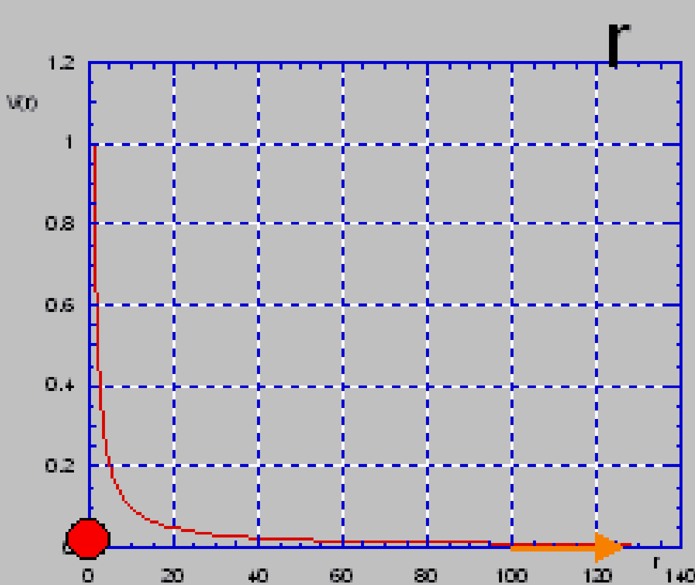
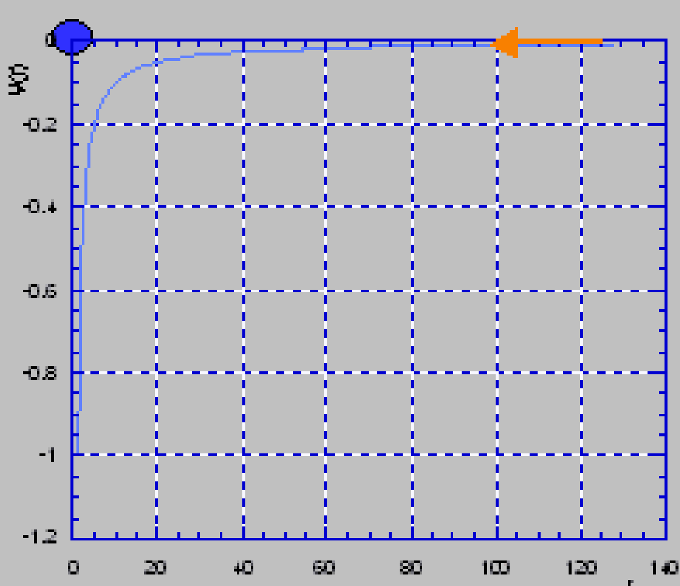
Campo elettrico e potenziale

$$\mathbf{E} = -\nabla V$$

Carica puntiforme



Carica +va:
V decrescente
Derivata -va
con segno - OK!



Carica -va:
V crescente
Derivata +va
con segno - OK!

Dal potenziale al campo

Es.: carica puntiforme

$$\boldsymbol{E} = -\nabla V$$

$$E_x = -\frac{\partial V}{\partial x}, \text{ etc}$$

$$V = \frac{q}{4\pi\varepsilon_0}\frac{1}{r}, r = \left(x^2 + y^2 + z^2\right)^{1/2}$$

$$\frac{\partial V}{\partial x} = \frac{dV}{dr}\frac{\partial r}{\partial x} = -\frac{q}{4\pi\varepsilon_0}\frac{1}{r^2}\frac{2x}{2\left(x^2 + y^2 + z^2\right)^{1/2}}$$

