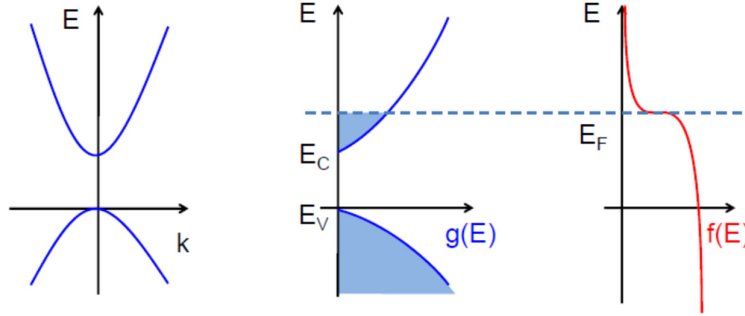


Metalli



Concentrazione portatori:

$$n = \int_{E_C}^{\infty} g(E) f(E) dE = \int_{E_C}^{\infty} \frac{(2m_n^*)^{3/2}}{2\pi^2 \hbar^3} \sqrt{E - E_C} \frac{1}{e^{\frac{E-E_F}{kT}} + 1} dE$$

Metallo \rightarrow Banda di valenza piena, di conduzione semipiena

\rightarrow Banda di valenza non interessata alla conduzione

\rightarrow Si puo' ridefinire lo 0 delle energie $\equiv E_C$

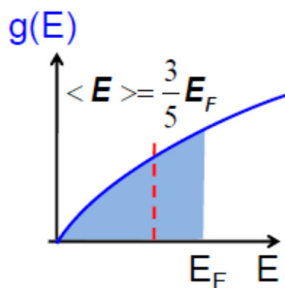
Assumendo Fermi-Dirac = Step:

$$n = \int_0^{E_F} \frac{(2m_n^*)^{3/2}}{2\pi^2 \hbar^3} \sqrt{E} dE = \frac{(2m_n^*)^{3/2}}{2\pi^2 \hbar^3} \frac{2}{3} E^{3/2} \Big|_0^{E_F} = \frac{(2m_n^*)^{3/2}}{3\pi^2 \hbar^3} E_F^{3/2}$$

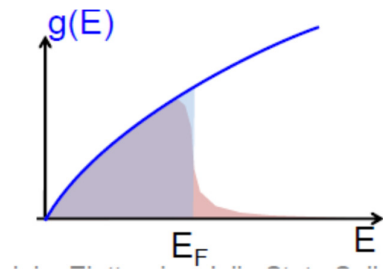
$$\rightarrow E_F = \frac{\hbar^3 (3n\pi^2)^{2/3}}{2m_n^*} \text{ a } T = 0 \text{ K}$$

$$\langle E \rangle = \frac{1}{n} \int_0^{\infty} E g(E) f(E) dE \approx \frac{1}{n} \int_0^{E_F} \frac{(2m_n^*)^{3/2}}{2\pi^2 \hbar^3} E^{3/2} dE = \frac{3}{5} E_F$$

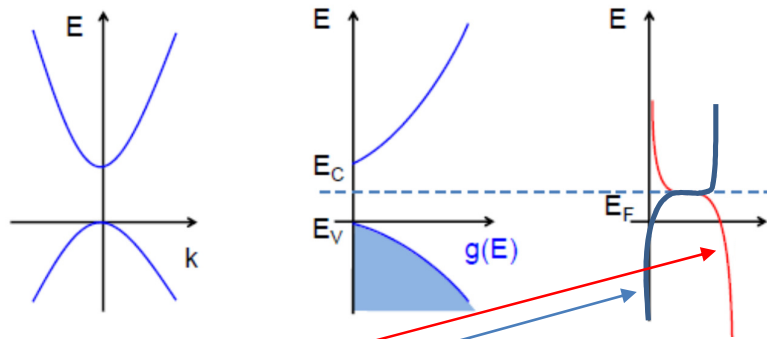
T = 0 K



T > 0 K



Semiconduttori/isolanti



$f(E)$ Fermi-Dirac (BC)

$1-f(E)$ Complemento a 1 di Fermi-Dirac (BV)

