1

Elementary Particles II

2 – Electroweak Interaction

Universal Current-Current Interaction, Intermediate Vector Bosons, Gauge Symmetry, Spontaneous Symmetry Breaking, Electroweak Unification, Neutral Currents, Discovery of W & Z, Precision Measurements, Higgs

E.Menichetti - Universita' di Torino

Fall 2017

Helicity/Chirality - I

With reference to Dirac equation:

$$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \boldsymbol{\gamma} = \begin{pmatrix} 0 & \boldsymbol{\sigma} \\ -\boldsymbol{\sigma} & 0 \end{pmatrix}, \gamma^5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$ Dirac representation

$$\mathbf{S} = \frac{\boldsymbol{\Sigma}}{2}, \boldsymbol{\Sigma} = \begin{pmatrix} \boldsymbol{\sigma} & 0 \\ 0 & \boldsymbol{\sigma} \end{pmatrix} = \underbrace{\gamma^0 \boldsymbol{\gamma}}_{\boldsymbol{\alpha}} \gamma^5 = \boldsymbol{\alpha}\gamma^5$$ Spin operator

$$\Lambda = \frac{\boldsymbol{\Sigma} \cdot \mathbf{p}}{|\mathbf{p}|}$$ Helicity operator

$$\left.\begin{aligned} \Lambda u^{(+)} &= +u^{(+)} \\ \Lambda u^{(-)} &= -u^{(-)} \end{aligned}\right\}$$ Helicity eigenstates

$$P_{\pm} = \frac{1 \pm \Lambda}{2}$$ Projection operators onto helicity eigenstates

Helicity/Chirality - II

Projectors, indeed:

$$P_+P_+ = \left(\frac{1+\Lambda}{2}\right)\left(\frac{1+\Lambda}{2}\right) = \frac{1}{4}\left(1+\Lambda+\Lambda+\Lambda^2\right)$$

$$\Lambda^2 = \frac{\left(\mathbf{\Sigma}\cdot\mathbf{p}\right)^2}{|\mathbf{p}|^2} = 1 \rightarrow P_+P_+ = \frac{1}{4}(1+2\Lambda+1) = \left(\frac{1+\Lambda}{2}\right) = P_+, \quad P_-P_- = P_-$$

$$P_+P_- = \left(\frac{1+\Lambda}{2}\right)\left(\frac{1-\Lambda}{2}\right) = \frac{1}{4}\left(1+\Lambda-\Lambda-\Lambda^2\right) = 0 = P_-P_+$$

$$1 = \frac{1-\Lambda}{2} + \frac{1+\Lambda}{2} = P_- + P_+ \rightarrow 1u = \left(P_+ + P_-\right)u = u_+ + u_-$$

$$\Lambda = \frac{\mathbf{\Sigma}\cdot\mathbf{p}}{|\mathbf{p}|} = \underbrace{\gamma^0\boldsymbol{\gamma}}_{\boldsymbol{\alpha}}\gamma^5\cdot\frac{\mathbf{p}}{|\mathbf{p}|} = \frac{\boldsymbol{\alpha}\cdot\mathbf{p}}{|\mathbf{p}|}\gamma^5 \rightarrow P_\pm = \frac{1\pm\boldsymbol{\alpha}\cdot\hat{\mathbf{p}}\gamma^5}{2}$$

Helicity/Chirality - III

γ^5 Chirality operator

$$P_L = \frac{1-\gamma^5}{2}, P_R = \frac{1+\gamma^5}{2}$$ Projectors onto chirality eigenstates

$$\begin{cases} P_L u = u_L \\ P_R u = u_R \end{cases} \rightarrow 1u = (P_L + P_R)u = u_L + u_R$$

A very important limit:

$$\boldsymbol{\alpha} \cdot \mathbf{p} = E - \beta m$$

$$\Lambda = \frac{E \cdot 1 - m \cdot \beta}{p} \gamma^5 \underset{E \gg m}{\rightarrow} \gamma^5$$

$$P_{\pm} = \frac{1 \pm \Lambda}{2} \underset{E \gg m}{\rightarrow} \frac{1 \pm \gamma^5}{2} = P_{R,L}$$

For high energy, or massless, particles:
Helicity projectors → *Chirality* projectors

Helicity/Chirality - IV

5

$$Eu=(\boldsymbol{\alpha}\cdot\mathbf{p}+\beta m)u$$

$$u=\begin{pmatrix}\phi\\ \chi\end{pmatrix},\quad \phi,\chi \quad \text{2 components spinors}$$

$$\boldsymbol{\alpha}=\begin{pmatrix}\boldsymbol{\sigma} & 0\\ 0 & -\boldsymbol{\sigma}\end{pmatrix},\quad \beta=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} \quad \text{Dirac matrices in chiral representation}$$

$$\rightarrow\begin{cases}E\phi=(\boldsymbol{\sigma}\cdot\mathbf{p})\phi+m\chi\\ E\chi=-(\boldsymbol{\sigma}\cdot\mathbf{p})\chi+m\phi\end{cases}$$

$$\rightarrow\begin{cases}E\phi=(\boldsymbol{\sigma}\cdot\mathbf{p})\phi\\ E\chi=-(\boldsymbol{\sigma}\cdot\mathbf{p})\chi\end{cases}, m=0\rightarrow\begin{cases}\dfrac{(\boldsymbol{\sigma}\cdot\mathbf{p})}{E=|\mathbf{p}|}\phi=\phi\\ \dfrac{(\boldsymbol{\sigma}\cdot\mathbf{p})}{E=|\mathbf{p}|}\chi=-\chi\end{cases}\rightarrow\phi,\chi \text{ Helicity eigenstates}$$

Helicity/Chirality - V

States with definite value of chirality, massive or massless particles

Particle	*Antiparticle*
$u_L = \frac{1}{2}\left(1-\gamma^5\right)u$	$v_L = \frac{1}{2}\left(1+\gamma^5\right)v$
$u_R = \frac{1}{2}\left(1+\gamma^5\right)u$	$v_R = \frac{1}{2}\left(1-\gamma^5\right)v$
$\overline{u}_L = \overline{u}\,\frac{1}{2}\left(1+\gamma^5\right)$	$\overline{v}_L = \overline{v}\,\frac{1}{2}\left(1-\gamma^5\right)$
$\overline{u}_R = \overline{u}\,\frac{1}{2}\left(1-\gamma^5\right)$	$\overline{v}_R = \overline{v}\,\frac{1}{2}\left(1+\gamma^5\right)$

Is it true? Try one example:

$$\gamma^5 u_L = \gamma^5\,\frac{1}{2}\left(1-\gamma^5\right)u = \frac{1}{2}\left(\gamma^5 - 1\right)u = -\frac{1}{2}\left(1-\gamma^5\right)u = -u_L \quad \text{OK}$$

Helicity/Chirality - VI

Reminder: Solutions of Dirac equation

$$\begin{cases} u_1, u_2 & +\text{ve energy} \\ u_3, u_4 & -\text{ve energy} \end{cases} \quad \begin{pmatrix} 1 \\ 0 \\ \frac{p_z}{E+m} \\ \frac{p_x+ip_y}{E+m} \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ \frac{p_x-ip_y}{E+m} \\ \frac{p_z}{E+m} \end{pmatrix}, \begin{pmatrix} \frac{p_z}{E-m} \\ \frac{p_x+ip_y}{E-m} \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} \frac{p_x-ip_y}{E-m} \\ \frac{-p_z}{E-m} \\ 0 \\ 1 \end{pmatrix} \cdot e^{i(\mathbf{p}\cdot\mathbf{r}-Et)}$$

Change to all $+$ve energy solutions by introducing antiparticle spinors v_1, v_2:

$$\begin{cases} u_1, u_2 & +\text{ve energy} \\ v_1, v_2 & +\text{ve energy} \end{cases} \begin{pmatrix} 1 \\ 0 \\ \frac{p_z}{E+m} \\ \frac{p_x+ip_y}{E+m} \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ \frac{p_x-ip_y}{E+m} \\ \frac{p_z}{E+m} \end{pmatrix} \cdot e^{i(\mathbf{p}\cdot\mathbf{r}-Et)}; \begin{pmatrix} \frac{p_x-ip_y}{E+m} \\ \frac{-p_z}{E+m} \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} \frac{p_z}{E+m} \\ \frac{p_x+ip_y}{E+m} \\ 1 \\ 0 \end{pmatrix} \cdot e^{-i(\mathbf{p}\cdot\mathbf{r}-Et)}$$

$\rightarrow$ Antiparticle spinors have *reversed momentum*

$$\rightarrow \begin{cases} v_R = P_L v = \dfrac{1-\gamma_5}{2} v \\[2ex] v_L = P_R v = \dfrac{1+\gamma_5}{2} v \end{cases}$$

Helicity/Chirality - VII

Helicity of chiral states:

Massless particle: $\begin{cases} u_L & \langle H \rangle = -1 \\ u_R & \langle H \rangle = +1 \end{cases}$

$\rightarrow$ Helicity defined $\equiv$ Full longitudinal polarization

Massive particle: $\begin{cases} u_L & \langle H \rangle = -\beta \\ u_R & \langle H \rangle = +\beta \end{cases}$

$\rightarrow$ Helicity undefined, superposition of ± 1 eigenstates

Massless particles: *Helicity is Lorentz invariant*

Massive particles: *Helicity is frame dependent*

Electroweak Interaction

Standard Model:
Electromagnetic and Weak Interaction unified into Electroweak

Energy scale where unification is evident:

$E \sim M_W, M_Z \sim 100$ GeV

At lower energies:

Can still find tiny traces of the unification (Electroweak interference, Parity violation in atomic processes,...)

Electroweak interaction split into two, almost not interfering, *effective* interactions:

↑ Non fundamental, useful low energy approximations

Electromagnetic

Weak

Weak Interaction: $V - A$ - I

After a long history of beta decay experiments: *Current-Current (Fermi) Interaction* including *Vector* & *Axial Vector* terms in order to account for P & C violation

$$H_{\text{int}} = \frac{G_F}{\sqrt{2}} \sum_{i=V,A} C_i \left(\bar{\psi}_p \Gamma_i \psi_n\right) \left(\bar{\psi}_e \Gamma^i \left(1 + \frac{C_i'}{C_i}\gamma^5\right)\psi_\nu\right)$$

Neutrino helicity = -1 yields lepton current = $V - A$

$$C_i' = -C_i \rightarrow \left(1 + \frac{C_i'}{C_i}\gamma^5\right)\psi_\nu = \left(1 - \gamma^5\right)\psi_\nu$$

$$= -\gamma^\mu\left(1-\gamma^5\right)$$

$$\rightarrow H_{\text{int}} = \frac{G_F}{\sqrt{2}}\left[C_V\left(\bar{\psi}_p\gamma_\mu\psi_n\right)\left(\bar{\psi}_e\gamma^\mu\left(1-\gamma^5\right)\psi_\nu\right) + C_A\left(\bar{\psi}_p\gamma_\mu\gamma_5\psi_n\right)\left(\bar{\psi}_e\gamma^\mu\gamma^5\left(1-\gamma^5\right)\psi_\nu\right)\right]$$

$$= \frac{G_F}{\sqrt{2}}\left[C_V\left(\bar{\psi}_p\gamma_\mu\psi_n\right)\left(\bar{\psi}_e\gamma^\mu\left(1-\gamma^5\right)\psi_\nu\right) - C_A\left(\bar{\psi}_p\gamma_\mu\gamma_5\psi_n\right)\left(\bar{\psi}_e\gamma^\mu\left(1-\gamma^5\right)\psi_\nu\right)\right]$$

$$= \frac{G_F}{\sqrt{2}}\left[C_V\left(\bar{\psi}_p\gamma_\mu\psi_n\right) - C_A\left(\bar{\psi}_p\gamma_\mu\gamma_5\psi_n\right)\right]\left(\bar{\psi}_e\gamma^\mu\left(1-\gamma^5\right)\psi_\nu\right)$$

Weak Interaction: $V-A$ - II

Therefore:

$$H_{\text{int}} = \frac{G_F}{\sqrt{2}} C_V \left(\bar{\psi}_p \gamma_\mu \left(1 - \frac{C_A}{C_V} \gamma_5 \right) \psi_n \right) \left(\bar{\psi}_e \gamma^\mu \left(1 - \gamma^5 \right) \psi_\nu \right)$$

Current-Current interaction:

Lepton current: $V-A$

Nucleon current: $V-\alpha A$ (Strong interaction corrections)

Many violations in weak processes:

Space Parity (large)

Charge Parity (large)

CP (very small)

T (very small)

Flavor conservation (Isospin, S, C, B, T) (larger + smaller)

Lepton numbers (Neutrino oscillations)

Weak Interaction: $V - A$ - III

Observe:

$$H_{\text{int}} = \frac{2G_F}{\sqrt{2}}\left(\bar{\psi}_p\gamma_\mu\left(1-\frac{C_A}{C_V}\gamma_5\right)\psi_n\right)\left(\bar{\psi}_e\gamma^\mu\frac{\left(1-\gamma^5\right)}{2}\psi_\nu\right)$$

$$\frac{1-\gamma^5}{2}\ \text{Projection operator} \rightarrow \left[\frac{\left(1-\gamma^5\right)}{2}\right]^2 = \frac{1-\gamma^5}{2}$$

$$\rightarrow H_{\text{int}} = \frac{2G_F}{\sqrt{2}}\left(\bar{\psi}_p\gamma_\mu\left(1-\frac{C_A}{C_V}\gamma_5\right)\psi_n\right)\left(\bar{\psi}_e\gamma^\mu\left[\frac{\left(1-\gamma^5\right)}{2}\right]^2\psi_\nu\right)$$

$$\rightarrow H_{\text{int}} = \sqrt{2}G_F\left(\bar{\psi}_p\gamma_\mu\left(1-\frac{C_A}{C_V}\gamma_5\right)\psi_n\right)\left(\bar{\psi}_e\left(\frac{1+\gamma^5}{2}\right)\gamma^\mu\left(\frac{1-\gamma^5}{2}\right)\psi_\nu\right)$$

Lepton current written as *pure vector* between *chiral parts* of ν,*e* states

→ The weak charged current is just the same as the e.m. current, except it operates between chiral projections with different charge $\Delta Q = \pm 1$

Weak Interaction: Universality - I

Extend *V-A* to muon weak interactions:

$\mu^+ \to e^+ + \overline{\nu}_\mu + \nu_e, \;\; \mu^- \to e^- + \nu_\mu + \overline{\nu}_e \quad \mu$ decay

$\mu^- + p \to n + \nu_\mu \quad \mu$ capture, involves nucleon current

μ decay purely leptonic:

Guess: *Current-Current*, *V-A* for both electron and muon charged currents

Lagrangian density:

$$L_{V-A} = -\frac{G_F}{\sqrt{2}}\left[\overline{\nu}_\mu \gamma_\mu \left(1-\gamma_5\right)\mu\right]\left[\overline{e}\gamma^\mu \left(1-\gamma_5\right)\nu\right] + H.C.$$

Compute:

μ *Lifetime*
Electron energy spectrum
Electron longitudinal polarization

Weak Interaction: Universality - II

Relativistic Golden Rule for 3-body μ decay:

$$d\Gamma = |M|^2 \frac{1}{2m_\mu} \frac{d^3\mathbf{p}_{\nu_\mu}}{(2\pi)^3 2E_{\nu_\mu}} \frac{d^3\mathbf{p}_{\bar{\nu}_e}}{(2\pi)^3 2E_{\bar{\nu}_e}} \frac{d^3\mathbf{p}_e}{(2\pi)^3 2E_e} (2\pi)^4 \delta^{(4)}\left(p_\mu - p_{\nu_\mu} - p_{\bar{\nu}_e} - p_e\right)$$

$|M|^2$ squared matrix element

$\frac{1}{2m_\mu}$ 'flux' factor

$\frac{d^3\mathbf{p}_{\nu_\mu}}{(2\pi)^3 2E_{\nu_\mu}} \frac{d^3\mathbf{p}_{\bar{\nu}_e}}{(2\pi)^3 2E_{\bar{\nu}_e}} \frac{d^3\mathbf{p}_e}{(2\pi)^3 2E_e}$ phase space factor

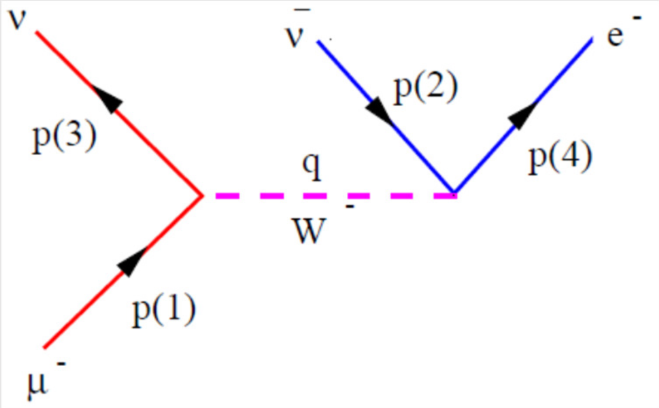
$\delta^{(4)}\left(p_\mu - p_{\nu_\mu} - p_{\bar{\nu}_e} - p_e\right)$ 4-momentum conservation

And:

Must average over initial / sum over final spin projections in $|M|^2$

Weak Interaction: Universality - III

Feynman diagram (tree level):



Amplitude:

$$M = i\int \underbrace{\bar{u}\left(\nu_{\mu}\right)\frac{ig_W}{2\sqrt{2}}\gamma^{\mu}\left(1-\gamma^5\right)u\left(\mu\right)}_{\text{Charged current}} \underbrace{\frac{ig_{\mu\nu}}{m_W^2}}_{W\text{ propagator}} \underbrace{\bar{u}\left(e\right)\frac{ig_W}{2\sqrt{2}}\gamma_{\nu}\left(1-\gamma^5\right)v\left(\bar{\nu}_e\right)}_{\text{Charged current}}\cdot$$

$$\cdot\underbrace{\left(2\pi\right)^4\delta^{(4)}\left(p_{\mu}-p_{\nu}-q\right)}_{\substack{\text{4-mom conservation}\\ \text{1st vertex}}}\underbrace{\left(2\pi\right)^4\delta^{(4)}\left(q-p_e-p_{\bar{\nu}}\right)}_{\substack{\text{4-mom conservation}\\ \text{2nd vertex}}}$$

Weak Interaction: Universality - IV

Group constants together

Integrate over internal W momentum, q

$\left(\leftarrow\right.$ get rid of 2 δ-functions$\left.\right)$

$\rightarrow$ Amplitude:

$$M=\frac{g_W^2}{8m_W^2}\overline{u}\left(\nu_\mu\right)\gamma^\mu\left(1-\gamma^5\right)u\left(\mu\right)\overline{u}\left(e\right)\gamma^\nu\left(1-\gamma^5\right)v\left(\overline{\nu}_e\right)$$

$\rightarrow$ Squared amplitude:

$$\left|M\right|^2=\left(\frac{g_W^2}{8m_W^2}\right)^2\left[\overline{u}\left(\nu_\mu\right)\gamma^\mu\left(1-\gamma^5\right)u\left(\mu\right)\right]\left[\overline{u}\left(\nu_\mu\right)\gamma^\mu\left(1-\gamma^5\right)u\left(\mu\right)\right]^*\cdot$$

$$\cdot\left[\overline{u}\left(e\right)\gamma^\nu\left(1-\gamma^5\right)v\left(\overline{\nu}_e\right)\right]\left[\overline{u}\left(e\right)\gamma^\nu\left(1-\gamma^5\right)v\left(\overline{\nu}_e\right)\right]^*$$

Identity:

$$\left[\overline{u}\left(a\right)\Gamma_1\left(1-\gamma^5\right)u\left(b\right)\right]\left[\overline{u}\left(a\right)\Gamma_2\left(1-\gamma^5\right)u\left(b\right)\right]^*=Tr\left[\Gamma_1\left(\not{p}_b+m_b\right)\right]Tr\left[\Gamma_2\left(\not{p}_a+m_a\right)\right]$$

Weak Interaction: Universality - V

Obtain:

$$\left\langle |M|^2 \right\rangle = 2\left(\frac{g_W}{m_W}\right)^4 \left(p_\mu p_{\bar{\nu}_e}\right)\left(p_{\nu_\mu} p_e\right)$$

Muon rest frame:

$$p_\mu = \left(m_\mu, 0\right)$$

$$\rightarrow p_\mu p_{\bar{\nu}_e} = m_\mu E_{\bar{\nu}_e}$$

$$p_\mu = p_{\nu_\mu} + p_{\bar{\nu}_e} + p_e$$

$$\rightarrow \left(p_\mu - p_{\bar{\nu}_e}\right)^2 = \left(p_{\nu_\mu} + p_e\right)^2$$

$$\rightarrow m_\mu^2 - 2m_\mu E_{\bar{\nu}_e} = m_e^2 + 2p_{\nu_\mu} p_e$$

$$\rightarrow p_{\nu_\mu} p_e = \frac{1}{2}\left(m_\mu^2 - m_e^2\right) - m_\mu E_{\bar{\nu}_e}$$

$$\rightarrow \left\langle |M|^2 \right\rangle \approx 2\left(\frac{g_W}{m_W}\right)^4 m_\mu^2 E_{\bar{\nu}_e}\left(\frac{1}{2}m_\mu - E_{\bar{\nu}_e}\right)$$

Weak Interaction: Universality - VI

$$d\Gamma = |M|^2 \frac{1}{2m_\mu} \frac{d^3\mathbf{p}_{\nu_\mu}}{(2\pi)^3 2E_{\nu_\mu}} \frac{d^3\mathbf{p}_{\bar{\nu}_e}}{(2\pi)^3 2E_{\bar{\nu}_e}} \frac{d^3\mathbf{p}_e}{(2\pi)^3 2E_e} (2\pi)^4 \delta^{(4)}\left(p_\mu - p_{\nu_\mu} - p_{\bar{\nu}_e} - p_e\right)$$

Choose μ rest frame as reference

Split 4-dim δ into Energy*Momentum

$$\delta^{(4)}\left(p_\mu - p_{\nu_\mu} - p_{\bar{\nu}_e} - p_e\right) = \delta\left(m_\mu - E_{\nu_\mu} - E_{\bar{\nu}_e} - E_e\right)\delta^{(3)}\left(\mathbf{p}_{\nu_\mu} + \mathbf{p}_{\bar{\nu}_e} + \mathbf{p}_e\right)$$

$$\rightarrow d\Gamma = \frac{\left\langle |M|^2 \right\rangle}{16(2\pi)^5 m_\mu} \int \frac{d^3\mathbf{p}_{\nu_\mu} d^3\mathbf{p}_{\bar{\nu}_e} d^3\mathbf{p}_e}{E_{\nu_\mu} E_{\bar{\nu}_e} E_e} \delta\left(m_\mu - E_{\nu_\mu} - E_{\bar{\nu}_e} - E_e\right)\delta^{(3)}\left(\mathbf{p}_{\nu_\mu} + \mathbf{p}_{\bar{\nu}_e} + \mathbf{p}_e\right)$$

Integrate over $\mathbf{p}_{\nu_\mu}, \mathbf{p}_{\bar{\nu}_e}, E_{\bar{\nu}_e}, E_{\nu_\mu}$:

$$\rightarrow d\Gamma = \left(\frac{g_W}{m_W}\right)^4 \frac{m_\mu}{16(2\pi)^4} \left(\frac{m_\mu}{2} - \frac{2E_e}{3}\right) d^3\mathbf{p}_e$$

$$d^3\mathbf{p}_e = |\mathbf{p}_e|^2 d|\mathbf{p}_e| d\Omega_e \approx E_e^2 dE_e d\Omega_e \rightarrow d\Gamma = \left(\frac{g_W}{m_W}\right)^4 \frac{m_\mu}{(4\pi)^3} \left(\frac{m_\mu}{2} - \frac{2E_e}{3}\right) E_e^2 dE_e$$

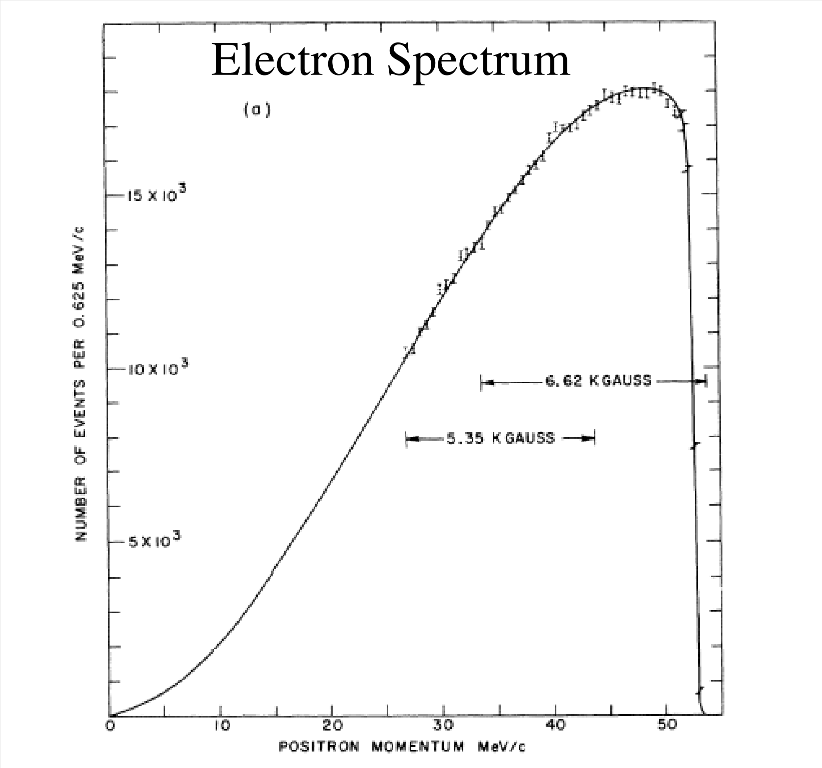
$$\rightarrow \Gamma = \int d\Gamma = \left(\frac{g_W}{m_W}\right)^4 \frac{m_\mu^5}{12(8\pi)^3} = \frac{1}{\tau_\mu}$$

$$\Gamma = \underbrace{\left(\frac{g_W}{m_W}\right)^4 \frac{1}{32}}_{G_F^2} \frac{32 m_\mu^5}{12(8\pi)^3} = G_F^2 \frac{8 m_\mu^5}{3(8\pi)^3} = \frac{G_F^2 m_\mu^5}{192\pi^3}$$

Fall 2017

E.Menichetti - Universita' di Torino

Weak Interaction: Universality - VII



Measure μ lifetime, get Fermi constant: $G_F^{(\mu)} = \sqrt{\dfrac{192\pi^3}{\tau_\mu m_\mu^5}} = 1.1638\ 10^{-5}\ \ GeV^{-2}$

After radiative corrections: $G_F^{(\mu)} = 1.16637\ 10^{-5}\ \ GeV^{-2}$

Measure τ lifetime and *BR* to electron, get Fermi constant:

$$G_F^{(\tau)} = \sqrt{\frac{\Gamma(\tau \to e\bar{\nu}_e \nu_\tau)}{\Gamma_{tot}} \frac{192\pi^3}{\tau_\tau m_\tau^5}} = 1.1642\ 10^{-5}\ \ GeV^{-2}$$

Compare to Fermi constant from β decay: $G_F^{(\beta)} = 1.1361\ 10^{-5}\ \ GeV^{-2}$

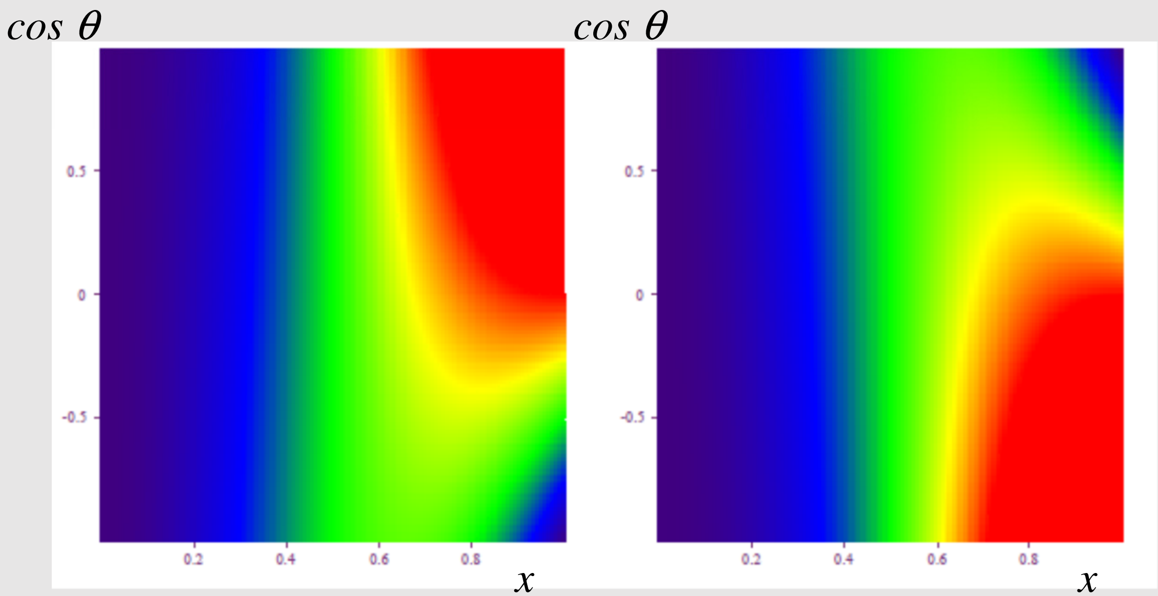
Weak Interaction: Universality - VIII

μ Decay : C & P Violations

Polarized $\mu^{\pm}$ decay: μ rest frame

$$\frac{dN^{\pm}}{dx\,d\cos\theta} = x^2(3-2x)\left[1 \pm \cos\theta \frac{2x-1}{3-2x}\right]$$

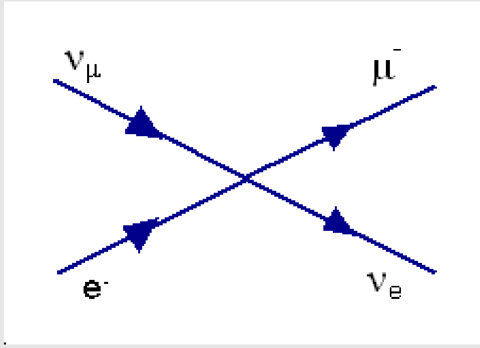
$$x = \frac{p_e}{p_e^{\max}},\ \theta \sphericalangle(\mathbf{s}_\mu, \mathbf{p}_e)$$



Weak Interaction: Universality - IX

Charged current:

$\nu_\mu + e^- \rightarrow \mu^- + \nu_e$, inverse μ decay



$$\sum_{spin} M_{fi} M_{fi}^* = \sum_{spin} \frac{G_F}{\sqrt{2}} \left[\bar{u}(3)\gamma^\mu\left(1-\gamma^5\right)u(1)\right]\left[\bar{u}(3)\gamma_\nu\left(1-\gamma_5\right)u(1)\right]^* \cdot$$

$$\left[\bar{u}(4)\gamma_\mu\left(1-\gamma_5\right)u(2)\right]\left[\bar{u}(4)\gamma^\nu\left(1-\gamma^5\right)u(2)\right]^*$$

$$\sum_{spin}\left[\bar{u}(a)\Gamma_1 u(b)\right]\left[\bar{u}(a)\Gamma_2 u(b)\right]^* = Tr\left[\Gamma_1\left(\not{p}_b + m_b\right)\Gamma_2\left(\not{p}_a + m_a\right)\right]$$

$$\sum_{spin}\left|M_{fi}\right|^2 = 64 G_F^2 \left(p_1 \cdot p_2\right)\left(p_3 \cdot p_4\right)$$

$$\sum_{spin}\left|M_{fi}\right|^2 = 256 G_F^2 E^4 \left[1 - \left(\frac{m_\mu}{2E}\right)^2\right]$$

Weak Interaction: Universality - X

$$\frac{d\sigma}{d\Omega} = \frac{G_F^2}{\pi^2} E^{*2} \left[1 - \left(\frac{m_\mu}{2E^*} \right)^2 \right]^2$$

$$\rightarrow \sigma = \frac{4G_F^2}{\pi} E^{*2} \left[1 - \left(\frac{m_\mu}{2E^*} \right)^2 \right]^2, \quad E^* \text{ CM energy of } e, v$$

$$E^* \simeq \sqrt{2mE_\nu}$$

$$\rightarrow \sigma = \frac{8G_F^2 m_\mu}{\pi} E_\nu \propto E_\nu \quad \text{at high energy}$$

σ badly divergent

$\rightarrow$ Unphysical

Weak Interaction: Universality - XI

Charged current $\nu_e/\overline{\nu}_e - e$ scattering:

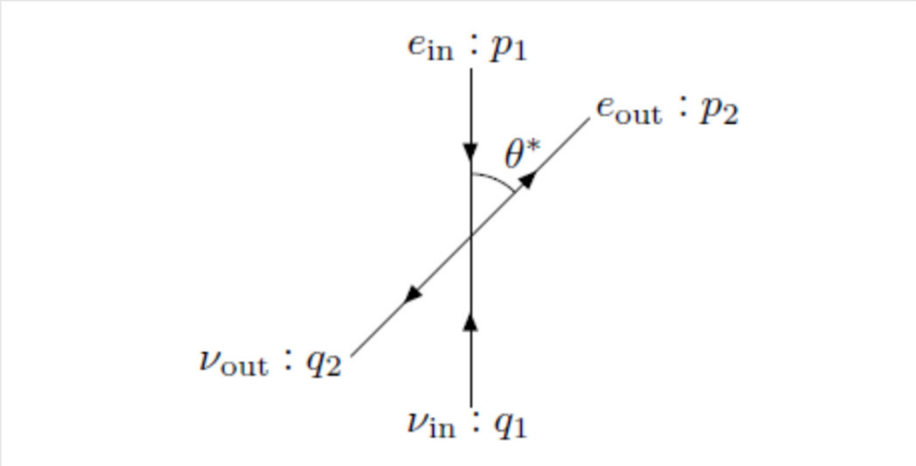
$$\nu_e + e^- \to e^- + \nu_e$$
$$\overline{\nu}_e + e^- \to \overline{\nu}_e + e^-$$

NB Actually incomplete:

Missing neutral current amplitude leading to the same final states

Cross sections must be avaluated by adding *all* the relevant amplitudes

Weak Interaction: Universality - XII



$$M_{fi} = -i\frac{G_F}{\sqrt{2}}\left[\bar{v}(\nu,q_1)\gamma_\mu(1-\gamma_5)u(e,p_1)\right]\cdot\left[\bar{u}(e,p_2)\gamma^\mu(1-\gamma_5)v(\nu,q_2)\right]$$

Weak Interaction: Universality -XIII

$\bar{\nu} e \to \bar{\nu} e$

$$\frac{d\sigma_{\bar{\nu}e}}{d\Omega^*} = \frac{\left\langle \left| M_{fi} \right|^2 \right\rangle}{64\pi^2 s} = \frac{G_F^2 2mE_\nu \left(1 - \cos\theta^*\right)^2}{16\pi^2}$$

Total cross section:

$$\sigma_{\bar{\nu}e} = \frac{G_F^2 2mE_\nu}{3\pi} \approx 0.574\ 10^{-41} E_\nu \left(GeV\right)\ cm^2$$

$\nu e \to \nu e$

$$\frac{d\sigma_{\nu e}}{d\Omega^*} = \frac{\left\langle \left| M_{fi} \right|^2 \right\rangle}{64\pi^2 s} = \frac{G_F^2 2mE_\nu}{4\pi^2}$$

Total cross section:

$$\sigma_{\nu e} = \frac{G_F^2 2mE_\nu}{\pi} \approx 1.72\ 10^{-41} E_\nu \left(GeV\right)\ cm^2$$

Weak Interaction: Universality - XIV

Total cross sections:

$$\sigma_{\overline{\nu}e} = \frac{G_F^2\, 2mE_\nu}{3\pi} \approx 0.574\ 10^{-41} E_\nu \left(GeV\right)\ cm^2$$

$$\sigma_{\nu e} = \frac{G_F^2\, 2mE_\nu}{\pi} \approx 1.72\ 10^{-41} E_\nu \left(GeV\right)\ cm^2$$

as divergent at high energy as the inverse muon decay

NB Cross sections only crude approximations:

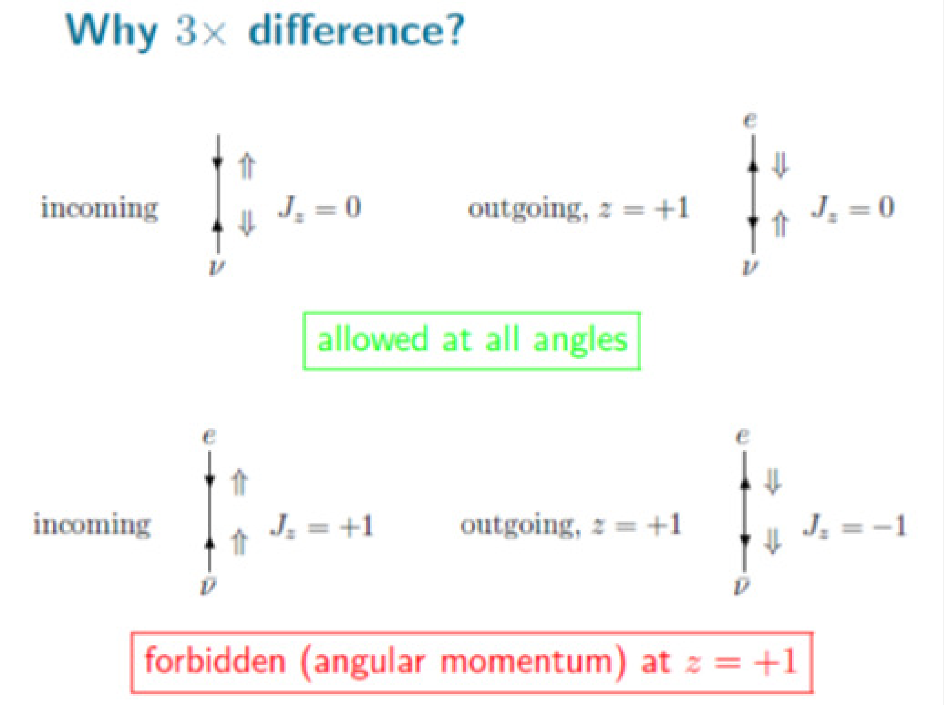
Neutral current contribute not included

Interesting factor $\times 3$ between νe and $\overline{\nu}e$

Weak Interaction: Universality - XV

Origin of factor $\times 3$:

$z = \cos\theta^*$

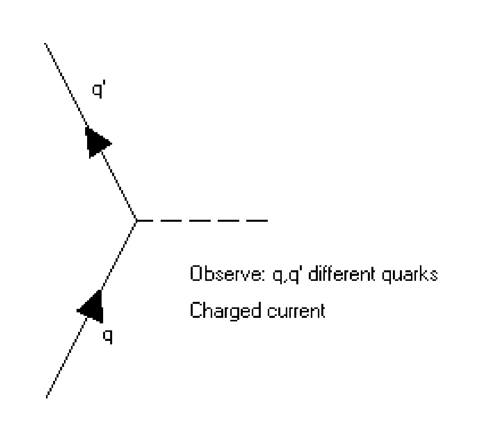


$\rightarrow$ Amplitude & Cross section suppressed at small angle

Weak Interaction: Universality - XVI

Semileptonic and non leptonic processes understood in terms of quarks

Coupling basically similar to leptonic charged currents:



Picture is slightly more complicated, however
Fundamental question:

Is the quark coupling identical to the lepton one?

