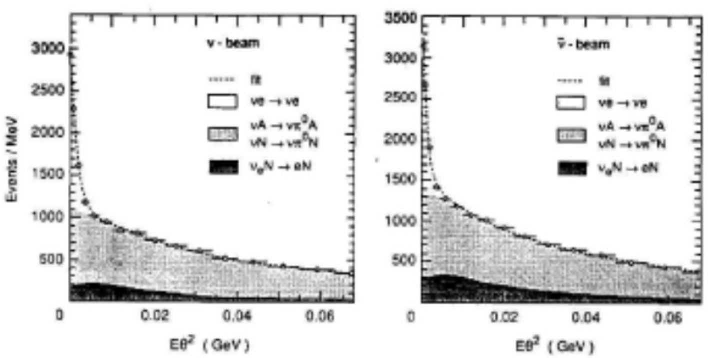
Hadronic shower

Neutral Currents Discovery- XXIV

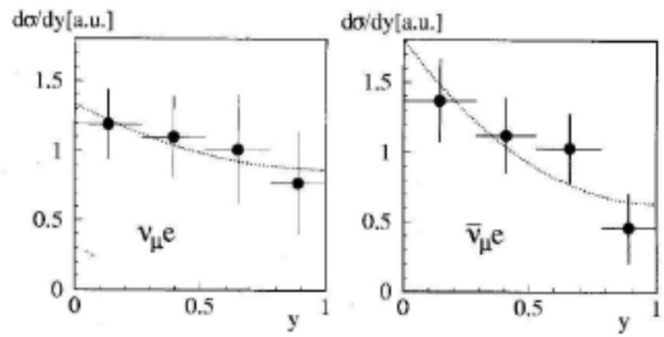
CHARM II data

Problem: discrimination of the NC events (~ 2500 for $\nu_\mu e^-$ and $\bar{\nu}_\mu e^-$ each) from the dominant background (CC scattering, inelastic scattering)

Solution: in processes of interest $\nu_\mu e \to \nu_\mu e$ and $\bar{\nu}_\mu e^- \to \bar{\nu}_\mu e^-$ the value $E_e\theta_e^2$ is kinematically restricted to small values



Phys.Lett. B335, 246 (1994)



Phys.Lett. B302, 351 (1993)

$$\sin^2\Theta_{\nu e} = 0.2324 \pm 0.0083$$

W & Z - I

Some reminescences about photons...

Free photons ($j^\mu = 0$): $\quad \Box^2 A^\mu - \partial^\mu \left(\partial_\nu A^\nu\right) = j^\mu$

Lorentz condition

$\partial_\mu A^\mu = 0 \rightarrow \Box^2 A^\mu = 0$

$\rightarrow A^\mu = \varepsilon^\mu (q) e^{-iqx} \rightarrow q^2 = 0$ massless quanta

4 components ε^μ ??

$a)$ $\partial_\mu A^\mu = 0 \rightarrow \varepsilon^\mu (q) q_\mu = 0 \rightarrow 3$ components

$b)$ Gauge freedom:

$A^\mu \rightarrow A^\mu + \partial^\mu \Lambda, \ \Box^2 \Lambda = 0$

$\Lambda = iae^{-iqx} \left(\leftarrow \ \Box^2 \Lambda = q^2 \Lambda = 0 \text{ OK}\right)$

$\rightarrow \partial^\mu \Lambda = ia\partial^\mu e^{-iqx}$

$A^\mu \rightarrow A^\mu + \partial^\mu \Lambda = \varepsilon^\mu (q) e^{-iqx} + ia\partial^\mu e^{-iqx} = \left[\varepsilon^\mu (q) + ia\left(-iq_\mu\right)\right] e^{-iqx} = \left[\varepsilon^\mu (q) + aq_\mu\right] e^{-iqx}$

$\rightarrow$ EM field unchanged by $\varepsilon^\mu (q) \rightarrow \varepsilon^\mu (q) + aq^\mu$

Choose a to make $\varepsilon^0 = 0$

$\rightarrow \varepsilon^\mu (q) q_\mu = 0 \rightarrow \boldsymbol{\varepsilon} \cdot \mathbf{q} = 0 \rightarrow 2$ components

W & Z - II

2 components $\rightarrow$ 2 independent ε^{μ}

Take photon momentum along z

$$\varepsilon_1^{\mu} = \begin{pmatrix} 0 & 1 & 0 & 0 \end{pmatrix} \quad x\text{ - linear polarization}$$

$$\varepsilon_2^{\mu} = \begin{pmatrix} 0 & 0 & 1 & 0 \end{pmatrix} \quad y\text{ - linear polarization}$$

or

$$\varepsilon_L^{\mu} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & -i & 0 \end{pmatrix} \text{ Left circular polarization: } S_z = -1$$

$$\varepsilon_R^{\mu} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & +i & 0 \end{pmatrix} \text{ Right circular polarization: } S_z = +1$$

W & Z - III

Original wave equation:

$$\Box^2 A^\mu - \partial^\mu \left(\partial_\nu A^\nu\right) = j^\mu$$

For a massive vector boson:

$$\left(\Box^2 + m^2\right) B^\mu - \partial^\mu \left(\partial_\nu B^\nu\right) = j^\mu$$

Free particle:

$$\left(\Box^2 + m^2\right) B^\mu - \partial^\mu \left(\partial_\nu B^\nu\right) = 0$$

But:

$$\partial_\mu \left(\Box^2 + m^2\right) B^\mu - \partial_\mu \partial^\mu \left(\partial_\nu B^\nu\right) = 0 \rightarrow \left(\Box^2 + m^2\right) \partial_\mu B^\mu - \Box^2 \left(\partial_\nu B^\nu\right) = 0$$

$$\rightarrow m^2 \partial_\mu B^\mu = 0 \rightarrow \partial_\mu B^\mu = 0$$

Bottom line: Not an extra condition...

$$\rightarrow \left(\Box^2 + m^2\right) B^\mu = 0$$

$$B^\mu = \varepsilon^\mu \left(p\right) e^{-ipx} \rightarrow \varepsilon^\mu p_\mu = 0 \rightarrow 3 \text{ independent components}$$

No gauge freedom...

W & Z - IV

3 independent components → 3 independent ε^μ

Take photon momentum along z

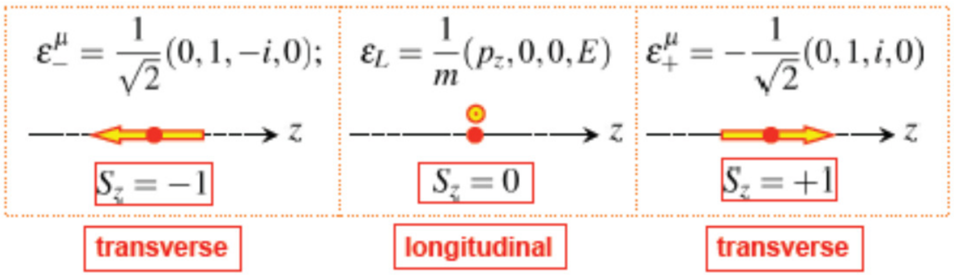
$$\varepsilon_L^\mu = \frac{1}{\sqrt{2}}\begin{pmatrix}0 & 1 & -i & 0\end{pmatrix} \text{ Left circular polarization: } S_z = -1$$

$$\varepsilon_R^\mu = -\frac{1}{\sqrt{2}}\begin{pmatrix}0 & 1 & +i & 0\end{pmatrix} \text{ Right circular polarization: } S_z = +1$$

To find 3rd polarization 4-vector:

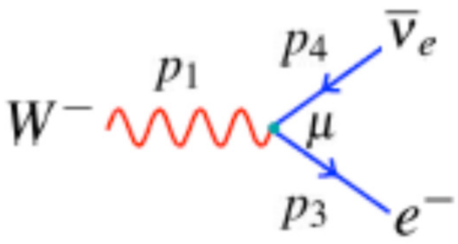
$$\varepsilon_0^\mu = \frac{1}{\sqrt{\alpha^2-\beta^2}}\begin{pmatrix}\alpha & 0 & 0 & \beta\end{pmatrix} \to \varepsilon^\mu p_\mu = 0 \to \alpha E - \beta p_z = 0, \quad \frac{1}{\sqrt{\alpha^2-\beta^2}} \text{ normalization}$$

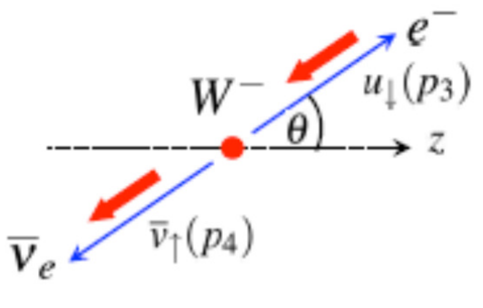
$$\to \varepsilon_0^\mu = \frac{1}{m}\begin{pmatrix}p_z & 0 & 0 & E\end{pmatrix} \text{ Longitudinal polarization: } S_z = 0$$



W & Z - V

Decay: $W^- \to e^- + \overline{\nu}_e$





Matrix element:

$$M_{fi} = \frac{g}{\sqrt{2}}\varepsilon_\mu(p_1)\overline{u}(p_3)\frac{1}{2}\gamma^\mu(1-\gamma_5)v(p_4) = \frac{g}{\sqrt{2}}\varepsilon_\mu(p_1)\, j^\mu$$

$\overline{u}(p_3)$: outgoing fermion, $v(p_4)$: outgoing antifermion

$$\overline{u}(p_3)\gamma^\mu\frac{1}{2}(1-\gamma_5)v(p_4) = \overline{u}(p_3)\frac{1}{2}\gamma^\mu(1-\gamma_5)\frac{1}{2}\gamma^\mu(1-\gamma_5)v(p_4)$$

$$= \underbrace{\overline{u}(p_3)\frac{1}{2}(1+\gamma_5)}_{\overline{e}_L}\gamma^\mu\underbrace{\frac{1}{2}(1-\gamma_5)v(p_4)}_{v_R} = \overline{e}_L\gamma^\mu v_R$$

LR current :

Build from rotated L, R spinors

$$j^\mu = \overline{u}_\downarrow(p_3)\frac{1}{2}\gamma^\mu v_\uparrow(p_4) = 2E(0, -\cos\theta, -i, \sin\theta)$$

W & Z - VI

W polarization states in the rest system:

$$\varepsilon_L^\mu = \frac{1}{\sqrt{2}}\begin{pmatrix}0 & 1 & -i & 0\end{pmatrix}$$

$$\varepsilon_R^\mu = -\frac{1}{\sqrt{2}}\begin{pmatrix}0 & 1 & +i & 0\end{pmatrix}$$

$$\varepsilon_0^\mu = \frac{1}{m}\begin{pmatrix}0 & 0 & 0 & m\end{pmatrix} = \begin{pmatrix}0 & 0 & 0 & 1\end{pmatrix}$$

Matrix elements for different W polarization states in the rest system:

$$\varepsilon_L^\mu : \quad \frac{g}{\sqrt{2}}\frac{1}{\sqrt{2}}\begin{pmatrix}0 & 1 & -i & 0\end{pmatrix} 2\frac{M_W}{2}\left(0, -\cos\theta, -i, \sin\theta\right) = -\frac{gM_W}{2}\left(1+\cos\theta\right) \rightarrow \left|M_L\right|^2 = \frac{g^2 M_W^2}{4}\left(1+\cos\theta\right)^2$$

$$\varepsilon_R^\mu : \quad \frac{g}{\sqrt{2}}\left[-\frac{1}{\sqrt{2}}\begin{pmatrix}0 & 1 & +i & 0\end{pmatrix}\right] 2\frac{M_W}{2}\left(0, -\cos\theta, -i, \sin\theta\right) = -\frac{gM_W}{2}\left(1-\cos\theta\right) \rightarrow \left|M_R\right|^2 = \frac{g^2 M_W^2}{4}\left(1-\cos\theta\right)^2$$

$$\varepsilon_0^\mu : \quad \frac{g}{\sqrt{2}}\frac{1}{m}\begin{pmatrix}0 & 0 & 0 & m\end{pmatrix} 2\frac{M_W}{2}\left(0, -\cos\theta, -i, \sin\theta\right) = \frac{gM_W}{\sqrt{2}}\sin\theta \rightarrow \left|M_0\right|^2 = \frac{g^2 M_W^2}{2}\sin^2\theta$$

W & Z - VII

$$|M_L|^2 = \frac{g^2 M_W^2}{4}(1+\cos\theta)^2$$

$$|M_R|^2 = \frac{g^2 M_W^2}{4}(1-\cos\theta)^2$$

$$|M_0|^2 = \frac{g^2 M_W^2}{2}\sin^2\theta$$

2-body differential decay rate:

$$\frac{d\Gamma_{L,0,R}}{d\Omega} = \frac{p}{32\pi^2 M_W^2}|M|^2 = \frac{1}{64\pi^2 M_W}|M|^2 = \frac{g^2 M_W}{64\pi^2}\begin{cases}\frac{1}{4}(1+\cos\theta)^2 \\ \frac{1}{2}\sin^2\theta \\ \frac{1}{4}(1-\cos\theta)^2\end{cases}$$

Total rates:

$$\int \frac{1}{4}(1\pm\cos\theta)^2\, d\cos\theta d\varphi = \int \frac{1}{2}\sin^2\theta d\cos\theta d\varphi = \frac{4\pi}{3}$$

