Elementary Particles II

3 – Flavor Physics and CP Violation

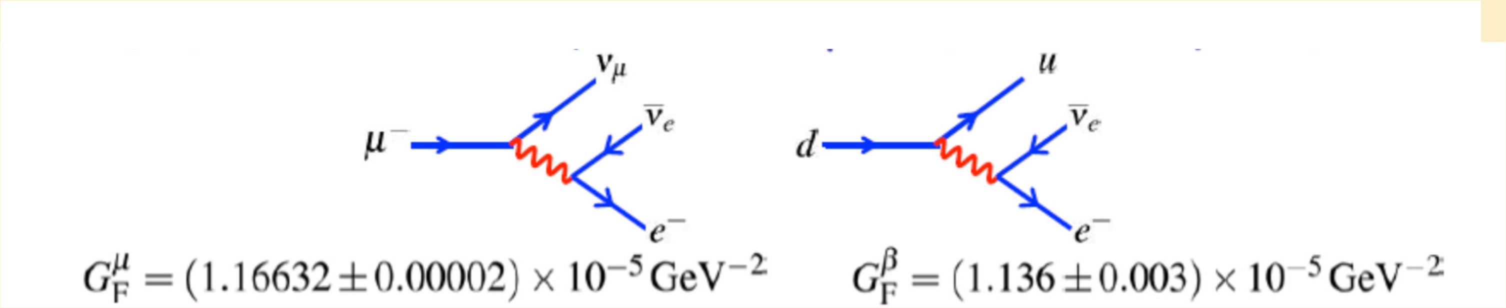
Quark Mixing – CKM – K^0 Strangeness oscillations CP violation – Extension to Bottom and Charm FCNC and Physics Beyond the Standard Model

Quark Mixing - I

Reminder:

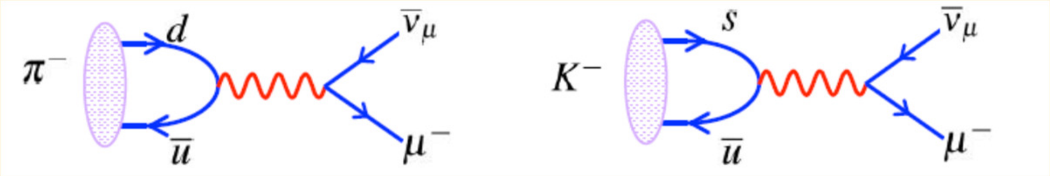
Fermi constant from μ decay $\simeq$ Fermi constant from β decay

Tiny difference:



$G_F^\mu = (1.16632 \pm 0.00002) \times 10^{-5}\,\mathrm{GeV}^{-2}$ $\quad$ $G_F^\beta = (1.136 \pm 0.003) \times 10^{-5}\,\mathrm{GeV}^{-2}$

Kaon decay suppressed by a factor ~ 20 as compared to π decay



Quark Mixing - II

Cabibbo explanation:

Weak eigenstates Strong (mass) eigenstates

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

Weak charged currents: Linear combinations of different flavors

$\theta_C \approx 13.1^0$

Unique value for Cabibbo angle explaining many strange particle decays

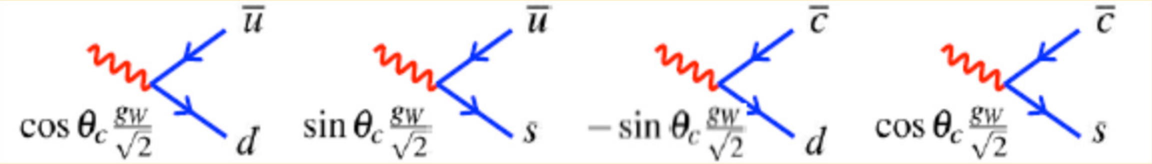
Strong support for universality of weak interaction

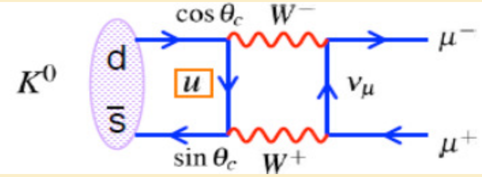
Quark Mixing - III

Another mistery

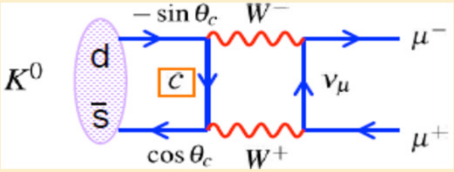
$$BR\left(K^0 \to \mu^+\mu^-\right) \sim 10^{-8}\, BR\left(K^+ \to \mu^+\nu_\mu\right)$$

GIM explaination:





$$M_1 \propto g_W^4 \cos\theta_c \sin\theta_c$$



$$M_2 \propto -g_W^4 \cos\theta_c \sin\theta_c$$

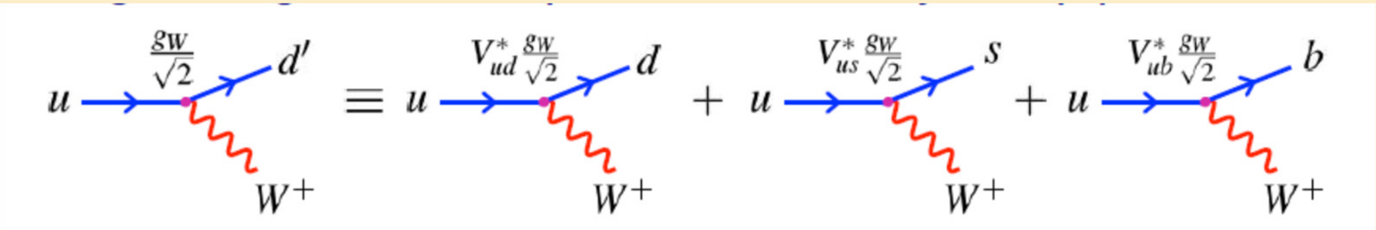
Tiny BR left due to $m_c \neq m_u$ in the virtual quark propagator

Quark Mixing - IV

Extend mixing to 3 families:

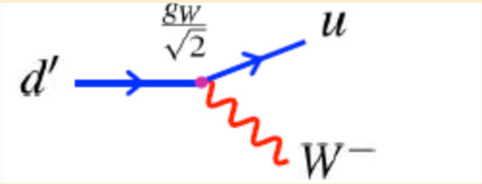
$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V^{\dagger}_{(d)} \begin{pmatrix} d \\ s \\ b \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}, \quad \begin{pmatrix} u' \\ c' \\ t' \end{pmatrix} = V^{\dagger}_{(u)} \begin{pmatrix} u \\ c \\ t \end{pmatrix}$$

→ Conventionally: Mixing of d-like quarks only



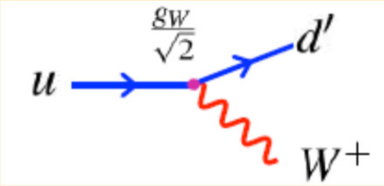
Quark Mixing - V

Encode mixing CKM matrix element into charged current



$$j_{d'u} = \bar{u}\left[-i\frac{g_W}{\sqrt{2}}\gamma^\mu \tfrac{1}{2}(1-\gamma^5)\right] d'$$

$$j_{du} = \bar{u}\left[-i\frac{g_W}{\sqrt{2}}\gamma^\mu \tfrac{1}{2}(1-\gamma^5)\right] V_{ud}\, d$$



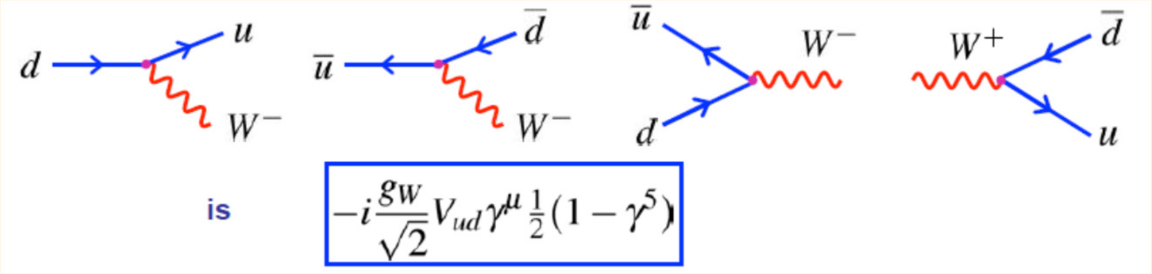
$$j_{ud'} = \bar{d}'\left[-i\frac{g_W}{\sqrt{2}}\gamma^\mu \tfrac{1}{2}(1-\gamma^5)\right] u$$

$$\bar{d}' = d'^\dagger\gamma^0 \to (V_{ud}d)^\dagger\gamma^0 = V^*_{ud}d^\dagger\gamma^0 = V^*_{ud}\bar{d}$$

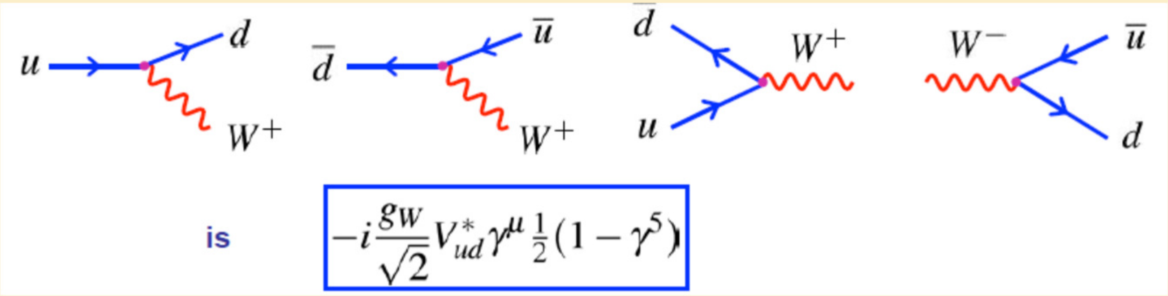
$$j_{ud} = \bar{d}V^*_{ud}\left[-i\frac{g_W}{\sqrt{2}}\gamma^\mu \tfrac{1}{2}(1-\gamma^5)\right] u$$

Quark Mixing - VI

Charged current : $qq, \overline{qq}, q\overline{q}$



is $\boxed{-i\frac{g_W}{\sqrt{2}}V_{ud}\gamma^\mu\frac{1}{2}(1-\gamma^5)}$



is $\boxed{-i\frac{g_W}{\sqrt{2}}V_{ud}^*\gamma^\mu\frac{1}{2}(1-\gamma^5)}$

CKM - I

Generic mixing matrix:

Mixing weak eigenstates into mass eigenstates (or the opposite)

3×3 Unitary matrix:

9 complex parameters → 18 real parameters

9 unitarity conditions: $\left.\begin{array}{r} UU^{\dagger}=1 \\ \left(U^{\dagger}\right)_{ij}=U^{*}_{ji} \end{array}\right\} \rightarrow \sum_{j=1}^{3} a_{ij}a^{*}_{jk}=\delta_{ik}, \quad i,k=1,..,3$

$\rightarrow 18-9=9$ free, real parameters

Observe: Cannot mix quarks carrying different electric charge

Indeed, Superselection rule:

Physical states always have sharp charge

CKM - II

Mixing matrix definition for 'up'- and 'down'-like quarks:

$$\begin{pmatrix} u \\ c \\ t \end{pmatrix} = V_{(u)} \begin{pmatrix} u_1' \\ u_2' \\ u_3' \end{pmatrix}; \qquad \begin{pmatrix} d \\ s \\ b \end{pmatrix} = V_{(d)} \begin{pmatrix} d_1' \\ d_2' \\ d_3' \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} u_1' \\ u_2' \\ u_3' \end{pmatrix} = V_{(u)}^{-1} \begin{pmatrix} u \\ c \\ t \end{pmatrix} = V_{(u)}^{\dagger} \begin{pmatrix} u \\ c \\ t \end{pmatrix} \rightarrow \begin{pmatrix} u & c & t \end{pmatrix} = \begin{pmatrix} u_1' & u_2' & u_3' \end{pmatrix} \left(V_{(u)}^{\dagger} \right)^{\dagger} = \begin{pmatrix} u_1' & u_2' & u_3' \end{pmatrix} V_{(u)}$$

$$\rightarrow \begin{pmatrix} d_1' \\ d_2' \\ d_3' \end{pmatrix} = V_{(d)}^{-1} \begin{pmatrix} d \\ s \\ b \end{pmatrix} = V_{(d)}^{\dagger} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$\rightarrow V_{CKM} = V_{(u)} V_{(d)}^{\dagger} \equiv V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Observe: Different quark flavors have independent, arbitrary phase

Indeed: No strong/electromagnetic transition connecting different flavors

CKM - III

Re-define arbitrary phases of quark mass eigenstates:

$$\begin{pmatrix} u \\ c \\ t \end{pmatrix} \to \begin{pmatrix} e^{i\varphi_u} & 0 & 0 \\ 0 & e^{i\varphi_c} & 0 \\ 0 & 0 & e^{i\varphi_t} \end{pmatrix} \begin{pmatrix} u \\ c \\ t \end{pmatrix}, \quad \begin{pmatrix} d \\ s \\ b \end{pmatrix} \to \begin{pmatrix} e^{i\varphi_d} & 0 & 0 \\ 0 & e^{i\varphi_s} & 0 \\ 0 & 0 & e^{i\varphi_b} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Translate into redefinition of weak eigenstates:

$$\begin{pmatrix} u' \\ c' \\ t' \end{pmatrix} \to V_u^\dagger \begin{pmatrix} e^{i\phi_u} & 0 & 0 \\ 0 & e^{i\phi_c} & 0 \\ 0 & 0 & e^{i\phi_t} \end{pmatrix} \begin{pmatrix} u \\ c \\ t \end{pmatrix}, \quad \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} \to V_d^\dagger \begin{pmatrix} e^{i\phi_d} & 0 & 0 \\ 0 & e^{i\phi_s} & 0 \\ 0 & 0 & e^{i\phi_b} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Redefinition of weak eigenstates equivalent to CKM redefinition:

$$V_{CKM} \to \begin{pmatrix} e^{-i\varphi_u} & 0 & 0 \\ 0 & e^{-i\varphi_c} & 0 \\ 0 & 0 & e^{-i\varphi_t} \end{pmatrix} V_{CKM} \begin{pmatrix} e^{i\varphi_d} & 0 & 0 \\ 0 & e^{i\varphi_s} & 0 \\ 0 & 0 & e^{i\varphi_b} \end{pmatrix} = \begin{pmatrix} V_{ud}e^{i(\varphi_d-\varphi_u)} & V_{us}e^{i(\varphi_s-\varphi_u)} & V_{ub}e^{i(\varphi_b-\varphi_u)} \\ V_{cd}e^{i(\varphi_d-\varphi_c)} & V_{cs}e^{i(\varphi_s-\varphi_c)} & V_{cb}e^{i(\varphi_b-\varphi_c)} \\ V_{td}e^{i(\varphi_d-\varphi_t)} & V_{ts}e^{i(\varphi_s-\varphi_t)} & V_{tb}e^{i(\varphi_b-\varphi_t)} \end{pmatrix}$$

CKM - IV

Factorize one (any) phase:

$$\rightarrow V_{CKM} = e^{-i\varphi_u}\begin{pmatrix} V_{ud}e^{i\varphi_d} & V_{us}e^{i\varphi_s} & V_{ub}e^{i\varphi_b} \\ V_{cd}e^{i(\varphi_u+\varphi_d-\varphi_c)} & V_{cs}e^{i(\varphi_u+\varphi_s-\varphi_c)} & V_{cb}e^{i(\varphi_u+\varphi_b-\varphi_c)} \\ V_{td}e^{i(\varphi_u+\varphi_d-\varphi_t)} & V_{ts}e^{i(\varphi_u+\varphi_s-\varphi_t)} & V_{tb}e^{i(\varphi_u+\varphi_b-\varphi_t)} \end{pmatrix}$$

Global field phase not relevant: Can't be used to fix one free V parameter

5 free relative phases

$\rightarrow$ Use to make 5 elements *real* in V_{CKM}

$\rightarrow$ Leave $9-5=4$ real, free parameters

Encode as:

3 'rotation angles' $\left(\leftarrow \text{Euler angles}\right)$

$\big[$In order to understand this:

Suppose the matrix is real $\rightarrow$ Any 3×3 real, unitary matrix = Orthogonal

Any 3×3 orthogonal matrix = 3D Rotation $\rightarrow$ 3 angles$\big]$

1 complex (irreducible) phase factor

CKM - IVbis

Original version by K&M (1973): 4 complex elements
Parameters:
3 ‘rotation’ angles $\theta_1, \theta_2, \theta_3$
1 complex phase δ

$s_i = \sin\theta_i$
$c_i = \cos\theta_i$

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} c_1 & -s_1c_3 & -s_1s_3 \\ s_1c_2 & c_1c_2c_3 - s_2s_3e^{i\delta} & c_1c_2s_3 + s_2c_3e^{i\delta} \\ s_1s_2 & c_1s_2c_3 + c_2s_3e^{i\delta} & c_1s_2s_3 - c_2c_3e^{i\delta} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

CKM - V

Standard form: 5 complex elements

Parameters:

3 'Mixing' angles $\theta_{12}, \theta_{13}, \theta_{23}$

1 Irreducible phase δ

$s_{ij} = \sin\theta_{ij}$

$c_{ij} = \cos\theta_{ij}$

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}e^{+i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{+i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{+i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{+i\delta} & c_{13}c_{23} \end{pmatrix}$$

Experiment:

$\sin\theta_{13} \ll \sin\theta_{23} \ll \sin\theta_{12} \ll 1$

CKM - VI

Visualizing CKM in standard representation:

Product of 3 independent 2D rotations + 1 Phase

$$U_{12} = \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$U_{13} = \begin{bmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{bmatrix}$$

$$U_{23} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix}$$

$$U_{\delta} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\delta_{13}} \end{bmatrix}$$

$$V_{\mathrm{CKM}} = U_{23} U_{\delta}^{\dagger} U_{13} U_{\delta} U_{12}$$

CKM - VII

Wolfenstein parametrization of V_{CKM}:

Based on experimental evidence of some hierarchy among angles

Define:

$$\lambda = \sin\theta_{12}$$

$$A\lambda^2 = \sin\theta_{23}$$

$$A\lambda^3(\rho - i\eta) = \sin\theta_{13}e^{-i\delta}$$

Then:

$$\cos\theta_{12} = \sqrt{1-\lambda^2} \simeq 1 - \frac{\lambda^2}{2} - \frac{\lambda^4}{8}$$

$$\rightarrow V_{CKM} \simeq \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

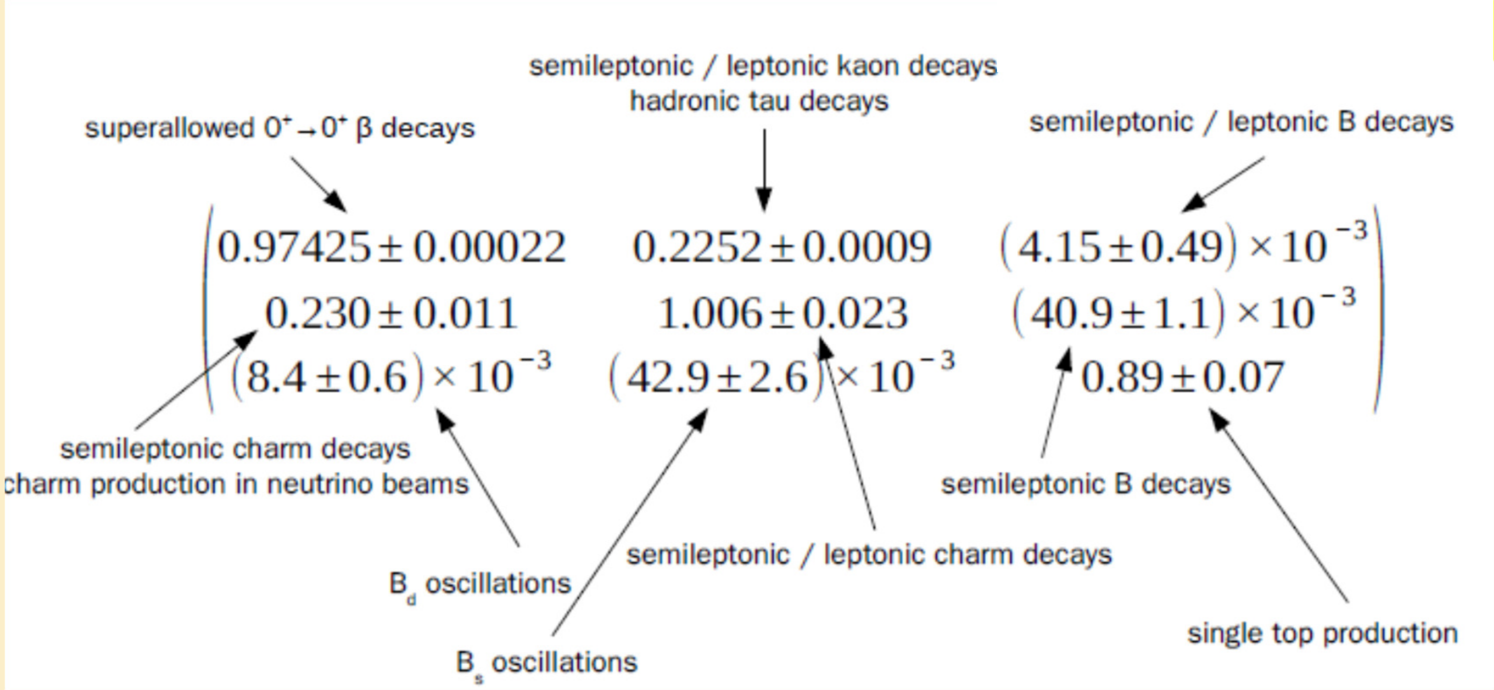
$$\lambda = 0.2259 \pm 0.0021 \approx \sin\theta_C$$

$$A = 0.82 \pm 0.02$$

$$\eta \neq 0 \rightarrow \cancel{CP}$$

Filling CKM - I

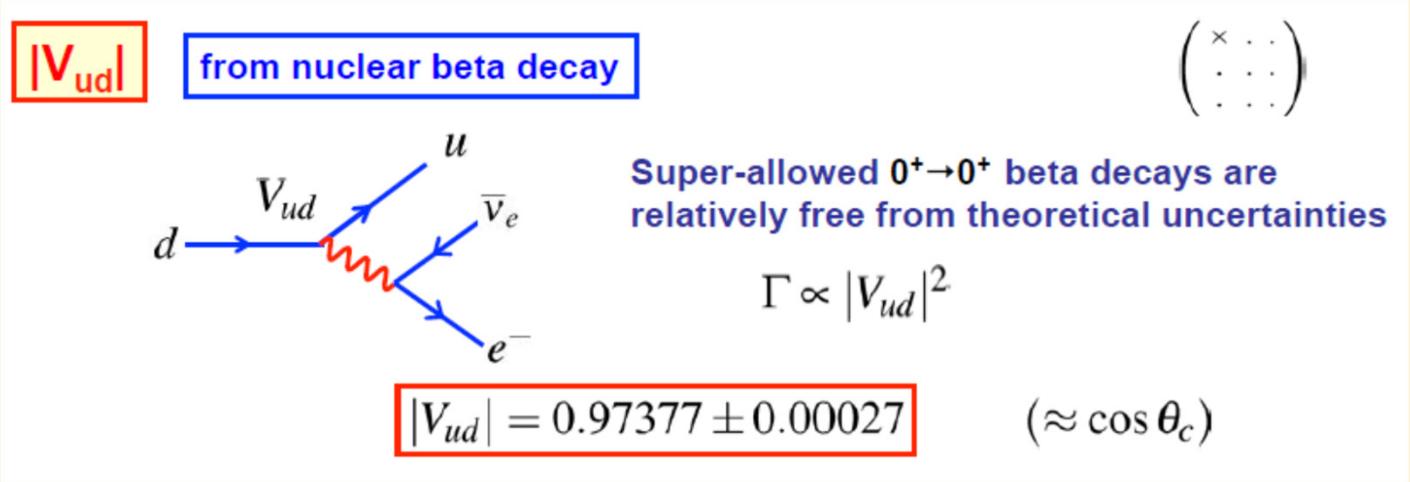
Filling CKM (PDG 2013)



$$\begin{pmatrix} 0.97425 \pm 0.00022 & 0.2252 \pm 0.0009 & (4.15 \pm 0.49) \times 10^{-3} \\ 0.230 \pm 0.011 & 1.006 \pm 0.023 & (40.9 \pm 1.1) \times 10^{-3} \\ (8.4 \pm 0.6) \times 10^{-3} & (42.9 \pm 2.6) \times 10^{-3} & 0.89 \pm 0.07 \end{pmatrix}$$

CKM elements involving *t* quark less well known

Filling CKM - II



$|V_{ud}|$

from nuclear beta decay

Super-allowed $0^+ \to 0^+$ beta decays are relatively free from theoretical uncertainties

$$\Gamma \propto |V_{ud}|^2$$

$$\boxed{|V_{ud}| = 0.97377 \pm 0.00027} \qquad (\approx \cos\theta_c)$$

Superallowed β transition: $\Delta J = 0,\ \Delta P = 0$

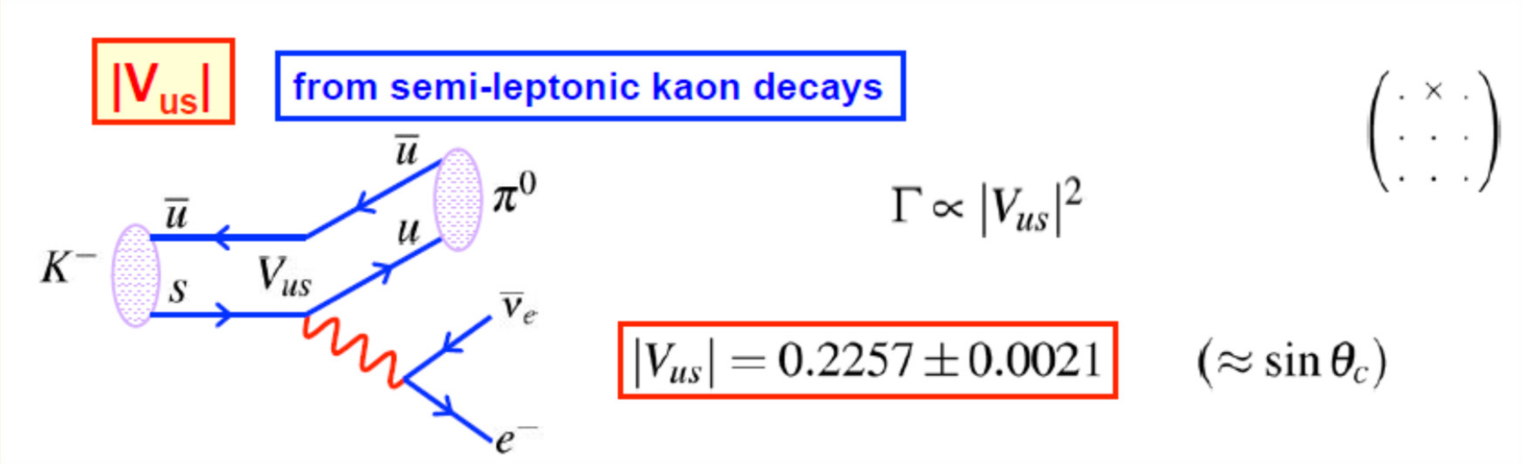
+ Same level structure for both initial and final nucleus, just $n \leftrightarrow p$

Global level shift, only due to Coulomb energy $\rightarrow$ No theoretical corrections!

Example: $n \rightarrow p + e^- + \overline{\nu}_e$

$\rightarrow$ High precision measurement of V_{ud} from transition rate

Filling CKM - III



$|V_{us}|$ **from semi-leptonic kaon decays**

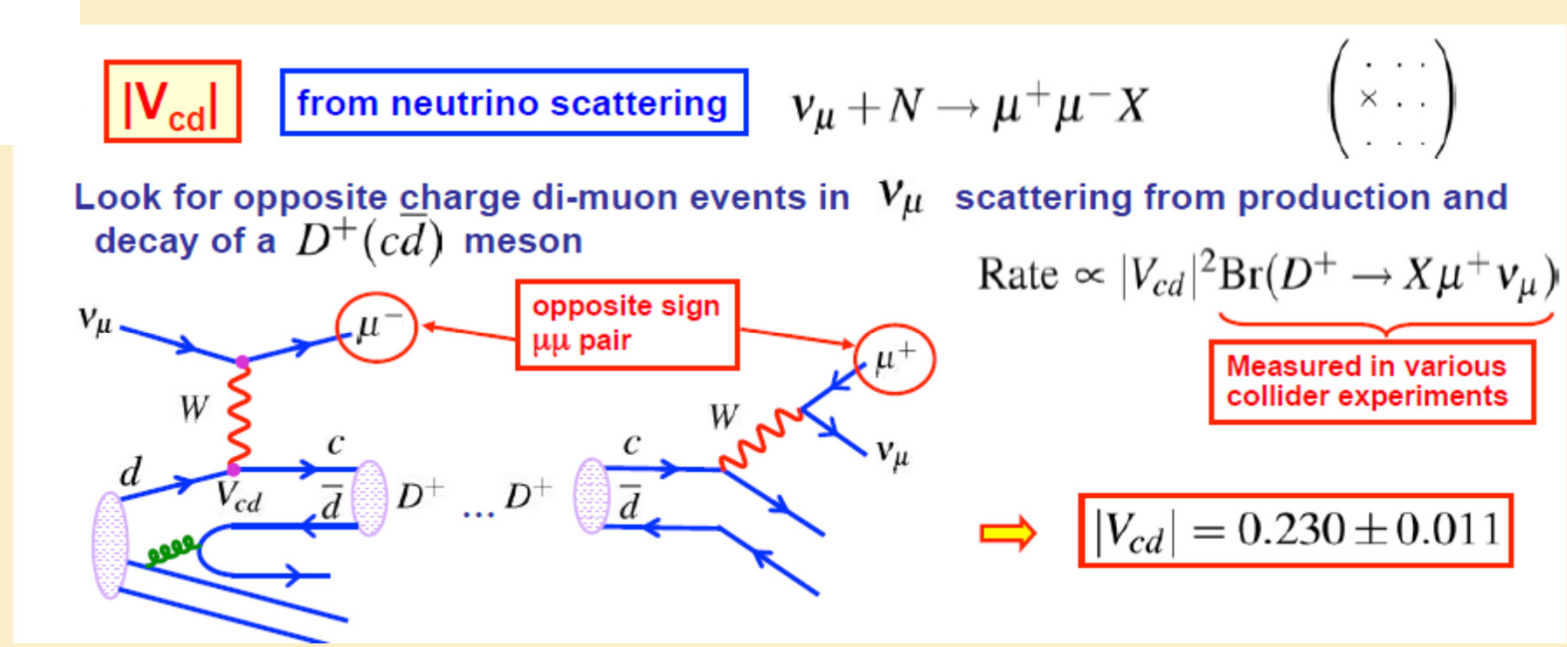
$$\Gamma \propto |V_{us}|^2$$

$$\boxed{|V_{us}| = 0.2257 \pm 0.0021} \qquad (\approx \sin\theta_c)$$

Differential decay rate:

$$\frac{d\Gamma(\overline{K^0} \to \pi^+ e^- \overline{\nu}_e)}{dx_\pi} = \underbrace{\frac{G_F^2 m_K^5}{192\pi^2}}_{\substack{\text{Standard 3-body}\\ \text{total rate}}} |V_{us}|^2 \underbrace{f(q^2)^2}_{\text{Form factor}} \underbrace{\left(x_\pi^2 - 4\frac{m_\pi^2}{m_K^2} \right)^{3/2}}_{\text{Phase space factor}}, \; x_\pi = \frac{2E_\pi}{m_K}$$

Filling CKM - IV



$|V_{cd}|$ **from neutrino scattering** $\nu_\mu + N \to \mu^+\mu^- X$ $\begin{pmatrix} \cdot & \cdot & \cdot \\ \times & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$

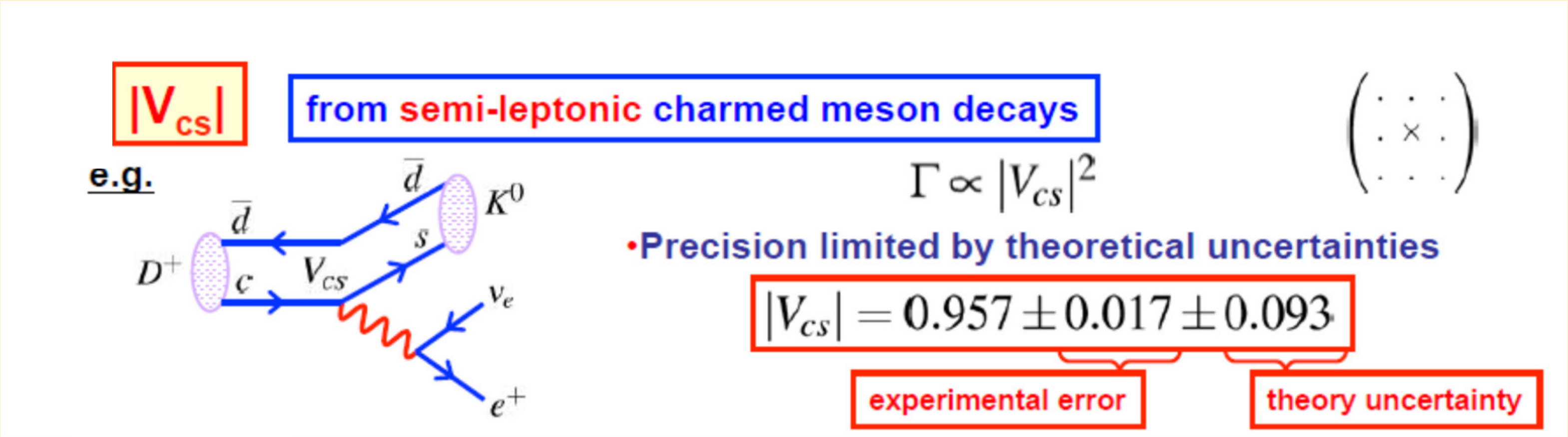
Look for opposite charge di-muon events in ν_μ scattering from production and decay of a $D^+(c\bar{d})$ meson

$$\text{Rate} \propto |V_{cd}|^2 \underbrace{\text{Br}(D^+ \to X\mu^+\nu_\mu)}_{\text{Measured in various collider experiments}}$$

opposite sign μμ pair

$$\Rightarrow \boxed{|V_{cd}| = 0.230 \pm 0.011}$$

Filling CKM - V



$|V_{cs}|$ **from semi-leptonic charmed meson decays**

$$\Gamma \propto |V_{cs}|^2$$

- **Precision limited by theoretical uncertainties**

$$\boxed{|V_{cs}| = 0.957 \pm \underbrace{0.017}_{\text{experimental error}} \pm \underbrace{0.093}_{\text{theory uncertainty}}}$$

Make D^+D^- pairs from $e^+e^- \rightarrow \psi(3770) \rightarrow D^+D^-$
BR $D^+ \rightarrow K^0\, e^+ \nu_e = (\, 8.83 \pm 0.22)\ \%$

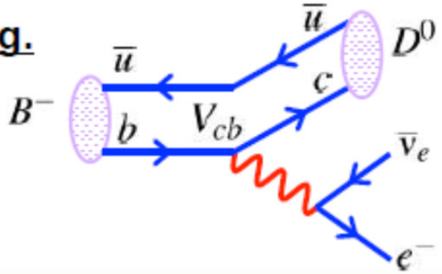
Filling CKM - VI

|V|

$|V_{cb}|$ **from semi-leptonic B hadron decays**

$$\begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \times \\ \cdot & \cdot & \cdot \end{pmatrix}$$

e.g.



