Elementary Particles II

4 – Quarkonium

Heavy Quarks, Quarkonium, Models,
Experiments, Renaissance

Quarks

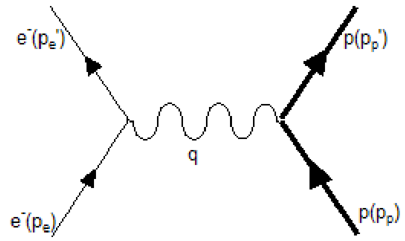
Flavor	*Mass*	*Q*	*I*	I_3	*S*	*C*	*B*	*T*
Up	5.6 MeV	2/3	½	+1/2	0	0	0	0
Down	9.9 MeV	-1/3	½	-1/2	0	0	0	0
strange	199 MeV	-1/3	0	0	-1	0	0	0
charm	1350 MeV	2/3	0	0	0	1	0	0
bottom	4400 MeV	-1/3	0	0	0	0	-1	0
Top	174 GeV	2/3	0	0	0	0	0	1

Physics case for flavourless, scalar/vector bound states:
Most interesting for heavy c,b quarks, where asymptotic freedom allows for semi-perturbative calculations; large mass hints for non-relativistic motion
Less interesting case for s quark, too close to confinement region

Hydrogen Atom - I

Start first from electron-proton interaction:

$$T_{fi} = e^2 \frac{\left(\overline{u}\left(p_e^{'}\right)\gamma^{\mu}u\left(p_e\right)\right)\left(\overline{u}\left(p_p{'}\right)\gamma_{\mu}u\left(p\right)\right)}{q^2}$$



Expand matrix element to low speed approximation

Get a non-relativistic matrix element, where χ, χ' are 2-dimensional (Pauli) spinors for electron and proton

The Bottom Line:

At low speed/energy we can neglect radiation, pair production (real & virtual)

$\rightarrow$Left with corrections:

Relativistic Energy/Momentum
Magnetic Moments
More

Hydrogen Atom - II

Transition matrix element:

$$T_{fi} \simeq -\frac{e^2}{q^2}\left[1-\frac{\mathbf{p}_e^2+\mathbf{p}_e^{'2}}{8m_e^2}\right]\left[1-\frac{\mathbf{p}_p^2+\mathbf{p}_p^{'2}}{8m_p^2}\right]\cdot$$

$$\left\{\underbrace{\tilde{\chi}'^{\dagger}\left[1+\frac{\mathbf{p}_p^{'}\cdot\mathbf{p}_p+i\boldsymbol{\sigma}\cdot\left(\mathbf{p}_p^{'}\times\mathbf{p}_p\right)}{4m_p^2}\right]\tilde{\chi}}_{\text{time section, p 4-current}}\underbrace{\chi'^{\dagger}\left[1+\frac{\mathbf{p}_e^{'}\cdot\mathbf{p}_e+i\boldsymbol{\sigma}\cdot\left(\mathbf{p}_e^{'}\times\mathbf{p}_e\right)}{4m_e^2}\right]\chi}_{\text{time section, e 4-current}}+\right.$$

$$\left.-\underbrace{\tilde{\chi}'^{\dagger}\left[\frac{\mathbf{p}_p^{'}+\mathbf{p}_p-i\boldsymbol{\sigma}\times\left(\mathbf{p}_p^{'}-\mathbf{p}_p\right)}{2m_p}\right]\tilde{\chi}}_{\text{space section. p 4-current}}\cdot\underbrace{\chi'^{\dagger}\left[\frac{\mathbf{p}_e^{'}+\mathbf{p}_e-i\boldsymbol{\sigma}\times\left(\mathbf{p}_e^{'}-\mathbf{p}_e\right)}{2m_e}\right]\chi}_{\text{space section, e 4-current}}\right\}$$

Hydrogen Atom - III

Reminder from potential scattering:
Transition amplitude = Fourier transform of the effective potential
$\rightarrow$Get effective *e-p* potential by anti-transforming the amplitude
Useful to calculate energy levels, atomic properties
Several terms:

$$V_C = -\frac{e^2}{r}\left(\tilde{\chi}'^{\dagger}\tilde{\chi}\right)\left(\chi'^{\dagger}\chi\right) \quad \text{Coulomb term}$$

$$\left.\begin{aligned} V_{SO} &= \frac{e^2}{4m_e^2 r^3}\left(\tilde{\chi}'^{\dagger}\tilde{\chi}\right)\left(\chi'^{\dagger}\boldsymbol{\sigma}\cdot\mathbf{L}\chi\right) \quad \text{Spin-orbit} \\ V_D &= \frac{e^2}{8m_e^2}4\pi\delta^{(3)}(\mathbf{r})\left(\tilde{\chi}'^{\dagger}\tilde{\chi}\right)\left(\chi'^{\dagger}\chi\right) \quad \text{'Darwin term'} \end{aligned}\right\} \text{Fine structure terms}$$

$$V_{dip-dip} = \underbrace{\frac{8\pi e^2}{3m_e m_p}g_p\mathbf{s}_e\cdot\mathbf{s}_p\delta^{(3)}(\mathbf{r})}_{\text{Hyperfine structure}} + \underbrace{\frac{e^2}{m_e m_p r^3}g_p\left[3\left(\mathbf{s}_e\cdot\mathbf{r}\right)\left(\mathbf{s}_p\cdot\mathbf{r}\right)-\mathbf{s}_e\cdot\mathbf{s}_p\right]}_{\text{Tensor interaction}} \quad \text{Dipole-dipole interaction}$$

Valid for *S* states

Astonishing: Everything included in our modest 1-photon diagram…

Hydrogen Atom - IV

Effect of hyperfine interaction on ground state energy:

$$\left\langle \frac{8\pi e^2}{3m_e m_p} g_p \mathbf{s}_e \cdot \mathbf{s}_p \delta^{(3)}(\mathbf{r}) \right\rangle_{\text{ground state}}$$

$$\rightarrow \Delta E_{hyp} = \frac{8\pi e^2}{3m_e m_p} g_p \cdot \frac{1}{2}\left[j(j+1) - s_e(s_e+1) - s_p(s_p+1) \right] \cdot \left|\psi(0)\right|^2$$

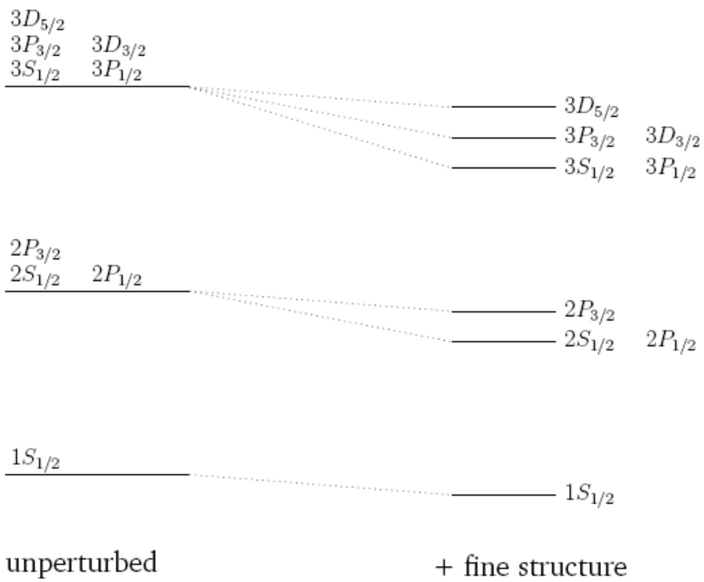
$$\left|\psi(0)\right|^2 = \frac{(m_e\alpha)^3}{\pi} \rightarrow \Delta E_{hyp} = \frac{8\pi e^2}{3m_e m_p} g_p \frac{1}{2} \frac{(m_e\alpha)^3}{\pi}\left[j(j+1) - \frac{3}{4} - \frac{3}{4} \right]$$

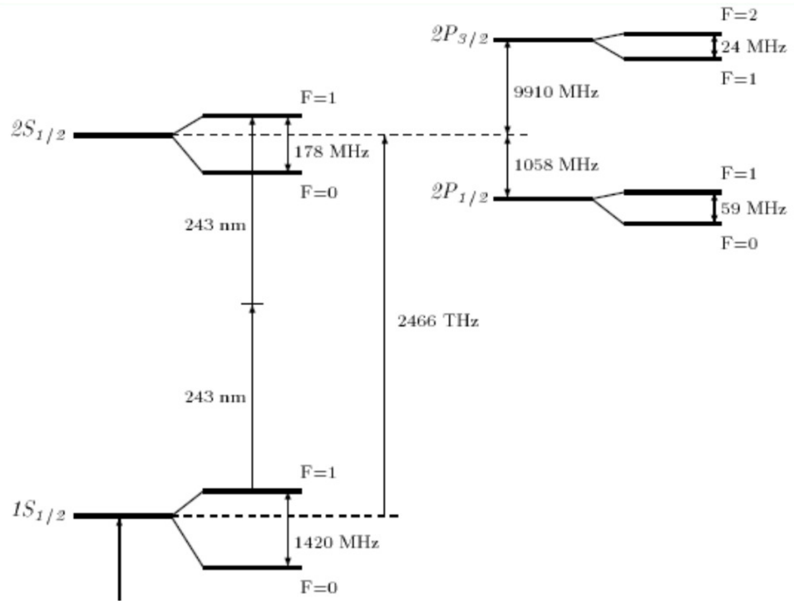
$$\rightarrow \Delta E_{hyp} = \frac{4}{3} g_p \frac{e^2}{m_e m_p}\left[j(j+1) - \frac{3}{2} \right](m_e\alpha)^3$$

$$\Delta E_{hyp} = \begin{cases} \dfrac{4}{3} g_p \dfrac{e^2 m_e}{m_p}(2-3/2)\left(m_e\alpha^3\right) = \dfrac{4}{3} g_p \dfrac{m_e}{m_p}\dfrac{1}{2}\left(m_e\alpha^4\right) & \text{hyperfine shift -triplet} \\ \dfrac{4}{3} g_p \dfrac{2e^2 m_e}{m_p}(0-3/2)\left(m_e\alpha^3\right) = -\dfrac{12}{3} g_p \dfrac{1}{2}\dfrac{m_e}{m_p}\left(m_e\alpha^4\right) & \text{hyperfine shift - singlet} \end{cases}$$

$$\rightarrow \Delta\left(\Delta E_{hyp}\right)_{\text{triplet-singlet}} = \frac{8}{3} g_p \frac{m_e}{m_p}\left(m_e\alpha^4\right)$$

Hydrogen Atom - V





$$\Delta E_{l,1/2;j,m_j} = E_n \frac{\alpha^2}{n^2}\left(\frac{n}{j+1/2} - \frac{3}{4}\right).$$

