Elementary Particles II

3 – Flavor Physics and CP Violation

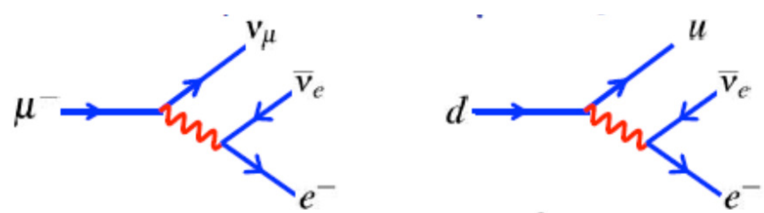
Quark Mixing – CKM – K^0 Strangeness oscillations – CP violation
Extension to Bottom and Charm – FCNC and Physics Beyond the Standard Model

Quark Mixing - I

Reminder:

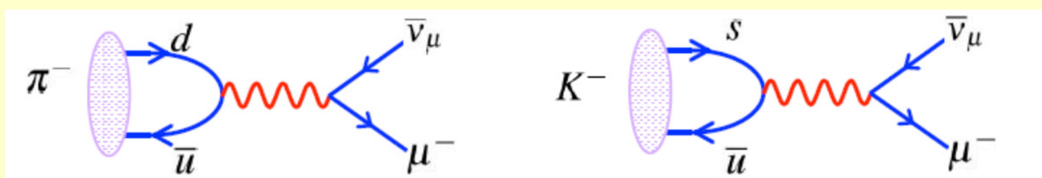
Fermi constant from μ decay $\simeq$ Fermi constant from β decay

Tiny difference:



$G_F^\mu = (1.16632 \pm 0.00002) \times 10^{-5}\,\mathrm{GeV}^{-2}$ $\quad$ $G_F^\beta = (1.136 \pm 0.003) \times 10^{-5}\,\mathrm{GeV}^{-2}$

Kaon decay suppressed by a factor ~ 20 as compared to π decay



Quark Mixing - II

Cabibbo explaination:

Weak eigenstates — Strong (mass) eigenstates

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

Weak charged currents: Linear combinations of different flavors

$\theta_C \approx 13.1^0$

Unique value for Cabibbo angle explaining many strange particle decays

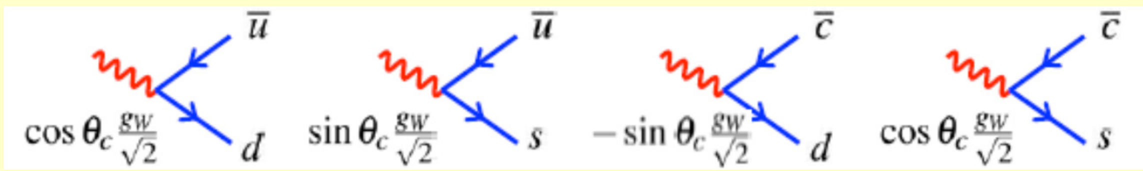
Strong support for universality of weak interaction

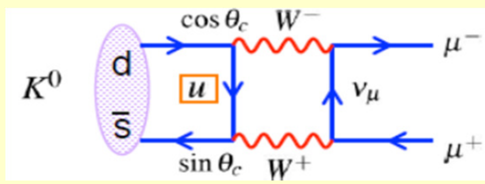
Quark Mixing - III

Another mistery

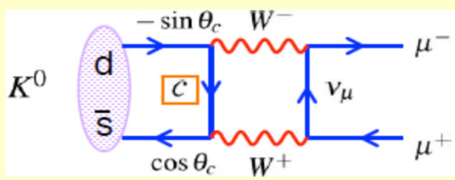
$$BR\left(K^0 \to \mu^+\mu^-\right) \sim 10^{-8}\, BR\left(K^+ \to \mu^+\nu_\mu\right)$$

GIM explaination:





$$M_1 \propto g_W^4 \cos\theta_c \sin\theta_c$$



$$M_2 \propto -g_W^4 \cos\theta_c \sin\theta_c$$

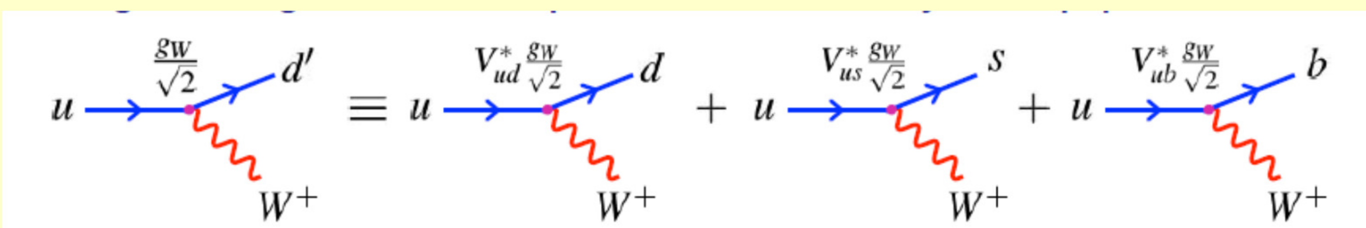
Tiny BR left due to $m_c \neq m_u$ in the virtual quark propagator

Quark Mixing - IV

Extend mixing to 3 families:

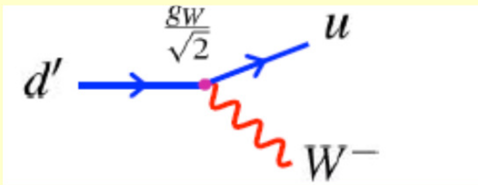
$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V^{\dagger}_{(d)} \begin{pmatrix} d \\ s \\ b \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}, \quad \begin{pmatrix} u' \\ c' \\ t' \end{pmatrix} = V^{\dagger}_{(u)} \begin{pmatrix} u \\ c \\ t \end{pmatrix}$$

$\rightarrow$ Conventionally: Mixing of d-like quarks only



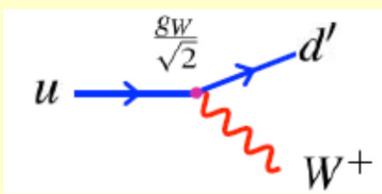
Quark Mixing - V

Encode mixing CKM matrix element into charged current



$$j_{d'u} = \bar{u}\left[-i\frac{g_W}{\sqrt{2}}\gamma^{\mu}\tfrac{1}{2}(1-\gamma^5)\right]d'$$

$$j_{du} = \bar{u}\left[-i\frac{g_W}{\sqrt{2}}\gamma^{\mu}\tfrac{1}{2}(1-\gamma^5)\right]V_{ud}d$$



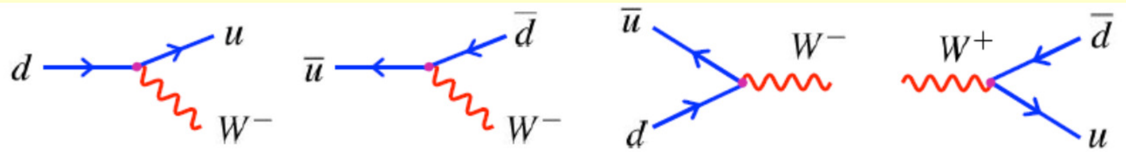
$$j_{ud'} = \bar{d}'\left[-i\frac{g_W}{\sqrt{2}}\gamma^{\mu}\tfrac{1}{2}(1-\gamma^5)\right]u$$

$$\bar{d}' = d'^{\dagger}\gamma^0 \rightarrow (V_{ud}d)^{\dagger}\gamma^0 = V_{ud}^{*}d^{\dagger}\gamma^0 = V_{ud}^{*}\bar{d}$$

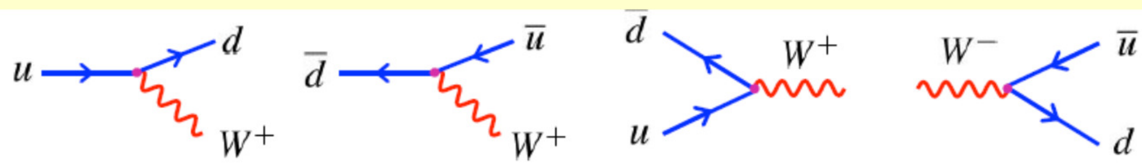
$$j_{ud} = \bar{d}V_{ud}^{*}\left[-i\frac{g_W}{\sqrt{2}}\gamma^{\mu}\tfrac{1}{2}(1-\gamma^5)\right]u$$

Quark Mixing - VI

Charged current : $qq, \overline{qq}, q\overline{q}$



is $$-i\frac{g_W}{\sqrt{2}}V_{ud}\gamma^\mu \tfrac{1}{2}(1-\gamma^5)$$



is $$-i\frac{g_W}{\sqrt{2}}V_{ud}^*\gamma^\mu \tfrac{1}{2}(1-\gamma^5)$$

Quark Mixing - VII

CKM elements involving *t* quark poorly known

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} \approx \begin{pmatrix} 0.974 & 0.226 & 0.004 \\ 0.23 & 0.96 & 0.04 \\ ? & ? & ? \end{pmatrix}$$

Fill missing elements by unitarity: $|V_{ub}|^2 + |V_{cb}|^2 + |V_{tb}|^2 = 1$ etc.

$$\begin{pmatrix} 0.97427 \pm 0.00015 & 0.22534 \pm 0.00065 & 0.00351^{+0.00015}_{-0.00014} \\ 0.22520 \pm 0.00065 & 0.97344 \pm 0.00016 & 0.0412^{+0.0011}_{-0.0005} \\ 0.00867^{+0.00029}_{-0.00031} & 0.0404^{+0.0011}_{-0.0005} & 0.999146^{+0.000021}_{-0.000046} \end{pmatrix}$$

CKM - I

Generic mixing matrix:

Mixing weak eigenstates into mass eigenstates (or the opposite)

3×3 Unitary matrix:

9 complex parameters $\rightarrow$ 18 real parameters

9 unitarity conditions: $\left.\begin{array}{r} UU^{\dagger} = 1 \\ \left(U^{\dagger}\right)_{ij} = U^{*}_{ji} \end{array}\right\} \rightarrow \sum_{j=1}^{3} a_{ij} a^{*}_{jk} = \delta_{ik}, \quad i,k = 1,..,3$

$\rightarrow 18 - 9 = 9$ free, real parameters

CKM - II

Mixing matrix definition for 'up'- and 'down'-like quarks:

$$\begin{pmatrix} u \\ c \\ t \end{pmatrix} = V_{(u)} \begin{pmatrix} u_1' \\ u_2' \\ u_3' \end{pmatrix}; \qquad \begin{pmatrix} d \\ s \\ b \end{pmatrix} = V_{(d)} \begin{pmatrix} d_1' \\ d_2' \\ d_3' \end{pmatrix}$$

$$V_{CKM} = V_{(u)} V_{(d)}^{\dagger} \equiv V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Re-define (arbitrary) phases of quark mass eigenstates:

$$\begin{pmatrix} u \\ c \\ t \end{pmatrix} \to \begin{pmatrix} e^{i\varphi_u} & 0 & 0 \\ 0 & e^{i\varphi_c} & 0 \\ 0 & 0 & e^{i\varphi_t} \end{pmatrix} \begin{pmatrix} u \\ c \\ t \end{pmatrix}, \quad \begin{pmatrix} d \\ s \\ b \end{pmatrix} \to \begin{pmatrix} e^{i\varphi_d} & 0 & 0 \\ 0 & e^{i\varphi_s} & 0 \\ 0 & 0 & e^{i\varphi_b} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

CKM - III

Translate into redefinition of weak eigenstates:

$$\begin{pmatrix} u' \\ c' \\ t' \end{pmatrix} \rightarrow V_u^\dagger \begin{pmatrix} e^{i\phi_u} & 0 & 0 \\ 0 & e^{i\phi_c} & 0 \\ 0 & 0 & e^{i\phi_t} \end{pmatrix} \begin{pmatrix} u \\ c \\ t \end{pmatrix}, \quad \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} \rightarrow V_d^\dagger \begin{pmatrix} e^{i\phi_d} & 0 & 0 \\ 0 & e^{i\phi_s} & 0 \\ 0 & 0 & e^{i\phi_b} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Redefinition of weak eigenstates equivalent to CKM redefinition:

$$V_{CKM} \rightarrow \begin{pmatrix} e^{-i\varphi_u} & 0 & 0 \\ 0 & e^{-i\varphi_c} & 0 \\ 0 & 0 & e^{-i\varphi_t} \end{pmatrix} V_{CKM} \begin{pmatrix} e^{i\varphi_d} & 0 & 0 \\ 0 & e^{i\varphi_s} & 0 \\ 0 & 0 & e^{i\varphi_b} \end{pmatrix} = \begin{pmatrix} V_{ud} e^{i(\varphi_d - \varphi_u)} & V_{us} e^{i(\varphi_s - \varphi_u)} & V_{ub} e^{i(\varphi_b - \varphi_u)} \\ V_{cd} e^{i(\varphi_d - \varphi_c)} & V_{cs} e^{i(\varphi_s - \varphi_c)} & V_{cb} e^{i(\varphi_b - \varphi_c)} \\ V_{td} e^{i(\varphi_d - \varphi_t)} & V_{ts} e^{i(\varphi_s - \varphi_t)} & V_{tb} e^{i(\varphi_b - \varphi_t)} \end{pmatrix}$$

CKM - IV

Factorize one (any) phase:

$$\rightarrow V_{CKM} = e^{-i\varphi_u} \begin{pmatrix} V_{ud} e^{i\varphi_d} & V_{us} e^{i\varphi_s} & V_{ub} e^{i\varphi_b} \\ V_{cd} e^{i(\varphi_u+\varphi_d-\varphi_c)} & V_{cs} e^{i(\varphi_u+\varphi_s-\varphi_c)} & V_{cb} e^{i(\varphi_u+\varphi_b-\varphi_c)} \\ V_{td} e^{i(\varphi_u+\varphi_d-\varphi_t)} & V_{ts} e^{i(\varphi_u+\varphi_s-\varphi_t)} & V_{tb} e^{i(\varphi_u+\varphi_b-\varphi_t)} \end{pmatrix}$$

Global field phase not relevant: Can't be used to fix one free V parameter

$\rightarrow$ 5 free relative phases $\rightarrow$ Set to 0

$\rightarrow 9 - 5 = 4$ real parameters

Encode as:

3 'rotation angles' ($\leftarrow$ Euler angles)

[In order to understand this:

Suppose the matrix is real $\rightarrow$ Any 3×3 real, unitary matrix = Orthogonal

Any 3×3 orthogonal matrix = 3D Rotation]

1 complex (irreducible) phase factor

CKM - V

→Generally *CKM* matrix *must* be complex, *3x3* real would require just 3 parameters
Standard form :

Parameters:

$\theta_{12}, \theta_{13}, \theta_{23}$ Rotation angles

δ Irreducible phase

$s_{ij} = \sin\theta_{ij}$

$c_{ij} = \cos\theta_{ij}$

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}e^{+i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{+i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{+i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{+i\delta} & c_{13}c_{23} \end{pmatrix}$$

Experiment:

$\sin\theta_{13} \ll \sin\theta_{23} \ll \sin\theta_{12} \ll 1$

CKM - VI

Wolfenstein parametrization of V_{CKM}:

Based on experimental evidence of some hierarchy among angles

Define:

$\lambda = \sin\theta_{12}$

$A\lambda^2 = \sin\theta_{23}$

$A\lambda^3(\rho - i\eta) = \sin\theta_{13} e^{-i\delta}$

Then:

$$\cos\theta_{12} = \sqrt{1-\lambda^2} \simeq 1 - \frac{\lambda^2}{2} - \frac{\lambda^4}{8}$$

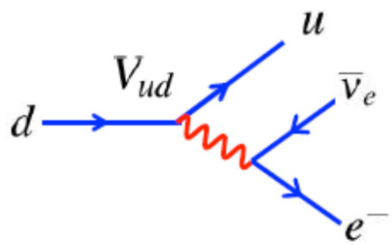
$$\rightarrow V_{CKM} \simeq \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

$$\lambda = 0.2259 \pm 0.0021 \approx \sin\theta_C$$
$$A = 0.82 \pm 0.02$$
$$\eta \neq 0 \rightarrow \not{CP}$$

Filling CKM - I

$|V_{ud}|$ **from nuclear beta decay**

$$\begin{pmatrix} \times & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$



Super-allowed $0^+ \rightarrow 0^+$ beta decays are relatively free from theoretical uncertainties

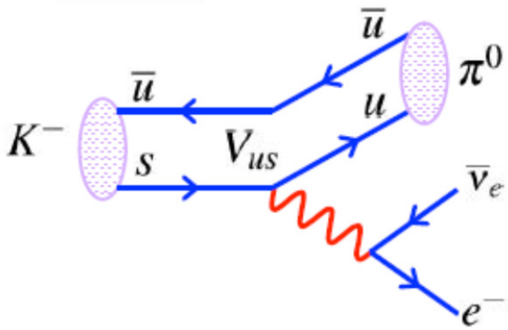
$$\Gamma \propto |V_{ud}|^2$$

$$|V_{ud}| = 0.97377 \pm 0.00027 \qquad (\approx \cos\theta_c)$$

Filling CKM - II

$|V_{us}|$ from semi-leptonic kaon decays

$$\begin{pmatrix} \cdot & \times & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$



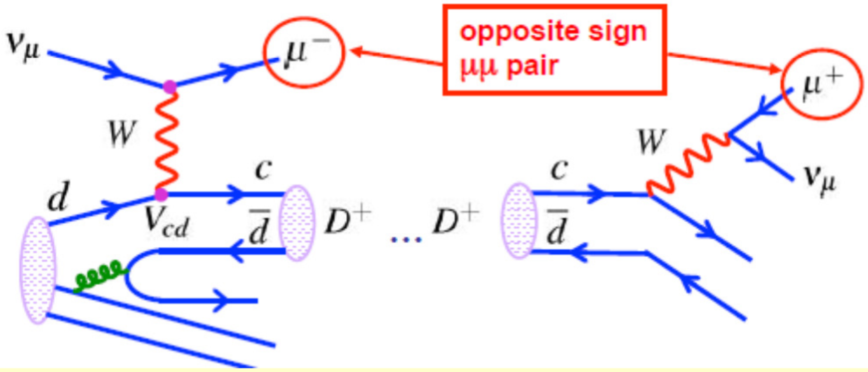
$$\Gamma \propto |V_{us}|^2$$

$$|V_{us}| = 0.2257 \pm 0.0021 \qquad (\approx \sin\theta_c)$$

Filling CKM - III

$|V_{cd}|$ **from neutrino scattering** $\nu_\mu + N \to \mu^+ \mu^- X$ $\begin{pmatrix} \cdot & \cdot & \cdot \\ \times & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$

Look for opposite charge di-muon events in ν_μ scattering from production and decay of a $D^+(c\bar{d})$ meson

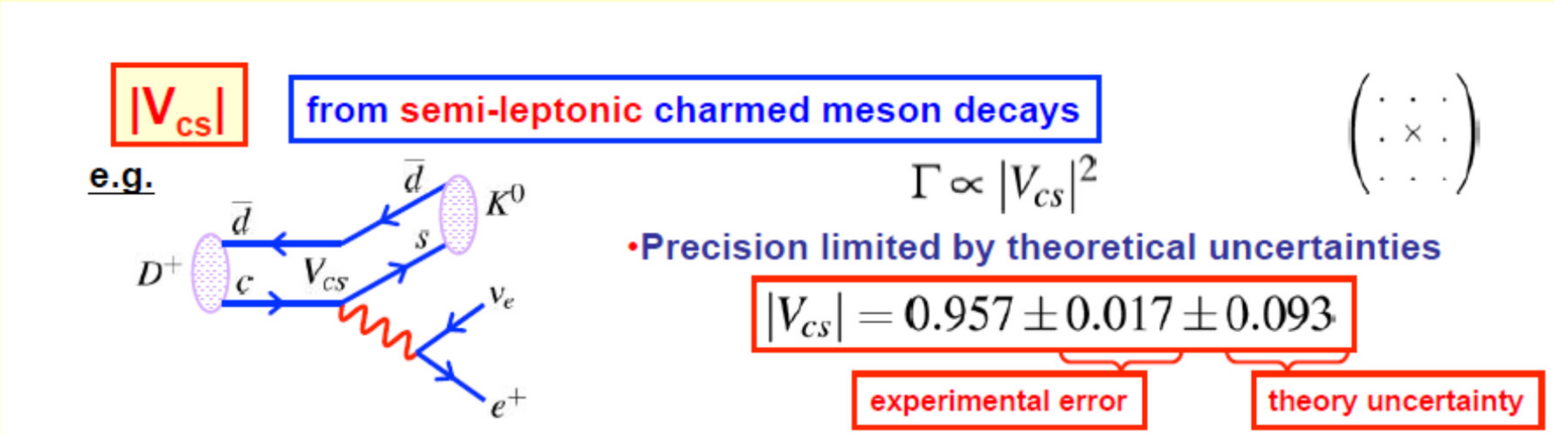


$$\text{Rate} \propto |V_{cd}|^2 \text{Br}(D^+ \to X \mu^+ \nu_\mu)$$

Measured in various collider experiments

⇨ $|V_{cd}| = 0.230 \pm 0.011$

Filling CKM - IV



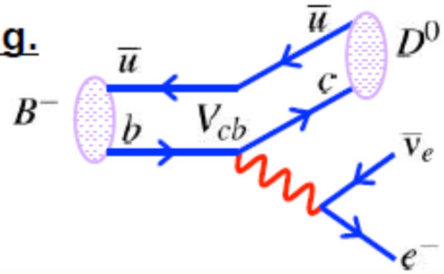
$K^0\, e^+ \nu_e$ *(8.83±0.22) %*

Filling CKM - V

$|V_{cb}|$ from semi-leptonic B hadron decays

$$\begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \times \\ \cdot & \cdot & \cdot \end{pmatrix}$$

e.g.



$$\Gamma \propto |V_{cb}|^2$$

$$|V_{cb}| = 0.0416 \pm 0.0006$$

$\overline{D}^0 \ell^+ \nu_\ell$ (2.23 ±0.11) %

$$L = 10^{33} cm^{-2} s^{-1} \rightarrow L_{\text{int}} \sim 10^{38} cm^{-2} d^{-1}$$

$$\sigma_{B\overline{B}} \sim 1\,nb = 10^{-33} cm^2 \rightarrow R \sim 10^5\, B\overline{B}\ \ d^{-1}$$

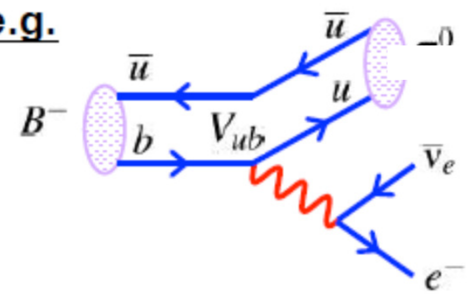
$$\rightarrow R_{dec} \sim 10^3\ \ D^0 e^- \overline{\nu}_e\ \ d^{-1} \rightarrow \frac{\sqrt{\sigma_{stat}}}{|V_{cb}|} \sim \frac{3\%}{\sqrt{T(days)}}$$

Filling CKM - VI

$|V_{ub}|$ from semi-leptonic B hadron decays

$$\begin{pmatrix} \cdot & \cdot & \times \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

e.g.



$$\Gamma \propto |V_{ub}|^2$$

$$|V_{ub}| = 0.0043 \pm 0.0003$$

$\pi^0 \ell^+ \nu_\ell$	$(\ 7.7\ \pm 1.2\ ) \times 10^{-5}$

$$L = 10^{33} cm^{-2} s^{-1} \rightarrow L_{\text{int}} \sim 10^{38} cm^{-2} d^{-1}$$

$$\sigma_{B\bar{B}} \sim 1\ nb = 10^{-33} cm^2 \rightarrow R \sim 10^5 B\bar{B}\ \ d^{-1}$$

$$\rightarrow R_{dec} \sim 10\ \ \pi^0 e^- \bar{\nu}_e\ \ d^{-1} \rightarrow \frac{\sigma_{stat}}{|V_{ub}|} \sim \frac{30\%}{\sqrt{T(days)}}$$

CKM Triangles - I

V_{CKM} unitary: 9 unitarity conditions

Take 6 'off-diagonal' conditions:

$$(1)\quad V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0; \qquad (2)\quad V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0;$$

$$(3)\quad V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0; \qquad (4)\quad V_{ud}^*V_{cd} + V_{us}^*V_{cs} + V_{ub}^*V_{cb} = 0;$$

$$(5)\quad V_{ud}^*V_{td} + V_{us}^*V_{ts} + V_{ub}^*V_{tb} = 0; \qquad (6)\quad V_{cd}^*V_{td} + V_{cs}^*V_{ts} + V_{cb}^*V_{tb} = 0;$$

Each condition:

Sum of 3 complex numbers = 0

Complex number $\triangleq$ Vector in the complex plane

$\rightarrow$ Each condition $\sim$ 3 numbers should add to a closed triangle

CKM Triangles - II

Sides & Angles from experiment

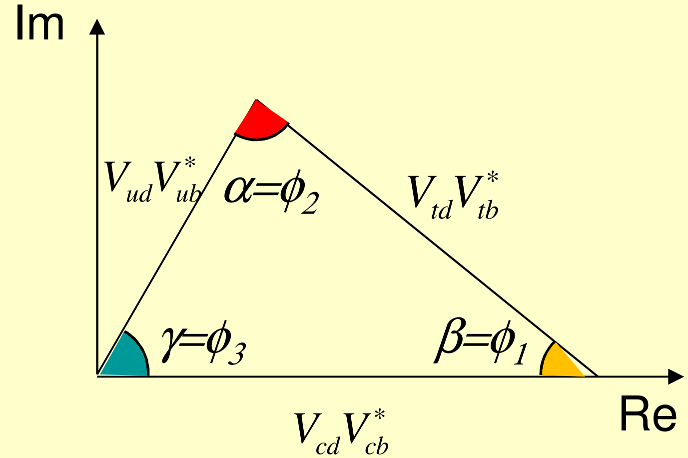
Area: Same for all 6

$$A_{triangle} = \frac{1}{2} J_{CP} = \frac{1}{2} \mathrm{Im}\left(V_{ij} V_{kl} V_{il}^* V_{kj}^*\right); \qquad i \neq k, j \neq l;$$

$$J_{CP} = s_{12} s_{13} s_{23} c_{12} c_{13}^2 c_{23} s_{\delta_{13}} \approx A^2 \lambda^6 \eta$$

$\rightarrow J_{CP}$ = Nice measure of $\not{CP}$

Example: Most common unitary triangle



$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$$

$$A\lambda^3 \left(1 - \frac{\lambda^2}{2}\right)(\rho + i\eta) \; - \; A\lambda^3 \left[1 + A^2 \lambda^4 (\rho + i\eta)\right] \; + \; A\lambda^3 \left[1 - (\rho + i\eta)\left(1 - \frac{\lambda^2}{2}\right)\right] = 0$$

CKM Triangles - III

$$\begin{cases} V_{ud}V_{ub}^* = A\lambda^3\left(1-\frac{\lambda^2}{2}\right)(\rho+i\eta) \\ V_{cd}V_{cb}^* = -A\lambda^3 \\ V_{td}V_{tb}^* = A\lambda^3\left[1-(\rho+i\eta)\right] \end{cases}$$

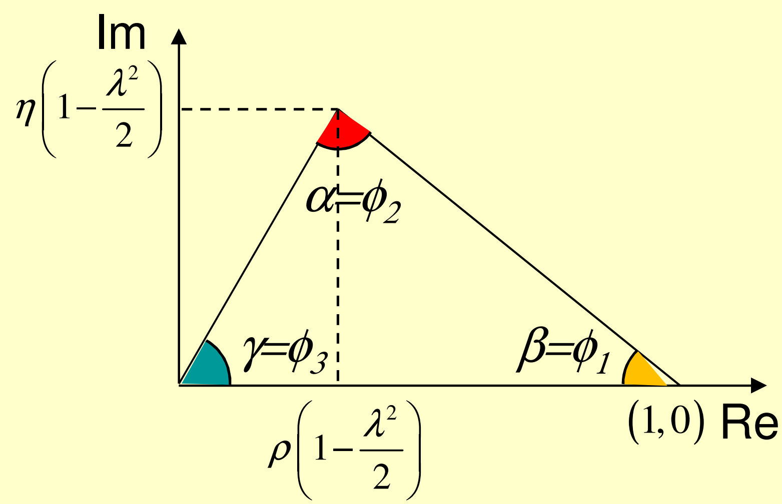
Normalize to $V_{cd}V_{cb}^* = -A\lambda^3 \equiv 1$;