Weak Interaction: Universality-XVII

Consider charged current of leptons:
Very natural to group charged and neutral leptons into *doublets*, or *families*

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix} \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix} \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}$$

Within each doublet, charged current transitions can be thought as due to emission/absorption of $W^\pm$ bosons , similar to (neutral) e.m. current transitions

$$\begin{matrix} W^- \rightarrow & & \nu_e & & \rightarrow W^+ \\ & \uparrow & & \downarrow & \\ W^- \leftarrow & & e^- & & \leftarrow W^+ \end{matrix}$$

Similar for 2nd, 3rd family

Weak Interaction: Universality-XVIII

Natural to extend this scheme to quarks

$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ s \end{pmatrix} \quad \begin{pmatrix} t \\ b \end{pmatrix}$$

$$\begin{matrix} W^- \rightarrow & & u & & \rightarrow W^+ \\ & \uparrow & & \downarrow & \\ W^- \leftarrow & & d & & \leftarrow W^+ \end{matrix}$$

Similar for 2nd, 3rd family

Almost correct, but incomplete:
Does not account for strangeness (more generally, → flavour) violating processes

Cabibbo's very ingenious idea:

Quark flavor eigenstates (i.e., quark model eigenstates) are not to be identified with quark weak currents
→Weak currents are mixtures of different flavors

By universal convention, mixing is assumed between *d* , *s, b* quarks

Weak Interaction: Universality-XIX

In terms of mixed "*d*-like" quarks, with just 2 families:

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix} \quad \theta_c \text{ Cabibbo's angle}$$

This explain *many* things….

How to extract θ_c?

Just one example: Get the angle from β decay

$$G_F^{(\beta)} = 0.975 G_F^{(\mu)} \text{(Remember that 2\% difference ?)}$$

$$\rightarrow G_F^{(\beta)} = \cos\theta_C G_F^{(\mu)}$$

$$\rightarrow \theta_c \simeq 13^0$$

Weak Interaction: Universality-XX

Extend the idea to 3 families:
From Cabibbo's angle to *Cabibbo-Kobayashi-Maskawa* matrix

$$\begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} |d\rangle \\ |s\rangle \\ |b\rangle \end{bmatrix} = \begin{bmatrix} |d'\rangle \\ |s'\rangle \\ |b'\rangle \end{bmatrix}$$

From unitarity:
3 mixing angles
1 complex phase This can account for CP violation

Experimental values:

$$\begin{bmatrix} 0.9753 & 0.221 & 0.003 \\ 0.221 & 0.9747 & 0.040 \\ 0.009 & 0.039 & 0.9991 \end{bmatrix}$$

Almost diagonal
Heavy quarks even more diagonal

Weak Interaction: Universality-XXI

Extend V-A to neutrino-nucleon scattering

$$\nu_\mu + N \rightarrow \mu^- + X$$

$$\bar{\nu}_\mu + N \rightarrow \mu^+ + X$$

Somewhat similar to *e-N*, μ-*N* deep inelastic scattering
Modeling similar to DIS: Parton elastic scattering

Deep ineastic neutrino scattering reveals the same structure as charged lepton DIS

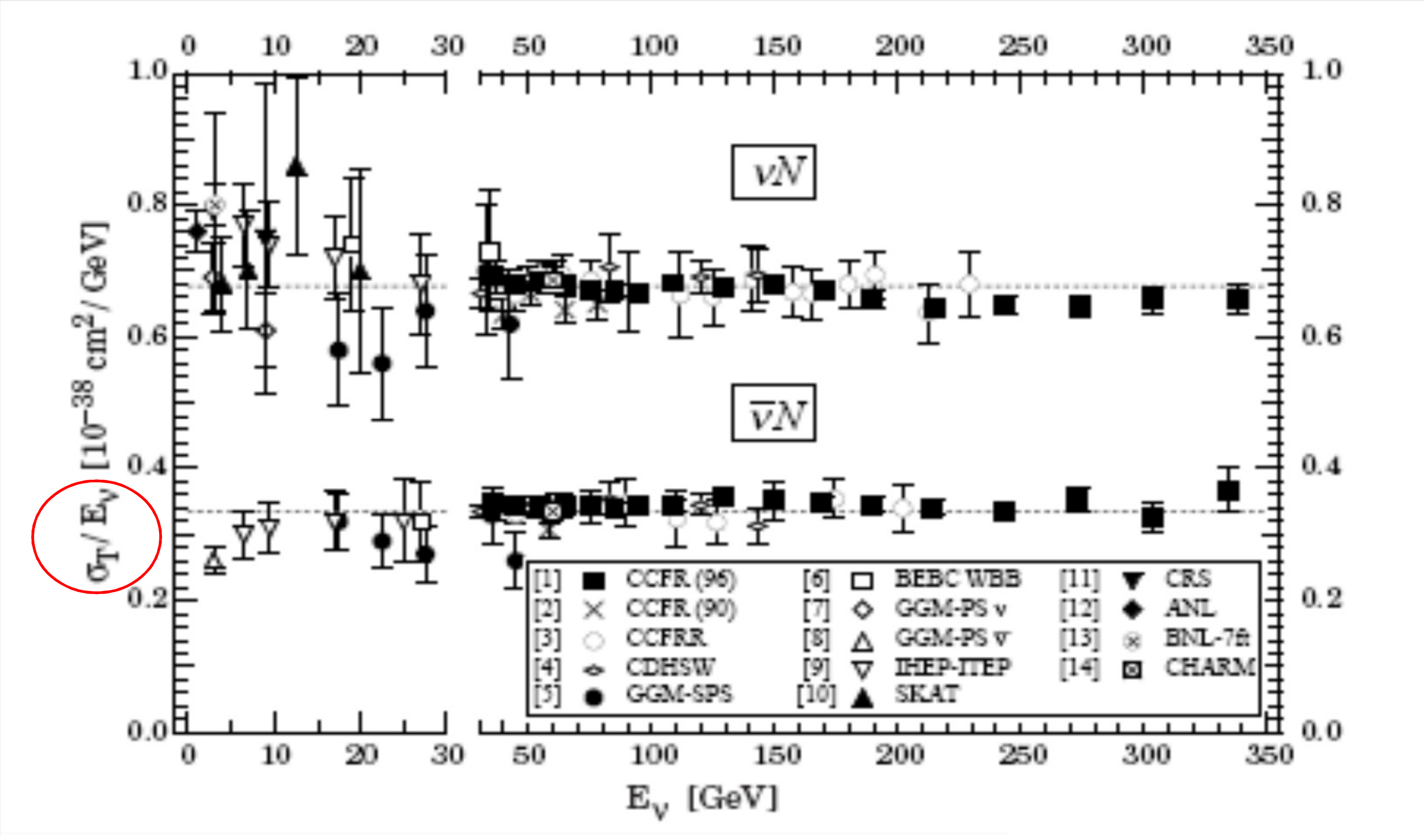
More information: Charged current sensitive to parton charge sign

→ Can separate quark/antiquark contribution

And: Yes, by looking at (anti)neutrino-nucleon DIS structure functions (probing the parton structure by charged – and neutral – weak currents) one concludes that quarks couple to weak currents exactly as leptons

Weak Interaction: Universality-XXII

$\nu, \overline{\nu}$



Linearly rising cross section confirmed...

Troubles: Unitarity - I

Divergence at high energy : Unitarity bound violated around $E_\nu^* \sim\ 300\ GeV$

$$\sigma \le \frac{4\pi}{k^2} = \frac{4\pi}{E^{*2}}$$ Unitarity bound for S-Wave scattering

Reminder: (Simpler) Spinless potential scattering

Expand incident (plane) wave into angular momentum eigenstates

$$\Psi_i = e^{ikz} = \frac{i}{2kr}\sum_{l=0}^{\infty}(2l+1)\left[(-1)^l e^{-ikr} - e^{ikr}\right]P_l(\cos\theta)$$

Outgoing spherical wave phase shifted by potential:

$$\Psi_{total} = \Psi_{scattered} + \Psi_i = \frac{i}{2kr}\sum_{l=0}^{\infty}(2l+1)\left[(-1)^l e^{-ikr} - \eta_l e^{2i\delta_l} e^{ikr}\right]P_l(\cos\theta)$$

$$\Psi_{scattered} = \Psi_{total} - \Psi_i = \frac{e^{ikr}}{kr}\sum_{l=0}^{\infty}(2l+1)\frac{\eta_l e^{2i\delta_l} - 1}{2i}P_l(\cos\theta) = \frac{e^{ikr}}{r} f\left(\theta\right)$$

Troubles: Unitarity - II

Outgoing elementary flux:

$$d\Phi_{out} = v_{out}\Psi_{scat}\Psi^*_{scat}r^2 d\Omega = v_{out}\left|F(\theta)\right|^2 d\Omega$$

Incident flux:

$$\Phi_{in} = \Psi_{in}\Psi^*_{in}v_{in} = v_{in}$$

$$\rightarrow d\sigma = \frac{\Phi_{out}}{\Phi_{in}} = \left|F(\theta)\right|^2 d\Omega$$

$$\sigma = \int \left|F(\theta)\right|^2 d\Omega$$

$$\sigma = \frac{1}{k^2}\sum_{l,m}(2l+1)\left[\frac{\eta_l e^{2i\delta_l}-1}{2i}\right](2m+1)\left[\frac{\eta_m e^{2i\delta_m}-1}{2i}\right]^*$$

$$\times\int P_l(\cos\theta)P_m(\cos\theta)d\Omega$$

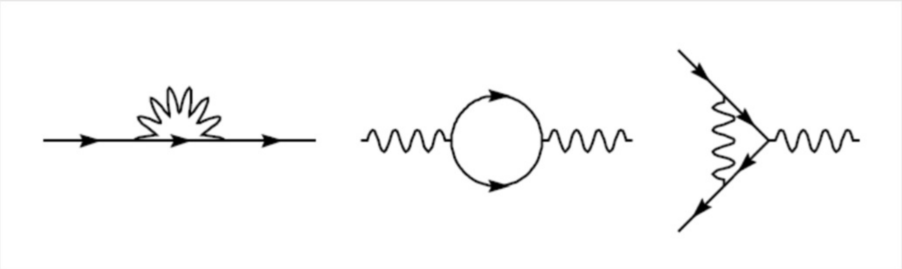
$$\sigma = \frac{4\pi}{k^2}\sum_l(2l+1)\left\|\left[\frac{\eta_l e^{2i\delta_l}-1}{2i}\right]\right\|^2 = \frac{4\pi}{k^2}\sum_l(2l+1)\sin^2\delta_l$$

$$\rightarrow \sigma_{l=0} = \frac{4\pi}{k^2}\sin^2\delta_0 \le \frac{4\pi}{k^2} = \frac{4\pi}{E^{*2}}$$ Unitarity bound for S-Wave scattering

Troubles: Renormalization - I

Suppose Fermi's theory can be saved by radiative corrections:
Assume divergent cross-section as due to our limited, tree-level approximation
Maybe higher orders could fix it

Take QED as an example

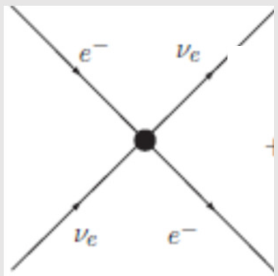


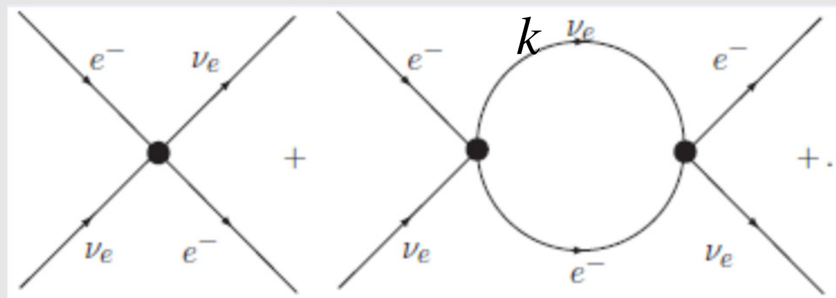
These diagrams (and higher orders) divergent:
However, nice fix available by renormalization procedure

Very successful program, leading to extraordinary accuracy & agreement between theory and experiment

Troubles: Renormalization - II

Higher order diagrams in Fermi's theory:





Cannot be fixed by renormalization:
Fermi's theory non-renormalizable

Indeed: Each vertex ~ G_F

Troubles: Renormalization - III

Lagrangian density (μ decay etc)

$$L_{Fermi} = -\frac{G_F}{\sqrt{2}} \bar{\nu}_\mu \gamma^\alpha \left(1-\gamma^5\right) \mu \bar{e} \gamma_\alpha \left(1-\gamma^5\right) \nu_e$$

Action dimension:

$$S = \int L_{Fermi} d^4x \to [S] = [E][T] = [E]\left[E^{-1}\right] = 0 \text{ Action dimensionless}$$

$$\to [L_{Fermi}] = \left[E^4\right]$$

$$[L_{Fermi}] = [G_F]\left[\psi^4\right]$$

Field dimension: $[\psi] = \left[E^{3/2}\right]$

$$\to [L_{Fermi}] = [G_F]\left[E^6\right] = \left[E^4\right]$$

$$\to [G_F] = \left[E^{-2}\right]$$

Amplitude dimensionless: $[A] = 0$

$\to$ Each G_F in the amplitude to be dimensionally compensated by some k^2 factor

$\to$ Loop diagrams of higher orders must include integrals of higher powers of k

$\to$ More and more divergent

Intermediate Vector Boson - I

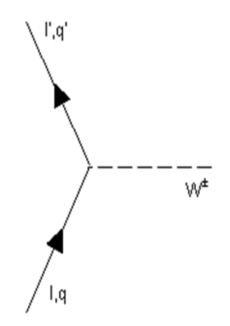
As anticipated:
Forced to go beyond Fermi's theory
Current-Current must be a *low energy effective theory:*
Low energy approximation of a more general theory

Replace *contact interaction* (current-current) by *boson exchange*:

Modeled after the electromagnetic interaction

Exchanged particle must be

- *Charged* (Charged current ±)
- *Chiral* (Only coupled to left chiral parts: Parity violation)
- *Heavy* (Fermi's point-like interaction OK at low energy)



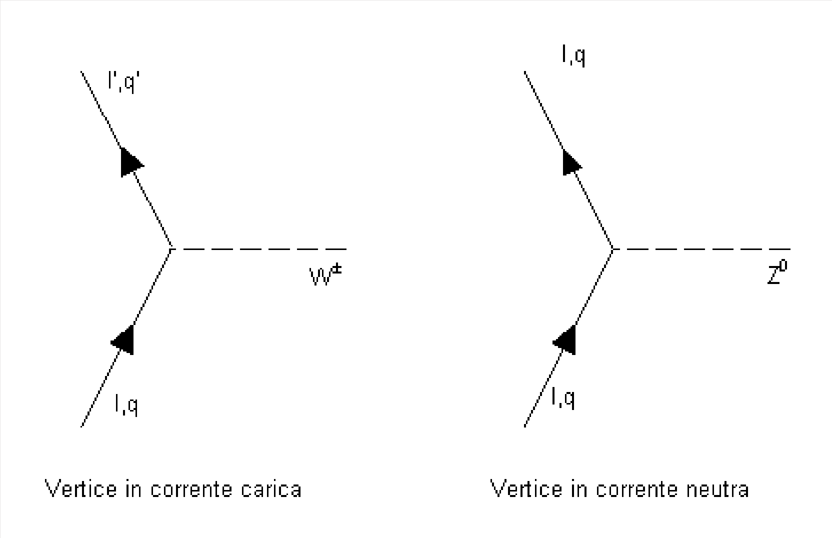
Vertice in corrente carica

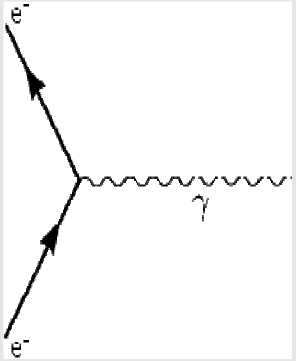
Intermediate Vector Boson - II

Some key points

A) (Quarks and) Leptons (both) interact through the exchange of *vector particles*

$\Delta Q = \pm 1$ $\Delta Q = 0$





Vertice elettromagnetico

Intermediate Vector Boson - III

B) Exchanged vector bosons are *(very)* massive

Range of weak interaction quite small:

Compare β-decay of nuclei, $R<R_{nucleus}$

Cannot tell how large is boson mass, just raw estimate $M \geq 1\ GeV$

Intermediate Vector Boson - IV

W propagator: $-i\dfrac{g_{\mu\nu} - q_\mu q_\nu / M_W^2}{q^2 - M_W^2}$ (Gauge dependent)

$q^2 \ll M_W^2 \rightarrow -i\dfrac{g_{\mu\nu} - q_\mu q_\nu / M_W^2}{q^2 - M_W^2} \approx i\dfrac{g_{\mu\nu}}{M_W^2}$ q^2-independent

$$T_{fi} \cong \left(\frac{1}{2\sqrt{2}}\right)^2 g_W^2 \left(\bar{u}_f^{(1)} \frac{1}{2}\gamma^\mu \left(1-\gamma^5\right) u_i^{(1)}\right) i\frac{g_{\mu\nu}}{M_W^2} \left(\bar{u}_f^{(2)} \frac{1}{2}\gamma_\nu \left(1-\gamma_5\right) u_i^{(2)}\right)$$

$$\rightarrow T_{fi} \cong i\frac{1}{8} 4\sqrt{2} G_F \;\; j^{\mu(1)} j_\mu^{(2)}$$

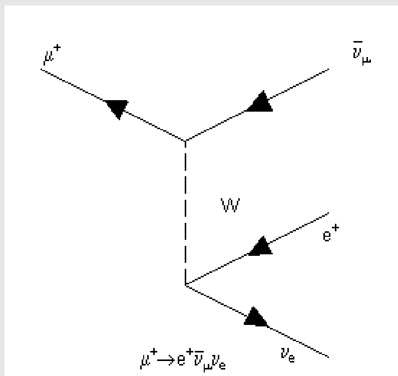
$g_W^2 \equiv \alpha_W$ Charged current coupling constant

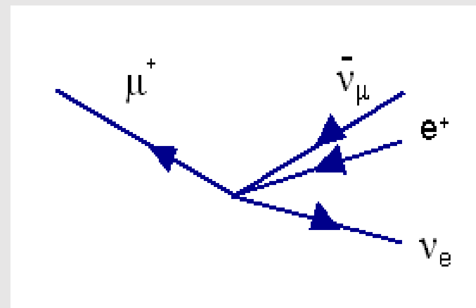
$G_F = \dfrac{\sqrt{2}}{8}\left(\dfrac{g_W}{M_W}\right)^2$ Fermi constant

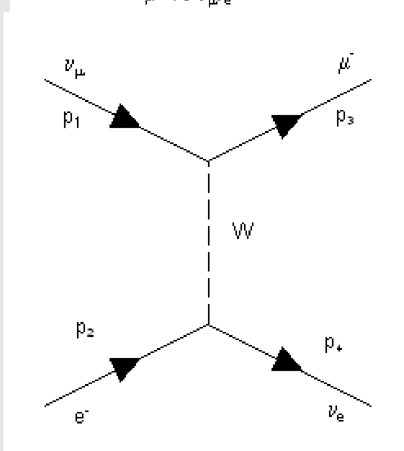
Intermediate Vector Boson - V

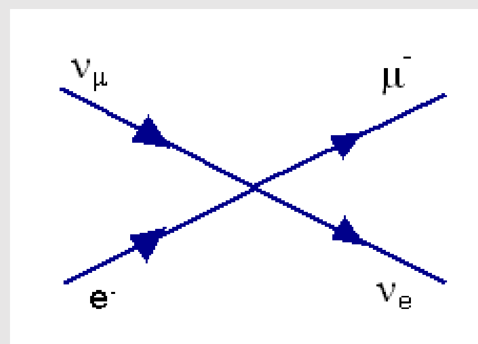
Showing how Standard Model diagrams collapse into current-current at low energy:

$$q^2 \ll M_W^2 \rightarrow -i\frac{g_{\mu\nu} - q_\mu q_\nu / M_W^2}{q^2 - M_W^2} \approx i\frac{g_{\mu\nu}}{M_W^2}$$







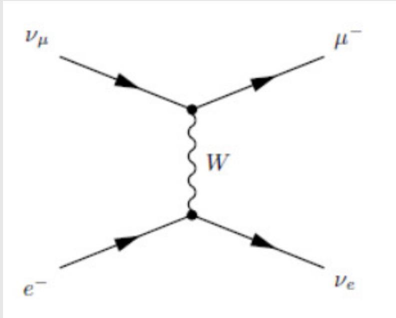


Intermediate Vector Boson - VI

Good fix for some problems:

Cross sections of several neutrino reactions

Inverse Muon Decay:

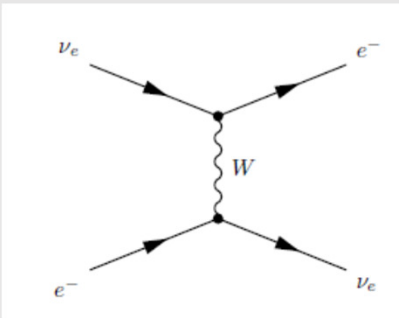


$$\frac{d\sigma}{d\Omega_{CM}} = \frac{G_F^2 M_W^4}{16\pi^2 k^2}\left(\frac{4k^2}{4k^2 - M_W^2}\right)^2 \rightarrow \frac{d\sigma}{d\Omega_{CM}} \approx \begin{cases} \dfrac{G_F^2 k^2}{\pi^2}, & 4k^2 \ll M_W^2 \\ \dfrac{G_F^2 M_W^4}{16\pi^2 k^2}, & 4k^2 \gg M_W^2 \end{cases} \rightarrow \sigma \sim \begin{cases} \dfrac{4G_F^2 k^2}{\pi}, & 4k^2 \ll M_W^2 \\ \dfrac{G_F^2 M_W^4}{4\pi k^2}, & 4k^2 \gg M_W^2 \end{cases}$$

No divergence!

Intermediate Vector Boson - VII

Charged current (only), tree level elastic (anti) electronic neutrino-electron cross sections:

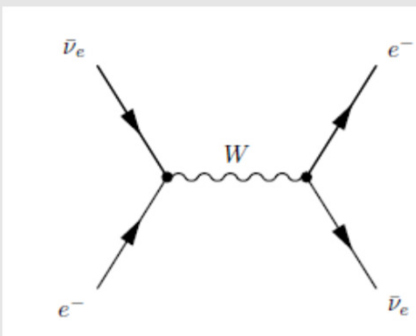


$$\nu_e + e \to e + \nu_e$$

$$\frac{d\sigma}{d\Omega_{CM}} \simeq \frac{16\mathrm{G}_F^2 M_W^2}{\pi^2} \frac{4k^2}{\left(q^2 - M_W^2\right)^2}, \quad k^2 \gg m_e^2$$

$$q^2 \simeq -2k^2\left(1-\cos\theta\right), \quad s \simeq 4k^2$$

$$\to \sigma \simeq \frac{G_F^2}{\pi} \frac{4k^2}{1+\frac{4k^2}{M_W^2}} \underset{k^2 \gg M_W^2}{\to} \frac{G_F^2 M_W^2}{\pi}, \quad \text{no divergence!}$$



$$\bar{\nu}_e + e \to \bar{\nu}_e + e$$

$$\sigma \simeq \frac{G_F^2 M_W^4}{3\pi} \frac{4k^2}{16k^4\left(1-\frac{M_W^2}{4k^2}\right)^2} = \frac{G_F^2 M_W^4}{3\pi} \frac{1}{4k^2\left(1-\frac{M_W^2}{4k^2}\right)^2}$$

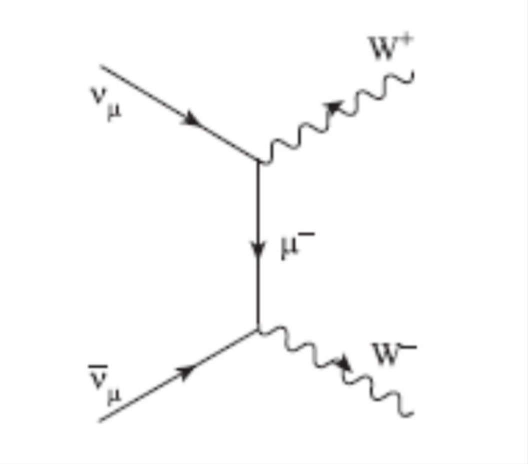
$$\to \sigma \underset{k^2 \gg M_W^2}{\to} \frac{G_F^2 M_W^4}{3\pi s}, \quad \text{no divergence!}$$

Intermediate Vector Boson - VIII

Another dark side of naive IVB model:

Take hypothetical reaction

$\nu_\mu + \overline{\nu}_\mu \rightarrow W^+ + W^-$



No question, not easy to realize in the lab...

Nevertheless, it should be possible to compute the cross section

Anyway, similar issues for the (realistic) reaction

$e^+ + e^- \rightarrow W^+ + W^-$

Intermediate Vector Boson - IX

Central issue:

Massive $W^{\pm}$ bosons in the final state

$\rightarrow$ 3 polarization states for a massive vector particle

Rest frame:

$$\left.\begin{array}{l}\varepsilon_x = (0,1,0,0) \\ \varepsilon_y = (0,0,1,0)\end{array}\right\} \varepsilon_T \quad \text{Transverse polarizatiom}$$

$$\varepsilon_z = (0,0,0,1) \quad \varepsilon_L \quad \text{Longitudinal polarization}$$

After a z-boost, carrying the W to 4-momentum $k^{\mu} = (k^0, 0, 0, k)$

$$\varepsilon_T(k) = \varepsilon_T(0)$$

$$\varepsilon_L(k) = \left(\frac{k}{M_W}, 0, 0, \frac{k_0}{M_W}\right) = \frac{k^{\mu}}{M_W} + O\left(\frac{M_W}{k_0}\right)$$

Intermediate Vector Boson - X

Matrix element $(1=\nu, W^+, 2=\bar{\nu}, W^-; p=\nu, k=W)$:

$$T_{fi}^{\lambda_1\lambda_2} = g^2 \varepsilon_\mu^{-*}(k_2,\lambda_2)\varepsilon_\mu^{+*}(k_1,\lambda_1)\bar{v}(p_2)\gamma^\mu(1-\gamma_5)\frac{(\not{p}_1-\not{k}_1+m_\mu)}{(p_1-k_1)^2-m_\mu^2}\gamma^\nu(1-\gamma_5)u(p_1)$$

By:

Neglecting μ mass,

Restricting to longitudinally polarized $W's$ $(\lambda=0)$,

Taking the high energy($\gg M_W$) limit for the polarization $4-$vectors,

commuting γ_5 :

$$\left|T_{fi}^{00}\right|^2 = \frac{g^4}{M_W^4(p_1-k_1)^4} Tr\left[k_2(1-\gamma_5)(\not{p}_1-\not{k}_1)\not{k}_1\not{p}_1\not{k}_1(\not{p}_1-\not{k}_1)\not{k}_2\not{p}_2\right]$$

By averaging/summing over initial/final spin projections:

$$\sum_{spin}\left|T_{fi}^{00}\right|^2 = \frac{g^4}{M_W^4}(p_1\cdot k_1)(p_2\cdot k_2) = \frac{g^4}{M_W^4}E^4\left(1-\cos^2\theta\right) \rightarrow \frac{d\sigma}{d\Omega^*} = \frac{G_F^2}{8\pi^2}E^2\sin^2\theta$$

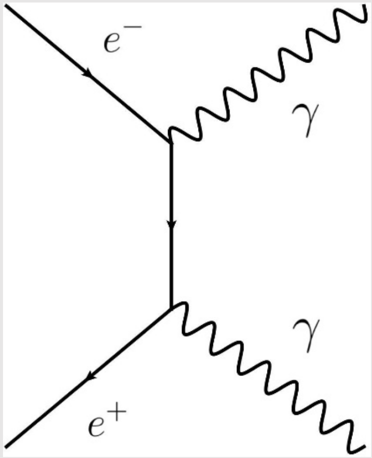
$\rightarrow$ Still divergent at high energy

No simple solution for this problem:

Massive vector particles cannot make it without 3 *polarization states*

Intermediate Vector Boson - XI

Compare to well known QED process



No contribution from longitudinal photons:

Real photons always transverse, as a consequence of *gauge invariance* of QED

(Compare to classical radiation field: $\mathbf{E},\mathbf{B}$ purely transverse)

Hope the gauge invariance benefits can be extended to weak interactions..

Intermediate Vector Boson - XII

Is that a single trouble, unrelated to the full IVB scheme?

Have a look at diagrams including *virtual W*:

Discover that a new divergence hits hard our naive IVB model..

Looking at virtual *W* propagator:

$$\frac{-g_{\mu\nu} + k_\mu k_\nu / M_W^2}{k^2 - M_W^2} \underset{k^2 \to \infty}{\longrightarrow} const$$

→ Will make diagrams with virtual *Ws* divergent at high energy

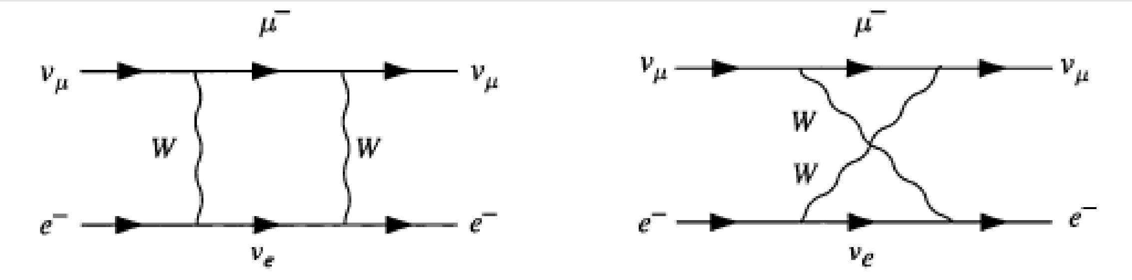
Serious illness of IVB model, particularly relevant for *neutral current* processes

Intermediate Vector Boson - XIII

Neutral current reactions like:

$$\nu_\mu + e^- \rightarrow \nu_\mu + e^-$$
$$\bar{\nu}_\mu + e^- \rightarrow \bar{\nu}_\mu + e^-$$

Not allowed at tree level by our IVB model, only by loop diagrams:



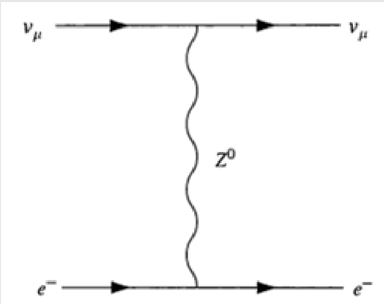
But we can't compute loop diagrams including virtual *W*:
Divergent, IVB theory *not renormalizable*
Basic requirement: Theory must be renormalizable

Intermediate Vector Boson - XIV

Aside: Expect strong suppression
Surprisingly, after first observations: NC Cross sections ≈ Allowed processes

$$\nu_e + e^- \to e^- + \nu_e$$
$$\nu_\mu + e^- \to \mu^- + \nu_e$$

Suggestion:
Maybe neutral currents do exist *at tree level*, e.g.



[Indeed, neutral currents are *required* in standard electroweak theory]

Gauge Symmetry - I

What makes QED so successful?

Renormalization program allows for computing observables with high accuracy, comparable to experimental resolution

QED is a renormalizable field theory

Fermi's theory is a non-renormalizable theory

And:

Naive IVB theory of weak interactions is a non-renormalizable theory

Try to discover what makes the difference

Gauge Symmetry - II

Back to QED for a while: Reconsider global and local gauge invariance

Free Dirac Lagrangian:

$$L_0 = \overline{\psi}(x)\left(i\gamma^\mu \partial_\mu - m\right)\psi(x)$$

Invariant upon global gauge transformation:

$$\begin{cases} \psi(x) \to \psi'(x) = e^{-i\alpha}\psi(x) \\ \overline{\psi}(x) \to \overline{\psi}'(x) = e^{+i\alpha}\overline{\psi}(x) \end{cases}, \quad \alpha \text{ constant}$$

Noether's theorem $\to$ Conserved current:

$$\partial_\mu s^\mu(x) = 0, \quad s^\mu(x) = q\overline{\psi}(x)\gamma^\mu\psi(x)$$

$\to$ Conserved charge:

$$Q = \int s^0(x)\, d^3\mathbf{r} = const$$

Gauge Symmetry - III

Non invariant under local gauge transformation:

$$\begin{cases} \psi(x) \rightarrow \psi'(x) = e^{-iqf(x)}\psi(x) \\ \overline{\psi}(x) \rightarrow \overline{\psi}'(x) = e^{+iqf(x)}\overline{\psi}(x) \end{cases}$$

$$L_0 \rightarrow L_0' = L_0 + q\overline{\psi}(x)\gamma^\mu\psi(x)\partial_\mu f(x)$$

Define then a *covariant derivative* as:

$$D_\mu\psi(x) = \left[\partial_\mu + iqA_\mu(x)\right]\psi(x)$$

where, upon the previous local gauge transformation:

$$A_\mu(x) \rightarrow A_\mu(x) = A_\mu(x) + \partial_\mu f(x)$$

Then the Lagrangian:

$$L = \overline{\psi}(x)\left(i\gamma^\mu D_\mu - m\right)\psi(x) = L_0 - q\overline{\psi}(x)\gamma^\mu\psi(x)A_\mu(x)$$

is invariant

L contains an *interaction* term $\left(\leftarrow j^\mu A_\mu\right)$

Gauge Symmetry - IV

Consider a single family of massless leptons:

$$L_0 = i\overline{\psi}(x)\gamma^\mu\partial_\mu\psi(x) + i\overline{\psi}_\nu(x)\gamma^\mu\partial_\mu\psi_\nu(x)$$

Chiral spinors:

$$\psi^L(x) = P_L\psi(x) = \frac{1}{2}(1-\gamma_5)\psi(x)$$

$$\psi^R(x) = P_R\psi(x) = \frac{1}{2}(1+\gamma_5)\psi(x)$$

$$\rightarrow L_0 = i\overline{\psi}^{\,L}(x)\gamma^\mu\partial_\mu\psi^{\,L}(x) + i\overline{\psi}_\nu{}^L(x)\gamma^\mu\partial_\mu\psi_\nu{}^L(x)$$

$$+i\overline{\psi}^{\,R}(x)\gamma^\mu\partial_\mu\psi^{\,R}(x) + i\overline{\psi}_\nu{}^R(x)\gamma^\mu\partial_\mu\psi_\nu{}^R(x)$$

Charged current: Connecting two leptons with $\Delta Q = \pm 1$

To encode this into a symmetry scheme, define the doublet:

$$\Psi^{\,L}(x) = \begin{pmatrix} \psi_\nu{}^L(x) \\ \psi^{\,L}(x) \end{pmatrix}, \overline{\Psi}^{\,L}(x) = \begin{pmatrix} \overline{\psi}_\nu{}^L(x) & \overline{\psi}^{\,L}(x) \end{pmatrix}$$

$$\rightarrow L_0 = i\overline{\Psi}^{\,L}(x)\gamma^\mu\partial_\mu\Psi^{\,L}(x) + i\overline{\psi}^{\,R}(x)\gamma^\mu\partial_\mu\psi^{\,R}(x) + i\overline{\psi}_\nu{}^R(x)\gamma^\mu\partial_\mu\psi_\nu{}^R(x)$$

Gauge Symmetry - V

Suppose the L-doublet realizes the fundamental representation of a $SU(2)$ (*gauge*) *symmetry* of the weak interaction , exactly as $U(1)$ is the (gauge) symmetry of QED

Then L-spinors will transform

$$\Psi^L(x) \to \Psi^L{}'(x) = U(\alpha)\Psi^L(x) = \exp\left[i\alpha_j\tau_j/2\right]\Psi^L(x)$$

$$\bar{\Psi}^L(x) \to \bar{\Psi}^L{}'(x) = \bar{\Psi}^L(x)U^\dagger(\alpha) = \bar{\Psi}^L(x)\exp\left[-i\alpha_j\tau_j/2\right]$$

$\alpha_1, \alpha_2, \alpha_3$ 3 continuous, real parameters

τ_1, τ_2, τ_3 Pauli matrices

$\left[\text{Reminder}: \left[\tau_i, \tau_j\right] = 2i\varepsilon_{ijk}\,\tau_k\right]$

Also take R-spinors as $SU(2)$ *singlets*:

$$\Psi^R(x) \to \Psi^R{}'(x) = \Psi^R(x), \Psi^R_\nu(x) \to \Psi^R_\nu{}'(x) = \Psi^R_\nu(x)$$

$$\bar{\Psi}^R(x) \to \bar{\Psi}^R{}'(x) = \bar{\Psi}^R(x), \bar{\Psi}^R_\nu(x) \to \bar{\Psi}^R_\nu{}'(x) = \bar{\Psi}^R_\nu(x)$$

Gauge Symmetry - VI

According to Noether's theorem :

Expect conserved current after L invariance

under infinitesimal $SU(2)$ transformations :

$$\Psi^L(x) \to \Psi^L{}'(x) = \left(1 + i\alpha_j \tau_j/2\right)\Psi^L(x)$$

$$\bar{\Psi}^L(x) \to \bar{\Psi}^L{}'(x) = \bar{\Psi}^L(x)\left(1 - i\alpha_j \tau_j/2\right)$$

$\to$ Identify 3 weak isospin, conserved currents / charges:

$$J_i^\mu(x) = \frac{1}{2}\bar{\Psi}^L(x)\gamma^\mu \tau_i \Psi^L(x)$$

$$I_i^W = \int d^3\mathbf{x} J_i^0(x) = \int \frac{1}{2}\Psi^{L\dagger}(x)\tau_i \Psi^L(x)$$

Make 2 non – Hermitian, linear combinations :

$$J^\mu(x) = 2\left[J_1^\mu(x) - iJ_2^\mu(x)\right] = \left[\bar{\Psi}^L(x)\gamma^\mu \tau_1 \Psi^L(x) - i\bar{\Psi}^L(x)\gamma^\mu \tau_2 \Psi^L(x)\right]$$

$$J^{\mu\dagger}(x) = 2\left[J_1^\mu(x) + iJ_2^\mu(x)\right] = \left[\bar{\Psi}^L(x)\gamma^\mu \tau_1 \Psi^L(x) + i\bar{\Psi}^L(x)\gamma^\mu \tau_2 \Psi^L(x)\right]$$

Gauge Symmetry - VII

Rememeber:

$$\Psi^L(x)=\begin{pmatrix}\psi_\nu{}^L(x)\\ \psi^L(x)\end{pmatrix},\bar{\Psi}^L(x)=\begin{pmatrix}\bar{\psi}_\nu{}^L(x) & \bar{\psi}^L(x)\end{pmatrix}$$

Then:

$$J_1(x)=\bar{\Psi}^L(x)\gamma^\mu\tau_1\Psi^L(x)=\begin{pmatrix}\bar{\psi}_\nu{}^L(x) & \bar{\psi}^L(x)\end{pmatrix}\gamma^\mu\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}\psi_\nu{}^L(x)\\ \psi^L(x)\end{pmatrix}$$

$$\bar{\Psi}^L(x)\gamma^\mu\tau_1\Psi^L(x)=\begin{pmatrix}\bar{\psi}_\nu{}^L(x) & \bar{\psi}^L(x)\end{pmatrix}\gamma^\mu\begin{pmatrix}\psi^L(x)\\ \psi_\nu{}^L(x)\end{pmatrix}$$

$$=\bar{\psi}_\nu{}^L(x)\gamma^\mu\psi^L(x)+\bar{\psi}^L(x)\gamma^\mu\psi_\nu{}^L(x)$$

$$iJ2(x)=i\bar{\Psi}^L(x)\gamma^\mu\tau_2\Psi^L(x)=i\begin{pmatrix}\bar{\psi}_\nu{}^L(x) & \bar{\psi}^L(x)\end{pmatrix}\gamma^\mu\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}\begin{pmatrix}\psi_\nu{}^L(x)\\ \psi^L(x)\end{pmatrix}$$

$$=\bar{\psi}_\nu{}^L(x)\gamma^\mu\psi^L(x)-\bar{\psi}^L(x)\gamma^\mu\psi_\nu{}^L(x)$$

Therefore:

$$\begin{cases}J^\mu(x)=2\bar{\psi}^L(x)\gamma^\mu\psi_\nu{}^L(x)\\ J^{\mu\dagger}(x)=2\bar{\psi}_\nu{}^L(x)\gamma^\mu\psi^L(x)\end{cases}$$

: Just our weak charged currents

Gauge Symmetry - VIII

$$J_3^\mu = \frac{1}{2}\bar{\Psi}^L(x)\gamma^\mu\tau_3\Psi^L(x)$$

$$\bar{\Psi}^L(x)\gamma^\mu\tau_3\Psi^L(x) = \left(\bar{\psi}_\nu{}^L(x) \quad \bar{\psi}^L(x)\right)\gamma^\mu\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}\psi_\nu{}^L(x)\\ \psi^L(x)\end{pmatrix}$$

$$\rightarrow \bar{\Psi}^L(x)\gamma^\mu\tau_3\Psi^L(x) = \left(\bar{\psi}_\nu{}^L(x) \quad \bar{\psi}^L(x)\right)\gamma^\mu\begin{pmatrix}\psi_\nu{}^L(x)\\ -\psi^L(x)\end{pmatrix}$$

$$\rightarrow J_3^\mu = \frac{1}{2}\left[\bar{\psi}_\nu{}^L(x)\gamma^\mu\psi_\nu{}^L(x) - \bar{\psi}^L(x)\gamma^\mu\psi^L(x)\right]$$

Neutral (as opposed to charged) current

Observe:

2nd term is actually a part of the electromagnetic current, up to a constant factor

$$J_{EM}^\mu = -e\bar{\psi}(x)\gamma^\mu\psi(x) = -e\left[\bar{\psi}^L(x)\gamma^\mu\psi^L(x) + \bar{\psi}^R(x)\gamma^\mu\psi^R(x)\right]$$

Might be possible to unify EM and weak interactions. But:

Count 3 weak isospin + 1 electromagnetic currents = 2 charged + 2 neutral

$\rightarrow$ Unified symmetry group must be larger that $SU(2)$, which has only 3 parameters

Gauge Symmetry - IX

Early models (between ’50s and ’60s...):

Neutral current ≡ 3rd weak isospin current

Symmetry group is $SU(2)_L \times U(1)_Q$

$SU(2)_L$ (Non Abelian) symmetry group of weak interactions of *L*-fermions
$U(1)_Q$ (Abelian) symmetry group of QED

Then:

Neutral current has same V-A structure of charged current
(Wrong: When finally observed, neutral current was found ≠ V-A)

Weak and Electromagnetic interactions stay independent
(Wrong: At high energy, proofs of unification easily found)

Gauge Symmetry - X

Rather assume the symmetry group of (unified) Electroweak interaction is

$$SU(2)_L \times U(1)_Y$$

where Y is a new observable called *weak hypercharge*

$$J_Y^\mu = \frac{1}{e} J_{EM}^\mu - J_3^\mu = -\bar{\psi}(x)\gamma^\mu\psi(x) - \frac{1}{2}\left[\bar{\psi}_\nu{}^L(x)\gamma^\mu\psi_\nu{}^L(x) - \bar{\psi}^L(x)\gamma^\mu\psi^L(x)\right]$$

$$\rightarrow J_Y^\mu = \frac{1}{2}\bar{\psi}^L(x)\gamma^\mu\psi^L(x) - \frac{1}{2}\bar{\psi}_\nu{}^L(x)\gamma^\mu\psi_\nu{}^L(x) - \bar{\psi}^R(x)\gamma^\mu\psi^R(x)$$

$\rightarrow$ Conserved charge:

$$Y_W = \int d^3\mathbf{x} J_Y^0(x) = \frac{Q}{e} - I_3^W$$

Fermion EW quantum numbers: Defined by I, I_3, Y

Different for different chiralities!

