Elementary Particles II

3 – Flavor Physics and CP Violation

Quark Mixing – CKM – K^o Strangeness oscillations
CP violation – Extension to Bottom and Charm
FCNC and Physics Beyond the Standard Model

Quark Mixing - I

Reminder:

Fermi constant from μ decay $\simeq$ Fermi constant from β decay

Tiny difference:
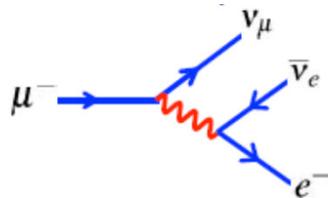
$$G_F^\mu = (1.16632 \pm 0.00002) \times 10^{-5}\,\text{GeV}^{-2} \qquad G_F^\beta = (1.136 \pm 0.003) \times 10^{-5}\,\text{GeV}^{-2}$$
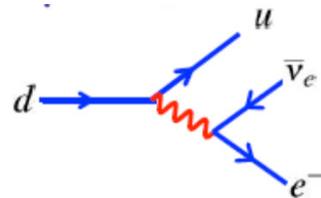
Kaon decay suppressed by a factor ~ 20 as compared to π decay
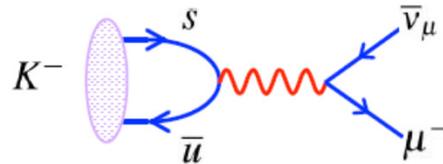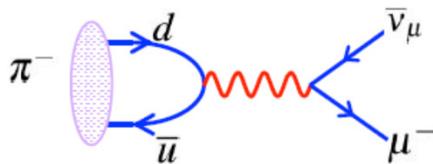
Quark Mixing - II

Cabibbo explanation:

Weak eigenstates   Strong (mass) eigenstates

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

Weak charged currents: Linear combinations of different flavors

$\theta_C \approx 13.1^0$

Unique value for Cabibbo angle explaining many strange particle decays

Strong support for universality of weak interaction

Quark Mixing - III

Another mistery

$$BR\left(K^0 \to \mu^+\mu^-\right) \sim 10^{-8}\, BR\left(K^+ \to \mu^+\nu_\mu\right)$$

GIM explaination:

$$\cos\theta_c \frac{g_W}{\sqrt{2}} \qquad \sin\theta_c \frac{g_W}{\sqrt{2}} \qquad -\sin\theta_c \frac{g_W}{\sqrt{2}} \qquad \cos\theta_c \frac{g_W}{\sqrt{2}}$$

$$M_1 \propto g_W^4 \cos\theta_c \sin\theta_c$$

$$M_2 \propto -g_W^4 \cos\theta_c \sin\theta_c$$
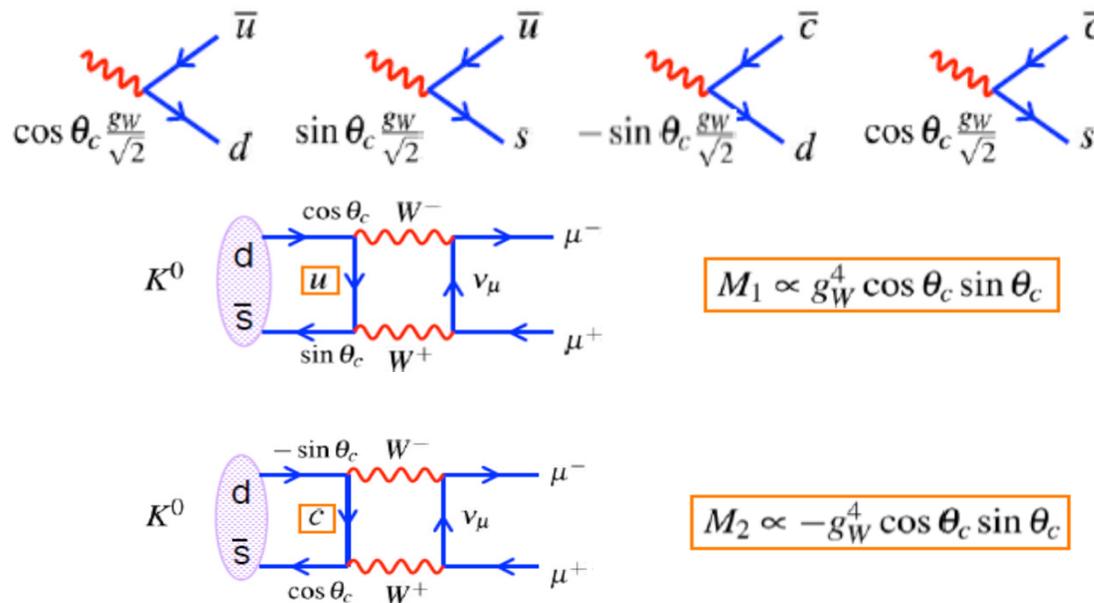
Tiny BR left due to $m_c \neq m_u$ in the virtual quark propagator

Quark Mixing - IV

Extend mixing to 3 families:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V_{(d)}^\dagger \begin{pmatrix} d \\ s \\ b \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}, \quad \begin{pmatrix} u' \\ c' \\ t' \end{pmatrix} = V_{(u)}^\dagger \begin{pmatrix} u \\ c \\ t \end{pmatrix}$$

$\rightarrow$ Conventionally: Mixing of d-like quarks only
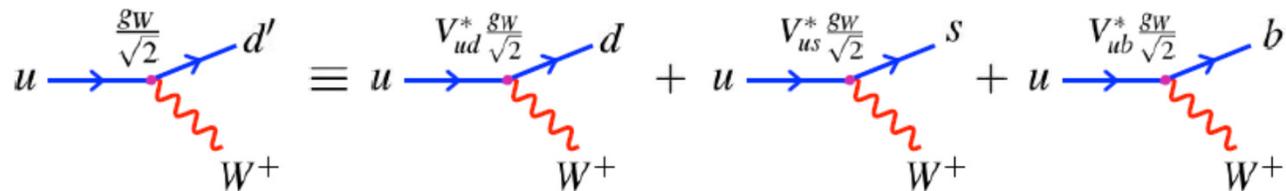
Quark Mixing - V

Encode mixing CKM matrix element into charged current
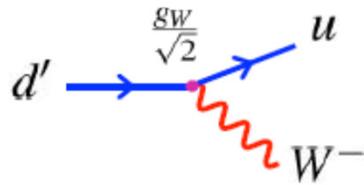
$$j_{d'u} = \overline{u} \left[-i \frac{g_W}{\sqrt{2}} \gamma^\mu \tfrac{1}{2} (1 - \gamma^5) \right] d'$$

$$j_{du} = \overline{u} \left[-i \frac{g_W}{\sqrt{2}} \gamma^\mu \tfrac{1}{2} (1 - \gamma^5) \right] V_{ud} d$$
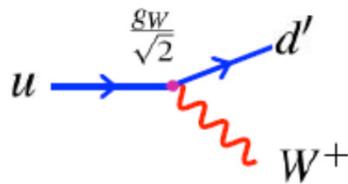
$$j_{ud'} = \overline{d}' \left[-i \frac{g_W}{\sqrt{2}} \gamma^\mu \tfrac{1}{2} (1 - \gamma^5) \right] u$$

$$\overline{d}' = d'^\dagger \gamma^0 \rightarrow (V_{ud} d)^\dagger \gamma^0 = V_{ud}^* d^\dagger \gamma^0 = V_{ud}^* \overline{d}$$

$$j_{ud} = \overline{d} V_{ud}^* \left[-i \frac{g_W}{\sqrt{2}} \gamma^\mu \tfrac{1}{2} (1 - \gamma^5) \right] u$$

Quark Mixing - VI

Charged current : $qq, \overline{q}\overline{q}, q\overline{q}$
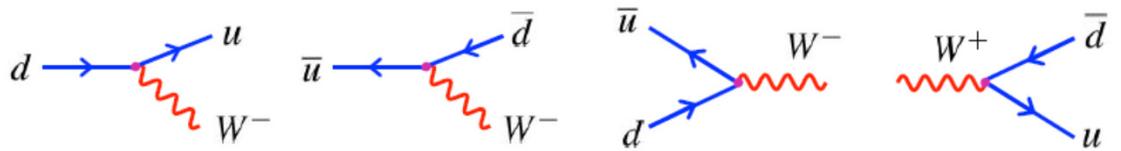
is $\boxed{-i\frac{g_w}{\sqrt{2}}V_{ud}\gamma^\mu\frac{1}{2}(1-\gamma^5)}$
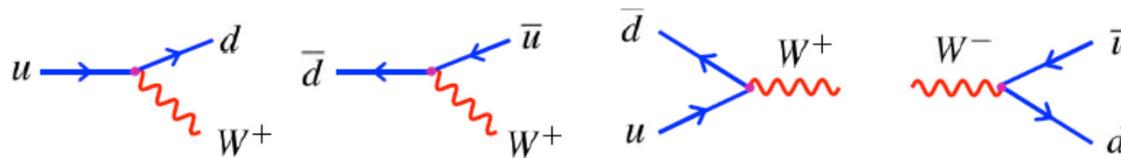
is $\boxed{-i\frac{g_w}{\sqrt{2}}V_{ud}^*\gamma^\mu\frac{1}{2}(1-\gamma^5)}$

CKM - I

Generic mixing matrix:

Mixing weak eigenstates into mass eigenstates (or the opposite)

3×3 Unitary matrix:

9 complex parameters $\rightarrow$ 18 real parameters

9 unitarity conditions:
$$\left.\begin{array}{l} UU^{\dagger}=1 \\ \left(U^{\dagger}\right)_{ij}=U^{*}_{ji} \end{array}\right\} \rightarrow \sum_{j=1}^{3}a_{ij}a^{*}_{jk}=\delta_{ik}, \quad i,k=1,..,3$$

$\rightarrow 18-9=9$ free, real parameters

CKM - II

Mixing matrix definition for 'up'- and 'down'-like quarks:

$$\begin{pmatrix} u \\ c \\ t \end{pmatrix} = V_{(u)} \begin{pmatrix} u_1' \\ u_2' \\ u_3' \end{pmatrix}; \qquad \begin{pmatrix} d \\ s \\ b \end{pmatrix} = V_{(d)} \begin{pmatrix} d_1' \\ d_2' \\ d_3' \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} u_1' \\ u_2' \\ u_3' \end{pmatrix} = V_{(u)}^{-1} \begin{pmatrix} u \\ c \\ t \end{pmatrix} = V_{(u)}^{\dagger} \begin{pmatrix} u \\ c \\ t \end{pmatrix} \rightarrow \begin{pmatrix} u & c & t \end{pmatrix} = \begin{pmatrix} u_1' & u_2' & u_3' \end{pmatrix} \left(V_{(u)}^{\dagger} \right)^{\dagger} = \begin{pmatrix} u_1' & u_2' & u_3' \end{pmatrix} V_{(u)}$$

$$\rightarrow \begin{pmatrix} d_1' \\ d_2' \\ d_3' \end{pmatrix} = V_{(d)}^{-1} \begin{pmatrix} d \\ s \\ b \end{pmatrix} = V_{(d)}^{\dagger} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$\rightarrow V_{CKM} = V_{(u)} V_{(d)}^{\dagger} \equiv V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Re-define (arbitrary) phases of quark mass eigenstates:

$$\begin{pmatrix} u \\ c \\ t \end{pmatrix} \rightarrow \begin{pmatrix} e^{i\varphi_u} & 0 & 0 \\ 0 & e^{i\varphi_c} & 0 \\ 0 & 0 & e^{i\varphi_t} \end{pmatrix} \begin{pmatrix} u \\ c \\ t \end{pmatrix}, \quad \begin{pmatrix} d \\ s \\ b \end{pmatrix} \rightarrow \begin{pmatrix} e^{i\varphi_d} & 0 & 0 \\ 0 & e^{i\varphi_s} & 0 \\ 0 & 0 & e^{i\varphi_b} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Translate into redefinition of weak eigenstates:

$$\begin{pmatrix} u' \\ c' \\ t' \end{pmatrix} \rightarrow V_u^\dagger \begin{pmatrix} e^{i\phi_u} & 0 & 0 \\ 0 & e^{i\phi_c} & 0 \\ 0 & 0 & e^{i\phi_t} \end{pmatrix} \begin{pmatrix} u \\ c \\ t \end{pmatrix}, \quad \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} \rightarrow V_d^\dagger \begin{pmatrix} e^{i\phi_d} & 0 & 0 \\ 0 & e^{i\phi_s} & 0 \\ 0 & 0 & e^{i\phi_b} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Redefinition of weak eigenstates equivalent to *CKM* redefinition:

$$V_{CKM} \rightarrow \begin{pmatrix} e^{-i\varphi_u} & 0 & 0 \\ 0 & e^{-i\varphi_c} & 0 \\ 0 & 0 & e^{-i\varphi_t} \end{pmatrix} V_{CKM} \begin{pmatrix} e^{i\varphi_d} & 0 & 0 \\ 0 & e^{i\varphi_s} & 0 \\ 0 & 0 & e^{i\varphi_b} \end{pmatrix} = \begin{pmatrix} V_{ud}e^{i(\varphi_d-\varphi_u)} & V_{us}e^{i(\varphi_s-\varphi_u)} & V_{ub}e^{i(\varphi_b-\varphi_u)} \\ V_{cd}e^{i(\varphi_d-\varphi_c)} & V_{cs}e^{i(\varphi_s-\varphi_c)} & V_{cb}e^{i(\varphi_b-\varphi_c)} \\ V_{td}e^{i(\varphi_d-\varphi_t)} & V_{ts}e^{i(\varphi_s-\varphi_t)} & V_{tb}e^{i(\varphi_b-\varphi_t)} \end{pmatrix}$$

CKM - IV

Factorize one (any) phase:

$$\rightarrow V_{CKM} = e^{-i\varphi_u} \begin{pmatrix} V_{ud}e^{i\varphi_d} & V_{us}e^{i\varphi_s} & V_{ub}e^{i\varphi_b} \\ V_{cd}e^{i(\varphi_u+\varphi_d-\varphi_c)} & V_{cs}e^{i(\varphi_u+\varphi_s-\varphi_c)} & V_{cb}e^{i(\varphi_u+\varphi_b-\varphi_c)} \\ V_{td}e^{i(\varphi_u+\varphi_d-\varphi_t)} & V_{ts}e^{i(\varphi_u+\varphi_s-\varphi_t)} & V_{tb}e^{i(\varphi_u+\varphi_b-\varphi_t)} \end{pmatrix}$$

Global field phase not relevant: Can't be used to fix one free V parameter

5 free relative phases

$\rightarrow 9-5=4$ real free parameters

Encode as:

3 'rotation angles' $(\leftarrow$ Euler angles$)$

$\big[$In order to understand this:

Suppose the matrix is real $\rightarrow$ Any 3×3 real, unitary matrix = Orthogonal

Any 3×3 orthogonal matrix = 3D Rotation $\rightarrow$ 3 angles$\big]$

1 complex (irreducible) phase factor

CKM - V

→Generally *CKM* matrix *must* be complex, *3x3* real would require just 3 parameters
Standard form : → Will have 5 complex V_{ij}

Parameters:

$\theta_{12}, \theta_{13}, \theta_{23}$ Rotation angles

δ Irreducible phase

$s_{ij} = \sin \theta_{ij}$

$c_{ij} = \cos \theta_{ij}$

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}e^{+i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{+i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{+i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{+i\delta} & c_{13}c_{23} \end{pmatrix}$$

Experiment:

$\sin \theta_{13} \ll \sin \theta_{23} \ll \sin \theta_{12} \ll 1$

CKM - VI

Visualizing CKM in standard representation:

Product of 3 independent 2D rotations + 1 Phase

$$U_{12} = \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$U_{13} = \begin{bmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{bmatrix}$$

$$V_{\text{CKM}} = U_{23} U_\delta^\dagger U_{13} U_\delta U_{12}$$

$$U_{23} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix}$$

$$U_\delta = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\delta_{13}} \end{bmatrix}$$

CKM - VII

Wolfenstein parametrization of V_{CKM}:

Based on experimental evidence of some hierarchy among angles

Define:

$$\lambda = \sin \theta_{12}$$

$$A\lambda^2 = \sin \theta_{23}$$

$$A\lambda^3 (\rho - i\eta) = \sin \theta_{13} e^{-i\delta}$$

Then:

$$\cos \theta_{12} = \sqrt{1 - \lambda^2} \simeq 1 - \frac{\lambda^2}{2} - \frac{\lambda^4}{8}$$

$$\rightarrow V_{CKM} \simeq \begin{pmatrix} 1 - \dfrac{\lambda^2}{2} & \lambda & A\lambda^3 (\rho - i\eta) \\ -\lambda & 1 - \dfrac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3 (1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

$$\lambda = 0.2259 \pm 0.0021 \approx \sin \theta_C$$

$$A = 0.82 \pm 0.02$$

$$\eta \neq 0 \rightarrow \cancel{CP}$$

Filling CKM - I

Filling CKM (PDG 2013)

superallowed $0^+ \to 0^+$ β decays

semileptonic / leptonic kaon decays
hadronic tau decays

semileptonic / leptonic B decays

$$\begin{pmatrix} 0.97425 \pm 0.00022 & 0.2252 \pm 0.0009 & (4.15 \pm 0.49) \times 10^{-3} \\ 0.230 \pm 0.011 & 1.006 \pm 0.023 & (40.9 \pm 1.1) \times 10^{-3} \\ (8.4 \pm 0.6) \times 10^{-3} & (42.9 \pm 2.6) \times 10^{-3} & 0.89 \pm 0.07 \end{pmatrix}$$

semileptonic charm decays
charm production in neutrino beams

semileptonic B decays

B_d oscillations

semileptonic / leptonic charm decays

B_s oscillations

single top production

CKM elements involving *t* quark less well known

Filling CKM - II

$|V_{ud}|$ from nuclear beta decay

$$\begin{pmatrix} \times & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$
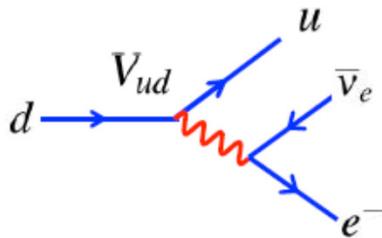
Super-allowed $0^+ \rightarrow 0^+$ beta decays are relatively free from theoretical uncertainties

$$\Gamma \propto |V_{ud}|^2$$

$$|V_{ud}| = 0.97377 \pm 0.00027 \qquad (\approx \cos\theta_c)$$

Superallowed β transition: $\Delta J = 0, \ \Delta P = 0$

+ Same level structure for both initial and final nucleus, just $n \leftrightarrow p$

Global level shift, only due to Coulomb energy $\rightarrow$ No theoretical corrections!

Example: $n \rightarrow p + e^- + \overline{\nu}_e$

$\rightarrow$ High precision measurement of V_{ud} from transition rate

Filling CKM - III

|V_us| **from semi-leptonic kaon decays**
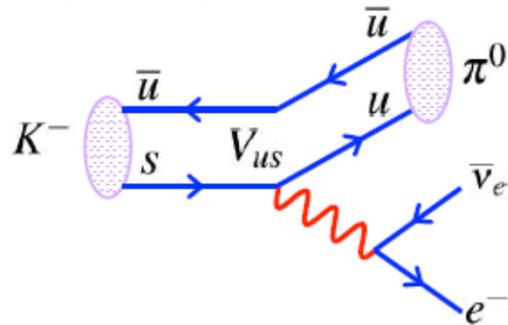
$$\begin{pmatrix} \cdot & \times & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

$$\Gamma \propto |V_{us}|^2$$

$$|V_{us}| = 0.2257 \pm 0.0021 \qquad (\approx \sin\theta_c)$$

Differential decay rate:

$$\frac{d\Gamma(\overline{K^0} \to \pi^+ e^- \overline{\nu}_e)}{dx_\pi} = \underbrace{\frac{G_F^2 m_K^5}{192\pi^2}}_{\substack{\text{Standard 3-body} \\ \text{total rate}}} |V_{us}|^2 \underbrace{f(q^2)^2}_{\text{Form factor}} \underbrace{\left(x_\pi^2 - 4\frac{m_\pi^2}{m_K^2} \right)^{3/2}}_{\text{Phase space factor}} , x_\pi = \frac{2E_\pi}{m_K}$$

Filling CKM - IV

$|V_{cd}|$ | from neutrino scattering | $\nu_\mu + N \to \mu^+ \mu^- X$

$$\begin{pmatrix} \cdot & \cdot & \cdot \\ \times & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

Look for opposite charge di-muon events in ν_μ scattering from production and decay of a $D^+(c\bar{d})$ meson

$$\text{Rate} \propto |V_{cd}|^2 \text{Br}(D^+ \to X\mu^+\nu_\mu)$$

opposite sign μμ pair
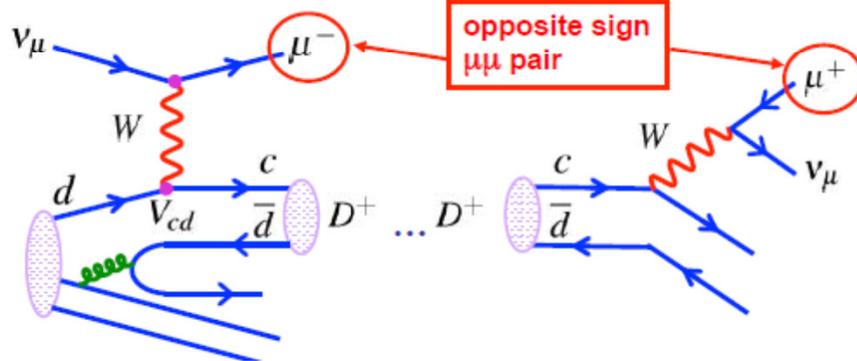
Measured in various collider experiments

$$|V_{cd}| = 0.230 \pm 0.011$$

Filling CKM - V

|V_cs| from semi-leptonic charmed meson decays

$$\begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \times & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

e.g.

$$\Gamma \propto |V_{cs}|^2$$

• **Precision limited by theoretical uncertainties**

$$|V_{cs}| = 0.957 \pm 0.017 \pm 0.093$$

experimental error   theory uncertainty

Make D^+D^- pairs from $e^+e^- \to \psi(3770) \to D^+D^-$
BR $D^+ \to K^0 e^+ \nu_e = (8.83 \pm 0.22) \%$

Filling CKM - VI

|V$_{cb}$| **from semi-leptonic B hadron decays**

$$\begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \times \\ \cdot & \cdot & \cdot \end{pmatrix}$$

e.g.
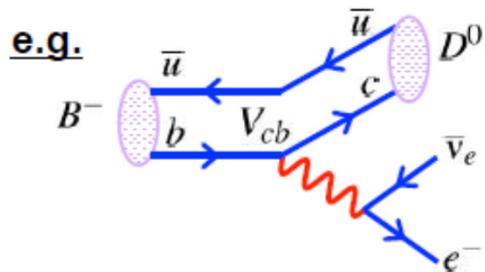
$$\Gamma \propto |V_{cb}|^2$$

$$\boxed{|V_{cb}| = 0.0416 \pm 0.0006}$$

$\overline{D}^0 \ell^+ \nu_\ell$ [a]   (2.23 ± 0.11) %

$$\frac{d\Gamma(b \to cl^-\overline{\nu}_l)}{dx} = \frac{G_F^2 m_b^5}{192\pi^2}|V_{\alpha b}|^2 \left[2x^2 \left(\frac{1-x-\zeta}{1-x}\right)^2 \left(3-2x+\zeta+\frac{2\zeta}{1-x}\right)\right], \zeta = \frac{m_c^2}{m_b^2}, x = \frac{2E_l}{m_b}$$

$$L = 10^{33} cm^{-2}s^{-1} \to L_{int} \sim 10^{38} cm^{-2}d^{-1}$$

$$\sigma_{B\overline{B}} \sim 1\, nb = 10^{-33} cm^2 \to R \sim 10^5 B\overline{B}\ d^{-1}$$

$$\to R_{dec} \sim 10^3\ D^0 e^- \overline{\nu}_e\ d^{-1} \to \frac{\sqrt{\sigma_{stat}}}{|V_{cb}|} \sim \frac{3\%}{\sqrt{T(days)}}$$

Filling CKM - VII

$|V_{ub}|$ **from semi-leptonic B hadron decays**

$$\begin{pmatrix} \cdot & \cdot & \times \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

e.g.
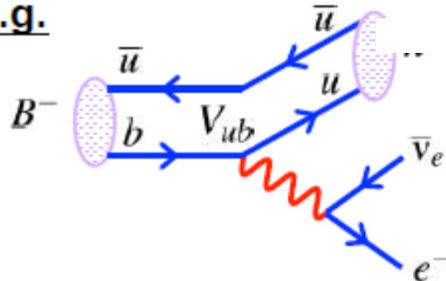
$$\Gamma \propto |V_{ub}|^2$$

$$|V_{ub}| = 0.0043 \pm 0.0003$$

$\pi^0 \ell^+ \nu_\ell$

$$(\quad 7.7 \quad \pm 1.2 \quad) \times 10^{-5}$$

$$\frac{\Gamma(b \rightarrow u l^- \overline{\nu}_l)}{\Gamma(b \rightarrow c l^- \overline{\nu}_l)} = \frac{|V_{ub}|^2}{|V_{cb}|^2} \left(\frac{f(m_u^2 / m_b^2)}{f(m_c^2 / m_b^2)} \right), \quad \text{compare rates}$$

$$L = 10^{33} cm^{-2} s^{-1} \rightarrow L_{\text{int}} \sim 10^{38} cm^{-2} d^{-1}$$

$$\sigma_{B\overline{B}} \sim 1\, nb = 10^{-33} cm^2 \rightarrow R \sim 10^5\, B\overline{B}\ \ d^{-1}$$

$$\rightarrow R_{dec} \sim 10\ \ \pi^0 e^- \overline{\nu}_e\ \ d^{-1} \rightarrow \frac{\sigma_{stat}}{|V_{ub}|} \sim \frac{30\%}{\sqrt{T(days)}}$$

Filling CKM - VIII

$$\left|V_{tb}\right|$$ **from single top production**
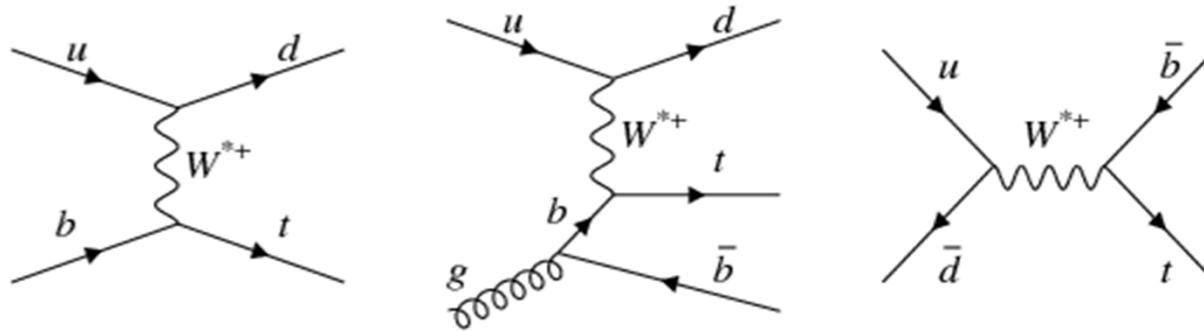
Filling CKM - IX

$$\boxed{|V_{td}|\ ,\ |V_{ts}|}$$ **from B_d , B_s oscillations**

Cannot rely on direct measurements of V_{td} , V_{ts} from t decays: Too small

Rather use loop diagrams of B_d , B_s oscillations
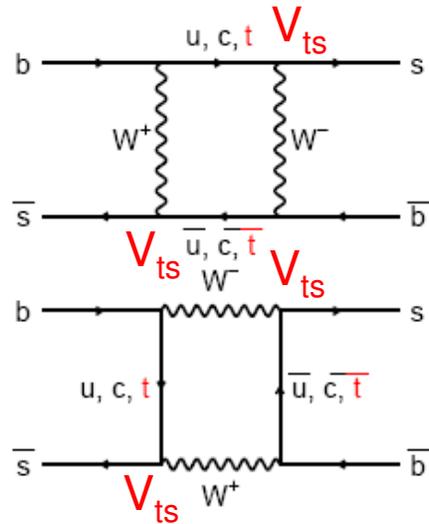
+   Similar for B_d , yielding V_{td}

CKM Triangles - I

V_{CKM} unitary: 9 unitarity conditions

Take 6 'off-diagonal' conditions:

(1) $V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0;$ (2) $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0;$

(3) $V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0;$ (4) $V_{ud}^*V_{cd} + V_{us}^*V_{cs} + V_{ub}^*V_{cb} = 0;$

(5) $V_{ud}^*V_{td} + V_{us}^*V_{ts} + V_{ub}^*V_{tb} = 0;$ (6) $V_{cd}^*V_{td} + V_{cs}^*V_{ts} + V_{cb}^*V_{tb} = 0;$

Each condition:

Sum of 3 complex numbers = 0

Complex number $\triangleq$ Vector in the complex plane

$\rightarrow$ Each condition $\sim$ 3 numbers should add to a closed triangle

CKM Triangles - II

Sides & Angles from experiment

Area: Same for all 6

$$A_{triangle} = \frac{1}{2} J_{CP} = \frac{1}{2} \text{Im}\left(V_{ij} V_{kl} V_{il}^* V_{kj}^*\right); \qquad i \neq k, j \neq l;$$

$$J_{CP} = s_{12} s_{13} s_{23} c_{12} c_{13}^2 c_{23} s_{\delta_{13}} \approx A^2 \lambda^6 \eta$$

$$\rightarrow J_{CP} = \text{Nice measure of } C\!\!\!/P$$

Example: Most common unitary triangle

Im

$V_{ud} V_{ub}^*$ $\alpha = \phi_2$ $V_{td} V_{tb}^*$

$\gamma = \phi_3$ $\beta = \phi_1$

$V_{cd} V_{cb}^*$

Re

$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$$

Wolfenstein approximation:

$$A\lambda^3 \left(1 - \frac{\lambda^2}{2}\right)(\rho + i\eta) \ - \ A\lambda^3 \left[1 + A^2 \lambda^4 (\rho + i\eta)\right] \ + \ A\lambda^3 \left[1 - (\rho + i\eta)\left(1 - \frac{\lambda^2}{2}\right)\right] = 0$$
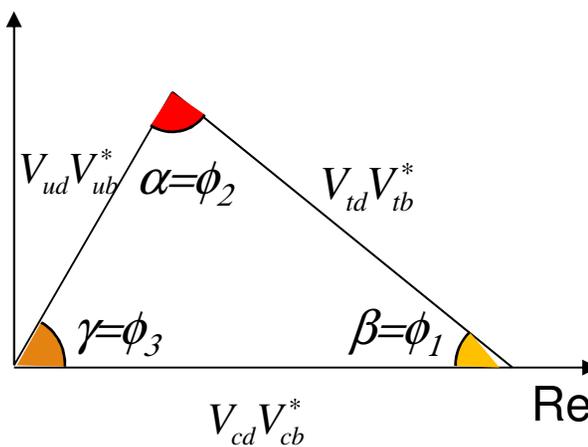
CKM Triangles - III

$$\begin{cases} V_{ud}V_{ub}^* = A\lambda^3\left(1-\dfrac{\lambda^2}{2}\right)(\rho+i\eta) \\ V_{cd}V_{cb}^* = -A\lambda^3 \\ V_{td}V_{tb}^* = A\lambda^3\left[1-(\rho+i\eta)\right] \end{cases}$$