Ignore overall $-$ signs

$$V_{ud}V_{ub}^* = \left(1-\frac{\lambda^2}{2}\right)(\rho+i\eta) \approx \rho+i\eta$$

$$V_{cd}V_{cb}^* = 1$$

$$V_{td}V_{tb}^* \approx 1-(\rho+i\eta)$$



CKM Triangles - IV

Recent fit from CKMFitter group:

Observable	Central ± 1 σ	± 2 σ	± 3 σ
A	0.812 [+0.015 -0.022]	0.812 [+0.025 -0.031]	0.812 [+0.035 -0.039]
λ	0.22543 [+0.00059 -0.00095]	0.2254 [+0.0010 -0.0019]	0.2254 [+0.0013 -0.0027]
ρbar	0.145 [+0.027 -0.027]	0.145 [+0.046 -0.040]	0.145 [+0.057 -0.050]
ηbar	0.343 [+0.015 -0.015]	0.343 [+0.030 -0.026]	0.343 [+0.044 -0.035]
J $[10^{-5}]$	2.96 [+0.18 -0.14]	2.96 [+0.32 -0.19]	2.96 [+0.46 -0.23]
α [deg]	91.1 [+4.3 -4.3]	91.1 [+7.1 -6.2]	91.1 [+8.8 -7.8]
α [deg] (meas. not in the fit)	95.9 [+2.2 -5.6]	95.9 [+3.6 -10.9]	95.9 [+5.0 -12.8]
α [deg] (dir. meas.)	88.7 [+4.6 -4.2]	88.7 [+9.4 -8.5]	89 [+21 -13]
β [deg]	21.85 [+0.80 -0.77]	21.9 [+1.6 -1.3]	21.9 [+2.5 -1.8]
β [deg] (meas. not in the fit)	27.5 [+1.2 -1.4]	27.5 [+1.9 -3.9]	27.5 [+2.6 -6.8]
β [deg] (dir. meas.)	21.38 [+0.79 -0.77]	21.4 [+1.6 -1.5]	21.4 [+2.4 -2.3]
γ [deg]	67.1 [+4.3 -4.3]	67.1 [+6.1 -7.0]	67.1 [+7.6 -8.5]
γ [deg] (meas. not in the fit)	67.2 [+4.4 -4.6]	67.2 [+6.1 -7.2]	67.2 [+7.6 -8.7]
γ [deg] (dir. meas.)	66 [+12 -12]	66 [+23 -22]	66 [+36 -30]

K Oscillations - I

$$\left|K^0\right\rangle = \left|d\bar{s}\right\rangle \qquad S = +1$$

$$\left|\bar{K}^0\right\rangle = \left|\bar{d}s\right\rangle \qquad S = -1$$

$$P\left|K^0\right\rangle = -\left|K^0\right\rangle \quad \text{Pseudoscalar}$$

$$P\left|\bar{K}^0\right\rangle = -\left|\bar{K}^0\right\rangle \quad \text{Pseudoscalar}$$

$$C\left|K^0\right\rangle = \left|\bar{K}^0\right\rangle \qquad \text{Not a C eigenstate}$$

$$C\left|\bar{K}^0\right\rangle = \left|K^0\right\rangle \qquad \text{Not a C eigenstate}$$

$\rightarrow$ Make C eigenstates:

$$\left|K_1^0\right\rangle = \frac{1}{\sqrt{2}}\left(\left|K^0\right\rangle - \left|\bar{K}^0\right\rangle\right) \rightarrow C\left|K_1^0\right\rangle = C\left[\frac{1}{\sqrt{2}}\left(\left|K^0\right\rangle - \left|\bar{K}^0\right\rangle\right)\right] = \frac{1}{\sqrt{2}}\left(\left|\bar{K}^0\right\rangle - \left|K^0\right\rangle\right) = -\left|K_1^0\right\rangle$$

$$\left|K_2^0\right\rangle = \frac{1}{\sqrt{2}}\left(\left|K^0\right\rangle + \left|\bar{K}^0\right\rangle\right) \rightarrow C\left|K_2^0\right\rangle = \left|K_2^0\right\rangle$$

K Oscillations - II

Kaons : Just weak decays $(\leftarrow$ Lightest strange hadron$)$

C, P not conserved by weak processes

CP *almost* conserved by weak processes $\rightarrow$ Take them as good for the moment

$\rightarrow$ Focus on CP as a symmetry for weak processes

CP eigenstates:

$$CP\left|K_1^0\right\rangle = CP\left[\frac{1}{\sqrt{2}}\left(\left|K^0\right\rangle - \left|\bar{K}^0\right\rangle\right)\right] = C\left[\frac{1}{\sqrt{2}}\left(-\left|K^0\right\rangle + \left|\bar{K}^0\right\rangle\right)\right] = +\left|K_1^0\right\rangle \quad CP = +1$$

$$CP\left|K_2^0\right\rangle = -\left|K_2^0\right\rangle \qquad CP = -1$$

Observe : K_1^0, K_2^0 CP eigenstates, like photon, π^0

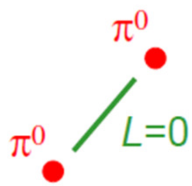
$\rightarrow$ Different particles

K Oscillations - III

K^0: Many different decay modes, including weak decays into pions

Consider first decays into 2 π's:

$K^0 \to \pi^0\pi^0$



J^P: $0^- \to 0^- + 0^-$

Ang. mom. conservation

$$\Rightarrow \quad L = 0$$

$$\Rightarrow \quad P(\pi^0\pi^0) = -1.-1.(-1)^L = (-1)^L = +1$$

π^0 is eigenstate of C : $\hat{C}|\pi^0\rangle = +|\pi^0\rangle$

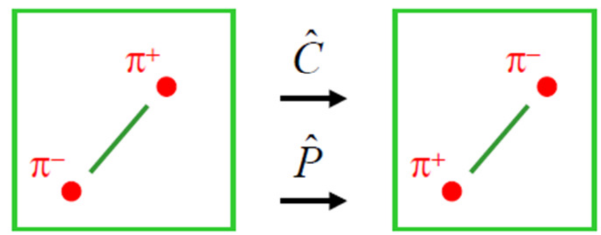
$$\Rightarrow \quad C(\pi^0\pi^0) = +1.+1 = +1$$

$K^0 \to \pi^+\pi^-$

Still have L = 0

$$\Rightarrow \quad P(\pi^+\pi^-) = -1.-1.(-1)^L = (-1)^L = +1$$

C and P operations have identical effect :



(no spins involved)

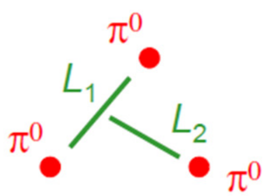
$$\Rightarrow \quad C(\pi^+\pi^-) = P(\pi^+\pi^-) = (-1)^L = +1$$

$\Rightarrow$ **CP = +1** for both $\pi^+\pi^-$ and $\pi^0\pi^0$

K Oscillations - IV

Consider then decays into 3 π's:

$K^0 \to \pi^0 \pi^0 \pi^0$



J^P: $0^- \to 0^- + 0^- + 0^-$

Ang. mom. conservation

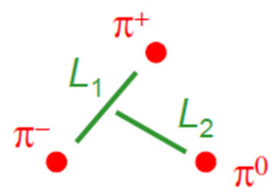
$\Rightarrow \quad L_1 \oplus L_2 = 0$

$\Rightarrow \quad L_1 = L_2$

$\Rightarrow \quad P(\pi^0\pi^0\pi^0) = -1.-1.-1.(-1)^{L_1}.(-1)^{L_2} = -1$

$C(\pi^0\pi^0\pi^0) = +1.+1.+1 = +1$

$K^0 \to \pi^+ \pi^- \pi^0$



As above: $L_1 = L_2$

$\Rightarrow \quad P(\pi^+\pi^-\pi^0) = -1.-1.-1.(-1)^{L_1}.(-1)^{L_2} = -1$

$C(\pi^+\pi^-\pi^0) = +1.C(\pi^+\pi^-) = +1.(-1)^{L_1} = +1$

Experimentally: $L_1 = 0$ from study of angular distributions of π^+, π^-

$\Rightarrow$ **CP = –1** for both $\pi^+ \pi^- \pi^0$ and $\pi^0 \pi^0 \pi^0$

K Oscillations - V

If CP is conserved in weak processes:

$$\left.\begin{array}{l} K_1^0 \to \pi\pi \\ K_2^0 \to \pi\pi\pi \end{array}\right\} \textit{Exclusively}$$

Summary so far about neutral K states:

Production (by strong interaction): $\left|K^0\right\rangle, \left|\bar{K}^0\right\rangle$

Decay (by weak interaction): $\left|K_1^0\right\rangle, \left|K_2^0\right\rangle$

$$m_{\left|K^0\right\rangle} = m_{\left|\bar{K}^0\right\rangle} \approx m_{\left|K_1^0\right\rangle} \approx m_{\left|K_2^0\right\rangle} \approx 498\ MeV$$

Expect, and find:

$K_1^0 \to \pi\pi$ Fast: Larger phase space etc $\to \tau_1 = 0.9\ 10^{-10} s$ 'K *short*'

$K_2^0 \to \pi\pi\pi$ Slow: Smaller phase space etc $\to \tau_2 = 0.5\ 10^{-7} s$ 'K *long*'

K Oscillations - VI

Provisionally identify:

$$K_S \equiv K_1^0 \left(\to \pi\pi\right) \qquad \tau_S = 0.9\ 10^{-10}\, s \qquad 'K\ short' \quad CP = +1$$
$$K_L \equiv K_2^0 \left(\to \pi\pi\pi\right) \qquad \tau_L = 0.5\ 10^{-7}\, s \qquad 'K\ long' \quad CP = -1$$

Therefore:

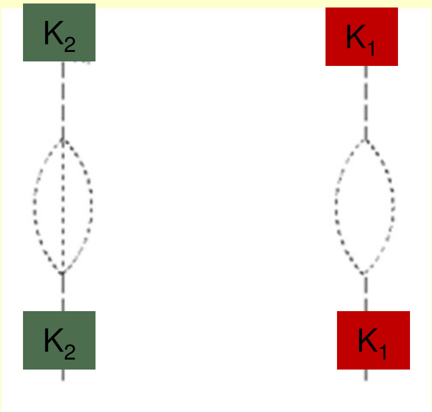
$$K_S \equiv K_1^0, K_L \equiv K_2^0 : \begin{array}{ll} \text{Different} & CP \\ \text{Different} & \text{lifetime} \end{array}$$

Also: Different mass!

Old fashioned (but simple) argument:

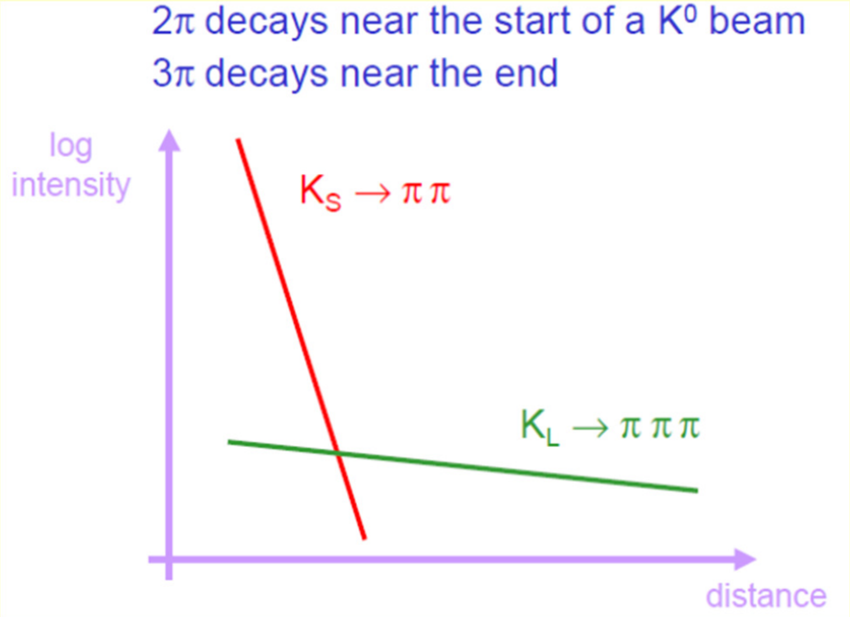
Different virtual weak couplings, 2π vs 3π

$\to$ Different corrections to the mass



K Oscillations - VII

Taking a neutral K beam produced by strong interaction, expect qualitatively



K Oscillations - VII

Production: Strong interaction $\rightarrow$ Strangeness conserved

Neglect weak interaction in production process

Strongly produced neutral K either K^0 or $\overline{K}^0$

$\rightarrow$ Either K^0 or $\overline{K}^0$ as *initial condition* for the K wave function

Time evolution : Weak interaction $\rightarrow$ Strangeness *not* conserved

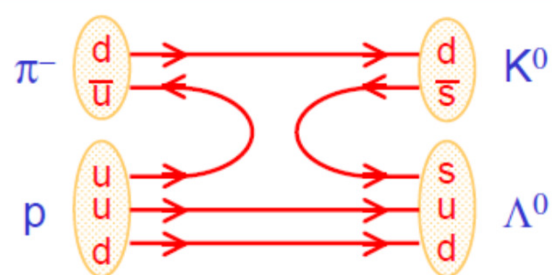
Neglect strong interaction in time evolution

Neither P, C conserved by weak interaction; CP $($*provisionally*$)$ conserved

$\rightarrow$ Propagate CP eigenstates K_S, K_L

Take a definite production process:

$$\pi^- p \rightarrow \Lambda^0 K^0$$



K Oscillations - VIII

$$\psi(t=0): \left|K^0\right\rangle = \frac{1}{\sqrt{2}}\left(\left|K_L^0\right\rangle + \left|K_S^0\right\rangle\right)$$

$$\begin{cases} \left|K_L^0(t)\right\rangle = \left|K_L^0\right\rangle e^{-i\left(m_L - i\frac{\Gamma_L}{2}\right)t}, \qquad \Gamma_L = \dfrac{1}{\tau_L} \\ \left|K_S^0(t)\right\rangle = \left|K_S^0\right\rangle e^{-i\left(m_S - i\frac{\Gamma_S}{2}\right)t}, \qquad \Gamma_S = \dfrac{1}{\tau_S} \end{cases}$$

$$\rightarrow \psi(t) = \frac{1}{\sqrt{2}}\left(\left|K_L^0\right\rangle e^{-i\left(m_L - i\frac{\Gamma_L}{2}\right)t} + \left|K_S^0\right\rangle e^{-i\left(m_S - i\frac{\Gamma_S}{2}\right)t}\right)$$

$$\rightarrow \psi(t) = \frac{1}{\sqrt{2}}\left(\left[\frac{1}{\sqrt{2}}\left(\left|K^0\right\rangle + \left|\bar{K}^0\right\rangle\right)\right] e^{-i\left(m_L - i\frac{\Gamma_L}{2}\right)t} + \frac{1}{\sqrt{2}}\left(\left|K^0\right\rangle - \left|\bar{K}^0\right\rangle\right)\frac{1}{\sqrt{2}} e^{-i\left(m_S - i\frac{\Gamma_S}{2}\right)t}\right)$$

$$\rightarrow \psi(t) = \frac{1}{2}\left(\left|K^0\right\rangle\left[e^{-i\left(m_L - i\frac{\Gamma_L}{2}\right)t} + e^{-i\left(m_S - i\frac{\Gamma_S}{2}\right)t}\right] + \left|\bar{K}^0\right\rangle\left[e^{-i\left(m_L - i\frac{\Gamma_L}{2}\right)t} - e^{-i\left(m_S - i\frac{\Gamma_S}{2}\right)t}\right]\right)$$

K Oscillations - IX

Time evolution of strangeness content of the beam:

$$\Delta m = m_L - m_S$$

$$\rightarrow \begin{cases} I\left(K^0\right) = \frac{1}{4}\left| e^{-i\left(m_L - i\frac{\Gamma_L}{2}\right)t} + e^{-i\left(m_S - i\frac{\Gamma_S}{2}\right)t} \right|^2 = \frac{1}{4}\left[e^{-\Gamma_L t} + e^{-\Gamma_S t} + 2e^{-\frac{(\Gamma_L + \Gamma_S)}{2}t} \cos \Delta m t \right] \\ I\left(\bar{K}^0\right) = \frac{1}{4}\left| e^{-i\left(m_L - i\frac{\Gamma_L}{2}\right)t} - e^{-i\left(m_S - i\frac{\Gamma_S}{2}\right)t} \right|^2 = \frac{1}{4}\left[e^{-\Gamma_L t} + e^{-\Gamma_S t} - 2e^{-\frac{(\Gamma_L + \Gamma_S)}{2}t} \cos \Delta m t \right] \end{cases}$$

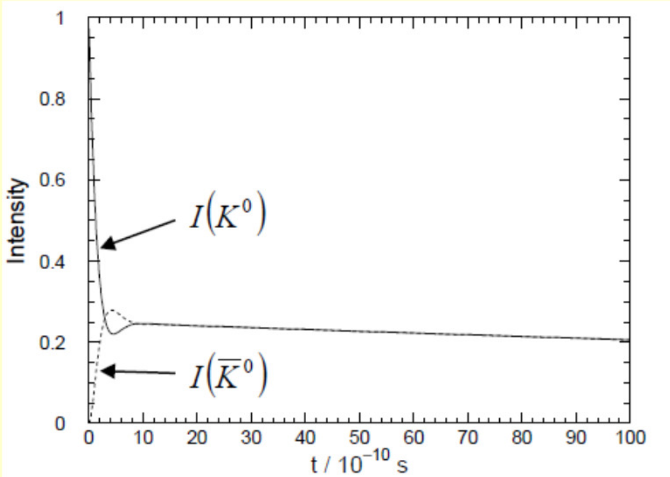
$\rightarrow$ Strangeness oscillations

Detected in many ways, for example by semileptonic modes:

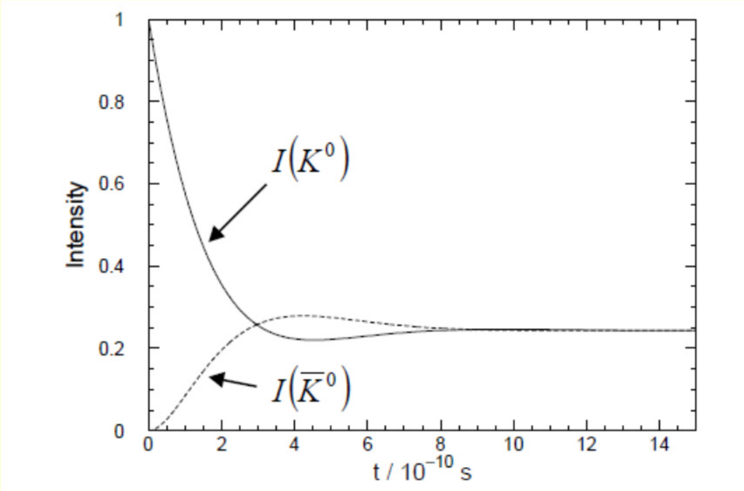
$$K^0 \rightarrow \pi^- e^+ \nu_e$$
$$\bar{K}^0 \rightarrow \pi^+ e^- \bar{\nu}_e$$

K Oscillations - IX

Interference!

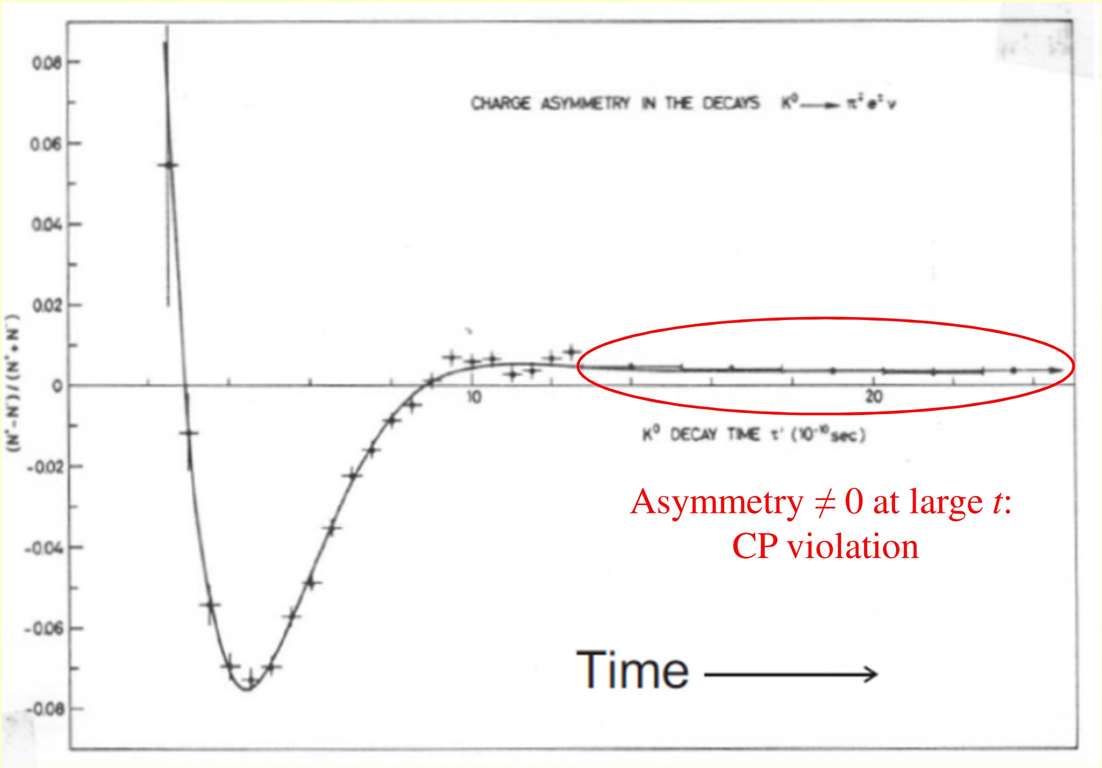


Expanded timescale:



K Oscillations - X

…And it's true!



!!!

K Oscillations - XI

$$R_+ \equiv \Gamma(K^0_{t=0} \to \pi^- e^+ \nu_e)$$

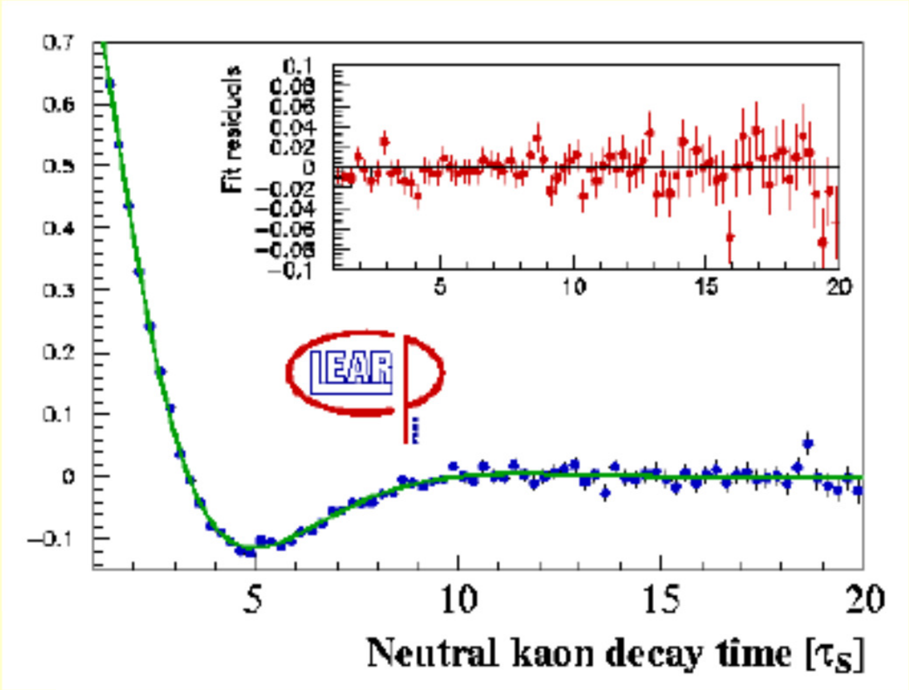
$$R_- \equiv \Gamma(K^0_{t=0} \to \pi^+ e^- \overline{\nu}_e)$$

$$\overline{R}_- \equiv \Gamma(\overline{K}^0_{t=0} \to \pi^+ e^- \overline{\nu}_e)$$

$$\overline{R}_+ \equiv \Gamma(\overline{K}^0_{t=0} \to \pi^- e^+ \nu_e)$$

$$A_{\Delta m} = \frac{(R_+ + \overline{R}_-) - (R_- + \overline{R}_+)}{(R_+ + \overline{R}_-) + (R_- + \overline{R}_+)}$$

$$A_{\Delta m} = \frac{2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t}{e^{-\Gamma_S t} + e^{-\Gamma_L t}}$$



$$\Delta m = 3.485 \times 10^{-15}\,\text{GeV}$$

!!!

K Oscillations - XII

Observe:

$$T\left(K^0 \leftrightarrow \bar{K}^0\right) = \frac{2\pi}{\Delta m}$$

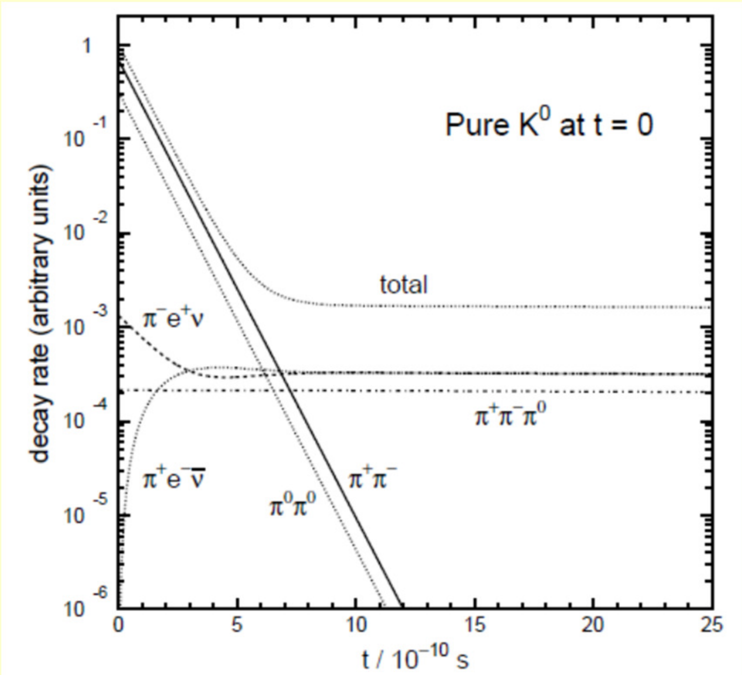
$$\rightarrow T\left(K^0 \leftrightarrow \bar{K}^0\right) = \frac{2\pi}{3.49\ 10^{-12} MeV} = \frac{2\pi}{3.49\ 10^{-12} MeV} \underbrace{6.5810^{-22} MeVs}_{\hbar} \simeq 1.18\ ns$$

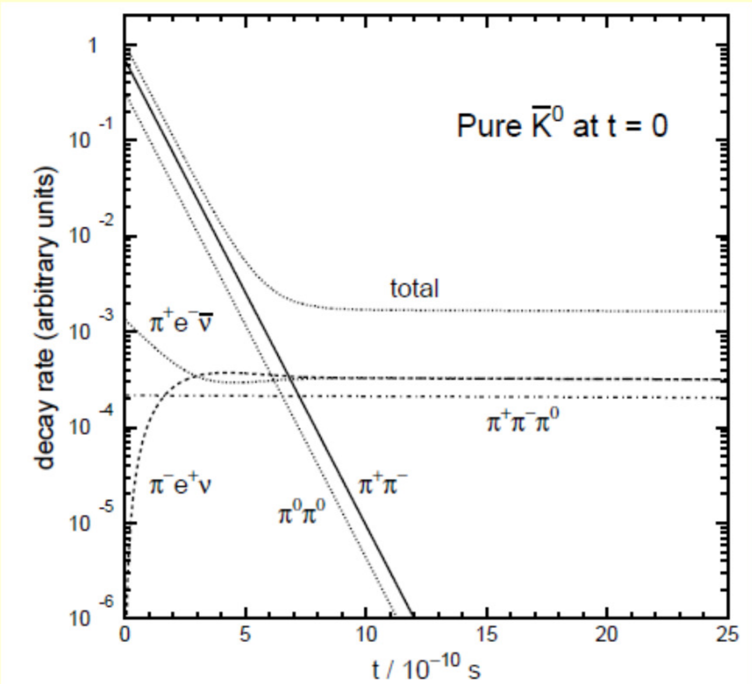
$$\rightarrow \frac{T}{\tau_S} \approx 13.3$$

$\rightarrow$ Just a fraction of a single oscillation within a K_S lifetime

K Oscillations - XIII

Summary of decay rates (CP conserved)





K_S Regeneration - I

Production reactions (e.g. at low energy):

$$K^0: \quad \pi^- + p \to K^0 + \Lambda^0$$
$$\bar{K}^0: \quad \pi^+ + p \to \bar{K}^0 + K^+ + p \quad \text{Higher threshold}$$

Take first reaction $\to$ Initially pure K^0 beam

After several τ_s: K_S component off $\to$ Pure K_L beam

Introduce some material in the beam path: Funny effect!