$$\Gamma \propto |V_{cb}|^2$$

$$\boxed{|V_{cb}| = 0.0416 \pm 0.0006}$$

$\overline{D}^0 \ell^+ \nu_\ell$ [a] (2.23 ±0.11) %

$$\frac{d\Gamma(b \to c l^- \overline{\nu_l})}{dx} = \frac{G_F^2 m_b^5}{192\pi^2} |V_{\alpha b}|^2 \left[2x^2 \left(\frac{1-x-\zeta}{1-x} \right)^2 \left(3 - 2x + \zeta + \frac{2\zeta}{1-x} \right) \right], \zeta = \frac{m_c^2}{m_b^2}, x = \frac{2E_l}{m_b}$$

$$L = 10^{33}\, cm^{-2} s^{-1} \to L_{\text{int}} \sim 10^{38}\, cm^{-2} d^{-1}$$

$$\sigma_{B\overline{B}} \sim 1\, nb = 10^{-33}\, cm^2 \to R \sim 10^5\, B\overline{B} \;\; d^{-1}$$

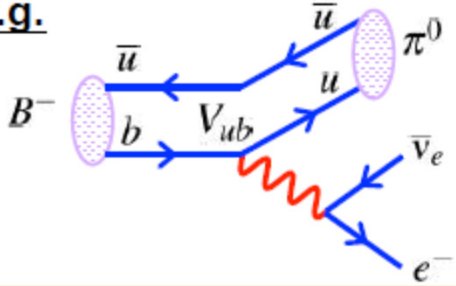
$$\to R_{dec} \sim 10^3 \;\; D^0 e^- \overline{\nu}_e \;\; d^{-1} \to \frac{\sqrt{\sigma_{stat}}}{|V_{cb}|} \sim \frac{3\%}{\sqrt{T\,(days)}}$$

Filling CKM - VII

$|V_{ub}|$ **from semi-leptonic B hadron decays**

$$\begin{pmatrix} \cdot & \cdot & \times \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

e.g.



$$\Gamma \propto |V_{ub}|^2$$

$$\boxed{|V_{ub}| = 0.0043 \pm 0.0003}$$

$\pi^0 \ell^+ \nu_\ell$ $(7.7 \pm 1.2) \times 10^{-5}$

$$\frac{\Gamma(b \to u l^- \overline{\nu}_l)}{\Gamma(b \to c l^- \overline{\nu}_l)} = \frac{|V_{ub}|^2}{|V_{cb}|^2} \left(\frac{f(m_u^2 / m_b^2)}{f(m_c^2 / m_b^2)} \right), \quad \text{compare rates}$$

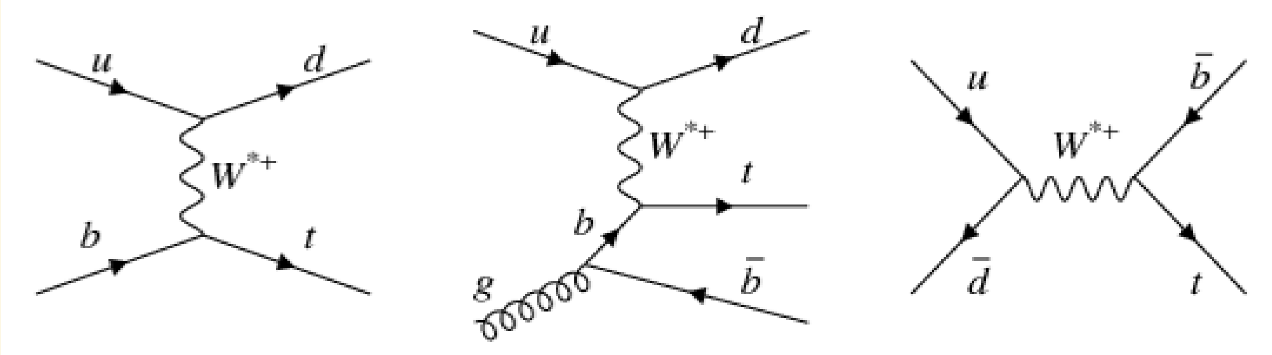
$$L = 10^{33}\, cm^{-2} s^{-1} \to L_{\text{int}} \sim 10^{38}\, cm^{-2} d^{-1}$$

$$\sigma_{B\bar{B}} \sim 1\ nb = 10^{-33}\, cm^2 \to R \sim 10^5\, B\bar{B}\ \ d^{-1}$$

$$\to R_{dec} \sim 10\ \ \pi^0 e^- \overline{\nu}_e\ \ d^{-1} \to \frac{\sigma_{stat}}{|V_{ub}|} \sim \frac{30\%}{\sqrt{T(days)}}$$

Filling CKM - VIII

$|V_{tb}|$ **from single top production**

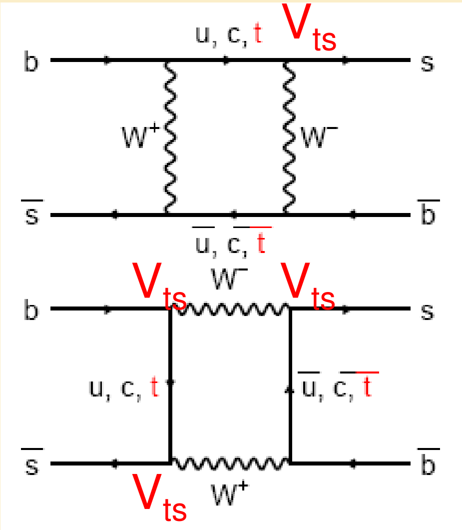


Filling CKM - IX

$|V_{td}|, |V_{ts}|$ **from B_d, B_s oscillations**

Cannot rely on direct measurements of V_{td}, V_{ts} from t decays: Too small

Rather use loop diagrams of B_d, B_s oscillations



\+ Similar for B_d, yielding V_{td}

CKM Triangles - I

V_{CKM} unitary: 9 unitarity conditions

Take 6 'off-diagonal' conditions:

$$(1)\quad V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0; \qquad (2)\quad V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0;$$

$$(3)\quad V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0; \qquad (4)\quad V_{ud}^*V_{cd} + V_{us}^*V_{cs} + V_{ub}^*V_{cb} = 0;$$

$$(5)\quad V_{ud}^*V_{td} + V_{us}^*V_{ts} + V_{ub}^*V_{tb} = 0; \qquad (6)\quad V_{cd}^*V_{td} + V_{cs}^*V_{ts} + V_{cb}^*V_{tb} = 0;$$

Each condition:

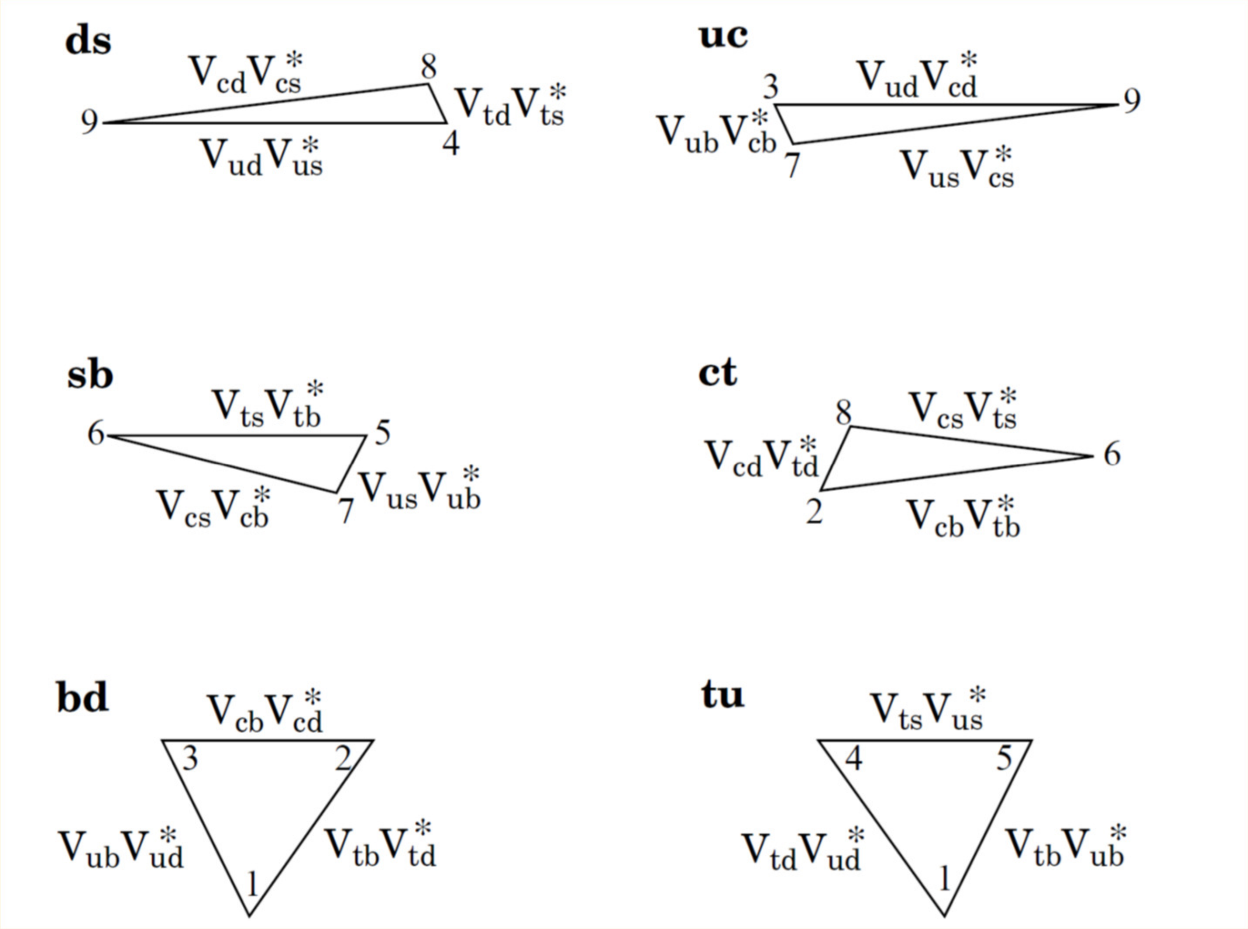
Sum of 3 complex numbers = 0

Complex number $\triangleq$ Vector in the complex plane

$\rightarrow$ Each condition $\sim$ 3 numbers should add to a closed triangle

CKM Triangles - Ibis

6 unitarity conditions, off-diagonal:



CKM Triangles - II

Sides & Angles from experiment

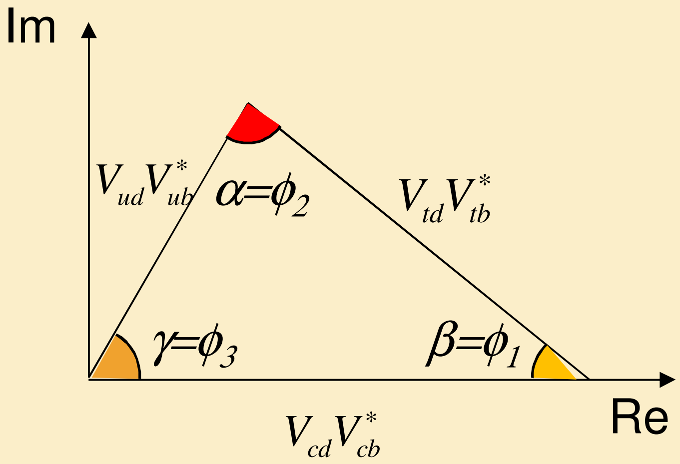
Area: Same for all 6

$$A_{triangle} = \frac{1}{2} J_{CP} = \frac{1}{2} \mathrm{Im}\left(V_{ij} V_{kl} V_{il}^* V_{kj}^*\right); \qquad i \neq k, j \neq l;$$

$$J_{CP} = s_{12} s_{13} s_{23} c_{12} c_{13}^2 c_{23} s_{\delta_{13}} \approx A^2 \lambda^6 \eta$$

$\rightarrow J_{CP} =$ Nice measure of $\cancel{CP}$

Example: Most common unitary triangle



$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$$

Wolfenstein approximation:

$$A\lambda^3 \left(1 - \frac{\lambda^2}{2}\right)(\rho + i\eta) \; - \; A\lambda^3 \left[1 + A^2 \lambda^4 (\rho + i\eta)\right] \; + \; A\lambda^3 \left[1 - (\rho + i\eta)\left(1 - \frac{\lambda^2}{2}\right)\right] = 0$$

CKM Triangles - III

$$\begin{cases} V_{ud}V_{ub}^* = A\lambda^3\left(1-\frac{\lambda^2}{2}\right)(\rho+i\eta) \\ V_{cd}V_{cb}^* = -A\lambda^3 \\ V_{td}V_{tb}^* = A\lambda^3\left[1-(\rho+i\eta)\right] \end{cases}$$

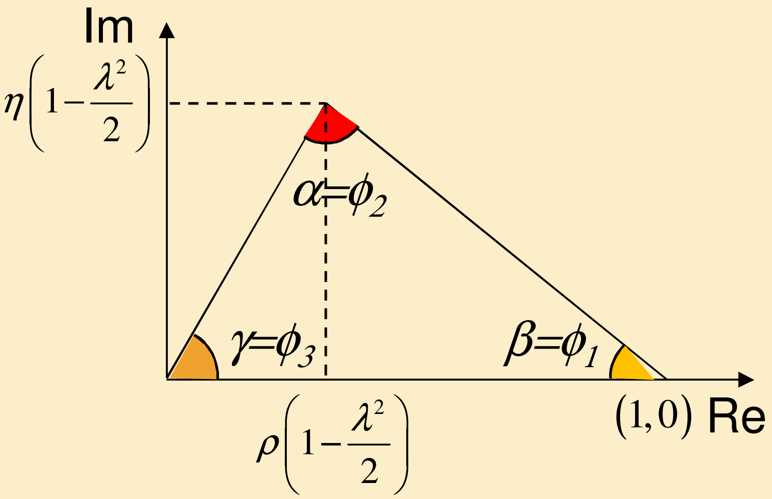
Normalize to $V_{cd}V_{cb}^* = -A\lambda^3 \equiv 1;$

Ignore overall $-$ signs

$$V_{ud}V_{ub}^* = \left(1-\frac{\lambda^2}{2}\right)(\rho+i\eta) \approx \rho+i\eta$$

$$V_{cd}V_{cb}^* = 1$$

$$V_{td}V_{tb}^* \approx 1-(\rho+i\eta)$$



CKM Triangles - IV

Recent fit by CKMFitter group:

Observable	Central ± 1 σ	± 2 σ	± 3 σ
A	0.812 [+0.015 -0.022]	0.812 [+0.025 -0.031]	0.812 [+0.035 -0.039]
λ	0.22543 [+0.00059 -0.00095]	0.2254 [+0.0010 -0.0019]	0.2254 [+0.0013 -0.0027]
ρbar	0.145 [+0.027 -0.027]	0.145 [+0.046 -0.040]	0.145 [+0.057 -0.050]
ηbar	0.343 [+0.015 -0.015]	0.343 [+0.030 -0.026]	0.343 [+0.044 -0.035]
J [10^{-5}]	2.96 [+0.18 -0.14]	2.96 [+0.32 -0.19]	2.96 [+0.46 -0.23]
α [deg]	91.1 [+4.3 -4.3]	91.1 [+7.1 -6.2]	91.1 [+8.8 -7.8]
α [deg] (meas. not in the fit)	95.9 [+2.2 -5.6]	95.9 [+3.6 -10.9]	95.9 [+5.0 -12.8]
α [deg] (dir. meas.)	88.7 [+4.6 -4.2]	88.7 [+9.4 -8.5]	89 [+21 -13]
β [deg]	21.85 [+0.80 -0.77]	21.9 [+1.6 -1.3]	21.9 [+2.5 -1.8]
β [deg] (meas. not in the fit)	27.5 [+1.2 -1.4]	27.5 [+1.9 -3.9]	27.5 [+2.6 -6.8]
β [deg] (dir. meas.)	21.38 [+0.79 -0.77]	21.4 [+1.6 -1.5]	21.4 [+2.4 -2.3]
γ [deg]	67.1 [+4.3 -4.3]	67.1 [+6.1 -7.0]	67.1 [+7.6 -8.5]
γ [deg] (meas. not in the fit)	67.2 [+4.4 -4.6]	67.2 [+6.1 -7.2]	67.2 [+7.6 -8.7]
γ [deg] (dir. meas.)	66 [+12 -12]	66 [+23 -22]	66 [+36 -30]

K Oscillations - I

First among a host of astonishing quantum mechanical oddities

$$\left|K^0\right\rangle = \left|d\bar{s}\right\rangle \qquad S=+1$$

$$\left|\bar{K}^0\right\rangle = \left|\bar{d}s\right\rangle \qquad S=-1$$

$P\left|K^0\right\rangle = -\left|K^0\right\rangle$ Pseudoscalar

$P\left|\bar{K}^0\right\rangle = -\left|\bar{K}^0\right\rangle$ Pseudoscalar

$C\left|K^0\right\rangle = \left|\bar{K}^0\right\rangle$ Not a C eigenstate

$C\left|\bar{K}^0\right\rangle = \left|K^0\right\rangle$ Not a C eigenstate

$\rightarrow$ Make C eigenstates:

$$\left|K_1^0\right\rangle = \frac{1}{\sqrt{2}}\left(\left|K^0\right\rangle - \left|\bar{K}^0\right\rangle\right) \rightarrow C\left|K_1^0\right\rangle = C\left[\frac{1}{\sqrt{2}}\left(\left|K^0\right\rangle - \left|\bar{K}^0\right\rangle\right)\right] = \frac{1}{\sqrt{2}}\left(\left|\bar{K}^0\right\rangle - \left|K^0\right\rangle\right) = -\left|K_1^0\right\rangle$$

$$\left|K_2^0\right\rangle = \frac{1}{\sqrt{2}}\left(\left|K^0\right\rangle + \left|\bar{K}^0\right\rangle\right) \rightarrow C\left|K_2^0\right\rangle = \left|K_2^0\right\rangle$$

K Oscillations - II

Kaons : Just weak decays $(\leftarrow$ Lightest strange hadron$)$

C, P not conserved by weak processes

CP *almost* conserved by weak processes → Take it as good for the moment

→ Focus on CP as a symmetry for weak processes

CP eigenstates:

$$CP\left|K_1^0\right\rangle = CP\left[\frac{1}{\sqrt{2}}\left(\left|K^0\right\rangle - \left|\bar{K}^0\right\rangle\right)\right] = C\left[\frac{1}{\sqrt{2}}\left(-\left|K^0\right\rangle + \left|\bar{K}^0\right\rangle\right)\right] = +\left|K_1^0\right\rangle \quad CP = +1$$

$$CP\left|K_2^0\right\rangle = -\left|K_2^0\right\rangle \qquad CP = -1$$

Observe : K_1^0, K_2^0 CP eigenstates, like photon, π^0

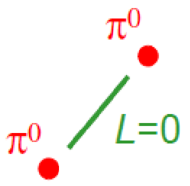
→ Different particles

K Oscillations - III

K^0: Many different decay modes, including weak decays into pions

Consider first decays into 2 π's:

$K^0 \to \pi^0\pi^0$



J^P : $0^- \to 0^- + 0^-$

Ang. mom. conservation

$\Rightarrow \quad L = 0$

$\Rightarrow \quad P(\pi^0\pi^0) = -1.-1.(-1)^L = (-1)^L = +1$

π^0 is eigenstate of C : $\hat{C}|\pi^0\rangle = +|\pi^0\rangle$

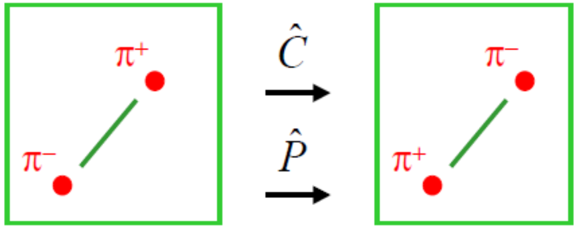
$\Rightarrow \quad C(\pi^0\pi^0) = +1.+1 = +1$

$K^0 \to \pi^+\pi^-$

Still have $L = 0$

$\Rightarrow \quad P(\pi^+\pi^-) = -1.-1.(-1)^L = (-1)^L = +1$

C and P operations have identical effect :



(no spins involved)

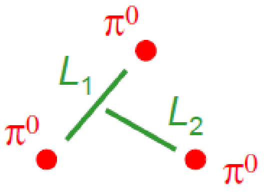
$\Rightarrow \quad C(\pi^+\pi^-) = P(\pi^+\pi^-) = (-1)^L = +1$

$\Rightarrow$ CP = +1 for both $\pi^+\pi^-$ and $\pi^0\pi^0$

K Oscillations - IV

Consider then decays into 3 π's:

$K^0 \to \pi^0 \pi^0 \pi^0$



JP : $0^- \to 0^- + 0^- + 0^-$

Ang. mom. conservation

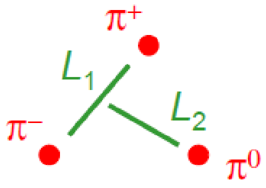
$\Rightarrow \quad L_1 \oplus L_2 = 0$

$\Rightarrow \quad L_1 = L_2$

$$\Rightarrow \quad P(\pi^0\pi^0\pi^0) = -1.-1.-1.(-1)^{L_1}.(-1)^{L_2} = -1$$

$$C(\pi^0\pi^0\pi^0) = +1.+1.+1 = +1$$

$K^0 \to \pi^+ \pi^- \pi^0$



As above: $L_1 = L_2$

$$\Rightarrow \quad P(\pi^+\pi^-\pi^0) = -1.-1.-1.(-1)^{L_1}.(-1)^{L_2} = -1$$

$$C(\pi^+\pi^-\pi^0) = +1.C(\pi^+\pi^-) = +1.(-1)^{L_1} = +1$$

Experimentally: $L_1 = 0$ from study of angular distributions of π^+, π^-

$\Rightarrow$ CP = –1 for both $\pi^+\pi^-\pi^0$ and $\pi^0\pi^0\pi^0$

K Oscillations - V

If *CP* is conserved in weak processes:

$$\left.\begin{array}{l} K_1^0 \to \pi\pi \\ K_2^0 \to \pi\pi\pi \end{array}\right\} \textit{Exclusively}$$

Summary so far about neutral K states:

Production (by strong interaction) : $\left|K^0\right\rangle, \left|\bar{K}^0\right\rangle$

Decay (by weak interaction): $\left|K_1^0\right\rangle, \left|K_2^0\right\rangle$

$$m_{\left|K^0\right\rangle} = m_{\left|\bar{K}^0\right\rangle} \approx m_{\left|K_1^0\right\rangle} \approx m_{\left|K_2^0\right\rangle} \approx 498\ MeV$$

Expect, and find:

$K_1^0 \to \pi\pi$ Fast: Larger phase space etc $\to \tau_1 = 0.9\ 10^{-10}\,s$ $'K\ short'$

$K_2^0 \to \pi\pi\pi$ Slow: Smaller phase space etc $\to \tau_2 = 0.5\ 10^{-7}\,s$ $'K\ long'$

K Oscillations - VI

Provisionally identify:

$$K_S \equiv K_1^0 (\rightarrow \pi\pi) \qquad \tau_S = 0.9\ 10^{-10}\,s \qquad 'K\ short' \qquad CP = +1$$

$$K_L \equiv K_2^0 (\rightarrow \pi\pi\pi) \qquad \tau_L = 0.5\ 10^{-7}\,s \qquad 'K\ long' \qquad CP = -1$$

Therefore:

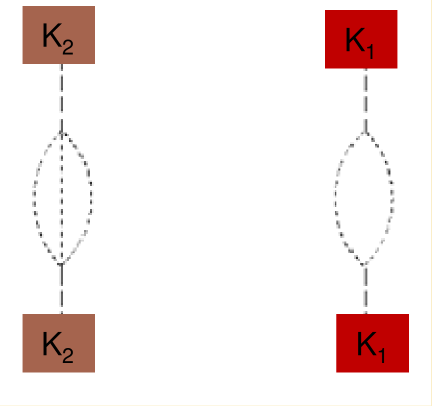
$$K_S \equiv K_1^0, K_L \equiv K_2^0 : \begin{array}{ll} \text{Different} & CP \\ \text{Different} & \text{lifetime} \end{array}$$

Also: Different mass!

Old fashioned (but simple) argument:

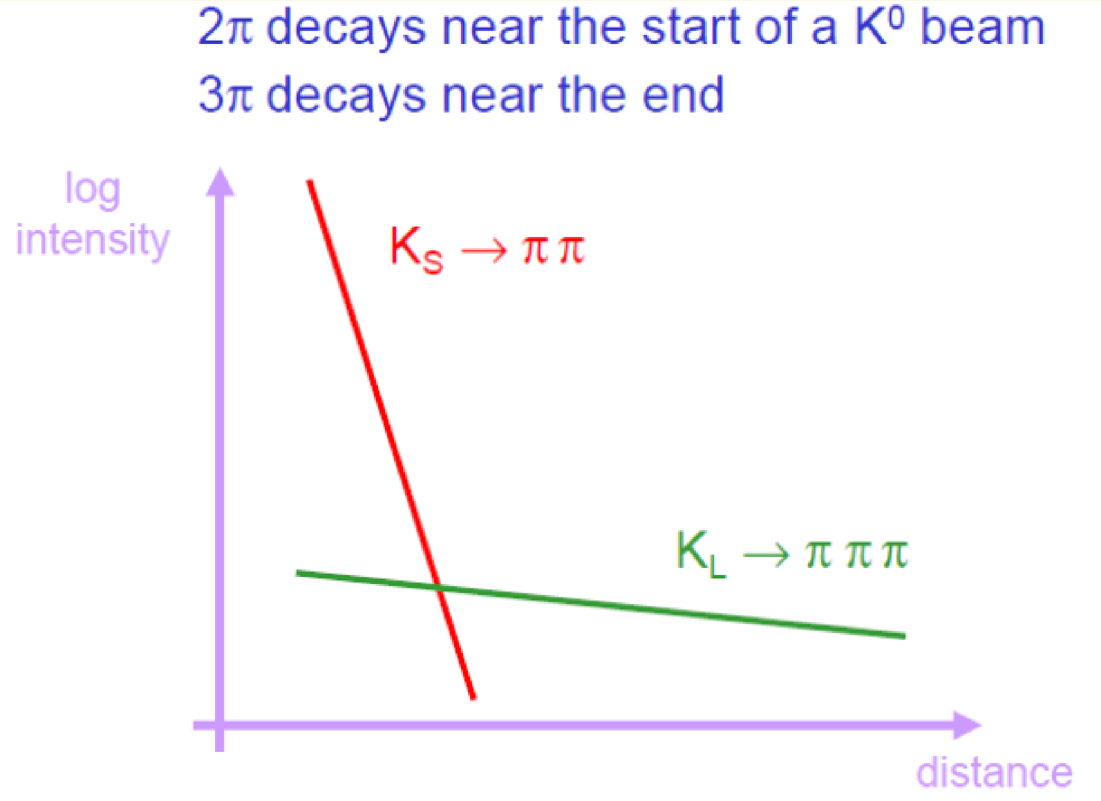
Different virtual weak couplings, 2π vs 3π

→ Different corrections to the mass



K Oscillations - VII

Taking a neutral *K* beam produced by strong interaction, expect qualitatively



2π decays near the start of a K^0 beam

3π decays near the end

K Oscillations - VIII

Production: Strong interaction $\rightarrow$ Strangeness conserved

Neglect weak interaction in production process

Strongly produced neutral K either K^0 or $\bar{K}^0$

$\rightarrow$ Either K^0 or $\bar{K}^0$ as *initial condition* for the K wave function

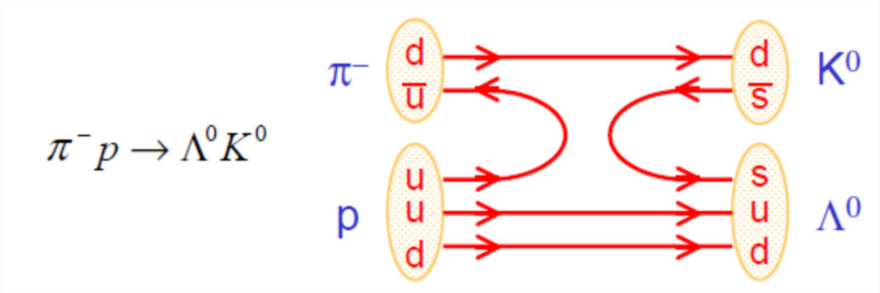
Time evolution : Weak interaction $\rightarrow$ Strangeness *not* conserved

Neglect strong interaction in time evolution

Neither P or C conserved by weak interaction; CP $\left(\textit{provisionally}\right)$ conserved

$\rightarrow$ Propagate CP eigenstates K_S, K_L

Take a definite production process:



K Oscillations - IX

$$\psi(t=0):\left|K^0\right\rangle=\frac{1}{\sqrt{2}}\left(\left|K_L^0\right\rangle+\left|K_S^0\right\rangle\right)$$

$$\begin{cases}\left|K_L^0(t)\right\rangle=\left|K_L^0\right\rangle e^{-i\left(m_L-i\frac{\Gamma_L}{2}\right)t}, \qquad \Gamma_L=\dfrac{1}{\tau_L}\\ \left|K_S^0(t)\right\rangle=\left|K_S^0\right\rangle e^{-i\left(m_S-i\frac{\Gamma_S}{2}\right)t}, \qquad \Gamma_S=\dfrac{1}{\tau_S}\end{cases}$$

$$\rightarrow\psi(t)=\frac{1}{\sqrt{2}}\left(\left|K_L^0\right\rangle e^{-i\left(m_L-i\frac{\Gamma_L}{2}\right)t}+\left|K_S^0\right\rangle e^{-i\left(m_S-i\frac{\Gamma_S}{2}\right)t}\right)$$

$$\rightarrow\psi(t)=\frac{1}{\sqrt{2}}\left(\left[\frac{1}{\sqrt{2}}\left(\left|K^0\right\rangle+\left|\bar{K}^0\right\rangle\right)\right]e^{-i\left(m_L-i\frac{\Gamma_L}{2}\right)t}+\frac{1}{\sqrt{2}}\left(\left|K^0\right\rangle-\left|\bar{K}^0\right\rangle\right)\frac{1}{\sqrt{2}}e^{-i\left(m_S-i\frac{\Gamma_S}{2}\right)t}\right)$$

$$\rightarrow\psi(t)=\frac{1}{2}\left(\left|K^0\right\rangle\left[e^{-i\left(m_L-i\frac{\Gamma_L}{2}\right)t}+e^{-i\left(m_S-i\frac{\Gamma_S}{2}\right)t}\right]+\left|\bar{K}^0\right\rangle\left[e^{-i\left(m_L-i\frac{\Gamma_L}{2}\right)t}-e^{-i\left(m_S-i\frac{\Gamma_S}{2}\right)t}\right]\right)$$

K Oscillations - X

Time evolution of strangeness content of the beam:

Initial condition K^0

$\Delta m = m_L - m_S$

$$\rightarrow \begin{cases} I\left(K^0\right) = \dfrac{1}{4}\left| e^{-i\left(m_L - i\frac{\Gamma_L}{2}\right)t} + e^{-i\left(m_S - i\frac{\Gamma_S}{2}\right)t} \right|^2 = \dfrac{1}{4}\left[e^{-\Gamma_L t} + e^{-\Gamma_S t} + 2e^{-\frac{(\Gamma_L+\Gamma_S)}{2}t} \cos\Delta m t \right] \\ I\left(\overline{K}^0\right) = \dfrac{1}{4}\left| e^{-i\left(m_L - i\frac{\Gamma_L}{2}\right)t} - e^{-i\left(m_S - i\frac{\Gamma_S}{2}\right)t} \right|^2 = \dfrac{1}{4}\left[e^{-\Gamma_L t} + e^{-\Gamma_S t} - 2e^{-\frac{(\Gamma_L+\Gamma_S)}{2}t} \cos\Delta m t \right] \end{cases}$$

$\rightarrow$ Strangeness oscillations

Detected in many ways, for example by semileptonic modes:

$$K^0 \rightarrow \pi^- e^+ \nu_e$$
$$\overline{K}^0 \rightarrow \pi^+ e^- \overline{\nu}_e$$

$\Delta Q = \Delta S$ rule: Unambiguous strangeness assignment from decay products

K Oscillations - Xbis

Reminder: Weak decays selection rules

$|\Delta Q| = 1$

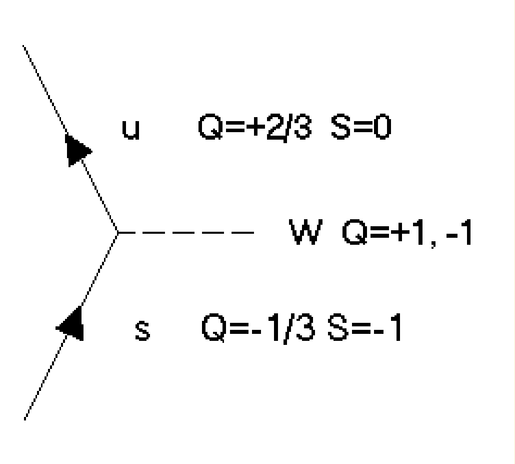
~~$\Xi^- \to n\ell^-\bar{\nu}, \quad \Xi^0 \to p\ell^-\bar{\nu},$~~

~~$\Omega^- \to \Lambda^0\ell^-\bar{\nu}, \quad \Omega^- \to \Sigma^0\ell^-\bar{\nu}, \quad \Omega^- \to n\ell^-\bar{\nu}$~~

$\Delta Q = \Delta S$

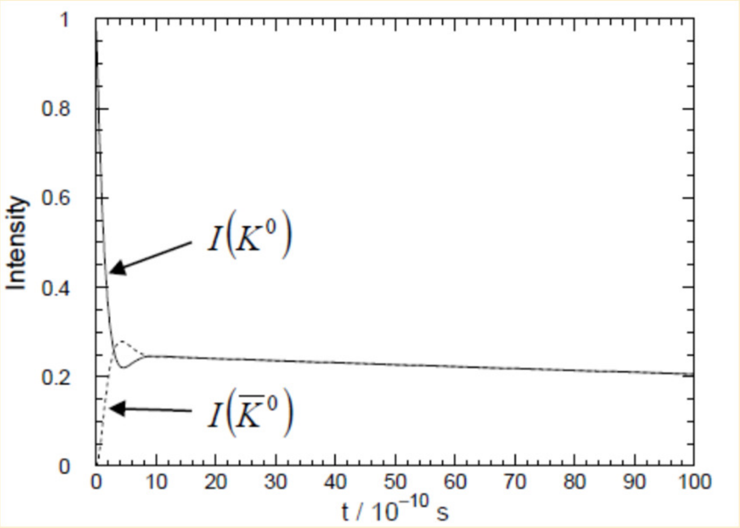
~~$K^0 \to \ell^-\bar{\nu}\pi^+, \quad \bar{K}^0 \to \ell^+\nu\pi^-, \quad K^+ \to \ell^-\nu\pi^+\pi^+, \quad \Sigma^+ \to n\ell^+\nu$~~

Both coming from 1st order diagram

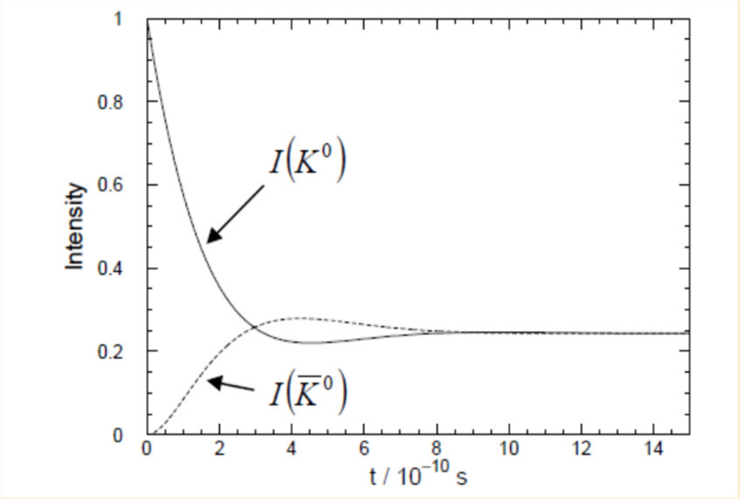


K Oscillations - XI

Interference!



