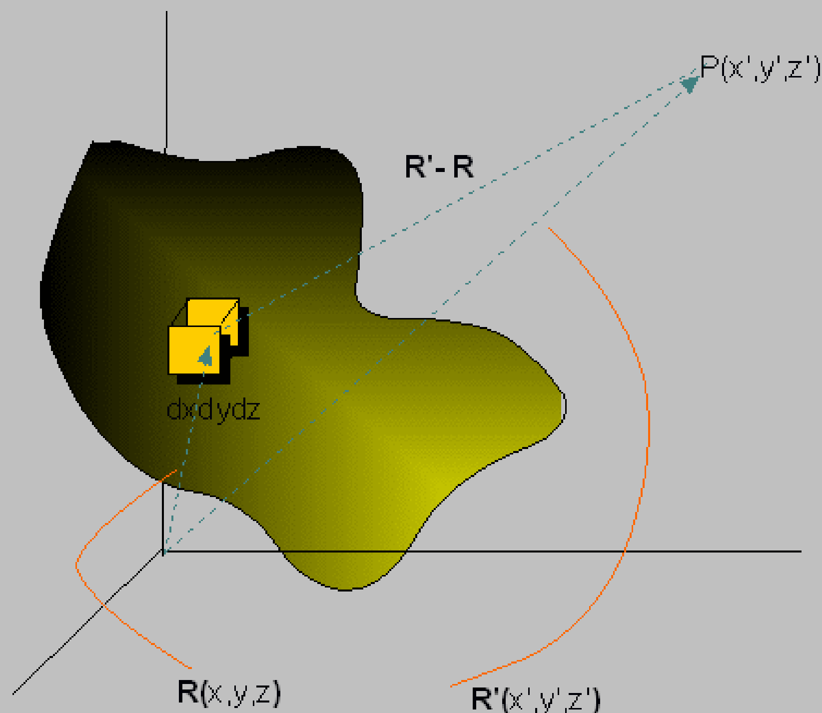
Distribuzione continua di carica

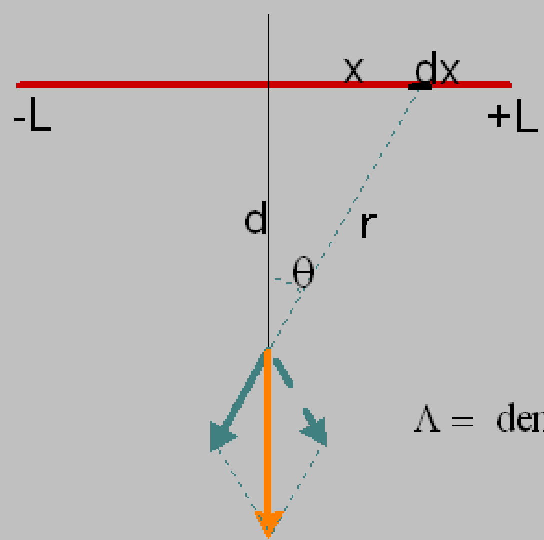


$$\mathbf{E} \approx \frac{1}{4\pi\varepsilon_0} \sum_{volumetti} \frac{dq}{\left|\mathbf{R}' - \mathbf{R}\right|^2} \frac{\mathbf{R}' - \mathbf{R}}{\left|\mathbf{R}' - \mathbf{R}\right|}$$

$$dq = \rho(x, y, z)\, dxdydz = \rho dV$$

$$\rightarrow \mathbf{E} = \frac{1}{4\pi\varepsilon_0} \iiint \frac{\rho dV}{\left|\mathbf{R}' - \mathbf{R}\right|^2} \frac{\mathbf{R}' - \mathbf{R}}{\left|\mathbf{R}' - \mathbf{R}\right|}$$

Filo carico



$\Lambda =$ densita' lineare di carica

$$d\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{d^2+x^2}\hat{\mathbf{r}} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\hat{\mathbf{r}}$$

$$dq = \Lambda dx$$

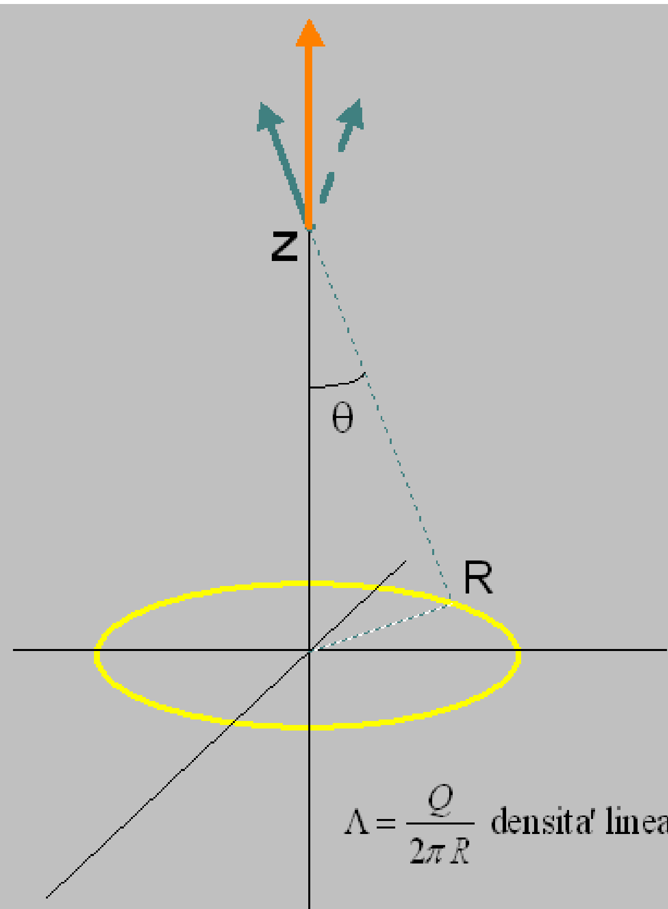
$$dE = \frac{1}{4\pi\varepsilon_0}\frac{\Lambda dx}{r^2}\cos\theta$$