Gauge Symmetry - XI

Find the EW quantum numbers of (chiral) leptons:

$$Y_W = 2\left(Q - I_3\right)$$

$$I_3^W \left| l^-, L \right\rangle = -\frac{1}{2}\left| l^-, L \right\rangle \quad Y_W \left| l^-, L \right\rangle = (-1)\left| l^-, L \right\rangle$$

$$I_3^W \left| \nu_l, L \right\rangle = +\frac{1}{2}\left| \nu_l, L \right\rangle \quad Y_W \left| \nu_l, L \right\rangle = (-1)\left| \nu_l, L \right\rangle$$

$$I_3^W \left| l^-, R \right\rangle = 0 \qquad Y_W \left| l^-, R \right\rangle = (-2)\left| l^-, R \right\rangle$$

$$I_3^W \left| \nu_l, R \right\rangle = 0 \qquad Y_W \left| \nu_l, R \right\rangle = 0\left| \nu_l, R \right\rangle$$

Gauge Symmetry - XII

Extend to *local* $SU(2)$ gauge transformations: Similar to QCD

L – doublet

$$\Psi^L(x) \to \Psi^L{}'(x) = U(\alpha)\Psi^L(x) = \exp\left[i\, g\omega_j(x)\tau_j/2\right]\Psi^L(x)$$

$$\bar{\Psi}^L(x) \to \bar{\Psi}^L{}'(x) = \bar{\Psi}^L(x)U^\dagger(\alpha) = \bar{\Psi}^L(x)\exp\left[-i\, g\omega_j(x)\tau_j/2\right]$$

R – singlet

$$\psi^R(x) \to \psi^R{}'(x) = \psi^R(x), \psi^R_\nu(x) \to \psi^R_\nu{}'(x) = \psi^R_\nu(x)$$

$$\bar{\psi}^R(x) \to \bar{\psi}^R{}'(x) = \bar{\psi}^R(x), \psi^R_\nu(x) \to \psi^R_\nu{}'(x) = \psi^R_\nu(x)$$

$\omega_j(x)$: 3 real parameters, functions of $(\mathbf{r}, t)$

As for QCD: L_0 not invariant

$$L_0 \to L_0{}' = L_0 + \delta L_0 \ = L_0 - \frac{1}{2} g \bar{\Psi}^L(x)\tau_j\gamma_\mu\partial^\mu\omega_j(x)\Psi^L(x)$$

Gauge Symmetry - XIII

$\rightarrow$ Define a covariant derivative for the doublet:

$$\partial^\mu \Psi^L(x) \rightarrow D^\mu \Psi^L(x) = \left[\partial^\mu + i\frac{g}{2}\tau_j W_j^\mu(x)\right]\Psi^L(x)$$

W_j^μ : triplet of (charged, massless) , gluon – like vector fields

Requiring suitable transformation rules:

[Repeated indexes summed, ω_i infinitesimal]

$$W_j^\mu(x) \rightarrow W_j^\mu{}'(x) = W_j^\mu(x) - \partial^\mu \omega_j(x) - g\varepsilon_{jik}\omega_i(x) W_k^\mu(x)$$

$\rightarrow L$ invariant

Gauge Symmetry - XIV

Now weak hypercharge $U(1)$ gauge transformations: Similar to QED

$$\psi(x) \rightarrow \psi'(x) = e^{-ig'Yf(x)}\psi(x)$$

$$\bar{\psi}(x) \rightarrow \bar{\psi}'(x) = \bar{\psi}(x)e^{ig'Yf(x)}$$

$\rightarrow$ Define a covariant derivative:

$$\partial^\mu\psi(x) \rightarrow D^\mu\psi(x) = \left[\partial^\mu + ig'B^\mu(x)\right]\psi(x)$$

B^μ : Neutral, massless photon – like vector field

g': New coupling constant

$$B^\mu(x) \rightarrow B^\mu{}'(x) = B^\mu(x) - \partial^\mu f(x)$$

$\rightarrow L$ invariant

Gauge Symmetry - XV

Collecting all pieces together:

$$L = i\left[\overline{\Psi}^L(x)\gamma_\mu D^\mu \Psi^L(x) + \overline{\psi}^R(x)\gamma_\mu D^\mu \psi^R(x) + \overline{\psi}^R_\nu(x)\gamma_\mu D^\mu \psi^R_\nu(x)\right]$$

$$D^\mu \Psi^L(x) = \left[\partial^\mu + i\frac{g}{2}\tau_j W^\mu_j(x) - i\frac{g'}{2}B^\mu(x)\right]\Psi^L(x)$$

$$D^\mu \psi^R(x) = \left[\partial^\mu - i\frac{g'}{2}B^\mu(x)\right]\psi^R(x)$$

$$D^\mu \psi^R_\nu(x) = \partial^\mu \psi^R_\nu(x)$$

Write it as:

$$L = L_0 + L_I$$

$$L_I = -gJ^\mu_1(x)W_{1\mu} - gJ^\mu_2(x)W_{2\mu} - gJ^\mu_3(x)W_{3\mu} - g' J^\mu_Y(x)B_\mu(x)$$

Gauge Symmetry - XVI

To understand the meaning of the interaction terms:

Re – write the interaction part

Define

$$W_\mu(x) = \frac{1}{\sqrt{2}}\left[W_{1\mu}(x) - iW_{2\mu}(x)\right]$$

$$W_\mu^\dagger(x) = \frac{1}{\sqrt{2}}\left[W_{1\mu}(x) + iW_{2\mu}(x)\right]$$

And get for the first 2 terms:

$$\rightarrow L_{I-ch} = -\frac{g}{2\sqrt{2}}\left[J^{\mu\dagger}(x)W_\mu(x) + J^\mu(x)W_\mu^\dagger(x)\right]$$

Charged current interaction of $L-$fermions

Gauge Symmetry - XVII

Define:

$$W_{3\mu}(x) = \cos\theta_W Z_\mu(x) + \sin\theta_W A_\mu(x)$$

$$B_\mu(x) = -\sin\theta_W Z_\mu(x) + \cos\theta_W A_\mu(x)$$

Reminder:

$$J_Y^\mu(x) = \frac{1}{e} J_{EM}^\mu(x) - J_3^\mu(x)$$

$$\rightarrow \begin{cases} -g' J_Y^\mu(x) B_\mu(x) = -g'\left[\frac{1}{e} J_{EM}^\mu(x) - J_3^\mu(x)\right]\left[-\sin\theta_W Z_\mu(x) + \cos\theta_W A_\mu(x)\right] \\ -g J_3^\mu(x) W_{3\mu} = -g J_3^\mu(x)\left[\cos\theta_W Z_\mu(x) + \sin\theta_W A_\mu(x)\right] \end{cases}$$

$\rightarrow$ Remaining terms:

$$-J_{EM}^\mu(x)\frac{g'}{e}\left[-\sin\theta_W Z_\mu(x) + \cos\theta_W A_\mu(x)\right] +$$

$$-J_3^\mu(x)\left\{g\left[\cos\theta_W Z_\mu(x) + \sin\theta_W A_\mu(x)\right] + g'\left[-\sin\theta_W Z_\mu(x) + \cos\theta_W A_\mu(x)\right]\right\}$$

Gauge Symmetry - XVIII

Most simple way of unifying the EM and weak interaction :

Require this condition on g', θ_W constants

$$g \sin\theta_W = g' \cos\theta_W = e$$

and contemplate the miracle:

$$L_I = -J^{\mu}_{EM}(x) A_\mu - \frac{g}{2\sqrt{2}} \left[J^{\mu\dagger}(x) W_\mu + J^{\mu}(x) W_\mu^{\dagger} \right] - \frac{g}{\cos\theta_W} \left[J^{\mu}_3(x) - \sin^2\theta_W \frac{J^{\mu}_{EM}}{e}(x) \right] Z_\mu(x)$$

Electromagnetic interaction

Charged current weak interaction

Neutral current weak interaction

Gauge Symmetry - XIX

As for *QED*:

Additional terms required in order to account for:

Energy, *Momentum*, *Angular Momentum*

carried over by the fields

Weak Hypercharge field:

$$-\frac{1}{4}B_{\mu\nu}(x)B^{\mu\nu}(x)$$

$$B^{\mu\nu}(x)=\partial^{\nu}B^{\mu}(x)-\partial^{\mu}B^{\nu}(x)$$

Similar to *QED*: Abelian symmetry $U(1)$

Weak Isospin fields:

$$-\frac{1}{4}\sum_{i=1}^{3}G^{(i)}{}_{\mu\nu}(x)G^{(i)\mu\nu}(x)$$

$$G^{(i)\mu\nu}(x)=\underbrace{\partial^{\nu}W^{(i)\mu}(x)-\partial^{\mu}W^{(i)\nu}(x)}_{F^{(i)\mu\nu}(x)}+g\sum_{i,j=1}^{3}\varepsilon_{ijk}W^{(j)\mu}(x)W^{(k)\nu}(x)$$

Similar to *QCD*: Non-Abelian symmetry $SU(2)_L$

Gauge Symmetry - XX

Gauge Boson Lagrangian:

$$L^B = -\frac{1}{4} B_{\mu\nu}(x) B^{\mu\nu}(x) - \frac{1}{4}\sum_{i=1}^{3} G^{(i)}{}_{\mu\nu}(x) G^{(i)\mu\nu}(x)$$

$$\rightarrow L^B = \underbrace{-\frac{1}{4} B_{\mu\nu}(x) B^{\mu\nu}(x) - \frac{1}{4}\sum_{i=1}^{3} F^{(i)}{}_{\mu\nu}(x) F^{(i)\mu\nu}(x)}_{L_0^B}$$

$$\underbrace{+ g \sum_{i,j,k=1}^{3} \varepsilon_{ijk} W^{(j)\mu}(x) W^{(k)\nu}(x) \partial^\mu W^{(k)\nu}(x) - \frac{1}{4} \sum_{i,j,k,l,m=1}^{3} \varepsilon_{ijk}\varepsilon_{ilm} g^2 W^{(j)\mu}(x) W^{(k)\nu}(x) W^{(l)}{}_{\mu}(x) W^{(m)}{}_{\nu}}_{L_{SI}^B}$$

L_0^B = Free term

L_{SI}^B = Self-Interaction term

Gauge Symmetry - XXI

Free term: Rewrite using $A^{\mu}, W^{\mu}, W^{\dagger\mu}, Z^{\mu}$:

$$L_0^B = -\frac{1}{4} F_{\mu\nu}(x) F^{\mu\nu}(x) - \frac{1}{2} F^W{}_{\mu\nu}(x) F^{W\dagger\mu\nu}(x) - \frac{1}{4} Z_{\mu\nu}(x) Z^{\mu\nu}(x)$$

Field tensors:

$F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x)$ Coupled to EM current

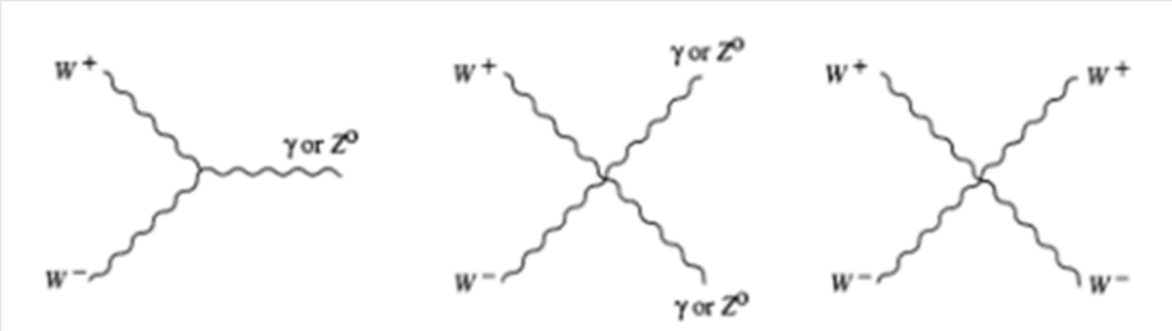
$F^W{}_{\mu\nu}(x) = \partial_\mu W_\nu(x) - \partial_\nu W_\mu(x)$

$F^{W\dagger\mu\nu}(x) = \partial^\mu W^{\dagger\nu}(x) - \partial^\nu W^{\dagger\mu}(x)$

} Coupled to Charged current

$Z^{\mu\nu}(x) = \partial^\mu Z^\nu(x) - \partial^\nu Z^\mu(x)$ Coupled to Neutral current

Self-Interaction term: Similar to 3- and 4-gluons terms of QCD



Gauge Symmetry - XXII

Massless leptons & gauge bosons not physical: Mass must be there

But: Putting 'by hand' a mass term in L would spoil gauge invariance

Gauge bosons:

$$m_W^2 W_\mu^\dagger W^\mu + \frac{1}{2} m_Z^2 Z_\mu Z^\mu$$

Take W as an example:

$W_i^\mu \to W_i^\mu - \partial^\mu \omega_i(x) - g\varepsilon_{ijk}\omega_j(x) W_k^\mu$ infinitesimal parameters

Then:

$$m_W^2 W_\mu^\dagger W^\mu \to m_W^2 \left(W_i^{\dagger\mu} - \partial^\mu \omega_i(x) - g\varepsilon_{ijk}\omega_j(x) W_k^{\dagger\mu}\right)\left(W_i^\mu - \partial^\mu \omega_i(x) - g\varepsilon_{ijk}\omega_j(x) W_k^\mu\right)$$

$$\neq m_W^2 W_\mu^\dagger W^\mu$$

Gauge Symmetry - XXIII

Leptons:

$-m\bar{\psi}(x)\psi(x)$

Write in terms of chiral parts:

$$-m\bar{\psi}(x)\psi(x) = -m\bar{\psi}(x)\left(\underbrace{P_R + P_L}_{=1}\right)\psi(x)$$

$$P_R = \frac{1+\gamma_5}{2}, \quad P_L = \frac{1-\gamma_5}{2}$$

$$\rightarrow -m\bar{\psi}(x)\left(\frac{1+\gamma_5}{2}+\frac{1-\gamma_5}{2}\right)\psi(x) = -m\bar{\psi}(x)\left(\left(\frac{1+\gamma_5}{2}\right)^2+\left(\frac{1-\gamma_5}{2}\right)^2\right)\psi(x)$$

$$\rightarrow -m\bar{\psi}(x)\left(\frac{1+\gamma_5}{2}+\frac{1-\gamma_5}{2}\right)\psi(x) = -m\bar{\psi}_L(x)\psi_R(x) - m\bar{\psi}_R(x)\psi_L(x)$$

Not invariant under $SU(2)$:

L, R chiral parts live in different $SU(2)$ representations

$\rightarrow$ Different gauge transformations

$\rightarrow$ Mass term not gauge invariant

Gauge Symmetry - XXIV

Bottom line: *Any* mass term not invariant

Glashow model (1961): Put mass by hand
→ Gauge invariance lost, back to naive IVB

Finally, discover a subtle mechanism to give mass to physical states, without spoiling gauge invariance:

Spontaneous Symmetry Breaking

Broad phenomenology, also remotely rooted in classical physics

SSB - I

Symmetries: Frequently approximate $\rightarrow$ Broken

Breaking modes:

(a) Explicit breaking

$H = H_0 + H_b$

H_0 invariant

H_b non-invariant

Ex: Hydrogen atom in a magnetic field $\mathbf{B}$

$H_0 = \frac{p^2}{2m} - \frac{Ze^2}{r}$ rotationally invariant

$H_b = -\boldsymbol{\mu} \cdot \mathbf{B}$ invariant wrt rotations around $\mathbf{B}$

$\rightarrow H_0$ degeneracies removed by H_b

(b) Spontaneous breaking

H symmetric, ground state non symmetric

Ex: Ferromagnetism

$T > T_c$: $\mathbf{M} = 0 \rightarrow$ Dipoles randomly oriented

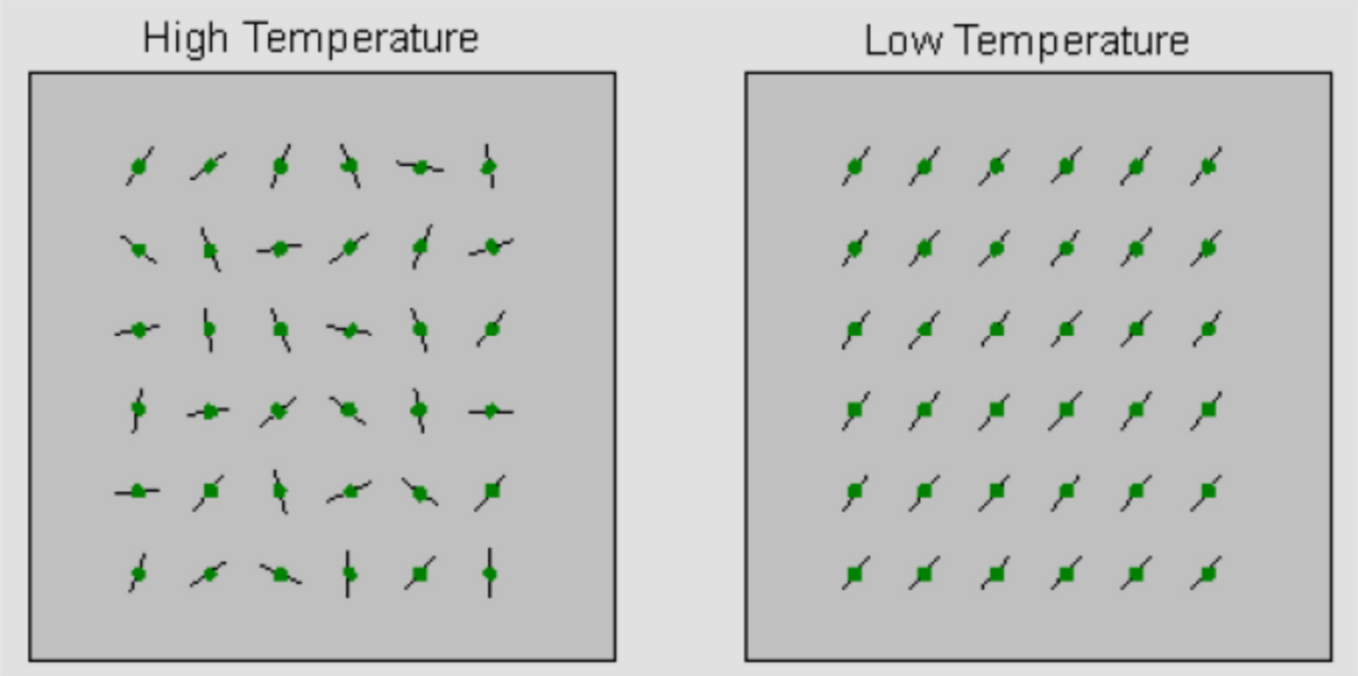
$\rightarrow$ Rotational symmetry

$T < T_c$: $\mathbf{M} \neq 0 \rightarrow$ Dipoles pick some direction

$\rightarrow H$ degeneracies not removed

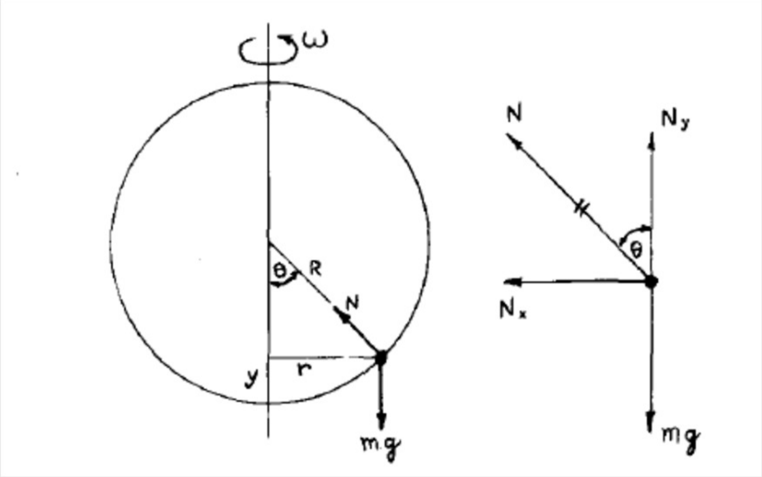
Ground state *degenerate*

SSB - II



SSB - III

Take a first year classical mechanics exercise:
Bead sliding frictionless along a spinning hoop
Find equilibrium angle



$$N = m\omega^2 R$$

$$N_y = N\cos\theta = m\omega^2 R\cos\theta = mg$$

$$\rightarrow \cos\theta_0 = \frac{g}{\omega^2 R}$$

Funny observation:

Critical frequency: $\cos\theta_0 = 1 = \frac{g}{\omega_0^2 R}$

For $\omega < \omega_0$: Different solution

$\theta_1 = 0$

SSB - IV

$$T = \frac{1}{2}mv^2 = \frac{1}{2}m\left(R^2\dot{\theta}^2 + \omega^2 R^2 \sin^2\theta\right)$$

$$V = mgy = mgR\left(1-\cos\theta\right)$$

$$L = T - V = \frac{1}{2}mR^2\dot{\theta}^2 + \frac{1}{2}m\omega^2 R^2 \sin^2\theta - mgR\left(1-\cos\theta\right)$$

$$L = \frac{1}{2}mR^2\dot{\theta}^2 - V_{eff}$$

Define effective potential, including centrifugal term:

$$V_{eff} \equiv -\frac{1}{2}m\omega^2 R^2 \sin^2\theta + mgR\left(1-\cos\theta\right) = mgR\left[\left(1-\cos\theta\right) - \frac{\omega^2 R \sin^2\theta}{2g}\right]$$

Define reduced effective potential, β parameter:

$$U \equiv \frac{V_{eff}}{mgR} = \left(1-\cos\theta\right) - \frac{1}{2}\frac{\omega^2 R}{g}\sin^2\theta$$

$$\beta \equiv \frac{\omega^2 R}{g}$$

$$\rightarrow U = \left(1-\cos\theta\right) - \frac{1}{2}\beta\sin^2\theta = 2\sin^2\frac{\theta}{2} - \frac{\beta}{2}\left(1-\cos^2\theta\right) = 2\sin^2\frac{\theta}{2}\left(1-\beta\cos^2\frac{\theta}{2}\right)$$

SSB - V

Find equilibrium angles, identify stable and unstable:

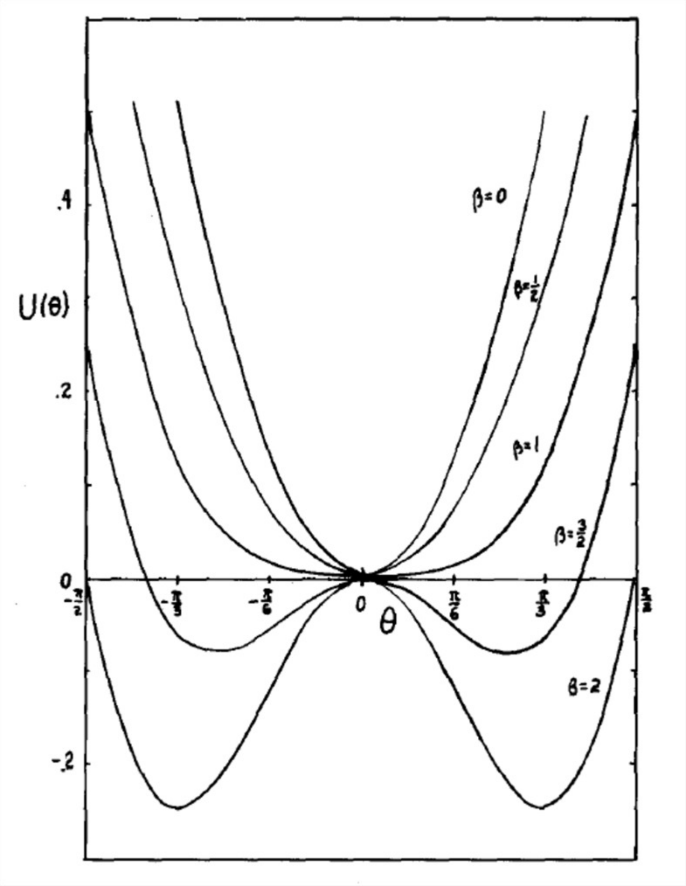
$$\frac{\partial U}{\partial \theta} = \sin\theta\left(1-\beta\cos\theta\right) = 0$$

$$\rightarrow \begin{cases} \cos\theta_0 = \dfrac{1}{\beta} \\ \theta_1 = 0 \end{cases}$$

$$\frac{\partial^2 U}{\partial \theta^2} = \cos\theta - \beta\cos 2\theta$$

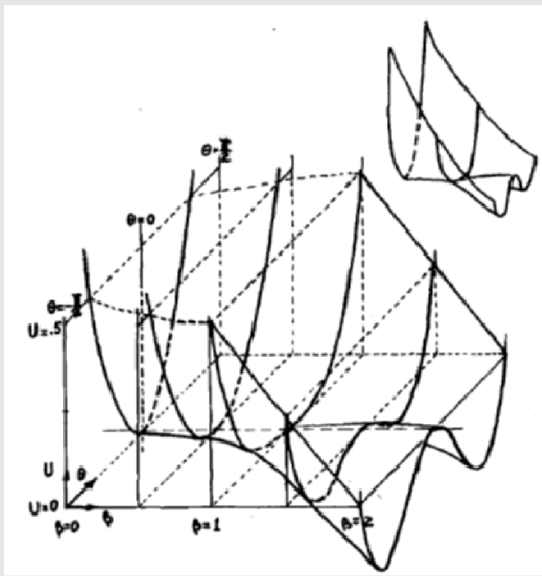
$$\left.\frac{\partial^2 U}{\partial \theta^2}\right|_{\theta=\theta_1} = 1-\beta \quad \text{stable for } \beta<1$$

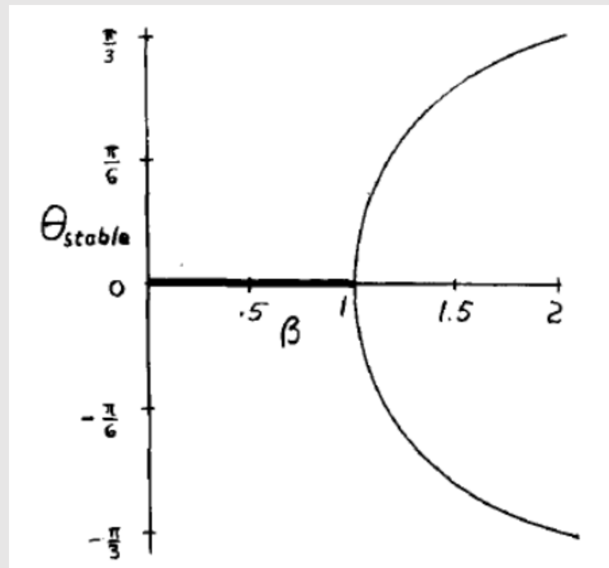
$$\left.\frac{\partial^2 U}{\partial \theta^2}\right|_{\theta=\theta_0} = \beta - \frac{1}{\beta} \quad \text{stable for } \beta>1$$



SSB - VI

Showing how shape of potential curve, equilibrium angle change with β





a) $\beta<1 \rightarrow 1$ eq. angle

$\beta>1 \rightarrow 2$ eq. angles: Cannot tell which one will be found

Reflection symmetry of V lost (← spontaneously broken) in the solution of eq. of motion

b) Small oscillations around equilibrium angle:

$\beta<1 \rightarrow OK$ Symmetrical wrt origin

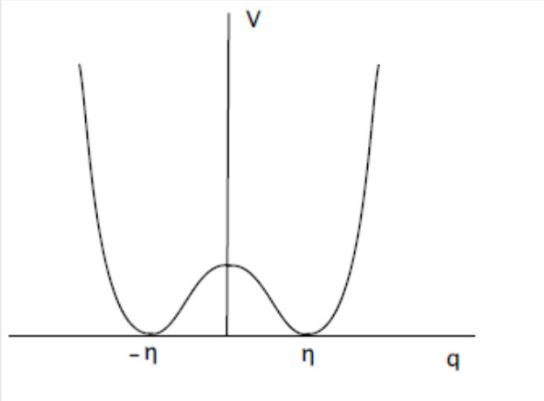
$\beta>1 \rightarrow KO$ Non symmetrical wrt origin

SSB - VII

Quantum Mechanics: Simple system with 1 degree of freedom:

$$L = \frac{1}{2} m \dot{x}^2 - \frac{1}{2} \lambda^2 \left(x^2 - \eta^2 \right)^2$$

$$V(x) = \frac{1}{2} \lambda^2 \left(x^2 - \eta^2 \right)^2$$



Potential: Parity symmetric

$$V(x) = V(-x)$$

Expand around $\pm\eta$ to quadratic terms only: Harmonic oscillator

$|+\rangle$ solution, centered on $+\eta$

$|-\rangle$ solution centered on $-\eta$

Naively:

Expect two degenerate ground states, both with undefined parity

SSB - VIII

But:

H not diagonal in this basis

$\langle + | H | + \rangle = \langle - | H | - \rangle = a, \langle + | H | - \rangle = \langle - | H | + \rangle = b$

Physical reason : Tunneling through central barrier

$\rightarrow$ Diagonalize, find:

Eigenstates	Energies
$\vert S\rangle = \vert +\rangle + \vert -\rangle$	$a+b$
$\vert A\rangle = \vert +\rangle - \vert -\rangle$	$a-b$

$|S\rangle, |A\rangle$: Parity eigenstates

$\rightarrow$ Degeneracy removed: Just 1 ground state

$E_{ground} = a - |b|$

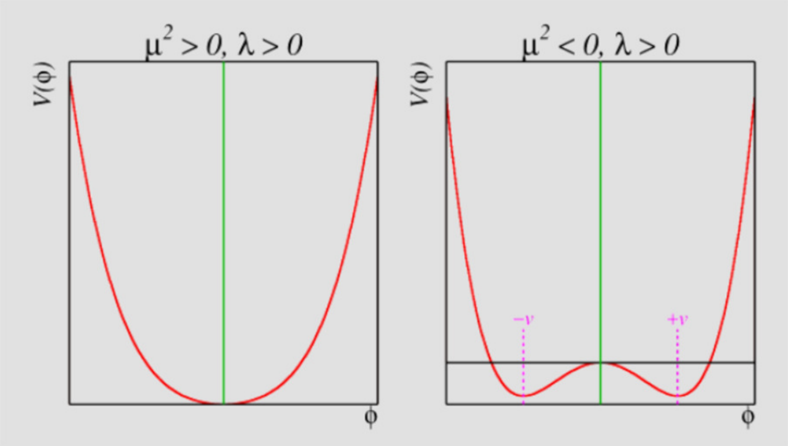
SSB - IX

Field theory: Real scalar field

$$L = \frac{1}{2}\left(\partial_\mu \phi\right)^2 - V(\varphi)$$

$$V(\varphi) = \frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda^2\phi^4, \lambda > 0$$

Reflection symmetric: $V(\varphi) = V(-\varphi)$



V Minima:

$$\mu^2 > 0 : \phi = 0$$

$$\mu^2 < 0 : \phi = \nu = \pm\sqrt{-\frac{\mu^2}{\lambda}}$$

V Minima: Defining *vacuum* state (← Cannot have less energy)

$\mu^2 > 0$: Vacuum (non degenerate) ≡ Zero field

$\mu^2 < 0$: Vacuum (degenerate!) = $\nu \neq$ Zero field !!

ν = Vacuum Expectation Value (VEV) of ϕ

SSB - X

Choose vacuum state:

$\langle \phi(x) \rangle_0 = v$ Spontaneous Symmetry Breakdown

Define: $\phi(x) = v + \eta(x)$

$$\rightarrow L = \frac{1}{2}(\partial_\mu \eta)^2 - \lambda\left(v^2\eta^2 - v\eta^3 - \frac{1}{4}\eta^4\right) = \left[\frac{1}{2}(\partial_\mu \eta)^2 - \lambda v^2 \eta^2\right] - \lambda v \eta^3 - \frac{1}{4}\lambda\eta^4$$

$$L = \frac{1}{2}(\partial_\mu \eta)^2 - \lambda v^2 \eta^2 \; \cancel{+\text{higher powers of } \eta}$$

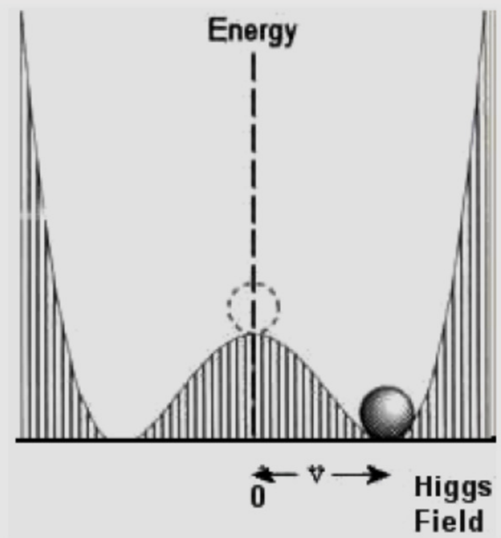
$\rightarrow$ Free Klein-Gordon equation $\rightarrow$ Scalar quantum field

$m^2 = 2\lambda v^2 \rightarrow m = \sqrt{-2\mu^2} > 0$

$\left[\text{Observe:} \mu^2 < 0 \rightarrow \text{Imaginary mass in original } L!\right]$

KO: $L(\eta) \neq L(-\eta)$

Reflection symmetry *spontaneously broken*



SSB - XI

What makes the difference between a single degree of freedom system and a field?

