



# Elementary Particles II

## 2 – Electroweak Interaction

Universal Current-Current Interaction,  
Intermediate Vector Bosons, Gauge Symmetry,  
Spontaneous Symmetry Breaking, Electroweak  
Unification, Neutral Currents, Discovery of  $W$  &  $Z$ ,  
Precision Measurements, Higgs

# Electroweak Interaction

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Standard Model:

Electromagnetic and Weak Interaction unified into Electroweak

Energy scale where unification is evident:

$$E \sim M_W, M_Z \sim 100 \text{ GeV}$$

At lower energies:

Can still find tiny traces of the unification (Electroweak interference, Parity violation in atomic processes,...)

Electroweak interaction split into two, almost not interfering, *effective* interactions:

*Electromagnetic*

*Weak*

Non fundamental, useful low energy approximations



# V - A - I

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After a long history of beta decay experiments: *Current-Current (Fermi) Interaction* including *Vector & Axial Vector* terms in order to account for P & C violation

$$H_{\text{int}} = \frac{G_F}{\sqrt{2}} \sum_{i=V,A} C_i (\bar{\psi}_p \Gamma_i \psi_n) \left( \bar{\psi}_e \Gamma^i \left( 1 + \frac{C_i'}{C_i} \gamma^5 \right) \psi_\nu \right)$$

Neutrino helicity =  $-1$  yields lepton current =  $V - A$

$$C_i' = -C_i \rightarrow \left( 1 + \frac{C_i'}{C_i} \gamma^5 \right) \psi_\nu = (1 - \gamma^5) \psi_\nu = -\gamma^\mu (1 - \gamma^5)$$

$$\rightarrow H_{\text{int}} = \frac{G_F}{\sqrt{2}} \left[ C_V (\bar{\psi}_p \gamma_\mu \psi_n) (\bar{\psi}_e \gamma^\mu (1 - \gamma^5) \psi_\nu) + C_A (\bar{\psi}_p \gamma_\mu \gamma_5 \psi_n) (\bar{\psi}_e \gamma^\mu \gamma^5 (1 - \gamma^5) \psi_\nu) \right]$$

$$= \frac{G_F}{\sqrt{2}} \left[ C_V (\bar{\psi}_p \gamma_\mu \psi_n) (\bar{\psi}_e \gamma^\mu (1 - \gamma^5) \psi_\nu) - C_A (\bar{\psi}_p \gamma_\mu \gamma_5 \psi_n) (\bar{\psi}_e \gamma^\mu (1 - \gamma^5) \psi_\nu) \right]$$

$$= \frac{G_F}{\sqrt{2}} \left[ C_V (\bar{\psi}_p \gamma_\mu \psi_n) - C_A (\bar{\psi}_p \gamma_\mu \gamma_5 \psi_n) \right] (\bar{\psi}_e \gamma^\mu (1 - \gamma^5) \psi_\nu)$$

# V - A - II

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Therefore:

$$H_{\text{int}} = \frac{G_F}{\sqrt{2}} C_V \left( \bar{\psi}_p \gamma_\mu \left( 1 - \frac{C_A}{C_V} \gamma_5 \right) \psi_n \right) \left( \bar{\psi}_e \gamma^\mu (1 - \gamma^5) \psi_\nu \right)$$

Current-Current interaction:

Lepton current:  $V - A$

Nucleon current:  $V - \alpha A$  (Strong interaction corrections)

Many violations in weak processes :

Space Parity (large)

Charge Parity (large)

CP (very small)

T (very small)

Flavor conservation (Isospin, S, C, B, T) (larger + smaller)

Lepton numbers (Neutrino oscillations)

# V - A - III

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Observe:

$$H_{\text{int}} = \frac{2G_F}{\sqrt{2}} \left( \bar{\psi}_p \gamma_\mu \left( 1 - \frac{C_A}{C_V} \gamma_5 \right) \psi_n \right) \left( \bar{\psi}_e \gamma^\mu \frac{(1 - \gamma^5)}{2} \psi_\nu \right)$$

$$\frac{1 - \gamma^5}{2} \text{ Projection operator} \rightarrow \left[ \frac{(1 - \gamma^5)}{2} \right]^2 = \frac{1 - \gamma^5}{2}$$

$$\rightarrow H_{\text{int}} = \frac{2G_F}{\sqrt{2}} \left( \bar{\psi}_p \gamma_\mu \left( 1 - \frac{C_A}{C_V} \gamma_5 \right) \psi_n \right) \left( \bar{\psi}_e \gamma^\mu \left[ \frac{(1 - \gamma^5)}{2} \right]^2 \psi_\nu \right)$$

$$\rightarrow H_{\text{int}} = \sqrt{2} G_F \left( \bar{\psi}_p \gamma_\mu \left( 1 - \frac{C_A}{C_V} \gamma_5 \right) \psi_n \right) \left( \bar{\psi}_e \left( \frac{1 + \gamma^5}{2} \right) \gamma^\mu \left( \frac{1 - \gamma^5}{2} \right) \psi_\nu \right)$$

Lepton current written as *pure vector* between *chiral parts* of  $\nu, e$  states

→ The weak charged current is just the same as the e.m. current, except it operates between chiral projections with different charge  $\Delta Q = \pm 1$

# Helicity/Chirality - I

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With reference to Dirac equation:

$$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \boldsymbol{\gamma} = \begin{pmatrix} 0 & \boldsymbol{\sigma} \\ -\boldsymbol{\sigma} & 0 \end{pmatrix}, \gamma^5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \text{Dirac representation}$$

$$\mathbf{S} = \frac{\boldsymbol{\Sigma}}{2}, \boldsymbol{\Sigma} = \begin{pmatrix} \boldsymbol{\sigma} & 0 \\ 0 & \boldsymbol{\sigma} \end{pmatrix} = \frac{\boldsymbol{\gamma}^0 \boldsymbol{\gamma}}{a} \gamma^5 = \boldsymbol{a} \gamma^5 \quad \text{Spin operator}$$

$$\Lambda = \frac{\boldsymbol{\Sigma} \cdot \mathbf{p}}{|\mathbf{p}|} \quad \text{Helicity operator}$$

$$\left. \begin{aligned} \Lambda u^{(+)} &= +u^{(+)} \\ \Lambda u^{(-)} &= -u^{(-)} \end{aligned} \right\} \quad \text{Helicity eigenstates}$$

$$P_{\pm} = \frac{1 \pm \Lambda}{2} \quad \text{Projection operators onto helicity eigenstates}$$

# Helicity/Chirality - II

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Projectors, indeed:

$$P_+ P_+ = \left( \frac{1+\Lambda}{2} \right) \left( \frac{1+\Lambda}{2} \right) = \frac{1}{4} (1 + \Lambda + \Lambda + \Lambda^2)$$

$$\Lambda^2 = \frac{(\boldsymbol{\Sigma} \cdot \mathbf{p})^2}{|\mathbf{p}|^2} = 1 \rightarrow P_+ P_+ = \frac{1}{4} (1 + 2\Lambda + 1) = \left( \frac{1+\Lambda}{2} \right) = P_+, \quad P_- P_- = P_-$$

$$P_+ P_- = \left( \frac{1+\Lambda}{2} \right) \left( \frac{1-\Lambda}{2} \right) = \frac{1}{4} (1 + \Lambda - \Lambda - \Lambda^2) = 0 = P_- P_+$$

$$1 = \frac{1-\Lambda}{2} + \frac{1+\Lambda}{2} = P_- + P_+ \rightarrow 1u = (P_+ + P_-)u = u_+ + u_-$$

$$\Lambda = \frac{\boldsymbol{\Sigma} \cdot \mathbf{p}}{|\mathbf{p}|} = \frac{\gamma^0 \boldsymbol{\gamma} \gamma^5 \cdot \mathbf{p}}{|\mathbf{p}|} = \frac{\boldsymbol{\alpha} \cdot \mathbf{p}}{|\mathbf{p}|} \gamma^5 \rightarrow P_{\pm} = \frac{1 \pm \boldsymbol{\alpha} \cdot \hat{\mathbf{p}} \gamma^5}{2}$$

# Helicity/Chirality - III

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$\gamma^5$  Chirality operator

$P_L = \frac{1-\gamma^5}{2}, P_R = \frac{1+\gamma^5}{2}$  Projectors onto chirality eigenstates

$$\begin{cases} P_L u = u_L \\ P_R u = u_R \end{cases} \rightarrow 1u = (P_L + P_R)u = u_L + u_R$$

A very important limit:

$$\boldsymbol{\alpha} \cdot \mathbf{p} = E - \beta m$$

$$\Lambda = \frac{E - m \cdot \beta}{p} \gamma^5 \xrightarrow{E \gg m} \gamma^5$$

$$P_{\pm} = \frac{1 \pm \Lambda}{2} \xrightarrow{E \gg m} \frac{1 \pm \gamma^5}{2} = P_{R,L}$$

For high energy, or massless, particles:

*Helicity* projectors  $\rightarrow$  *Chirality* projectors



# Helicity/Chirality - IV

$$Eu = (\boldsymbol{\alpha} \cdot \mathbf{p} + \beta m)u$$

$$u = \begin{pmatrix} \phi \\ \chi \end{pmatrix}, \quad \phi, \chi \text{ 2 components spinors}$$

$$\boldsymbol{\alpha} = \begin{pmatrix} \boldsymbol{\sigma} & 0 \\ 0 & -\boldsymbol{\sigma} \end{pmatrix}, \quad \beta = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \text{ Dirac matrices in chiral representation}$$

$$\rightarrow \begin{cases} E\phi = (\boldsymbol{\sigma} \cdot \mathbf{p})\phi + m\chi \\ E\chi = -(\boldsymbol{\sigma} \cdot \mathbf{p})\chi + m\phi \end{cases}$$

$$\rightarrow \begin{cases} E\phi = (\boldsymbol{\sigma} \cdot \mathbf{p})\phi \\ E\chi = -(\boldsymbol{\sigma} \cdot \mathbf{p})\chi \end{cases}, m=0 \rightarrow \begin{cases} \frac{(\boldsymbol{\sigma} \cdot \mathbf{p})}{E = |\mathbf{p}|} \phi = \phi \\ \frac{(\boldsymbol{\sigma} \cdot \mathbf{p})}{E = |\mathbf{p}|} \chi = -\chi \end{cases} \rightarrow \phi, \chi \text{ Helicity eigenstates}$$

# Helicity/Chirality - V

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States with definite value of chirality, massive or massless particles

*Particle*

$$u_L = \frac{1}{2}(1 - \gamma^5)u$$

$$u_R = \frac{1}{2}(1 + \gamma^5)u$$

$$\bar{u}_L = \bar{u} \frac{1}{2}(1 + \gamma^5)$$

$$\bar{u}_R = \bar{u} \frac{1}{2}(1 - \gamma^5)$$

*Antiparticle*

$$v_L = \frac{1}{2}(1 + \gamma^5)v$$

$$v_R = \frac{1}{2}(1 - \gamma^5)v$$

$$\bar{v}_L = \bar{v} \frac{1}{2}(1 - \gamma^5)$$

$$\bar{v}_R = \bar{v} \frac{1}{2}(1 + \gamma^5)$$

Is it true? Try one example:

$$\gamma^5 u_L = \gamma^5 \frac{1}{2}(1 - \gamma^5)u = \frac{1}{2}(\gamma^5 - 1)u = -\frac{1}{2}(1 - \gamma^5)u = -u_L \quad \text{OK}$$

# Helicity/Chirality - VI

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Reminder: Solutions of Dirac equation

$$\begin{cases} u_1, u_2 & \text{+ve energy} \\ u_3, u_4 & \text{-ve energy} \end{cases} \begin{pmatrix} 1 \\ 0 \\ \frac{p_z}{E+m} \\ \frac{p_x+ip_y}{E+m} \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ \frac{p_x-ip_y}{E+m} \\ \frac{p_z}{E+m} \end{pmatrix}, \begin{pmatrix} \frac{p_z}{E-m} \\ \frac{p_x+ip_y}{E-m} \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} \frac{p_x-ip_y}{E-m} \\ \frac{-p_z}{E-m} \\ 0 \\ 1 \end{pmatrix} \cdot e^{i(\mathbf{p}\cdot\mathbf{r}-Et)}$$

Change to all +ve energy solutions by introducing antiparticle spinors  $v_1, v_2$ :

$$\begin{cases} u_1, u_2 & \text{+ve energy} \\ v_1, v_2 & \text{+ve energy} \end{cases} \begin{pmatrix} 1 \\ 0 \\ \frac{p_z}{E+m} \\ \frac{p_x+ip_y}{E+m} \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ \frac{p_x-ip_y}{E+m} \\ \frac{p_z}{E+m} \end{pmatrix} \cdot e^{i(\mathbf{p}\cdot\mathbf{r}-Et)}; \begin{pmatrix} \frac{p_x-ip_y}{E+m} \\ \frac{-p_z}{E+m} \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} \frac{p_z}{E+m} \\ \frac{p_x+ip_y}{E+m} \\ 1 \\ 0 \end{pmatrix} \cdot e^{-i(\mathbf{p}\cdot\mathbf{r}-Et)}$$

→ Antiparticle spinors have *reversed momentum*

$$\rightarrow \begin{cases} v_R = P_L v = \frac{1-\gamma_5}{2} v \\ v_L = P_R v = \frac{1+\gamma_5}{2} v \end{cases}$$

# Helicity/Chirality - VII

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Helicity of chiral states:

$$\text{Massless particle: } \begin{cases} u_L & \langle H \rangle = -1 \\ u_R & \langle H \rangle = +1 \end{cases}$$

→ Helicity defined  $\equiv$  Full longitudinal polarization

$$\text{Massive particle: } \begin{cases} u_L & \langle H \rangle = -\beta \\ u_R & \langle H \rangle = +\beta \end{cases}$$

→ Helicity undefined, superposition of  $\pm 1$  eigenstates

Massless particles: *Helicity is Lorentz invariant*

Massive particles: *Helicity is frame dependent*

# Universality: Leptons

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Extend V-A to muon weak interactions:

$$\mu^+ \rightarrow e^+ + \bar{\nu}_\mu + \nu_e, \quad \mu^- \rightarrow e^- + \nu_\mu + \bar{\nu}_e \quad \mu \text{ decay}$$

$$\mu^- + p \rightarrow n + \nu_\mu \quad \mu \text{ capture, involves nucleon current}$$

$\mu$  decay purely leptonic:

Guess: *Current-Current*, V-A for both electron and muon charged currents

Lagrangian density:

$$L_{V-A} = -\frac{G_F}{\sqrt{2}} [\bar{\nu}_\mu \gamma_\mu (1 - \gamma_5) \mu] [\bar{e} \gamma^\mu (1 - \gamma_5) \nu] + H.C.$$

Compute:

$\mu$  Lifetime

Electron energy spectrum

Electron longitudinal polarization

# Universal Weak Coupling

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Measure  $\mu$  lifetime, get Fermi constant:

$$G_F^{(\mu)} = \sqrt{\frac{192\pi^3}{\tau_\mu m_\mu^5}} = 1.1638 \cdot 10^{-5} \text{ GeV}^{-2}$$

After careful (radiative) corrections:

$$G_F^{(\mu)} = 1.16637 \cdot 10^{-5} \text{ GeV}^{-2}$$

Measure  $\tau$  lifetime and  $BR$  to electron, get Fermi constant:

$$G_F^{(\tau)} = \sqrt{\frac{\Gamma(\tau \rightarrow e\bar{\nu}_e\nu_\tau)}{\Gamma_{tot}} \frac{192\pi^3}{\tau_\tau m_\tau^5}} = 1.1642 \cdot 10^{-5} \text{ GeV}^{-2}$$

Compare to Fermi constant from  $\beta$  decay:

$$G_F^{(\beta)} = 1.1361 \cdot 10^{-5} \text{ GeV}^{-2}$$

→ (Almost) Universal leptonic charged current!

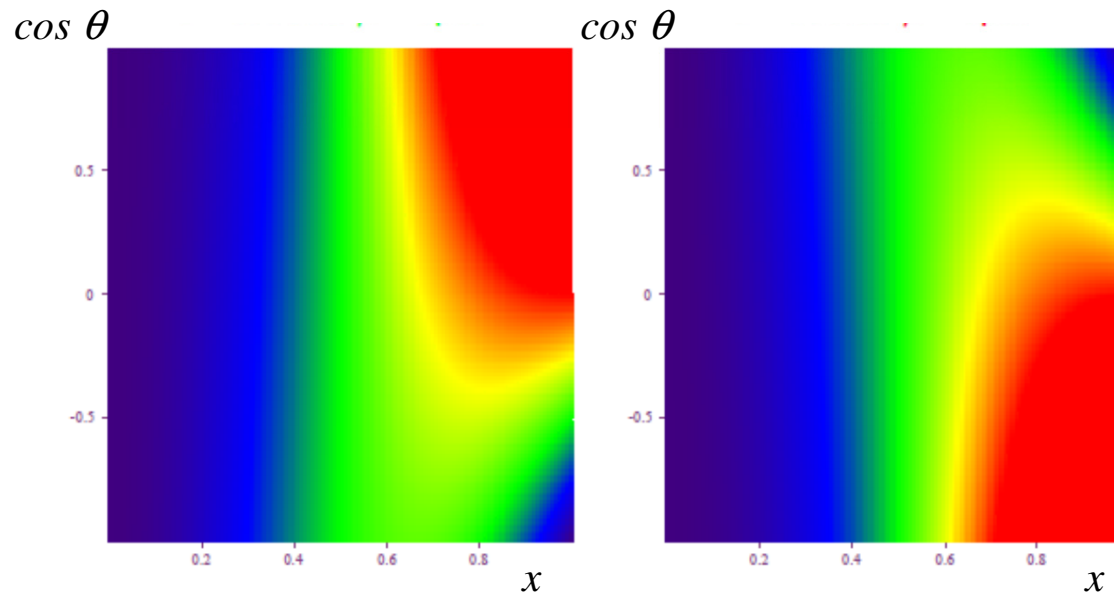
# $\mu$ Decay: $C$ & $P$ Violations

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Polarized  $\mu^\pm$  decay:  $\mu$  rest frame

$$\frac{dN^\pm}{dx d\cos\theta} = x^2(3-2x) \left[ 1 \pm \cos\theta \frac{2x-1}{3-2x} \right]$$

$$x = \frac{p_e}{p_e^{\max}}, \quad \theta \triangleq (\mathbf{s}_\mu, \mathbf{p}_e)$$

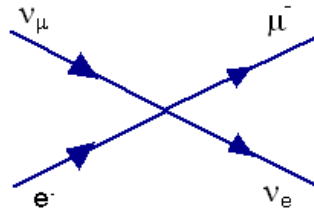


# Inverse Muon Decay - I

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Charged current:

$$\nu_\mu + e^- \rightarrow \mu^- + \nu_e$$



$$\sum_{spin} T_{fi} T_{fi}^* = \sum_{spin} \frac{G_F}{\sqrt{2}} [\bar{u}(3) \gamma^\mu (1 - \gamma^5) u(1)] [\bar{u}(3) \gamma_\nu (1 - \gamma_5) u(1)]^* \cdot$$

$$[\bar{u}(4) \gamma_\mu (1 - \gamma_5) u(2)] [\bar{u}(4) \gamma^\nu (1 - \gamma^5) u(2)]^*$$

$$\sum_{spin} [\bar{u}(a) \Gamma_1 u(b)] [\bar{u}(a) \Gamma_2 u(b)]^* = Tr[\Gamma_1 (\not{p}_b + m_b) \Gamma_2 (\not{p}_a + m_a)]$$

$$\sum_{spin} |T_{fi}|^2 = 64 G_F^2 (p_1 \cdot p_2) (p_3 \cdot p_4)$$

$$\sum_{spin} |T_{fi}|^2 = 256 G_F^2 E^4 \left[ 1 - \left( \frac{m_\mu}{2E} \right)^2 \right]$$



# Inverse Muon Decay - II

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$$\frac{d\sigma}{d\Omega} = \frac{G_F^2}{\pi^2} E^{*2} \left[ 1 - \left( \frac{m_\mu}{2E^*} \right)^2 \right]^2$$

$$\rightarrow \sigma = \frac{4G_F^2}{\pi} E^{*2} \left[ 1 - \left( \frac{m_\mu}{2E^*} \right)^2 \right]^2, \quad E^* \text{ CM energy of } e, \nu$$

$$E^* \simeq \sqrt{2mE_\nu}$$

$$\rightarrow \sigma = \frac{8G_F^2 m_\mu}{\pi} E_\nu \propto E_\nu \quad \text{at high energy}$$

$\sigma$  badly divergent

→ Unphysical

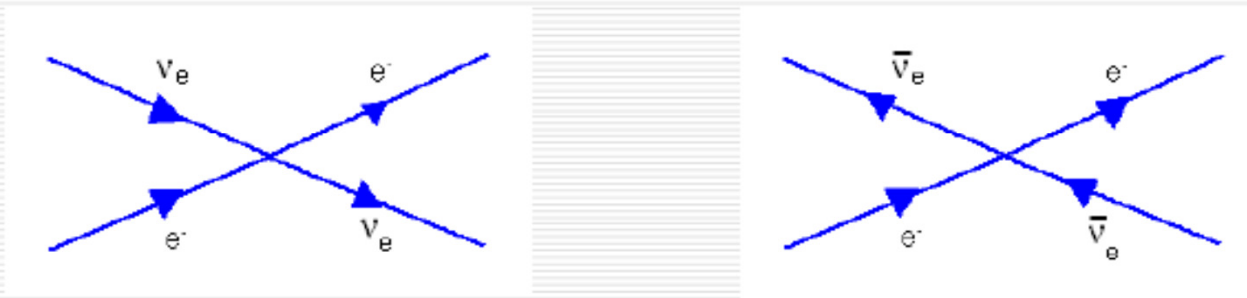
# Neutrino – Lepton Scattering - I

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Charged current  $\nu_e/\bar{\nu}_e - e$  scattering:

$$\nu_e + e^- \rightarrow e^- + \nu_e$$

$$\bar{\nu}_e + e^- \rightarrow \bar{\nu}_e + e^-$$



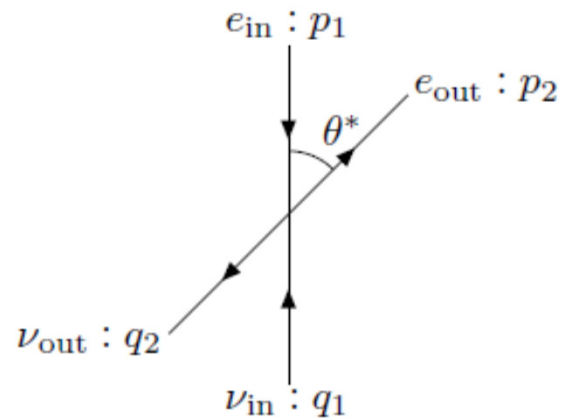
NB Actually incomplete:

Missing neutral current amplitude leading to the same final states

Cross sections must be evaluated by adding *all* the relevant amplitudes

# Neutrino – Lepton Scattering - II

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$$T_{fi} = -i \frac{G_F}{\sqrt{2}} [\bar{v}(\nu, q_1) \gamma_\mu (1 - \gamma_5) u(e, p_1)] \cdot [\bar{u}(e, p_2) \gamma^\mu (1 - \gamma_5) v(\nu, q_2)]$$

# Neutrino – Lepton Scattering - III

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$\bar{\nu}e \rightarrow \bar{\nu}e$

$$\frac{d\sigma_{\bar{\nu}e}}{d\Omega^*} = \frac{\langle |T_{fi}|^2 \rangle}{64\pi^2 s} = \frac{G_F^2 2mE_\nu (1 - \cos\theta^*)^2}{16\pi^2}$$

Total cross section:

$$\sigma_{\bar{\nu}e} = \frac{G_F^2 2mE_\nu}{3\pi} \approx 0.574 \cdot 10^{-41} E_\nu (\text{GeV}) \text{ cm}^2$$

$\nu e \rightarrow \nu e$

$$\frac{d\sigma_{\nu e}}{d\Omega^*} = \frac{\langle |T_{fi}|^2 \rangle}{64\pi^2 s} = \frac{G_F^2 2mE_\nu}{4\pi^2}$$

Total cross section:

$$\sigma_{\nu e} = \frac{G_F^2 2mE_\nu}{\pi} \approx 1.72 \cdot 10^{-41} E_\nu (\text{GeV}) \text{ cm}^2$$

# Neutrino – Lepton Scattering - IV

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Total cross sections:

$$\sigma_{\bar{\nu}e} = \frac{G_F^2 2mE_\nu}{3\pi} \approx 0.574 \cdot 10^{-41} E_\nu (\text{GeV}) \text{ cm}^2$$

$$\sigma_{\nu e} = \frac{G_F^2 2mE_\nu}{\pi} \approx 1.72 \cdot 10^{-41} E_\nu (\text{GeV}) \text{ cm}^2$$

as divergent at high energy as the inverse muon decay

NB Cross sections only crude approximations:

Neutral current contribute not included

Interesting factor  $\times 3$  between  $\nu e$  and  $\bar{\nu}e$

# Neutrino – Lepton Scattering - V

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Origin of factor  $\times 3$ :

$$z = \cos \theta^*$$



allowed at all angles



forbidden (angular momentum) at  $z = +1$

→ Amplitude & Cross section suppressed at small angle

# Unitarity Troubles - I

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Divergence at high energy : Unitarity bound violated around  $E_\nu^* \sim 300 \text{ GeV}$

$$\sigma \leq \frac{4\pi}{k^2} = \frac{4\pi}{E^{*2}} \quad \text{Unitarity bound for S-Wave scattering}$$

Reminder: (Simpler) Spinless potential scattering

Expand incident (plane) wave into angular momentum eigenstates

$$\Psi_i = e^{ikz} = \frac{i}{2kr} \sum_{l=0}^{\infty} (2l+1) [(-1)^l e^{-ikr} - e^{ikr}] P_l(\cos \theta)$$

Outgoing spherical wave phase shifted by potential:

$$\Psi_{total} = \Psi_{scattered} + \Psi_i = \frac{i}{2kr} \sum_{l=0}^{\infty} (2l+1) [(-1)^l e^{-ikr} - \eta_l e^{2i\delta_l} e^{ikr}] P_l(\cos \theta)$$

$$\Psi_{scattered} = \Psi_{total} - \Psi_i = \frac{e^{ikr}}{kr} \sum_{l=0}^{\infty} (2l+1) \frac{\eta_l e^{2i\delta_l} - 1}{2i} P_l(\cos \theta) = \frac{e^{ikr}}{r} f(\theta)$$

# Unitarity Troubles - II

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Outgoing elementary flux:

$$d\Phi_{out} = v_{out} \Psi_{scat} \Psi_{scat}^* r^2 d\Omega = v_{out} |F(\theta)|^2 d\Omega$$

Incident flux:

$$\Phi_{in} = \Psi_{in} \Psi_{in}^* v_{in} = v_{in}$$

$$\rightarrow d\sigma = \frac{\Phi_{out}}{\Phi_{in}} = |F(\theta)|^2 d\Omega$$

$$\sigma = \int |F(\theta)|^2 d\Omega$$

$$\sigma = \frac{1}{k^2} \sum_{l,m} (2l+1) \left[ \frac{\eta_l e^{2i\delta_l} - 1}{2i} \right] (2m+1) \left[ \frac{\eta_m e^{2i\delta_m} - 1}{2i} \right]^*$$

$$\times \int P_l(\cos\theta) P_m(\cos\theta) d\Omega$$

$$\sigma = \frac{4\pi}{k^2} \sum_l (2l+1) \left[ \left| \frac{\eta_l e^{2i\delta_l} - 1}{2i} \right| \right]^2 = \frac{4\pi}{k^2} \sum_l (2l+1) \sin^2 \delta_l$$

$$\rightarrow \sigma_{l=0} = \frac{4\pi}{k^2} \sin^2 \delta_0 \leq \frac{4\pi}{k^2} = \frac{4\pi}{E^{*2}} \quad \text{Unitarity bound for S-Wave scattering}$$

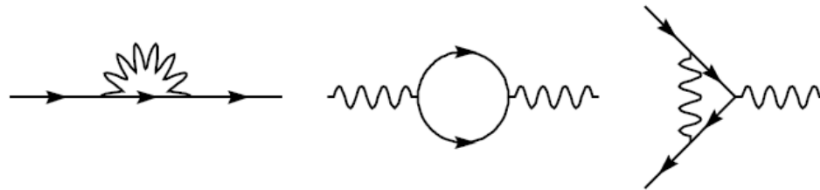


# Renormalization Troubles - I

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Suppose Fermi's theory can be saved by radiative corrections:  
Assume divergent cross-section as due to our limited, tree-level approximation  
Maybe higher orders could fix it

Take QED as an example



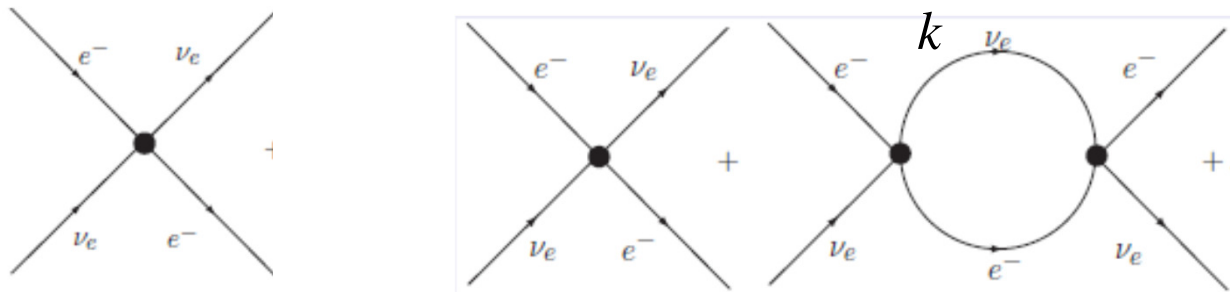
These diagrams (and higher orders) divergent:  
However, nice fix available by renormalization procedure

Very successful program, leading to extraordinary accuracy & agreement  
between theory and experiment

# Renormalization Troubles - II

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Higher order diagrams in Fermi's theory:



Cannot be fixed by renormalization: Fermi's theory non-renormalizable

Indeed: Each vertex  $\sim G_F$

# Renormalization Troubles - III

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Lagrangian density ( $\mu$  decay etc)

$$L_{Fermi} = -\frac{G_F}{\sqrt{2}} \bar{\nu}_\mu \gamma^\alpha (1 - \gamma^5) \mu \bar{e} \gamma_\alpha (1 - \gamma^5) \nu_e$$

Action dimension:

$$S = \int L_{Fermi} d^4x \rightarrow [S] = [E][T] = [E][E^{-1}] = 0 \quad \text{Action dimensionless}$$

$$\rightarrow [L_{Fermi}] = [E^4]$$

$$[L_{Fermi}] = [G_F][\psi^4]$$

$$\text{Field dimension: } [\psi] = [E^{3/2}]$$

$$\rightarrow [L_{Fermi}] = [G_F][E^6] = [E^4]$$

$$\rightarrow [G_F] = [E^{-2}]$$

Amplitude dimensionless:  $[A] = 0$

→ Each  $G_F$  in the amplitude to be dimensionally compensated by some  $k^2$  factor

→ Loop diagrams of higher orders must include integrals of higher powers of  $k$

→ More and more divergent

# Beyond Fermi's Theory

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As anticipated:

*Current-Current* must be a *low energy effective theory*:  
Low energy approximation of a more general theory

Replace *contact interaction* (current-current) by *boson exchange*:

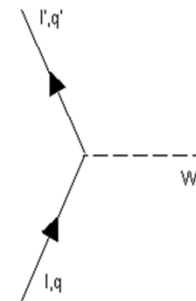
Modeled after the electromagnetic interaction

Exchanged particle must be

*Charged* (Charged current  $\pm$ )

*Chiral* (Only coupled to left chiral parts: Parity violation)

*Heavy* (Fermi's point-like interaction OK at low energy)



Vertice in corrente carica

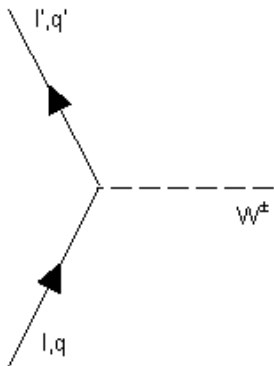
# Intermediate Vector Boson - I

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Some key points

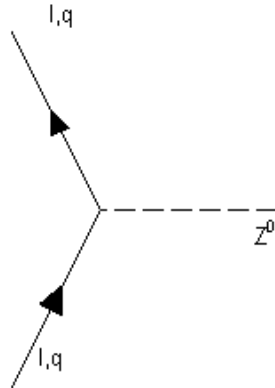
A) (Quarks and) Leptons (both) interact through the exchange of *vector particles*

$$\Delta Q = \pm 1$$

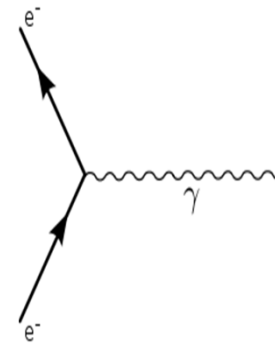


Vertice in corrente carica

$$\Delta Q = 0$$



Vertice in corrente neutra



Vertice elettromagnetico

# Intermediate Vector Boson - II

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B) Exchanged vector bosons are (*very*) massive

Range of weak interaction quite small:

Compare  $\beta$ -decay of nuclei,  $R < R_{nucleus}$

Cannot tell how large is boson mass, just raw estimate  $M \geq 1 \text{ GeV}$

C) Exchanged vector bosons have undefined parity

Parity violation

# Intermediate Vector Boson - III

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W propagator:  $-i \frac{g_{\mu\nu} - q_\mu q_\nu / M_W^2}{q^2 - M_W^2}$  (Gauge dependent)

$q^2 \ll M_W^2 \rightarrow -i \frac{g_{\mu\nu} - q_\mu q_\nu / M_W^2}{q^2 - M_W^2} \approx i \frac{g_{\mu\nu}}{M_W^2}$   $q^2$ -independent

$$T_{fi} \cong \left( \frac{1}{2\sqrt{2}} \right)^2 g_W^2 \left( \bar{u}_f^{(1)} \frac{1}{2} \gamma^\mu (1 - \gamma_5) u_i^{(1)} \right) i \frac{g_{\mu\nu}}{M_W^2} \left( \bar{u}_f^{(2)} \frac{1}{2} \gamma_\nu (1 - \gamma_5) u_i^{(2)} \right)$$

$$\rightarrow T_{fi} \cong i \frac{1}{8} 4\sqrt{2} G_F j^{\mu(1)} j_\mu^{(2)}$$

$g_W^2 \equiv \alpha_W$  Charged current coupling constant

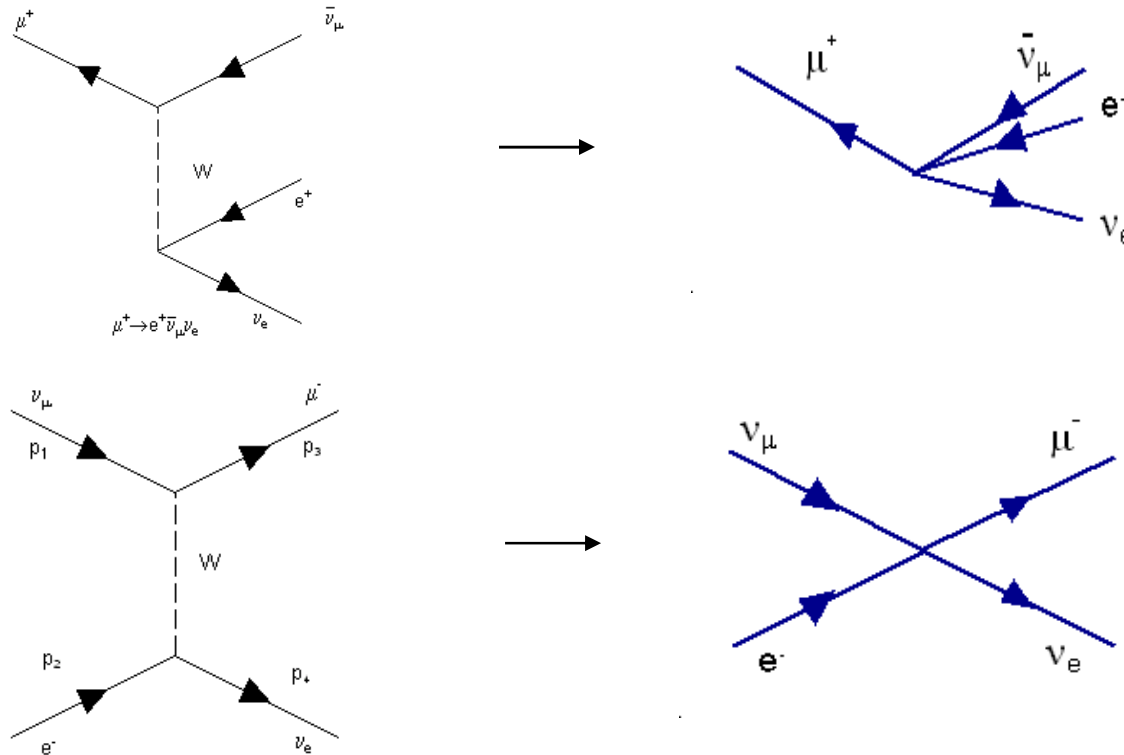
$$G_F = \frac{\sqrt{2}}{8} \left( \frac{g_W}{M_W} \right)^2 \quad \text{Fermi constant}$$

# Intermediate Vector Boson - IV

32

Showing how Standard Model diagrams collapse into current-current at low energy:

$$q^2 \ll M_W^2 \rightarrow -i \frac{g_{\mu\nu} - q_\mu q_\nu / M_W^2}{q^2 - M_W^2} \approx i \frac{g_{\mu\nu}}{M_W^2}$$





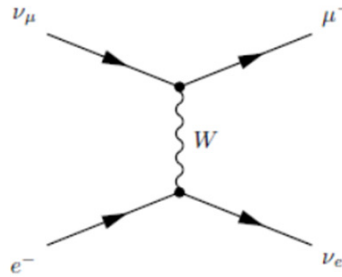
# Intermediate Vector Boson - V

33

Good fix for some problems:

Cross sections of several neutrino reactions

Inverse Muon Decay:



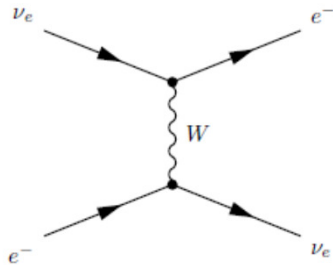
$$\frac{d\sigma}{d\Omega_{CM}} = \frac{G_F^2 M_W^4}{16\pi^2 k^2} \left( \frac{4k^2}{4k^2 - M_W^2} \right)^2 \rightarrow \frac{d\sigma}{d\Omega_{CM}} \approx \begin{cases} \frac{G_F^2 k^2}{\pi^2}, & 4k^2 \ll M_W^2 \\ \frac{G_F^2 M_W^4}{16\pi^2 k^2}, & 4k^2 \gg M_W^2 \end{cases} \rightarrow \sigma \sim \begin{cases} \frac{4G_F^2 k^2}{\pi}, & 4k^2 \ll M_W^2 \\ \frac{G_F^2 M_W^4}{4\pi k^2}, & 4k^2 \gg M_W^2 \end{cases}$$

No divergence!