Normalize to $V_{cd}V_{cb}^* = -A\lambda^3 \equiv 1$;

Ignore overall $-$ signs

$$V_{ud}V_{ub}^* = \left(1-\frac{\lambda^2}{2}\right)(\rho+i\eta) \approx \rho+i\eta$$

$$V_{cd}V_{cb}^* = 1$$

$$V_{td}V_{tb}^* \approx 1-(\rho+i\eta)$$
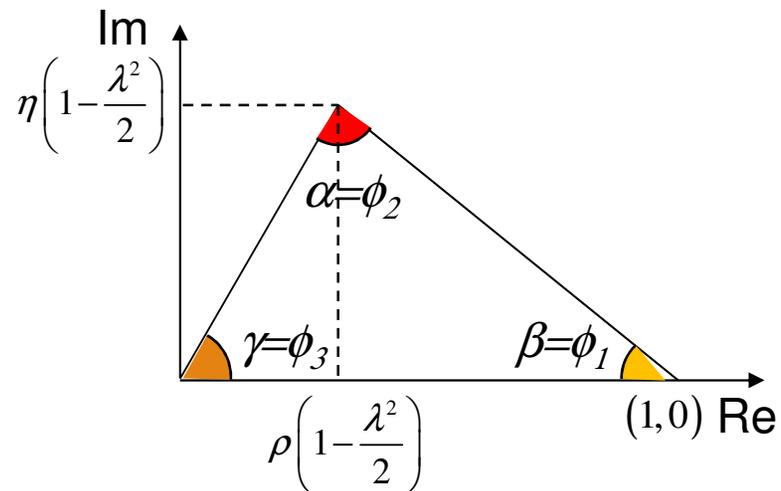
CKM Triangles - IV

Recent fit by CKMFitter group:

Observable	Central ± 1 σ	± 2 σ	± 3 σ
A	0.812 [+0.015 -0.022]	0.812 [+0.025 -0.031]	0.812 [+0.035 -0.039]
λ	0.22543 [+0.00059 -0.00095]	0.2254 [+0.0010 -0.0019]	0.2254 [+0.0013 -0.0027]
ρbar	0.145 [+0.027 -0.027]	0.145 [+0.046 -0.040]	0.145 [+0.057 -0.050]
ηbar	0.343 [+0.015 -0.015]	0.343 [+0.030 -0.026]	0.343 [+0.044 -0.035]
J [10^{-5}]	2.96 [+0.18 -0.14]	2.96 [+0.32 -0.19]	2.96 [+0.46 -0.23]
α [deg]	91.1 [+4.3 -4.3]	91.1 [+7.1 -6.2]	91.1 [+8.8 -7.8]
α [deg] (meas. not in the fit)	95.9 [+2.2 -5.6]	95.9 [+3.6 -10.9]	95.9 [+5.0 -12.8]
α [deg] (dir. meas.)	88.7 [+4.6 -4.2]	88.7 [+9.4 -8.5]	89 [+21 -13]
β [deg]	21.85 [+0.80 -0.77]	21.9 [+1.6 -1.3]	21.9 [+2.5 -1.8]
β [deg] (meas. not in the fit)	27.5 [+1.2 -1.4]	27.5 [+1.9 -3.9]	27.5 [+2.6 -6.8]
β [deg] (dir. meas.)	21.38 [+0.79 -0.77]	21.4 [+1.6 -1.5]	21.4 [+2.4 -2.3]
γ [deg]	67.1 [+4.3 -4.3]	67.1 [+6.1 -7.0]	67.1 [+7.6 -8.5]
γ [deg] (meas. not in the fit)	67.2 [+4.4 -4.6]	67.2 [+6.1 -7.2]	67.2 [+7.6 -8.7]
γ [deg] (dir. meas.)	66 [+12 -12]	66 [+23 -22]	66 [+36 -30]

K Oscillations - I

First among a host of astonishing quantum mechanical oddities

$$\left|K^0\right\rangle = \left|d\bar{s}\right\rangle \qquad S = +1$$

$$\left|\bar{K}^0\right\rangle = \left|\bar{d}s\right\rangle \qquad S = -1$$

$$P\left|K^0\right\rangle = -\left|K^0\right\rangle \quad \text{Pseudoscalar}$$

$$P\left|\bar{K}^0\right\rangle = -\left|\bar{K}^0\right\rangle \quad \text{Pseudoscalar}$$

$$C\left|K^0\right\rangle = \left|\bar{K}^0\right\rangle \qquad \text{Not a C eigenstate}$$

$$C\left|\bar{K}^0\right\rangle = \left|K^0\right\rangle \qquad \text{Not a C eigenstate}$$

$\rightarrow$ Make C eigenstates:

$$\left|K_1^0\right\rangle = \frac{1}{\sqrt{2}}\left(\left|K^0\right\rangle - \left|\bar{K}^0\right\rangle\right) \rightarrow C\left|K_1^0\right\rangle = C\left[\frac{1}{\sqrt{2}}\left(\left|K^0\right\rangle - \left|\bar{K}^0\right\rangle\right)\right] = \frac{1}{\sqrt{2}}\left(\left|\bar{K}^0\right\rangle - \left|K^0\right\rangle\right) = -\left|K_1^0\right\rangle$$

$$\left|K_2^0\right\rangle = \frac{1}{\sqrt{2}}\left(\left|K^0\right\rangle + \left|\bar{K}^0\right\rangle\right) \rightarrow C\left|K_2^0\right\rangle = \left|K_2^0\right\rangle$$

K Oscillations - II

Kaons : Just weak decays $(\leftarrow$ Lightest strange hadron$)$

C, P not conserved by weak processes

CP *almost* conserved by weak processes $\rightarrow$ Take it as good for the moment

$\rightarrow$ Focus on CP as a symmetry for weak processes

CP eigenstates:

$$CP\left|K_1^0\right\rangle = CP\left[\frac{1}{\sqrt{2}}\left(\left|K^0\right\rangle - \left|\bar{K}^0\right\rangle\right)\right] = C\left[\frac{1}{\sqrt{2}}\left(-\left|K^0\right\rangle + \left|\bar{K}^0\right\rangle\right)\right] = +\left|K_1^0\right\rangle \quad CP = +1$$

$$CP\left|K_2^0\right\rangle = -\left|K_2^0\right\rangle \qquad\qquad\qquad CP = -1$$

Observe : K_1^0, K_2^0 CP eigenstates, like photon, π^0

$\rightarrow$ Different particles

K Oscillations - III

K^0: Many different decay modes, including weak decays into pions

Consider first decays into 2 π's:

$$K^0 \rightarrow \pi^0 \pi^0$$
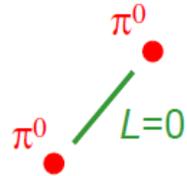
$$J^P: \quad 0^- \rightarrow 0^- + 0^-$$

Ang. mom. conservation

$$\Rightarrow \quad L = 0$$

$$\Rightarrow \quad P(\pi^0\pi^0) = -1.-1.(-1)^L = (-1)^L = +1$$

π^0 is eigenstate of C : $\quad \hat{C}|\pi^0\rangle = +|\pi^0\rangle$

$$\Rightarrow \quad C(\pi^0\pi^0) = +1.+1 = +1$$

$$K^0 \rightarrow \pi^+ \pi^-$$

Still have $L = 0$

$$\Rightarrow \quad P(\pi^+\pi^-) = -1.-1.(-1)^L = (-1)^L = +1$$

C and P operations have identical effect :

(no spins involved)
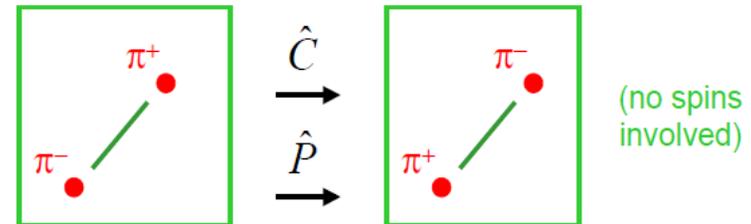
$$\Rightarrow \quad C(\pi^+\pi^-) = P(\pi^+\pi^-) = (-1)^L = +1$$

$$\Rightarrow \quad \boxed{CP = +1} \quad \text{for both } \pi^+\pi^- \text{ and } \pi^0\pi^0$$

K Oscillations - IV

Consider then decays into 3 π's:

$K^0 \to \pi^0 \pi^0 \pi^0$
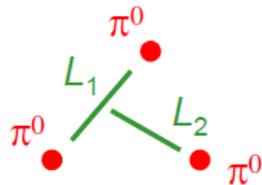
$J^P: \quad 0^- \to 0^- + 0^- + 0^-$

Ang. mom. conservation

$$\Longrightarrow \quad L_1 \oplus L_2 = 0$$

$$\Longrightarrow \quad L_1 = L_2$$

$$\Longrightarrow \quad P(\pi^0 \pi^0 \pi^0) = -1.-1.-1.(-1)^{L_1}.(-1)^{L_2} = -1$$

$$C(\pi^0 \pi^0 \pi^0) = +1.+1.+1 = +1$$

$K^0 \to \pi^+ \pi^- \pi^0$
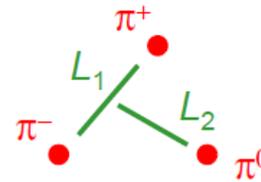
As above: $\quad L_1 = L_2$

$$\Longrightarrow \quad P(\pi^+ \pi^- \pi^0) = -1.-1.-1.(-1)^{L_1}.(-1)^{L_2} = -1$$

$$C(\pi^+ \pi^- \pi^0) = +1.C(\pi^+ \pi^-) = +1.(-1)^{L_1} = +1$$

Experimentally: $L_1 = 0$ from study
of angular distributions of π^+, π^-

$$\Longrightarrow \quad \boxed{CP = -1} \quad \text{for both} \quad \pi^+ \pi^- \pi^0 \quad \text{and} \quad \pi^0 \pi^0 \pi^0$$

K Oscillations - V

If *CP* is conserved in weak processes:

$$\left.\begin{array}{l} K_1^0 \to \pi\pi \\ K_2^0 \to \pi\pi\pi \end{array}\right\} \; Exclusively$$

Summary so far about neutral K states:

Production (by strong interaction): $\left|K^0\right\rangle, \left|\bar{K}^0\right\rangle$

Decay (by weak interaction): $\left|K_1^0\right\rangle, \left|K_2^0\right\rangle$

$$m_{\left|K^0\right\rangle} = m_{\left|\bar{K}^0\right\rangle} \approx m_{\left|K_1^0\right\rangle} \approx m_{\left|K_2^0\right\rangle} \approx 498 \; MeV$$

Expect, and find:

$K_1^0 \to \pi\pi$ Fast: Larger phase space etc $\to \tau_1 = 0.9 \; 10^{-10} s \;\; 'K \; short'$

$K_2^0 \to \pi\pi\pi$ Slow: Smaller phase space etc $\to \tau_2 = 0.5 \; 10^{-7} s \;\; 'K \; long'$

K Oscillations - VI

Provisionally identify:

$$K_S \equiv K_1^0 \left(\to \pi\pi \right) \qquad \tau_S = 0.9 \; 10^{-10} \, s \quad \text{'} K \; short \text{'} \quad CP = +1$$

$$K_L \equiv K_2^0 \left(\to \pi\pi\pi \right) \qquad \tau_L = 0.5 \; 10^{-7} \, s \quad \text{'} K \; long \text{'} \quad CP = -1$$

Therefore:

$$K_S \equiv K_1^0, K_L \equiv K_2^0 : \begin{array}{ll} \text{Different} & CP \\ \text{Different} & \text{lifetime} \end{array}$$

Also: Different mass!

Old fashioned (but simple) argument:

Different virtual weak couplings, 2π vs 3π

$\to$ Different corrections to the mass
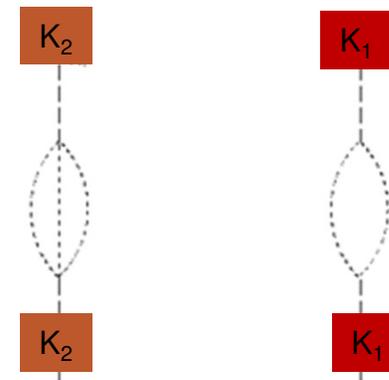
K Oscillations - VII

Taking a neutral *K* beam produced by strong interaction, expect qualitatively

2π decays near the start of a K^0 beam
3π decays near the end

log intensity

$K_S \to \pi\pi$

$K_L \to \pi\pi\pi$

distance
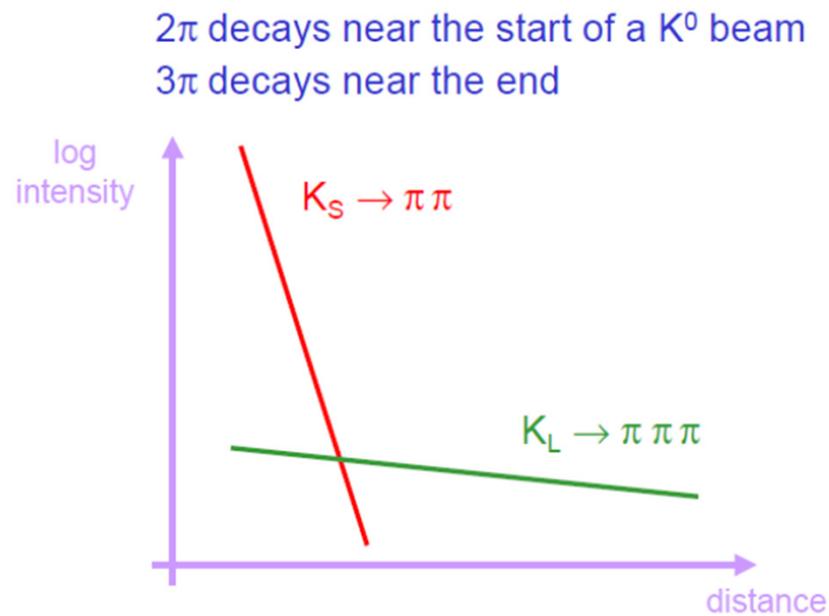
K Oscillations - VIII

Production: Strong interaction $\rightarrow$ Strangeness conserved

Neglect weak interaction in production process

Strongly produced neutral K either K^0 or $\overline{K}^0$

$\rightarrow$ Either K^0 or $\overline{K}^0$ as *initial condition* for the K wave function

Time evolution : Weak interaction $\rightarrow$ Strangeness *not* conserved

Neglect strong interaction in time evolution

Neither P or C conserved by weak interaction; $CP\ (provisionally)$ conserved

$\rightarrow$ Propagate CP eigenstates K_S, K_L

Take a definite production process:

$$\pi^- p \rightarrow \Lambda^0 K^0$$
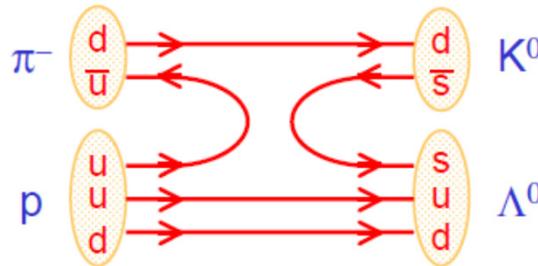
K Oscillations - IX

$$\psi(t=0):\left|K^0\right\rangle=\frac{1}{\sqrt{2}}\left(\left|K_L^0\right\rangle+\left|K_S^0\right\rangle\right)$$

$$\begin{cases}\left|K_L^0(t)\right\rangle=\left|K_L^0\right\rangle e^{-i\left(m_L-i\frac{\Gamma_L}{2}\right)t}, & \Gamma_L=\frac{1}{\tau_L}\\[4mm]\left|K_S^0(t)\right\rangle=\left|K_S^0\right\rangle e^{-i\left(m_S-i\frac{\Gamma_S}{2}\right)t}, & \Gamma_S=\frac{1}{\tau_S}\end{cases}$$

$$\rightarrow\psi(t)=\frac{1}{\sqrt{2}}\left(\left|K_L^0\right\rangle e^{-i\left(m_L-i\frac{\Gamma_L}{2}\right)t}+\left|K_S^0\right\rangle e^{-i\left(m_S-i\frac{\Gamma_S}{2}\right)t}\right)$$

$$\rightarrow\psi(t)=\frac{1}{\sqrt{2}}\left(\left[\frac{1}{\sqrt{2}}\left(\left|K^0\right\rangle+\left|\bar{K}^0\right\rangle\right)\right]e^{-i\left(m_L-i\frac{\Gamma_L}{2}\right)t}+\frac{1}{\sqrt{2}}\left(\left|K^0\right\rangle-\left|\bar{K}^0\right\rangle\right)\frac{1}{\sqrt{2}}e^{-i\left(m_S-i\frac{\Gamma_S}{2}\right)t}\right)$$

$$\rightarrow\psi(t)=\frac{1}{2}\left(\left|K^0\right\rangle\left[e^{-i\left(m_L-i\frac{\Gamma_L}{2}\right)t}+e^{-i\left(m_S-i\frac{\Gamma_S}{2}\right)t}\right]+\left|\bar{K}^0\right\rangle\left[e^{-i\left(m_L-i\frac{\Gamma_L}{2}\right)t}-e^{-i\left(m_S-i\frac{\Gamma_S}{2}\right)t}\right]\right)$$

K Oscillations - X

Time evolution of strangeness content of the beam:

Initial condition K^0

$$\Delta m = m_L - m_S$$

$$\rightarrow \begin{cases} I\left(K^0\right) = \dfrac{1}{4}\left| e^{-i\left(m_L - i\frac{\Gamma_L}{2}\right)t} + e^{-i\left(m_S - i\frac{\Gamma_S}{2}\right)t} \right|^2 = \dfrac{1}{4}\left[e^{-\Gamma_L t} + e^{-\Gamma_S t} + 2e^{-\frac{(\Gamma_L + \Gamma_S)}{2}t} \cos \Delta m t \right] \\[4mm] I\left(\bar{K}^0\right) = \dfrac{1}{4}\left| e^{-i\left(m_L - i\frac{\Gamma_L}{2}\right)t} - e^{-i\left(m_S - i\frac{\Gamma_S}{2}\right)t} \right|^2 = \dfrac{1}{4}\left[e^{-\Gamma_L t} + e^{-\Gamma_S t} - 2e^{-\frac{(\Gamma_L + \Gamma_S)}{2}t} \cos \Delta m t \right] \end{cases}$$

$\rightarrow$ Strangeness oscillations

Detected in many ways, for example by semileptonic modes:

$K^0 \rightarrow \pi^- e^+ \nu_e$
$\bar{K}^0 \rightarrow \pi^+ e^- \bar{\nu}_e$
$\quad \Delta Q = \Delta S$ rule: Unambiguous strangeness assignment from decay products

# K Oscillations - XI

Interference!
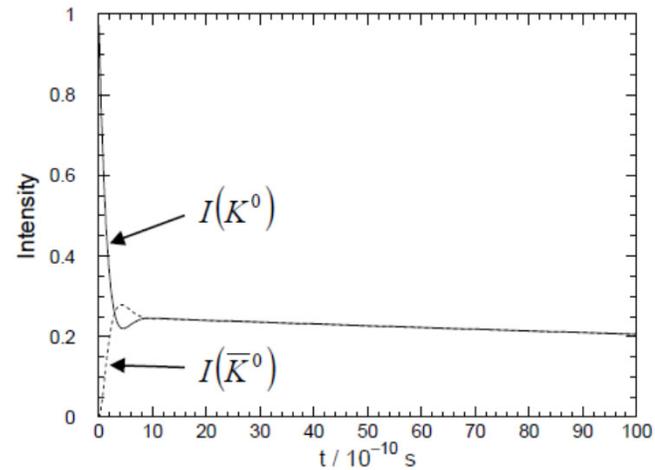
Expanded timescale:
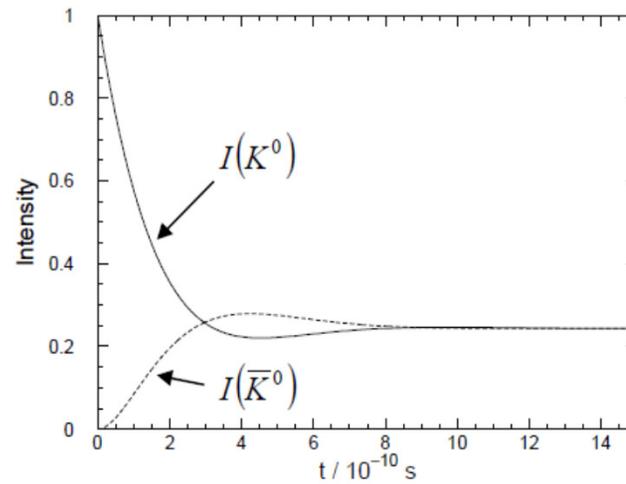
K Oscillations - XII

…And it's true!

Semileptonic decays

$$\frac{N_{\pi^+} - N_{\pi^-}}{N_{\pi^+} + N_{\pi^-}}$$

CHARGE ASYMMETRY IN THE DECAYS $K^0 \longrightarrow \pi^{\pm} e^{\mp} \nu$

Asymmetry $\neq 0$ at large t:
CP violation

!!!
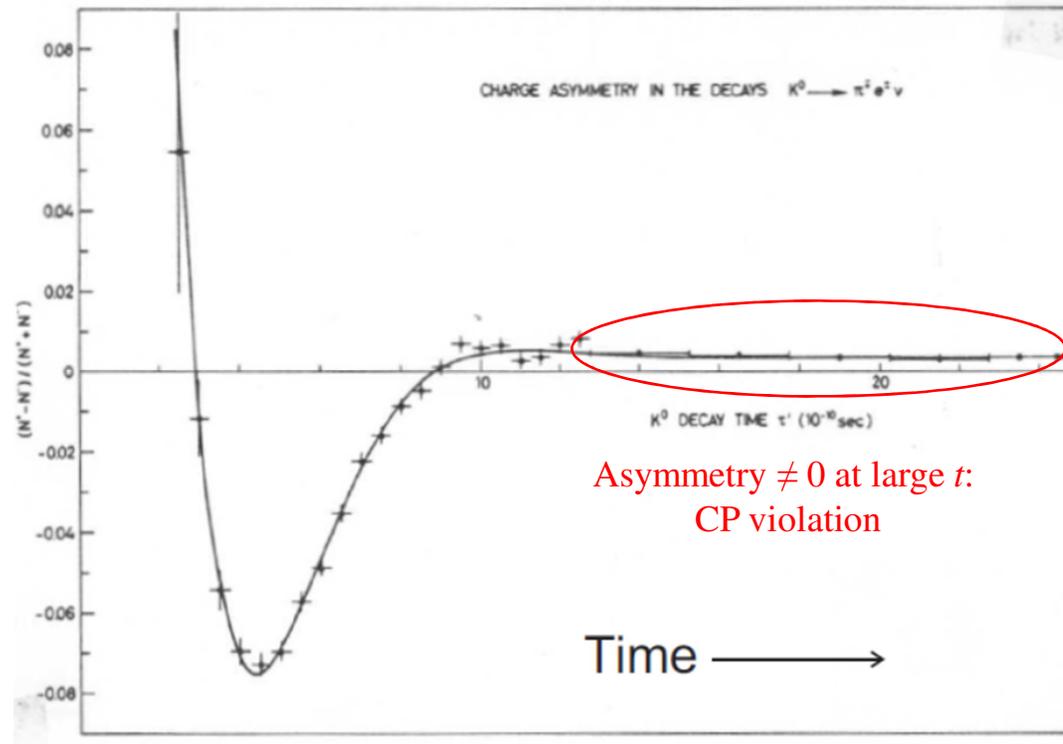
K Oscillations - XIII

CPLEAR experiment fit to Δm

$$R_+ \equiv \Gamma(K^0_{t=0} \to \pi^- e^+ \nu_e)$$

$$R_- \equiv \Gamma(K^0_{t=0} \to \pi^+ e^- \overline{\nu}_e)$$

$$\overline{R}_- \equiv \Gamma(\overline{K}^0_{t=0} \to \pi^+ e^- \overline{\nu}_e)$$

$$\overline{R}_+ \equiv \Gamma(\overline{K}^0_{t=0} \to \pi^- e^+ \nu_e)$$

$$A_{\Delta m} = \frac{(R_+ + \overline{R}_-) - (R_- + \overline{R}_+)}{(R_+ + \overline{R}_-) + (R_- + \overline{R}_+)}$$

$$A_{\Delta m} = \frac{2e^{-(\Gamma_S + \Gamma_L)t/2} \cos \Delta mt}{e^{-\Gamma_S t} + e^{-\Gamma_L t}}$$

$$\boxed{\Delta m = 3.485 \times 10^{-15}\,\text{GeV}}$$

!!!

Neutral kaon decay time [τ_S]
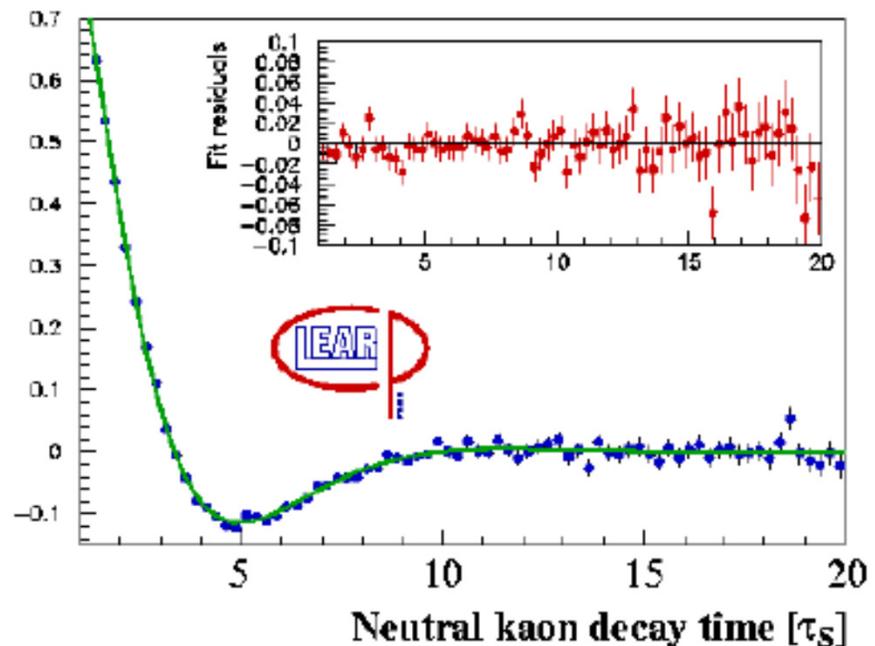
K Oscillations - XIV

Observe:

$$T\left(K^0 \leftrightarrow \bar{K}^0\right) = \frac{2\pi}{\Delta m}$$

$$\rightarrow T\left(K^0 \leftrightarrow \bar{K}^0\right) = \frac{2\pi}{3.49\ 10^{-12}\ MeV} = \frac{2\pi}{3.49\ 10^{-12}\ MeV} \underbrace{6.58 10^{-22}\ MeVs}_{\hbar} \simeq 1.18\ ns$$

$$\tau_S \simeq 8.95 10^{-11}\ s$$

$$\rightarrow \frac{T}{\tau_S} \approx 13.3$$

$\rightarrow$ Just a fraction of a single oscillation within a K_S lifetime

K Oscillations - XV

Summary of decay rates (CP conserved):
2 lifetimes (*2π, 3π*)
Strangeness oscillations (Semileptonic)
2 masses   (*2π, 3π*)
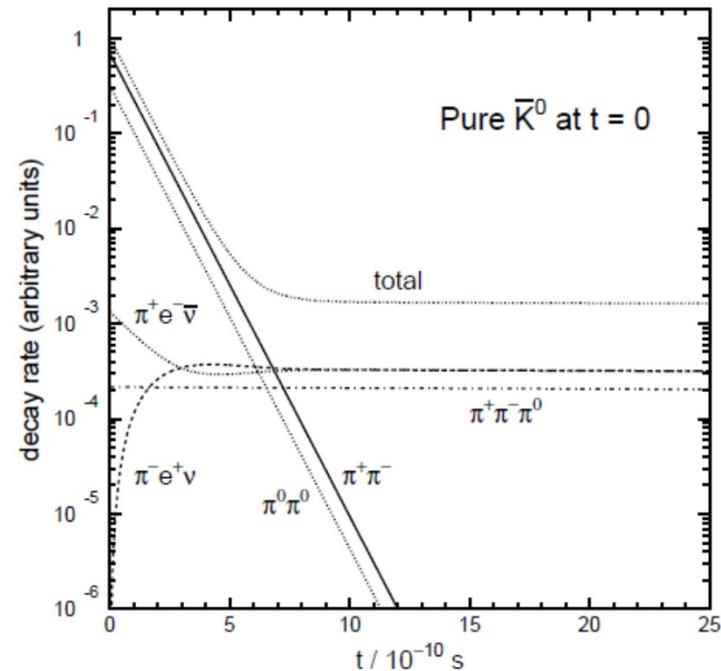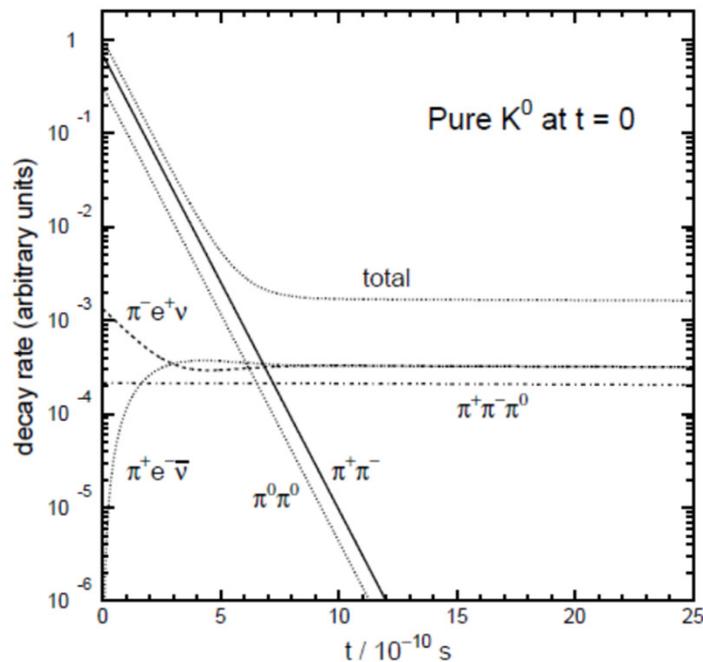
K_S Regeneration - I

Production reactions (e.g. at low energy):

$K^0:\quad \pi^- + p \rightarrow K^0 + \Lambda^0$

$\bar{K}^0:\quad \pi^+ + p \rightarrow \bar{K}^0 + K^+ + p$ Higher threshold
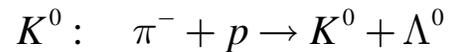
Take first reaction $\rightarrow$ Initially pure K^0 beam

After several τ_s: K_S component off $\rightarrow$ Pure K_L beam

Introduce some material in the beam path: Funny effect!