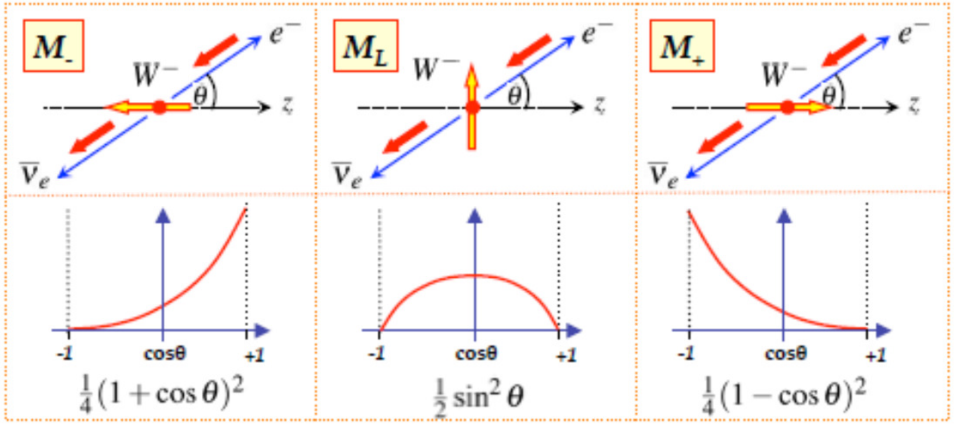
$$\rightarrow \Gamma_L = \Gamma_R = \Gamma_0 = \frac{g^2 M_W}{48\pi}$$

W & Z - VIII

$$\left|M_L\right|^2 = \frac{g^2 M_W^2}{4}(1+\cos\theta)^2$$

$$\left|M_R\right|^2 = \frac{g^2 M_W^2}{4}(1-\cos\theta)^2$$

$$\left|M_0\right|^2 = \frac{g^2 M_W^2}{2}\sin^2\theta$$



Averaging over the initial spin states:

$$\left\langle |M|^2 \right\rangle = \frac{1}{3} g^2 M_W^2 \left[\frac{1}{4}(1+\cos\theta)^2 + \frac{1}{4}(1-\cos\theta)^2 + \frac{1}{2}\sin^2\theta \right]$$

$$\to \left\langle |M|^2 \right\rangle = \frac{1}{3} g^2 M_W^2$$

Isotropic: OK for an unpolarized mother particle

$$\to \Gamma\left(W^- \to e^- + \overline{\nu}_e\right) = \frac{g^2 M_W}{48\pi}$$

W & Z - IX

Considering all the others decay modes: Large W mass $\rightarrow$ All fermions $\approx$ massless

Do *not* count Top: Too heavy, decay energetically forbidden

Color factor = 3

Similar to $e^{+}e= \rightarrow q\overline{q}$: Take quarks as free, on shell particles

Taking into account CKM mixing:

$W^{-} \rightarrow e^{-}\overline{\nu}_{e}$ $\quad W^{-} \rightarrow d\overline{u} \quad \times 3|V_{ud}|^{2}$ $\quad W^{-} \rightarrow d\overline{c} \quad \times 3|V_{cd}|^{2}$

$W^{-} \rightarrow \mu^{-}\overline{\nu}_{\mu}$ $\quad W^{-} \rightarrow s\overline{u} \quad \times 3|V_{us}|^{2}$ $\quad W^{-} \rightarrow s\overline{c} \quad \times 3|V_{cs}|^{2}$

$W^{-} \rightarrow \tau^{-}\overline{\nu}_{\tau}$ $\quad W^{-} \rightarrow b\overline{u} \quad \times 3|V_{ub}|^{2}$ $\quad W^{-} \rightarrow b\overline{c} \quad \times 3|V_{cb}|^{2}$

CKM Unitarity:

$e.g.\ |V_{ud}|^{2} + |V_{us}|^{2} + |V_{ub}|^{2} = 1$ etc

$$\rightarrow \Gamma_{W} = 9\Gamma_{W\rightarrow e\nu} = \frac{3g^{2}M_{W}}{16\pi} = 2.07\ GeV$$

Experiment :

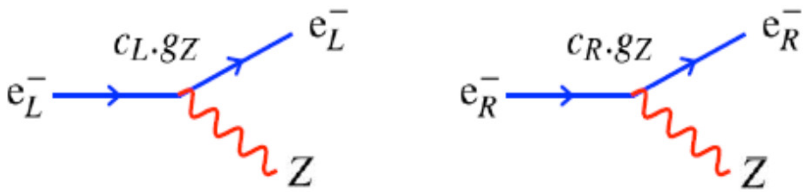
$2.14 \pm 0.04\ GeV$

QCD corrections..

W & Z - X

Z couplings:

$$c_L = I_3 - Q\sin^2\theta_W$$
$$c_R = -Q\sin^2\theta_W$$
$$j_\mu^Z = g_Z \bar{u}\gamma_\mu \left[c_L \frac{1}{2}(1-\gamma_5) + c_R \frac{1}{2}(1+\gamma_5) \right] u$$



$$c_V = c_L + c_R = I_3 - 2Q\sin^2\theta_W$$
$$c_A = c_L - c_R = I_3$$

W & Z - XI

Therefore:

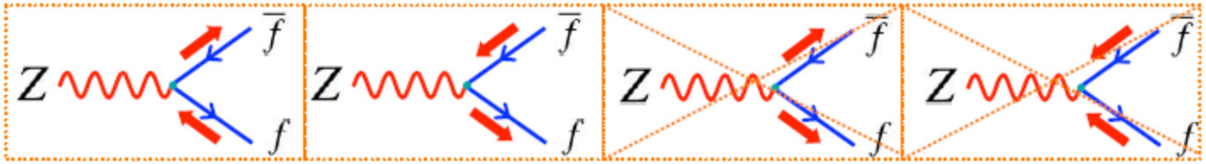
$$\sin^2\theta_W \approx 0.23$$

Fermion	Q	I_W^3	c_L	c_R	c_V	c_A
ν_e, ν_μ, ν_τ	0	$+\frac{1}{2}$	$+\frac{1}{2}$	0	$+\frac{1}{2}$	$+\frac{1}{2}$
e^-, μ^-, τ^-	-1	$-\frac{1}{2}$	-0.27	0.23	-0.04	$-\frac{1}{2}$
u, c, t	$+\frac{2}{3}$	$+\frac{1}{2}$	0.35	-0.15	$+0.19$	$+\frac{1}{2}$
d, s, b	$-\frac{1}{3}$	$-\frac{1}{2}$	-0.42	0.08	-0.35	$-\frac{1}{2}$

W & *Z* - XII

Z couplings: Both to *L* and *R* fermions

Nevertheless:
Only 2 vertexes, remaining 2 = 0



To show that RR vertex is 0 (LL similar):

$$\bar{u}_R = u_R^\dagger \gamma^0 = u^\dagger \frac{1+\gamma^5}{2}\gamma^0, \quad v_R = \frac{1-\gamma^5}{2} v$$

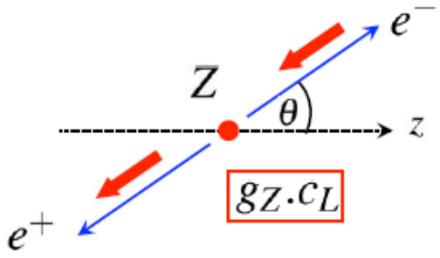
$$\bar{u}_R \gamma^\mu (c_V + c_A\gamma_5) v_R = u^\dagger \frac{1+\gamma^5}{2}\gamma^0\gamma^\mu (c_V + c_A\gamma_5)\frac{1-\gamma^5}{2} v$$

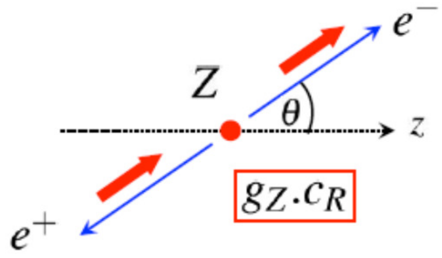
$$= \frac{1}{4} u^\dagger \gamma^0 (1-\gamma^5)(1-\gamma^5)(c_V + c_A\gamma_5) v$$

$$= \bar{u}\gamma^\mu \underbrace{\frac{1+\gamma^5}{2}\frac{1-\gamma^5}{2}}_{=0}(c_V + c_A\gamma_5) v = 0$$

W & Z - XIII

Decay: $Z^0 \to e^+ + e^-$





$$\left\langle |M|^2 \right\rangle = \frac{2}{3} g^2 \cos^2\theta_W M_Z^2 \left[c_L^2 + c_R^2\right]$$

$$2\left[c_L^2 + c_R^2\right] = \left[c_V^2 + c_A^2\right]$$

$$\to \Gamma\left(Z \to e^+ e^-\right) = \frac{g^2 \cos^2\theta_W M_Z}{48\pi}\left[c_V^2 + c_A^2\right]$$

W & Z - XIV

$$Br(Z \to e^+e^-) = Br(Z \to \mu^+\mu^-) = Br(Z \to \tau^+\tau^-) \approx 3.5\%$$

$$Br(Z \to \nu_1\bar{\nu}_1) = Br(Z \to \nu_2\bar{\nu}_2) = Br(Z \to \nu_3\bar{\nu}_3) \approx 6.9\%$$

$$Br(Z \to d\bar{d}) = Br(Z \to s\bar{s}) = Br(Z \to b\bar{b}) \approx 15\%$$

$$Br(Z \to u\bar{u}) = Br(Z \to c\bar{c}) \approx 12\%$$

$$Br(Z \to \text{hadrons}) \approx 69\%$$

$$\to \Gamma_Z = \sum_i \Gamma_i = 2.5 \quad GeV$$

$$\text{Experiment: } \Gamma_Z = 2.4952 \pm 0.0023 \quad GeV$$

!!!

W & Z - XV

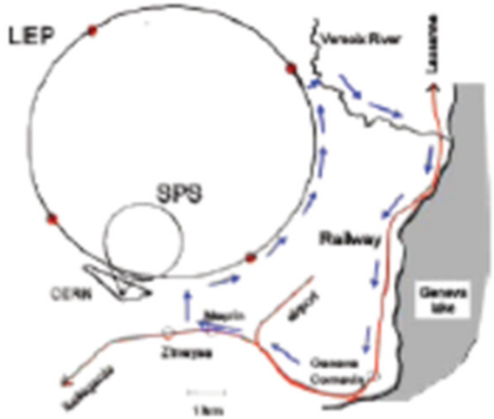
Ultimate systematics….

Moon:

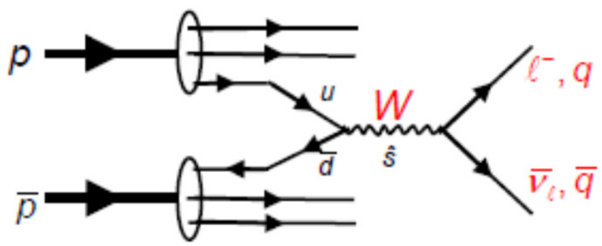
- As the moon orbits the Earth it distorts the rock in the Geneva area very slightly !
- The nominal radius of the accelerator of 4.3 km varies by ±0.15 mm
- Changes beam energy by ~10 MeV : need to correct for tidal effects !

Trains:

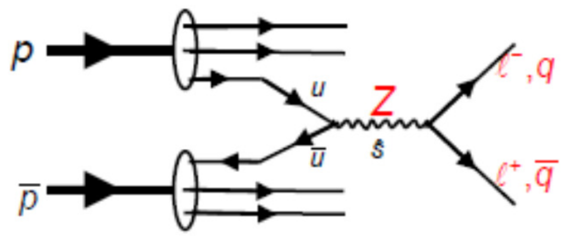
- Leakage currents from the TGV railway line return to Earth following the path of least resistance.
- Travelling via the Versoix river and using the LEP ring as a conductor.
- Each time a TGV train passed by, a small current circulated LEP slightly changing the magnetic field in the accelerator
- LEP beam energy changes by ~10 MeV