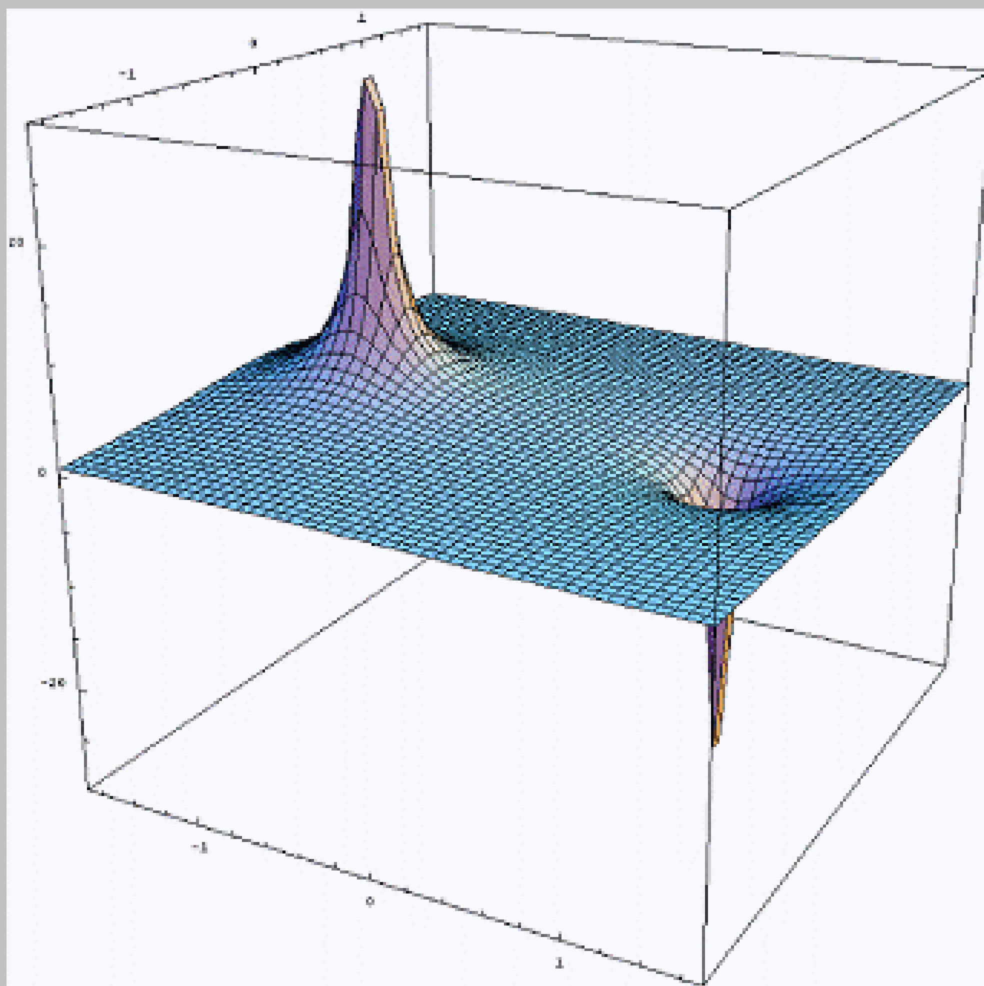
$$E_x = -\left(-\frac{q}{4\pi\varepsilon_0}\frac{x}{r^3}\right), E_y, E_z \text{ analoghi}$$

$$\mathbf{E} = E_x\hat{\mathbf{i}} + E_y\hat{\mathbf{j}} + E_z\hat{\mathbf{k}} = \frac{q}{4\pi\varepsilon_0}\frac{x}{r^3}\hat{\mathbf{i}} + \frac{q}{4\pi\varepsilon_0}\frac{y}{r^3}\hat{\mathbf{j}} + \frac{q}{4\pi\varepsilon_0}\frac{z}{r^3}\hat{\mathbf{k}}$$

$$\mathbf{E} = \frac{q}{4\pi\varepsilon_0}\frac{\mathbf{r}}{r^3} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{\mathbf{r}}$$