K_S Regeneration - II

Total cross section different for $K^0, \bar{K}^0$:

Indeed, e.g.

$\bar{K}^0 + p \rightarrow \pi^+ + \Lambda^0$

is strictly forbidden for K^0

$\rightarrow \sigma_{K^0} \neq \sigma_{\bar{K}^0}$

Remembering the "OpticalTheorem":

$\sigma_{tot} = \frac{4\pi}{k} \operatorname{Im} f(0),\ f(0)$ forward scattering amplitude

$\rightarrow f_{K^0}(0) \neq f_{\bar{K}^0}(0)$

Take forward scattering $(= propagation)$ of our pure K_L beam:

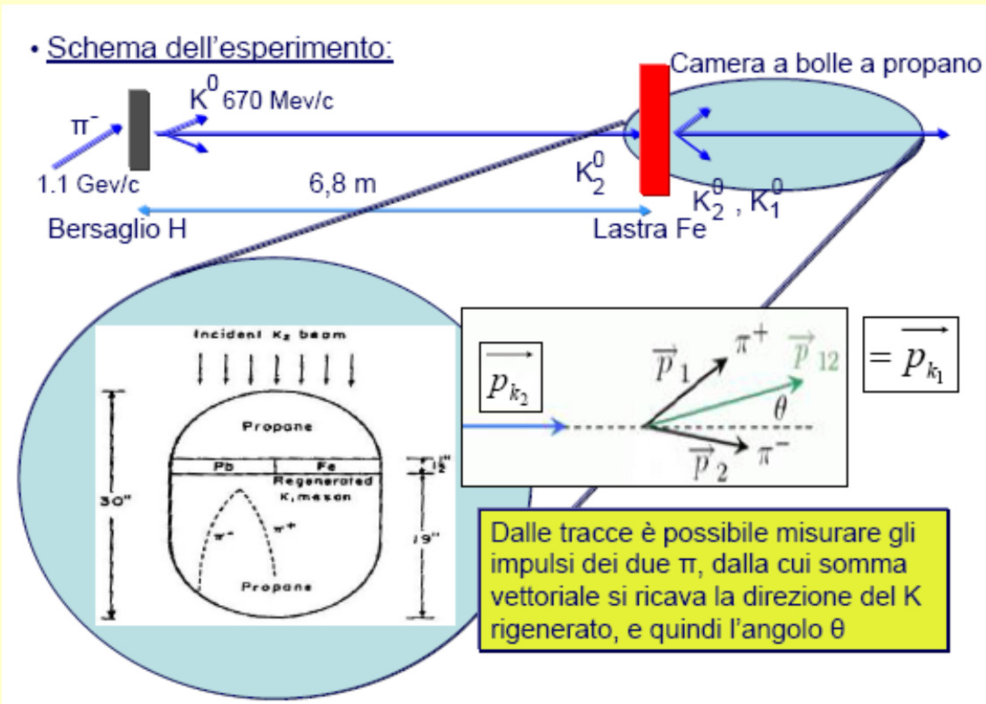
$$\left|K_L\right\rangle = \frac{1}{\sqrt{2}}\left(\left|K^0\right\rangle + \left|\bar{K}^0\right\rangle\right) \rightarrow \frac{1}{\sqrt{2}}\left(A\left(f_{K^0}\right)\left|K^0\right\rangle + B\left(f_{\bar{K}^0}\right)\left|\bar{K}^0\right\rangle\right) \neq \left|K_L\right\rangle$$

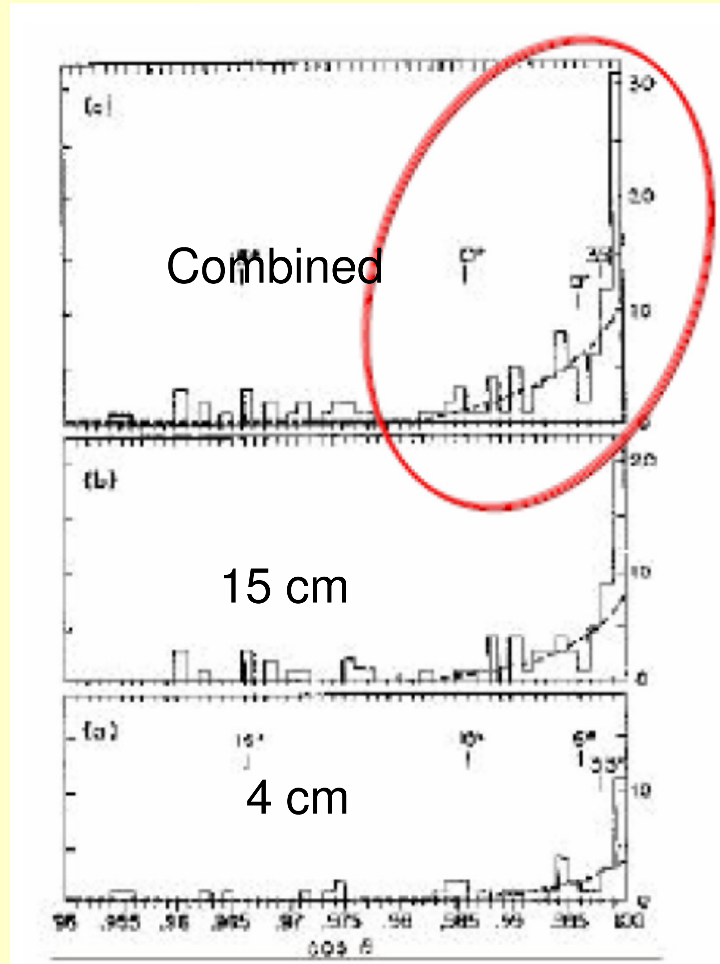
$$\left|K_L\right\rangle \rightarrow a\left|K_L\right\rangle + b\left|K_S\right\rangle, \quad |a|^2 + |b|^2 = 1$$

$\rightarrow$ A $\left|K_S\right\rangle$ component has been regenerated by the material!

K_S Regeneration - III

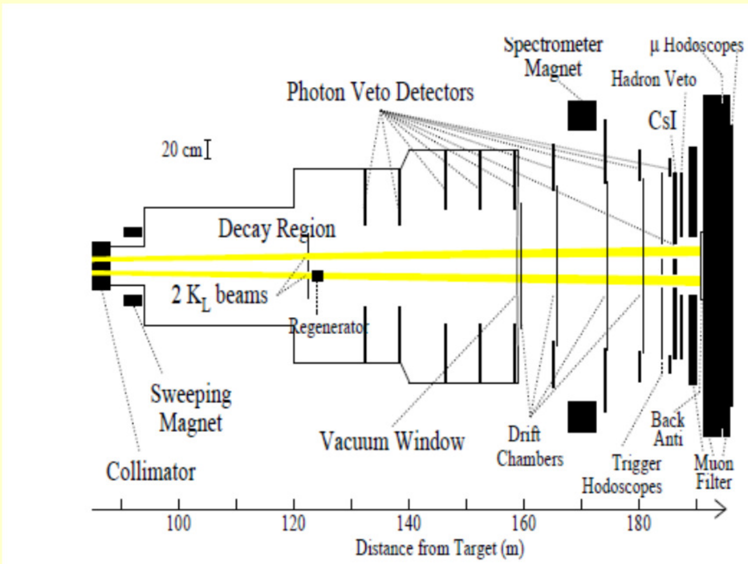
Piccioni et al. – Berkeley ≈ 1956

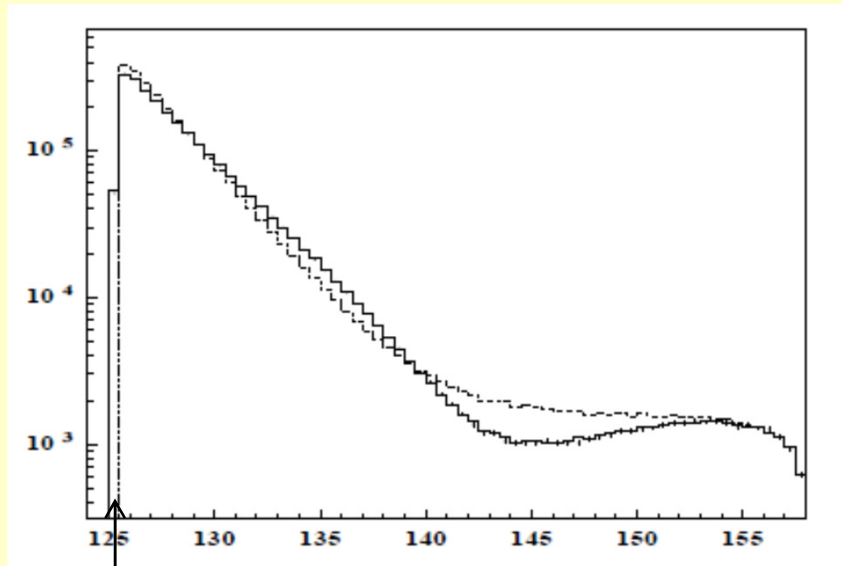




K_S Regeneration - IV

KTEV - Fermilab ≈ 2000





Strong K_S regeneration signalled by 2π decays with τ_s lifetime

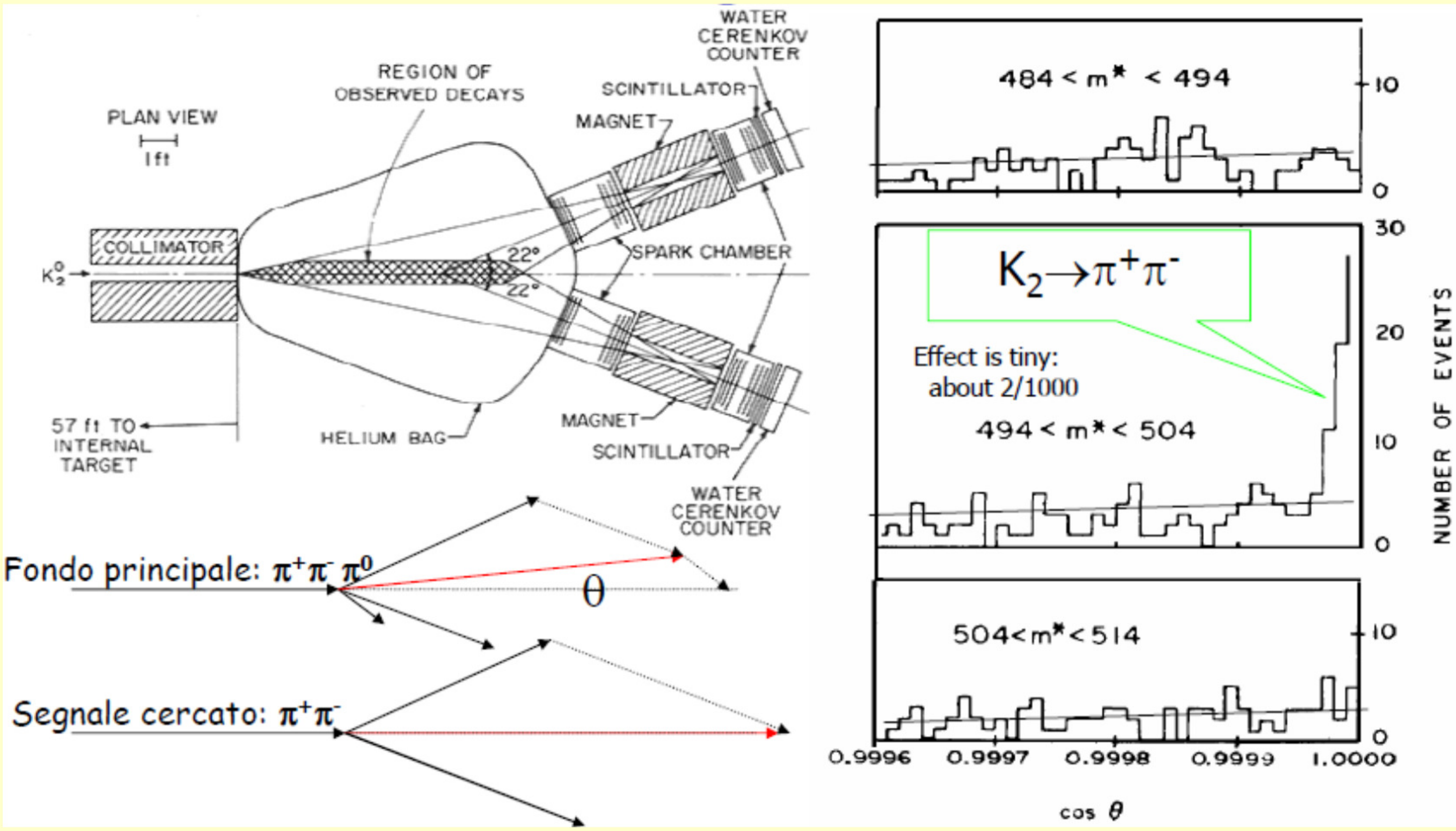
[Large interference observed in 2π rate:

Regenerated K_S component interfering with CP violating, 2π decay of K_L beam]

CP Violation - I

$K_2 \to \pi^+\pi^-$ decay observed in 1964; $K_2 \to \pi^0\pi^0$ decay also observed

Small $BR \sim 10^{-3}$



CP Violation - II

Two possible explanations:

1) K_L, K_S not *CP* eigenstates - Decay *CP* conserving

$$\rightarrow \left|K_L^0\right\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left(\left|K_2^0\right\rangle + \varepsilon\left|K_1^0\right\rangle\right), \left|K_S^0\right\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left(\left|K_1^0\right\rangle - \varepsilon\left|K_2^0\right\rangle\right)$$

$K_L^0 \rightarrow \pi\pi$ accounted for by

$$\left|K_L^0\right\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left(\left|K_2^0\right\rangle + \varepsilon\left|K_1^0\right\rangle\right)$$

$$\begin{array}{cc} \downarrow & \downarrow \\ \pi\pi\pi & \pi\pi \end{array}$$

Mixing $\not{CP}$: Measured by small parameter ε

2) Decay *CP* violating - K_L, K_S *CP* eigenstates

$$\left|K_L^0\right\rangle = \left|K_2^0\right\rangle$$

Direct $\not{CP}$: Measured by *very* small parameter ε'

CP Violation - III

Define:

$$|\eta_{+-}| \equiv \frac{\Gamma(K_L^0 \to \pi^+\pi^-)}{\Gamma(K_S^0 \to \pi^+\pi^-)} = (2.276 \pm 0.017) \times 10^{-3}$$

$$|\eta_{00}| \equiv \frac{\Gamma(K_L^0 \to \pi^0\pi^0)}{\Gamma(K_S^0 \to \pi^0\pi^0)} = (2.262 \pm 0.017) \times 10^{-3}$$

Expect:

Case 1) $\rightarrow \eta_{+-} = \eta_{00}$

Case 2) $\rightarrow \eta_{+-} \neq \eta_{00}$

Generally:

$\eta_{+-} = \varepsilon + \varepsilon'$

$\eta_{+-} = \varepsilon - 2\varepsilon'$

$\rightarrow \varepsilon' \ll \varepsilon$, must be very small

CP Violation - IV

Focus on mixing $\not{CP}$, ignore direct $\not{CP}$ for the moment

$$\varepsilon = |\varepsilon| e^{i\varphi}$$

For a neutral beam initially pure $K^0 / \bar{K}^0$:

Measure $\pi\pi$ decay rate as a function of distance

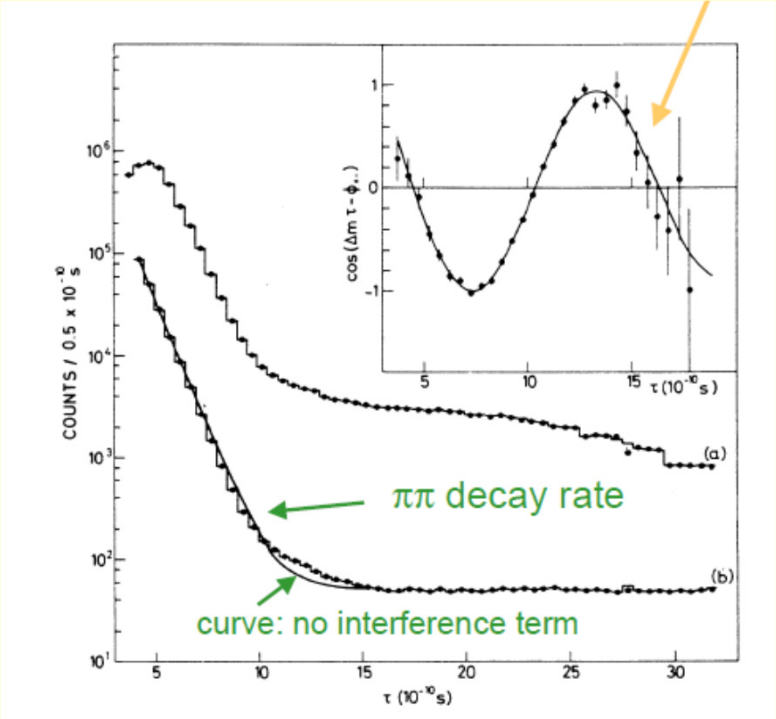
$$I\left(K^0;t\right) = \frac{N}{2}\left(1 - 2\,\mathrm{Re}(\varepsilon)\right)\left[e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t} + 2|\varepsilon| e^{-\frac{\Gamma_L+\Gamma_S}{2}t} \cos(\Delta m t - \varphi)\right]$$

$$I\left(\bar{K}^0;t\right) = \frac{N}{2}\left(1 + 2\,\mathrm{Re}(\varepsilon)\right)\left[e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t} - 2|\varepsilon| e^{-\frac{\Gamma_L+\Gamma_S}{2}t} \cos(\Delta m t - \varphi)\right]$$

$\rightarrow$ Get $|\varepsilon|, \varphi, \Delta m$

$$|\varepsilon| = (2.285 \pm 0.019) \times 10^{-3}$$
$$\phi = (43.5 \pm 0.6)^\circ$$

CP Violation - V



$$\text{Similar to regeneration data, but: No regenerator!}$$

Interference between K_L and K_S in 2π decay

$\rightarrow K_L$ and K_S states not orthogonal: Both have a K_1 component

CP Violation - VI

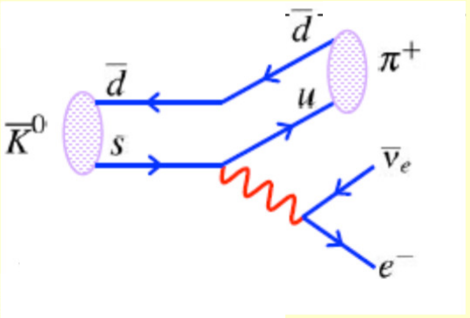
Neutral beam at large distance from production target: Pure K_L

$$\left|K_L^0\right\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left(\left|K_2^0\right\rangle + \varepsilon\left|K_1^0\right\rangle\right)$$

$$\rightarrow \left|K_L^0\right\rangle = \frac{1}{\sqrt{2\left(1+|\varepsilon|^2\right)}}\left[(1+\varepsilon)\left|K^0\right\rangle + (1-\varepsilon)\left|\bar{K}^0\right\rangle\right]$$

Take semileptonic decays, e.g. K_{e3}:

$$K^0 \rightarrow \pi^- e^+ \nu_e$$
$$\bar{K}^0 \rightarrow \pi^+ e^- \bar{\nu}_e$$



Observe:

$$CP\left|K^0\right\rangle = \left|\bar{K}^0\right\rangle$$
$$CP\left|\pi^- e^+ \nu_e\right\rangle = \left|\pi^+ e^- \bar{\nu}_e\right\rangle$$

$\rightarrow$ No *CP* eigenstates

CP Violation - VII

$$\Gamma(K_L \to \pi^+ e^- \overline{\nu}_e) \propto |\langle \overline{K}^0 | K_L \rangle|^2 \propto |1-\varepsilon|^2 \approx 1 - 2\Re\{\varepsilon\}$$
$$\Gamma(K_L \to \pi^- e^+ \nu_e) \propto |\langle K^0 | K_L \rangle|^2 \propto |1+\varepsilon|^2 \approx 1 + 2\Re\{\varepsilon\}$$

$$|1 \pm \epsilon|^2 = (1 \pm \epsilon)(1 \pm \epsilon^*) \approx 1 \pm 2\mathrm{Re}(\epsilon) \ .$$

$$\delta \approx \frac{(1+2\mathrm{Re}(\epsilon)) - (1-2\mathrm{Re}(\epsilon))}{(1+2\mathrm{Re}(\epsilon)) + (1-2\mathrm{Re}(\epsilon))}$$

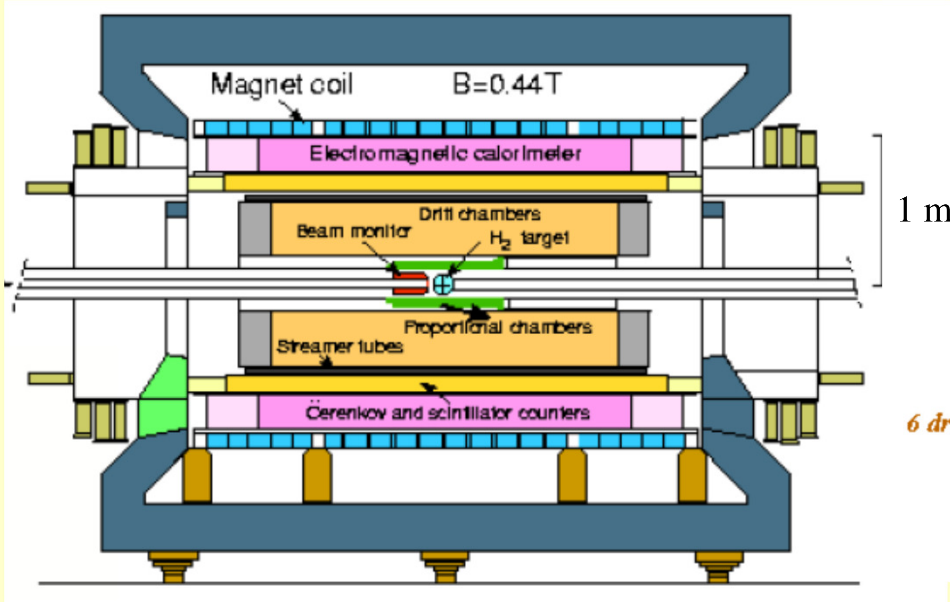
$$\boxed{\delta \approx 2\mathrm{Re}(\epsilon) = 2|\epsilon|\cos\phi}$$

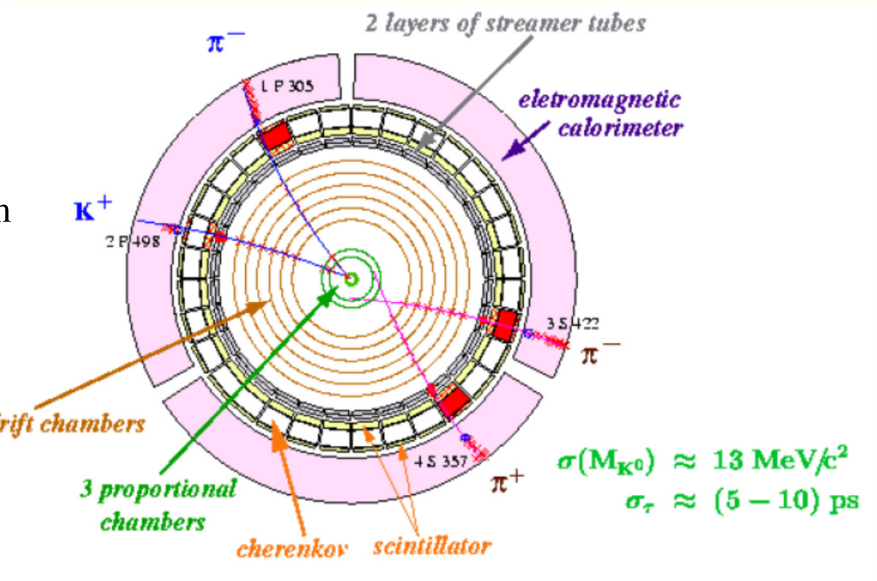
$$\delta \equiv \frac{\Gamma(\mathrm{K_L} \to \pi^- \mathrm{e}^+ \nu_e) - \Gamma(\mathrm{K_L} \to \pi^+ \mathrm{e}^- \overline{\nu}_e)}{\Gamma(\mathrm{K_L} \to \pi^- \mathrm{e}^+ \nu_e) + \Gamma(\mathrm{K_L} \to \pi^+ \mathrm{e}^- \overline{\nu}_e)} = (0.327 \pm 0.012)\,\%$$

Good agreement between prediction and measurement of δ

CP Violation - VIII

CPLEAR – CERN ’90s





CP Violation - IX

Experiment CPLEAR

(CERN $\overline{p}$ Low Energy Accumulator Ring - LEAR) $\rightarrow \sim 1995$

Use reactions:

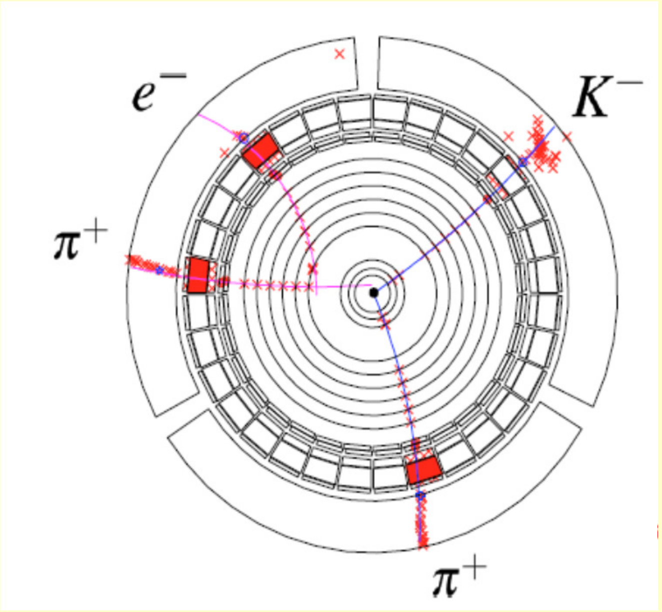
$p\overline{p} \rightarrow K^- \pi^+ K^0$

$p\overline{p} \rightarrow K^+ \pi^- \overline{K}^0$

and semileptonic decays

$K^0 \rightarrow \pi^- e^+ \nu_e$

$\overline{K}^0 \rightarrow \pi^+ e^- \overline{\nu}_e$



Strangeness of $\begin{matrix} produced \\ decaying \end{matrix}$ K state: Tagged *unambiguously*