1 degree of freedom: Vacuum not degenerate

$\leftarrow$ Tunneling

∞ degrees of freedom: Vacuum degenerate

$\leftarrow$ Tunneling not effective

Indeed, it can be shown that:

$A_{tunnel} \propto e^{-aV}$, V system volume

$\rightarrow A_{tunnel} \sim 0$ for a (infinite) field

SSB - XII

Field theory: Complex scalar field

$$\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2), \phi^* = \frac{1}{\sqrt{2}}(\phi_1 - i\phi_2)$$

$$L = (\partial_\mu \phi)(\partial^\mu \phi)^* - V(\phi)$$

$$V(\phi) = \mu^2 \phi^* \phi + \lambda (\phi^* \phi)^2, \lambda > 0, \mu^2 < 0$$

$U(1)$ symmetric: $\phi \to \phi' = e^{i\alpha}\phi$

Observe: $U(1)$ *continuous* symmetry

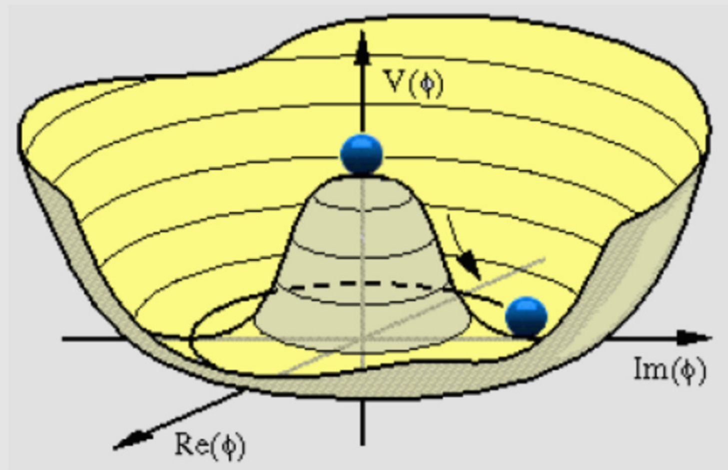
V Minima:

$$\mu^2 < 0 : \phi_1^2 + \phi_2^2 = -\frac{\mu^2}{\lambda}$$

$\rightarrow$ Vacuum infinitely degenerate

Choose vacuum = $(v, 0)$

$\rightarrow U(1)$ symmetry *spontaneously broken*



SSB - XIII

Define:

$$\phi(x) = \frac{1}{\sqrt{2}}\left[v + \xi(x) + i\eta(x)\right]$$

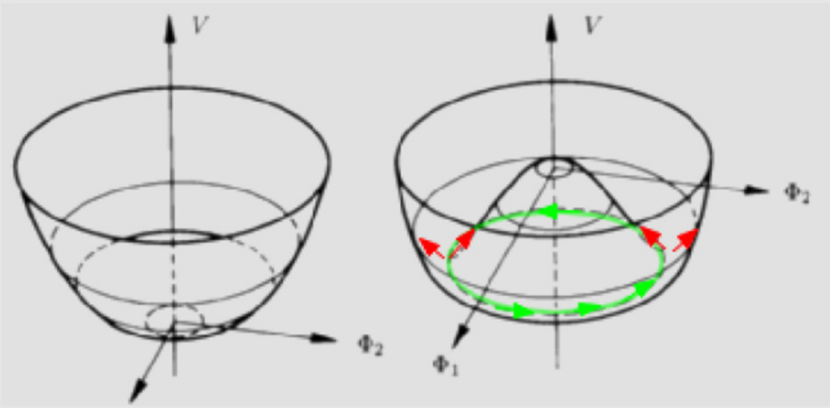
$$L = \frac{1}{2}\left[\left(\partial_\mu \xi\right)^2 + \left(\partial_\mu \eta\right)^2\right] + \mu^2\eta^2 \text{ ~~+higher powers of } \eta\text{~~}$$

Free Klein-Gordon equations for (ξ, η)

But: "Kinetic energy" terms for both ξ, η; Mass term only for η

$\rightarrow$ η field excitations: *massive* scalar particles

ξ field excitations: *massless* scalar particles, aka *Goldstone Bosons*



SSB - XIV

SSB of a continuous, global $(\leftarrow \text{non local})$ symmetry

$\rightarrow \infty$ degenerate vacuum states

Symmetry generators transform any vacuum state into another one

Indeed, for a non-degenerate vacuum: $\langle \phi \rangle_0$ Invariant under G

$\rightarrow e^{i\alpha G} \langle \phi \rangle_0 \simeq (1 + i\alpha G) \langle \phi \rangle_0 = \langle \phi \rangle_0 \leftrightarrow G \langle \phi \rangle_0 = 0$

$\rightarrow G$ does annihilate a non-degenerate vacuum

$\rightarrow$ Goldstone Theorem:

n generators not annihilating the vacuum $\rightarrow$ Appearance of n massless scalars

Also called *Goldstone bosons*

Example from condensed matter physics:

Ferromagnet $\rightarrow$ Rotational symmetry lost $\rightarrow$ Spin waves = Goldstones

SSB - XV

Example from condensed matter physics: Ferromagnet

Interaction rotationally invariant

But, below Curie temperature

$\longrightarrow$ Spontaneous magnetization in a random orientation

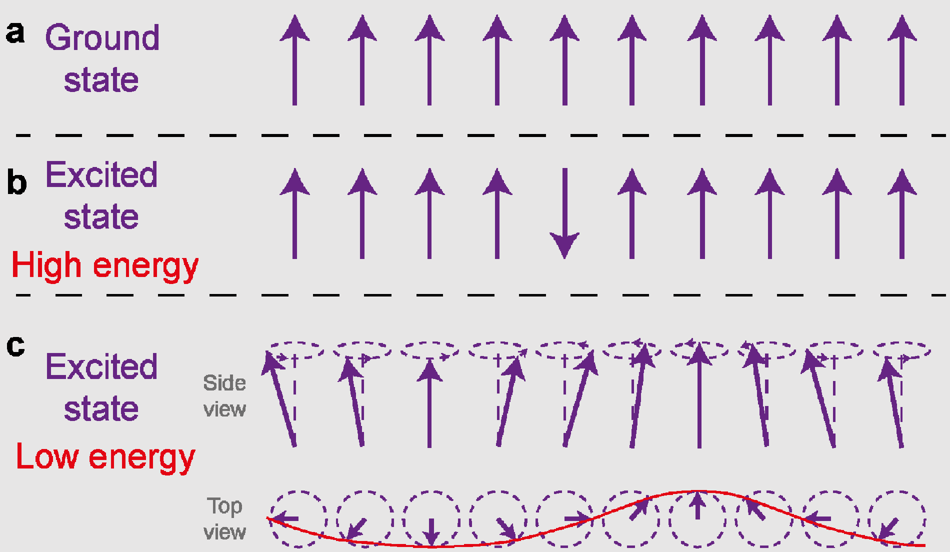
$\longrightarrow$ Rotational symmetry lost

$\longrightarrow$ Degenerate vacuum

$\longrightarrow$ Goldstones

Non-relativistic version of *massless*

Spin waves $\equiv$ Zero energy-gap *quasi - particles* (i.e. lattice excitation)



SSB - XVI

Would seem to definitely destroy our hint of a Standard Model:

$3+1$ massless gauge bosons, only one observed

4 massless scalar bosons, none observed

Local gauge invariance + SSB: Higgs mechanism, evading Goldstone's theorem

Simple, yet subtle way of giving mass to gauge bosons

without spoiling gauge invariance (and renormalizability)

Example by Higgs :

$U(1)$ gauge group, require *local* symmetry:

Gauge vector boson A_μ to be introduced, coupling to some current as usual

Now: Add "sombrero" potential for a complex, scalar field $\phi = \phi_1 + i\phi_2$

$$L = \underbrace{\left[\left(\partial_\mu - ieA_\mu\right)\phi^*\right]\left[\left(\partial^\mu + ieA^\mu\right)\phi\right]}_{EM \text{ interaction of (charged) } \phi} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \mu^2\phi^*\phi - \lambda\left(\phi^*\phi\right)^2 + \underbrace{\ldots\ldots}_{\substack{\text{Current-field} \\ \text{interaction etc}}}$$

As found before:

Degenerate vacuum state $\rightarrow$ SSB picks as vacuum state $(v,0)$

SSB - XVII

$\rightarrow \phi = v + \eta_1 + i\eta_2$

L written in terms of η_1, η_2 :

Upon quantization, 2 scalar particles $\rightarrow m_1 = \sqrt{2\lambda v^2}, m_2 = 0$

Plugging $\phi = v + \eta_1 + i\eta_2$ into L:

$$L = \frac{1}{2}\left(\partial_\mu \eta_1\right)\left(\partial^\mu \eta_1\right) - \frac{1}{2}\overbrace{2\lambda v^2}^{m_1^2}\eta_1^2 + \frac{1}{2}\left(\partial_\mu \eta_2\right)\left(\partial^\mu \eta_2\right)$$

$$+\underbrace{\frac{1}{2}\overbrace{(ev)^2}^{m_V^2} A^\mu A_\mu}_{Massive\ vector!} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \underbrace{evA^\mu \partial_\mu \eta_2}_{??} + \ldots$$

Attempting to understand L:

Massive vector field A^μ + Massive scalar field η_1 + Massless scalar field η_2

Troubling term coupling A^μ and η_2

SSB - XVIII

Use gauge invariance:

$$\begin{cases} \phi \to \phi' = e^{-ie\theta(x)}\phi \\ A^\mu \to A'^\mu = A^\mu + \partial_\mu \theta \end{cases}$$

Choose θ to make ϕ real: Then $\eta_2 \equiv 0$ $\left(\leftarrow \text{Unitary gauge}\right)$

$$\to L = \frac{1}{2}\left(\partial_\mu \eta_1\right)\left(\partial^\mu \eta_1\right) - \frac{1}{2}2\lambda v^2 \eta_1^2 + \underbrace{\frac{1}{2}\overbrace{(ev)^2}^{M_V^2} A^\mu A_\mu}_{Massive\ vector!} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \ldots$$

Massive vector field A^μ + Massive scalar field η_1

Counting degrees of freedom:

$$\underbrace{2}_{A_\mu} + \underbrace{1 + 1}_{\phi} = \underbrace{3}_{A_\mu} + \underbrace{1}_{\eta_1} \quad \text{OK}$$

Standard picture:

By effect of a smart gauge transformation, the massless vector field A_μ has eaten the Goldstone boson η_2 to become massive

SSB - XIX

Attempting to dissipate some misunderstandings likely to sneak in:

Mostly related to our naive perception of what is really a 'particle'

1) Where is the mass?

To identify mass terms in *L*: Not necessarily a trivial task

Key point: Particle content *only meaningful in perturbative expansion*

Example: $L = (\partial_\mu \phi)(\partial^\mu \phi)^* - (-\mu^2 \phi^* \phi) - \lambda (\phi^* \phi)^2, \lambda > 0, \mu^2 > 0$

$-\mu^2 \to$ Imaginary mass $\to$ Nonsense $\to$???

But: To use this form of *L* to extract Feynman rules, should expand around $|\phi| = 0$

Unstable extremum $\to$ Can't make it

Rewrite by expanding around $\eta = 0 : L = \frac{1}{2}(\partial_\mu \eta)^2 - \lambda \nu^2 \eta^2 + \cdots$

Stable extremum $\to$ OK

$\to$ Particle content should be identified in this form

SSB - XX

2)What's so special in unitary gauge?

Nothing: L invariant under local gauge transformations,

including the one to unitary gauge:

$\rightarrow$ L describe the same physics before and after the gauge transofrmation

But: Particle content much easier to extract in the unitary gauge

3) Disappearing Goldstones !?

Indeed: And re-appearing as extra degrees of freedom for massive gauge bosons

See comment above on the tricky business of defining what is a particle...

4)What decides which vacuum is selected among the many?

Not really relevant: Any choice yields identical results

5) Could we make it with the SM without SSB and all that complicated swapping of degrees of freedom?

Actually no: SSB is an *intrinsic* feature of certain quantum systems

More on Higgs field and particle later (last part of the lectures)

Standard Model - I

Higgs mechanism fixes troublesome, massless gauge bosons in the unified EW interaction

Boson counting:

Local gauge symmetry $SU(2)_L \otimes U(1)_Y \rightarrow$ 4 vector bosons

Will need 3 symmetries spontaneously broken to give mass to 3 weak bosons

Photon *is* massless

Extend Abelian Higgs model to non-Abelian gauge symmetry:

Introduce a doublet of complex, scalar fields:

$$\phi(x) = \begin{pmatrix} \phi^+(x) \\ \phi^0(x) \end{pmatrix}$$

Assuming $Y_W = 1 \rightarrow \begin{cases} Q\left[\phi^+(x)\right] = +1 \\ Q\left[\phi^0(x)\right] = 0 \end{cases}$

Standard Model - II

$SU(2)_L \otimes U(1)$ Gauge transformation of doublet:

$$\phi \rightarrow \phi' = \exp\left\{-i\left[\frac{g}{2}\boldsymbol{\alpha}(x)\cdot\boldsymbol{\tau} + \frac{g'}{2}y\theta(x)I\right]\right\}\phi$$

$SU(2)_L \otimes U(1)$ Covariant derivative:

$$D^\mu = \partial^\mu + i\left[\frac{g}{2}\boldsymbol{\tau}\cdot\mathbf{W}^\mu + \frac{g'}{2}yB^\mu\right]$$

$\rightarrow$ Additional term to EW lagrangian:

$$L_H = D_\mu\phi^\dagger D^\mu\phi - \mu^2\phi^\dagger\phi - \lambda\left(\phi^\dagger\phi\right)^2$$

Take $\mu^2 < 0, \;\; \lambda > 0$:

$$\left|\phi\right|^2_{\min} = -\frac{\mu^2}{2\lambda} = \frac{v^2}{2} \rightarrow v = \sqrt{-\frac{\mu^2}{\lambda}}$$

Pick ground state (= vacuum) as

$$\left\langle\phi\right\rangle_0 = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \rightarrow \text{SSB of Electroweak gauge symmetry}$$

Standard Model - III

Goldstone boson: Associated with each generator not annihilating the vacuum

Take generators of $SU(2)_L \otimes U(1)_Y$:

$$\tau_1 \langle \phi \rangle_0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} v \\ 0 \end{pmatrix} \neq 0$$

$$\tau_2 \langle \phi \rangle_0 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} = -\frac{i}{\sqrt{2}} \begin{pmatrix} v \\ 0 \end{pmatrix} \neq 0$$

$$\tau_3 \langle \phi \rangle_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ -v \end{pmatrix} \neq 0$$

$$Y \langle \phi \rangle_0 = +1 \langle \phi \rangle_0 \neq 0$$

But: $Q \langle \phi \rangle_0 = \frac{1}{2} (Y + \tau_3) \langle \phi \rangle_0 = 0$

$\rightarrow \langle \phi \rangle_0 : U(1)_Q$ Invariant $\rightarrow U(1)_Q$ symmetry unbroken $\rightarrow$ Photon stays massless

Standard Model - IV

As before for the Higgs model, rewrite:

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} \sigma_1 + i\sigma_2 \\ v + \eta_1 + i\eta_2 \end{pmatrix}$$

$$\rightarrow V = -\frac{\mu^4}{4\lambda} + \lambda v^2 \eta_1^2 + \lambda v \eta_1 \left(\sigma_1^2 + \sigma_2^2 + \eta_1^2 + \eta_2^2\right) + \frac{\lambda}{4}\left(\sigma_1^2 + \sigma_2^2 + \eta_1^2 + \eta_2^2\right)^2$$

3 massless scalars: $\sigma_1, \sigma_2, \eta_2 \leftarrow$ The Goldstones

1 massive scalar: $\eta_1, m_{\eta_1} \equiv m_H = \sqrt{2\lambda v^2} \leftarrow$ The Higgs

Gauge transformation suitable to get rid of 3 Goldstones:

$$\phi \rightarrow \phi' = U\phi = U\frac{1}{\sqrt{2}}\begin{pmatrix} \sigma_1 + i\sigma_2 \\ v + \eta_1 + i\eta_2 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + \eta_1 \end{pmatrix}$$

$$\rightarrow \begin{cases} SU(2)_L \text{ rotation of doublet to make it 'down'} \\ U(1)_Y \text{ re-phasing of doublet to make it real} \end{cases} \left(\leftarrow \text{Unitary gauge}\right)$$

Standard Model - V

Re-write gauge terms of L in the unitary gauge, in terms of the physical fields:

$$L_B + L_H$$

$$= -\frac{1}{4} F_{\mu\nu}(x) F^{\mu\nu}(x) \qquad \text{Photon}$$

$$-\frac{1}{2} F^{W}{}_{\mu\nu}(x) F^{W\dagger\mu\nu}(x) + \frac{1}{2} m_W^2 W_\mu^\dagger W^\mu \qquad W^\pm \text{ boson}$$

$$-\frac{1}{4} Z_{\mu\nu}(x) Z^{\mu\nu}(x) + \frac{1}{2} m_Z^2 Z_\mu Z^\mu \qquad Z^0 \text{ boson}$$

$$+\left(\partial_\mu \sigma\right)\left(\partial^\mu \sigma\right) - \frac{1}{2} m_H^2 \sigma^2 \qquad H \text{ Higgs boson}$$

$$+L_{BB}^I + L_{HH}^I + L_{HB}^I \qquad \text{Gauge-Higgs, Higgs self-, Gauge self-interactions}$$

Standard Model - VI

Finding the acquired mass of gauge bosons in terms of couplings and VEV of the Higgs field:

$$\left.\begin{aligned} m_{W^\pm} &= \frac{g}{2}\sqrt{-\frac{\mu^2}{\lambda}} \\ m_{Z^0} &= \frac{\sqrt{\left(g^2+g'^2\right)}}{2}\sqrt{-\frac{\mu^2}{\lambda}} \end{aligned}\right\} \rightarrow m_{Z^0} = m_{W^\pm}\sqrt{1+\frac{g'^2}{g^2}}$$

$$m_\gamma = 0$$

$$m_H = \sqrt{-2\mu^2} = ???$$

Model parameters v, λ and θ_W :

$$v = \sqrt{-\frac{\mu^2}{\lambda}}$$

$$\lambda = ???$$

$$g\sin\theta_W = g'\cos\theta_W = e$$

Standard Model - VII

Lepton masses: Different mechanism required

$\rightarrow$ Yukawa $\left(\text{scalar}\right)$ coupling

Describing interaction between Dirac and scalar fields:

$V \approx g\overline{\Psi}\phi\Psi$, Static limit:

$$V = -\frac{g}{4\pi}\frac{e^{-\mu r}}{r}$$

$$L_{HL} = -g_l\left[\overline{\Psi}_l^L\Phi\psi_l^R + \overline{\psi}_l^R\Phi^\dagger\Psi_l^L\right] - \underbrace{g_{\nu_l}}_{=0 \text{ for massless neutrino}}\left[\overline{\Psi}_l^L\tilde{\Phi}\psi_{\nu_l}^R + \overline{\psi}_{\nu_l}^R\tilde{\Phi}^\dagger\Psi_l^L\right], \quad \tilde{\Phi} = \begin{pmatrix} \phi_b^* \\ -\phi_a^* \end{pmatrix}$$

In the unitary gauge:

$$L_{HL} = -\frac{1}{v}m_l\overline{\psi}_l\,\psi_l\,\sigma - \frac{1}{v}m_{\nu_l}\overline{\psi}_{\nu_l}\psi_{\nu_l}\sigma$$

Lepton masses in terms of model parameters:

$$m_l = \frac{vg_l}{\sqrt{2}}, m_{\nu_l} = \frac{vg_{\nu_l}}{\sqrt{2}}$$

Standard Model - VIII

Model parameters:

$$g, g', -\mu^2, \lambda, g_l, g_{\nu_l}$$

Quite remarkably, get m_W, m_Z by measured constants:

$$\begin{cases} \alpha = \dfrac{1}{137.04} \\ G_F = 1.166\ 10^{-5}\, GeV^{-2} \\ \sin^2\theta_W = 0.23122 \end{cases}$$

$$\rightarrow m_W = 77.5\ \ GeV, m_Z = 88.4\ \ GeV$$

Experimental values:

$$m_W = 80.40\ GeV, m_Z = 90.19\ GeV$$

Difference originating from radiative corrections

Higgs:

$$m_H = \sqrt{-2\mu^2} = \sqrt{2\lambda v^2} \rightarrow ???$$

Standard Model - IX

Relating model parameters to measured constants $e, G_F, \sin\theta_W$:

$$g = \frac{e}{\sin\theta_W}, \quad g' = \frac{e}{\cos\theta_W}$$

$$G_F = \frac{\sqrt{2}}{8}\left(\frac{g}{M_W}\right)^2 \to \frac{8G_F}{\sqrt{2}} = \left(\frac{g}{M_W}\right)^2 \to \frac{M_W}{g} = \sqrt{\frac{\sqrt{2}}{8G_F}}$$

$$\to M_W = \sqrt{\frac{\sqrt{2}g^2}{8G_F}} = \sqrt{\frac{\sqrt{2}e^2}{8G_F\sin^2\theta_W}} = \frac{37.3}{\sin\theta_W} \quad GeV$$

$$M_Z = \frac{M_W}{\cos\theta_W} = \frac{37.3}{\sin\theta_W\cos\theta_W} \quad GeV$$

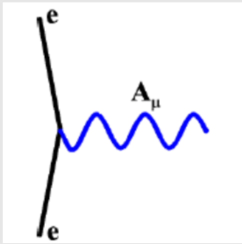
$$v = \frac{2M_W}{g} = 2\sqrt{\frac{\sqrt{2}}{8G_F}} = \frac{1}{\sqrt{\sqrt{2}G_F}} \simeq 246\ GeV \quad \text{VEV of the Higgs field}$$

$$\to \frac{v}{\sqrt{2}} = \frac{1}{\sqrt{\sqrt{2}G_F}}\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{\sqrt{8}G_F}} \approx 174\ GeV \to \langle\phi\rangle_0 = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \approx \begin{pmatrix} 0 \\ 174\ GeV \end{pmatrix}$$

No clues on $\lambda \to$ No $($direct$)$ prediction of $M_H = \sqrt{2v^2\lambda}$

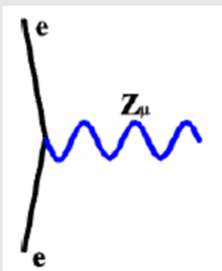
Standard Model - X

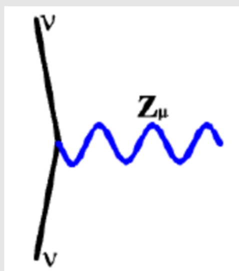
Lepton-Gauge Boson vertexes:



EM

$$-iQ_e\,\bar{e}\gamma^\mu e\,A_\mu$$

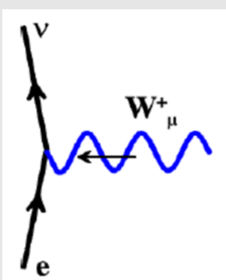


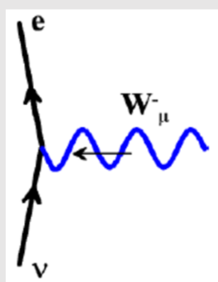


Neutral Current - NC

$$-i\frac{g}{4\cos\theta_W}\bar{e}\gamma^\mu\left[2\sin^2\theta_W\left(1+\gamma_5\right)+\left(2\sin^2\theta_W-1\right)\left(1-\gamma_5\right)\right]e\,Z_\mu.$$

$$-i\frac{g}{4\cos\theta_W}\bar{\nu}\gamma^\mu\left(1-\gamma_5\right)\nu\,Z_\mu.$$





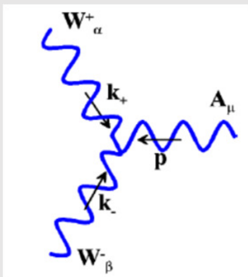
Charged Current - CC

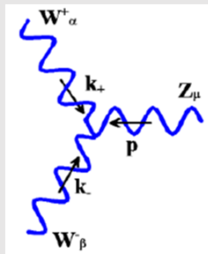
$$-i\frac{g}{2\sqrt{2}}\bar{\nu}\gamma^\mu\left(1-\gamma_5\right)eW_\mu^+$$

$$-i\frac{g}{2\sqrt{2}}\bar{e}\gamma^\mu\left(1-\gamma_5\right)\nu\,W_\mu^-$$

Standard Model - XI

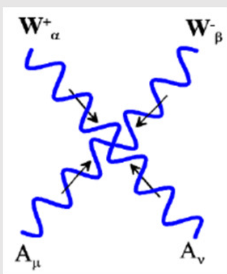
Gauge bosons self-interaction vertexes:

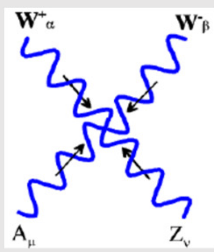




$$ig\sin\theta_W\left[g^{\alpha\beta}(k_+-k_-)^\mu+g^{\alpha\mu}(p-k_+)^\beta+g^{\beta\mu}(k_--p)^\alpha\right]W^+_\alpha W^-_\beta A_\mu.$$

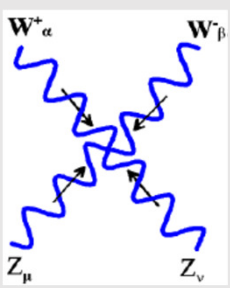
$$ig\cos\theta_W\left[g^{\alpha\beta}(k_+-k_-)^\mu+g^{\alpha\mu}(p-k_+)^\beta+g^{\beta\mu}(k_--p)^\alpha\right]W^+_\alpha W^-_\beta Z_\mu.$$

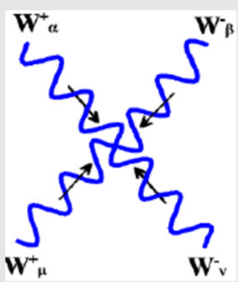




$$-ig^2\sin^2\theta_W\left[2g^{\mu\nu}g^{\alpha\beta}-g^{\mu\alpha}g^{\nu\beta}-g^{\mu\beta}g^{\alpha\nu}\right]\times W^+_\alpha W^-_\beta A_\mu A_\nu,$$

$$-ig^2\sin\theta_W\cos\theta_W\left[2g^{\mu\nu}g^{\alpha\beta}-g^{\mu\alpha}g^{\nu\beta}-g^{\mu\beta}g^{\alpha\nu}\right]W^+_\alpha W^-_\beta A_\mu Z_\nu.$$



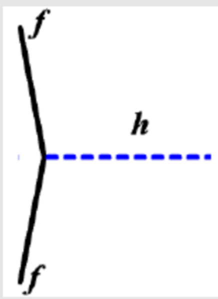


$$-ig^2\cos^2\theta_W\left[2g^{\mu\nu}g^{\alpha\beta}-g^{\mu\alpha}g^{\nu\beta}-g^{\mu\beta}g^{\alpha\nu}\right]W^+_\alpha W^-_\beta Z_\mu Z_\nu,$$

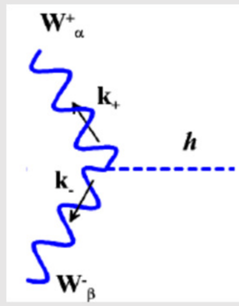
$$-ig^2\left[2g^{\mu\nu}g^{\alpha\beta}-g^{\mu\alpha}g^{\nu\beta}-g^{\mu\beta}g^{\alpha\nu}\right]W^+_\alpha W^-_\beta W^+_\mu W^-_\nu$$

Standard Model - XII

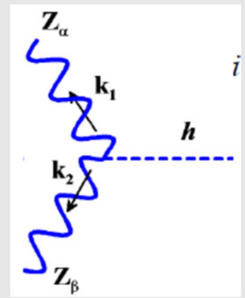
Higgs vertexes:



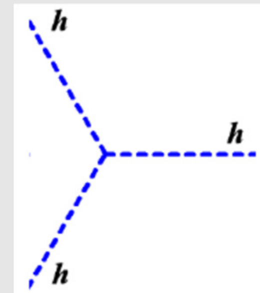
$-im_f \sqrt{\sqrt{2}G_F}\, \bar{f}\, f$



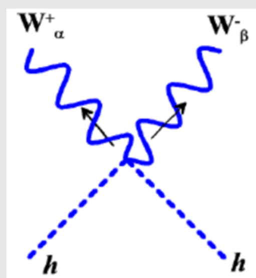
$2iM_W^2 \sqrt{\sqrt{2}G_F}\, g^{\alpha\beta} W_\alpha^{+\dagger} W_\beta^{-\dagger} h$



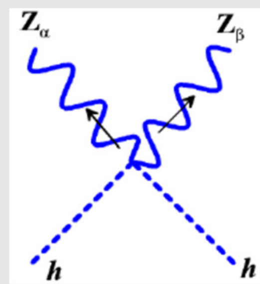
$2iM_Z^2 \sqrt{\sqrt{2}G_F}\, g^{\alpha\beta} Z_\alpha^\dagger Z_\beta^\dagger h$



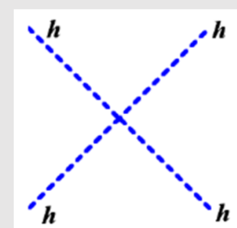
$-3im_h^2 \sqrt{\sqrt{2}G_F}\, hhh$



$2iM_W^2 \sqrt{2}G_F\, g^{\alpha\beta} hh W_\alpha^{+\dagger} W_\beta^{-\dagger}$



$2iM_Z^2 \sqrt{2}G_F\, g^{\alpha\beta} hh Z_\alpha^\dagger Z_\beta^\dagger$



$-3im_h^2 \sqrt{2}G_F\, hhhh$

Standard Model - XIII

Extension to 2nd, 3rd lepton family: Straightforward

Will need 2+2 = 4 new parameters (Yukawa couplings)

'Minimal' Standard Model:

Massless neutrinos $\rightarrow g_{\nu_l}^{(i)} = 0$

'Non Minimal' Standard Model:

Neutrino mixing $\left(\leftarrow \text{Require massive neutrinos, mixing matrix}\right)$:

Account for observed neutrino oscillations

May indicate physics beyond Standard Model

Extension to 3 quark families: Similar to leptons

Will need 6 more parameters

Will require CKM 'flavor rotation' (see later)

Strong interaction effects

} Flavor physics

Standard Model - XIV

Fermion electroweak quantum numbers:

helicity	Generations 1.	2.	3.	Quantum Numbers Q	T_3	Y_W
L	$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$	$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$	$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$	0 -1	$1/2$ $-1/2$	-1 -1
	$\begin{pmatrix} u \\ d' \end{pmatrix}_L$	$\begin{pmatrix} c \\ s' \end{pmatrix}_L$	$\begin{pmatrix} t \\ b' \end{pmatrix}_L$	$2/3$ $-1/3$	$1/2$ $-1/2$	$1/3$ $1/3$
R	e_R	μ_R	τ_R	-1	0	-2
	u_R d_R	c_R s_R	t_R b_R	$2/3$ $-1/3$	0 0	$4/3$ $-2/3$

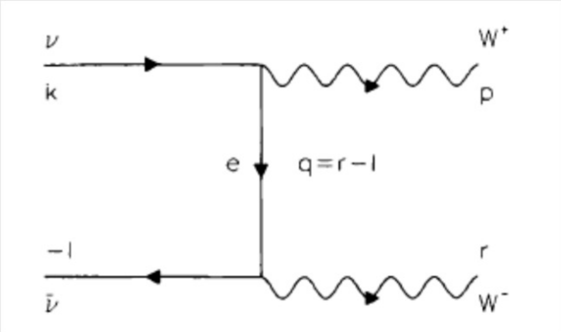
Standard Model - XV

Internal consistency of SM:

Reconsidering hypothetical, troublesome reaction

$\nu\bar{\nu} \to W_L^+ W_L^-$

at very high energy



Polarization 4-vectors of longitudinally polarized *W*s:

$$\varepsilon_L^\mu(p) = \frac{p^\mu}{m_W} + O\left(\frac{m_W}{p^0}\right) \sim \frac{p^\mu}{m_W}$$

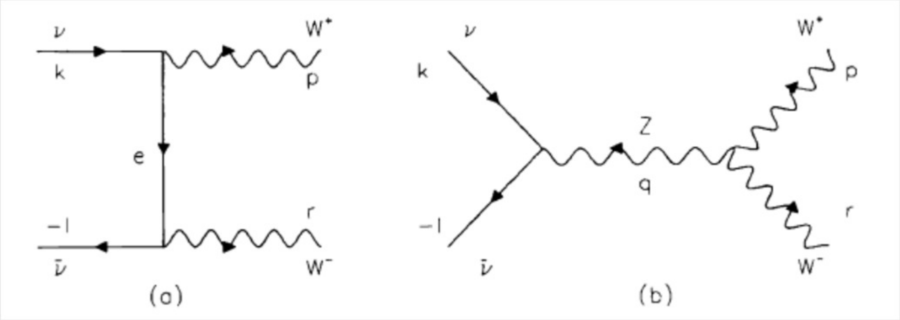
Divergent term of matrix element:

$$M_{fi} \approx -\frac{g^2}{8}\bar{v}(l)(1-\gamma_5)\frac{1}{\not{q}-m}(1-\gamma_5)u(k)\frac{r^\mu}{m_W}\frac{p^\nu}{m_W}$$

$$M_{fi} \approx -\frac{g^2}{4m_W^2}\bar{v}(l)\not{p}(1-\gamma_5)u(k) - \frac{g^2}{8m_W^2}m\bar{v}(l)(1+\gamma_5)\frac{\not{q}+m}{q^2-m^2}\not{p}(1-\gamma_5)u(k)$$

$$M_{fi}^a \approx -\frac{g^2}{4m_W^2}\bar{v}(l)\not{p}(1-\gamma_5)u(k)$$

Standard Model - XVI



Standard Model: Neutral Current → Two diagrams instead of one

$M_{fi}^{b} : Z^{0}$ matrix element:

$\nu\nu Z, WWZ$ vertexes, Z propagator

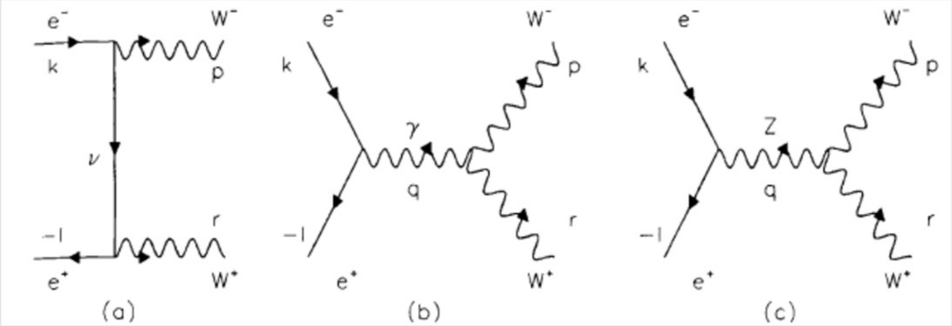
After quite intense calculations....

$$M_{fi}^{b} \approx \frac{g^{2}}{4m_{W}^{2}} \overline{v}(l) \not{p} (1-\gamma_{5}) u(k)$$

$$\rightarrow M_{fi}^{b} + M_{fi}^{b} = 0$$

Divergence fixed in a gauge theory!

Standard Model - XVII



Another, similar reaction

$e^+e^- \rightarrow W_L^+ W_L^-$

Quite realistic!

$$M_{fi}^a \approx -\frac{g^2}{4m_W^2}\bar{v}(l)\not{p}(1-\gamma_5)u(k)$$

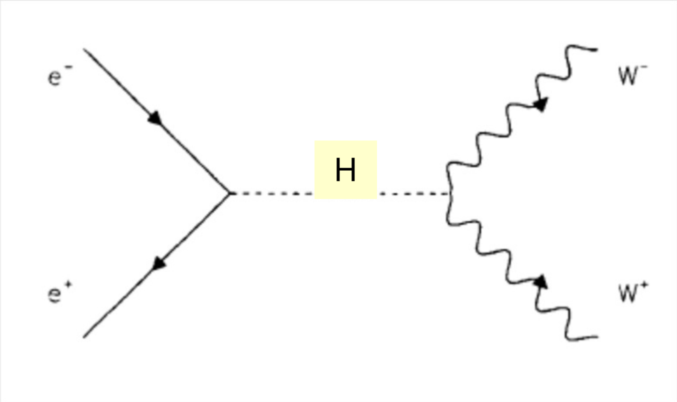
$$M_{fi}^b \approx \frac{e^2}{m_W^2}\bar{v}(l)\not{p}u(k)$$

$$M_{fi}^c \approx -\frac{g_{WWZ}}{2m_W^2}\bar{v}(l)\left[g_L\not{p}(1-\gamma_5)+g_R\not{p}(1+\gamma_5)\right]u(k)$$

$\rightarrow M_{fi}^a + M_{fi}^b + M_{fi}^c \approx --\frac{g^2}{4m_W^2}m\bar{v}(l)u(k)$ Still (weakly) divergent at high energy

Standard Model - XVIII

Reason of extra divergence: *R* chiral parts of massive fermions



Higgs diagram:

$$M_{fi}^{H} \approx -\frac{1}{2m_W^2} g_{eeH}\, g_{WWH} \overline{v}(l) u(k)$$

$\rightarrow$ Correct compensation with gauge theory & SSB

Strong support for the Standard Model:

Higgs *must* be there

(or something really new must happen at $\sim$ 1 TeV to save unitarity)

Neutral Current - I

As a result of the weak-electromagnetic unification, neutral currents are *different* from charged

Lorentz structure not $V-A$

$$-i\frac{g_W}{\sqrt{2}}\gamma^\mu\frac{\left(1-\gamma^5\right)}{2} \quad \text{Charged}$$

$$-ig_Z\gamma^\mu\frac{\left(C_V^f-C_A^f\gamma^5\right)}{2} \quad \text{Neutral}$$

Coupling

Fermion	C_V	C_A
ν_e,ν_μ,ν_τ	$+1/2$	$+1/2$
e,μ,τ	$-1/2+2\sin\theta_W$	$-1/2$
u,c,t	$+1/2-4/3\sin^2\theta_W$	$+1/2$
d,s,b	$-1/2+2/3\sin^2\theta_W$	$-1/2$

θ_W new fundamental constant

What about interaction strength?

Neutral Current - II

Tight relationship between weak and electromagnetic interactions

Coupling constants:

$$g_w = \frac{e}{\sin\theta_W} \qquad \text{Charged currents}$$

$$g_z = \frac{e}{\sin\theta_W \cos\theta_W} \qquad \text{Neutral currents}$$

Charged, neutral and electromagnetic couplings fixed by 2 universal constants:

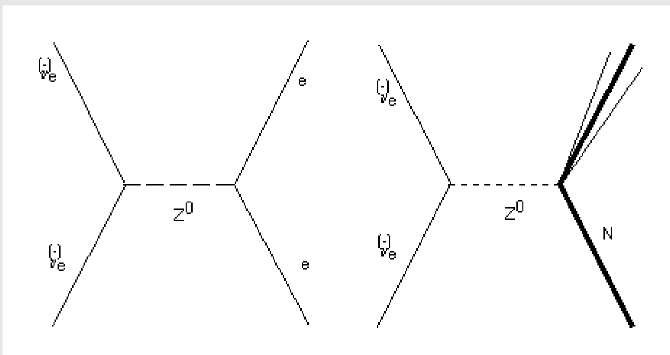
e : Elementary charge

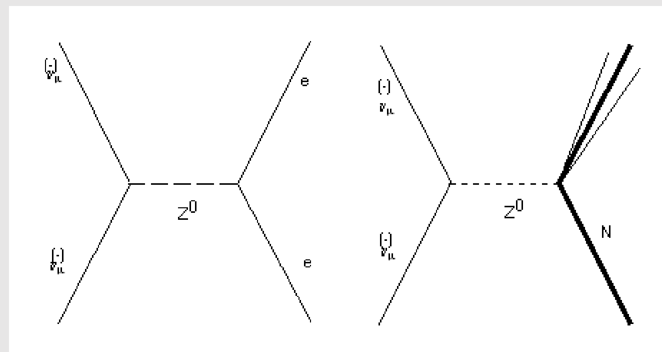
θ_W: Weinberg angle, new fundamental constant

$\sin^2\theta_W = 0.23122 \pm 0.00015$

Neutral Current - III

Expect to observe typical neutrino processes like:





$(\nu_e, \overline{\nu}_e) + e \rightarrow (\nu_e, \overline{\nu}_e) + e$ Contributing to elastic scattering

$$\left.\begin{aligned}&(\nu_\mu, \overline{\nu}_\mu) + e \rightarrow (\nu_\mu, \overline{\nu}_\mu) + e \\ &(\nu_e, \overline{\nu}_e) + N \rightarrow (\nu_e, \overline{\nu}_e) + hadron\ \ shower \\ &(\nu_\mu, \overline{\nu}_\mu) + N \rightarrow (\nu_\mu, \overline{\nu}_\mu) + hadron\ \ shower\end{aligned}\right\}\quad \text{New}$$

W & Z - I

Some reminescences about photons...

Free photons ($j^\mu = 0$): $\quad \Box^2 A^\mu - \partial^\mu \left(\partial_\nu A^\nu\right) = j^\mu$

Lorentz condition

$\partial_\mu A^\mu = 0 \rightarrow \Box^2 A^\mu = 0$

$\rightarrow A^\mu = \varepsilon^\mu (q) e^{-iqx} \rightarrow q^2 = 0$ massless quanta

4 components ε^μ ??