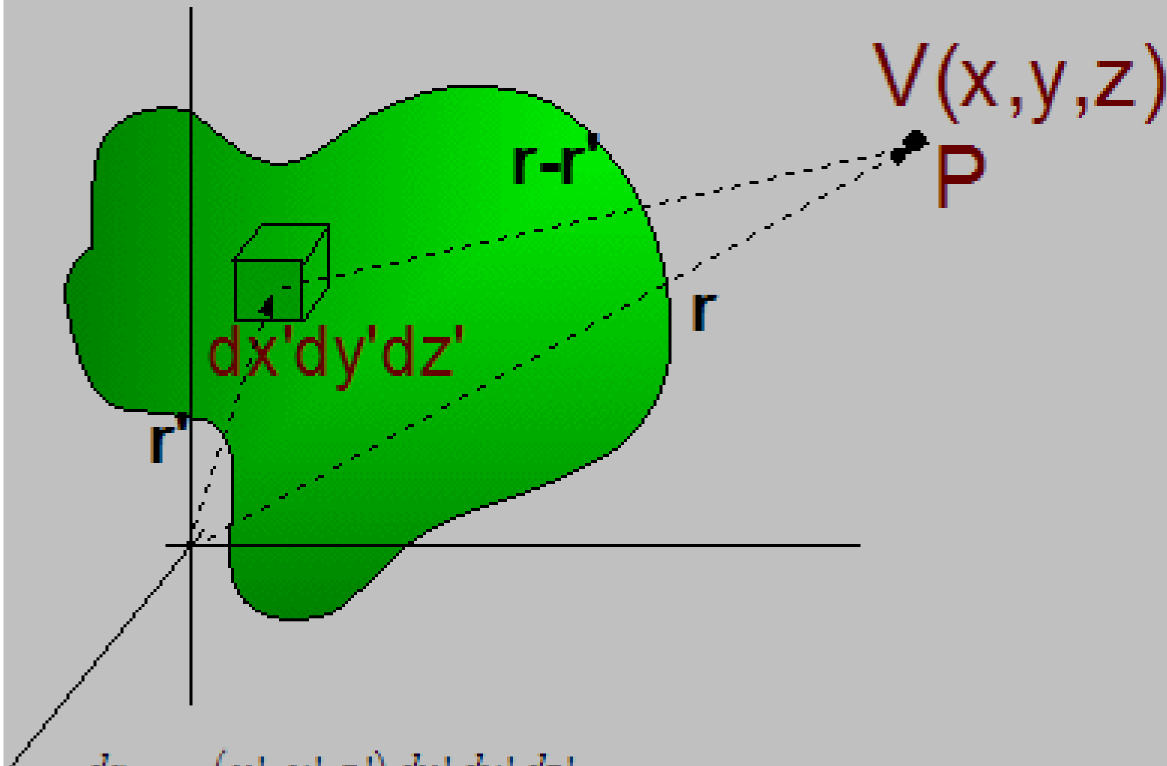
Campo coulombiano

Potenziale: 2 cariche opposte



Potenziale in funzione di (x,y)
(da V.Gracco, Fis. generale II)