# Intermediate Vector Boson - VI

34

Charged current (only), tree level elastic (anti) electronic neutrino-electron cross sections:



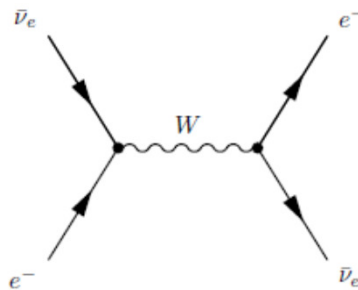
$$\nu_e + e \rightarrow e + \nu_e$$

$$\frac{d\sigma}{d\Omega_{CM}} \simeq \frac{16G_F^2 M_W^2}{\pi^2} \frac{4k^2}{(q^2 - M_W^2)^2}, \quad k^2 \gg m_e^2$$

$$q^2 \simeq -2k^2(1 - \cos\theta)$$

$$s \simeq 4k^2$$

$$\rightarrow \sigma \simeq \frac{G_F^2}{\pi} \frac{4k^2}{1 + \frac{M_W^2}{4k^2}} \xrightarrow{k^2 \gg M_W^2} \frac{G_F^2 M_W^2}{\pi}, \quad \text{no divergence!}$$



$$\bar{\nu}_e + e \rightarrow \bar{\nu}_e + e$$

$$\sigma \simeq \frac{G_F^2 M_W^4}{3\pi} \frac{4k^2}{16k^4 \left(1 - \frac{M_W^2}{4k^2}\right)^2} = \frac{G_F^2 M_W^4}{3\pi} \frac{1}{4k^2 \left(1 - \frac{M_W^2}{4k^2}\right)^2}$$

$$\sigma \xrightarrow{k^2 \gg M_W^2} \frac{G_F^2 M_W^4}{3\pi s} \quad \text{no divergence!}$$

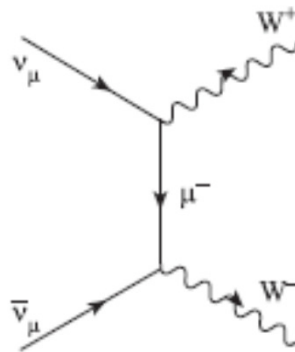
# Intermediate Vector Boson - VII

35

Another dark side of naive IVB model:

Take hypothetical reaction

$$\nu_{\mu} + \bar{\nu}_{\mu} \rightarrow W^{+} + W^{-}$$



No question, not easy to realize in the lab...

Nevertheless, it should be possible to compute cross section

Anyway, similar issues for the (realistic) reaction

$$e^{+} + e^{-} \rightarrow W^{+} + W^{-}$$

# Intermediate Vector Boson - VIII

36

Central issue:

Massive  $W^\pm$  bosons in the final state

→ 3 polarization states for a massive vector particle

Rest frame:

$$\left. \begin{aligned} \varepsilon_x &= (0, 1, 0, 0) \\ \varepsilon_y &= (0, 0, 1, 0) \end{aligned} \right\} \varepsilon_T \text{ Transverse polarization}$$

$$\varepsilon_z = (0, 0, 0, 1) \quad \varepsilon_L \text{ Longitudinal polarization}$$

After a  $z$ -boost, carrying the  $W$  to 4-momentum  $k^\mu = (k^0, 0, 0, k)$

$$\varepsilon_T(k) = \varepsilon_T(0)$$

$$\varepsilon_L(k) = \left( \frac{k}{M_W}, 0, 0, \frac{k_0}{M_W} \right) = \frac{k^\mu}{M_W} + O\left(\frac{M_W}{k_0}\right)$$

# Intermediate Vector Boson - IX

37

Matrix element ( $1 = \nu, W^+, 2 = \bar{\nu}, W^-; p = \nu, k = W$ ):

$$T_{fi}^{\lambda_1 \lambda_2} = g^2 \varepsilon_{\mu}^{-*}(k_2, \lambda_2) \varepsilon_{\mu}^{+*}(k_1, \lambda_1) \bar{v}(p_2) \gamma^{\mu} (1 - \gamma_5) \frac{(\not{p}_1 - \not{k}_1 + m_{\mu})}{(p_1 - k_1)^2 - m_{\mu}^2} \gamma^{\nu} (1 - \gamma_5) u(p_1)$$

By:

Neglecting  $\mu$  mass,

restricting to longitudinally polarized  $W$ 's ( $\lambda = 0$ ),

taking the high energy ( $\gg M_W$ ) limit for the polarization 4-vectors,

commuting  $\gamma_5$ :

$$|T_{fi}^{00}|^2 = \frac{g^4}{M_W^4 (p_1 - k_1)^4} \text{Tr} \left[ k_2 (1 - \gamma_5) (\not{p}_1 - \not{k}_1) \not{k}_1 \not{p}_1 \not{k}_1 (\not{p}_1 - \not{k}_1) \not{k}_2 \not{p}_2 \right]$$

Summing over initial spins:

$$\sum_{spin} |T_{fi}^{00}|^2 = \frac{g^4}{M_W^4} (p_1 \cdot k_1) (p_2 \cdot k_2) = \frac{g^4}{M_W^4} E^4 (1 - \cos^2 \theta) \rightarrow \frac{d\sigma}{d\Omega^*} = \frac{G_F^2}{8\pi^2} E^2 \sin^2 \theta$$

→ Still divergent at high energy

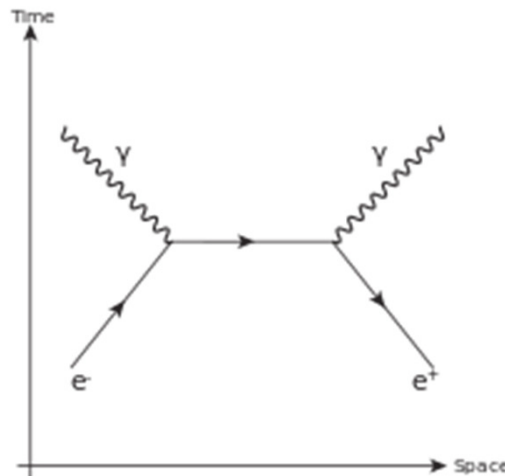
No simple solution for this problem:

*Massive vector particles cannot make it without 3 polarization states*

# Intermediate Vector Boson - X

38

Compare to well known QED process



No contribution from longitudinal photons:

Real photons always transverse, as a consequence of *gauge invariance* of QED

(Compare to classical radiation field:  $\mathbf{E}, \mathbf{B}$  purely transverse)

Hope the gauge invariance benefits can be extended to weak interactions..

# Intermediate Vector Boson - XI

39

Is that a single trouble, unrelated to the full IVB scheme?

Have a look at diagrams including *virtual W*:

Discover that a new divergence hits hard our naive IVB model..

Looking at virtual *W* propagator:

$$\frac{-g_{\mu\nu} + k_{\mu}k_{\nu}/M_W^2}{k^2 - M_W^2} \xrightarrow{k^2 \rightarrow \infty} \text{const}$$

→ Will make diagrams with virtual *Ws* divergent at high energy

Serious illness of IVB model, particularly relevant for *neutral current* processes

# Intermediate Vector Boson - XII

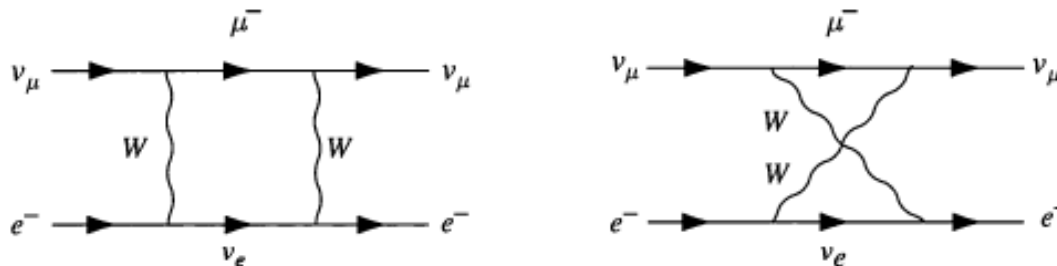
40

Neutral current reactions like:

$$\nu_\mu + e^- \rightarrow \nu_\mu + e^-$$

$$\bar{\nu}_\mu + e^- \rightarrow \bar{\nu}_\mu + e^-$$

Not allowed at tree level by our IVB model, only by loop diagrams:



But we can't compute loop diagrams including virtual  $W$ :

Divergent, IVB theory *not renormalizable*

Basic requirement: Theory must be renormalizable



# Intermediate Vector Boson - XIII

41

Aside: Expect strong suppression

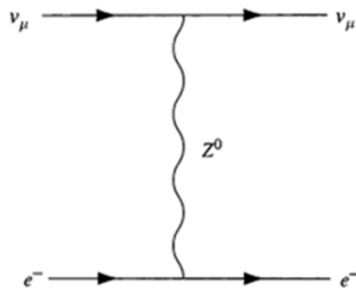
Surprisingly, after first observations: NC Cross sections  $\approx$  Allowed processes

$$\nu_e + e^- \rightarrow e^- + \nu_e$$

$$\nu_\mu + e^- \rightarrow \mu^- + \nu_e$$

Suggestion:

Maybe neutral currents do exist *at tree level*, e.g.



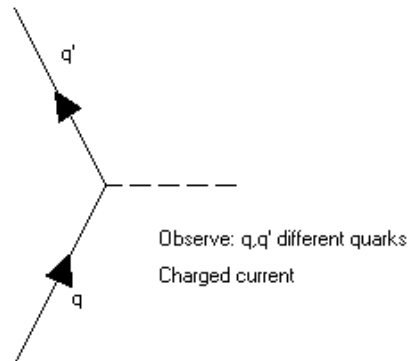
[Indeed, neutral currents are *required* in standard electroweak theory]

# Universality: Quarks

42

Semileptonic and non leptonic processes understood in terms of quarks

Coupling basically similar to leptonic charged currents:



Picture is slightly more complicated, however  
Fundamental question:

*Is the quark coupling identical to the lepton one?*

# Families - I

43

Consider charged current of leptons:

Very natural to group charged and neutral leptons into *doublets*, or *families*

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix} \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix} \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}$$

Within each doublet, charged current transitions can be thought as due to emission/absorption of  $W^\pm$  bosons, similar to (neutral) e.m. current transitions

$$\begin{array}{ccccc} W^- & \rightarrow & \nu_e & \rightarrow & W^+ \\ & & \uparrow & & \downarrow \\ W^- & \leftarrow & e^- & \leftarrow & W^+ \end{array}$$

Similar for 2nd, 3rd family

# Families - II

44

Natural to extend this scheme to quarks

$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ s \end{pmatrix} \quad \begin{pmatrix} t \\ b \end{pmatrix}$$

$$\begin{array}{ccccc} W^- & \rightarrow & u & \rightarrow & W^+ \\ & & \uparrow & & \\ W^- & \leftarrow & d & \leftarrow & W^+ \end{array}$$

Similar for 2nd, 3rd family

Almost correct, but incomplete:

Does not account for strangeness (more generally,  $\rightarrow$  flavour) violating processes

Cabibbo's very ingenious idea:

*Quark flavor eigenstates (i.e., quark model eigenstates) are not to be identified with quark weak currents*

*$\rightarrow$ Weak currents are mixtures of different flavors*

By universal convention, mixing is assumed between  $d$ ,  $s$ ,  $b$  quarks

# Families - III

45

In terms of mixed “ $d$ -like” quarks, with just 2 families:

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos \theta_c & \sin \theta_c \\ -\sin \theta_c & \cos \theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix} \quad \theta_c \text{ Cabibbo's angle}$$

This explain *many* things....

How to extract  $\theta_c$ ?

Just one example: Get the angle from  $\beta$  decay

$$G_F^{(\beta)} = 0.975 G_F^{(\mu)} \text{ (Remember that 2\% difference ?)}$$

$$\rightarrow G_F^{(\beta)} = \cos \theta_c G_F^{(\mu)}$$

$$\rightarrow \theta_c \simeq 13^\circ$$

# Families - IV

46

Extend the idea to 3 families:

From Cabibbo's angle to *Cabibbo-Kobayashi-Maskawa* matrix

$$\begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} |d\rangle \\ |s\rangle \\ |b\rangle \end{bmatrix} = \begin{bmatrix} |d'\rangle \\ |s'\rangle \\ |b'\rangle \end{bmatrix}$$

From unitarity:

*3 mixing angles*

*1 complex phase*      This can account for CP violation

Experimental values:

$$\begin{bmatrix} 0.9753 & 0.221 & 0.003 \\ 0.221 & 0.9747 & 0.040 \\ 0.009 & 0.039 & 0.9991 \end{bmatrix}$$

Almost diagonal

Heavy quarks even more diagonal

# $\nu, \bar{\nu}$ - Nucleon Cross Section - I

47

Extend V-A to neutrino-nucleon scattering

$$\nu_{\mu} + N \rightarrow \mu^{-} + X$$

$$\bar{\nu}_{\mu} + N \rightarrow \mu^{+} + X$$

Somewhat similar to  $e$ - $N$ ,  $\mu$ - $N$  deep inelastic scattering

Modeling similar to DIS: Parton elastic scattering

Deep inelastic neutrino scattering reveals the same structure as charged lepton DIS

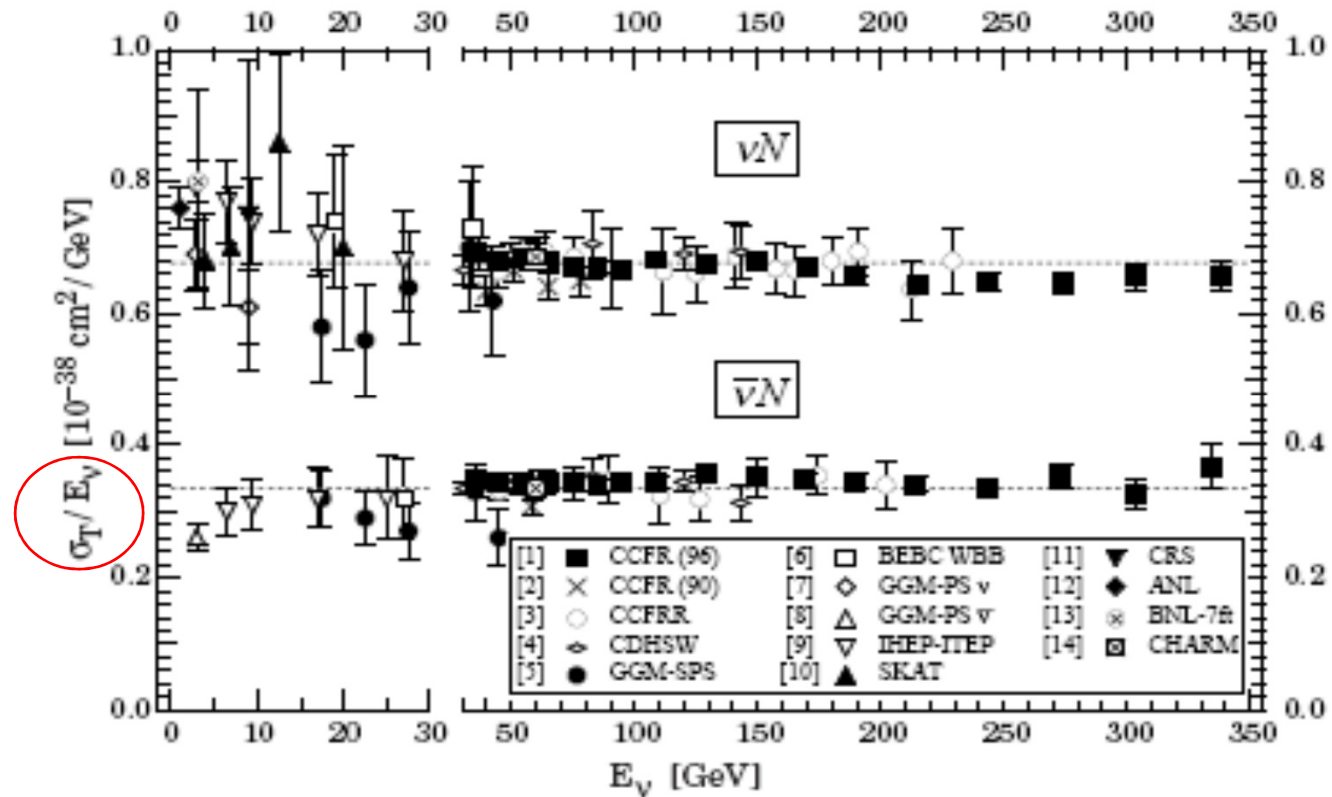
More information: Charged current sensitive to parton charge sign

→ Can separate quark/antiquark contribution

And: Yes, by looking at (anti)neutrino-nucleon DIS structure functions (probing the parton structure by charged – and neutral – weak currents) one concludes that quarks couple to weak currents exactly as leptons

# $\nu, \bar{\nu}$ - Nucleon Cross Section - II

48



Linearly rising cross section confirmed...



# Gauge Symmetry - I

49

What makes QED so successful?

Renormalization program allows for computing observables with high accuracy, comparable to experimental resolution

QED is a renormalizable field theory

Fermi's theory is a non-renormalizable theory

And:

Naive IVB theory of weak interactions is a non-renormalizable theory

Try to discover what makes the difference

# Gauge Symmetry - II

50

Back to QED for a while: Reconsider global and local gauge invariance

Free Dirac Lagrangian:

$$L_0 = \bar{\psi}(x)(i\gamma^\mu \partial_\mu - m)\psi(x)$$

Invariant upon global gauge transformation:

$$\begin{cases} \psi(x) \rightarrow \psi'(x) = e^{-i\alpha}\psi(x) \\ \bar{\psi}(x) \rightarrow \bar{\psi}'(x) = e^{+i\alpha}\bar{\psi}(x) \end{cases}, \quad \alpha \text{ constant}$$

Noether's theorem  $\rightarrow$  Conserved current:

$$\partial_\mu s^\mu(x) = 0, \quad s^\mu(x) = q\bar{\psi}(x)\gamma^\mu\psi(x)$$

$\rightarrow$  Conserved charge:

$$Q = \int s^0(x) d^3\mathbf{r} = \text{const}$$

# Gauge Symmetry - III

51

Non invariant under local gauge transformation:

$$\begin{cases} \psi(x) \rightarrow \psi'(x) = e^{-iqf(x)}\psi(x) \\ \bar{\psi}(x) \rightarrow \bar{\psi}'(x) = e^{+iqf(x)}\bar{\psi}(x) \end{cases}$$

$$L_0 \rightarrow L_0' = L_0 + q\bar{\psi}(x)\gamma^\mu\psi(x)\partial_\mu f(x)$$

Define then a *covariant derivative* as:

$$D_\mu\psi(x) = [\partial_\mu + iqA_\mu(x)]\psi(x)$$

where, upon the previous local gauge transformation:

$$A_\mu(x) \rightarrow A_\mu(x) = A_\mu(x) + \partial_\mu f(x)$$

Then the Lagrangian:

$$L = \bar{\psi}(x)(i\gamma^\mu D_\mu - m)\psi(x) = L_0 - q\bar{\psi}(x)\gamma^\mu\psi(x)A_\mu(x)$$

is invariant

$L$  contains an *interaction* term ( $\leftarrow j^\mu A_\mu$ )

# Gauge Symmetry - IV

52

Consider a single family of massless leptons:

$$L_0 = i\bar{\psi}(x)\gamma^\mu\partial_\mu\psi(x) + i\bar{\psi}_\nu(x)\gamma^\mu\partial_\mu\psi_\nu(x)$$

Chiral spinors:

$$\psi^L(x) = P_L\psi(x) = \frac{1}{2}(1 - \gamma_5)\psi(x)$$

$$\psi^R(x) = P_R\psi(x) = \frac{1}{2}(1 + \gamma_5)\psi(x)$$

$$\rightarrow L_0 = i\bar{\psi}^L(x)\gamma^\mu\partial_\mu\psi^L(x) + i\bar{\psi}_\nu^L(x)\gamma^\mu\partial_\mu\psi_\nu^L(x) + i\bar{\psi}^R(x)\gamma^\mu\partial_\mu\psi^R(x) + i\bar{\psi}_\nu^R(x)\gamma^\mu\partial_\mu\psi_\nu^R(x)$$

Charged current: Connecting two leptons with  $\Delta Q = \pm 1$

To encode this into a symmetry scheme, define the doublet:

$$\Psi^L(x) = \begin{pmatrix} \psi_\nu^L(x) \\ \psi^L(x) \end{pmatrix}, \bar{\Psi}^L(x) = (\bar{\psi}_\nu^L(x) \quad \bar{\psi}^L(x))$$

$$\rightarrow L_0 = i\bar{\Psi}^L(x)\gamma^\mu\partial_\mu\Psi^L(x) + i\bar{\psi}^R(x)\gamma^\mu\partial_\mu\psi^R(x) + i\bar{\psi}_\nu^R(x)\gamma^\mu\partial_\mu\psi_\nu^R(x)$$

# Gauge Symmetry - V

53

Suppose the  $L$ -doublet realizes the fundamental representation of a  $SU(2)$  (*gauge*) symmetry of the weak interaction, exactly as  $U(1)$  is the (*gauge*) symmetry of QED

Then  $L$ -spinors will transform

$$\Psi^L(x) \rightarrow \Psi'^L(x) = U(\alpha) \Psi^L(x) = \exp[i\alpha_j \tau_j / 2] \Psi^L(x)$$

$$\bar{\Psi}^L(x) \rightarrow \bar{\Psi}'^L(x) = \bar{\Psi}^L(x) U^\dagger(\alpha) = \bar{\Psi}^L(x) \exp[-i\alpha_j \tau_j / 2]$$

$\alpha_1, \alpha_2, \alpha_3$  3 continuous, real parameters

$\tau_1, \tau_2, \tau_3$  Pauli matrices

$$[\text{Reminder: } [\tau_i, \tau_j] = 2i\epsilon_{ijk} \tau_k]$$

Also take  $R$ -spinors as  $SU(2)$  singlets:

$$\Psi^R(x) \rightarrow \Psi'^R(x) = \Psi^R(x), \Psi_\nu^R(x) \rightarrow \Psi_\nu'^R(x) = \Psi_\nu^R(x)$$

$$\bar{\Psi}^R(x) \rightarrow \bar{\Psi}'^R(x) = \bar{\Psi}^R(x), \bar{\Psi}_\nu^R(x) \rightarrow \bar{\Psi}_\nu'^R(x) = \bar{\Psi}_\nu^R(x)$$

# Gauge Symmetry - VI

54

According to Noether's theorem :

Expect conserved current after  $L$  invariance under infinitesimal  $SU(2)$  transformations :

$$\Psi^L(x) \rightarrow \Psi^{L'}(x) = \left(1 + i\alpha_j \tau_j / 2\right) \Psi^L(x)$$

$$\bar{\Psi}^L(x) \rightarrow \bar{\Psi}^{L'}(x) = \bar{\Psi}^L(x) \left(1 - i\alpha_j \tau_j / 2\right)$$

Identify 3 weak isospin, conserved currents / charges:

$$J_i^\mu(x) = \frac{1}{2} \bar{\Psi}^L(x) \gamma^\mu \tau_i \Psi^L(x)$$

$$I_i^W = \int d^3\mathbf{x} J_i^0(x) = \int \frac{1}{2} \Psi^{L\dagger}(x) \tau_i \Psi^L(x)$$

Make 2 non-Hermitian, linear combinations :

$$J^\mu(x) = 2[J_1^\mu(x) - iJ_2^\mu(x)] = [\bar{\Psi}^L(x) \gamma^\mu \tau_1 \Psi^L(x) - i\bar{\Psi}^L(x) \gamma^\mu \tau_2 \Psi^L(x)]$$

$$J^{\mu\dagger}(x) = 2[J_1^\mu(x) + iJ_2^\mu(x)] = [\bar{\Psi}^L(x) \gamma^\mu \tau_1 \Psi^L(x) + i\bar{\Psi}^L(x) \gamma^\mu \tau_2 \Psi^L(x)]$$

# Gauge Symmetry - VII

55

Remember:

$$\Psi^L(x) = \begin{pmatrix} \psi_\nu^L(x) \\ \psi^L(x) \end{pmatrix}, \bar{\Psi}^L(x) = (\bar{\psi}_\nu^L(x) \quad \bar{\psi}^L(x))$$

Then:

$$\begin{aligned} \bar{\Psi}^L(x) \gamma^\mu \tau_1 \Psi^L(x) &= (\bar{\psi}_\nu^L(x) \quad \bar{\psi}^L(x)) \gamma^\mu \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \psi_\nu^L(x) \\ \psi^L(x) \end{pmatrix} = (\bar{\psi}_\nu^L(x) \quad \bar{\psi}^L(x)) \gamma^\mu \begin{pmatrix} \psi^L(x) \\ \psi_\nu^L(x) \end{pmatrix} \\ &= \bar{\psi}_\nu^L(x) \gamma^\mu \psi^L(x) + \bar{\psi}^L(x) \gamma^\mu \psi_\nu^L(x) \end{aligned}$$

$$\begin{aligned} i \bar{\Psi}^L(x) \gamma^\mu \tau_2 \Psi^L(x) &= i (\bar{\psi}_\nu^L(x) \quad \bar{\psi}^L(x)) \gamma^\mu \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} \psi_\nu^L(x) \\ \psi^L(x) \end{pmatrix} = i (\bar{\psi}_\nu^L(x) \quad \bar{\psi}^L(x)) \gamma^\mu \begin{pmatrix} -i \psi^L(x) \\ i \psi_\nu^L(x) \end{pmatrix} \\ &= \bar{\psi}_\nu^L(x) \gamma^\mu \psi^L(x) - \bar{\psi}^L(x) \gamma^\mu \psi_\nu^L(x) \end{aligned}$$

Therefore:

$$\begin{cases} J^\mu(x) = 2 \bar{\psi}^L(x) \gamma^\mu \psi_\nu^L(x) \\ J^{\mu\dagger}(x) = 2 \bar{\psi}_\nu^L(x) \gamma^\mu \psi^L(x) \end{cases} : \text{Just our weak charged currents}$$

# Gauge Symmetry - VIII

56

$$J_3^\mu = \frac{1}{2} \bar{\Psi}^L(x) \gamma^\mu \tau_3 \Psi^L(x)$$

$$\bar{\Psi}^L(x) \gamma^\mu \tau_3 \Psi^L(x) = (\bar{\psi}_\nu^L(x) \quad \bar{\psi}^L(x)) \gamma^\mu \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \psi_\nu^L(x) \\ \psi^L(x) \end{pmatrix} = (\bar{\psi}_\nu^L(x) \quad \bar{\psi}^L(x)) \gamma^\mu \begin{pmatrix} \psi_\nu^L(x) \\ -\psi^L(x) \end{pmatrix}$$

$$\rightarrow J_3^\mu = \frac{1}{2} [\bar{\psi}_\nu^L(x) \gamma^\mu \psi_\nu^L(x) - \bar{\psi}^L(x) \gamma^\mu \psi^L(x)]$$

Neutral (as opposed to charged) current

Observe:

2nd term is actually a part of the electromagnetic current, up to a constant factor

$$J_{EM}^\mu = -e \bar{\psi}(x) \gamma^\mu \psi(x)$$

Might be possible to unify EM and weak interactions.

But:

Count 3 weak isospin + 1 electromagnetic = 4 currents = 2 charged + 2 neutral

→ Unified symmetry group must be larger than  $SU(2)$ , which has only 3 parameters



# Gauge Symmetry - IX

57

Early models ( between '50s and '60s...):

Neutral current  $\equiv$  3rd weak isospin current

Symmetry group is  $SU(2)_L \times U(1)_Q$

$SU(2)_L$  (Non Abelian) symmetry group of weak interactions of  $L$ -fermions

$U(1)_Q$  (Abelian) symmetry group of QED

Then:

*Neutral current has same V-A structure of charged current*

(Wrong: When finally observed, neutral current was found  $\neq$  V-A)

*Weak and Electromagnetic interactions stay independent*

(Wrong: At high energy, proofs of unification easily found)

# Gauge Symmetry - X

58

Rather assume the symmetry group of (unified) Electroweak interaction is

$$SU(2)_L \times U(1)_Y$$

where  $Y$  is a new observable called *weak hypercharge*

$$J_Y^\mu = \frac{1}{e} J_{EM}^\mu - J_3^\mu = -\bar{\psi}(x) \gamma^\mu \psi(x) - \frac{1}{2} [\bar{\psi}_\nu^L(x) \gamma^\mu \psi_\nu^L(x) - \bar{\psi}^L(x) \gamma^\mu \psi^L(x)]$$

$$\rightarrow J_Y^\mu = \frac{1}{2} \bar{\psi}^L(x) \gamma^\mu \psi^L(x) - \frac{1}{2} \bar{\psi}_\nu^L(x) \gamma^\mu \psi_\nu^L(x) - \bar{\psi}^R(x) \gamma^\mu \psi^R(x)$$

→ Conserved charge:

$$Y_W = \int d^3\mathbf{x} J_Y^0(x) = \frac{Q}{e} - I_3^W$$

Fermion EW quantum numbers: Defined by  $I, I_3, Y$

Different for different chiralities!