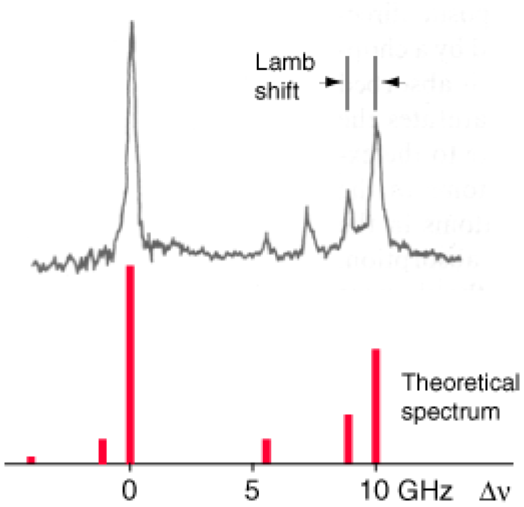
$$\Delta E = \frac{\mu_0 \, g_p \, e^2}{8\pi \, m_p \, m_e}\left\langle \frac{3\,(\mathbf{S}_p\cdot\hat{\mathbf{r}})\,(\mathbf{S}_e\cdot\hat{\mathbf{r}}) - \mathbf{S}_p\cdot\mathbf{S}_e}{r^3}\right\rangle + \frac{\mu_0 \, g_p \, e^2}{3\, m_p \, m_e}\langle \mathbf{S}_p\cdot\mathbf{S}_e\rangle\,|\psi(0)|^2.$$

Fine structure:
Spin-Orbit+Relativistic+Darwin
Splits *j* sublevels

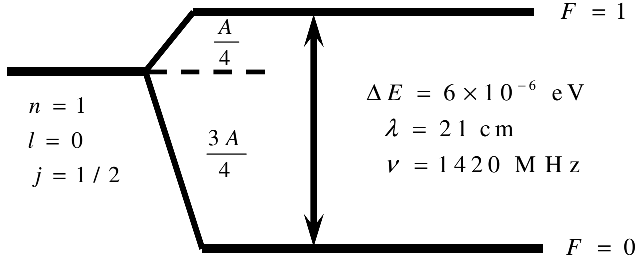
Hyperfine structure:
Dipole-Dipole
Splits *F* sublevels

Hyperfine Splitting of Hydrogen

Ground state

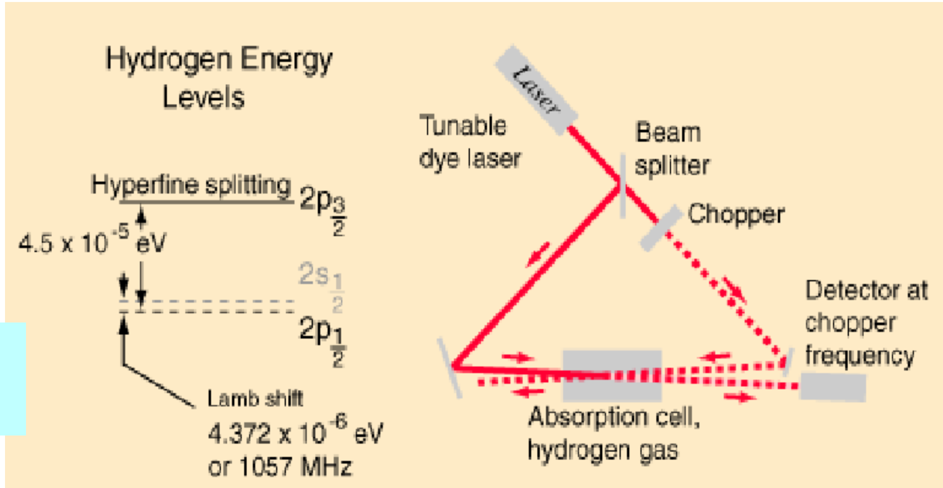


Hydrogen fine structure and hyperfine structure for the n=3 ► 2 transition.
(After Ohanian, Modern Physics, Ch 7., spectrum from T. W. Hansch, Stanford Univ.)



Ground state, ***L=0***

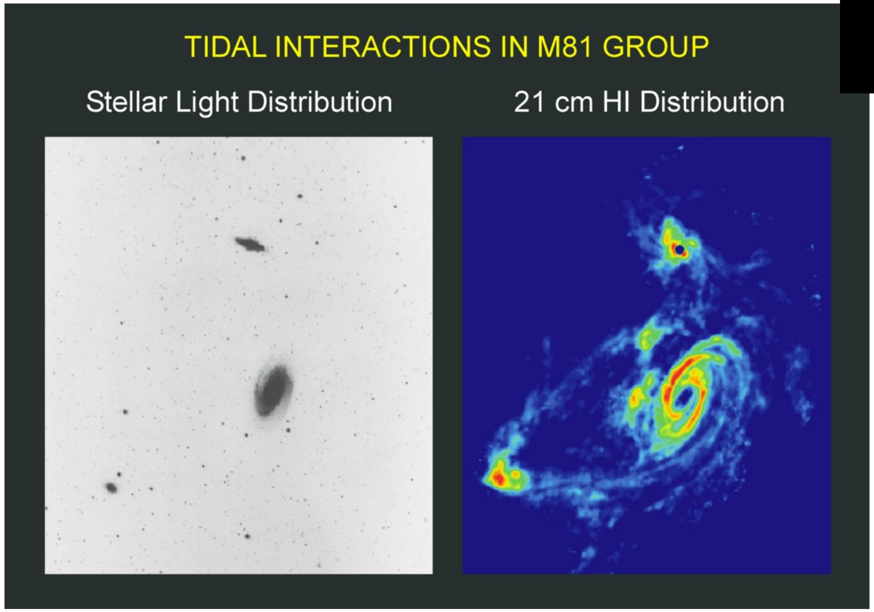
@TBA



The 21 cm *H* Line: A Cosmic Tune

Important radio-astronomical tool: *Mapping the Hydrogen content of the Universe*

@TBA

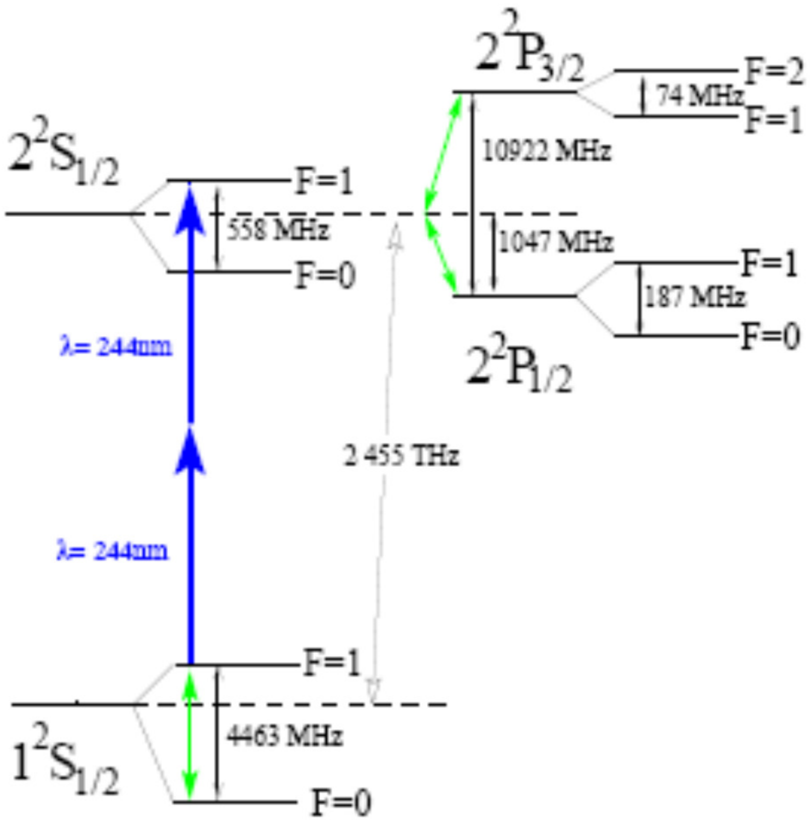


Lots of physics and cosmology..
Example:
How is the transition excited?

A measurement of the galactic/ intergalactic temperature

More Hydrogens: Muonium

μ^+-e^- 'atom'

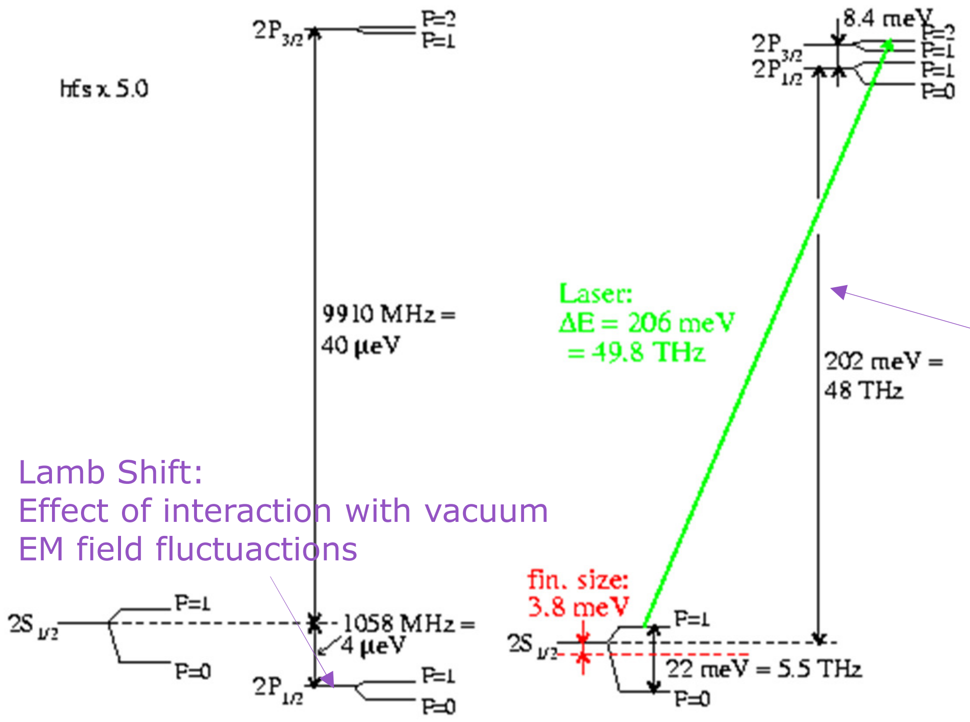


Similar to Hydrogen

Different:
Reduced mass
Muon magnetic moment

@TBA

And More: Muonic Hydrogen



Standard *H*

Muonic *H*
Replace e^- by μ^-

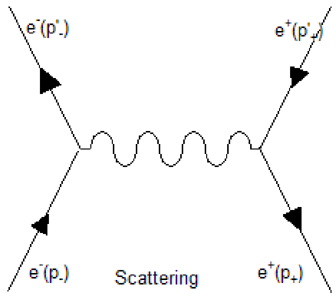
Simplest μ-mesic atom
Made by stopping μ^- in hydrogenated matter

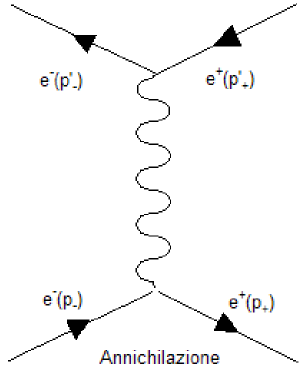
Huge Lamb shift:
~ 45000 x Hydrogen!
~ $(m_m/m_e)^2$

Positronium - I

Take e^+ - e^- interaction

There are now 2 diagrams:





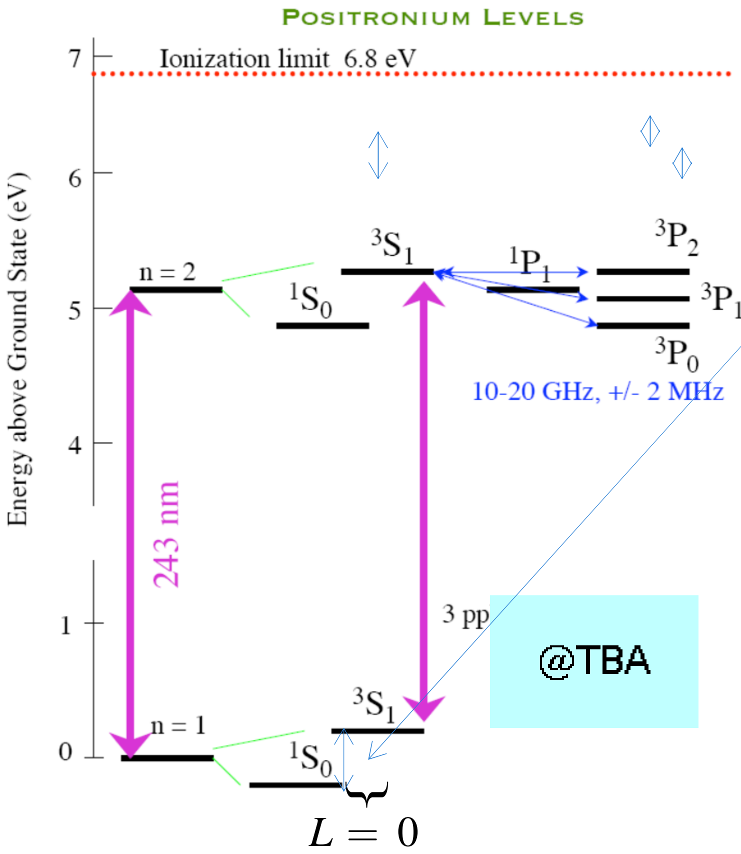
$$T_{fi} = \alpha \left[-\frac{\left(\bar{u}\left(p_-'\right)\gamma^\mu u\left(p_-\right)\right)\left(\bar{v}\left(p_+\right)\gamma_\mu v\left(p_+'\right)\right)}{\left(p_- - p_-'\right)^2} + \frac{\left(\bar{v}\left(p_+\right)\gamma^\mu u\left(p_-\right)\right)\left(\bar{u}\left(p_-'\right)\gamma_\mu v\left(p_+'\right)\right)}{\left(p_+ + p_-\right)^2} \right]$$

Net effect: Add another term to the effective interaction

$$V_A = \frac{e^2\pi}{2m^2}\left(3 + \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2\right)\delta^{(3)}(\mathbf{r})$$ Same structure as hyperfine term

Positronium - II

(Unstable) Electron-Positron bound state: Positronium
Annihilating into 2,3 γ-rays



Hyperfine splitting:

$$\Delta E_{hyp} = \left\langle \frac{8\pi e^2}{3m_e^2} \mathbf{s}_{e^-} \cdot \mathbf{s}_{e^+} \delta^{(3)}(\mathbf{r}) \right\rangle$$

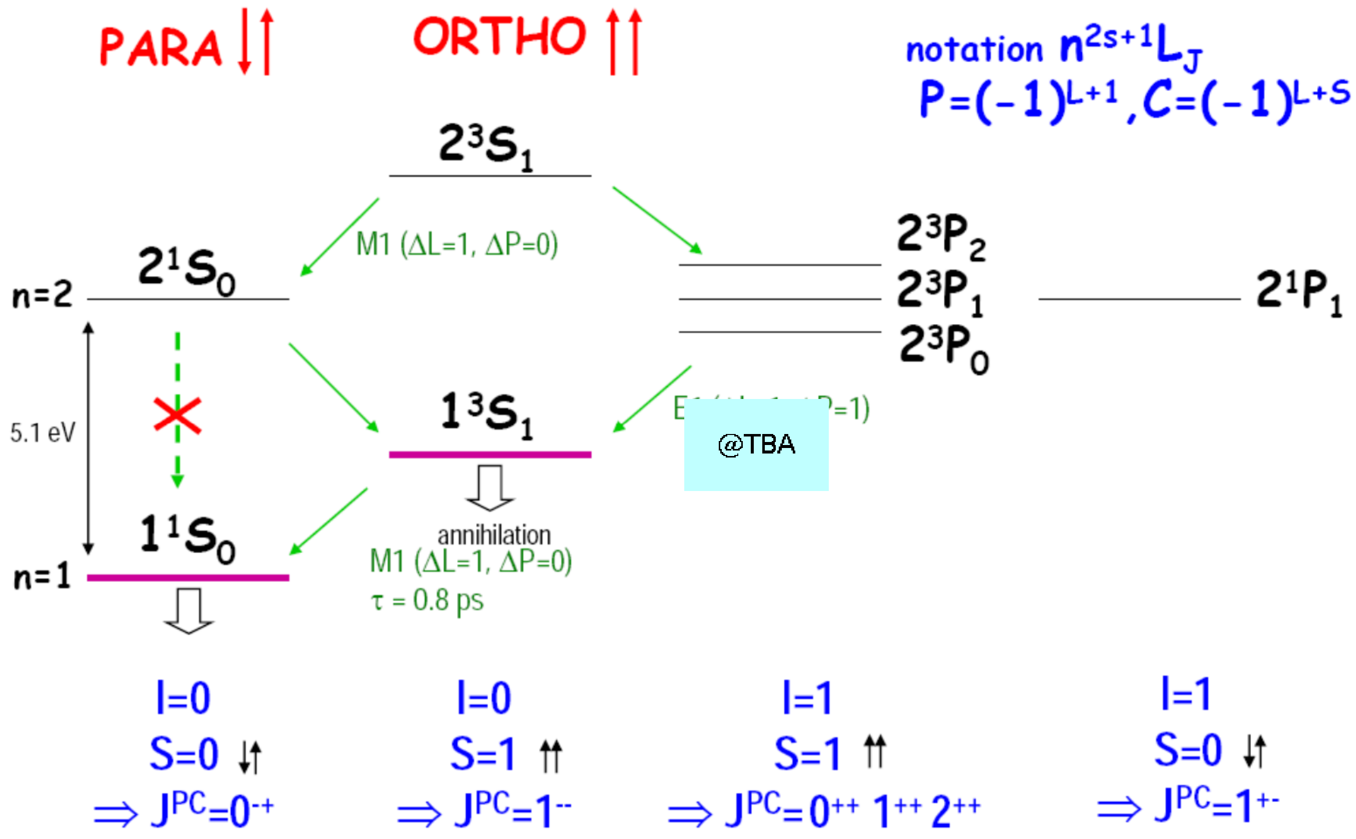
Ground state

More complicated for *n>1, l>0*

Observe:
Levels labeled by $^S L_J$
S: *Total* spin

Previous pictures:
Levels labeled by $^S L_J$
S: *Electron* spin
Proton spin only in hyperfine term

Positronium - III



Breit-Fermi Hamiltonian - I

Consider the effective interaction previously introduced:

Add kinetic term, including relativistic corrections

$$H = \frac{1}{2m}\left(\mathbf{p}_1^2 + \mathbf{p}_2^2\right) - \frac{1}{8m^3}\left(\mathbf{p}_1^4 + \mathbf{p}_2^4\right) + U\left(\mathbf{p}_1, \mathbf{p}_2, \mathbf{r}\right)$$

$$U\left(\mathbf{p}_1, \mathbf{p}_2, \mathbf{r}\right) = \frac{q^2}{r}$$

$$-\frac{q^2}{(2m)^2} 4\pi\delta^{(3)}(\mathbf{r}) - 2\frac{q^2}{(2m)^2}\left(\mathbf{p}_1 \cdot \mathbf{p}_2 + \frac{\mathbf{r}\cdot\mathbf{p}_1 + \mathbf{r}\cdot\mathbf{p}_2}{r^2}\right)$$

$$+\frac{q^2}{4m^2 r^3}\left[-\left(\boldsymbol{\sigma}_1 + 2\boldsymbol{\sigma}_2\right)\cdot\left(\mathbf{r}\times\mathbf{p}_1\right) + \left(\boldsymbol{\sigma}_2 + 2\boldsymbol{\sigma}_1\right)\cdot\left(\mathbf{r}\times\mathbf{p}_2\right)\right]$$

$$+\frac{q^2}{(2m)^2}\left[\frac{\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2}{r^3} - 3\frac{\left(\boldsymbol{\sigma}_1\cdot\mathbf{r}\right)\left(\boldsymbol{\sigma}_2\cdot\mathbf{r}\right)}{r^5} - \frac{8\pi}{3}\left(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\right)\delta^{(3)}(\mathbf{r})\right]$$

$$-2\pi\frac{q^2}{(2m)^2}\left(3 + \boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\right)\delta^{(3)}(\mathbf{r})$$

Breit-Fermi Hamiltonian - II

Rearrange to:

$$H = \frac{\mathbf{p}^2}{m} - \frac{q^2}{r} + V_1 + V_2 + V_3$$

$$V_1 = -\frac{\mathbf{p}^4}{4m^3} + 4\pi\left(\frac{q}{2m}\right)^2 \delta^{(3)}(\mathbf{r}) - 2\left(\frac{q}{2m}\right)^2 \frac{1}{r}\left(\mathbf{p}^2 + \frac{(\mathbf{r}\cdot\mathbf{p})^2}{r^2}\right)$$

$$V_2 = 6\left(\frac{q}{2m}\right)^2 \frac{1}{r^3}\mathbf{L}\cdot\mathbf{S}$$

$$V_3 = 6\left(\frac{q}{2m}\right)^2 \frac{1}{r^3}\left[\frac{(\mathbf{S}\cdot\mathbf{r})^2}{r^2} - \frac{1}{3}\mathbf{S}^2\right] + 4\pi\left(\frac{q}{2m}\right)^2\left(\frac{7}{3}\mathbf{S}^2 - 2\right)\delta^{(3)}(\mathbf{r})$$

Breit-Fermi Hamiltonian - III

Level splitting due to spin:

$$\Delta E = \frac{\alpha}{2m^2 r^3}\left(\mathbf{L}\cdot\mathbf{s}_1 + \mathbf{L}\cdot\mathbf{s}_2 + 2\mathbf{L}\cdot\mathbf{S}\right)$$

$$+\frac{8\pi}{3}\frac{\alpha}{m^2}\mathbf{s}_1\cdot\mathbf{s}_2\delta^{(3)}\left(\mathbf{r}\right)$$

$$+\frac{\alpha}{m^2 r^3}\left[3\left(\mathbf{s}_1\cdot\hat{\mathbf{r}}\right)\left(\mathbf{s}_2\cdot\hat{\mathbf{r}}\right)-\mathbf{s}_1\cdot\mathbf{s}_2\right]$$

Breit-Fermi Hamiltonian - IV

Extend to quark-antiquark, as coming from one gluon exchange diagram:

$$\Delta E = \frac{4}{3}\frac{\alpha_S}{2m^2 r^3}\left(\mathbf{L}\cdot\mathbf{s}_1 + \mathbf{L}\cdot\mathbf{s}_2 + 2\mathbf{L}\cdot\mathbf{S}\right)$$

$$+\frac{32\pi}{9}\frac{\alpha_S}{m^2}\mathbf{s}_1\cdot\mathbf{s}_2\delta^{(3)}(\mathbf{r})$$

$$+\frac{4}{3}\frac{\alpha_S}{m^2 r^3}\left[3(\mathbf{s}_1\cdot\hat{\mathbf{r}})(\mathbf{s}_2\cdot\hat{\mathbf{r}}) - \mathbf{s}_1\cdot\mathbf{s}_2\right]$$

$$-\frac{k}{2m^2 r}\left(\mathbf{L}\cdot\mathbf{s}_1 + \mathbf{L}\cdot\mathbf{s}_2\right)$$