Expanded timescale:

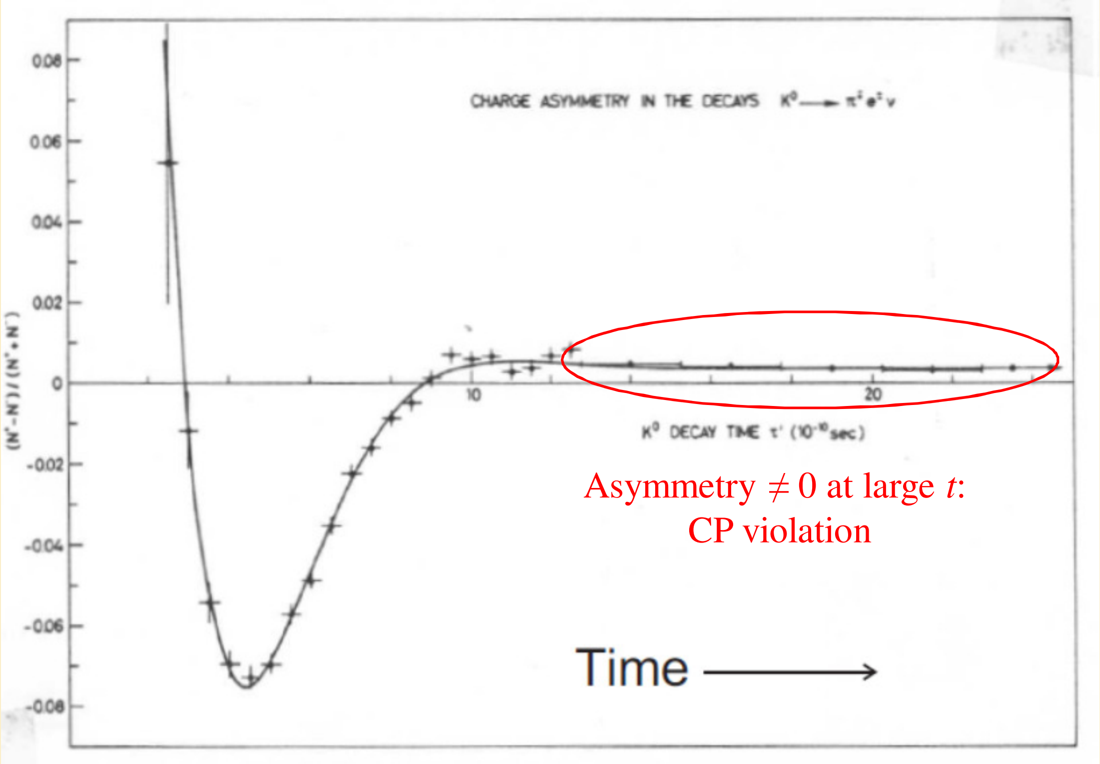


K Oscillations - XII

…And it's true!

Semileptonic decays

$$\frac{N_{\pi^+} - N_{\pi^-}}{N_{\pi^+} + N_{\pi^-}}$$



Asymmetry $\neq 0$ at large t:
CP violation

!!!

K Oscillations - XIII

CPLEAR experiment fit to Δm

$$R_{+} \equiv \Gamma(K^0_{t=0} \to \pi^- e^+ \nu_e)$$

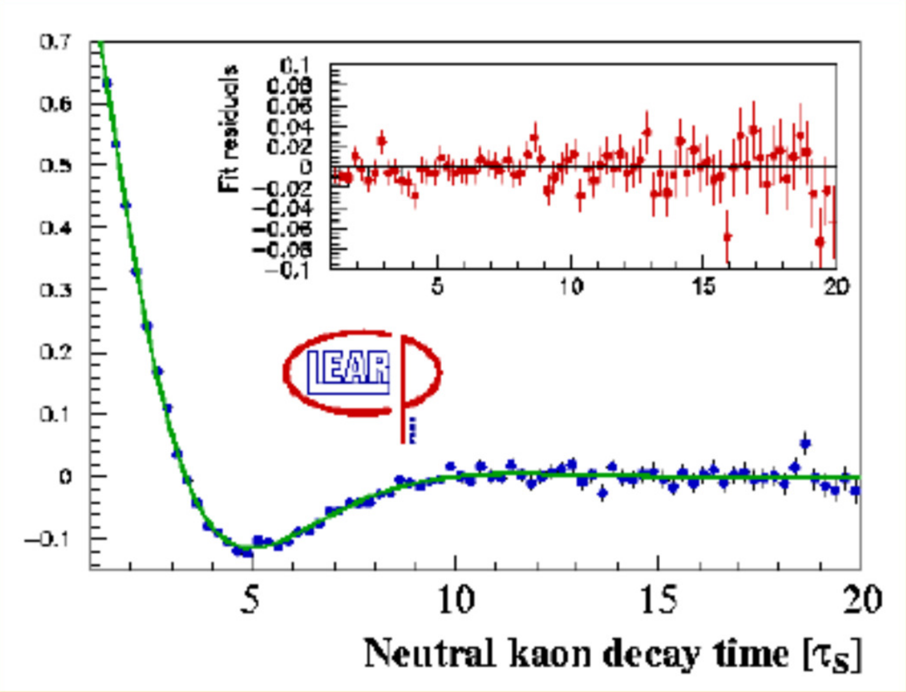
$$R_{-} \equiv \Gamma(K^0_{t=0} \to \pi^+ e^- \overline{\nu}_e)$$

$$\overline{R}_{-} \equiv \Gamma(\overline{K}^0_{t=0} \to \pi^+ e^- \overline{\nu}_e)$$

$$\overline{R}_{+} \equiv \Gamma(\overline{K}^0_{t=0} \to \pi^- e^+ \nu_e)$$

$$A_{\Delta m} = \frac{(R_+ + \overline{R}_-) - (R_- + \overline{R}_+)}{(R_+ + \overline{R}_-) + (R_- + \overline{R}_+)}$$

$$A_{\Delta m} = \frac{2e^{-(\Gamma_S+\Gamma_L)t/2}\cos \Delta m t}{e^{-\Gamma_S t} + e^{-\Gamma_L t}}$$



$$\boxed{\Delta m = 3.485 \times 10^{-15}\,\text{GeV}}$$

!!!

K Oscillations - XIV

Observe:

$$T\left(K^0 \leftrightarrow \bar{K}^0\right) = \frac{2\pi}{\Delta m}$$

$$\rightarrow T\left(K^0 \leftrightarrow \bar{K}^0\right) = \frac{2\pi}{3.49\ 10^{-12}\, MeV} = \frac{2\pi}{3.49\ 10^{-12}\, MeV} \underbrace{6.5810^{-22}\, MeVs}_{\hbar} \simeq 1.18\ ns$$

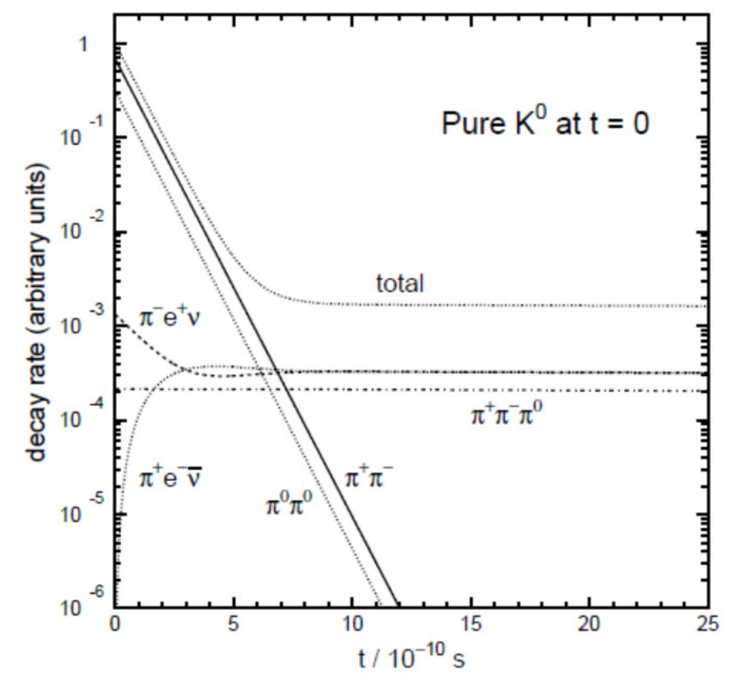
$$\tau_S \simeq 8.9510^{-11}\ s$$

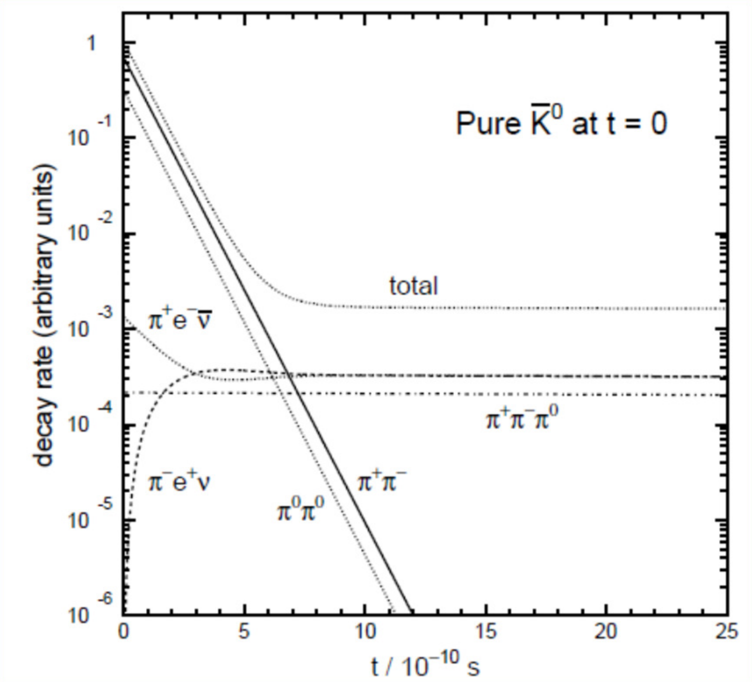
$$\rightarrow \frac{T}{\tau_S} \approx 13.3$$

→ Just a fraction of a single oscillation within a K_s lifetime

K Oscillations - XV

Summary of decay rates (CP conserved):
2 lifetimes (*2π, 3π*)
Strangeness oscillations (Semileptonic)
2 masses (*2π, 3π*)





K_S Regeneration - I

Production reactions (e.g. at low energy):

$$K^0: \quad \pi^- + p \to K^0 + \Lambda^0$$
$$\bar{K}^0: \quad \pi^+ + p \to \bar{K}^0 + K^+ + p \quad \text{Higher threshold}$$

Take first reaction → Initially pure K^0 beam

After several τ_s: K_S component off → Pure K_L beam

Introduce some material in the beam path: Funny effect!

K_S Regeneration - II

Total cross section different for $K^0, \bar{K}^0$:

Indeed, e.g.

$\bar{K}^0 + p \to \pi^+ + \Lambda^0$

is strictly forbidden for K^0

$\to \sigma_{K^0} \neq \sigma_{\bar{K}^0}$

Remembering the "OpticalTheorem":

$\sigma_{tot} = \frac{4\pi}{k} \operatorname{Im} f(0)$, $f(0)$ forward scattering amplitude

$\to f_{K^0}(0) \neq f_{\bar{K}^0}(0)$

Take forward scattering $(= \textit{propagation})$ of our pure K_L beam:

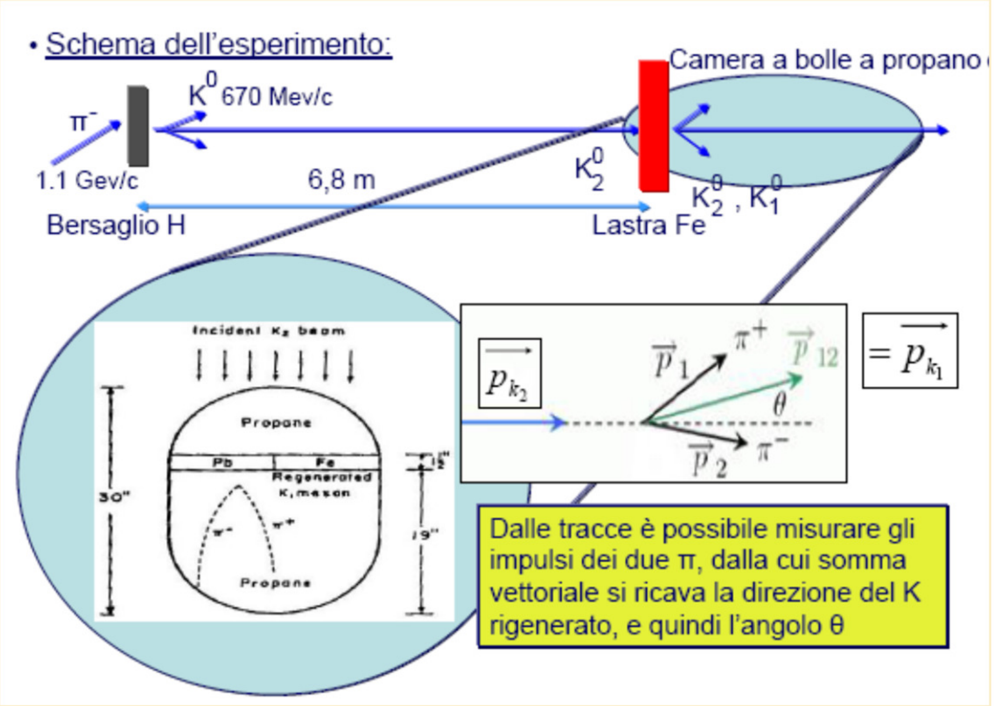
$$|K_L\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\bar{K}^0\rangle\right) \to \frac{1}{\sqrt{2}}\left(A\left(f_{K^0}\right)|K^0\rangle + B\left(f_{\bar{K}^0}\right)|\bar{K}^0\rangle\right) \neq |K_L\rangle$$

$$|K_L\rangle \to a|K_L\rangle + b|K_S\rangle, \quad |a|^2 + |b|^2 = 1$$

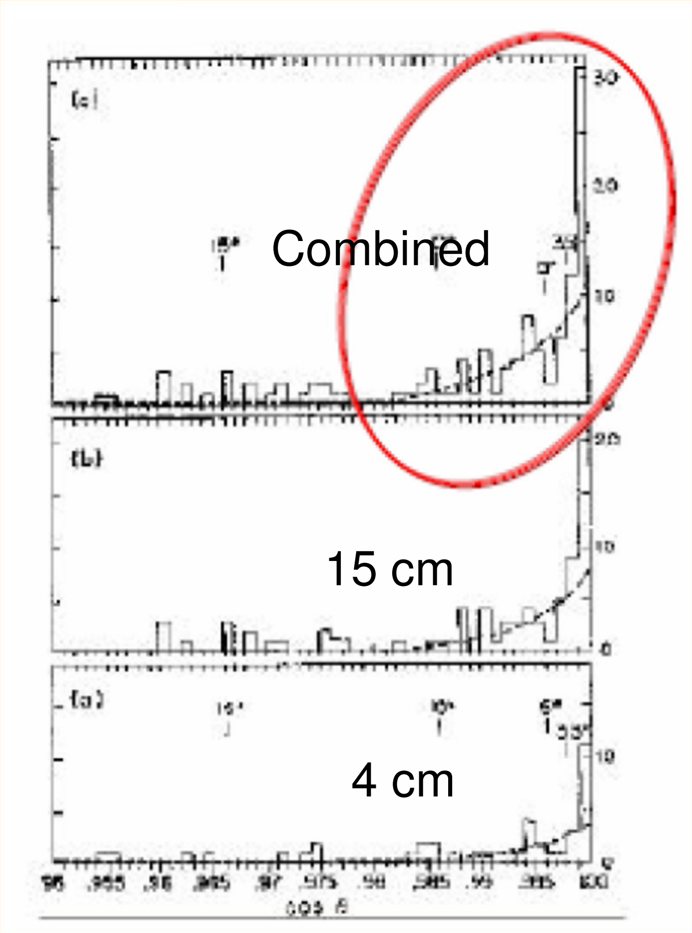
$\to$ A $|K_S\rangle$ component has been regenerated by the material!

K_S Regeneration - III

Piccioni et al. – Berkeley ≈ 1956

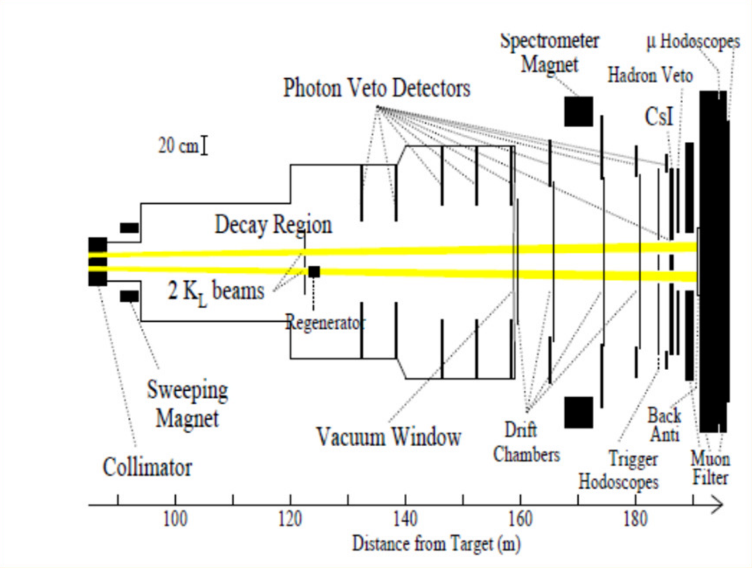


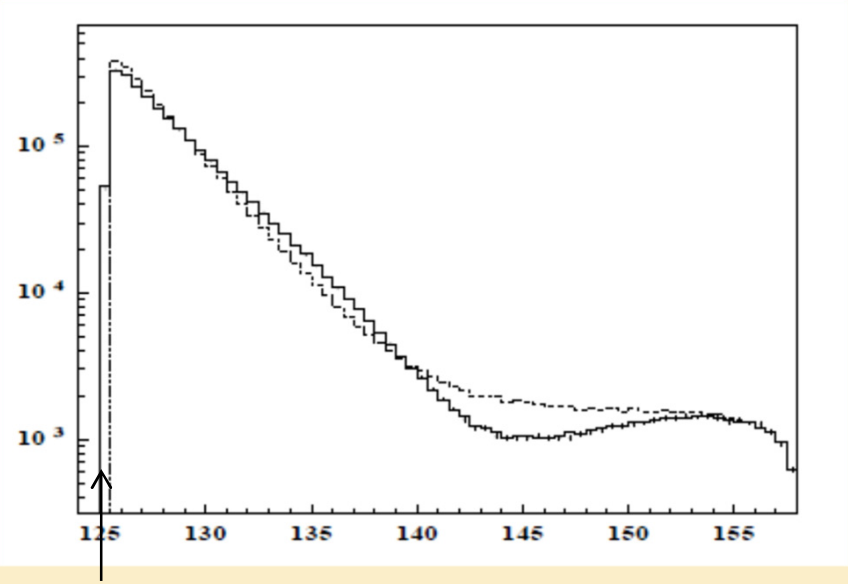
Dalle tracce è possibile misurare gli impulsi dei due π, dalla cui somma vettoriale si ricava la direzione del K rigenerato, e quindi l'angolo θ



K_S Regeneration - IV

KTEV - Fermilab ≈ 2000





Regenerator

Strong K_S regeneration signalled by 2π decays with τ_s lifetime

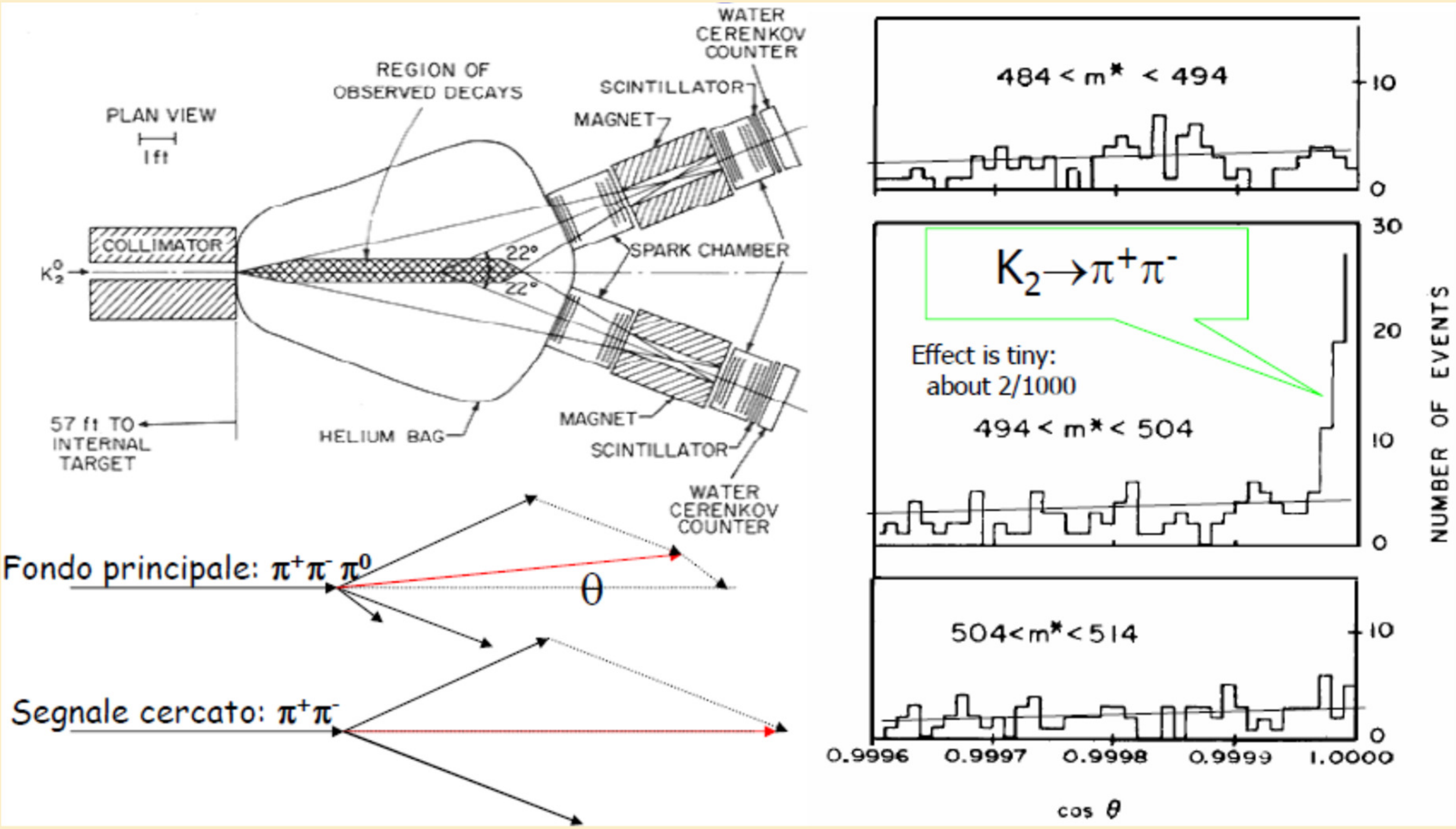
[Large interference observed in 2π rate:

Regenerated K_S component interfering with CP violating, 2π decay of K_L beam]

K *CP* Violation - I

$K_2 \rightarrow \pi^+\pi^-$ decay observed in 1964; $K_2 \rightarrow \pi^0\pi^0$ decay also observed

Small *BR* $\sim\ 10^{-3}$



Fondo principale: $\pi^+\pi^-\pi^0$

Segnale cercato: $\pi^+\pi^-$

$K_2 \rightarrow \pi^+\pi^-$

Effect is tiny: about 2/1000

K *CP* Violation - II

Three possible mechanisms:

1) K_L, K_S not *CP* eigenstates - Decay *CP* conserving

$$\rightarrow \left|K_L^0\right\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left(\left|K_2^0\right\rangle + \varepsilon\left|K_1^0\right\rangle\right), \left|K_S^0\right\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left(\left|K_1^0\right\rangle - \varepsilon\left|K_2^0\right\rangle\right)$$

$K_L^0 \rightarrow \pi\pi$ accounted for by

$$\left|K_L^0\right\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left(\underset{\pi\pi\pi}{\underset{\downarrow}{\left|K_2^0\right\rangle}} + \varepsilon\underset{\pi\pi}{\underset{\downarrow}{\left|K_1^0\right\rangle}}\right)$$

Mixing $\cancel{CP}$: Measured by small parameter ε

2) Decay *CP* violating - K_L, K_S *CP* eigenstates

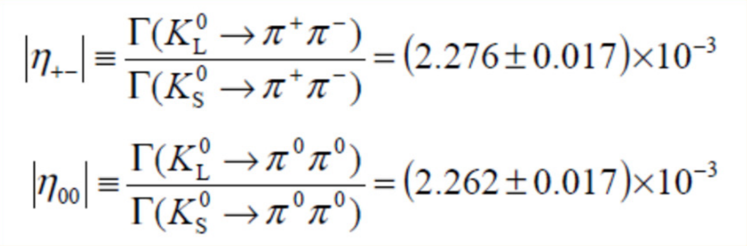
$$\left|K_L^0\right\rangle = \left|K_2^0\right\rangle$$

Direct $\cancel{CP}$: Measured by *very* small parameter ε'

3) Interference between mixing and decay

K *CP* Violation - III

Define:



$$|\eta_{+-}| \equiv \frac{\Gamma(K_L^0 \to \pi^+\pi^-)}{\Gamma(K_S^0 \to \pi^+\pi^-)} = (2.276 \pm 0.017) \times 10^{-3}$$

$$|\eta_{00}| \equiv \frac{\Gamma(K_L^0 \to \pi^0\pi^0)}{\Gamma(K_S^0 \to \pi^0\pi^0)} = (2.262 \pm 0.017) \times 10^{-3}$$

Expect:

Case 1) $\to \eta_{+-} = \eta_{00}$

Case 2) $\to \eta_{+-} \neq \eta_{00}$

Generally (see later):

$\eta_{+-} = \varepsilon + \varepsilon'$

$\eta_{+-} = \varepsilon - 2\varepsilon'$

$\to \varepsilon' \ll \varepsilon$, must be very small

K *CP* Violation - IV

Focus on mixing $\cancel{CP}$, ignore direct $\cancel{CP}$ for the moment

$\varepsilon = |\varepsilon| e^{i\varphi}$ Measuring mixing $\cancel{CP}$

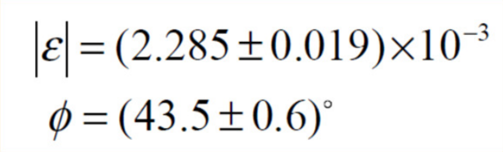
For a neutral beam initially pure $K^0 / \bar{K}^0$:

$\pi\pi$ decay rate as a function of distance

$$I\left(K^0;t\right) = \frac{N}{2}\left(1 - 2\,\mathrm{Re}\left(\varepsilon\right)\right)\left[e^{-\Gamma_S t} + \underbrace{|\varepsilon|^2 e^{-\Gamma_L t}}_{K_L \text{ contribution}} + \underbrace{2|\varepsilon| e^{-\frac{\Gamma_L+\Gamma_S}{2}t}\cos\left(\Delta m t - \varphi\right)}_{\text{Interference}}\right]$$

$$I\left(\bar{K}^0;t\right) = \frac{N}{2}\left(1 + 2\,\mathrm{Re}\left(\varepsilon\right)\right)\left[e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t} - 2|\varepsilon| e^{-\frac{\Gamma_L+\Gamma_S}{2}t}\cos\left(\Delta m t - \varphi\right)\right]$$

$\rightarrow$ Get $|\varepsilon|, \varphi, \Delta m$

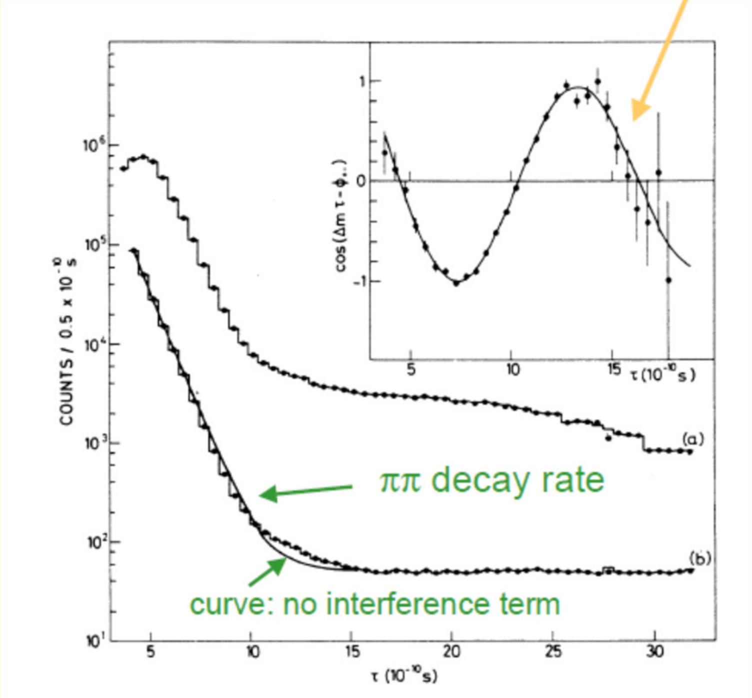


$$|\varepsilon| = (2.285 \pm 0.019) \times 10^{-3}$$
$$\phi = (43.5 \pm 0.6)^{\circ}$$

K *CP* Violation - V

...Quite correct!

Pure interference term



$\pi\pi$ decay rate

curve: no interference term

Similar to regeneration data, but : No regenerator !

Interference between K_L and K_S in 2π decay

$\rightarrow K_L$ and K_S states not orthogonal: Both have a K_1 component

K *CP* Violation - VI

Neutral beam at large distance from production target: Pure K_L

$$\left|K_L^0\right\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left(\left|K_2^0\right\rangle + \varepsilon\left|K_1^0\right\rangle\right)$$

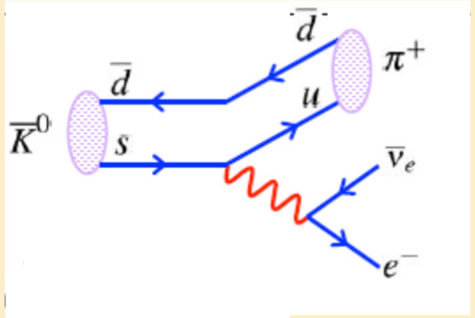
$$\rightarrow \left|K_L^0\right\rangle = \frac{1}{\sqrt{2\left(1+|\varepsilon|^2\right)}}\left[(1+\varepsilon)\left|K^0\right\rangle + (1-\varepsilon)\left|\bar{K}^0\right\rangle\right]$$

$$\rightarrow \left|K_S^0\right\rangle = \frac{1}{\sqrt{2\left(1+|\varepsilon|^2\right)}}\left[(1+\varepsilon)\left|K^0\right\rangle - (1-\varepsilon)\left|\bar{K}^0\right\rangle\right]$$

Take semileptonic decays, e.g. K_{e3}:

$K^0 \rightarrow \pi^- e^+ \nu_e$

$\bar{K}^0 \rightarrow \pi^+ e^- \bar{\nu}_e$



Observe:

$CP\left|K^0\right\rangle = \left|\bar{K}^0\right\rangle$

$CP\left|\pi^- e^+ \nu_e\right\rangle = \left|\pi^+ e^- \bar{\nu}_e\right\rangle$

→ No *CP* eigenstates

K *CP* Violation - VII

Define CP violation asymmetry for semileptonic decays (K_{e3})

$$\delta = \frac{\Gamma\left(K_L \to \pi^- e^+ \nu_e\right) - \Gamma\left(K_L \to \pi^+ e^- \overline{\nu}_e\right)}{\Gamma\left(K_L \to \pi^- e^+ \nu_e\right) + \Gamma\left(K_L \to \pi^+ e^- \overline{\nu}_e\right)}$$

$$\Gamma\left(K_L \to \pi^+ e^- \overline{\nu}_e\right) \propto \left|\left\langle K^0 \middle| K_L \right\rangle\right|^2 \propto \left|1 - \varepsilon\right|^2$$

$$\Gamma\left(K_L \to \pi^+ e^- \overline{\nu}_e\right) \propto \left|\left\langle \overline{K}^0 \middle| K_L \right\rangle\right|^2 \propto \left|1 + \varepsilon\right|^2$$

$$\left|1 \pm \varepsilon\right|^2 = \left(1 \pm \varepsilon\right)\left(1 \pm \varepsilon^*\right) \approx 1 + \varepsilon + \varepsilon^* = 1 \pm 2\,\mathrm{Re}\,\varepsilon$$

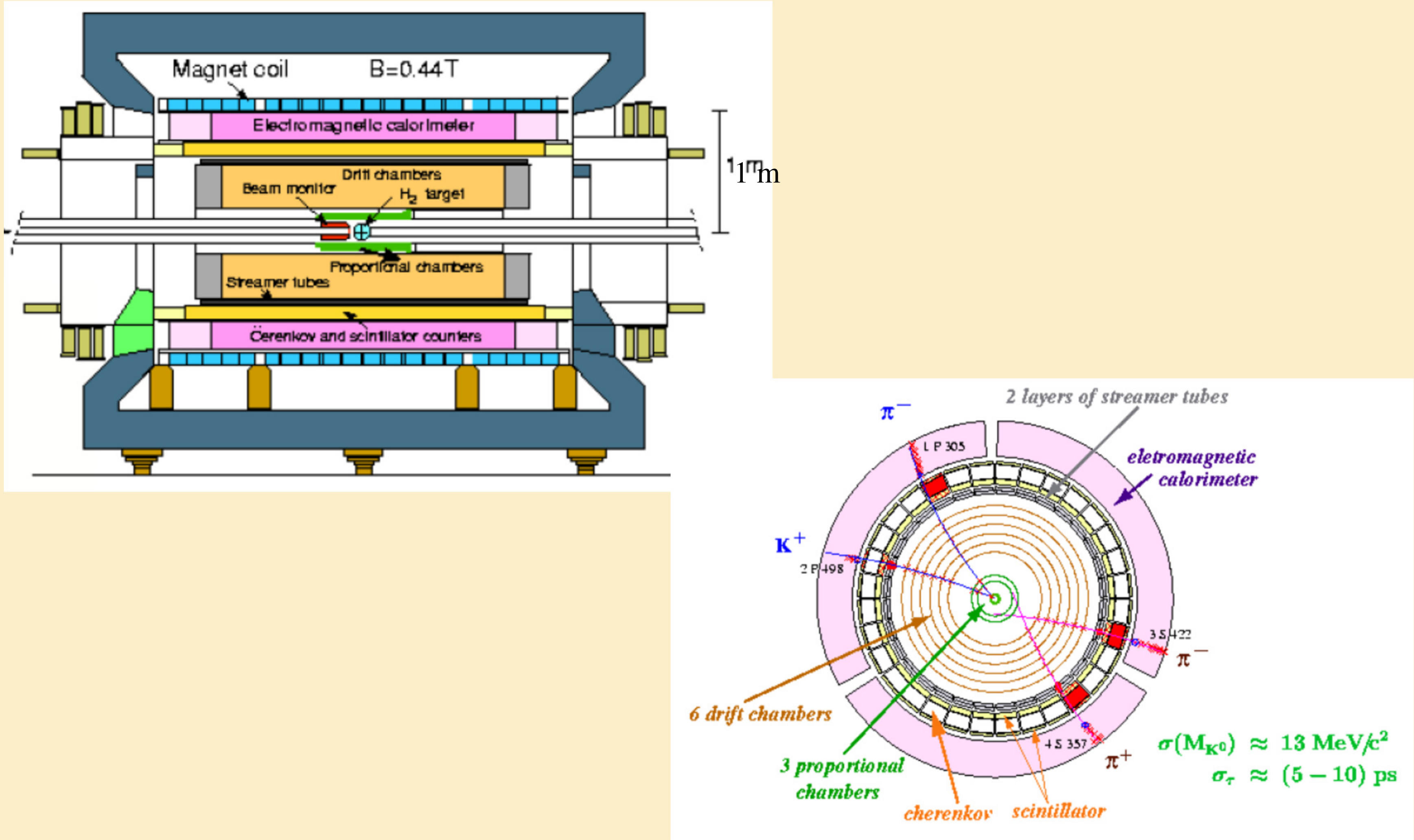
$$\to \delta \approx \frac{\left(1 + 2\,\mathrm{Re}\,\varepsilon\right) - \left(1 - 2\,\mathrm{Re}\,\varepsilon\right)}{\left(1 + 2\,\mathrm{Re}\,\varepsilon\right) + \left(1 - 2\,\mathrm{Re}\,\varepsilon\right)} = \frac{4\,\mathrm{Re}\,\varepsilon}{2} = 2\,\mathrm{Re}\,\varepsilon = 2\left|\varepsilon\right|\cos\phi$$