# Gauge Symmetry - XI

59

Find the  $EW$  quantum numbers of (chiral) leptons :

$$I_3^W |l^-, L\rangle = -\frac{1}{2} |l^-, L\rangle \quad Y_W |l^-, L\rangle = -\frac{1}{2} |l^-, L\rangle$$

$$I_3^W |\nu_l, L\rangle = +\frac{1}{2} |\nu_l, L\rangle \quad Y_W |\nu_l, L\rangle = -\frac{1}{2} |\nu_l, L\rangle$$

$$I_3^W |l^-, R\rangle = 0 \quad Y_W |l^-, R\rangle = -Y_W |l^-, L\rangle$$

$$I_3^W |\nu_l, R\rangle = 0 \quad Y_W |\nu_l, R\rangle = 0$$

# Gauge Symmetry - XII

60

Extend to *local*  $SU(2)$  gauge transformations: Similar to QCD

$L$  – doublet

$$\Psi^L(x) \rightarrow \Psi^{L'}(x) = U(\alpha)\Psi^L(x) = \exp[i g \omega_j(x) \tau_j / 2] \Psi^L(x)$$

$$\bar{\Psi}^L(x) \rightarrow \bar{\Psi}^{L'}(x) = \bar{\Psi}^L(x) U^\dagger(\alpha) = \bar{\Psi}^L(x) \exp[-i g \omega_j(x) \tau_j / 2]$$

$R$  – singlet

$$\psi^R(x) \rightarrow \psi^{R'}(x) = \psi^R(x), \psi_\nu^R(x) \rightarrow \psi_\nu^{R'}(x) = \psi_\nu^R(x)$$

$$\bar{\psi}^R(x) \rightarrow \bar{\psi}^{R'}(x) = \bar{\psi}^R(x), \bar{\psi}_\nu^R(x) \rightarrow \bar{\psi}_\nu^{R'}(x) = \bar{\psi}_\nu^R(x)$$

$\omega_j(x)$ : 3 real parameters, functions of  $(\mathbf{r}, t)$

As for QCD:  $L_0$  not invariant

$$L_0 \rightarrow L_0' = L_0 + \delta L_0 = L_0 - \frac{1}{2} g \bar{\Psi}^L(x) \tau_j \gamma_\mu \partial^\mu \omega_j(x) \Psi^L(x)$$

# Gauge Symmetry - XIII

61

→ Define a covariant derivative for the doublet:

$$\partial^\mu \Psi^L(x) \rightarrow D^\mu \Psi^L(x) = \left[ \partial^\mu + i \frac{g}{2} \tau_j W_j^\mu(x) \right] \Psi^L(x)$$

$W_j^\mu$  : triplet of (charged, massless) , gluon – like vector fields

Requiring suitable transformation rules:

[Repeated indexes summed,  $\omega_i$  infinitesimal]

$$W_j^\mu(x) \rightarrow W_j^{\mu'}(x) = W_j^\mu(x) - \partial^\mu \omega_j(x) - g \varepsilon_{jik} \omega_i(x) W_k^\mu(x)$$

→  $L$  invariant

# Gauge Symmetry - XIV

62

Now consider weak hypercharge  $U(1)$  gauge transformations: Similar to QED

$$\psi(x) \rightarrow \psi'(x) = e^{-ig'Yf(x)}\psi(x)$$

$$\bar{\psi}(x) \rightarrow \bar{\psi}'(x) = \bar{\psi}(x)e^{ig'Yf(x)}$$

→ Define a covariant derivative:

$$\partial^\mu \psi(x) \rightarrow D^\mu \psi(x) = [\partial^\mu + ig' B^\mu(x)]\psi(x)$$

$B^\mu$  : Neutral, massless photon – like vector field

$g'$  : New coupling constant

$$B^\mu(x) \rightarrow B^{\mu'}(x) = B^\mu(x) - \partial^\mu f(x)$$

→  $L$  invariant

# Gauge Symmetry - XV

63

Collecting all pieces together:

$$L = i \left[ \bar{\Psi}^L(x) \gamma_\mu D^\mu \Psi^L(x) + \bar{\psi}^R(x) \gamma_\mu D^\mu \psi^R(x) + \bar{\psi}_\nu^R(x) \gamma_\mu D^\mu \psi_\nu^R(x) \right]$$

$$D^\mu \Psi^L(x) = \left[ \partial^\mu + i \frac{g}{2} \tau_j W_j^\mu(x) - i \frac{g'}{2} B^\mu(x) \right] \Psi^L(x)$$

$$D^\mu \psi^R(x) = \left[ \partial^\mu - i \frac{g'}{2} B^\mu(x) \right] \psi^R(x)$$

$$D^\mu \psi_\nu^R(x) = \partial^\mu \psi_\nu^R(x)$$

Write it as:

$$L = L_0 + L_I$$

$$L_I = -g J_1^\mu(x) W_{1\mu} - g J_2^\mu(x) W_{2\mu} - g J_3^\mu(x) W_{3\mu} - g' J_Y^\mu(x) B_\mu(x)$$

# Gauge Symmetry - XVI

64

To understand the meaning of the interaction terms :

Re – write the interaction part

Define

$$W_{\mu}(x) = \frac{1}{\sqrt{2}} [W_{1\mu}(x) - iW_{2\mu}(x)]$$

$$W_{\mu}^{\dagger}(x) = \frac{1}{\sqrt{2}} [W_{1\mu}(x) + iW_{2\mu}(x)]$$

And get for the first 2 terms:

$$\rightarrow L_{I-ch} = -\frac{g}{2\sqrt{2}} [J^{\mu\dagger}(x)W_{\mu}(x) + J^{\mu}(x)W_{\mu}^{\dagger}(x)]$$

Charged current interaction of  $L$  – fermions



# Gauge Symmetry - XVII

65

Define:

$$W_{3\mu}(x) = \cos\theta_W Z_\mu(x) + \sin\theta_W A_\mu(x)$$

$$B_\mu(x) = -\sin\theta_W Z_\mu(x) + \cos\theta_W A_\mu(x)$$

Reminder:

$$J_Y^\mu(x) = \frac{1}{e} J_{EM}^\mu(x) - J_3^\mu(x)$$

$$\rightarrow \begin{cases} -g' J_Y^\mu(x) B_\mu(x) = -g' \left[ \frac{1}{e} J_{EM}^\mu(x) - J_3^\mu(x) \right] \left[ -\sin\theta_W Z_\mu(x) + \cos\theta_W A_\mu(x) \right] \\ -g J_3^\mu(x) W_{3\mu} = -g J_3^\mu(x) \left[ \cos\theta_W Z_\mu(x) + \sin\theta_W A_\mu(x) \right] \end{cases}$$

→ Remaining terms:

$$\begin{aligned} & -J_{EM}^\mu(x) \frac{g'}{e} \left[ -\sin\theta_W Z_\mu(x) + \cos\theta_W A_\mu(x) \right] + \\ & -J_3^\mu(x) \left\{ g \left[ \cos\theta_W Z_\mu(x) + \sin\theta_W A_\mu(x) \right] + g' \left[ -\sin\theta_W Z_\mu(x) + \cos\theta_W A_\mu(x) \right] \right\} \end{aligned}$$

# Gauge Symmetry - XVIII

66

Most simple way of unifying the EM and weak interaction :

Require this condition on  $g', \theta_w$  constants

$$g \sin \theta_w = g' \cos \theta_w = e$$

and contemplate the miracle:

$$L_I = -J_{EM}^\mu(x) A_\mu - \frac{g}{2\sqrt{2}} [J^{\mu\dagger}(x) W_\mu + J^\mu(x) W_\mu^\dagger] - \frac{g}{\cos \theta_w} \left[ J_3^\mu(x) - \sin^2 \theta_w \frac{J_{EM}^\mu(x)}{e} \right] Z_\mu(x)$$

Electromagnetic interaction

Charged current weak interaction

Neutral current weak interaction

# Neutral Current - I

67

As a result of the weak-electromagnetic unification, neutral currents are *different* from charged

Lorentz structure not  $V - A$

$$-i \frac{g_W}{\sqrt{2}} \gamma^\mu \frac{(1 - \gamma^5)}{2} \quad \text{Charged}$$

$$-i g_Z \gamma^\mu \frac{(C_V^f - C_A^f \gamma^5)}{2} \quad \text{Neutral}$$

	Fermion	$C_V$	$C_A$
Coupling	$\nu_e, \nu_\mu, \nu_\tau$	+1/2	+1/2
	$e, \mu, \tau$	$-1/2 + 2 \sin^2 \theta_W$	-1/2
	$u, c, t$	$+1/2 - 4/3 \sin^2 \theta_W$	+1/2
	$d, s, b$	$-1/2 + 2/3 \sin^2 \theta_W$	-1/2

$\theta_W$  new fundamental constant

What about interaction strength?

# Neutral Current - II

68

Tight relationship between weak and electromagnetic interactions

Coupling constants:

$$g_w = \frac{e}{\sin \theta_w} \quad \text{Charged currents}$$

$$g_z = \frac{e}{\sin \theta_w \cos \theta_w} \quad \text{Neutral currents}$$

Charged, neutral and electromagnetic couplings fixed by 2 universal constants:

$e$  : Elementary charge

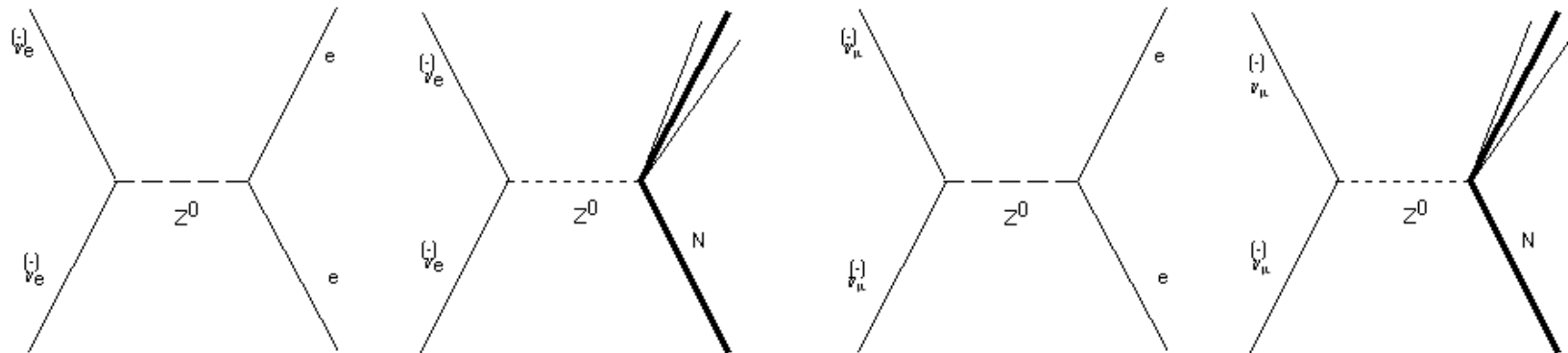
$\theta_w$  : Weinberg angle, new fundamental constant

$$\sin^2 \theta_w = 0.23122 \pm 0.00015$$

# Neutral Current - III

69

Expect to observe typical processes like:



$(\nu_e, \bar{\nu}_e) + e \rightarrow (\nu_e, \bar{\nu}_e) + e$  Contributing to elastic scattering

$(\nu_\mu, \bar{\nu}_\mu) + e \rightarrow (\nu_\mu, \bar{\nu}_\mu) + e$

$(\nu_e, \bar{\nu}_e) + N \rightarrow (\nu_e, \bar{\nu}_e) + \text{hadron shower}$

$(\nu_\mu, \bar{\nu}_\mu) + N \rightarrow (\nu_\mu, \bar{\nu}_\mu) + \text{hadron shower}$

} New

# Gauge Boson Terms - I

70

As for *QED*:

Additional terms required in order to account for:

*Energy, Momentum, Angular Momentum*

carried over by the fields

Weak Hypercharge field:

$$-\frac{1}{4} B_{\mu\nu}(x) B^{\mu\nu}(x)$$

$$B^{\mu\nu}(x) = \partial^\nu B^\mu(x) - \partial^\mu B^\nu(x)$$

Similar to *QED*: Abelian symmetry  $U(1)$

Weak Isospin fields:

$$-\frac{1}{4} \sum_{i=1}^3 G_{\mu\nu}^{(i)}(x) G^{(i)\mu\nu}(x)$$

$$G^{(i)\mu\nu}(x) = \underbrace{\partial^\nu W^{(i)\mu}(x) - \partial^\mu W^{(i)\nu}(x)}_{F^{(i)\mu\nu}(x)} + g \sum_{i,j=1}^3 \varepsilon_{ijk} W^{(j)\mu}(x) W^{(k)\nu}(x)$$

Similar to *QCD*: Non-Abelian symmetry  $SU(2)_L$

# Gauge Boson Terms - II

71

Gauge Boson Lagrangian:

$$L^B = -\frac{1}{4} B_{\mu\nu}(x) B^{\mu\nu}(x) - \frac{1}{4} \sum_{i=1}^3 G_{\mu\nu}^{(i)}(x) G^{(i)\mu\nu}(x)$$

$$\rightarrow L^B = \underbrace{-\frac{1}{4} B_{\mu\nu}(x) B^{\mu\nu}(x) - \frac{1}{4} \sum_{i=1}^3 F_{\mu\nu}^{(i)}(x) F^{(i)\mu\nu}(x)}_{L_0^B}$$

$$+ \underbrace{g \sum_{i,j,k=1}^3 \varepsilon_{ijk} W^{(j)\mu}(x) W^{(k)\nu}(x) \partial^\mu W^{(k)\nu}(x) - \frac{1}{4} \sum_{i,j,k,l,m=1}^3 \varepsilon_{ijk} \varepsilon_{ilm} g^2 W^{(j)\mu}(x) W^{(k)\nu}(x) W^{(l)\mu}(x) W^{(m)\nu}}_{L_{SI}^B}$$

$L_0^B$  = Free term

$L_{SI}^B$  = Self-Interaction term

# Gauge Boson Terms - III

72

Free term: Rewrite using  $A^\mu, W^\mu, W^{\dagger\mu}, Z^\mu$  :

$$L_0^B = -\frac{1}{4} F_{\mu\nu}(x) F^{\mu\nu}(x) - \frac{1}{2} F_{\mu\nu}^W(x) F^{W\dagger\mu\nu}(x) - \frac{1}{4} Z_{\mu\nu}(x) Z^{\mu\nu}(x)$$

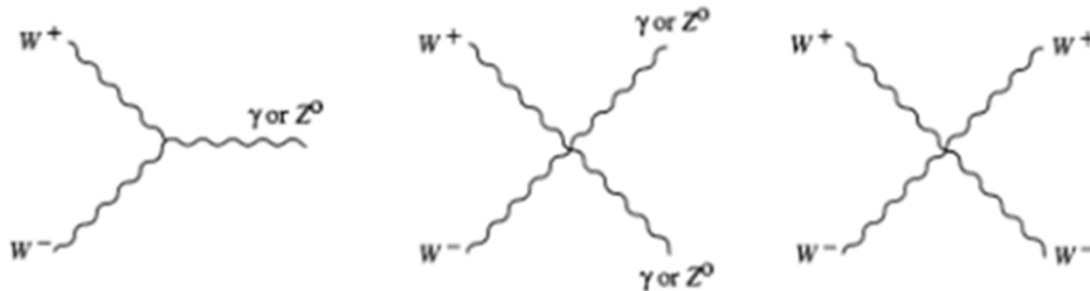
Field tensors:

$$F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x) \quad \text{Coupled to EM current}$$

$$\left. \begin{aligned} F_{\mu\nu}^W(x) &= \partial_\mu W_\nu(x) - \partial_\nu W_\mu(x) \\ F^{W\dagger\mu\nu}(x) &= \partial^\mu W^{\dagger\nu}(x) - \partial^\nu W^{\dagger\mu}(x) \end{aligned} \right\} \text{Coupled to Charged current}$$

$$Z^{\mu\nu}(x) = \partial^\mu Z^\nu(x) - \partial^\nu Z^\mu(x) \quad \text{Coupled to Neutral current}$$

Self-Interaction term: Similar to 3- and 4-gluons terms of QCD





# Mass - I

73

Massless leptons & gauge bosons not physical: Mass must be there

But: Putting 'by hand' a mass term in  $L$  would spoil gauge invariance

Gauge bosons:

$$m_W^2 W_\mu^\dagger W^\mu + \frac{1}{2} m_Z^2 Z_\mu Z^\mu$$

Take  $W$  as an example:

$$W_i^\mu \rightarrow W_i^\mu - \partial^\mu \omega_i(x) - g \varepsilon_{ijk} \omega_j(x) W_k^\mu \quad \text{infinitesimal parameters}$$

Then:

$$\begin{aligned} m_W^2 W_\mu^\dagger W^\mu &\rightarrow m_W^2 \left( W_i^{\dagger\mu} - \partial^\mu \omega_i(x) - g \varepsilon_{ijk} \omega_j(x) W_k^{\dagger\mu} \right) \left( W_i^\mu - \partial^\mu \omega_i(x) - g \varepsilon_{ijk} \omega_j(x) W_k^\mu \right) \\ &\rightarrow \neq m_W^2 W_\mu^\dagger W^\mu \end{aligned}$$

# Mass - II

74

Leptons :

$$-m\bar{\psi}(x)\psi(x)$$

Write in terms of chiral parts :

$$-m\bar{\psi}(x)\psi(x) = -m\bar{\psi}(x)\left(\underbrace{P_R + P_L}_{=1}\right)\psi(x)$$

$$P_R = \frac{1+\gamma_5}{2}, \quad P_L = \frac{1-\gamma_5}{2}$$

$$\rightarrow -m\bar{\psi}(x)\left(\frac{1+\gamma_5}{2} + \frac{1-\gamma_5}{2}\right)\psi(x) = -m\bar{\psi}(x)\left(\left(\frac{1+\gamma_5}{2}\right)^2 + \left(\frac{1-\gamma_5}{2}\right)^2\right)\psi(x)$$

$$\rightarrow -m\bar{\psi}(x)\left(\frac{1+\gamma_5}{2} + \frac{1-\gamma_5}{2}\right)\psi(x) = -m\bar{\psi}_L(x)\psi_R(x) - m\bar{\psi}_R(x)\psi_L(x)$$

Not invariant under  $SU(2)$ :

$L, R$  chiral parts live in different  $SU(2)$  representations

→ Different gauge transformations

→ Mass term not gauge invariant

# Mass - III

75

Bottom line: *Any* mass term not invariant

Glashow model (1961): Put mass by hand  
→ Gauge invariance lost, back to naive IVB

Finally, discover a subtle mechanism to give mass to physical states,  
without spoiling gauge invariance:

*Spontaneous Symmetry Breaking*

Broad phenomenology, also remotely rooted in classical physics

# SSB - I

76

Symmetries: Frequently approximate  $\rightarrow$  Broken

Breaking modes:

(a) Explicit breaking

$$H = H_0 + H_b$$

$H_0$  invariant

$H_b$  non-invariant

Ex: Hydrogen atom in a magnetic field  $\mathbf{B}$

$$H_0 = \frac{p^2}{2m} - \frac{Ze^2}{r} \quad \text{rotationally invariant}$$

$H_b = -\boldsymbol{\mu} \cdot \mathbf{B}$  invariant wrt rotations around  $\mathbf{B}$

$\rightarrow H_0$  degeneracies removed by  $H_b$

(b) Spontaneous breaking

$H$  symmetric, ground state non symmetric

Ex: Ferromagnetism

$T > T_c$ :  $\mathbf{M} = 0 \rightarrow$  Dipoles randomly oriented

$\rightarrow$  Rotational symmetry

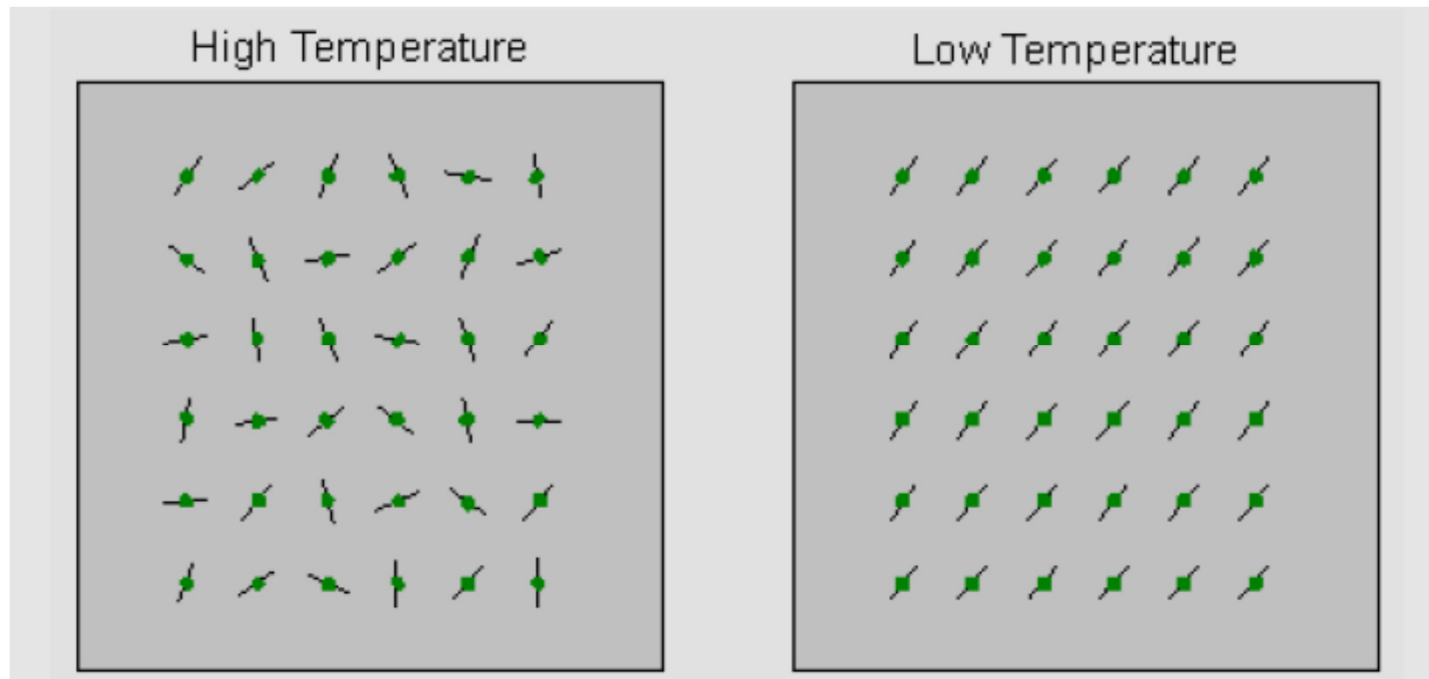
$T < T_c$ :  $\mathbf{M} \neq 0 \rightarrow$  Dipoles pick some direction

$\rightarrow H$  degeneracies not removed

Ground state *degenerate*

# SSB - II

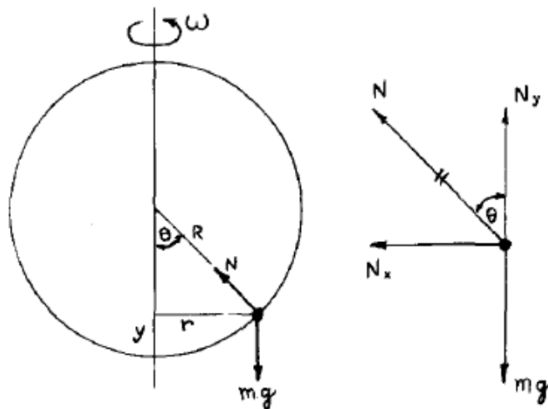
77



# SSB - III

78

Take a first year classical mechanics exercise:  
Bead sliding frictionless along a spinning hoop  
Find equilibrium angle



$$N = m\omega^2 R$$

$$N_y = N \cos \theta = m\omega^2 R \cos \theta = mg$$

$$\rightarrow \cos \theta_0 = \frac{g}{\omega^2 R}$$

Funny observation:

$$\text{Critical frequency: } \cos \theta_0 = 1 = \frac{g}{\omega_0^2 R}$$

For  $\omega < \omega_0$  : Different solution

$$\theta_1 = 0$$

# SSB - IV

79

$$T = \frac{1}{2}mv^2 = \frac{1}{2}m(R^2\dot{\theta}^2 + \omega^2 R^2 \sin^2 \theta)$$

$$V = mgy = mgR(1 - \cos \theta)$$

$$L = T - V = \frac{1}{2}mR^2\dot{\theta}^2 + \frac{1}{2}m\omega^2 R^2 \sin^2 \theta - mgR(1 - \cos \theta)$$

$$L = \frac{1}{2}mR^2\dot{\theta}^2 - V_{\text{eff}}$$

Define effective potential, including centrifugal term :

$$V_{\text{eff}} \equiv -\frac{1}{2}m\omega^2 R^2 \sin^2 \theta + mgR(1 - \cos \theta) = mgR \left[ (1 - \cos \theta) - \frac{\omega^2 R \sin^2 \theta}{2g} \right]$$

Define reduced effective potential,  $\beta$  parameter :

$$U \equiv \frac{V_{\text{eff}}}{mgR} = (1 - \cos \theta) - \frac{1}{2} \frac{\omega^2 R}{g} \sin^2 \theta$$

$$\beta \equiv \frac{\omega^2 R}{g}$$

$$\rightarrow U = (1 - \cos \theta) - \frac{1}{2}\beta \sin^2 \theta = 2 \sin^2 \frac{\theta}{2} - \frac{\beta}{2} (1 - \cos^2 \theta) = 2 \sin^2 \frac{\theta}{2} \left( 1 - \beta \cos^2 \frac{\theta}{2} \right)$$

# SSB - V

80

Find equilibrium angles, identify stable and unstable:

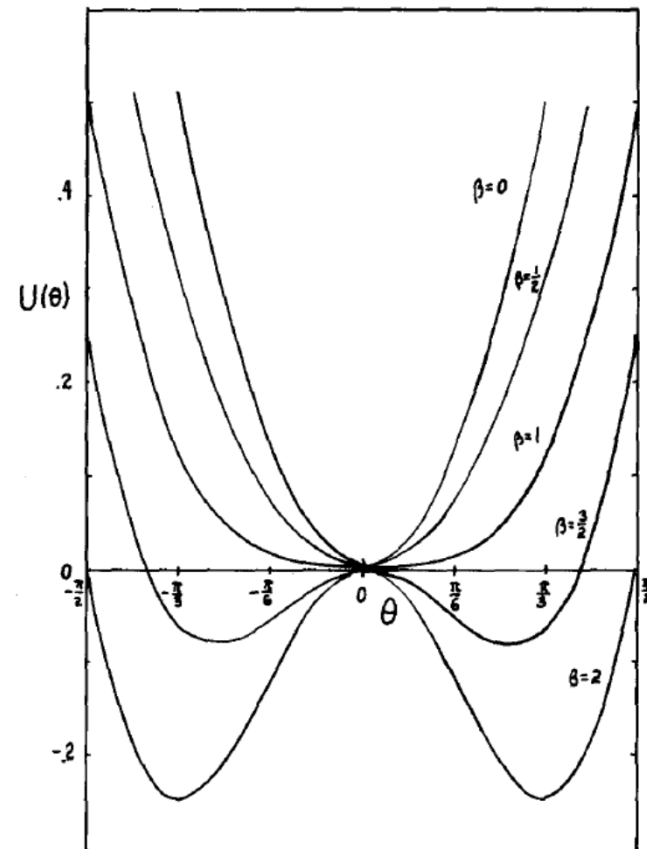
$$\frac{\partial U}{\partial \theta} = \sin \theta (1 - \beta \cos \theta) = 0$$

$$\rightarrow \begin{cases} \cos \theta_0 = \frac{1}{\beta} \\ \theta_1 = 0 \end{cases}$$

$$\frac{\partial^2 U}{\partial \theta^2} = \cos \theta - \beta \cos 2\theta$$

$$\left. \frac{\partial^2 U}{\partial \theta^2} \right|_{\theta=\theta_1} = 1 - \beta \quad \text{stable for } \beta < 1$$

$$\left. \frac{\partial^2 U}{\partial \theta^2} \right|_{\theta=\theta_0} = \beta - \frac{1}{\beta} \quad \text{stable for } \beta > 1$$

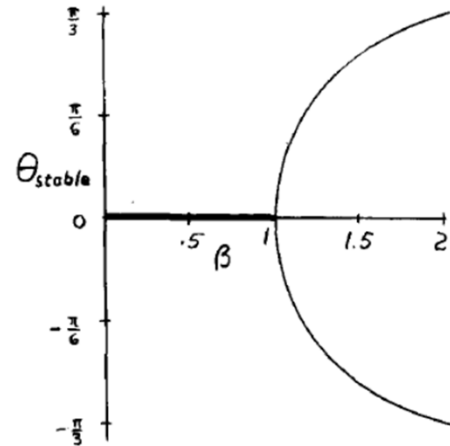
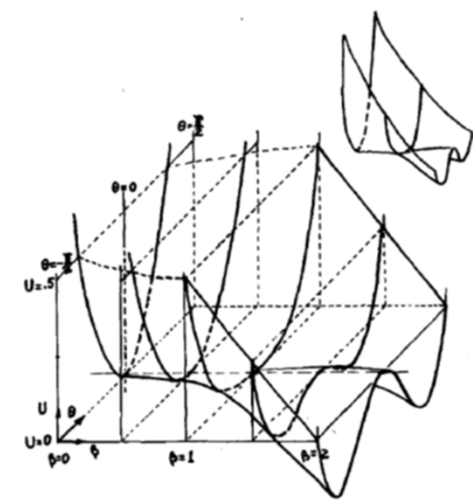




# SSB - VI

81

Showing how shape of potential curve, equilibrium angle change with  $\beta$



a)  $\beta < 1 \rightarrow 1$  eq. angle

$\beta > 1 \rightarrow 2$  eq. angles: Cannot tell which one will be found

Reflection symmetry of  $V$  lost ( $\leftarrow$  spontaneously broken) in the solution of eq. of motion

b) Small oscillations around equilibrium angle:

$\beta < 1 \rightarrow OK$  Symmetrical wrt origin

$\beta > 1 \rightarrow KO$  Non symmetrical wrt origin

# SSB - VII

82

Quantum Mechanics: Simple system with 1 degree of freedom:

$$L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}\lambda^2(x^2 - \eta^2)^2$$

$$V(x) = \frac{1}{2}\lambda^2(x^2 - \eta^2)^2$$

Potential: Parity symmetric

$$V(x) = V(-x)$$

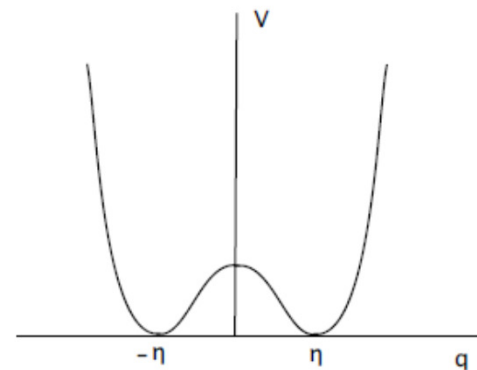
Expand around  $\pm\eta$  to quadratic terms only: Harmonic oscillator

$|+\rangle$  solution, centered on  $+\eta$

$|-\rangle$  solution centered on  $-\eta$

Naively:

Expect two degenerate ground states, both with undefined parity



# SSB - VIII

83

But:

$H$  not diagonal in this basis

$$\langle + | H | + \rangle = \langle - | H | - \rangle = a, \langle + | H | - \rangle = \langle - | H | + \rangle = b$$

Physical reason : Tunneling through central barrier

→ Diagonalize, find:

Eigenstates                  Energies

$$|S\rangle = |+\rangle + |-\rangle \quad a + b$$

$$|A\rangle = |+\rangle - |-\rangle \quad a - b$$

$|S\rangle, |A\rangle$  : Parity eigenstates

→ Degeneracy removed: Just 1 ground state

$$E_{ground} = a - |b|$$

# SSB - IX

84

Field theory: Real scalar field

$$L = \frac{1}{2}(\partial_\mu \phi)^2 - V(\phi)$$

$$V(\phi) = \frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda^2\phi^4, \lambda > 0$$

Reflection symmetric:  $V(\phi) = V(-\phi)$

$V$  Minima:

$$\mu^2 > 0: \phi = 0$$

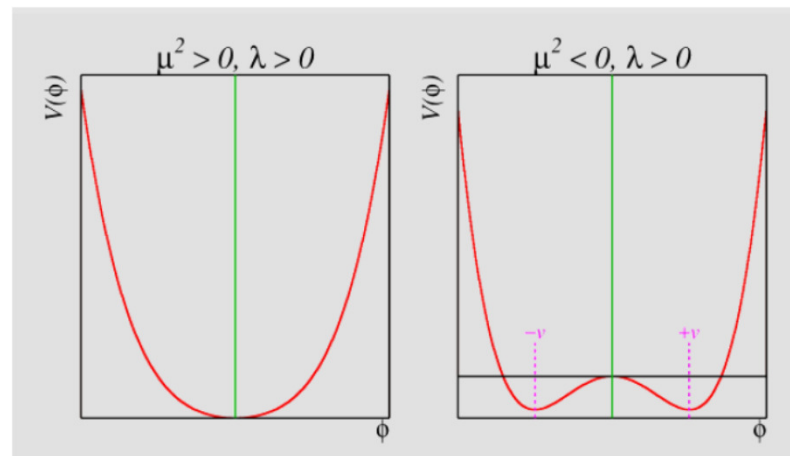
$$\mu^2 < 0: \phi = v = \pm \sqrt{-\frac{\mu^2}{\lambda}}$$

$V$  Minima: Defining *vacuum* state ( $\leftarrow$  Cannot have less energy)

$\mu^2 > 0$ : Vacuum (non degenerate)  $\equiv$  Zero field

$\mu^2 < 0$ : Vacuum (degenerate!) =  $v \neq$  Zero field !!

$v$  = Vacuum Expectation Value (VEV) of  $\phi$



# SSB - X

85

Choose vacuum state:

$$\langle \phi(x) \rangle_0 = v \quad \text{Spontaneous Symmetry Breakdown}$$

$$\text{Define: } \phi(x) = v + \eta(x)$$

$$\rightarrow L = \frac{1}{2}(\partial_\mu \eta)^2 - \lambda \left( v^2 \eta^2 - v \eta^3 - \frac{1}{4} \eta^4 \right) = \left[ \frac{1}{2}(\partial_\mu \eta)^2 - \lambda v^2 \eta^2 \right] - \lambda v \eta^3 - \frac{1}{4} \lambda \eta^4$$

$$L = \frac{1}{2}(\partial_\mu \eta)^2 - \lambda v^2 \eta^2 + \text{higher powers of } \eta$$

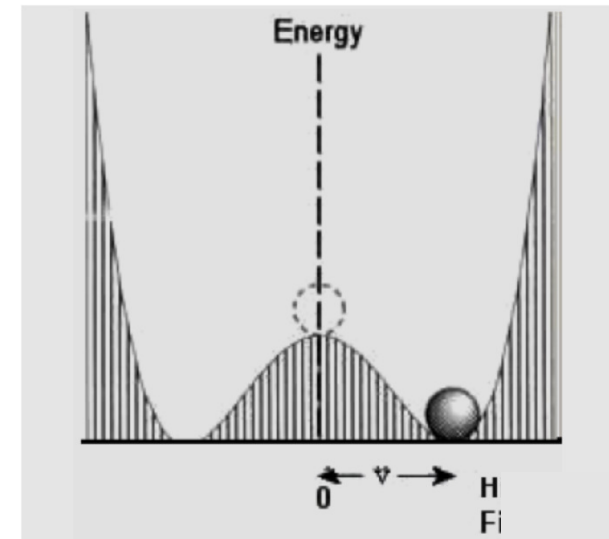
$\rightarrow$  Free Klein-Gordon equation  $\rightarrow$  Scalar quantum field

$$m^2 = 2\lambda v^2 \rightarrow m = \sqrt{-2\mu^2} > 0$$

[ Observe:  $\mu^2 < 0 \rightarrow$  Imaginary mass in original  $L!$  ]

$$\text{KO: } L(\eta) \neq L(-\eta)$$

Reflection symmetry *spontaneously broken*



# SSB - XI

86

What makes the difference between a single degree of freedom system and a field?