$a)$ $\partial_\mu A^\mu = 0 \rightarrow \varepsilon^\mu (q) q_\mu = 0 \rightarrow 3$ components

$b)$ Gauge freedom:

$A^\mu \rightarrow A^\mu + \partial^\mu \Lambda, \ \Box^2 \Lambda = 0$

$\Lambda = iae^{-iqx} \left(\leftarrow \ \Box^2 \Lambda = q^2 \Lambda = 0 \text{ OK}\right)$

$\rightarrow \partial^\mu \Lambda = ia\partial^\mu e^{-iqx}$

$A^\mu \rightarrow A^\mu + \partial^\mu \Lambda = \varepsilon^\mu (q) e^{-iqx} + ia\partial^\mu e^{-iqx} = \left[\varepsilon^\mu (q) + ia\left(-iq_\mu\right)\right] e^{-iqx} = \left[\varepsilon^\mu (q) + aq_\mu\right] e^{-iqx}$

$\rightarrow$ EM field unchanged by $\varepsilon^\mu (q) \rightarrow \varepsilon^\mu (q) + aq^\mu$

Choose a to make $\varepsilon^0 = 0$

$\rightarrow \varepsilon^\mu (q) q_\mu = 0 \rightarrow \boldsymbol{\varepsilon} \cdot \mathbf{q} = 0 \rightarrow 2$ components

W & Z - II

2 components $\rightarrow$ 2 independent ε^{μ}

Take photon momentum along z

$\varepsilon_1^{\mu} = \begin{pmatrix} 0 & 1 & 0 & 0 \end{pmatrix}$ x - linear polarization

$\varepsilon_2^{\mu} = \begin{pmatrix} 0 & 0 & 1 & 0 \end{pmatrix}$ y - linear polarization

or

$\varepsilon_L^{\mu} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & -i & 0 \end{pmatrix}$ Left circular polarization: $S_z = -1$

$\varepsilon_R^{\mu} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & +i & 0 \end{pmatrix}$ Right circular polarization: $S_z = +1$

W & Z - III

Original wave equation:

$$\Box^2 A^\mu - \partial^\mu \left(\partial_\nu A^\nu\right) = j^\mu$$

For a massive vector boson:

$$\left(\Box^2 + m^2\right) B^\mu - \partial^\mu \left(\partial_\nu B^\nu\right) = j^\mu$$

Free particle:

$$\left(\Box^2 + m^2\right) B^\mu - \partial^\mu \left(\partial_\nu B^\nu\right) = 0$$

But:

$$\partial_\mu \left(\Box^2 + m^2\right) B^\mu - \partial_\mu \partial^\mu \left(\partial_\nu B^\nu\right) = 0 \rightarrow \left(\Box^2 + m^2\right) \partial_\mu B^\mu - \Box^2 \left(\partial_\nu B^\nu\right) = 0$$

$$\rightarrow m^2 \partial_\mu B^\mu = 0 \rightarrow \partial_\mu B^\mu = 0$$

Bottom line: Not an extra condition...

$$\rightarrow \left(\Box^2 + m^2\right) B^\mu = 0$$

$B^\mu = \varepsilon^\mu \left(p\right) e^{-ipx} \rightarrow \varepsilon^\mu p_\mu = 0 \rightarrow$ 3 independent components

No gauge freedom...

W & Z - IV

3 independent components $\to$ 3 independent ε^μ

Take photon momentum along z

$$\varepsilon_L^\mu = \frac{1}{\sqrt{2}}\begin{pmatrix}0 & 1 & -i & 0\end{pmatrix} \text{ Left circular polarization: } S_z = -1$$

$$\varepsilon_R^\mu = -\frac{1}{\sqrt{2}}\begin{pmatrix}0 & 1 & +i & 0\end{pmatrix} \text{ Right circular polarization: } S_z = +1$$

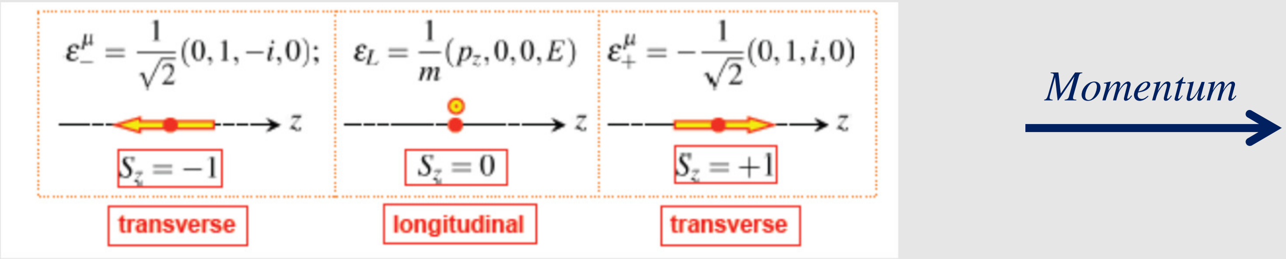
To find 3rd polarization 4-vector:

$$\varepsilon_0^\mu = \frac{1}{\sqrt{\left|\alpha^2-\beta^2\right|}}\begin{pmatrix}\alpha & 0 & 0 & \beta\end{pmatrix}, \quad \frac{1}{\sqrt{\left|\alpha^2-\beta^2\right|}} \quad \text{normalization}: \varepsilon^\mu p_\mu = 0 \to \alpha E - \beta p_z = 0$$

$$\to \alpha = p_z, \beta = E \to \varepsilon_0^\mu = \frac{1}{m}\begin{pmatrix}p_z & 0 & 0 & E\end{pmatrix} \text{ Longitudinal polarization: } S_z = 0$$

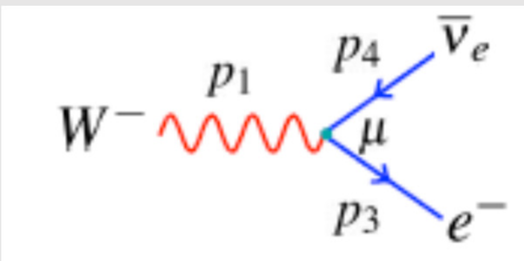
Observe: Longitudinal/Transverse boson $\to$ Transverse/Longitudinal spin...

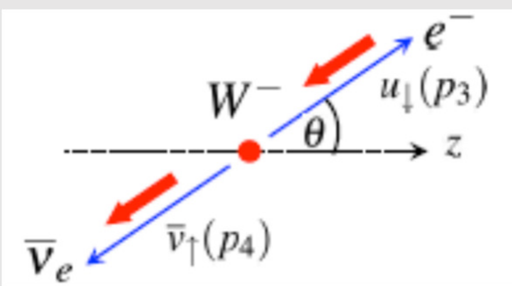
As for photons: Attribute refers to oscillating 'electric/magnetic field', rather than spin



W & Z - V

Decay: $W^- \to e^- + \overline{\nu}_e$





Matrix element:

$$M_{fi} = \frac{g}{\sqrt{2}} \varepsilon_\mu (p_1) \overline{u}(p_3) \gamma^\mu \frac{1}{2}(1-\gamma_5) v(p_4) = \frac{g}{\sqrt{2}} \varepsilon_\mu (p_1) j^\mu$$

$\overline{u}(p_3)$: outgoing fermion, $v(p_4)$: outgoing antifermion

$$\overline{u}(p_3)\gamma^\mu \frac{1}{2}(1-\gamma_5)v(p_4) = \overline{u}(p_3)\frac{1}{2}\gamma^\mu(1-\gamma_5)\frac{1}{2}\gamma^\mu(1-\gamma_5)v(p_4)$$

$$= \underbrace{\overline{u}(p_3)\frac{1}{2}(1+\gamma_5)}_{\overline{e}_L}\gamma^\mu \underbrace{\frac{1}{2}(1-\gamma_5)v(p_4)}_{\nu_R} = \overline{e}_L \gamma^\mu \nu_R$$

LR current :

Build from rotated L, R spinors

$$j^\mu = \overline{u}_\downarrow (p_3) \frac{1}{2} \gamma^\mu v_\uparrow (p_4) = 2E(0, -\cos\theta, -i, \sin\theta)$$

W & Z - VI

W polarization states in the rest system:

$$\varepsilon_L^\mu = \frac{1}{\sqrt{2}}\begin{pmatrix}0 & 1 & -i & 0\end{pmatrix}$$

$$\varepsilon_R^\mu = -\frac{1}{\sqrt{2}}\begin{pmatrix}0 & 1 & +i & 0\end{pmatrix}$$

$$\varepsilon_0^\mu = \frac{1}{m}\begin{pmatrix}0 & 0 & 0 & m\end{pmatrix} = \begin{pmatrix}0 & 0 & 0 & 1\end{pmatrix}$$

Matrix elements for different W polarization states in the rest system:

$$\varepsilon_L^\mu : \quad \frac{g}{\sqrt{2}}\frac{1}{\sqrt{2}}\begin{pmatrix}0 & 1 & -i & 0\end{pmatrix} 2\frac{M_W}{2}\left(0,-\cos\theta,-i,\sin\theta\right) = -\frac{gM_W}{2}\left(1+\cos\theta\right)$$

$$\rightarrow \left|M_L\right|^2 = \frac{g^2 M_W^2}{4}\left(1+\cos\theta\right)^2$$

$$\varepsilon_R^\mu : \quad \frac{g}{\sqrt{2}}\left[-\frac{1}{\sqrt{2}}\begin{pmatrix}0 & 1 & +i & 0\end{pmatrix}\right] 2\frac{M_W}{2}\left(0,-\cos\theta,-i,\sin\theta\right) = -\frac{gM_W}{2}\left(1-\cos\theta\right)$$

$$\rightarrow \left|M_R\right|^2 = \frac{g^2 M_W^2}{4}\left(1-\cos\theta\right)^2$$

$$\varepsilon_0^\mu : \quad \frac{g}{\sqrt{2}}\frac{1}{m}\begin{pmatrix}0 & 0 & 0 & m\end{pmatrix} 2\frac{M_W}{2}\left(0,-\cos\theta,-i,\sin\theta\right) = \frac{gM_W}{\sqrt{2}}\sin\theta$$

$$\rightarrow \left|M_0\right|^2 = \frac{g^2 M_W^2}{2}\sin^2\theta$$

W & Z - VII

$$|M_L|^2 = \frac{g^2 M_W^2}{4}(1+\cos\theta)^2$$

$$|M_R|^2 = \frac{g^2 M_W^2}{4}(1-\cos\theta)^2$$

$$|M_0|^2 = \frac{g^2 M_W^2}{2}\sin^2\theta$$

2-body differential decay rate:

$$\frac{d\Gamma_{L,0,R}}{d\Omega} = \frac{p}{32\pi^2 M_W^2}|M|^2 = \frac{1}{64\pi^2 M_W}|M|^2 = \frac{g^2 M_W}{64\pi^2}\begin{cases}\frac{1}{4}(1+\cos\theta)^2\\ \frac{1}{2}\sin^2\theta\\ \frac{1}{4}(1-\cos\theta)^2\end{cases}$$

Total rates:

$$\int \frac{1}{4}(1\pm\cos\theta)^2\, d\cos\theta d\varphi = \int \frac{1}{2}\sin^2\theta d\cos\theta d\varphi = \frac{4\pi}{3}$$

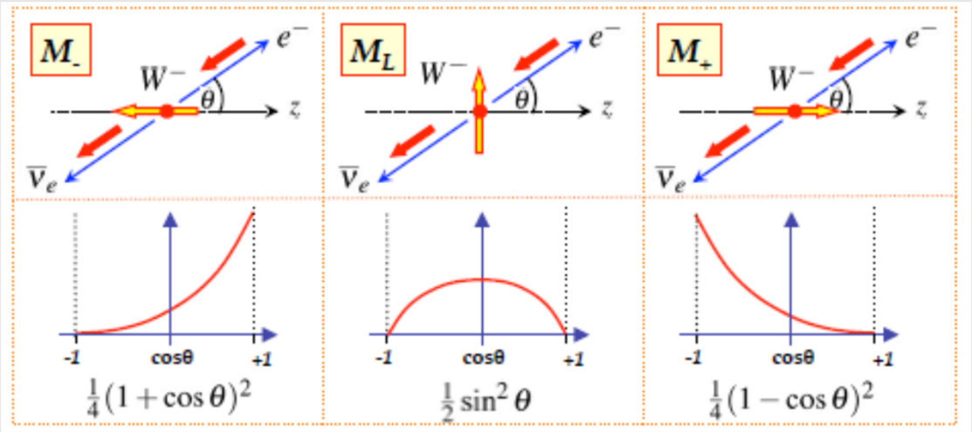
$$\rightarrow \Gamma_L = \Gamma_R = \Gamma_0 = \frac{g^2 M_W}{48\pi}$$

W & Z - VIII

$$|M_L|^2 = \frac{g^2 M_W^2}{4}(1+\cos\theta)^2$$

$$|M_R|^2 = \frac{g^2 M_W^2}{4}(1-\cos\theta)^2$$

$$|M_0|^2 = \frac{g^2 M_W^2}{2}\sin^2\theta$$



Averaging over the initial spin states:

$$\left\langle |M|^2 \right\rangle = \frac{1}{3} g^2 M_W^2 \left[\frac{1}{4}(1+\cos\theta)^2 + \frac{1}{4}(1-\cos\theta)^2 + \frac{1}{2}\sin^2\theta \right]$$

$$\rightarrow \left\langle |M|^2 \right\rangle = \frac{1}{3} g^2 M_W^2$$

Isotropic: OK for an unpolarized mother particle

$$\rightarrow \Gamma\left(W^- \rightarrow e^- + \bar{\nu}_e\right) = \frac{g^2 M_W}{48\pi}$$

W & Z - IX

Considering all the others decay modes: Large W mass $\rightarrow$ All fermions $\approx$ massless

Do *not* count Top: Too heavy, decay energetically forbidden

Color factor = 3

Similar to $e^+e = \rightarrow q\overline{q}$: Take quarks as free, on shell particles

Taking into account CKM mixing:

$$W^- \rightarrow e^- \overline{\nu}_e \qquad W^- \rightarrow d\overline{u} \quad \times 3|V_{ud}|^2 \qquad W^- \rightarrow d\overline{c} \quad \times 3|V_{cd}|^2$$

$$W^- \rightarrow \mu^- \overline{\nu}_\mu \qquad W^- \rightarrow s\overline{u} \quad \times 3|V_{us}|^2 \qquad W^- \rightarrow s\overline{c} \quad \times 3|V_{cs}|^2$$

$$W^- \rightarrow \tau^- \overline{\nu}_\tau \qquad W^- \rightarrow b\overline{u} \quad \times 3|V_{ub}|^2 \qquad W^- \rightarrow b\overline{c} \quad \times 3|V_{cb}|^2$$

CKM Unitarity:

$$e.g. \quad |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 \quad \text{etc}$$

$$\rightarrow \Gamma_W = 9\Gamma_{W\rightarrow e\nu} = \frac{3g^2 M_W}{16\pi} = 2.07 \ GeV$$

Experiment :

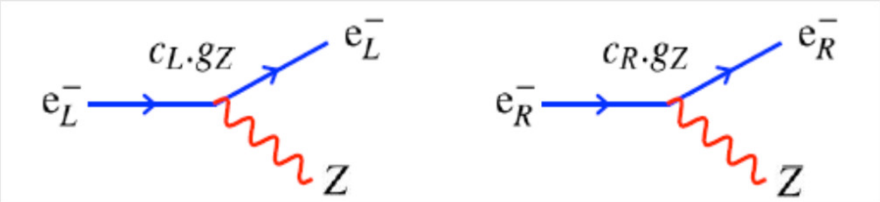
$2.14 \pm 0.04 \ GeV$

QCD corrections..

W & Z - X

Z couplings:

$$c_L = I_3 - Q\sin^2\theta_W$$
$$c_R = -Q\sin^2\theta_W$$
$$j_\mu^Z = g_Z \bar{u}\gamma_\mu \left[c_L \frac{1}{2}(1-\gamma_5) + c_R \frac{1}{2}(1+\gamma_5)\right]u$$



$$c_V = c_L + c_R = I_3 - 2Q\sin^2\theta_W$$
$$c_A = c_L - c_R = I_3$$

W & Z - XI

Therefore:

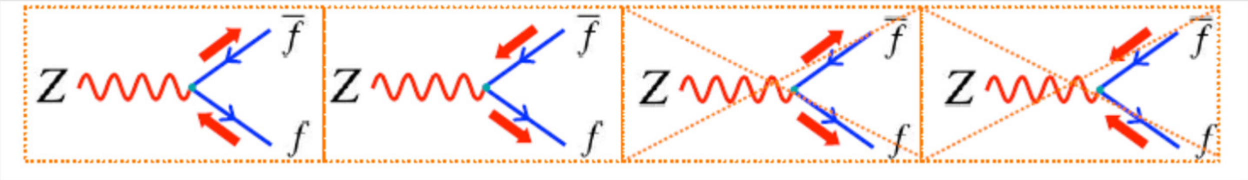
$\sin^2\theta_W \approx 0.23$ ⇨

Fermion	Q	I_W^3	c_L	c_R	c_V	c_A
ν_e, ν_μ, ν_τ	0	$+\frac{1}{2}$	$+\frac{1}{2}$	0	$+\frac{1}{2}$	$+\frac{1}{2}$
e^-, μ^-, τ^-	-1	$-\frac{1}{2}$	-0.27	0.23	-0.04	$-\frac{1}{2}$
u, c, t	$+\frac{2}{3}$	$+\frac{1}{2}$	0.35	-0.15	$+0.19$	$+\frac{1}{2}$
d, s, b	$-\frac{1}{3}$	$-\frac{1}{2}$	-0.42	0.08	-0.35	$-\frac{1}{2}$

W & Z - XII

Z couplings: Both to L and R fermions

Nevertheless:
Only 2 vertexes, remaining 2 = 0



To show that RR vertex is 0 (LL similar):

$$\bar{u}_R = u_R^\dagger \gamma^0 = u^\dagger \frac{1+\gamma^5}{2}\gamma^0, \quad v_R = \frac{1-\gamma^5}{2} v$$

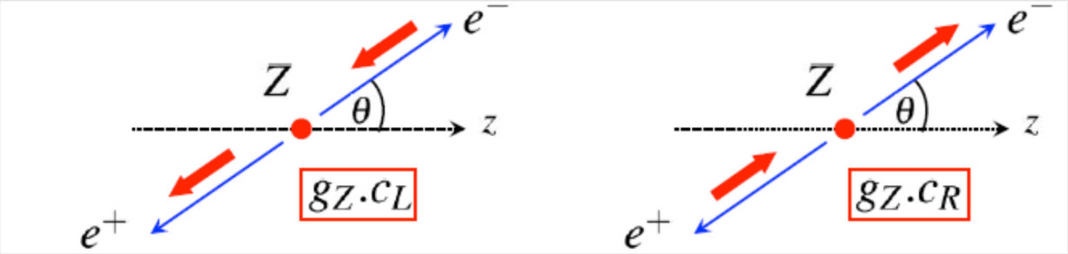
$$\bar{u}_R \gamma^\mu (c_V + c_A\gamma_5) v_R = u^\dagger \frac{1+\gamma^5}{2}\gamma^0\gamma^\mu (c_V + c_A\gamma_5)\frac{1-\gamma^5}{2} v$$

$$= u^\dagger \gamma^0 \frac{1-\gamma^5}{2}\gamma^\mu \frac{1-\gamma^5}{2}(c_V + c_A\gamma_5) v$$

$$= \bar{u}\gamma^\mu \underbrace{\frac{1+\gamma^5}{2}\frac{1-\gamma^5}{2}}_{=0}(c_V + c_A\gamma_5) v = 0$$

W & Z - XIII

Decay: $Z^0 \to e^+ + e^-$



$$\left\langle \left|M\right|^2 \right\rangle = \frac{2}{3} g^2 \cos^2\theta_W M_Z^2 \left[c_L^2 + c_R^2\right]$$

$$2\left[c_L^2 + c_R^2\right] = \left[c_V^2 + c_A^2\right]$$

$$\to \Gamma\left(Z \to e^+ e^-\right) = \frac{g^2 \cos^2\theta_W M_Z}{48\pi}\left[c_V^2 + c_A^2\right]$$

W & Z - XIV

$$Br(Z \to e^+e^-) = Br(Z \to \mu^+\mu^-) = Br(Z \to \tau^+\tau^-) \approx 3.5\%$$
$$Br(Z \to \nu_1\bar{\nu}_1) = Br(Z \to \nu_2\bar{\nu}_2) = Br(Z \to \nu_3\bar{\nu}_3) \approx 6.9\%$$
$$Br(Z \to d\bar{d}) = Br(Z \to s\bar{s}) = Br(Z \to b\bar{b}) \approx 15\%$$
$$Br(Z \to u\bar{u}) = Br(Z \to c\bar{c}) \approx 12\%$$

$$Br(Z \to \text{hadrons}) \approx 69\%$$

$$\to \Gamma_Z = \sum_i \Gamma_i = 2.5 \quad GeV$$

$$\text{Experiment: } \Gamma_Z = 2.4952 \pm 0.0023 \quad GeV$$

!!!

Fall 2017

E.Menichetti - Universita' di Torino

W & *Z* - XV

Ultimate systematics….

Moon:

- As the moon orbits the Earth it distorts the rock in the Geneva area very slightly !
- The nominal radius of the accelerator of 4.3 km varies by ±0.15 mm
- Changes beam energy by ~10 MeV : need to correct for tidal effects !

Trains:

- Leakage currents from the TGV railway line return to Earth following the path of least resistance.
- Travelling via the Versoix river and using the LEP ring as a conductor.
- Each time a TGV train passed by, a small current circulated LEP slightly changing the magnetic field in the accelerator
- LEP beam energy changes by ~10 MeV

Neutral Currents Discovery - I

Predicted by Glashow-Salam-Weinberg model ('60s)

Not really accepted for a long time:

Mostly because of strong suppression of strangeness changing decays like:

$K^0 \rightarrow \mu^+\mu^- \quad BR \ < 10^{-8}$

not accounted for. Compare:

$K^+ \rightarrow \mu^+\nu_\mu \quad BR \ 63.4\ \%$

Also because it was not clearly demonstrated that GSW was renormalizable

Two breakthroughs:

GIM prediction of charm to solve the $K^0 \rightarrow \mu^+\mu^-$ puzzle ('70)

GSW model shown to be renormalizable by 't Hooft ('71)

$\rightarrow$ Sudden wave of interest in gauge theories

Neutral Currents Discovery - II

Main interest = Prediction of new phenomena

Most shocking prediction of GSW: neutral currents, never seen before

$\rightarrow$ Try to find neutral currents to validate GSW

Best opportunity :

High energy neutrino interactions

Larger cross sections

No EM background

Drawback:

Neutrino experiments difficult

Neutral Currents Discovery - III

Neutrino beams

Take 2 body decays of π, K obtained from a high energy proton machine

Kinematics:

$$\pi^{\pm}, K^{\pm} \to \mu^{\pm} + \overset{(-)}{\nu}_{\mu}$$

$\beta, \gamma, M, \lvert\mathbf{p}\rvert$	K, π	LAB
$p^*, E_\mu{}^*, \theta_\mu{}^*$	μ	CM
p_μ, E_μ, θ_μ	μ	LAB

$$\lvert\mathbf{p}_\nu{}^*\rvert = E_\nu^* = \frac{M_{\pi,K}^2 - m_\mu^2}{2M_{\pi,K}} \simeq \begin{cases} 30 \\ 236 \end{cases} \text{MeV}$$

CM : Isotropic decay

$$\frac{dP}{d\left(\cos\theta^*\right)} = \frac{1}{2} \to \frac{dP}{dE} = \frac{dP}{d\left(\cos\theta^*\right)} \frac{d\left(\cos\theta^*\right)}{dE}$$

Fall 2017

E.Menichetti - Universita' di Torino

Neutral Currents Discovery - IV

Lorentz transform to LAB:

$$E = \gamma\left(E^* + \beta p^* \cos\theta^*\right) \rightarrow dE = \gamma\beta p^* d\left(\cos\theta^*\right) \rightarrow d\left(\cos\theta^*\right) = \frac{dE}{\gamma\beta p^*}$$

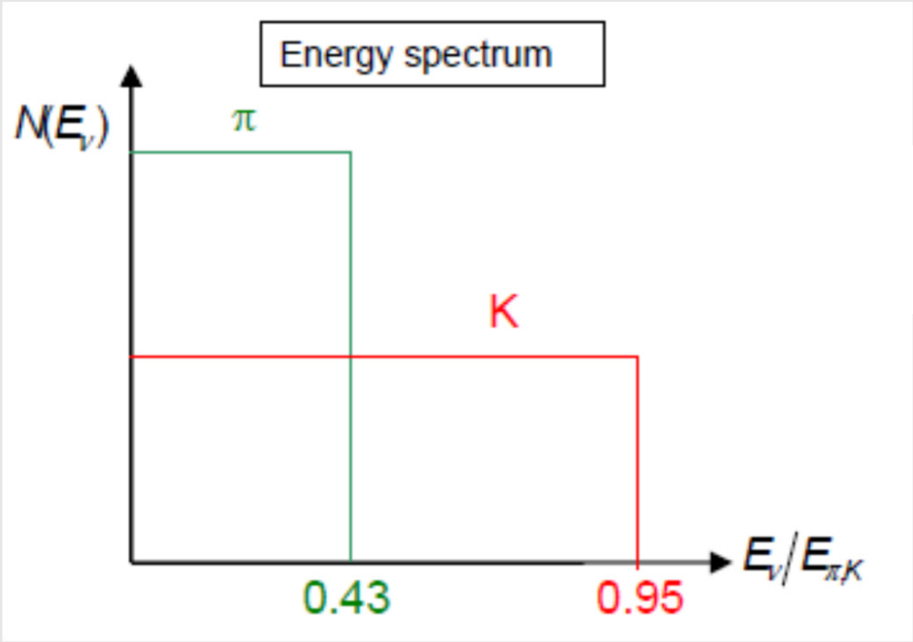
$$\rightarrow \frac{dP}{dE} = \frac{1}{2\gamma\beta p^*}$$

Flat distribution over wide interval:
$$\begin{cases} \gamma(1+\beta) E^* = \gamma(1+\beta)\dfrac{M^2_{\pi,K} - m^2_\mu}{2M_{\pi,K}} & \text{max} \\ \gamma(1-\beta) E^* = \gamma(1-\beta)\dfrac{M^2_{\pi,K} - m^2_\mu}{2M_{\pi,K}} & \text{min} \end{cases}$$

$$\frac{dN}{dE} = \frac{M_{\pi,K}}{\gamma\beta\left(M^2_{\pi,K} - m^2_\mu\right)} = \frac{1}{\underbrace{\gamma\beta M_{\pi,K}}_{|\mathbf{p}|}\left(1 - \dfrac{m^2_\mu}{M^2_{\pi,K}}\right)} = \frac{1}{|\mathbf{p}|\left(1 - \dfrac{m^2_\mu}{M^2_{\pi,K}}\right)}$$

Neutral Currents Discovery - V

→ Broad LAB ν_μ energy distribution

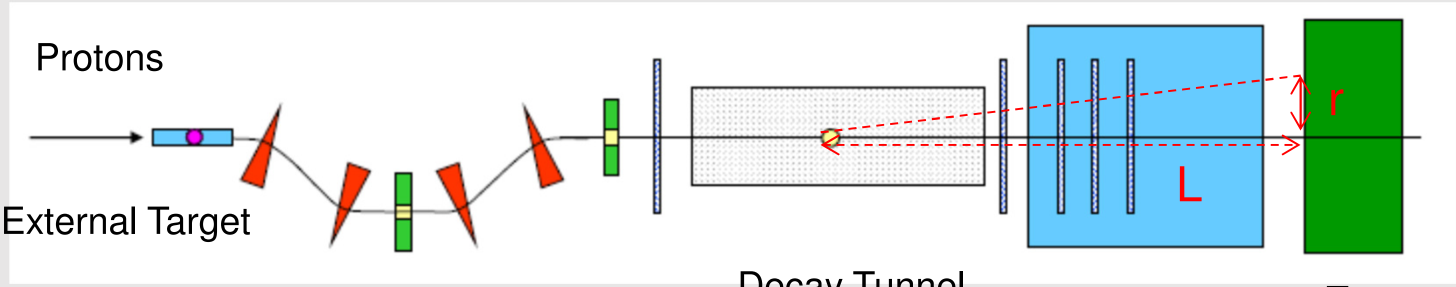


Neutral Currents Discovery - VI

a) Narrow Band Beam: ν energy known, low intensity
Magnetic selection of a narrow π/K momentum window

Magnetic Analysis & Selection:
Dipoles, Quads, Momentum Slits

Absorber
Muon Monitor



Decay Tunnel
Hadron Monitor

ν Target
&
Detector

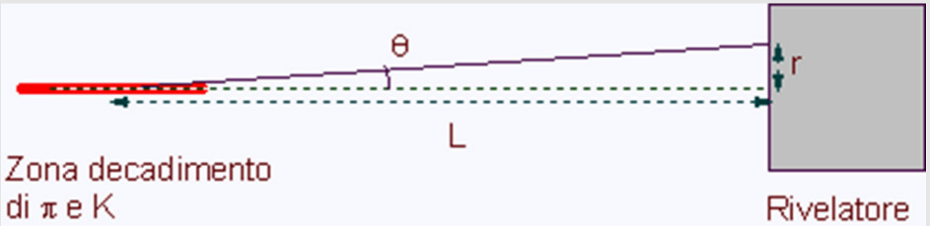
π/K momentum well defined

Beam/Detector geometry: $r<<L$

Neutral Currents Discovery - VI

Measure ν energy by direction:

Exploit hadron beam $\sim$ monocromaticity



$$p_\pi = p_\mu + p_\nu \rightarrow p_\mu = p_\pi - p_\nu \rightarrow \left(p_\mu\right)^2 = \left(p_\pi - p_\nu\right)^2$$

$$m_\mu^2 = m_\pi^2 - 2p_\pi \cdot p_\nu \rightarrow m_\pi^2 - m_\mu^2 = 2\left(E_\pi E_\nu - \mathbf{p}_\pi \cdot \mathbf{p}_\nu\right)$$

$$E_\nu = \frac{m_{\pi,K}^2 - m_\mu^2}{2\left(E_{\pi,K} - p_{\pi,K}\cos\theta_\nu\right)} = \frac{m_\pi^2 - m_\mu^2}{2E_{\pi,K}\left(1 - \beta\cos\theta_\nu\right)}$$

$$\tan\theta_\nu = \frac{\sin\theta_\nu^{\ *}}{\gamma\left(\cos\theta_\nu^{\ *} + \beta\right)} \rightarrow \tan\theta_{\max} = \frac{\sin\frac{\pi}{2}}{\gamma\left(\cos\frac{\pi}{2} + \beta\right)} = \frac{1}{\beta\gamma} \approx \frac{1}{\gamma} = \frac{m_{\pi,K}}{|\mathbf{p}|} \ll 1$$

$$E_\nu = \frac{m_{\pi,K}^2 - m_\mu^2}{2E_{\pi,K}\left(1 - \beta\cos\theta\right)} \approx \frac{m_{\pi,K}^2 - m_\mu^2}{2E_{\pi,K}\left(1 - \beta\left(1 - \frac{\theta^2}{2}\right)\right)}$$

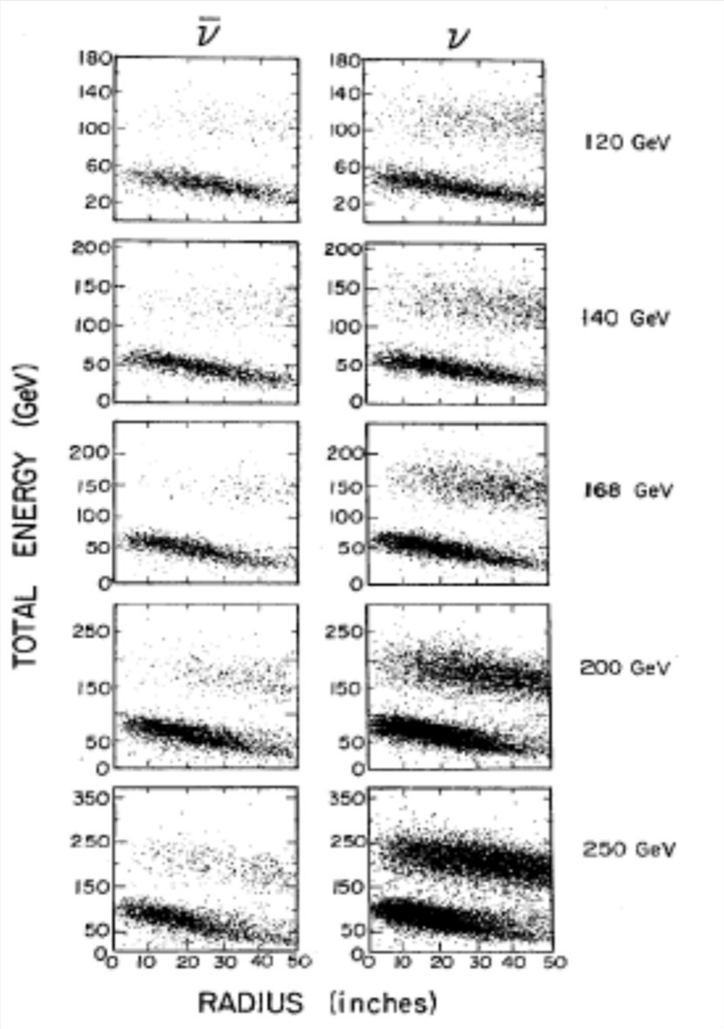
Neutral Currents Discovery - VII

$$E_\nu = \frac{m_{\pi,K}^2 - m_\mu^2}{2E_{\pi,K}\left(1-\beta+\beta\frac{\theta^2}{2}\right)} = \frac{m_{\pi,}^2 - m_\mu^2}{2E_{\pi,K}\left(\frac{1-\beta^2}{1+\beta}+\beta\frac{\theta^2}{2}\right)}$$

$$E_\nu \simeq \frac{m_{\pi,K}^2 - m_\mu^2}{2E_{\pi,K}\left(\frac{1-\beta^2}{2}+\frac{\theta^2}{2}\right)}, \beta \approx 1$$

$$E_\nu = \frac{m_{\pi,K}^2 - m_\mu^2}{E_{\pi,K}\left(\frac{1}{\gamma^2}+\theta^2\right)} = \frac{m_{\pi,K}^2\left(1-\frac{m_\mu^2}{m_{\pi,K}^2}\right)E_{\pi,K}}{E_{\pi,K}^2\left(\frac{1}{\gamma^2}+\theta^2\right)}$$

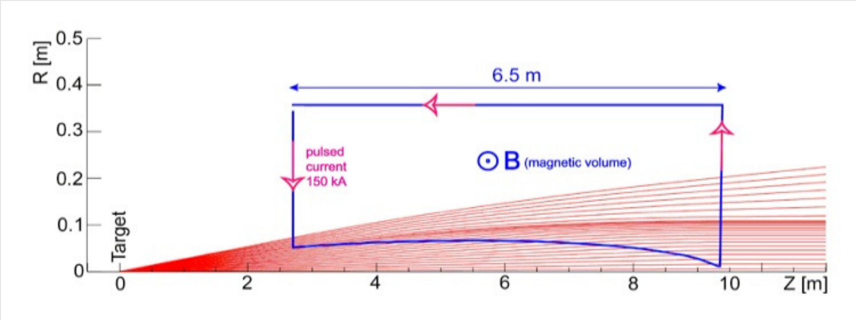
$$E_\nu = \frac{m_{\pi,K}^2\left(1-\frac{m_\mu^2}{m_{\pi,K}^2}\right)E_{\pi,K}}{\gamma^2 m_{\pi,K}^2\left(\frac{1}{\gamma^2}+\theta^2\right)} = E_{\pi,K}\frac{\left(1-\frac{m_\mu^2}{m_{\pi,K}^2}\right)}{\left(1+\gamma^2\theta^2\right)}$$

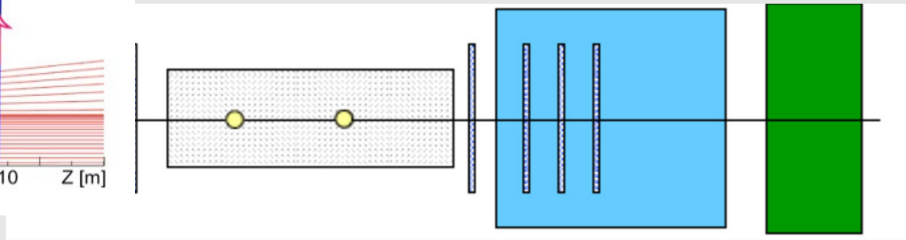


Neutral Currents Discovery - VIII

b) Wide Band Beam: ν energy unknown, high intensity

Replace magnetic selection by a special focussing device, suitable to make a low divergence hadron beam out of an uncollimated, divergent source: Van der Meer Horn

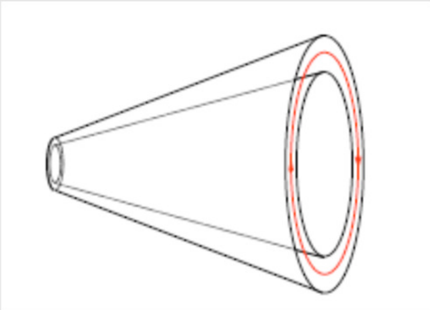


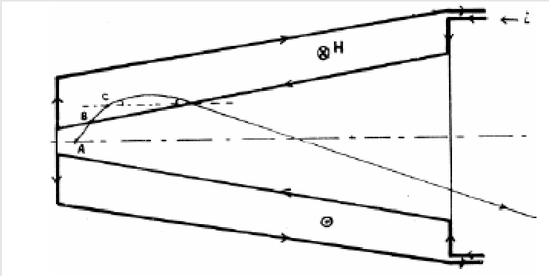


Collect a wide momentum window, focus into a narrow, intense beam

Neutral Currents Discovery - IX

2 conical (high) current sheets:
Equivalent to many trapezoidal current loops symmetrically placed around the axis
→ Circular magnetic field (red circumference)





Trajectory in the B field: $\sim$ Circular arc

$$|\mathbf{p}| = 0.3BR \quad \begin{cases} p & GeV \\ B & T \\ R & m \end{cases}$$

Deflection after a path length l in the field:

$$\Delta\theta = \frac{\Delta l}{R} = 0.3B\frac{\Delta l}{|\mathbf{p}|}$$

Deflection should compensate $\langle p_T \rangle$ of hadrons coming out of the target:

$\langle p_T \rangle \sim p\Delta\theta \sim 0.2\ GeV$ at PS energies

$\rightarrow 0.2\ GeV \sim |\mathbf{p}|\Delta\theta = 0.3B\Delta l$

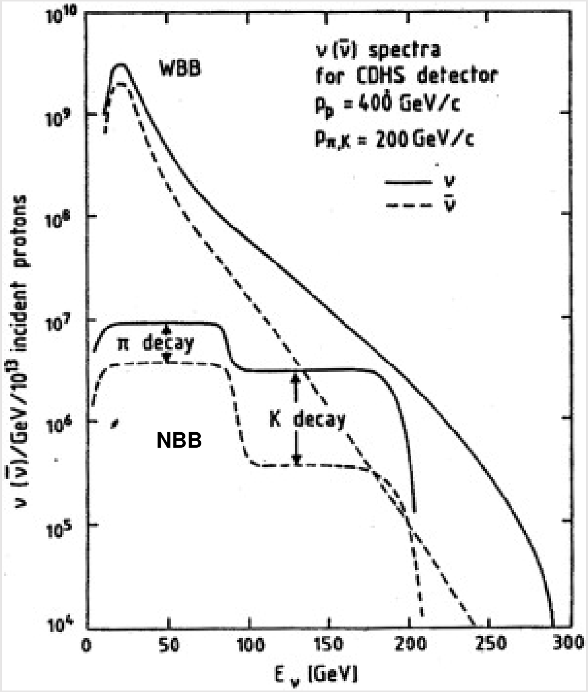
Simple guess:

$$B = \frac{\mu_0 I}{2\pi r} \rightarrow 0.2\ GeV \sim |\mathbf{p}|\Delta\theta \sim 0.3\frac{\mu_0 I}{2\pi r}\Delta l$$

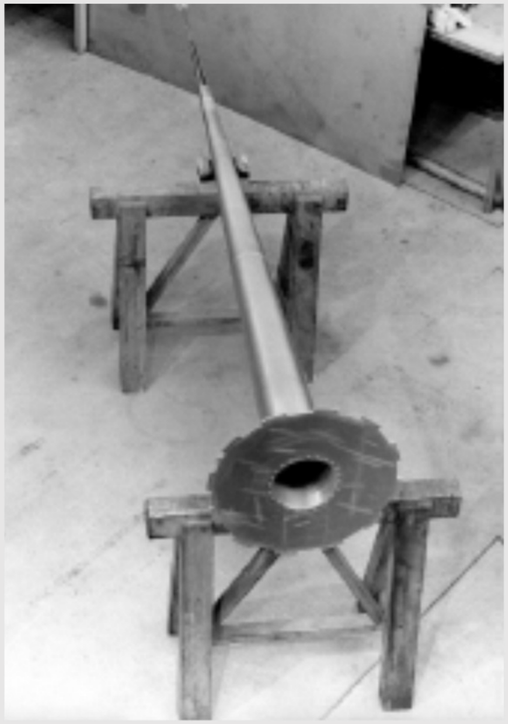
$$\rightarrow I \sim \frac{|\mathbf{p}|\Delta\theta 2\pi r}{0.3\mu_0 \Delta l} \sim 10^5 A!$$