Last term coming from linear (confining) potential contributing to orbital motion

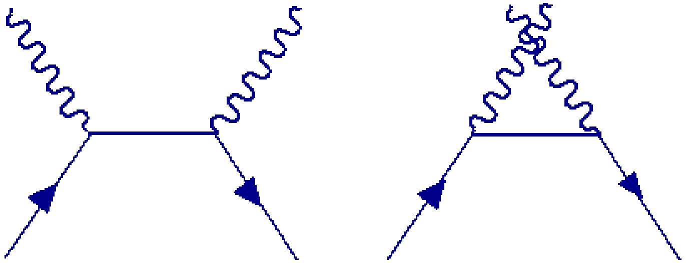
Breit-Fermi Hamiltonian - V

State	M_0	ΔM_S	ΔM_l	M_{th}	M_{exp}
$1\,^3P_2$	3521	+45 a)	−13	3553	3551
$1\,^3P_1$	3521	−32	+13	3502	3507
$1\,^3P_0$	3521	−128	+26	3419	3414
$1\,^1P_1$	3521	0	0	3521	–

Sample prediction

Annihilation - I

Annihilation into two photons:
Transition amplitude in the small speed limit ($\beta \to 0$):
2 diagrams, similar to (rotated) Compton scattering



Permutations of 2 photons
$\to$2 diagrams altogether

$$T_{fi} = T_1 + T_2$$

$$T_1 = \frac{e^2}{(p_1 - p_3)^2 - m^2} \bar{v}(2) \not{\varepsilon}_4 (\not{p}_1 - \not{p}_3 + m) \not{\varepsilon}_3 u(1), \quad T_2 = \frac{e^2}{(p_1 - p_4)^2 - m^2} \bar{v}(2) \not{\varepsilon}_3 (\not{p}_1 - \not{p}_4 + m) \not{\varepsilon}_4 u(1)$$

$p_1 = m(1,0,0,0),\ p_2 = m(1,0,0,0),\ p_3 = m(1,0,0,1),\ p_4 = m(1,0,0,-1)$ γ rays emitted along z

$$(p_1 - p_3)^2 - m^2 = (p_1 - p_4)^2 - m^2 = -2m^2$$

$\to T = -4e^2$ Averaged over initial, summed over final spin projections

Annihilation - II

Cross-section from amplitude: 2-body reaction

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi s}\frac{\left|\mathbf{p}_f\right|}{\left|\mathbf{p}_i\right|}\left|T\right|^2$$

$$\left|\mathbf{p}_f\right| = m, \quad \left|\mathbf{p}_i\right| \simeq m\beta, \quad s = \left(2m\right)^2 = 4m^2$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi 4m^2}\frac{m}{m\beta}16\alpha^2 = \frac{\alpha^2}{16\pi m^2\beta}$$

$$\rightarrow \sigma = 4\pi\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4m^2\beta}$$

Fast increasing at low speed

Annihilation - III

Selection rule for bound state annihilation into 2,3 photons

$$U_C\left|2\gamma\right\rangle=\left(-1\right)^2\left|2\gamma\right\rangle\to\eta_c\left(2\gamma\right)=+1$$

$$\to\left(-1\right)^{L+S}=+1$$

$$\to L=0\Rightarrow S=0$$

S-wave: Singlet only

$$U_C\left|3\gamma\right\rangle=\left(-1\right)^3=-1$$

$$\to\left(-1\right)^{L+S}=-1$$

$$\to L=0\Rightarrow S=1$$

S-wave: Triplet only

Annihilation - IV

2 γ Annihilation : Initial state not a plane wave → Expand into plane waves

$$A_{pos} = \sum_p \underbrace{\langle \gamma\gamma | T | p \rangle}_{A(\mathbf{p})} \underbrace{\langle p | \Pi \rangle}_{\psi(\mathbf{p})}$$

$A(\mathbf{p})$ plane wave amplitude, $\psi(\mathbf{p})$ momentum space wave function

$$\to A_{pos} = \int d^3\mathbf{p} A(\mathbf{p}) \psi(\mathbf{p}) = \frac{1}{(2\pi)^{3/2}} \int d^3\mathbf{p} A(\mathbf{p}) \int \psi(\mathbf{r}) e^{i\mathbf{p}\cdot\mathbf{r}} d^3\mathbf{r}$$

$$\to A_{pos} = \frac{1}{(2\pi)^{3/2}} \int d^3\mathbf{r} \psi(\mathbf{r}) \int A(\mathbf{p}) e^{i\mathbf{p}\cdot\mathbf{r}} d^3\mathbf{r}$$

Take $A(\mathbf{p}) \approx A = const$ (can be shown to be true)

$$\to A_{pos} \approx \frac{A}{(2\pi)^{3/2}} \int d^3\mathbf{r} \psi(\mathbf{r}) \underbrace{\int e^{i\mathbf{p}\cdot\mathbf{r}} d^3\mathbf{p}}_{(2\pi)^3 \delta^3(\mathbf{r})} = (2\pi)^{3/2} A \psi(0)$$

$$\to \Gamma_{pos} = |A_{pos}|^2 \approx (2\pi)^3 |A|^2 |\psi(0)|^2$$

Annihilation - V

$$\sigma = \frac{\alpha^2}{4m^2\beta} = |A|^2 \frac{(2\pi)^3}{\beta} \rightarrow |A|^2 = \frac{\alpha^2}{4m^2\beta}\frac{\beta}{(2\pi)^3} = \frac{\alpha^2}{(2\pi)^3 4m^2}$$

$$\rightarrow \Gamma_{pos} \approx (2\pi)^3 |A|^2 |\psi(0)|^2 = \frac{\alpha^2}{4m^2}|\psi(0)|^2$$

Computed by using averaged matrix element: 3+1 = 4 spin states
Since only singlet initial state contributes:

$$\Gamma_{pos} \rightarrow \frac{1}{1/4}\cdot\frac{\alpha^2}{4m^2}|\psi(0)|^2 = \frac{\alpha^2}{m^2}|\psi(0)|^2$$

Ground state wave function required:
Use scaled Hydrogen w.f.

Annihilation - VI

Taking scaled Hydrogen wave function

$$\psi(r) = \frac{2}{(a_0)^{3/2}} e^{-\frac{2r}{a_0}}, a_0 = \frac{1}{\alpha m}, m \text{ reduced mass}$$

$$Hyd: \quad m \simeq m_e \to a_0 \approx \frac{1}{\alpha m_e}$$

$$Pos: \ m = \frac{m_e}{2} \to a_0 = \frac{2}{\alpha m_e}$$

$$\to \psi_{pos}(0) = \frac{2}{(a_0)^{3/2}} = \frac{2(\alpha m_e)^{3/2}}{(2)^{3/2}} \to |\psi_{pos}(0)|^2 = \frac{4\alpha^3 m_e^3}{8} = \frac{\alpha^3 m_e^3}{2}$$

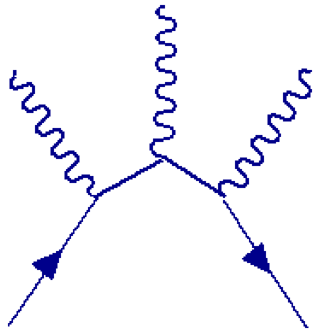
$$\to \Gamma_{pos} \approx (2\pi)^3 |A|^2 |\psi(0)|^2 = \frac{\alpha^2}{m_e^2} \frac{\alpha^3 m_e^3}{2} = \frac{\alpha^5 m_e}{2}$$

Observe: α^5 dependence not obvious from 2 vertex diagram

Initially a bound state, not perturbative

Annihilation - VII

3 γ Annihilation



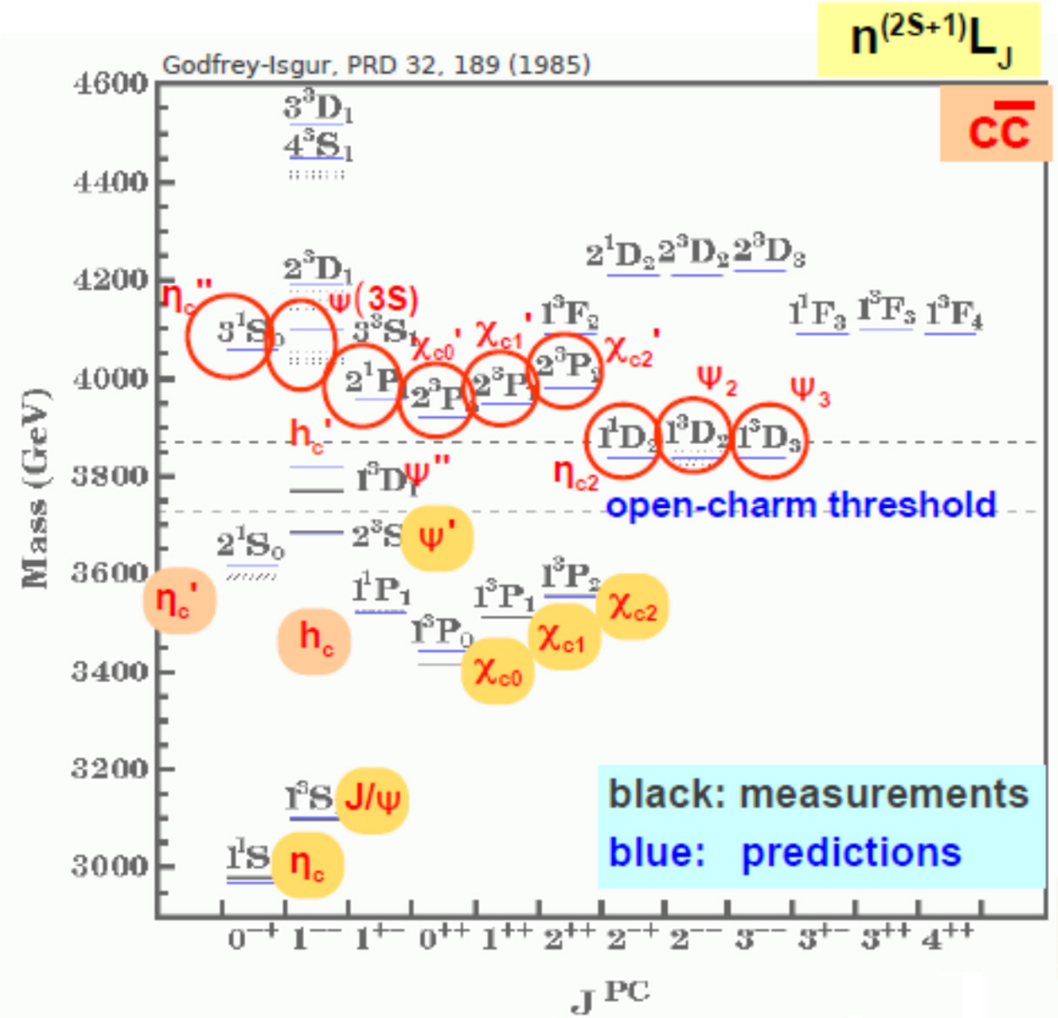
Permutations of 3 photons
→6 diagrams altogether