1 degree of freedom: Vacuum not degenerate

← Tunneling

$\infty$  degrees of freedom: Vacuum degenerate

← Tunneling not effective

Indeed, it can be shown that:

$A_{tunnel} \propto e^{-aV}$ ,  $V$  system volume

→  $A_{tunnel} \sim 0$  for a (infinite) field

# SSB - XII

87

Field theory: Complex scalar field

$$\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2), \phi^* = \frac{1}{\sqrt{2}}(\phi_1 - i\phi_2)$$

$$L = (\partial_\mu \phi)(\partial^\mu \phi)^* - V(\phi)$$

$$V(\phi) = \mu^2 \phi^* \phi + \lambda (\phi^* \phi)^2, \lambda > 0, \mu^2 < 0$$

$U(1)$  symmetric:  $\phi \rightarrow \phi' = e^{i\alpha} \phi$

Observe:  $U(1)$  continuous symmetry

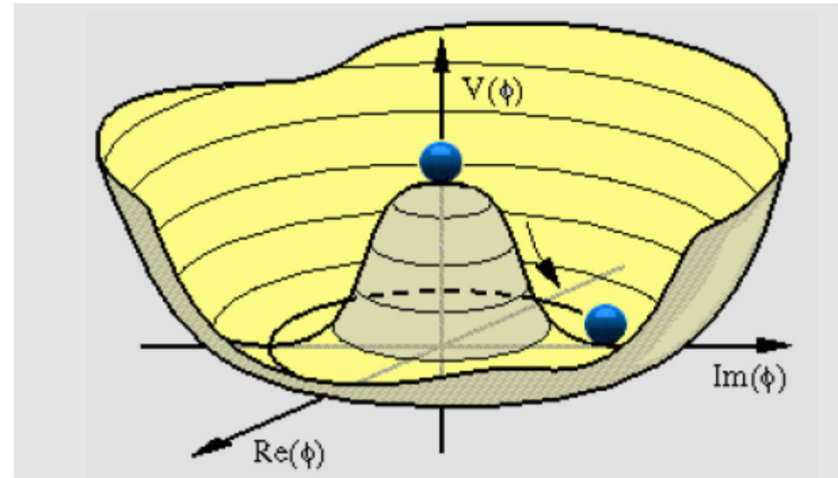
$V$  Minima:

$$\mu^2 < 0: \phi_1^2 + \phi_2^2 = -\frac{\mu^2}{\lambda}$$

→ Vacuum infinitely degenerate

Choose vacuum =  $(v, 0)$

→  $U(1)$  symmetry spontaneously broken



# SSB - XIII

88

Define:

$$\phi(x) = \frac{1}{\sqrt{2}} [v + \xi(x) + i\eta(x)]$$

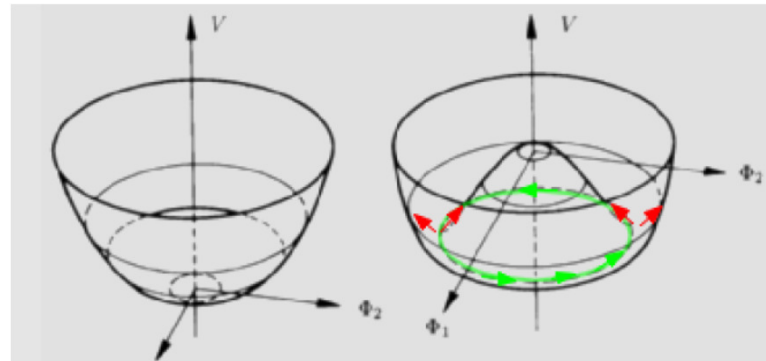
$$L = \frac{1}{2} [(\partial_\mu \xi)^2 + (\partial_\mu \eta)^2] + \mu^2 \eta^2 + \text{higher powers of } \eta$$

Free Klein-Gordon equations for  $(\xi, \eta)$

But: "Kinetic energy" terms for both  $\xi, \eta$ ; Mass term only for  $\eta$

→  $\eta$  field excitations: *massive* scalar particles

→  $\xi$  field excitations: *massless* scalar particles, aka *Goldstone Bosons*





# SSB - XIV

89

SSB of a continuous, global ( $\leftarrow$  non local) symmetry

$\rightarrow \infty$  degenerate vacuum states

Symmetry generators transform any vacuum state into another one

Indeed, for a non-degenerate vacuum:  $\langle \phi \rangle_0$  Invariant under  $G$

$\rightarrow e^{i\alpha G} \langle \phi \rangle_0 \simeq (1 + i\alpha G) \langle \phi \rangle_0 = \langle \phi \rangle_0 \leftrightarrow G \langle \phi \rangle_0 = 0$

$\rightarrow G$  does annihilate a non-degenerate vacuum

$\rightarrow$  Goldstone Theorem:

$n$  generators not annihilating the vacuum  $\rightarrow$  Appearance of  $n$  massless scalars

Also called *Goldstone bosons*

Example from condensed matter physics:

Ferromagnet  $\rightarrow$  Rotational symmetry lost  $\rightarrow$  Spin waves = Goldstones

# Higgs Mechanism - I

90

Would seem to definitely destroy our hint of a Standard Model:

3 + 1 massless gauge bosons, only one observed

4 massless scalar bosons, none observed

Local gauge invariance + SSB: Higgs mechanism, evading Goldstone's theorem

Simple, yet subtle way of giving mass to gauge bosons

without spoiling gauge invariance (and renormalizability)

Example by Higgs :

$U(1)$  gauge group, require *local* symmetry:

Gauge vector boson  $A_\mu$  to be introduced, coupling to some current as usual

Now: Add "sombbrero" potential for a complex, scalar field  $\phi = \phi_1 + i\phi_2$

$$L = \underbrace{\left[ (\partial_\mu - ieA_\mu) \phi^* \right] \left[ (\partial^\mu + ieA^\mu) \phi \right]}_{EM \text{ interaction of (charged) } \phi} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \mu^2 \phi^* \phi - \lambda (\phi^* \phi)^2 + \underbrace{\dots}_{\text{Current-field interaction etc}}$$

As found before:

Degenerate vacuum state  $\rightarrow$  SSB picks as vacuum state  $(v, 0)$

# Higgs Mechanism - II

91

$$\rightarrow \phi = v + \eta_1 + i\eta_2$$

$L$  written in terms of  $\eta_1, \eta_2$ :

Upon quantization, 2 scalar particles  $\rightarrow m_1 = \sqrt{2\lambda v^2}, m_2 = 0$

Plugging  $\phi = v + \eta_1 + i\eta_2$  into  $L$ :

$$L = \frac{1}{2}(\partial_\mu \eta_1)(\partial^\mu \eta_1) - \frac{1}{2} \overbrace{2\lambda v^2}^{m_1^2} \eta_1^2 + \frac{1}{2}(\partial_\mu \eta_2)(\partial^\mu \eta_2) \\ + \frac{1}{2} \overbrace{(ev)^2}^{m_V^2} A^\mu A_\mu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \underbrace{evA^\mu \partial_\mu \eta_2}_{??} + \dots$$

*Massive vector!*

Attempting to understand  $L$ :

Massive vector field  $A^\mu$  + Massive scalar field  $\eta_1$  + Massless scalar field  $\eta_2$

Troubling term coupling  $A^\mu$  and  $\eta_2$

# Higgs Mechanism - III

92

Use gauge invariance:

$$\begin{cases} \phi \rightarrow \phi' = e^{-ie\theta(x)} \phi \\ A^\mu \rightarrow A'^\mu = A^\mu + \partial_\mu \theta \end{cases}$$

Choose  $\theta$  to make  $\phi$  real: Then  $\eta_2 \equiv 0$  ( $\leftarrow$  Unitary gauge)

$$\rightarrow L = \frac{1}{2} (\partial_\mu \eta_1) (\partial^\mu \eta_1) - \frac{1}{2} 2\lambda v^2 \eta_1^2 + \underbrace{\frac{1}{2} \overbrace{(ev)^2}^{M_V^2} A^\mu A_\mu}_{\text{Massive vector!}} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \dots$$

Massive vector field  $A^\mu$  + Massive scalar field  $\eta_1$

Counting degrees of freedom:

$$\underbrace{2}_{A_\mu} + \underbrace{1+1}_\phi = \underbrace{3}_{A_\mu} + \underbrace{1}_{\eta_1} \quad \text{OK}$$

Standard picture:

By effect of a smart gauge transformation, the massless vector field  $A_\mu$  has eaten the Goldstone boson  $\eta_2$  to become massive

# Higgs Mechanism - IV

93

Attempting to dissipate some misunderstandings likely to sneak in:

Mostly related to our naive perception of what is really a 'particle'

1) Where is the mass?

To identify mass terms in  $L$ : Not necessarily a trivial task

Key point: Particle content *only meaningful in perturbative expansion*

Example:  $L = (\partial_\mu \phi)(\partial^\mu \phi)^* - (-\mu^2 \phi^* \phi) - \lambda(\phi^* \phi)^2, \lambda > 0, \mu^2 > 0$

$-\mu^2 \rightarrow$  Imaginary mass  $\rightarrow$  Nonsense  $\rightarrow$  ???

But: To use this form of  $L$  to extract Feynman rules, should expand around  $|\phi| = 0$

Unstable extremum  $\rightarrow$  Can't make it

Rewrite by expanding around  $\eta = 0$ :  $L = \frac{1}{2}(\partial_\mu \eta)^2 - \lambda \nu^2 \eta^2 + \dots$

Stable extremum  $\rightarrow$  OK

$\rightarrow$  Particle content should be identified in this form

# Higgs Mechanism - V

94

2) What's so special in unitary gauge?

Nothing:  $L$  invariant under local gauge transformations, including the one to unitary gauge:

→  $L$  describe the same physics before and after the gauge transformation

But: Particle content much easier to extract in the unitary gauge

3) Disappearing Goldstones !?

Indeed: And re-appearing as extra degrees of freedom for massive gauge bosons

See comment above on the tricky business of defining what is a particle...

4) What decides which vacuum is selected among the many?

Not really relevant: Any choice yields identical results

5) Could we make it with the SM without SSB and all that complicated swapping of degrees of freedom?

Actually no: SSB is an *intrinsic* feature of certain quantum systems

More on Higgs field and particle later (last part of the lectures)

# SM - I

95

Higgs mechanism fixes troublesome, massless gauge bosons in the unified EW interaction

Boson counting:

Local gauge symmetry  $SU(2)_L \otimes U(1)_Y \rightarrow 4$  vector bosons

Will need 3 symmetries spontaneously broken to give mass to 3 weak bosons

Photon *is* massless

Extend Abelian Higgs model to non-Abelian gauge symmetry:

Introduce a doublet of complex, scalar fields:

$$\phi(x) = \begin{pmatrix} \phi^+(x) \\ \phi^0(x) \end{pmatrix}$$

$$\text{Assuming } y = 1 \rightarrow \begin{cases} Q[\phi^+(x)] = +1 \\ Q[\phi^0(x)] = 0 \end{cases}$$

# SM - II

96

$SU(2)_L \otimes U(1)$  Gauge transformation of doublet:

$$\phi \rightarrow \phi' = \exp \left\{ -i \left[ \frac{g}{2} \mathbf{\alpha}(x) \cdot \boldsymbol{\tau} + \frac{g'}{2} y \theta(x) I \right] \right\} \phi$$

$SU(2)_L \otimes U(1)$  Covariant derivative:

$$D^\mu = \partial^\mu + i \left[ \frac{g}{2} \boldsymbol{\tau} \cdot \mathbf{W}^\mu + \frac{g'}{2} y B^\mu \right]$$

→ Additional term to EW lagrangian:

$$L_H = D_\mu \phi^\dagger D^\mu \phi - \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2$$

Take  $\mu^2 < 0$ ,  $\lambda > 0$ :

$$|\phi|_{\min}^2 = -\frac{\mu^2}{2\lambda} = \frac{v^2}{2} \rightarrow v = \sqrt{-\frac{\mu^2}{\lambda}}$$

Pick ground state (= vacuum) as

$$\langle \phi \rangle_0 = \begin{pmatrix} 0 \\ v \\ \sqrt{2} \end{pmatrix} \rightarrow \text{SSB of Electroweak gauge symmetry}$$



# SM - III

97

Goldstone boson: Associated with each generator not annihilating the vacuum

Take generators of  $SU(2)_L \otimes U(1)_Y$  :

$$\tau_1 \langle \phi \rangle_0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} v \\ 0 \end{pmatrix} \neq 0$$

$$\tau_2 \langle \phi \rangle_0 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} = -\frac{i}{\sqrt{2}} \begin{pmatrix} v \\ 0 \end{pmatrix} \neq 0$$

$$\tau_3 \langle \phi \rangle_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ -v \end{pmatrix} \neq 0$$

$$Y \langle \phi \rangle_0 = +1 \langle \phi \rangle_0 \neq 0$$

$$\text{But: } Q \langle \phi \rangle_0 = \frac{1}{2} (Y + \tau_3) \langle \phi \rangle_0 = 0$$

$\rightarrow \langle \phi \rangle_0 : U(1)_Q$  Invariant  $\rightarrow U(1)_Q$  symmetry unbroken  $\rightarrow$  Photon stays massless

# SM - IV

98

As before for the Higgs model, rewrite:

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \sigma_1 + i\sigma_2 \\ v + \eta_1 + i\eta_2 \end{pmatrix}$$

$$\rightarrow V = -\frac{\mu^4}{4\lambda} + \lambda v^2 \eta_1^2 + \lambda v \eta_1 (\sigma_1^2 + \sigma_2^2 + \eta_1^2 + \eta_2^2) + \frac{\lambda}{4} (\sigma_1^2 + \sigma_2^2 + \eta_1^2 + \eta_2^2)^2$$

3 massless scalars:  $\sigma_1, \sigma_2, \eta_2 \leftarrow$  The Goldstones

1 massive scalar:  $\eta_1, m_{\eta_1} \equiv m_H = \sqrt{2\lambda v^2} \leftarrow$  The Higgs

Gauge transformation suitable to get rid of 3 Goldstones:

$$\phi \rightarrow \phi' = U\phi = U \frac{1}{\sqrt{2}} \begin{pmatrix} \sigma_1 + i\sigma_2 \\ v + \eta_1 + i\eta_2 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + \eta_1 \end{pmatrix}$$

$\rightarrow \begin{cases} SU(2)_L \text{ rotation of doublet to make it 'down'} \\ U(1)_Y \text{ re-phasing of doublet to make it real} \end{cases} \leftarrow \text{Unitary gauge}$

# SM - V

99

Re-write gauge terms of  $L$  in the unitary gauge, in terms of the physical fields:

$$L_B + L_H$$

$$= -\frac{1}{4} F_{\mu\nu}(x) F^{\mu\nu}(x) \quad \text{Photon}$$

$$-\frac{1}{2} F_{\mu\nu}^W(x) F^{W\mu\nu}(x) + \frac{1}{2} m_W^2 W_\mu^\dagger W^\mu \quad W^\pm \text{ boson}$$

$$-\frac{1}{4} Z_{\mu\nu}(x) Z^{\mu\nu}(x) + \frac{1}{2} m_Z^2 Z_\mu Z^\mu \quad Z^0 \text{ boson}$$

$$+(\partial_\mu \sigma)(\partial^\mu \sigma) - \frac{1}{2} m_H^2 \sigma^2 \quad H \text{ Higgs boson}$$

$$+L_{BB}^I + L_{HH}^I + L_{HB}^I \quad \text{Gauge-Higgs, Higgs self-, Gauge self-interactions}$$

# SM - VI

100

Finding the acquired mass of gauge bosons in terms of couplings and VEV of the Higgs field:

$$\left. \begin{aligned} m_{W^\pm} &= \frac{g}{2} \sqrt{-\frac{\mu^2}{\lambda}} \\ m_{Z^0} &= \frac{\sqrt{(g^2 + g'^2)}}{2} \sqrt{-\frac{\mu^2}{\lambda}} \end{aligned} \right\} \rightarrow m_{Z^0} = m_{W^\pm} \sqrt{1 + \frac{g'^2}{g^2}}$$

$$m_\gamma = 0$$

$$m_H = \sqrt{-2\mu^2} = ???$$

Model parameters  $v, \lambda$  and  $\theta_W$  :

$$v = \sqrt{-\frac{\mu^2}{\lambda}}$$

$$\lambda = ???$$

$$g \sin \theta_W = g' \cos \theta_W = e$$

# SM - VII

101

Lepton masses: Different mechanism required

→ Yukawa (scalar) coupling

Describing interaction between Dirac and scalar fields:

$V \approx g \bar{\Psi} \phi \Psi$ , Static limit:

$$V = -\frac{g}{4\pi} \frac{e^{-\mu r}}{r}$$

$$L_{HL} = -g_l \left[ \bar{\Psi}_l^L \Phi \psi_l^R + \bar{\psi}_l^R \Phi^\dagger \Psi_l^L \right] - \underbrace{g_{\nu_l}}_{=0 \text{ for massless neutrino}} \left[ \bar{\Psi}_l^L \tilde{\Phi} \psi_{\nu_l}^R + \bar{\psi}_{\nu_l}^R \tilde{\Phi}^\dagger \Psi_l^L \right], \quad \tilde{\Phi} = \begin{pmatrix} \phi_b^* \\ -\phi_a^* \end{pmatrix}$$

In the unitary gauge:

$$L_{HL} = -\frac{1}{v} m_l \bar{\psi}_l \psi_l \sigma - \frac{1}{v} m_{\nu_l} \bar{\psi}_{\nu_l} \psi_{\nu_l} \sigma$$

Lepton masses in terms of model parameters:

$$m_l = \frac{v g_l}{\sqrt{2}}, m_{\nu_l} = \frac{v g_{\nu_l}}{\sqrt{2}}$$

# SM - VIII

102

Model parameters:

$$g, g', -\mu^2, \lambda, g_l, g_{\nu_l}$$

Quite remarkably, get  $m_W, m_Z$  by measured constants:

$$\left\{ \begin{array}{l} \alpha = \frac{1}{137.04} \\ G_F = 1.166 \cdot 10^{-5} \text{ GeV}^{-2} \\ \sin^2 \theta_W = 0.23122 \end{array} \right.$$

$$\rightarrow m_W = 77.5 \text{ GeV}, m_Z = 88.4 \text{ GeV}$$

Experimental values:

$$m_W = 80.40 \text{ GeV}, m_Z = 90.19 \text{ GeV}$$

Difference originating from radiative corrections

Higgs:

$$m_H = \sqrt{-2\mu^2} = \sqrt{2\lambda v^2} \rightarrow ???$$

# SM - IX

103

Relating model parameters to measured constants  $e, G_F, \sin \theta_W$ :

$$g = \frac{e}{\sin \theta_W}, \quad g' = \frac{e}{\cos \theta_W}$$

$$G_F = \frac{\sqrt{2}}{8} \left( \frac{g}{M_W} \right)^2 \rightarrow \frac{8G_F}{\sqrt{2}} = \left( \frac{g}{M_W} \right)^2 \rightarrow \frac{M_W}{g} = \sqrt{\frac{\sqrt{2}}{8G_F}}$$

$$\rightarrow M_W = \sqrt{\frac{\sqrt{2}g^2}{8G_F}} = \sqrt{\frac{\sqrt{2}e^2}{8G_F \sin^2 \theta_W}} = \frac{37.3}{\sin \theta_W} \text{ GeV}$$

$$M_Z = \frac{M_W}{\cos \theta_W} = \frac{37.3}{\sin \theta_W \cos \theta_W} \text{ GeV}$$

$$v = \frac{2M_W}{g} = 2 \sqrt{\frac{\sqrt{2}}{8G_F}} = \frac{1}{\sqrt{\sqrt{2}G_F}} \simeq 246 \text{ GeV} \quad \text{VEV of the Higgs field}$$

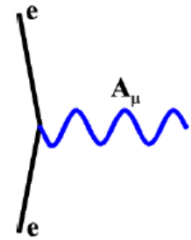
$$\rightarrow \frac{v}{\sqrt{2}} = \frac{1}{\sqrt{\sqrt{2}G_F}} \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{\sqrt{8}G_F}} \simeq 174 \text{ GeV} \rightarrow \langle \phi \rangle_0 = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \simeq \begin{pmatrix} 0 \\ 174 \text{ GeV} \end{pmatrix}$$

No clues on  $\lambda \rightarrow$  No (direct) prediction of  $M_H = \sqrt{2v^2\lambda}$

# SM - X

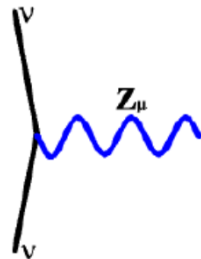
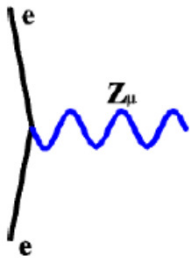
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Lepton-Gauge Boson vertexes:



EM

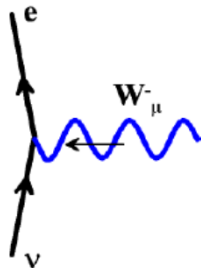
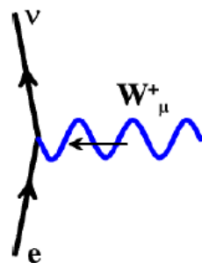
$$-iQ_e \bar{e} \gamma^\mu e A_\mu$$



Neutral Current - NC

$$-i \frac{g}{4 \cos \theta_W} \bar{e} \gamma^\mu \left[ 2 \sin^2 \theta_W (1 + \gamma_5) + (2 \sin^2 \theta_W - 1)(1 - \gamma_5) \right] e Z_\mu$$

$$-i \frac{g}{4 \cos \theta_W} \bar{\nu} \gamma^\mu (1 - \gamma_5) \nu Z_\mu$$



Charged Current - CC

$$-i \frac{g}{2\sqrt{2}} \bar{\nu} \gamma^\mu (1 - \gamma_5) e W_\mu^+$$

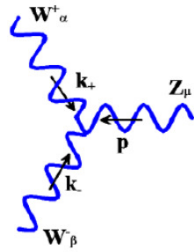
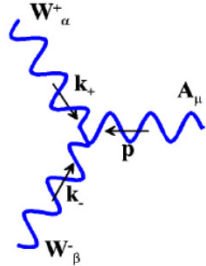
$$-i \frac{g}{2\sqrt{2}} \bar{e} \gamma^\mu (1 - \gamma_5) \nu W_\mu^-$$



# SM - XI

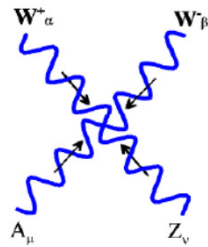
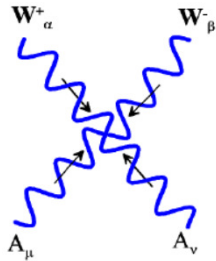
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Gauge bosons self-interaction vertexes:



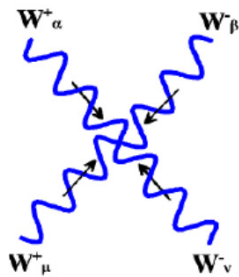
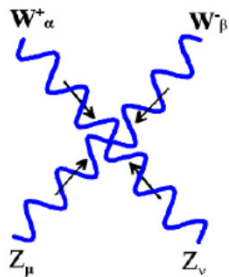
$$ig \sin \theta_W \left[ g^{\alpha\beta} (k_+ - k_-)^\mu + g^{\alpha\mu} (p - k_+)^\beta + g^{\beta\mu} (k_- - p)^\alpha \right] W_\alpha^+ W_\beta^- A_\mu.$$

$$ig \cos \theta_W \left[ g^{\alpha\beta} (k_+ - k_-)^\mu + g^{\alpha\mu} (p - k_+)^\beta + g^{\beta\mu} (k_- - p)^\alpha \right] W_\alpha^+ W_\beta^- Z_\mu.$$



$$-ig^2 \sin^2 \theta_W \left[ 2g^{\mu\nu} g^{\alpha\beta} - g^{\mu\alpha} g^{\nu\beta} - g^{\mu\beta} g^{\alpha\nu} \right] \times W_\alpha^+ W_\beta^- A_\mu A_\nu,$$

$$-ig^2 \sin \theta_W \cos \theta_W \left[ 2g^{\mu\nu} g^{\alpha\beta} - g^{\mu\alpha} g^{\nu\beta} - g^{\mu\beta} g^{\alpha\nu} \right] W_\alpha^+ W_\beta^- A_\mu Z_\nu.$$



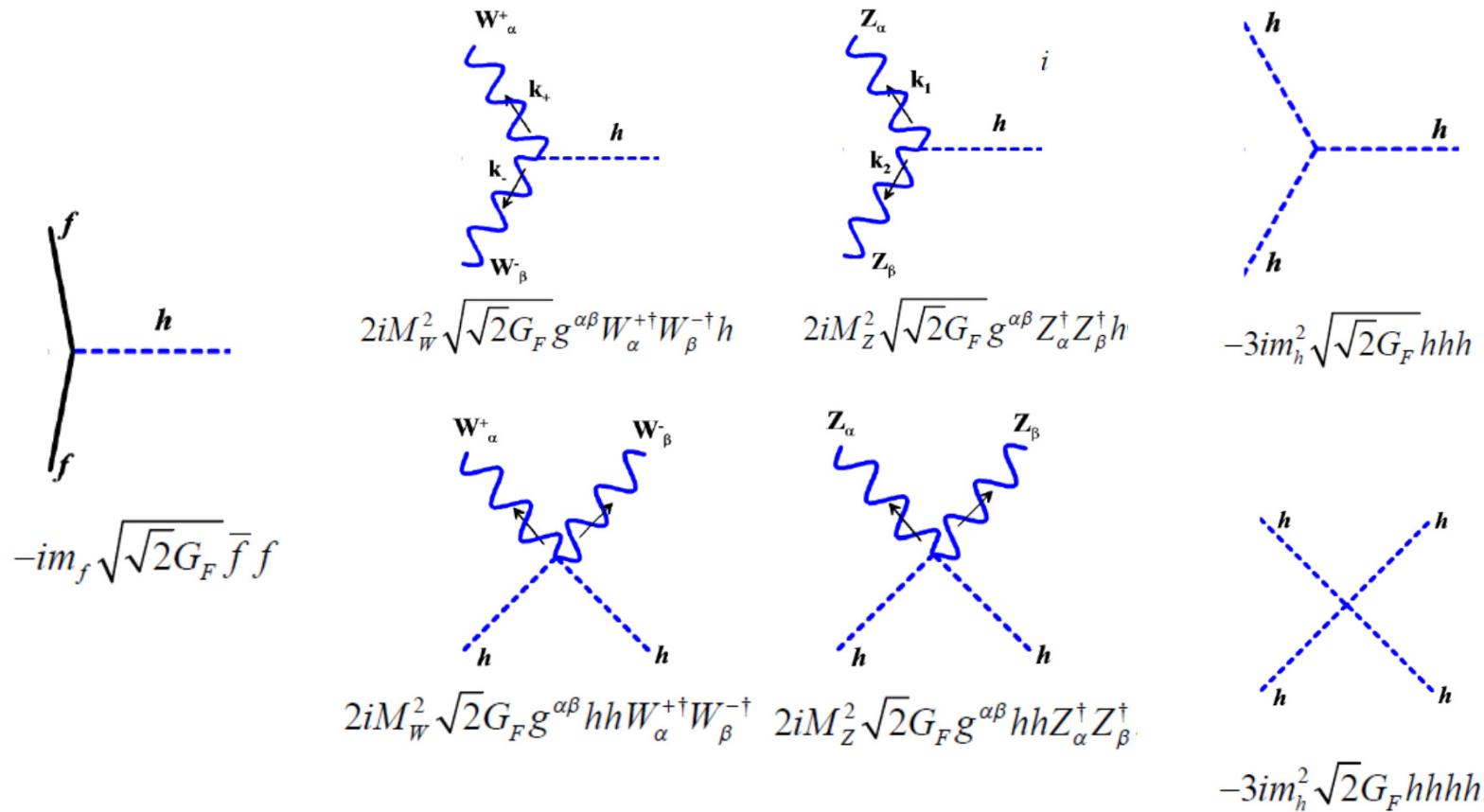
$$-ig^2 \cos^2 \theta_W \left[ 2g^{\mu\nu} g^{\alpha\beta} - g^{\mu\alpha} g^{\nu\beta} - g^{\mu\beta} g^{\alpha\nu} \right] W_\alpha^+ W_\beta^- Z_\mu Z_\nu,$$

$$-ig^2 \left[ 2g^{\mu\nu} g^{\alpha\beta} - g^{\mu\alpha} g^{\nu\beta} - g^{\mu\beta} g^{\alpha\nu} \right] W_\alpha^+ W_\beta^- W_\mu^+ W_\nu^-.$$

# SM - XII



Higgs vertexes:



# SM - XIII

107

Extension to 2nd, 3rd lepton family: Straightforward  
Will need  $2+2 = 4$  new parameters (Yukawa couplings)

'Minimal' Standard Model:

Massless neutrinos  $\rightarrow g_{\nu_i}^{(i)} = 0$

'Non Minimal' Standard Model:

Neutrino mixing ( $\leftarrow$  Require massive neutrinos, mixing matrix):

Account for observed neutrino oscillations

May indicate physics beyond Standard Model

Extension to 3 quark families: Similar to leptons

Will need 6 more parameters

Will require CKM 'flavor rotation' (see later)

Strong interaction effects

} Flavor physics

# SM - XIV

108

Fermion electroweak quantum numbers:

helicity	Generations			Quantum Numbers		
	1.	2.	3.	Q	$T_3$	$Y_W$
L	$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$	$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$	$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$	0 -1	1/2 -1/2	-1 -1
	$\begin{pmatrix} u \\ d' \end{pmatrix}_L$	$\begin{pmatrix} c \\ s' \end{pmatrix}_L$	$\begin{pmatrix} t \\ b' \end{pmatrix}_L$	2/3 -1/3	1/2 -1/2	1/3 1/3
R	$e_R$	$\mu_R$	$\tau_R$	-1	0	-2
	$u_R$ $d_R$	$c_R$ $s_R$	$t_R$ $b_R$	2/3 -1/3	0 0	4/3 -2/3

# W & Z - I

109

Some reminiscences about photons...

$$\text{Free photons } (j^\mu = 0): \quad \square^2 A^\mu - \partial^\mu (\partial_\nu A^\nu) = j^\mu$$

Lorentz condition

$$\partial_\mu A^\mu = 0 \rightarrow \square^2 A^\mu = 0$$

$$\rightarrow A^\mu = \varepsilon^\mu(q) e^{-iqx} \rightarrow q^2 = 0 \text{ massless quanta}$$

4 components  $\varepsilon^\mu$  ??

$$a) \partial_\mu A^\mu = 0 \rightarrow \varepsilon^\mu(q) q_\mu = 0 \rightarrow 3 \text{ components}$$

b) Gauge freedom:

$$A^\mu \rightarrow A^\mu + \partial^\mu \Lambda, \quad \square^2 \Lambda = 0$$

$$\Lambda = iae^{-iqx} \left( \leftarrow \square^2 \Lambda = q^2 \Lambda = 0 \text{ OK} \right)$$

$$\rightarrow \partial^\mu \Lambda = ia \partial^\mu e^{-iqx}$$

$$A^\mu \rightarrow A^\mu + \partial^\mu \Lambda = \varepsilon^\mu(q) e^{-iqx} + ia \partial^\mu e^{-iqx} = \left[ \varepsilon^\mu(q) + ia(-iq_\mu) \right] e^{-iqx} = \left[ \varepsilon^\mu(q) + aq_\mu \right] e^{-iqx}$$

$$\rightarrow \text{EM field unchanged by } \varepsilon^\mu(q) \rightarrow \varepsilon^\mu(q) + aq^\mu$$

Choose  $a$  to make  $\varepsilon^0 = 0$

$$\rightarrow \varepsilon^\mu(q) q_\mu = 0 \rightarrow \boldsymbol{\varepsilon} \cdot \mathbf{q} = 0 \rightarrow 2 \text{ components}$$

# W & Z - II

110

2 components  $\rightarrow$  2 independent  $\varepsilon^\mu$

Take photon momentum along  $z$

$$\varepsilon_1^\mu = (0 \ 1 \ 0 \ 0) \quad x\text{-linear polarization}$$

$$\varepsilon_2^\mu = (0 \ 0 \ 1 \ 0) \quad y\text{-linear polarization}$$

or

$$\varepsilon_L^\mu = \frac{1}{\sqrt{2}}(0 \ 1 \ -i \ 0) \quad \text{Left circular polarization: } S_z = -1$$

$$\varepsilon_R^\mu = \frac{1}{\sqrt{2}}(0 \ 1 \ +i \ 0) \quad \text{Right circular polarization: } S_z = +1$$

# W & Z - III

111

Original wave equation:

$$\square^2 A^\mu - \partial^\mu (\partial_\nu A^\nu) = j^\mu$$

For a massive vector boson:

$$(\square^2 + m^2) B^\mu - \partial^\mu (\partial_\nu B^\nu) = j^\mu$$

Free particle:

$$(\square^2 + m^2) B^\mu - \partial^\mu (\partial_\nu B^\nu) = 0$$

But:

$$\partial_\mu (\square^2 + m^2) B^\mu - \partial_\mu \partial^\mu (\partial_\nu B^\nu) = 0 \rightarrow (\square^2 + m^2) \partial_\mu B^\mu - \square^2 (\partial_\nu B^\nu) = 0$$

$$\rightarrow m^2 \partial_\mu B^\mu = 0 \rightarrow \partial_\mu B^\mu = 0$$

Bottom line: Not an extra condition...

$$\rightarrow (\square^2 + m^2) B^\mu = 0$$

$$B^\mu = \varepsilon^\mu(p) e^{-ipx} \rightarrow \varepsilon^\mu p_\mu = 0 \rightarrow 3 \text{ independent components}$$

No gauge freedom...

# W & Z - IV

112

3 independent components  $\rightarrow$  3 independent  $\varepsilon^\mu$

Take photon momentum along  $z$

$$\varepsilon_L^\mu = \frac{1}{\sqrt{2}}(0 \ 1 \ -i \ 0) \text{ Left circular polarization: } S_z = -1$$

$$\varepsilon_R^\mu = -\frac{1}{\sqrt{2}}(0 \ 1 \ +i \ 0) \text{ Right circular polarization: } S_z = +1$$

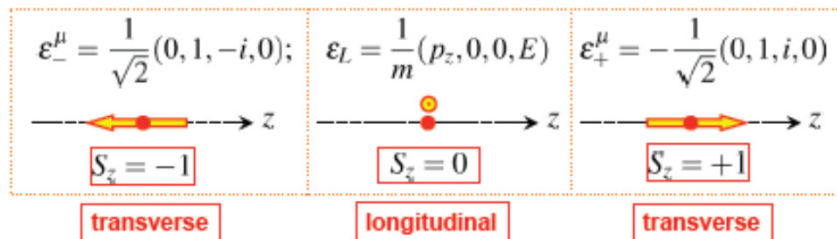
To find 3rd polarization 4-vector:

$$\varepsilon_0^\mu = \frac{1}{\sqrt{|\alpha^2 - \beta^2|}}(\alpha \ 0 \ 0 \ \beta), \quad \frac{1}{\sqrt{|\alpha^2 - \beta^2|}} \text{ normalization: } \varepsilon^\mu p_\mu = 0 \rightarrow \alpha E - \beta p_z = 0$$

$$\rightarrow \alpha = p_z, \beta = E \rightarrow \varepsilon_0^\mu = \frac{1}{m}(p_z \ 0 \ 0 \ E) \text{ Longitudinal polarization: } S_z = 0$$

Observe: Longitudinal/Transverse boson  $\rightarrow$  Transverse/Longitudinal spin...