Caso intrinseco \equiv puro

Concentrazioni:

$$n = \int_{E_C}^{\infty} g(E) f(E) dE = \int_{E_C}^{\infty} \frac{(2m_n^*)^{3/2}}{2\pi^2 \hbar^3} \sqrt{E - E_C} \frac{1}{e^{\frac{E-E_F}{kT}} + 1} dE$$

T basse $\rightarrow e^{\frac{E-E_F}{kT}} \gg 1 \rightarrow$ Fermi-Dirac \sim Maxwell-Boltzmann

$$\rightarrow n \approx \int_{E_C}^{\infty} \frac{(2m_n^*)^{3/2}}{2\pi^2 \hbar^3} \sqrt{E - E_C} e^{-\frac{E-E_F}{kT}} dE = \frac{(2m_n^*)^{3/2}}{2\pi^2 \hbar^3} e^{-\frac{E_C-E_F}{kT}} \int_{E_C}^{\infty} \sqrt{\frac{E - E_C}{kT}} e^{-\frac{E-E_C}{kT}} dE$$

$$x = \frac{E - E_C}{kT} \rightarrow dE = kT dx$$

$$\rightarrow n \approx \frac{(2m_n^* kT)^{3/2}}{2\pi^2 \hbar^3} e^{-\frac{E_C-E_F}{kT}} \int_0^{\infty} \sqrt{x} e^{-x} dx$$

$$y = \sqrt{x} \rightarrow x = y^2 \rightarrow dx = 2y dy$$

$$\int_0^{\infty} \sqrt{x} e^{-x} dx = \int_0^{\infty} y e^{-y^2} dy^2 = -\int_0^{\infty} y d(e^{-y^2}) = -y e^{-y^2} \Big|_0^{\infty} + \int_0^{\infty} e^{-y^2} dy = \frac{\sqrt{\pi}}{2}$$

$$\rightarrow n \approx \frac{(2m_n^* kT)^{3/2}}{2\pi^2 \hbar^3} e^{-\frac{E_C-E_F}{kT}} \frac{\sqrt{\pi}}{2} = \frac{1}{4\hbar^3} \left(\frac{2m_n^* kT}{\pi} \right)^{3/2} e^{-\frac{E_C-E_F}{kT}} = N_C e^{-\frac{E_C-E_F}{kT}}$$

N_C = densita' efficace di portatori in BC

$$p = \int_{-\infty}^{E_V} g(E)[1 - f(E)] dE = \int_{-\infty}^{E_V} \frac{(2m_p^*)^{3/2}}{2\pi^2 \hbar^3} \sqrt{E_V - E} \left[1 - \frac{1}{e^{\frac{E - E_F}{kT}} + 1} \right] dE$$

$$1 - \frac{1}{e^{\frac{E - E_F}{kT}} + 1} = \frac{e^{\frac{E - E_F}{kT}}}{e^{\frac{E - E_F}{kT}} + 1} = \frac{1}{e^{-\frac{E - E_F}{kT}} + 1}$$

$$\rightarrow p = \int_{-\infty}^{E_V} \frac{(2m_p^*)^{3/2}}{2\pi^2 \hbar^2} \sqrt{E_V - E} \frac{1}{e^{-\frac{E - E_F}{kT}} + 1} dE$$

T basse $\rightarrow e^{\frac{E_F - E}{kT}} \gg 1 \rightarrow$ Fermi-Dirac \sim Maxwell-Boltzmann

$$\rightarrow p \approx \int_{-\infty}^{E_V} \frac{(2m_p^*)^{3/2}}{2\pi^2 \hbar^3} \sqrt{E_V - E} e^{\frac{E - E_F}{kT}} dE = \frac{(2m_p^*)^{3/2}}{2\pi^2 \hbar^3} e^{\frac{E_V - E_F}{kT}} \int_{-\infty}^{E_V} \sqrt{E_V - E} e^{\frac{E - E_V}{kT}} dE$$

$$\rightarrow p \approx \frac{(2m_p^* kT)^{3/2}}{2\pi^2 \hbar^3} e^{\frac{E_V - E_F}{kT}} \int_0^{\infty} \sqrt{x} e^{-x} dx = \frac{1}{4\hbar^3} \left(\frac{2m_p^* kT}{\pi} \right)^{3/2} e^{\frac{E_V - E_F}{kT}} = N_V e^{\frac{E_V - E_F}{kT}}$$