K_S Regeneration - II

Total cross section different for $K^0, \bar{K}^0$:

Indeed, e.g.

$$\bar{K}^0 + p \rightarrow \pi^+ + \Lambda^0$$
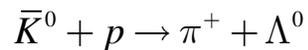
is strictly forbidden for K^0

$$\rightarrow \sigma_{K^0} \neq \sigma_{\bar{K}^0}$$

Remembering the "OpticalTheorem":

$$\sigma_{tot} = \frac{4\pi}{k} \operatorname{Im} f(0), \ f(0) \text{ forward scattering amplitude}$$

$$\rightarrow f_{K^0}(0) \neq f_{\bar{K}^0}(0)$$

Take forward scattering $(= propagation)$ of our pure K_L beam:

$$|K_L\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\bar{K}^0\rangle\right) \rightarrow \frac{1}{\sqrt{2}}\left(A(f_{K^0})|K^0\rangle + B(f_{\bar{K}^0})|\bar{K}^0\rangle\right) \neq |K_L\rangle$$

$$|K_L\rangle \rightarrow a|K_L\rangle + b|K_S\rangle, \quad |a|^2 + |b|^2 = 1$$

$\rightarrow$ A $|K_S\rangle$ component has been regenerated by the material!

K_S Regeneration - III

Piccioni et al. – Berkeley ≈ 1956

- Schema dell'esperimento:

K^0 670 Mev/c

π⁻

1.1 Gev/c   6,8 m

Bersaglio H

Camera a bolle a propano

K^0_2

Lastra Fe K^0_2, K^0_1

Incident K₂ beam

Propane

Pb   Fe
       Regenerated
       K, meson

30"   19"

Propane

$\overrightarrow{p_{k_2}}$ $\overrightarrow{p_1}$ π⁺ $\overrightarrow{p}_{12}$ $= \overrightarrow{p_{k_1}}$

θ

$\overrightarrow{p_2}$ π⁻

Dalle tracce è possibile misurare gli impulsi dei due π, dalla cui somma vettoriale si ricava la direzione del K rigenerato, e quindi l'angolo θ

Combined

15 cm

4 cm

cos θ
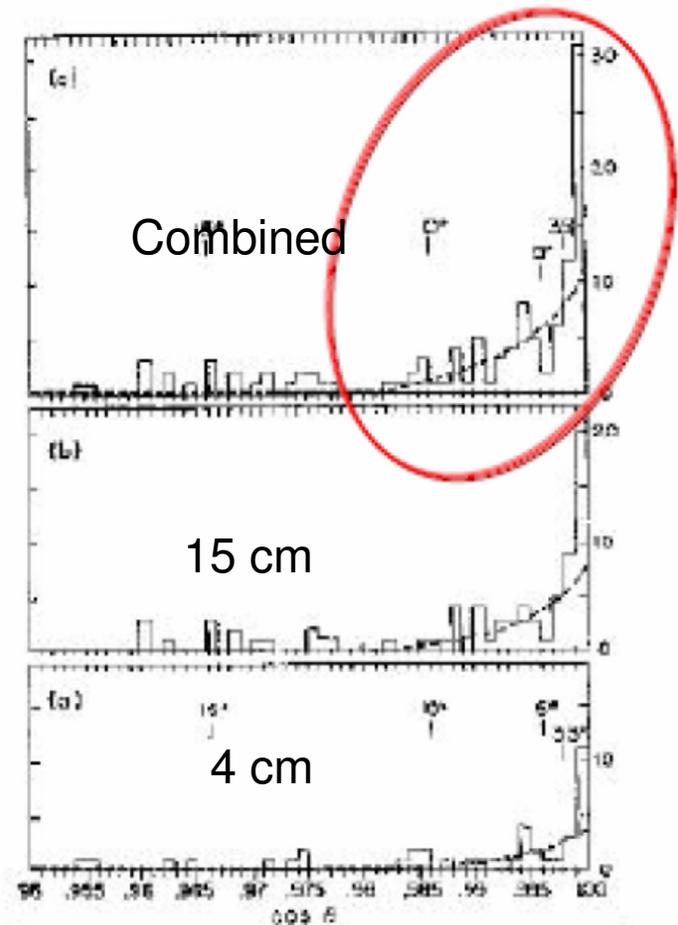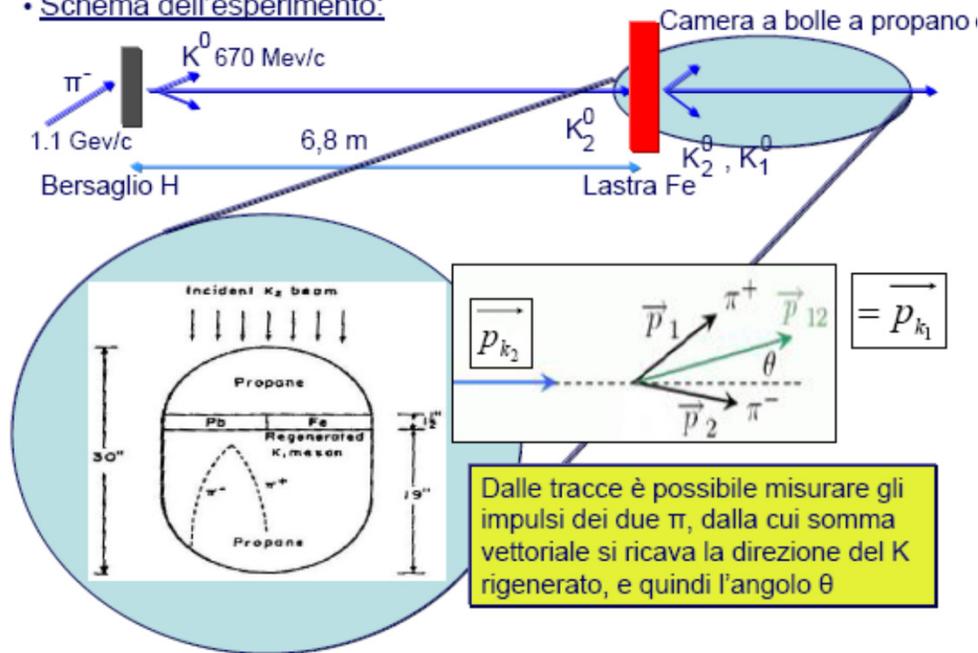
K_S Regeneration - IV

KTEV - Fermilab ≈ 2000
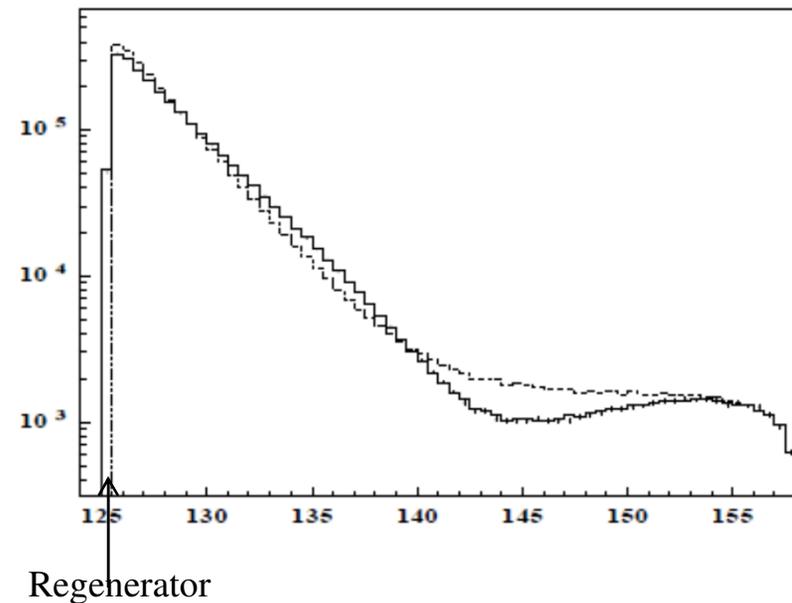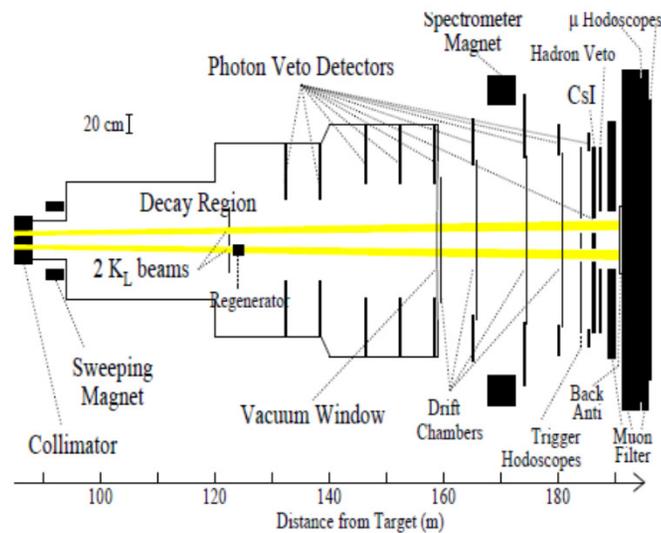
Regenerator

Strong K_S regeneration signalled by 2π decays with τ_s lifetime

[Large interference observed in 2π rate:

Regenerated K_S component interfering with CP violating, 2π decay of K_L beam]

K *CP* Violation - I

$K_2 \to \pi^+ \pi^-$ decay observed in 1964; $K_2 \to \pi^0 \pi^0$ decay also observed

Small $BR \sim 10^{-3}$

PLAN VIEW
1 ft

REGION OF
OBSERVED DECAYS

WATER
CERENKOV
COUNTER

SCINTILLATOR

MAGNET

K_2^0

COLLIMATOR

22°
22°

SPARK CHAMBER

57 ft TO
INTERNAL
TARGET

HELIUM BAG

MAGNET
SCINTILLATOR

WATER
CERENKOV
COUNTER

Fondo principale: $\pi^+ \pi^- \pi^0$

θ

Segnale cercato: $\pi^+ \pi^-$

$484 < m^* < 494$

$K_2 \to \pi^+ \pi^-$

Effect is tiny:
about 2/1000

$494 < m^* < 504$

$504 < m^* < 514$

NUMBER OF EVENTS

0.9996   0.9997   0.9998   0.9999   1.0000

cos θ
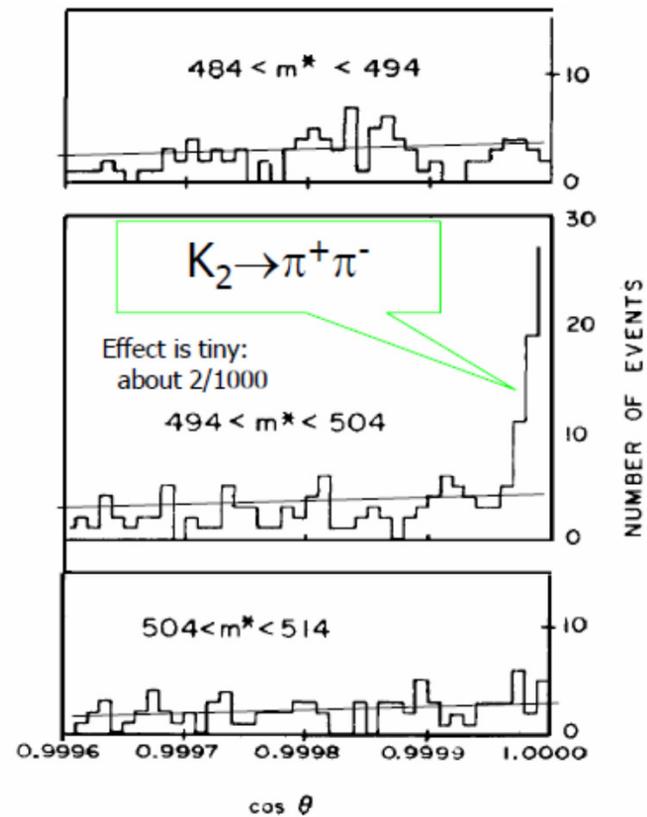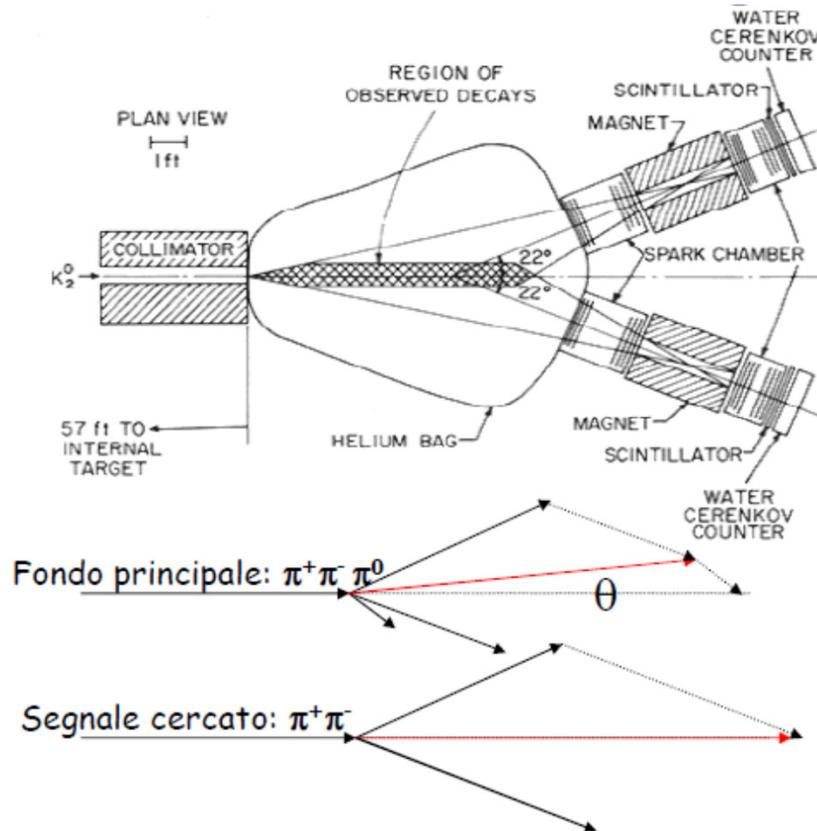
K *CP* Violation - II

Three possible mechanisms:

1) K_L, K_S not *CP* eigenstates - Decay *CP* conserving

$$\rightarrow \left| K_L^0 \right\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}} \left(\left| K_2^0 \right\rangle + \varepsilon \left| K_1^0 \right\rangle \right), \left| K_S^0 \right\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}} \left(\left| K_1^0 \right\rangle - \varepsilon \left| K_2^0 \right\rangle \right)$$

$K_L^0 \rightarrow \pi\pi$ accounted for by

$$\left| K_L^0 \right\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}} \left(\left| K_2^0 \right\rangle + \varepsilon \left| K_1^0 \right\rangle \right)$$
$$\underset{\pi\pi\pi}{\downarrow} \qquad \underset{\pi\pi}{\downarrow}$$

Mixing $\cancel{CP}$: Measured by small parameter ε

2) Decay *CP* violating - K_L, K_S *CP* eigenstates

$$\left| K_L^0 \right\rangle = \left| K_2^0 \right\rangle$$

Direct $\cancel{CP}$: Measured by *very* small parameter ε'

3) Interference between mixing and decay

K *CP* Violation - III

Define:

$$\left|\eta_{+-}\right| \equiv \frac{\Gamma(K_L^0 \to \pi^+\pi^-)}{\Gamma(K_S^0 \to \pi^+\pi^-)} = (2.276 \pm 0.017) \times 10^{-3}$$

$$\left|\eta_{00}\right| \equiv \frac{\Gamma(K_L^0 \to \pi^0\pi^0)}{\Gamma(K_S^0 \to \pi^0\pi^0)} = (2.262 \pm 0.017) \times 10^{-3}$$

Expect:

Case 1) $\to \eta_{+-} = \eta_{00}$

Case 2) $\to \eta_{+-} \neq \eta_{00}$

Generally (see later):

$$\eta_{+-} = \varepsilon + \varepsilon'$$

$$\eta_{+-} = \varepsilon - 2\varepsilon'$$

$$\to \varepsilon' \ll \varepsilon, \text{ must be very small}$$

K *CP* Violation - IV

Focus on mixing $\cancel{CP}$, ignore direct $\cancel{CP}$ for the moment

$\varepsilon = |\varepsilon| e^{i\varphi}$ Measuring mixing $\cancel{CP}$

For a neutral beam initially pure $K^0 / \overline{K}^0$:

$\pi\pi$ decay rate as a function of distance

$$I\left(K^0;t\right) = \frac{N}{2}\left(1 - 2\operatorname{Re}(\varepsilon)\right)\left[e^{-\Gamma_S t} + \underbrace{|\varepsilon|^2 e^{-\Gamma_L t}}_{K_L \text{ contribution}} + \underbrace{2|\varepsilon| e^{-\frac{\Gamma_L + \Gamma_S}{2} t} \cos\left(\Delta m t - \varphi\right)}_{\text{Interference}}\right]$$

$$I\left(\overline{K}^0;t\right) = \frac{N}{2}\left(1 + 2\operatorname{Re}(\varepsilon)\right)\left[e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t} - 2|\varepsilon| e^{-\frac{\Gamma_L + \Gamma_S}{2} t} \cos\left(\Delta m t - \varphi\right)\right]$$

$\rightarrow$ Get $|\varepsilon|, \varphi, \Delta m$

$|\varepsilon| = (2.285 \pm 0.019) \times 10^{-3}$

$\phi = (43.5 \pm 0.6)°$

K *CP* Violation - V

...Quite correct!

Pure interference term

ππ decay rate

curve: no interference term
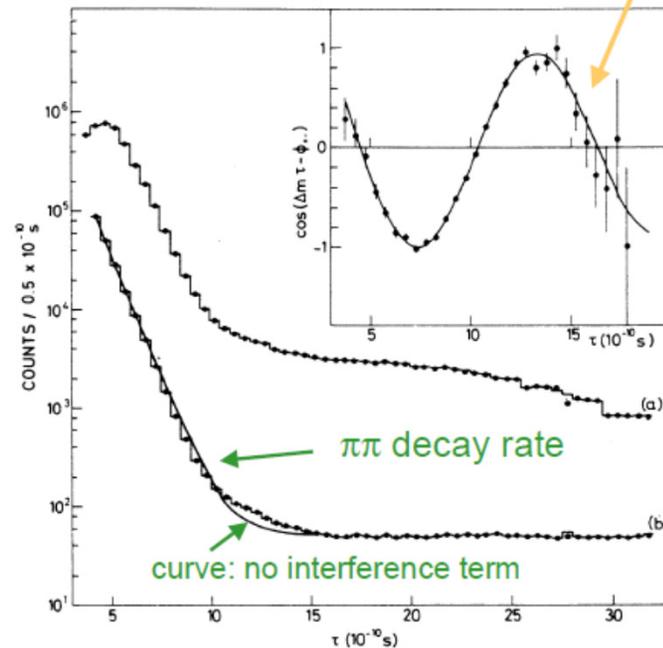
Similar to regeneration data, but : No regenerator!

Interference between K_L and K_S in 2π decay

$\rightarrow K_L$ and K_S states not orthogonal: Both have a K_1 component

K *CP* Violation - VI

Neutral beam at large distance from production target: Pure K_L

$$\left|K_L^0\right\rangle = \frac{1}{\sqrt{1+\left|\varepsilon\right|^2}}\left(\left|K_2^0\right\rangle + \varepsilon\left|K_1^0\right\rangle\right)$$

$$\rightarrow \left|K_L^0\right\rangle = \frac{1}{\sqrt{2\left(1+\left|\varepsilon\right|^2\right)}}\left[(1+\varepsilon)\left|K^0\right\rangle + (1-\varepsilon)\left|\bar{K}^0\right\rangle\right]$$

$$\rightarrow \left|K_S^0\right\rangle = \frac{1}{\sqrt{2\left(1+\left|\varepsilon\right|^2\right)}}\left[(1+\varepsilon)\left|K^0\right\rangle - (1-\varepsilon)\left|\bar{K}^0\right\rangle\right]$$

Take semileptonic decays, e.g. K_{e3}:

$$K^0 \rightarrow \pi^- e^+ \nu_e$$
$$\bar{K}^0 \rightarrow \pi^+ e^- \bar{\nu}_e$$
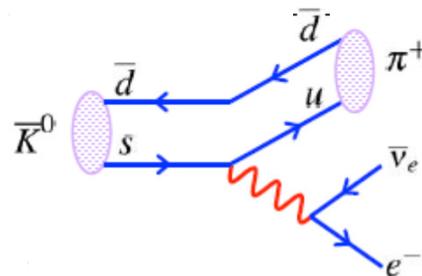
Observe:

$$CP\left|K^0\right\rangle = \left|\bar{K}^0\right\rangle$$
$$CP\left|\pi^- e^+ \nu_e\right\rangle = \left|\pi^+ e^- \bar{\nu}_e\right\rangle$$

$\rightarrow$ No *CP* eigenstates

Define CP violation asymmetry for semileptonic decays (K_{e3})

$$\delta = \frac{\Gamma\left(K_L \to \pi^- e^+ \nu_e\right) - \Gamma\left(K_L \to \pi^+ e^- \overline{\nu}_e\right)}{\Gamma\left(K_L \to \pi^- e^+ \nu_e\right) + \Gamma\left(K_L \to \pi^+ e^- \overline{\nu}_e\right)}$$

$$\Gamma\left(K_L \to \pi^+ e^- \overline{\nu}_e\right) \propto \left|\left\langle K^0 | K_L\right\rangle\right|^2 \propto |1-\varepsilon|^2$$

$$\Gamma\left(K_L \to \pi^+ e^- \overline{\nu}_e\right) \propto \left|\left\langle \overline{K}^0 | K_L\right\rangle\right|^2 \propto |1+\varepsilon|^2$$

$$|1\pm\varepsilon|^2 = (1\pm\varepsilon)(1\pm\varepsilon^*) \approx 1 + \varepsilon + \varepsilon^* = 1 \pm 2\,\mathrm{Re}\,\varepsilon$$

$$\to \delta \approx \frac{(1+2\,\mathrm{Re}\,\varepsilon) - (1-2\,\mathrm{Re}\,\varepsilon)}{(1+2\,\mathrm{Re}\,\varepsilon) + (1-2\,\mathrm{Re}\,\varepsilon)} = \frac{4\,\mathrm{Re}\,\varepsilon}{2} = 2\,\mathrm{Re}\,\varepsilon = 2|\varepsilon|\cos\phi$$

$$\to \delta \approx 3.21\,10^{-3} \quad \text{calculated by taking } \varepsilon \text{ from } \pi\pi$$

Measured: $(3.27 \pm 0.012)10^{-3}$
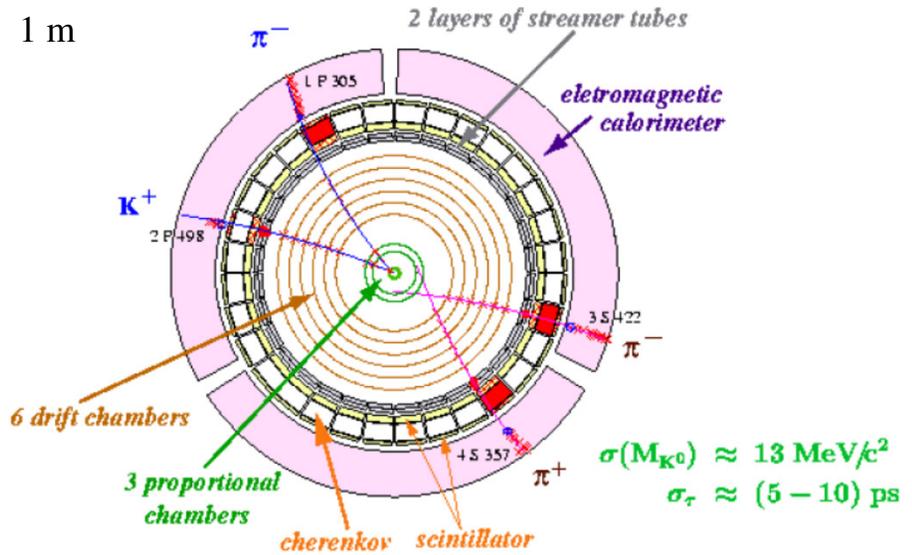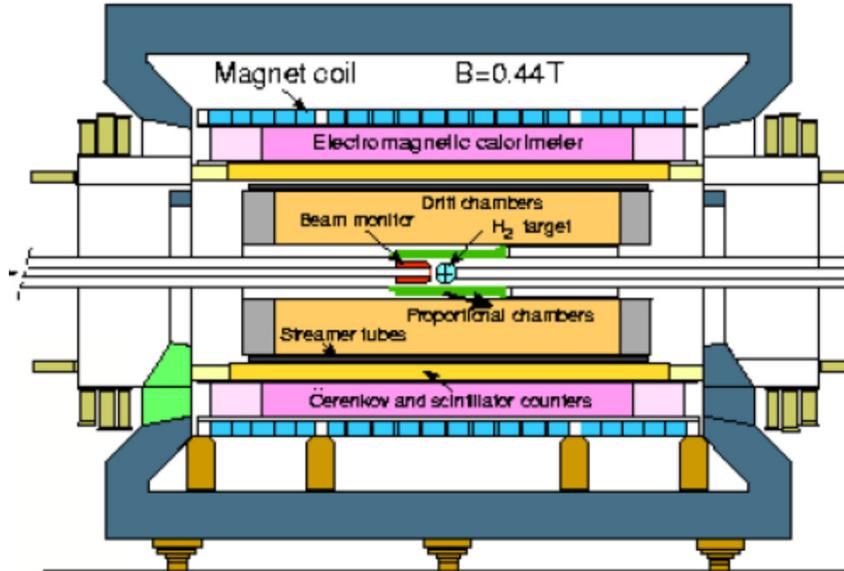
CPLEAR – CERN '90s

K *CP* Violation - IX

Experiment CPLEAR

(CERN $\bar{p}$ Low Energy Accumulator Ring - LEAR) $\rightarrow \sim 1995$

Use reactions:

$$p\bar{p} \to K^- \pi^+ K^0$$

$$p\bar{p} \to K^+ \pi^- \bar{K}^0$$

and semileptonic decays

$$K^0 \to \pi^- e^+ \nu_e$$

$$\bar{K}^0 \to \pi^+ e^- \bar{\nu}_e$$
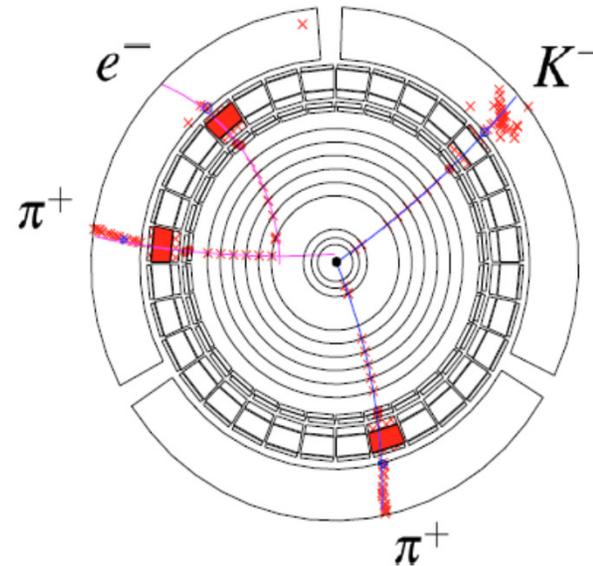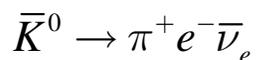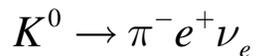
Strangeness of $\dfrac{produced}{decaying}$ *K* state: Tagged *unambiguously*

K CP Violation - X

Strangeness oscillations in presence of $\cancel{CP}$:

$$R_+ = \Gamma\left(K^0_{t=0} \to \pi^- e^+ \nu_e\right) = N\frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-\frac{\Gamma_S + \Gamma_L}{2}t}\cos\Delta mt\right]$$

$$R_- = \Gamma\left(K^0_{t=0} \to \pi^+ e^- \bar{\nu}_e\right) = \boxed{N\left(1 - 4\,\mathrm{Re}\,\varepsilon\right)}\frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-\frac{\Gamma_S + \Gamma_L}{2}t}\cos\Delta mt\right]$$

$$\bar{R}_+ = \Gamma\left(\bar{K}^0_{t=0} \to \pi^- e^+ \nu_e\right) = \boxed{N\left(1 + 4\,\mathrm{Re}\,\varepsilon\right)}\frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-\frac{\Gamma_S + \Gamma_L}{2}t}\cos\Delta mt\right]$$

$$\bar{R}_- = \Gamma\left(\bar{K}^0_{t=0} \to \pi^+ e^- \bar{\nu}_e\right) = N\frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-\frac{\Gamma_S + \Gamma_L}{2}t}\cos\Delta mt\right]$$

$$A_{\Delta m} = \frac{\left(R_+ + \bar{R}_-\right) - \left(R_- + \bar{R}_+\right)}{\left(R_+ + \bar{R}_-\right) + \left(R_- + \bar{R}_+\right)} = \frac{2e^{-\frac{\Gamma_S + \Gamma_L}{2}t}\cos\Delta mt}{e^{-\Gamma_S t} + e^{-\Gamma_L t}}$$

$$\to A_{\Delta m}\left(\cancel{CP}\right) = A_{\Delta m}\left(CP\right)$$

Asymmetry not sensitive to CP violation

Decays into 2 π's: Time dependent asymmetry

$$A(\pi\pi) = \frac{\Gamma(\bar{K}^0 \to \pi\pi) - \Gamma(K^0 \to \pi\pi)}{\Gamma(\bar{K}^0 \to \pi\pi) + \Gamma(K^0 \to \pi\pi)}$$

$$I(K^0;t) = \frac{N}{2}(1 - 2\,\mathrm{Re}(\varepsilon))\left[e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t} + 2|\varepsilon| e^{-\frac{\Gamma_L + \Gamma_S}{2}t} \cos(\Delta mt - \varphi) \right]$$

$$I(\bar{K}^0;t) = \frac{N}{2}(1 + 2\,\mathrm{Re}(\varepsilon))\left[e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t} - 2|\varepsilon| e^{-\frac{\Gamma_L + \Gamma_S}{2}t} \cos(\Delta mt - \varphi) \right]$$

$$I(\bar{K}^0;t) \approx \frac{N}{2}\left[e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t} - 2|\varepsilon| e^{-\frac{\Gamma_L + \Gamma_S}{2}t} \cos(\Delta mt - \varphi) + 2\,\mathrm{Re}(\varepsilon)e^{-\Gamma_S t} - 4\,\mathrm{Re}(\varepsilon)|\varepsilon| e^{-\frac{\Gamma_L + \Gamma_S}{2}t} \cos(\Delta mt - \varphi) \right]$$

$$I(K^0;t) \approx \frac{N}{2}\left[e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t} + 2|\varepsilon| e^{-\frac{\Gamma_L + \Gamma_S}{2}t} \cos(\Delta mt - \varphi) - 2\,\mathrm{Re}(\varepsilon)e^{-\Gamma_S t} - 4\,\mathrm{Re}(\varepsilon)|\varepsilon| e^{-\frac{\Gamma_L + \Gamma_S}{2}t} \cos(\Delta mt - \varphi) \right]$$

$$I(\bar{K}^0;t) - I(K^0;t) \approx N\left[-2|\varepsilon| e^{-\frac{\Gamma_L + \Gamma_S}{2}t} \cos(\Delta mt - \varphi) + 2\,\mathrm{Re}(\varepsilon)e^{-\Gamma_S t} \right]$$

$$I(\bar{K}^0;t) + I(K^0;t) \approx N\left[e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t} - 4\,\mathrm{Re}(\varepsilon)|\varepsilon| e^{-\frac{\Gamma_L + \Gamma_S}{2}t} \right]$$

$$A(\pi\pi) \approx \frac{-2|\varepsilon|e^{-\frac{\Gamma_L+\Gamma_S}{2}t}\cos(\Delta mt - \varphi) + 2\operatorname{Re}(\varepsilon)e^{-\Gamma_S t}}{e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t} - 4\operatorname{Re}(\varepsilon)|\varepsilon|e^{-\frac{\Gamma_L+\Gamma_S}{2}t}}$$