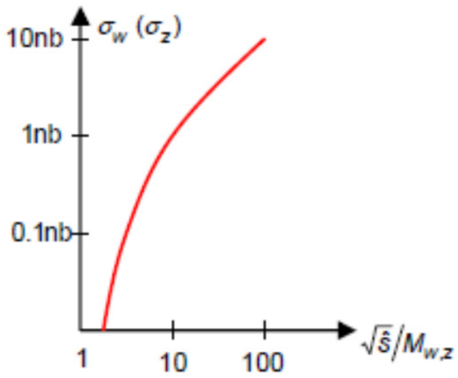
W & Z Discovery - I



$p\bar{p} \to W \to \ell \bar{\nu}_\ell + X$



$p\bar{p} \to Z \to f\bar{f} + X$



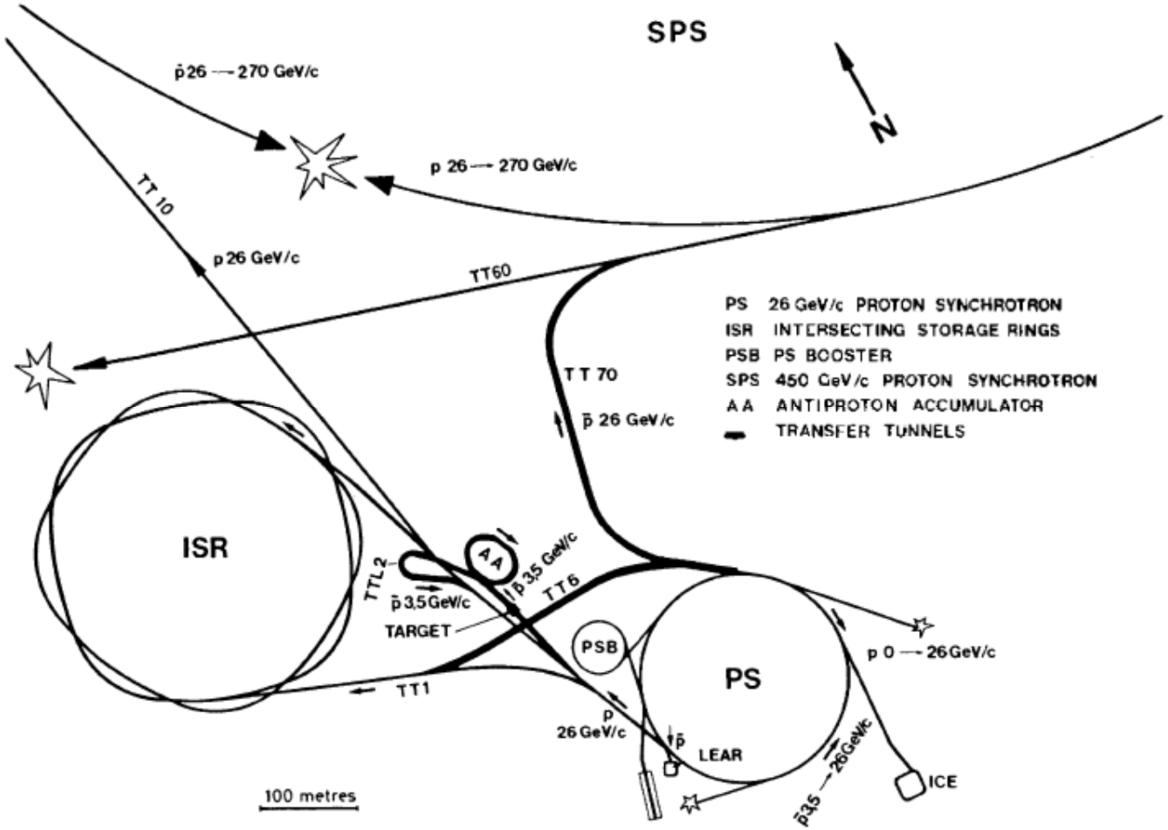
Similar to Drell-Yan: (photon instead of W)

$\hat{s} = x_q x_{\bar{q}} s \qquad \langle x_q \rangle \approx 0.12$

$\hat{s} = \langle x_q \rangle^2 s \approx 0.014 s = (65\ \text{GeV})^2$

$\to$ Cross section is small !

W & *Z* Discovery - II



W & Z Discovery - III

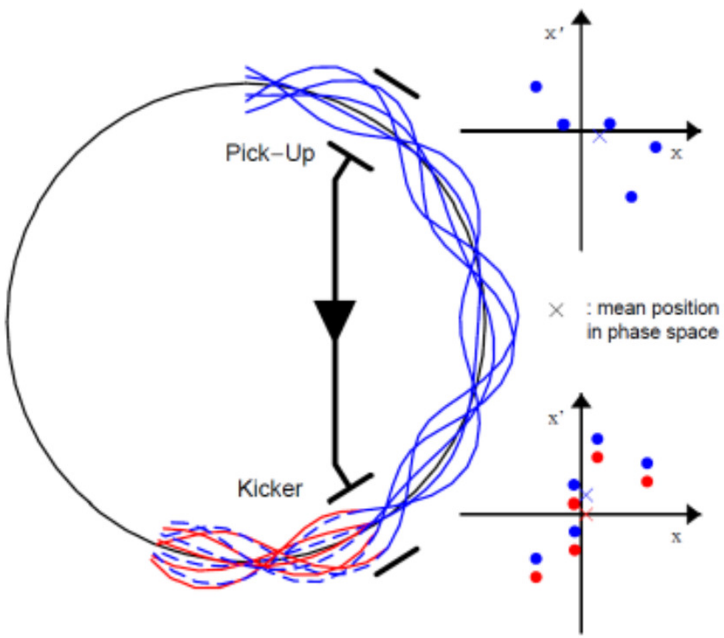
$S\bar{p}pS$ Collider main parameters

- $\sqrt{s}$ = 540 GeV
- 3 bunches protons, 3 bunches antiprotons, 10^{11} particles per bunch
- Luminosity = 5 x 10^{27} $cm^{-2}sec^{-1}$
- First collisions in December 1981

W & *Z* Discovery - IV

Stochastical cooling system

Basic principle



- 10^7 antiprotons with $p = 3.5$ GeV/c gets in outer part of toroidale vacuum chamber
- Inductor measures discrepancy of particles
- Correction signal is send to opposite side
- Magnet deflects particles

W & *Z* Discovery - V

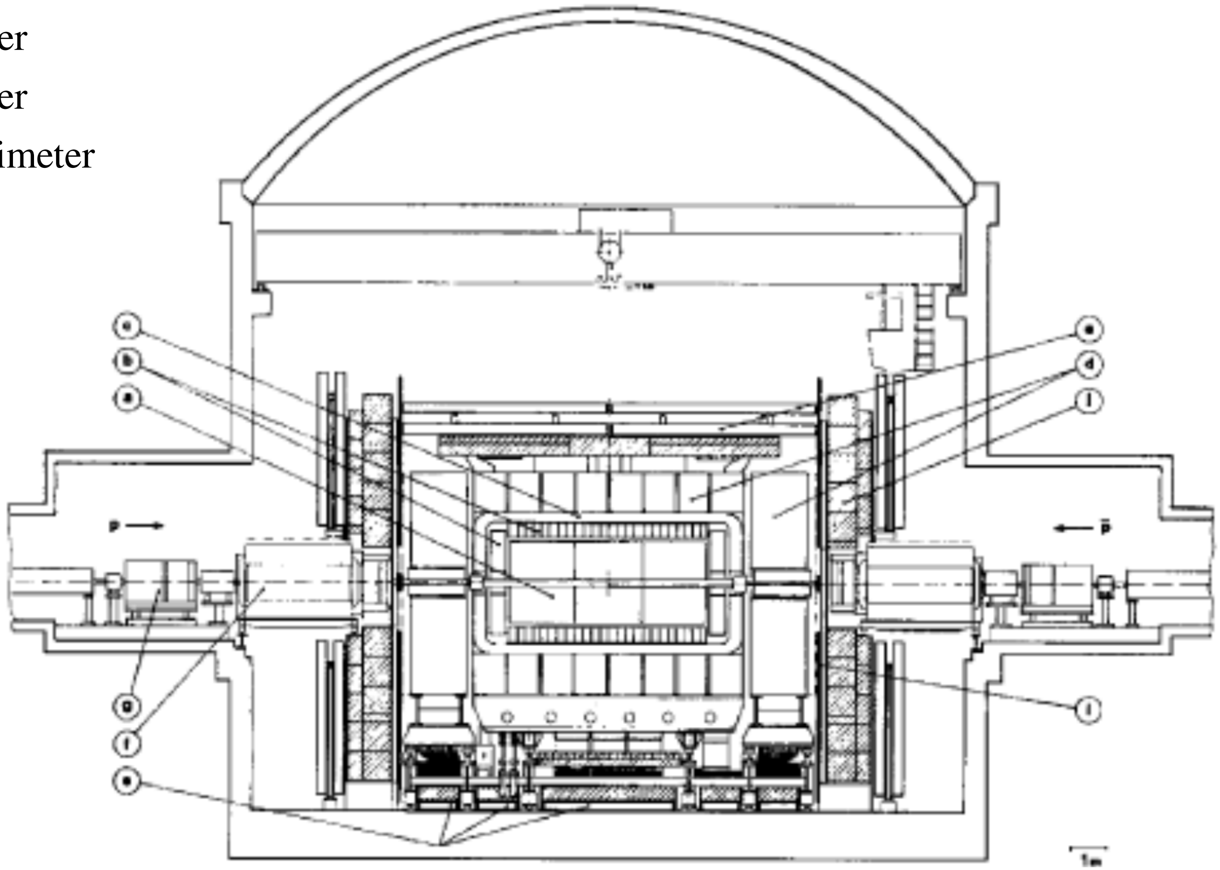
*UA*1 Detector

Central Tracker

EM calorimeter

Hadron Calorimeter

Muon System



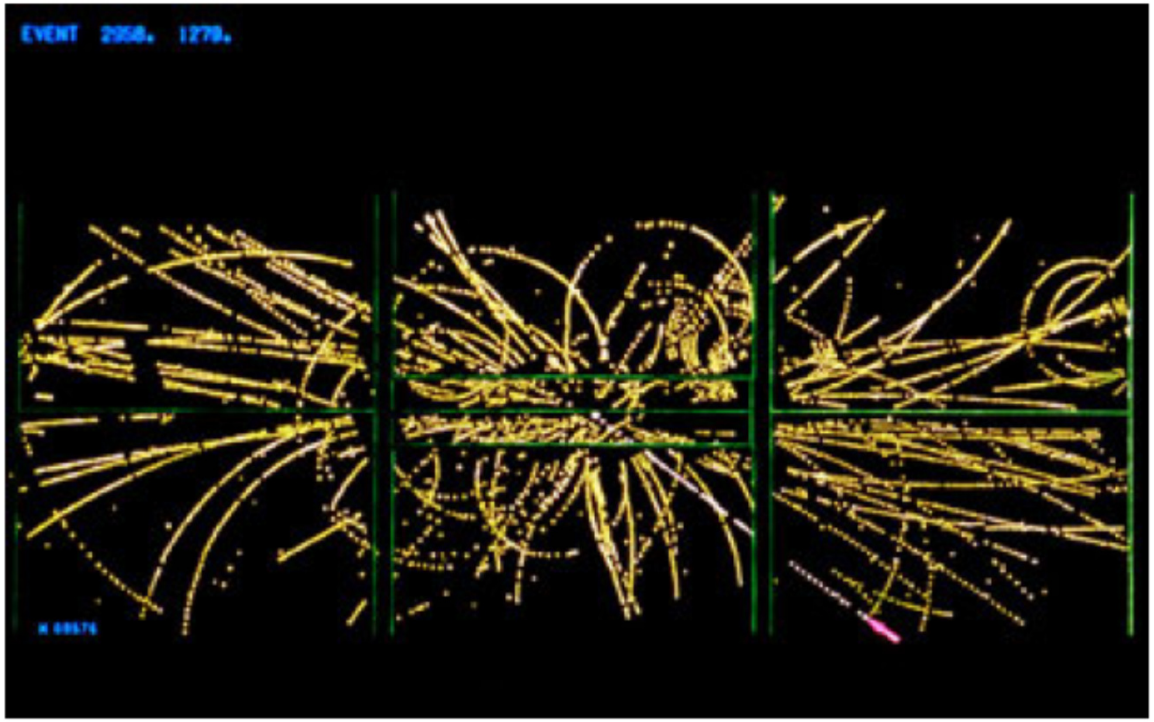
W & Z Discovery - VI

Several conditions to select events

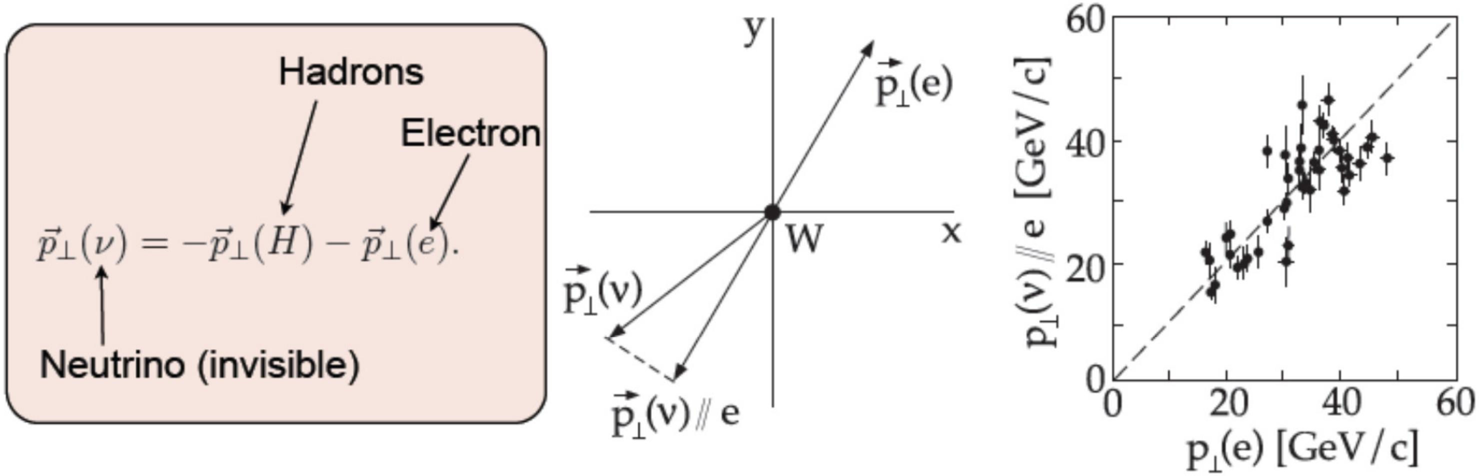
- More than 10000 events/s, most of them selected by a trigger
- Trace in central detector must point into center of electromagnetic shower
- Transversal momentum in central detector > 7 GeV
- Trace must be isolated (only other traces with transversal momentum <2.5 GeV allowed)
- Missing energy >15 GeV, has to point contrary to trace of electron