$$x = d\tan\theta \rightarrow dx = d\frac{1}{\cos^2\theta}d\theta$$

$$d = r\cos\theta \rightarrow r^2 = \frac{d^2}{\cos^2\theta}$$

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{\Lambda d\frac{1}{\cos^2\theta}d\theta}{\frac{d^2}{\cos^2\theta}}\cos\theta = \frac{1}{4\pi\varepsilon_0}\frac{\Lambda}{d}\cos\theta d\theta$$

$$E = \int dE = \int_{-\theta_{max}}^{+\theta_{max}}\frac{1}{4\pi\varepsilon_0}\frac{\Lambda}{d}\cos\theta d\theta = \frac{1}{4\pi\varepsilon_0}\frac{2\Lambda}{d}\sin\theta_{max} = \frac{1}{2\pi\varepsilon_0}\frac{\Lambda}{d}\sin\theta_{max}$$



$$\Lambda = \frac{Q}{2\pi R} \quad \text{densita' lineare di carica}$$

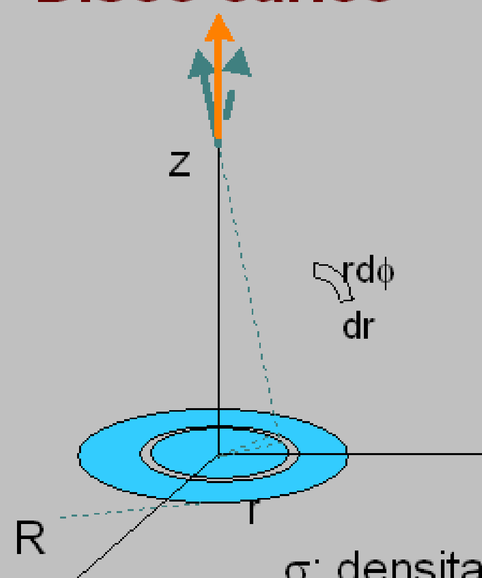
$$d\boldsymbol{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{R^2+z^2}\hat{\boldsymbol{r}}$$

$$dq = \Lambda ds = \frac{Q}{2\pi R}ds$$

$$dE_{\parallel} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{2\pi R\left(R^2+z^2\right)}\cos\theta ds = \frac{1}{4\pi\varepsilon_0}\frac{Q}{2\pi R\left(R^2+z^2\right)}\frac{z}{\left(R^2+z^2\right)^{1/2}}ds$$

$$E = \int dE_{\parallel} = \int_0^{2\pi R}\frac{1}{4\pi\varepsilon_0}\frac{Qz}{2\pi R\left(R^2+z^2\right)^{3/2}}ds = \frac{1}{4\pi\varepsilon_0}\frac{Qz}{\left(R^2+z^2\right)^{3/2}}$$

Disco carico



σ: densita' superficiale di carica

$$\sigma = \frac{Q}{\pi R^2}$$

$$dq = \sigma r dr d\varphi$$

$$d\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{d^2}\hat{\mathbf{r}}$$

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{\sigma r dr d\varphi}{d^2}\cos\theta$$

$$d^2 = r^2 + z^2, \cos\theta = \frac{z}{d}$$

$$\rightarrow dE = \frac{1}{4\pi\varepsilon_0}\frac{\sigma r dr d\varphi}{r^2 + z^2}\frac{z}{\left(r^2 + z^2\right)^{1/2}}$$

$$E = \int dE = \iint \frac{1}{4\pi\varepsilon_0}\frac{\sigma r dr d\varphi}{r^2 + z^2}\frac{z}{\left(r^2 + z^2\right)^{1/2}}$$

$$E = \frac{1}{4\pi\varepsilon_0}\int_0^{2\pi} d\varphi \int_o^R \frac{\sigma r dr}{r^2 + z^2}\frac{z}{\left(r^2 + z^2\right)^{1/2}} = \frac{\sigma z}{2\varepsilon_0}\int_o^R \frac{r dr}{\left(r^2 + z^2\right)^{3/2}}$$

$$E = \frac{\sigma z}{2\varepsilon_0}\frac{1}{2}\int_o^R \frac{2r dr}{\left(r^2 + z^2\right)^{3/2}} = \frac{\sigma z}{4\varepsilon_0}\left(-2\right)\left(r^2 + z^2\right)^{-1/2}\Big|_0^R = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{z}{\sqrt{R^2 + z^2}}\right]$$

Piano carico

Caso del disco

$$E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{z}{\sqrt{R^2 + z^2}}\right]$$

Se $R \to \infty, \dfrac{z}{R} \to 0$

Disco $\to$ Piano infinito

$$E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{z}{\sqrt{R^2 + z^2}}\right] = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{\frac{z}{R}}{\sqrt{1 + \left(\frac{z}{R}\right)^2}}\right] \xrightarrow[\frac{z}{R}\to 0]{} \frac{\sigma}{2\varepsilon_0}$$

Campo elettrostatico uniforme

Piano infinito carico +vamente