CP Violation - X

Strangeness oscillations in presence of $\not{CP}$:

$$\Gamma\left(K^0_{t=0} \to K^0\right) \propto \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-\frac{\Gamma_S+\Gamma_L}{2}t}\cos\Delta mt\right], \quad \Gamma\left(K^0_{t=0} \to \bar{K}^0\right) \propto \frac{1}{4}(1-4\operatorname{Re}\varepsilon)\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-\frac{\Gamma_S+\Gamma_L}{2}t}\cos\Delta mt\right]$$

$$\Gamma\left(\bar{K}^0_{t=0} \to \bar{K}^0\right) \propto \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-\frac{\Gamma_S+\Gamma_L}{2}t}\cos\Delta mt\right], \quad \Gamma\left(\bar{K}^0_{t=0} \to K^0\right) \propto \frac{1}{4}(1+4\operatorname{Re}\varepsilon)\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-\frac{\Gamma_S+\Gamma_L}{2}t}\cos\Delta mt\right]$$

$$\to R_+ = \Gamma\left(K^0_{t=0} \to \pi^- e^+ \nu_e\right) = N\frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-\frac{\Gamma_S+\Gamma_L}{2}t}\cos\Delta mt\right]$$

$$R_- = \Gamma\left(K^0_{t=0} \to \pi^+ e^- \bar{\nu}_e\right) = N(1-4\operatorname{Re}\varepsilon)\frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-\frac{\Gamma_S+\Gamma_L}{2}t}\cos\Delta mt\right]$$

$$\bar{R}_+ = \Gamma\left(\bar{K}^0_{t=0} \to \pi^- e^+ \nu_e\right) = N(1+4\operatorname{Re}\varepsilon)\frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-\frac{\Gamma_S+\Gamma_L}{2}t}\cos\Delta mt\right]$$

$$\bar{R}_- = \Gamma\left(\bar{K}^0_{t=0} \to \pi^+ e^- \bar{\nu}_e\right) = N\frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-\frac{\Gamma_S+\Gamma_L}{2}t}\cos\Delta mt\right]$$

$$A_{\Delta m} = \frac{\left(R_+ + \bar{R}_-\right) - \left(R_- + \bar{R}_+\right)}{\left(R_+ + \bar{R}_-\right) + \left(R_- + \bar{R}_+\right)} \to A_{\Delta m}\left(\not{CP}\right) = A_{\Delta m}(CP)$$

Astonishing T-violation:

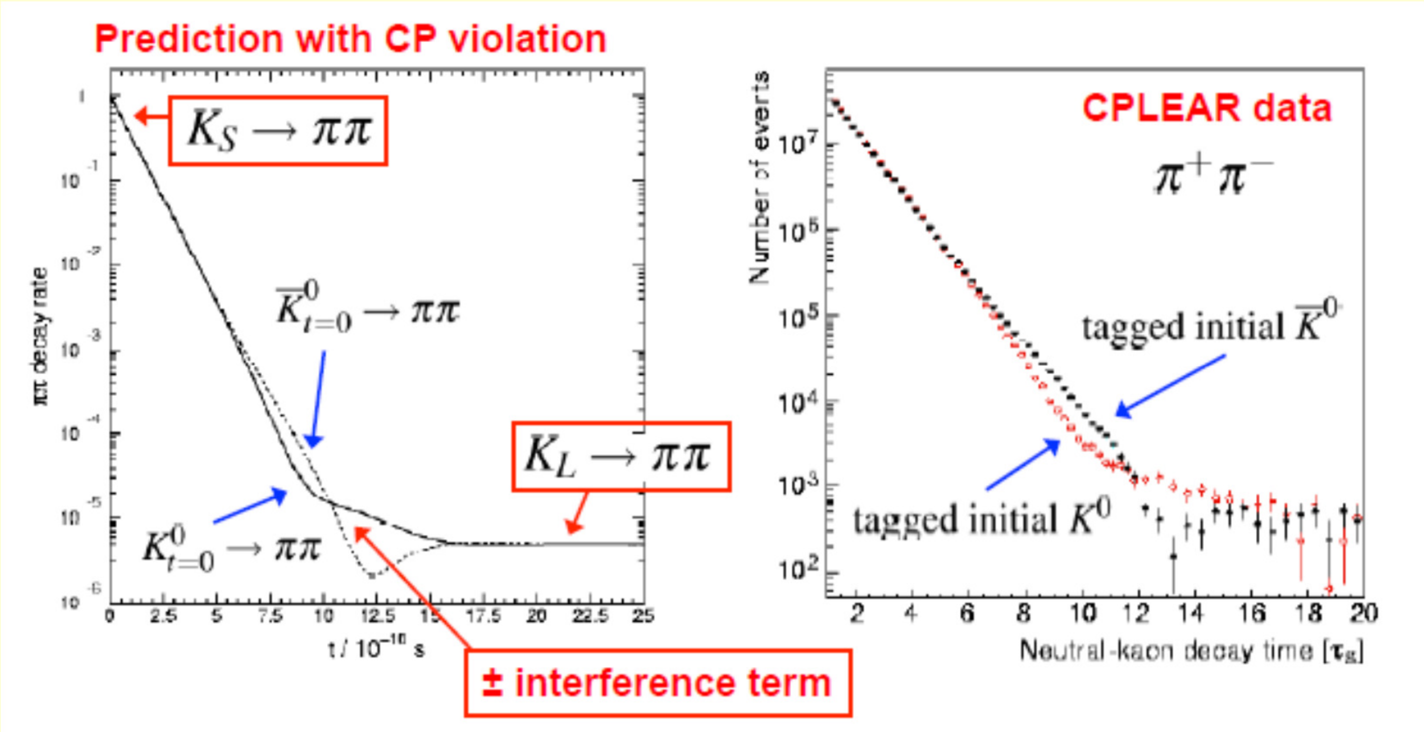
$$\Gamma\left(K^0_{t=0} \to \bar{K}^0\right) \neq \Gamma\left(\bar{K}^0_{t=0} \to K^0\right)$$

CP Violation - XI

CPLEAR on 2π decays

Expected decay rates for K^0, $\bar{K}^0$ initial state

Observed decay rates for K^0, $\bar{K}^0$ initial state



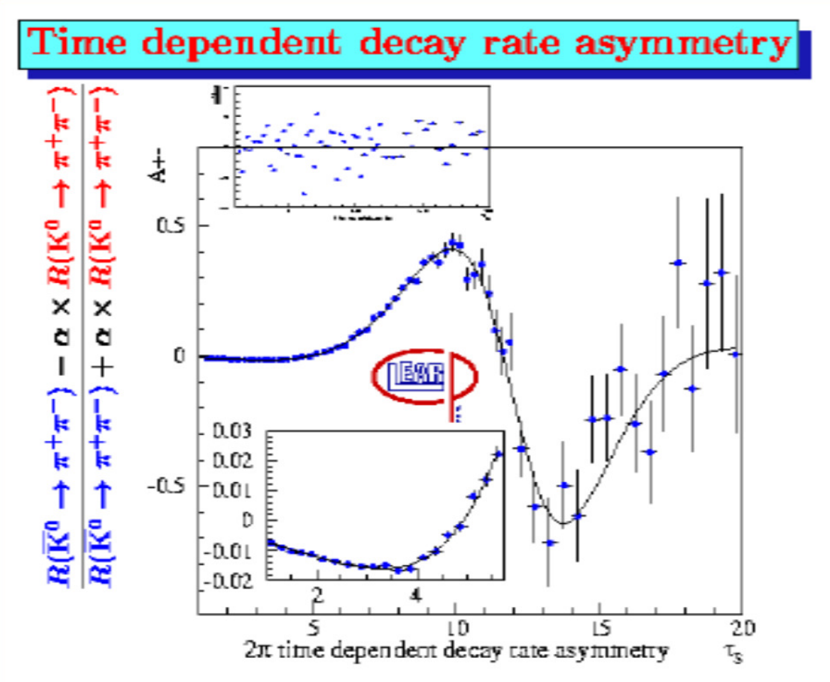
CP Violation - XII

Time dependent asymmetry:

$$A(\pi\pi) = \frac{\Gamma(\bar{K}^0) - \Gamma(K^0)}{\Gamma(\bar{K}^0) + \Gamma(K^0)}$$

$$\rightarrow A(\pi\pi) \approx 2\,\mathrm{Re}(\varepsilon) - \frac{2|\varepsilon| e^{\frac{(\Gamma_S - \Gamma_L)}{2}t}}{1 + |\varepsilon|^2 e^{\frac{(\Gamma_S - \Gamma_L)}{2}t}} \cos(\Delta m t - \phi)$$

$$\rightarrow \begin{cases} |\varepsilon| = (2.264 \pm 0.035)10^{-3} \\ \varphi = (43.19 \pm 0.073)^0 \\ \Delta m = (3.4852 \pm 0.013)10^{-15}\,GeV \end{cases}$$



CP Violation - XIII

CP violation in 3π decays

Expect, by swapping $K_S \leftrightarrow K_L$:

$$I\left(K^0;t\right)=\frac{N'}{2}\left(1-2\,\mathrm{Re}\left(\varepsilon\right)\right)\left[e^{-\Gamma_L t}+\left|\varepsilon\right|^2 e^{-\Gamma_S t}+2\left|\varepsilon\right|e^{-\frac{\Gamma_L+\Gamma_S}{2}t}\cos\left(\Delta m t-\varphi\right)\right]$$

Very different experimental conditions as compared to 2π :

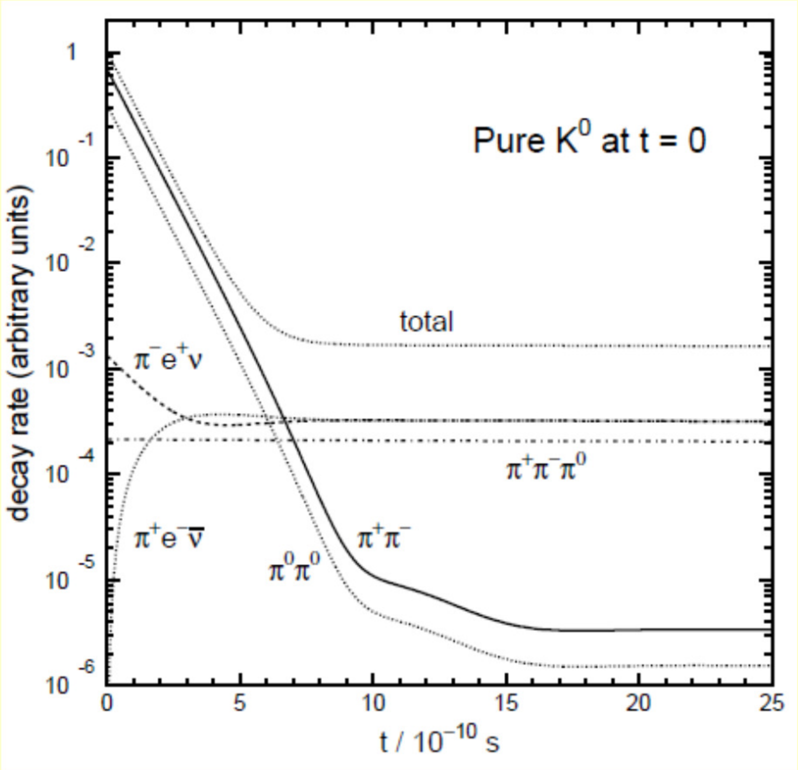
Lots of CP conserving 3π decays from K_L component of the beam

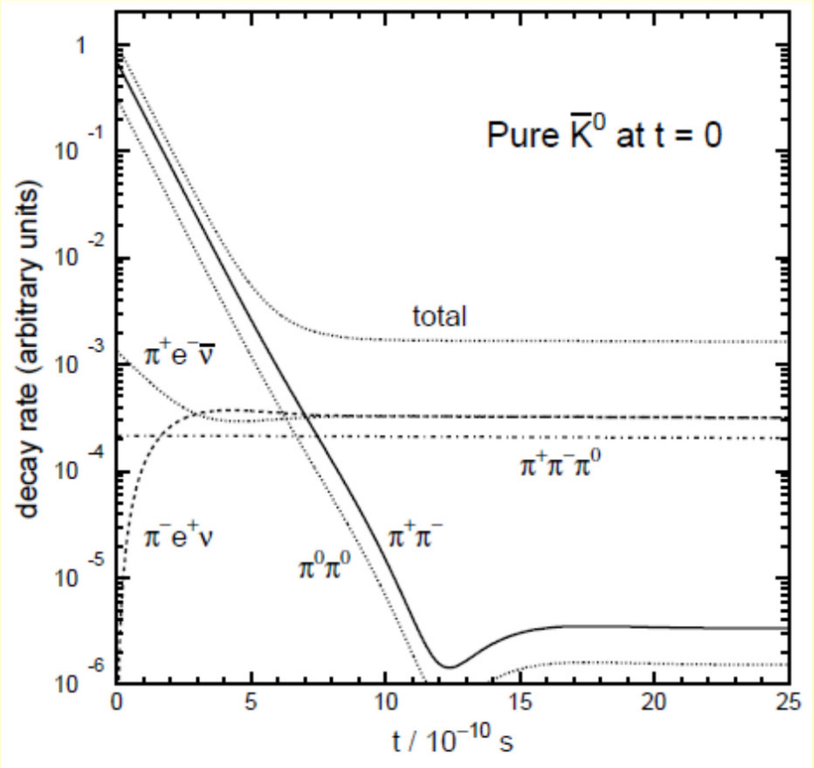
(Compare: No CP conserving 2π from K_S component, which just dies out at large distance)

$\rightarrow$ Measurement difficult, large errors

CP Violation - XIV

Summary of decay rates (CP violated)





T, CPT Tests - I

From previous conclusions on CP violation:

$$\Gamma\left(K^0_{t=0} \to \bar{K}^0\right) \neq \Gamma\left(\bar{K}^0_{t=0} \to K^0\right)$$

Also *time reversal* violation:

Amplitude of direct process $\neq$ Amplitude of reverse process

To be expected if CPT is a good symmetry

$$A_T \equiv \frac{\Gamma(\overline{\mathrm{K}}^0 \to \mathrm{K}^0) - \Gamma(\mathrm{K}^0 \to \overline{\mathrm{K}}^0)}{\Gamma(\overline{\mathrm{K}}^0 \to \mathrm{K}^0) + \Gamma(\mathrm{K}^0 \to \overline{\mathrm{K}}^0)}.$$

Taking semileptonic decays as a benchmark:

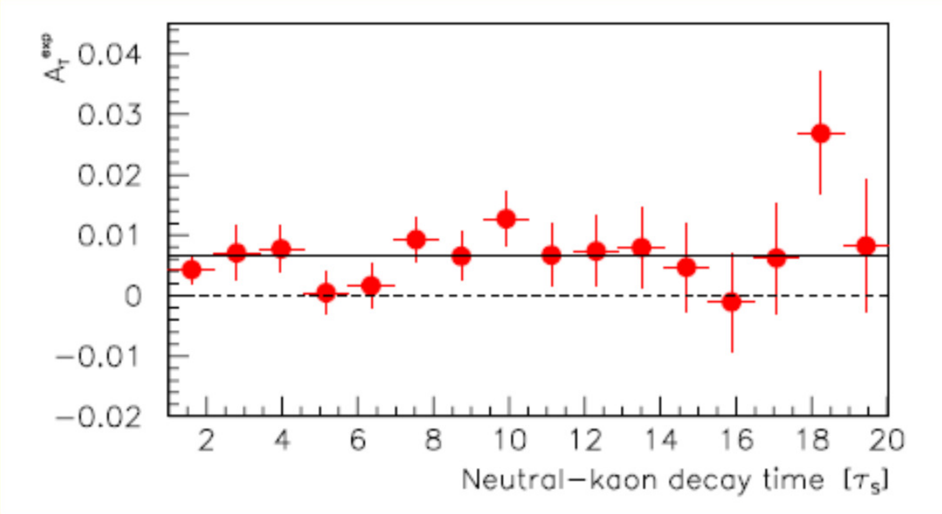
$$A_T = \frac{\Gamma(\overline{\mathrm{K}}^0_{t=0} \to \pi^- \mathrm{e}^+ \nu_\mathrm{e}) - \Gamma(\mathrm{K}^0_{t=0} \to \pi^+ \mathrm{e}^- \overline{\nu}_e)}{\Gamma(\overline{\mathrm{K}}^0_{t=0} \to \pi^- \mathrm{e}^+ \nu_\mathrm{e}) + \Gamma(\mathrm{K}^0_{t=0} \to \pi^+ \mathrm{e}^- \overline{\nu}_e)}$$

T, CPT Tests - II

$$A_T \approx \text{Time independent constant} \approx 4\,\mathrm{Re}(\varepsilon) = 4|\varepsilon|\cos\phi$$

$\rightarrow$ Expect

$$A_T \approx 6.6\ 10^{-3}$$



$$A_T = (6.2 \pm 1.7) \times 10^{-3}\,,$$

T, *CPT* Tests - III

Semileptonic decays also used to test *CPT*

Simple test:

$$\Gamma\left(K^0 \to K^0\right) = \Gamma\left(\bar{K}^0 \to \bar{K}^0\right)$$

Define *CPT* asymmetry :

$$A_{CPT} = \frac{\Gamma\left(K^0 \to K^0\right) - \Gamma\left(\bar{K}^0 \to \bar{K}^0\right)}{\Gamma\left(K^0 \to K^0\right) + \Gamma\left(\bar{K}^0 \to \bar{K}^0\right)}$$

Measure by:

$$A_{CPT} = \frac{\Gamma\left(K^0_{t=0} \to \pi^- e^+ \nu_e\right) - \Gamma\left(\bar{K}^0_{t=0} \to \pi^+ e^- \bar{\nu}_e\right)}{\Gamma\left(K^0_{t=0} \to \pi^- e^+ \nu_e\right) + \Gamma\left(\bar{K}^0_{t=0} \to \pi^+ e^- \bar{\nu}_e\right)}$$

T, CPT Tests - IV

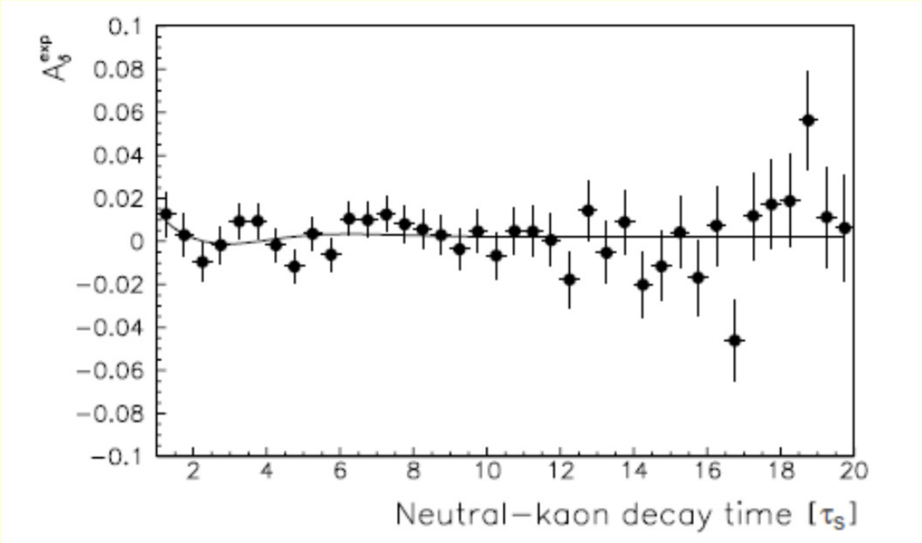
Since:

$$\Gamma\left(K^0_{t=0} \to \pi^- e^+ \nu_e\right) = N \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-\frac{\Gamma_S + \Gamma_L}{2}t} \cos \Delta m t\right]$$

$$\Gamma\left(\bar{K}^0_{t=0} \to \pi^+ e^- \bar{\nu}_e\right) = N \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-\frac{\Gamma_S + \Gamma_L}{2}t} \cos \Delta m t\right]$$

$\rightarrow$ Expect:

$A_{CPT} = 0, \quad t$ independent



Direct *CP* Violation - I

Another side of $\not{CP}$: K^0 *decays* *CP* violating

$\rightarrow$ Direct $\not{CP}$

Amplitude ratios:

$$\eta_{+-} = \frac{\left\langle \pi^+\pi^- \left| T \right| K_L^0 \right\rangle}{\left\langle \pi^+\pi^- \left| T \right| K_S^0 \right\rangle} = \left|\eta_{+-}\right| e^{i\phi_{+-}}, \qquad \eta_{00} = \frac{\left\langle \pi^0\pi^0 \left| T \right| K_L^0 \right\rangle}{\left\langle \pi^0\pi^0 \left| T \right| K_S^0 \right\rangle} = \left|\eta_{00}\right| e^{i\phi_{00}}$$

In order to relate η, ϕ parameters to $\varepsilon, \varepsilon'$

a)Decompose 2π states into isospin eigenstates:

$$\begin{cases} \left\langle \pi^+\pi^- \right| = \frac{1}{\sqrt{3}}\left\langle I=2\right| + \sqrt{\frac{2}{3}}\left\langle I=0\right| \\ \left\langle \pi^0\pi^0 \right| = \sqrt{\frac{2}{3}}\left\langle I=2\right| - \frac{1}{\sqrt{3}}\left\langle I=0\right| \end{cases}$$

$I=1$ absent due to Bose statistics of π's in a S-wave

Direct *CP* Violation - II

Full 2π states include proper phase factors
originating from S-wave $\pi\pi$ scattering

$$\langle \pi^+\pi^- | = \frac{1}{\sqrt{3}}\langle 2| e^{i\delta_2} + \sqrt{\frac{2}{3}}\langle 0| e^{i\delta_0}$$

$$\langle \pi^0\pi^0 | = \sqrt{\frac{2}{3}}\langle 2| e^{i\delta_2} - \frac{1}{\sqrt{3}}\langle 0| e^{i\delta_0}$$

Define decay amplitudes into isospin states:

$$A_0 = \langle 0 | H_w | K^0 \rangle$$
$$A_2 = \langle 2 | H_w | K^0 \rangle$$

Direct *CP* Violation - III

$$CP\left|\pi\pi\right\rangle = +1 \rightarrow CPT\left|0\right\rangle = \left\langle 0\right|, CPT\left|2\right\rangle = \left\langle 2\right|$$

$$CP\left|K^0\right\rangle = -\left|\bar{K}^0\right\rangle \rightarrow CPT\left|K^0\right\rangle = -\left\langle \bar{K}^0\right|$$

$$\left[H_w, CPT\right] = 0$$

$$\rightarrow \begin{cases} \left\langle 0\right| H_w \left|\bar{K}^0\right\rangle \overset{CPT}{\rightarrow} -\left\langle K^0\right| H_w \left|0\right\rangle = -A_0^* \\ \left\langle 2\right| H_w \left|\bar{K}^0\right\rangle \overset{CPT}{\rightarrow} -\left\langle K^0\right| H_w \left|2\right\rangle = -A_2^* \end{cases}$$

$$\left|K_L^0\right\rangle = \frac{1}{\sqrt{2\left(1+\left|\varepsilon\right|^2\right)}}\left[\left(1+\varepsilon\right)\left|K^0\right\rangle + \left(1-\varepsilon\right)\left|\bar{K}^0\right\rangle\right]$$

$$\left|K_S^0\right\rangle = \frac{1}{\sqrt{2\left(1+\left|\varepsilon\right|^2\right)}}\left[\left(1+\varepsilon\right)\left|K^0\right\rangle - \left(1-\varepsilon\right)\left|\bar{K}^0\right\rangle\right]$$

Direct *CP* Violation - IV

Transition matrix elements:

$$\left\langle \pi^+\pi^- \middle| H \middle| K_L^0 \right\rangle = \frac{2}{\sqrt{3}} \frac{1}{\sqrt{2\left(1+|\varepsilon|^2\right)}} \varepsilon \left[\operatorname{Re} A_2 e^{i\delta_2} + \sqrt{2} A_0 e^{i\delta_0} \right] + \operatorname{Im} A_2 e^{i\delta_2}$$

$$\left\langle \pi^+\pi^- \middle| H \middle| K_S^0 \right\rangle = \frac{2}{\sqrt{3}} \frac{1}{\sqrt{2\left(1+|\varepsilon|^2\right)}} \left[\operatorname{Re} A_2 e^{i\delta_2} + \sqrt{2} A_0 e^{i\delta_0} + \varepsilon \operatorname{Im} A_2 e^{i\delta_2} \right]$$

$$\left\langle \pi^0\pi^0 \middle| H \middle| K_L^0 \right\rangle = \frac{2}{\sqrt{3}} \frac{1}{\sqrt{2\left(1+|\varepsilon|^2\right)}} \varepsilon \left[\sqrt{2} \operatorname{Re} A_2 e^{i\delta_2} - A_0 e^{i\delta_0} \right] + \sqrt{2} \operatorname{Im} A_2 e^{i\delta_2}$$

$$\left\langle \pi^0\pi^0 \middle| H \middle| K_S^0 \right\rangle = \frac{2}{\sqrt{3}} \frac{1}{\sqrt{2\left(1+|\varepsilon|^2\right)}} \left[\sqrt{2} \operatorname{Re} A_2 e^{i\delta_2} - A_0 e^{i\delta_0} + \varepsilon \sqrt{2} \operatorname{Im} A_2 e^{i\delta_2} \right]$$

After some complex algebra:

$$\eta_{+-} = \frac{\left\langle \pi^+\pi^- \middle| T \middle| K_L^0 \right\rangle}{\left\langle \pi^+\pi^- \middle| T \middle| K_S^0 \right\rangle} \simeq \varepsilon + \underbrace{\frac{1}{\sqrt{2}} \frac{\operatorname{Im} A_2}{A_0} e^{i(\delta_2 - \delta_0)}}_{\varepsilon'} = \varepsilon + \varepsilon'$$

$$\eta_{00} = \frac{\left\langle \pi^0\pi^0 \middle| T \middle| K_L^0 \right\rangle}{\left\langle \pi^0\pi^0 \middle| T \middle| K_S^0 \right\rangle} \simeq \varepsilon - \underbrace{\sqrt{2} \frac{\operatorname{Im} A_2}{A_0} e^{i(\delta_2 - \delta_0)}}_{\varepsilon'} = \varepsilon - 2\varepsilon'$$

Direct *CP* Violation - V

Double ratio magic:

$$\eta_{+-} = \frac{\left\langle \pi^+\pi^- \middle| T \middle| K_L^0 \right\rangle}{\left\langle \pi^+\pi^- \middle| T \middle| K_S^0 \right\rangle} \simeq \varepsilon + \varepsilon'$$

$$\eta_{00} = \frac{\left\langle \pi^0\pi^0 \middle| T \middle| K_L^0 \right\rangle}{\left\langle \pi^0\pi^0 \middle| T \middle| K_S^0 \right\rangle} \simeq \varepsilon - 2\varepsilon'$$

$$\rightarrow \left|\frac{\eta_{00}}{\eta_{+-}}\right|^2 \simeq \frac{\left|\varepsilon - 2\varepsilon'\right|^2}{\left|\varepsilon + \varepsilon'\right|^2} = \frac{(\varepsilon - 2\varepsilon')(\varepsilon - 2\varepsilon')^*}{(\varepsilon + \varepsilon')(\varepsilon + \varepsilon')^*} \simeq \frac{|\varepsilon|^2 - 4\,\mathrm{Re}(\varepsilon'\varepsilon)}{|\varepsilon|^2 + 2\,\mathrm{Re}(\varepsilon'\varepsilon)}$$

$$\rightarrow \left|\frac{\eta_{00}}{\eta_{+-}}\right|^2 \approx \frac{1 - 4\dfrac{\mathrm{Re}(\varepsilon'\varepsilon)}{|\varepsilon|^2}}{1 + 2\dfrac{\mathrm{Re}(\varepsilon'\varepsilon)}{|\varepsilon|^2}} \approx \left[1 - 2\frac{\mathrm{Re}(\varepsilon'\varepsilon)}{|\varepsilon|^2}\right]\left[1 - 4\frac{\mathrm{Re}(\varepsilon'\varepsilon)}{|\varepsilon|^2}\right] \approx 1 - 6\frac{\mathrm{Re}(\varepsilon'\varepsilon)}{|\varepsilon|^2}$$

$$\rightarrow \left|\frac{\eta_{00}}{\eta_{+-}}\right|^2 = \frac{\dfrac{N_L^{00}}{N_S^{00}}}{\dfrac{N_L^{+-}}{N_S^{+-}}} \approx 1 - 6\,\mathrm{Re}\frac{(\varepsilon')}{|\varepsilon|}$$

Direct *CP* Violation - VI

Actually a very important question:
Does weak interaction violate CP?
$\varepsilon' \neq 0$ yes
$\varepsilon' = 0$ don't know

'80s:

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = (23.0 \pm 6.5) \times 10^{-4} (NA31) \qquad > 3\sigma$$

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = (7.4 \pm 5.9) \times 10^{-4} (E731) \qquad \sim 1.5\sigma$$

'90s:

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = (15.3 \pm 2.6) \times 10^{-4} (NA48) \qquad \sim 6\sigma$$

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = (20.7 \pm 2.8) \times 10^{-4} (KTEV) \qquad > 7\sigma$$