After *some* algebra…

$$\Gamma_{pos}^{3\gamma} = \frac{4}{9\pi}\left(\pi^2 - 9\right)\frac{\alpha^3}{m_e^2}\left|\psi(0)\right|^2 = \frac{2}{9\pi}\left(\pi^2 - 9\right)m_e\alpha^6$$

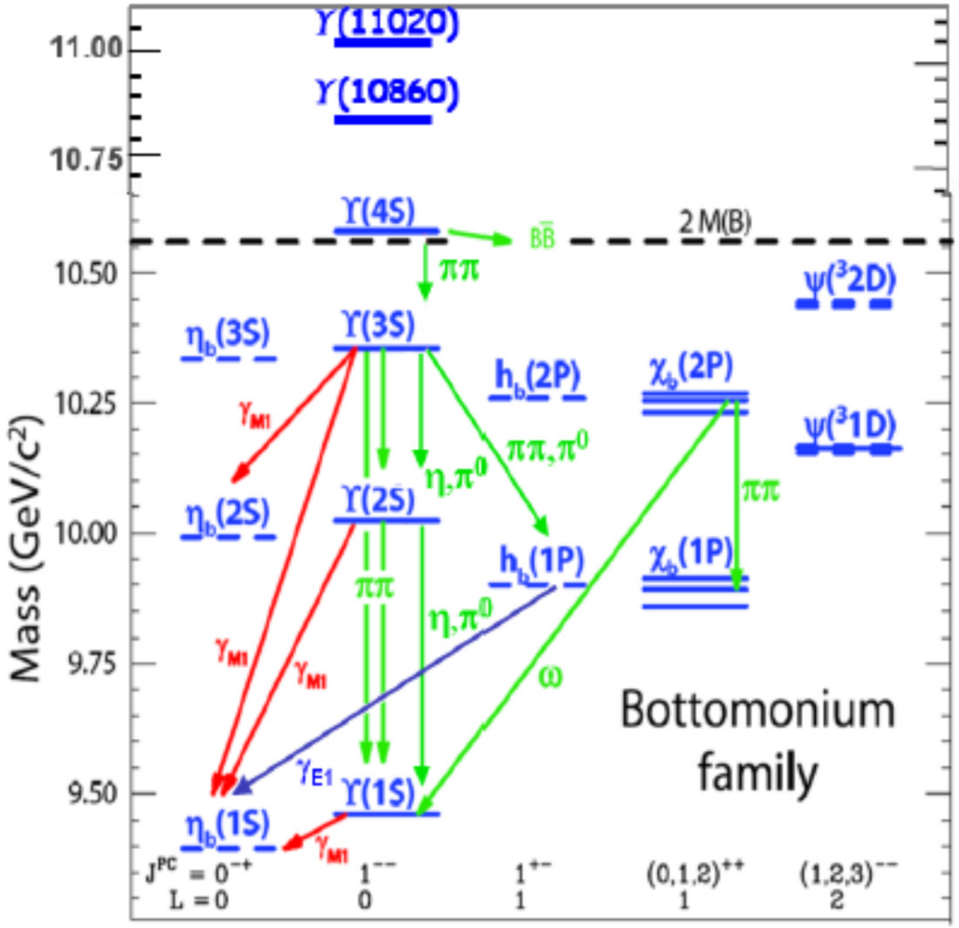
$$\rightarrow \frac{\Gamma_{pos}^{3\gamma}}{\Gamma_{pos}^{2\gamma}} = \frac{\frac{2}{9\pi}\left(\pi^2 - 9\right)m_e\alpha^6}{\frac{\alpha^5 m_e}{2}} = \frac{4\left(\pi^2 - 9\right)}{9\pi}\alpha \sim 10^{-3}$$

Charmonium



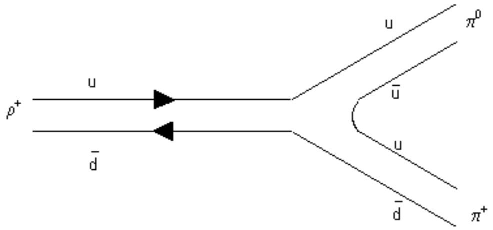
n(2S+1)LJ
cc̄
Godfrey-Isgur, PRD 32, 189 (1985)
4600
4400
4200
4000
3800
3600
3400
3200
3000
Mass (GeV)
0−+ 1−− 1+− 0++ 1++ 2++ 2−+ 2−− 3−− 3+− 3++ 4++
J PC
3³D1
4³S1
2³D1
3¹S0
3³S1
2¹P1
2³P0
2³P1
2³P2
1³F2
2¹D2 2³D2 2³D3
1¹F3 1³F3 1³F4
1¹D2 1³D2 1³D3
1³D1
2¹S0
2³S1
1¹P1
1³P0
1³P1
1³P2
1³S1
1¹S0
ηc''
ψ(3S)
χc0'
χc1'
χc2'
ψ2
ψ3
hc'
ψ''
ηc2
open-charm threshold
ψ'
ηc'
hc
χc0
χc1
χc2
J/ψ
ηc
black: measurements
blue: predictions

Bottomonium

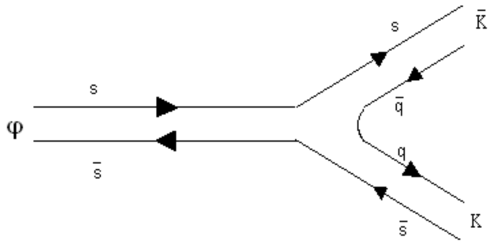


OZI Rule - I

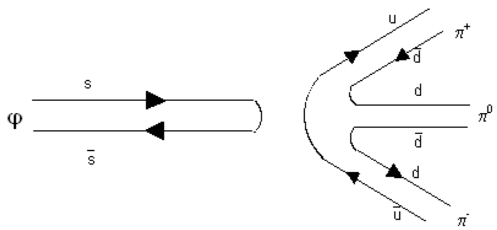
Okubo-Zweig-Iizuka Rule: *Disconnected diagrams are suppressed*



This diagram is connected

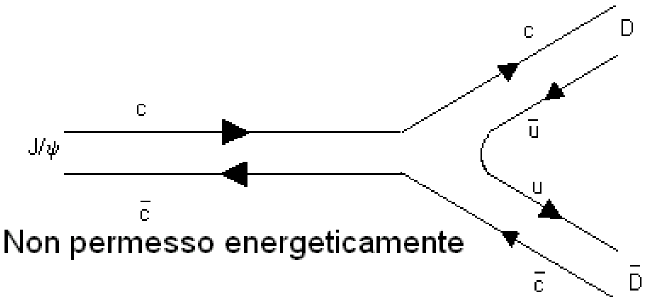


This diagram is connected: *BR 83 %*
(with smallish phase space)

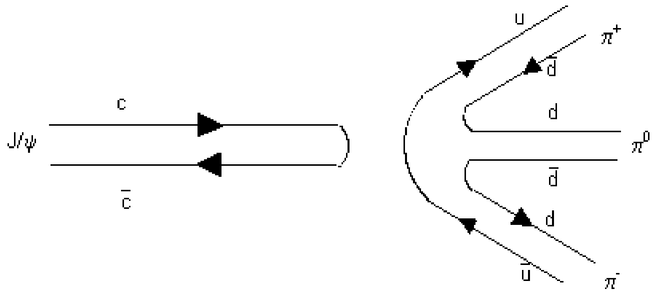


This diagram is disconnected: *BR 15 %*
(with much larger phase space)

OZI Rule - II



Non permesso energeticamente



Compare mass and width

$$m_{J/\psi} = 3097 \ \text{MeV}, \ \Gamma_{J/\psi} = 90 \ \text{keV} \qquad J^{PC}=1^{--}$$

$$m_{\psi'} = 3686 \ \text{MeV}, \ \Gamma_{\psi'} = 250 \ \text{keV} \qquad J^{PC}=1^{--}$$

Explaining the small width:

$$m_{D^0} = 1865 \ MeV \rightarrow 2\times m_{D^0} = 3730 \ MeV > m_{J/\psi}, m_{\psi'}$$

Therefore J/ψ, ψ' decay to open charm is energetically forbidden

$\rightarrow$ Decay diagrams are disconnected

$\rightarrow$ OZI rule: Decay is suppressed

$\rightarrow$ *States are very narrow*

OZI Rule - III

As a general rule

$\rightarrow A \propto \alpha_s^n \quad n =$ number of gluons

Connected diagrams: Small number of soft gluons $\rightarrow A$ = *large*
Disconnected diagrams: Large number of hard gluons $\rightarrow A$ = *small*

Indeed:

Single gluon annihilation is forbidden for mesons by color conservation (meson = ***1***, gluon = ***8***)

Annihilation of massive quarks yields hard gluons$\rightarrow \alpha_s$ is small

Connected diagrams involve softer gluons $\rightarrow \alpha_s$ is large

OZI Rule - IV

Consider quarkonium annihilation into gluons:

$q\bar{q} \to g$ Excluded: $(q\bar{q})_1 \not\to (1g)_8$

$q\bar{q} \to gg$ Allowed

$q\bar{q} \to ggg$ Allowed

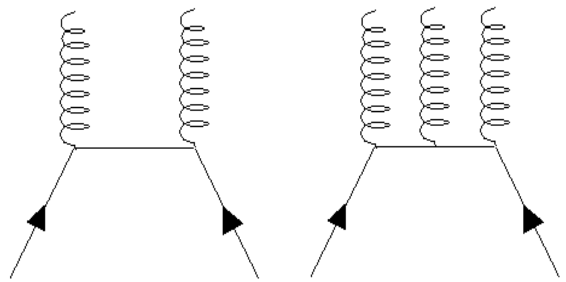
Decompose the direct product of 2 octets:

$\boldsymbol{8 \otimes 8 = 1 \oplus 8 \oplus 8 \oplus 10 \oplus 10^* \oplus 27}$

Charge Parity:

$J^{PC} = 0^{-+} \to C = +1 \to 2g$ OK

$J^{PC} = 1^{--} \to C = -1 \to 3g$ OK



Perturbative regime: $A(2g)>A(3g)$

$\to$Pseudoscalars wider than vectors

Quarkonium Decays - I

From the roaring '60s

Attempting to calculate the vector meson decay rate to lepton pairs

$\Gamma_V = |A_V|^2, \quad A_V = \langle f|T|V\rangle$ Transition amplitude between V(initial), f (final) state

Meson is a bound state $\rightarrow$ Initial state *not* a plane wave

Then expand the amplitude into plane waves:

$$A_V = \sum_p \underbrace{\langle f|T|p\rangle}_{A(\mathbf{p})} \underbrace{\langle p|V\rangle}_{\psi(\mathbf{p})}$$

$A(\mathbf{p})$ plane wave amplitude, $\psi(\mathbf{p})$ momentum space wave function

$$\rightarrow A_V = \int d^3\mathbf{p} A(\mathbf{p})\psi(\mathbf{p}) = \frac{1}{(2\pi)^{3/2}} \int d^3\mathbf{p} A(\mathbf{p}) \int \psi(\mathbf{r}) e^{i\mathbf{p}\cdot\mathbf{r}} d^3\mathbf{r} = \frac{1}{(2\pi)^{3/2}} \int d^3\mathbf{r}\,\psi(\mathbf{r}) \int A(\mathbf{p}) e^{i\mathbf{p}\cdot\mathbf{r}} d^3\mathbf{r}$$