Neutral Currents Discovery - X

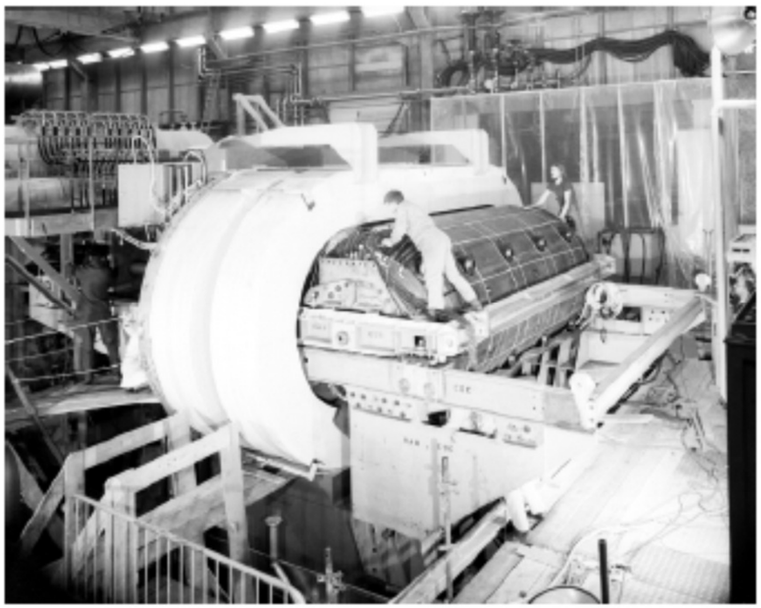
Narrow/Wideband beam spectra
SPS beam



The Gargamelle horn



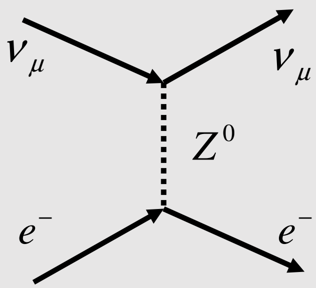
Neutral Currents Discovery - XI



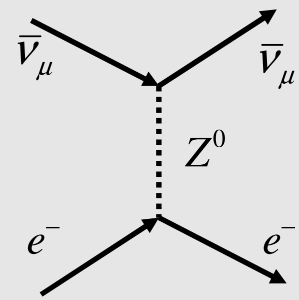
Length: 4.8 m
Diameter: 2 m
Liquid Freon: 12 m^3

Neutral Currents Discovery - XII

ν-e processes



Pure NC

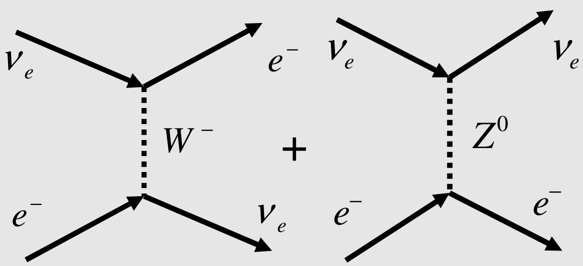


$$\sigma^{NC}_{\nu_\mu e^-}(E) = \frac{G_F^{\,2} s}{\pi}\left[\left(-\frac{1}{2}+\sin^2\theta_W\right)^2+\frac{1}{3}\sin^4\theta_W\right]$$

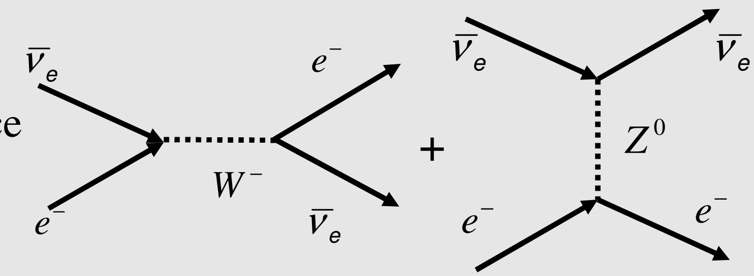
0.16x10^{-41} E(GeV) cm^2

$$\sigma^{NC}_{\bar{\nu}_\mu e^-}(E) = \frac{G_F^{\,2} s}{\pi}\left[\frac{1}{3}\left(-\frac{1}{2}+\sin^2\theta_W\right)^2+\sin^4\theta_W\right]$$

0.13x10^{-41} E(GeV) cm^2



Interference
CC+NC



$$\sigma^{CC+NC}_{\nu_e e^-}(E) = \frac{G_F^{\,2} s}{\pi}\left[\left(\frac{1}{2}+\sin^2\theta_W\right)^2+\frac{1}{3}\sin^4\theta_W\right]$$

0.96x10^{-41} E(GeV)

$$\sigma^{CC+NC}_{\bar{\nu}_e e^-}(E) = \frac{G_F^{\,2} s}{\pi}\left[\frac{1}{3}\left(\frac{1}{2}+\sin^2\theta_W\right)^2+\sin^4\theta_W\right]$$

0.40x10^{-41} E(GeV) cm^2

Neutral Currents Discovery - XIII

Effective couplings for several reactions

Reaction	ε	Electroweak theory g_V	Electroweak theory g_A	V–A theory g_V	V–A theory g_A
$\nu_\mu + e^- \to \nu_\mu + e^-$	+1	$-\frac{1}{2} + 2s^2$	$-\frac{1}{2}$	0	0
$\bar{\nu}_\mu + e^- \to \bar{\nu}_\mu + e^-$	−1	$-\frac{1}{2} + 2s^2$	$-\frac{1}{2}$	0	0
$\nu_e + e^- \to \nu_e + e^-$	+1	$+\frac{1}{2} + 2s^2$	$+\frac{1}{2}$	1	1
$\bar{\nu}_e + e^- \to \bar{\nu}_e + e^-$	−1	$+\frac{1}{2} + 2s^2$	$+\frac{1}{2}$	1	1
$\nu_\mu + e^- \to \mu^- + \nu_e$	+1	1	1	1	1

Neutral Currents Discovery - XIV

Differential cross sections:

$$y = 1 - \frac{E_\nu '}{E_\nu} \simeq \frac{E_e}{E_\nu} \quad \text{Bjorken } y$$

$$\frac{d\sigma_{\nu_\mu e}}{dy} = \frac{G_F^2 m_e}{2\pi} E_\nu \left[\left(g_V + g_A\right)^2 + \left(g_V - g_A\right)^2 (1-y)^2 + \frac{m_e}{E_\nu}\left(g_A^2 - g_V^2\right) y \right]$$

$$\frac{d\sigma_{\bar{\nu}_\mu e}}{dy} = \frac{G_F^2 m_e}{2\pi} E_\nu \left[\left(g_V - g_A\right)^2 + \left(g_V + g_A\right)^2 (1-y)^2 + \frac{m_e}{E_\nu}\left(g_A^2 - g_V^2\right) y \right]$$

$$\int_0^1 (1-y)^2 dy = \frac{1}{3}, \quad \int_0^1 y dy = \frac{1}{2}$$

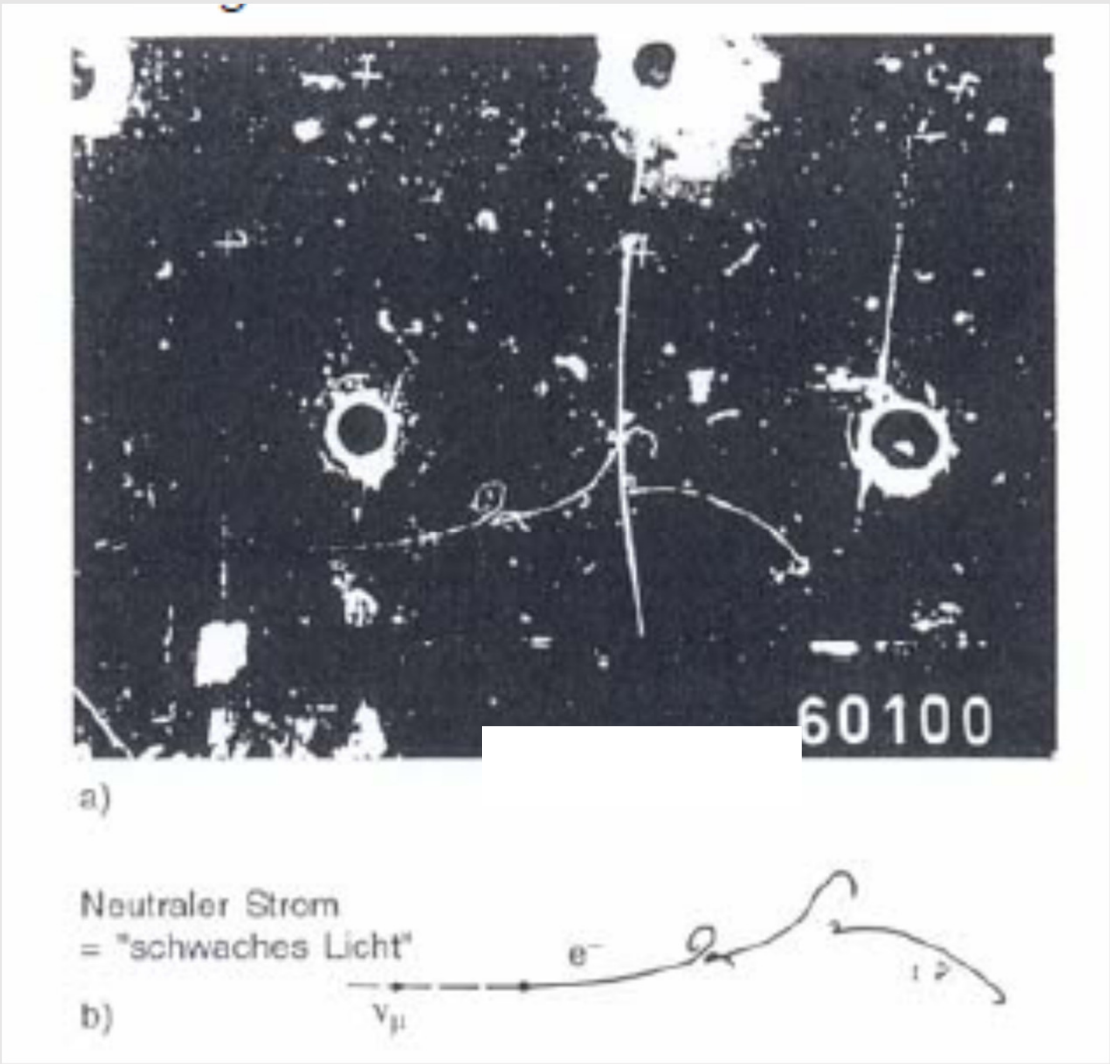
Total cross sections:

$$\rightarrow \sigma_{\nu_\mu e} = \frac{G_F^2 m_e}{2\pi} E_\nu \left[\left(g_V + g_A\right)^2 + \frac{1}{3}\left(g_V - g_A\right)^2 + \frac{m_e}{E_\nu}\frac{1}{2}\left(g_A^2 - g_V^2\right) \right]$$

$$\rightarrow \sigma_{\bar{\nu}_\mu e} = \frac{G_F^2 m_e}{2\pi} E_\nu \left[\left(g_V - g_A\right)^2 + \frac{1}{3}\left(g_V + g_A\right)^2 + \frac{m_e}{E_\nu}\frac{1}{2}\left(g_A^2 - g_V^2\right) \right]$$

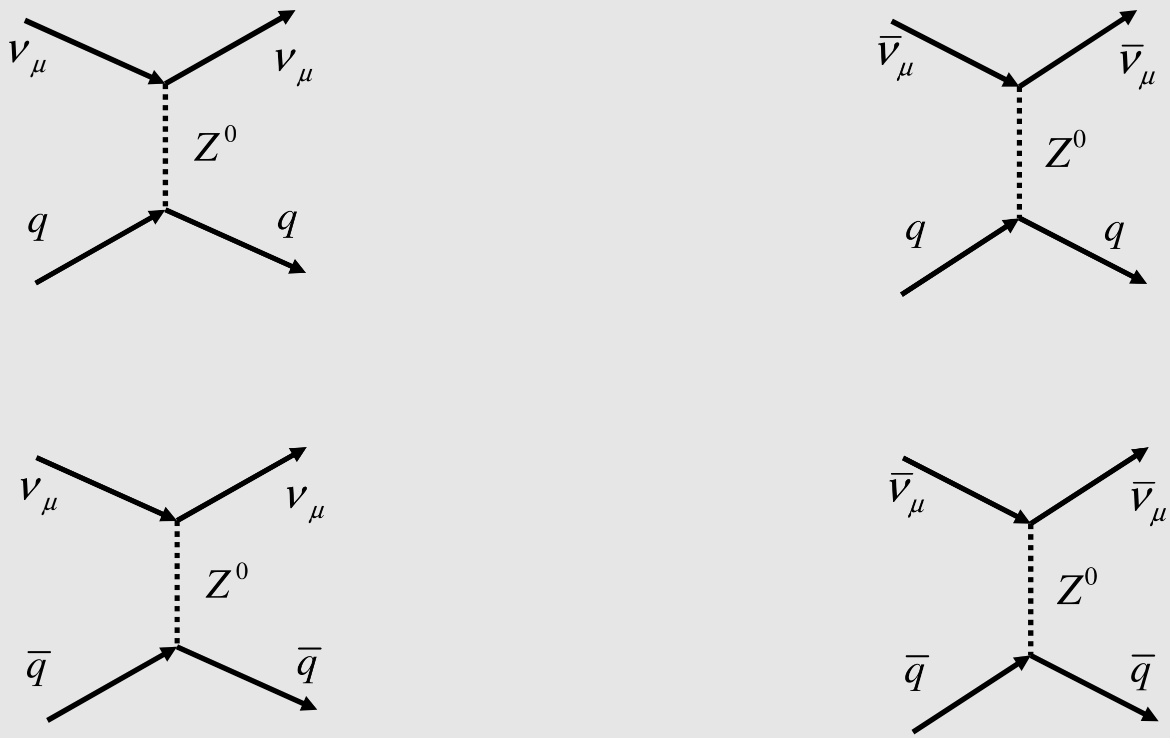
Neutral Currents Discovery - XV

First Gargamelle leptonic neutral current event



Neutral Currents Discovery - XVI

$\overset{(-)}{\nu} - q, \bar{q}$ processes



Neutral Currents Discovery - XVII

$$g_V = \frac{1}{2} - \frac{4}{3}\sin^2\theta_W \qquad g_A = \frac{1}{2} \qquad u,c,t$$

$$g'_V = -\frac{1}{2} + \frac{4}{3}\sin^2\theta_W \qquad g'_A = \frac{1}{2} \qquad d,s,b$$

$$g_L = \frac{1}{2} - \frac{2}{3}\sin^2\theta_W \qquad g_R = -\frac{2}{3}\sin^2\theta_W \qquad u,c,t$$

$$g'_L = -\frac{1}{2} + \frac{1}{3}\sin^2\theta_W \qquad g_R = \frac{1}{3}\sin^2\theta_W \qquad d,s,b$$

$$\frac{d\sigma_{\nu_\mu q}}{dy} = \frac{d\sigma_{\bar{\nu}_\mu \bar{q}}}{dy} = \frac{G_F^2 m_q}{2\pi} E_\nu \left[\left(g_V + g_A\right)^2 + \left(g_V - g_A\right)^2 \left(1-y\right)^2 + \frac{m_q}{E}\left(g_A^2 - g_V^2\right) y \right]$$

$$\frac{d\sigma_{\bar{\nu}_\mu q}}{dy} = \frac{d\sigma_{\nu_\mu \bar{q}}}{dy} = \frac{G_F^2 m_q}{2\pi} E_\nu \left[\left(g_V - g_A\right)^2 + \left(g_V + g_A\right)^2 \left(1-y\right)^2 + \frac{m_q}{E_\nu}\left(g_A^2 - g_V^2\right) y \right]$$

Neutral Currents Discovery -XVIII

$$\frac{d\sigma_{\overset{(-)}{\nu}_\mu}}{dxdy} = \sum_q q(x)\frac{d\sigma_{\overset{(-)}{\nu}_\mu q}}{dy} + \sum_{\bar q} \bar q(x)\frac{d\sigma_{\overset{(-)}{\nu}_\mu \bar q}}{dy}$$

$$\rightarrow \frac{d\sigma_{\nu_\mu N}}{dxdy} = \frac{G_F^2 m_N}{2\pi} xE_\nu \left[\left(g_L{}^2 + g_L{}'^2\right)\left(q + \bar q(1-y)^2\right) + \left(g_R{}^2 + g_R{}'^2\right)\left(\bar q + q(1-y)^2\right)\right]$$

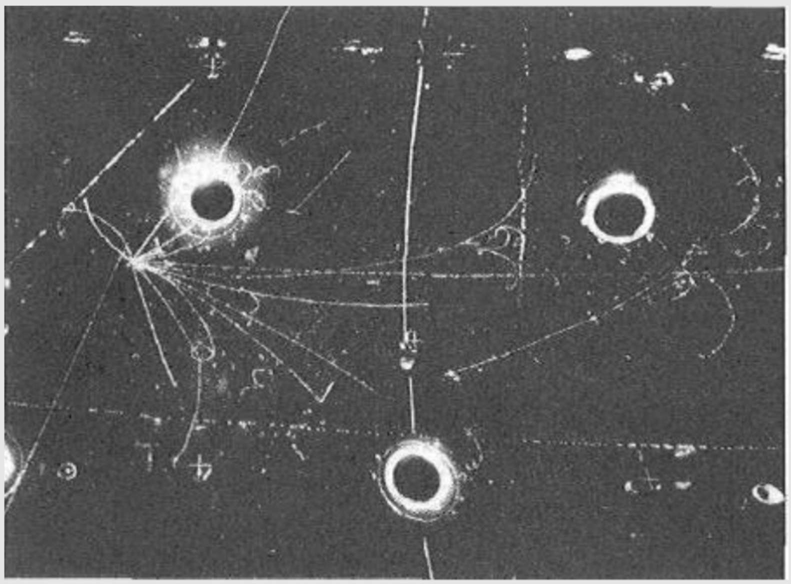
$$\rightarrow \frac{d\sigma_{\tilde\nu_\mu N}}{dxdy} = \frac{G_F^2 m_N}{2\pi} xE_\nu \left[\left(g_R{}^2 + g_R{}'^2\right)\left(q + \bar q(1-y)^2\right) + \left(g_L{}^2 + g_L{}'^2\right)\left(\bar q + q(1-y)^2\right)\right]$$

$$R_\nu^N = \frac{\sigma_{NC}(\nu)}{\sigma_{CC}(\nu)} \quad R_{\bar\nu}^N = \frac{\sigma_{NC}(\bar\nu)}{\sigma_{CC}(\bar\nu)} \quad r = \frac{\sigma_{CC}(\bar\nu)}{\sigma_{CC}(\nu)}$$

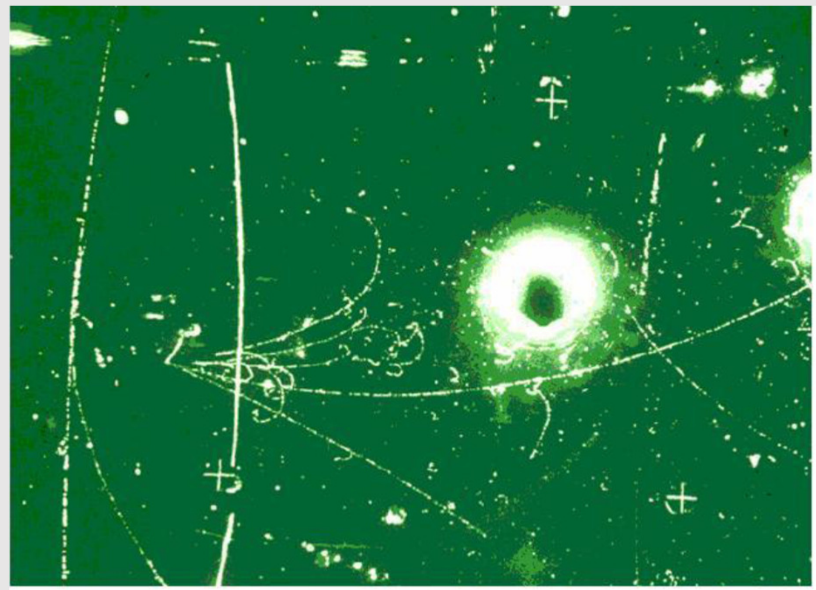
$$\rightarrow g_L{}^2 + g_L{}'^2 = \frac{R_\nu^N - r^2 R_{\bar\nu}^N}{1-r^2} \qquad g_R{}^2 + g_R{}'^2 = \frac{r\left(R_\nu^N - R_{\bar\nu}^N\right)}{1-r^2}$$

Neutral Currents Discovery - XIX

Gargamelle
charged current



Gargamelle
hadronic neutral current event

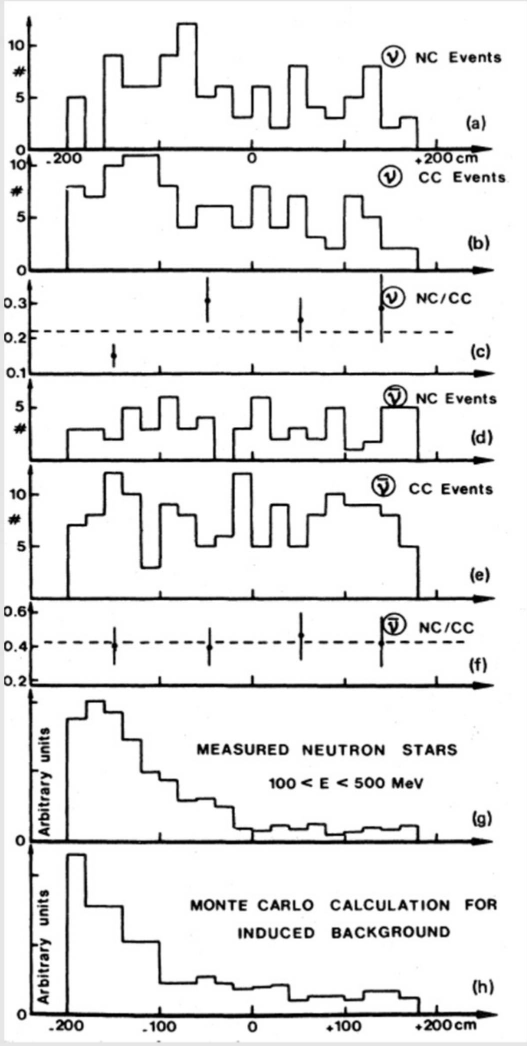


Neutral Currents Discovery - XX

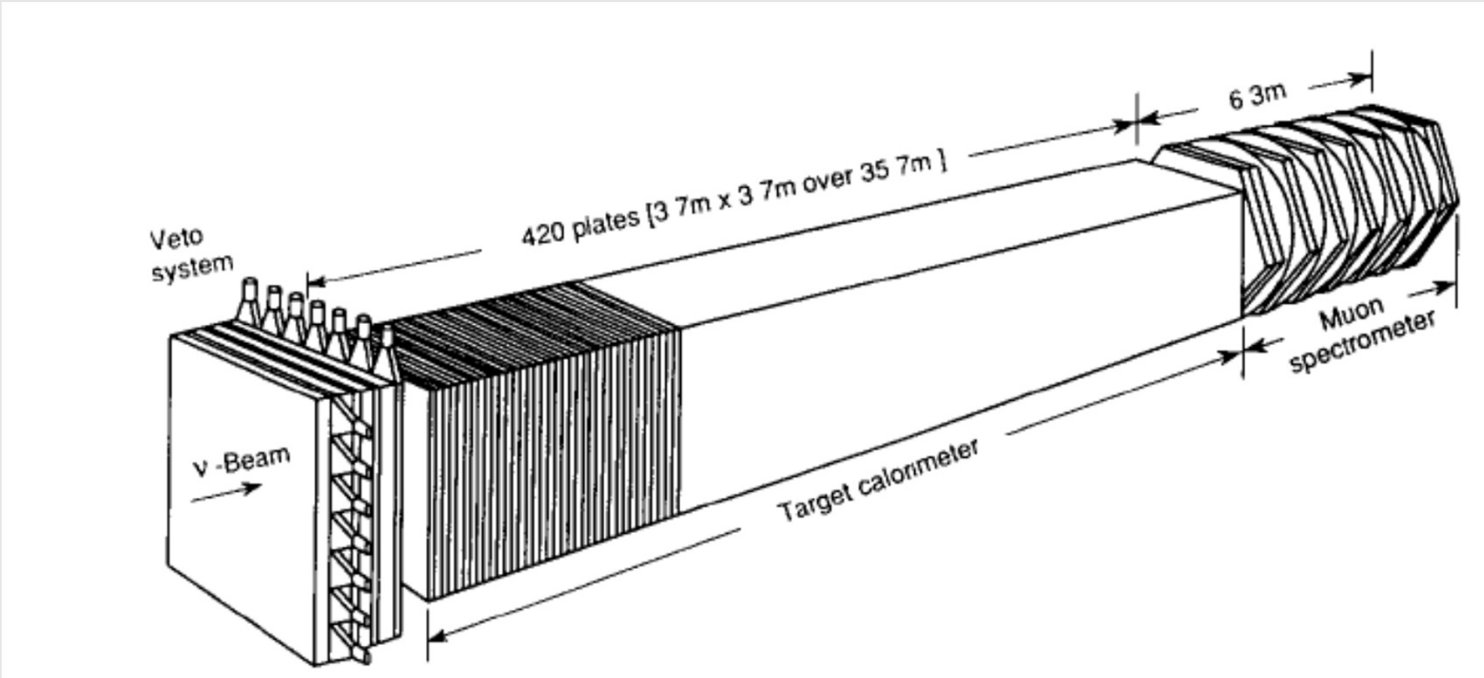
Background

Result

$$\sin^2\theta_W = 0.3 \div 0.4$$



Neutral Currents Discovery - XXI

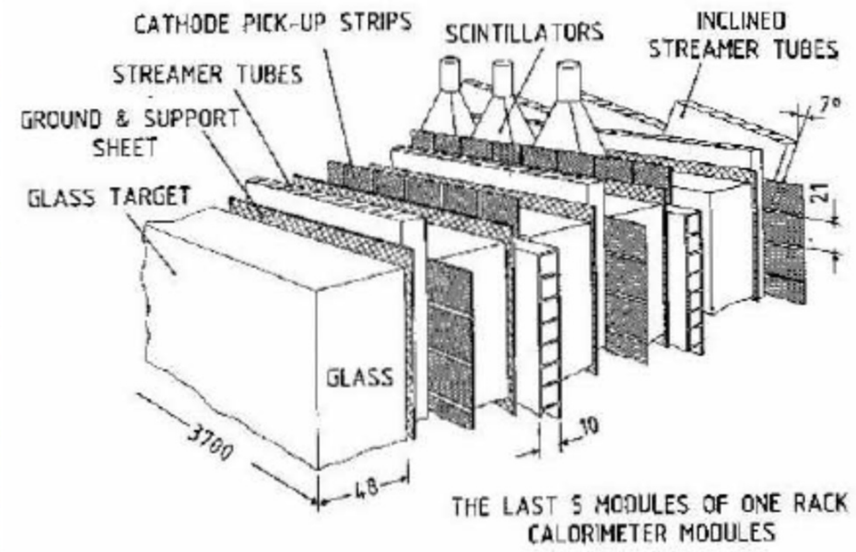


CHARM II

Neutral Currents Discovery - XXII

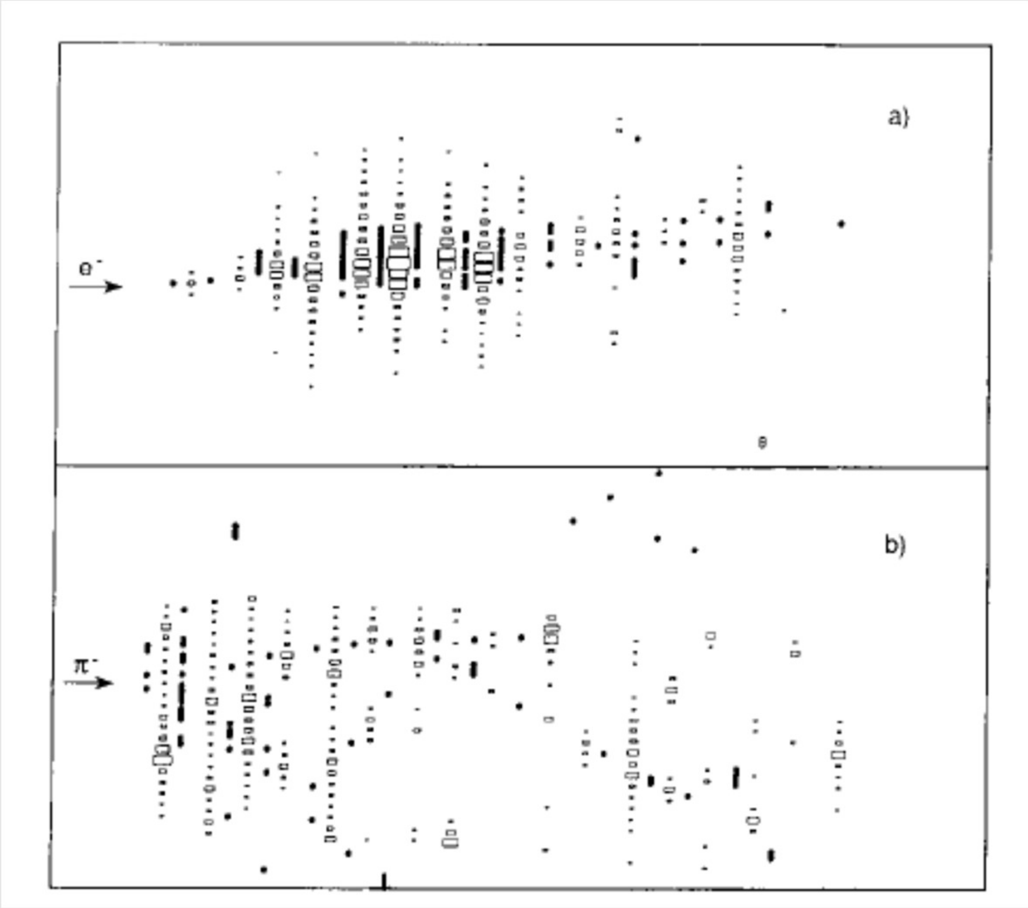
Experimental setup (CHARM II, 1987-1991)

electronic tracking detector



$\sim 700\,t$ calorimeter, digital readout of energy and direction of produced particles

Neutral Currents Discovery - XXIII



Electromagnetic shower

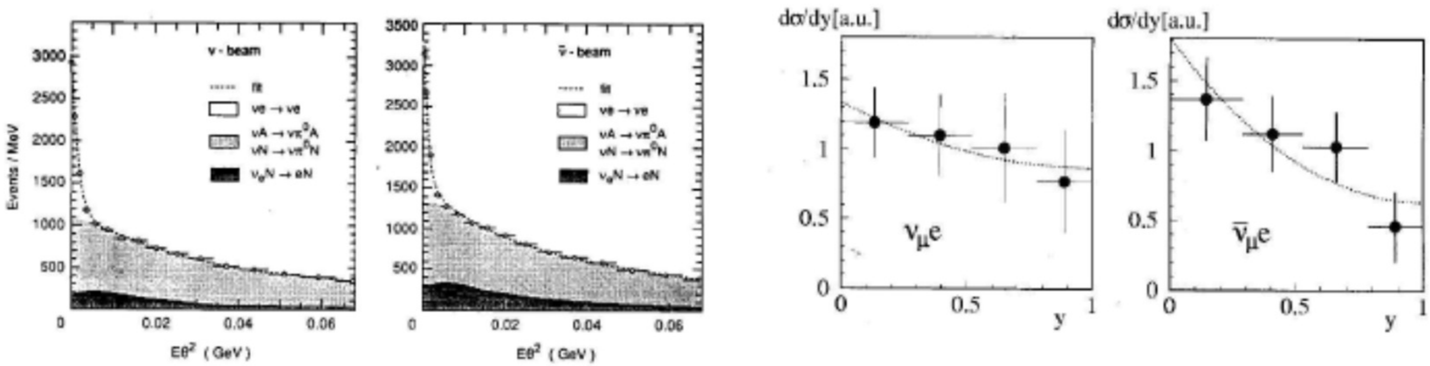
Hadronic shower

Neutral Currents Discovery- XXIV

CHARM II data

Problem: discrimination of the NC events (~ 2500 for $\nu_\mu e^-$ and $\bar{\nu}_\mu e^-$ each) from the dominant background (CC scattering, inelastic scattering)

Solution: in processes of interest $\nu_\mu e \to \nu_\mu e$ and $\bar{\nu}_\mu e^- \to \bar{\nu}_\mu e^-$ the value $E_e \theta_e^2$ is kinematically restricted to small values

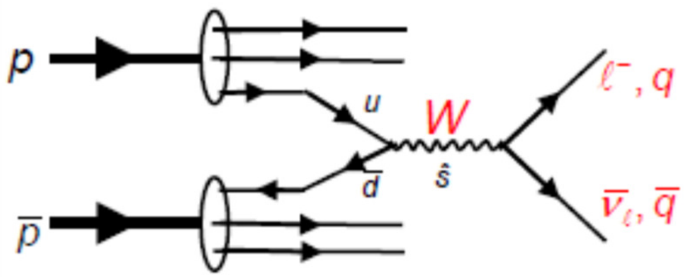


Phys.Lett. B335, 246 (1994)

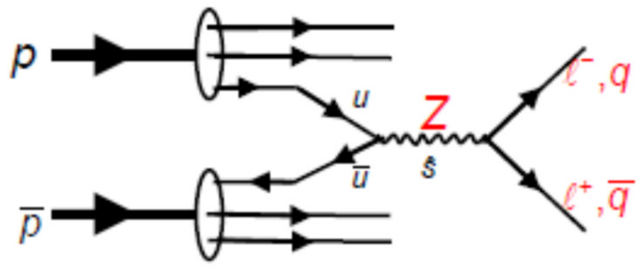
Phys.Lett. B302, 351 (1993)

$$\sin^2 \Theta_W = 0.2324 \pm 0.0083$$

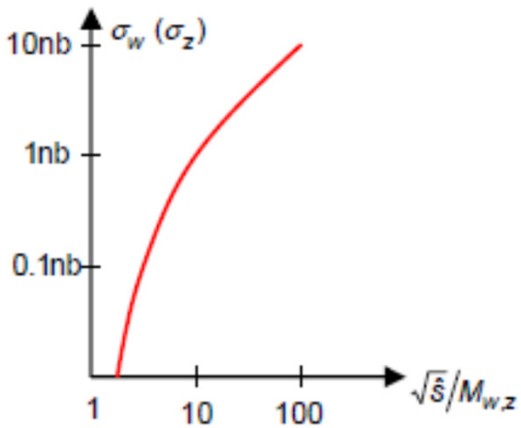
W & Z Discovery - I



$$p\overline{p} \to W \to \ell \overline{\nu}_\ell + X$$



$$p\overline{p} \to Z \to f\bar{f} + X$$



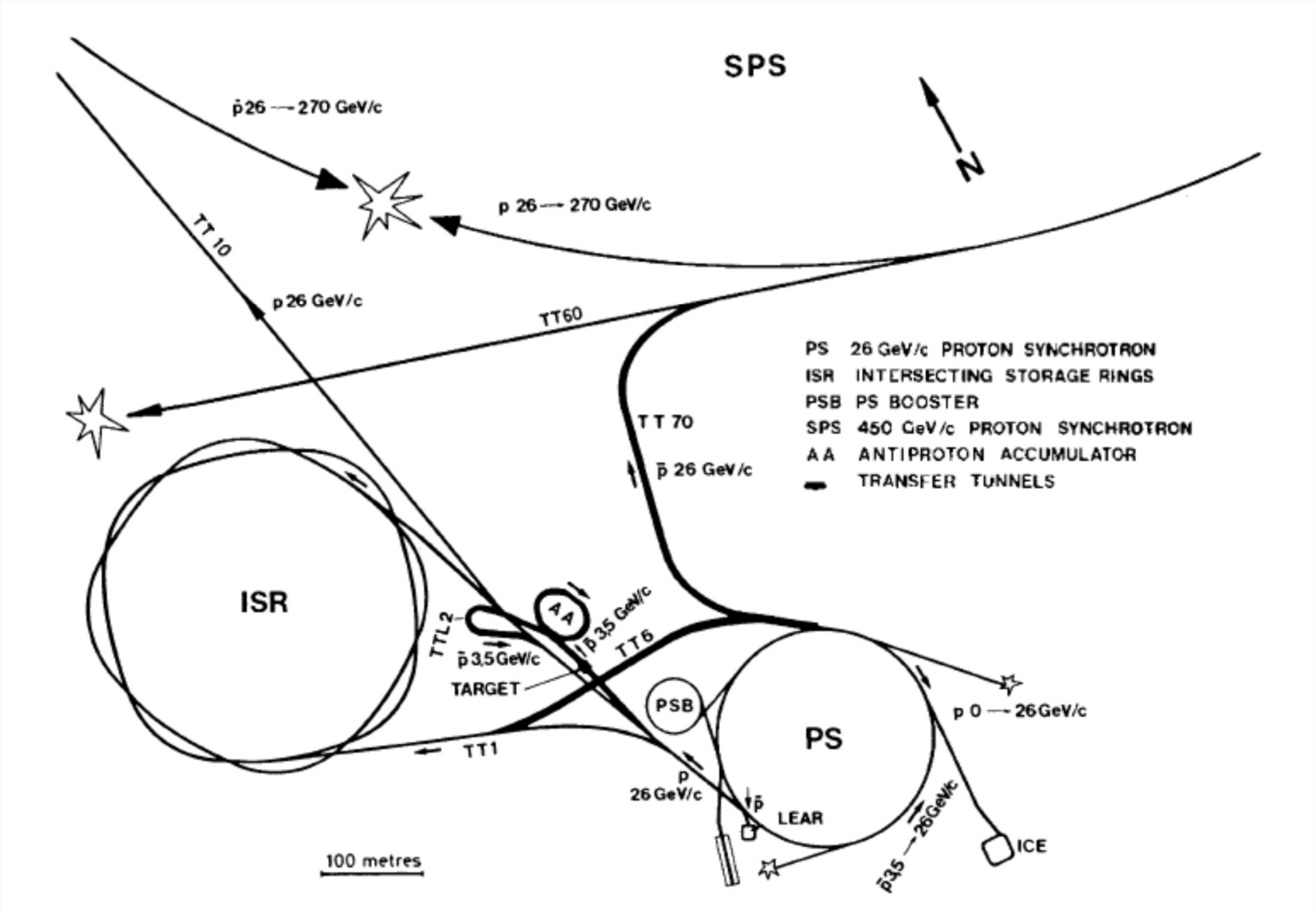
<u>Similar to Drell-Yan:</u> (photon instead of W)

$\hat{s} = x_q x_{\overline{q}} s$ it $\langle x_q \rangle \approx 0.12$

$\hat{s} = \langle x_q \rangle^2 s \approx 0.014 s = (65\ \text{GeV})^2$

$\rightarrow$ Cross section is small !