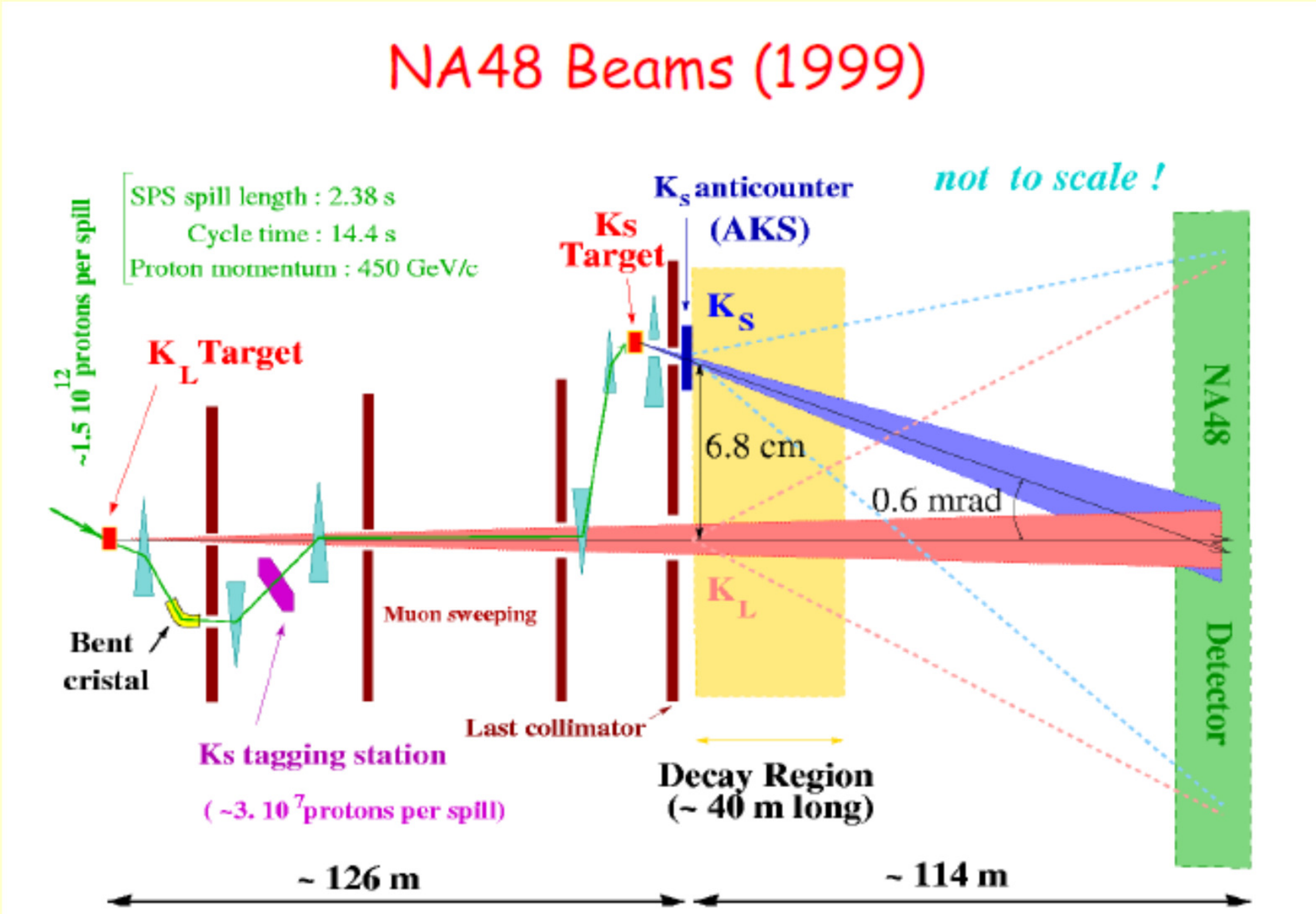
Mostly systematics

Direct *CP* Violation - VII

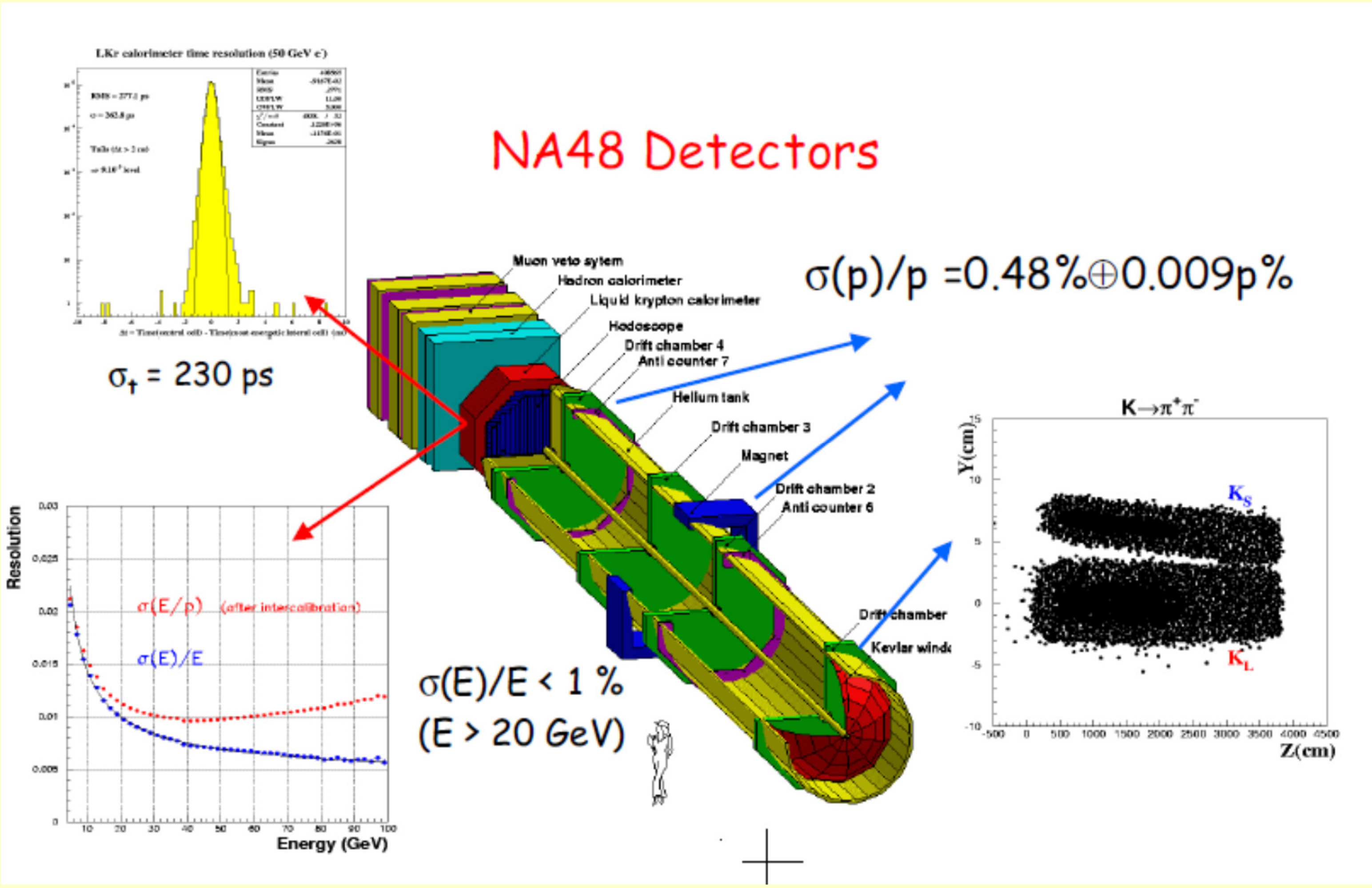
NA48 technique

- Employ two almost collinear neutral beams
- Collect the four decay modes simoultaneously, in the same detector and from the same decay region
- Keep the acceptance correction small by weighting the K_L events according to the ratio of K_S/K_L decay intensities as a function of proper time
- Distinguish K_S and K_L events by tagging the protons upstream of the K_S target
- Use precise and stable liquid krypton (LKr) calorimetry to control the relative momentum scale

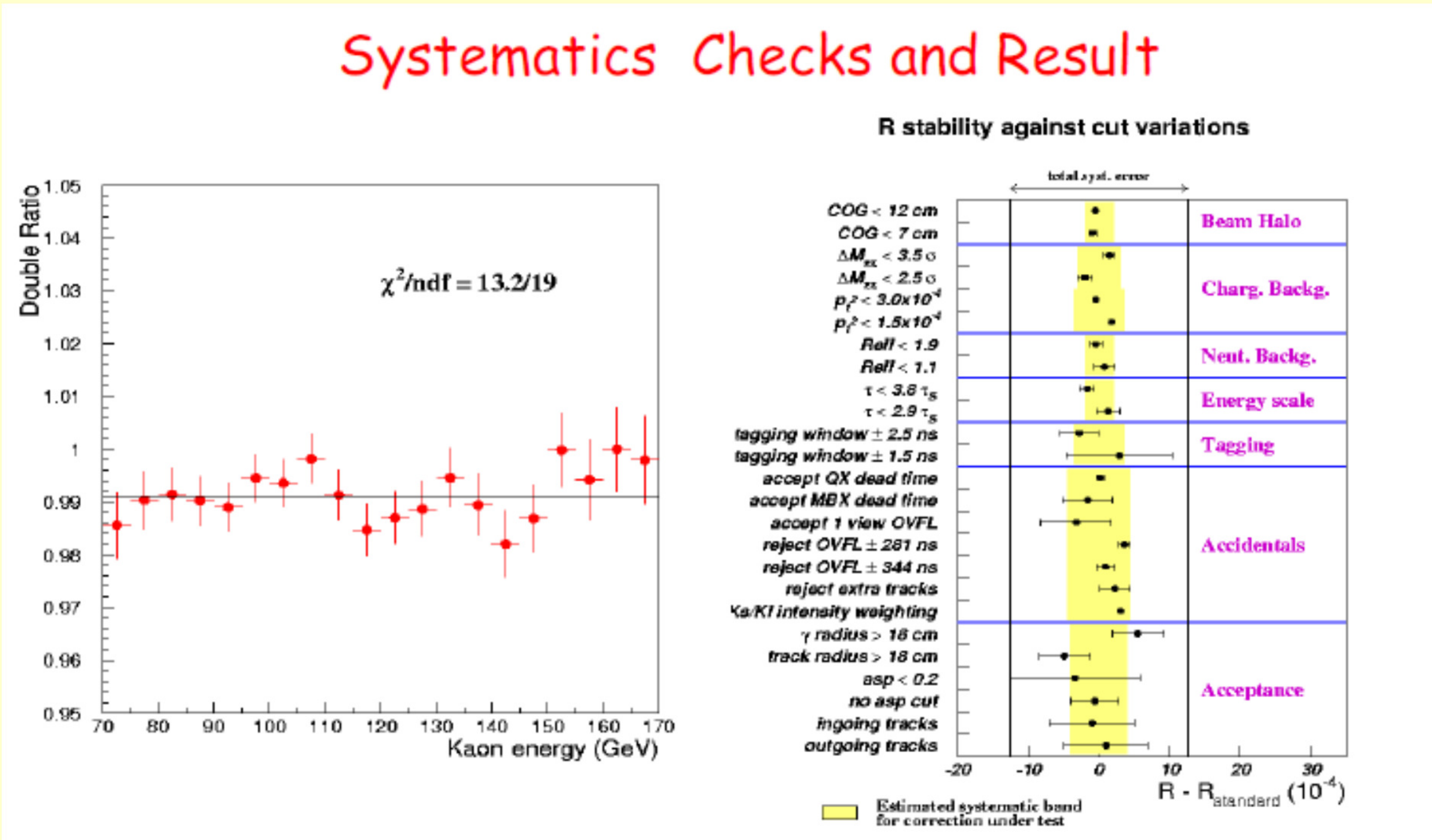
Direct *CP* Violation - VIII



Direct *CP* Violation - IX



Direct *CP* Violation - X

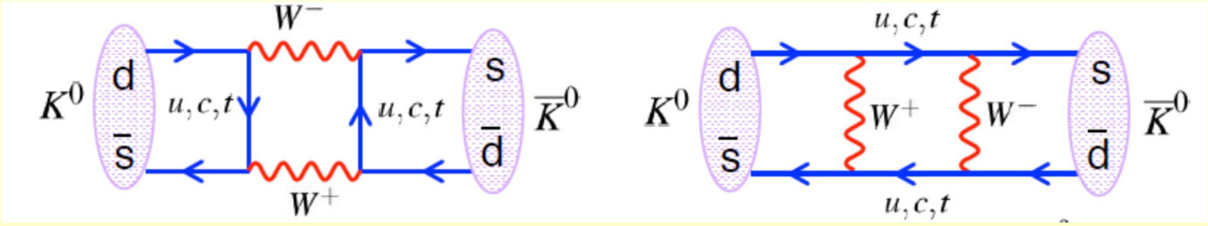


CP Violation and SM - I

So far: Phenomenological description $\rightarrow$ ~ Just symmetries

Now: Try to connect to SM

Mixing : *Box* diagrams



Mass difference between mass eigenstates:

$$\Delta m_K \simeq \frac{G_F^2}{3\pi^2} f_K^2 m_K \left| V_{qd} V_{qs}^* V_{q'd} V_{q's}^* \right| m_q m_{q'}$$

CP Violation and SM - II

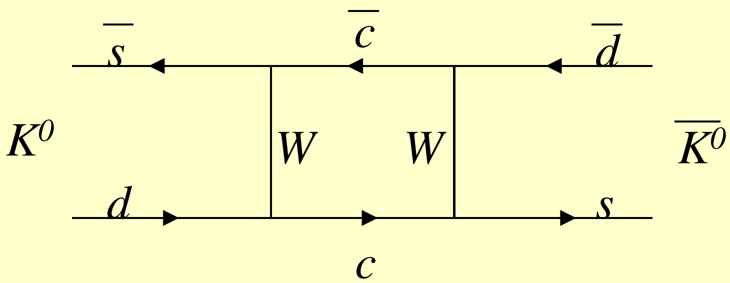
Go to *CKM*, find:

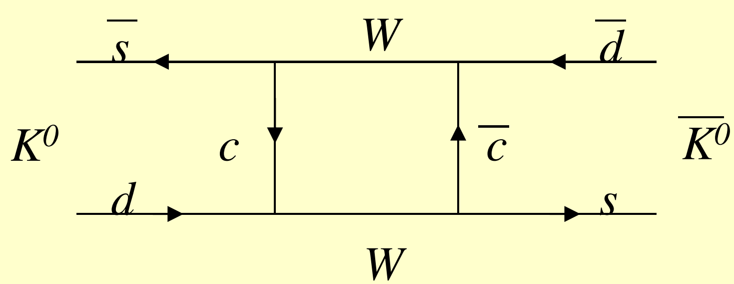
u,u	$\sin^2\theta_c \cos^2\theta_c m_u^2$	$\sim 0.048 m_u^2$	$\sim .005$
u,c	$\sin^2\theta_c \cos^2\theta_c m_u m_c$	$\sim 0.048 m_u m_c$	$\sim .022$
u,t	$\lvert V_{td}\rvert \lvert V_{ts}\rvert \sin\theta_c \cos\theta_c m_u m_t$	$\sim 0.220\ 4 10^{-5} m_u m_t$	$\sim .0005$
c,c	$\sin^2\theta_c \cos^2\theta_c m_c^2$	$\sim 0.048 m_c^2$	$\sim .095$
c,t	$\lvert V_{td}\rvert \lvert V_{ts}\rvert \sin\theta_c \cos\theta_c m_c m_t$	$\sim 0.220\ 4 10^{-5} m_c m_t$	$\sim .0021$
t,t	$\lvert V_{td}\rvert^2 \lvert V_{ts}\rvert^2 m_t^2$	$\sim 1.6 10^{-10} m_t^2$	~ 0

$\rightarrow$ Diagrams with c quark dominant

CP Violation and SM - IV

Just for the fun: Oversimplify, take only charm contribution





$$A_{box} \propto \left(V_{cs}V_{cd}^*\right)^2 m_c^2 \approx \left(\lambda^2 + i2A^2\lambda^6\eta\right)m_c^2$$

$$|\varepsilon| \approx \frac{\mathrm{Im}\left(A_{box}\right)}{\mathrm{Re}\left(A_{box}\right)} \approx \frac{2A^2\lambda^6\eta}{\lambda^2} = 2A^2\lambda^4\eta$$

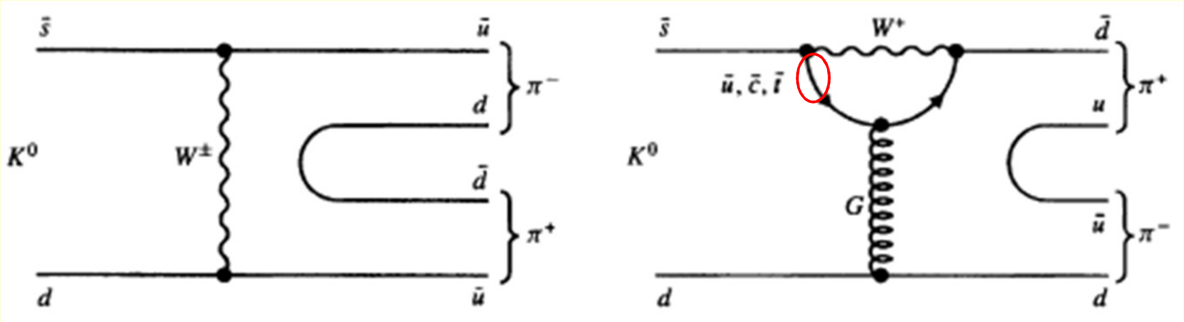
$$|\varepsilon| \sim 2\ 0.81\ 0.0025\ 0.343 \sim 1.410^{-3}$$

Not that bad...

CP Violation and SM - V

Fun again: 2π decays
Must take into account two diagrams:

'Tree' 'Penguin'



$$\propto \left|V_{us}V_{ud}\right|^2 \approx \lambda^2$$

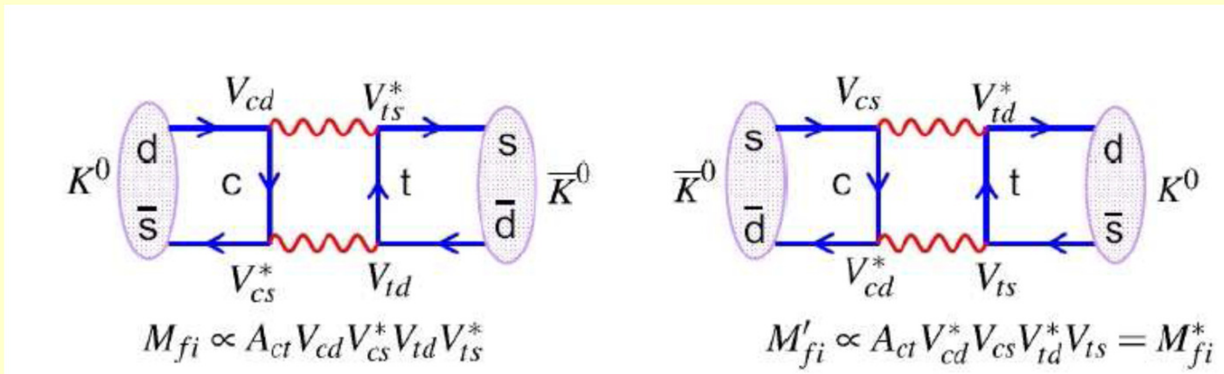
Top dominating:

$$\propto \mathrm{Im}\left(V_{ts}V_{td}\right) \approx A^2\lambda^5\eta$$

$\varepsilon' \propto$ Interference between the two above

CP Violation and SM - VI

Reconsidering box diagrams:



$$\Gamma(K^0_{t=0} \to \bar{K}^0) - \Gamma(\bar{K}^0_{t=0} \to K^0) \propto M_{fi} - M^*_{fi} = 2Im(M_{fi})$$

$$A_T = \frac{\Gamma(\overline{\mathrm{K}}^0 \to \mathrm{K}^0) - \Gamma(\mathrm{K}^0 \to \overline{\mathrm{K}}^0)}{\Gamma(\overline{\mathrm{K}}^0 \to \mathrm{K}^0) + \Gamma(\mathrm{K}^0 \to \overline{\mathrm{K}}^0)}.$$

Rememebering:

$$A_T \approx 4\,\mathrm{Re}(\varepsilon)$$

$$\to 4\,\mathrm{Re}(\varepsilon) \propto 2\,\mathrm{Im}\, M_{fi}$$

Two State System - I

Electron in a magnetic field $\boldsymbol{B}$ along $\hat{z}$:

$$H = -\boldsymbol{\mu}\cdot\boldsymbol{B} = \frac{1}{2}a\boldsymbol{\sigma}\cdot\boldsymbol{B} \quad \boldsymbol{B} = B\hat{k}, \quad \boldsymbol{\sigma} = \left(\sigma_x, \sigma_y, \sigma_z\right)\text{: Pauli spin matrices}$$

$$\rightarrow H = \frac{1}{2}a\sigma_z B = \begin{pmatrix} \frac{1}{2}aB & 0 \\ 0 & -\frac{1}{2}aB \end{pmatrix} = \begin{pmatrix} +E & 0 \\ 0 & -E \end{pmatrix}$$

2 state system: Choose as base states

$$\left|+\right\rangle \equiv \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \left|-\right\rangle \equiv \begin{pmatrix} 0 \\ 1 \end{pmatrix} \text{ Eigenstates of } \sigma_z \rightarrow \text{Generic state:} \left|\psi\right\rangle = \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} \rightarrow \sigma_z\left|\psi\right\rangle = \begin{pmatrix} \psi_+ \\ -\psi_- \end{pmatrix}$$

Schrodinger equation:

$$i\hbar\frac{\partial\left|\psi\right\rangle}{\partial t} = H\left|\psi\right\rangle \rightarrow \left|\psi(t)\right\rangle = e^{-i\frac{Ht}{\hbar}}\left|\psi\right\rangle = e^{-i\frac{aB}{2\hbar}t\sigma_z}\left|\psi\right\rangle \rightarrow i\hbar\frac{\partial}{\partial t}\begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} = \frac{1}{2}aB\sigma_z\begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} = \frac{1}{2}aB\begin{pmatrix} \psi_+ \\ -\psi_- \end{pmatrix}$$

$$\rightarrow \begin{cases} \psi_+(t) = A_+ e^{-i\frac{aB}{\hbar}t} \\ \psi_-(t) = A_- e^{+i\frac{aB}{\hbar}t} \end{cases}, \quad \left|A_+\right|^2 + \left|A_-\right|^2 = 1 \rightarrow \begin{cases} \left|+,t\right\rangle = \begin{pmatrix} \psi_+(t) \\ 0 \end{pmatrix} \\ \left|-,t\right\rangle = \begin{pmatrix} 0 \\ \psi_-(t) \end{pmatrix} \end{cases} \text{ Stationary states}$$

Two State System - II

Introduce another $\boldsymbol{B}$ component along x:

$$\boldsymbol{B} = B\hat{\boldsymbol{k}} + B'\hat{\boldsymbol{i}}$$

$$H = -\boldsymbol{\mu}\cdot\boldsymbol{B} = \frac{1}{2}a\boldsymbol{\sigma}\cdot\boldsymbol{B} = \frac{1}{2}a\left(\sigma_z B + \sigma_x B'\right)$$

$$\rightarrow H = \begin{pmatrix} \frac{1}{2}aB & 0 \\ 0 & -\frac{1}{2}aB \end{pmatrix} + \begin{pmatrix} 0 & \frac{1}{2}aB' \\ \frac{1}{2}aB' & 0 \end{pmatrix} = \begin{pmatrix} +E & E' \\ E' & -E \end{pmatrix}$$

$$\rightarrow i\hbar\frac{\partial}{\partial t}\begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} = \frac{1}{2}a\left(B\sigma_z + B'\sigma_x\right)\begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} = \frac{1}{2}a\left[B\begin{pmatrix} \psi_+ \\ -\psi_- \end{pmatrix} + B'\begin{pmatrix} \psi_- \\ \psi_+ \end{pmatrix}\right]$$

$$\rightarrow \begin{cases} i\hbar\dfrac{\partial\psi_+}{\partial t} = \dfrac{1}{2}aB\psi_+ + B'\psi_- \\ i\hbar\dfrac{\partial\psi_-}{\partial t} = -\dfrac{1}{2}aB\psi_- + B'\psi_+ \end{cases} \rightarrow \text{Coupled equations}$$

$$\rightarrow \begin{cases} \left|+,t\right\rangle \\ \left|-,t\right\rangle \end{cases} \text{ Non -stationary states}$$

Mixing and Oscillations - I

Use symbol B everywhere for B^0, but:

Most of the formalism suitable for K^0, D^0, B_S^0

Neutral meson time evolution: Two-state system

$$\left|B^0\right\rangle = \begin{pmatrix}1\\0\end{pmatrix}, \left|\overline{B}^0\right\rangle = \begin{pmatrix}0\\1\end{pmatrix}$$

$$i\frac{\partial}{\partial t}\psi = H\psi \quad \text{Schrodinger equation}$$

$$\psi(t) = a(t)\left|B^0\right\rangle + b(t)\left|\overline{B}^0\right\rangle \equiv \begin{pmatrix}a(t)\\b(t)\end{pmatrix} \quad \text{Two-component state vector}$$

Just free evolution for both components, no decay:

$$H = \underbrace{\begin{pmatrix}M & 0\\0 & M\end{pmatrix}}_{\text{hermitian}} \quad \text{Effective Hamiltonian}$$

Free evolution for both components, with decay:

$$H = \underbrace{\begin{pmatrix}M & 0\\0 & M\end{pmatrix}}_{\text{hermitian}} - \frac{i}{2}\underbrace{\begin{pmatrix}\Gamma & 0\\0 & \Gamma\end{pmatrix}}_{\text{hermitian}}$$

Mixing and Oscillations - II

Observe:

$$H^{\dagger} = \begin{pmatrix} M & 0 \\ 0 & M \end{pmatrix}^{\dagger} + \frac{i}{2}\begin{pmatrix} \Gamma & 0 \\ 0 & \Gamma \end{pmatrix}^{\dagger} = \begin{pmatrix} M & 0 \\ 0 & M \end{pmatrix} + \frac{i}{2}\begin{pmatrix} \Gamma & 0 \\ 0 & \Gamma \end{pmatrix} \neq H$$

$\rightarrow H$ non-Hermitian $\rightarrow e^{-iHt}$ non-unitary $\rightarrow$ State norm not conserved: Decreasing $\leftrightarrow \Gamma>0$

$$H = \underbrace{\begin{pmatrix} M & A \\ B & M \end{pmatrix}}_{\text{hermitian}} - \frac{i}{2}\underbrace{\begin{pmatrix} \Gamma & C \\ D & \Gamma \end{pmatrix}}_{\text{hermitian}}$$

Include mixing

$$\begin{pmatrix} M & A \\ B & M \end{pmatrix}^{\dagger} = \begin{pmatrix} M & A \\ B & M \end{pmatrix} \rightarrow \begin{pmatrix} M & B^* \\ A^* & M \end{pmatrix} = \begin{pmatrix} M & A \\ B & M \end{pmatrix} \rightarrow A^* = B, \text{ same for } \Gamma$$

$$\rightarrow H = \underbrace{\begin{pmatrix} M & M_{12} \\ M_{12}^* & M \end{pmatrix}}_{\text{hermitian}} - \frac{i}{2}\underbrace{\begin{pmatrix} \Gamma & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma \end{pmatrix}}_{\text{hermitian}}$$

$$\rightarrow i\frac{\partial}{\partial t}\psi(t) = \begin{pmatrix} M - \frac{i}{2}\Gamma & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M - \frac{i}{2}\Gamma \end{pmatrix}\psi(t)$$

Mixing and Oscillations - III

Eigenvalues:

$$\begin{vmatrix} M - \frac{i}{2}\Gamma - \lambda & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M - \frac{i}{2}\Gamma - \lambda \end{vmatrix} = 0$$

Define $F = \sqrt{\left(M_{12} - \frac{i}{2}\Gamma_{12}\right)\left(M_{12}^* - \frac{i}{2}\Gamma_{12}^*\right)}$

$$\rightarrow \begin{cases} m_1 + i\frac{\Gamma_1}{2} = M - \mathrm{Re}(F) - i\frac{\Gamma}{2} - \mathrm{Im}(F) \\ m_2 + i\frac{\Gamma_2}{2} = M + \mathrm{Re}(F) - i\frac{\Gamma}{2} + \mathrm{Im}(F) \end{cases}$$

$$\rightarrow \begin{cases} \Delta m = 2\,\mathrm{Re}(F) = 2\,\mathrm{Re}\sqrt{\left(M_{12} - \frac{i}{2}\Gamma_{12}\right)\left(M_{12}^* - \frac{i}{2}\Gamma_{12}^*\right)} \\ \Delta\Gamma = 4\,\mathrm{Im}(F) = 4\,\mathrm{Im}\sqrt{\left(M_{12} - \frac{i}{2}\Gamma_{12}\right)\left(M_{12}^* - \frac{i}{2}\Gamma_{12}^*\right)} \end{cases}$$