Keeping only terms linear in $|\varepsilon|$:

$$A \approx \frac{4\operatorname{Re}(\varepsilon)e^{-\Gamma_S t} - 4|\varepsilon|e^{-\frac{\Gamma_L+\Gamma_S}{2}t}\cos(\Delta mt - \varphi)}{2e^{-\Gamma_S t}} = 2\operatorname{Re}(\varepsilon) - 2|\varepsilon|e^{\frac{\Gamma_S-\Gamma_L}{2}t}\cos(\Delta mt - \varphi)$$

$$\rightarrow A(\pi\pi) \approx 2\left[\operatorname{Re}(\varepsilon) - |\varepsilon|e^{\frac{(\Gamma_S-\Gamma_L)}{2}t}\cos(\Delta mt - \phi)\right]$$

K *CP* Violation - XIII

CPLEAR on 2π decays

Expected decay rates
for K^0, $\overline{K}^0$ initial state

Observed decay rates
for K^0, $\overline{K}^0$ initial state

Prediction with CP violation

$K_S \rightarrow \pi\pi$

$\overline{K}^0_{t=0} \rightarrow \pi\pi$

$K^0_{t=0} \rightarrow \pi\pi$

$K_L \rightarrow \pi\pi$

± interference term

$\pi\pi$ decay rate

$t / 10^{-10}$ s

CPLEAR data

$\pi^+\pi^-$

Number of events

tagged initial $\overline{K}^0$

tagged initial K^0

Neutral-kaon decay time [τ_S]
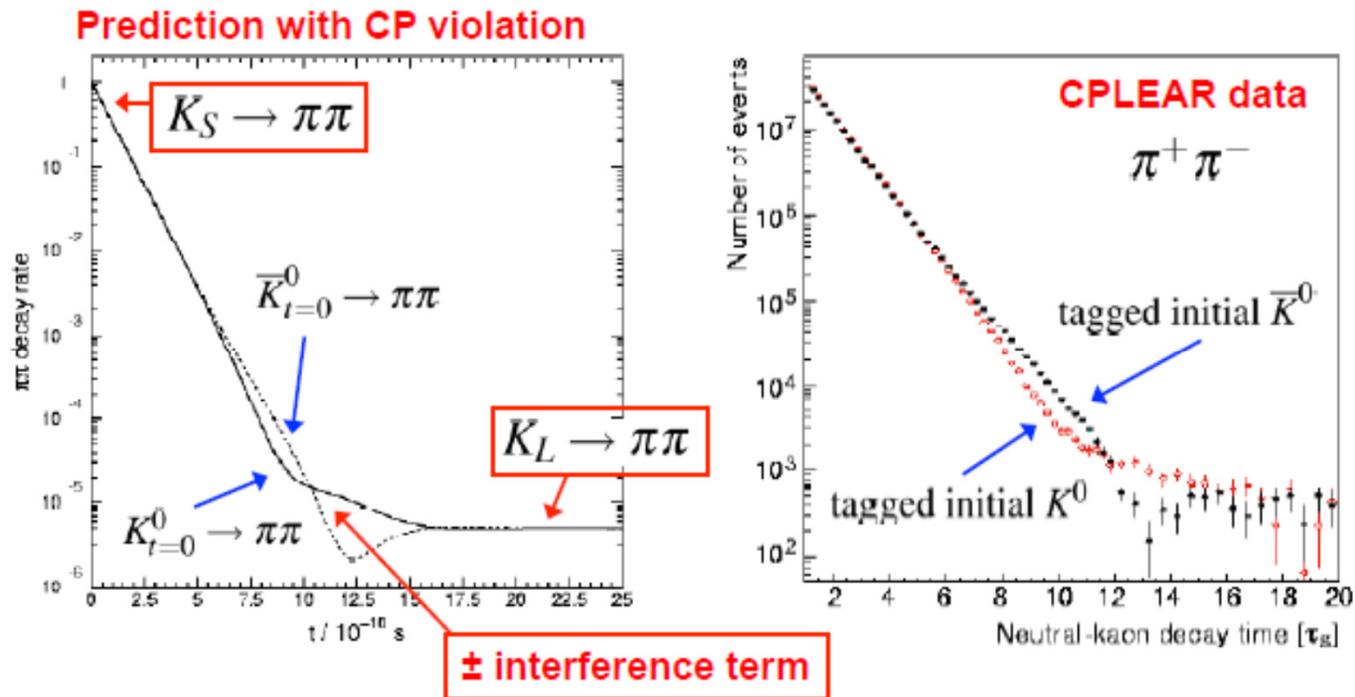
K *CP* Violation - XIV

Time dependent asymmetry:

$$A(\pi\pi) = \frac{\Gamma(\bar{K}^0) - \Gamma(K^0)}{\Gamma(\bar{K}^0) + \Gamma(K^0)}$$

$$\rightarrow A(\pi\pi) \approx 2\,\mathrm{Re}(\varepsilon) - 2|\varepsilon|e^{\frac{(\Gamma_S - \Gamma_L)}{2}t}\cos(\Delta mt - \phi)$$

$$\rightarrow \begin{cases} |\varepsilon| = (2.264 \pm 0.035)10^{-3} \\ \varphi = (43.19 \pm 0.073)^0 \\ \Delta m = (3.4852 \pm 0.013)10^{-15}\,GeV \end{cases}$$

Time dependent decay rate asymmetry

2π time dependent decay rate asymmetry
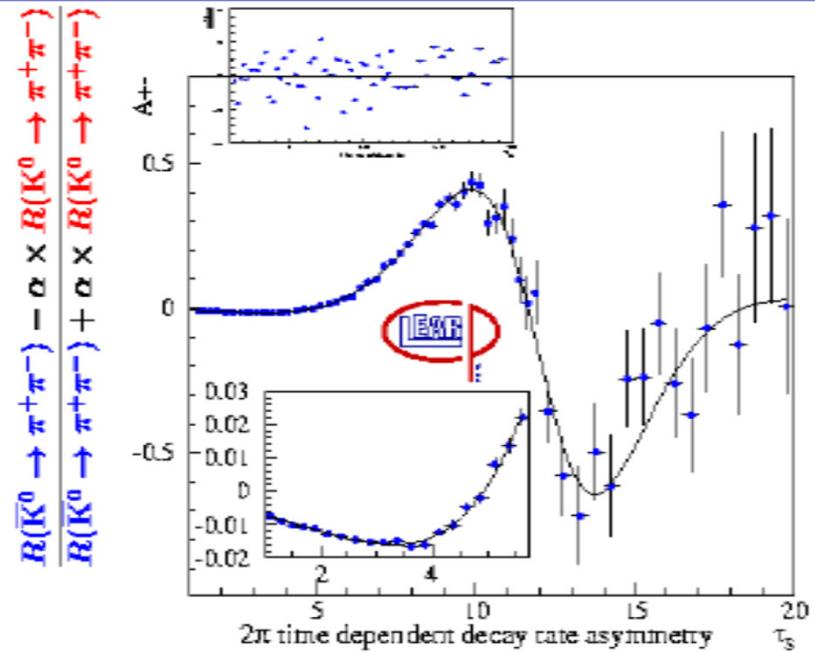
CP violation in 3π decays

Expect, by swapping $K_S \leftrightarrow K_L$:

$$I\left(K^0;t\right) = \frac{N'}{2}\left(1 - 2\operatorname{Re}(\varepsilon)\right)\left[e^{-\Gamma_L t} + |\varepsilon|^2 e^{-\Gamma_S t} + 2|\varepsilon|e^{-\frac{\Gamma_L+\Gamma_S}{2}t}\cos\left(\Delta mt - \varphi\right)\right]$$

Very different experimental conditions as compared to 2π :

Lots of *CP* conserving 3π decays from K_L component of the beam

$\left(\text{Compare: No } CP \text{ conserving } 2\pi \text{ from } K_S \text{ component, which just dies out at large distance}\right)$

$\rightarrow$ Measurement difficult, large errors

K *CP* Violation - XVI

Summary of decay rates (CP violated)
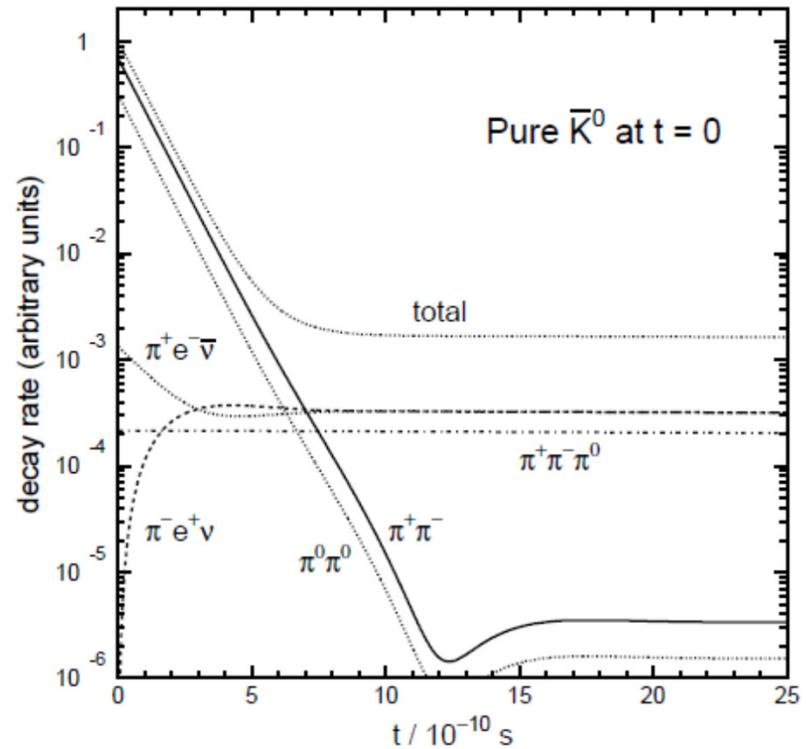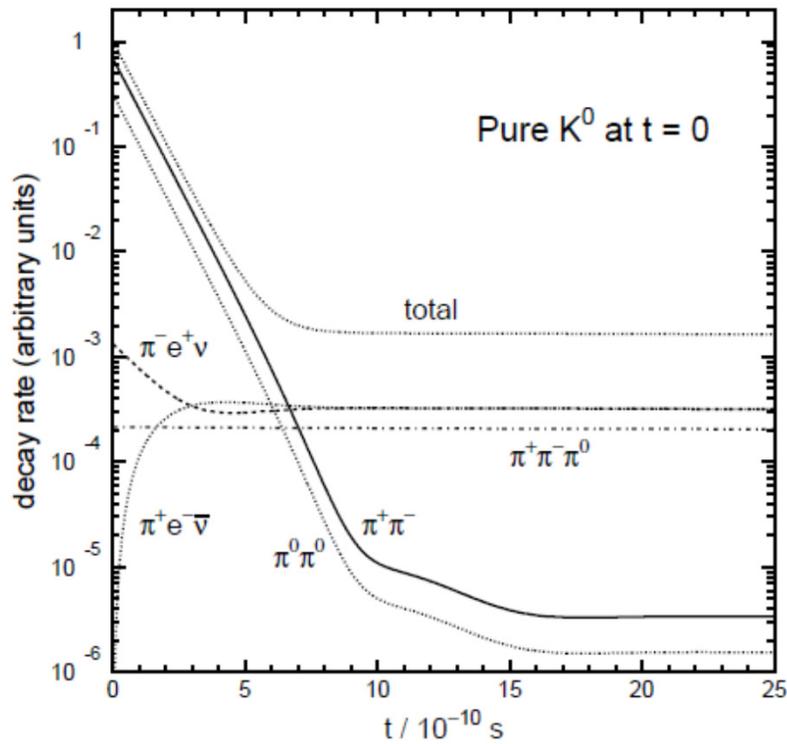
K T, CPT Tests - I

From previous conclusions on *CP* violation:

$$R_- = \Gamma\left(K^0_{t=0} \to \pi^+ e^- \overline{\nu}_e\right) = N\left(1 - 4\,\mathrm{Re}\,\varepsilon\right)\frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-\frac{\Gamma_S + \Gamma_L}{2}t}\cos\Delta mt\right]$$

$$\overline{R}_+ = \Gamma\left(\overline{K}^0_{t=0} \to \pi^- e^+ \nu_e\right) = N\left(1 + 4\,\mathrm{Re}\,\varepsilon\right)\frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-\frac{\Gamma_S + \Gamma_L}{2}t}\cos\Delta mt\right]$$

$$\to \Gamma\left(K^0_{t=0} \to \overline{K}^0\right) \ne \Gamma\left(\overline{K}^0_{t=0} \to K^0\right)$$

$\to$ Amplitude of direct process $\ne$ Amplitude of reverse process

$\to$ CP violation $\leftrightarrow$ *Time Reversal* violation

To be expected if *CPT* is a good symmetry

Define T asymmetry:

$$A_T = \frac{\Gamma\left(\overline{K}^0_{t=0} \to K^0\right) - \Gamma\left(K^0_{t=0} \to \overline{K}^0\right)}{\Gamma\left(\overline{K}^0_{t=0} \to K^0\right) + \Gamma\left(K^0_{t=0} \to \overline{K}^0\right)}$$

K T, CPT Tests - II

Measure by taking semileptonic:

$$A_T = \frac{\Gamma\left(\bar{K}^0_{t=0} \to \pi^- e^+ \nu_e\right) - \Gamma\left(K^0_{t=0} \to \pi^+ e^- \bar{\nu}_e\right)}{\Gamma\left(\bar{K}^0_{t=0} \to \pi^- e^+ \nu_e\right) + \Gamma\left(K^0_{t=0} \to \pi^+ e^- \bar{\nu}_e\right)}$$

$A_T \approx 4\,\mathrm{Re}(\varepsilon) = 4|\varepsilon|\cos\phi$ Time independent constant

$\to$ Expect from $\pi\pi$ CP violation

$A_T \approx 6.6\ 10^{-3}$

Measure:

$A_T = (6.2 \pm 1.7)\ 10^{-3}$
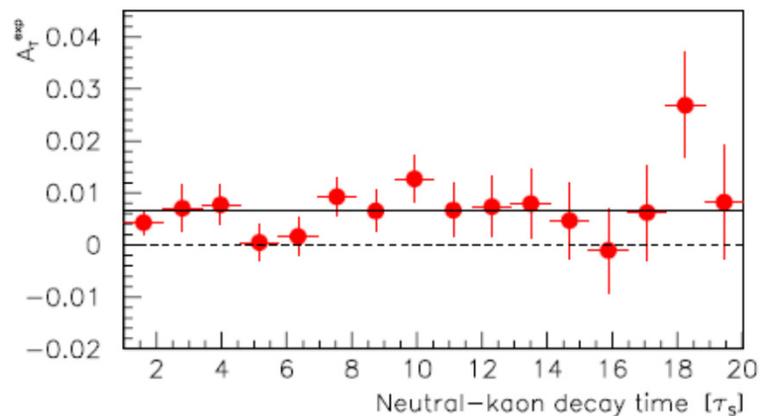
K T, CPT Tests - III

Semileptonic decays also used to test CPT

Simple test:

$$\Gamma\left(K^0 \to K^0\right) = \Gamma\left(\bar{K}^0 \to \bar{K}^0\right)$$

Define CPT asymmetry :

$$A_{CPT} = \frac{\Gamma\left(K^0 \to K^0\right) - \Gamma\left(\bar{K}^0 \to \bar{K}^0\right)}{\Gamma\left(K^0 \to K^0\right) + \Gamma\left(\bar{K}^0 \to \bar{K}^0\right)}$$

Measure by:

$$A_{CPT} = \frac{\Gamma\left(K^0_{t=0} \to \pi^- e^+ \nu_e\right) - \Gamma\left(\bar{K}^0_{t=0} \to \pi^+ e^- \bar{\nu}_e\right)}{\Gamma\left(K^0_{t=0} \to \pi^- e^+ \nu_e\right) + \Gamma\left(\bar{K}^0_{t=0} \to \pi^+ e^- \bar{\nu}_e\right)}$$

K *T, CPT* Tests - IV

Since:

$$\Gamma\left(K^0_{t=0} \to \pi^- e^+ \nu_e\right) = N\frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-\frac{\Gamma_S + \Gamma_L}{2}t}\cos\Delta mt\right]$$

$$\Gamma\left(\overline{K}^0_{t=0} \to \pi^+ e^- \overline{\nu}_e\right) = N\frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-\frac{\Gamma_S + \Gamma_L}{2}t}\cos\Delta mt\right]$$

$\to$ Expect:

$A_{CPT} = 0,$ t independent

Measure:
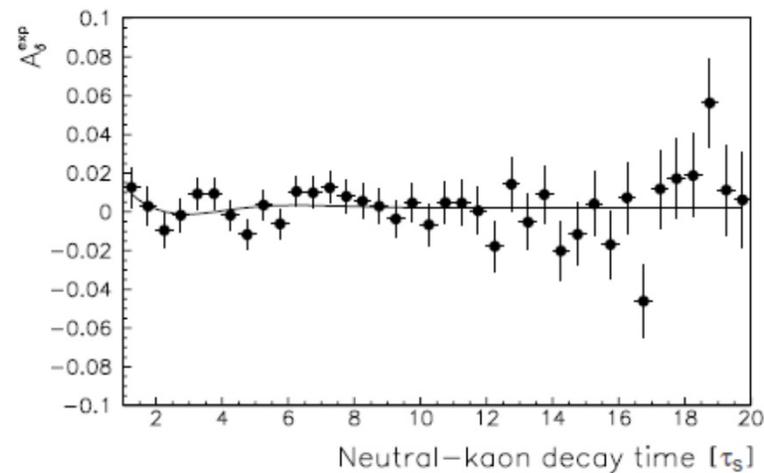
K Direct *CP* Violation - I

Another side of $\cancel{CP}: K^0\ decays\ CP$ violating

$\rightarrow$ Direct $\cancel{CP}$

Amplitude ratios :

$$\eta_{+-} = \frac{\left\langle \pi^+\pi^- \left| T \right| K_L^0 \right\rangle}{\left\langle \pi^+\pi^- \left| T \right| K_S^0 \right\rangle} = \left| \eta_{+-} \right| e^{i\phi_{+-}}, \qquad \eta_{00} = \frac{\left\langle \pi^0\pi^0 \left| T \right| K_L^0 \right\rangle}{\left\langle \pi^0\pi^0 \left| T \right| K_S^0 \right\rangle} = \left| \eta_{00} \right| e^{i\phi_{00}}$$

In order to relate η, ϕ parameters to $\varepsilon, \varepsilon'$

a)Decompose 2π states into isospin eigenstates:

$$\begin{cases} \left\langle \pi^+\pi^- \right| = \dfrac{1}{\sqrt{3}} \left\langle I=2 \right| + \sqrt{\dfrac{2}{3}} \left\langle I=0 \right| \\ \left\langle \pi^0\pi^0 \right| = \sqrt{\dfrac{2}{3}} \left\langle I=2 \right| - \dfrac{1}{\sqrt{3}} \left\langle I=0 \right| \end{cases} \qquad I=1 \text{ absent due to Bose statistics of } \pi\text{'s in a } S\text{-wave}$$

K Direct *CP* Violation - II

Full 2π states should include proper phase factors
originating from S-wave $\pi\pi$ scattering

$$\left\langle \pi^+\pi^- \right| = \frac{1}{\sqrt{3}}\left\langle 2\right| e^{i\delta_2} + \sqrt{\frac{2}{3}}\left\langle 0\right| e^{i\delta_0}$$

$$\left\langle \pi^0\pi^0 \right| = \sqrt{\frac{2}{3}}\left\langle 2\right| e^{i\delta_2} - \frac{1}{\sqrt{3}}\left\langle 0\right| e^{i\delta_0}$$

Define decay amplitudes into isospin states:

$$A_0 = \left\langle 0 \right| H_w \left| K^0 \right\rangle$$

$$A_2 = \left\langle 2 \right| H_w \left| K^0 \right\rangle$$

K Direct *CP* Violation - III

$$CP|\pi\pi\rangle = +1 \rightarrow CPT|0\rangle = \langle 0|, CPT|2\rangle = \langle 2|$$

$$CP|K^0\rangle = -|\bar{K}^0\rangle \rightarrow CPT|K^0\rangle = -\langle \bar{K}^0|$$

$$[H_w, CPT] = 0$$

$$\rightarrow \begin{cases} \langle 0|H_w|\bar{K}^0\rangle \xrightarrow{CPT} -\langle K^0|H_w|0\rangle = -A_0^* \\ \langle 2|H_w|\bar{K}^0\rangle \xrightarrow{CPT} -\langle K^0|H_w|2\rangle = -A_2^* \end{cases}$$

$$|K_L^0\rangle = \frac{1}{\sqrt{2(1+|\varepsilon|^2)}}\left[(1+\varepsilon)|K^0\rangle + (1-\varepsilon)|\bar{K}^0\rangle\right]$$

$$|K_S^0\rangle = \frac{1}{\sqrt{2(1+|\varepsilon|^2)}}\left[(1+\varepsilon)|K^0\rangle - (1-\varepsilon)|\bar{K}^0\rangle\right]$$

K Direct *CP* Violation - IV

Transition matrix elements:

$$\left\langle \pi^+ \pi^- \left| H \right| K_L^0 \right\rangle = \frac{2}{\sqrt{3}} \frac{1}{\sqrt{2\left(1+|\varepsilon|^2\right)}} \varepsilon \left[\mathrm{Re}\, A_2 e^{i\delta_2} + \sqrt{2} A_0 e^{i\delta_0} \right] + \mathrm{Im}\, A_2 e^{i\delta_2}$$

$$\left\langle \pi^+ \pi^- \left| H \right| K_S^0 \right\rangle = \frac{2}{\sqrt{3}} \frac{1}{\sqrt{2\left(1+|\varepsilon|^2\right)}} \left[\mathrm{Re}\, A_2 e^{i\delta_2} + \sqrt{2} A_0 e^{i\delta_0} + \varepsilon\, \mathrm{Im}\, A_2 e^{i\delta_2} \right]$$

$$\left\langle \pi^0 \pi^0 \left| H \right| K_L^0 \right\rangle = \frac{2}{\sqrt{3}} \frac{1}{\sqrt{2\left(1+|\varepsilon|^2\right)}} \varepsilon \left[\sqrt{2}\, \mathrm{Re}\, A_2 e^{i\delta_2} - A_0 e^{i\delta_0} \right] + \sqrt{2}\, \mathrm{Im}\, A_2 e^{i\delta_2}$$

$$\left\langle \pi^0 \pi^0 \left| H \right| K_S^0 \right\rangle = \frac{2}{\sqrt{3}} \frac{1}{\sqrt{2\left(1+|\varepsilon|^2\right)}} \left[\sqrt{2}\, \mathrm{Re}\, A_2 e^{i\delta_2} - A_0 e^{i\delta_0} + \varepsilon \sqrt{2}\, \mathrm{Im}\, A_2 e^{i\delta_2} \right]$$

After some complex algebra:

$$\eta_{+-} = \frac{\left\langle \pi^+ \pi^- \left| T \right| K_L^0 \right\rangle}{\left\langle \pi^+ \pi^- \left| T \right| K_S^0 \right\rangle} \simeq \varepsilon + \underbrace{\frac{1}{\sqrt{2}} \frac{\mathrm{Im}\, A_2}{A_0} e^{i(\delta_2 - \delta_0)}}_{\varepsilon'} = \varepsilon + \varepsilon'$$

$$\eta_{00} = \frac{\left\langle \pi^0 \pi^0 \left| T \right| K_L^0 \right\rangle}{\left\langle \pi^0 \pi^0 \left| T \right| K_S^0 \right\rangle} \simeq \varepsilon - \underbrace{\sqrt{2} \frac{\mathrm{Im}\, A_2}{A_0} e^{i(\delta_2 - \delta_0)}}_{2\varepsilon'} = \varepsilon - 2\varepsilon'$$

Double ratio magic:

$$\eta_{+-} = \frac{\left\langle \pi^+ \pi^- \left| T \right| K_L^0 \right\rangle}{\left\langle \pi^+ \pi^- \left| T \right| K_S^0 \right\rangle} \simeq \varepsilon + \varepsilon'$$

$$\eta_{00} = \frac{\left\langle \pi^0 \pi^0 \left| T \right| K_L^0 \right\rangle}{\left\langle \pi^0 \pi^0 \left| T \right| K_S^0 \right\rangle} \simeq \varepsilon - 2\varepsilon'$$

$$\rightarrow \left| \frac{\eta_{00}}{\eta_{+-}} \right|^2 \simeq \frac{|\varepsilon - 2\varepsilon'|^2}{|\varepsilon + \varepsilon'|^2} = \frac{(\varepsilon - 2\varepsilon')(\varepsilon - 2\varepsilon')^*}{(\varepsilon + \varepsilon')(\varepsilon + \varepsilon')^*} \simeq \frac{|\varepsilon|^2 - 4\,\mathrm{Re}(\varepsilon'\varepsilon)}{|\varepsilon|^2 + 2\,\mathrm{Re}(\varepsilon'\varepsilon)}$$

$$\rightarrow \left| \frac{\eta_{00}}{\eta_{+-}} \right|^2 \approx \frac{1 - 4\dfrac{\mathrm{Re}(\varepsilon'\varepsilon)}{|\varepsilon|^2}}{1 + 2\dfrac{\mathrm{Re}(\varepsilon'\varepsilon)}{|\varepsilon|^2}} \approx \left[1 - 2\frac{\mathrm{Re}(\varepsilon'\varepsilon)}{|\varepsilon|^2} \right]\left[1 - 4\frac{\mathrm{Re}(\varepsilon'\varepsilon)}{|\varepsilon|^2} \right] \approx 1 - 6\frac{\mathrm{Re}(\varepsilon'\varepsilon)}{|\varepsilon|^2}$$

$$\rightarrow \left| \frac{\eta_{00}}{\eta_{+-}} \right|^2 = \frac{\dfrac{N_L^{00}}{N_S^{00}}}{\dfrac{N_L^{+-}}{N_S^{+-}}} \approx 1 - 6\,\mathrm{Re}\frac{(\varepsilon')}{|\varepsilon|}$$

Actually a very important question:
Does weak interaction violate CP?
$\varepsilon' \neq 0$ yes
$\varepsilon' = 0$ don't know

'80s :

$$\text{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = (23.0 \pm 6.5) \times 10^{-4} (NA31) \qquad > 3\sigma$$

$$\text{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = (7.4 \pm 5.9) \times 10^{-4} (E731) \qquad \sim 1.5\sigma$$

Mostly systematics

'90s :

$$\text{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = (15.3 \pm 2.6) \times 10^{-4} (NA48) \qquad \sim 6\sigma$$

$$\text{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = (20.7 \pm 2.8) \times 10^{-4} (KTEV) \qquad > 7\sigma$$

NA48 technique

- Employ two almost collinear neutral beams
- Collect the four decay modes simoultaneously, in the same detector and from the same decay region
- Keep the acceptance correction small by weighting the K_L events according to the ratio of K_S/K_L decay intensities as a function of proper time
- Distinguish K_S and K_L events by tagging the protons upstream of the K_S target
- Use precise and stable liquid krypton (LKr) calorimetry to control the relative momentum scale

NA48 Beams (1999)

SPS spill length : 2.38 s
Cycle time : 14.4 s
Proton momentum : 450 GeV/c

~1.5 10^{12} protons per spill

K_L Target

K_s Target

K_s anticounter (AKS)

not to scale !