Take $A(\mathbf{p}) \approx A = const \rightarrow A_V \approx \dfrac{A}{(2\pi)^{3/2}} \int d^3\mathbf{r}\,\psi(\mathbf{r}) \underbrace{\int e^{i\mathbf{p}\cdot\mathbf{r}} d^3\mathbf{p}}_{(2\pi)^3 \delta^3(\mathbf{r})} = (2\pi)^{3/2} A\psi(0)$

$$\rightarrow \Gamma_V = |A_V|^2 \approx (2\pi)^3 |A|^2 |\psi(0)|^2$$

Quarkonium Decays - II

Why is *A(p)* ≈ *const*?
Consider the process involving free quarks (plane wave):

$$q\bar{q} \to e^+ e^-$$

Cross section, averaged over initial, summed over final spin projections:

$$\sigma_{q\bar{q}\to e^+e^-}(p) = |A(p)|^2 \frac{1}{\underbrace{\frac{v}{(2\pi)^3}}_{\text{flux}}}, v \; q, \; \bar{q} \text{ relative velocity} \to \sigma_{q\bar{q}\to e^+e^-}(p) = |A(p)|^2 \frac{(2\pi)^3}{v}$$

Take 1-photon annihilation QED diagram:

$$\sigma_{q\bar{q}\to e^+e^-}(p) = \frac{\pi\alpha^2 Q^2}{s} \frac{p_e}{p_q}\left(1 + \frac{v^2}{3} + \frac{4m_q^2}{s}\right)$$

Just the same as $e^+ + e^- \to \mu^+ + \mu^-$
But: Do not neglect rest mass

For small initial velocity:

$$s \approx (2m_q)^2, p_q \approx m_q \frac{v}{2}$$

$$\sigma_{q\bar{q}\to e^+e^-}(p) = \frac{\pi\alpha^2 Q^2}{s} \frac{p_e}{p_q}\left(1 + \frac{v^2}{3} + \frac{4m_q^2}{s}\right) \approx \frac{\pi\alpha^2 Q^2}{4m_q^2} \frac{p_e}{m_q \frac{v}{2}}\left(1 + \frac{v^2}{3} + 1\right) \approx \frac{\pi\alpha^2 Q^2}{4m_q^2} \frac{p_e}{m_q v} 4 \approx \frac{\pi\alpha^2 Q^2}{m_q^3} \frac{p_e}{v}$$

Quarkonium Decays - III

Obtain the decay rate:

$$\rightarrow \sigma_{q\bar{q}\rightarrow e^+e^-}(p) = |A(p)|^2 \frac{(2\pi)^3}{v} = \frac{\pi\alpha^2 Q^2}{m_q^3}\frac{p_e}{v}$$

$$\rightarrow |A(p)|^2 = \frac{\pi\alpha^2 Q^2}{m_q^3}\frac{p_e}{v}\frac{v}{(2\pi)^3} = \frac{\pi\alpha^2 Q^2}{m_q^3}\frac{p_e}{(2\pi)^3}$$

$p_e \approx m_q \rightarrow |A(p)|^2 \approx \dfrac{\pi\alpha^2 Q^2}{(2\pi)^3 m_q^2}$ Neglect quark momentum, electron mass

$m_q \approx \dfrac{M_V}{2} \rightarrow |A(p)|^2 \approx \dfrac{4\pi\alpha^2 Q^2}{(2\pi)^3 M_V^2}$ p-independent OK

We took the average over initial spins for our plane wave cross section, resulting in 3 (triplet) + 1 (singlet) = 4 states
Vector mesons have spin 1, so we should not count spin 0
$\rightarrow$Get a further factor *4/3*:

$\Gamma_V \approx \dfrac{4}{3}(2\pi)^3 \dfrac{4\pi\alpha^2 Q^2}{(2\pi)^3 M_V^2}|\psi(0)|^2 = \dfrac{16}{3}\dfrac{\pi\alpha^2 Q^2}{M_V^2}|\psi(0)|^2$ Van Royen-Weisskopf formula

Quarkonium Decays - IV

For Bottomonium and Charmonium:

$$|\psi_q(0)|^2 \sim (\mu|\lambda|)^3 = \left(\frac{m_q}{2}|\lambda|\right)^3 = \frac{m_q^3}{8}|\lambda|^3$$

$$\rightarrow \Gamma_q \approx \frac{4}{3}(2\pi)^3 \frac{4\pi\alpha^2 Q_q^2}{(2\pi)^3 M_V^2}|\psi(0)|^2 \sim \frac{4}{3}\frac{\pi\alpha^2 Q_q^2}{m_q^2}\frac{m_q^3}{8}|\lambda|^3 = \frac{\pi\alpha^2 Q_q^2 m_q}{6}|\lambda|^3$$

$$\rightarrow \frac{\Gamma_\Upsilon}{\Gamma_\psi} \approx \frac{Q_b^2}{Q_c^2}\frac{m_b}{m_c} \approx \frac{Q_b^2}{Q_c^2}\frac{9.46}{3.10}$$

$$\Gamma_\psi(ee) \simeq 5.55\ KeV$$

DORIS (DESY) results (1978):

$$\Gamma_\Upsilon(ee) \simeq 1.26 KeV$$

$$\rightarrow \left|\frac{Q_b}{Q_c}\right| \approx \sqrt{\frac{\Gamma_\Upsilon}{\Gamma_\psi}\frac{m_c}{m_b}} \approx \sqrt{\frac{1.26}{5.55}\frac{3.10}{9.46}} \sim 0.28 \rightarrow |Q_b| = \frac{1}{3} \text{ strongly preferred}$$

Quarkonium Decays - V

By comparison with positronium:

$$\left(e^+e^-\right)_{positronium} \rightarrow \gamma\gamma$$

$$\Gamma\left[\left(e^+e^-\right) \rightarrow \gamma\gamma\right] = \frac{\alpha^2}{m^2}\left|\psi(0)\right|^2$$

$$\left(c\overline{c}\right)_{charmonium} \rightarrow \gamma\gamma$$

$$\begin{cases} e \rightarrow \frac{2}{3}e \rightarrow \alpha \rightarrow \frac{4}{9}\alpha \text{ Quark charge} \\ \times 9 \quad \text{Sum amplitude over colors} \end{cases}$$

$$\Gamma\left[\left(c\overline{c}\right) \rightarrow \gamma\gamma\right] = \frac{48\alpha^2}{27m_c^2}\left|\psi_{c\overline{c}}(0)\right|^2$$

$$\left(c\overline{c}\right)_{charmonium} \rightarrow gg$$

From SU(3) algebra: 2 g in a color singlet state

Color factor $= \dfrac{9}{8}$

$$\Gamma\left[\left(c\overline{c}\right) \rightarrow gg\right] = \frac{2\alpha_s^2}{3m_c^2}\left|\psi_{c\overline{c}}(0)\right|^2$$

But:

Positronium rate was obtained by taking the low speed limit of scattering amplitude to 1-photon approx

Is it granted for $c\overline{c}$?

Another Way to Charmonium - I

Try to avoid limitations inherent to electron positron annihilation:

(1) 1-photon tree diagram: Forming just 1^{--} states

(2) 2-photon process: Forming just C = + states
Difficult to manage

Go for $\overline{p}p$ annihilation: No constraints on quantum numbers
High luminosity possible by special techniques

$$p+\overline{p}\rightarrow \underbrace{c\overline{c}}_{Charmonium} \rightarrow Electromagnetic\ \ decay$$

Another Way to Charmonium - II

Fermilab antiproton accumulator
Built around 1990 in order to provide
intense, cooled antiproton beam to the Tevatron

$$\beta c = L_{REF} f_{REV} \rightarrow E_{\bar{p}} = \frac{m_{\bar{p}} c^2}{\sqrt{1-\beta^2}}$$

$$\Delta E_{\bar{p}} = m_{\bar{p}} c^2 \gamma^3 \beta^2 \sqrt{\left[\left(\frac{\Delta f}{f}\right)^2 + \left(\frac{\Delta L}{L}\right)^2\right]}$$

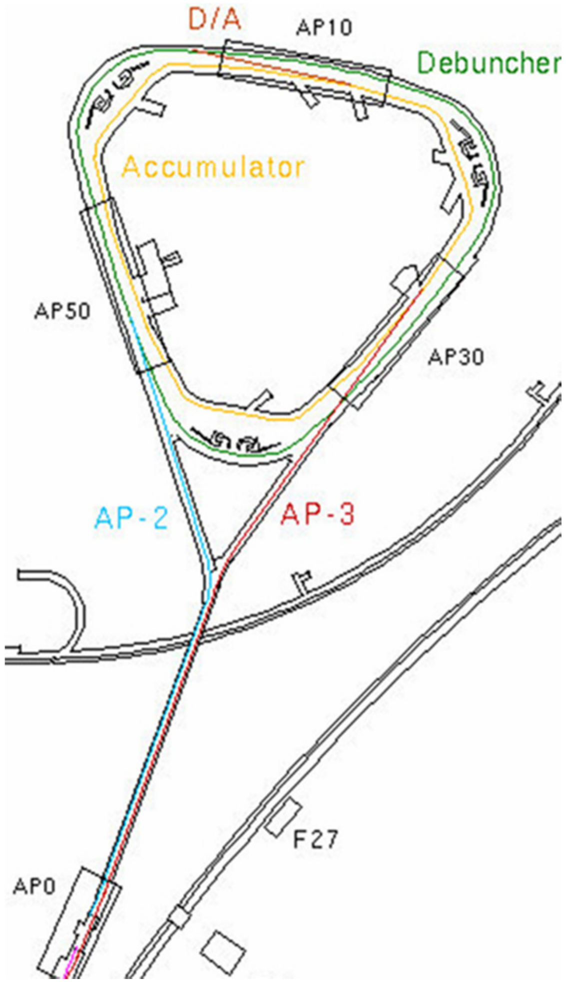
$$\frac{\Delta f}{f} \approx few \;\; 10^{-7}$$

