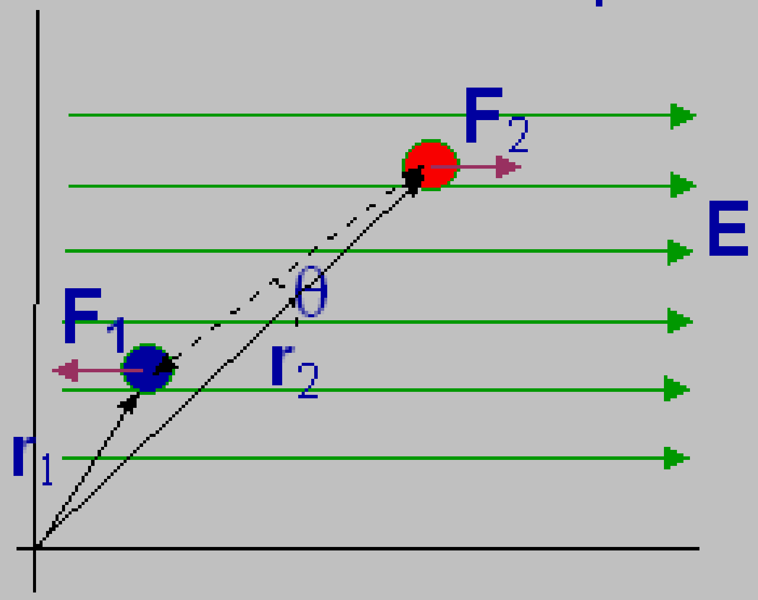
Forza su un dipolo elettrico

Dipolo immerso in campo uniforme:



Forza totale = 0

Coppia: momento meccanico

$$\mathbf{M} = \mathbf{r}_1 \times \mathbf{F}_1 + \mathbf{r}_2 \times \mathbf{F}_2 = \mathbf{r}_1 \times (-\mathbf{F}) + \mathbf{r}_2 \times \mathbf{F}$$

$$\rightarrow \mathbf{M} = (\mathbf{r}_2 - \mathbf{r}_1) \times \mathbf{F} = \mathbf{a} \times (q\mathbf{E}) = (q\mathbf{a}) \times \mathbf{E} = \mathbf{p} \times \mathbf{E}$$

Posizione di equilibrio: **M**=0

$$\rightarrow \mathbf{p} \parallel \mathbf{E}$$

Energia potenziale di un dipolo in un campo esterno

Spostamento angolare infinitesimo lavoro del momento meccanico:

$$dL = \mathbf{F}_1 \cdot d\mathbf{s}_1 + \mathbf{F}_2 \cdot d\mathbf{s}_2$$

$$= \mathbf{F}_1 \cdot \mathbf{r}_1 d\theta + \mathbf{F}_2 \cdot \mathbf{r}_2 d\theta$$

$$= \mathbf{F} \cdot \left(\mathbf{r}_1 d\theta - \mathbf{r}_2 d\theta\right) = M d\theta$$

Spostamento finito da θ_0 a θ:

$$L = \int dL = \int_{\theta_0}^{\theta} M d\theta = \int_{\theta_0}^{\theta} -pE \sin\theta d\theta$$

$$\rightarrow L = pE\left(\cos\theta - \cos\theta_0\right)$$

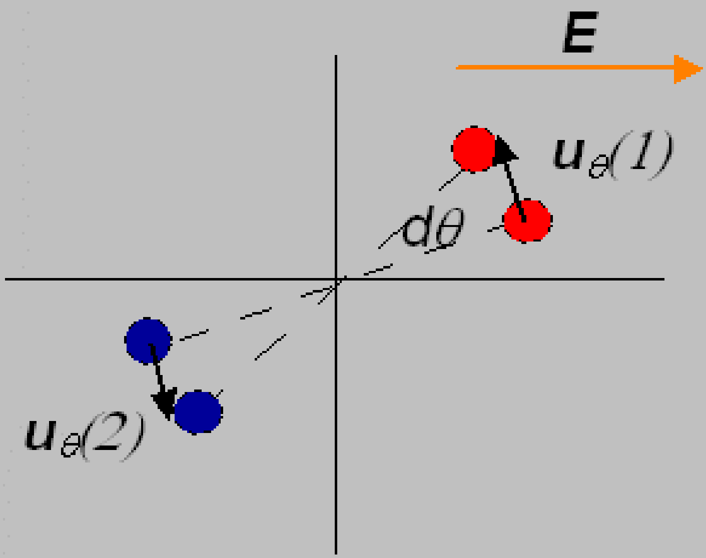
Variazione en. potenziale:

$$L = pE\left(\cos\theta - \cos\theta_0\right)$$

$$L = -\Delta U = -\left(U(\theta) - U(\theta_0)\right)$$

$$\rightarrow U(\theta) = -pE\cos\theta = -\mathbf{p} \cdot \mathbf{E}$$

Calcolo lavoro elementare



$$dL = dL_1 + dL_2 = \boldsymbol{F}_1 \cdot d\boldsymbol{s}_1 + \boldsymbol{F}_2 \cdot d\boldsymbol{s}_2$$

$$\boldsymbol{F}_1 = -\boldsymbol{F}_2 = \boldsymbol{F} = q\boldsymbol{E}$$

$$d\boldsymbol{s}_1 = |\boldsymbol{r}_1| d\theta \hat{\boldsymbol{u}}_\theta = \frac{d}{2} d\theta \hat{\boldsymbol{u}}_\theta(1)$$