As for photons: Attribute refers to oscillating 'electric/magnetic field', rather than spin, direction



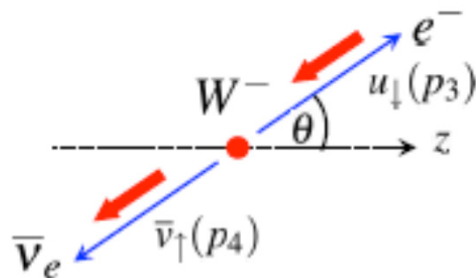
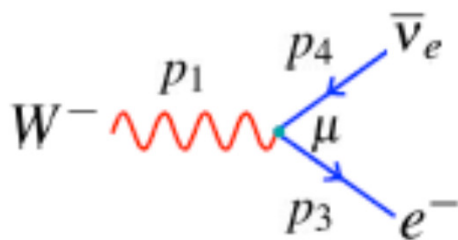
Momentum  $\rightarrow$



# W & Z - V

113

Decay:  $W^- \rightarrow e^- + \bar{\nu}_e$



Matrix element:

$$M_{fi} = \frac{g}{\sqrt{2}} \varepsilon_\mu(p_1) \bar{u}(p_3) \gamma^\mu \frac{1}{2} (1 - \gamma_5) v(p_4) = \frac{g}{\sqrt{2}} \varepsilon_\mu(p_1) j^\mu$$

$\bar{u}(p_3)$ : outgoing fermion,  $v(p_4)$ : outgoing antifermion

$$\begin{aligned} \bar{u}(p_3) \gamma^\mu \frac{1}{2} (1 - \gamma_5) v(p_4) &= \bar{u}(p_3) \frac{1}{2} \gamma^\mu (1 - \gamma_5) \frac{1}{2} \gamma^\mu (1 - \gamma_5) v(p_4) \\ &= \underbrace{\bar{u}(p_3) \frac{1}{2} (1 + \gamma_5)}_{\bar{e}_L} \gamma^\mu \frac{1}{2} \underbrace{(1 - \gamma_5) v(p_4)}_{\nu_R} = \bar{e}_L \gamma^\mu \nu_R \end{aligned}$$

LR current :

Build from rotated  $L, R$  spinors

$$j^\mu = \bar{u}_\downarrow(p_3) \frac{1}{2} \gamma^\mu v_\uparrow(p_4) = 2E(0, -\cos \theta, -i, \sin \theta)$$

# W & Z - VI

114

W polarization states in the rest system:

$$\varepsilon_L^\mu = \frac{1}{\sqrt{2}}(0 \ 1 \ -i \ 0)$$

$$\varepsilon_R^\mu = -\frac{1}{\sqrt{2}}(0 \ 1 \ +i \ 0)$$

$$\varepsilon_0^\mu = \frac{1}{m}(0 \ 0 \ 0 \ m) = (0 \ 0 \ 0 \ 1)$$

Matrix elements for different W polarization states in the rest system:

$$\varepsilon_L^\mu : \frac{g}{\sqrt{2}} \frac{1}{\sqrt{2}}(0 \ 1 \ -i \ 0) 2 \frac{M_W}{2}(0, -\cos\theta, -i, \sin\theta) = -\frac{gM_W}{2}(1 + \cos\theta) \rightarrow |M_L|^2 = \frac{g^2 M_W^2}{4}(1 + \cos\theta)^2$$

$$\varepsilon_R^\mu : \frac{g}{\sqrt{2}} \left[ -\frac{1}{\sqrt{2}}(0 \ 1 \ +i \ 0) \right] 2 \frac{M_W}{2}(0, -\cos\theta, -i, \sin\theta) = -\frac{gM_W}{2}(1 - \cos\theta) \rightarrow |M_R|^2 = \frac{g^2 M_W^2}{4}(1 - \cos\theta)^2$$

$$\varepsilon_0^\mu : \frac{g}{\sqrt{2}} \frac{1}{m}(0 \ 0 \ 0 \ m) 2 \frac{M_W}{2}(0, -\cos\theta, -i, \sin\theta) = \frac{gM_W}{\sqrt{2}} \sin\theta \rightarrow |M_0|^2 = \frac{g^2 M_W^2}{2} \sin^2\theta$$

# W & Z - VII

115

$$|M_L|^2 = \frac{g^2 M_W^2}{4} (1 + \cos \theta)^2$$

$$|M_R|^2 = \frac{g^2 M_W^2}{4} (1 - \cos \theta)^2$$

$$|M_0|^2 = \frac{g^2 M_W^2}{2} \sin^2 \theta$$

2-body differential decay rate:

$$\frac{d\Gamma_{L,O,R}}{d\Omega} = \frac{p}{32\pi^2 M_W^2} |M|^2 = \frac{1}{64\pi^2 M_W} |M|^2 = \frac{g^2 M_W}{64\pi^2} \begin{cases} \frac{1}{4} (1 + \cos \theta)^2 \\ \frac{1}{2} \sin^2 \theta \\ \frac{1}{4} (1 - \cos \theta)^2 \end{cases}$$

Total rates:

$$\int \frac{1}{4} (1 \pm \cos \theta)^2 d \cos \theta d\varphi = \int \frac{1}{2} \sin^2 \theta d \cos \theta d\varphi = \frac{4\pi}{3}$$

$$\rightarrow \Gamma_L = \Gamma_R = \Gamma_0 = \frac{g^2 M_W}{48\pi}$$

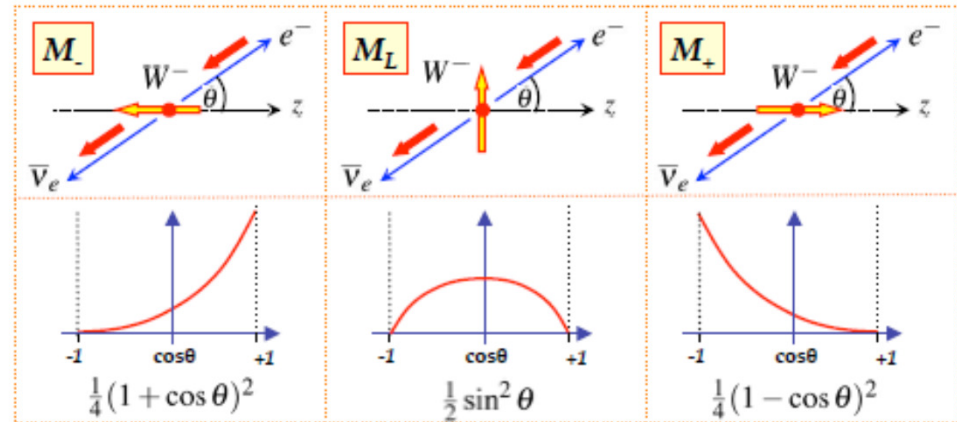
# W & Z - VIII

116

$$|M_L|^2 = \frac{g^2 M_W^2}{4} (1 + \cos \theta)^2$$

$$|M_R|^2 = \frac{g^2 M_W^2}{4} (1 - \cos \theta)^2$$

$$|M_0|^2 = \frac{g^2 M_W^2}{2} \sin^2 \theta$$



Averaging over the initial spin states:

$$\langle |M|^2 \rangle = \frac{1}{3} g^2 M_W^2 \left[ \frac{1}{4} (1 + \cos \theta)^2 + \frac{1}{4} (1 - \cos \theta)^2 + \frac{1}{2} \sin^2 \theta \right]$$

$$\rightarrow \langle |M|^2 \rangle = \frac{1}{3} g^2 M_W^2$$

Isotropic: OK for an unpolarized mother particle

$$\rightarrow \Gamma(W^- \rightarrow e^- + \bar{\nu}_e) = \frac{g^2 M_W}{48\pi}$$

# W & Z - IX

117

Considering all the others decay modes: Large  $W$  mass  $\rightarrow$  All fermions  $\approx$  massless

Do *not* count Top: Too heavy, decay energetically forbidden

Color factor = 3

Similar to  $e^+e^- \rightarrow q\bar{q}$ : Take quarks as free, on shell particles

Taking into account CKM mixing:

$$W^- \rightarrow e^- \bar{\nu}_e \quad W^- \rightarrow d\bar{u} \times 3|V_{ud}|^2 \quad W^- \rightarrow d\bar{c} \times 3|V_{cd}|^2$$

$$W^- \rightarrow \mu^- \bar{\nu}_\mu \quad W^- \rightarrow s\bar{u} \times 3|V_{us}|^2 \quad W^- \rightarrow s\bar{c} \times 3|V_{cs}|^2$$

$$W^- \rightarrow \tau^- \bar{\nu}_\tau \quad W^- \rightarrow b\bar{u} \times 3|V_{ub}|^2 \quad W^- \rightarrow b\bar{c} \times 3|V_{cb}|^2$$

*CKM* Unitarity:

$$e.g. \quad |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 \quad \text{etc}$$

$$\rightarrow \Gamma_W = 9\Gamma_{W \rightarrow e\nu} = \frac{3g^2 M_W}{16\pi} = 2.07 \text{ GeV}$$

*Experiment*:

$$2.14 \pm 0.04 \text{ GeV}$$

*QCD* corrections..

# W & Z - X

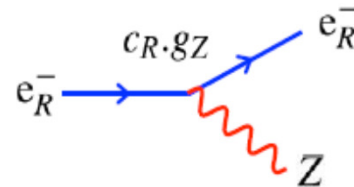
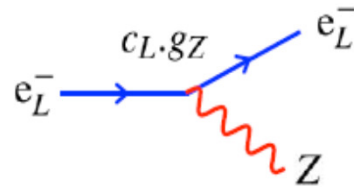
118

Z couplings:

$$c_L = I_3 - Q \sin^2 \theta_W$$

$$c_R = -Q \sin^2 \theta_W$$

$$j_\mu^Z = g_Z \bar{u} \gamma_\mu \left[ c_L \frac{1}{2} (1 - \gamma_5) + c_R \frac{1}{2} (1 + \gamma_5) \right] u$$



$$c_V = c_L + c_R = I_3 - 2Q \sin^2 \theta_W$$

$$c_A = c_L - c_R = I_3$$

# W & Z - XI

119

Therefore:

$$\sin^2 \theta_W \approx 0.23$$



Fermion	$Q$	$I_W^3$	$c_L$	$c_R$	$c_V$	$c_A$
$\nu_e, \nu_\mu, \nu_\tau$	0	$+\frac{1}{2}$	$+\frac{1}{2}$	0	$+\frac{1}{2}$	$+\frac{1}{2}$
$e^-, \mu^-, \tau^-$	-1	$-\frac{1}{2}$	-0.27	0.23	-0.04	$-\frac{1}{2}$
$u, c, t$	$+\frac{2}{3}$	$+\frac{1}{2}$	0.35	-0.15	+0.19	$+\frac{1}{2}$
$d, s, b$	$-\frac{1}{3}$	$-\frac{1}{2}$	-0.42	0.08	-0.35	$-\frac{1}{2}$

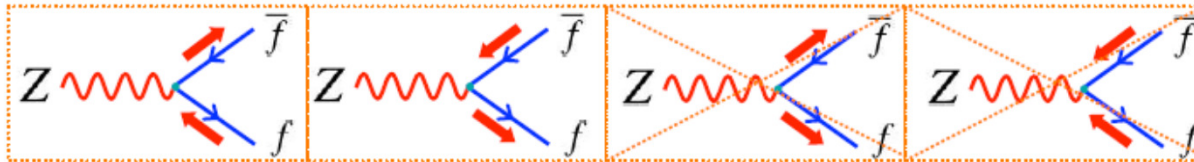
# W & Z - XII

120

Z couplings: Both to  $L$  and  $R$  fermions

Nevertheless:

Only 2 vertexes, remaining 2 = 0



To show that RR vertex is 0 (LL similar):

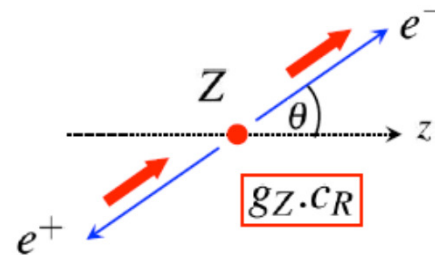
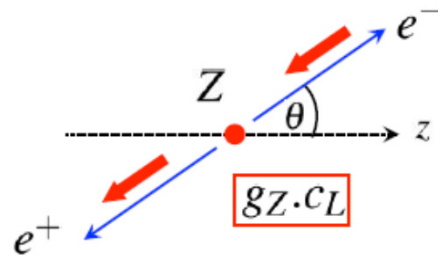
$$\begin{aligned} \bar{u}_R &= u_R^\dagger \gamma^0 = u^\dagger \frac{1+\gamma^5}{2} \gamma^0, & v_R &= \frac{1-\gamma^5}{2} v \\ \bar{u}_R \gamma^\mu (c_V + c_A \gamma_5) v_R &= u^\dagger \frac{1+\gamma^5}{2} \gamma^0 \gamma^\mu (c_V + c_A \gamma_5) \frac{1-\gamma^5}{2} v \\ &= u^\dagger \gamma^0 \frac{1-\gamma^5}{2} \gamma^\mu \frac{1-\gamma^5}{2} (c_V + c_A \gamma_5) v \\ &= \bar{u} \gamma^\mu \underbrace{\frac{1+\gamma^5}{2} \frac{1-\gamma^5}{2}}_{=0} (c_V + c_A \gamma_5) v = 0 \end{aligned}$$



# W & Z - XIII

121

Decay:  $Z^0 \rightarrow e^+ + e^-$



$$\langle |M|^2 \rangle = \frac{2}{3} g^2 \cos^2 \theta_W M_Z^2 [c_L^2 + c_R^2]$$

$$2[c_L^2 + c_R^2] = [c_V^2 + c_A^2]$$

$$\rightarrow \Gamma(Z \rightarrow e^+ e^-) = \frac{g^2 \cos^2 \theta_W M_Z}{48\pi} [c_V^2 + c_A^2]$$

# W & Z - XIV

122

$$Br(Z \rightarrow e^+e^-) = Br(Z \rightarrow \mu^+\mu^-) = Br(Z \rightarrow \tau^+\tau^-) \approx 3.5\%$$

$$Br(Z \rightarrow \nu_1\bar{\nu}_1) = Br(Z \rightarrow \nu_2\bar{\nu}_2) = Br(Z \rightarrow \nu_3\bar{\nu}_3) \approx 6.9\%$$

$$Br(Z \rightarrow d\bar{d}) = Br(Z \rightarrow s\bar{s}) = Br(Z \rightarrow b\bar{b}) \approx 15\%$$

$$Br(Z \rightarrow u\bar{u}) = Br(Z \rightarrow c\bar{c}) \approx 12\%$$

$$Br(Z \rightarrow \text{hadrons}) \approx 69\%$$

$$\rightarrow \Gamma_Z = \sum_i \Gamma_i = 2.5 \text{ GeV}$$

$$\text{Experiment: } \Gamma_Z = 2.4952 \pm 0.0023 \text{ GeV}$$



# W & Z - XV

123

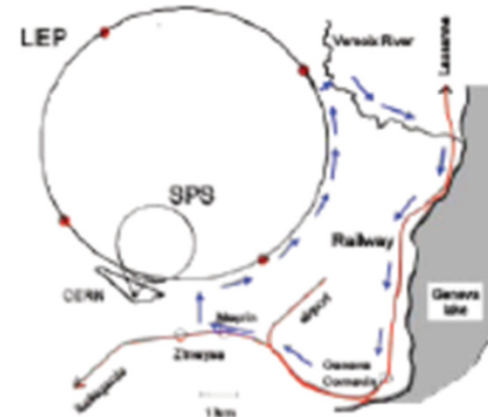
Ultimate systematics....

## Moon:

- ♦ As the moon orbits the Earth it distorts the rock in the Geneva area very slightly !
- ♦ The nominal radius of the accelerator of 4.3 km varies by  $\pm 0.15$  mm
- ♦ Changes beam energy by  $\sim 10$  MeV : need to correct for tidal effects !

## Trains:

- ♦ Leakage currents from the TGV railway line return to Earth following the path of least resistance.
- ♦ Travelling via the Versoix river and using the LEP ring as a conductor.
- ♦ Each time a TGV train passed by, a small current circulated LEP slightly changing the magnetic field in the accelerator
- ♦ LEP beam energy changes by  $\sim 10$  MeV



# SM Internal Consistency - I

124

Reconsidering hypothetical, troublesome reaction

$$\nu\bar{\nu} \rightarrow W_L^+ W_L^-$$

at very high energy

Polarization 4-vectors of longitudinally polarized Ws:

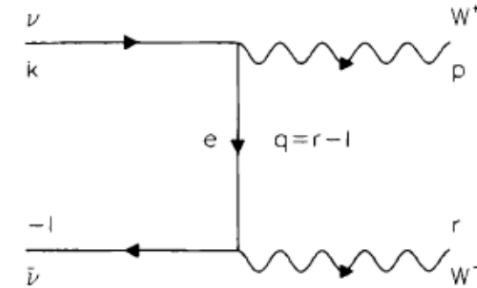
$$\varepsilon_L^\mu(p) = \frac{p^\mu}{m_W} + O\left(\frac{m_W}{p^0}\right) \sim \frac{p^\mu}{m_W}$$

Divergent term of matrix element:

$$M_{fi} \approx -\frac{g^2}{8} \bar{\nu}(l)(1-\gamma_5) \frac{1}{\not{q}-m} (1-\gamma_5) u(k) \frac{r^\mu}{m_W} \frac{p^\nu}{m_W}$$

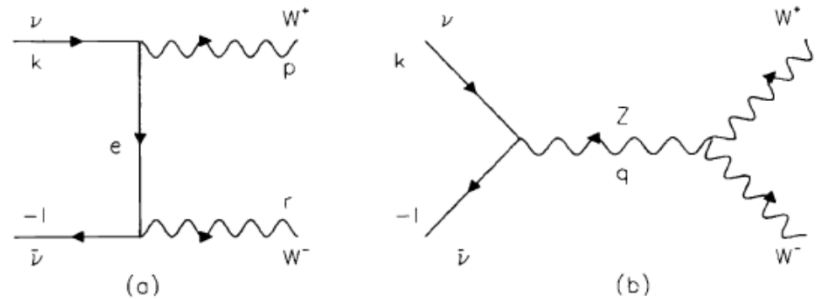
$$M_{fi} \approx -\frac{g^2}{4m_W^2} \bar{\nu}(l) \not{p}' (1-\gamma_5) u(k) - \frac{g^2}{8m_W^2} m \bar{\nu}(l) (1+\gamma_5) \frac{\not{q}+m}{q^2-m^2} \not{p}' (1-\gamma_5) u(k)$$

$$M_{fi}^a \approx -\frac{g^2}{4m_W^2} \bar{\nu}(l) \not{p}' (1-\gamma_5) u(k)$$



# SM Internal Consistency - II

125



Standard Model: Neutral Current  $\rightarrow$  Two diagrams instead of one

$M_{fi}^b : Z^0$  matrix element:

$\nu\nu Z, WWZ$  vertexes,  $Z$  propagator

After quite intense calculations....

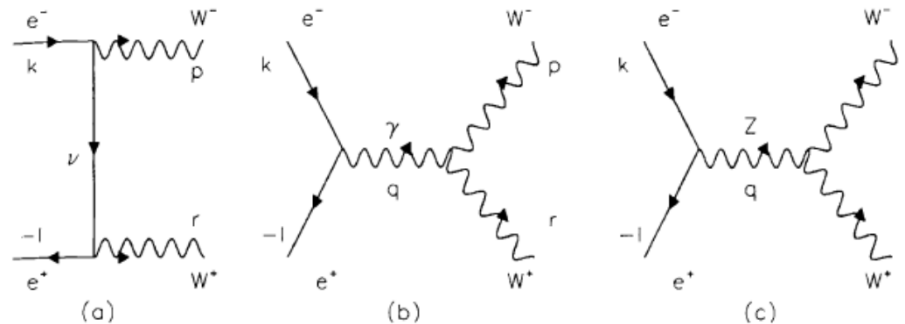
$$M_{fi}^b \approx \frac{g^2}{4m_W^2} \bar{v}(l) \not{\epsilon} (1 - \gamma_5) u(k)$$

$$\rightarrow M_{fi}^b + M_{fi}^b = 0$$

Divergence fixed in a gauge theory!

# SM Internal Consistency - III

126



Another, similar reaction

$$e^+e^- \rightarrow W_L^+W_L^-$$

Quite realistic!

$$M_{fi}^a \approx -\frac{g^2}{4m_W^2} \bar{v}(l) \not{p} (1 - \gamma_5) u(k)$$

$$M_{fi}^b \approx \frac{e^2}{m_W^2} \bar{v}(l) \not{p} u(k)$$

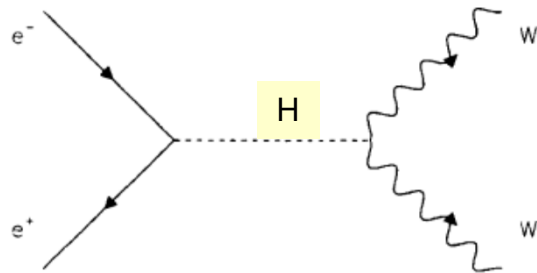
$$M_{fi}^c \approx -\frac{g_{WWZ}}{2m_W^2} \bar{v}(l) [g_L \not{p} (1 - \gamma_5) + g_R \not{p} (1 + \gamma_5)] u(k)$$

$$\rightarrow M_{fi}^a + M_{fi}^b + M_{fi}^c \approx -\frac{g^2}{4m_W^2} m \bar{v}(l) u(k) \text{ Still (weakly) divergent at high energy}$$

# SM Internal Consistency - IV

127

Reason of extra divergence:  $R$  chiral parts of massive fermions



Higgs diagram:

$$M_{fi}^H \approx -\frac{1}{2m_W^2} g_{eeH} g_{WWH} \bar{v}(l) u(k)$$

→ Correct compensation with gauge theory & SSB

Strong support for the Standard Model:

Higgs *must* be there

(or something really new must happen at  $\sim 1$  TeV to save unitarity)

# Neutral Currents Discovery - I

128

Predicted by Glashow-Salam-Weinberg model ('60s)

Not really accepted for a long time:

Mostly because of strong suppression of strangeness changing decays like:

$$K^0 \rightarrow \mu^+ \mu^- \quad BR < 10^{-8}$$

not accounted for. Compare:

$$K^+ \rightarrow \mu^+ \nu_\mu \quad BR \ 63.4 \%$$

Also because it was not clearly demonstrated that GSW was renormalizable

Two breakthroughs:

GIM prediction of charm to solve the  $K^0 \rightarrow \mu^+ \mu^-$  puzzle ('70)

GSW model shown to be renormalizable by 't Hooft ('71)

→ Sudden wave of interest in gauge theories



# Neutral Currents Discovery - II

129

Main interest = Prediction of new phenomena

Most shocking prediction of GSW: neutral currents, never seen before

→ Try to find neutral currents to validate GSW

Best opportunity :

High energy neutrino interactions

Larger cross sections

No EM background

Drawback:

Neutrino experiments difficult

# Neutral Currents Discovery - III

130

Neutrino beams

Take 2 body decays of  $\pi, K$  obtained from a high energy proton machine

Kinematics:

$$\pi^\pm, K^\pm \rightarrow \mu^\pm + \nu_\mu^{(-)}$$

$$\beta, \gamma, M, |\mathbf{p}| \quad K, \pi \quad \text{LAB}$$

$$p^*, E_\mu^*, \theta_\mu^* \quad \mu \quad \text{CM}$$

$$p_\mu, E_\mu, \theta_\mu \quad \mu \quad \text{LAB}$$

$$|\mathbf{p}_\nu^*| = E_\nu^* = \frac{M_{\pi,K}^2 - m_\mu^2}{2M_{\pi,K}} \simeq \begin{cases} 30 \\ 236 \end{cases} \text{ MeV}$$

CM: Isotropic decay

$$\frac{dP}{d(\cos \theta^*)} = \frac{1}{2} \rightarrow \frac{dP}{dE} = \frac{dP}{d(\cos \theta^*)} \frac{d(\cos \theta^*)}{dE}$$

# Neutral Currents Discovery - IV

131

Lorentz transform to LAB:

$$E = \gamma(E^* + \beta p^* \cos \theta^*) \rightarrow dE = \gamma \beta p^* d(\cos \theta^*) \rightarrow d(\cos \theta^*) = \frac{dE}{\gamma \beta p^*}$$

$$\rightarrow \frac{dP}{dE} = \frac{1}{2\gamma \beta p^*}$$

Flat distribution over wide interval:

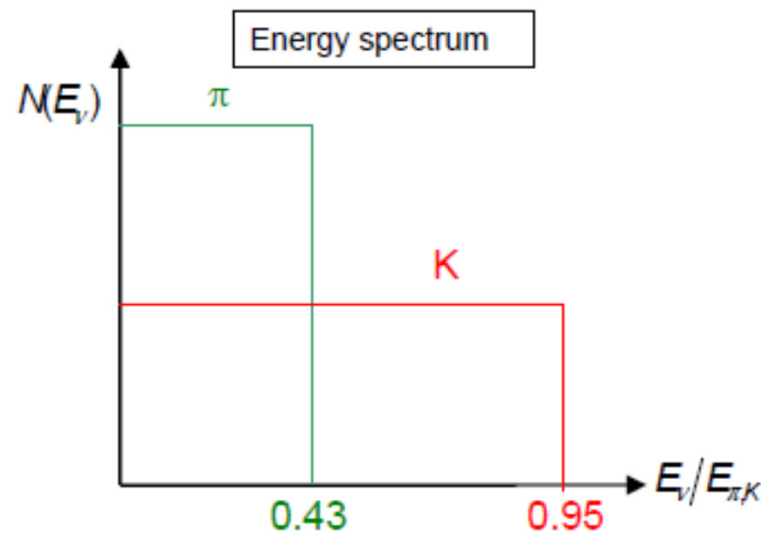
$$\begin{cases} \gamma(1+\beta) E^* = \gamma(1+\beta) \frac{M_{\pi,K}^2 - m_\mu^2}{2M_{\pi,K}} & \text{max} \\ \gamma(1-\beta) E^* = \gamma(1-\beta) \frac{M_{\pi,K}^2 - m_\mu^2}{2M_{\pi,K}} & \text{min} \end{cases}$$

$$\frac{dN}{dE} = \frac{M_{\pi,K}}{\gamma \beta (M_{\pi,K}^2 - m_\mu^2)} = \frac{1}{\underbrace{\gamma \beta M_{\pi,K}}_{|\mathbf{p}|} \left(1 - \frac{m_\mu^2}{M_{\pi,K}^2}\right)} = \frac{1}{|\mathbf{p}| \left(1 - \frac{m_\mu^2}{M_{\pi,K}^2}\right)}$$

# Neutral Currents Discovery - V

132

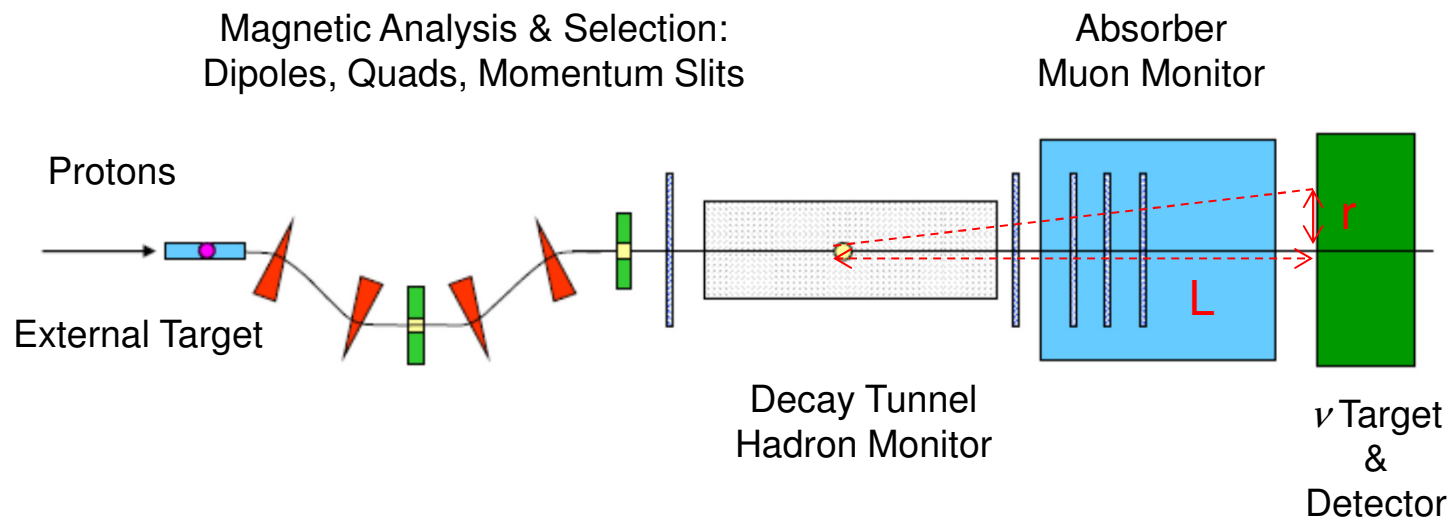
→ Broad LAB  $\nu_\mu$  energy distribution



# Neutral Currents Discovery - VI

133

- a) Narrow Band Beam:  $\nu$  energy known, low intensity  
Magnetic selection of a narrow  $\pi/K$  momentum window



$\pi/K$  momentum well defined

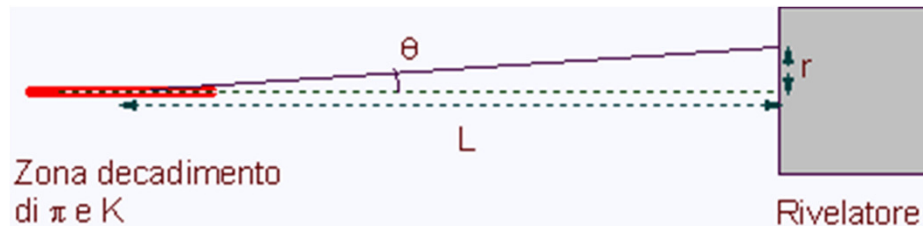
Beam/Detector geometry:  $r \ll L$

# Neutral Currents Discovery - VI

134

Measure  $\nu$  energy by direction:

Exploit hadron beam  $\sim$  monocromaticity



$$p_\pi = p_\mu + p_\nu \rightarrow p_\mu = p_\pi - p_\nu \rightarrow (p_\mu)^2 = (p_\pi - p_\nu)^2$$

$$m_\mu^2 = m_\pi^2 - 2p_\pi \cdot p_\nu \rightarrow m_\pi^2 - m_\mu^2 = 2(E_\pi E_\nu - \mathbf{p}_\pi \cdot \mathbf{p}_\nu)$$

$$E_\nu = \frac{m_{\pi,K}^2 - m_\mu^2}{2(E_{\pi,K} - p_{\pi,K} \cos \theta_\nu)} = \frac{m_\pi^2 - m_\mu^2}{2E_{\pi,K} (1 - \beta \cos \theta_\nu)}$$

$$\tan \theta_\nu = \frac{\sin \theta_\nu^*}{\gamma (\cos \theta_\nu^* + \beta)} \rightarrow \tan \theta_{\max} = \frac{\sin \frac{\pi}{2}}{\gamma \left( \cos \frac{\pi}{2} + \beta \right)} = \frac{1}{\beta \gamma} \approx \frac{1}{\gamma} = \frac{m_{\pi,K}}{|\mathbf{p}|} \ll 1$$

$$E_\nu = \frac{m_{\pi,K}^2 - m_\mu^2}{2E_{\pi,K} (1 - \beta \cos \theta)} \approx \frac{m_{\pi,K}^2 - m_\mu^2}{2E_{\pi,K} \left( 1 - \beta \left( 1 - \frac{\theta^2}{2} \right) \right)}$$

# Neutral Currents Discovery - VII

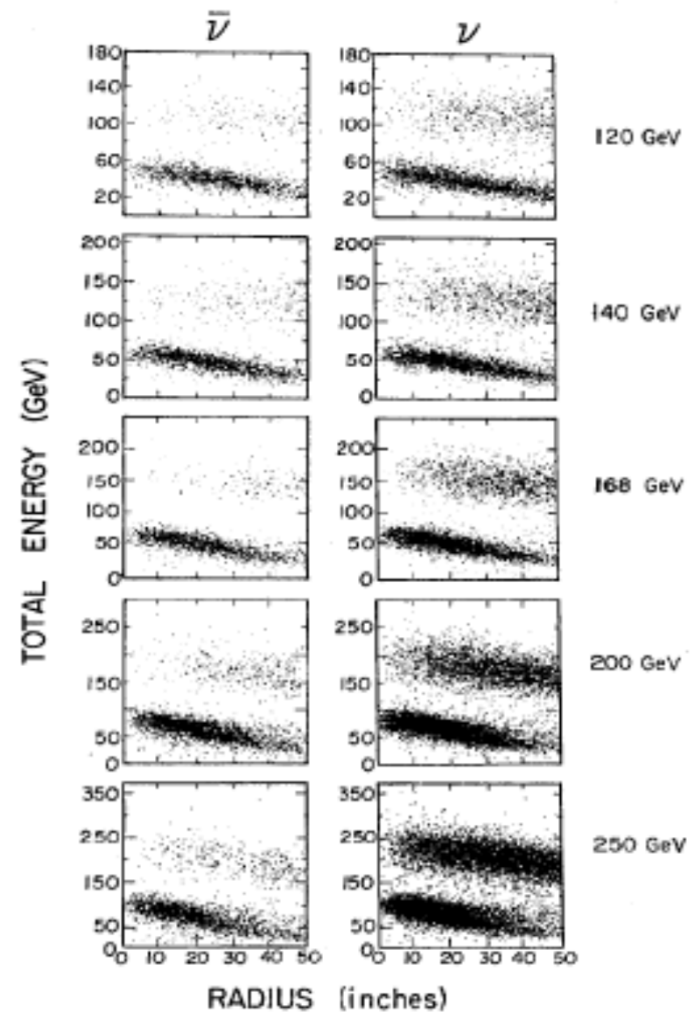
135

$$E_\nu = \frac{m_{\pi,K}^2 - m_\mu^2}{2E_{\pi,K} \left(1 - \beta + \beta \frac{\theta^2}{2}\right)} = \frac{m_{\pi,K}^2 - m_\mu^2}{2E_{\pi,K} \left(\frac{1 - \beta^2}{1 + \beta} + \beta \frac{\theta^2}{2}\right)}$$

$$E_\nu \simeq \frac{m_{\pi,K}^2 - m_\mu^2}{2E_{\pi,K} \left(\frac{1 - \beta^2}{2} + \frac{\theta^2}{2}\right)}, \beta \approx 1$$

$$E_\nu = \frac{m_{\pi,K}^2 - m_\mu^2}{E_{\pi,K} \left(\frac{1}{\gamma^2} + \theta^2\right)} = \frac{m_{\pi,K}^2 \left(1 - \frac{m_\mu^2}{m_{\pi,K}^2}\right) E_{\pi,K}}{E_{\pi,K}^2 \left(\frac{1}{\gamma^2} + \theta^2\right)}$$

$$E_\nu = \frac{m_{\pi,K}^2 \left(1 - \frac{m_\mu^2}{m_{\pi,K}^2}\right) E_{\pi,K}}{\gamma^2 m_{\pi,K}^2 \left(\frac{1}{\gamma^2} + \theta^2\right)} = E_{\pi,K} \frac{\left(1 - \frac{m_\mu^2}{m_{\pi,K}^2}\right)}{\left(1 + \gamma^2 \theta^2\right)}$$