$N_V =$ densita' efficace di portatori in BV

Origine delle concentrazioni in BC e in BV:

"Code" delle distribuzioni di Fermi-Dirac a $T > 0$

$n(E) = f(E) g_C(E)$ coda dist. Fermi-Dirac * densita' stati in BC

$p(E) = [1 - f(E)] g_V(E)$ coda (1-dist. Fermi-Dirac) * densita' stati in BV

\rightarrow Attivazione termica dei portatori

Livello di Fermi nel semiconduttore intrinseco:

$$n = p = n_i \rightarrow N_C e^{-\frac{E_C - E_F}{kT}} = N_V e^{\frac{E_V - E_F}{kT}}$$

$$\rightarrow 1 = \frac{N_V e^{\frac{E_V - E_F}{kT}}}{N_C e^{-\frac{E_C - E_F}{kT}}} = \frac{N_V}{N_C} e^{\frac{E_V - E_F}{kT}} e^{\frac{E_C - E_F}{kT}} = \frac{N_V}{N_C} e^{-\frac{2E_F}{kT}} e^{\frac{E_C + E_V}{kT}}$$

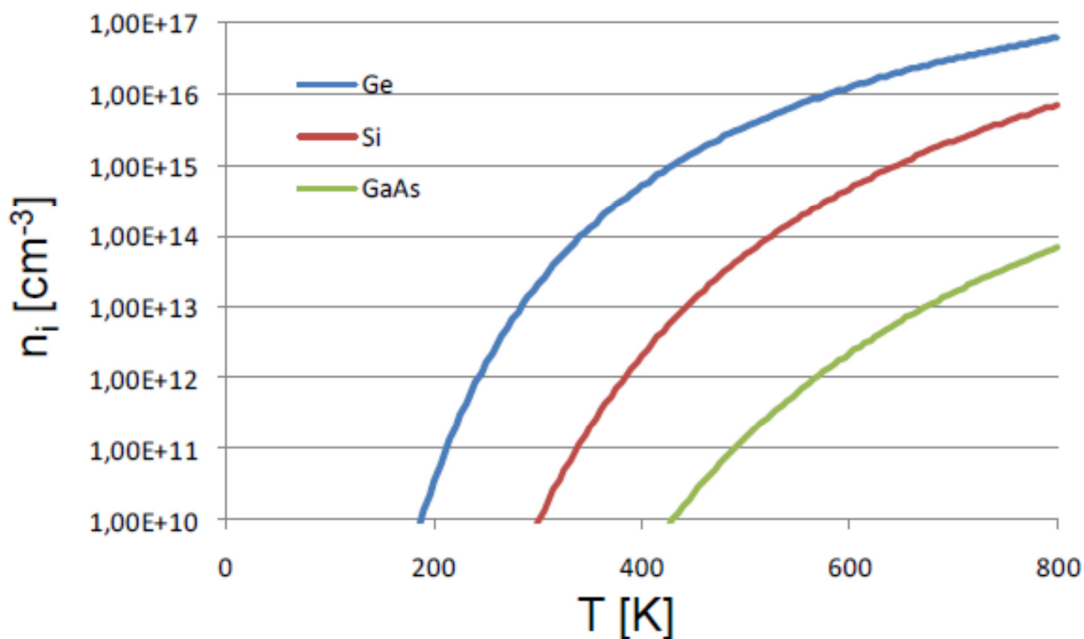
$$\rightarrow e^{\frac{2E_F}{kT}} = \frac{N_V}{N_C} e^{\frac{E_C + E_V}{kT}} \rightarrow E_F = \frac{kT}{2} \ln \frac{N_V}{N_C} + \frac{E_C + E_V}{2} \equiv E_i$$

$$\rightarrow E_i \approx \frac{E_G}{2} + \frac{kT}{2} \ln \frac{N_V}{N_C} = \frac{E_G}{2} + \frac{kT}{2} \ln \left(\frac{m_p^*}{m_n^*} \right)^{3/2} = \frac{E_G}{2} + \frac{3kT}{4} \ln \left(\frac{m_p^*}{m_n^*} \right)$$