$\to \delta \approx 3.2110^{-3}$ calculated by taking ε from $\pi\pi$

Measured: $\left(3.27 \pm 0.012\right)10^{-3}$

K *CP* Violation - VIII

CPLEAR – CERN ’90s



K *CP* Violation - IX

Experiment CPLEAR
(CERN $\overline{p}$ Low Energy Accumulator Ring - LEAR) →~1995

Use reactions:

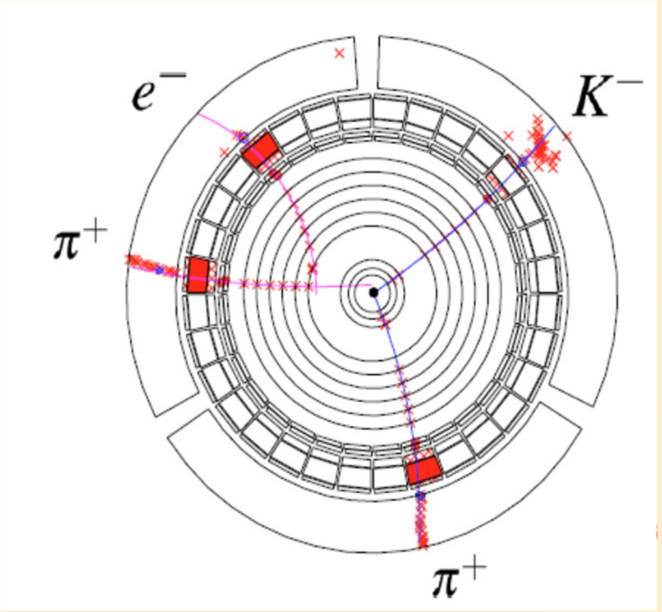
$p\overline{p} \rightarrow K^{-}\pi^{+}K^{0}$

$p\overline{p} \rightarrow K^{+}\pi^{-}\overline{K}^{0}$

and semileptonic decays

$K^{0} \rightarrow \pi^{-}e^{+}\nu_{e}$

$\overline{K}^{0} \rightarrow \pi^{+}e^{-}\overline{\nu}_{e}$



Strangeness of $\begin{matrix} \textit{produced} \\ \textit{decaying} \end{matrix}$ K state: Tagged *unambiguously*

K *CP* Violation - X

Strangeness oscillations in presence of $\cancel{CP}$:

$$R_+ = \Gamma\left(K^0_{t=0} \to \pi^- e^+ \nu_e\right) = N\frac{1}{4}\left[e^{-\Gamma_s t} + e^{-\Gamma_L t} + 2e^{-\frac{\Gamma_S+\Gamma_L}{2}t}\cos\Delta m t\right]$$

$$R_- = \Gamma\left(K^0_{t=0} \to \pi^+ e^- \bar{\nu}_e\right) = \boxed{N\left(1-4\operatorname{Re}\varepsilon\right)}\frac{1}{4}\left[e^{-\Gamma_s t} + e^{-\Gamma_L t} - 2e^{-\frac{\Gamma_S+\Gamma_L}{2}t}\cos\Delta m t\right]$$

$$\bar{R}_+ = \Gamma\left(\bar{K}^0_{t=0} \to \pi^- e^+ \nu_e\right) = \boxed{N\left(1+4\operatorname{Re}\varepsilon\right)}\frac{1}{4}\left[e^{-\Gamma_s t} + e^{-\Gamma_L t} - 2e^{-\frac{\Gamma_S+\Gamma_L}{2}t}\cos\Delta m t\right]$$

$$\bar{R}_- = \Gamma\left(\bar{K}^0_{t=0} \to \pi^+ e^- \bar{\nu}_e\right) = N\frac{1}{4}\left[e^{-\Gamma_s t} + e^{-\Gamma_L t} + 2e^{-\frac{\Gamma_S+\Gamma_L}{2}t}\cos\Delta m t\right]$$

$$A_{\Delta m} = \frac{\left(R_+ + \bar{R}_-\right) - \left(R_- + \bar{R}_+\right)}{\left(R_+ + \bar{R}_-\right) + \left(R_- + \bar{R}_+\right)} = \frac{2e^{-\frac{\Gamma_S+\Gamma_L}{2}t}\cos\Delta m t}{e^{-\Gamma_s t} + e^{-\Gamma_L t}}$$

$$\to A_{\Delta m}\left(\cancel{CP}\right) = A_{\Delta m}\left(CP\right)$$

Asymmetry not sensitive to CP violation

K *CP* Violation - XI

Decays into 2 π's: Time dependent asymmetry

$$A(\pi\pi)=\frac{\Gamma\left(\bar{K}^0\to\pi\pi\right)-\Gamma\left(K^0\to\pi\pi\right)}{\Gamma\left(\bar{K}^0\to\pi\pi\right)+\Gamma\left(K^0\to\pi\pi\right)}$$

$$I\left(K^0;t\right)=\frac{N}{2}\left(1-2\,\mathrm{Re}(\varepsilon)\right)\left[e^{-\Gamma_s t}+|\varepsilon|^2 e^{-\Gamma_L t}+2|\varepsilon|e^{-\frac{\Gamma_L+\Gamma_S}{2}t}\cos\left(\Delta mt-\varphi\right)\right]$$

$$I\left(\bar{K}^0;t\right)=\frac{N}{2}\left(1+2\,\mathrm{Re}(\varepsilon)\right)\left[e^{-\Gamma_s t}+|\varepsilon|^2 e^{-\Gamma_L t}-2|\varepsilon|e^{-\frac{\Gamma_L+\Gamma_S}{2}t}\cos\left(\Delta mt-\varphi\right)\right]$$

$$I\left(\bar{K}^0;t\right)\approx\frac{N}{2}\left[e^{-\Gamma_s t}+|\varepsilon|^2 e^{-\Gamma_L t}-2|\varepsilon|e^{-\frac{\Gamma_L+\Gamma_S}{2}t}\cos\left(\Delta mt-\varphi\right)+2\,\mathrm{Re}(\varepsilon)e^{-\Gamma_s t}-4\,\mathrm{Re}(\varepsilon)|\varepsilon|e^{-\frac{\Gamma_L+\Gamma_S}{2}t}\cos\left(\Delta mt-\varphi\right)\right]$$

$$I\left(K^0;t\right)\approx\frac{N}{2}\left[e^{-\Gamma_s t}+|\varepsilon|^2 e^{-\Gamma_L t}+2|\varepsilon|e^{-\frac{\Gamma_L+\Gamma_S}{2}t}\cos\left(\Delta mt-\varphi\right)-2\,\mathrm{Re}(\varepsilon)e^{-\Gamma_s t}-4\,\mathrm{Re}(\varepsilon)|\varepsilon|e^{-\frac{\Gamma_L+\Gamma_S}{2}t}\cos\left(\Delta mt-\varphi\right)\right]$$

$$I\left(\bar{K}^0;t\right)-I\left(K^0;t\right)\approx N\left[-2|\varepsilon|e^{-\frac{\Gamma_L+\Gamma_S}{2}t}\cos\left(\Delta mt-\varphi\right)+2\,\mathrm{Re}(\varepsilon)e^{-\Gamma_s t}\right]$$

$$I\left(\bar{K}^0;t\right)+I\left(K^0;t\right)\approx N\left[e^{-\Gamma_s t}+|\varepsilon|^2 e^{-\Gamma_L t}-4\,\mathrm{Re}(\varepsilon)|\varepsilon|e^{-\frac{\Gamma_L+\Gamma_S}{2}t}\right]$$

K *CP* Violation - XII

$$A(\pi\pi) \approx \frac{-2|\varepsilon| e^{-\frac{\Gamma_L+\Gamma_S}{2}t} \cos(\Delta m t - \varphi) + 2\,\mathrm{Re}(\varepsilon) e^{-\Gamma_s t}}{e^{-\Gamma_s t} + |\varepsilon|^2 e^{-\Gamma_L t} - 4\,\mathrm{Re}(\varepsilon)|\varepsilon| e^{-\frac{\Gamma_L+\Gamma_S}{2}t}}$$

Keeping only terms linear in $|\varepsilon|$:

$$A \approx \frac{4\,\mathrm{Re}(\varepsilon) e^{-\Gamma_s t} - 4|\varepsilon| e^{-\frac{\Gamma_L+\Gamma_S}{2}t} \cos(\Delta m t - \varphi)}{2e^{-\Gamma_s t}} = 2\,\mathrm{Re}(\varepsilon) - 2|\varepsilon| e^{\frac{\Gamma_S-\Gamma_L}{2}t} \cos(\Delta m t - \varphi)$$

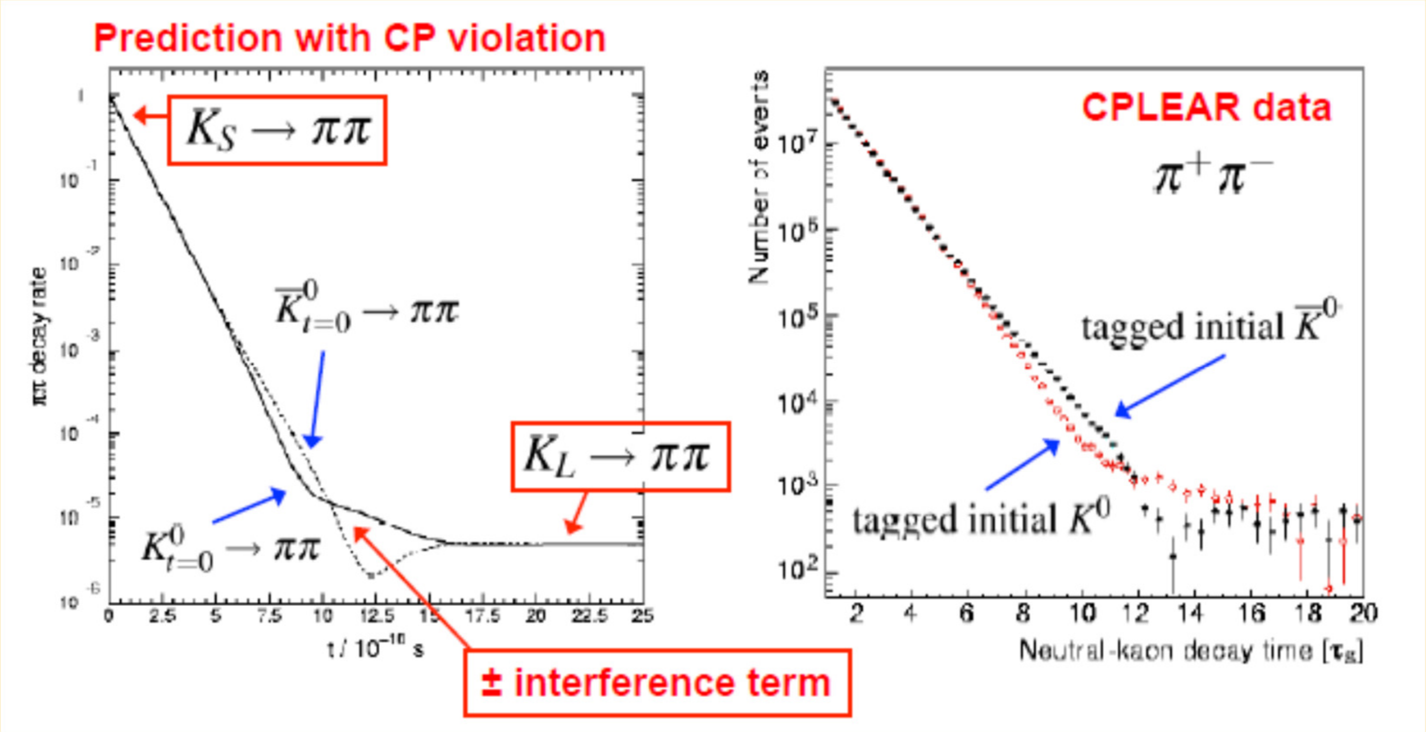
$$\rightarrow A(\pi\pi) \approx 2\left[\mathrm{Re}(\varepsilon) - |\varepsilon| e^{\frac{(\Gamma_S-\Gamma_L)}{2}t} \cos(\Delta m t - \phi)\right]$$

K *CP* Violation - XIII

CPLEAR on 2π decays

Expected decay rates for $K^0,\ \overline{K}^0$ initial state

Observed decay rates for $K^0,\ \overline{K}^0$ initial state



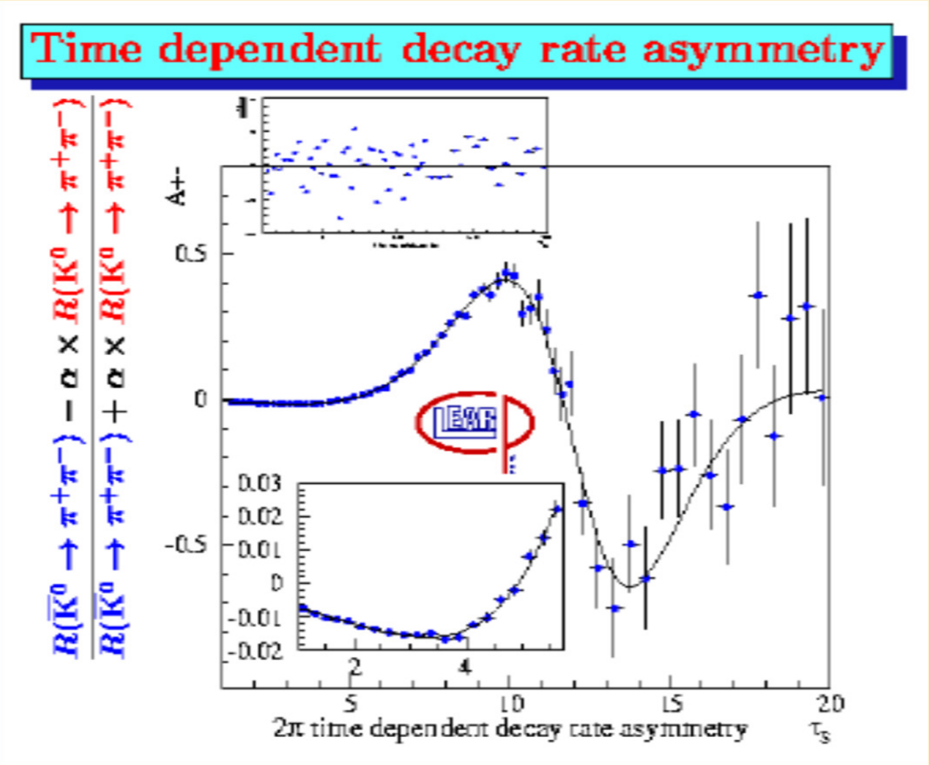
K *CP* Violation - XIV

Time dependent asymmetry:

$$A(\pi\pi)=\frac{\Gamma\left(\bar{K}^0\right)-\Gamma\left(K^0\right)}{\Gamma\left(\bar{K}^0\right)+\Gamma\left(K^0\right)}$$

$$\to A(\pi\pi)\approx 2\,\mathrm{Re}(\varepsilon)-2|\varepsilon|\,e^{\frac{(\Gamma_S-\Gamma_L)}{2}t}\cos(\Delta m t-\phi)$$

$$\to\begin{cases}|\varepsilon|=(2.264\pm0.035)10^{-3}\\ \varphi=(43.19\pm0.073)^{0}\\ \Delta m=(3.4852\pm0.013)10^{-15}\,GeV\end{cases}$$



K *CP* Violation - XV

CP violation in 3π decays

Expect, by swapping $K_S \leftrightarrow K_L$:

$$I\left(K^0;t\right)=\frac{N'}{2}\left(1-2\,\mathrm{Re}(\varepsilon)\right)\left[e^{-\Gamma_L t}+|\varepsilon|^2 e^{-\Gamma_S t}+2|\varepsilon|e^{-\frac{\Gamma_L+\Gamma_S}{2}t}\cos\left(\Delta m t-\varphi\right)\right]$$

Very different experimental conditions as compared to 2π :

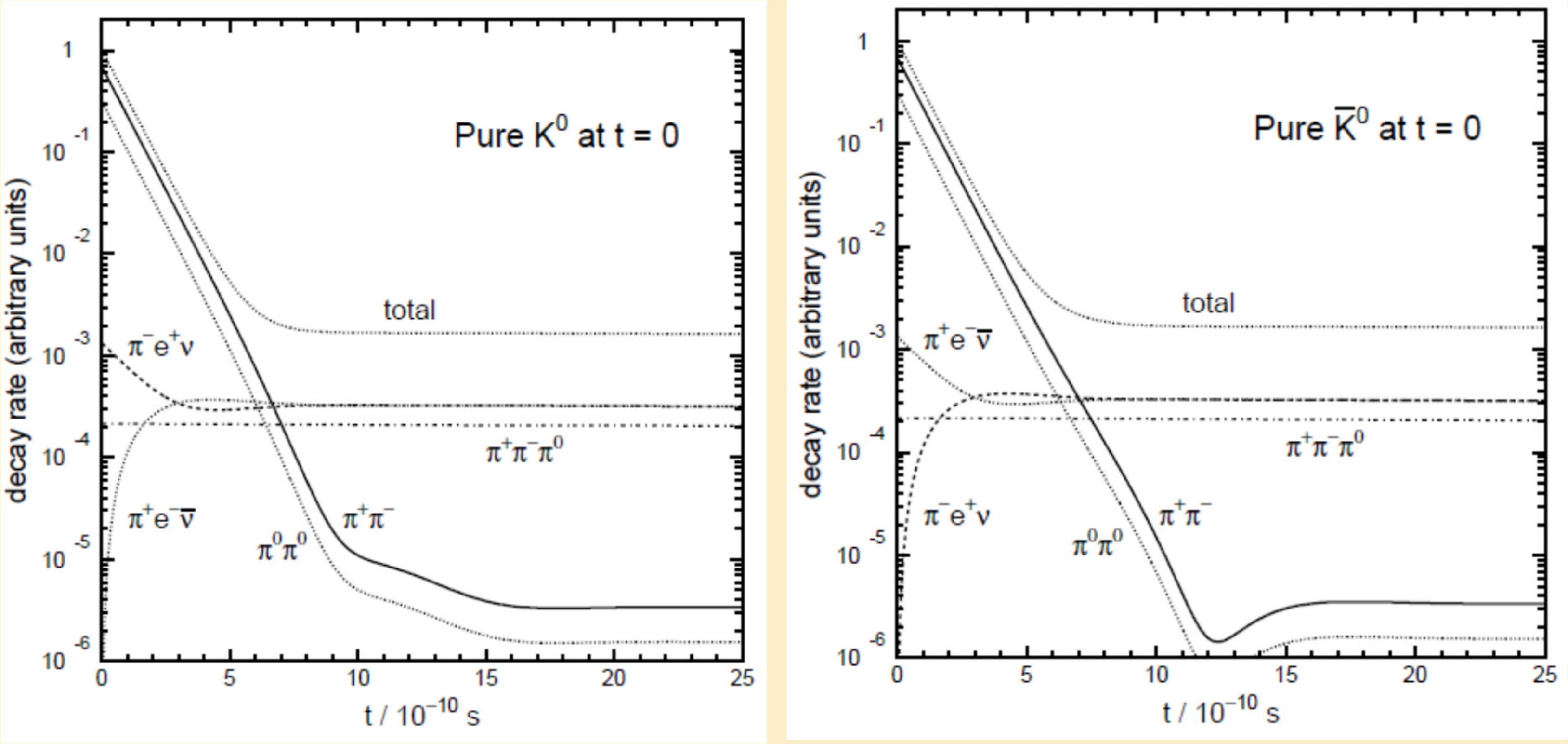
Lots of CP conserving 3π decays from K_L component of the beam

(Compare: No CP conserving 2π from K_S component, which just dies out at large distance)

→ Measurement difficult, large errors

K *CP* Violation - XVI

Summary of decay rates (CP violated)



K *T*, *CPT* Tests - I

From previous conclusions on *CP* violation:

$$R_- = \Gamma\left(K^0_{t=0} \to \pi^+ e^- \bar{\nu}_e\right) = N\left(1-4\,\mathrm{Re}\,\varepsilon\right)\frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-\frac{\Gamma_S+\Gamma_L}{2}t}\cos\Delta m t\right]$$

$$\bar{R}_+ = \Gamma\left(\bar{K}^0_{t=0} \to \pi^- e^+ \nu_e\right) = N\left(1+4\,\mathrm{Re}\,\varepsilon\right)\frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-\frac{\Gamma_S+\Gamma_L}{2}t}\cos\Delta m t\right]$$

$$\to \Gamma\left(K^0_{t=0} \to \bar{K}^0\right) \neq \Gamma\left(\bar{K}^0_{t=0} \to K^0\right)$$

→ Amplitude of direct process $\neq$ Amplitude of reverse process

→ CP violation ↔ *Time Reversal* violation

To be expected if *CPT* is a good symmetry

Define *T* asymmetry:

$$A_T = \frac{\Gamma\left(\bar{K}^0_{t=0} \to K^0\right) - \Gamma\left(K^0_{t=0} \to \bar{K}^0\right)}{\Gamma\left(\bar{K}^0_{t=0} \to K^0\right) + \Gamma\left(K^0_{t=0} \to \bar{K}^0\right)}$$

K *T*, *CPT* Tests - II

Measure by taking semileptonic:

$$A_T = \frac{\Gamma\left(\bar{K}^0_{t=0} \to \pi^- e^+ \nu_e\right) - \Gamma\left(K^0_{t=0} \to \pi^+ e^- \bar{\nu}_e\right)}{\Gamma\left(\bar{K}^0_{t=0} \to \pi^- e^+ \nu_e\right) + \Gamma\left(K^0_{t=0} \to \pi^+ e^- \bar{\nu}_e\right)}$$

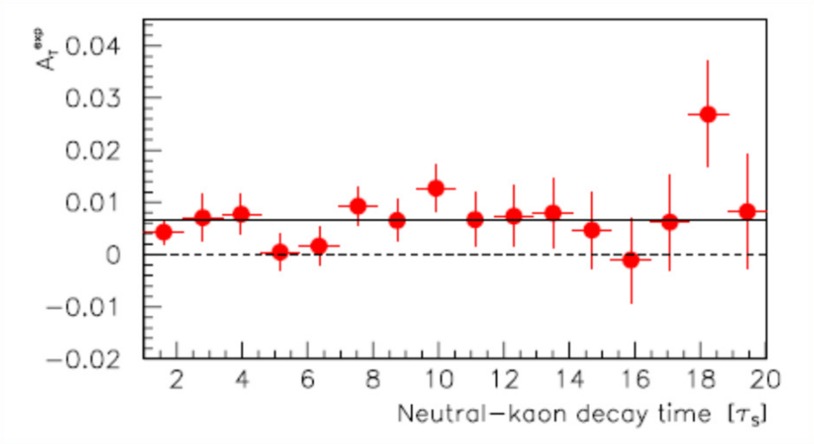
$A_T \approx 4\,\mathrm{Re}(\varepsilon) = 4|\varepsilon|\cos\phi$ Time independent constant

→ Expect from $\pi\pi$ CP violation

$A_T \approx 6.6\ 10^{-3}$

Measure:

$A_T = (6.2 \pm 1.7)\ 10^{-3}$



K *T*, *CPT* Tests - III

Semileptonic decays also used to test *CPT*

Simple test:

$$\Gamma\left(K^0 \to K^0\right) = \Gamma\left(\bar{K}^0 \to \bar{K}^0\right)$$

Define *CPT* asymmetry :

$$A_{CPT} = \frac{\Gamma\left(K^0 \to K^0\right) - \Gamma\left(\bar{K}^0 \to \bar{K}^0\right)}{\Gamma\left(K^0 \to K^0\right) + \Gamma\left(\bar{K}^0 \to \bar{K}^0\right)}$$

Measure by:

$$A_{CPT} = \frac{\Gamma\left(K^0_{t=0} \to \pi^- e^+ \nu_e\right) - \Gamma\left(\bar{K}^0_{t=0} \to \pi^+ e^- \bar{\nu}_e\right)}{\Gamma\left(K^0_{t=0} \to \pi^- e^+ \nu_e\right) + \Gamma\left(\bar{K}^0_{t=0} \to \pi^+ e^- \bar{\nu}_e\right)}$$

K *T*, *CPT* Tests - IV

Since:

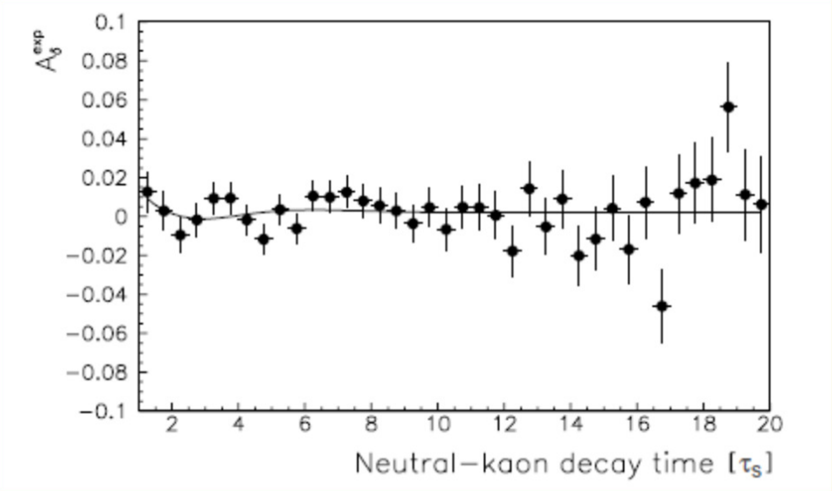
$$\Gamma\left(K^0_{t=0} \to \pi^- e^+ \nu_e\right) = N\frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-\frac{\Gamma_S+\Gamma_L}{2}t}\cos\Delta m t\right]$$

$$\Gamma\left(\bar{K}^0_{t=0} \to \pi^+ e^- \bar{\nu}_e\right) = N\frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-\frac{\Gamma_S+\Gamma_L}{2}t}\cos\Delta m t\right]$$

→ Expect:

$A_{CPT} = 0, \quad t \quad$ independent

Measure:



K Direct *CP* Violation - I

Another side of $\cancel{CP}$: K^0 *decays* *CP* violating

→ Direct $\cancel{CP}$

Amplitude ratios:

$$\eta_{+-}=\frac{\langle\pi^+\pi^-|T|K_L^0\rangle}{\langle\pi^+\pi^-|T|K_S^0\rangle}=|\eta_{+-}|e^{i\phi_{+-}},\qquad \eta_{00}=\frac{\langle\pi^0\pi^0|T|K_L^0\rangle}{\langle\pi^0\pi^0|T|K_S^0\rangle}=|\eta_{00}|e^{i\phi_{00}}$$

In order to relate η,ϕ parameters to ε,ε'

a)Decompose 2π states into isospin eigenstates:

$$\begin{cases}\langle\pi^+\pi^-|=\frac{1}{\sqrt{3}}\langle I=2|+\sqrt{\frac{2}{3}}\langle I=0| \\ \langle\pi^0\pi^0|=\sqrt{\frac{2}{3}}\langle I=2|-\frac{1}{\sqrt{3}}\langle I=0|\end{cases}$$

$I=1$ absent due to Bose statistics of π's in a S-wave

K Direct *CP* Violation - II

Full 2π states should include proper phase factors
originating from S-wave $\pi\pi$ scattering

$$\left\langle \pi^+\pi^- \right| = \frac{1}{\sqrt{3}} \langle 2| e^{i\delta_2} + \sqrt{\frac{2}{3}} \langle 0| e^{i\delta_0}$$

$$\left\langle \pi^0\pi^0 \right| = \sqrt{\frac{2}{3}} \langle 2| e^{i\delta_2} - \frac{1}{\sqrt{3}} \langle 0| e^{i\delta_0}$$

Define decay amplitudes into isospin states:

$$A_0 = \langle 0| H_w \left| K^0 \right\rangle$$

$$A_2 = \langle 2| H_w \left| K^0 \right\rangle$$

K Direct *CP* Violation - III

$$CP\left|\pi\pi\right\rangle = +1 \rightarrow CPT\left|0\right\rangle = \left\langle 0\right|, CPT\left|2\right\rangle = \left\langle 2\right|$$

$$CP\left|K^0\right\rangle = -\left|\bar{K}^0\right\rangle \rightarrow CPT\left|K^0\right\rangle = -\left\langle \bar{K}^0\right|$$

$$\left[H_w, CPT\right] = 0$$

$$\rightarrow \begin{cases} \left\langle 0\right|H_w\left|\bar{K}^0\right\rangle \overset{CPT}{\rightarrow} -\left\langle K^0\right|H_w\left|0\right\rangle = -A_0^* \\ \left\langle 2\right|H_w\left|\bar{K}^0\right\rangle \overset{CPT}{\rightarrow} -\left\langle K^0\right|H_w\left|2\right\rangle = -A_2^* \end{cases}$$

$$\left|K_L^0\right\rangle = \frac{1}{\sqrt{2\left(1+\left|\varepsilon\right|^2\right)}}\left[\left(1+\varepsilon\right)\left|K^0\right\rangle + \left(1-\varepsilon\right)\left|\bar{K}^0\right\rangle\right]$$

$$\left|K_S^0\right\rangle = \frac{1}{\sqrt{2\left(1+\left|\varepsilon\right|^2\right)}}\left[\left(1+\varepsilon\right)\left|K^0\right\rangle - \left(1-\varepsilon\right)\left|\bar{K}^0\right\rangle\right]$$

K Direct *CP* Violation - IV

Transition matrix elements:

$$\left\langle \pi^+\pi^- \middle| H \middle| K_L^0 \right\rangle = \frac{2}{\sqrt{3}} \frac{1}{\sqrt{2\left(1+|\varepsilon|^2\right)}} \varepsilon\left[\operatorname{Re} A_2 e^{i\delta_2} + \sqrt{2} A_0 e^{i\delta_0}\right] + \operatorname{Im} A_2 e^{i\delta_2}$$

$$\left\langle \pi^+\pi^- \middle| H \middle| K_S^0 \right\rangle = \frac{2}{\sqrt{3}} \frac{1}{\sqrt{2\left(1+|\varepsilon|^2\right)}} \left[\operatorname{Re} A_2 e^{i\delta_2} + \sqrt{2} A_0 e^{i\delta_0} + \varepsilon \operatorname{Im} A_2 e^{i\delta_2}\right]$$

$$\left\langle \pi^0\pi^0 \middle| H \middle| K_L^0 \right\rangle = \frac{2}{\sqrt{3}} \frac{1}{\sqrt{2\left(1+|\varepsilon|^2\right)}} \varepsilon\left[\sqrt{2} \operatorname{Re} A_2 e^{i\delta_2} - A_0 e^{i\delta_0}\right] + \sqrt{2} \operatorname{Im} A_2 e^{i\delta_2}$$

$$\left\langle \pi^0\pi^0 \middle| H \middle| K_S^0 \right\rangle = \frac{2}{\sqrt{3}} \frac{1}{\sqrt{2\left(1+|\varepsilon|^2\right)}} \left[\sqrt{2} \operatorname{Re} A_2 e^{i\delta_2} - A_0 e^{i\delta_0} + \varepsilon \sqrt{2} \operatorname{Im} A_2 e^{i\delta_2}\right]$$

After some complex algebra:

$$\eta_{+-} = \frac{\left\langle \pi^+\pi^- \middle| T \middle| K_L^0 \right\rangle}{\left\langle \pi^+\pi^- \middle| T \middle| K_S^0 \right\rangle} \simeq \varepsilon + \underbrace{\frac{1}{\sqrt{2}} \frac{\operatorname{Im} A_2}{A_0} e^{i\left(\delta_2-\delta_0\right)}}_{\varepsilon'} = \varepsilon + \varepsilon'$$

$$\eta_{00} = \frac{\left\langle \pi^0\pi^0 \middle| T \middle| K_L^0 \right\rangle}{\left\langle \pi^0\pi^0 \middle| T \middle| K_S^0 \right\rangle} \simeq \varepsilon - \underbrace{\sqrt{2} \frac{\operatorname{Im} A_2}{A_0} e^{i\left(\delta_2-\delta_0\right)}}_{2\varepsilon'} = \varepsilon - 2\varepsilon'$$

K Direct *CP* Violation - V

Double ratio magic:

$$\eta_{+-} = \frac{\langle \pi^+\pi^- | T | K_L^0 \rangle}{\langle \pi^+\pi^- | T | K_S^0 \rangle} \simeq \varepsilon + \varepsilon'$$

$$\eta_{00} = \frac{\langle \pi^0\pi^0 | T | K_L^0 \rangle}{\langle \pi^0\pi^0 | T | K_S^0 \rangle} \simeq \varepsilon - 2\varepsilon'$$

$$\rightarrow \left|\frac{\eta_{00}}{\eta_{+-}}\right|^2 \simeq \frac{|\varepsilon - 2\varepsilon'|^2}{|\varepsilon + \varepsilon'|^2} = \frac{(\varepsilon - 2\varepsilon')(\varepsilon - 2\varepsilon')^*}{(\varepsilon + \varepsilon')(\varepsilon + \varepsilon')^*} \simeq \frac{|\varepsilon|^2 - 4\,\mathrm{Re}(\varepsilon'\varepsilon)}{|\varepsilon|^2 + 2\,\mathrm{Re}(\varepsilon'\varepsilon)}$$

$$\rightarrow \left|\frac{\eta_{00}}{\eta_{+-}}\right|^2 \approx \frac{1 - 4\frac{\mathrm{Re}(\varepsilon'\varepsilon)}{|\varepsilon|^2}}{1 + 2\frac{\mathrm{Re}(\varepsilon'\varepsilon)}{|\varepsilon|^2}} \approx \left[1 - 2\frac{\mathrm{Re}(\varepsilon'\varepsilon)}{|\varepsilon|^2}\right]\left[1 - 4\frac{\mathrm{Re}(\varepsilon'\varepsilon)}{|\varepsilon|^2}\right] \approx 1 - 6\frac{\mathrm{Re}(\varepsilon'\varepsilon)}{|\varepsilon|^2}$$

$$\rightarrow \left|\frac{\eta_{00}}{\eta_{+-}}\right|^2 = \frac{\frac{N_L^{00}}{N_S^{00}}}{\frac{N_L^{+-}}{N_S^{+-}}} \approx 1 - 6\,\mathrm{Re}\frac{(\varepsilon')}{|\varepsilon|}$$

K Direct *CP* Violation - VI

Actually a very important question:
Does weak interaction violate CP?
$\varepsilon' \neq 0$ yes
$\varepsilon' = 0$ don't know

$'80s:$

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = (23.0 \pm 6.5) \times 10^{-4} (NA31) \qquad >3\sigma$$

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = (7.4 \pm 5.9) \times 10^{-4} (E731) \qquad \sim 1.5\sigma$$