W & *Z* Discovery - VII

UA1 $W \to e\nu$ candidate event



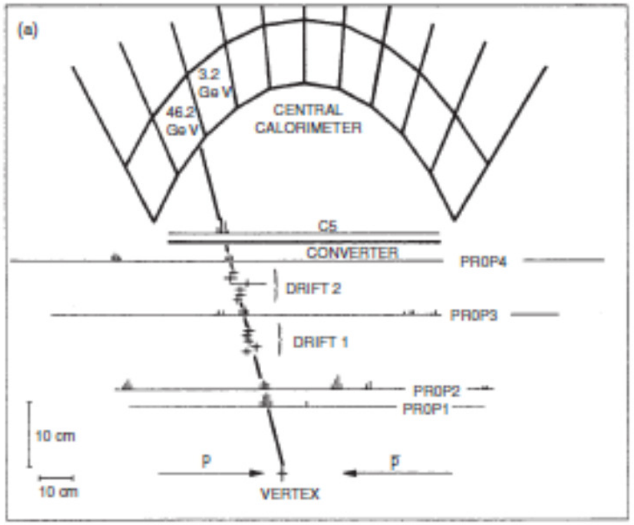
W & *Z* Discovery - VIII

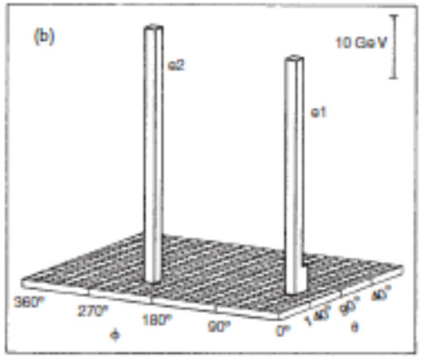


- Calculate sum of all hadron momenta in the **transverse plane** (to avoid leaks along the beam lines)
- **$p_T(\nu)$** not exactly antiparallel to **$p_T(e)$**
  - W boson not always produced at rest, finite detector resolution

W & *Z* Discovery - IX

UA2 Candidate Z event





W & Z Discovery - X

$$\frac{d\sigma}{d\cos\theta^*} = \text{const} \qquad \text{Just an approximation}$$

$$\frac{d\sigma}{dp_T} = \frac{d\sigma}{d\cos\theta^*}\frac{d\cos\theta^*}{dp_T}$$

$$p_T = p^*\sin\theta^* = \frac{M_W}{2}\sin\theta^*$$

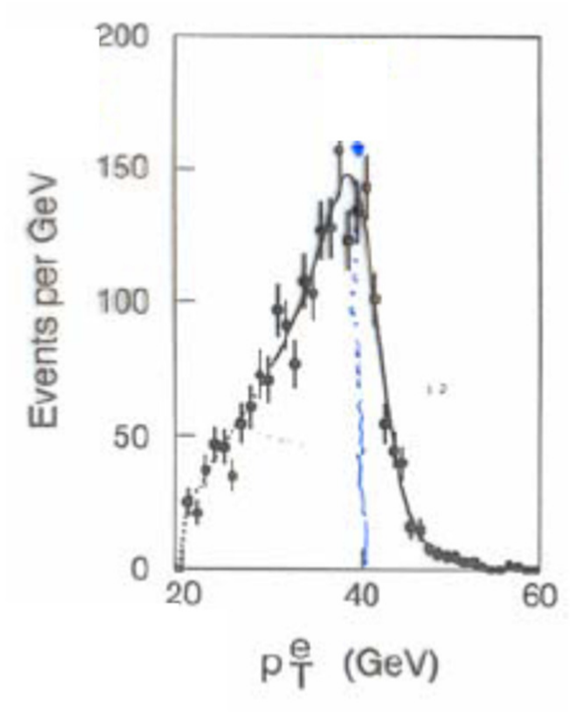
$$\rightarrow \sin\theta^* = \frac{2p_T}{M_W}$$

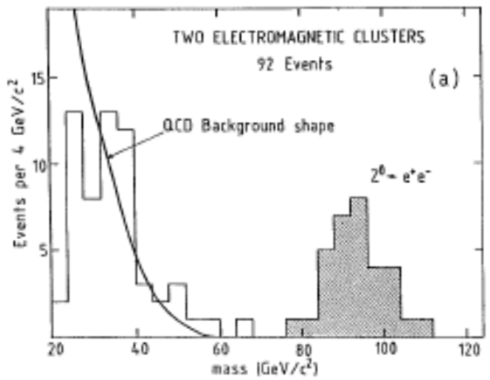
$$\rightarrow \cos\theta^* = \sqrt{1-\sin^2\theta^*} = \sqrt{1-\left(\frac{2p_T}{M_W}\right)^2}$$

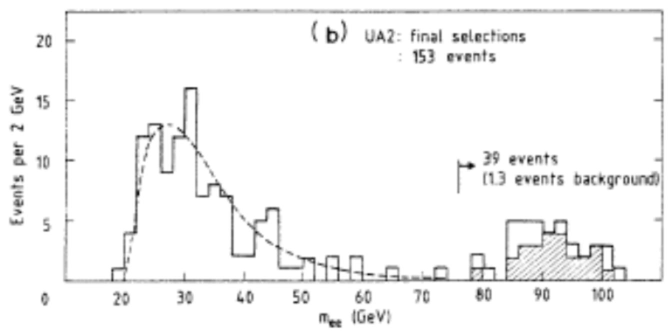
$$\rightarrow \frac{d\cos\theta^*}{dp_T} = \frac{\frac{4p_T}{M_W}}{2\sqrt{1-\left(\frac{2p_T}{M_W}\right)^2}} = \frac{2p_T}{M_W\sqrt{1-\left(\frac{2p_T}{M_W}\right)^2}}$$

$$\rightarrow \frac{d\sigma}{dp_T} = A\left(\cos\theta^*\right)\frac{d\cos\theta^*}{dp_T} \approx K\frac{2p_T}{M_W\sqrt{1-\left(\frac{2p_T}{M_W}\right)^2}} \quad \text{Jacobian peak at } \frac{M_W}{2}$$

W & *Z* Discovery - XI

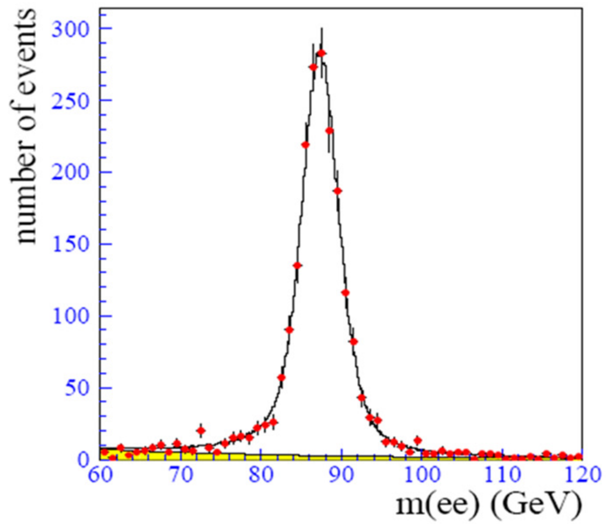




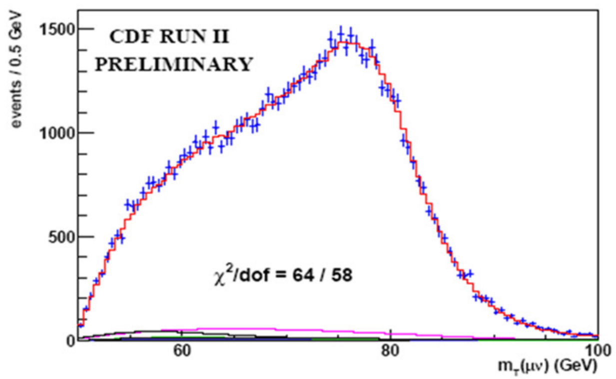


W & Z Discovery - XII

D0 $Z \to e^+e^-$



CDF $W \to \mu\nu$



$m_{W^\pm} = 82.1 \pm 1.7$ GeV

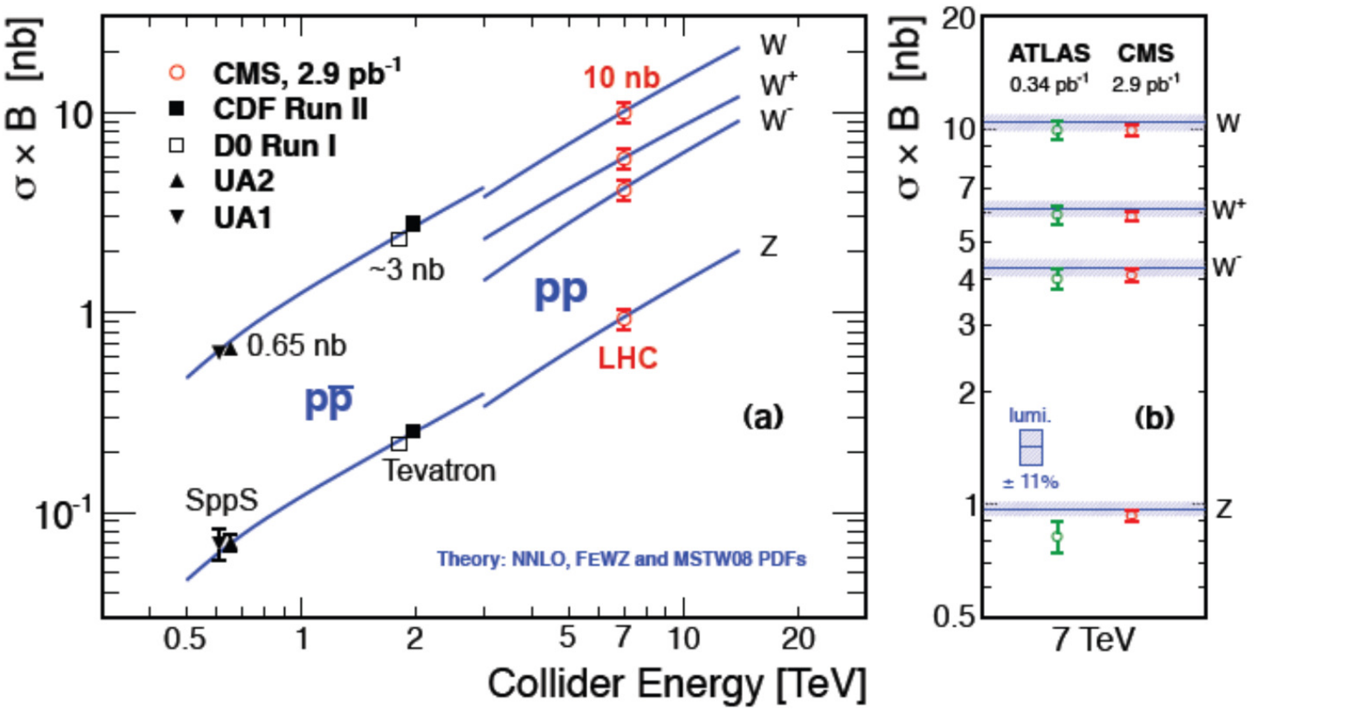
$m_{Z^0} = 93.0 \pm 1.7$ GeV

Current values (Particle Data Group 2006):

$m_{W^\pm} = 80.403 \pm 0.029$ GeV

$m_{Z^0} = 91.1876 \pm 0.0021$ GeV

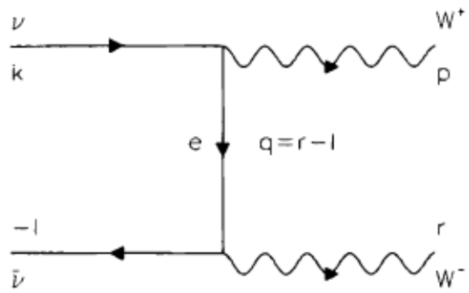
W & *Z* Discovery - XIII



Source: arXiv:1012.2466

- Z^0 cross section ~10 times smaller than $W^\pm$ boson production
- W^+ cross section ~43% larger than W^- at LHC (pp collider!)