K_S

6.8 cm

0.6 mrad

NA48

Bent cristal

Muon sweeping

K_L

Detector

Ks tagging station

(~3. 10^7 protons per spill)

Last collimator

Decay Region
(~ 40 m long)

~ 126 m

~ 114 m

K Direct *CP* Violation - IX

NA48 Detectors

$\sigma(p)/p = 0.48\% \oplus 0.009p\%$

$\sigma_t = 230$ ps

$\sigma(E)/E < 1\%$
$(E > 20 \text{ GeV})$

LKr calorimeter time resolution (50 GeV e⁻)

Muon veto sytem
Hadron calorimeter
Liquid krypton calorimeter
Hodoscope
Drift chamber 4
Anti counter 7
Helium tank
Drift chamber 3
Magnet
Drift chamber 2
Anti counter 6
Drift chamber
Kevlar window

$\sigma(E/p)$ (after intercalibration)
$\sigma(E)/E$

Resolution

Energy (GeV)

$K \rightarrow \pi^+\pi^-$

K_S
K_L

Y(cm)

Z(cm)

K Direct *CP* Violation - X

Systematics Checks and Result
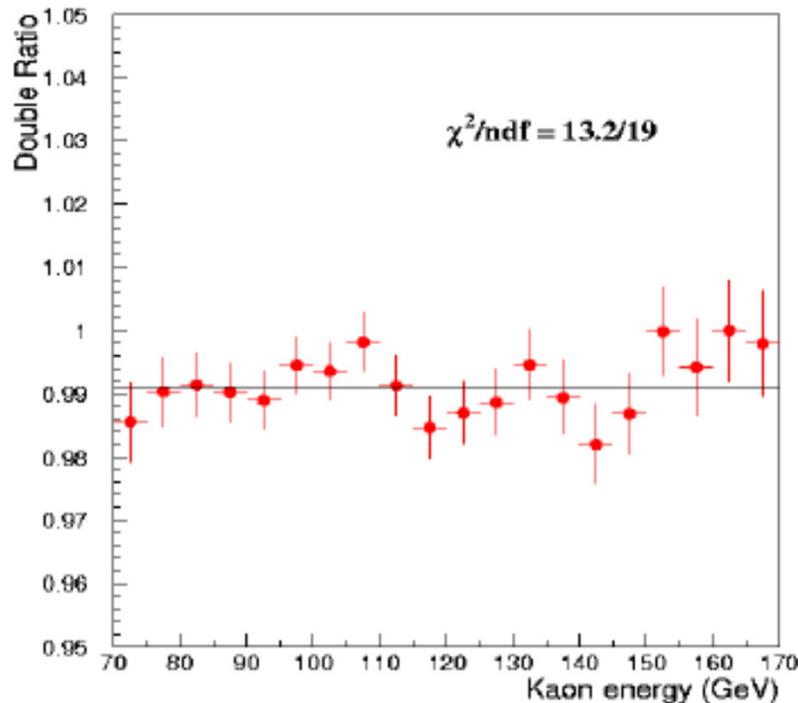
R stability against cut variations
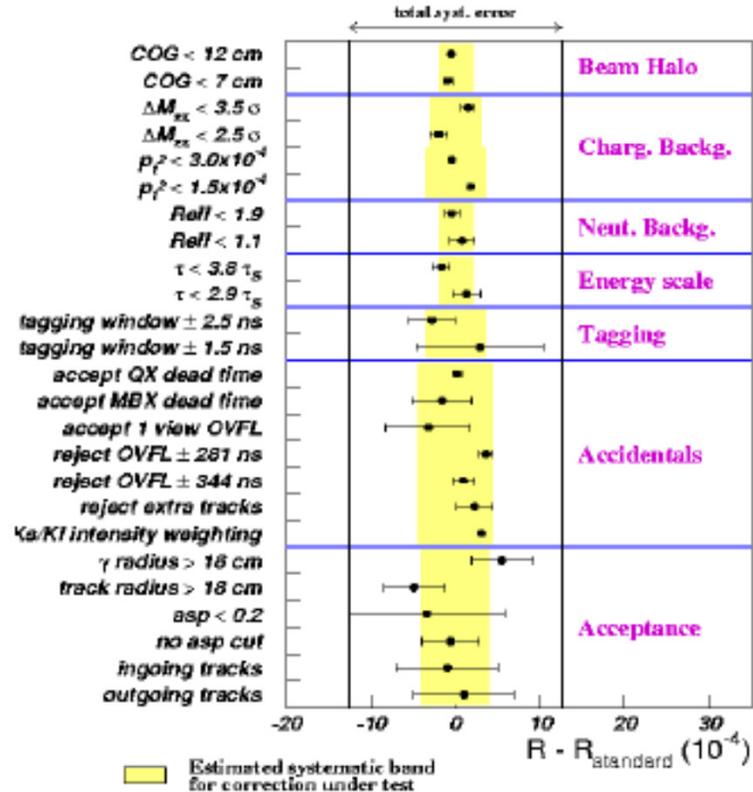
Mixing and Oscillations - I

Two-state system: Electron in a magnetic field $\boldsymbol{B}$ along $\hat{z}$

$$H = -\boldsymbol{\mu} \cdot \boldsymbol{B} = \frac{1}{2} a \boldsymbol{\sigma} \cdot \boldsymbol{B} \quad \boldsymbol{B} = B\hat{k}, \quad \boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z): \text{ Pauli spin matrices}$$

$$\rightarrow H = \frac{1}{2} a \sigma_z B = \begin{pmatrix} \frac{1}{2} aB & 0 \\ 0 & -\frac{1}{2} aB \end{pmatrix} = \begin{pmatrix} +E & 0 \\ 0 & -E \end{pmatrix}$$

2 state system: Choose as base states

$$|+\rangle \equiv \begin{pmatrix} 1 \\ 0 \end{pmatrix}, |-\rangle \equiv \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \text{Eigenstates of } \sigma_z \rightarrow \text{Generic state:} |\psi\rangle = \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} \rightarrow \sigma_z |\psi\rangle = \begin{pmatrix} \psi_+ \\ -\psi_- \end{pmatrix}$$

Schrodinger equation:

$$i\hbar \frac{\partial |\psi\rangle}{\partial t} = H |\psi\rangle \rightarrow |\psi(t)\rangle = e^{-i\frac{Ht}{\hbar}} |\psi\rangle = e^{-i\frac{aB}{2\hbar} t \sigma_z} |\psi\rangle \rightarrow i\hbar \frac{\partial}{\partial t} \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} = \frac{1}{2} aB \sigma_z \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} = \frac{1}{2} aB \begin{pmatrix} \psi_+ \\ -\psi_- \end{pmatrix}$$

Uncoupled equations:

$$\rightarrow \begin{cases} i\hbar \dfrac{\partial \psi_+}{\partial t} = \dfrac{1}{2} aB\psi_+ \\ i\hbar \dfrac{\partial \psi_-}{\partial t} = -\dfrac{1}{2} aB\psi_- \end{cases} \rightarrow \begin{cases} \psi_+(t) = A_+ e^{-i\frac{aB}{2\hbar}t} \\ \psi_-(t) = A_- e^{+i\frac{aB}{2\hbar}t} \end{cases}, \quad |A_+|^2 + |A_-|^2 = 1 \rightarrow \begin{cases} |+,t\rangle = \begin{pmatrix} \psi_+(t) \\ 0 \end{pmatrix} \\ |-,t\rangle = \begin{pmatrix} 0 \\ \psi_-(t) \end{pmatrix} \end{cases} \text{Stationary states}$$

Mixing and Oscillations - II

Introduce another **B** component along x:

$$\boldsymbol{B} = B\hat{\boldsymbol{k}} + B'\hat{\boldsymbol{i}}$$

$$H = -\boldsymbol{\mu} \cdot \boldsymbol{B} = \frac{1}{2}a\boldsymbol{\sigma} \cdot \boldsymbol{B} = \frac{1}{2}a(\sigma_z B + \sigma_x B')$$

$$\rightarrow H = \begin{pmatrix} \frac{1}{2}aB & 0 \\ 0 & -\frac{1}{2}aB \end{pmatrix} + \begin{pmatrix} 0 & \frac{1}{2}aB' \\ \frac{1}{2}aB' & 0 \end{pmatrix} = \begin{pmatrix} +E & E' \\ E' & -E \end{pmatrix}$$

$$\rightarrow i\hbar\frac{\partial}{\partial t}\begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} = \frac{1}{2}a(B\sigma_z + B'\sigma_x)\begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} = \frac{1}{2}a\left[B\begin{pmatrix} \psi_+ \\ -\psi_- \end{pmatrix} + B'\begin{pmatrix} \psi_- \\ \psi_+ \end{pmatrix}\right]$$

Coupled equations:

$$\rightarrow \begin{cases} i\hbar\dfrac{\partial\psi_+}{\partial t} = \dfrac{1}{2}aB\psi_+ + aB'\psi_- \\ i\hbar\dfrac{\partial\psi_-}{\partial t} = -\dfrac{1}{2}aB\psi_- + aB'\psi_+ \end{cases} \rightarrow \begin{cases} |+,t\rangle \\ |-,t\rangle \end{cases} \text{Non -stationary states}$$

Mixing and Oscillations - III

Build a phenomenological framework suitable to describe flavor oscillations

Use symbol M^0 for neutral, flavored mesons: Most of the formalism suitable for K^0, D^0, B^0, B_s^0

Neutral meson time evolution: Two-state system

$$\left|M^0\right\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \left|\overline{M}^0\right\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$i\frac{\partial}{\partial t}\psi = H\psi \quad \text{Schrodinger equation}$$

$$\psi(t) = a(t)\left|M^0\right\rangle + b(t)\left|\overline{M}^0\right\rangle \equiv \begin{pmatrix} a(t) \\ b(t) \end{pmatrix} \quad \text{Two-component state vector}$$

Just free evolution for both components, no decay:

$$H = \underbrace{\begin{pmatrix} M & 0 \\ 0 & M \end{pmatrix}}_{\text{hermitian}} \quad \text{Effective Hamiltonian, } M = \text{mass}$$

Free evolution for both components, with decay:

$$H = \underbrace{\begin{pmatrix} M & 0 \\ 0 & M \end{pmatrix}}_{\text{hermitian}} - \frac{i}{2}\underbrace{\begin{pmatrix} \Gamma & 0 \\ 0 & \Gamma \end{pmatrix}}_{\text{hermitian}}, \quad \Gamma = \text{total decay width}$$

Mixing and Oscillations - IV

Observe:

$$H^\dagger = \begin{pmatrix} M & 0 \\ 0 & M \end{pmatrix}^\dagger + \frac{i}{2}\begin{pmatrix} \Gamma & 0 \\ 0 & \Gamma \end{pmatrix}^\dagger = \begin{pmatrix} M & 0 \\ 0 & M \end{pmatrix} + \frac{i}{2}\begin{pmatrix} \Gamma & 0 \\ 0 & \Gamma \end{pmatrix} \neq H$$

$\rightarrow H$ non-Hermitian $\rightarrow e^{-iHt}$ non-unitary $\rightarrow$ State norm not conserved: Decreasing $\leftrightarrow \Gamma > 0$

$$H = \underbrace{\begin{pmatrix} M & A \\ B & M \end{pmatrix}}_{\text{hermitian}} - \frac{i}{2}\underbrace{\begin{pmatrix} \Gamma & C \\ D & \Gamma \end{pmatrix}}_{\text{hermitian}} \qquad \text{Include mixing}$$

$$\begin{pmatrix} M & A \\ B & M \end{pmatrix}^\dagger = \begin{pmatrix} M & A \\ B & M \end{pmatrix} \rightarrow \begin{pmatrix} M & B^* \\ A^* & M \end{pmatrix} = \begin{pmatrix} M & A \\ B & M \end{pmatrix} \rightarrow A^* = B, \text{ same for } \Gamma$$

$$\rightarrow H = \underbrace{\begin{pmatrix} M & M_{12} \\ M_{12}^* & M \end{pmatrix}}_{\text{hermitian}} - \frac{i}{2}\underbrace{\begin{pmatrix} \Gamma & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma \end{pmatrix}}_{\text{hermitian}}$$

$$\rightarrow i\frac{\partial}{\partial t}\psi(t) = \begin{pmatrix} M - \dfrac{i}{2}\Gamma & M_{12} - \dfrac{i}{2}\Gamma_{12} \\[2ex] M_{12}^* - \dfrac{i}{2}\Gamma_{12}^* & M - \dfrac{i}{2}\Gamma \end{pmatrix}\psi(t)$$

Mixing and Oscillations - V

Eigenvalues:

Define $F \equiv \operatorname{Re} F + i \operatorname{Im} F = \sqrt{\left(M_{12} - \dfrac{i}{2}\Gamma_{12}\right)\left(M_{12}^{*} - \dfrac{i}{2}\Gamma_{12}^{*}\right)}$

$$\begin{vmatrix} M - \dfrac{i}{2}\Gamma - \lambda & M_{12} - \dfrac{i}{2}\Gamma_{12} \\ M_{12}^{*} - \dfrac{i}{2}\Gamma_{12}^{*} & M - \dfrac{i}{2}\Gamma - \lambda \end{vmatrix} = 0 \rightarrow \begin{array}{l} \lambda_{1} \equiv m_{1} - i\dfrac{\Gamma_{1}}{2} = M - i\dfrac{\Gamma}{2} - F \\[2ex] \lambda_{2} \equiv m_{2} - i\dfrac{\Gamma_{2}}{2} = M - i\dfrac{\Gamma}{2} + F \end{array}$$

$$\rightarrow \begin{cases} m_{1} - i\dfrac{\Gamma_{1}}{2} = M - \operatorname{Re}(F) - i\left(\dfrac{\Gamma}{2} + \operatorname{Im}(F)\right) \\[3ex] m_{2} - i\dfrac{\Gamma_{2}}{2} = M + \operatorname{Re}(F) - i\left(\dfrac{\Gamma}{2} - \operatorname{Im}(F)\right) \end{cases}$$

$$\rightarrow \begin{cases} \Delta m = m_{2} - m_{1} = 2\operatorname{Re}(F) = 2\operatorname{Re}\sqrt{\left(M_{12} - \dfrac{i}{2}\Gamma_{12}\right)\left(M_{12}^{*} - \dfrac{i}{2}\Gamma_{12}^{*}\right)} \\[4ex] \Delta\Gamma = \Gamma_{2} - \Gamma_{1} = 4\operatorname{Im}(F) = 4\operatorname{Im}\sqrt{\left(M_{12} - \dfrac{i}{2}\Gamma_{12}\right)\left(M_{12}^{*} - \dfrac{i}{2}\Gamma_{12}^{*}\right)} \end{cases}$$

Mixing and Oscillations - VI

Eigenvectors:

$$\begin{pmatrix} M - \dfrac{i}{2}\Gamma & M_{12} - \dfrac{i}{2}\Gamma_{12} \\ M_{12}^* - \dfrac{i}{2}\Gamma_{12}^* & M - \dfrac{i}{2}\Gamma \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix} = \lambda_{1,2} \begin{pmatrix} p \\ q \end{pmatrix} \rightarrow \eta \equiv \frac{q}{p} = \sqrt{\frac{M_{12}^* - \dfrac{i}{2}\Gamma_{12}^*}{M_{12} - \dfrac{i}{2}\Gamma_{12}}}, |q|^2 + |p|^2 = 1$$

Mass eigenstates, similar to K_S, K_L:

Named "Heavy" and "Light" because for heavy quarks $\cong$ same lifetime

$$\begin{cases} |M_H\rangle = p|M^0\rangle + q|\overline{M^0}\rangle, & m_H = m_1, \Gamma_H = \Gamma_1 \\ |M_L\rangle = p|M^0\rangle - q|\overline{M^0}\rangle, & m_L = m_2, \Gamma_L = \Gamma_2 \end{cases}$$

Flavor eigenstates:

$$\begin{cases} |M^0\rangle = \dfrac{1}{2p}\left(|M_H\rangle + |M_L\rangle\right) \\ |\overline{M}^0\rangle = \dfrac{1}{2q}\left(|M_H\rangle - |M_L\rangle\right) \end{cases}$$

Mixing and Oscillations - VII

Define:

$$\omega_+ = m_H - i\frac{\Gamma_H}{2}, \omega_- = m_L - i\frac{\Gamma_L}{2}$$

Time evolution of mass eigenstates:

$$\begin{cases} \left|M_H(t)\right\rangle = e^{-i\omega_+ t}\left|M_H(0)\right\rangle \\ \left|M_L(t)\right\rangle = e^{-i\omega_- t}\left|M_L(0)\right\rangle \end{cases}$$

$\rightarrow$ Straightforward free propagation & decay

Observe:

$$\Gamma_H \cong \Gamma_L \rightarrow \tau_H \cong \tau_L$$

Generally true for heavy mesons, due to a large number of decay modes

Mixing and Oscillations - VIII

Time evolution of flavor eigenstates:

Flavor oscillations

$$\left|M^0(t)\right\rangle = \frac{1}{2p}\left[\left|M_H\right\rangle e^{-i\omega_+ t} + \left|M_L\right\rangle e^{-i\omega_- t}\right] = \frac{1}{2p}\left[\left(\left|M^0\right\rangle + \eta\left|\bar{M}^0\right\rangle\right)e^{-i\omega_+ t} + \left(\left|M^0\right\rangle - \eta\left|\bar{M}^0\right\rangle\right)e^{-i\omega_- t}\right]$$

$$\left|M^0(t)\right\rangle = \frac{1}{2p}\left[\left|M^0\right\rangle\frac{e^{-i\omega_+ t} + e^{-i\omega_- t}}{2} + \eta\left|\bar{M}^0\right\rangle\frac{e^{-i\omega_+ t} - e^{-i\omega_- t}}{2}\right]$$

Define: $g_\pm(t) = \dfrac{e^{-i\omega_+ t} \pm e^{-i\omega_- t}}{2}$

$$\rightarrow \begin{cases} \left|M^0(t)\right\rangle \propto g_+(t)\left|M^0\right\rangle + \dfrac{q}{p}g_-(t)\left|\bar{M}^0\right\rangle \\[3mm] \left|\bar{M}^0(t)\right\rangle \propto g_+(t)\left|\bar{M}^0\right\rangle + \dfrac{p}{q}g_-(t)\left|M^0\right\rangle \end{cases}$$

$$\rightarrow \begin{cases} \left|M^0(t)\right\rangle = \dfrac{1}{\sqrt{1+|\eta|^2}}\left(g_+(t)\left|M^0\right\rangle + \eta g_-(t)\left|\bar{M}^0\right\rangle\right) \\[3mm] \left|\bar{M}^0(t)\right\rangle = \dfrac{1}{\sqrt{1+|\eta|^2}}\left(\eta g_+(t)\left|\bar{M}^0\right\rangle + g_-(t)\left|M^0\right\rangle\right) \end{cases}$$

$M^0 \longrightarrow \dfrac{q}{p}g_-(t) \longrightarrow \bar{M}^0$ Mixed $\rightarrow \ell^-\bar{\nu}_\ell X^+$

$M^0 \longrightarrow g_+(t) \longrightarrow M^0$ Unmixed $\leftarrow \ell^+\nu_\ell X^-$

$\bar{M}^0 \longrightarrow g_+(t) \longrightarrow \bar{M}^0$ Unmixed $\rightarrow \ell^-\bar{\nu}_\ell X^+$

$\bar{M}^0 \longrightarrow \dfrac{p}{q}g_-(t) \longrightarrow M^0$ Mixed $\leftarrow \ell^+\nu_\ell X^-$
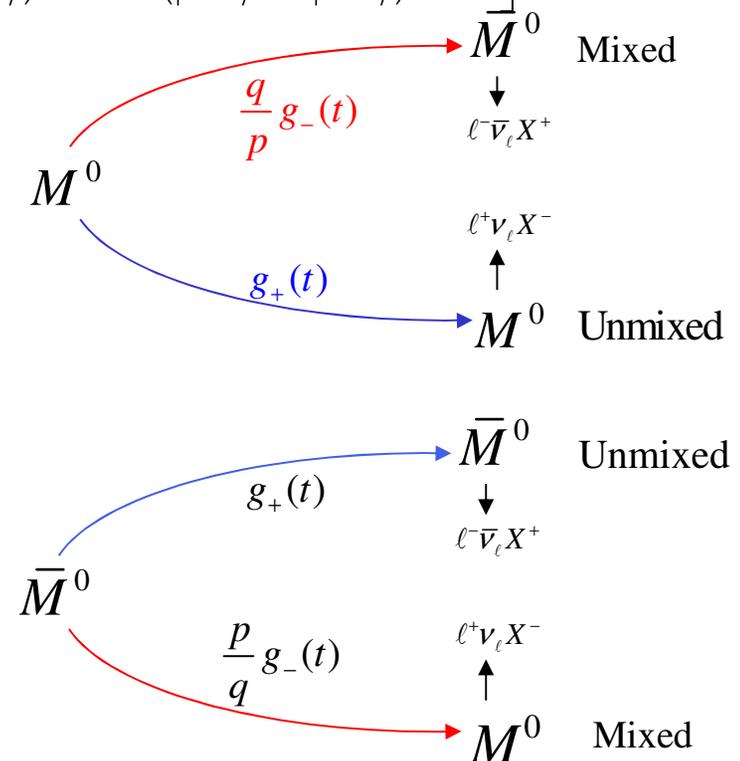
Mixing and Oscillations - IX

Compare different ratios

$$x = \frac{\Delta m}{\Gamma}$$
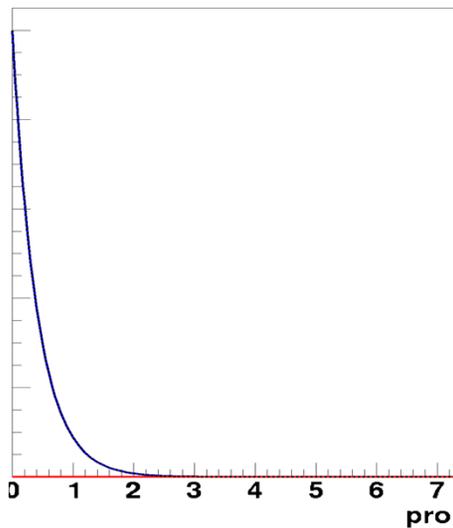
$$x \equiv \frac{\Delta m}{\Gamma} = 0$$
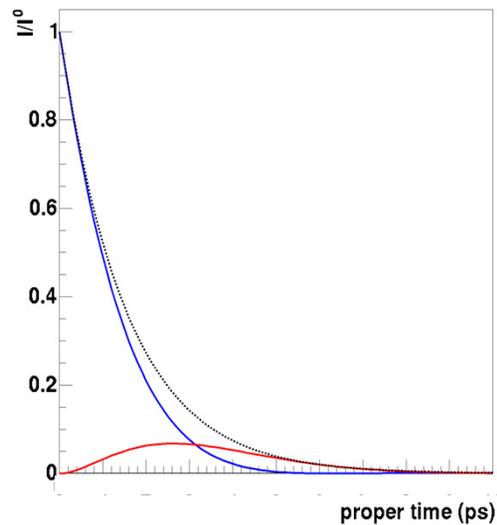
$$x \equiv \frac{\Delta m}{\Gamma} \approx 1$$
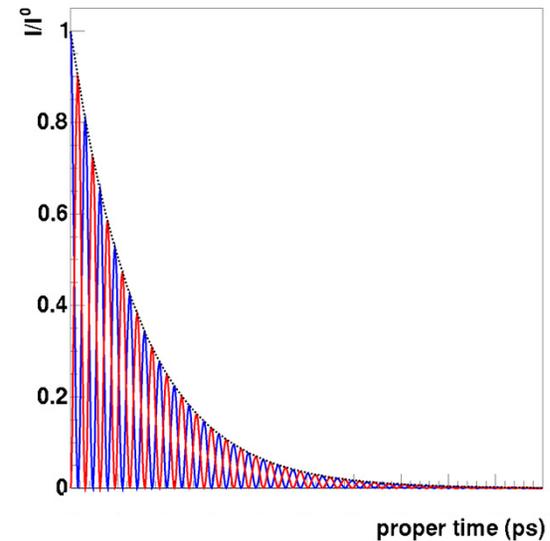
$$x \equiv \frac{\Delta m}{\Gamma} >> 1$$

CP Violation and SM - I

So far: Phenomenological description $\rightarrow$ ~ Just symmetries

Now: Try to connect to SM

K Mixing : *Box* diagrams
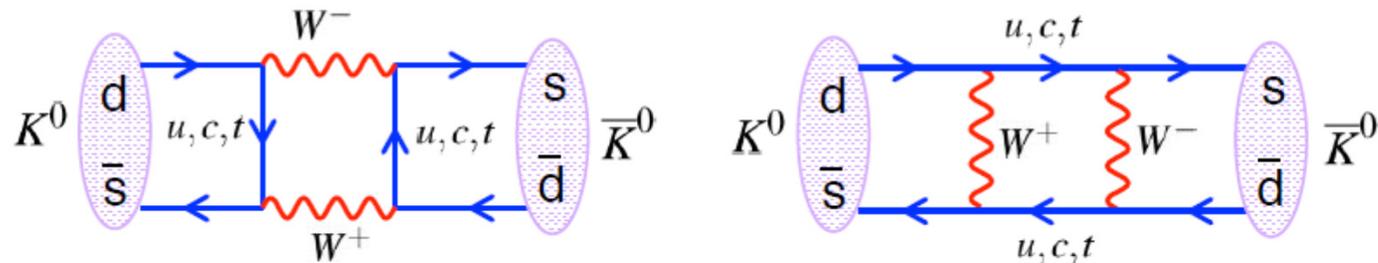
Mass difference between mass eigenstates:

$$\Delta m_K \approx \frac{G_F^2}{3\pi^2} f_K^2 m_K \left| V_{qd} V_{qs}^* V_{q'd} V_{q's}^* \right| m_q m_{q'}, \quad q, q' = u, c, t$$

CP Violation and SM - II

Go to *CKM* , find:

$$u,u \quad \sin^2\theta_c \cos^2\theta_c m_u^2 \qquad\qquad \sim 0.048 m_u^2 \qquad\qquad \sim .005$$

$$u,c \quad \sin^2\theta_c \cos^2\theta_c m_u m_c \qquad\qquad \sim 0.048 m_u m_c \qquad\qquad \sim .022$$

$$u,t \quad |V_{td}||V_{ts}|\sin\theta_c \cos\theta_c m_u m_t \qquad \sim 0.220\ 4 10^{-5} m_u m_t \quad \sim .0005$$

$$c,c \quad \sin^2\theta_c \cos^2\theta_c m_c^2 \qquad\qquad \sim 0.048 m_c^2 \qquad\qquad \sim .095$$

$$c,t \quad |V_{td}||V_{ts}|\sin\theta_c \cos\theta_c m_c m_t \qquad \sim 0.220\ 4 10^{-5} m_c m_t \quad \sim .0021$$

$$t,t \quad |V_{td}|^2 |V_{ts}|^2 m_t^2 \qquad\qquad \sim 1.6 10^{-10} m_t^2 \qquad\qquad \sim 0$$

$\rightarrow$ Diagrams with *c* quark : Most relevant

CP Violation and SM - III

Just for the fun: Oversimplify, take only charm contribution
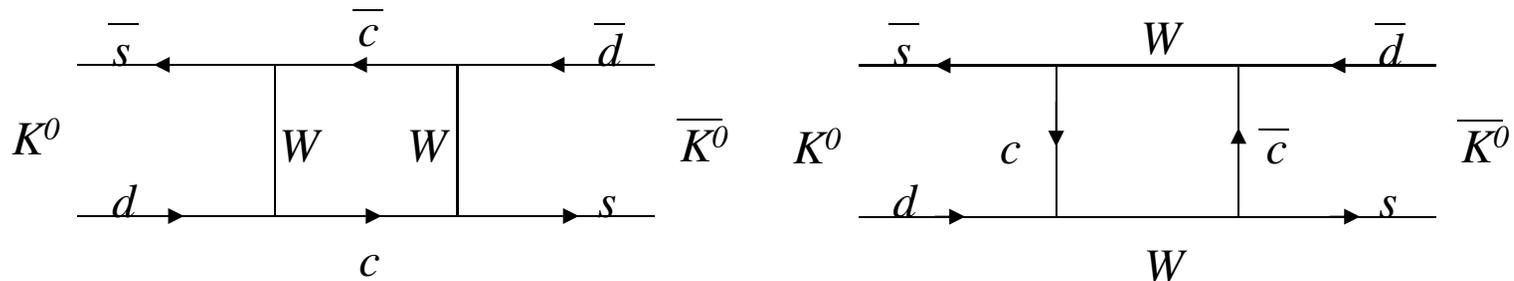
$$A_{box} \propto \left(V_{cs}V_{cd}^*\right)^2 m_c^2 \approx \left(\lambda^2 + i2A^2\lambda^6\eta\right)m_c^2$$

$$|\varepsilon| \approx \frac{\mathrm{Im}\left(A_{box}\right)}{\mathrm{Re}\left(A_{box}\right)} \approx \frac{2A^2\lambda^6\eta}{\lambda^2} = 2A^2\lambda^4\eta$$

$$|\varepsilon| \sim 2\ 0.81\ 0.0025\ 0.343 \sim 1.4\,10^{-3}$$

Not that bad...

CP Violation and SM - IV

Fun again: 2π decays and ε'

Must take into account two diagrams:

'Tree'   'Penguin'

$$\propto \left|V_{us}V_{ud}\right|^2 \approx \lambda^2$$

Top dominating:

$$\propto \mathrm{Im}\left(V_{ts}V_{td}\right) \approx A^2\lambda^5\eta$$
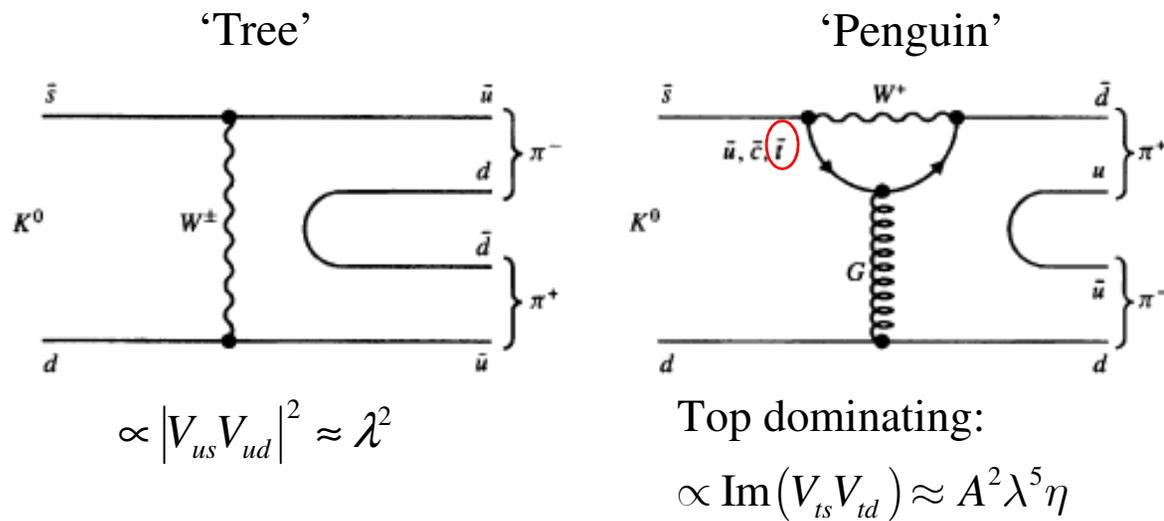
$\varepsilon' \propto$ Interference between the two above $\sim \lambda^2 A^2 \lambda^5 \eta \sim \lambda^7$

$$\rightarrow \frac{\varepsilon'}{\varepsilon} \sim \frac{A^2\lambda^7\eta}{2A^2\lambda^4\eta} \sim \frac{\lambda^3}{2} \sim 5\,10^{-3} \quad \text{Not that bad too...}$$

CP Violation and SM - V

Reconsidering box diagrams:

$$M_{fi} \propto A_{ct} V_{cd} V_{cs}^* V_{td} V_{ts}^* \qquad M'_{fi} \propto A_{ct} V_{cd}^* V_{cs} V_{td}^* V_{ts} = M_{fi}^*$$
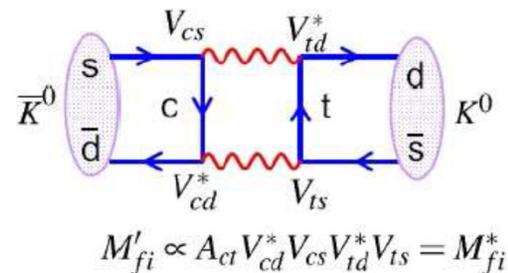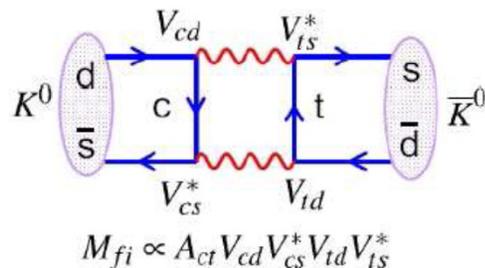
$$\Gamma(K_{t=0}^0 \to \bar{K}^0) - \Gamma(\bar{K}_{t=0}^0 \to K^0) \propto M_{fi} - M_{fi}^* = 2 Im(M_{fi})$$

$$A_T \equiv \frac{\Gamma(\overline{K}^0 \to K^0) - \Gamma(K^0 \to \overline{K}^0)}{\Gamma(\overline{K}^0 \to K^0) + \Gamma(K^0 \to \overline{K}^0)}.$$

Rememebering:

$$A_T \approx 4 \operatorname{Re}(\varepsilon) \to 4 \operatorname{Re}(\varepsilon) \propto 2 \operatorname{Im} M_{fi}$$

$\to$ No $C\!\!\!/P$ from mixing unless some *CKM* elements are complex

Summary about neutral kaons:

Lifetime, width, mass:

$$m_1 \simeq m_2 \rightarrow \Delta m \simeq 4.1\,10^{-6}\,eV$$

$$\tau_1 \ll \tau_2 \sim 0.09 - 52 \; ns$$

$$\Gamma_2 \ll \Gamma_1 \sim 11.1\,ns^{-1} \sim 0.038\,10^{-3}\,eV$$

$$\Gamma = \frac{\Gamma_1 + \Gamma_2}{2} \simeq \frac{\Gamma_1}{2}$$

$$x \equiv \frac{\Delta m}{\Gamma} \sim 0.5, \quad y \equiv \frac{\Delta \Gamma}{2\Gamma} \sim 0.5$$

$$\omega_+ = m_H - i\frac{\Gamma_H}{2}, \omega_- = m_L - i\frac{\Gamma_L}{2}$$

$$\rightarrow \quad g_+(t) = \frac{e^{-i\left(m_L + \Delta m - i\frac{\Gamma_H}{2}\right)t} + e^{-i\left(m_L - i\frac{\Gamma_L}{2}\right)t}}{2} = \frac{e^{-im_L t}e^{-\frac{\Gamma_L}{2}t}\left(e^{i\Delta mt}e^{-\frac{\Gamma_H - \Gamma_L}{2}t} + 1\right)}{2} \approx e^{-imt}\frac{e^{i\Delta mt}e^{-\frac{\Gamma}{2}t} + 1}{2}$$

$$g_-(t) = \frac{e^{-i\left(m_L + \Delta m - i\frac{\Gamma_H}{2}\right)t} - e^{-i\left(m_L - i\frac{\Gamma_L}{2}\right)t}}{2} = \frac{e^{-im_L t}e^{-\frac{\Gamma_L}{2}t}\left(e^{i\Delta mt}e^{-\frac{\Gamma_H - \Gamma_L}{2}t} - 1\right)}{2} \approx e^{-imt}\frac{e^{i\Delta mt}e^{-\frac{\Gamma}{2}t} - 1}{2}$$