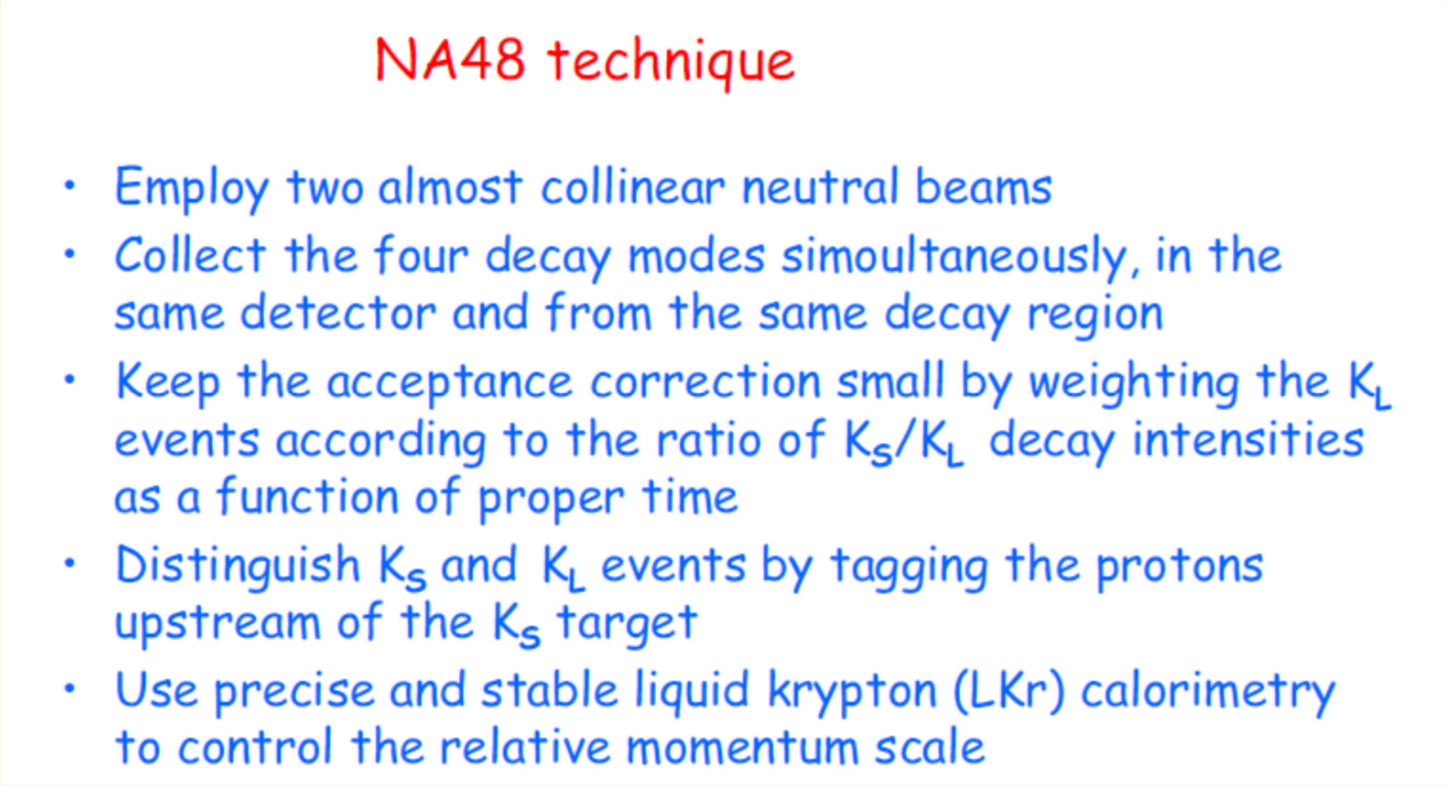
Mostly systematics

$'90s:$

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = (15.3 \pm 2.6) \times 10^{-4} (NA48) \qquad \sim 6\sigma$$

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = (20.7 \pm 2.8) \times 10^{-4} (KTEV) \qquad >7\sigma$$

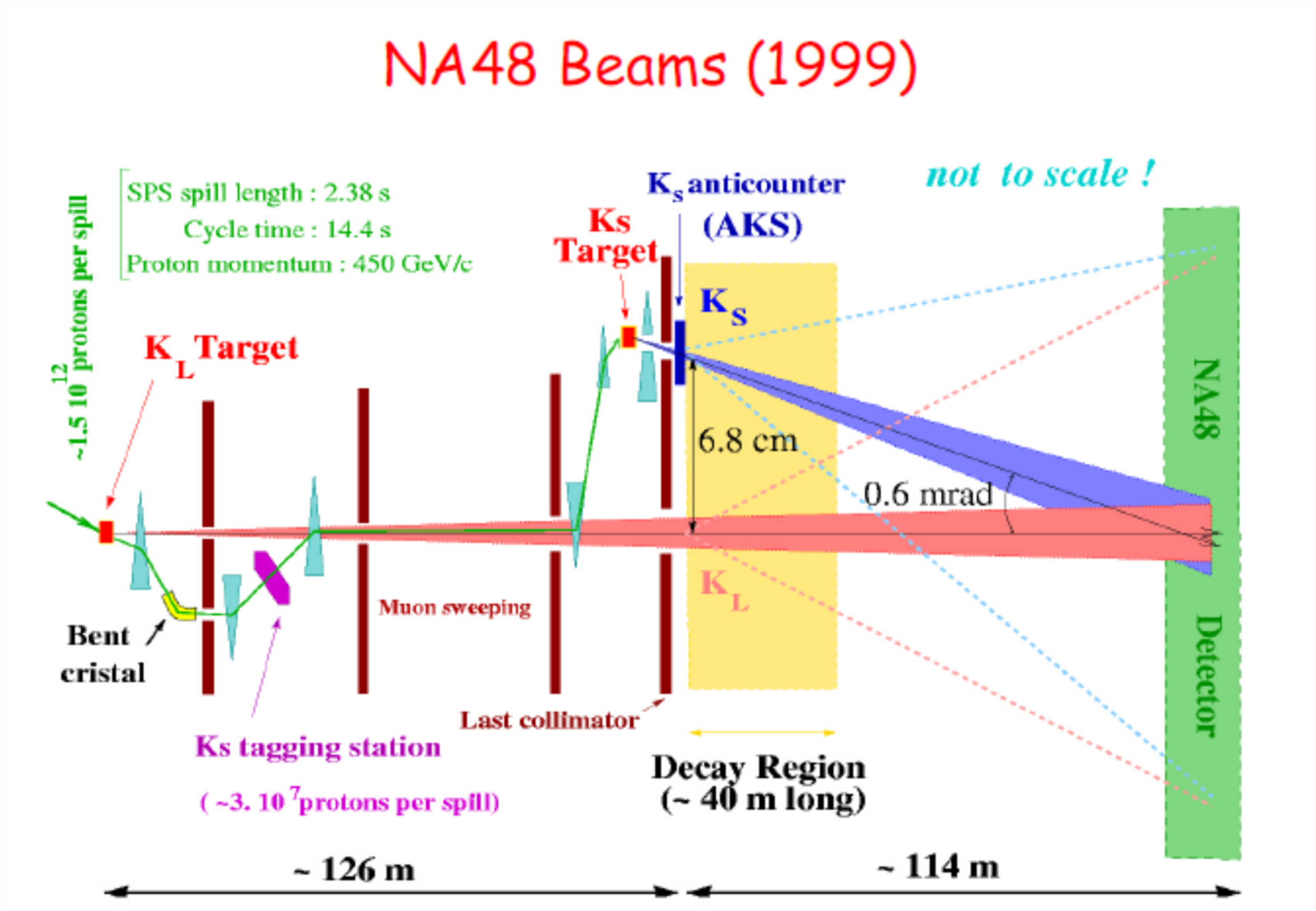
K Direct *CP* Violation - VII



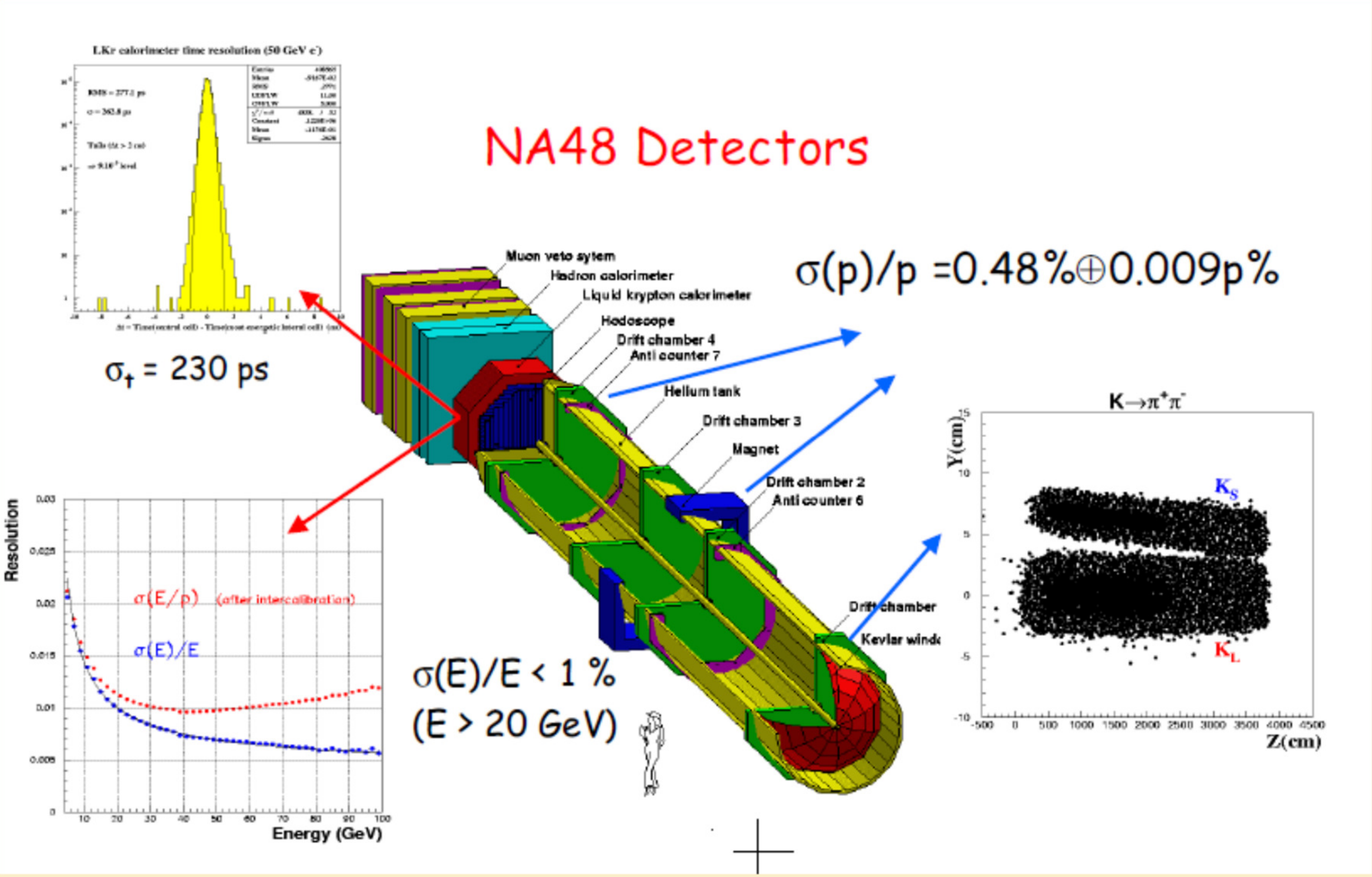
NA48 technique

- Employ two almost collinear neutral beams
- Collect the four decay modes simoultaneously, in the same detector and from the same decay region
- Keep the acceptance correction small by weighting the K_L events according to the ratio of K_S/K_L decay intensities as a function of proper time
- Distinguish K_S and K_L events by tagging the protons upstream of the K_S target
- Use precise and stable liquid krypton (LKr) calorimetry to control the relative momentum scale

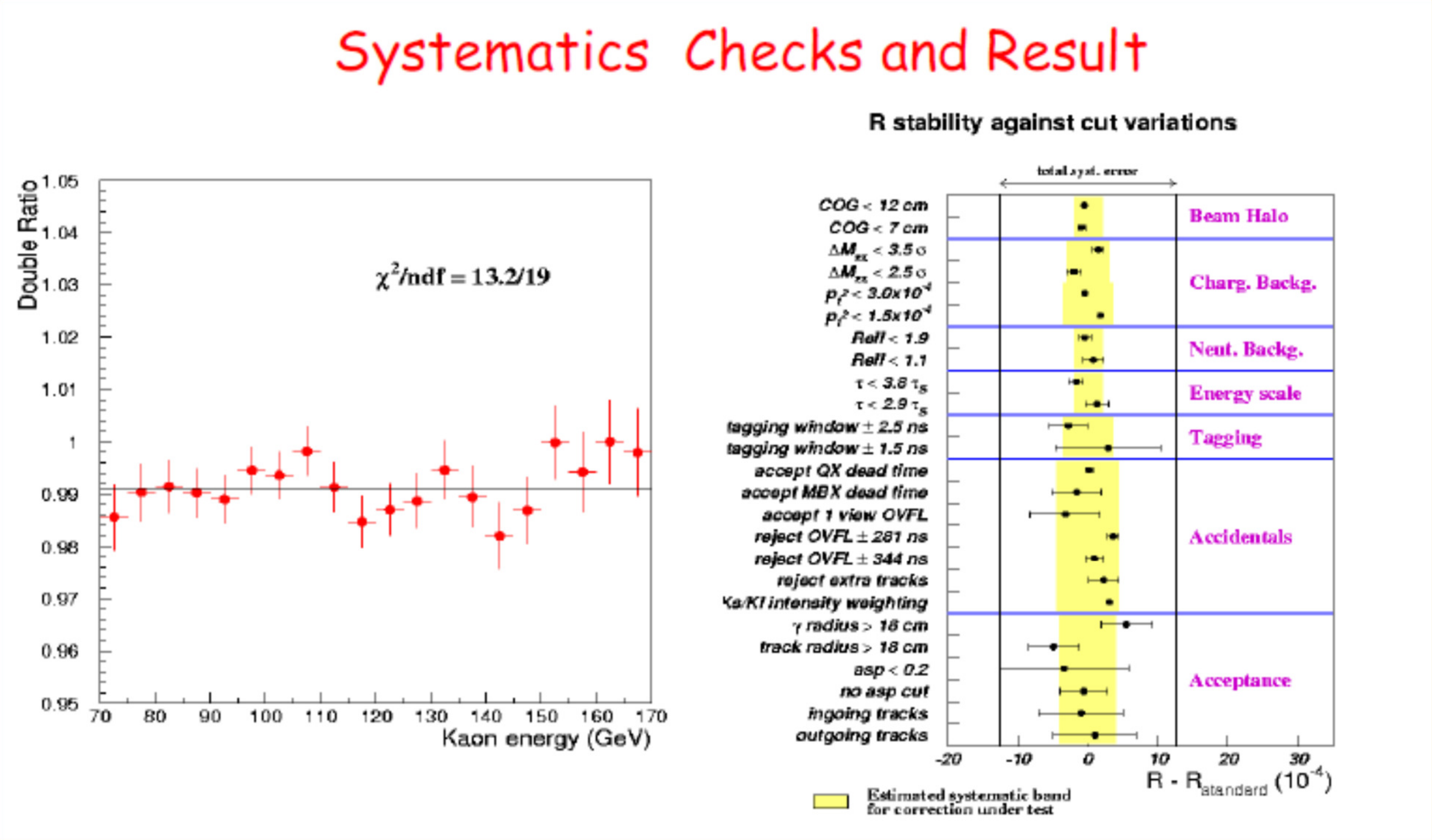
K Direct *CP* Violation - VIII



K Direct *CP* Violation - IX



K Direct *CP* Violation - X



Systematics Checks and Result

Mixing and Oscillations - I

Two-state system: Electron in a magnetic field $\boldsymbol{B}$ along $\hat{z}$

$$H = -\boldsymbol{\mu}\cdot\boldsymbol{B} = \frac{1}{2}a\boldsymbol{\sigma}\cdot\boldsymbol{B} \qquad \boldsymbol{B} = B\hat{k}, \quad \boldsymbol{\sigma} = \left(\sigma_x, \sigma_y, \sigma_z\right): \text{ Pauli spin matrices}$$

$$\rightarrow H = \frac{1}{2}a\sigma_z B = \begin{pmatrix} \frac{1}{2}aB & 0 \\ 0 & -\frac{1}{2}aB \end{pmatrix} = \begin{pmatrix} +E & 0 \\ 0 & -E \end{pmatrix}$$

2 state system: Choose as base states

$$|+\rangle \equiv \begin{pmatrix} 1 \\ 0 \end{pmatrix}, |-\rangle \equiv \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \text{Eigenstates of } \sigma_z \rightarrow \text{Generic state:} |\psi\rangle = \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} \rightarrow \ \sigma_z|\psi\rangle = \begin{pmatrix} \psi_+ \\ -\psi_- \end{pmatrix}$$

Schrodinger equation:

$$i\hbar\frac{\partial|\psi\rangle}{\partial t} = H|\psi\rangle \rightarrow |\psi(t)\rangle = e^{-i\frac{Ht}{\hbar}}|\psi\rangle = e^{-i\frac{aB}{2\hbar}t\sigma_z}|\psi\rangle \rightarrow i\hbar\frac{\partial}{\partial t}\begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} = \frac{1}{2}aB\sigma_z\begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} = \frac{1}{2}aB\begin{pmatrix} \psi_+ \\ -\psi_- \end{pmatrix}$$

Uncoupled equations:

$$\rightarrow \begin{cases} i\hbar\dfrac{\partial\psi_+}{\partial t} = \dfrac{1}{2}aB\psi_+ \\ i\hbar\dfrac{\partial\psi_-}{\partial t} = -\dfrac{1}{2}aB\psi_- \end{cases} \rightarrow \begin{cases} \psi_+(t) = A_+e^{-i\frac{aB}{2\hbar}t} \\ \psi_-(t) = A_-e^{+i\frac{aB}{2\hbar}t} \end{cases}, \quad |A_+|^2 + |A_-|^2 = 1 \rightarrow \begin{cases} |+,t\rangle = \begin{pmatrix} \psi_+(t) \\ 0 \end{pmatrix} \\ |-,t\rangle = \begin{pmatrix} 0 \\ \psi_-(t) \end{pmatrix} \end{cases} \text{ Stationary states}$$

Mixing and Oscillations - II

Introduce another $\boldsymbol{B}$ component along x:

$$\boldsymbol{B} = B\hat{\boldsymbol{k}} + B'\hat{\boldsymbol{i}}$$

$$H = -\boldsymbol{\mu}\cdot\boldsymbol{B} = \frac{1}{2}a\boldsymbol{\sigma}\cdot\boldsymbol{B} = \frac{1}{2}a\left(\sigma_z B + \sigma_x B'\right)$$

$$\to H = \begin{pmatrix} \frac{1}{2}aB & 0 \\ 0 & -\frac{1}{2}aB \end{pmatrix} + \begin{pmatrix} 0 & \frac{1}{2}aB' \\ \frac{1}{2}aB' & 0 \end{pmatrix} = \begin{pmatrix} +E & E' \\ E' & -E \end{pmatrix}$$

$$\to i\hbar\frac{\partial}{\partial t}\begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} = \frac{1}{2}a\left(B\sigma_z + B'\sigma_x\right)\begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} = \frac{1}{2}a\left[B\begin{pmatrix} \psi_+ \\ -\psi_- \end{pmatrix} + B'\begin{pmatrix} \psi_- \\ \psi_+ \end{pmatrix}\right]$$

Coupled equations:

$$\to \begin{cases} i\hbar\dfrac{\partial\psi_+}{\partial t} = \dfrac{1}{2}aB\psi_+ + aB'\psi_- \\ i\hbar\dfrac{\partial\psi_-}{\partial t} = -\dfrac{1}{2}aB\psi_- + aB'\psi_+ \end{cases} \to \begin{cases} \left|+,t\right\rangle \\ \left|-,t\right\rangle \end{cases} \text{Non -stationary states}$$

Mixing and Oscillations - III

Build a phenomenological framework suitable to describe flavor oscillations

Use symbol M^0 for neutral, flavored mesons: Most of the formalism suitable for K^0, D^0, B^0, B_S^0

Neutral meson time evolution: Two-state system

$$\left|M^0\right\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \left|\bar{M}^0\right\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$i\frac{\partial}{\partial t}\psi = H\psi \quad \text{Schrodinger equation}$$

$$\psi(t) = a(t)\left|M^0\right\rangle + b(t)\left|\bar{M}^0\right\rangle \equiv \begin{pmatrix} a(t) \\ b(t) \end{pmatrix} \quad \text{Two-component state vector}$$

Just free evolution for both components, no decay:

$$H = \underbrace{\begin{pmatrix} M & 0 \\ 0 & M \end{pmatrix}}_{\text{hermitian}} \quad \text{Effective Hamiltonian, } M = \text{mass}$$

Free evolution for both components, with decay:

$$H = \underbrace{\begin{pmatrix} M & 0 \\ 0 & M \end{pmatrix}}_{\text{hermitian}} - \frac{i}{2}\underbrace{\begin{pmatrix} \Gamma & 0 \\ 0 & \Gamma \end{pmatrix}}_{\text{hermitian}}, \ \Gamma = \text{total decay width}$$

Mixing and Oscillations - IV

Observe:

$$H^\dagger = \begin{pmatrix} M & 0 \\ 0 & M \end{pmatrix}^\dagger + \frac{i}{2}\begin{pmatrix} \Gamma & 0 \\ 0 & \Gamma \end{pmatrix}^\dagger = \begin{pmatrix} M & 0 \\ 0 & M \end{pmatrix} + \frac{i}{2}\begin{pmatrix} \Gamma & 0 \\ 0 & \Gamma \end{pmatrix} \neq H$$

$\rightarrow H$ non-Hermitian $\rightarrow e^{-iHt}$ non-unitary $\rightarrow$ State norm not conserved: Decreasing $\leftrightarrow$ $\Gamma>0$

$$H = \underbrace{\begin{pmatrix} M & A \\ B & M \end{pmatrix}}_{\text{hermitian}} - \frac{i}{2}\underbrace{\begin{pmatrix} \Gamma & C \\ D & \Gamma \end{pmatrix}}_{\text{hermitian}}$$

Include mixing

$$\begin{pmatrix} M & A \\ B & M \end{pmatrix}^\dagger = \begin{pmatrix} M & A \\ B & M \end{pmatrix} \rightarrow \begin{pmatrix} M & B^* \\ A^* & M \end{pmatrix} = \begin{pmatrix} M & A \\ B & M \end{pmatrix} \rightarrow A^* = B, \text{ same for } \Gamma$$

$$\rightarrow H = \underbrace{\begin{pmatrix} M & M_{12} \\ M_{12}^* & M \end{pmatrix}}_{\text{hermitian}} - \frac{i}{2}\underbrace{\begin{pmatrix} \Gamma & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma \end{pmatrix}}_{\text{hermitian}}$$

$$\rightarrow i\frac{\partial}{\partial t}\psi(t) = \begin{pmatrix} M - \frac{i}{2}\Gamma & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M - \frac{i}{2}\Gamma \end{pmatrix}\psi(t)$$

Mixing and Oscillations - V

Eigenvalues:

Define $F \equiv \operatorname{Re} F + i \operatorname{Im} F = \sqrt{\left(M_{12} - \frac{i}{2}\Gamma_{12}\right)\left(M_{12}^* - \frac{i}{2}\Gamma_{12}^*\right)}$

$$\begin{vmatrix} M - \frac{i}{2}\Gamma - \lambda & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M - \frac{i}{2}\Gamma - \lambda \end{vmatrix} = 0 \rightarrow \begin{array}{l} \lambda_1 \equiv m_1 - i\frac{\Gamma_1}{2} = M - i\frac{\Gamma}{2} - F \\ \lambda_2 \equiv m_2 - i\frac{\Gamma_2}{2} = M - i\frac{\Gamma}{2} + F \end{array}$$

$$\rightarrow \begin{cases} m_1 - i\frac{\Gamma_1}{2} = M - \operatorname{Re}(F) - i\left(\frac{\Gamma}{2} + \operatorname{Im}(F)\right) \\ m_2 - i\frac{\Gamma_2}{2} = M + \operatorname{Re}(F) - i\left(\frac{\Gamma}{2} - \operatorname{Im}(F)\right) \end{cases}$$

$$\rightarrow \begin{cases} \Delta m = m_2 - m_1 = 2\operatorname{Re}(F) = 2\operatorname{Re}\sqrt{\left(M_{12} - \frac{i}{2}\Gamma_{12}\right)\left(M_{12}^* - \frac{i}{2}\Gamma_{12}^*\right)} \\ \Delta\Gamma = \Gamma_2 - \Gamma_1 = 4\operatorname{Im}(F) = 4\operatorname{Im}\sqrt{\left(M_{12} - \frac{i}{2}\Gamma_{12}\right)\left(M_{12}^* - \frac{i}{2}\Gamma_{12}^*\right)} \end{cases}$$

Mixing and Oscillations - VI

Eigenvectors:

$$\begin{pmatrix} M-\frac{i}{2}\Gamma & M_{12}-\frac{i}{2}\Gamma_{12} \\ M_{12}^{*}-\frac{i}{2}\Gamma_{12}^{*} & M-\frac{i}{2}\Gamma \end{pmatrix}\begin{pmatrix} p \\ q \end{pmatrix}=\lambda_{1,2}\begin{pmatrix} p \\ q \end{pmatrix}\rightarrow\eta\equiv\frac{q}{p}=\sqrt{\frac{M_{12}^{*}-\frac{i}{2}\Gamma_{12}^{*}}{M_{12}-\frac{i}{2}\Gamma_{12}}},|q|^{2}+|p|^{2}=1$$

Mass eigenstates, similar to K_S, K_L:

Named "Heavy" and "Light" because for heavy quarks ≅ same lifetime

$$\begin{cases} |M_H\rangle = p|M^0\rangle + q|\overline{M^0}\rangle, & m_H = m_1, \Gamma_H = \Gamma_1 \\ |M_L\rangle = p|M^0\rangle - q|\overline{M^0}\rangle, & m_L = m_2, \Gamma_L = \Gamma_2 \end{cases}$$

Flavor eigenstates:

$$\begin{cases} |M^0\rangle = \frac{1}{2p}\left(|M_H\rangle + |M_L\rangle\right) \\ |\overline{M}^0\rangle = \frac{1}{2q}\left(|M_H\rangle - |M_L\rangle\right) \end{cases}$$

Mixing and Oscillations - VII

Define:

$$\omega_+ = m_H - i\frac{\Gamma_H}{2}, \omega_- = m_L - i\frac{\Gamma_L}{2}$$

Time evolution of mass eigenstates:

$$\begin{cases} \left|M_H(t)\right\rangle = e^{-i\omega_+ t}\left|M_H(0)\right\rangle \\ \left|M_L(t)\right\rangle = e^{-i\omega_- t}\left|M_L(0)\right\rangle \end{cases}$$

→ Straightforward free propagation & decay

Observe:

$\Gamma_H \cong \Gamma_L \rightarrow \tau_H \cong \tau_L$

Generally true for heavy mesons, due to a large number of decay modes

Mixing and Oscillations - VIII

Time evolution of flavor eigenstates:

Flavor oscillations

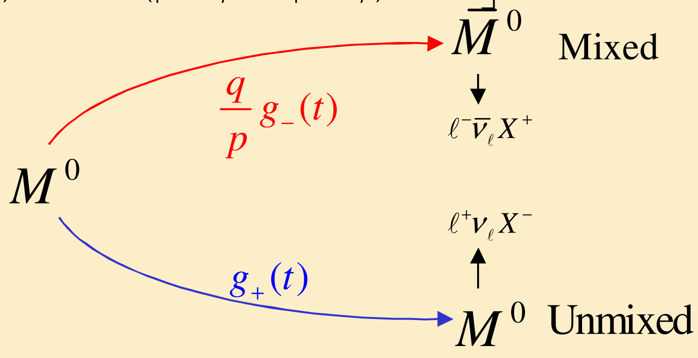
$$\left|M^0(t)\right\rangle = \frac{1}{2p}\left[\left|M_H\right\rangle e^{-i\omega_+ t} + \left|M_L\right\rangle e^{-i\omega_- t}\right] = \frac{1}{2p}\left[\left(\left|M^0\right\rangle + \eta\left|\bar{M}^0\right\rangle\right)e^{-i\omega_+ t} + \left(\left|M^0\right\rangle - \eta\left|\bar{M}^0\right\rangle\right)e^{-i\omega_- t}\right]$$

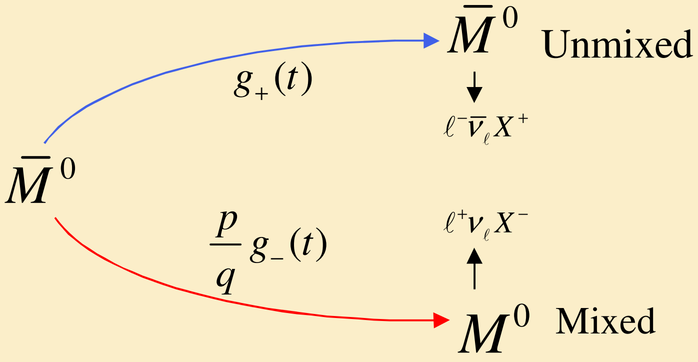
$$\left|M^0(t)\right\rangle = \frac{1}{2p}\left[\left|M^0\right\rangle \frac{e^{-i\omega_+ t} + e^{-i\omega_- t}}{2} + \eta\left|\bar{M}^0\right\rangle \frac{e^{-i\omega_+ t} - e^{-i\omega_- t}}{2}\right]$$

Define: $g_\pm(t) = \dfrac{e^{-i\omega_+ t} \pm e^{-i\omega_- t}}{2}$

$$\rightarrow \begin{cases} \left|M^0(t)\right\rangle \propto g_+(t)\left|M^0\right\rangle + \dfrac{q}{p} g_-(t)\left|\overline{M^0}\right\rangle \\ \left|\overline{M^0}(t)\right\rangle \propto g_+(t)\left|\overline{M^0}\right\rangle + \dfrac{p}{q} g_-(t)\left|M^0\right\rangle \end{cases}$$

$$\rightarrow \begin{cases} \left|M^0(t)\right\rangle = \dfrac{1}{\sqrt{1+|\eta|^2}}\left(g_+(t)\left|M^0\right\rangle + \eta g_-(t)\left|\overline{M^0}\right\rangle\right) \\ \left|\overline{M^0}(t)\right\rangle = \dfrac{1}{\sqrt{1+|\eta|^2}}\left(\eta g_+(t)\left|\overline{M^0}\right\rangle + g_-(t)\left|M^0\right\rangle\right) \end{cases}$$

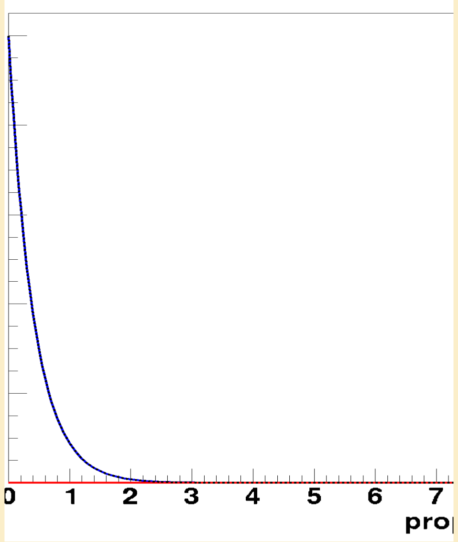




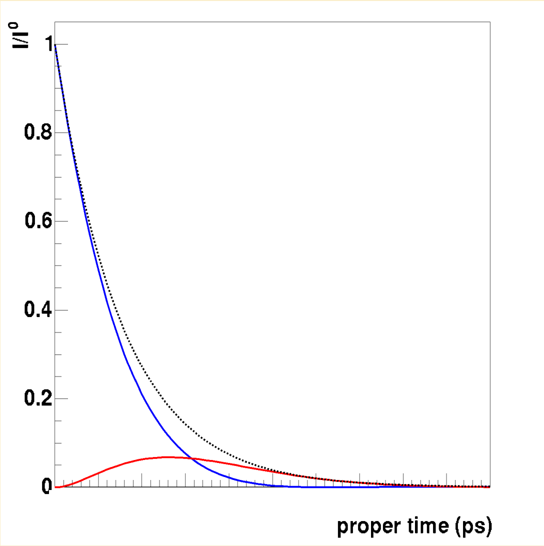
Mixing and Oscillations - IX

Compare different ratios

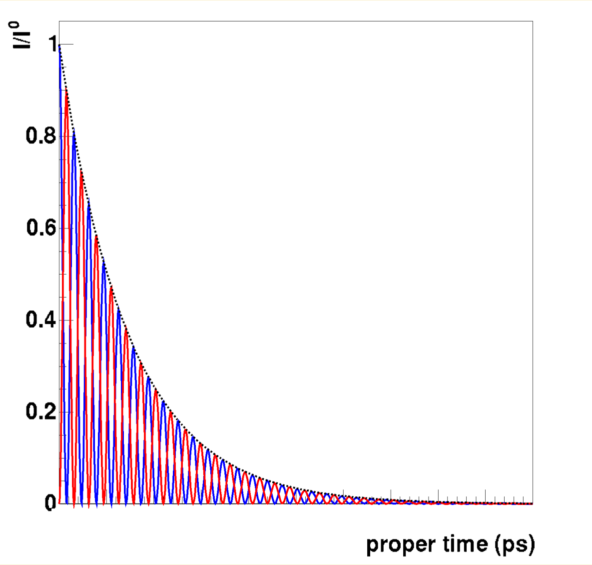
$$x = \frac{\Delta m}{\Gamma}$$



$$x \equiv \frac{\Delta m}{\Gamma} = 0$$



$$x \equiv \frac{\Delta m}{\Gamma} \approx 1$$



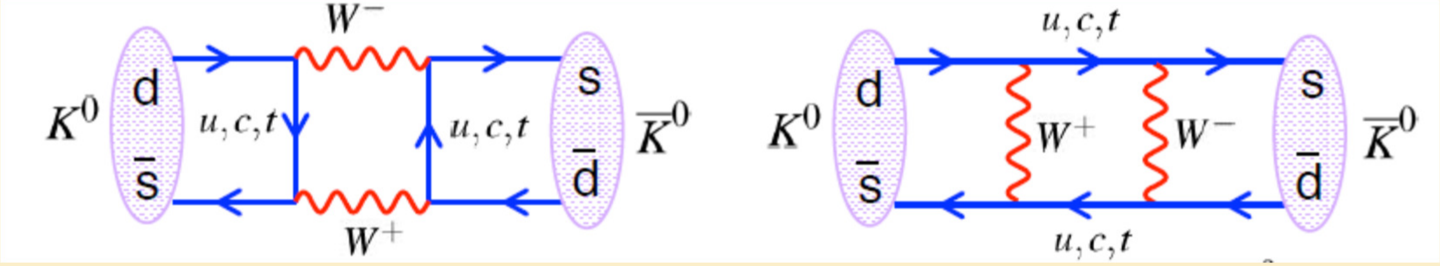
$$x \equiv \frac{\Delta m}{\Gamma} >> 1$$

CP Violation and SM - I

So far: Phenomenological description → ~ Just symmetries

Now: Try to connect to SM

K Mixing : *Box* diagrams



Mass difference between mass eigenstates:

$$\Delta m_K \approx \frac{G_F^2}{3\pi^2} f_K^2 m_K \left| V_{qd} V_{qs}^* V_{q'd} V_{q's}^* \right| m_q m_{q'}, \quad q,q' = u,c,t$$

CP Violation and SM - II

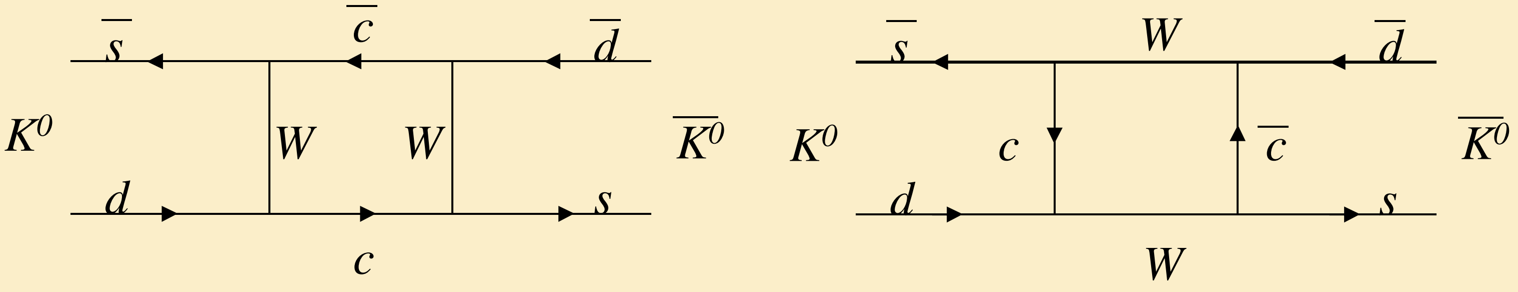
Go to *CKM*, find:

u,u	$\sin^2\theta_c \cos^2\theta_c m_u^2$	$\sim 0.048 m_u^2$	$\sim .005$
u,c	$\sin^2\theta_c \cos^2\theta_c m_u m_c$	$\sim 0.048 m_u m_c$	$\sim .022$
u,t	$\lvert V_{td}\rvert \lvert V_{ts}\rvert \sin\theta_c \cos\theta_c m_u m_t$	$\sim 0.220\ 410^{-5} m_u m_t$	$\sim .0005$
c,c	$\sin^2\theta_c \cos^2\theta_c m_c^2$	$\sim 0.048 m_c^2$	$\sim .095$
c,t	$\lvert V_{td}\rvert \lvert V_{ts}\rvert \sin\theta_c \cos\theta_c m_c m_t$	$\sim 0.220\ 410^{-5} m_c m_t$	$\sim .0021$
t,t	$\lvert V_{td}\rvert^2 \lvert V_{ts}\rvert^2 m_t^2$	$\sim 1.610^{-10} m_t^2$	~ 0

→ Diagrams with *c* quark : Most relevant

CP Violation and SM - III

Just for the fun: Oversimplify, take only charm contribution



$$A_{box} \propto \left(V_{cs}V_{cd}^{*}\right)^{2} m_c^2 \approx \left(\lambda^2 + i2A^2\lambda^6\eta\right) m_c^2$$

$$|\varepsilon| \approx \frac{\mathrm{Im}\left(A_{box}\right)}{\mathrm{Re}\left(A_{box}\right)} \approx \frac{2A^2\lambda^6\eta}{\lambda^2} = 2A^2\lambda^4\eta$$

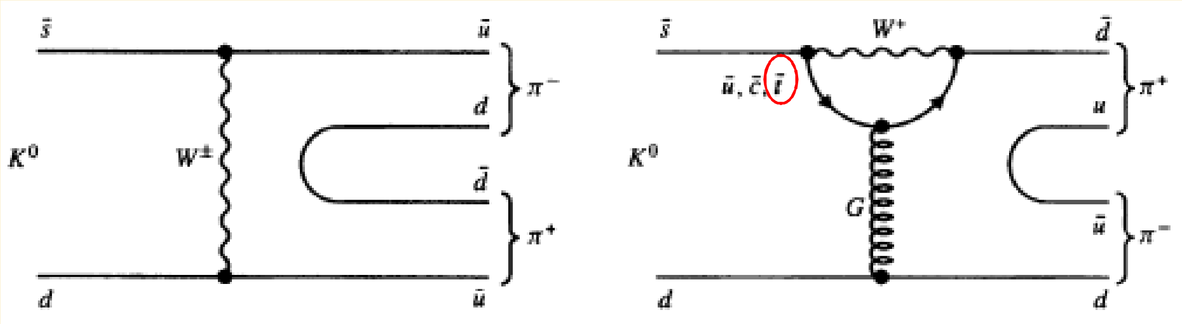
$$|\varepsilon| \sim 2\ 0.81\ 0.0025\ 0.343 \sim 1.410^{-3}$$

Not that bad...