SM Internal Consistency - I



Reconsidering hypothetical, troublesome reaction

$\nu\bar{\nu} \to W_L^+ W_L^-$

at very high energy

Polarization 4-vectors of longitudinally polarized *W*s:

$$\varepsilon_L^\mu(p) = \frac{p^\mu}{m_W} + O\left(\frac{m_W}{p^0}\right) \sim \frac{p^\mu}{m_W}$$

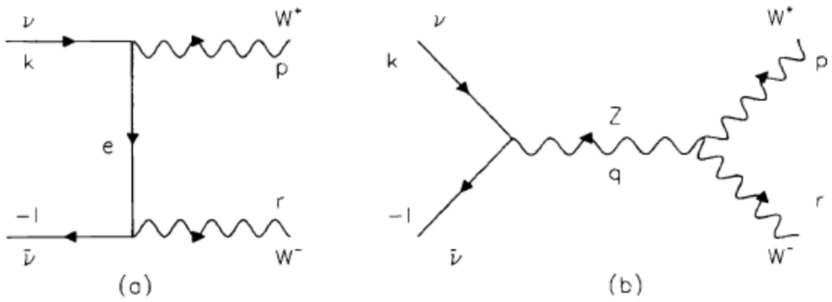
Divergent term of matrix element:

$$M_{fi} \approx -\frac{g^2}{8}\bar{v}(l)(1-\gamma_5)\frac{1}{\not{q}-m}(1-\gamma_5)u(k)\frac{r^\mu}{m_W}\frac{p^\nu}{m_W}$$

$$M_{fi} \approx -\frac{g^2}{4m_W^2}\bar{v}(l)\not{p}(1-\gamma_5)u(k) - \frac{g^2}{8m_W^2}m\bar{v}(l)(1+\gamma_5)\frac{\not{q}+m}{q^2-m^2}\not{p}(1-\gamma_5)u(k)$$

$$M_{fi}^a \approx -\frac{g^2}{4m_W^2}\bar{v}(l)\not{p}(1-\gamma_5)u(k)$$

SM Internal Consistency - II



Standard Model: Neutral Current → Two diagrams instead of one

M_{fi}^{b} : Z^0 matrix element:

$\nu\nu Z, WWZ$ vertexes, Z propagator

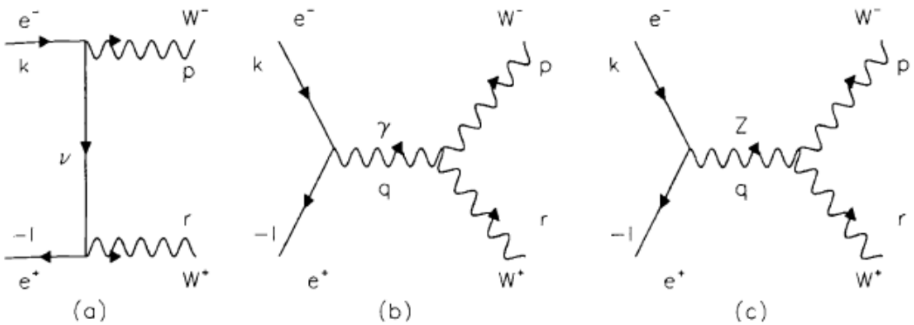
After quite intense calculations....

$$M_{fi}^{b} \approx \frac{g^2}{4m_W^2}\overline{v}(l)\not{p}(1-\gamma_5)u(k)$$

$$\rightarrow M_{fi}^{b} + M_{fi}^{b} = 0$$

Divergence fixed in a gauge theory!

SM Internal Consistency - III



Another, similar reaction

$e^+e^- \to W_L^+W_L^-$

Quite realistic!

$$M_{fi}^{a} \approx -\frac{g^2}{4m_W^2}\bar{v}(l)\not{p}(1-\gamma_5)u(k)$$

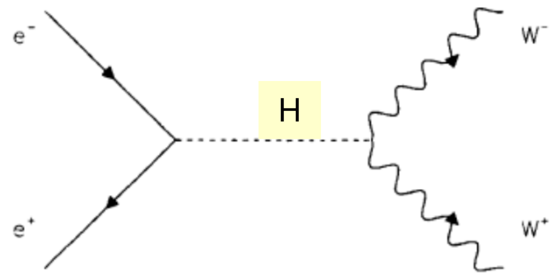
$$M_{fi}^{b} \approx \frac{e^2}{m_W^2}\bar{v}(l)\not{p}u(k)$$

$$M_{fi}^{c} \approx -\frac{g_{WWZ}}{2m_W^2}\bar{v}(l)\left[g_L\not{p}(1-\gamma_5)+g_R\not{p}(1+\gamma_5)\right]u(k)$$

$\to M_{fi}^{a}+M_{fi}^{b}+M_{fi}^{c} \approx --\frac{g^2}{4m_W^2}m\bar{v}(l)u(k)$ Still (weakly) divergent at high energy

SM Internal Consistency - IV

Reason of extra divergence: *R* chiral parts of massive fermions



Higgs diagram:

$$M_{fi}^{H} \approx -\frac{1}{2m_W^2} g_{eeH}\, g_{WWH}\, \overline{v}(l)\, u(k)$$

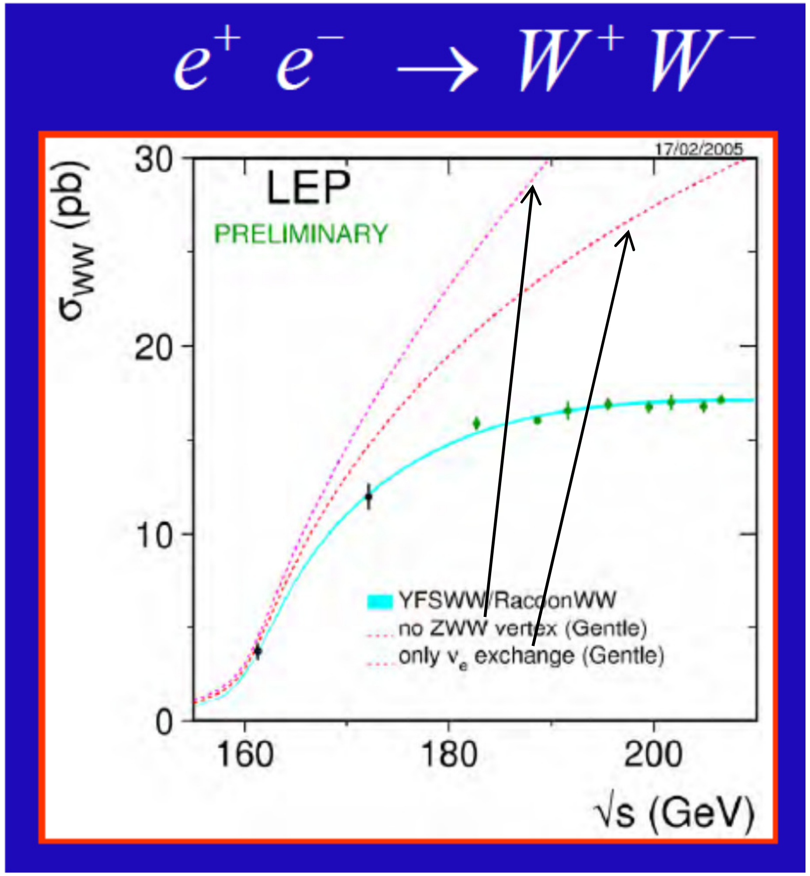
$\rightarrow$ Correct compensation with gauge theory & SSB

Strong support for the Standard Model:

Higgs *must* be there

(or something really new must happen at $\sim$ 1 TeV to save unitarity)