Mixing and Oscillations - IV

Eigenvectors:

$$\begin{pmatrix} M - \frac{i}{2}\Gamma & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M - \frac{i}{2}\Gamma \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix} = \lambda_\pm \begin{pmatrix} p \\ q \end{pmatrix}$$

$$\frac{q}{p} = \sqrt{\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}}$$

$$\rightarrow \begin{cases} |B_H\rangle = p|B^0\rangle + q|\overline{B^0}\rangle \\ |B_L\rangle = p|B^0\rangle - q|\overline{B^0}\rangle \end{cases}$$

$$\rightarrow \begin{cases} |B^0\rangle = \frac{1}{2p}\left(|B_H\rangle + |B_L\rangle\right) \\ |\overline{B}^0\rangle = \frac{1}{2q}\left(|B_H\rangle - |B_L\rangle\right) \end{cases}$$

Mixing and Oscillations - V

Time evolution of mass eigenstates:

Define

$$\omega_+ = m_H - \frac{1}{2}\Gamma_H, \omega_- = m_L - \frac{1}{2}\Gamma_L$$

$$\rightarrow \begin{array}{l} \left|B_H(t)\right\rangle = e^{-i\omega_+ t}\left|B_H(0)\right\rangle \\ \left|B_L(t)\right\rangle = e^{-i\omega_- t}\left|B_L(0)\right\rangle \end{array}$$

$\rightarrow$ Straightforward free propagation & decay

Mixing and Oscillations - VI

Time evolution of flavor eigenstates: Flavor oscillations

Define

$$g_\pm(t) = \frac{e^{-i\omega_+ t} \pm e^{-i\omega_- t}}{2}$$

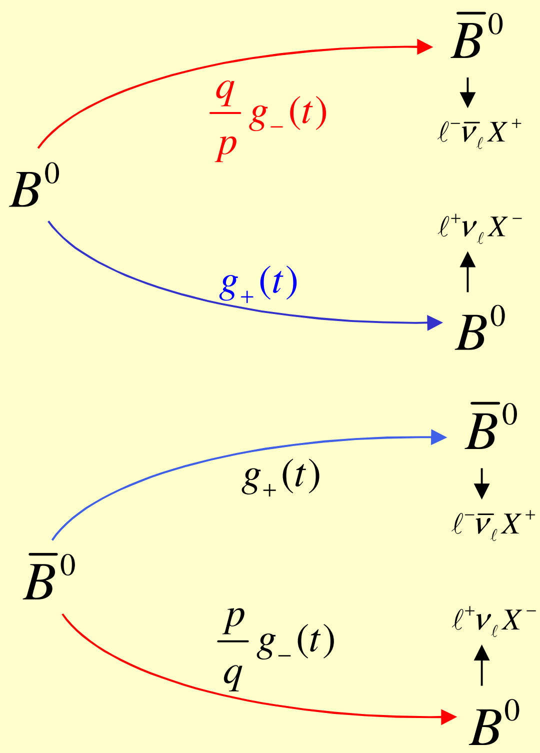
$$\rightarrow \begin{cases} \left|B^0(t)\right\rangle = g_+(t)\left|B^0\right\rangle + \dfrac{q}{p} g_-(t)\left|\overline{B^0}\right\rangle \\ \left|\overline{B^0}(t)\right\rangle = g_+(t)\left|\overline{B^0}\right\rangle + \dfrac{p}{q} g_-(t)\left|B^0\right\rangle \end{cases}$$

For B:

$$\Delta\Gamma \sim 0, \left|\frac{q}{p}\right| \sim 1$$

$$\rightarrow \begin{cases} g_+(t) = e^{-imt} e^{-\Gamma t/2} \cos\dfrac{\Delta m t}{2} \\ g_-(t) = e^{-imt} e^{-\Gamma t/2} \sin\dfrac{\Delta m t}{2} \end{cases}$$

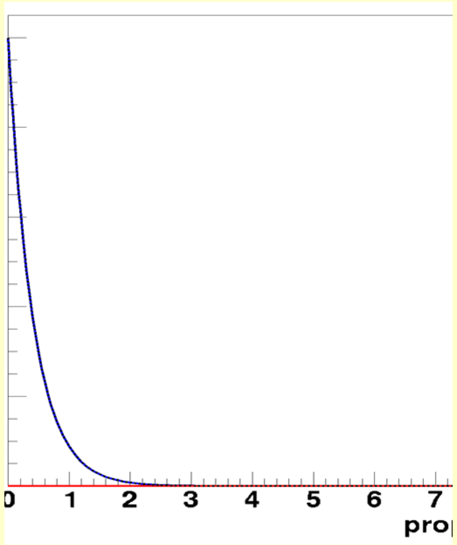
Observe by:



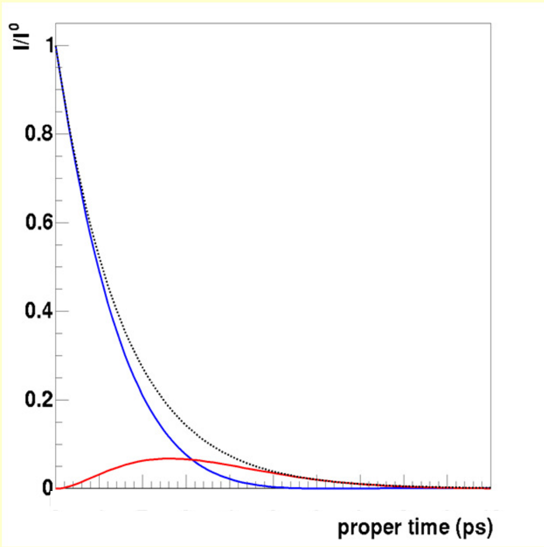
Mixing and Oscillations - VII

Compare different ratios

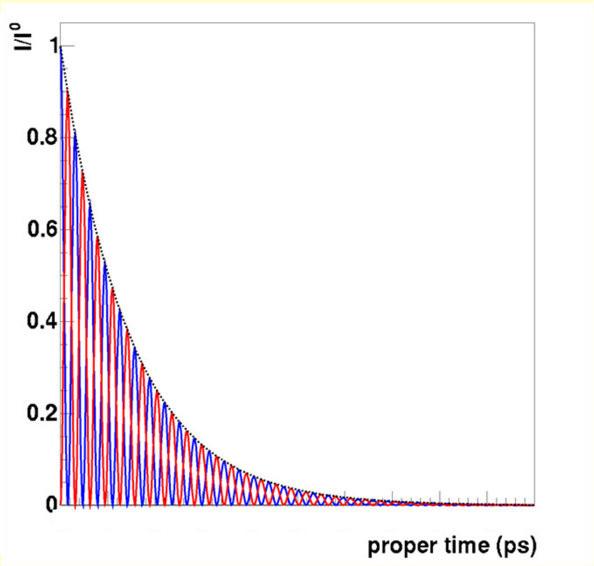
$$x = \frac{\Delta m}{\Gamma}$$



$$x \equiv \frac{\Delta m}{\Gamma} = 0$$



$$x \equiv \frac{\Delta m}{\Gamma} \approx 1$$



$$x \equiv \frac{\Delta m}{\Gamma} \gg 1$$

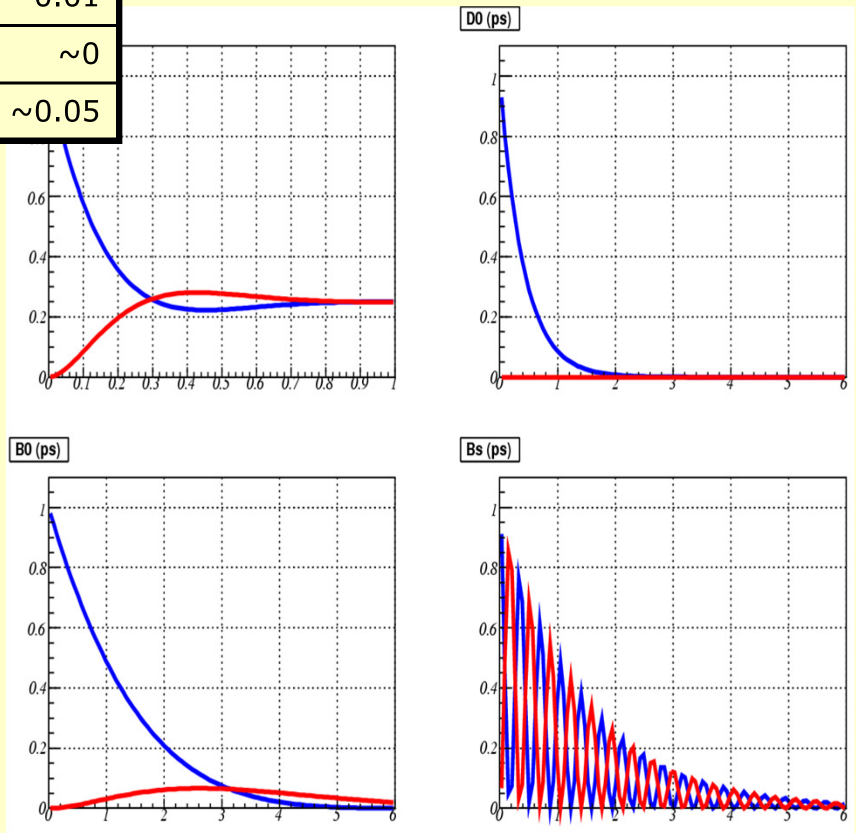
Mixing and Oscillations - VIII

Neutral, flavored mesons: Lightest states

$$K^0 : d\bar{s}$$
$$\bar{K}^0 : \bar{d}s$$
$$D^0 : c\bar{u}$$
$$\bar{D}^0 : \bar{c}u$$
$$B^0 : \bar{b}d$$
$$\bar{B}^0 : b\bar{d}$$
$$B_S^0 : \bar{b}s$$
$$\bar{B}_S^0 : b\bar{s}$$

Mixing and Oscillations - IX

	$<\tau>$	Δm	$x=\Delta m/\Gamma$	$y=\Delta\Gamma/2\Gamma$
K^0	$2.6\ 10^{-8}$ s	$5.29\ ns^{-1}$	$\Delta m/\Gamma_S=0.49$	~1
D^0	$0.41\ 10^{-12}$ s	$0.001\ fs^{-1}$	~0	0.01
B^0	$1.53\ 10^{-12}$ s	$0.507\ ps^{-1}$	0.78	~0
B_s^0	$1.47\ 10^{-12}$ s	$17.8\ ps^{-1}$	12.1	~0.05



B vs K - I

Rationale:

$\not{CP}$ observed in neutral kaon decays

Ascribed to mixing, decay, or both

Accounted for by a *single* complex phase in *CKM*

$\rightarrow$ Expect $\not{CP}$ to occur in other neutral, flavored meson decays

$\rightarrow$ Heavy quarks involved

$\rightarrow$ B mesons best bet

Looking again at unitarity triangles:

$(1)\ \ V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0;$ $\qquad (2)\ \ V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0;$

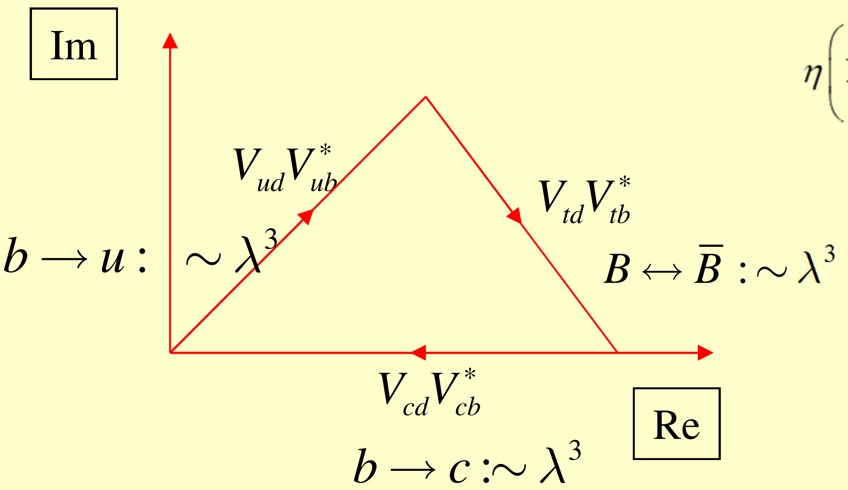
$(3)\ \ V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0;$ $\qquad (4)\ \ V_{ud}^*V_{cd} + V_{us}^*V_{cs} + V_{ub}^*V_{cb} = 0;$

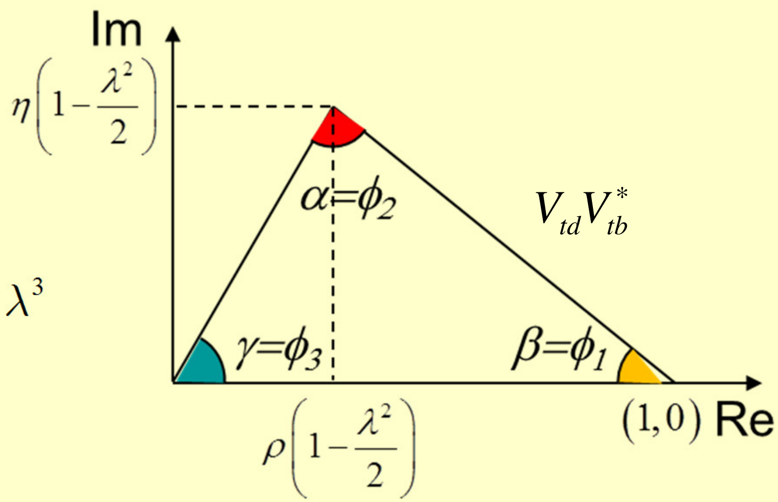
$(5)\ \ V_{ud}^*V_{td} + V_{us}^*V_{ts} + V_{ub}^*V_{tb} = 0;$ $\qquad (6)\ \ V_{cd}^*V_{td} + V_{cs}^*V_{ts} + V_{cb}^*V_{tb} = 0$

Not all equally useful: *Shape, Easy to measure*

B vs K - II

$$(2)\quad V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0;$$





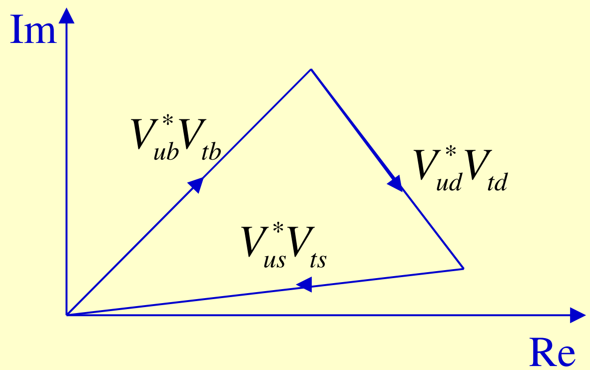
The unitarity triangle: Somewhat 'equilateral'→Large angles

B vs K - III

$$V_{ud}^* V_{td} + V_{us}^* V_{ts} + V_{ub}^* V_{tb} = \delta_{tu} = 0 \quad \hat{=} \quad tu \text{ triangle}$$

Another $\approx$ equilateral one

Each side $\propto \lambda^3$

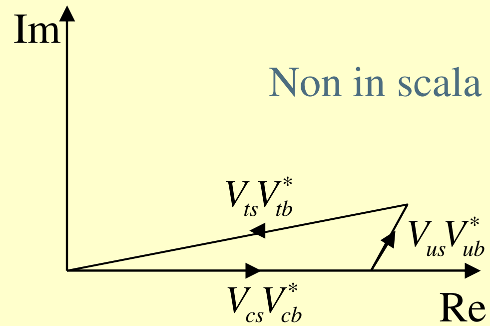


B vs K - IV

Two 'squashed' triangles…

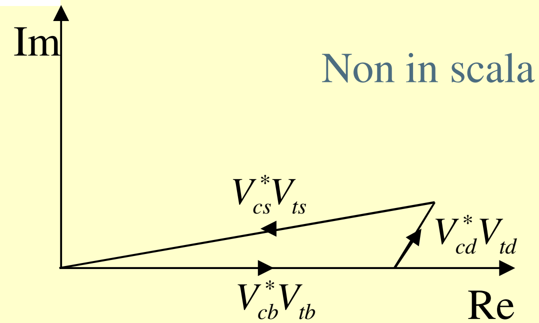
2 sides $\propto \lambda^2$

1 side $\propto \lambda^4$



$$V_{us}^* V_{ub} + V_{cs}^* V_{cb} + V_{ts}^* V_{tb} = \delta_{bs} = 0 \quad \hat{=} \quad bs \text{ triangle}$$

$$V_{td}^* V_{cd} + V_{ts}^* V_{cs} + V_{tb}^* V_{cb} = \delta_{tc} = 0 \quad \hat{=} \quad tc \text{ triangle}$$



Difficult to use to test $\not{CP}$

$\not{CP} \propto$ Height with base normalized to 1

B vs *K* - V

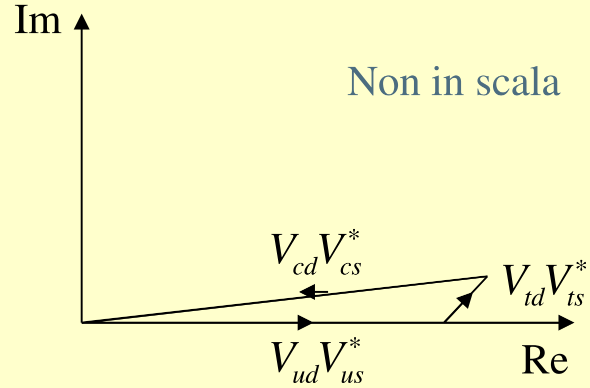
…and two even more squashed

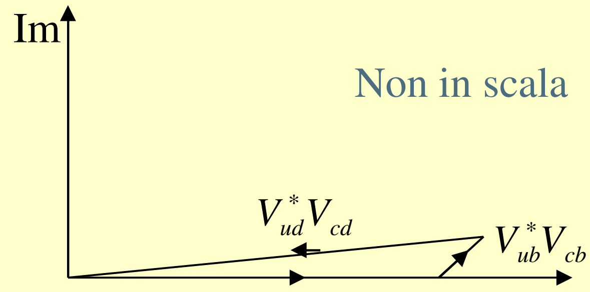
2 sides $\propto \lambda$

1 side $\propto \lambda^5$

$$V_{ud}^*V_{us} + V_{cd}^*V_{cs} + V_{td}^*V_{ts} = \delta_{sd} = 0 \quad \hat{=} \quad sd \text{ triangle}$$

$$V_{ud}^*V_{cd} + V_{us}^*V_{cs} + V_{ub}^*V_{cb} = \delta_{cu} = 0 \quad \hat{=} \quad cu \text{ triangle}$$





B vs K - VI

$B^0 - \overline{B}^0$: Most promising sector for validation of CKM

'Large' $\not{CP}$ expected

Similar to $K^0 - \overline{K}^0$, but:

$\Delta M_B = 0.489 \pm 0.008\ ps^{-1} \sim 0.489\ 10^{12} s^{-1}\ 6.582\ 10^{-16} eV\ s \sim 3.22\ 10^{-4} eV$

$\tau_{B_1} \approx \tau_{B_2} = 1.56 \pm 0.06\ \ ps$

Compare to K:

$\Delta M_K = 5.29\ ns^{-1} \sim 5.29\ 10^9 s^{-1}\ 6.582\ 10^{-16} eV\ s \sim 3.4\ 10^{-6} eV$

$\tau_L \sim 600\ \tau_S \sim 600\ 89\ ps$

B_H, B_L states cannot be physically separated

B Mixing: CP - I

$$\begin{cases} |B_H\rangle = \dfrac{1}{\sqrt{1+|\eta|^2}}\left(|B^0\rangle + \eta|\overline{B^0}\rangle\right) \\ |B_L\rangle = \dfrac{1}{\sqrt{1+|\eta|^2}}\left(|B^0\rangle - \eta|\overline{B^0}\rangle\right) \end{cases}, \quad \begin{cases} |B^0\rangle = \dfrac{1}{2}\sqrt{1+|\eta|^2}\left(|B_H\rangle + |B_L\rangle\right) \\ |\overline{B}^0\rangle = \dfrac{1}{2\eta}\sqrt{1+|\eta|^2}\left(|B_H\rangle - |B_L\rangle\right) \end{cases}$$

Compare to:

$$|K_L^0\rangle = \frac{1}{\sqrt{2\left(1+|\varepsilon|^2\right)}}\left[(1+\varepsilon)|K^0\rangle + (1-\varepsilon)|\bar{K}^0\rangle\right], \quad |K_S^0\rangle = \frac{1}{\sqrt{2\left(1+|\varepsilon|^2\right)}}\left[(1-\varepsilon)|K^0\rangle - (1+\varepsilon)|\bar{K}^0\rangle\right]$$

$$\rightarrow \eta = \frac{1-\varepsilon_B}{1+\varepsilon_B}$$

$$\rightarrow \begin{cases} |B_H(t)\rangle = |B_H\rangle e^{-im_H t - \frac{\Gamma_H}{2}t} \\ |B_L(t)\rangle = |B_L\rangle e^{-im_L t - \frac{\Gamma_L}{2}t} \end{cases}, \quad \Gamma_H \simeq \Gamma_L = \Gamma, \ \Delta m \ll M$$

B Mixing: CP - II

$$\rightarrow \begin{cases} \left|B^0(t)\right\rangle = \left|B^0\right\rangle f_+(t) + \eta\left|\overline{B^0}\right\rangle f_-(t) \\ \left|\overline{B}^0(t)\right\rangle = \left|B^0\right\rangle \frac{1}{\eta} f_-(t) + \left|\overline{B^0}\right\rangle f_+(t) \end{cases}$$

$$f_\pm(t) \approx \frac{1}{2} e^{-\frac{1}{2}\Gamma t} e^{-iMt}\left[e^{-i\frac{\Delta m}{2}t} \pm e^{+i\frac{\Delta m}{2}t}\right], \quad \Delta m > 0, \quad \Gamma_H \simeq \Gamma_L = \Gamma$$

$$\rightarrow \begin{cases} f_+(t) \approx \frac{1}{2} e^{-\frac{1}{2}\Gamma t} e^{-iMt} \cos\left(\frac{\Delta m}{2} t\right) \\ f_-(t) \approx -i\frac{1}{2} e^{-\frac{1}{2}\Gamma t} e^{-iMt} \sin\left(\frac{\Delta m}{2} t\right) \end{cases}$$

$$\rightarrow \begin{cases} \left|B^0(t)\right\rangle = e^{-iMt} e^{-\frac{1}{2}\Gamma t}\left[\cos\left(\frac{\Delta m}{2} t\right)\left|B_0\right\rangle - i\eta \sin\left(\frac{\Delta m}{2} t\right)\left|\overline{B_0}\right\rangle\right] \\ \left|\overline{B}^0(t)\right\rangle = e^{-iMt} e^{-\frac{1}{2}\Gamma t}\left[-\frac{i}{\eta}\sin\left(\frac{\Delta m}{2} t\right)\left|B_0\right\rangle + \cos\left(\frac{\Delta m}{2} t\right)\left|\overline{B_0}\right\rangle\right] \end{cases}$$

B Mixing: CP - III

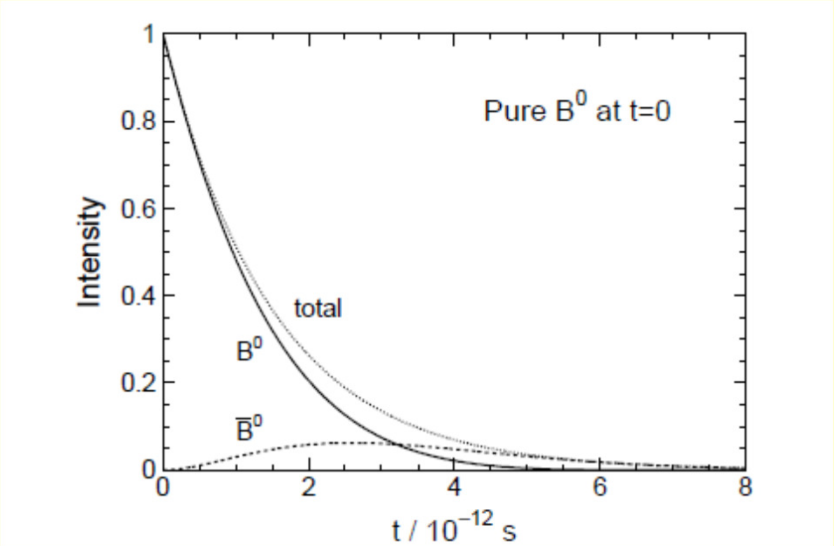
$\rightarrow$ Expect:

$$\Gamma\left(B^0\left(t=0\right)\rightarrow B^0\right)=e^{-\Gamma t}\cos^2\left(\frac{\Delta m}{2}t\right)$$

$$\Gamma\left(B^0\left(t=0\right)\rightarrow \bar{B}^0\right)=\left|\eta\right|^2 e^{-\Gamma t}\sin^2\left(\frac{\Delta m}{2}t\right)$$

$$\Gamma\left(\bar{B}^0\left(t=0\right)\rightarrow \bar{B}^0\right)=e^{-\Gamma t}\cos^2\left(\frac{\Delta m}{2}t\right)$$

$$\Gamma\left(\bar{B}^0\left(t=0\right)\rightarrow \bar{B}^0\right)=\left|\frac{1}{\eta}\right|^2 e^{-\Gamma t}\sin^2\left(\frac{\Delta m}{2}t\right)$$



Unlike $K^0/\bar{K}^0$:

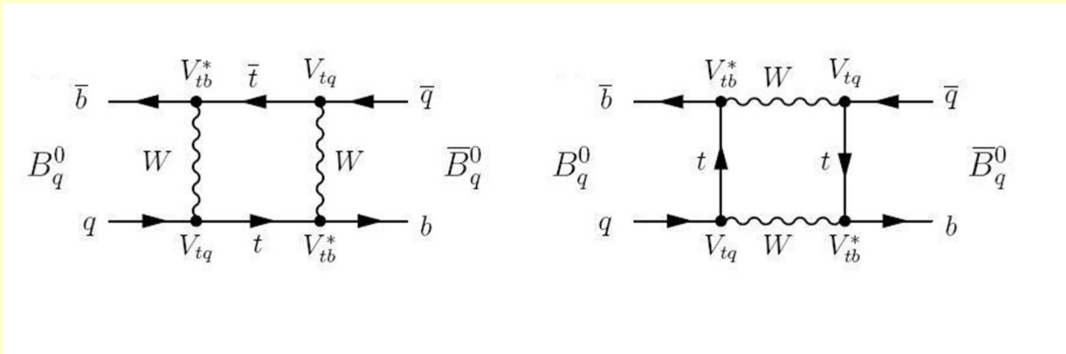
$|\eta|\simeq 1\rightarrow\ \sim$ No $\not{CP}$ effect observable by looking at flavor oscillations

By restricting to decays to *CP* eigenstates: OK! See below..