CP Violation and SM - IV

Fun again: 2π decays and ε'
Must take into account two diagrams:

‘Tree’ ‘Penguin’



$$\propto \left|V_{us}V_{ud}\right|^2 \approx \lambda^2$$

Top dominating:

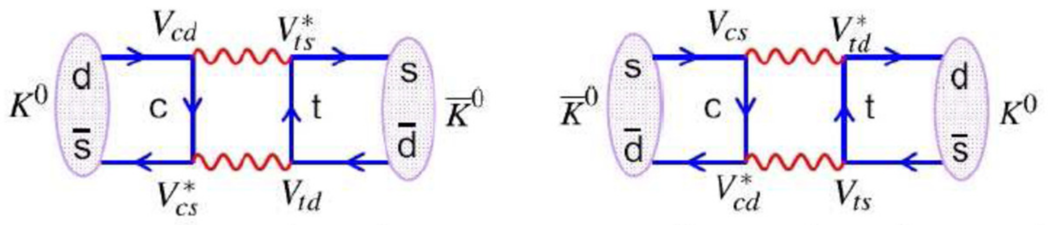
$$\propto \mathrm{Im}\left(V_{ts}V_{td}\right) \approx A^2\lambda^5\eta$$

$\varepsilon' \propto$ Interference between the two above $\sim \lambda^2 A^2 \lambda^5 \eta \sim \lambda^7$

$$\rightarrow \frac{\varepsilon'}{\varepsilon} \sim \frac{A^2\lambda^7\eta}{2A^2\lambda^4\eta} \sim \frac{\lambda^3}{2} \sim 5\,10^{-3}$$ Not that bad too...

CP Violation and SM - V

Reconsidering box diagrams:



$M_{fi} \propto A_{ct} V_{cd} V_{cs}^* V_{td} V_{ts}^*$ $\qquad$ $M'_{fi} \propto A_{ct} V_{cd}^* V_{cs} V_{td}^* V_{ts} = M_{fi}^*$

$$\Gamma(K^0_{t=0} \to \bar{K}^0) - \Gamma(\bar{K}^0_{t=0} \to K^0) \propto M_{fi} - M_{fi}^* = 2Im(M_{fi})$$

$$A_T = \frac{\Gamma(\overline{\mathrm{K}}^0 \to \mathrm{K}^0) - \Gamma(\mathrm{K}^0 \to \overline{\mathrm{K}}^0)}{\Gamma(\overline{\mathrm{K}}^0 \to \mathrm{K}^0) + \Gamma(\mathrm{K}^0 \to \overline{\mathrm{K}}^0)}.$$

Rememebering:

$A_T \approx 4\,\mathrm{Re}(\varepsilon) \to 4\,\mathrm{Re}(\varepsilon) \propto 2\,\mathrm{Im}\,M_{fi}$

→ No $\cancel{CP}$ from mixing unless some *CKM* elements are complex

CP Violation and SM - VI

Summary about neutral kaons:

Lifetime, width, mass:

$$m_1 \simeq m_2 \to \Delta m \simeq 4.1\,10^{-6}\,eV$$

$$\tau_1 \ll \tau_2 \sim 0.09 \;-\; 52 \;\; ns$$

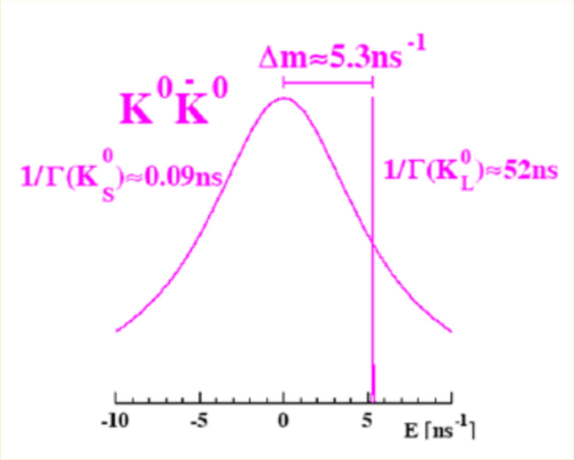
$$\Gamma_2 \ll \Gamma_1 \sim 11.1\,ns^{-1} \sim 0.038 \;\; 10^{-3} \;\; eV$$

$$\Gamma = \frac{\Gamma_1 + \Gamma_2}{2} \simeq \frac{\Gamma_1}{2}$$

$$x \equiv \frac{\Delta m}{\Gamma} \sim 0.5, \quad y \equiv \frac{\Delta\Gamma}{2\Gamma} \sim 0.5$$

$$\omega_+ = m_H - i\frac{\Gamma_H}{2}, \omega_- = m_L - i\frac{\Gamma_L}{2}$$

$$\to \begin{array}{l} g_+(t) = \dfrac{e^{-i\left(m_L+\Delta m - i\frac{\Gamma_H}{2}\right)t} + e^{-i\left(m_L - i\frac{\Gamma_L}{2}\right)t}}{2} = \dfrac{e^{-im_L t} e^{-\frac{\Gamma_L}{2}t}\left(e^{i\Delta m t} e^{-\frac{\Gamma_H - \Gamma_L}{2}t} + 1\right)}{2} \approx e^{-imt}\,\dfrac{e^{i\Delta m t} e^{-\frac{\Gamma}{2}t} + 1}{2} \\ g_-(t) = \dfrac{e^{-i\left(m_L+\Delta m - i\frac{\Gamma_H}{2}\right)t} - e^{-i\left(m_L - i\frac{\Gamma_L}{2}\right)t}}{2} = \dfrac{e^{-im_L t} e^{-\frac{\Gamma_L}{2}t}\left(e^{i\Delta m t} e^{-\frac{\Gamma_H - \Gamma_L}{2}t} - 1\right)}{2} \approx e^{-imt}\,\dfrac{e^{i\Delta m t} e^{-\frac{\Gamma}{2}t} - 1}{2} \end{array}$$



CP Violation and SM - VII

For K: $\Delta\Gamma = \Gamma_S - \Gamma_L \approx \Gamma_S, \left|\frac{q}{p}\right| = |\eta| \neq 1$

$$\begin{cases} \left|K^0(t)\right\rangle = g_+(t)\left|K^0\right\rangle + \eta g_-(t)\left|\overline{K^0}\right\rangle \approx e^{-imt}\left[\dfrac{e^{i\Delta mt}e^{-\frac{\Gamma}{2}t}+1}{2}\left|K^0\right\rangle + \eta\,\dfrac{e^{i\Delta mt}e^{-\frac{\Gamma}{2}t}-1}{2}\left|\overline{K^0}\right\rangle\right] \\ \left|\overline{K^0}(t)\right\rangle = g_+(t)\left|\overline{K^0}\right\rangle + \dfrac{1}{\eta}g_-(t)\left|K^0\right\rangle \approx e^{-imt}\left[\dfrac{e^{i\Delta mt}e^{-\frac{\Gamma}{2}t}+1}{2}\left|\overline{K^0}\right\rangle + \dfrac{1}{\eta}\,\dfrac{e^{i\Delta mt}e^{-\frac{\Gamma}{2}t}-1}{2}\left|K^0\right\rangle\right] \end{cases}$$

$$P_{K^0}\left(K^0,t\right) = \left|g_+(t)\right|^2 = g_+g_+^* \approx \frac{e^{i\Delta mt}e^{-\frac{\Gamma}{2}t}+1}{2}\,\frac{e^{-i\Delta mt}e^{-\frac{\Gamma}{2}t}+1}{2} = \frac{1}{4}\left(1+e^{-\frac{\Gamma}{2}t}\left(1+2\cos\Delta mt\right)\right)$$

$$P_{K^0}\left(\overline{K}^0,t\right) \approx \frac{e^{i\Delta mt}e^{-\frac{\Gamma}{2}t}-1}{2}\,\frac{e^{-i\Delta mt}e^{-\frac{\Gamma}{2}t}-1}{2} = \frac{1}{4}|\eta|^2\left(1+e^{-\frac{\Gamma}{2}t}\left(1-2\cos\Delta mt\right)\right)$$

$$P_{\overline{K}^0}\left(K^0,t\right) \approx \frac{1}{4|\eta|^2}\left(1+e^{-\frac{\Gamma}{2}t}\left(1-2\cos\Delta mt\right)\right)$$

$$P_{\overline{K}^0}\left(\overline{K}^0,t\right) \approx \frac{1}{4}\left(1+e^{-\frac{\Gamma}{2}t}\left(1+2\cos\Delta mt\right)\right)$$

CP Violation and SM - VIII

Transition probabilities

$K^0 \to K^0 \,/\, \bar{K}^0 \to \bar{K}^0$:

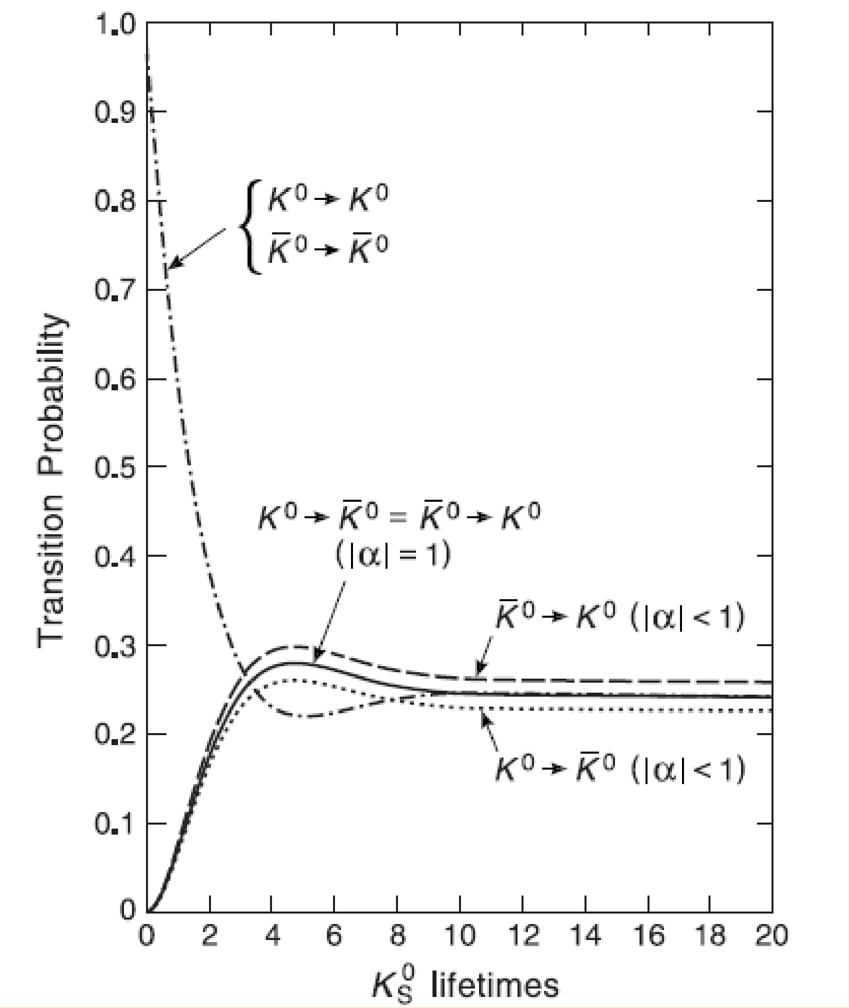
Identical (CPT)

$K^0 \to \bar{K}^0 \,/\, \bar{K}^0 \to K^0$:

Identical if $|\eta| = 1 \to$ No $\cancel{CP}$ in mixing: Full line

Different if $|\eta| \neq 1$: Dashed + Point lines

As shown prediction for $1-|\eta| = 10 \times$ Exp. value



CP Violation and SM - IX

Rationale:

$\not{CP}$ observed in neutral kaon decays

Ascribed to mixing, decay, or both

Accounted for by a *single* complex phase in *CKM*

→ Expect $\not{CP}$ to occur in other neutral, flavored meson decays

→ Heavy quarks involved

Looking again at unitarity triangles:

$$(1)\quad V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0; \qquad (2)\quad V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0;$$

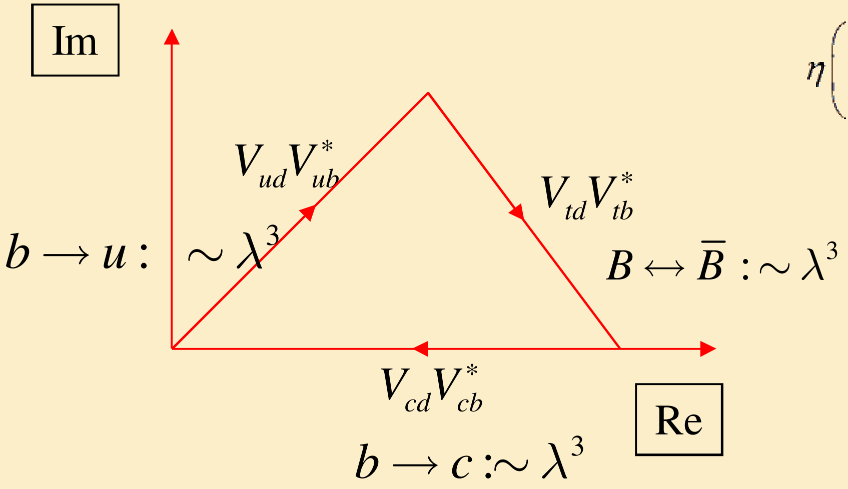
$$(3)\quad V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0; \qquad (4)\quad V_{ud}^*V_{cd} + V_{us}^*V_{cs} + V_{ub}^*V_{cb} = 0;$$

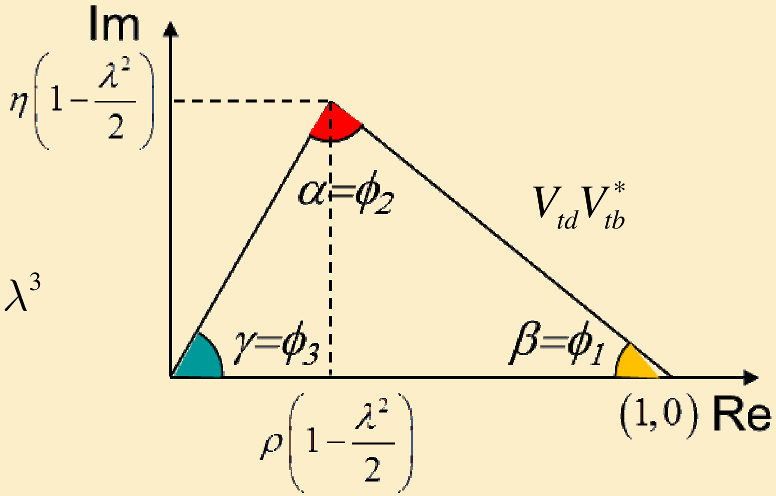
$$(5)\quad V_{ud}^*V_{td} + V_{us}^*V_{ts} + V_{ub}^*V_{tb} = 0; \qquad (6)\quad V_{cd}^*V_{td} + V_{cs}^*V_{ts} + V_{cb}^*V_{tb} = 0$$

Not all equally useful: *Shape, Easy to measure*

CP Violation and SM - X

$$V_{ud}V_{ub}^{*}+V_{cd}V_{cb}^{*}+V_{td}V_{tb}^{*}=0$$





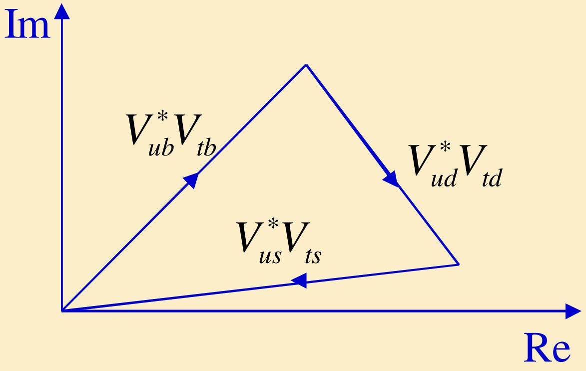
The unitarity triangle: Somewhat ‘equilateral’→Large angles

CP Violation and SM - XI

$$V_{ud}^* V_{td} + V_{us}^* V_{ts} + V_{ub}^* V_{tb} = \delta_{tu} = 0 \quad \hat{=} \quad tu \ \text{triangle}$$

Another $\approx$ equilateral one

Each side $\propto \lambda^3$



CP Violation and SM - XII

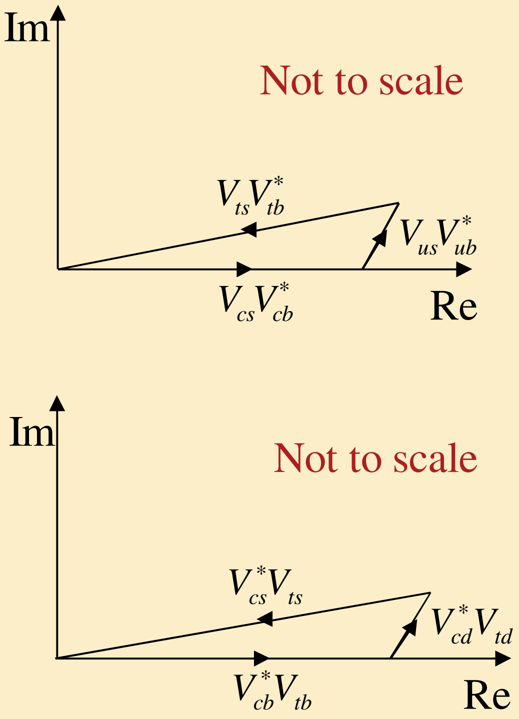
Two ‘squashed’ triangles…

2 sides $\propto \lambda^2$

1 side $\propto \lambda^4$

$$V_{us}^*V_{ub} + V_{cs}^*V_{cb} + V_{ts}^*V_{tb} = \delta_{bs} = 0 \quad \hat{=} \quad bs\ \text{triangle}$$

$$V_{td}^*V_{cd} + V_{ts}^*V_{cs} + V_{tb}^*V_{cb} = \delta_{tc} = 0 \quad \hat{=} \quad tc\ \text{triangle}$$



Not to scale

Not to scale

Difficult to use to test $\cancel{CP}$

$\cancel{CP} \propto$ Height with base normalized to 1

CP Violation and SM - XIII

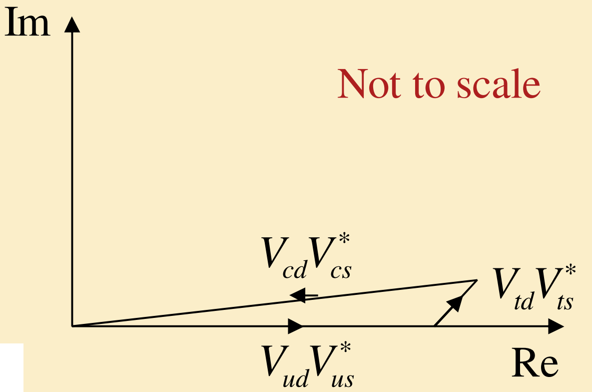
…and two even more squashed

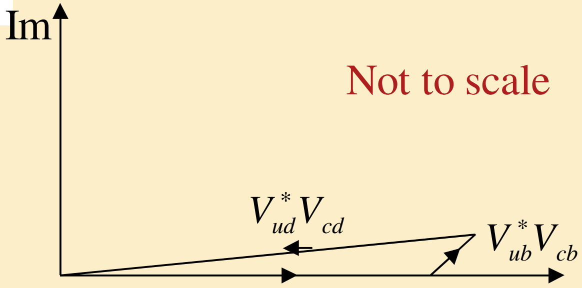
2 sides $\propto \lambda$

1 side $\propto \lambda^5$

$$V_{ud}^* V_{us} + V_{cd}^* V_{cs} + V_{td}^* V_{ts} = \delta_{sd} = 0 \quad \hat{=} \quad sd \text{ triangle}$$

$$V_{ud}^* V_{cd} + V_{us}^* V_{cs} + V_{ub}^* V_{cb} = \delta_{cu} = 0 \quad \hat{=} \quad cu \text{ triangle}$$





CP Violation and SM - XIV

Neutral, flavored mesons: Lightest states

$K^0 : d\overline{s}$

$\overline{K}^0 : \overline{d}s$

$D^0 : c\overline{u}$

$\overline{D}^0 : \overline{c}u$

$B^0 : \overline{b}d$

$\overline{B}^0 : b\overline{d}$

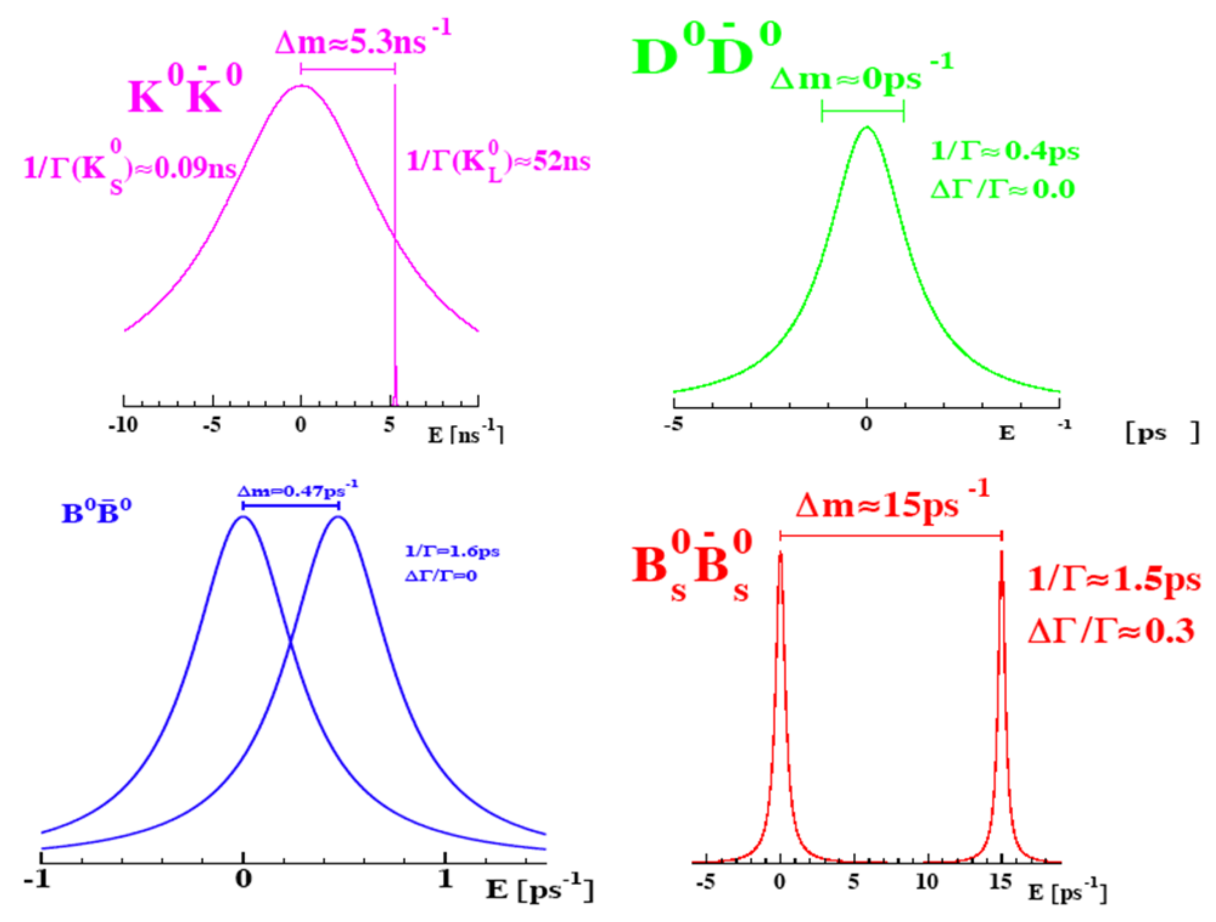
$B_S^0 : \overline{b}s$

$\overline{B}_S^0 : b\overline{s}$

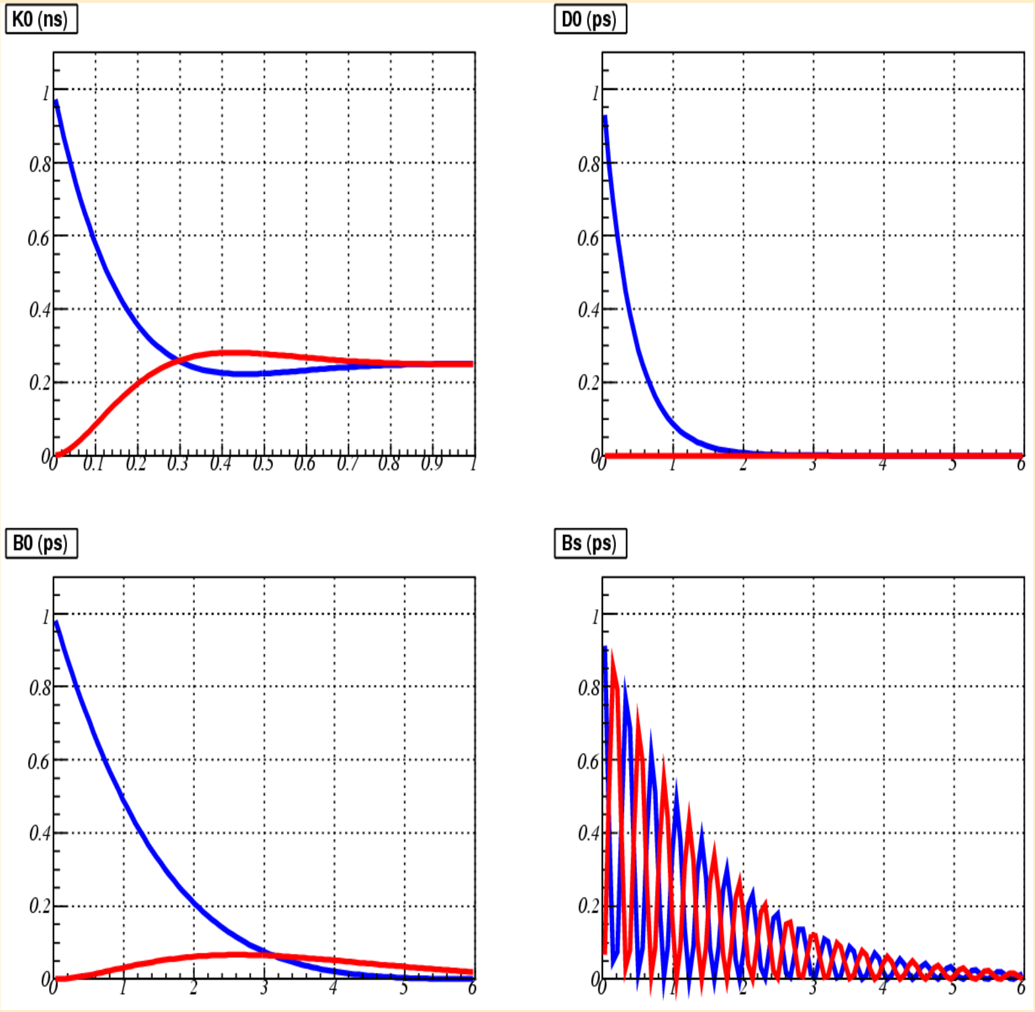
	$<\tau>$	Δm	$x=\Delta m/\Gamma$	$y=\Delta\Gamma/2\Gamma$
K^0	$2.6\ 10^{-8}$ s	$5.29\ ns^{-1}$	$\Delta m/\Gamma_S=0.49$	~1
D^0	$0.41\ 10^{-12}$ s	$0.001\ fs^{-1}$	~0	0.01
B^0	$1.53\ 10^{-12}$ s	$0.507\ ps^{-1}$	0.78	~0
B_s^0	$1.47\ 10^{-12}$ s	$17.8\ ps^{-1}$	12.1	~0.05

CP Violation and SM - XV

Lineshapes of mass eigenstates of neutral, flavored meson systems

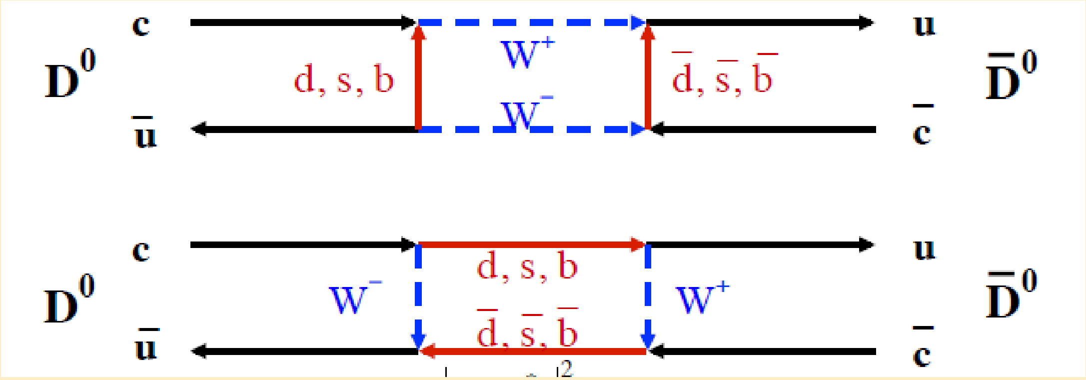


CP Violation and SM - XVI



CP Violation and SM - XVII

Extend box diagrams to other neutral systems: $D^0 - \bar{D}^0$



b loop: Strong CKM suppression

$$M \propto \left|V_{ub}V_{cb}^*\right|^2 \ll 1$$

Indeed, go to Wolfenstein parametrization:

$$\left|V_{ub}V_{cb}^*\right|^2 \sim \left|\lambda^3\lambda^2\right|^2 \sim 10^{-7}$$

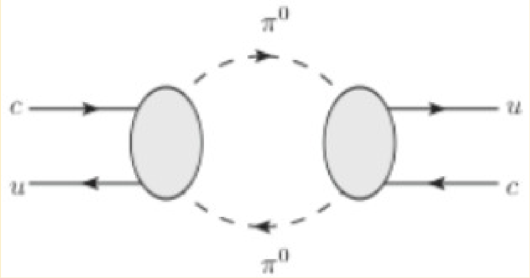
s, d loops: Strong GIM suppression

$M \propto \left(m_s^2 - m_d^2\right)$ small!

→ Expect very small mixing

CP Violation and SM - XVIII

Long distance effects (← Meson exchange, rather than quarks) important



Lifetime, width, mass: Very different from K^0, difficult to compute

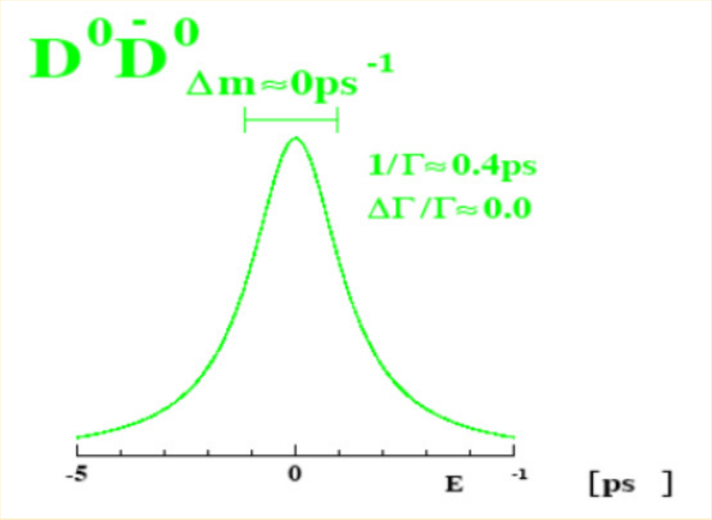
$$m_1 \simeq m_2$$

$$\tau_1 \simeq \tau_2 = (4.15 \pm 0.04) 10^{-13}\ s$$

$$\Gamma_2 \simeq \Gamma_1 = (1.59 \pm 0.01) 10^{-12}\ GeV$$

$$\Gamma = \frac{\Gamma_1 + \Gamma_2}{2} \simeq \Gamma_2 \simeq \Gamma_1$$

$$\left.\begin{aligned} x &\equiv \frac{\Delta m}{\Gamma} = \frac{m_2 - m_1}{\Gamma} \\ y &\equiv \frac{\Delta\Gamma}{2\Gamma} = \frac{\Gamma_2 - \Gamma_1}{2\Gamma} \end{aligned}\right\} \text{Estimate } x,\ \ y \sim 10^{-4} - 10^{-3}$$



CP Violation and SM - XIX

$\sim$ Same mass: Oscillation frequency small

$\sim$ Same (small) lifetime : $D_H,\ D_L$ cannot be physically separated (Compare to $K_S\ /\ K_L$..)