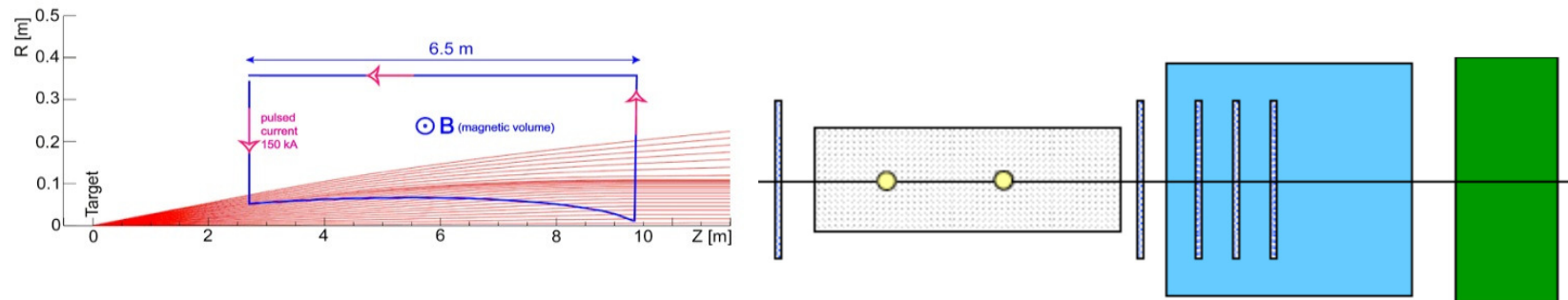


# Neutral Currents Discovery - VIII

136

b) Wide Band Beam:  $\nu$  energy unknown, high intensity

Replace magnetic selection by a special focussing device, suitable to make a low divergence hadron beam out of an uncollimated, divergent source: Van der Meer Horn



Collect a wide momentum window, focus into a narrow, intense beam

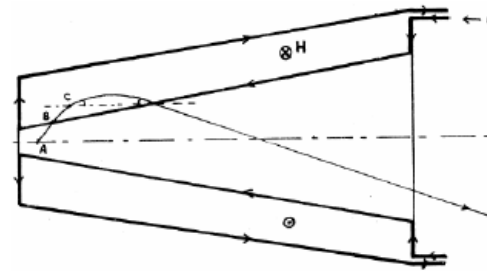
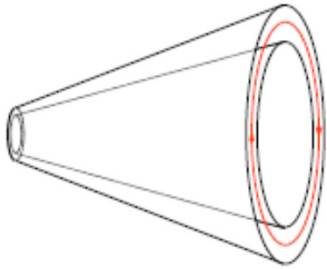


# Neutral Currents Discovery - IX

137

2 conical (high) current sheets:

Equivalent to many trapezoidal current loops symmetrically placed around the axis  
 → Circular magnetic field (red circumference)



Trajectory in the  $B$  field:  $\sim$  Circular arc

$$|\mathbf{p}| = 0.3BR \begin{cases} p & \text{GeV} \\ B & \text{T} \\ R & \text{m} \end{cases}$$

Deflection after a path length  $l$  in the field:

$$\Delta\theta = \frac{\Delta l}{R} = 0.3B \frac{\Delta l}{|\mathbf{p}|}$$

Deflection should compensate  $\langle p_T \rangle$

of hadrons coming out of the target:

$\langle p_T \rangle \sim p\Delta\theta \sim 0.2 \text{ GeV}$  at PS energies

$$\rightarrow 0.2 \text{ GeV} \sim |\mathbf{p}| \Delta\theta = 0.3B\Delta l$$

Simple guess:

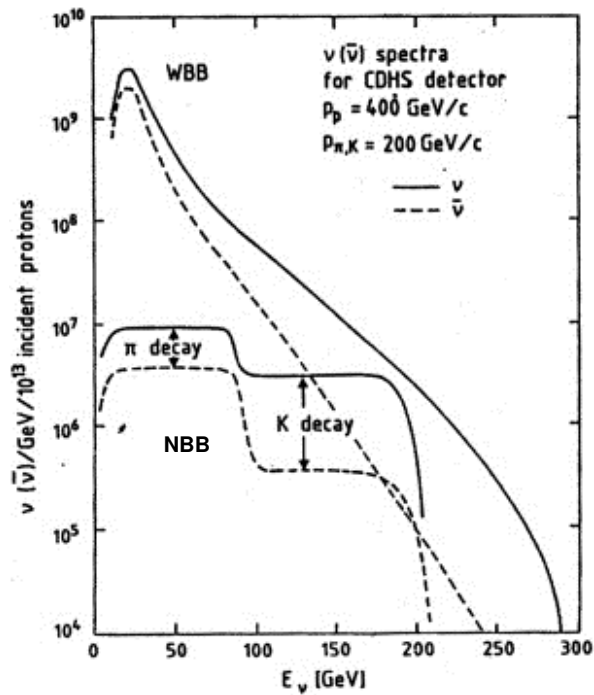
$$B = \frac{\mu_0 I}{2\pi r} \rightarrow 0.2 \text{ GeV} \sim |\mathbf{p}| \Delta\theta \sim 0.3 \frac{\mu_0 I}{2\pi r} \Delta l$$

$$\rightarrow I \sim \frac{|\mathbf{p}| \Delta\theta 2\pi r}{0.3\mu_0 \Delta l} \sim 10^5 \text{ A!}$$

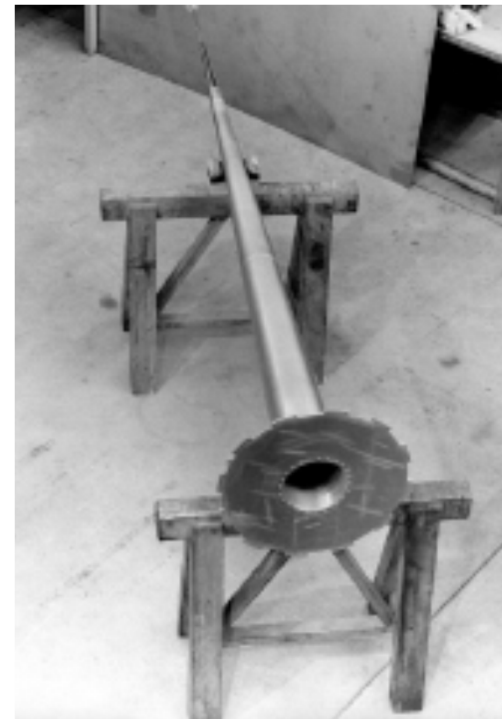
# Neutral Currents Discovery - X

138

Narrow/Wideband beam spectra  
SPS beam



The Gargamelle horn



# Neutral Currents Discovery - XI

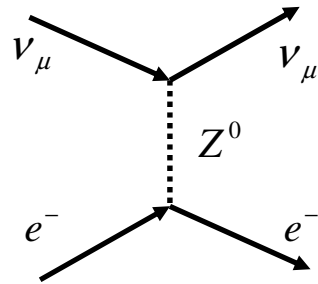
139



Length: 4.8 m  
Diameter: 2 m  
Liquid Freon: 12 m<sup>3</sup>

# Neutral Currents Discovery - XII

140

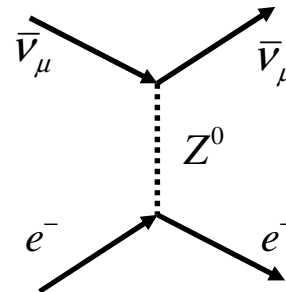


$\nu$ - $e$  processes

Pure NC

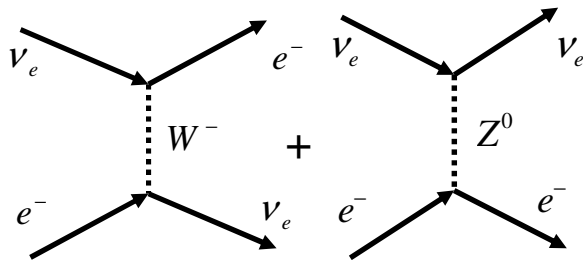
$$\sigma_{\nu_\mu e^-}^{NC}(E) = \frac{G_F^2 s}{\pi} \left[ \left( -\frac{1}{2} + \sin^2 \theta_W \right)^2 + \frac{1}{3} \sin^4 \theta_W \right]$$

$0.16 \times 10^{-41} E(\text{GeV}) \text{ cm}^2$



$$\sigma_{\bar{\nu}_\mu e^-}^{NC}(E) = \frac{G_F^2 s}{\pi} \left[ \frac{1}{3} \left( -\frac{1}{2} + \sin^2 \theta_W \right)^2 + \sin^4 \theta_W \right]$$

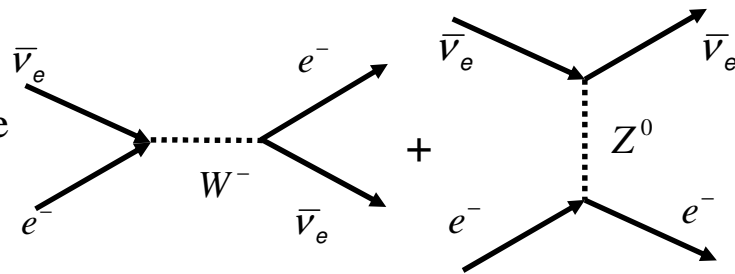
$0.13 \times 10^{-41} E(\text{GeV}) \text{ cm}^2$



Interference  
CC+NC

$$\sigma_{\nu_e e^-}^{CC+NC}(E) = \frac{G_F^2 s}{\pi} \left[ \left( \frac{1}{2} + \sin^2 \theta_W \right)^2 + \frac{1}{3} \sin^4 \theta_W \right]$$

$0.96 \times 10^{-41} E(\text{GeV})$



$$\sigma_{\bar{\nu}_e e^-}^{CC+NC}(E) = \frac{G_F^2 s}{\pi} \left[ \frac{1}{3} \left( \frac{1}{2} + \sin^2 \theta_W \right)^2 + \sin^4 \theta_W \right]$$

$0.40 \times 10^{-41} E(\text{GeV}) \text{ cm}^2$

# Neutral Currents Discovery - XIII

141

*Effective couplings for several reactions*

Reaction	$\varepsilon$	Electroweak theory		V-A theory	
		$g_V$	$g_A$	$g_V$	$g_A$
$\nu_\mu + e^- \rightarrow \nu_\mu + e^-$	+1	$-\frac{1}{2} + 2s^2$	$-\frac{1}{2}$	0	0
$\bar{\nu}_\mu + e^- \rightarrow \bar{\nu}_\mu + e^-$	-1	$-\frac{1}{2} + 2s^2$	$-\frac{1}{2}$	0	0
$\nu_e + e^- \rightarrow \nu_e + e^-$	+1	$+\frac{1}{2} + 2s^2$	$+\frac{1}{2}$	1	1
$\bar{\nu}_e + e^- \rightarrow \bar{\nu}_e + e^-$	-1	$+\frac{1}{2} + 2s^2$	$+\frac{1}{2}$	1	1
$\nu_\mu + e^- \rightarrow \mu^- + \nu_e$	+1	1	1	1	1

# Neutral Currents Discovery - XIV

142

Differential cross sections:

$$y = 1 - \frac{E_\nu'}{E_\nu} \simeq \frac{E_e}{E_\nu} \quad \text{Bjorken } y$$

$$\frac{d\sigma_{\nu_\mu e}}{dy} = \frac{G_F^2 m_e}{2\pi} E_\nu \left[ (g_V + g_A)^2 + (g_V - g_A)^2 (1-y)^2 + \frac{m_e}{E_\nu} (g_A^2 - g_V^2) y \right]$$

$$\frac{d\sigma_{\bar{\nu}_\mu e}}{dy} = \frac{G_F^2 m_e}{2\pi} E_\nu \left[ (g_V - g_A)^2 + (g_V + g_A)^2 (1-y)^2 + \frac{m_e}{E_\nu} (g_A^2 - g_V^2) y \right]$$

$$\int_0^1 (1-y)^2 dy = \frac{1}{3}, \quad \int_0^1 y dy = \frac{1}{2}$$

Total cross sections:

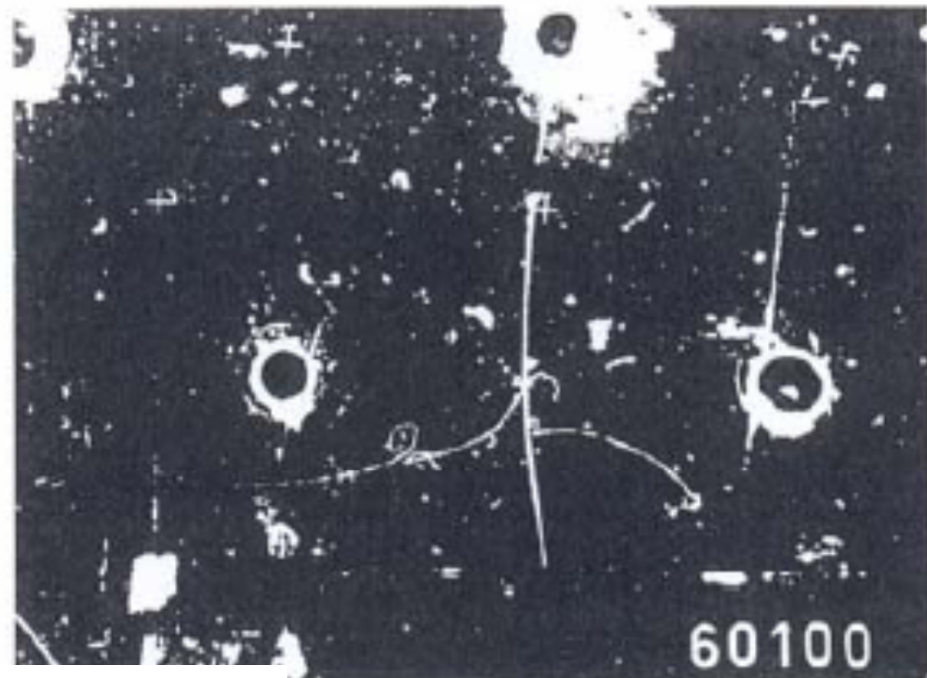
$$\rightarrow \sigma_{\nu_\mu e} = \frac{G_F^2 m_e}{2\pi} E_\nu \left[ (g_V + g_A)^2 + \frac{1}{3} (g_V - g_A)^2 + \frac{m_e}{E_\nu} \frac{1}{2} (g_A^2 - g_V^2) \right]$$

$$\rightarrow \sigma_{\bar{\nu}_\mu e} = \frac{G_F^2 m_e}{2\pi} E_\nu \left[ (g_V - g_A)^2 + \frac{1}{3} (g_V + g_A)^2 + \frac{m_e}{E_\nu} \frac{1}{2} (g_A^2 - g_V^2) \right]$$

# Neutral Currents Discovery - XV

143

First Gargamelle  
leptonic neutral current event



Neutraler Strom  
= "schwaches Licht"

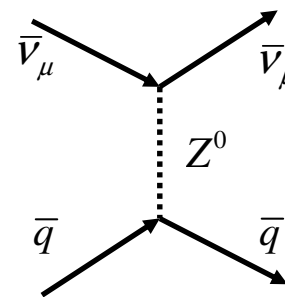
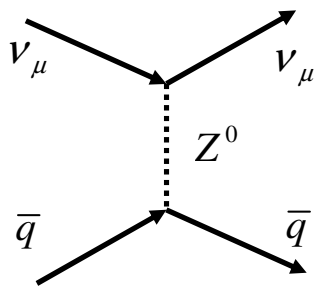
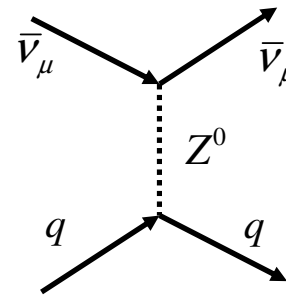
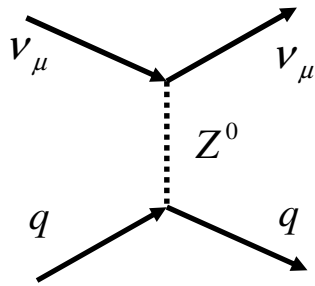
b)



# Neutral Currents Discovery - XVI

144

(-)  
 $\nu - q, \bar{q}$  processes





# Neutral Currents Discovery - XVII

145

$$g_V = \frac{1}{2} - \frac{4}{3} \sin^2 \theta_W \quad g_A = \frac{1}{2} \quad u, c, t$$

$$g'_V = -\frac{1}{2} + \frac{4}{3} \sin^2 \theta_W \quad g'_A = \frac{1}{2} \quad d, s, b$$

$$g_L = \frac{1}{2} - \frac{2}{3} \sin^2 \theta_W \quad g_R = -\frac{2}{3} \sin^2 \theta_W \quad u, c, t$$

$$g'_L = -\frac{1}{2} + \frac{1}{3} \sin^2 \theta_W \quad g_R = \frac{1}{3} \sin^2 \theta_W \quad d, s, b$$

$$\frac{d\sigma_{\nu_\mu q}}{dy} = \frac{d\sigma_{\bar{\nu}_\mu \bar{q}}}{dy} = \frac{G_F^2 m_q}{2\pi} E_\nu \left[ (g_V + g_A)^2 + (g_V - g_A)^2 (1-y)^2 + \frac{m_q}{E} (g_A^2 - g_V^2) y \right]$$

$$\frac{d\sigma_{\bar{\nu}_\mu q}}{dy} = \frac{d\sigma_{\nu_\mu \bar{q}}}{dy} = \frac{G_F^2 m_q}{2\pi} E_\nu \left[ (g_V - g_A)^2 + (g_V + g_A)^2 (1-y)^2 + \frac{m_q}{E_\nu} (g_A^2 - g_V^2) y \right]$$

# Neutral Currents Discovery - XVIII

146

$$\frac{d\sigma_{\nu\mu}^{(-)}}{dx dy} = \sum_q q(x) \frac{d\sigma_{\nu\mu q}^{(-)}}{dy} + \sum_{\bar{q}} \bar{q}(x) \frac{d\sigma_{\nu\mu\bar{q}}^{(-)}}{dy}$$

$$\rightarrow \frac{d\sigma_{\nu\mu N}}{dx dy} = \frac{G_F^2 m_N}{2\pi} x E_\nu \left[ (g_L^2 + g_L'^2) (q + \bar{q} (1-y)^2) + (g_R^2 + g_R'^2) (\bar{q} + q (1-y)^2) \right]$$

$$\rightarrow \frac{d\sigma_{\bar{\nu}\mu N}}{dx dy} = \frac{G_F^2 m_N}{2\pi} x E_\nu \left[ (g_R^2 + g_R'^2) (q + \bar{q} (1-y)^2) + (g_L^2 + g_L'^2) (\bar{q} + q (1-y)^2) \right]$$

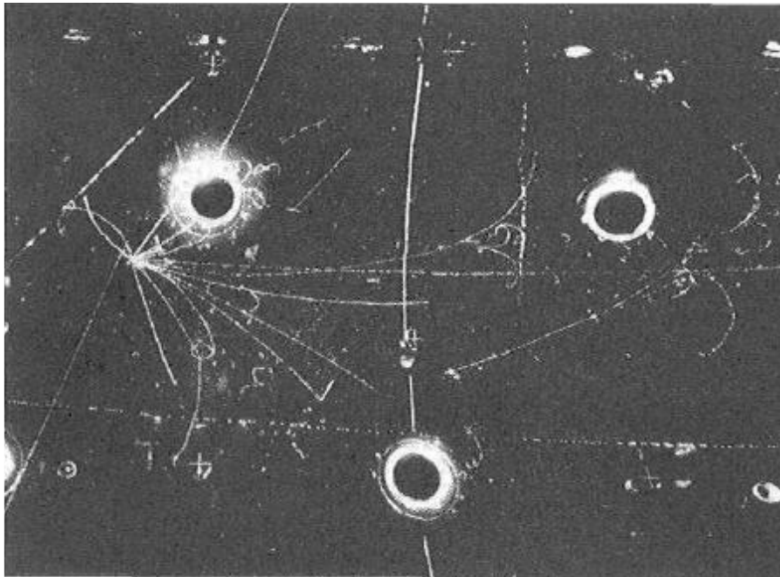
$$R_\nu^N = \frac{\sigma_{NC}(\nu)}{\sigma_{CC}(\nu)} \quad R_{\bar{\nu}}^N = \frac{\sigma_{NC}(\bar{\nu})}{\sigma_{CC}(\bar{\nu})} \quad r = \frac{\sigma_{CC}(\bar{\nu})}{\sigma_{CC}(\nu)}$$

$$\rightarrow g_L^2 + g_L'^2 = \frac{R_\nu^N - r^2 R_{\bar{\nu}}^N}{1 - r^2} \quad g_R^2 + g_R'^2 = \frac{r(R_\nu^N - R_{\bar{\nu}}^N)}{1 - r^2}$$

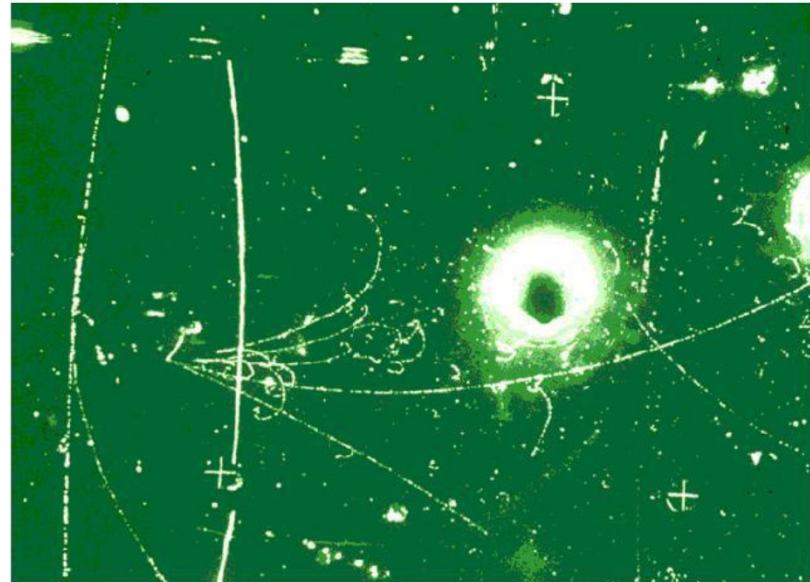
# Neutral Currents Discovery - XIX

147

Gargamelle  
charged current



Gargamelle  
hadronic neutral current event



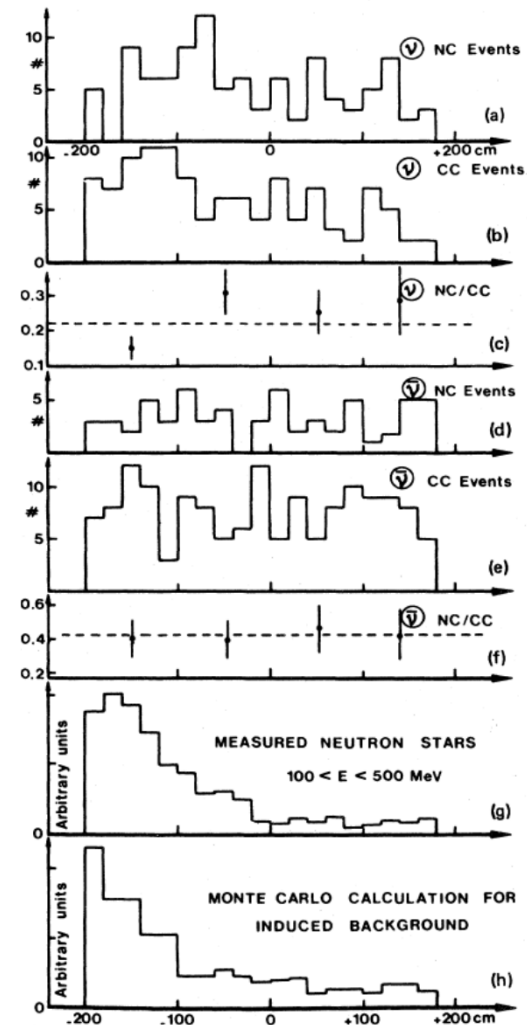
# Neutral Currents Discovery - XX

148

Background

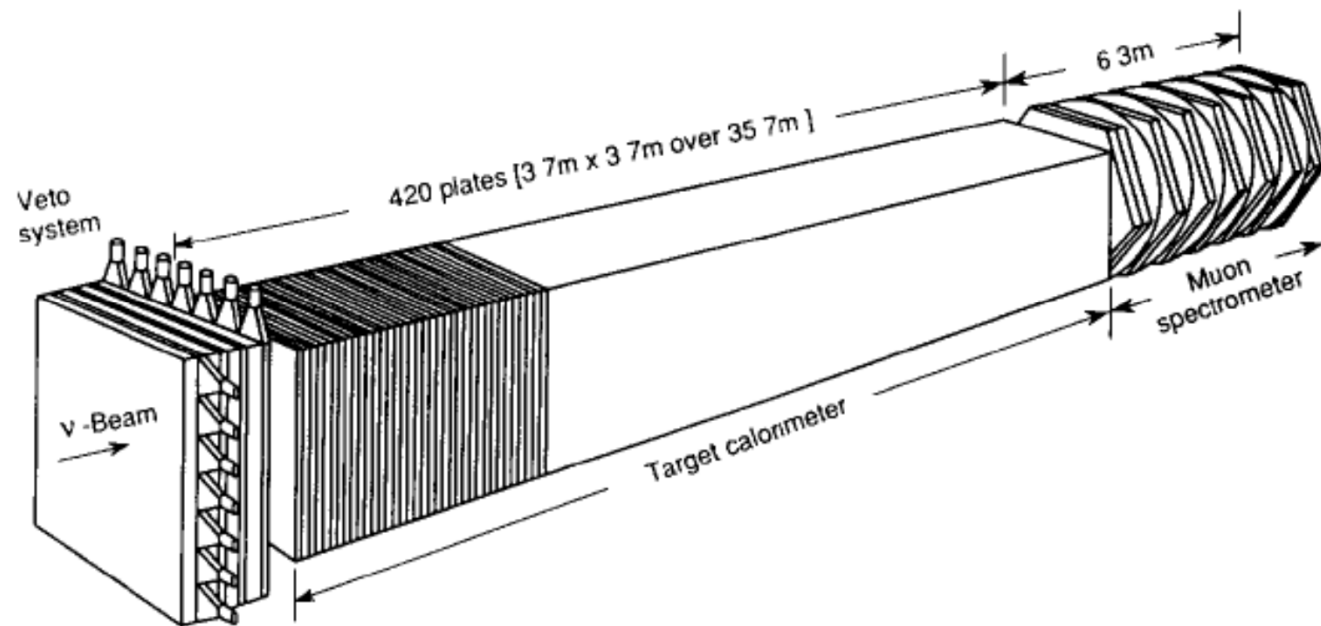
Result

$$\sin^2 \theta_w = 0.3 \div 0.4$$



# Neutral Currents Discovery - XXI

149



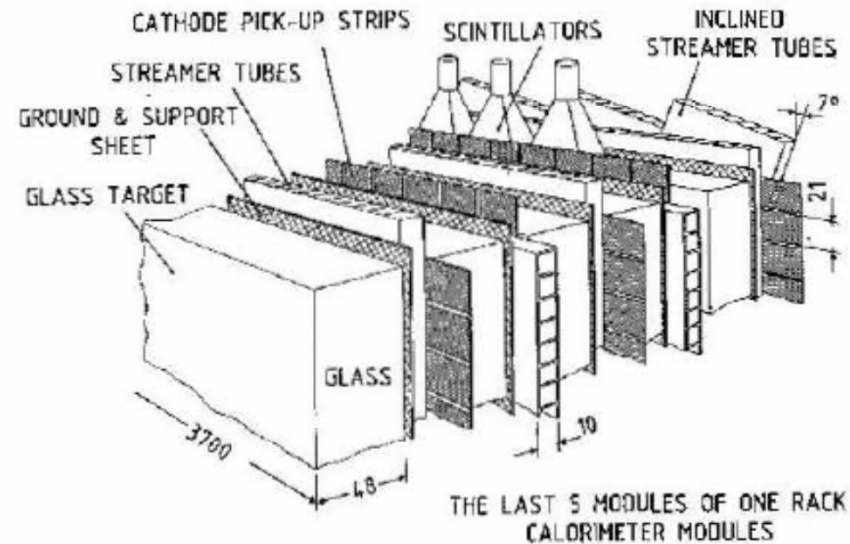
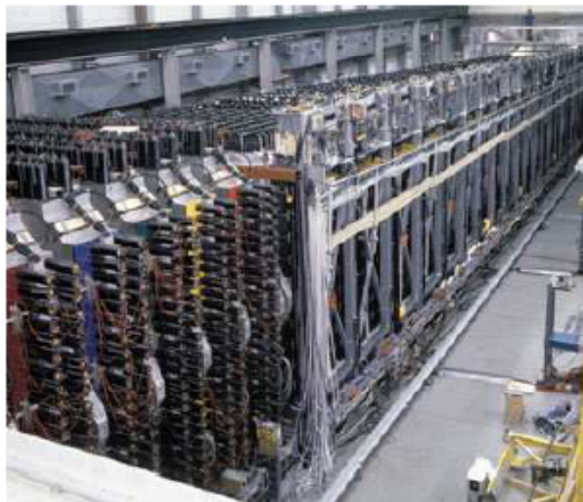
CHARM II

# Neutral Currents Discovery - XXII

150

## Experimental setup (CHARM II, 1987-1991)

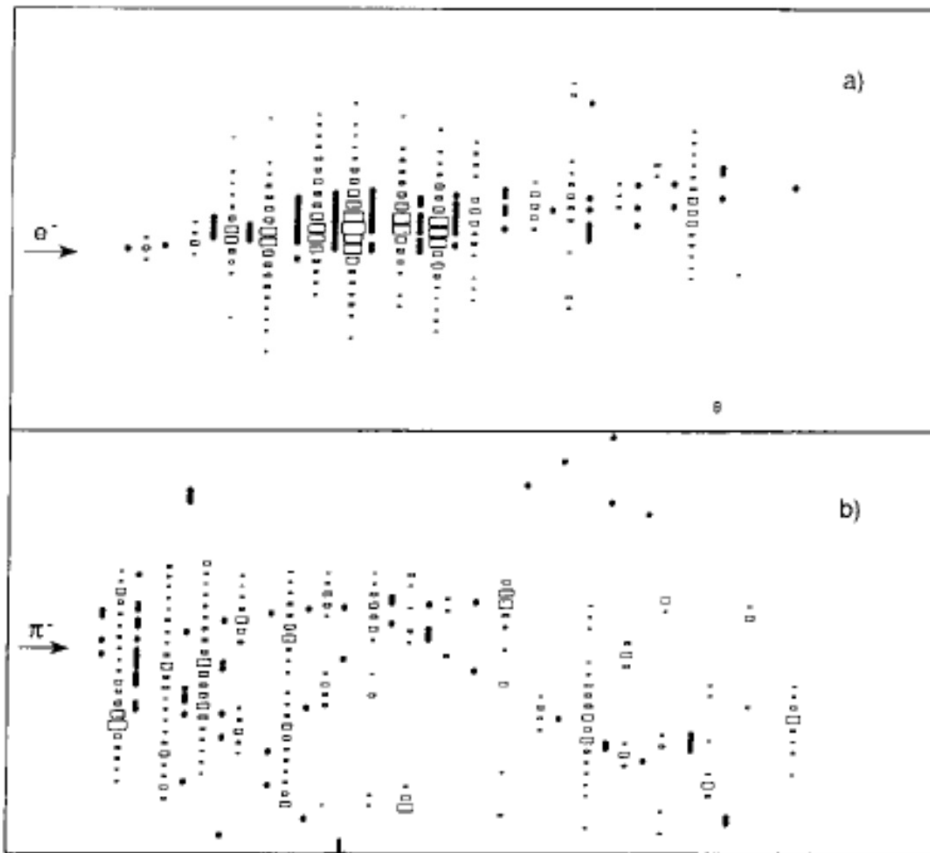
electronic tracking detector



~ 700 t calorimeter, digital readout of energy and direction of produced particles

# Neutral Currents Discovery - XXIII

151



Electromagnetic shower

Hadronic shower

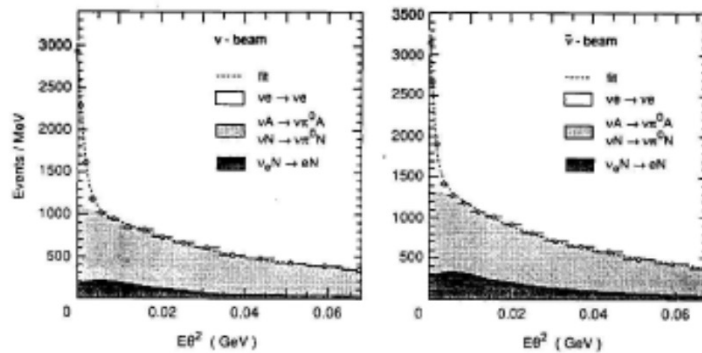
# Neutral Currents Discovery- XXIV

152

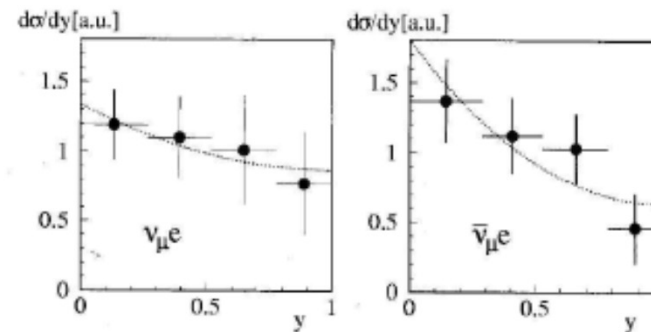
## CHARM II data

**Problem:** discrimination of the NC events ( $\sim 2500$  for  $\nu_\mu e^-$  and  $\bar{\nu}_\mu e^-$  each) from the dominant background (CC scattering, inelastic scattering)

**Solution:** in processes of interest  $\nu_\mu e \rightarrow \nu_\mu e$  and  $\bar{\nu}_\mu e^- \rightarrow \bar{\nu}_\mu e^-$  the value  $E_e \theta_e^2$  is kinematically restricted to small values



Phys.Lett. B335, 246 (1994)



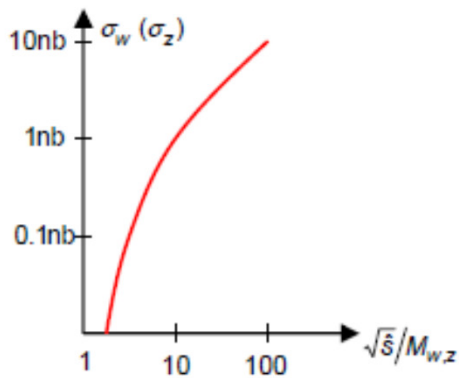
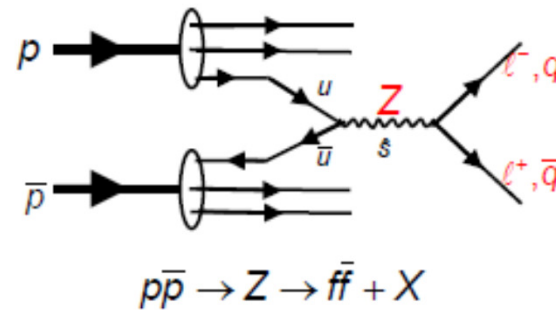
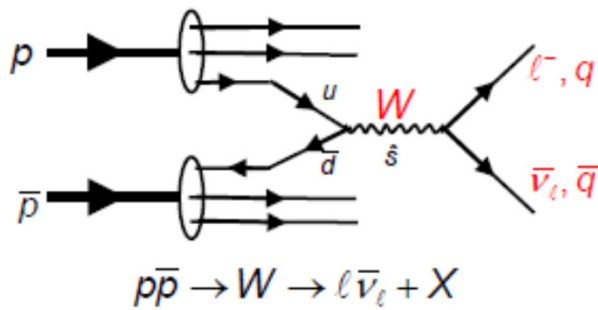
Phys.Lett. B302, 351 (1993)

$$\sin^2 \Theta_{\nu e} = 0.2324 \pm 0.0083$$



# W & Z Discovery - I

153



Similar to Drell-Yan: (photon instead of W)

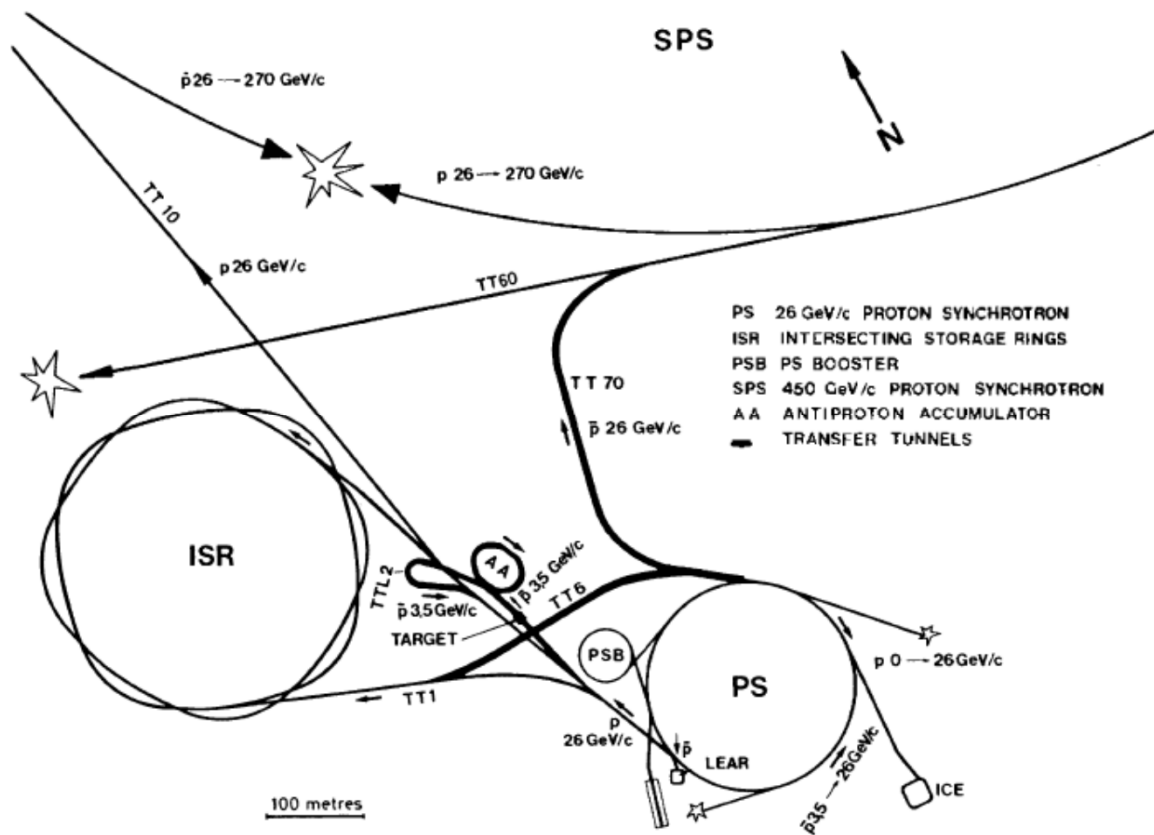
$$\hat{s} = x_q x_{\bar{q}} s \quad \langle x_q \rangle \approx 0.12$$

$$\hat{s} = \langle x_q \rangle^2 s \approx 0.014 s = (65 \text{ GeV})^2$$

→ Cross section is small !