SM Internal Consistency - V



Precision Tests - I

LEP – Precision tests of SM 1989-2000

$1989-1995$

$\sqrt{s}=91.2\ GeV$

$17\ 10^6\ Z^0$ detected

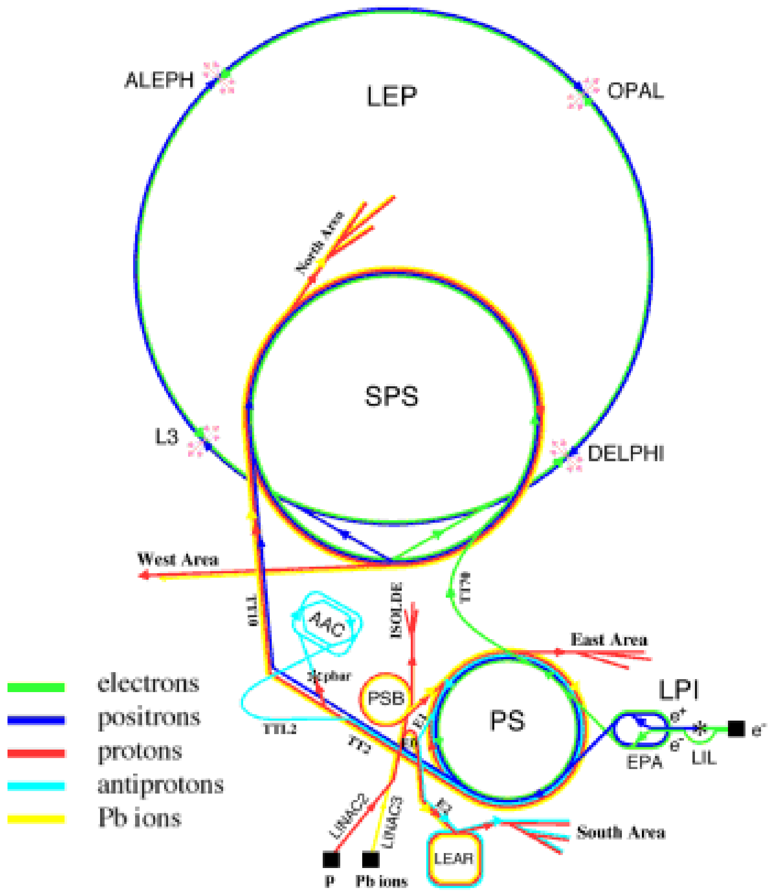
$1996-2000$

$\sqrt{s}=161-208\ GeV$

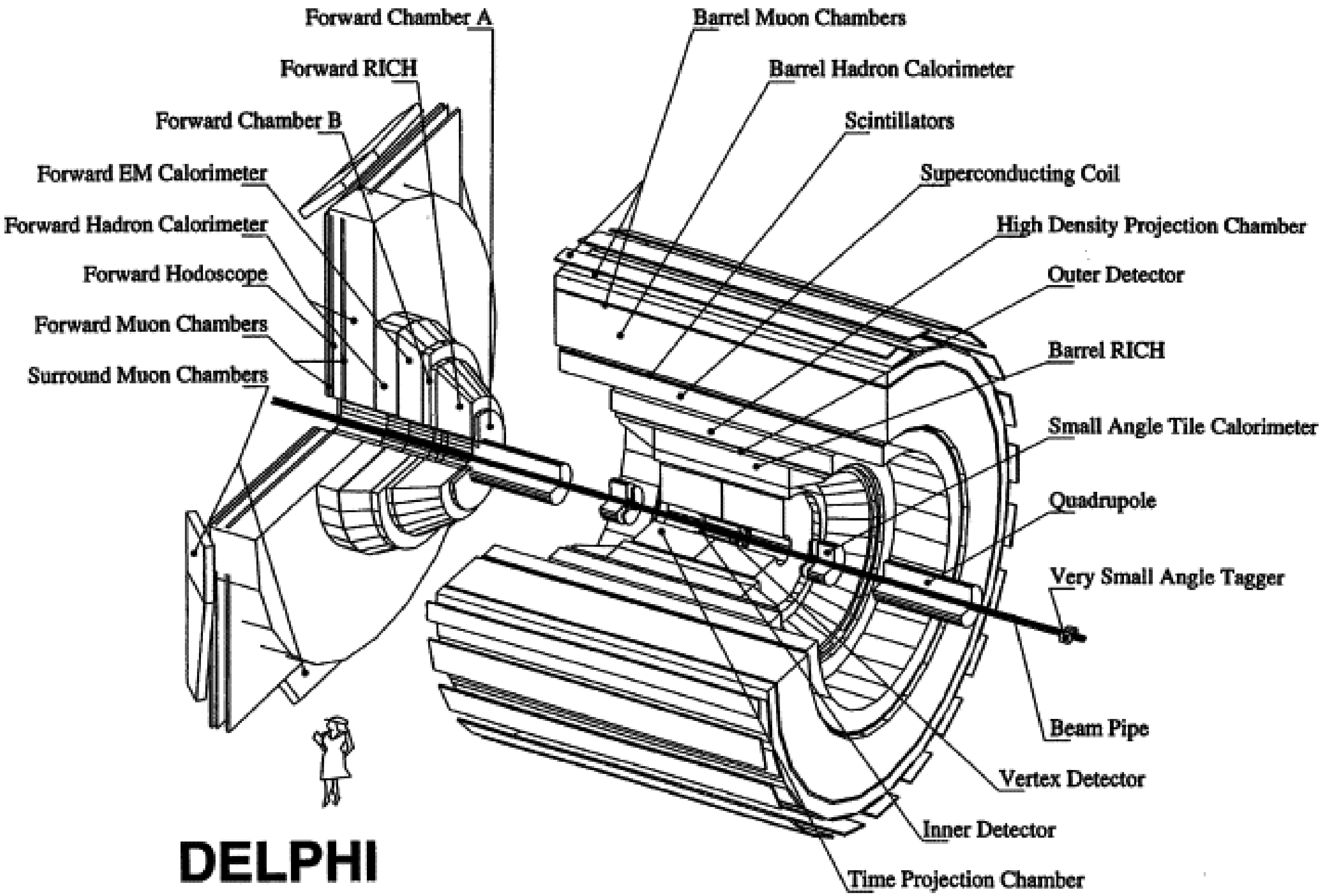
$30\ 10^3\ \ WW$ detected

26 km circumference
4 large experiments: ALEPH, DELPHI, L3, OPAL

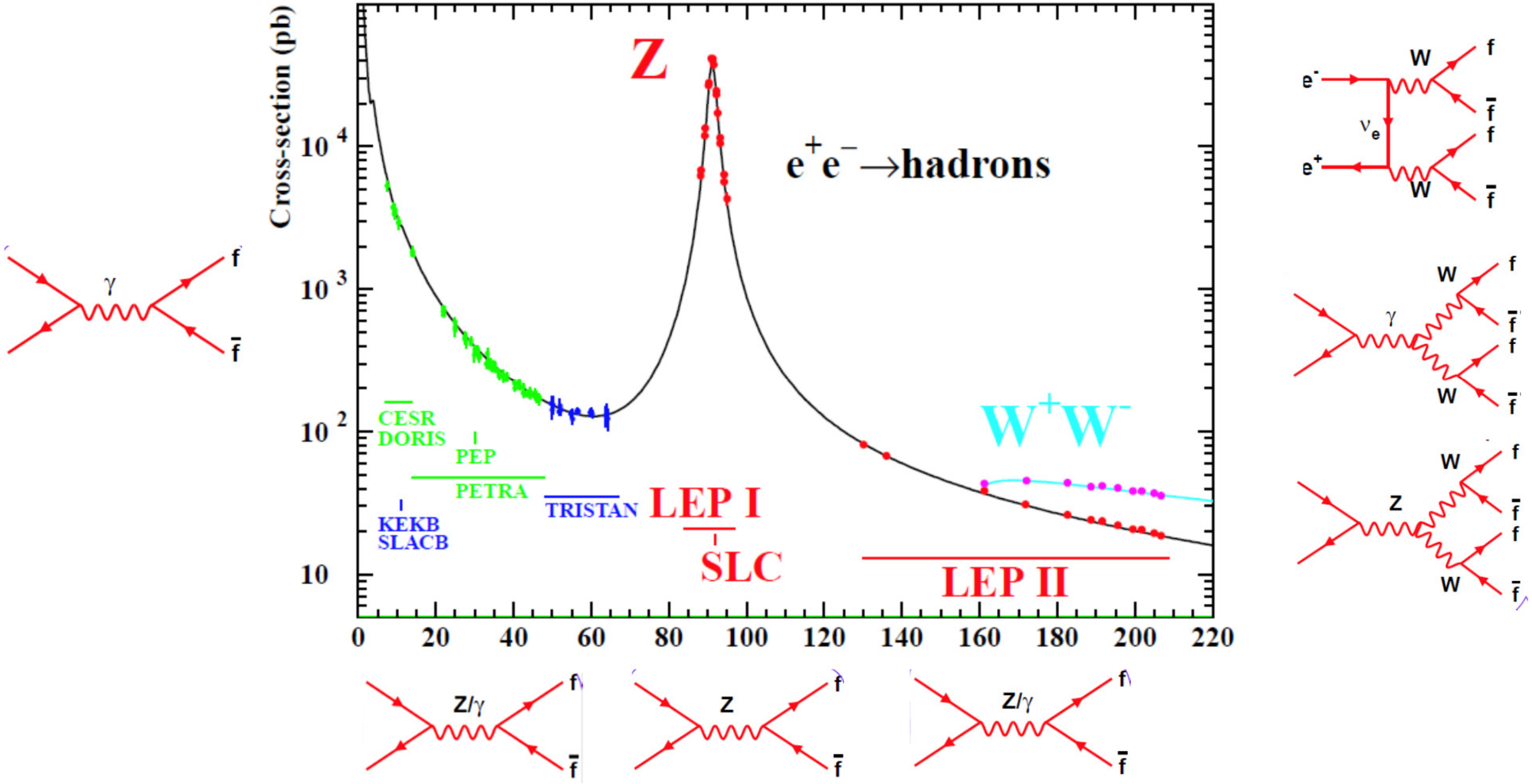
Precision Tests - II



Precision Tests - III



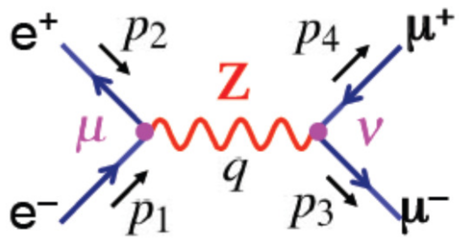
Precision Tests - IV



Precision Tests - V

$e^+e^- \to \mu^+\mu^-$ at the Z peak

Only consider (dominant) Z diagram



Electron vertex:

$$\bar{v}(p_2)\left(-ig_Z\gamma^\mu\right)\frac{1}{2}\left(c_V - c_A\gamma^5\right)u(p_1) \text{ Electron } c_V, c_A$$

Z propagator :

$$-i\frac{g_{\mu\nu}}{q^2 - m_Z^2} \text{ Approximate, see later}$$

Muon vertex:

$$\bar{u}(p_3)\left(-ig_Z\gamma^\nu\right)\frac{1}{2}\left(c_V - c_A\gamma^5\right)v(p_4) \text{ Muon } c_V, c_A$$

Precision Tests- VI

Ultrarelativistic limit $\rightarrow$ Chirality $\simeq$ Helicity

$\rightarrow$ Use helicity eigenstates for electron, muon vertexes

$$c_L = c_V + c_A, c_R = c_V - c_A$$

$$\rightarrow c_V = \frac{1}{2}(c_L + c_R), c_A = \frac{1}{2}(c_L - c_R)$$

$$\frac{1}{2}(c_V - c_A\gamma^5) \rightarrow \frac{1}{2}c_L(1-\gamma^5) + \frac{1}{2}c_R(1+\gamma^5)$$

$\rightarrow$ Matrix element:

$$\left[c_L\bar{v}(p_2)\gamma^\mu\frac{1}{2}(1-\gamma^5)u(p_1) + c_R\bar{v}(p_2)\gamma^\mu\frac{1}{2}(1+\gamma^5)u(p_1)\right]$$

$$\times\left(-\frac{g_Z}{q^2 - m_Z^2}g_{\mu\nu}\right)$$

$$\times\left[c_L\bar{u}(p_3)\gamma^\nu\frac{1}{2}(1-\gamma^5)v(p_4) + c_R\bar{u}(p_3)\gamma^\nu\frac{1}{2}(1+\gamma^5)v(p_4)\right]$$

Introduce chirality $\simeq$ helicity projectors:

$$\frac{1}{2}(1-\gamma^5)u \simeq u_\downarrow, \frac{1}{2}(1+\gamma^5)u \simeq u_\uparrow, \frac{1}{2}(1-\gamma^5)v \simeq v_\uparrow, \frac{1}{2}(1+\gamma^5)v \simeq v_\downarrow$$

Precision Tests- VII

$\rightarrow$ Matrix element:

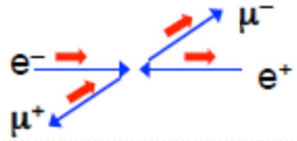
$$\left[c_L \bar{v}(p_2)\gamma^\mu u_\downarrow(p_1) + c_R \bar{v}(p_2)\gamma^\mu u_\uparrow(p_1)\right]\left(-\frac{g_Z}{q^2-m_Z^2}g_{\mu\nu}\right)\left[c_L \bar{u}(p_3)\gamma^\nu v_\uparrow(p_4) + c_R \bar{u}(p_3)\gamma^\nu v_\downarrow(p_4)\right]$$

$$\bar{v}(p_2) = \bar{v}_\uparrow(p_2) + \bar{v}_\downarrow(p_2), \bar{u}(p_3) = \bar{u}_\uparrow(p_3) + \bar{u}_\downarrow(p_3)$$

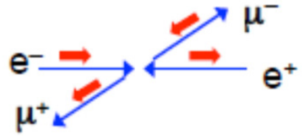
Surviving terms in both e, μ currents: LR, RL only

$$\rightarrow \left[c_L \bar{v}_\uparrow(p_2)\gamma^\mu u_\downarrow(p_1) + c_R \bar{v}_\downarrow(p_2)\gamma^\mu u_\uparrow(p_1)\right]\left(-\frac{g_Z}{q^2-m_Z^2}g_{\mu\nu}\right)\left[c_L \bar{u}_\downarrow(p_3)\gamma^\nu v_\uparrow(p_4) + c_R \bar{u}_\uparrow(p_3)\gamma^\nu v_\downarrow(p_4)\right]$$

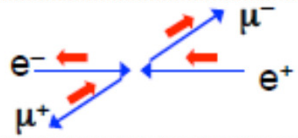
$$M_{RR} = -\frac{g_Z^2}{q^2-m_Z^2} c_R^e c_R^\mu g_{\mu\nu}[\bar{v}_\downarrow(p_2)\gamma^\mu u_\uparrow(p_1)][\bar{u}_\uparrow(p_3)\gamma^\nu v_\downarrow(p_4)]$$



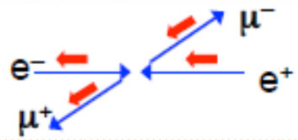
$$M_{RL} = -\frac{g_Z^2}{q^2-m_Z^2} c_R^e c_L^\mu g_{\mu\nu}[\bar{v}_\downarrow(p_2)\gamma^\mu u_\uparrow(p_1)][\bar{u}_\downarrow(p_3)\gamma^\nu v_\uparrow(p_4)]$$



$$M_{LR} = -\frac{g_Z^2}{q^2-m_Z^2} c_L^e c_R^\mu g_{\mu\nu}[\bar{v}_\uparrow(p_2)\gamma^\mu u_\downarrow(p_1)][\bar{u}_\uparrow(p_3)\gamma^\nu v_\downarrow(p_4)]$$



$$M_{LL} = -\frac{g_Z^2}{q^2-m_Z^2} c_L^e c_L^\mu g_{\mu\nu}[\bar{v}_\uparrow(p_2)\gamma^\mu u_\downarrow(p_1)][\bar{u}_\downarrow(p_3)\gamma^\nu v_\uparrow(p_4)]$$



Precision Tests- VIII

Almost 'Cut & Paste' from QED case:

$$|M_{RR}|^2 = s^2 \left| \frac{g_Z^2}{s - m_Z^2} \right|^2 (c_R^e)^2 (c_R^\mu)^2 (1 + \cos\theta)^2$$

$$|M_{RL}|^2 = s^2 \left| \frac{g_Z^2}{s - m_Z^2} \right|^2 (c_R^e)^2 (c_L^\mu)^2 (1 - \cos\theta)^2$$

$$|M_{LR}|^2 = s^2 \left| \frac{g_Z^2}{s - m_Z^2} \right|^2 (c_L^e)^2 (c_R^\mu)^2 (1 - \cos\theta)^2$$

$$|M_{LR}|^2 = s^2 \left| \frac{g_Z^2}{s - m_Z^2} \right|^2 (c_L^e)^2 (c_L^\mu)^2 (1 + \cos\theta)^2$$

Precision Tests- IX

Now take correct Z propagator: Z *unstable*

$$-i\frac{g_{\mu\nu}}{q^2-m_Z^2}=-i\frac{g_{\mu\nu}}{s-m_Z^2}\to -i\frac{g_{\mu\nu}}{s-m_Z^2+im_Z\Gamma_Z}$$

$$\to\left|-i\frac{1}{s-m_Z^2+im_Z\Gamma_Z}\right|^2=\frac{1}{\left(s-m_Z^2\right)^2+m_Z^2\Gamma_Z^2}$$

$$\frac{d\sigma}{d\Omega}=\frac{\left|M_{fi}\right|^2}{64\pi^2 s}$$

$\to$ Differential cross-section for the 4 combinations:

$$\frac{d\sigma_{RR}}{d\Omega}=\frac{1}{64\pi^2}\frac{g_Z^4 s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}(c_R^e)^2(c_R^\mu)^2(1+\cos\theta)^2$$

$$\frac{d\sigma_{LL}}{d\Omega}=\frac{1}{64\pi^2}\frac{g_Z^4 s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}(c_L^e)^2(c_L^\mu)^2(1+\cos\theta)^2$$

$$\frac{d\sigma_{LR}}{d\Omega}=\frac{1}{64\pi^2}\frac{g_Z^4 s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}(c_L^e)^2(c_R^\mu)^2(1-\cos\theta)^2$$

$$\frac{d\sigma_{RL}}{d\Omega}=\frac{1}{64\pi^2}\frac{g_Z^4 s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}(c_L^e)^2(c_R^\mu)^2(1-\cos\theta)^2$$

Precision Tests- X

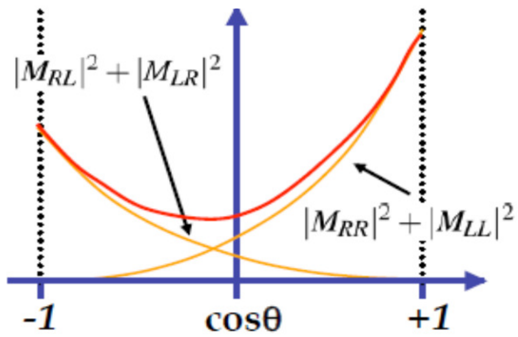
Most interesting difference wrt *QED* case:

$$|M_{LL}|^2 + |M_{RR}|^2 \neq |M_{LR}|^2 + |M_{RL}|^2$$

Unpolarized cross section: Average & Sum over spins

$$\rightarrow \frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2}\frac{1}{4}\frac{g_Z^4 s}{\left(s-m_Z^2\right)^2 + m_Z^2\Gamma_Z^2}\frac{1}{4}\left(c_V^2+c_A^2\right)_e\left(c_V^2+c_A^2\right)_\mu\left(1+\cos^2\theta\right)+2\left(c_Vc_A\right)_e\left(c_Vc_A\right)_\mu\cos\theta$$

Sizeable forward-backward asymmetry!