$\Delta m \approx 5.3\,ns^{-1}$

$K^0 \bar{K}^0$

$1/\Gamma(K^0_S) \approx 0.09ns$ $1/\Gamma(K^0_L) \approx 52ns$

-10   -5   0   5 $E\,[ns^{-1}]$
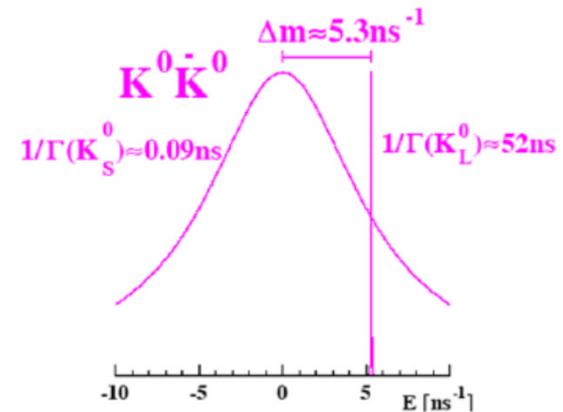
For K: $\Delta\Gamma = \Gamma_S - \Gamma_L \approx \Gamma_S, \left|\dfrac{q}{p}\right| = |\eta| \neq 1$

$$\begin{cases} \left|K^0(t)\right\rangle = g_+(t)\left|K^0\right\rangle + \eta g_-(t)\left|\overline{K^0}\right\rangle \approx e^{-imt}\left[\dfrac{e^{i\Delta mt}e^{-\frac{\Gamma}{2}t}+1}{2}\left|K^0\right\rangle + \eta\dfrac{e^{i\Delta mt}e^{-\frac{\Gamma}{2}t}-1}{2}\left|\overline{K^0}\right\rangle\right] \\[2em] \left|\overline{K^0}(t)\right\rangle = g_+(t)\left|\overline{K^0}\right\rangle + \dfrac{1}{\eta}g_-(t)\left|K^0\right\rangle \approx e^{-imt}\left[\dfrac{e^{i\Delta mt}e^{-\frac{\Gamma}{2}t}+1}{2}\left|\overline{K^0}\right\rangle + \dfrac{1}{\eta}\dfrac{e^{i\Delta mt}e^{-\frac{\Gamma}{2}t}-1}{2}\left|K^0\right\rangle\right] \end{cases}$$

$$P_{K^0}\left(K^0,t\right) = \left|g_+(t)\right|^2 = g_+ g_+^* \approx \frac{e^{i\Delta mt}e^{-\frac{\Gamma}{2}t}+1}{2}\frac{e^{-i\Delta mt}e^{-\frac{\Gamma}{2}t}+1}{2} = \frac{1}{4}\left(1+e^{-\frac{\Gamma}{2}t}\left(1+2\cos\Delta mt\right)\right)$$

$$P_{K^0}\left(\overline{K^0},t\right) \approx \frac{e^{i\Delta mt}e^{-\frac{\Gamma}{2}t}-1}{2}\frac{e^{-i\Delta mt}e^{-\frac{\Gamma}{2}t}-1}{2} = \frac{1}{4}|\eta|^2\left(1+e^{-\frac{\Gamma}{2}t}\left(1-2\cos\Delta mt\right)\right)$$

$$P_{\overline{K^0}}\left(K^0,t\right) \approx \frac{1}{4|\eta|^2}\left(1+e^{-\frac{\Gamma}{2}t}\left(1-2\cos\Delta mt\right)\right)$$

$$P_{\overline{K^0}}\left(\overline{K^0},t\right) \approx \frac{1}{4}\left(1+e^{-\frac{\Gamma}{2}t}\left(1+2\cos\Delta mt\right)\right)$$

CP Violation and SM - VIII

Transition probabilities

$K^0 \to K^0 \,/\, \bar{K}^0 \to \bar{K}^0$:

Identical (CPT)

$K^0 \to \bar{K}^0 \,/\, \bar{K}^0 \to K^0$:

Identical if $|\eta| = 1 \to$ No $\cancel{CP}$ in mixing: Full line

Different if $|\eta| \neq 1$: Dashed + Point lines

As shown prediction for $1 - |\eta| = 10 \times$ Exp. value
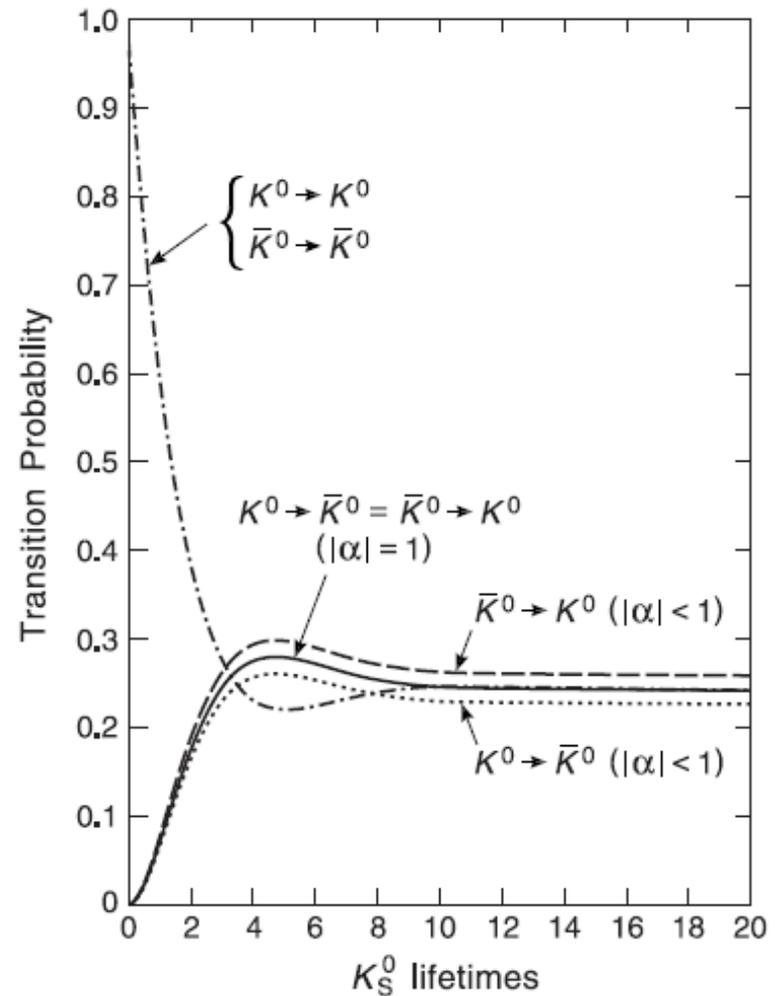
CP Violation and SM - IX

Rationale:

$\cancel{CP}$ observed in neutral kaon decays

Ascribed to mixing, decay, or both

Accounted for by a *single* complex phase in *CKM*

$\rightarrow$ Expect $\cancel{CP}$ to occur in other neutral, flavored meson decays

$\rightarrow$ Heavy quarks involved

Looking again at unitarity triangles:

$(1) \quad V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0;$ $\qquad (2) \quad V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0;$

$(3) \quad V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0;$ $\qquad (4) \quad V_{ud}^*V_{cd} + V_{us}^*V_{cs} + V_{ub}^*V_{cb} = 0;$

$(5) \quad V_{ud}^*V_{td} + V_{us}^*V_{ts} + V_{ub}^*V_{tb} = 0;$ $\qquad (6) \quad V_{cd}^*V_{td} + V_{cs}^*V_{ts} + V_{cb}^*V_{tb} = 0$

Not all equally useful: *Shape, Easy to measure*

CP Violation and SM - X

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

Im

$V_{ud}V_{ub}^*$

$V_{td}V_{tb}^*$

$b \to u : \quad \sim \lambda^3$

$B \leftrightarrow \bar{B} : \sim \lambda^3$

$V_{cd}V_{cb}^*$

Re

$b \to c :\sim \lambda^3$

Im

$\eta\left(1 - \dfrac{\lambda^2}{2}\right)$

$\alpha = \phi_2$

$V_{td}V_{tb}^*$

$\gamma = \phi_3$

$\beta = \phi_1$

$\rho\left(1 - \dfrac{\lambda^2}{2}\right)$

$(1,0)$ Re
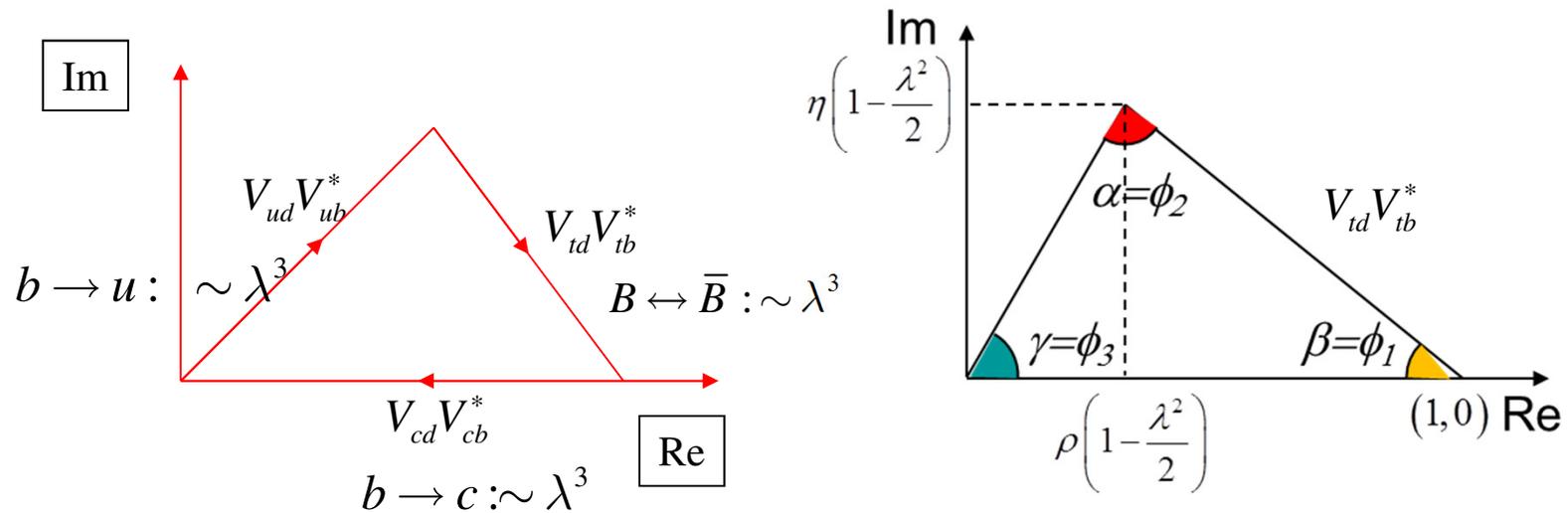
The unitarity triangle: Somewhat 'equilateral' →Large angles

$$V_{ud}^* V_{td} + V_{us}^* V_{ts} + V_{ub}^* V_{tb} = \delta_{tu} = 0 \quad \hat{=} \quad tu \ \text{triangle}$$

Another $\approx$ equilateral one

Each side $\propto \lambda^3$
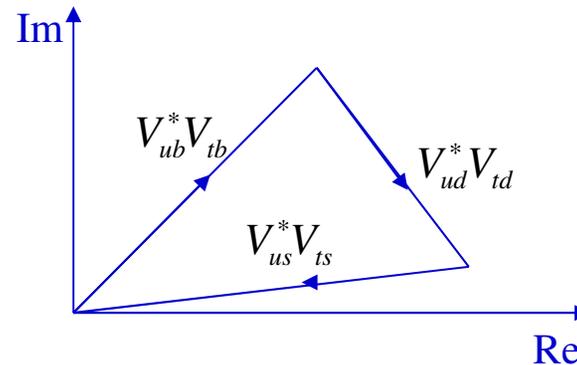
Two 'squashed' triangles…

2 sides $\propto \lambda^2$

1 side $\propto \lambda^4$

$$V_{us}^* V_{ub} + V_{cs}^* V_{cb} + V_{ts}^* V_{tb} = \delta_{bs} = 0 \quad \hat{=} \quad bs \text{ triangle}$$

$$V_{td}^* V_{cd} + V_{ts}^* V_{cs} + V_{tb}^* V_{cb} = \delta_{tc} = 0 \quad \hat{=} \quad tc \text{ triangle}$$

Im

Not to scale

$V_{ts} V_{tb}^*$

$V_{us} V_{ub}^*$

$V_{cs} V_{cb}^*$

Re

Im

Not to scale

$V_{cs}^* V_{ts}$

$V_{cd}^* V_{td}$

$V_{cb}^* V_{tb}$

Re

Difficult to use to test $\cancel{CP}$

$\cancel{CP} \propto$ Height with base normalized to 1
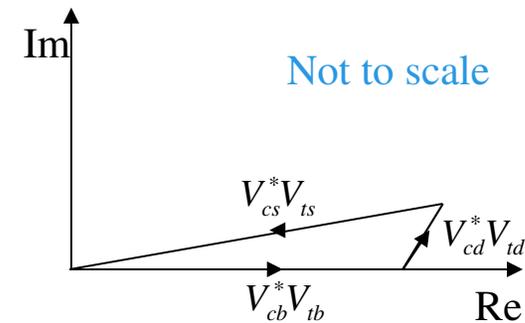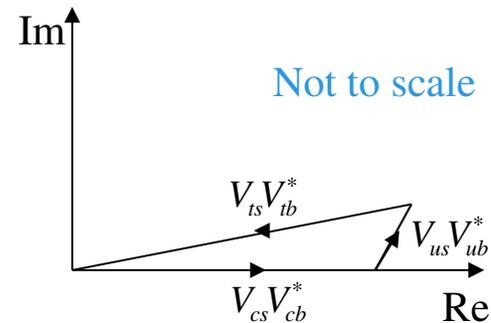
CP Violation and SM - XIII

…and two even more squashed

2 sides $\propto \lambda$

1 side $\propto \lambda^5$

$V_{ud}^* V_{us} + V_{cd}^* V_{cs} + V_{td}^* V_{ts} = \delta_{sd} = 0 \quad \hat{=} \quad sd$ triangle

$V_{ud}^* V_{cd} + V_{us}^* V_{cs} + V_{ub}^* V_{cb} = \delta_{cu} = 0 \quad \hat{=} \quad cu$ triangle

Im

Not to scale

$V_{cd}V_{cs}^*$

$V_{td}V_{ts}^*$

$V_{ud}V_{us}^*$

Re

Im

Not to scale

$V_{ud}^*V_{cd}$

$V_{ub}^*V_{cb}$
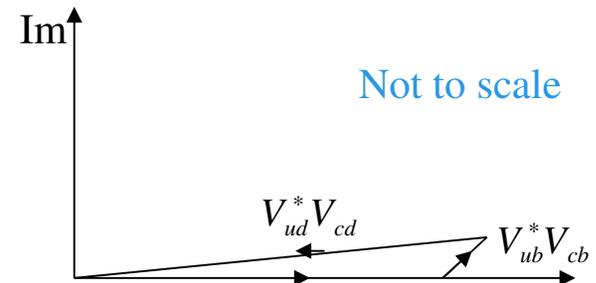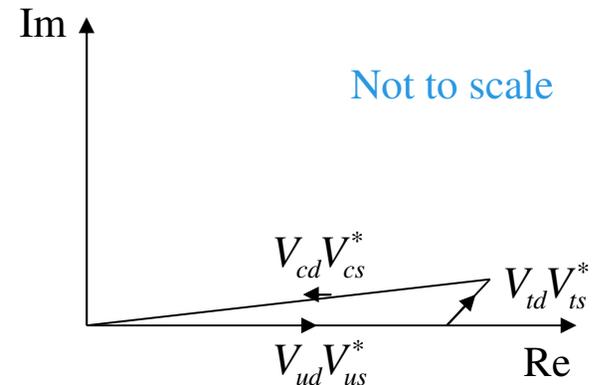
CP Violation and SM - XIV

Neutral, flavored mesons: Lightest states

$K^0 : d\bar{s}$

$\bar{K}^0 : \bar{d}s$

$D^0 : c\bar{u}$

$\bar{D}^0 : \bar{c}u$

$B^0 : \bar{b}d$

$\bar{B}^0 : b\bar{d}$

$B_S^0 : \bar{b}s$

$\bar{B}_S^0 : b\bar{s}$

	$<\tau>$	Δm	$x=\Delta m/\Gamma$	$y=\Delta\Gamma/2\Gamma$
K^0	$2.6 \ 10^{-8}$ s	5.29 ns^{-1}	$\Delta m/\Gamma_S=0.49$	~1
D^0	$0.41 \ 10^{-12}$ s	0.001 fs^{-1}	~0	0.01
B^0	$1.53 \ 10^{-12}$ s	0.507 ps^{-1}	0.78	~0
B_s^0	$1.47 \ 10^{-12}$ s	17.8 ps^{-1}	12.1	~0.05

CP Violation and SM - XV

Lineshapes of mass eigenstates of neutral, flavored meson systems
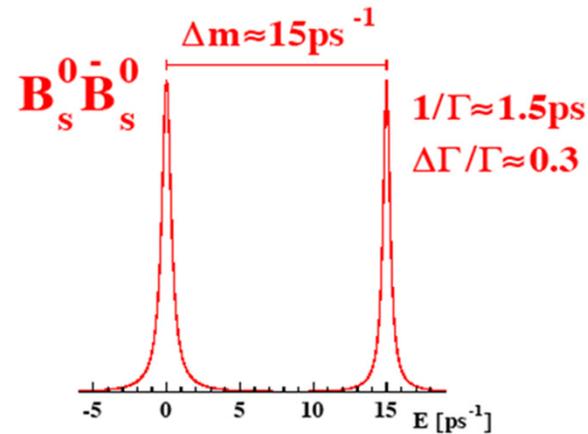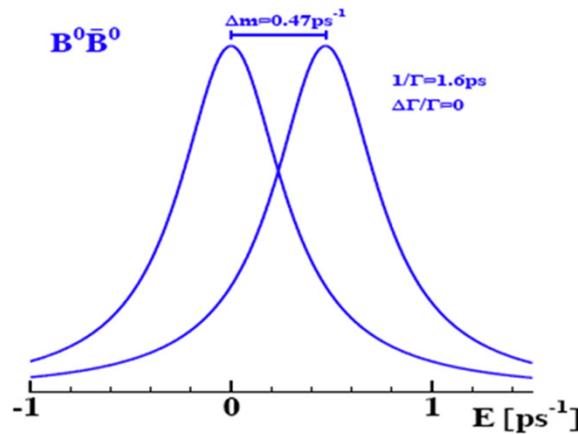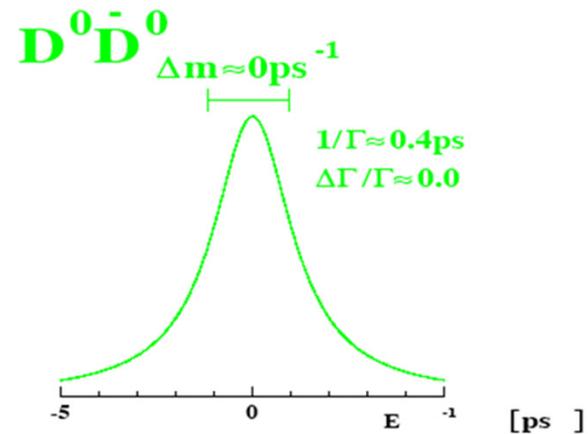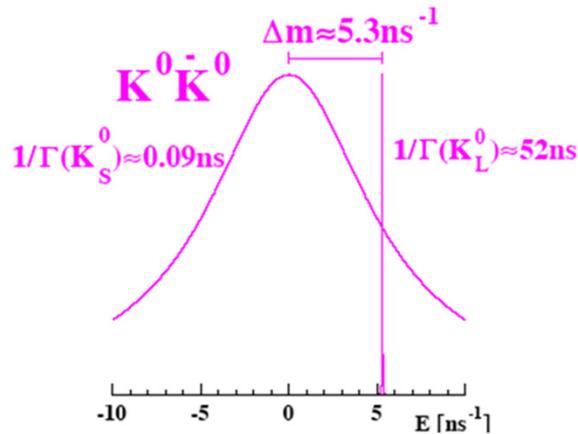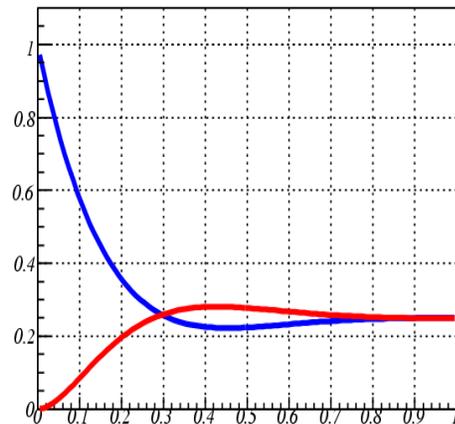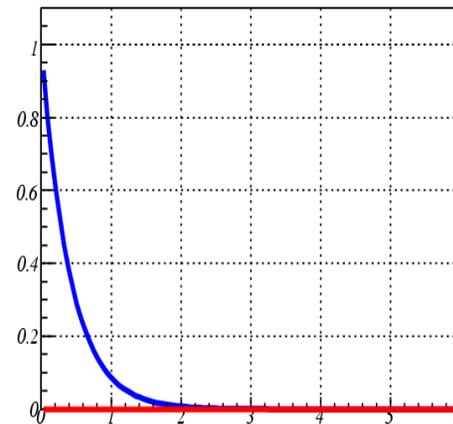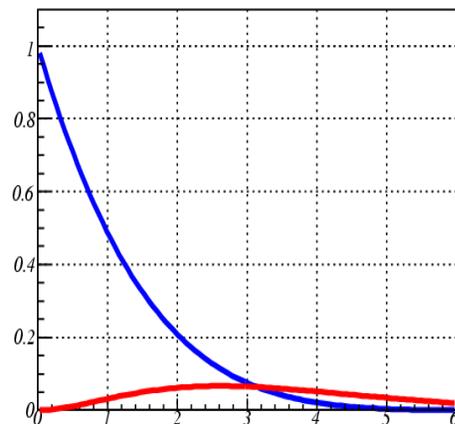
CP Violation and SM - XVI
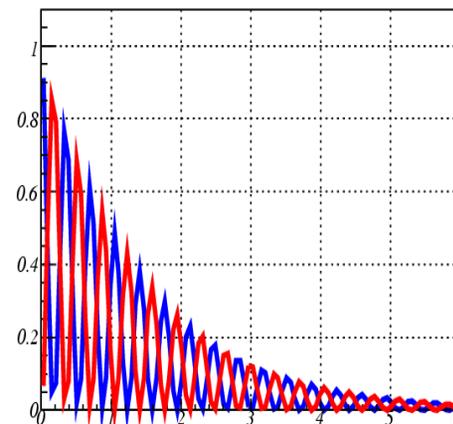
CP Violation and SM - XVII

Extend box diagrams to other neutral systems: $D^0 - \bar{D}^0$
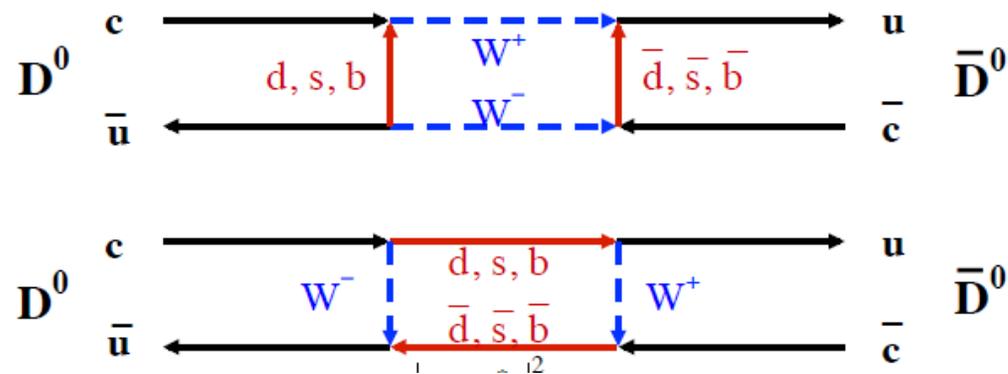
b loop: Strong CKM suppression

$$M \propto \left| V_{ub} V_{cb}^* \right|^2 \ll 1$$

Indeed, go to Wolfenstein parametrization:

$$\left| V_{ub} V_{cb}^* \right|^2 \sim \left| \lambda^3 \lambda^2 \right|^2 \sim 10^{-7}$$

s, d loops: Strong GIM suppression

$$M \propto \left(m_s^2 - m_d^2 \right) \text{ small!}$$

$\rightarrow$ Expect very small mixing

Long distance effects ($\leftarrow$ Meson exchange, rather than quarks) important
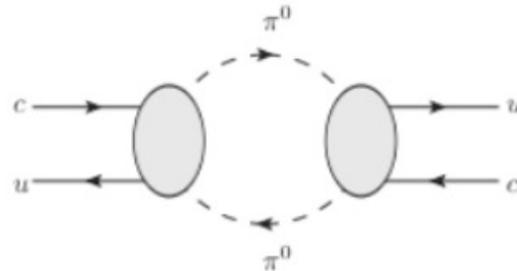
Lifetime, width, mass: Very different from K^0, difficult to compute

$$m_1 \simeq m_2$$

$$\tau_1 \simeq \tau_2 = (4.15 \pm 0.04)10^{-13} \; s$$

$$\Gamma_2 \simeq \Gamma_1 = (1.59 \pm 0.01)10^{-12} \; GeV$$

$$\Gamma = \frac{\Gamma_1 + \Gamma_2}{2} \simeq \Gamma_2 \simeq \Gamma_1$$

$$\left. \begin{aligned} x &\equiv \frac{\Delta m}{\Gamma} = \frac{m_2 - m_1}{\Gamma} \\ y &\equiv \frac{\Delta \Gamma}{2\Gamma} = \frac{\Gamma_2 - \Gamma_1}{2\Gamma} \end{aligned} \right\} \text{Estimate } x, \; y \sim 10^{-4} - 10^{-3}$$

$\mathbf{D^0\bar{D}^0}$ $_{\Delta m \approx 0 ps^{-1}}$

$1/\Gamma \approx 0.4 ps$
$\Delta \Gamma / \Gamma \approx 0.0$

-5   0   E $^{-1}$ [ps]
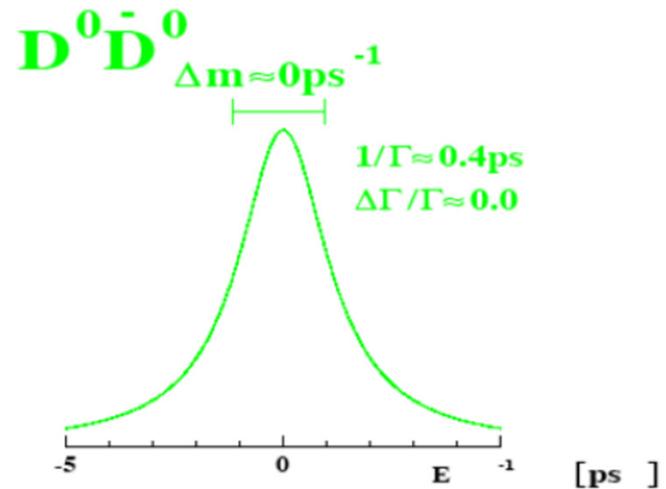
$\sim$ Same mass: Oscillation frequency small

$\sim$ Same (small) lifetime : D_H, D_L cannot be physically separated (Compare to K_S / K_L..)