Potenziale: distribuzione continua



$$dq = \rho\left(x', y', z'\right) dx' dy' dz'$$

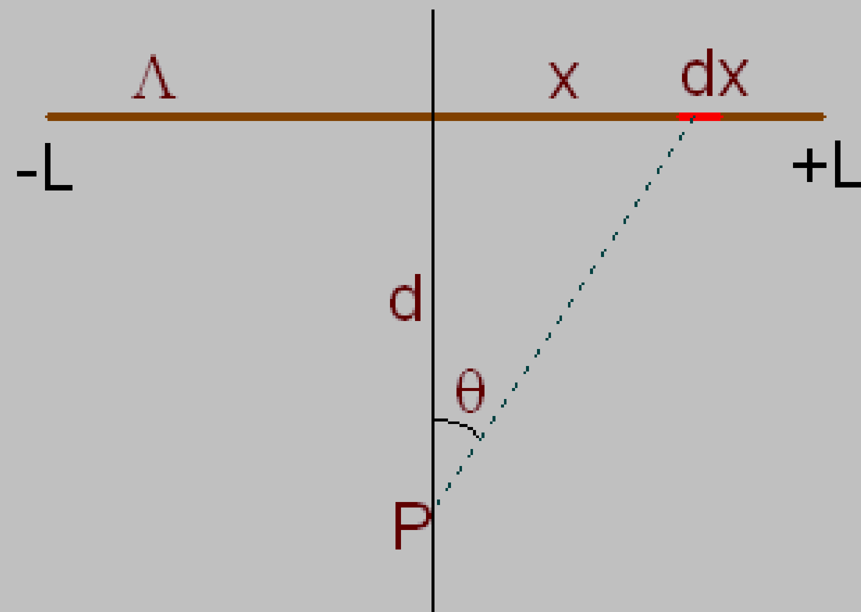
$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{|\mathbf{r}-\mathbf{r}'|} = \frac{1}{4\pi\varepsilon_0}\frac{\rho dx' dy' dz'}{|\mathbf{r}-\mathbf{r}'|}$$

$$|\mathbf{r}-\mathbf{r}'|^2 = (x-x')^2 + (y-y')^2 + (z-z')^2$$

$$V = \int dV = \int_{\substack{\text{volume}\\\text{della}\\\text{carica}}} \frac{1}{4\pi\varepsilon_0}\frac{\rho dx' dy' dz'}{\left[(x-x')^2 + (y-y')^2 + (z-z')^2\right]^{1/2}}$$

Simile a c.elettrico, ma piu' semplice: una sola funzione (V= scalare)

Esempio: filo finito



$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\frac{\Lambda dx}{r}$$

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{\Lambda dx}{\left(d^2 + x^2\right)^{1/2}}$$

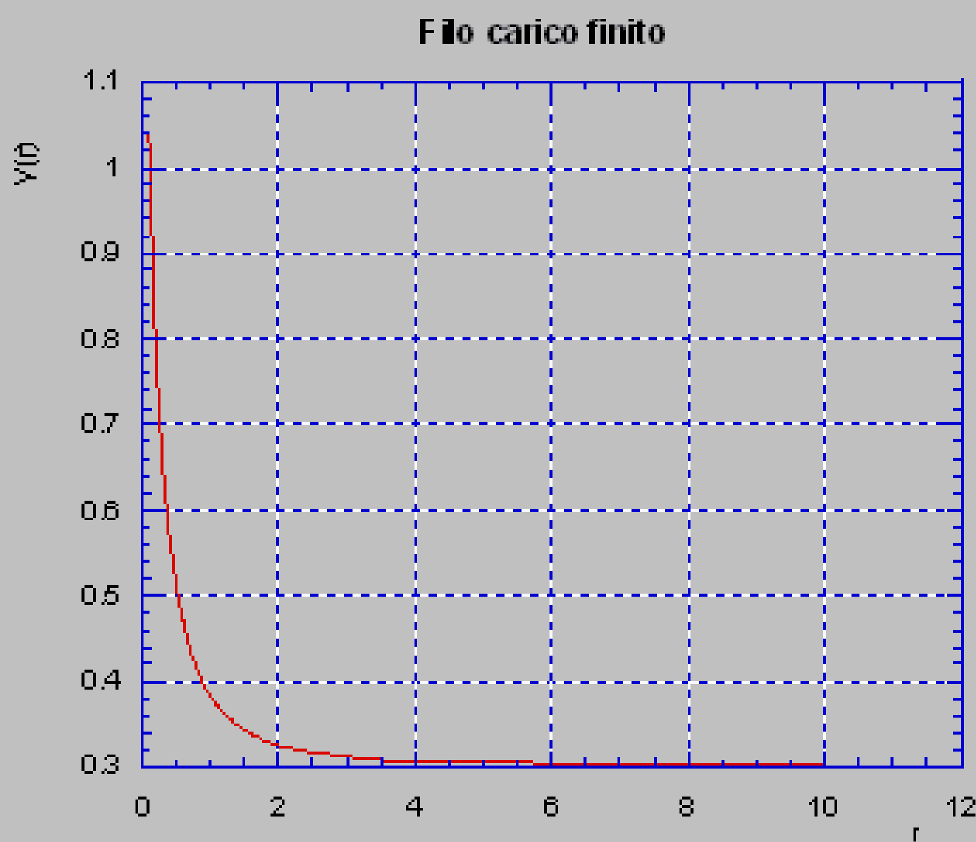
$$\rightarrow V = \frac{2\Lambda}{4\pi\varepsilon_0}\int_0^{+L}\frac{dx}{\left(d^2 + x^2\right)^{1/2}} = \frac{2\Lambda}{4\pi\varepsilon_0}\ln\left(x + \sqrt{x^2 + d^2}\right)\Big|_0^L$$