W & Z Discovery - II



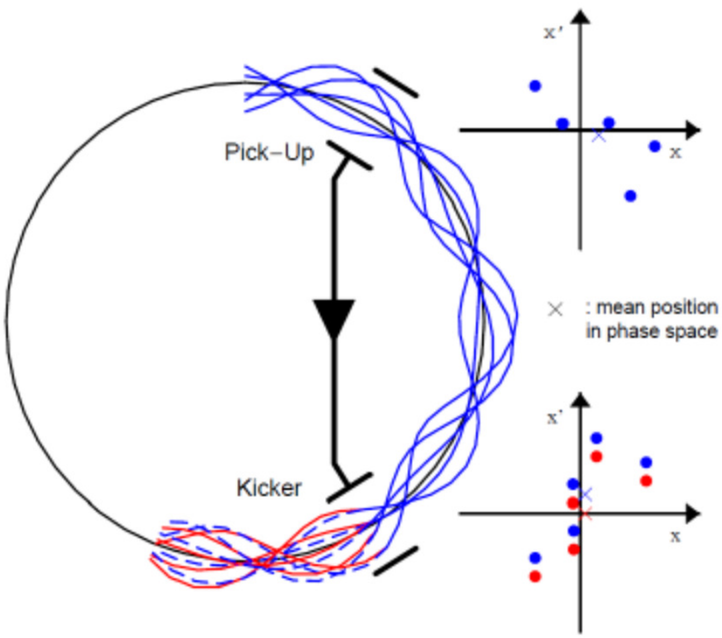
W & Z Discovery - III

$S\bar{p}pS$ Collider main parameters

- $\sqrt{s}$ = 540 GeV
- 3 bunches protons, 3 bunches antiprotons, 10^{11} particles per bunch
- Luminosity = 5 x 10^{27} $cm^{-2}sec^{-1}$
- First collisions in December 1981

W & Z Discovery - IV

Stochastical cooling system



- 10^7 antiprotons with $p = 3.5$ GeV/c gets in outer part of toroidale vacuum chamber
- Inductor measures discrepancy of particles
- Correction signal is send to opposite side
- Magnet deflects particles
- After 2*sec* aperture is opened

W & *Z* Discovery - V

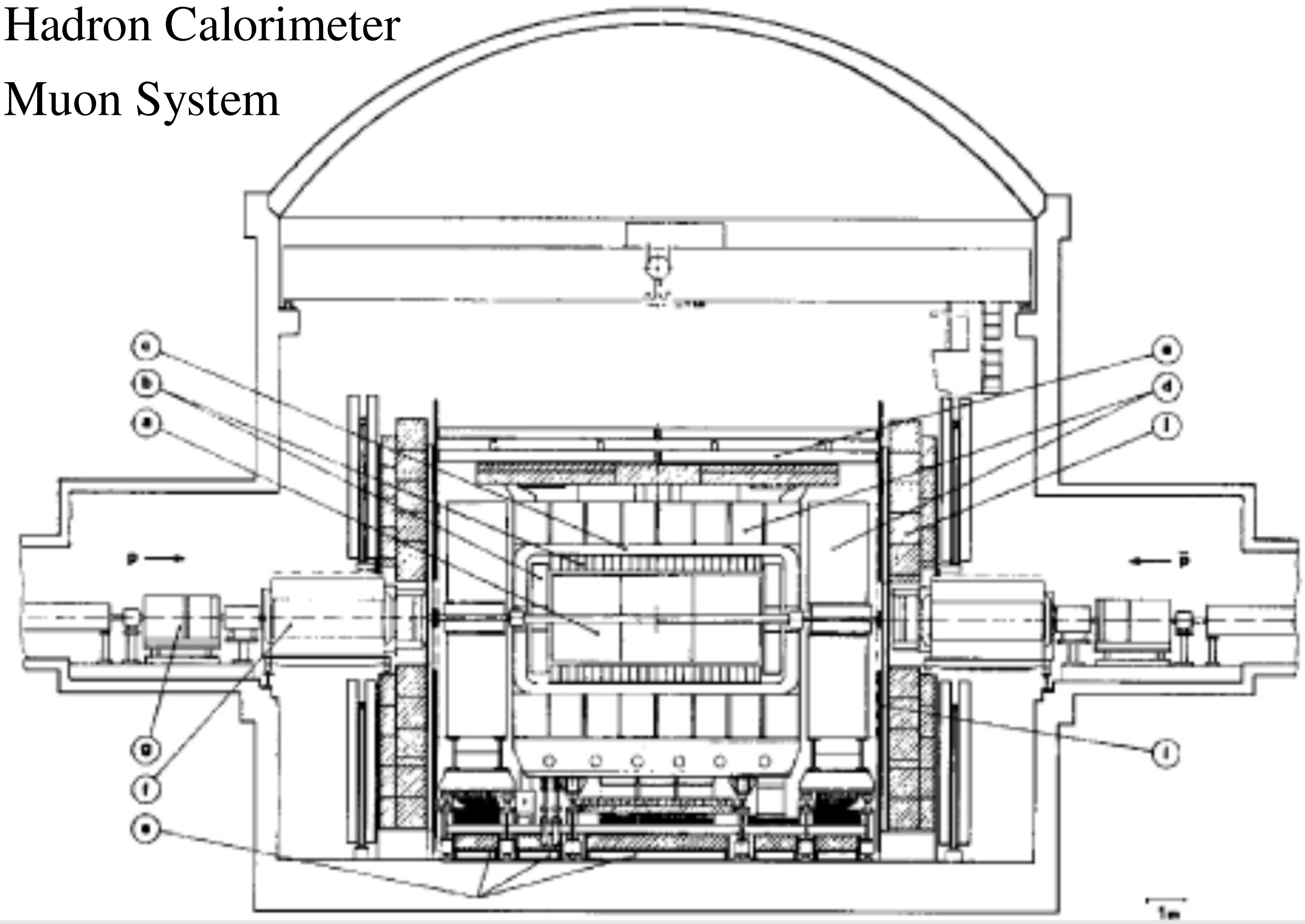
*UA*1 Detector

Central Tracker

EM calorimeter

Hadron Calorimeter

Muon System



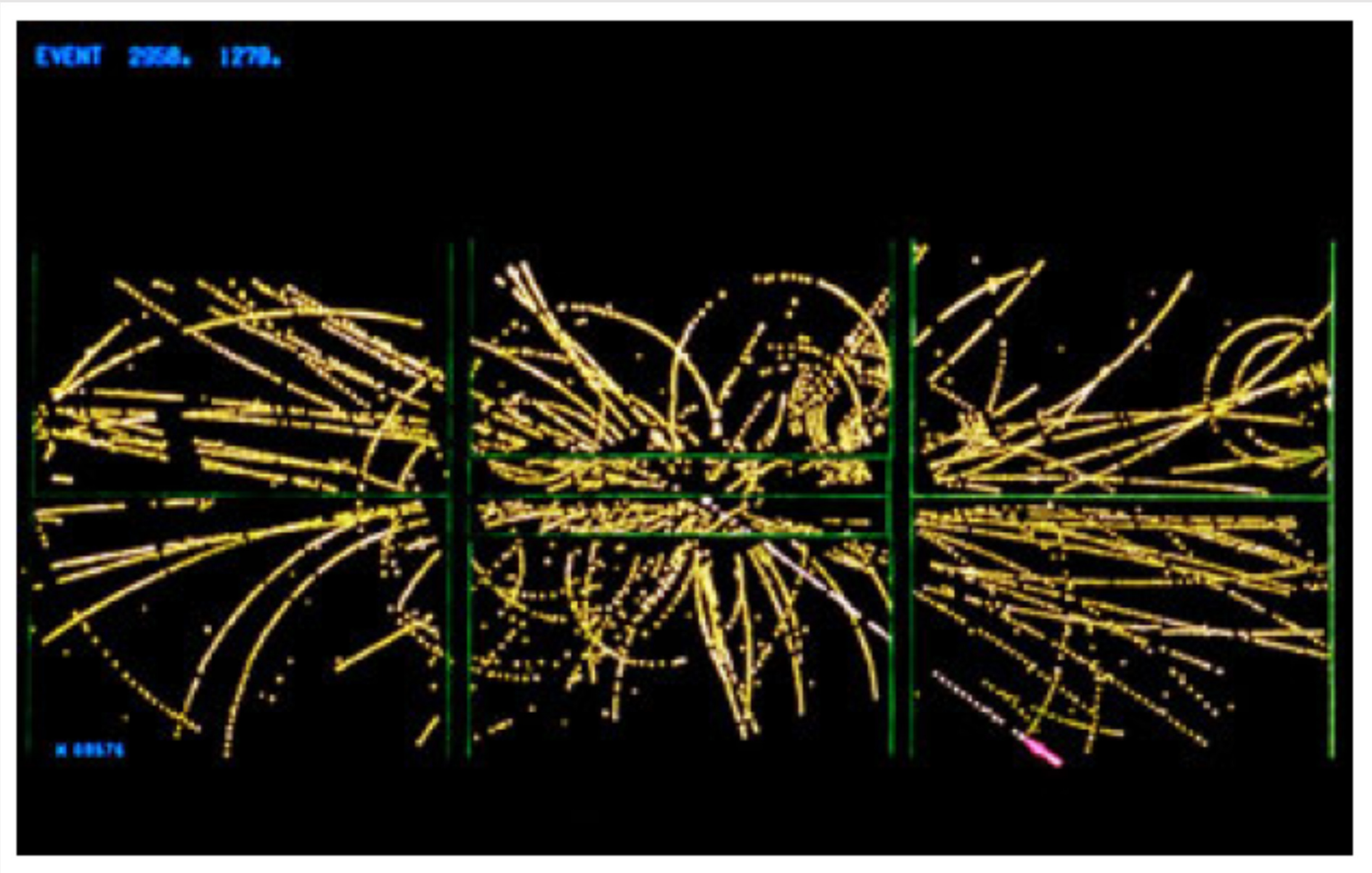
W & Z Discovery - VI

Several conditions to select events

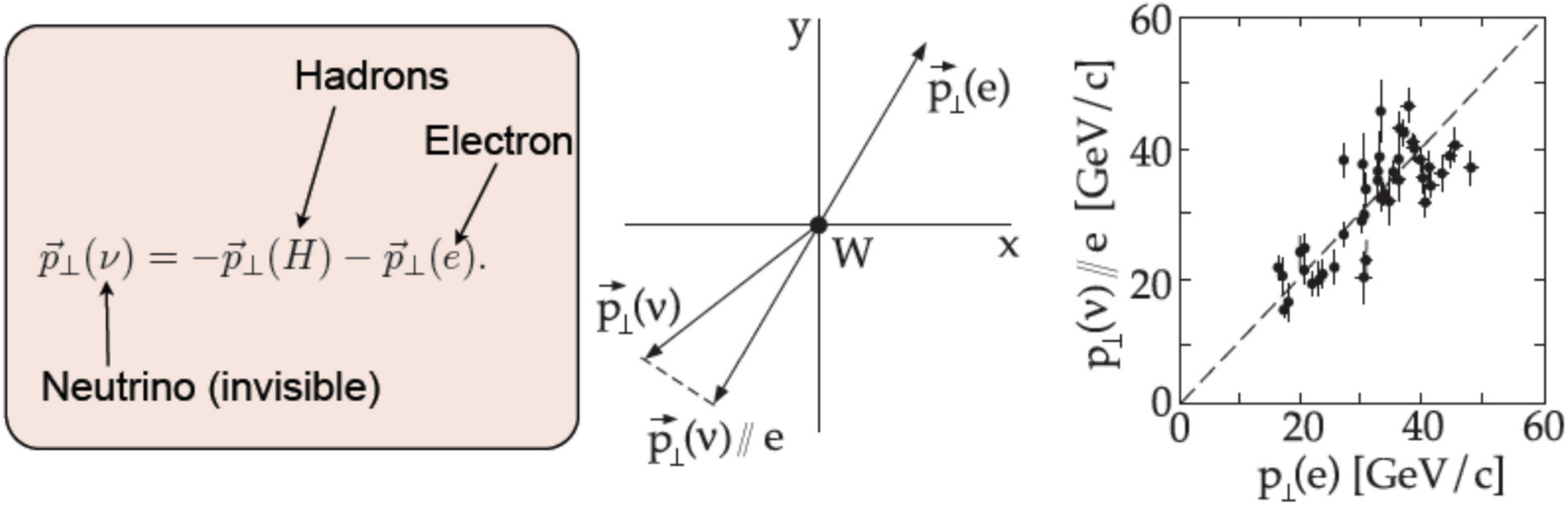
- More than 10000 events/s, most of them selected by a trigger
- Trace in central detector must point into center of electromagnetic shower
- Transversal momentum in central detector > 7 GeV
- Trace must be isolated (only other traces with transversal momentum <2.5 GeV allowed)
- Missing energy >15 GeV, has to point contrary to trace of electron

W & Z Discovery - VII

UA1 $W \to e\nu$ candidate event



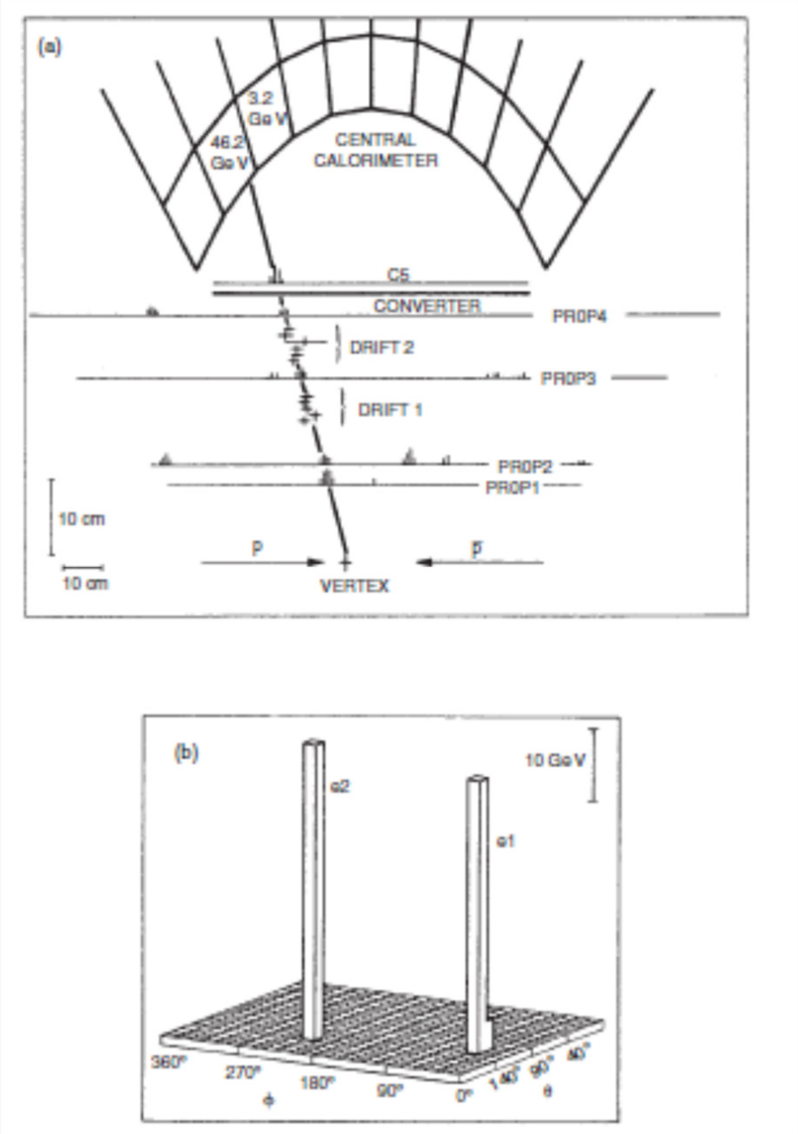
W & Z Discovery - VIII



- Calculate sum of all hadron momenta in the **transverse plane** (to avoid leaks along the beam lines)
- **p_T**(ν) not exactly antiparallel to **p_T**(e)
  - W boson not always produced at rest, finite detector resolution

W & *Z* Discovery - IX

UA2 Candidate Z event



W & Z Discovery - X

$$\frac{d\sigma}{d\cos\theta^*} = \text{const} \qquad \text{Just an approximation}$$

$$\frac{d\sigma}{dp_T} = \frac{d\sigma}{d\cos\theta^*}\frac{d\cos\theta^*}{dp_T}$$

$$p_T = p^*\sin\theta^* = \frac{M_W}{2}\sin\theta^*$$

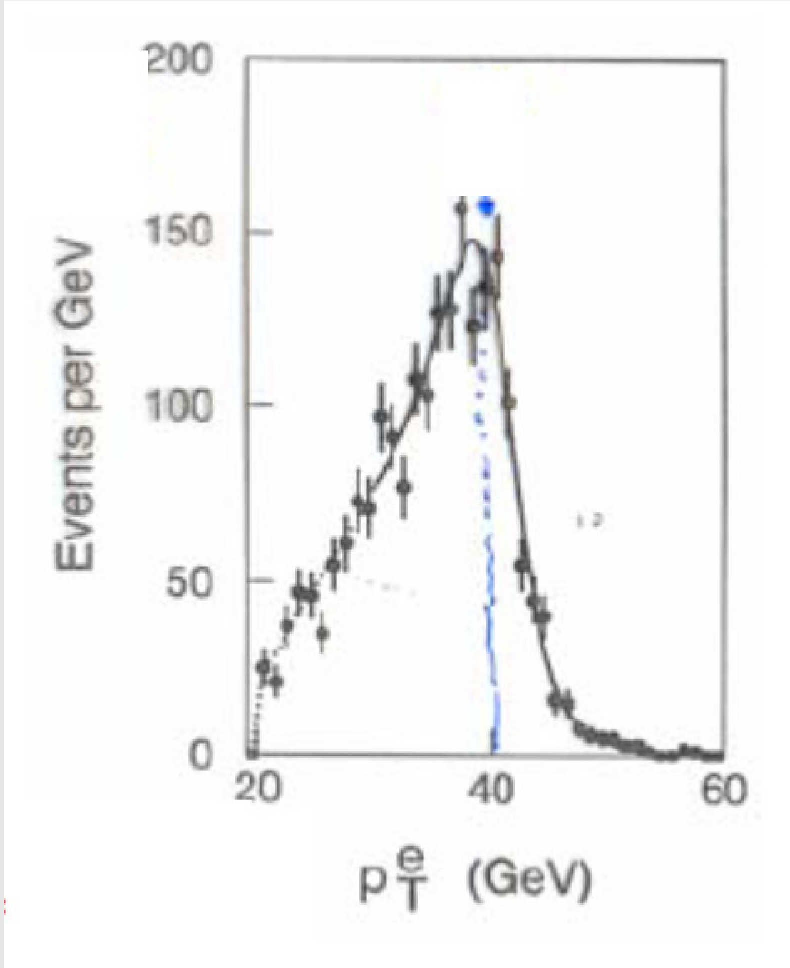
$$\rightarrow \sin\theta^* = \frac{2p_T}{M_W}$$

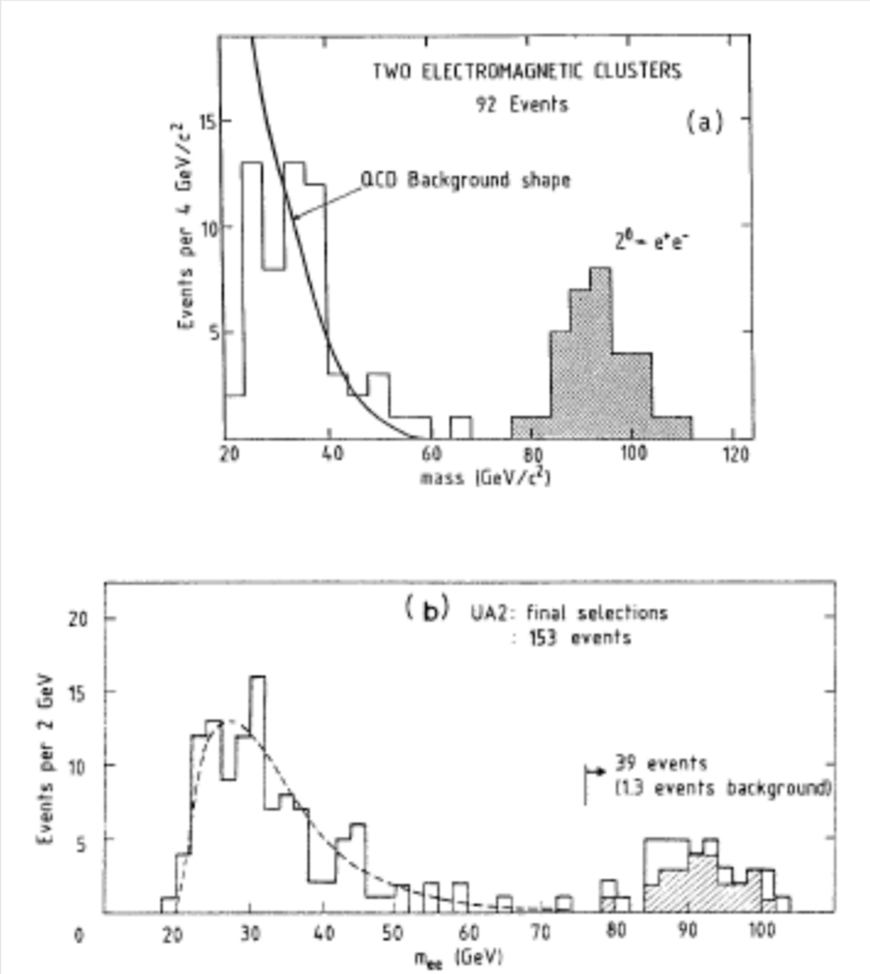
$$\rightarrow \cos\theta^* = \sqrt{1-\sin^2\theta^*} = \sqrt{1-\left(\frac{2p_T}{M_W}\right)^2}$$

$$\rightarrow \frac{d\cos\theta^*}{dp_T} = \frac{\frac{4p_T}{M_W}}{2\sqrt{1-\left(\frac{2p_T}{M_W}\right)^2}} = \frac{2p_T}{M_W\sqrt{1-\left(\frac{2p_T}{M_W}\right)^2}}$$

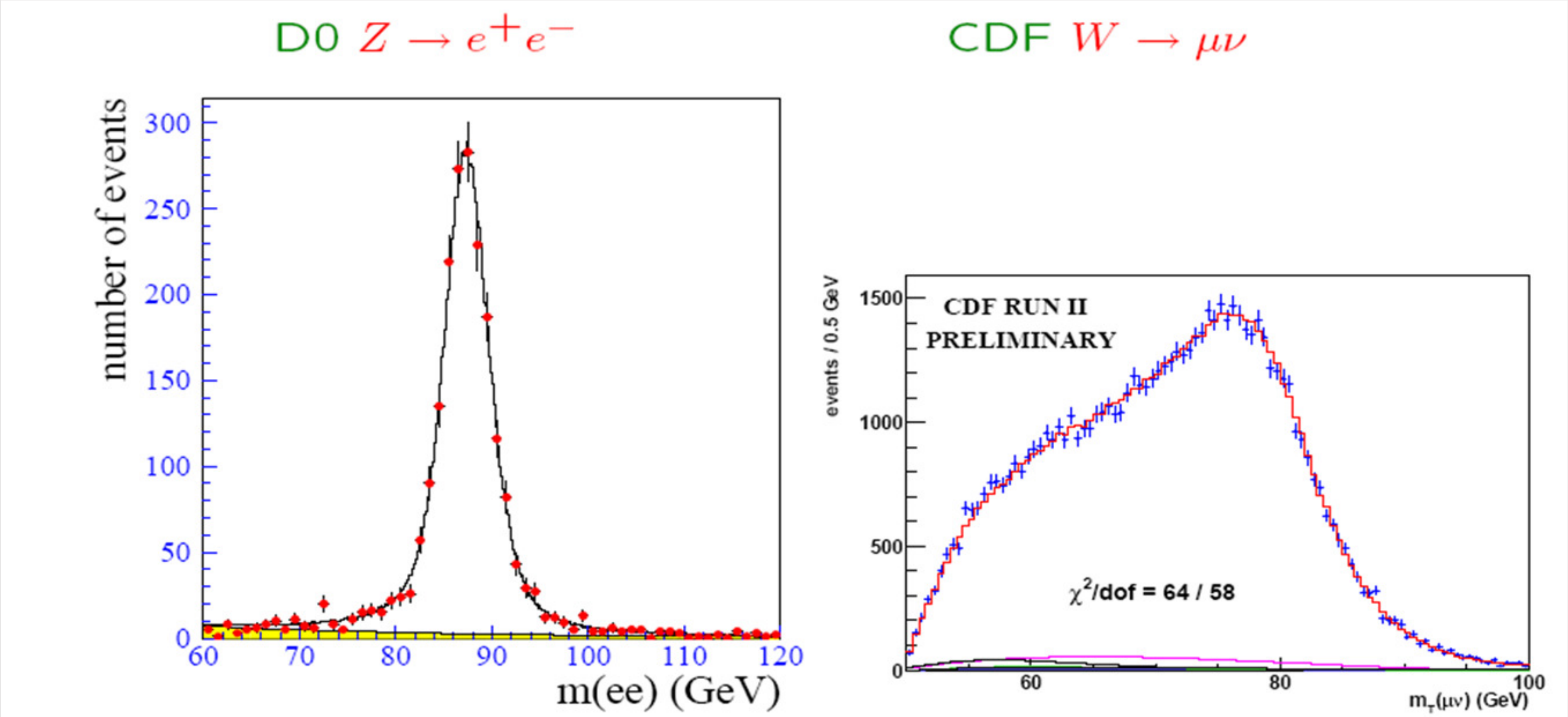
$$\rightarrow \frac{d\sigma}{dp_T} = A\left(\cos\theta^*\right)\frac{d\cos\theta^*}{dp_T} \approx K\frac{2p_T}{M_W\sqrt{1-\left(\frac{2p_T}{M_W}\right)^2}} \quad \text{Jacobian peak at } \frac{M_W}{2}$$

W & Z Discovery - XI





W & Z Discovery - XII



$m_{W^\pm} = 82.1 \pm 1.7$ GeV

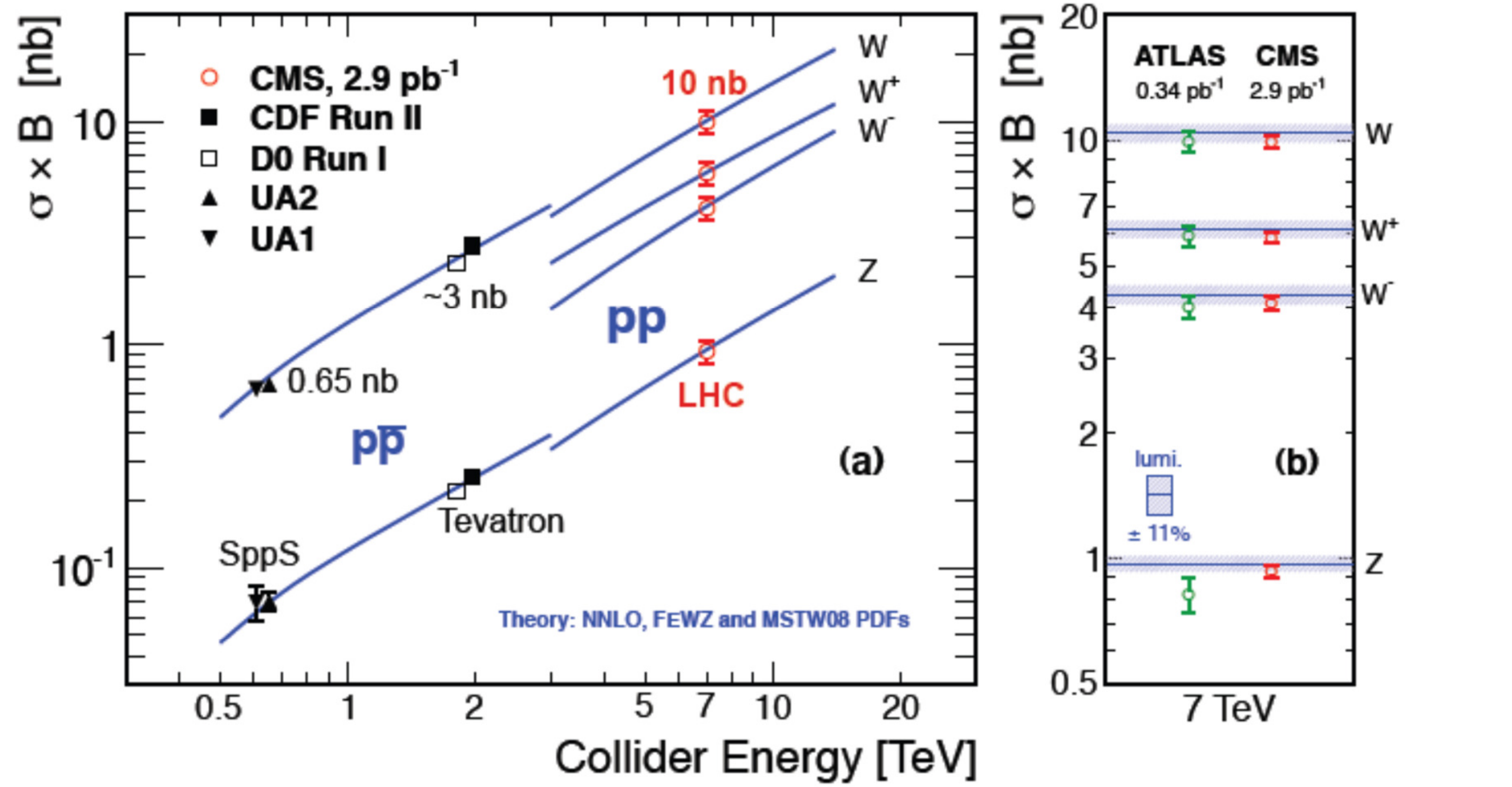
$m_{Z^0} = 93.0 \pm 1.7$ GeV

Current values (Particle Data Group 2006):

$m_{W\pm} = 80.403 \pm 0.029$ GeV

$m_{Z0} = 91.1876 \pm 0.0021$ GeV

W & Z Discovery - XIII



Source: arXiv:1012.2466

- Z^0 cross section ~10 times smaller than $W^{\pm}$ boson production
- W^+ cross section ~43% larger than W^- at LHC (pp collider!)

Precision Tests - I

LEP – Precision tests of SM 1989-2000

$1989-1995$

$\sqrt{s} = 91.2\ GeV$

$17\ 10^{6}\ Z^{0}$ detected

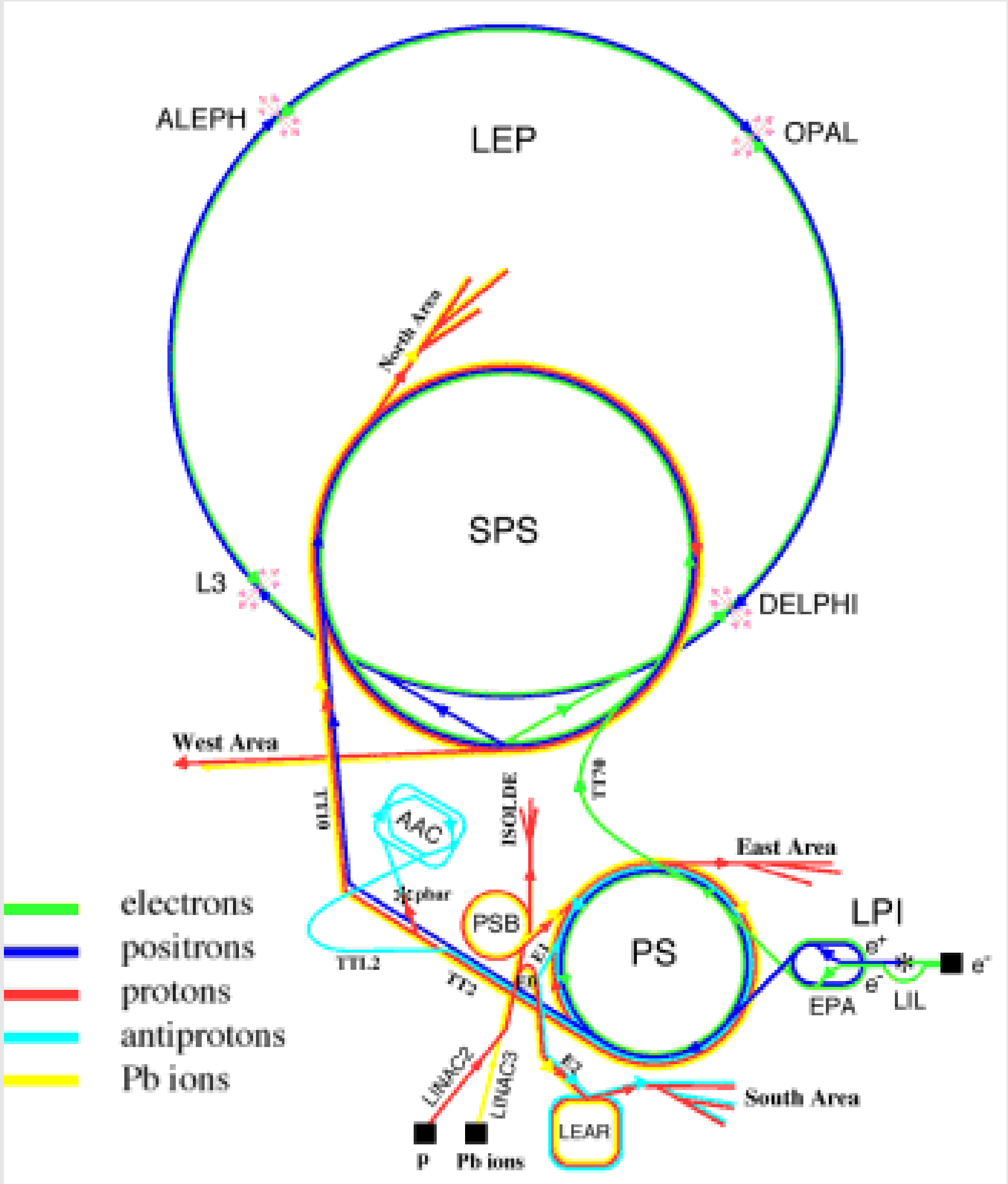
$1996-2000$

$\sqrt{s} = 161-208\ GeV$

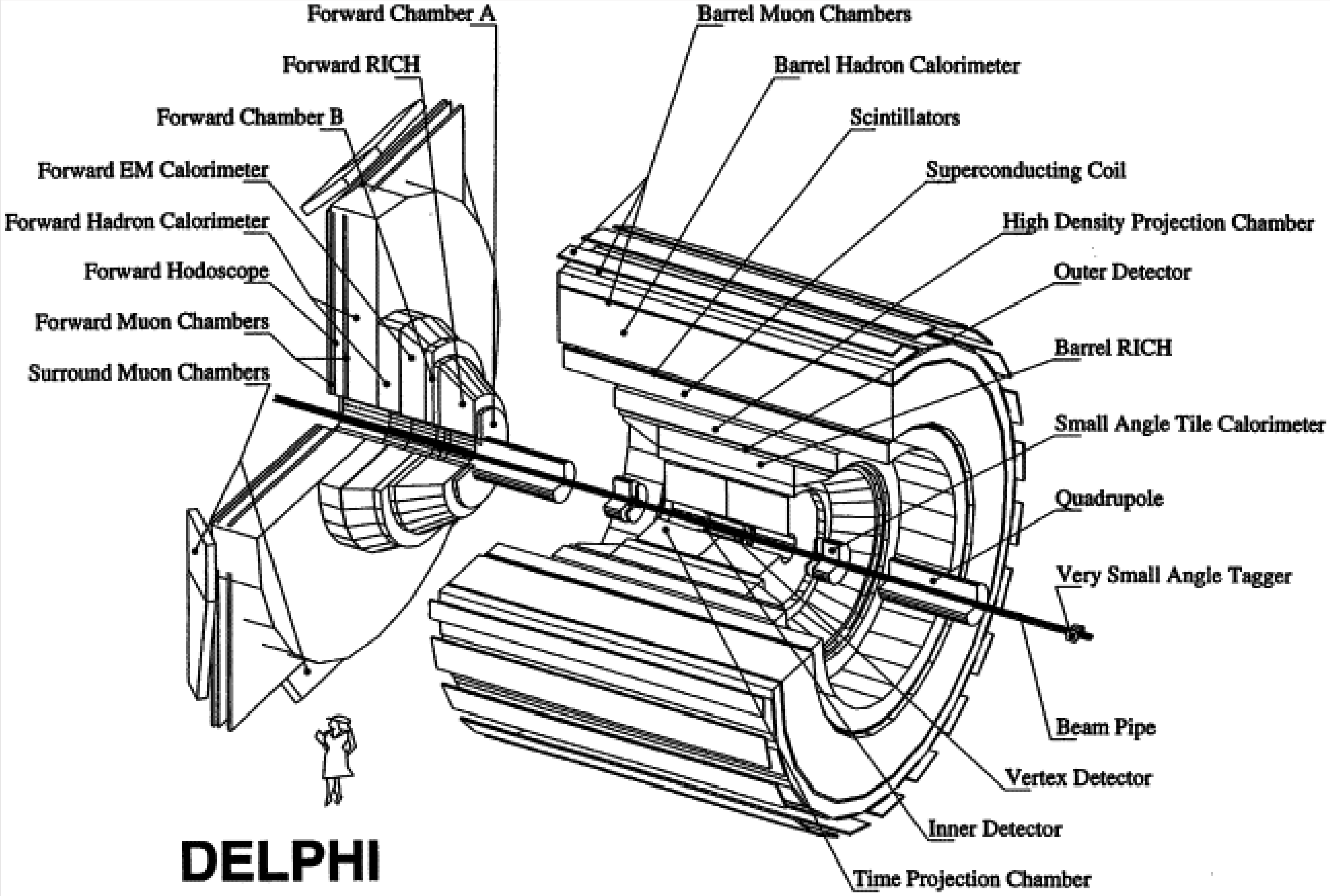
$30\ 10^{3}\ \ WW$ detected

26 km circumference
4 large experiments: ALEPH, DELPHI, L3, OPAL

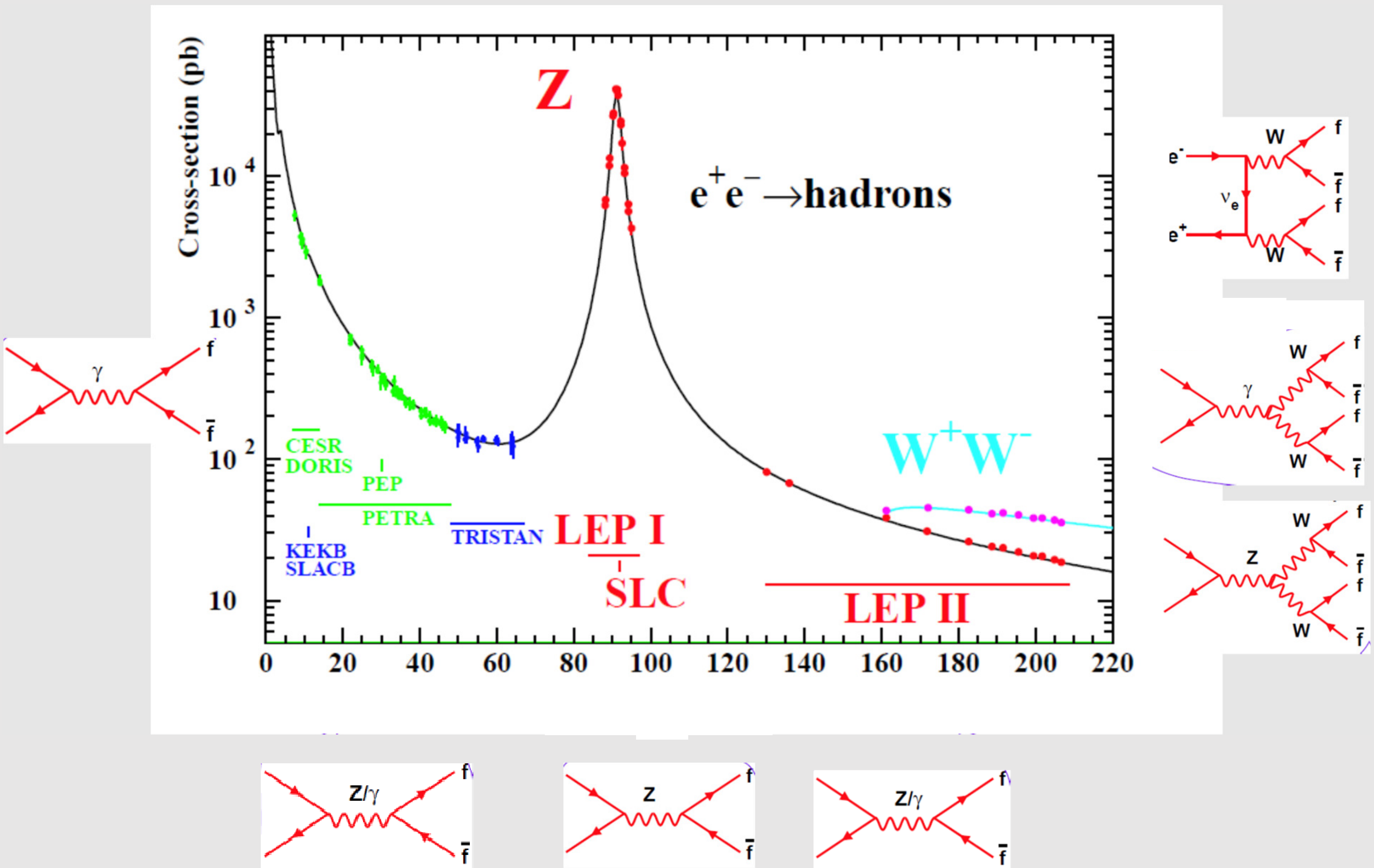
Precision Tests - II



Precision Tests - III



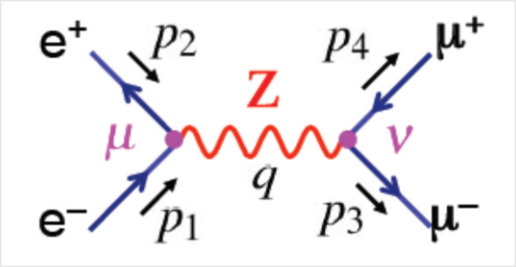
Precision Tests - IV



Precision Tests - V

$e^+e^- \to \mu^+\mu^-$ at the Z peak

Only consider (dominant) Z diagram



Electron vertex:

$$\bar{v}(p_2)\left(-ig_Z\gamma^\mu\right)\frac{1}{2}\left(c_V - c_A\gamma^5\right)u(p_1) \quad \text{Electron } c_V, c_A$$

