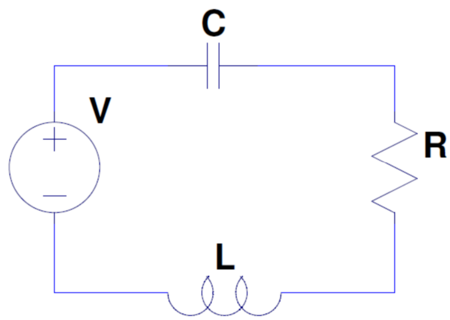
Circuiti risonanti: Due casi tipici

a) Risonanza serie



$$V = Z_T i$$

$$Z_T = R + j\omega L + \frac{1}{j\omega C} = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$T(\omega) = \frac{i}{V} = \frac{1}{Z_T} = \frac{1}{R + j\left(\omega L - \frac{1}{\omega C}\right)} = \frac{R - j\left(\omega L - \frac{1}{\omega C}\right)}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \qquad [T] = R^{-1}$$

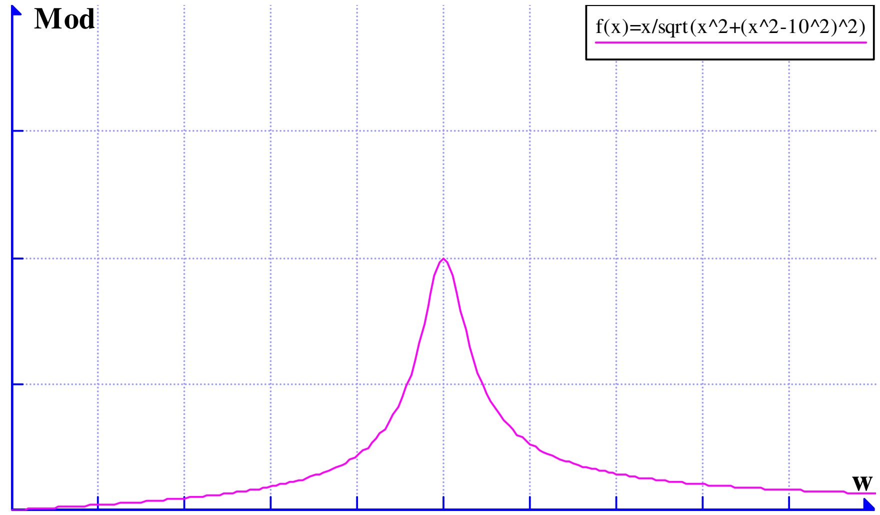
$$\rightarrow |T(\omega)|^2 = \frac{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^2} = \frac{1}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \frac{\omega^2 C^2}{\omega^2 C^2 R^2 + \left(\omega^2 LC - 1\right)^2}$$

$$\rightarrow |T(\omega)| = \frac{\omega C}{\sqrt{\omega^2 C^2 R^2 + \left(\omega^2 LC - 1\right)^2}}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \rightarrow \omega^2 C^2 R^2 + \left(\omega^2 LC - 1\right)^2 = \omega^2 C^2 R^2 + \left(\frac{\omega^2}{\omega_o^2} - 1\right)^2$$

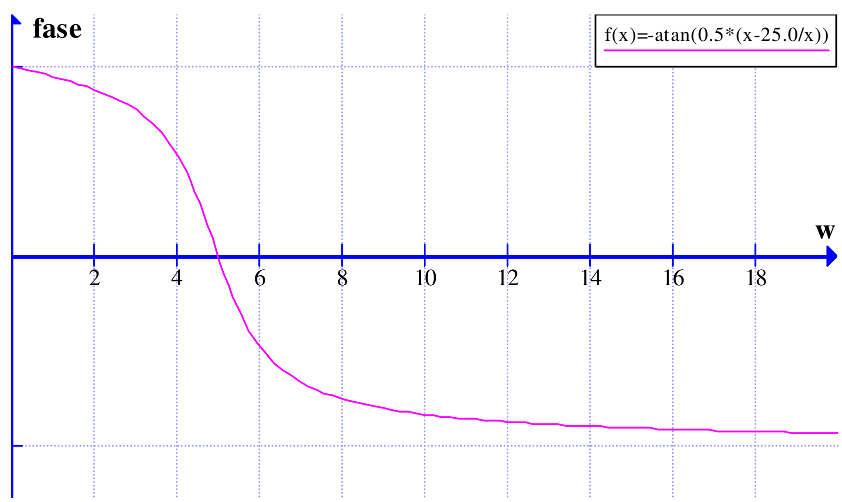
$$= \frac{\omega^2 C^2 L^2 R^2}{L^2} + \left(\frac{\omega^2 - \omega_o^2}{\omega_o^2}\right)^2 = \frac{\omega^2 \frac{R^2}{L^2}}{\omega_o^4} + \frac{\left(\omega^2 - \omega_o^2\right)^2}{\omega_o^4} = \frac{\omega^2 \frac{R^2}{L^2} + \left(\omega^2 - \omega_o^2\right)^2}{\omega_o^4}$$

$$\rightarrow |T(\omega)| = \frac{\omega C}{\sqrt{\omega^2 C^2 R^2 + \left(\omega^2 LC - 1\right)^2}} = \frac{\omega \omega_o^2 C}{\sqrt{\omega^2 \frac{R^2}{L^2} + \left(\omega^2 - \omega_o^2\right)^2}} = \frac{\omega}{R\sqrt{\omega^2 + \frac{L^2}{R^2}\left(\omega^2 - \omega_o^2\right)^2}}$$