$\rightarrow \Delta L$ dominant error source

From $m_{\psi'}$ get L_{REF}

Then $\Delta m_{\psi'} = 100 KeV \rightarrow \Delta L = 0.7 mm$

$$\frac{\Delta L}{L} \approx 10^{-6}$$

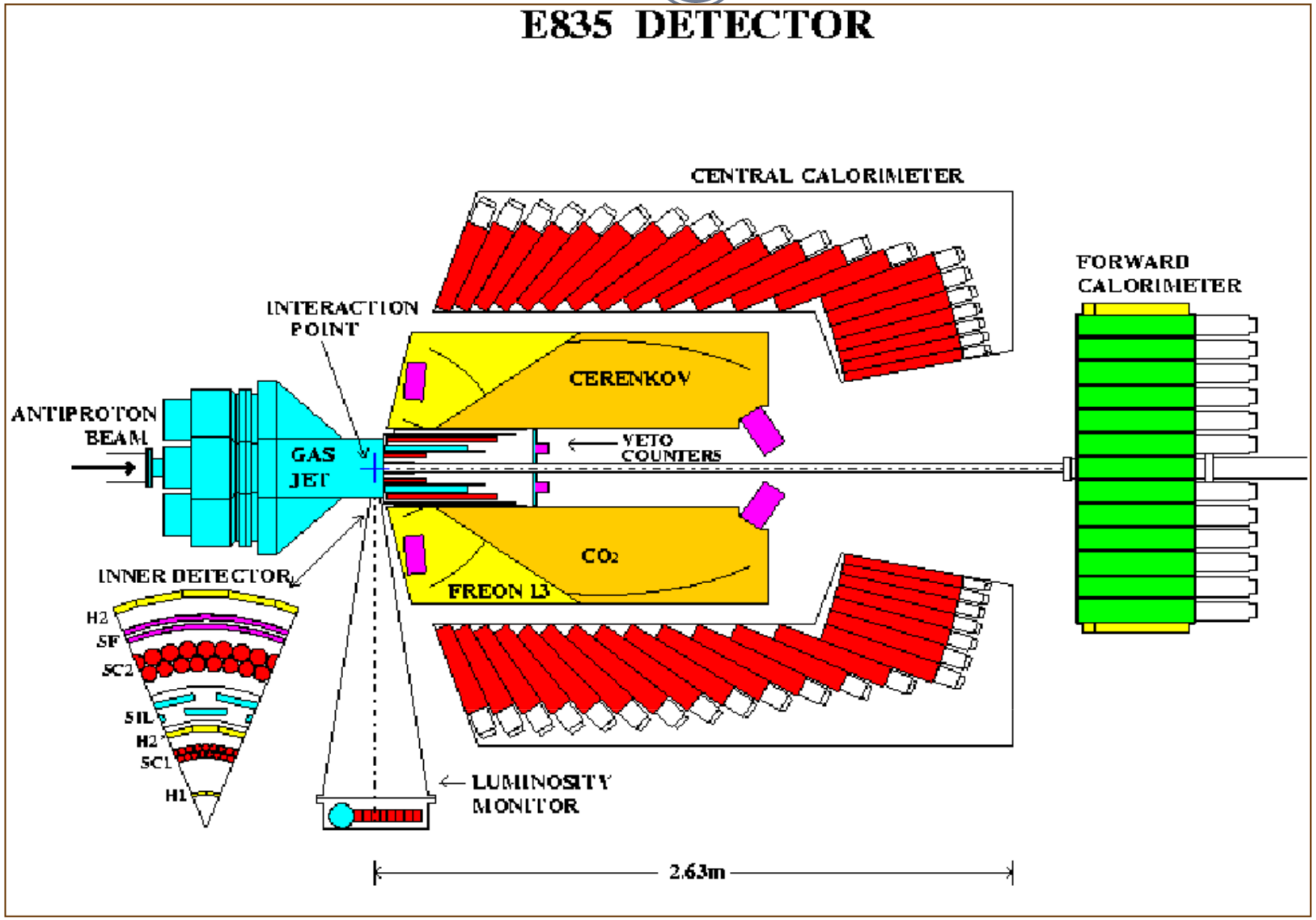


Another Way to Charmonium - III

Target

- Molecular hydrogen jet into the machine vacuum(lots of pumping power!)
- Microdroplets formation @ 1bar, 35 K
- High density $>10^{14}$ at/cm^3
- Instantaneous $L \approx 2.5\ 10^{31}\ \mathrm{cm}^{-2}\ \mathrm{s}^{-1}$ (DAQ limited)
- Several beneficial side effects (small source size, effective use of pbar, ...)

Another Way to Charmonium - IV

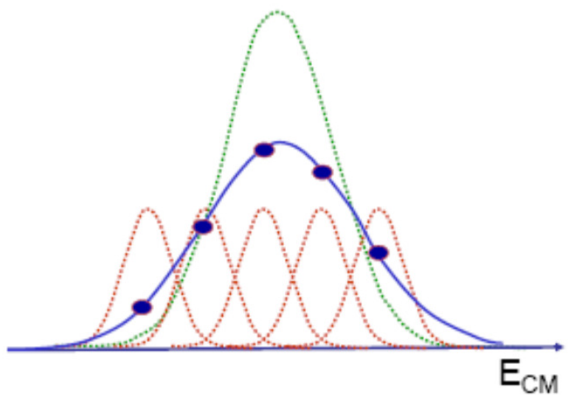


Another Way to Charmonium - V

Concept of resonance scan: A fixed target, formation experiment

Move the beam energy in small steps across the energy range of any given resonant state

Measure the decay rate of the state at each step



Rate

Resonance profile

Typical width $\Gamma < 1$ MeV for $c\bar{c}$

Beam profile

Typical resolution $\sigma\left(E_{CM}\right) \sim 0.2\ MeV$

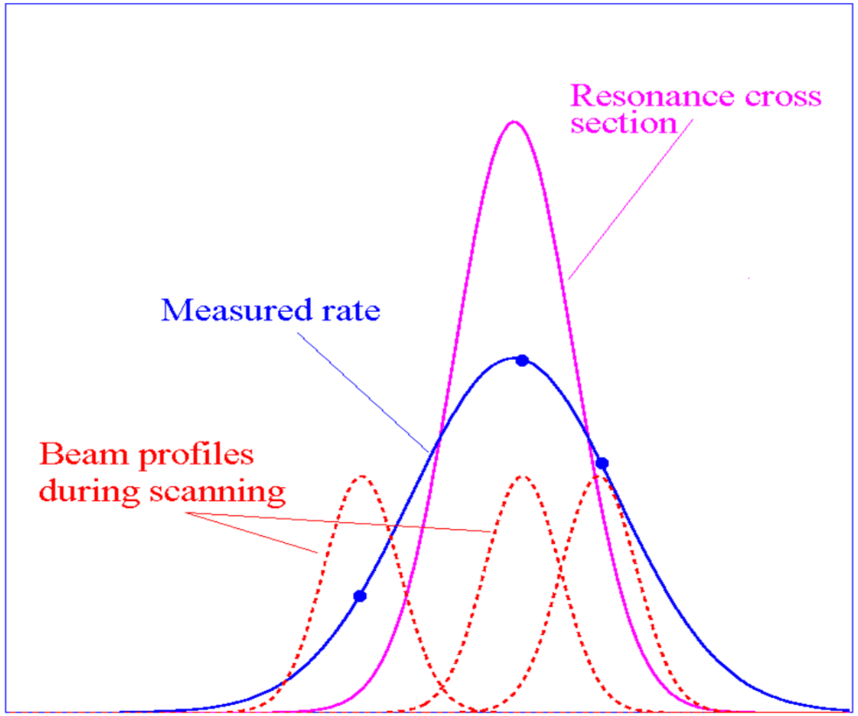
Get resonance mass, width, coupling by deconvolving the beam profile from the observed rate

Another Way to Charmonium - VI

$$\frac{\#ev.}{\int Ldt} = \varepsilon\alpha\int_0^\infty f_B\left(E-\sqrt{s}\right)\sigma_{BW}\left(E\right)dE + \sigma_B$$

$$\sigma_{BW} = \frac{(2J+1)\pi}{4k^2}\frac{B\left(p\overline{p}\to R\right)B\left(R\to f\right)\Gamma_R^2}{\left(\sqrt{s}-M_R\right)^2+\frac{\Gamma_R^2}{4}}$$

$$p\overline{p}\to c\overline{c}\to e^+e^-$$
$$p\overline{p}\to c\overline{c}\to J/\psi X\to e^+e^-X$$
$$p\overline{p}\to c\overline{c}\to \gamma\gamma$$
$$p\overline{p}\to multi\quad\gamma$$
$$p\overline{p}\to \varphi\varphi\to K^+K^-K^+K^-$$
$$p\overline{p}\to p\overline{p}$$



Another Way to Charmonium - VII

Electrons: *Cerenkov* + *Calorimeter* + *Tracking*
$\rightarrow$ Very low background to $e^+ e^-$

$\psi' \rightarrow J/\psi + X \qquad \psi' \rightarrow e^+e^-$

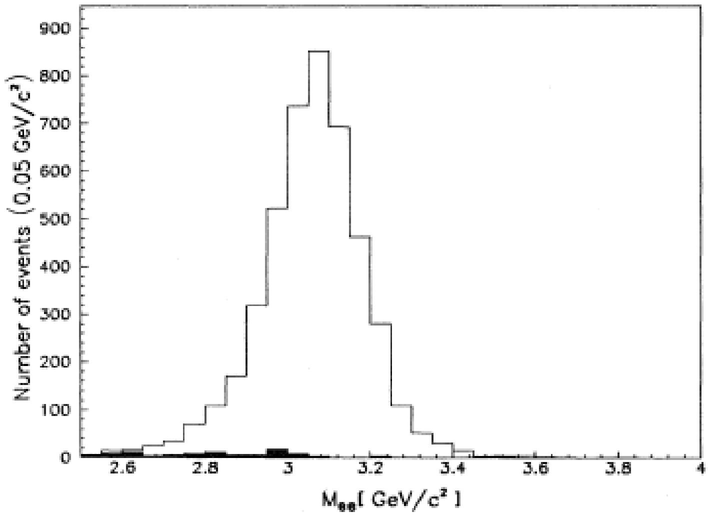


FIG. 5. Invariant mass distribution of electron pairs for the 1991 J/ψ scan (open area) and for the off-resonance background normalized to the same luminosity (shaded area). For this figure only the level of background has been multiplied by a factor of 10 to make it discernible.

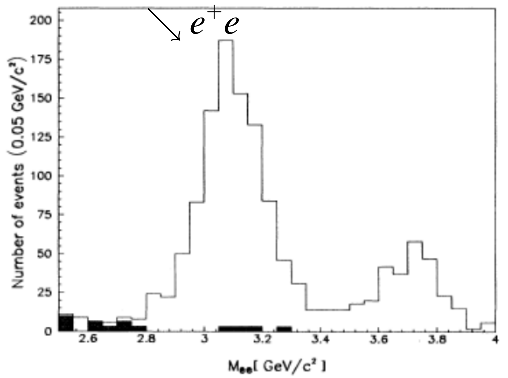


FIG. 6. Invariant mass distribution of electron pairs for the 1991 ψ' scan (open area) and for the off-resonance background normalized to the same luminosity (shaded area).

@TBA

$M_{e^+e^-}$ from scan across J/ψ

$M_{e^+e^-}$ from scan across ψ'

Another Way to Charmonium - VIII

A few results..

@TBA

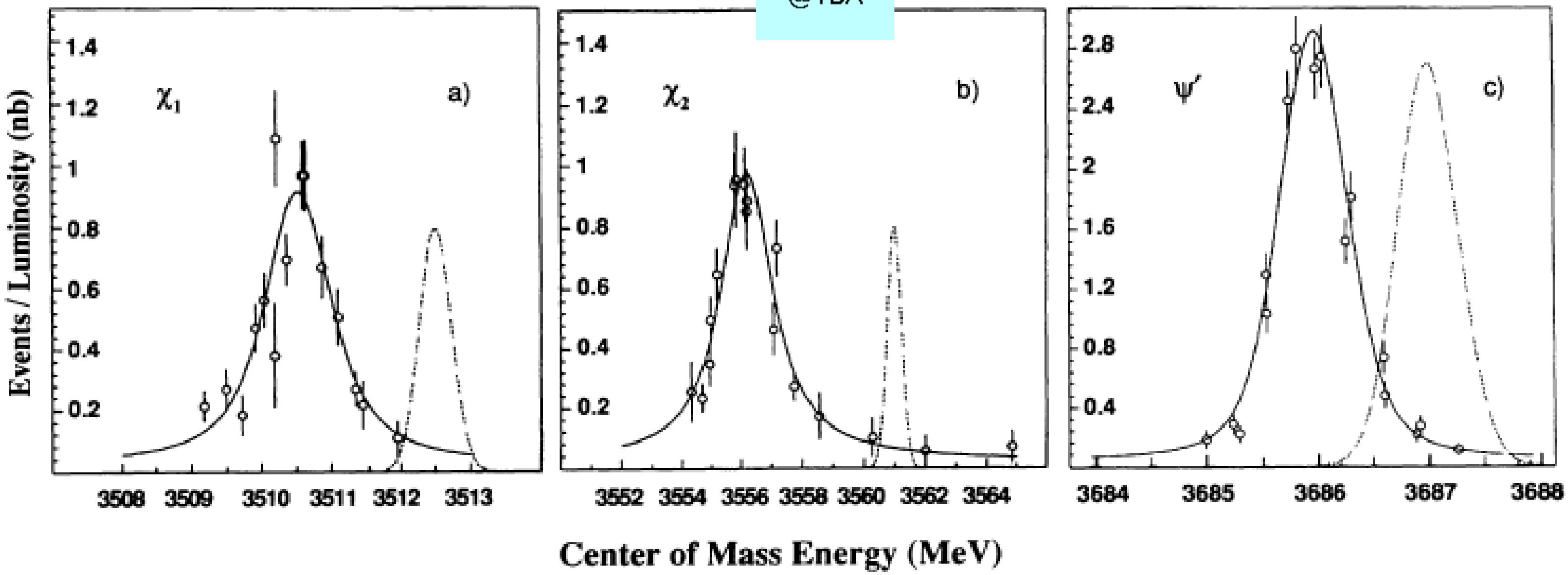


FIG. 3. Events per unit luminosity for the energy scan at (a) the χ_{c1}, (b) the χ_{c2}, and (c) the ψ'. The solid line represents the best fit with the data. The dashed line shows a typical resolution in the center-of-mass energy (arbitrary vertical units).