# W & Z Discovery - II

154



# W & Z Discovery - III

155

$S\bar{p}pS$  Collider main parameters

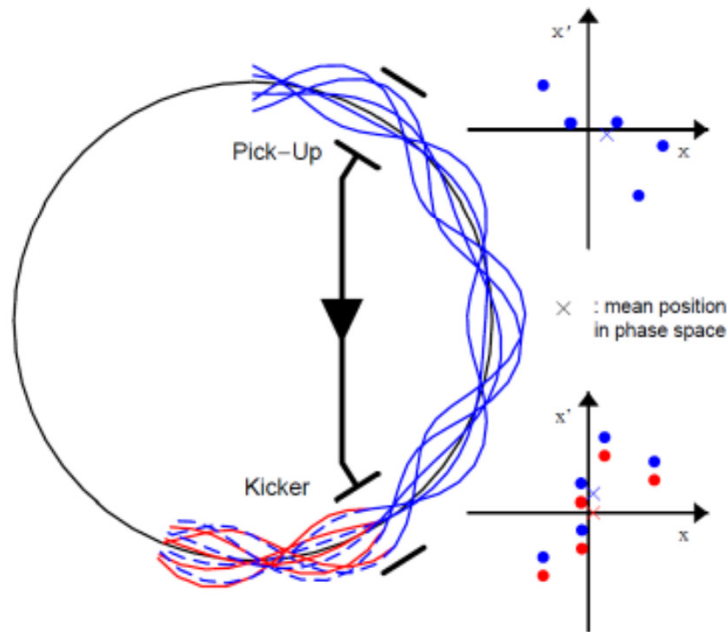
- $\sqrt{s} = 540 \text{ GeV}$
- 3 bunches protons, 3 bunches antiprotons,  
 $10^{11}$  particles per bunch
- Luminosity =  $5 \times 10^{27} \text{ cm}^{-2}\text{sec}^{-1}$
- First collisions in December 1981

# W & Z Discovery - IV

156

## Stochastical cooling system

### Basic principle

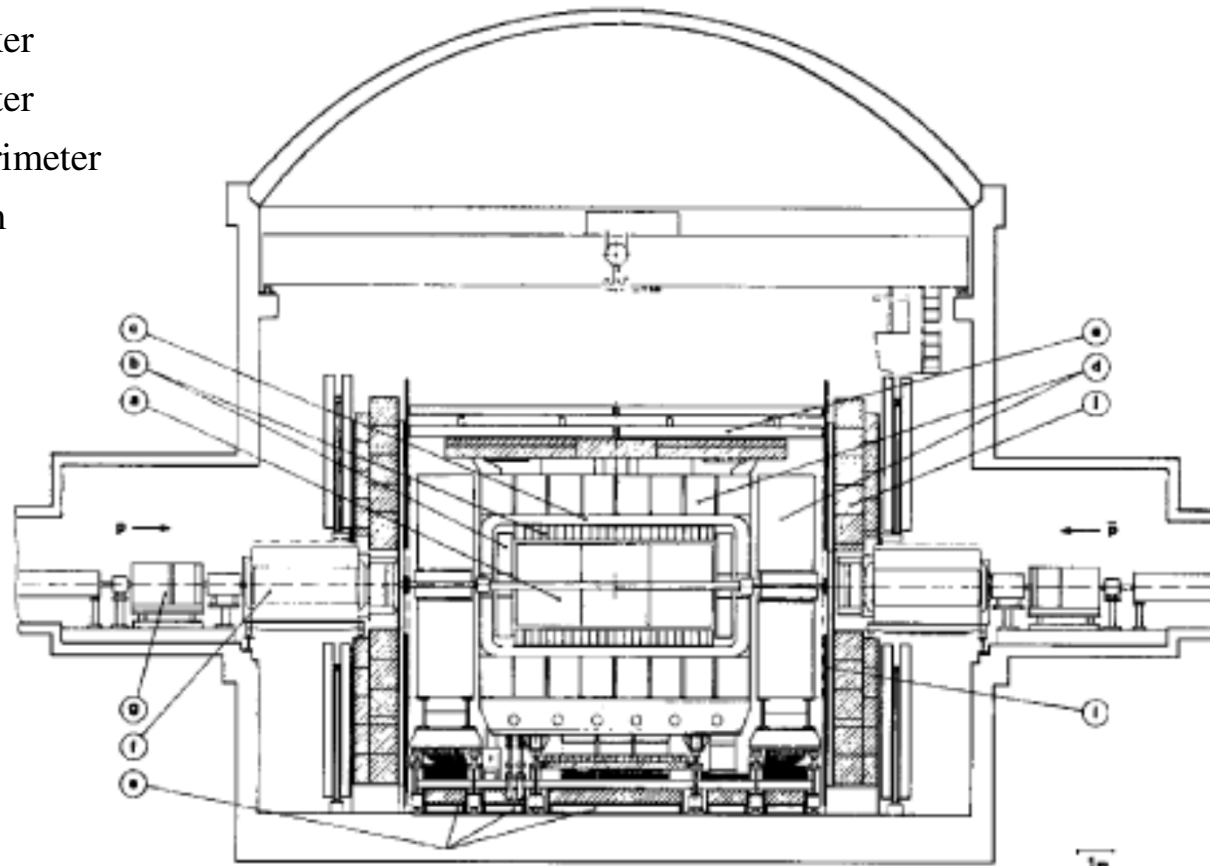


- $10^7$  antiprotons with  $p = 3.5 \text{ GeV}/c$  gets in outer part of toroidal vacuum chamber
- Inductor measures discrepancy of particles
- Correction signal is send to opposite side
- Magnet deflects particles

# W & Z Discovery - V

157

UA1 Detector  
Central Tracker  
EM calorimeter  
Hadron Calorimeter  
Muon System



# W & Z Discovery - VI

158

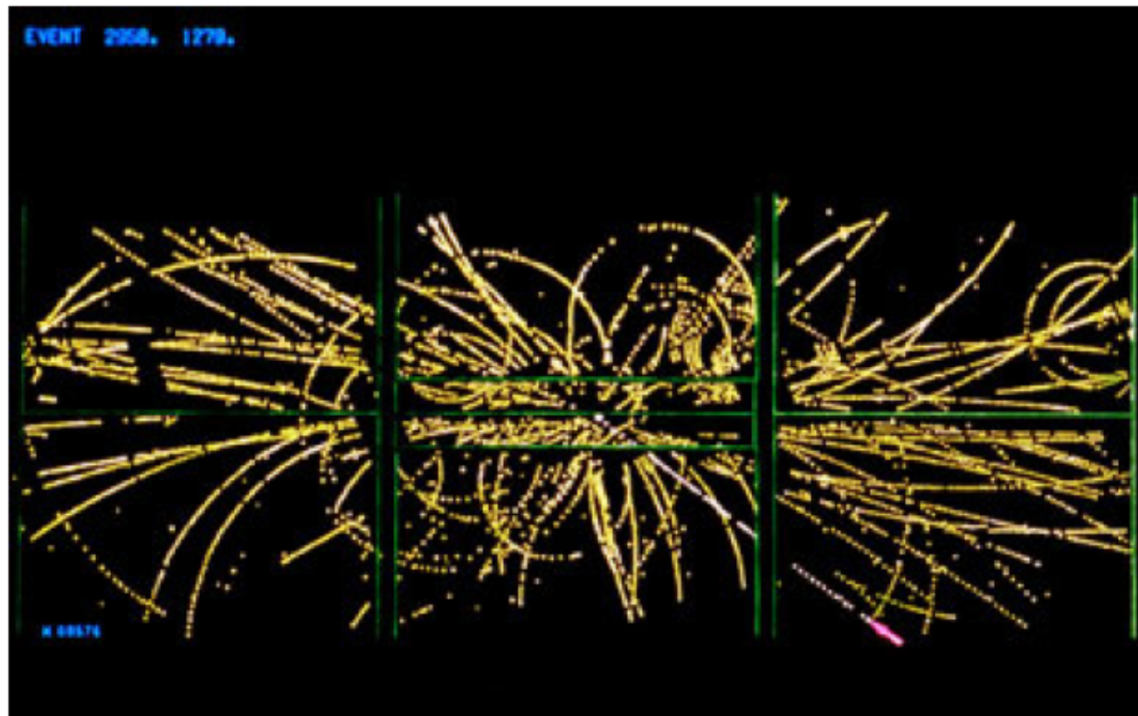
Several conditions to select events

- More than 10000 events/s, most of them selected by a trigger
- Trace in central detector must point into center of electromagnetic shower
- Transversal momentum in central detector  $> 7$  GeV
- Trace must be isolated (only other traces with transversal momentum  $< 2.5$  GeV allowed)
- Missing energy  $> 15$  GeV, has to point contrary to trace of electron

# W & Z Discovery - VII

159

UA1  $W \rightarrow e\nu$  candidate event



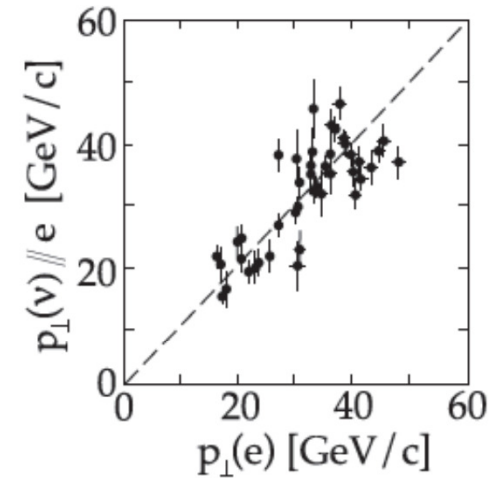
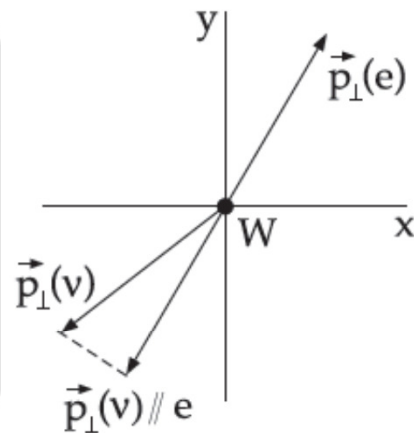
# W & Z Discovery - VIII

160

Hadrons  
Electron

$$\vec{p}_\perp(\nu) = -\vec{p}_\perp(H) - \vec{p}_\perp(e).$$

Neutrino (invisible)



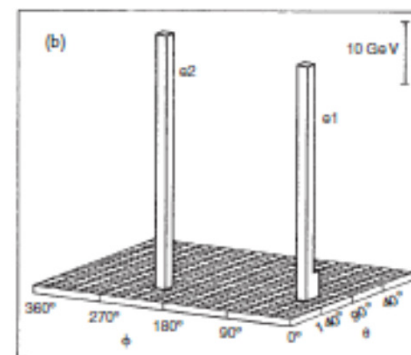
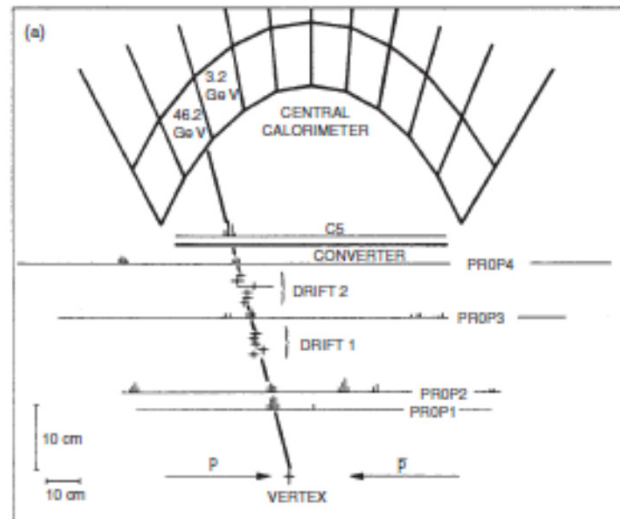
- Calculate sum of all hadron momenta in the **transverse plane** (to avoid leaks along the beam lines)
- $\mathbf{p}_T(\nu)$  not exactly antiparallel to  $\mathbf{p}_T(e)$ 
  - ◆ W boson not always produced at rest, finite detector resolution



# W & Z Discovery - IX

161

UA2 Candidate Z event



# W & Z Discovery - X

162

$$\frac{d\sigma}{d\cos\theta^*} = \text{const} \quad \text{Just an approximation}$$

$$\frac{d\sigma}{dp_T} = \frac{d\sigma}{d\cos\theta^*} \frac{d\cos\theta^*}{dp_T}$$

$$p_T = p^* \sin\theta^* = \frac{M_W}{2} \sin\theta^*$$

$$\rightarrow \sin\theta^* = \frac{2p_T}{M_W}$$

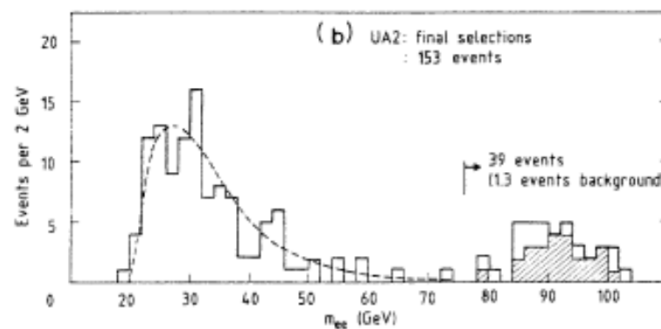
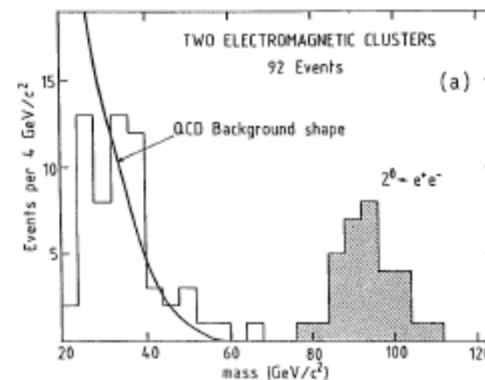
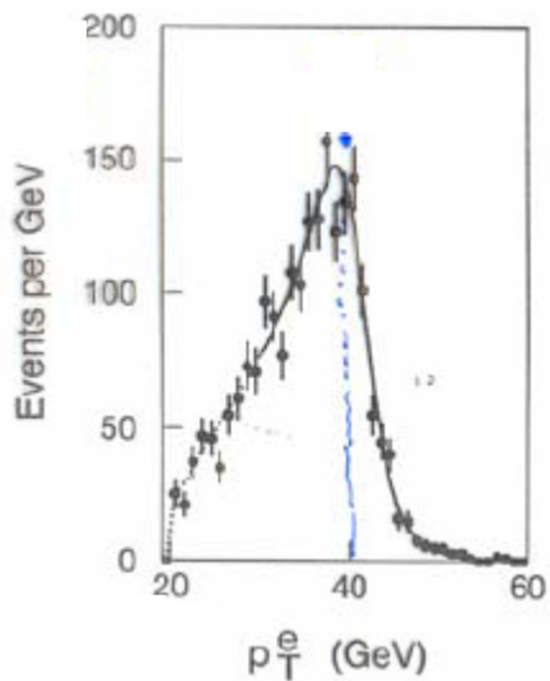
$$\rightarrow \cos\theta^* = \sqrt{1 - \sin^2\theta^*} = \sqrt{1 - \left(\frac{2p_T}{M_W}\right)^2}$$

$$\rightarrow \frac{d\cos\theta^*}{dp_T} = \frac{\frac{4p_T}{M_W}}{2\sqrt{1 - \left(\frac{2p_T}{M_W}\right)^2}} = \frac{2p_T}{M_W \sqrt{1 - \left(\frac{2p_T}{M_W}\right)^2}}$$

$$\rightarrow \frac{d\sigma}{dp_T} = A(\cos\theta^*) \frac{d\cos\theta^*}{dp_T} \approx K \frac{2p_T}{M_W \sqrt{1 - \left(\frac{2p_T}{M_W}\right)^2}} \quad \text{Jacobian peak at } \frac{M_W}{2}$$

# W & Z Discovery - XI

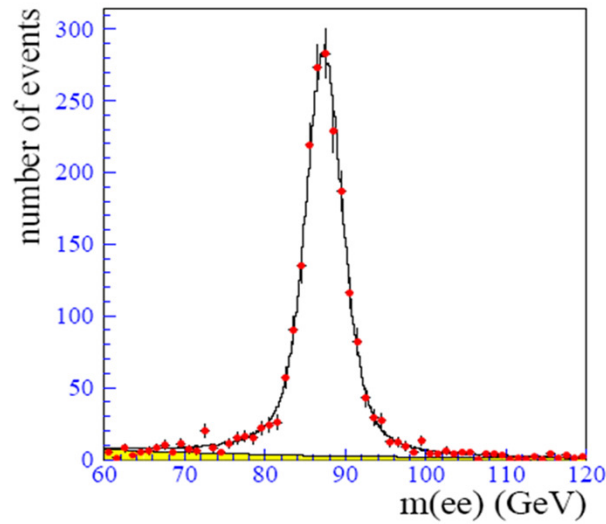
163



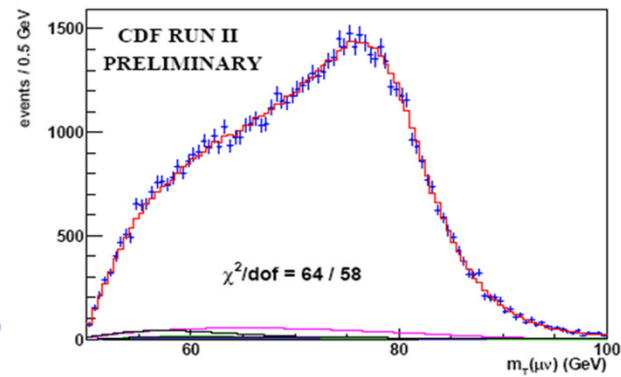
# W & Z Discovery - XII

164

D0  $Z \rightarrow e^+e^-$



CDF  $W \rightarrow \mu\nu$



$$m_{W^\pm} = 82.1 \pm 1.7 \text{ GeV}$$

$$m_{Z^0} = 93.0 \pm 1.7 \text{ GeV}$$

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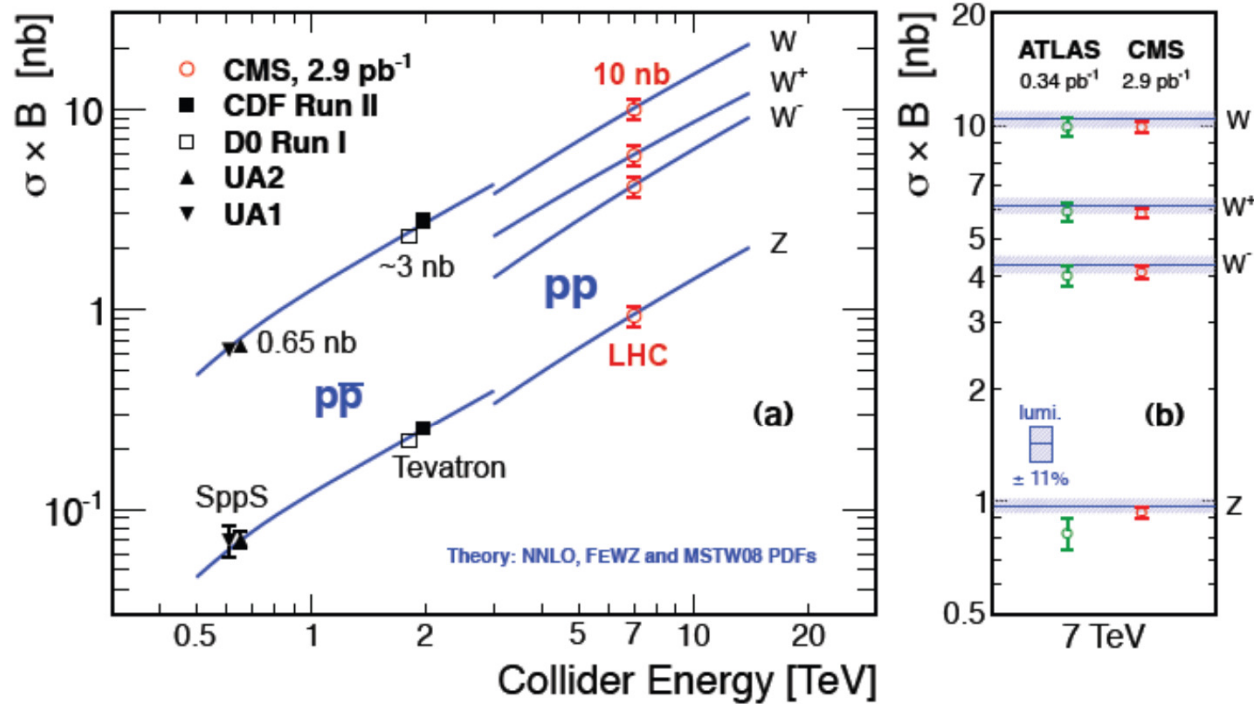
Current values (Particle Data Group 2006):

$$m_{W^\pm} = 80.403 \pm 0.029 \text{ GeV}$$

$$m_{Z^0} = 91.1876 \pm 0.0021 \text{ GeV}$$

# W & Z Discovery - XIII

165



Source: arXiv:1012.2466

- $Z^0$  cross section  $\sim 10$  times smaller than  $W^\pm$  boson production
- $W^+$  cross section  $\sim 43\%$  larger than  $W^-$  at LHC (pp collider!)

# Precision Tests - I

166

LEP – Precision tests of SM 1989-2000



26 km circumference

4 large experiments: ALEPH, DELPHI, L3, OPAL

1989 – 1995

$\sqrt{s} = 91.2 \text{ GeV}$

$17 \cdot 10^6 \text{ } Z^0$  detected

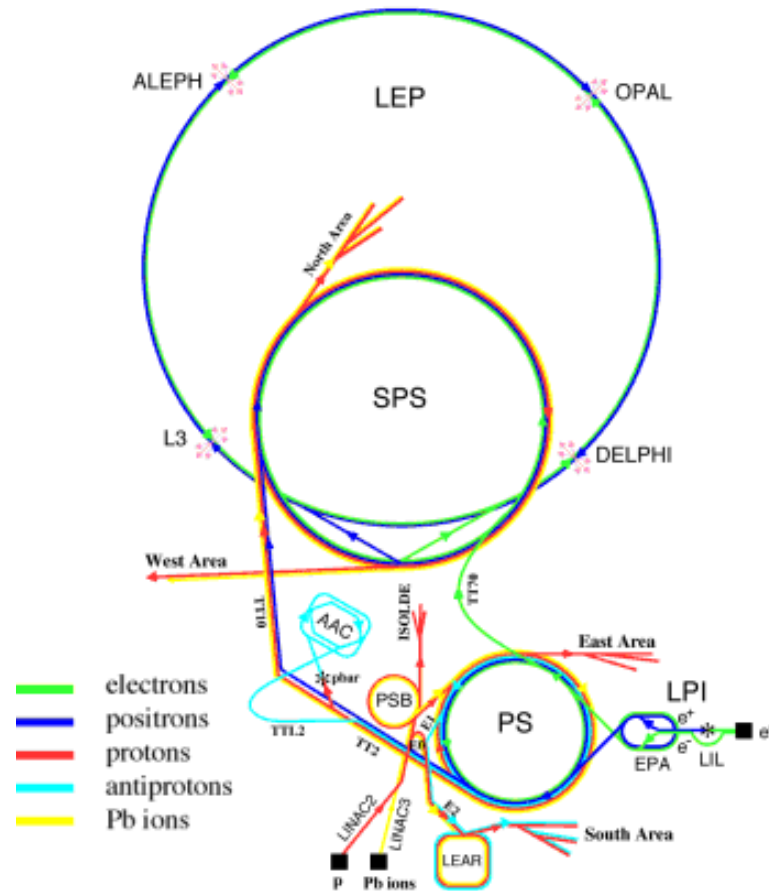
1996 – 2000

$\sqrt{s} = 161 - 208 \text{ GeV}$

$30 \cdot 10^3 \text{ } WW$  detected

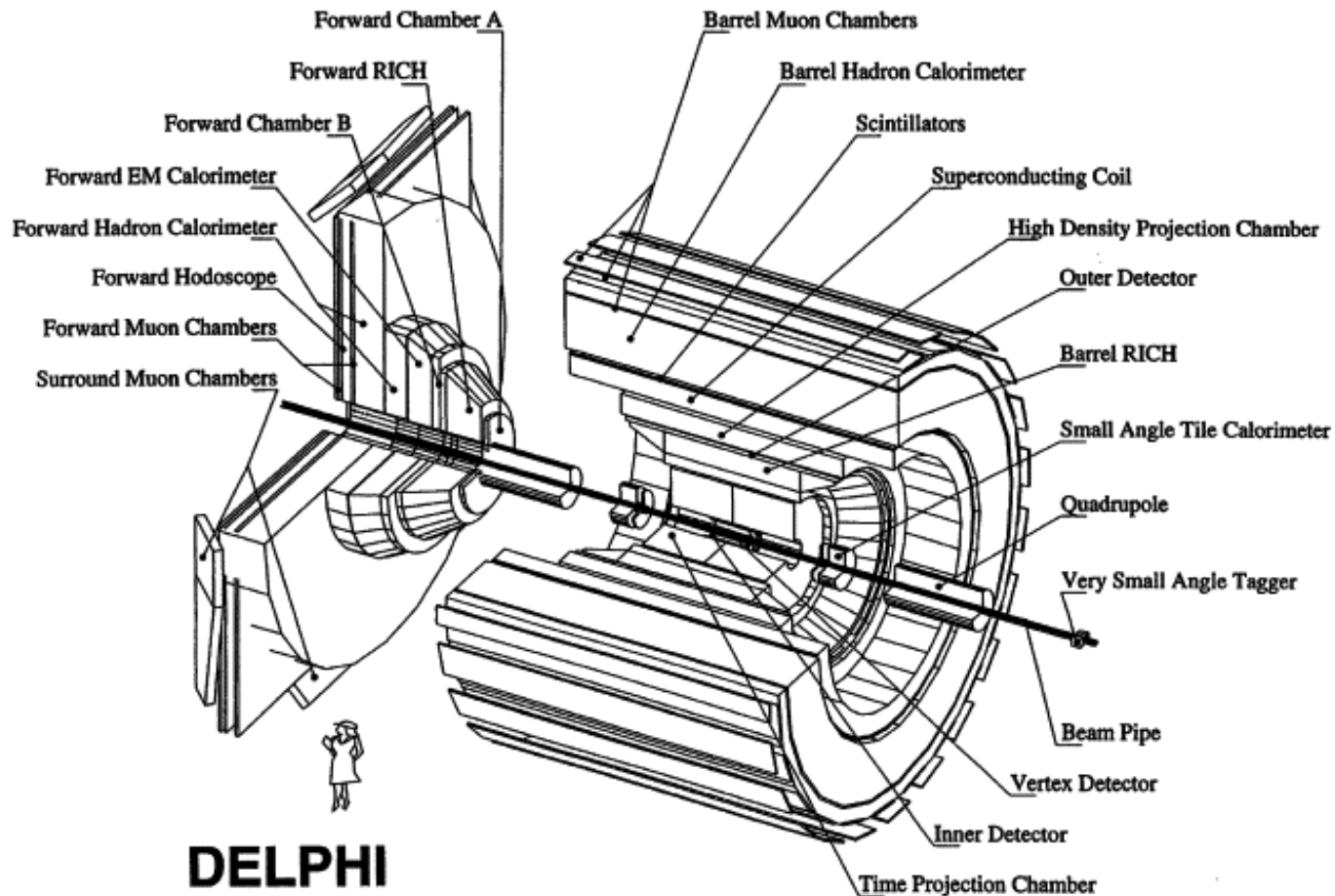
# Precision Tests - II

167



# Precision Tests - III

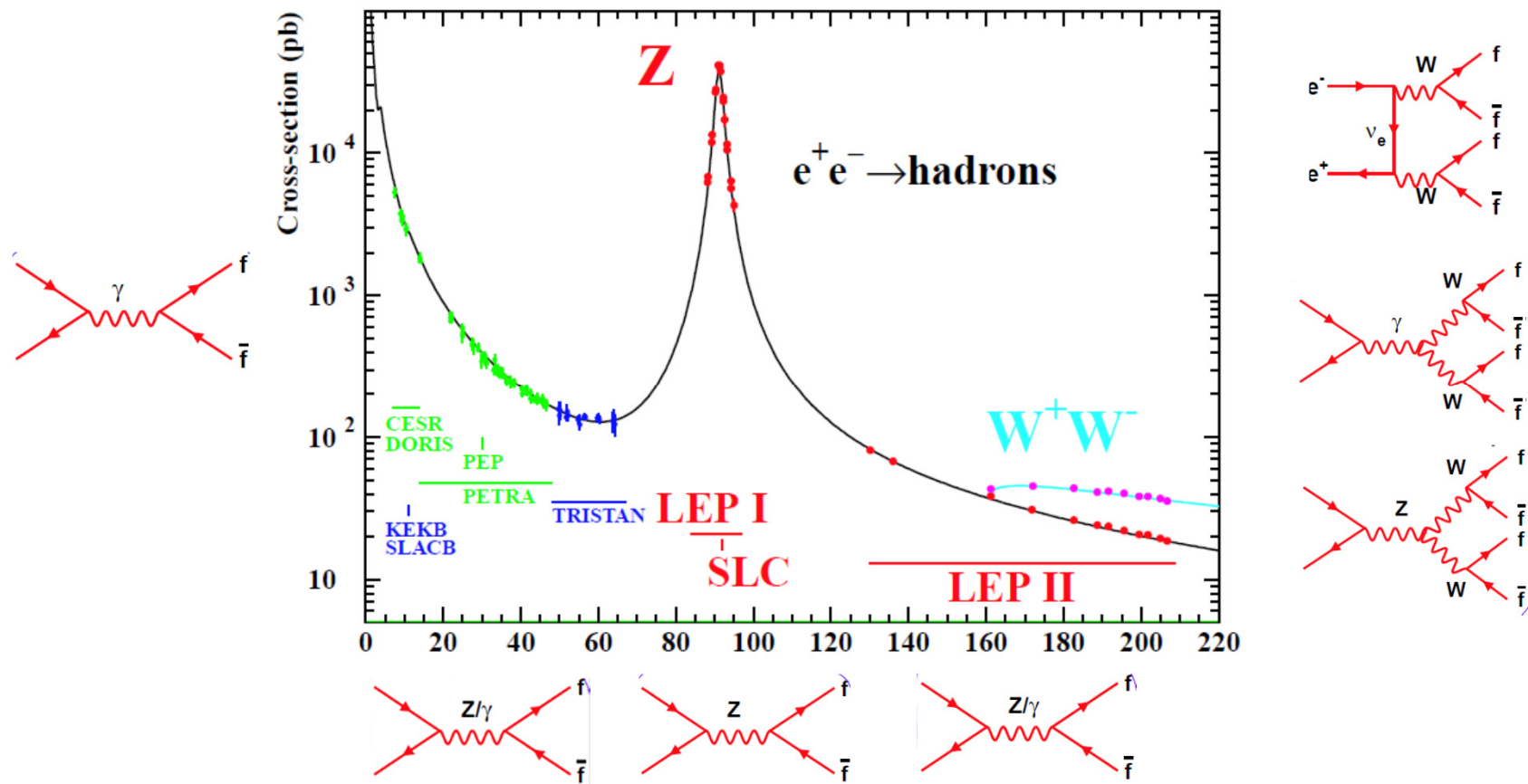
168





# Precision Tests - IV

169

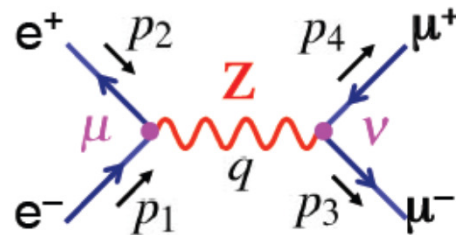


# Precision Tests - V

170

$e^+e^- \rightarrow \mu^+\mu^-$  at the Z peak

Only consider (dominant) Z diagram



Electron vertex:

$$\bar{v}(p_2)(-ig_Z\gamma^\mu)\frac{1}{2}(c_V - c_A\gamma^5)u(p_1) \quad \text{Electron } c_V, c_A$$