→ Only chance to observe mixing by time integrated measurement

Tag D flavor at both production and decay

Production: Take strong decays

$D^{*+} \rightarrow D^0\ \pi^+, \bar{D}^{*-} \rightarrow \bar{D}^0\ \pi^-$

Decay: Take two modes

$D^0 \rightarrow K^+\mu^-\bar{\nu}_\mu$ forbidden, only accessed by mixing $D^0 \rightarrow \bar{D}^0$

$D^0 \rightarrow K^-\mu^+\nu_\mu$ allowed

$$\rightarrow R = \frac{N\left(K^+\mu^-\bar{\nu}_\mu\right)}{N\left(K^-\mu^+\nu_\mu\right)} \cong \frac{x^2+y^2}{2}$$

Measurement difficult, large samples required

Mixing & $\cancel{CP}$ observed since 2007 by BaBar, Belle, LHCb

CP Violation and SM - XX

Most promising sector for validation of $CKM : B^0 - \overline{B}^0$

'Large' $\cancel{CP}$ expected

Similar to $K^0 - \overline{K}^0$, but:

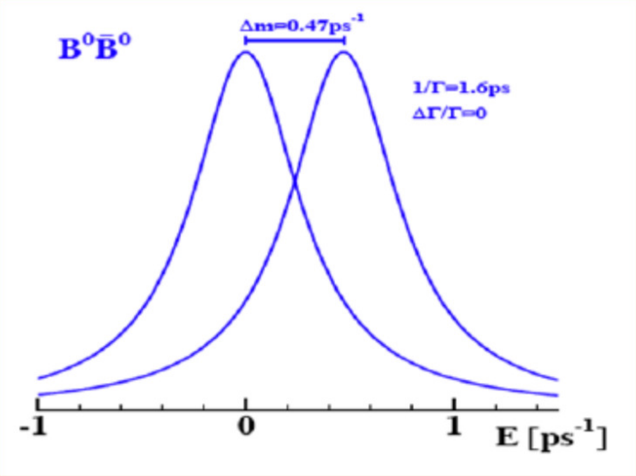
$\Delta M_B = 0.489 \pm 0.008 \; ps^{-1}$

$\rightarrow \Delta M_B \sim 0.489 \; 10^{12} s^{-1}$

$\hbar \approx \; 6.582 \; 10^{-16} eV \;\; s \rightarrow \Delta M_B \sim 3.22 \; 10^{-4} eV \sim 100 \Delta M_K$

$\tau_{B_1} \approx \tau_{B_2} = 1.56 \pm 0.06 \;\; ps$

$\rightarrow \dfrac{\Delta M_B}{\Gamma_B} \sim 1$



Compare to K:

$\Delta M_K = 5.29 \; ns^{-1} \sim 5.29 \; 10^9 s^{-1} \;\; 6.582 \; 10^{-16} eV \;\; s \sim 3.4 \; 10^{-6} eV$

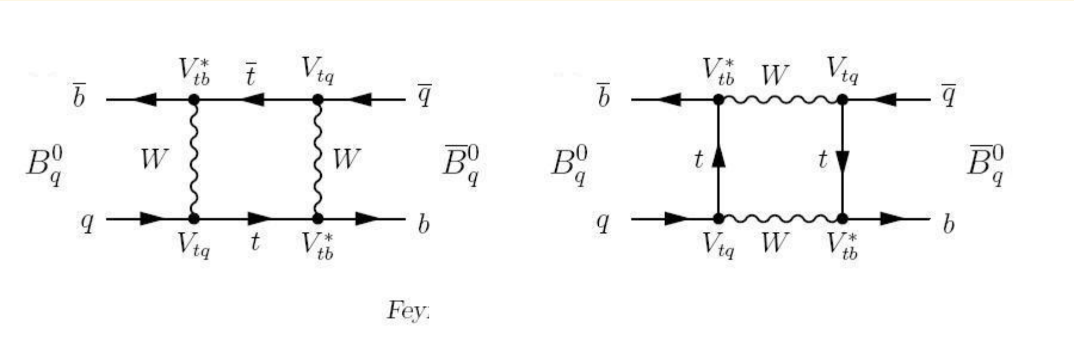
$\rightarrow \dfrac{\Delta M_K}{\Gamma_{K_S}} \sim 1$

$\tau_L \sim 600 \; \tau_S \sim 600 \; \cdot \; 89 \; ps$

B_H, B_L states cannot be physically separated

B Mixing: CP - I

Box diagrams, *t* dominated for B^0



$$\begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ \boxed{A\lambda^3(1-\rho-i\eta)} & -A\lambda^2 & \boxed{1} \end{pmatrix}$$

Mixing parameter:

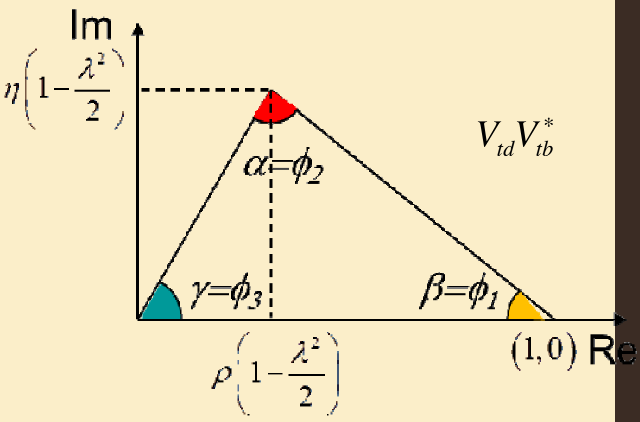
$$\eta = \frac{\left(V^*_{tb}V_{td}\right)}{\left(V_{tb}V^*_{td}\right)} \approx \frac{V_{td}}{V^*_{td}} = e^{-2i\varphi_{td}}$$

From UT:

$$\varphi_{td} = \beta$$

$$\rightarrow \eta = e^{-2i\beta}$$

$$\rightarrow |\eta| \cong 1$$



B Mixing: CP - II

Mass eigenstates:

$$\begin{cases} \left|B_H\right\rangle = \dfrac{1}{\sqrt{1+\left|\eta\right|^2}}\left(\left|B^0\right\rangle + \eta\left|\overline{B^0}\right\rangle\right) \\ \left|B_L\right\rangle = \dfrac{1}{\sqrt{1+\left|\eta\right|^2}}\left(\left|B^0\right\rangle - \eta\left|\overline{B^0}\right\rangle\right) \end{cases}, \quad \begin{cases} \left|B^0\right\rangle = \dfrac{1}{2}\sqrt{1+\left|\eta\right|^2}\left(\left|B_H\right\rangle + \left|B_L\right\rangle\right) \\ \left|\overline{B}^0\right\rangle = \dfrac{1}{2\eta}\sqrt{1+\left|\eta\right|^2}\left(\left|B_H\right\rangle - \left|B_L\right\rangle\right) \end{cases}$$

$\rightarrow \eta = \dfrac{1-\varepsilon_B}{1+\varepsilon_B}$, ε_B analog to ε used for kaons

Time evolution of mass eigenstates:

$$\rightarrow \begin{cases} \left|B_H\left(t\right)\right\rangle = \left|B_H\right\rangle e^{-i\left(m_H - i\frac{\Gamma_H}{2}\right)t} \\ \left|B_L\left(t\right)\right\rangle = \left|B_L\right\rangle e^{-i\left(m_L - i\frac{\Gamma_L}{2}\right)t} \end{cases}, \quad \Gamma_H \square \Gamma_L = \Gamma, \ \Delta m \square \ M$$

B Mixing: CP - III

Time evolution of flavor eigenstates:

$$\begin{cases} \left|B^0(t)\right\rangle = \left|B^0\right\rangle f_+(t) + \eta\left|\overline{B^0}\right\rangle f_-(t) \\ \left|\overline{B}^0(t)\right\rangle = \left|B^0\right\rangle \dfrac{1}{\eta} f_-(t) + \left|\overline{B^0}\right\rangle f_+(t) \end{cases}$$

$$f_+(t) = \frac{e^{-i\left(M+\frac{\Delta m}{2}-i\frac{\Gamma_H}{2}\right)t} + e^{-i\left(M-\frac{\Delta m}{2}-i\frac{\Gamma_L}{2}\right)t}}{2}, \; f_-(t) = \frac{e^{-i\left(M+\frac{\Delta m}{2}-i\frac{\Gamma_H}{2}\right)t} - e^{-i\left(M-\frac{\Delta m}{2}-i\frac{\Gamma_L}{2}\right)t}}{2}$$

$$f_\pm(t) \approx \frac{1}{2} e^{-\frac{1}{2}\Gamma t} e^{-iMt}\left[e^{-i\frac{\Delta m}{2}t} \pm e^{+i\frac{\Delta m}{2}t}\right], \;\; M = \frac{m_H + m_L}{2}, \;\; \Delta m > 0, \;\; \Gamma_H \simeq \Gamma_L = \Gamma$$

$$\rightarrow \begin{cases} f_+(t) \approx \dfrac{1}{2} e^{-\frac{1}{2}\Gamma t} e^{-iMt} \cos\left(\dfrac{\Delta m}{2} t\right) \\ f_-(t) \approx -i\dfrac{1}{2} e^{-\frac{1}{2}\Gamma t} e^{-iMt} \sin\left(\dfrac{\Delta m}{2} t\right) \end{cases}$$

$$\rightarrow \begin{cases} \left|B^0(t)\right\rangle = e^{-iMt} e^{-\frac{1}{2}\Gamma t}\left[\cos\left(\dfrac{\Delta m}{2} t\right)\left|B_0\right\rangle - i\eta \sin\left(\dfrac{\Delta m}{2} t\right)\left|\overline{B_0}\right\rangle\right] \\ \left|\overline{B}^0(t)\right\rangle = e^{-iMt} e^{-\frac{1}{2}\Gamma t}\left[-\dfrac{i}{\eta}\sin\left(\dfrac{\Delta m}{2} t\right)\left|B_0\right\rangle + \cos\left(\dfrac{\Delta m}{2} t\right)\left|\overline{B_0}\right\rangle\right] \end{cases}$$

B Mixing: CP - IV

→ Expect:

$$\Gamma\left(B^0(t=0)\to B^0\right)=e^{-\Gamma t}\cos^2\left(\frac{\Delta m}{2}t\right)$$

$$\Gamma\left(B^0(t=0)\to \overline{B}^0\right)=\left|\eta\right|^2 e^{-\Gamma t}\sin^2\left(\frac{\Delta m}{2}t\right)$$

$$\Gamma\left(\overline{B}^0(t=0)\to \overline{B}^0\right)=e^{-\Gamma t}\cos^2\left(\frac{\Delta m}{2}t\right)$$

$$\Gamma\left(\overline{B}^0(t=0)\to \overline{B}^0\right)=\left|\frac{1}{\eta}\right|^2 e^{-\Gamma t}\sin^2\left(\frac{\Delta m}{2}t\right)$$

Similar for B_s^0 :

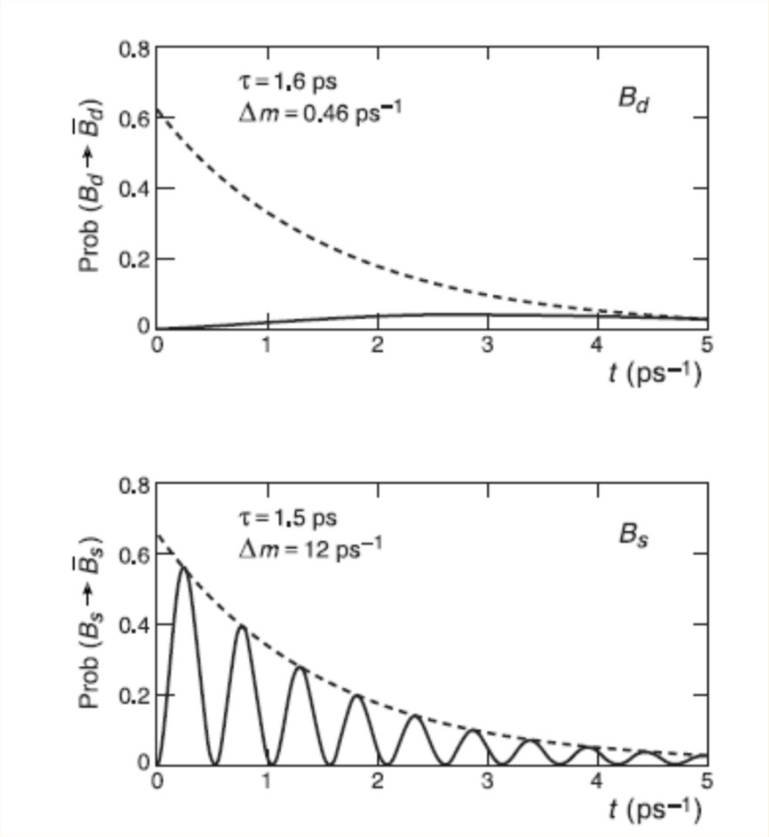
Important difference $\Delta m \gg \Gamma$

→ Many oscillations in a lifetime

B Mixing: CP - V

Unlike $K^0 / \bar{K}^0$:

$|\eta| \simeq 1 \rightarrow$ $\sim$ No $\cancel{CP}$ effect observable by looking at flavor oscillations



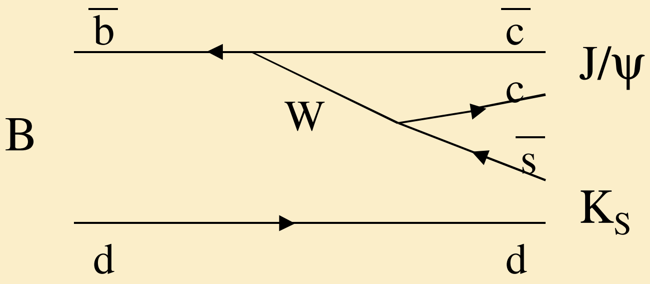
B Mixing: CP - VI

By restricting to decays to CP eigenstates: OK!

Main disadvantage: Statistics $\left(\text{Tiny BR}\right)$

Golden final state: $J/\psi K_S^0$ (or K_L^0: Experimentally less attractive)

$B^0, \overline{B}^0 \to J/\psi K_S^0$



Angular momentum balance:

$0 = 1 \oplus 0 \oplus L \to L = 1 \quad \text{Pure } P\text{-wave}$

$CP\left(J/\psi\right) = \left(-1\right)\left(-1\right) = +1$

$CP\left(K_S^0\right) = +1,\ \text{neglect } \cancel{CP} \text{ in } K^0$

$P_{orb} = \left(-1\right)^L = -1$

$\to CP\left(J/\psi K_S^0\right) = -1$

$\to CP\left(J/\psi K_L^0\right) = +1$

B Mixing: CP - VII

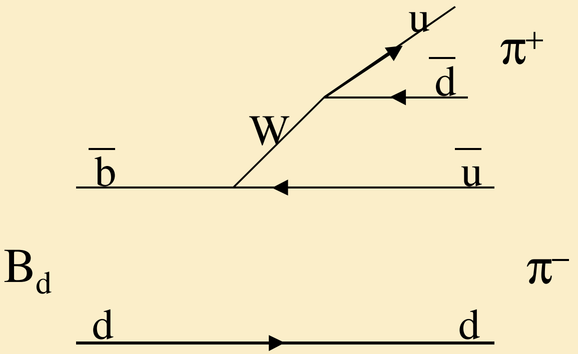
Another golden : $\pi\pi$

$B^0, \overline{B}^0 \to \pi\pi$

Angular momentum balance:

$L = 0$ Pure S - wave

$CP(\pi\pi) = +1$



B Mixing: CP - VIII

Taking decays into (golden) *CP* eigenstates:

$$A\left(B^0\to J/\psi K_S^0\right)=\left\langle J/\psi K_S^0\middle|H_{eff}\middle|B^0(t)\right\rangle$$

$$\to A\left(B^0\to J/\psi K_S^0\right)=f_+(t)\left\langle J/\psi K_S^0\middle|H_{eff}\middle|B^0\right\rangle+\eta f_-(t)\left\langle J/\psi K_S^0\middle|H_{eff}\middle|\overline{B}^0\right\rangle$$

$$\to A\left(B^0\to J/\psi K_S^0\right)=\left\langle J/\psi K_S^0\middle|H_{eff}\middle|B^0\right\rangle\left[f_+(t)+\eta f_-(t)\frac{\left\langle J/\psi K_S^0\middle|H_{eff}\middle|\overline{B}^0\right\rangle}{\left\langle J/\psi K_S^0\middle|H_{eff}\middle|B^0\right\rangle}\right]$$

Considering $B^0,\overline{B}^0$ decay: Must occur in two steps

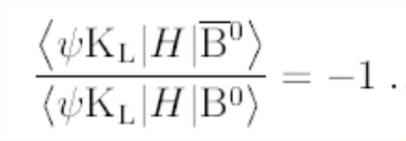
$B^0\to J/\psi\ K^0\to J/\psi\ K_S^0$ $\qquad$ $\overline{B}^0\to J/\psi\ \overline{K}^0\to J/\psi\ K_S^0$

because at the quark level:

$\overline{b}\to\overline{c}\ c\overline{s}$ $\qquad$ $b\to c\ \overline{c}s$

$A\left(B^0\to J/\psi K^0\right)\propto V_{cb}^*V_{cs}$ $\qquad$ $A\left(\overline{B}^0\to J/\psi\overline{K}^0\right)\propto V_{cb}V_{cs}^*$

$$\to\frac{\left\langle J/\psi K_S^0\middle|H_{eff}\middle|\overline{B}^0\right\rangle}{\left\langle J/\psi K_S^0\middle|H_{eff}\middle|B^0\right\rangle}=+1$$ *CKM* elements involved *real*



$$\frac{\langle\psi\mathrm{K_L}|H|\overline{\mathrm{B}}^0\rangle}{\langle\psi\mathrm{K_L}|H|\mathrm{B}^0\rangle}=-1\,.$$

B Mixing: CP - IX

$$\Gamma\left(B^0(t=0)\to J/\psi K_S\right)\propto\left|f_+(t)+\eta f_-(t)\right|^2$$

$$\to\Gamma\left(B^0(t=0)\to J/\psi K_S\right)\propto e^{-\Gamma t}\left|\cos\left(\frac{\Delta m}{2}t\right)-ie^{-2i\beta}\sin\left(\frac{\Delta m}{2}t\right)\right|^2$$

$$\to\Gamma\left(B^0(t=0)\to J/\psi K_S\right)\propto e^{-\Gamma t}\left(1-\sin\Delta mt\sin 2\beta\right)$$

$$\to\Gamma\left(\overline{B}^0(t=0)\to J/\psi K_S\right)\propto e^{-\Gamma t}\left(1+\sin\Delta mt\sin 2\beta\right)$$

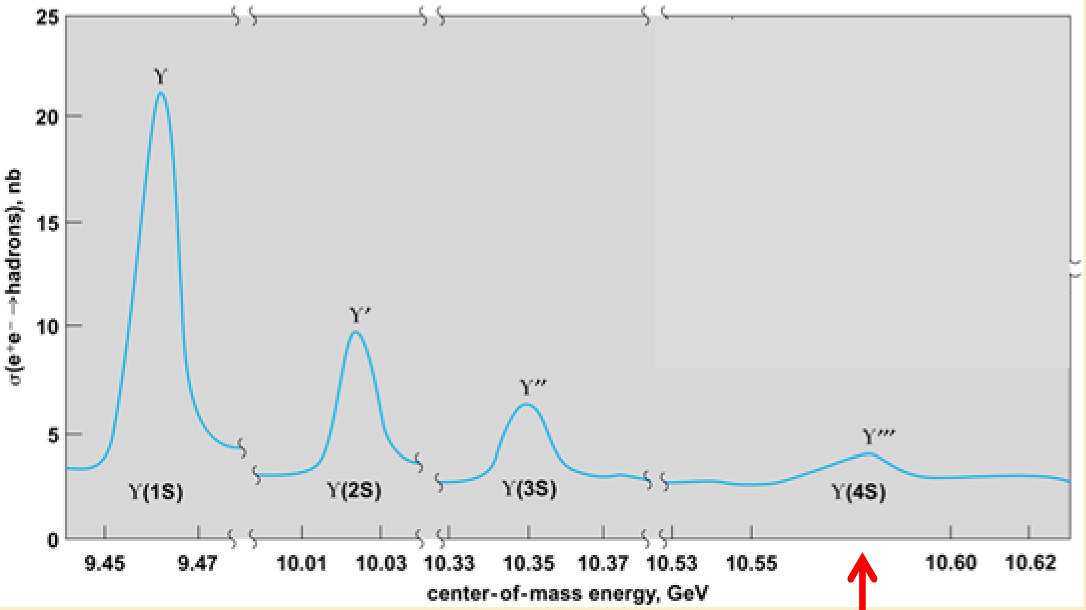
Time dependent asymmetry:

$$A_{J/\psi K_S}=\frac{\Gamma\left(B^0(t=0)\to J/\psi K_S\right)-\Gamma\left(\overline{B}^0(t=0)\to J/\psi K_S\right)}{\Gamma\left(B^0(t=0)\to J/\psi K_S\right)+\Gamma\left(\overline{B}^0(t=0)\to J/\psi K_S\right)}=\sin\Delta mt\sin 2\beta$$

$$A_{J/\psi K_L}=-\sin\Delta mt\sin 2\beta$$

B Factories - I

Total e^+e^- annihilation cross section:

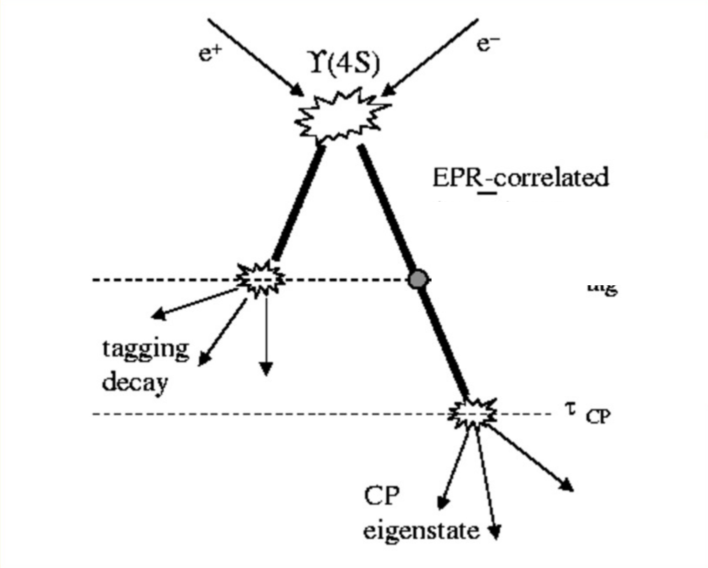


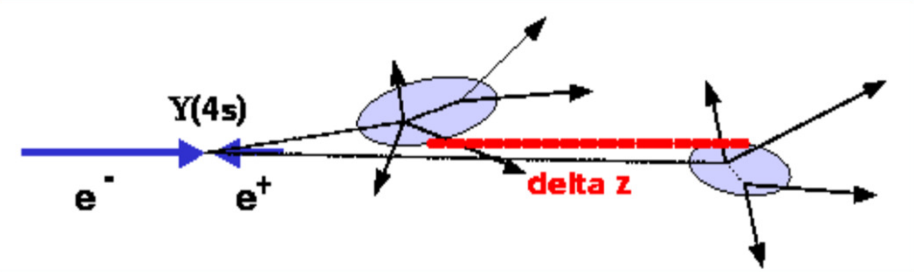
Main decay mode for $\Upsilon(4S)$:

$\Upsilon(4S) \to B\bar{B}$, including $B^0\bar{B}^0$

B Factories - II

Basic idea to measure time dependent asymmetry:





Measure time difference between 'tag' meson and 'CP' meson decays:

Use space distance between vertexes

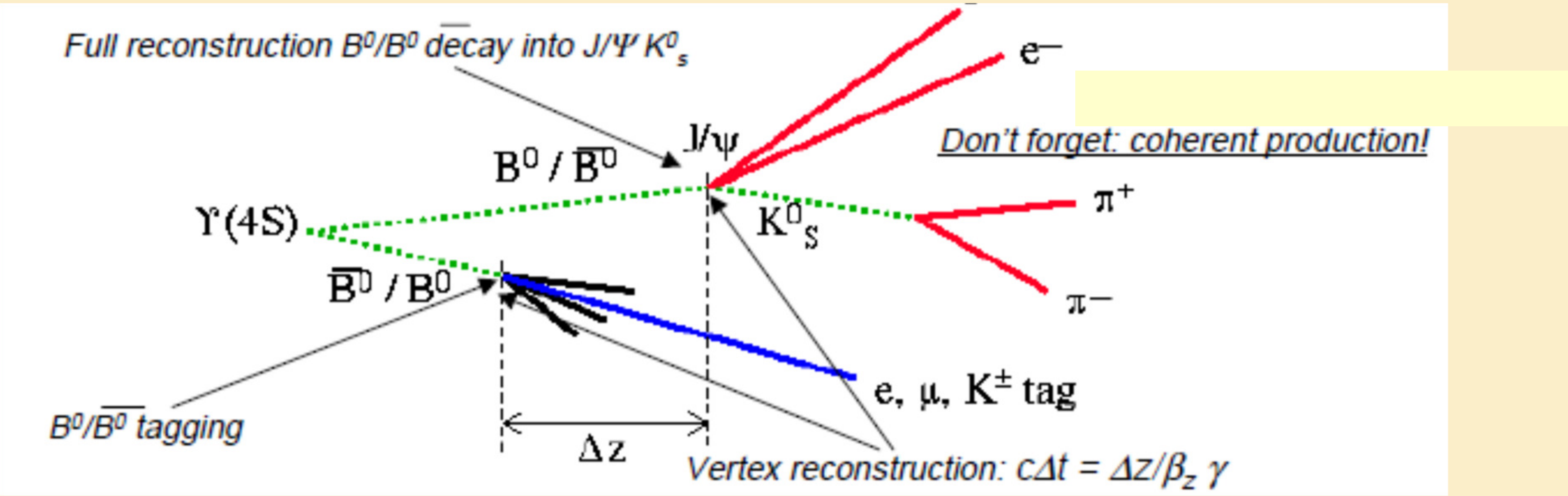
Measurement difficult in CM, due to short lifetime ($d \sim 30\ \mu m$)

→ Boost mesons in lab by making collider *asymmetric*

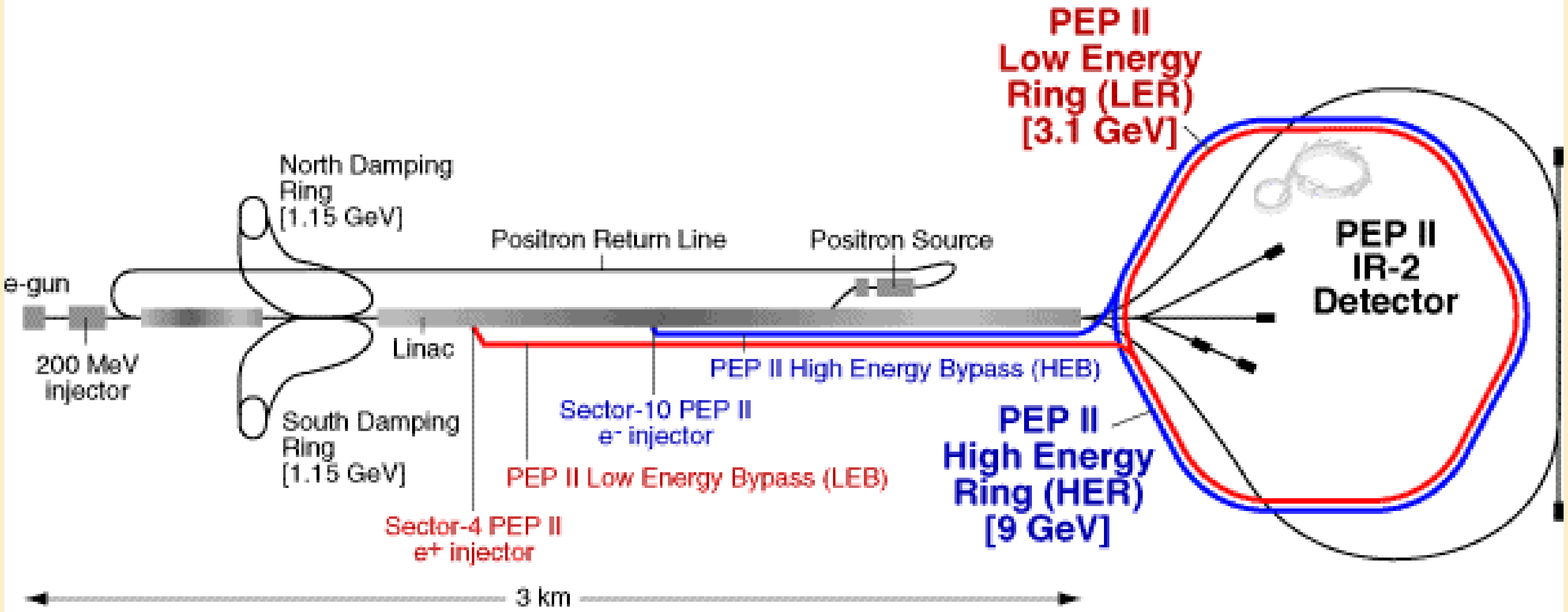
→ $\Upsilon(4S)$ moving in the lab system

B Factories - III

Example:

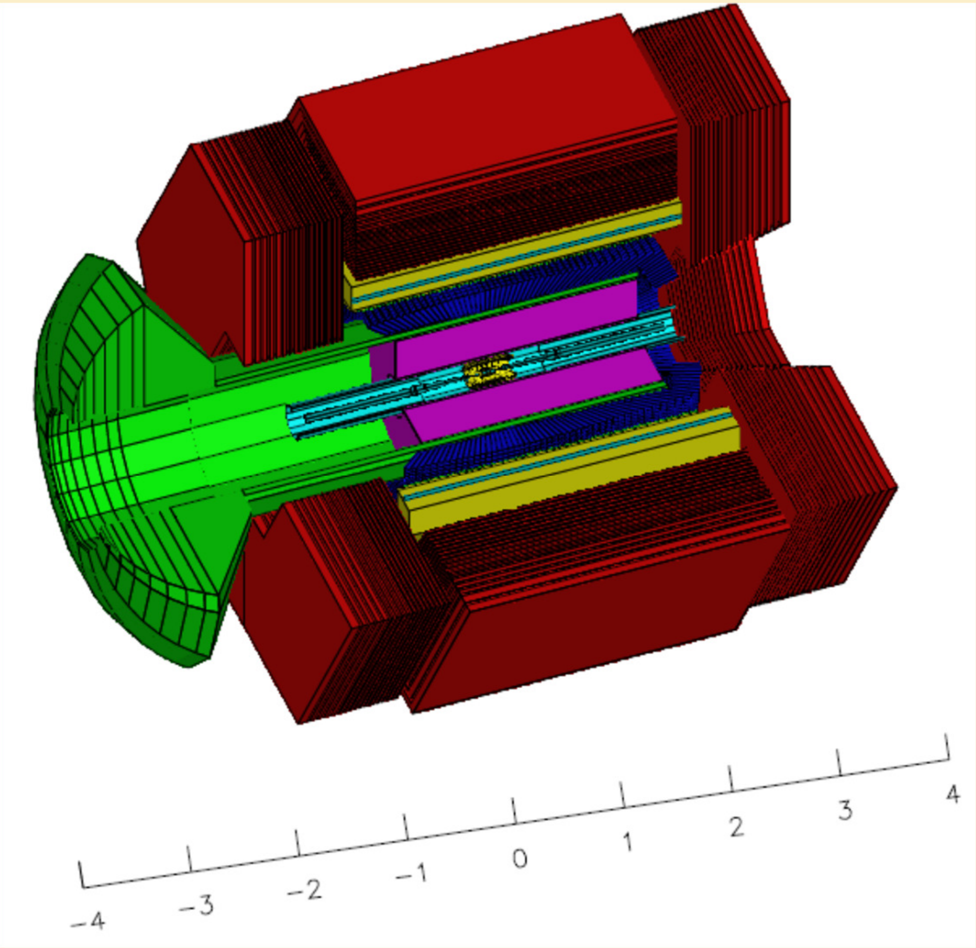


B Factories - IV

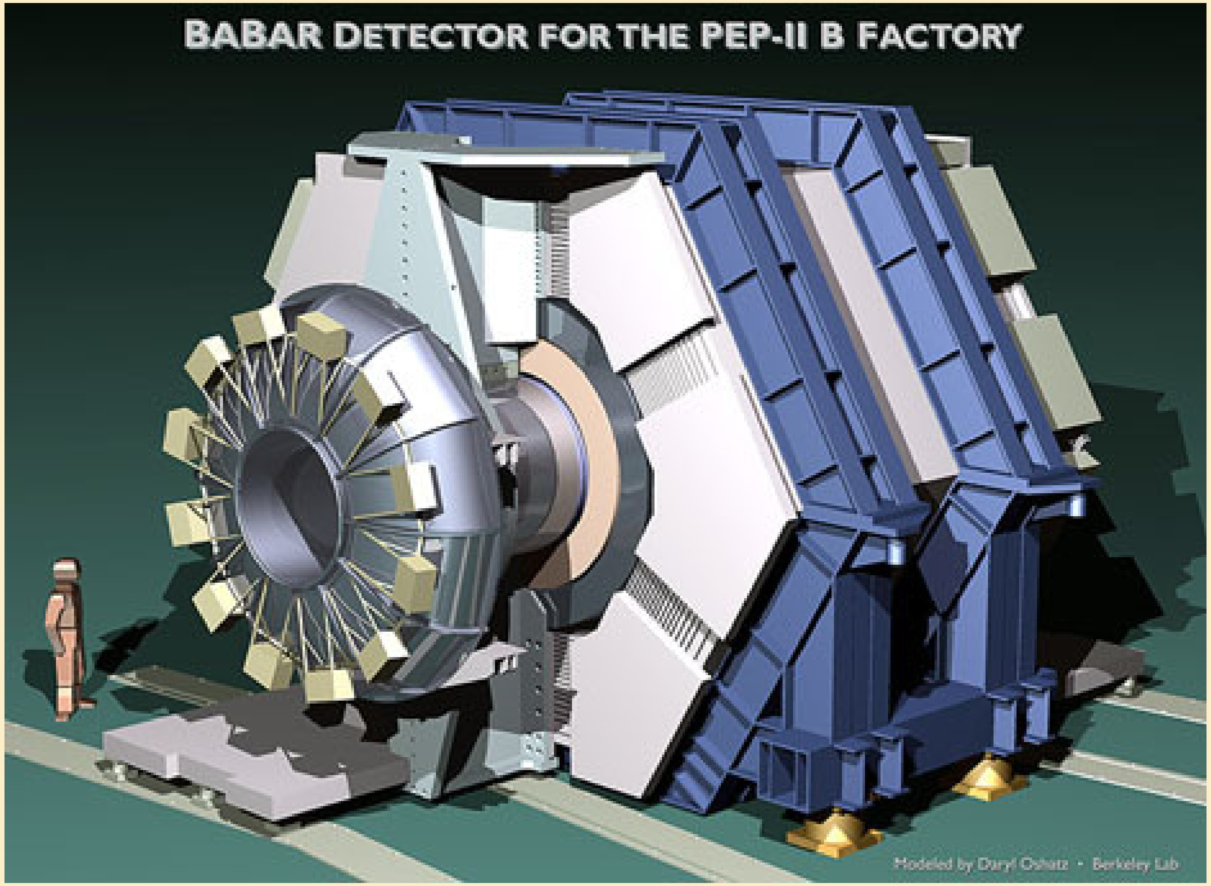


B Factories - V

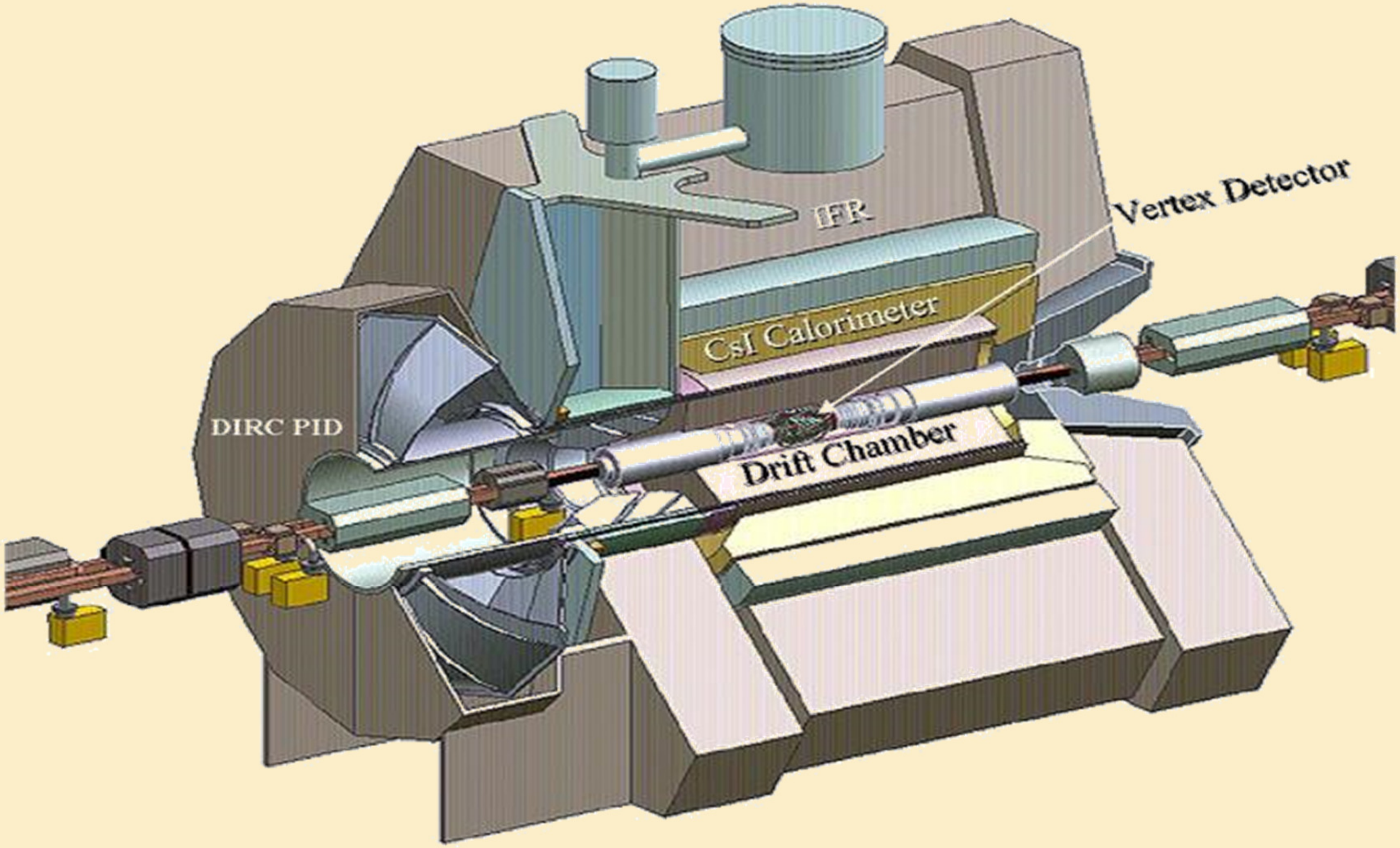
BABAR



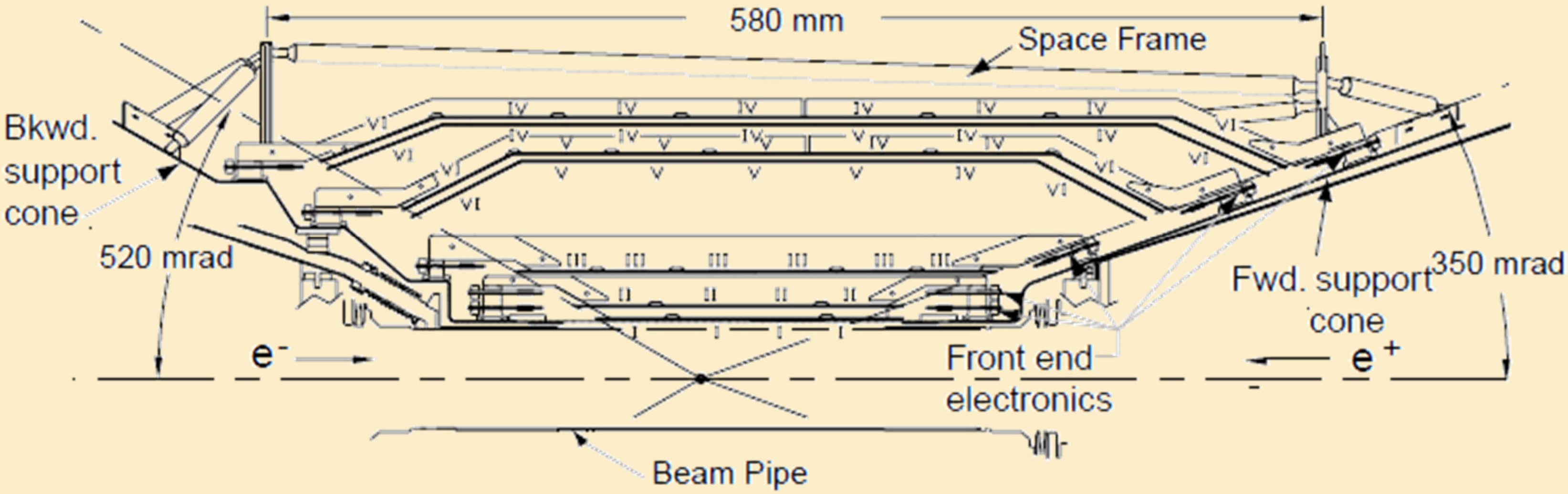
B Factories - VI



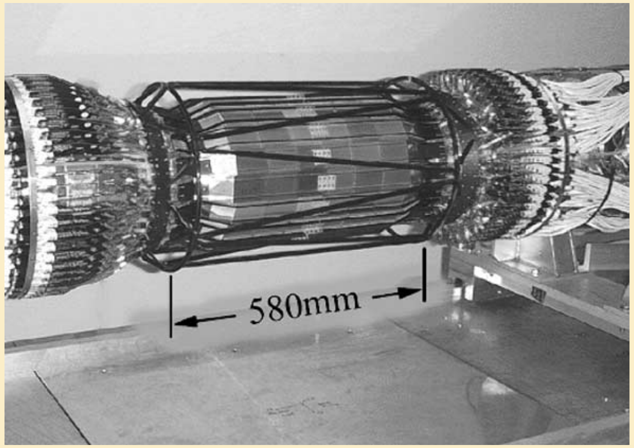
B Factories - VII



B Factories - VIII

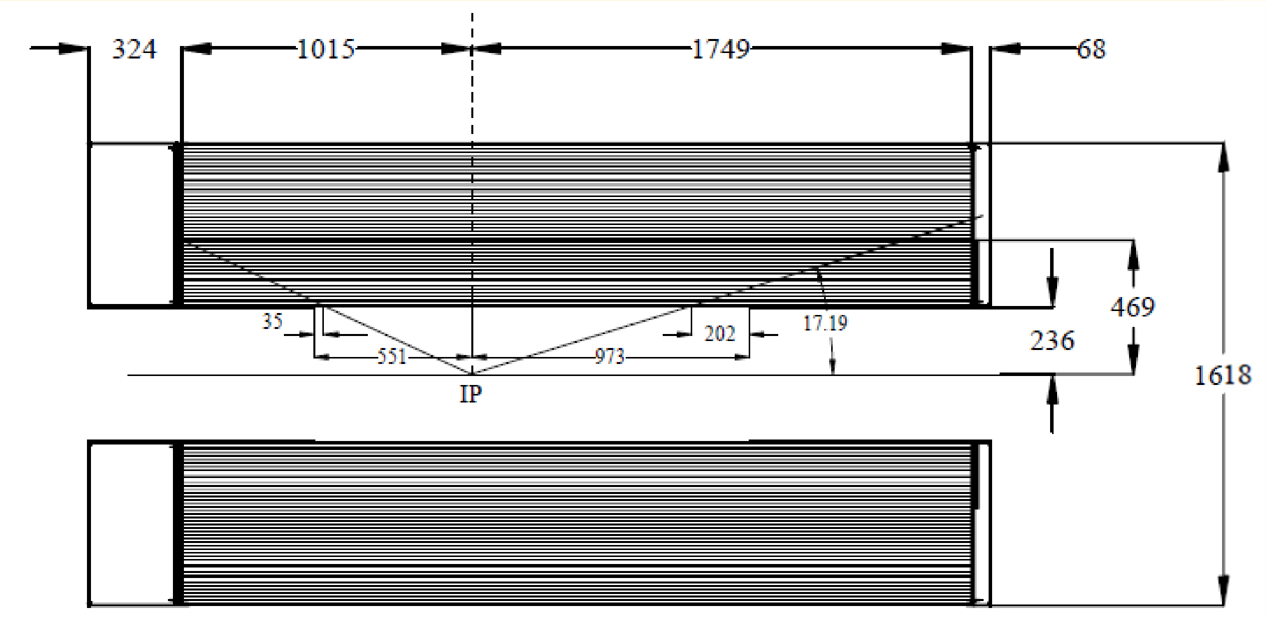


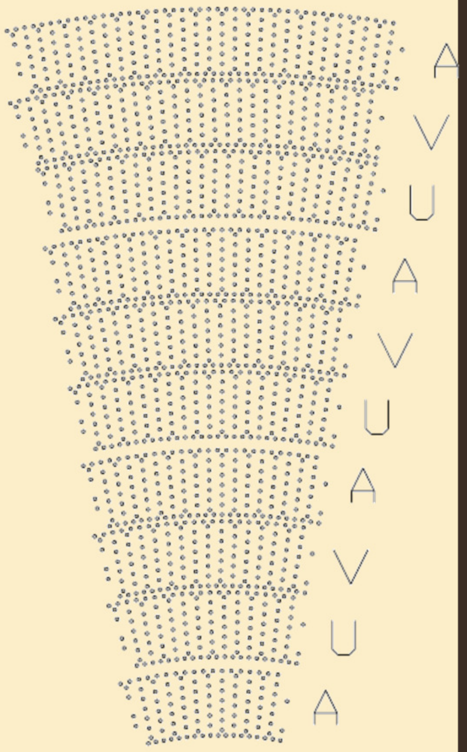
Vertex detector



B Factories - IX

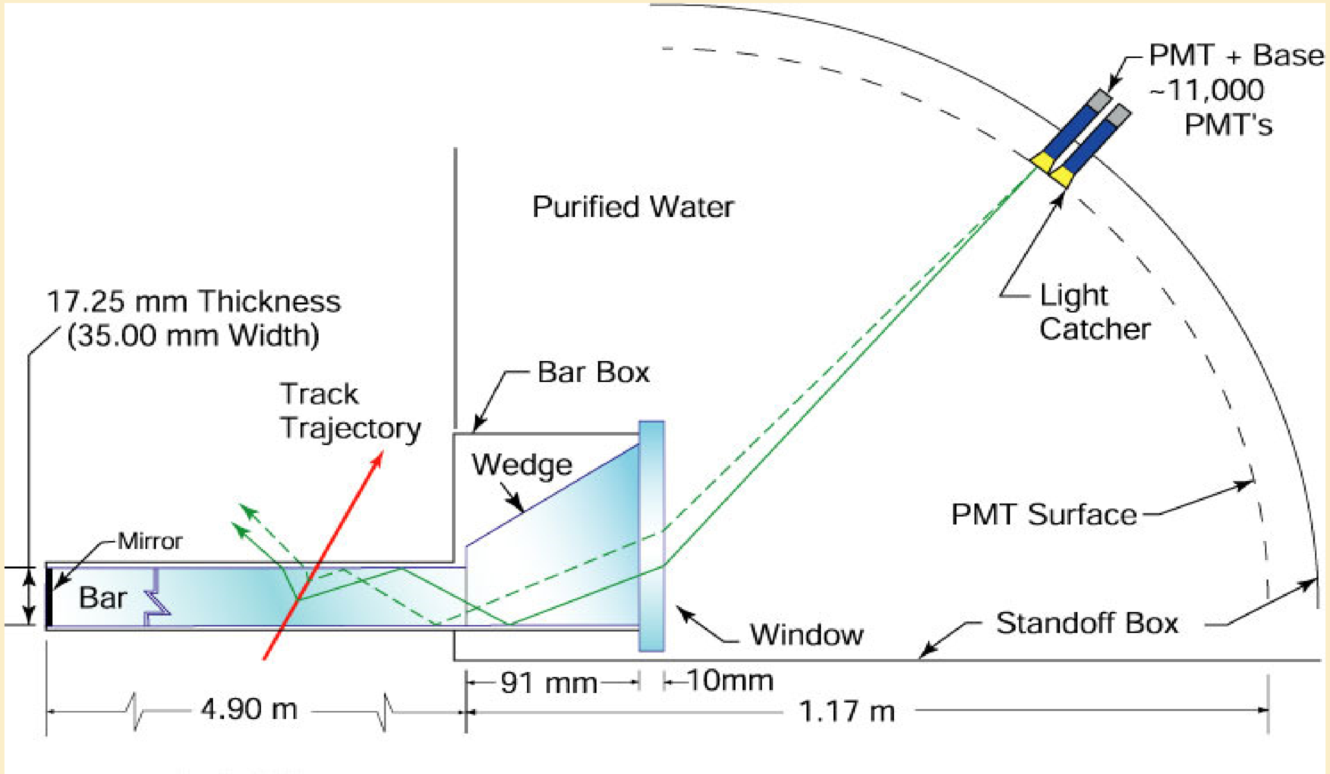
Drift chamber





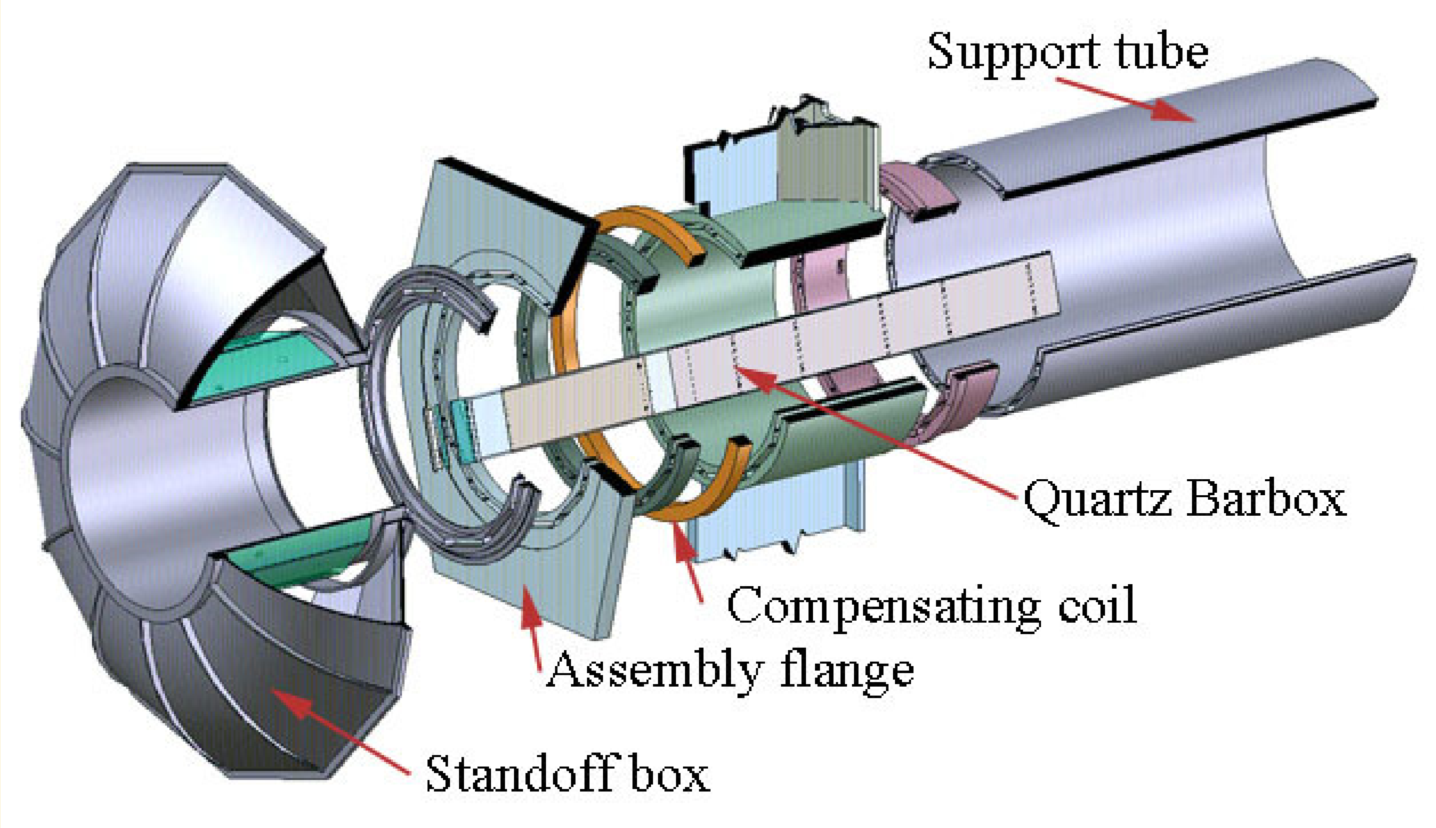
B Factories - X

DIRC : Particle Id



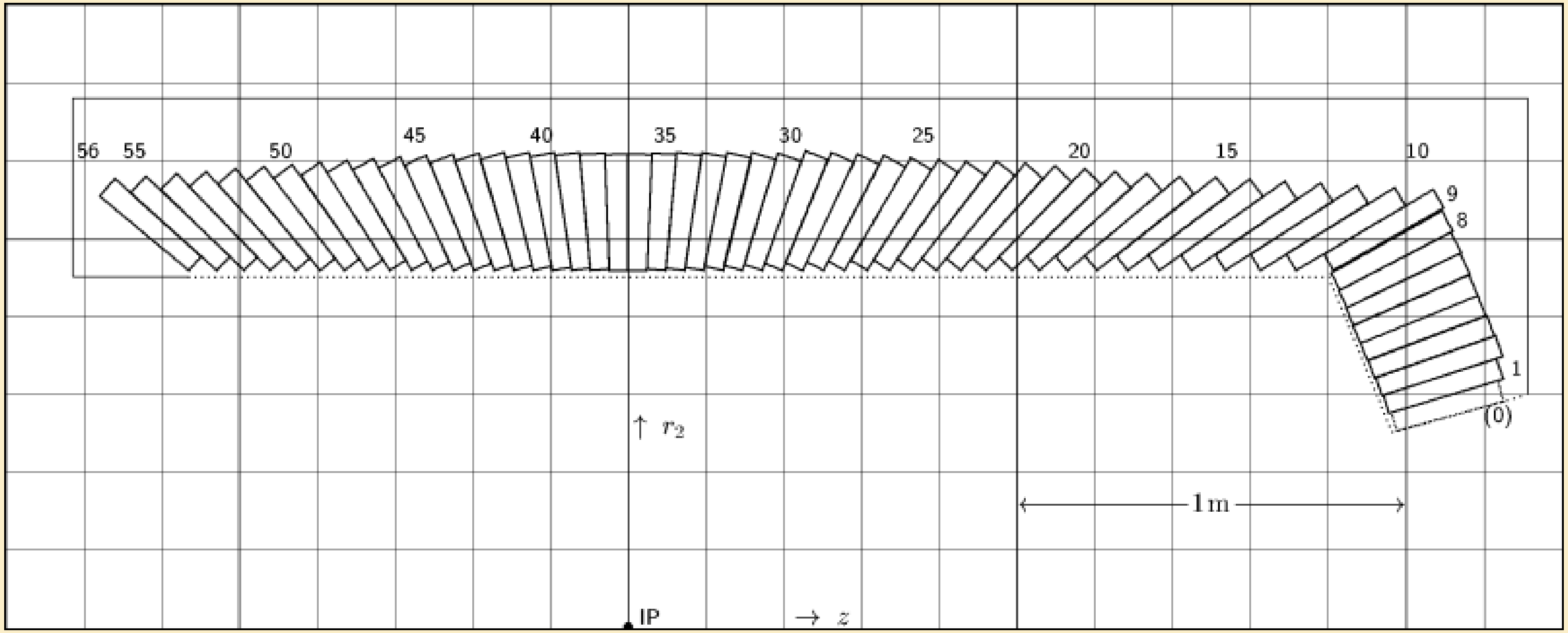
B Factories - XI

DIRC



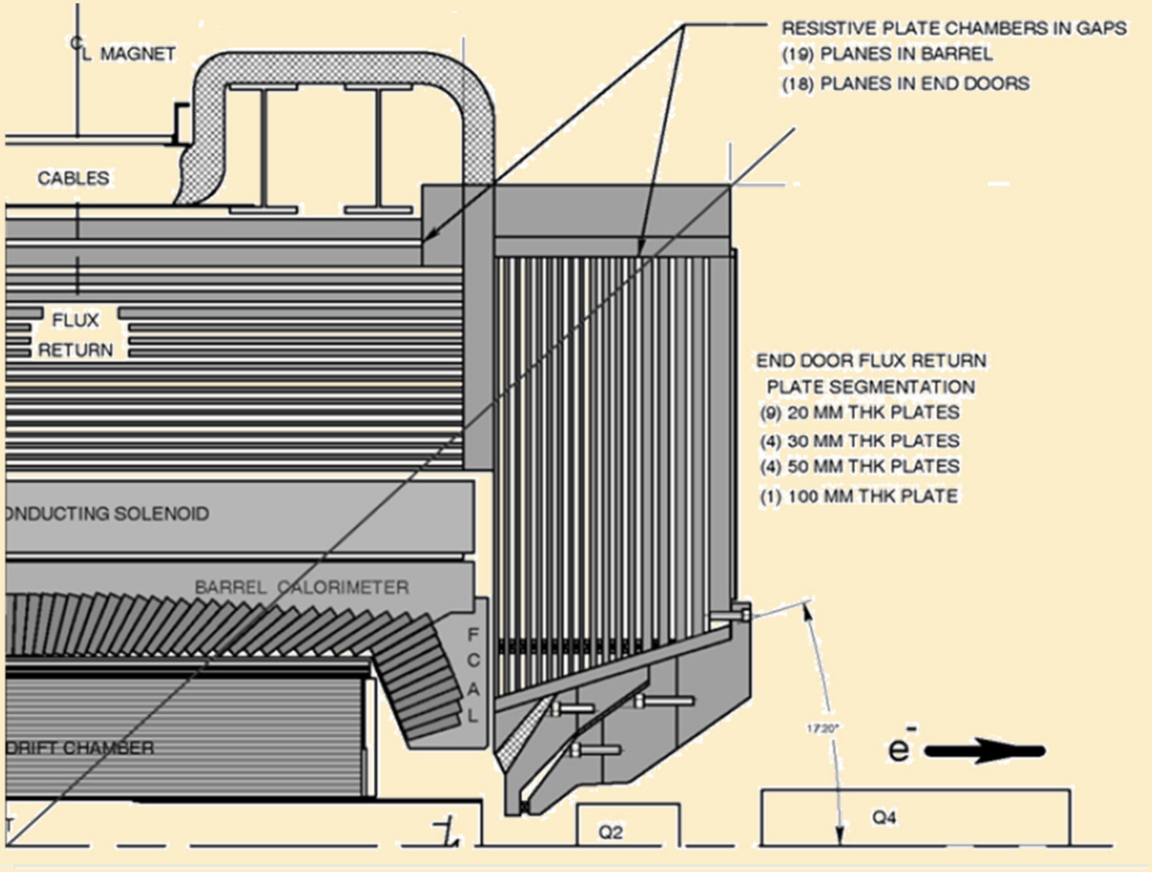
B Factories - XII

EM Calorimeter

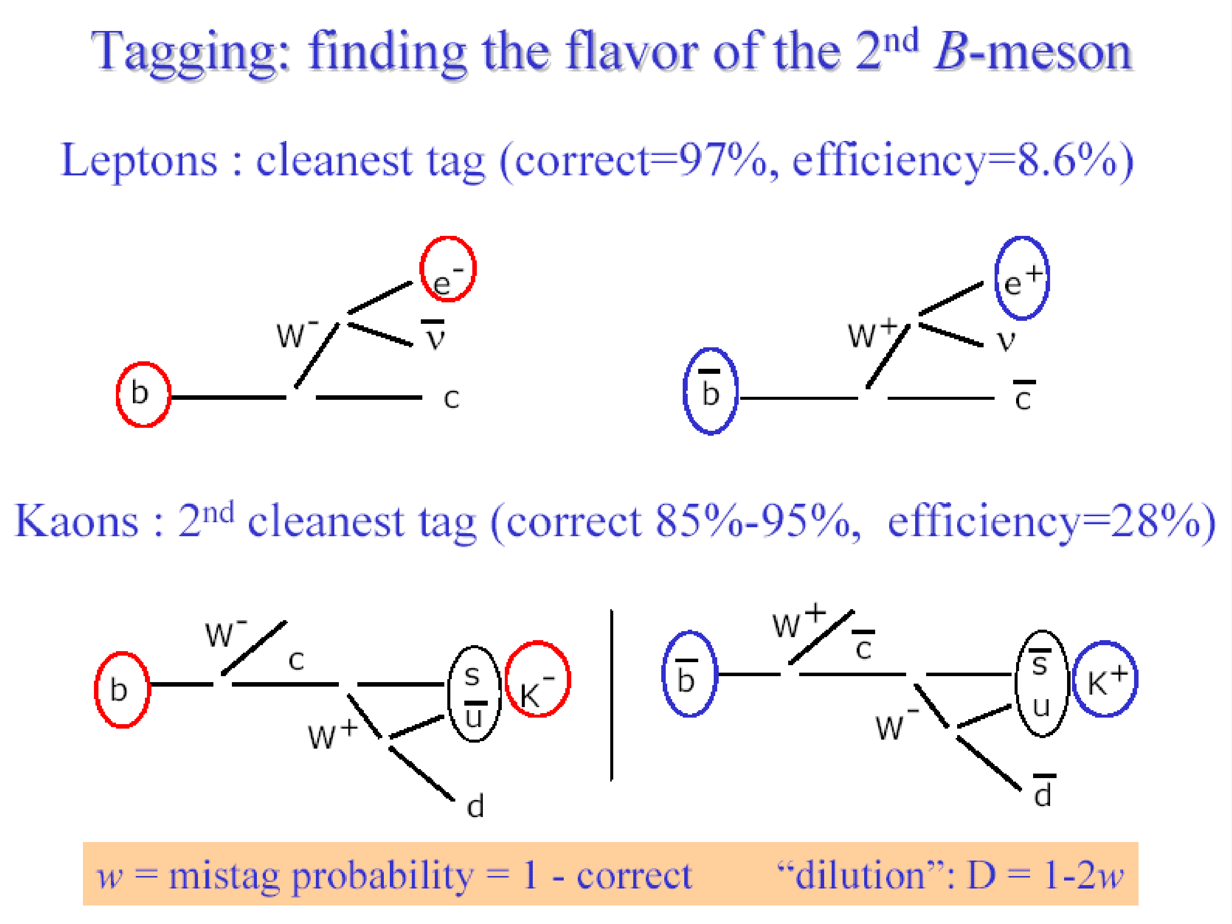


B Factories - XIII

Instrumented Flux Return: Muon detector & (coarse) hadron calorimeter



B Factories - XIV



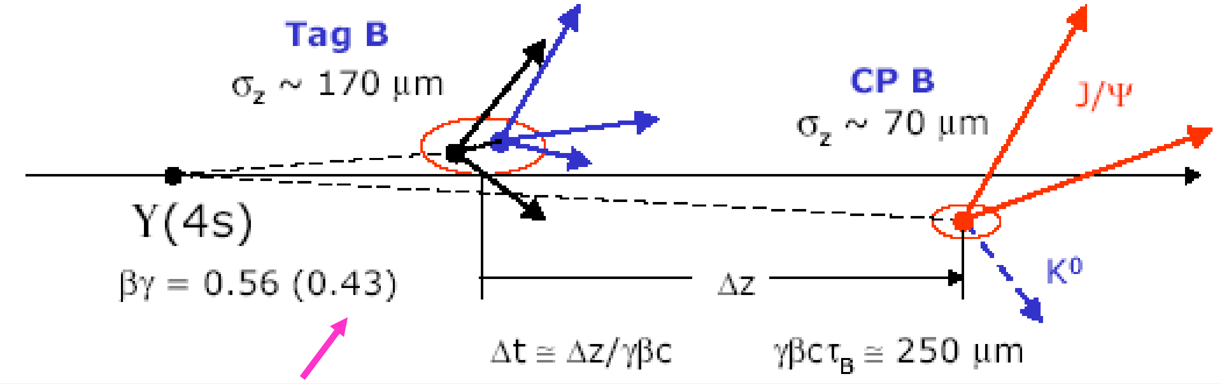
Tagging: finding the flavor of the 2nd *B*-meson

Leptons : cleanest tag (correct=97%, efficiency=8.6%)

Kaons : 2nd cleanest tag (correct 85%-95%, efficiency=28%)

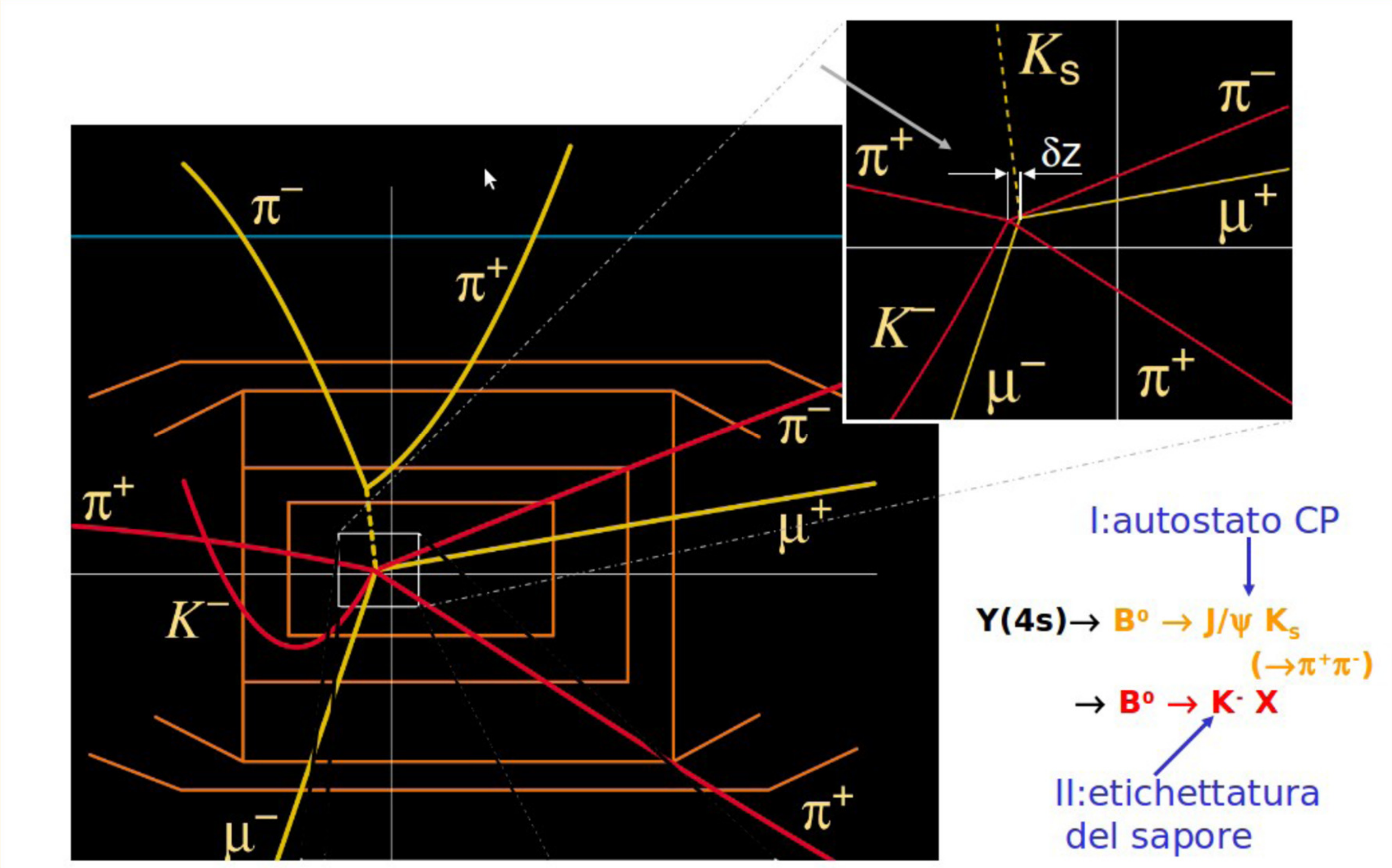
w = mistag probability = 1 - correct “dilution”: D = 1-2w

B Factories - XV



Belle

B Factories - XVI



I:autostato CP

Y(4s)→ $B^o \to J/\psi\ K_s$

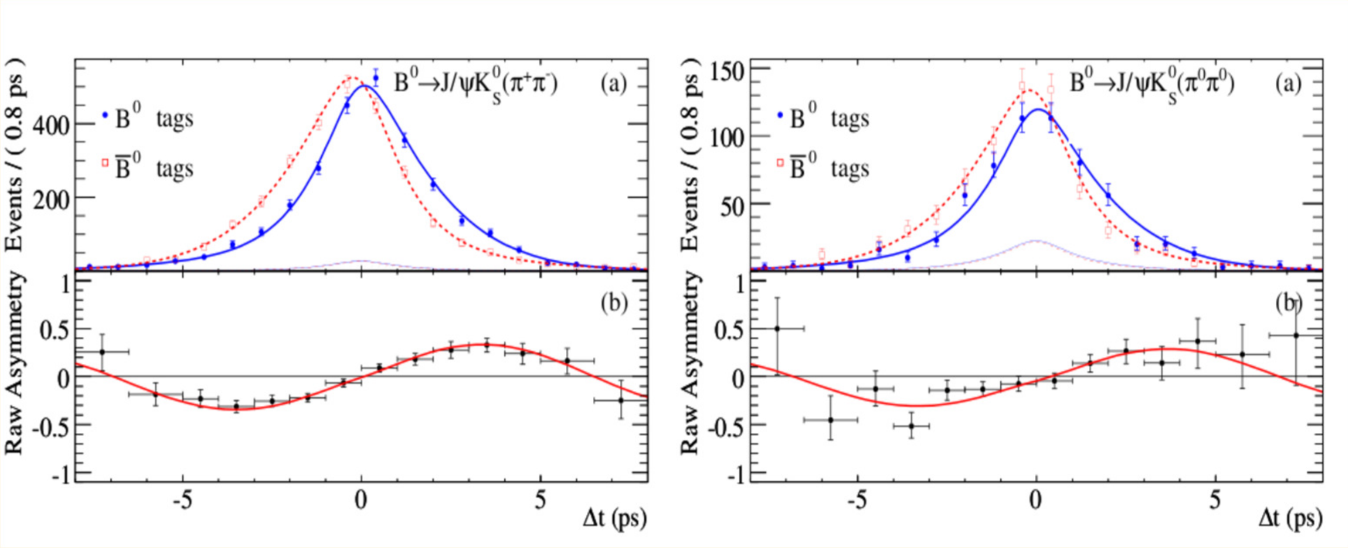
($\to\pi^+\pi^-$)

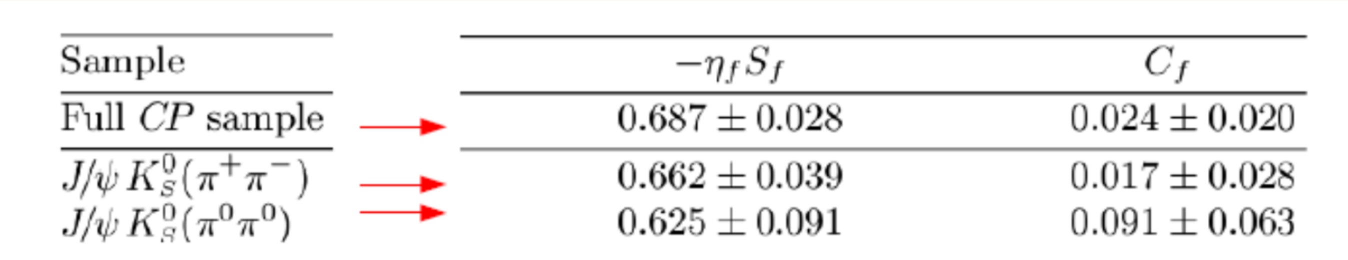
$\to B^o \to K^- X$

II:etichettatura del sapore

B Factories - XVII

BABAR





Sample	$-\eta_f S_f$	C_f
Full *CP* sample	0.687 ± 0.028	0.024 ± 0.020
$J/\psi\, K^0_S(\pi^+\pi^-)$	0.662 ± 0.039	0.017 ± 0.028
$J/\psi\, K^0_S(\pi^0\pi^0)$	0.625 ± 0.091	0.091 ± 0.063