Precision Tests - XI

Integrate over solid angle, get total cross section:

$$\sigma_{e^+e^-\to Z\to\mu^+\mu^-} = \frac{1}{192\pi}\frac{g_Z^4 s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}[(c_V^e)^2+(c_A^e)^2][(c_V^\mu)^2+(c_A^\mu)^2]]$$

Recall partial Z widths:

$$\Gamma(Z\to e^+e^-) = \frac{g_Z^2 m_Z}{48\pi}[(c_V^e)^2+(c_A^e)^2] \qquad \Gamma(Z\to\mu^+\mu^-) = \frac{g_Z^2 m_Z}{48\pi}[(c_V^\mu)^2+(c_A^\mu)^2]$$

$$\sigma = \frac{12\pi}{m_Z^2}\frac{s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}\Gamma(Z\to e^+e^-)\Gamma(Z\to\mu^+\mu^-)$$

$$\to \sigma_{peak} \simeq \frac{12\pi(BR)^2}{m_Z^2} \approx \frac{37.7\left(3.5\,10^{-2}\right)^2}{(91.2)^2} \approx 55\ 10^{-7}\,GeV^{-2}$$

$$(\hbar c)^2 \simeq 0.389\,GeV^2 mb$$

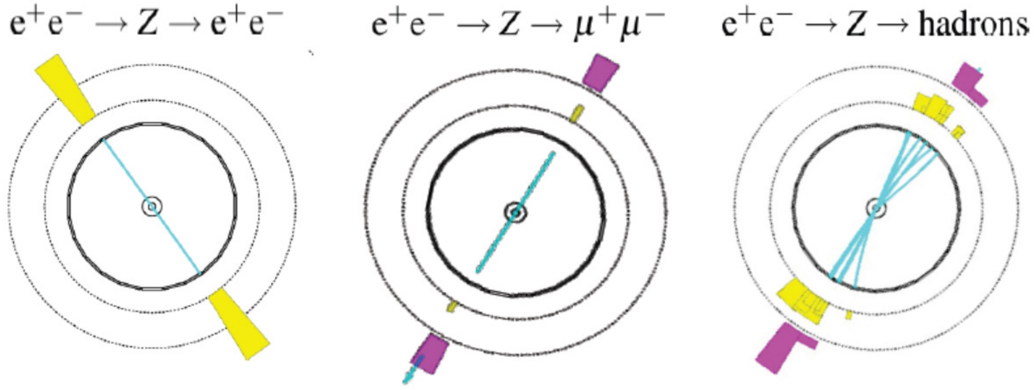
$$\to \sigma_{peak} \simeq 55\ 10^{-7}\,GeV^{-2}\,0.389\,GeV^2 mb \approx 2.14\,10^{-6}\,mb = 2.14\,nb$$

Precision Tests - XII

Z peak: Essentially 4 types of events

$e^+e^- \to e^+e^-, \mu^+\mu^-, \tau^+\tau^-, q\bar{q}\,(\to hadrons)$

Different topologies in the detectors:



Measuring cross sections:

Count events (!)

Subtract background

Correct for inefficiency

Get integrated luminosity (Most of the time from independent counting of Bhabha events)

$$\to \sigma = \frac{N - N_{bck}}{\varepsilon} \frac{1}{L_{\text{int}}}$$

Precision Tests - XIII

Among other results at the peak: Z^0 lineshape

Meaning in practice:

m_Z Z mass

Γ_Z Z total width

Γ_f Z partial width to fermion type f

N_ν Number of (SM) neutrino species

Obtained by 'scanning' the Z^0 peak:

Move $E_{beam} = \frac{\sqrt{s}}{2}$ in steps through the peak

Measure relevant σ at each step

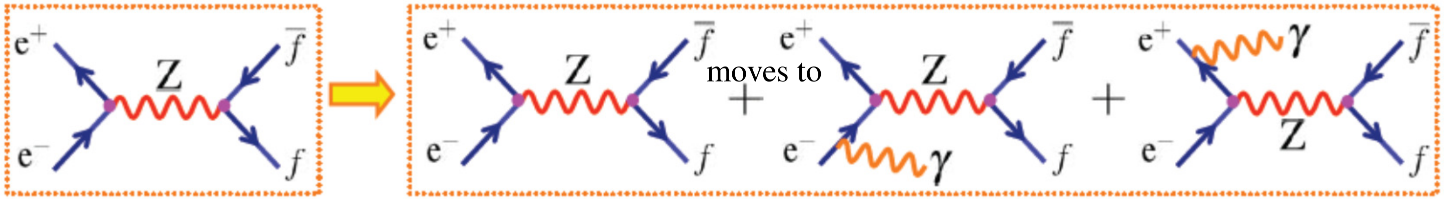
Fit profile:

$$\sigma\left(e^+e^- \to Z \to f\bar{f}\right) = \frac{12\pi}{m_Z^2} \frac{s\Gamma_{ee}\Gamma_{ff}}{\left(s - m_Z^2\right)^2 + m_Z^2\Gamma_Z^2}$$

Precision Tests - XIV

Lineshape quite distorted by several effects

Main effect: Initial State Radiation (*ISR*)



Result:

Collision CM energy $\neq 2E_{beam}$

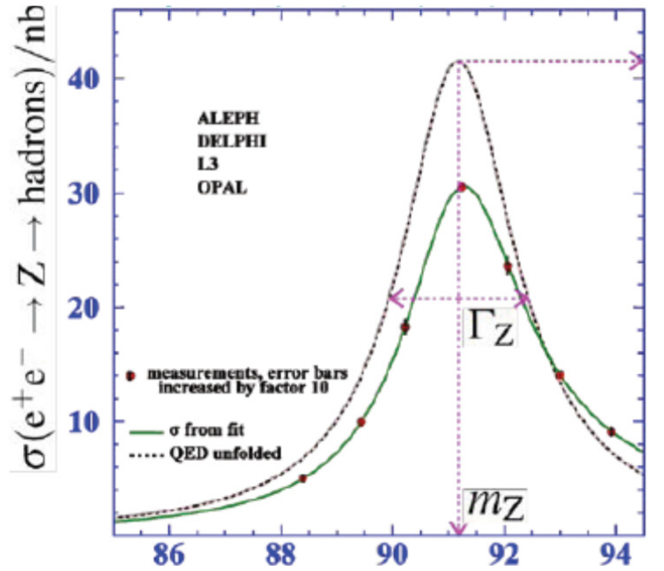
$$e^+ \xrightarrow{E} \xleftarrow{E} e^- \qquad \sqrt{s} = 2E$$

moves to

$$\xrightarrow{E} \xleftarrow{E-E_\gamma} \qquad \sqrt{s'} \approx 2E\left(1 - \frac{E_\gamma}{2E}\right)$$

$$m_Z = 91.1875 \pm 0.0021\,\mathrm{GeV}$$

$$\Gamma_Z = 2.4952 \pm 0.0023\,\mathrm{GeV}$$



Precision Tests - XV

Finding the number of Standard Model neutrinos

(Meaning: With standard coupling to Z)

Total width:

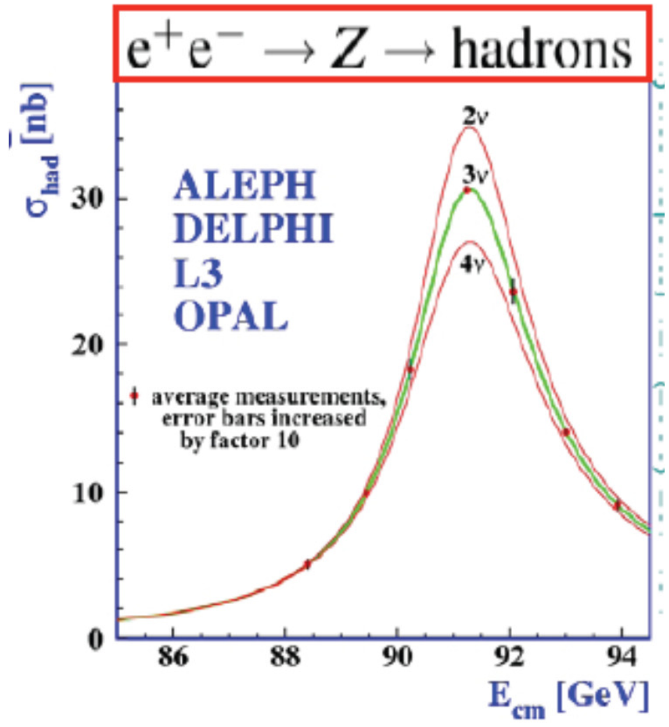
$$\Gamma = \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + \Gamma_{had} + \Gamma_{\nu_1\nu_1} + \Gamma_{\nu_2\nu_2} + \Gamma_{\nu_3\nu_3} + ?$$

$$\Gamma = 3\Gamma_{ll} + \Gamma_{had} + N_\nu \Gamma_{\nu\nu}$$

Measure partial widths from peak cross sections:

$$\sigma_0^{ff} = \frac{12\pi}{m_Z^2} \frac{\Gamma_{ee}\Gamma_{ff}}{\Gamma_Z^2}$$

$$N_\nu = 2.9840 \pm 0.0082$$



Precision Tests - XVI

Write differential cross section $\left(\text{e.g. for } e^+e^- \to \mu^+\mu^- \right)$ as:

$$\frac{d\sigma}{d\Omega} = k\left[A\left(1+\cos^2\theta\right) + B\cos\theta\right]$$

$$A = \left[\left(c_L^e\right)^2 + \left(c_R^e\right)^2\right]\left[\left(c_L^\mu\right)^2 + \left(c_R^\mu\right)^2\right], \quad B = \left[\left(c_L^e\right)^2 - \left(c_R^e\right)^2\right]\left[\left(c_L^\mu\right)^2 - \left(c_R^\mu\right)^2\right]$$

Forward/Backward cross sections:

$$\sigma_F = \int_0^1 \frac{d\sigma}{d\cos\theta} d\cos\theta, \quad \sigma_B = \int_{-1}^0 \frac{d\sigma}{d\cos\theta} d\cos\theta$$

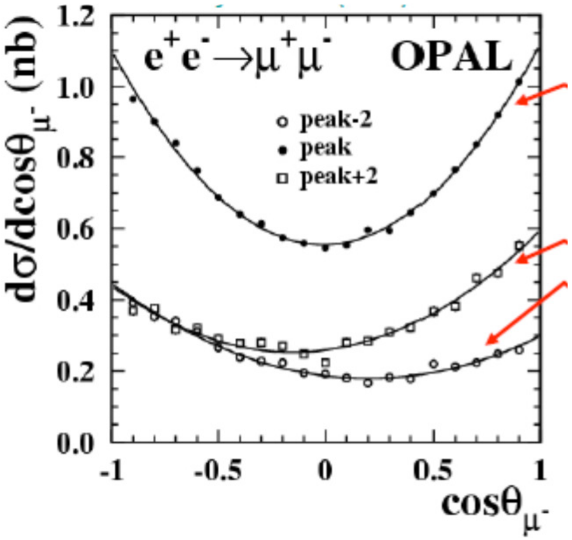
FB Asymmetry:

$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$

$$\sigma_F = k\left(\frac{4}{3}A + \frac{1}{2}B\right), \sigma_B = k\left(\frac{4}{3}A - \frac{1}{2}B\right)$$

$$\to A_{FB} = \frac{3}{8}\frac{B}{A} = \frac{3}{4}\frac{\left[\left(c_L^e\right)^2 - \left(c_R^e\right)^2\right]}{\left[\left(c_L^e\right)^2 + \left(c_R^e\right)^2\right]}\frac{\left[\left(c_L^\mu\right)^2 - \left(c_R^\mu\right)^2\right]}{\left[\left(c_L^\mu\right)^2 + \left(c_R^\mu\right)^2\right]} = \frac{3}{4}A_e A_\mu$$

Precision Tests - XVII



$A_{FB}(peak) \sim 0$ for leptons ($\sin^2\theta_W \approx 0.25$)