Another Way to Charmonium - IX

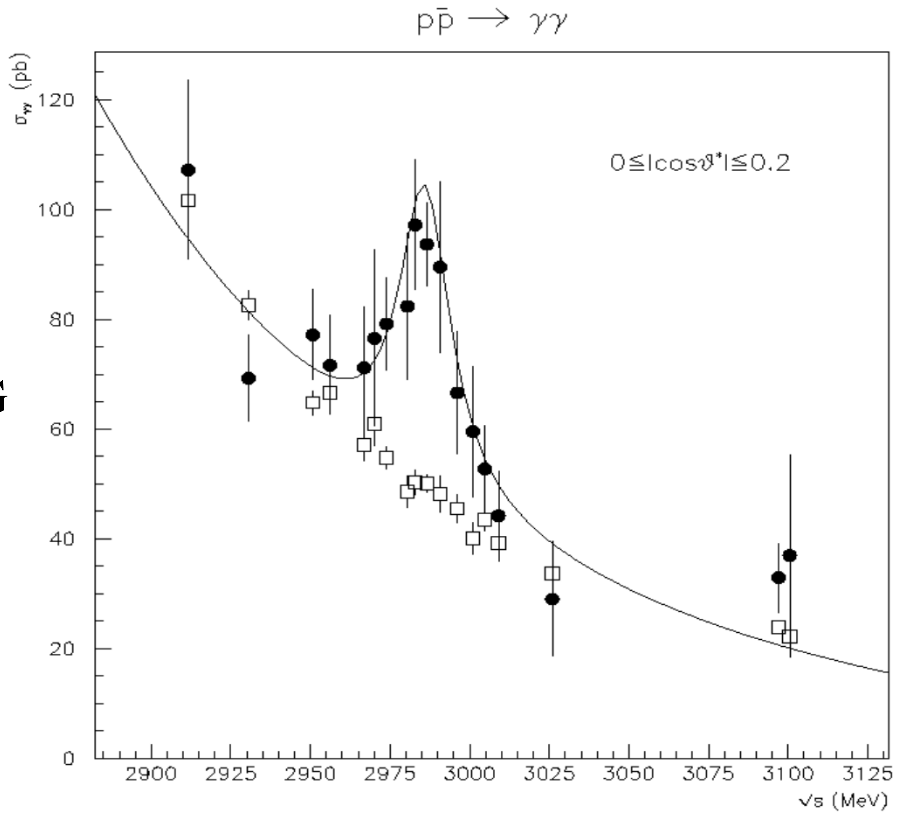
Back in 1999:

$$M(\eta_C) = 2984.8 \pm 1.9\ MeV$$

$$\Gamma_{TOT}(\eta_C) = 17.8^{+7.2}_{-5.9} MeV$$

$$\Gamma_{\gamma\gamma}(\eta_C) = 3.7^{+1.5}_{-1.3} \pm 1.2\ KeV$$

assuming $BR_{p\bar{p}} = (1.2 \pm 0.4)10^{-4}$ from PDG



Another Way to Charmonium - X

Repeat for χ_2 :

$\Gamma_{\gamma\gamma}(\chi_2) = 0.31 \pm 0.04 \pm 0.03$ KeV

by taking mass and total width from $\psi\gamma$ data

$\rightarrow$ Get a measurement of α_s close to confinement region

$$\frac{\Gamma_{\gamma\gamma}}{\Gamma_{gg}}(\eta_c) \cong \frac{8\alpha^2}{9\alpha_s^2}\left(\frac{1-3.4\frac{\alpha_s}{\pi}}{1+4.8\frac{\alpha_s}{\pi}}\right)$$

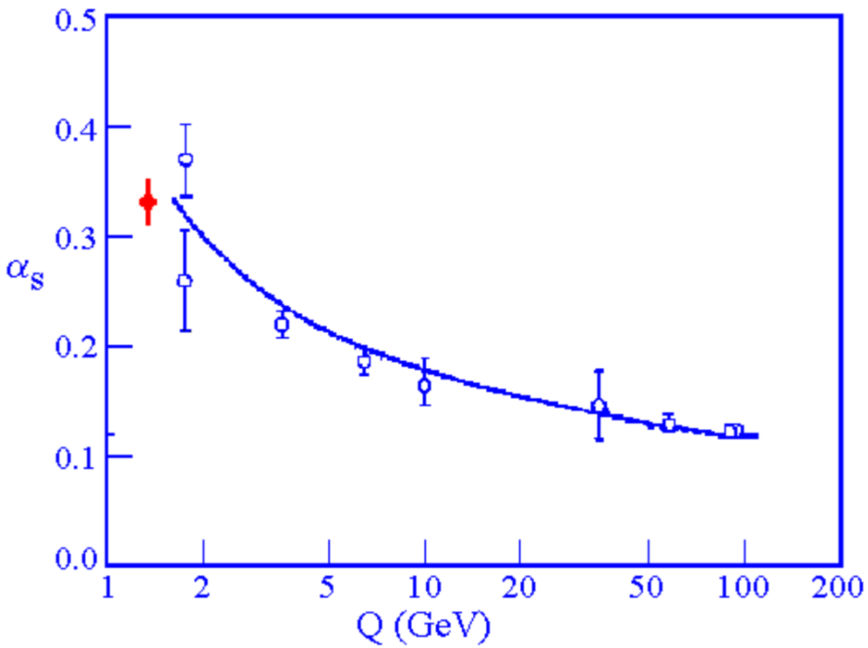
$$\frac{\Gamma_{\gamma\gamma}}{\Gamma_{gg}}(\chi_2) \cong \frac{8\alpha^2}{9\alpha_s^2}\left(\frac{1-16\frac{\alpha_s}{3\pi}}{1-2.2\frac{\alpha_s}{\pi}}\right)$$

Another Way to Charmonium - XI

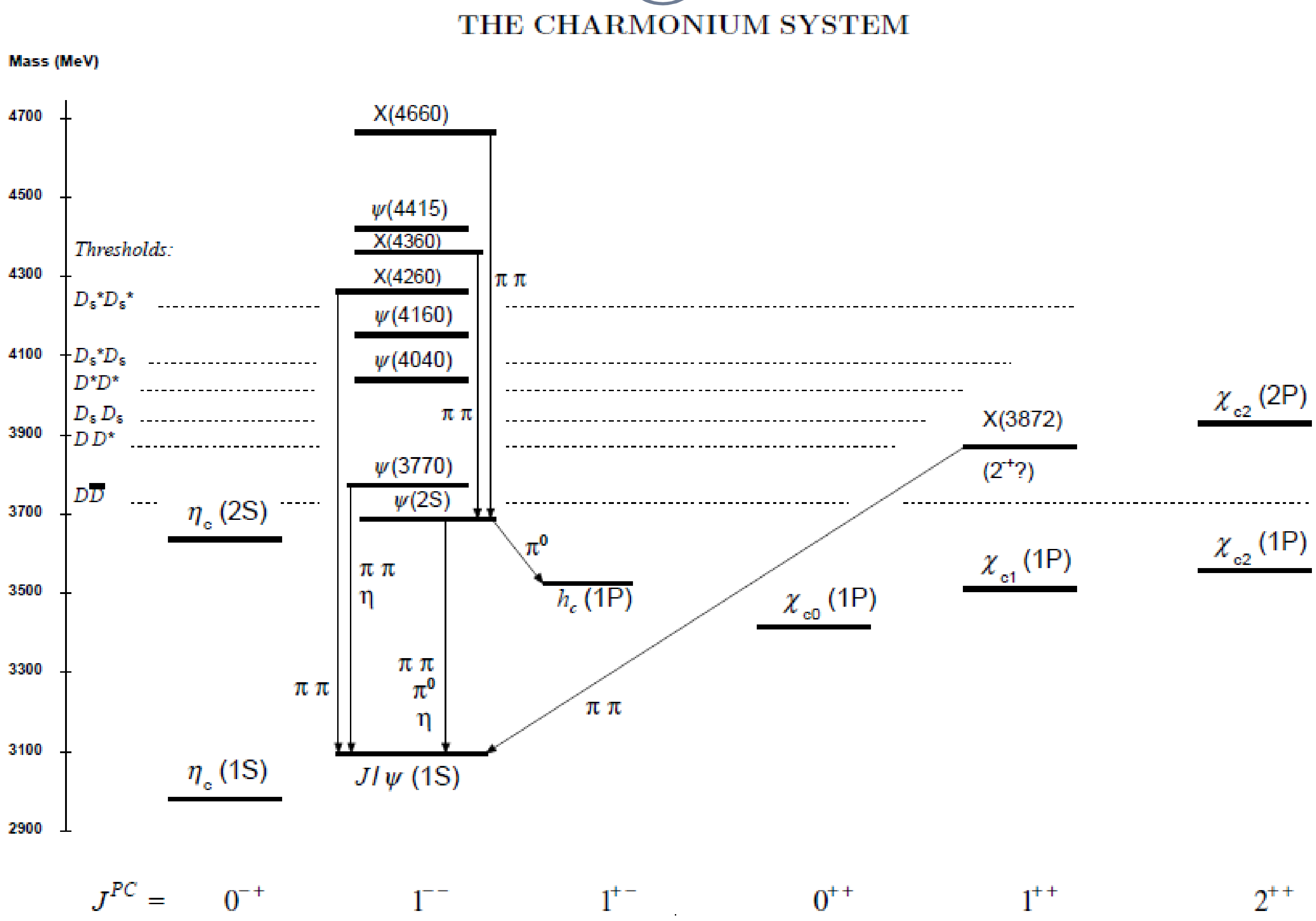
Get

$$\alpha_S(m_C) = 0.32^{+0.05}_{-0.04} \ \ from \ \ \eta_C$$

$$\alpha_S(m_C) = 0.36 \ \pm 0.02 \ \ from \ \ \chi_2$$



Charmonium on PDG



Bottomonium on PDG