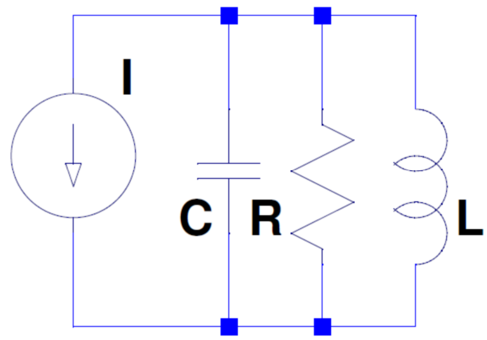


$$T(\omega)=\frac{R-j\left(\omega L-\frac{1}{\omega C}\right)}{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}$$

$$\rightarrow \varphi_T=\arctan\left[-\frac{L}{R}\left(\omega-\frac{1}{\omega CL}\right)\right]=-\arctan\left[\frac{L}{R}\left(\omega-\frac{\omega_0^2}{\omega}\right)\right]$$



b) Risonanza parallelo



$$V = Z_T i$$

$$Y_T = \frac{1}{R} + \frac{1}{j\omega L} + j\omega C = \frac{R + j\omega L + j\omega C j\omega LR}{j\omega LR} = \frac{R - \omega^2 RLC + j\omega L}{j\omega LR}$$

$$\rightarrow Z_T = \frac{1}{Y_T} = \frac{j\omega L}{1 - \omega^2 LC + j\omega \frac{L}{R}}$$

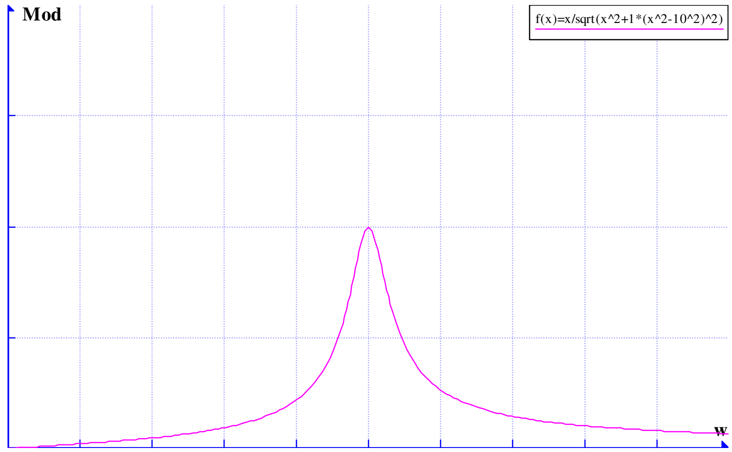
$$T(\omega) = \frac{V}{i} = Z_T = \frac{j\omega L}{1 - \omega^2 LC + j\omega \frac{L}{R}} \qquad [T] = R$$

$$\rightarrow T(\omega) = \frac{j\omega L\left(1 - \omega^2 LC - j\omega \frac{L}{R}\right)}{\left(1 - \omega^2 LC\right)^2 + \omega^2 \left(\frac{L}{R}\right)^2} = \omega L \frac{\omega \frac{L}{R} + j\left(1 - \omega^2 LC\right)}{\left(1 - \omega^2 LC\right)^2 + \omega^2 \left(\frac{L}{R}\right)^2}$$

$$\rightarrow \left|T(\omega)\right|^2 = \omega^2 L^2 \frac{\omega^2 \left(\frac{L}{R}\right)^2 + \left(1 - \omega^2 LC\right)^2}{\left[\left(1 - \omega^2 LC\right)^2 + \omega^2 \left(\frac{L}{R}\right)^2\right]^2} = \frac{\omega^2 L^2}{\left(1 - \omega^2 LC\right)^2 + \omega^2 \left(\frac{L}{R}\right)^2}$$

$$\rightarrow \left|T(\omega)\right| = \frac{\omega L}{\sqrt{\omega^2 \left(\frac{L}{R}\right)^2 + \left(1 - \frac{\omega^2}{\omega_0^2}\right)^2}} = \frac{\omega L}{\sqrt{\frac{\omega^2 \left(\frac{L}{R}\right)^2 \omega_0^4}{\omega_0^4} + \frac{\left(\omega_0^2 - \omega^2\right)^2}{\omega_0^4}}} = \frac{\omega L \omega_0^2}{\sqrt{\omega^2 \left(\frac{L}{R}\right)^2 \omega_0^4 + \left(\omega_0^2 - \omega^2\right)^2}}$$

$$\rightarrow \left|T(\omega)\right| = \frac{\omega L \frac{1}{LC}}{\sqrt{\omega^2 \left(\frac{L}{R}\right)^2 \frac{1}{L^2 C^2} + \left(\omega_0^2 - \omega^2\right)^2}} = \frac{\omega}{C\sqrt{\frac{\omega^2}{R^2 C^2} + \left(\omega_0^2 - \omega^2\right)^2}} = \frac{\omega R}{\sqrt{\omega^2 + R^2 C^2 \left(\omega_0^2 - \omega^2\right)^2}}$$



$$T(\omega) = \omega L \frac{\omega \frac{L}{R} + j\left(1 - \omega^2 LC\right)}{\left(1 - \omega^2 LC\right)^2 + \omega^2 \left(\frac{L}{R}\right)^2}$$

$$\rightarrow \varphi_T = \arctan\left[\frac{1 - \omega^2 LC}{\omega \frac{L}{R}}\right]$$