Z propagator:

$$-i\frac{g_{\mu\nu}}{q^2 - m_Z^2} \quad \text{Approximate, see later}$$

Muon vertex:

$$\bar{u}(p_3)(-ig_Z\gamma^\nu)\frac{1}{2}(c_V - c_A\gamma^5)v(p_4) \quad \text{Muon } c_V, c_A$$

# Precision Tests- VI

171

Ultrarelativistic limit  $\rightarrow$  Chirality  $\simeq$  Helicity

$\rightarrow$  Use helicity eigenstates for electron, muon vertexes

$$c_L = c_V + c_A, c_R = c_V - c_A$$

$$\rightarrow c_V = \frac{1}{2}(c_L + c_R), c_A = \frac{1}{2}(c_L - c_R)$$

$$\frac{1}{2}(c_V - c_A \gamma^5) \rightarrow \frac{1}{2}c_L(1 - \gamma^5) + \frac{1}{2}c_R(1 + \gamma^5)$$

$\rightarrow$  Matrix element:

$$\left[ c_L \bar{v}(p_2) \gamma^\mu \frac{1}{2}(1 - \gamma^5) u(p_1) + c_R \bar{v}(p_2) \gamma^\mu \frac{1}{2}(1 + \gamma^5) u(p_1) \right]$$
$$\times \left( -\frac{g_Z}{q^2 - m_Z^2} g_{\mu\nu} \right)$$
$$\times \left[ c_L \bar{u}(p_3) \gamma^\nu \frac{1}{2}(1 - \gamma^5) v(p_4) + c_R \bar{u}(p_3) \gamma^\nu \frac{1}{2}(1 + \gamma^5) v(p_4) \right]$$

Introduce chirality  $\simeq$  helicity projectors:

$$\frac{1}{2}(1 - \gamma^5) u \simeq u_\downarrow, \frac{1}{2}(1 + \gamma^5) u \simeq u_\uparrow, \frac{1}{2}(1 - \gamma^5) v \simeq v_\uparrow, \frac{1}{2}(1 + \gamma^5) v \simeq v_\downarrow$$

# Precision Tests- VII

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→ Matrix element:

$$\left[ c_L \bar{\nu}(p_2) \gamma^\mu u_\downarrow(p_1) + c_R \bar{\nu}(p_2) \gamma^\mu u_\uparrow(p_1) \right] \left( -\frac{g_Z}{q^2 - m_Z^2} g_{\mu\nu} \right) \left[ c_L \bar{u}(p_3) \gamma^\nu v_\uparrow(p_4) + c_R \bar{u}(p_3) \gamma^\nu v_\downarrow(p_4) \right]$$

$$\bar{\nu}(p_2) = \bar{\nu}_\uparrow(p_2) + \bar{\nu}_\downarrow(p_2), \bar{u}(p_3) = \bar{u}_\uparrow(p_3) + \bar{u}_\downarrow(p_3)$$

Surviving terms in both  $e, \mu$  currents:  $LR, RL$  only

$$\rightarrow \left[ c_L \bar{\nu}_\uparrow(p_2) \gamma^\mu u_\downarrow(p_1) + c_R \bar{\nu}_\downarrow(p_2) \gamma^\mu u_\uparrow(p_1) \right] \left( -\frac{g_Z}{q^2 - m_Z^2} g_{\mu\nu} \right) \left[ c_L \bar{u}_\downarrow(p_3) \gamma^\nu v_\uparrow(p_4) + c_R \bar{u}_\uparrow(p_3) \gamma^\nu v_\downarrow(p_4) \right]$$

$M_{RR} = -\frac{g_Z^2}{q^2 - m_Z^2} c_R^e c_R^\mu g_{\mu\nu} [\bar{\nu}_\downarrow(p_2) \gamma^\mu u_\uparrow(p_1)] [\bar{u}_\uparrow(p_3) \gamma^\nu v_\downarrow(p_4)]$	
$M_{RL} = -\frac{g_Z^2}{q^2 - m_Z^2} c_R^e c_L^\mu g_{\mu\nu} [\bar{\nu}_\downarrow(p_2) \gamma^\mu u_\uparrow(p_1)] [\bar{u}_\downarrow(p_3) \gamma^\nu v_\uparrow(p_4)]$	
$M_{LR} = -\frac{g_Z^2}{q^2 - m_Z^2} c_L^e c_R^\mu g_{\mu\nu} [\bar{\nu}_\uparrow(p_2) \gamma^\mu u_\downarrow(p_1)] [\bar{u}_\uparrow(p_3) \gamma^\nu v_\downarrow(p_4)]$	
$M_{LL} = -\frac{g_Z^2}{q^2 - m_Z^2} c_L^e c_L^\mu g_{\mu\nu} [\bar{\nu}_\uparrow(p_2) \gamma^\mu u_\downarrow(p_1)] [\bar{u}_\downarrow(p_3) \gamma^\nu v_\uparrow(p_4)]$	

# Precision Tests- VIII

173

Almost 'Cut & Paste' from QED case:

$$\begin{aligned} |M_{RR}|^2 &= s^2 \left| \frac{g_Z^2}{s - m_Z^2} \right|^2 (c_R^e)^2 (c_R^\mu)^2 (1 + \cos \theta)^2 \\ |M_{RL}|^2 &= s^2 \left| \frac{g_Z^2}{s - m_Z^2} \right|^2 (c_R^e)^2 (c_L^\mu)^2 (1 - \cos \theta)^2 \\ |M_{LR}|^2 &= s^2 \left| \frac{g_Z^2}{s - m_Z^2} \right|^2 (c_L^e)^2 (c_R^\mu)^2 (1 - \cos \theta)^2 \\ |M_{LR}|^2 &= s^2 \left| \frac{g_Z^2}{s - m_Z^2} \right|^2 (c_L^e)^2 (c_L^\mu)^2 (1 + \cos \theta)^2 \end{aligned}$$

# Precision Tests- IX

174

Now take correct  $Z$  propagator:  $Z$  unstable

$$-i \frac{g_{\mu\nu}}{q^2 - m_Z^2} = -i \frac{g_{\mu\nu}}{s - m_Z^2} \rightarrow -i \frac{g_{\mu\nu}}{s - m_Z^2 + im_Z \Gamma_Z}$$

$$\rightarrow \left| -i \frac{1}{s - m_Z^2 + im_Z \Gamma_Z} \right|^2 = \frac{1}{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2}$$

$$\frac{d\sigma}{d\Omega} = \frac{|M_{fi}|^2}{64\pi^2 s}$$

→ Differential cross-section for the 4 combinations:

$$\frac{d\sigma_{RR}}{d\Omega} = \frac{1}{64\pi^2} \frac{g_Z^4 s}{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2} (c_R^e)^2 (c_R^\mu)^2 (1 + \cos \theta)^2$$

$$\frac{d\sigma_{LL}}{d\Omega} = \frac{1}{64\pi^2} \frac{g_Z^4 s}{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2} (c_L^e)^2 (c_L^\mu)^2 (1 + \cos \theta)^2$$

$$\frac{d\sigma_{LR}}{d\Omega} = \frac{1}{64\pi^2} \frac{g_Z^4 s}{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2} (c_L^e)^2 (c_R^\mu)^2 (1 - \cos \theta)^2$$

$$\frac{d\sigma_{RL}}{d\Omega} = \frac{1}{64\pi^2} \frac{g_Z^4 s}{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2} (c_L^e)^2 (c_R^\mu)^2 (1 - \cos \theta)^2$$

# Precision Tests- X

175

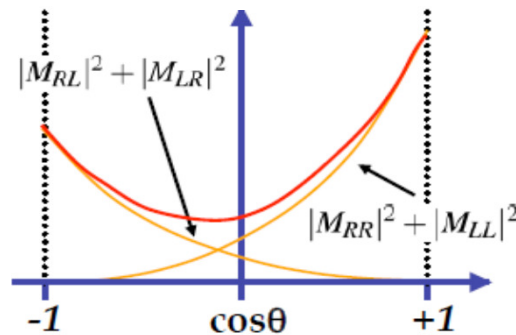
Most interesting difference wrt *QED* case:

$$|M_{LL}|^2 + |M_{RR}|^2 \neq |M_{LR}|^2 + |M_{RL}|^2$$

Unpolarized cross section: Average & Sum over spins

$$\rightarrow \frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2} \frac{1}{4} \frac{g_Z^4 s}{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2} \frac{1}{4} (c_V^2 + c_A^2)_e (c_V^2 + c_A^2)_\mu (1 + \cos^2 \theta) + 2(c_V c_A)_e (c_V c_A)_\mu \cos \theta$$

Sizeable forward-backward asymmetry!



# Precision Tests - XI

176

Integrate over solid angle, get total cross section:

$$\sigma_{e^+e^- \rightarrow Z \rightarrow \mu^+\mu^-} = \frac{1}{192\pi} \frac{g_Z^4 s}{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2} [(c_V^e)^2 + (c_A^e)^2][(c_V^\mu)^2 + (c_A^\mu)^2]$$

Recall partial Z widths:

$$\Gamma(Z \rightarrow e^+e^-) = \frac{g_Z^2 m_Z}{48\pi} [(c_V^e)^2 + (c_A^e)^2] \quad \Gamma(Z \rightarrow \mu^+\mu^-) = \frac{g_Z^2 m_Z}{48\pi} [(c_V^\mu)^2 + (c_A^\mu)^2]$$

$$\sigma = \frac{12\pi}{m_Z^2} \frac{s}{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2} \Gamma(Z \rightarrow e^+e^-) \Gamma(Z \rightarrow \mu^+\mu^-)$$

$$\rightarrow \sigma_{peak} \simeq \frac{12\pi (BR)^2}{m_Z^2} \simeq \frac{37.7 (3.510^{-2})^2}{(91.2)^2} \simeq 55 \cdot 10^{-7} GeV^{-2}$$

$$(\hbar c)^2 \simeq 0.389 GeV^2 mb$$

$$\rightarrow \sigma_{peak} \simeq 55 \cdot 10^{-7} GeV^{-2} \cdot 0.389 GeV^2 mb \simeq 2.14 \cdot 10^{-6} mb = 2.14 nb$$



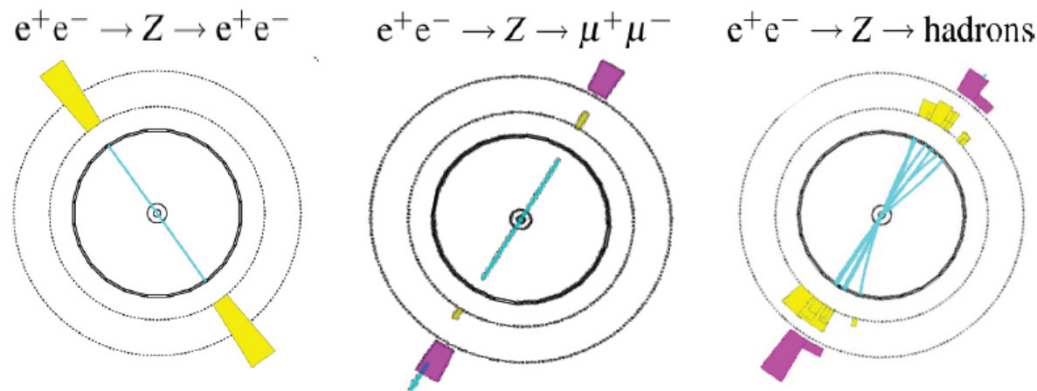
# Precision Tests - XII

177

Z peak: Essentially 4 types of events

$e^+e^- \rightarrow e^+e^-, \mu^+\mu^-, \tau^+\tau^-, q\bar{q} (\rightarrow \text{hadrons})$

Different topologies in the detectors:



Measuring cross sections:

Count events (!)

Subtract background

Correct for inefficiency

Get integrated luminosity (Most of the time from independent counting of Bhabha events)

$$\rightarrow \sigma = \frac{N - N_{bck}}{\varepsilon} \frac{1}{L_{int}}$$

# Precision Tests - XIII

178

Among other results at the peak:  $Z^0$  lineshape

Meaning in practice:

$m_Z$   $Z$  mass

$\Gamma_Z$   $Z$  total width

$\Gamma_f$   $Z$  partial width to fermion type  $f$

$N_\nu$  Number of (SM) neutrino species

Obtained by 'scanning' the  $Z^0$  peak:

Move  $E_{beam} = \frac{\sqrt{s}}{2}$  in steps through the peak

Measure relevant  $\sigma$  at each step

Fit profile:

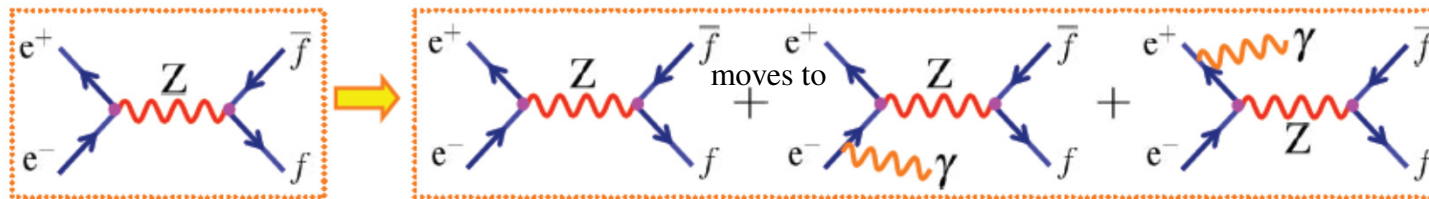
$$\sigma(e^+e^- \rightarrow Z \rightarrow f\bar{f}) = \frac{12\pi}{m_Z^2} \frac{s\Gamma_{ee}\Gamma_{ff}}{(s - m_Z^2)^2 + m_Z^2\Gamma_Z^2}$$

# Precision Tests - XIV

179

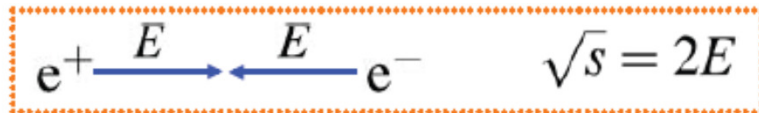
Lineshape quite distorted by several effects

Main effect: Initial State Radiation (ISR)

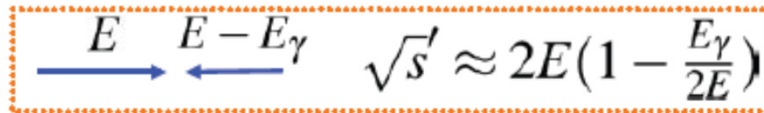


Result:

Collision  $CM$  energy  $\neq 2E_{beam}$

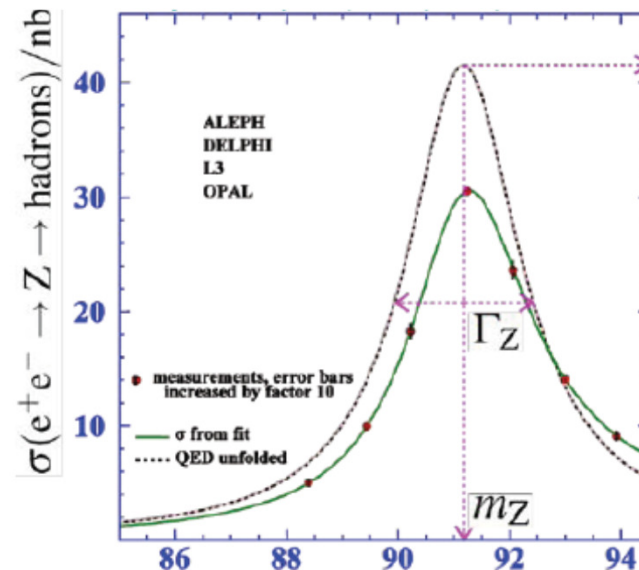


moves to



$$m_Z = 91.1875 \pm 0.0021 \text{ GeV}$$

$$\Gamma_Z = 2.4952 \pm 0.0023 \text{ GeV}$$



# Precision Tests - XV

180

Finding the number of Standard Model neutrinos  
(Meaning: With standard coupling to Z)

Total width:

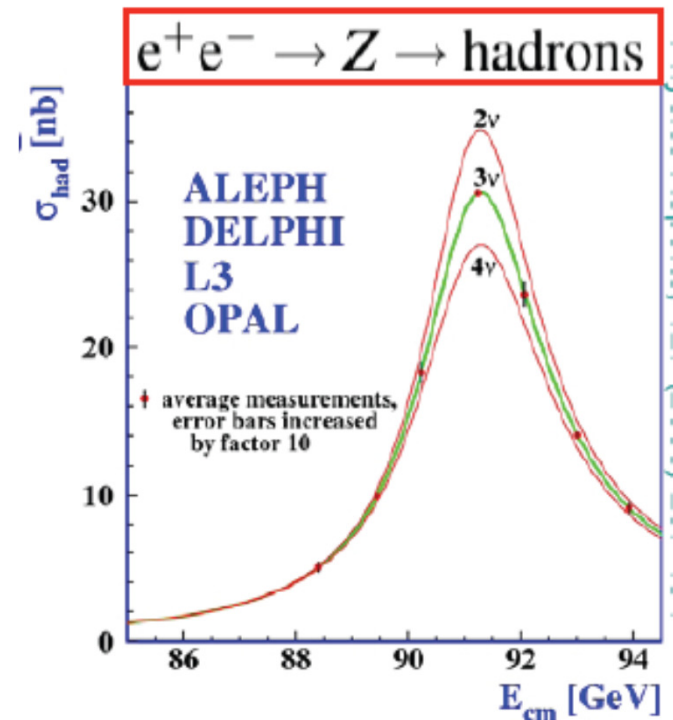
$$\Gamma = \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + \Gamma_{had} + \Gamma_{\nu_1\nu_1} + \Gamma_{\nu_2\nu_2} + \Gamma_{\nu_3\nu_3} + ?$$

$$\Gamma = 3\Gamma_{ll} + \Gamma_{had} + N_\nu \Gamma_{\nu\nu}$$

Measure partial widths from peak cross sections:

$$\sigma_0^{ff} = \frac{12\pi}{m_Z^2} \frac{\Gamma_{ee} \Gamma_{ff}}{\Gamma_Z^2}$$

$$N_\nu = 2.9840 \pm 0.0082$$



# Precision Tests - XVI

181

Write differential cross section ( e.g. for  $e^+e^- \rightarrow \mu^+\mu^-$  ) as:

$$\frac{d\sigma}{d\Omega} = k \left[ A(1 + \cos^2 \theta) + B \cos \theta \right]$$

$$A = \left[ (c_L^e)^2 + (c_R^e)^2 \right] \left[ (c_L^\mu)^2 + (c_R^\mu)^2 \right], \quad B = \left[ (c_L^e)^2 - (c_R^e)^2 \right] \left[ (c_L^\mu)^2 - (c_R^\mu)^2 \right]$$

Forward/Backward cross sections:

$$\sigma_F = \int_0^1 \frac{d\sigma}{d \cos \theta} d \cos \theta, \quad \sigma_B = \int_{-1}^0 \frac{d\sigma}{d \cos \theta} d \cos \theta$$

*FB* Asymmetry:

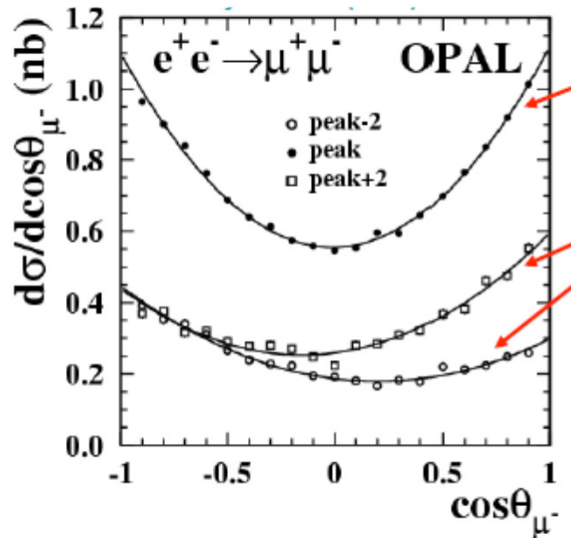
$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$

$$\sigma_F = k \left( \frac{4}{3} A + \frac{1}{2} B \right), \quad \sigma_B = k \left( \frac{4}{3} A - \frac{1}{2} B \right)$$

$$\rightarrow A_{FB} = \frac{3 B}{8 A} = \frac{3 \left[ (c_L^e)^2 - (c_R^e)^2 \right] \left[ (c_L^\mu)^2 - (c_R^\mu)^2 \right]}{4 \left[ (c_L^e)^2 + (c_R^e)^2 \right] \left[ (c_L^\mu)^2 + (c_R^\mu)^2 \right]} = \frac{3}{4} A_e A_\mu$$

# Precision Tests - XVII

182



$A_{FB}(peak) \sim 0$  for leptons ( $\sin^2\theta_W \approx 0.25$ )

$$A_{FB}(peak \pm 2 \text{ GeV}) \neq 0: = 2 \frac{c_V/c_A}{1 + (c_V/c_A)^2}$$

Interference with QED

$$A_{FB}^{0,e} = 0.0145 \pm 0.0025$$

$$A_e = 0.1514 \pm 0.0019$$

$$A_{FB}^{0,\mu} = 0.0169 \pm 0.0013$$

$$A_\mu = 0.1456 \pm 0.0091$$

$$A_{FB}^{0,\tau} = 0.0188 \pm 0.0017$$

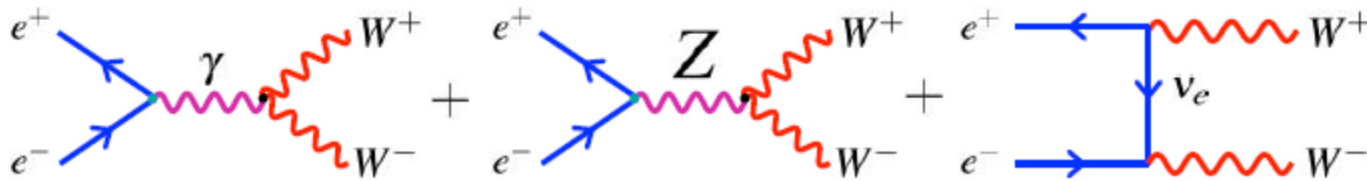
$$A_\tau = 0.1449 \pm 0.0040$$

$$\sin^2\theta_W = 0.23154 \pm 0.00016$$

# Precision Tests - XVIII

183

LEP2: Study of WW production

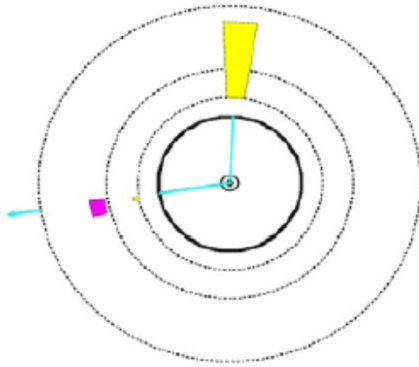


$$Br(W^- \rightarrow \text{hadrons}) \approx 0.67$$

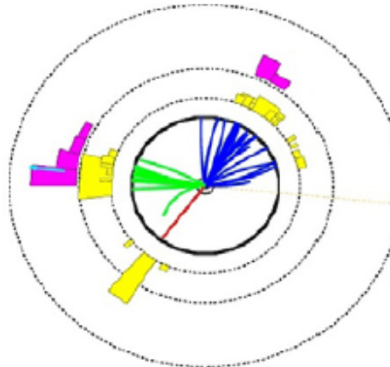
$$Br(W^- \rightarrow e^- \bar{\nu}_e) \approx 0.11$$

$$Br(W^- \rightarrow \mu^- \bar{\nu}_\mu) \approx 0.11$$

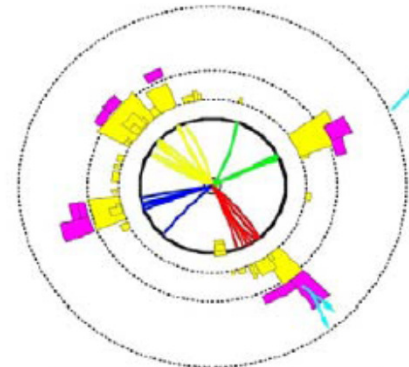
$$Br(W^- \rightarrow \tau^- \bar{\nu}_\tau) \approx 0.11$$



$$W^+ W^- \rightarrow l^+ \nu l^- \bar{\nu}$$



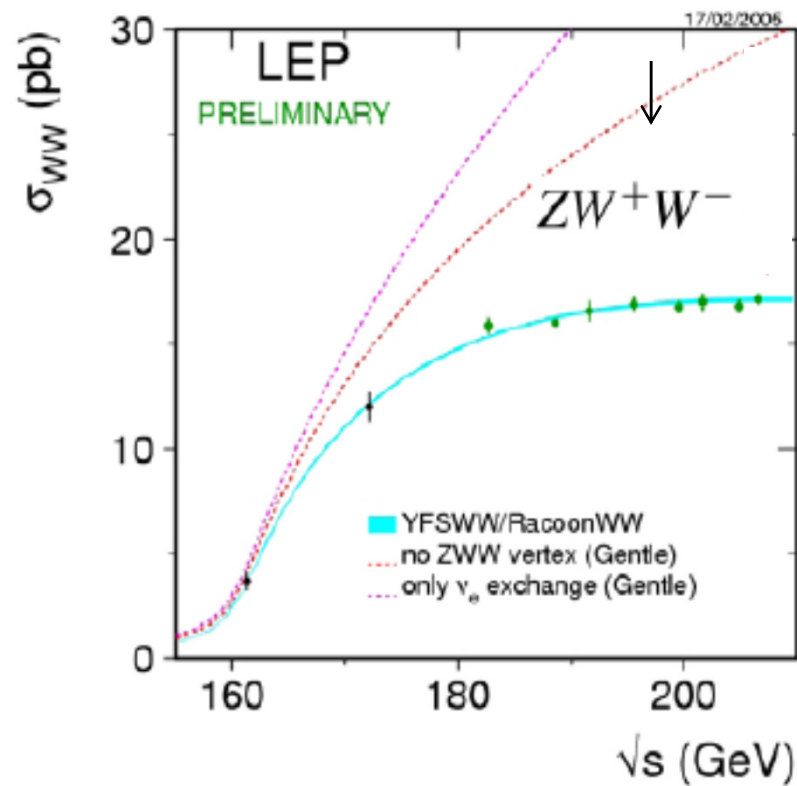
$$W^+ W^- \rightarrow q \bar{q} l \nu$$



$$W^+ W^- \rightarrow q \bar{q} q \bar{q}$$

# Precision Tests - XIX

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Maybe one of the best results of the whole LEP saga



# Precision Tests - XX

185

Measurement of  $m_W$  : Kinematical fit

Example:

$$W^+W^- \rightarrow q\bar{q}e^-\bar{\nu}$$

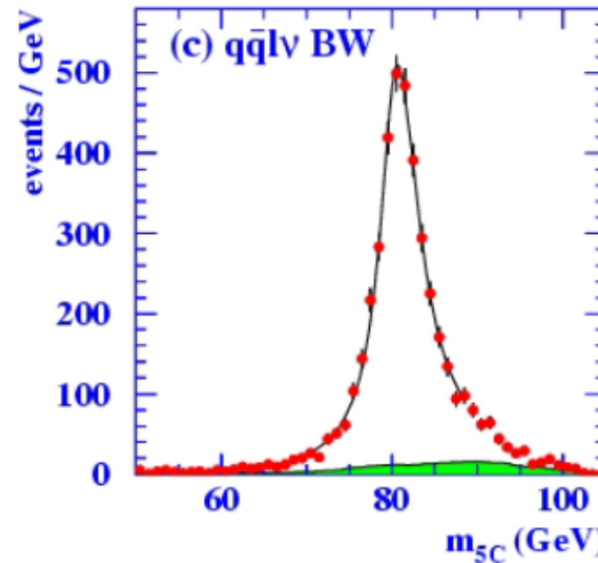
Get  $\bar{\nu}$  4-momentum from:

$$p_q + p_{\bar{q}} + p_{e^-} + p_{\bar{\nu}} = (\sqrt{s}, 0)$$

Make  $W$  bosons masses :

$$M_{W^+} = (p_q + p_{\bar{q}})^2$$

$$M_{W^-} = (p_{e^-} + p_{\bar{\nu}})^2$$



$$m_W = 80.376 \pm 0.033 \text{ GeV}$$

$$\Gamma_W = 2.196 \pm 0.083 \text{ GeV}$$

# Loopology - I

186

Standard Model :

$$M_W = M_Z \cos \theta_W$$

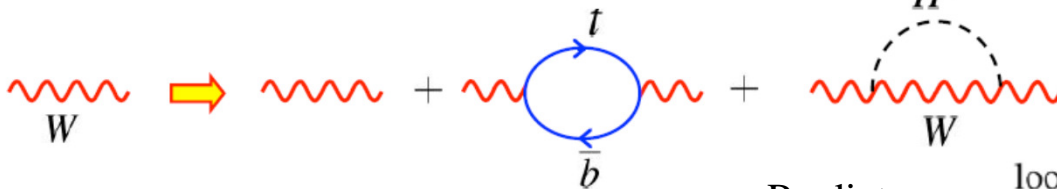
$$\text{Measure: } \begin{cases} M_Z = 91.1875 \pm 0.0021 \text{ GeV} \\ \sin^2 \theta_W = 0.23154 \pm 0.00016 \end{cases}$$

→ Predict  $M_W = 79.946 \pm 0.008 \text{ GeV}$

Measure

$$M_W = 80.376 \pm 0.033 \text{ GeV}$$

Discrepancy: Virtual loops (including Higgs..)



$$m_W = m_W^0 + a m_t^2 + b \ln \left( \frac{m_H}{m_W} \right)$$

$$\text{Predict: } m_t^{\text{loop}} = 173 \pm 11 \text{ GeV}$$

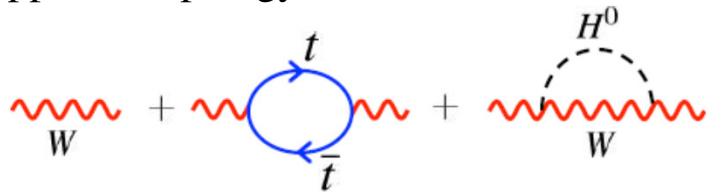
$$\text{Observe: } m_t^{\text{meas}} = 174.2 \pm 3.3 \text{ GeV}$$

!!!

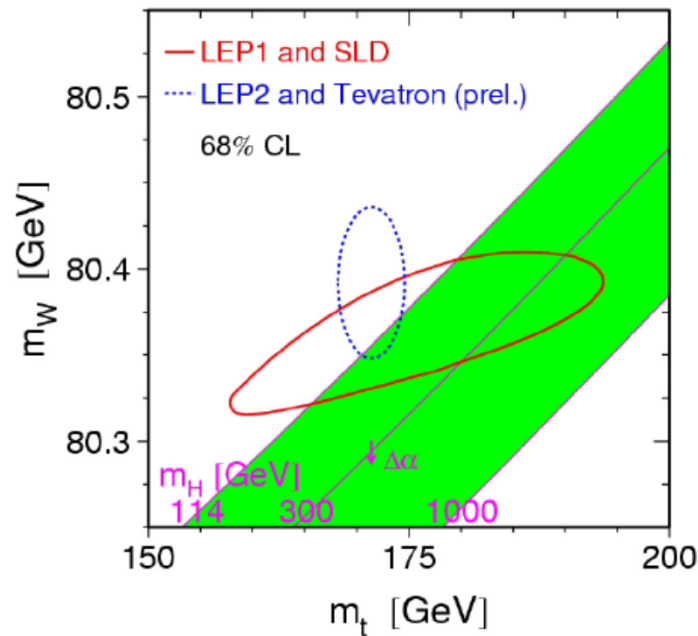
# Loopology - II

187

Applied loopology:



$$m_W = m_W^0 + am_t^2 + b \ln \left( \frac{m_H}{m_W} \right)$$

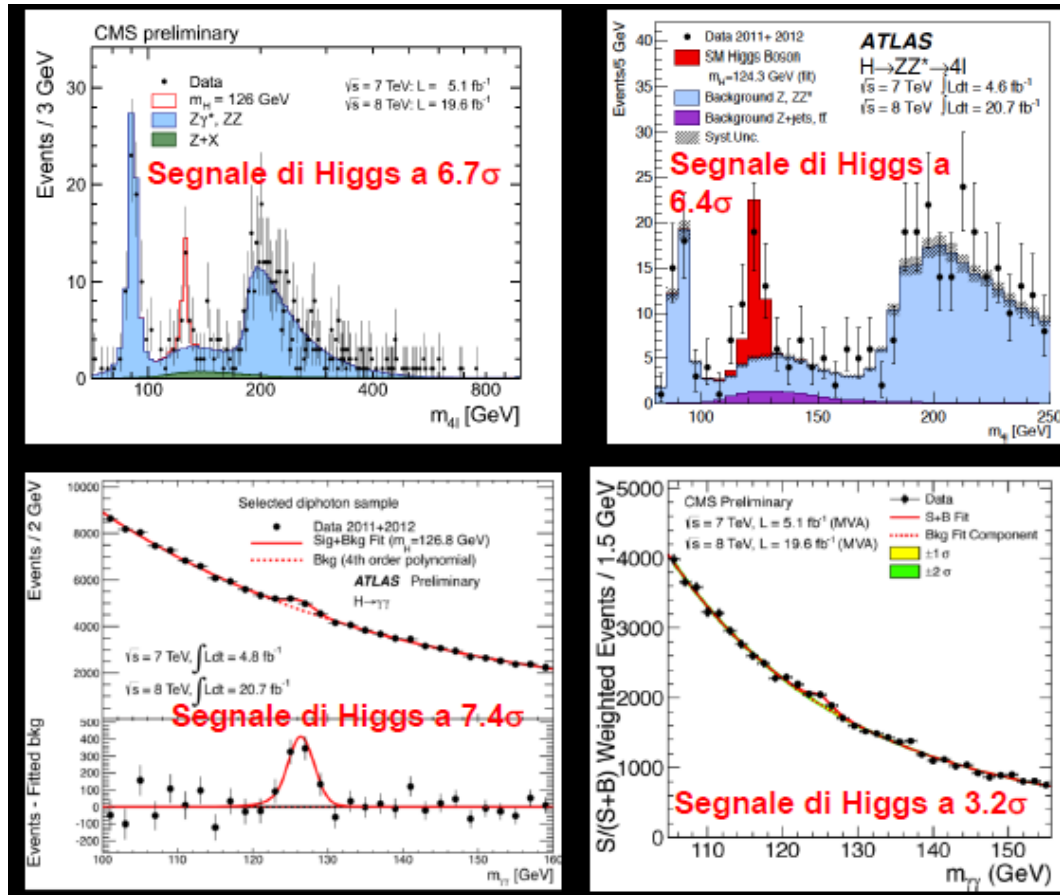


$$m_H < 200 \text{ GeV}$$

# The Happy End

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All is well that ends well:  
And finally...



... Mr. Higgs  
and Mr. Englert  
went to Stockholm