$$\rightarrow V = \frac{\Lambda}{2\pi\varepsilon_0}\left[\ln\left(L + \sqrt{L^2 + d^2}\right) - \ln d\right]$$

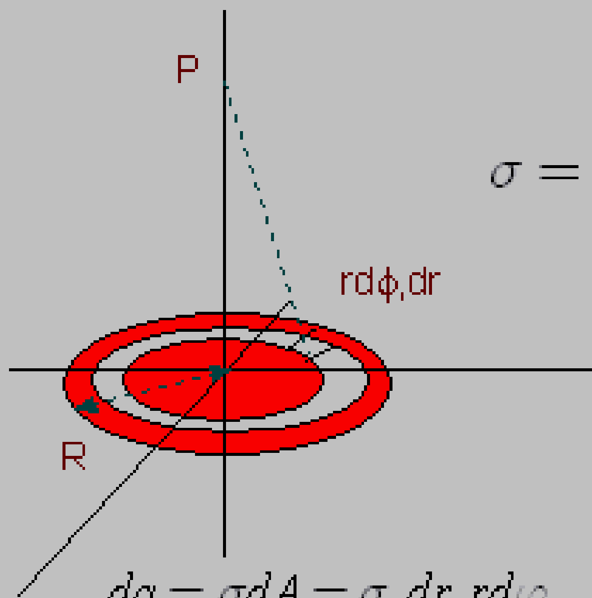
$$\rightarrow V = \frac{\Lambda}{2\pi\varepsilon_0}\left[\ln\frac{L + \sqrt{L^2 + d^2}}{d}\right]$$

Filo carico finito

Grafico del potenziale: *V(r)* vs *r*



Esempio: disco carico



$$\sigma = \frac{Q}{\pi R^2}$$

$$dq = \sigma dA = \sigma \ dr \ rd\varphi$$

$$u^2 = r^2 + z^2$$

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{u} = \frac{1}{4\pi\varepsilon_0}\frac{\sigma \ dr \ rd\varphi}{\left(r^2+z^2\right)^{1/2}}$$

$$\rightarrow V = \frac{1}{4\pi\varepsilon_0}\iint_{disco}\frac{\sigma \ dr \ rd\varphi}{\left(r^2+z^2\right)^{1/2}}$$

$$\rightarrow V = \frac{2\pi\sigma}{4\pi\varepsilon_0}\int_0^R \frac{r \ dr}{\left(r^2+z^2\right)^{1/2}} =$$

$$V = \frac{\sigma}{2\varepsilon_0}\left(r^2+z^2\right)^{1/2}\Big|_0^R = \frac{\sigma}{2\varepsilon_0}\left[\left(R^2+z^2\right)^{1/2} - z\right]$$

Disco carico

Grafico del potenziale: *V(r)* vs *r*