Dipendenza da T delle concentrazioni intrinseche:

$$n_i^2 = N_C N_V e^{\frac{E_V - E_F}{kT}} e^{-\frac{E_C - E_F}{kT}} = N_C N_V e^{-\frac{E_G}{kT}} \rightarrow n_i = (N_C N_V)^{1/2} e^{-\frac{E_G}{2kT}}$$

	N_C [cm ⁻³]	N_V [cm ⁻³]	E_G [eV]	n_i [cm ⁻³]
Ge	1,03E+19	5,35E+18	0,66	2,15E+13
Si	3,22E+19	1,83E+19	1,12	9,68E+09
GaAs	4,21E+17	9,52E+18	1,42	2,42E+06



In presenza di impurita' (v.dopo):

Spostamento del livello di Fermi

→ Cambiamento delle popolazioni nelle code delle distribuzioni

→ Non cambiano le espressioni delle concentrazioni, solo $E_i \rightarrow E_F$

Tuttavia: Legge di azione di massa:

$$\left. \begin{aligned} n &= N_C e^{-\frac{E_C - E_F}{kT}} \\ n_i &= N_C e^{-\frac{E_C - E_i}{kT}} \\ p &= N_V e^{-\frac{E_V - E_F}{kT}} \\ p_i &= N_V e^{-\frac{E_V - E_i}{kT}} \equiv n_i \end{aligned} \right\} \rightarrow \begin{aligned} n &= N_C e^{-\frac{E_C - E_i}{kT}} e^{-\frac{E_i - E_F}{kT}} = n_i e^{-\frac{E_i - E_F}{kT}} \\ p &= N_V e^{-\frac{E_V - E_i}{kT}} e^{-\frac{E_i - E_F}{kT}} = p_i e^{-\frac{E_i - E_F}{kT}} \end{aligned}$$

→ $np = n_i p_i = n_i^2$ Legge di azione di massa

Ricordo di Chimica:

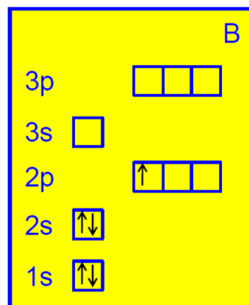
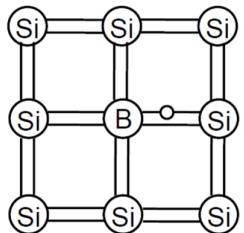
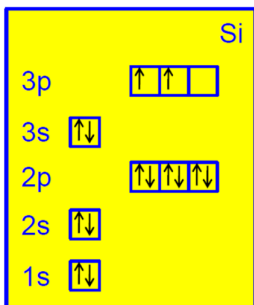
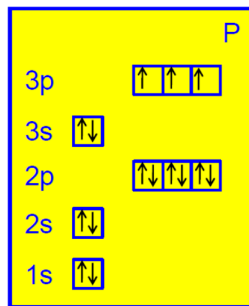
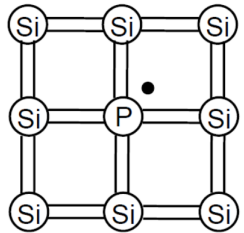
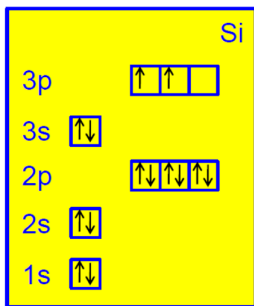
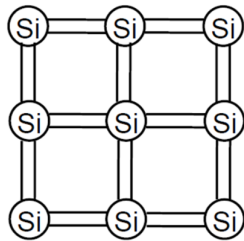
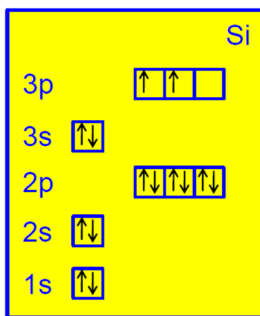
$$\frac{[\text{H}_2]^2 \cdot [\text{O}_2]}{[\text{H}_2\text{O}]^2} = K(T, p)$$