$$d\boldsymbol{s}_2 = |\boldsymbol{r}_2| d\theta \hat{\boldsymbol{u}}_\theta = \frac{d}{2} d\theta \hat{\boldsymbol{u}}_\theta(2)$$

$$\rightarrow dL = \boldsymbol{F} \cdot |\boldsymbol{r}_1| d\theta \hat{\boldsymbol{u}}_\theta(1) - \boldsymbol{F} \cdot |\boldsymbol{r}_2| d\theta \hat{\boldsymbol{u}}_\theta(2) = \boldsymbol{F} \cdot [\hat{\boldsymbol{u}}_\theta(1) - \hat{\boldsymbol{u}}_\theta(2)] \frac{d}{2} d\theta$$

$$\rightarrow dL = q\boldsymbol{E} \cdot [\hat{\boldsymbol{u}}_\theta(1) - \hat{\boldsymbol{u}}_\theta(2)] \frac{d}{2} d\theta$$

$$\boldsymbol{E} \cdot \hat{\boldsymbol{u}}_\theta(1) = |\boldsymbol{E}| \cos(\theta + \pi/2) = -|\boldsymbol{E}| \sin\theta$$

$$\boldsymbol{E} \cdot \hat{\boldsymbol{u}}_\theta(2) = |\boldsymbol{E}| \cos(\theta + 3\pi/2) = +|\boldsymbol{E}| \sin\theta$$

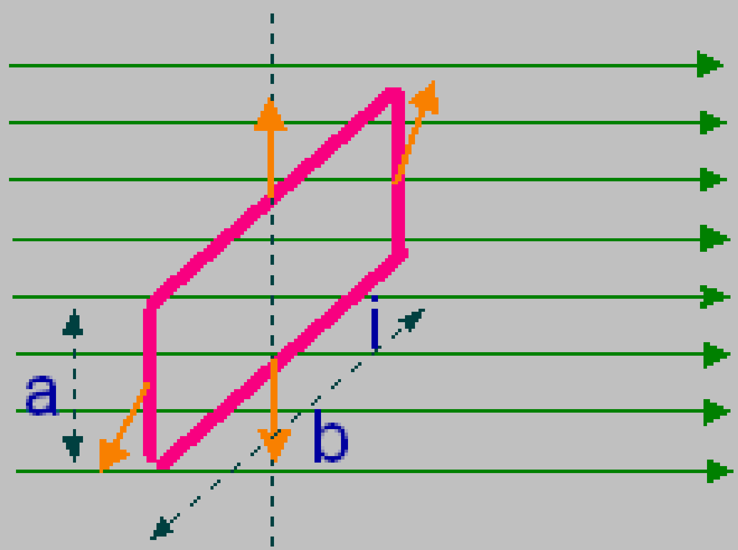
$$\rightarrow dL = q(-2|\boldsymbol{E}|\sin\theta) \frac{d}{2} d\theta = -|\boldsymbol{p}||\boldsymbol{E}| \sin\theta d\theta = -|\boldsymbol{p} \times \boldsymbol{E}| d\theta$$

$$\rightarrow \boldsymbol{dL} = -(\boldsymbol{p} \times \boldsymbol{E}) \boldsymbol{d\theta} = |\boldsymbol{M}| \boldsymbol{d\theta}$$

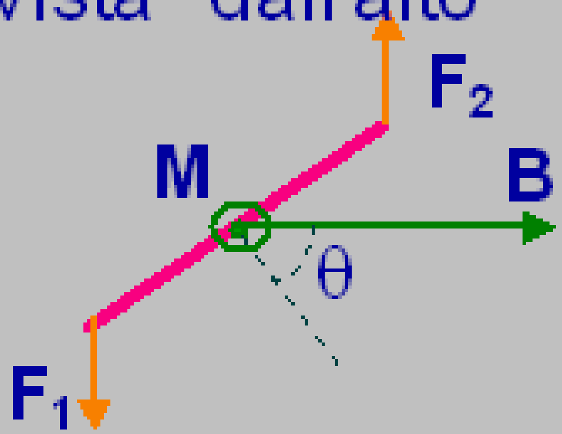
Dipolo magnetico

Spira rettangolare immersa in campo esterno uniforme:

vista "prospettica"



vista "dall'alto"



$$\mathbf{F}_{tot} = 0 \text{ sui lati lunghi}$$

$$|\mathbf{F}| = iaB \quad \text{sui lati corti}$$

Momento meccanico

$$\mathbf{M} = \mathbf{F}_1 \times \mathbf{r}_1 + \mathbf{F}_2 \times \mathbf{r}_2 = \mathbf{F} \times (\mathbf{r}_2 - \mathbf{r}_1)$$

$$\rightarrow \mathbf{M} = \mathbf{F} \times \mathbf{b} \rightarrow |\mathbf{M}| = iaBb\sin\theta$$

$$\rightarrow M = iAB\sin\theta$$

Momento di dipolo magnetico:

$$\mathbf{m} = iA\mathbf{u}_n, \quad \mathbf{u}_n \text{ versore normale alla spira}$$

Allora:

$$\mathbf{M} = \mathbf{m} \times \mathbf{B}$$

$$\text{posizione di equilibrio}: \mathbf{m} \parallel \mathbf{B}$$

Energia potenziale magnetica: analogia al caso del dipolo elettrico

$$U = -\mathbf{m} \cdot \mathbf{B}$$