Z propagator:

$$-i\frac{g_{\mu\nu}}{q^2 - m_Z^2} \quad \text{Approximate, see later}$$

Muon vertex:

$$\bar{u}(p_3)\left(-ig_Z\gamma^\nu\right)\frac{1}{2}\left(c_V - c_A\gamma^5\right)v(p_4) \quad \text{Muon } c_V, c_A$$

Precision Tests- VI

Ultrarelativistic limit $\rightarrow$ Chirality $\simeq$ Helicity

$\rightarrow$ Use helicity eigenstates for electron, muon vertexes

$$c_L = c_V + c_A, c_R = c_V - c_A$$

$$\rightarrow c_V = \frac{1}{2}(c_L + c_R), c_A = \frac{1}{2}(c_L - c_R)$$

$$\frac{1}{2}\left(c_V - c_A\gamma^5\right) \rightarrow \frac{1}{2}c_L\left(1-\gamma^5\right) + \frac{1}{2}c_R\left(1+\gamma^5\right)$$

$\rightarrow$ Matrix element:

$$\left[c_L\overline{v}(p_2)\gamma^\mu \frac{1}{2}\left(1-\gamma^5\right)u(p_1) + c_R\overline{v}(p_2)\gamma^\mu \frac{1}{2}\left(1+\gamma^5\right)u(p_1)\right]$$

$$\times\left(-\frac{g_Z}{q^2 - m_Z^2} g_{\mu\nu}\right)$$

$$\times\left[c_L\overline{u}(p_3)\gamma^\nu \frac{1}{2}\left(1-\gamma^5\right)v(p_4) + c_R\overline{u}(p_3)\gamma^\nu \frac{1}{2}\left(1+\gamma^5\right)v(p_4)\right]$$

Introduce chirality $\simeq$ helicity projectors:

$$\frac{1}{2}\left(1-\gamma^5\right)u \simeq u_\downarrow, \frac{1}{2}\left(1+\gamma^5\right)u \simeq u_\uparrow, \frac{1}{2}\left(1-\gamma^5\right)v \simeq v_\uparrow, \frac{1}{2}\left(1+\gamma^5\right)v \simeq v_\downarrow$$

Precision Tests- VII

$\rightarrow$ Matrix element:

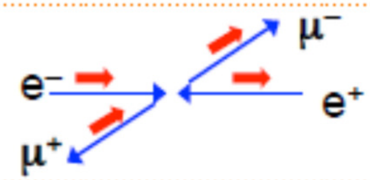
$$\left[c_L\bar{v}(p_2)\gamma^\mu u_\downarrow(p_1)+c_R\bar{v}(p_2)\gamma^\mu u_\uparrow(p_1)\right]\left(-\frac{g_Z}{q^2-m_Z^2}g_{\mu\nu}\right)\left[c_L\bar{u}(p_3)\gamma^\nu v_\uparrow(p_4)+c_R\bar{u}(p_3)\gamma^\nu v_\downarrow(p_4)\right]$$

$$\bar{v}(p_2)=\bar{v}_\uparrow(p_2)+\bar{v}_\downarrow(p_2),\bar{u}(p_3)=\bar{u}_\uparrow(p_3)+\bar{u}_\downarrow(p_3)$$

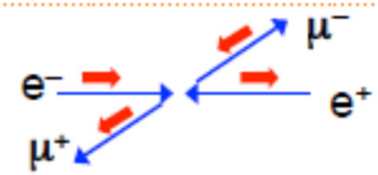
Surviving terms in both e,μ currents: LR, RL only

$$\rightarrow\left[c_L\bar{v}_\uparrow(p_2)\gamma^\mu u_\downarrow(p_1)+c_R\bar{v}_\downarrow(p_2)\gamma^\mu u_\uparrow(p_1)\right]\left(-\frac{g_Z}{q^2-m_Z^2}g_{\mu\nu}\right)\left[c_L\bar{u}_\downarrow(p_3)\gamma^\nu v_\uparrow(p_4)+c_R\bar{u}_\uparrow(p_3)\gamma^\nu v_\downarrow(p_4)\right]$$

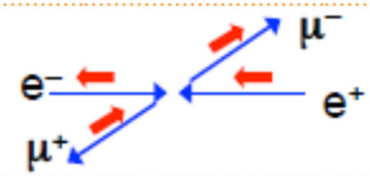
$$M_{RR}=-\frac{g_Z^2}{q^2-m_Z^2}c_R^e c_R^\mu g_{\mu\nu}[\bar{v}_\downarrow(p_2)\gamma^\mu u_\uparrow(p_1)][\bar{u}_\uparrow(p_3)\gamma^\nu v_\downarrow(p_4)]$$



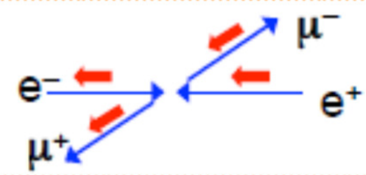
$$M_{RL}=-\frac{g_Z^2}{q^2-m_Z^2}c_R^e c_L^\mu g_{\mu\nu}[\bar{v}_\downarrow(p_2)\gamma^\mu u_\uparrow(p_1)][\bar{u}_\downarrow(p_3)\gamma^\nu v_\uparrow(p_4)]$$



$$M_{LR}=-\frac{g_Z^2}{q^2-m_Z^2}c_L^e c_R^\mu g_{\mu\nu}[\bar{v}_\uparrow(p_2)\gamma^\mu u_\downarrow(p_1)][\bar{u}_\uparrow(p_3)\gamma^\nu v_\downarrow(p_4)]$$



$$M_{LL}=-\frac{g_Z^2}{q^2-m_Z^2}c_L^e c_L^\mu g_{\mu\nu}[\bar{v}_\uparrow(p_2)\gamma^\mu u_\downarrow(p_1)][\bar{u}_\downarrow(p_3)\gamma^\nu v_\uparrow(p_4)]$$



Precision Tests- VIII

Almost ‘Cut & Paste’ from QED case:

$$|M_{RR}|^2 = s^2 \left| \frac{g_Z^2}{s - m_Z^2} \right|^2 (c_R^e)^2 (c_R^\mu)^2 (1 + \cos\theta)^2$$

$$|M_{RL}|^2 = s^2 \left| \frac{g_Z^2}{s - m_Z^2} \right|^2 (c_R^e)^2 (c_L^\mu)^2 (1 - \cos\theta)^2$$

$$|M_{LR}|^2 = s^2 \left| \frac{g_Z^2}{s - m_Z^2} \right|^2 (c_L^e)^2 (c_R^\mu)^2 (1 - \cos\theta)^2$$

$$|M_{LR}|^2 = s^2 \left| \frac{g_Z^2}{s - m_Z^2} \right|^2 (c_L^e)^2 (c_L^\mu)^2 (1 + \cos\theta)^2$$

Precision Tests- IX

Now take correct Z propagator: Z *unstable*

$$-i\frac{g_{\mu\nu}}{q^2-m_Z^2}=-i\frac{g_{\mu\nu}}{s-m_Z^2}\rightarrow -i\frac{g_{\mu\nu}}{s-m_Z^2+im_Z\Gamma_Z}$$

$$\rightarrow\left|-i\frac{1}{s-m_Z^2+im_Z\Gamma_Z}\right|^2=\frac{1}{\left(s-m_Z^2\right)^2+m_Z^2\Gamma_Z^2}$$

$$\frac{d\sigma}{d\Omega}=\frac{\left|M_{fi}\right|^2}{64\pi^2 s}$$

$\rightarrow$ Differential cross-section for the 4 combinations:

$$\frac{d\sigma_{RR}}{d\Omega}=\frac{1}{64\pi^2}\frac{g_Z^4 s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}(c_R^e)^2(c_R^\mu)^2(1+\cos\theta)^2$$

$$\frac{d\sigma_{LL}}{d\Omega}=\frac{1}{64\pi^2}\frac{g_Z^4 s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}(c_L^e)^2(c_L^\mu)^2(1+\cos\theta)^2$$

$$\frac{d\sigma_{LR}}{d\Omega}=\frac{1}{64\pi^2}\frac{g_Z^4 s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}(c_L^e)^2(c_R^\mu)^2(1-\cos\theta)^2$$

$$\frac{d\sigma_{RL}}{d\Omega}=\frac{1}{64\pi^2}\frac{g_Z^4 s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}(c_L^e)^2(c_R^\mu)^2(1-\cos\theta)^2$$

Precision Tests- X

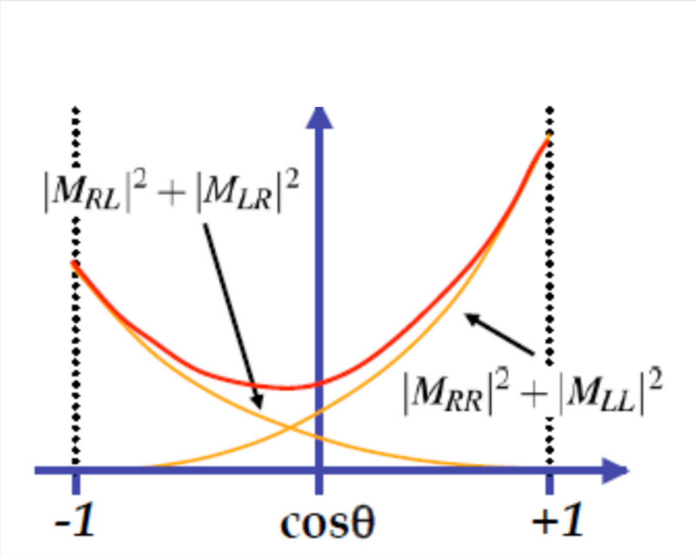
Most interesting difference wrt *QED* case:

$$|M_{LL}|^2 + |M_{RR}|^2 \neq |M_{LR}|^2 + |M_{RL}|^2$$

Unpolarized cross section: Average & Sum over spins

$$\rightarrow \frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2}\frac{1}{4}\frac{g_Z^4 s}{\left(s-m_Z^2\right)^2 + m_Z^2\Gamma_Z^2}\frac{1}{4}\left(c_V^2 + c_A^2\right)_e\left(c_V^2 + c_A^2\right)_\mu\left(1+\cos^2\theta\right) + 2\left(c_V c_A\right)_e\left(c_V c_A\right)_\mu \cos\theta$$

Sizeable forward-backward asymmetry!



Precision Tests - XI

Integrate over solid angle, get total cross section:

$$\sigma_{e^+e^-\to Z\to\mu^+\mu^-} = \frac{1}{192\pi}\frac{g_Z^4 s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}\left[(c_V^e)^2+(c_A^e)^2\right]\left[(c_V^\mu)^2+(c_A^\mu)^2\right]$$

Recall partial Z widths:

$$\Gamma(Z\to e^+e^-) = \frac{g_Z^2 m_Z}{48\pi}\left[(c_V^e)^2+(c_A^e)^2\right] \qquad \Gamma(Z\to \mu^+\mu^-) = \frac{g_Z^2 m_Z}{48\pi}\left[(c_V^\mu)^2+(c_A^\mu)^2\right]$$

$$\sigma = \frac{12\pi}{m_Z^2}\frac{s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}\Gamma(Z\to e^+e^-)\Gamma(Z\to\mu^+\mu^-)$$

$$\to \sigma_{peak} \simeq \frac{12\pi(BR)^2}{m_Z^2} \approx \frac{37.7\left(3.5 10^{-2}\right)^2}{(91.2)^2} \approx 55\ 10^{-7}\,GeV^{-2}$$

$$(\hbar c)^2 \simeq 0.389 GeV^2 mb$$

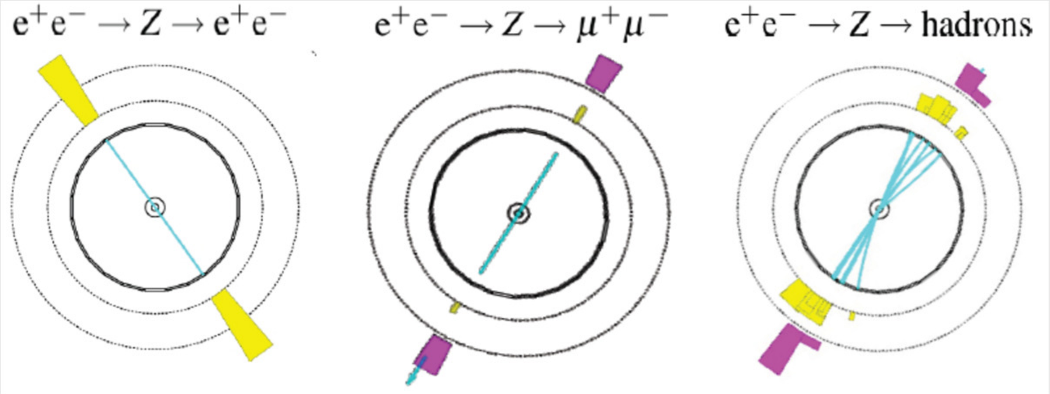
$$\to \sigma_{peak} \simeq 55\ 10^{-7}\,GeV^{-2}\,0.389 GeV^2 mb \approx 2.14 10^{-6} mb = 2.14 nb$$

Precision Tests - XII

Z peak: Essentially 4 types of events

$e^+e^- \to e^+e^-, \mu^+\mu^-, \tau^+\tau^-, q\overline{q} \left(\to hadrons\right)$

Different topologies in the detectors:



Measuring cross sections:

Count events (!)

Subtract background

Correct for inefficiency

Get integrated luminosity (Most of the time from independent counting of Bhabha events)

$$\to \sigma = \frac{N - N_{bck}}{\varepsilon} \frac{1}{L_{\text{int}}}$$

Precision Tests - XIII

Among other results at the peak: Z^0 lineshape

Meaning in practice:

m_Z Z mass

Γ_Z Z total width

Γ_f Z partial width to fermion type f

N_ν Number of (SM) neutrino species

Obtained by 'scanning' the Z^0 peak:

Move $E_{beam} = \frac{\sqrt{s}}{2}$ in steps through the peak

Measure relevant σ at each step

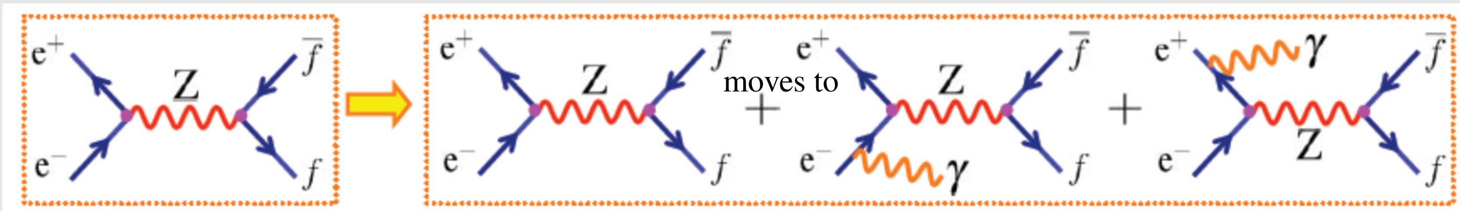
Fit profile:

$$\sigma\left(e^+e^- \to Z \to f\bar{f}\right) = \frac{12\pi}{m_Z^2} \frac{s\Gamma_{ee}\Gamma_{ff}}{\left(s - m_Z^2\right)^2 + m_Z^2\Gamma_Z^2}$$

Precision Tests - XIV

Lineshape quite distorted by several effects

Main effect: Initial State Radiation (*ISR*)



Result:

Collision *CM* energy $\neq 2E_{beam}$

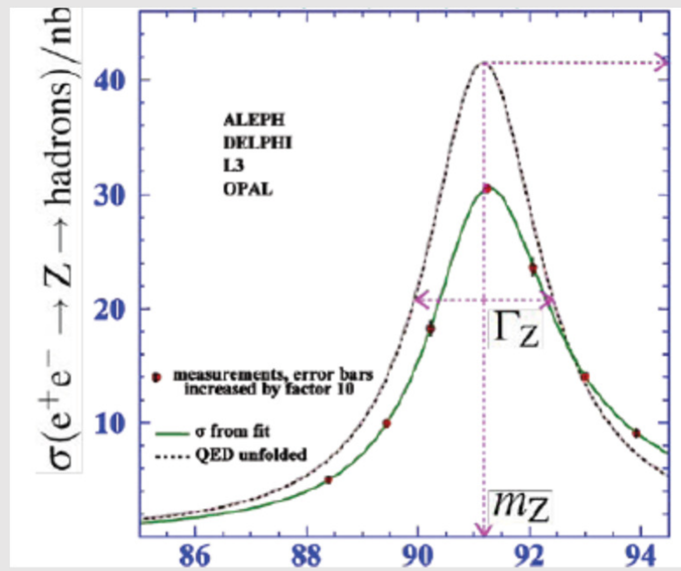
$$e^+ \xrightarrow{E} \quad \xleftarrow{E} e^- \qquad \sqrt{s} = 2E$$

moves to

$$\xrightarrow{E} \quad \xleftarrow{E - E_\gamma} \qquad \sqrt{s'} \approx 2E\left(1 - \frac{E_\gamma}{2E}\right)$$

$$m_Z = 91.1875 \pm 0.0021\,\mathrm{GeV}$$

$$\Gamma_Z = 2.4952 \pm 0.0023\,\mathrm{GeV}$$



Precision Tests - XV

Finding the number of Standard Model neutrinos

(Meaning: With standard coupling to Z)

Total width:

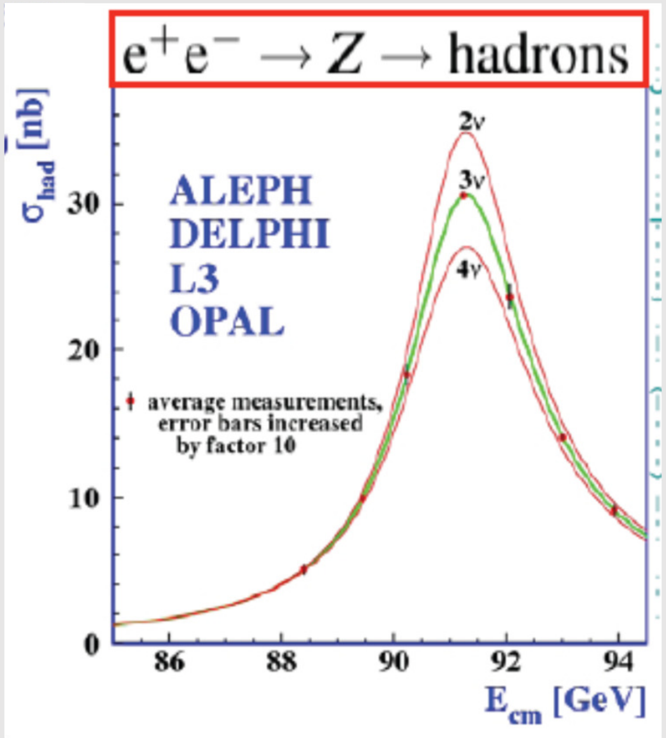
$$\Gamma = \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + \Gamma_{had} + \Gamma_{\nu_1\nu_1} + \Gamma_{\nu_2\nu_2} + \Gamma_{\nu_3\nu_3} + ?$$

$$\Gamma = 3\Gamma_{ll} + \Gamma_{had} + N_\nu \Gamma_{\nu\nu}$$

Measure partial widths from peak cross sections:

$$\sigma_0^{ff} = \frac{12\pi}{m_Z^2} \frac{\Gamma_{ee}\Gamma_{ff}}{\Gamma_Z^2}$$

$$N_\nu = 2.9840 \pm 0.0082$$



Precision Tests - XVI

Write differential cross section $\left(\text{ e.g. for } e^+e^- \to \mu^+\mu^-\right)$ as:

$$\frac{d\sigma}{d\Omega} = k\left[A\left(1+\cos^2\theta\right)+B\cos\theta\right]$$

$$A = \left[\left(c_L^e\right)^2 + \left(c_R^e\right)^2\right]\left[\left(c_L^\mu\right)^2 + \left(c_R^\mu\right)^2\right], \quad B = \left[\left(c_L^e\right)^2 - \left(c_R^e\right)^2\right]\left[\left(c_L^\mu\right)^2 - \left(c_R^\mu\right)^2\right]$$

Forward/Backward cross sections:

$$\sigma_F = \int_0^1 \frac{d\sigma}{d\cos\theta} d\cos\theta, \quad \sigma_B = \int_{-1}^0 \frac{d\sigma}{d\cos\theta} d\cos\theta$$

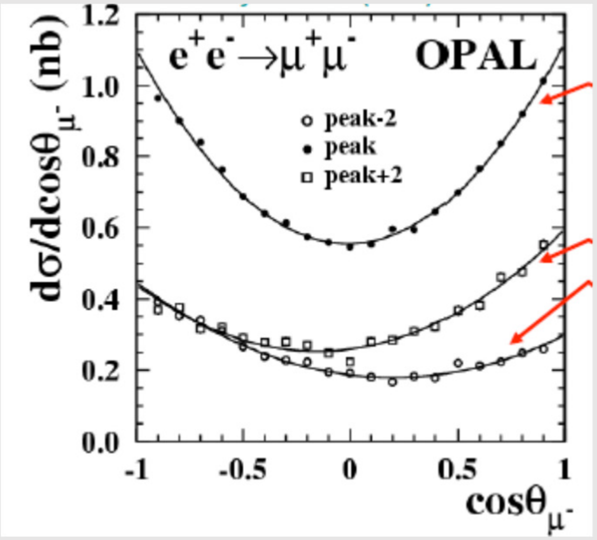
FB Asymmetry:

$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$

$$\sigma_F = k\left(\frac{4}{3}A + \frac{1}{2}B\right), \sigma_B = k\left(\frac{4}{3}A - \frac{1}{2}B\right)$$

$$\to A_{FB} = \frac{3}{8}\frac{B}{A} = \frac{3}{4}\frac{\left[\left(c_L^e\right)^2 - \left(c_R^e\right)^2\right]}{\left[\left(c_L^e\right)^2 + \left(c_R^e\right)^2\right]}\frac{\left[\left(c_L^\mu\right)^2 - \left(c_R^\mu\right)^2\right]}{\left[\left(c_L^\mu\right)^2 + \left(c_R^\mu\right)^2\right]} = \frac{3}{4}A_e A_\mu$$

Precision Tests - XVII



$A_{FB}(peak) \sim 0$ for leptons ($\sin^2\theta_W \approx 0.25$)

$A_{FB}(peak \pm 2\text{ GeV}) \neq 0 : = 2\dfrac{c_V/c_A}{1+(c_V/c_A)^2}$

Interference with QED

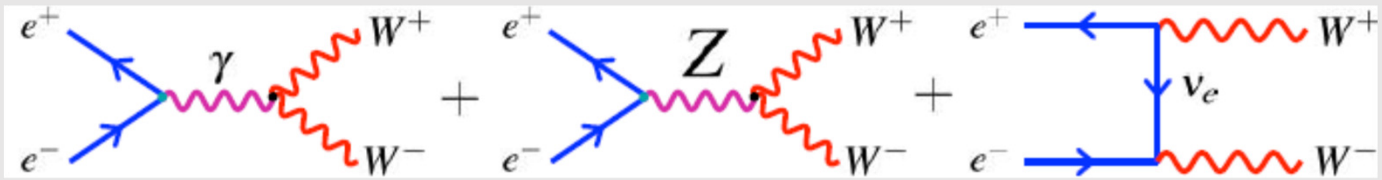
$$A_{FB}^{0,e} = 0.0145 \pm 0.0025$$
$$A_{FB}^{0,\mu} = 0.0169 \pm 0.0013$$
$$A_{FB}^{0,\tau} = 0.0188 \pm 0.0017$$

$$A_e = 0.1514 \pm 0.0019$$
$$A_\mu = 0.1456 \pm 0.0091$$
$$A_\tau = 0.1449 \pm 0.0040$$

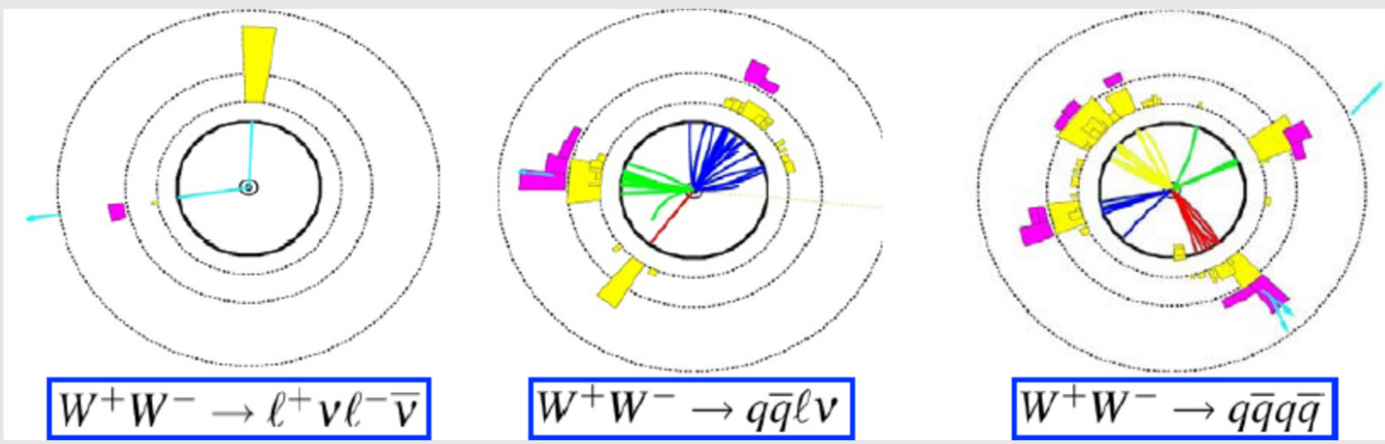
$$\sin^2\theta_W = 0.23154 \pm 0.00016$$

Precision Tests - XVIII

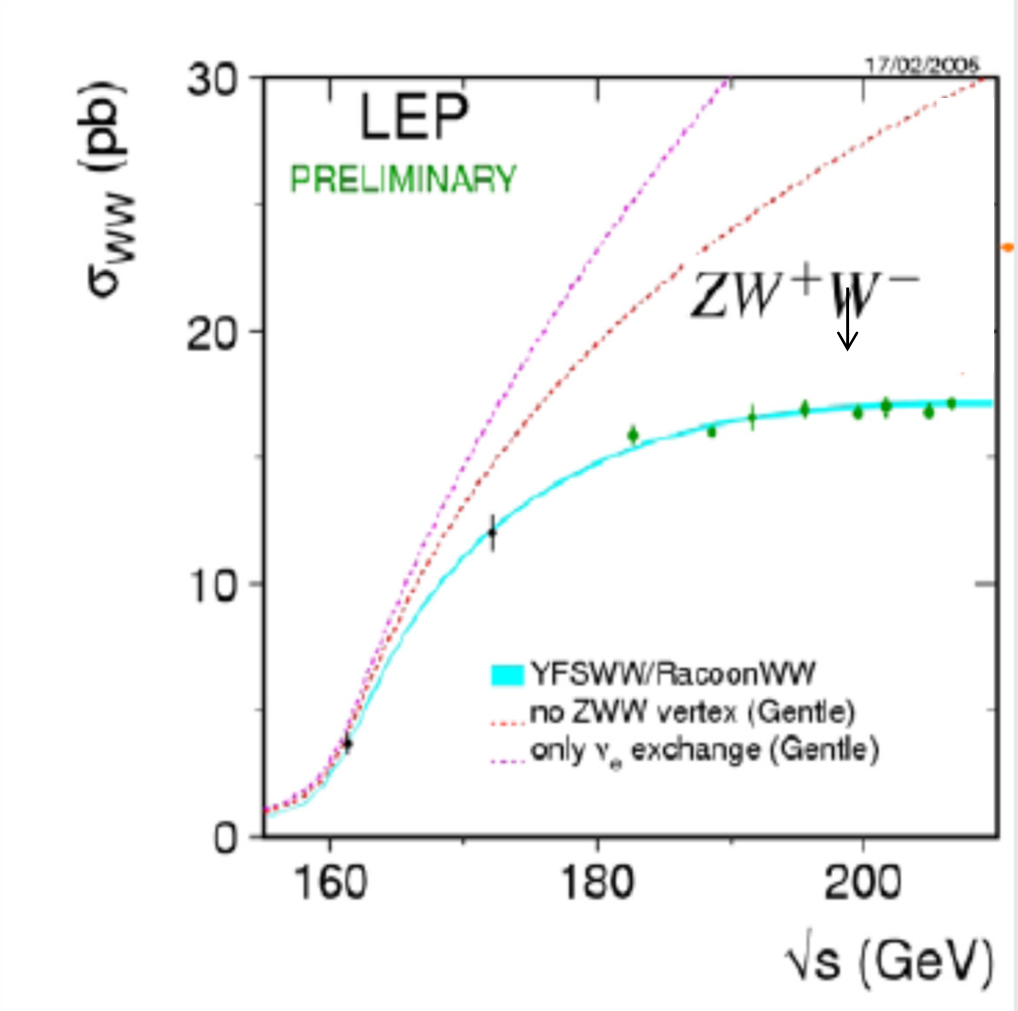
LEP2: Study of WW production



$$Br(W^- \to \text{hadrons}) \approx 0.67 \qquad Br(W^- \to e^- \overline{\nu}_e) \approx 0.11$$
$$Br(W^- \to \mu^- \overline{\nu}_\mu) \approx 0.11 \qquad Br(W^- \to \tau^- \overline{\nu}_\tau) \approx 0.11$$



Precision Tests - XIX



Maybe one of the best results of the whole LEP saga

Precision Tests - XX

Measurement of m_W : Kinematical fit

Example:

$$W^+W^- \to q\bar{q}e^-\bar{\nu}$$

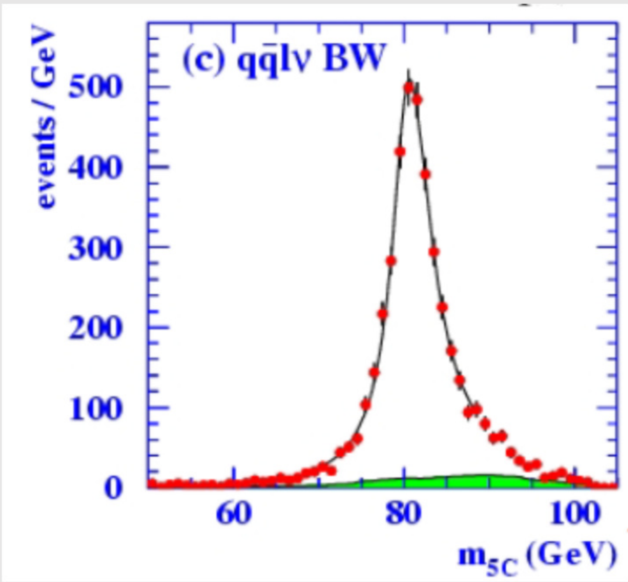
Get $\bar{\nu}$ 4-momentum from:

$$p_q + p_{\bar{q}} + p_{e^-} + p_{\bar{\nu}} = \left(\sqrt{s}, 0\right)$$

Make W bosons masses :

$$M_{W^+} = \left(p_q + p_{\bar{q}}\right)^2$$

$$M_{W^-} = \left(p_{e^-} + p_{\bar{\nu}}\right)^2$$



$$m_W = 80.376 \pm 0.033\,\text{GeV}$$

$$\Gamma_W = 2.196 \pm 0.083\,\text{GeV}$$

Loopology - I

Standard Model :

$M_W = M_Z \cos\theta_W$

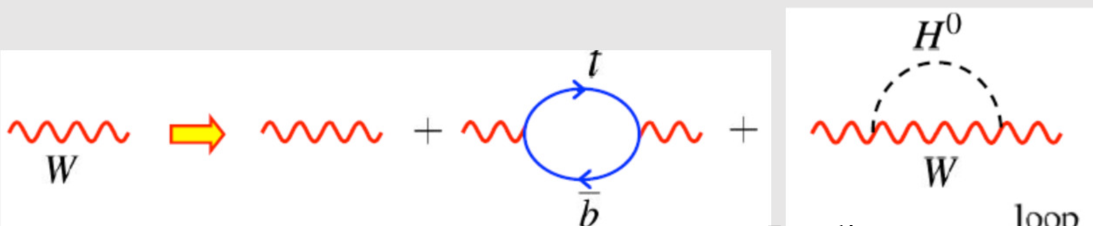
Measure: $\begin{cases} M_Z = 91.1875 \pm 0.0021 \;\; GeV \\ \sin^2\theta_W = 0.23154 \pm 0.00016 \end{cases}$

$\rightarrow$ Predict $M_W = 79.946 \pm 0.008 \;\; GeV$

Measure

$M_W = 80.376 \pm 0.033 \;\; GeV$

Discrepancy: Virtual loops (including Higgs..)



$$m_W = m_W^0 + a m_t^2 + b \ln\left(\frac{m_H}{m_W}\right)$$

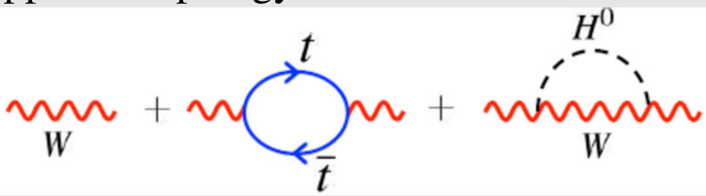
Predict: $m_t^{\text{loop}} = 173 \pm 11\,\text{GeV}$

Observe: $m_t^{\text{meas}} = 174.2 \pm 3.3\,\text{GeV}$

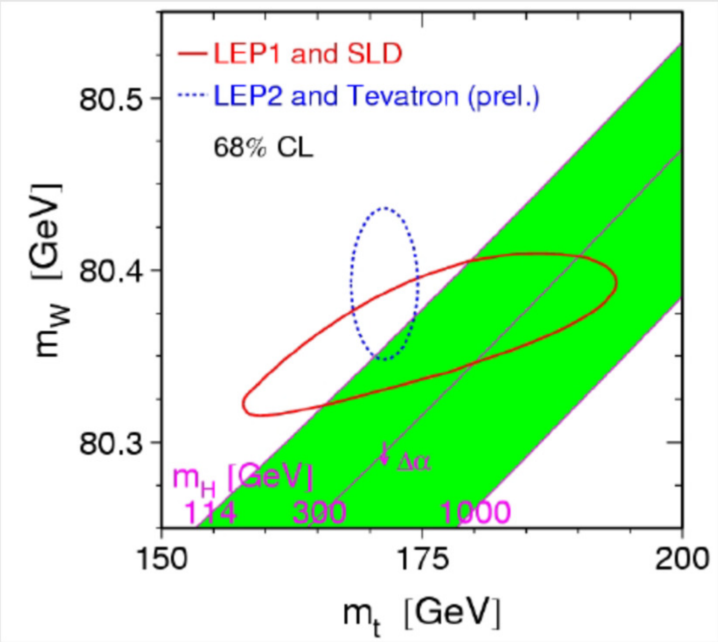
!!!

Loopology - II

Applied loopology:



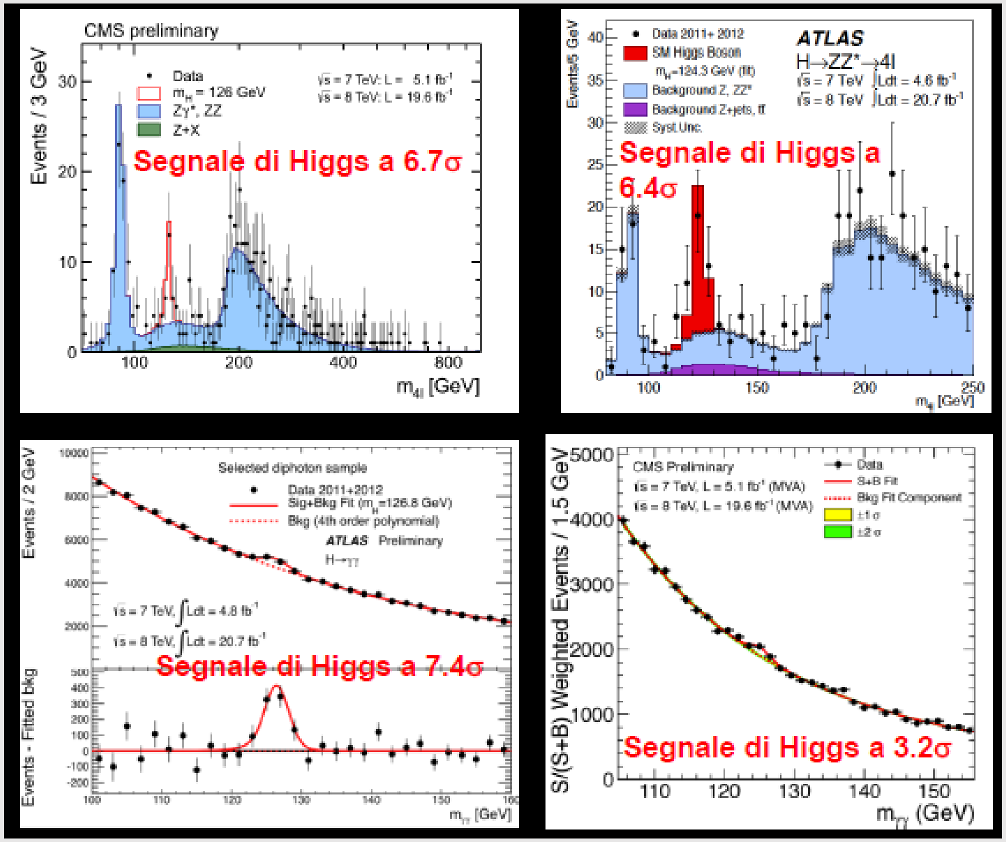
$$m_W = m_W^0 + a m_t^2 + b \ln\left(\frac{m_H}{m_W}\right)$$



$$m_H < 200\,\mathrm{GeV}$$

The Happy End

All is well that ends well:
And finally…



… Mr. Higgs
and Mr. Englert
went to Stockholm