$\rightarrow$ Only chance to observe mixing by time integrated measurement

Tag D flavor at both production and decay

Production: Take strong decays

$$D^{*+} \rightarrow D^0 \pi^+ , \bar{D}^{*-} \rightarrow \bar{D}^0 \pi^-$$

Decay: Take two modes

$$D^0 \rightarrow K^+ \mu^- \bar{\nu}_\mu \quad \text{forbidden, only accessed by mixing } D^0 \rightarrow \bar{D}^0$$

$$D^0 \rightarrow K^- \mu^+ \nu_\mu \quad \text{allowed}$$

$$\rightarrow R = \frac{N\left(K^+ \mu^- \bar{\nu}_\mu\right)}{N\left(K^- \mu^+ \nu_\mu\right)} \cong \frac{x^2 + y^2}{2}$$

Measurement difficult, large samples required

Mixing & $C\!\!\!/P$ observed since 2007 by BaBar, Belle, LHCb

Most promising sector for validation of $CKM : B^0 - \bar{B}^0$

'Large' $\cancel{CP}$ expected

Similar to $K^0 - \bar{K}^0$, but:

$\Delta M_B = 0.489 \pm 0.008 \ ps^{-1}$

$\rightarrow \Delta M_B \sim 0.489 \ 10^{12} \ s^{-1}$

$\hbar \approx 6.582 \ 10^{-16} eV \ s \rightarrow \Delta M_B \sim 3.22 \ 10^{-4} eV \sim 100\Delta$

$\tau_{B_1} \approx \tau_{B_2} = 1.56 \pm 0.06 \ ps$

$\rightarrow \dfrac{\Delta M_B}{\Gamma_B} \sim 1$

Compare to K:

$\Delta M_K = 5.29 \ ns^{-1} \sim 5.29 \ 10^9 \ s^{-1} \ 6.582 \ 10^{-16} eV \ s \sim 3.4 \ 10^{-6} eV$

$\rightarrow \dfrac{\Delta M_K}{\Gamma_{K_S}} \sim 1$

$\tau_L \sim 600 \ \tau_S \sim 600 \cdot 89 \ ps$

B_H, B_L states cannot be physically separated

Figure labels: $B^0\bar{B}^0$, $\Delta m = 0.47 ps^{-1}$, $1/\Gamma = 1.6 ps$, $\Delta\Gamma/\Gamma = 0$, E [ps^{-1}]
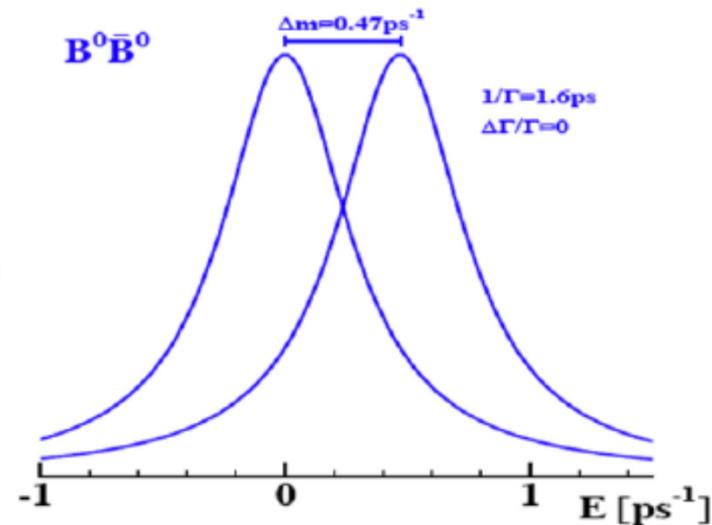
B Mixing: CP - I

Box diagrams, *t* dominated for B^0
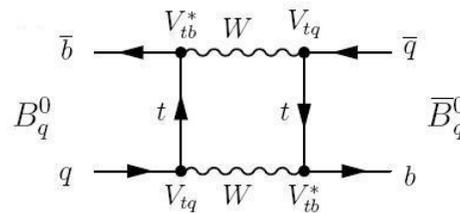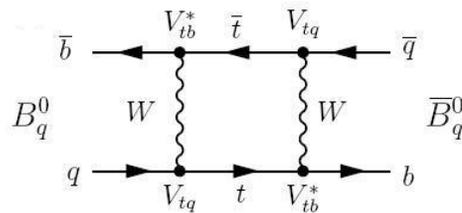
$$\begin{pmatrix} 1-\dfrac{\lambda^2}{2} & \lambda & A\lambda^3(\rho-i\eta) \\[2mm] -\lambda & 1-\dfrac{\lambda^2}{2} & A\lambda^2 \\[2mm] A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

Mixing parameter:

$$\eta = \frac{\left(V_{tb}^* V_{td}\right)}{\left(V_{tb} V_{td}^*\right)} \approx \frac{V_{td}}{V_{td}^*} = e^{-2i\varphi_{td}}$$

From UT:

$$\varphi_{td} = \beta$$

$$\rightarrow \eta = e^{-2i\beta}$$

$$\rightarrow |\eta| \cong 1$$
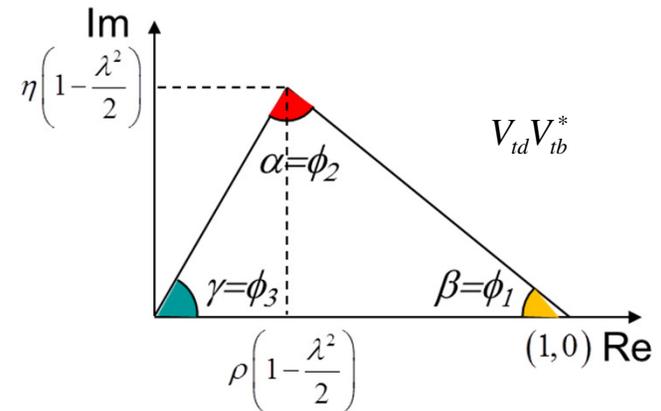
B Mixing: CP - II

Mass eigenstates:

$$
\begin{cases}
\left| B_H \right\rangle = \dfrac{1}{\sqrt{1+|\eta|^2}} \left(\left| B^0 \right\rangle + \eta \left| \overline{B^0} \right\rangle \right) \\[2em]
\left| B_L \right\rangle = \dfrac{1}{\sqrt{1+|\eta|^2}} \left(\left| B^0 \right\rangle - \eta \left| \overline{B^0} \right\rangle \right)
\end{cases}
,\qquad
\begin{cases}
\left| B^0 \right\rangle = \dfrac{1}{2} \sqrt{1+|\eta|^2} \left(\left| B_H \right\rangle + \left| B_L \right\rangle \right) \\[2em]
\left| \overline{B}^0 \right\rangle = \dfrac{1}{2\eta} \sqrt{1+|\eta|^2} \left(\left| B_H \right\rangle - \left| B_L \right\rangle \right)
\end{cases}
$$

$$
\rightarrow \eta = \frac{1-\varepsilon_B}{1+\varepsilon_B}, \ \ \varepsilon_B \ \text{analog to } \varepsilon \text{ used for kaons}
$$

Time evolution of mass eigenstates:

$$
\rightarrow
\begin{cases}
\left| B_H(t) \right\rangle = \left| B_H \right\rangle e^{-i\left(m_H - i\frac{\Gamma_H}{2} \right)t} \\[2em]
\left| B_L(t) \right\rangle = \left| B_L \right\rangle e^{-i\left(m_L - i\frac{\Gamma_L}{2} \right)t}
\end{cases}
,\ \ \Gamma_H \simeq \Gamma_L = \Gamma, \ \Delta m \ll M
$$

B Mixing: CP - III

Time evolution of flavor eigenstates:

$$\begin{cases} \left|B^0(t)\right\rangle = \left|B^0\right\rangle f_+(t) + \eta\left|\overline{B^0}\right\rangle f_-(t) \\ \left|\overline{B}^0(t)\right\rangle = \left|B^0\right\rangle \frac{1}{\eta} f_-(t) + \left|\overline{B^0}\right\rangle f_+(t) \end{cases}$$

$$f_+(t) = \frac{e^{-i\left(M+\frac{\Delta m}{2}-i\frac{\Gamma_H}{2}\right)t} + e^{-i\left(M-\frac{\Delta m}{2}-i\frac{\Gamma_L}{2}\right)t}}{2}, f_-(t) = \frac{e^{-i\left(M+\frac{\Delta m}{2}-i\frac{\Gamma_H}{2}\right)t} - e^{-i\left(M-\frac{\Delta m}{2}-i\frac{\Gamma_L}{2}\right)t}}{2}$$

$$f_\pm(t) \approx \frac{1}{2} e^{-\frac{1}{2}\Gamma t} e^{-iMt}\left[e^{-i\frac{\Delta m}{2}t} \pm e^{+i\frac{\Delta m}{2}t}\right], \quad M = \frac{m_H + m_L}{2}, \quad \Delta m > 0, \quad \Gamma_H \simeq \Gamma_L = \Gamma$$

$$\rightarrow \begin{cases} f_+(t) \approx \frac{1}{2} e^{-\frac{1}{2}\Gamma t} e^{-iMt} \cos\left(\frac{\Delta m}{2}t\right) \\ f_-(t) \approx -i\frac{1}{2} e^{-\frac{1}{2}\Gamma t} e^{-iMt} \sin\left(\frac{\Delta m}{2}t\right) \end{cases}$$

$$\rightarrow \begin{cases} \left|B^0(t)\right\rangle = e^{-iMt} e^{-\frac{1}{2}\Gamma t}\left[\cos\left(\frac{\Delta m}{2}t\right)\left|B_0\right\rangle - i\eta\sin\left(\frac{\Delta m}{2}t\right)\left|\overline{B_0}\right\rangle\right] \\ \left|\overline{B}^0(t)\right\rangle = e^{-iMt} e^{-\frac{1}{2}\Gamma t}\left[-\frac{i}{\eta}\sin\left(\frac{\Delta m}{2}t\right)\left|B_0\right\rangle + \cos\left(\frac{\Delta m}{2}t\right)\left|\overline{B_0}\right\rangle\right] \end{cases}$$

B Mixing: CP - IV

→ Expect:

$$\Gamma\left(B^0\left(t=0\right)\to B^0\right)=e^{-\Gamma t}\cos^2\left(\frac{\Delta m}{2}t\right)$$

$$\Gamma\left(B^0\left(t=0\right)\to \overline{B}^0\right)=\left|\eta\right|^2 e^{-\Gamma t}\sin^2\left(\frac{\Delta m}{2}t\right)$$

$$\Gamma\left(\overline{B}^0\left(t=0\right)\to \overline{B}^0\right)=e^{-\Gamma t}\cos^2\left(\frac{\Delta m}{2}t\right)$$

$$\Gamma\left(\overline{B}^0\left(t=0\right)\to \overline{B}^0\right)=\left|\frac{1}{\eta}\right|^2 e^{-\Gamma t}\sin^2\left(\frac{\Delta m}{2}t\right)$$

Similar for B_s^0 :

Important difference $\Delta m \gg \Gamma$

→ Many oscillations in a lifetime

B Mixing: CP - V

Unlike $K^0 / \bar{K}^0$:

$|\eta| \simeq 1 \rightarrow \ \sim$ No $\cancel{CP}$ effect observable by looking at flavor oscillations
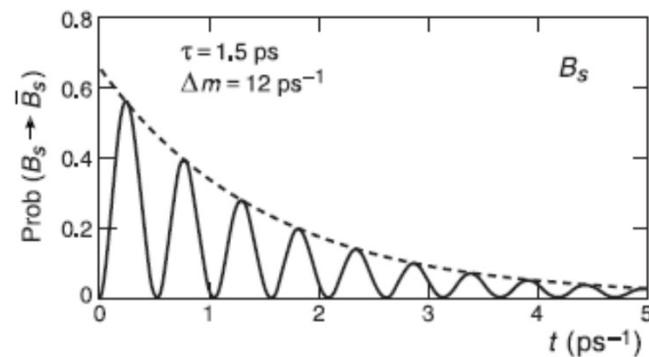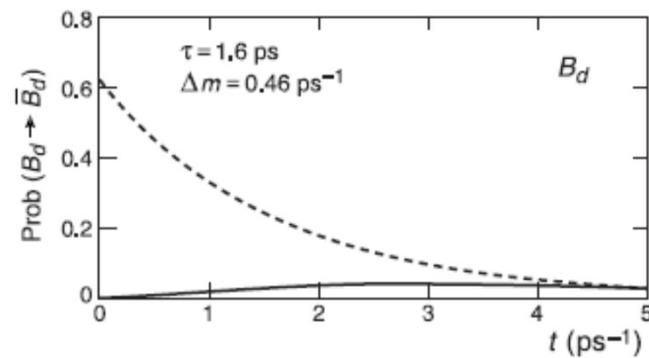
B Mixing: CP - VI

By restricting to decays to *CP* eigenstates: OK!

Main disadvantage: Statistics (Tiny BR)

Golden final state : $J/\psi K_S^0$ (or K_L^0 : Experimentally less attractive)

$B^0, \bar{B}^0 \to J/\psi K_S^0$

Angular momentum balance:

$0 = 1 \oplus 0 \oplus L \to L = 1$ Pure P-wave

$CP(J/\psi) = (-1)(-1) = +1$

$CP(K_S^0) = +1,$ neglect $\cancel{CP}$ in K^0

$P_{orb} = (-1)^L = -1$

$\to CP(J/\psi K_S^0) = -1$

$\to CP(J/\psi K_L^0) = +1$
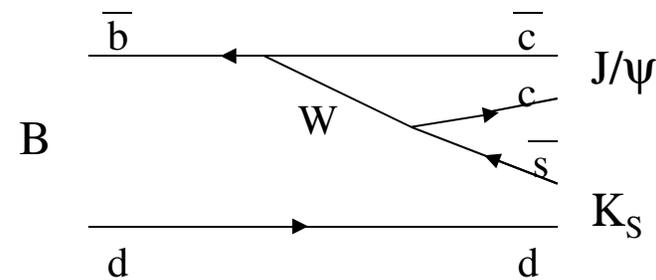
Another golden : $\pi\pi$

$B^0, \bar{B}^0 \to \pi\pi$

Angular momentum balance:

$L = 0$ Pure S - wave

$CP(\pi\pi) = +1$
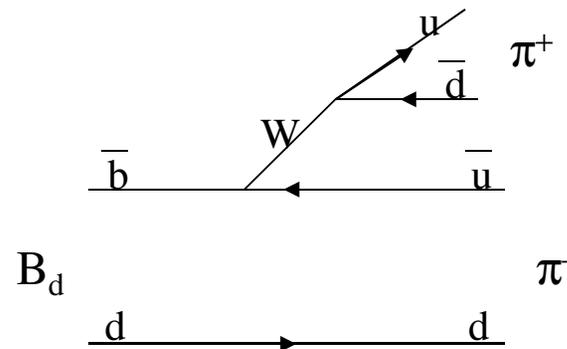
Taking decays into (golden) *CP* eigenstates:

$$A\left(B^0 \to J/\psi K_S^0\right) = \left\langle J/\psi K_S^0 \left| H_{eff} \right| B^0(t) \right\rangle$$

$$\to A\left(B^0 \to J/\psi K_S^0\right) = f_+(t)\left\langle J/\psi K_S^0 \left| H_{eff} \right| B^0 \right\rangle + \eta f_-(t)\left\langle J/\psi K_S^0 \left| H_{eff} \right| \overline{B}^0 \right\rangle$$

$$\to A\left(B^0 \to J/\psi K_S^0\right) = \left\langle J/\psi K_S^0 \left| H_{eff} \right| B^0 \right\rangle \left[f_+(t) + \eta f_-(t)\frac{\left\langle J/\psi K_S^0 \left| H_{eff} \right| \overline{B}^0 \right\rangle}{\left\langle J/\psi K_S^0 \left| H_{eff} \right| B^0 \right\rangle} \right]$$

Considering $B^0, \overline{B}^0$ decay: Must occur in two steps

$$B^0 \to J/\psi\, K^0 \to J/\psi\, K_S^0 \qquad \overline{B}^0 \to J/\psi\, \overline{K}^0 \to J/\psi\, K_S^0$$

because at the quark level:

$$\overline{b} \to \overline{c}\, c\overline{s} \qquad\qquad b \to c\, \overline{c}s$$

$$A\left(B^0 \to J/\psi K^0\right) \propto V_{cb}^* V_{cs} \qquad A\left(\overline{B}^0 \to J/\psi \overline{K}^0\right) \propto V_{cb} V_{cs}^*$$

$$\to \frac{\left\langle J/\psi K_S^0 \left| H_{eff} \right| \overline{B}^0 \right\rangle}{\left\langle J/\psi K_S^0 \left| H_{eff} \right| B^0 \right\rangle} = +1 \quad CKM \text{ elements involved } real \qquad \frac{\left\langle \psi K_L \left| H \right| \overline{B}^0 \right\rangle}{\left\langle \psi K_L \left| H \right| B^0 \right\rangle} = -1\,.$$

B Mixing: CP - IX

$$\Gamma\left(B^0\left(t=0\right)\to J/\psi K_S\right)\propto\left|f_+\left(t\right)+\eta f_-\left(t\right)\right|^2$$

$$\to\Gamma\left(B^0\left(t=0\right)\to J/\psi K_S\right)\propto e^{-\Gamma t}\left|\cos\left(\frac{\Delta m}{2}t\right)-ie^{-2i\beta}\sin\left(\frac{\Delta m}{2}t\right)\right|^2$$

$$\to\Gamma\left(B^0\left(t=0\right)\to J/\psi K_S\right)\propto e^{-\Gamma t}\left(1-\sin\Delta mt\sin 2\beta\right)$$

$$\to\Gamma\left(\bar{B}^0\left(t=0\right)\to J/\psi K_S\right)\propto e^{-\Gamma t}\left(1+\sin\Delta mt\sin 2\beta\right)$$

Time dependent asymmetry:

$$A_{J/\psi K_S}=\frac{\Gamma\left(B^0\left(t=0\right)\to J/\psi K_S\right)-\Gamma\left(\bar{B}^0\left(t=0\right)\to J/\psi K_S\right)}{\Gamma\left(B^0\left(t=0\right)\to J/\psi K_S\right)+\Gamma\left(\bar{B}^0\left(t=0\right)\to J/\psi K_S\right)}=\sin\Delta mt\sin 2\beta$$

$$A_{J/\psi K_L}=-\sin\Delta mt\sin 2\beta$$

B Factories - I

Total e^+e^- annihilation cross section:
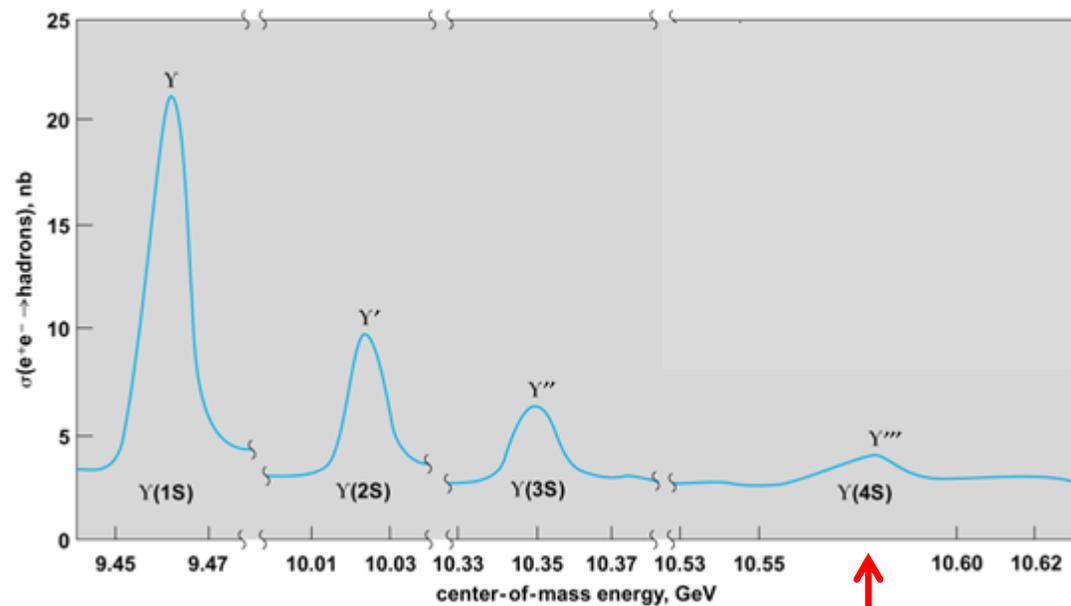
Main decay mode for $\Upsilon(4S)$:

$\Upsilon(4S) \rightarrow B\bar{B},$ including $B^0\bar{B}^0$

B Factories - II

Basic idea to measure time dependent asymmetry:
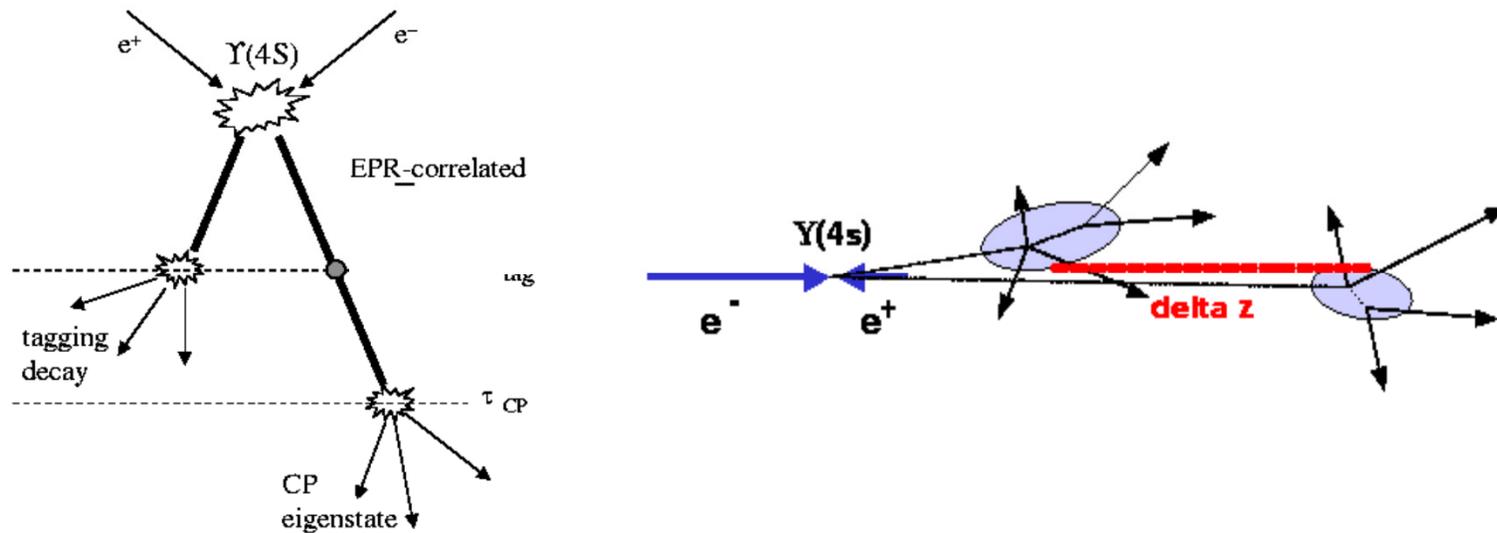
Measure time difference between 'tag' meson and 'CP' meson decays:

Use space distance between vertexes

Measurement difficult in CM, due to short lifetime ($d \sim 30 \ \mu m$)

$\rightarrow$ Boost mesons in lab by making collider *asymmetric*

$\rightarrow \Upsilon(4S)$ moving in the lab system

B Factories - III
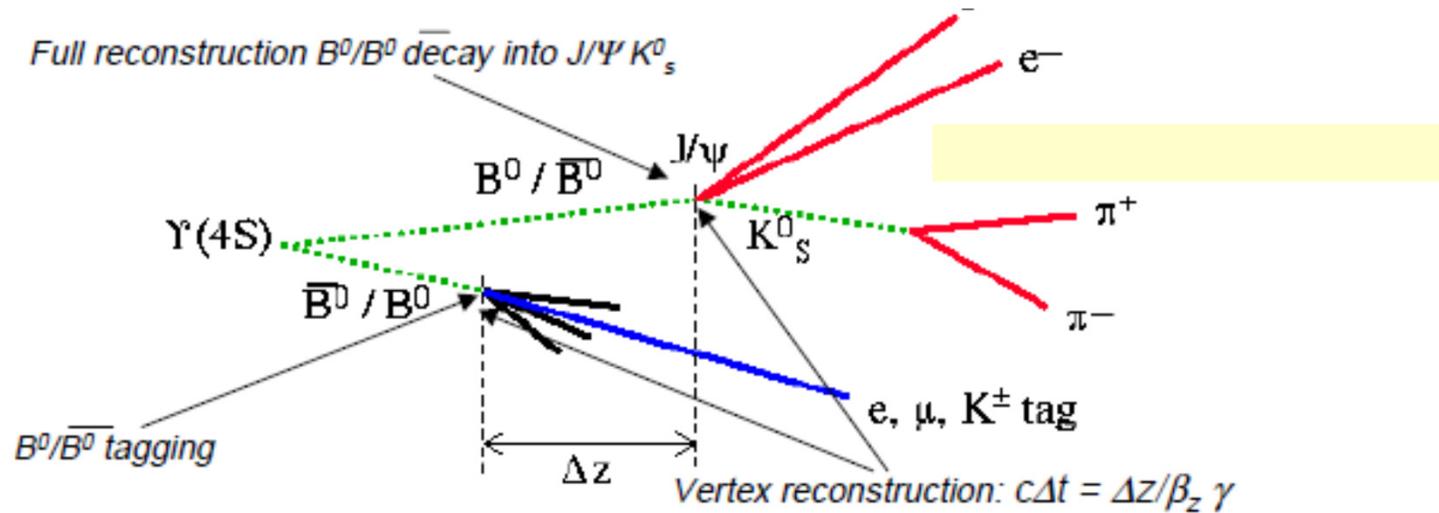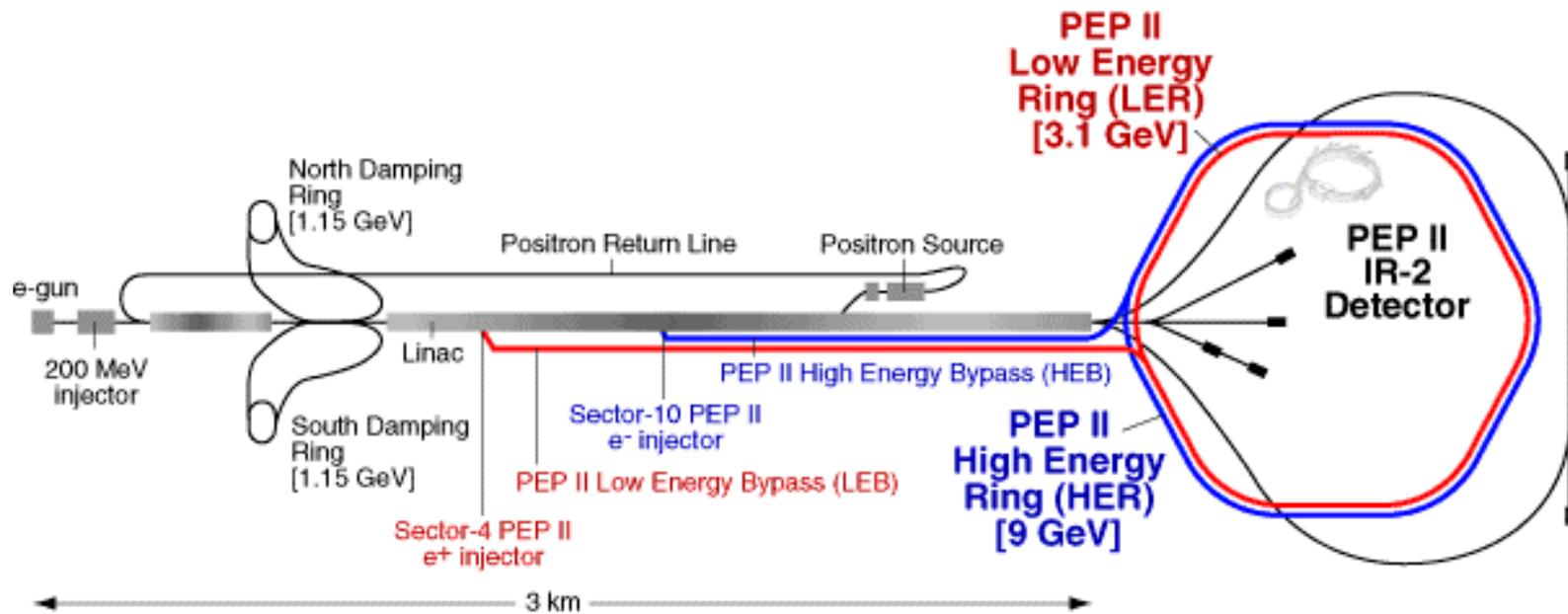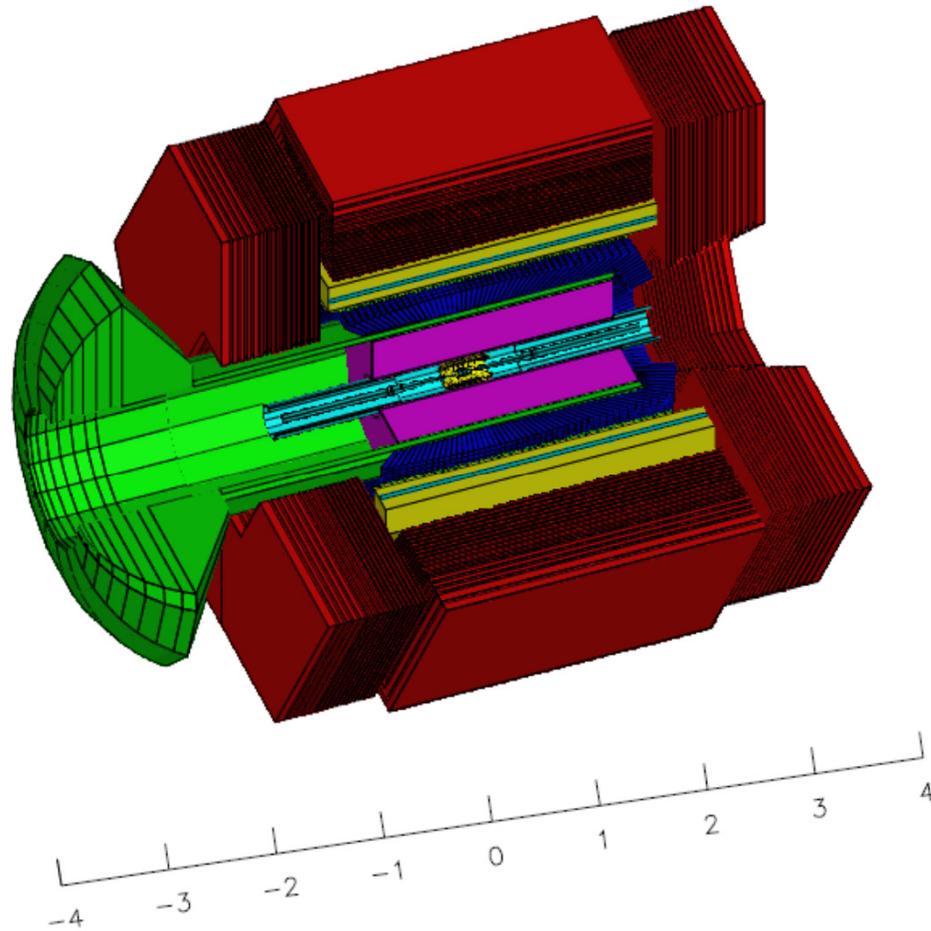
Example:

Full reconstruction B⁰/B̄⁰ decay into J/Ψ K⁰ₛ

$\Upsilon(4S)$

$B^0 / \overline{B^0}$

J/ψ

K^0_S

e⁻

π⁺

π⁻

$\overline{B^0} / B^0$

e, μ, K± tag

B⁰/B̄⁰ tagging

Δz

Vertex reconstruction: $c\Delta t = \Delta z/\beta_z \gamma$

B Factories - IV

B Factories - V

BABAR

B Factories - VI

BABAR DETECTOR FOR THE PEP-II B FACTORY

B Factories - VII

B Factories - VIII

580 mm — Space Frame

Bkwd. support cone

520 mrad

IV IV IV IV IV IV VI
VI IV IV IV IV IV
VI V V V V IV VI VI
VI V V V V IV VI
III III III III III III
II II II II II
I

Fwd. support cone   350 mrad

e^- e^+

Front end electronics

Beam Pipe

Vertex detector

580mm
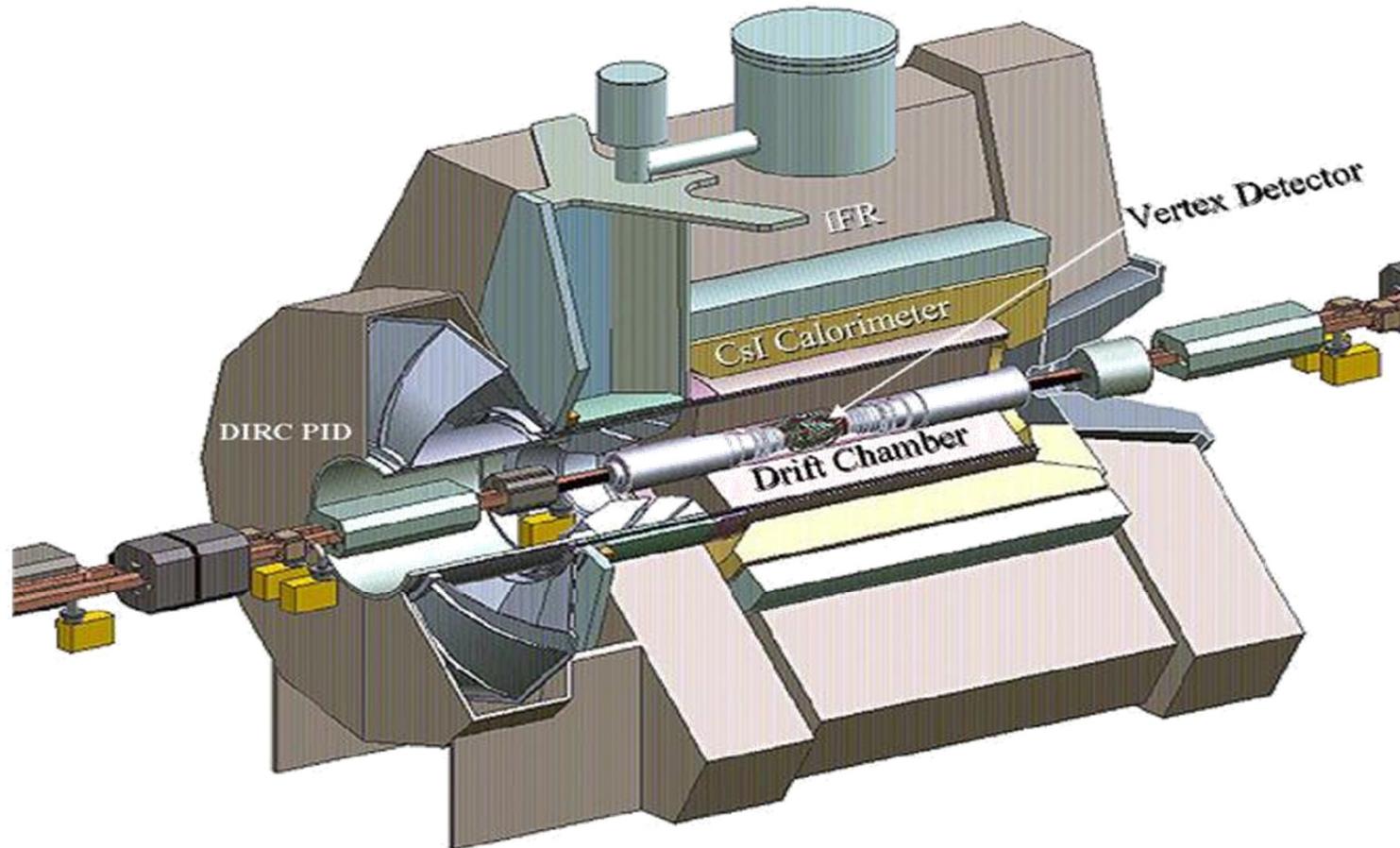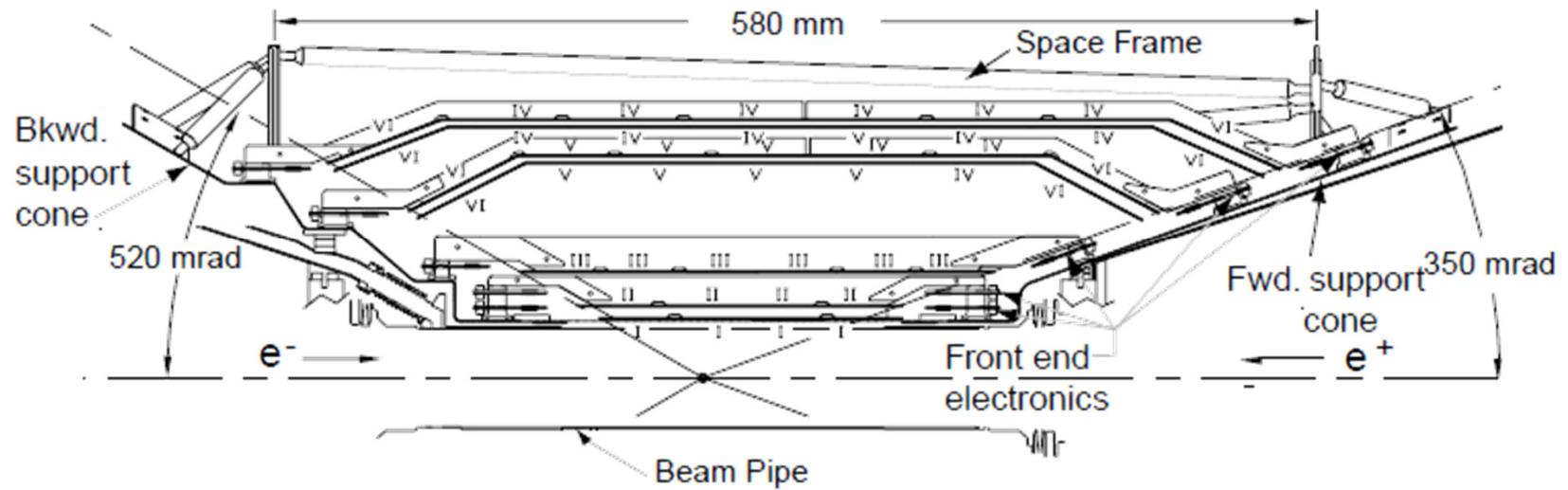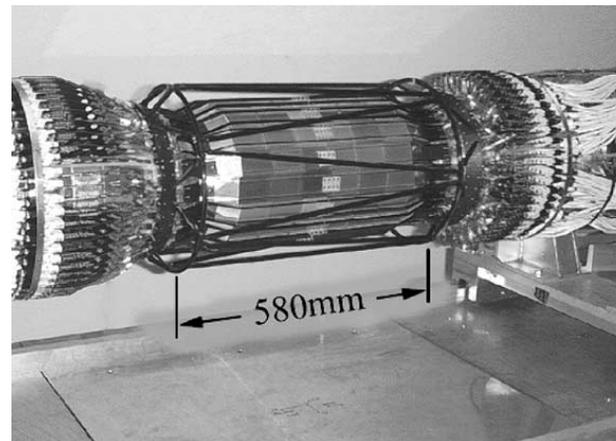
Drift chamber
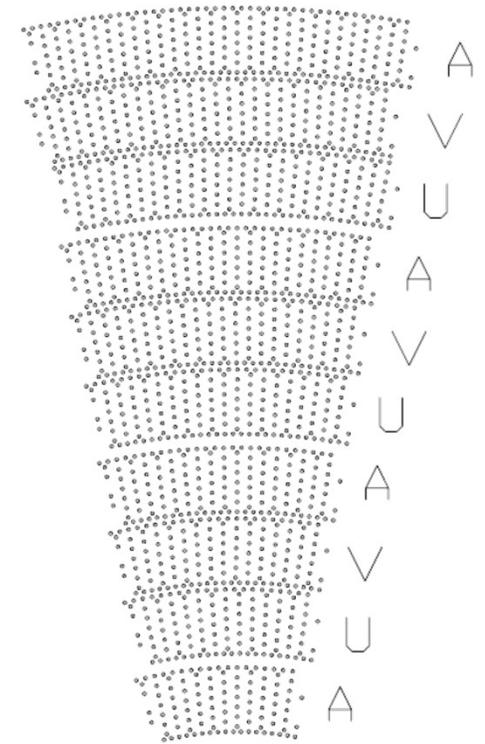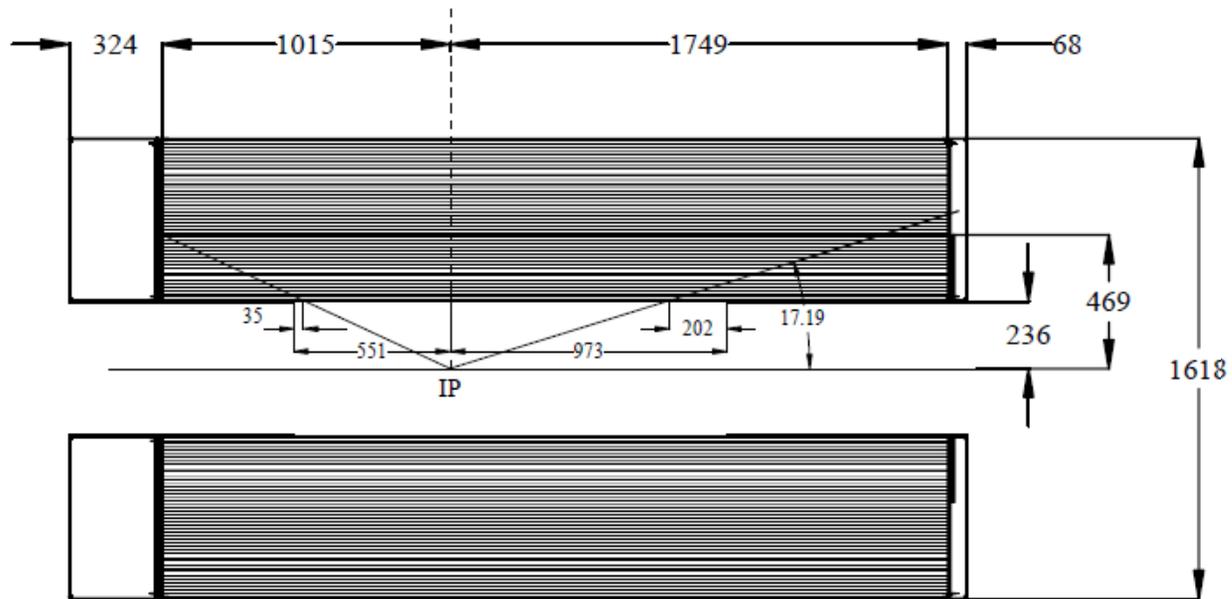
B Factories - X

DIRC : Particle Id

B Factories - XI

DIRC

Support tube

Quartz Barbox

Compensating coil

Assembly flange

Standoff box
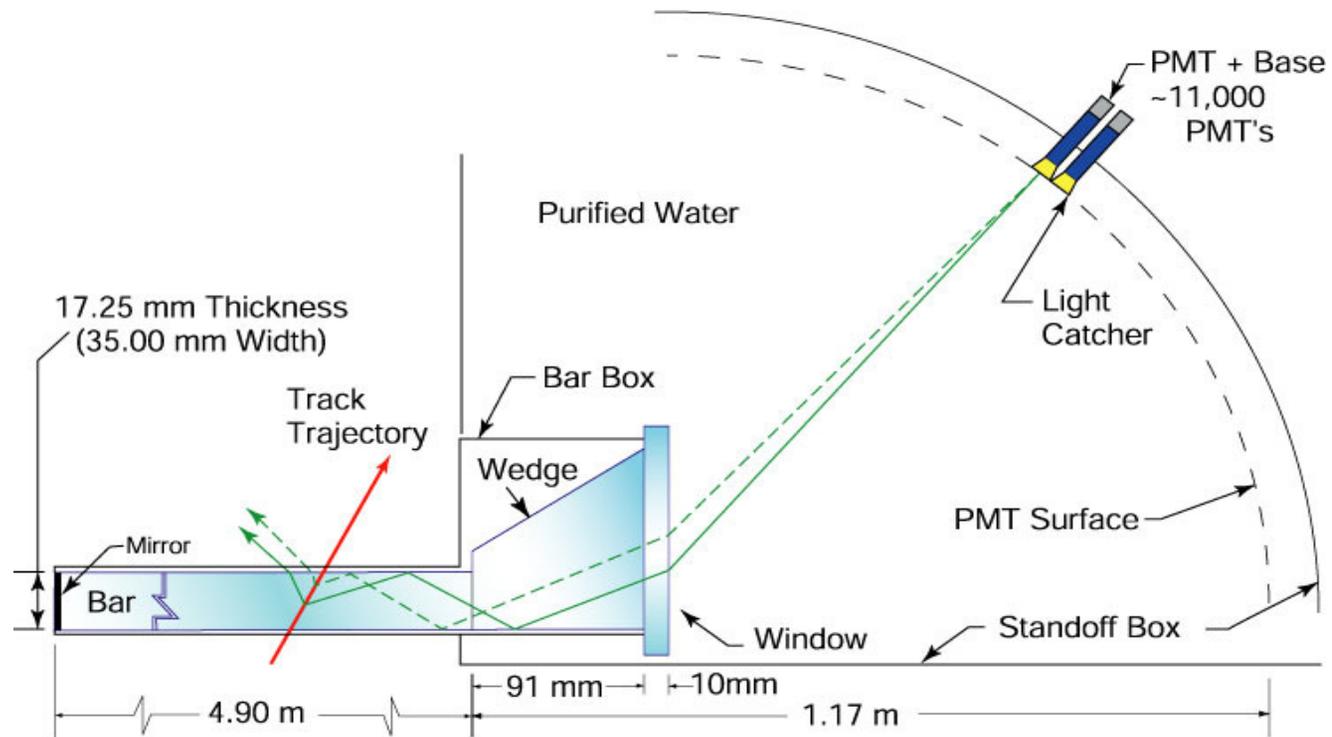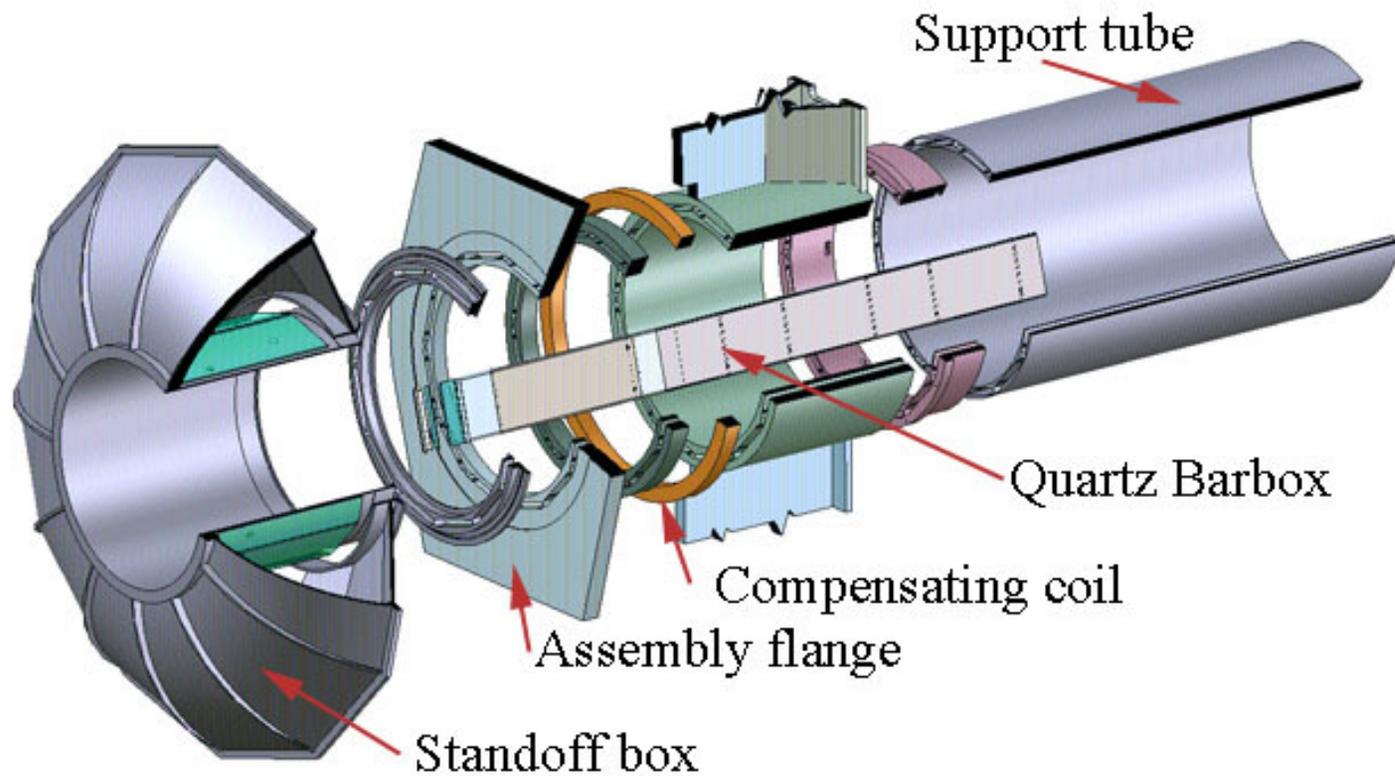
B Factories - XII
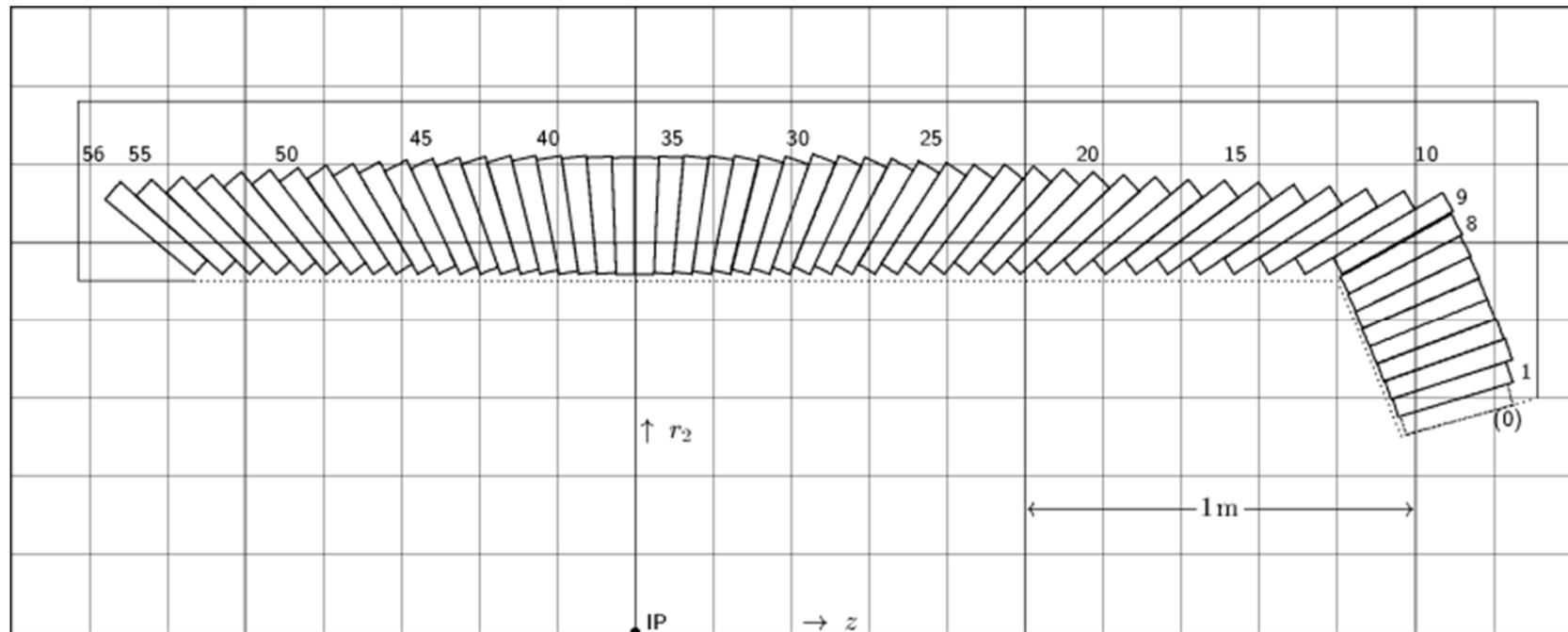
EM Calorimeter

B Factories - XIII

Instrumented Flux Return: Muon detector & (coarse) hadron calorimeter
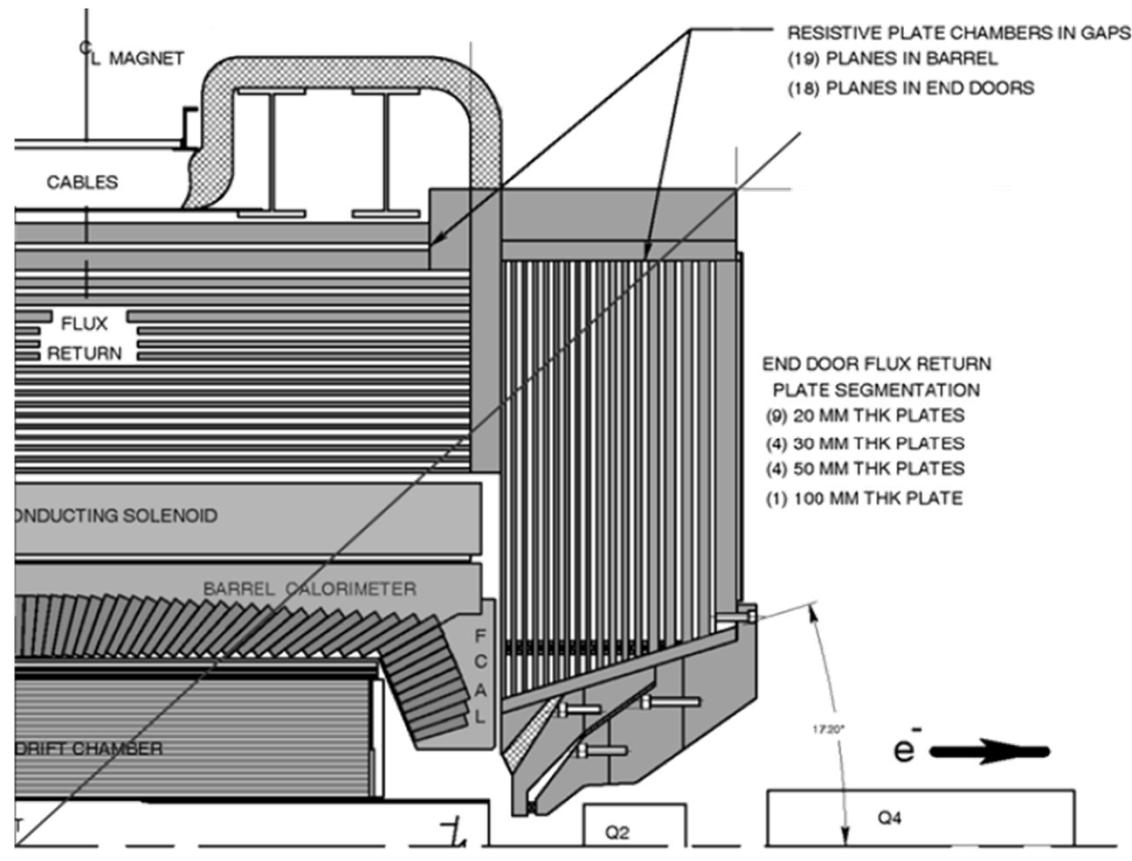
Tagging: finding the flavor of the 2nd *B*-meson

Leptons : cleanest tag (correct=97%, efficiency=8.6%)
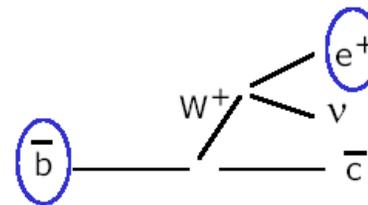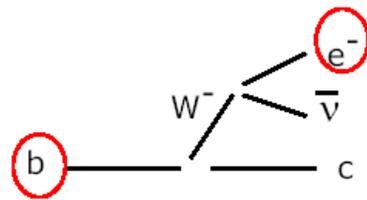
Kaons : 2nd cleanest tag (correct 85%-95%, efficiency=28%)
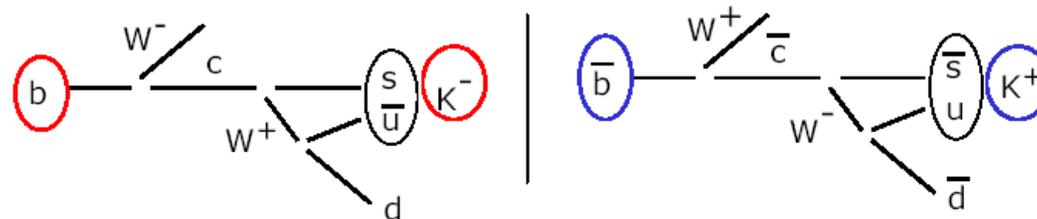
w = mistag probability = 1 - correct   "dilution": D = 1-2w

B Factories - XV

Tag B
$\sigma_z \sim 170~\mu m$

CP B
$\sigma_z \sim 70~\mu m$

J/Ψ

Y(4s)

$\beta\gamma = 0.56~(0.43)$

Belle

$\Delta t \cong \Delta z/\gamma\beta c$

$\gamma\beta c\tau_B \cong 250~\mu m$

Δz

K⁰

I:autostato CP

Y(4s)→ B⁰ → J/ψ K_s

(→π⁺π⁻)

→ B⁰ → K⁻ X

II:etichettatura del sapore
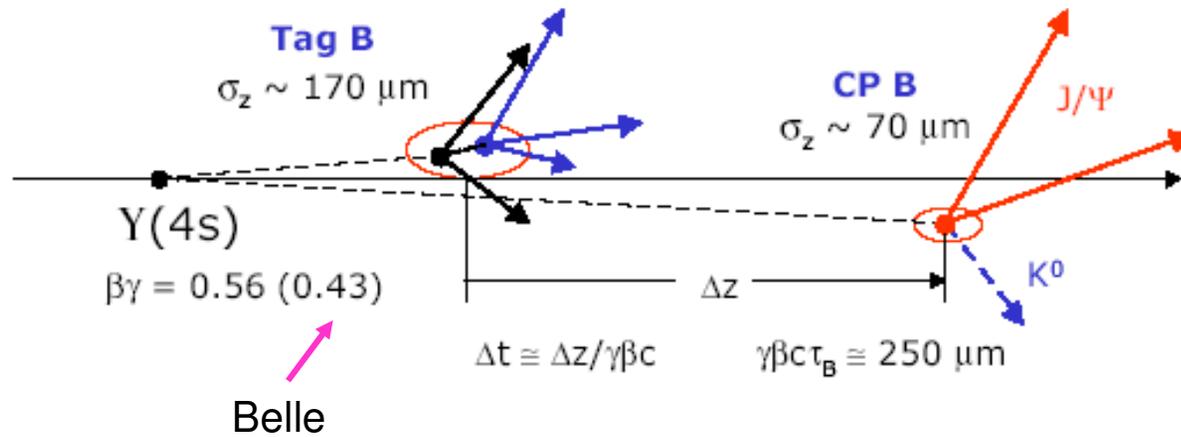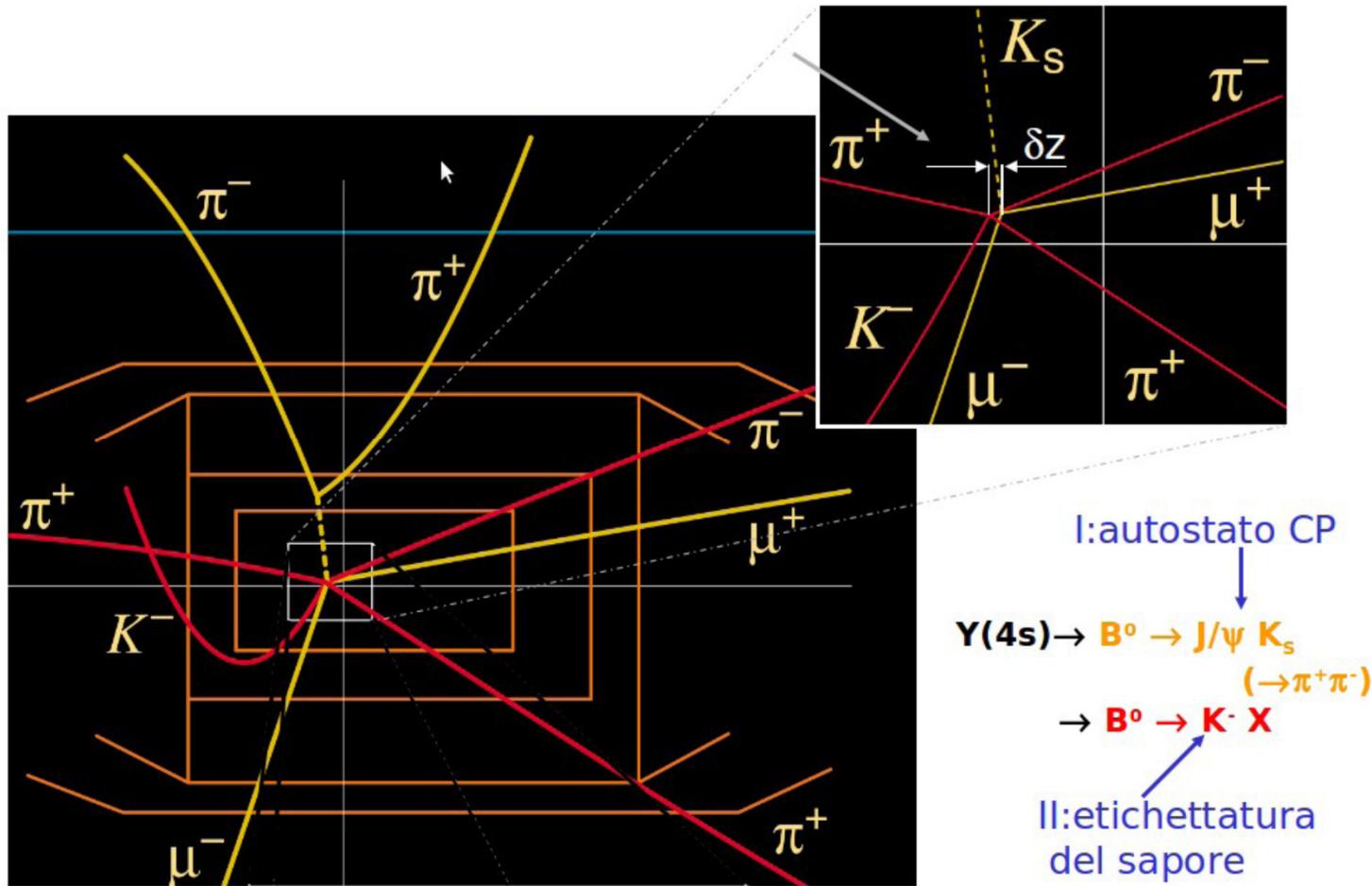
B Factories - XVII

BABAR
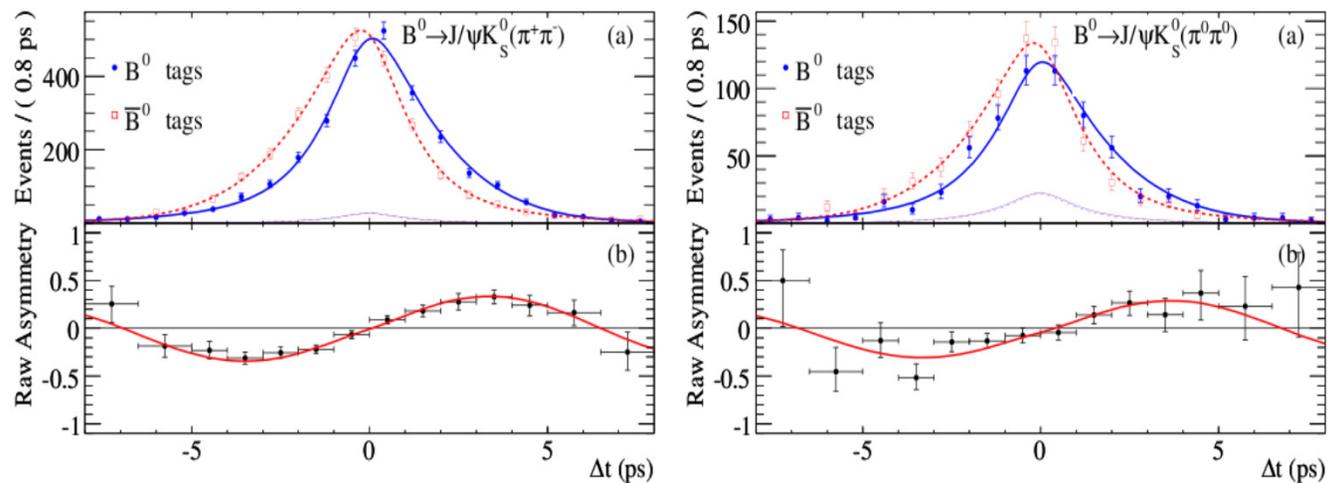
Sample	$-\eta_f S_f$	C_f
Full CP sample	0.687 ± 0.028	0.024 ± 0.020
$J/\psi K_S^0(\pi^+\pi^-)$	0.662 ± 0.039	0.017 ± 0.028
$J/\psi K_S^0(\pi^0\pi^0)$	0.625 ± 0.091	0.091 ± 0.063