$A_{FB}(peak \pm 2\ \text{GeV}) \neq 0: \quad = 2\dfrac{c_V/c_A}{1+(c_V/c_A)^2}$

Interference with QED

$A_{FB}^{0,e} = 0.0145 \pm 0.0025$

$A_{FB}^{0,\mu} = 0.0169 \pm 0.0013$

$A_{FB}^{0,\tau} = 0.0188 \pm 0.0017$

$A_e = 0.1514 \pm 0.0019$

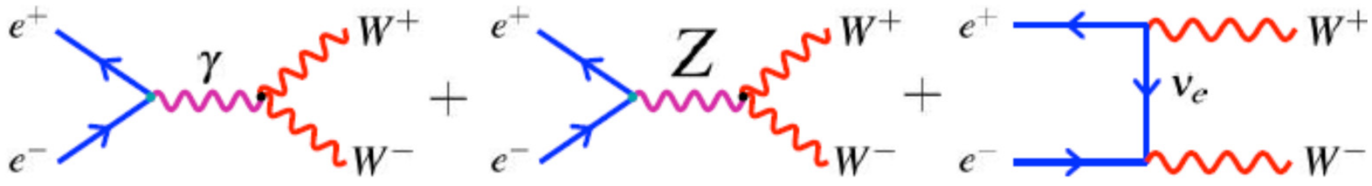
$A_\mu = 0.1456 \pm 0.0091$

$A_\tau = 0.1449 \pm 0.0040$

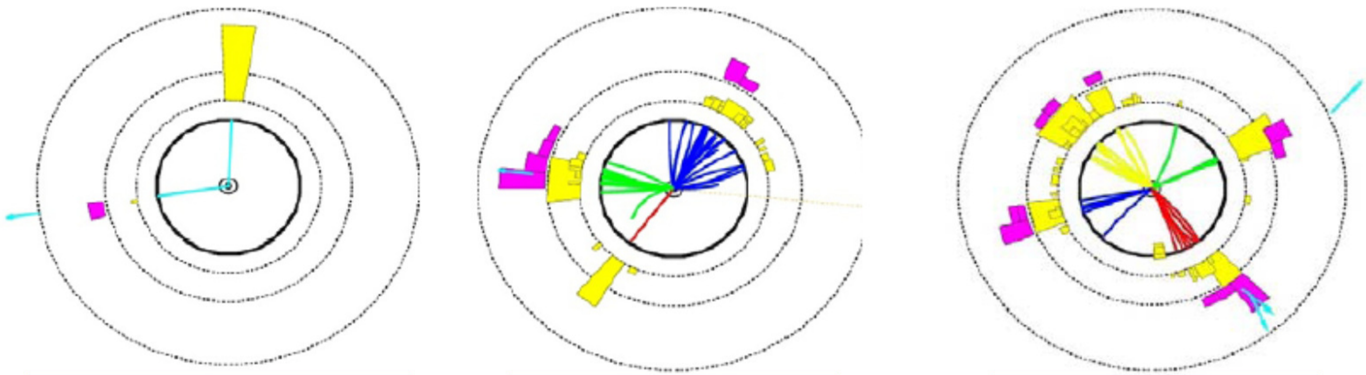
$\sin^2\theta_W = 0.23154 \pm 0.00016$

Precision Tests - XVIII

LEP2: Study of WW production

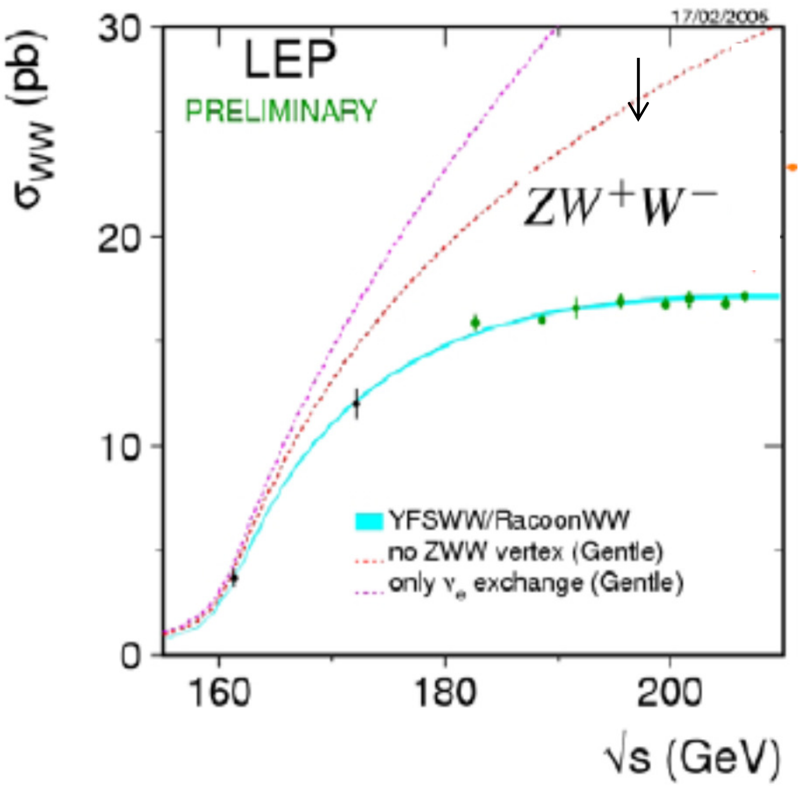


$$Br(W^- \to \text{hadrons}) \approx 0.67 \qquad Br(W^- \to e^- \overline{\nu}_e) \approx 0.11$$
$$Br(W^- \to \mu^- \overline{\nu}_\mu) \approx 0.11 \qquad Br(W^- \to \tau^- \overline{\nu}_\tau) \approx 0.11$$



$W^+W^- \to \ell^+\nu\ell^-\overline{\nu}$ $\qquad$ $W^+W^- \to q\bar{q}\ell\nu$ $\qquad$ $W^+W^- \to q\bar{q}q\bar{q}$

Precision Tests - XIX



Maybe one of the best results of the whole LEP saga

Precision Tests - XX

Measurement of m_W : Kinematical fit

Example:

$W^+W^- \to q\bar{q}e^-\bar{\nu}$

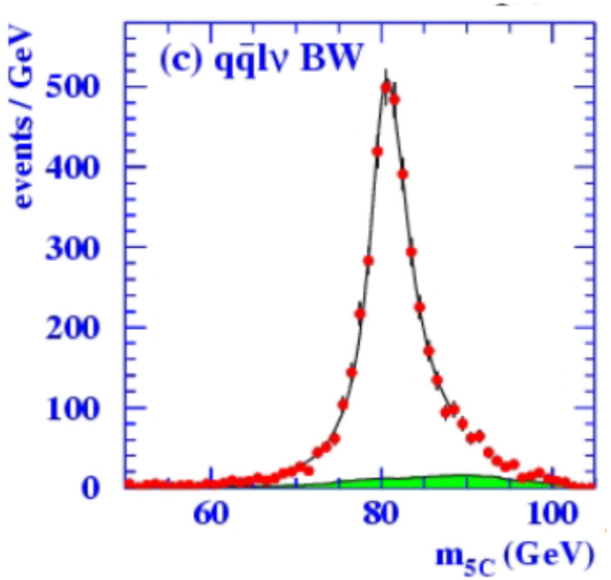
Get $\bar{\nu}$ 4-momentum from:

$$p_q + p_{\bar{q}} + p_{e^-} + p_{\bar{\nu}} = \left(\sqrt{s}, 0\right)$$

Make W bosons masses :

$$M_{W^+} = \left(p_q + p_{\bar{q}}\right)^2$$

$$M_{W^-} = \left(p_{e^-} + p_{\bar{\nu}}\right)^2$$



$$m_W = 80.376 \pm 0.033\,\text{GeV}$$

$$\Gamma_W = 2.196 \pm 0.083\,\text{GeV}$$

Loopology - I

Standard Model :

$M_W = M_Z \cos\theta_W$

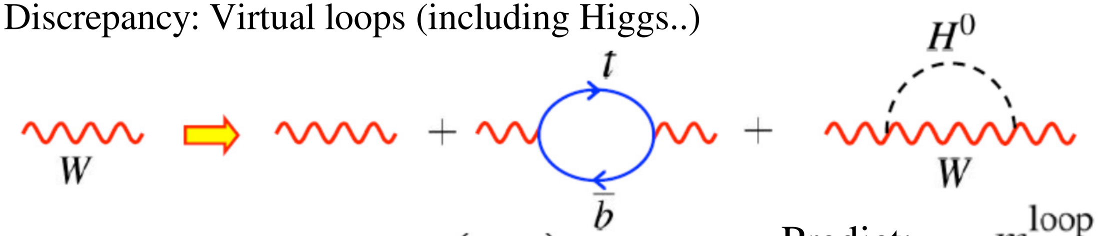
Measure: $\begin{cases} M_Z = 91.1875 \pm 0.0021 \;\; GeV \\ \sin^2\theta_W = 0.23154 \pm 0.00016 \end{cases}$

$\rightarrow$ Predict $M_W = 79.946 \pm 0.008 \;\; GeV$

Measure

$M_W = 80.376 \pm 0.033 \;\; GeV$

Discrepancy: Virtual loops (including Higgs..)



$$m_W = m_W^0 + a m_t^2 + b \ln\left(\frac{m_H}{m_W}\right)$$

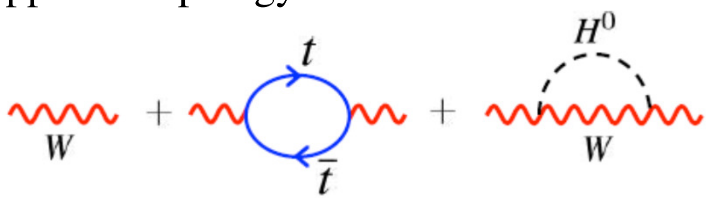
Predict: $m_t^{\text{loop}} = 173 \pm 11\,\text{GeV}$

Observe: $m_t^{\text{meas}} = 174.2 \pm 3.3\,\text{GeV}$

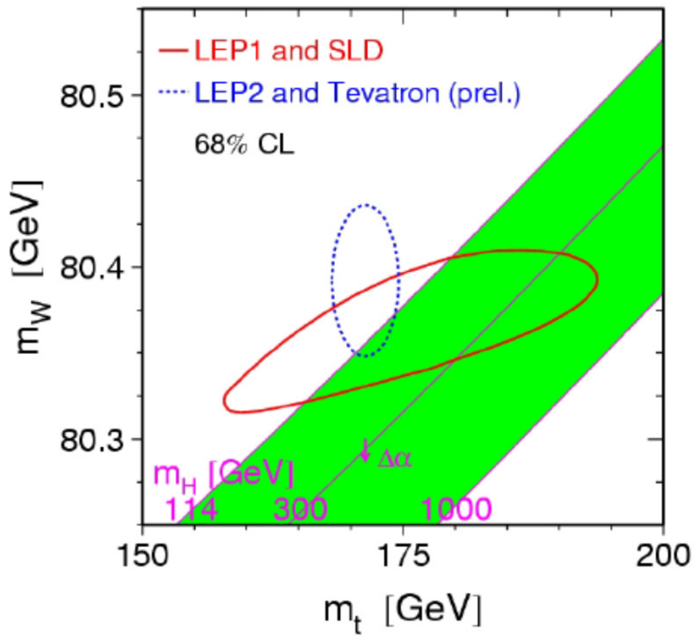
!!!

Loopology - II

Applied loopology:



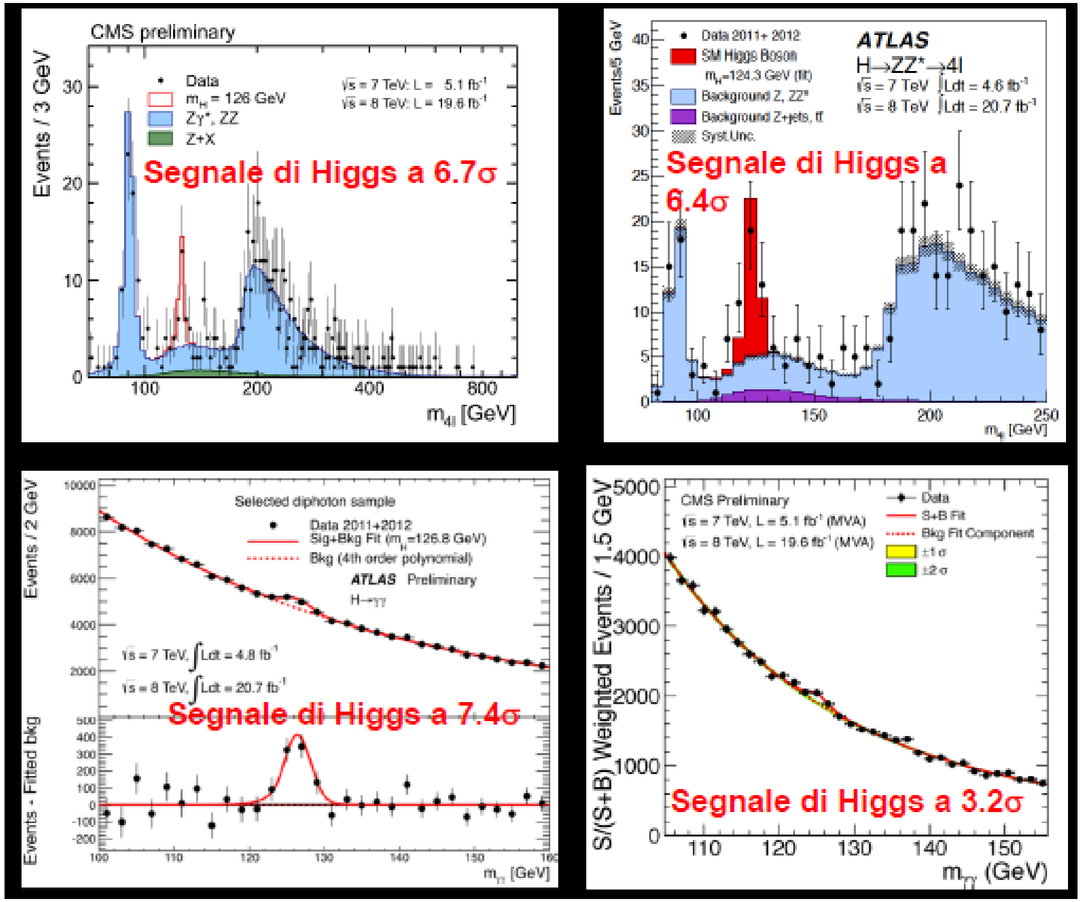
$$m_W = m_W^0 + a m_t^2 + b \ln\left(\frac{m_H}{m_W}\right)$$



$$m_H < 200\,\text{GeV}$$

The Happy End

All is well that ends well:
And finally…



… Mr. Higgs
and Mr. Englert
went to Stockholm