Main disadvantage: Statistics (Tiny BR)

B Mixing: CP - IV

Mixing in the SM: Box diagrams, t dominated for B^0



$$\begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

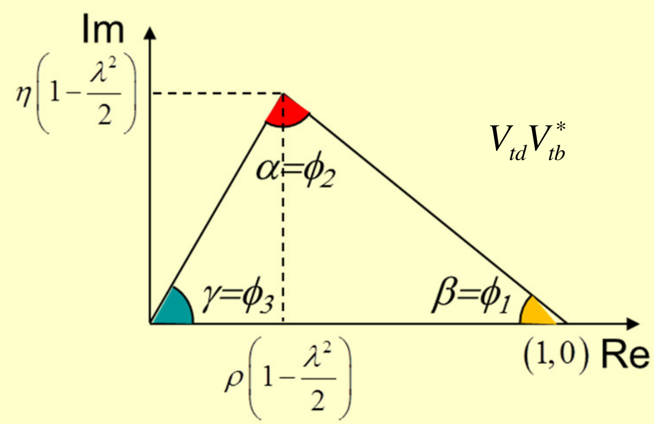
Mixing parameter:

$$\eta = \frac{\left(V_{tb}^* V_{td}\right)}{\left(V_{tb} V_{td}^*\right)} \approx \frac{V_{td}}{V_{td}^*} = e^{-2i\varphi_{td}}$$

From UT:

$$\varphi_{td} = \beta$$

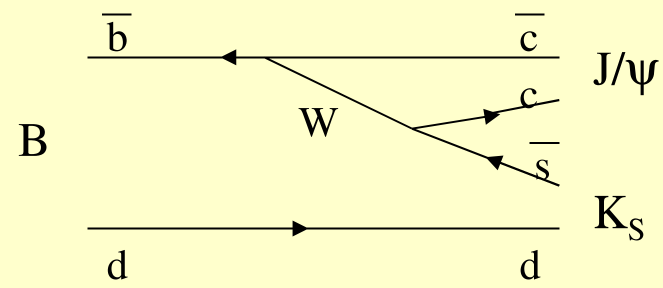
$$\to \eta = e^{-2i\beta}$$



B Mixing: CP - V

Golden final state: $J/\psi K_S^0$ (or K_L^0: Experimentally less attractive)

$B^0, \overline{B}^0 \to J/\psi K_S^0$



Angular momentum balance:

$0 = 1 \oplus 0 \oplus L \to L = 1$ Pure P - wave

$CP(J/\psi) = (-1)(-1) = +1$

$CP(K_S^0) = +1$, neglect $\not{CP}$ in K^0

$P_{orb} = (-1)^L = -1$

$\to CP(J/\psi K_S^0) = -1$

$\to CP(J/\psi K_L^0) = +1$

B Mixing: CP - VI

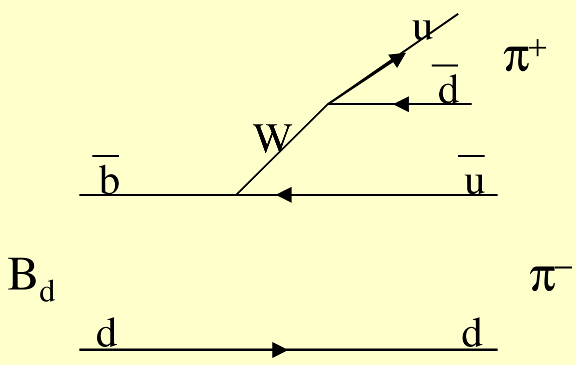
Another golden : $\pi\pi$

$B^0, \bar{B}^0 \to \pi\pi$

Angular momentum balance:

$L = 0$ Pure S - wave

$CP(\pi\pi) = +1$



B Mixing: CP - VII

Taking decays into (golden) *CP* eigenstates:

$$A\left(B^0 \to J/\psi K_S^0\right) = \left\langle J/\psi K_S^0 \right| H_{eff} \left| B^0(t) \right\rangle$$

$$\to A\left(B^0 \to J/\psi K_S^0\right) = f_+(t) \left\langle J/\psi K_S^0 \right| H_{eff} \left| B^0 \right\rangle + \eta f_-(t) \left\langle J/\psi K_S^0 \right| H_{eff} \left| \overline{B}^0 \right\rangle$$

$$\to A\left(B^0 \to J/\psi K_S^0\right) = \left\langle J/\psi K_S^0 \right| H_{eff} \left| B^0 \right\rangle \left[f_+(t) + \eta f_-(t) \frac{\left\langle J/\psi K_S^0 \right| H_{eff} \left| \overline{B}^0 \right\rangle}{\left\langle J/\psi K_S^0 \right| H_{eff} \left| B^0 \right\rangle} \right]$$

Considering $B^0, \overline{B}^0$ decay: Must occur in two steps

$$B^0 \to J/\psi\ K^0 \to J/\psi\ K_S^0 \qquad \overline{B}^0 \to J/\psi\ \overline{K}^0 \to J/\psi\ K_S^0$$

because at the quark level:

$$\overline{b} \to \overline{c}\ cs \qquad b \to c\ \overline{c}s$$

$$A\left(B^0 \to J/\psi K^0\right) \propto V_{cb}^* V_{cs} \qquad A\left(\overline{B}^0 \to J/\psi \overline{K}^0\right) \propto V_{cb} V_{cs}^*$$

$$\to \frac{\left\langle J/\psi K_S^0 \right| H_{eff} \left| \overline{B}^0 \right\rangle}{\left\langle J/\psi K_S^0 \right| H_{eff} \left| B^0 \right\rangle} = +1$$ *CKM* elements involved *real*

$$\frac{\langle \psi \mathrm{K_L} | H | \overline{\mathrm{B}}^0 \rangle}{\langle \psi \mathrm{K_L} | H | \mathrm{B}^0 \rangle} = -1\ .$$

B Mixing: CP - VIII

$$\Gamma\left(B^0(t=0)\to J/\psi K_S\right)\propto\left|f_+(t)+\eta f_-(t)\right|^2$$

$$\to\Gamma\left(B^0(t=0)\to J/\psi K_S\right)\propto e^{-\Gamma t}\left|\cos\left(\frac{\Delta m}{2}t\right)-ie^{-2i\beta}\sin\left(\frac{\Delta m}{2}t\right)\right|^2$$

$$\to\Gamma\left(B^0(t=0)\to J/\psi K_S\right)\propto e^{-\Gamma t}\left(1-\sin\Delta mt\sin 2\beta\right)$$

$$\to\Gamma\left(\overline{B}^0(t=0)\to J/\psi K_S\right)\propto e^{-\Gamma t}\left(1+\sin\Delta mt\sin 2\beta\right)$$

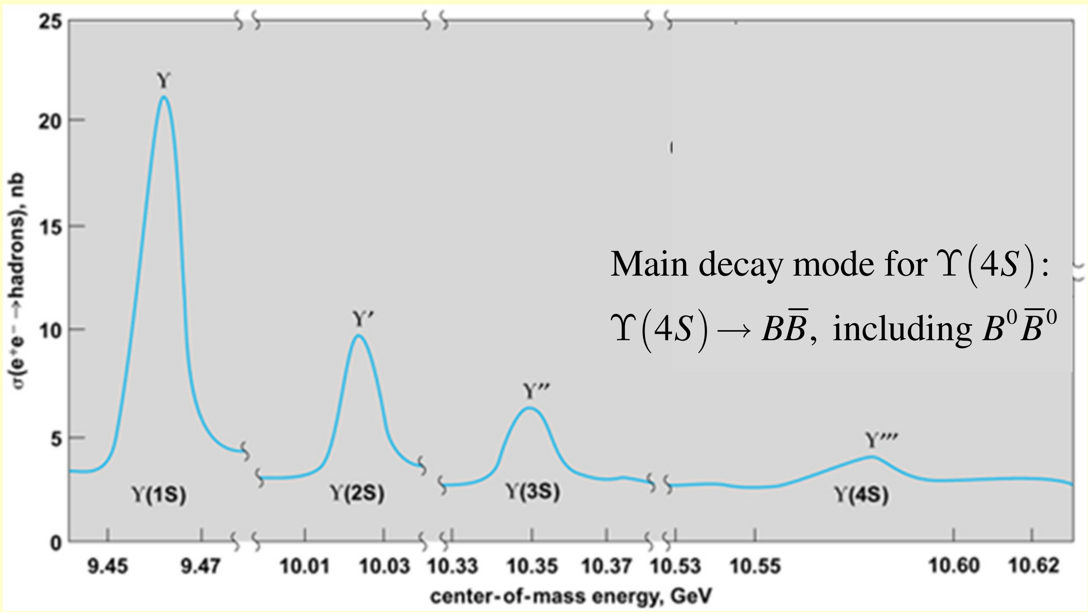
Time dependent asymmetry:

$$A_{J/\psi K_S}=\frac{\Gamma\left(B^0(t=0)\to J/\psi K_S\right)-\Gamma\left(\overline{B}^0(t=0)\to J/\psi K_S\right)}{\Gamma\left(B^0(t=0)\to J/\psi K_S\right)+\Gamma\left(\overline{B}^0(t=0)\to J/\psi K_S\right)}=\sin\Delta mt\sin 2\beta$$

$$A_{J/\psi K_L}=-\sin\Delta mt\sin 2\beta$$

B Factories - I

Total e^+e^- annihilation cross section:

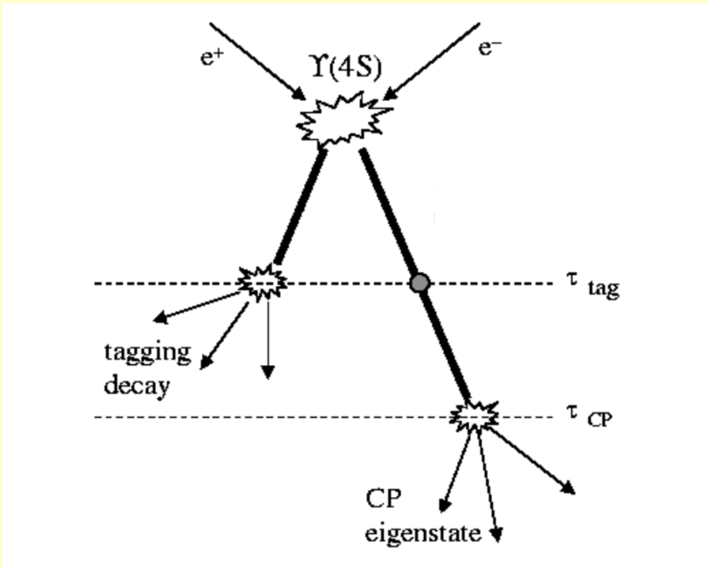


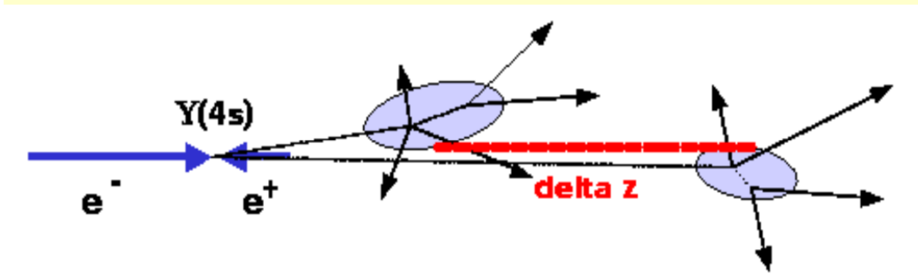
Main decay mode for $\Upsilon(4S)$:

$\Upsilon(4S) \rightarrow B\overline{B}$, including $B^0\overline{B}^0$

B Factories - II

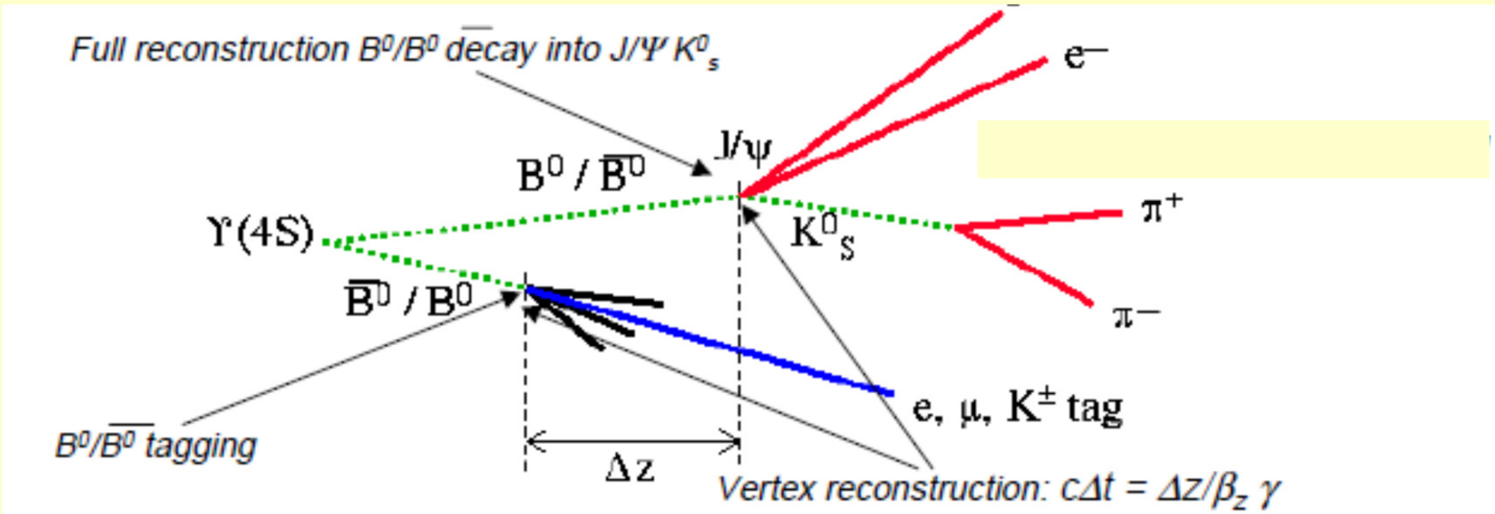
Basic idea to measure time dependent asymmetry:



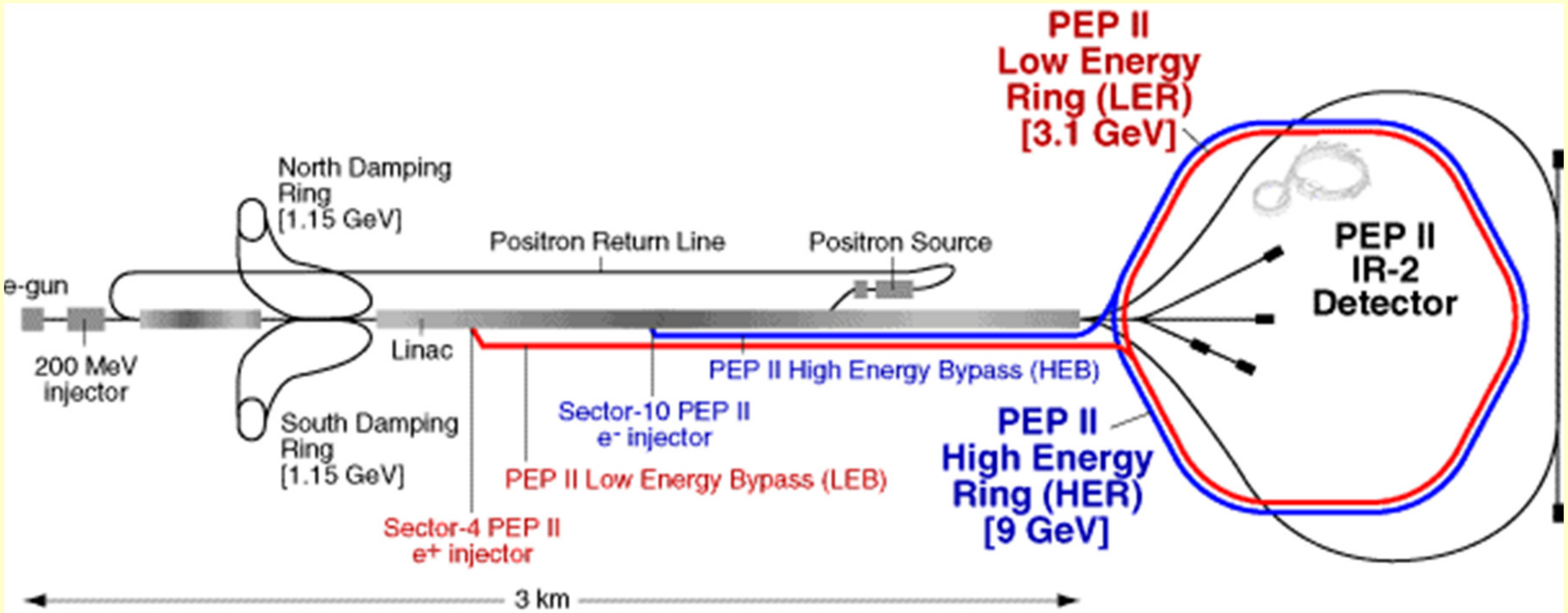


B Factories - III

Example:

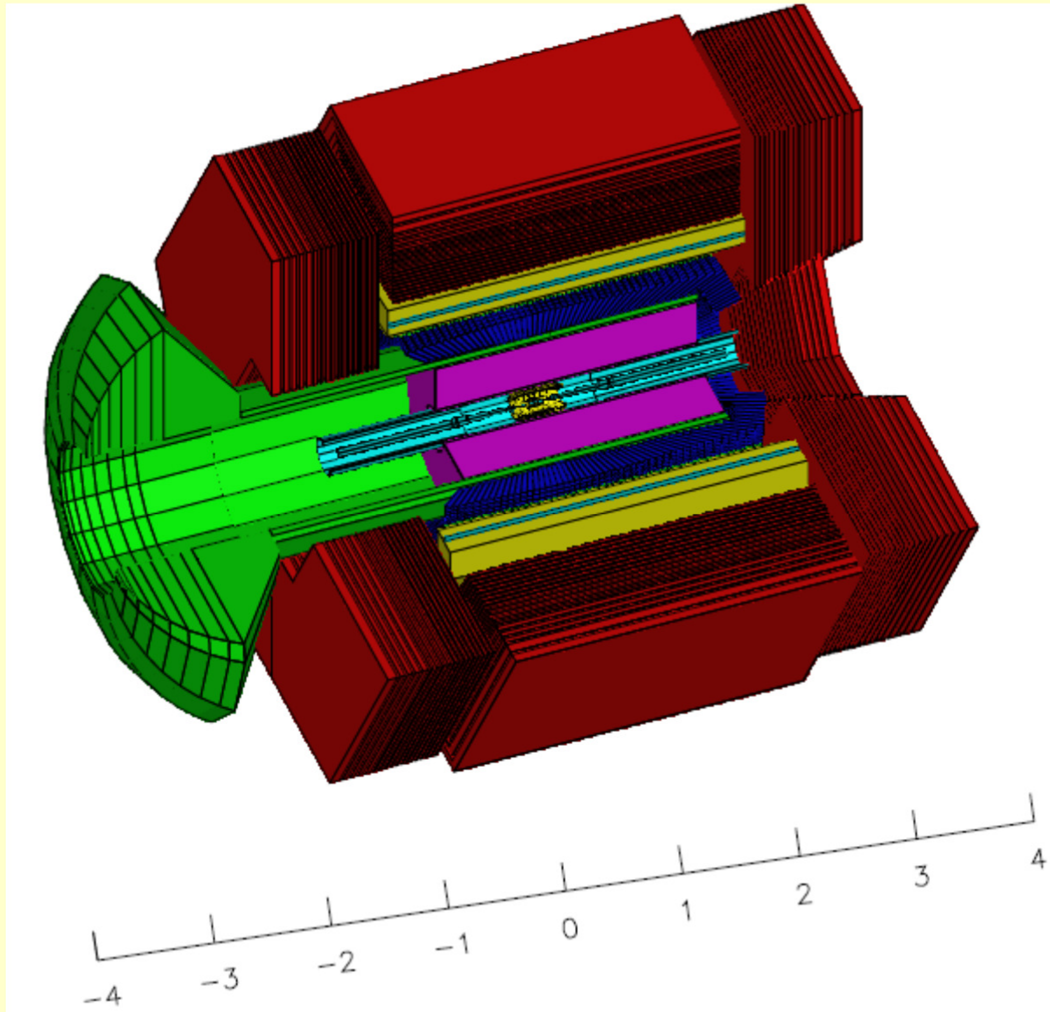


B Factories - IV

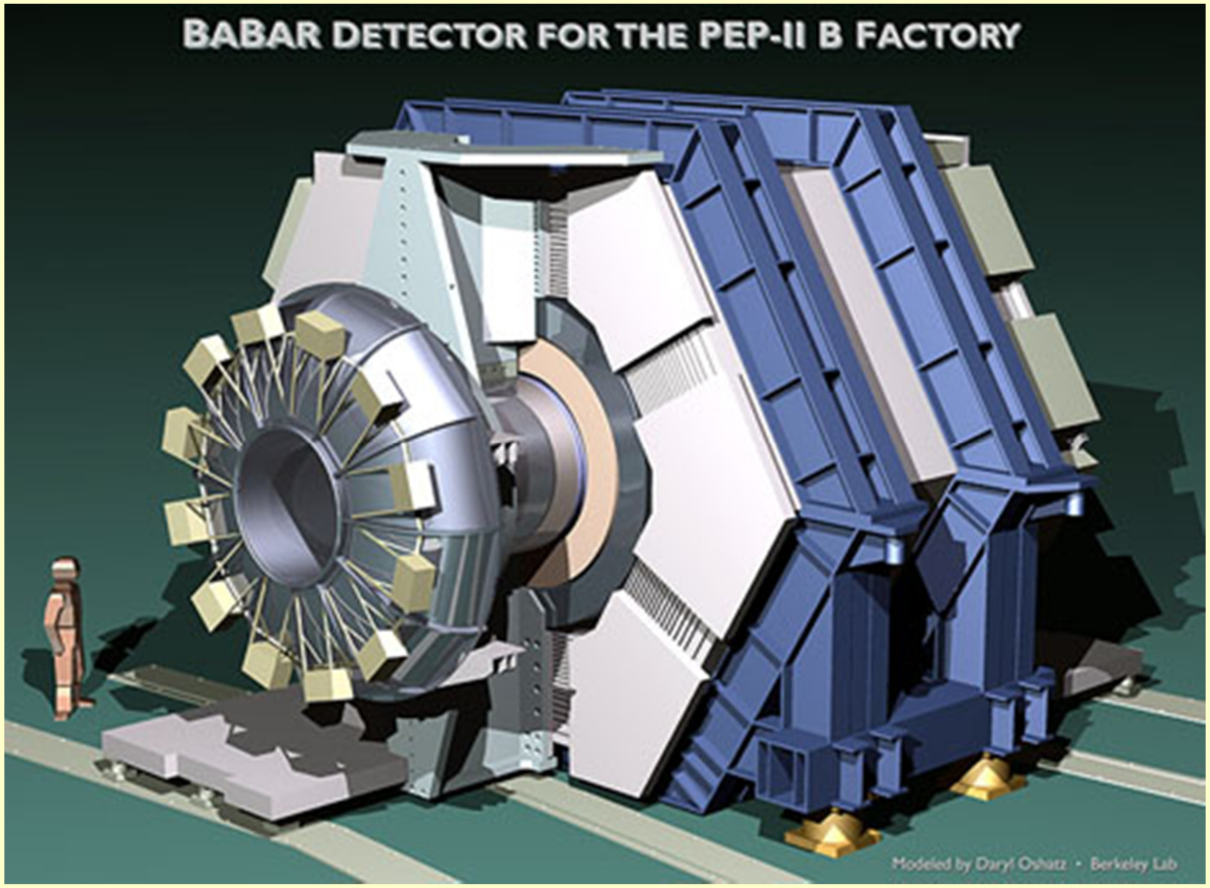


B Factories - V

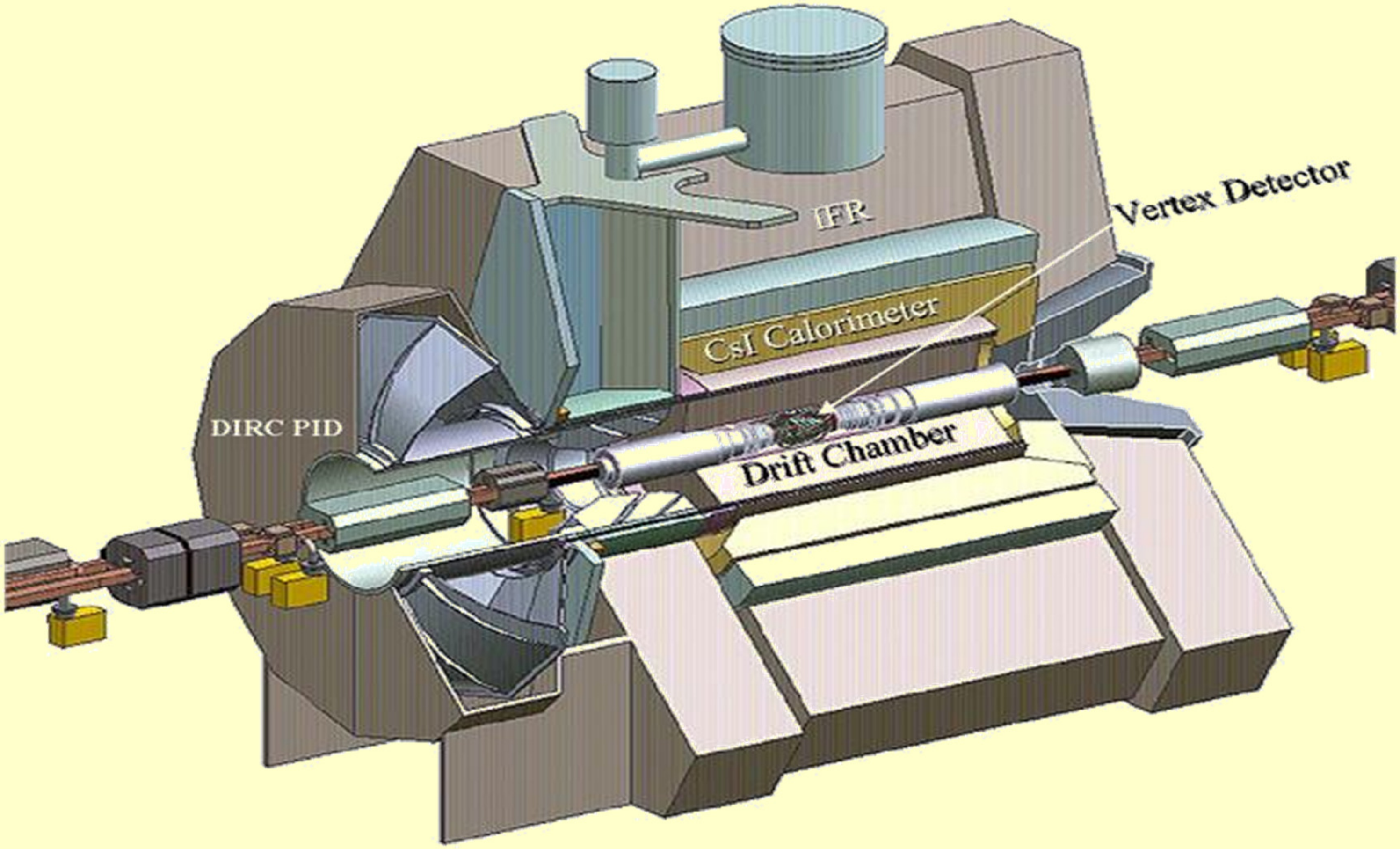
BABAR



B Factories - VI

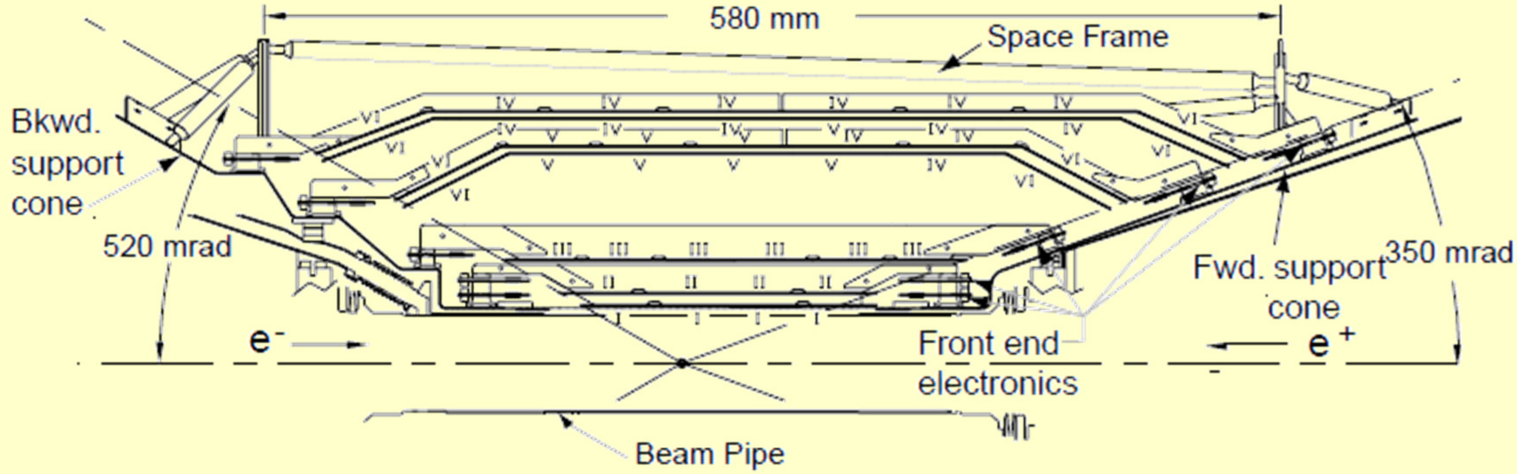


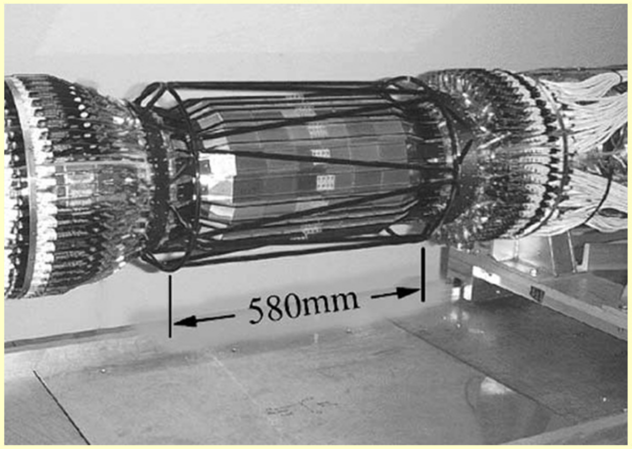
B Factories - VII



B Factories - VIII

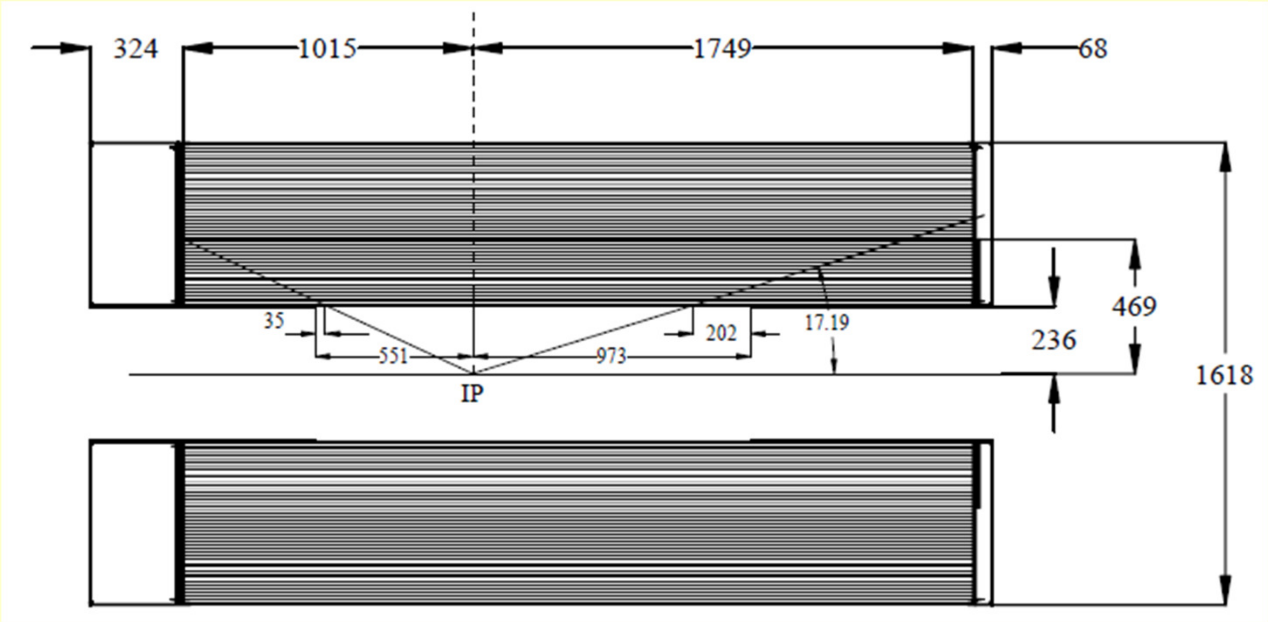
Vertex detector

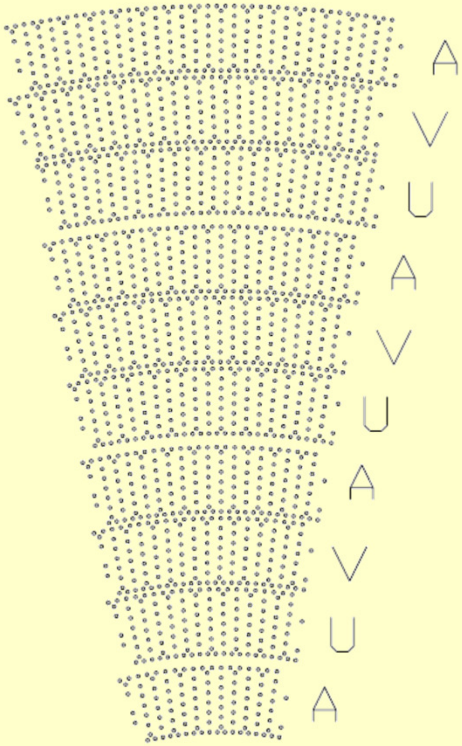




B Factories - IX

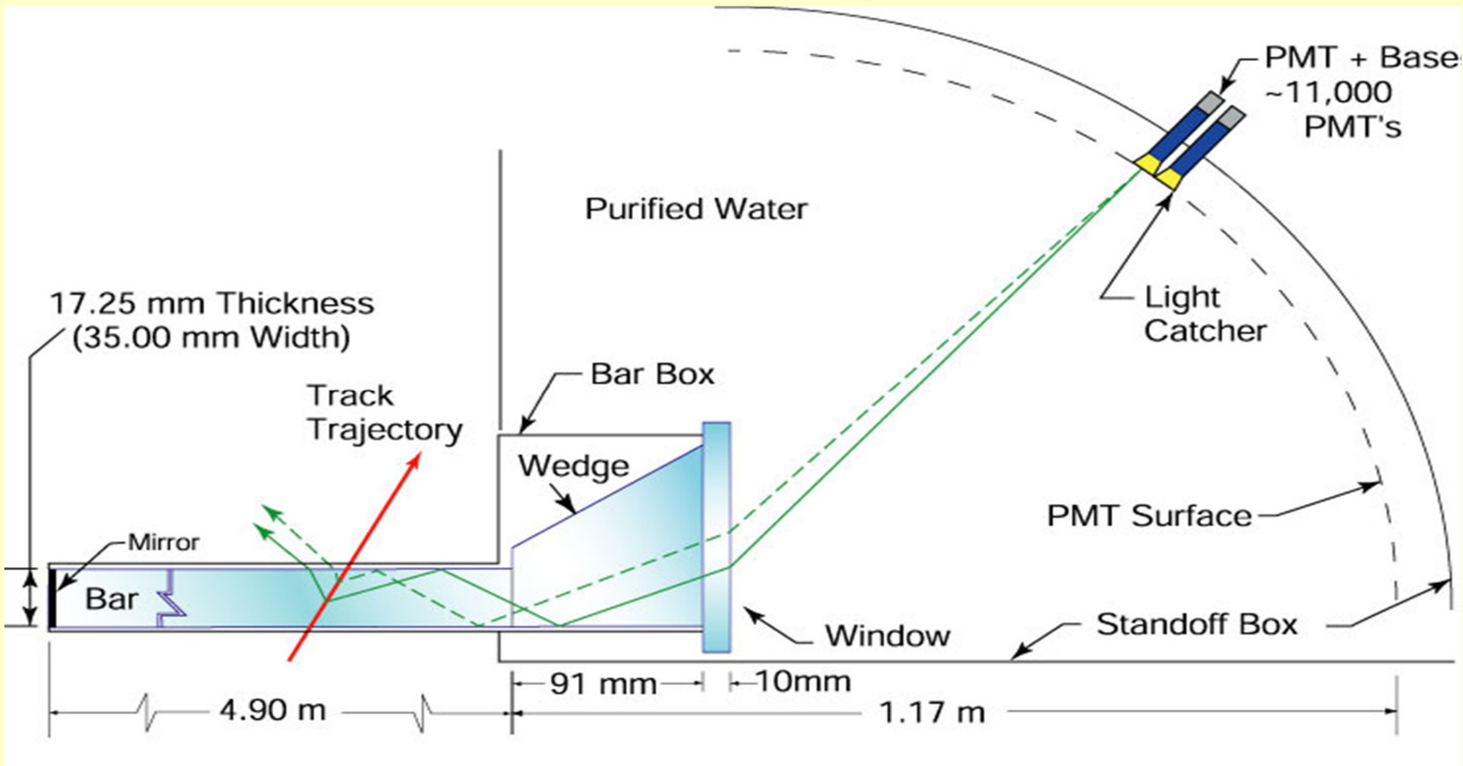
Drift chamber





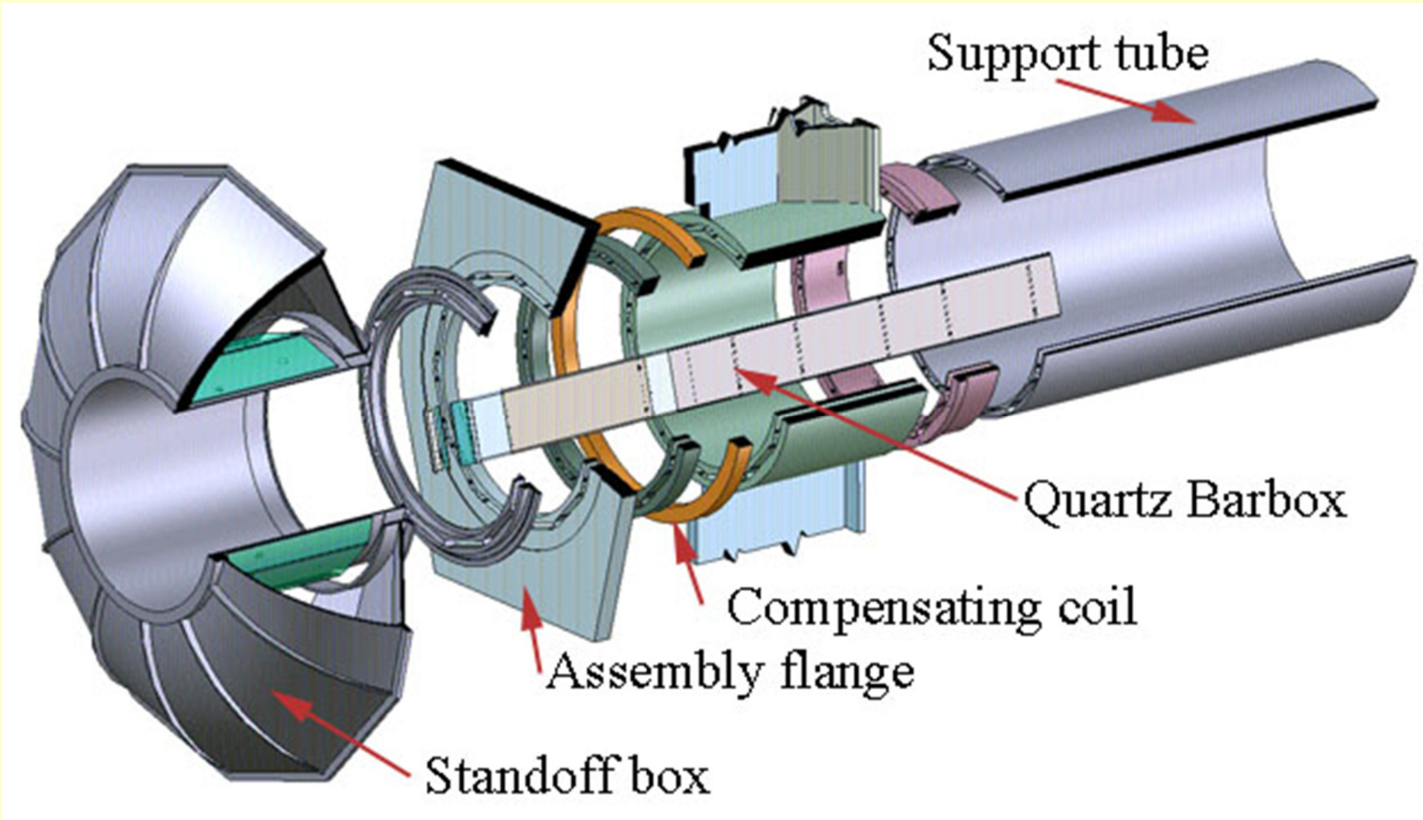
B Factories - X

DIRC : Particle Id



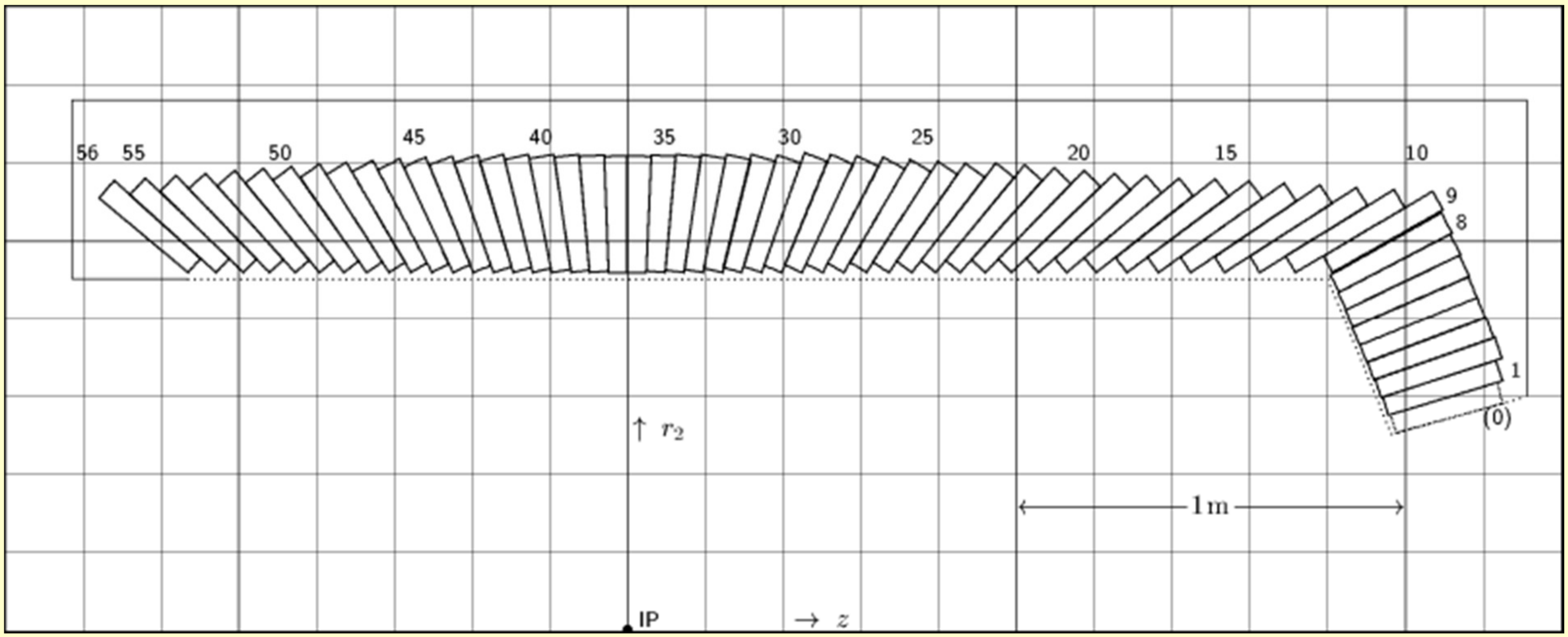
B Factories - XI

DIRC



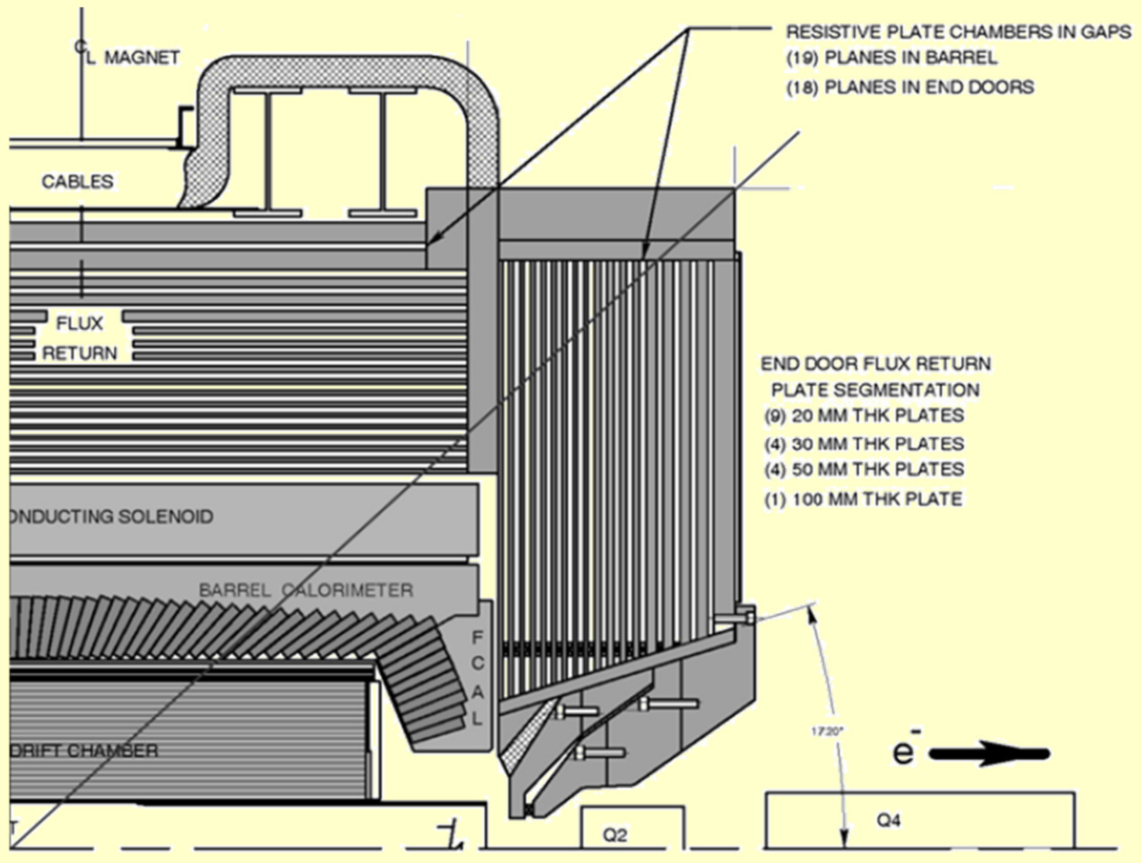
B Factories - XII

EM Calorimeter



B Factories - XIII

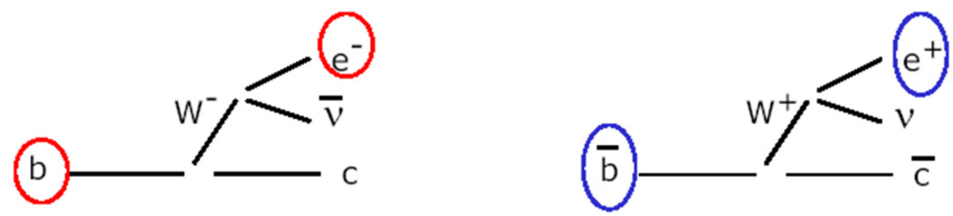
Instrumented Flux Return: Muon detector & (coarse) hadron calorimeter



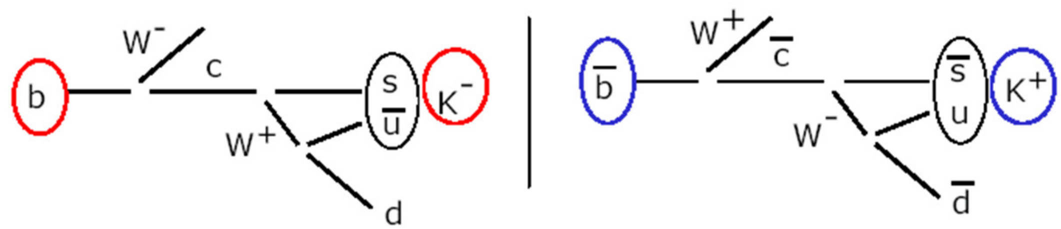
B Factories - XIV

Tagging: finding the flavor of the 2nd B-meson

Leptons : cleanest tag (correct=97%, efficiency=8.6%)

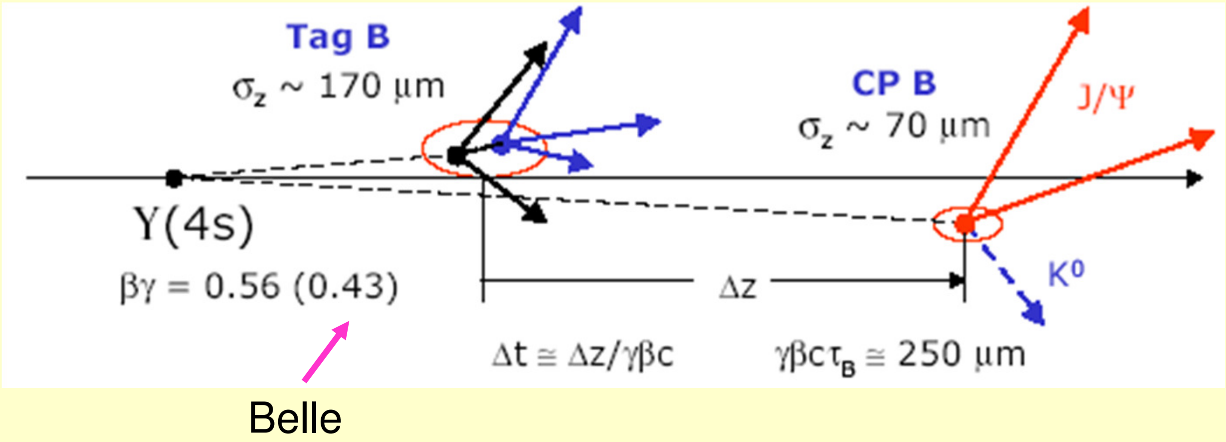


Kaons : 2nd cleanest tag (correct 85%-95%, efficiency=28%)

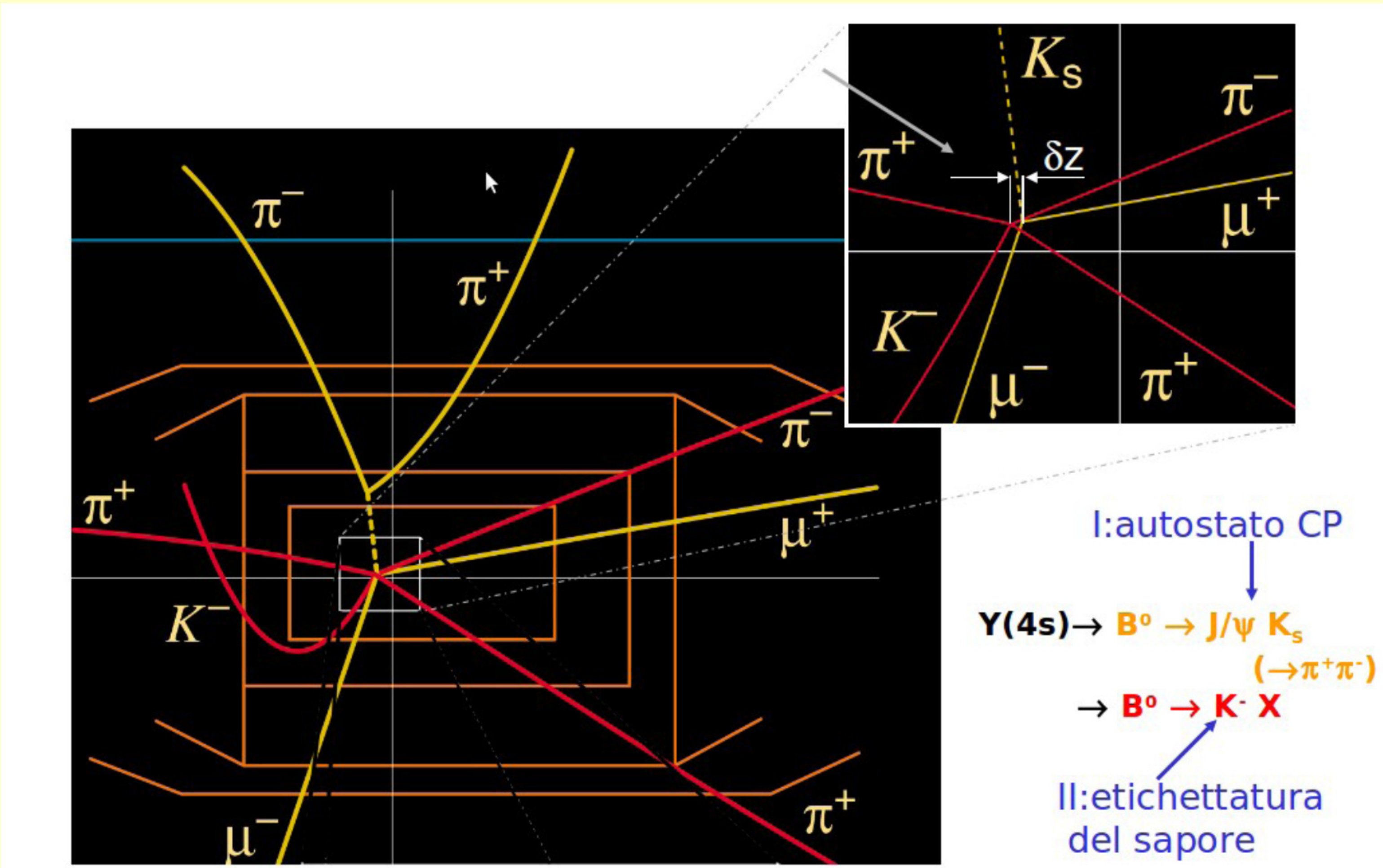


w = mistag probability = 1 - correct "dilution": D = 1-2w

B Factories - XV

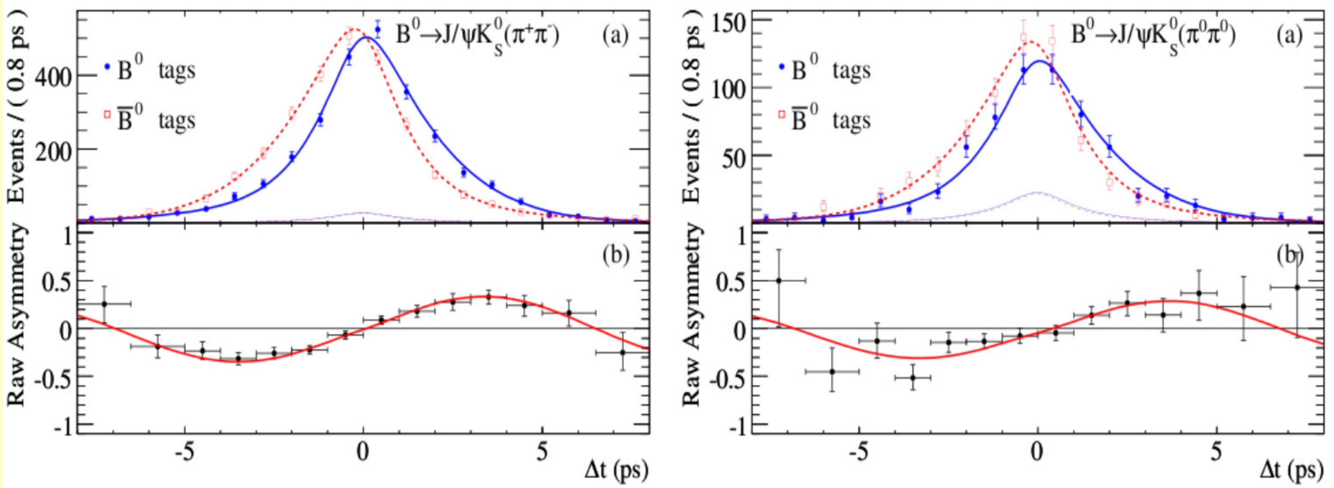


B Factories - XVI



B Factories - XVII

BABAR



Sample		$-\eta_f S_f$	C_f
Full *CP* sample	→	0.687 ± 0.028	0.024 ± 0.020
$J/\psi\, K_S^0(\pi^+\pi^-)$	→	0.662 ± 0.039	0.017 ± 0.028
$J/\psi\, K_S^0(\pi^0\pi^0)$	→	0.625 ± 0.091	0.091 ± 0.063