Prodotto np indipendente dalla presenza di droganti

Estrinseco = impuro, drogato

Introduzione di impurezze: popola la banda di conduzione o di valenza a seconda del tipo di drogante, donore o accettore

Il livello di Fermi si sposta dal livello intrinseco per assecondare lo sbilanciamento delle concentrazioni



Per confronto, i valori misurati in eV per donori ($E_C - E_D$) e accettori ($E_A - E_V$) in Si sono:

Li	Sb	P	As	B	Al	Ga	In
0,033	0,039	0,045	0,054	0,045	0,067	0,072	0,16

Donori e accettori \sim Interamente ionizzati

$$np = n_i^2$$

\rightarrow Spostamento livello di Fermi

Drogaggio n :

$$\left. \begin{array}{l} n \approx N_D \\ p = \frac{n_i^2}{N_D} \ll n \end{array} \right\} \rightarrow n = n_i e^{-\frac{E_i - E_F}{kT}} \rightarrow E_F = kT \ln \frac{n}{n_i} + E_i \approx kT \ln \frac{N_D}{n_i} + E_i$$

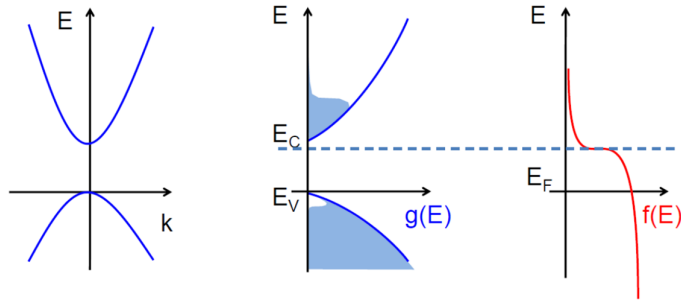
$$E_F = kT \ln \frac{n_i}{p} + E_i \quad \text{da l. di azione di massa}$$

Drogaggio p :

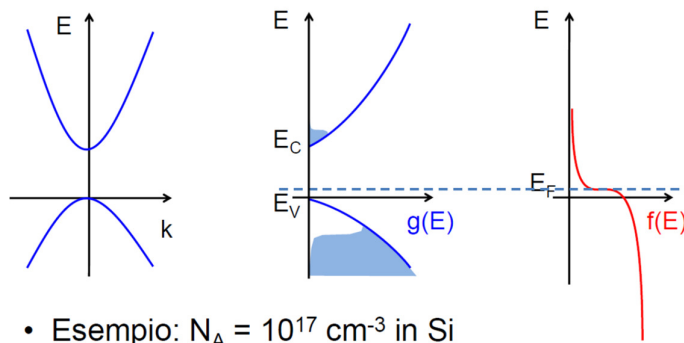
$$\left. \begin{array}{l} p \approx N_A \\ n = \frac{n_i^2}{N_A} \ll p \end{array} \right\} \rightarrow p = n_i e^{\frac{E_i - E_F}{kT}} \rightarrow E_F = -kT \ln \frac{p}{n_i} + E_i \approx -kT \ln \frac{N_A}{n_i} + E_i$$

$$E_F = -kT \ln \frac{n_i}{n} + E_i \quad \text{c.s.}$$

$$\rightarrow \left\{ \begin{array}{l} n = n_i e^{-\frac{\Delta E_F}{kT}} \\ p = p_i e^{\frac{\Delta E_F}{kT}} \end{array} \right.$$



- Esempio: $N_D = 10^{18} \text{ cm}^{-3}$ in Si
- $p = n_i^2/N_D \approx 10^2 \text{ cm}^{-3}$
- $E_F = E_i + kT \log N_D/n_i = E_i + 0.48 \text{ eV} \approx 1.04 \text{ eV}$



- Esempio: $N_A = 10^{17} \text{ cm}^{-3}$ in Si
- $n = n_i^2/N_A \approx 10^3 \text{ cm}^{-3}$
- $E_F = E_i - kT \log N_A/n_i = E_i - 0.42 \text{ eV} \approx 0.14 \text{ eV}$

