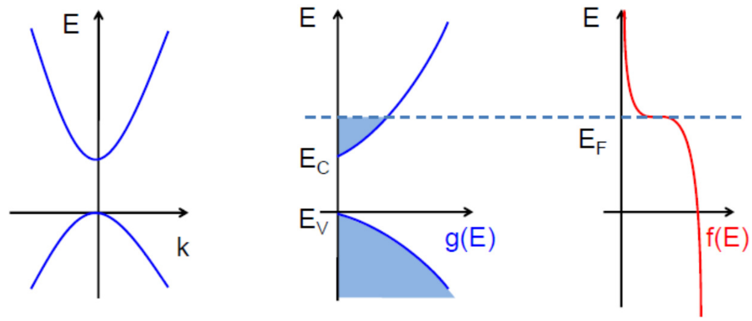
Metalli



Concentrazione portatori:

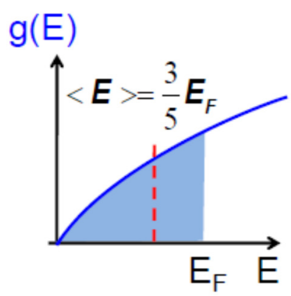
$$n = \int_{E_C}^{\infty} g(E) f(E)\, dE = \int_{E_C}^{\infty} \frac{\left(2m_n^*\right)^{3/2}}{2\pi^2\hbar^3} \sqrt{E - E_C}\, \frac{1}{e^{\frac{E-E_F}{kT}} + 1}\, dE$$

Assumendo Fermi-Dirac = Step:

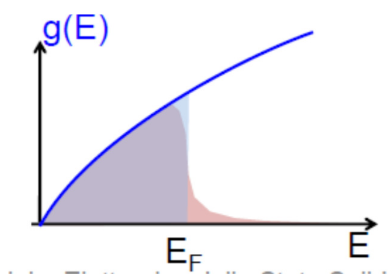
$$n = \int_{0}^{E_F} \frac{\left(2m_n^*\right)^{3/2}}{2\pi^2\hbar^3} \sqrt{E}\, dE = \frac{\left(2m_n^*\right)^{3/2}}{3\pi^2\hbar^3} E_F^{3/2}$$

$$\rightarrow E_F = \frac{\hbar^3 \left(3n\pi^2\right)^{2/3}}{2m_n^*} \quad \text{a } T = 0\ K$$

$$\langle E \rangle = \frac{1}{n} \int_{0}^{\infty} E g(E) f(E)\, dE \approx \frac{1}{n} \int_{0}^{E_F} \frac{\left(2m_n^*\right)^{3/2}}{2\pi^2\hbar^3} E^{3/2}\, dE = \frac{3}{5} E_F$$



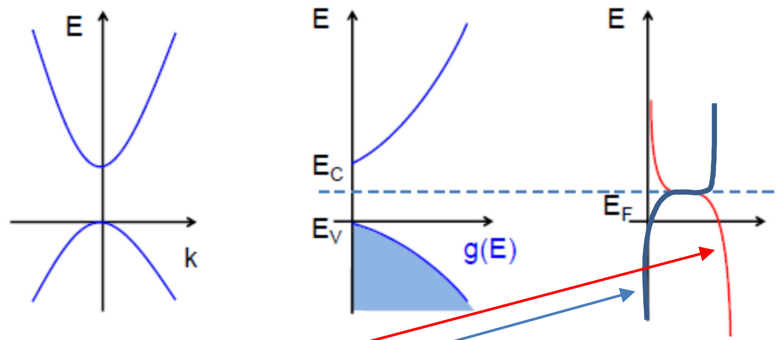
T = 0 K



T > 0 K

E_F poco dipendente da T

Semiconduttori/isolanti



$f(E)$ Fermi-Dirac (BC)

$1-f(E)$ Complemento a 1 di Fermi-Dirac (BV)

Caso intrinseco $\equiv$ puro

Concentrazioni:

$$n=\int_{E_C}^{\infty} g(E)f(E)\,dE=\int_{E_C}^{\infty}\frac{\left(2m_n^*\right)^{3/2}}{2\pi^2\hbar^3}\sqrt{E-E_C}\,\frac{1}{e^{\frac{E-E_F}{kT}}+1}\,dE$$

T basse $\rightarrow e^{\frac{E-E_F}{kT}}\gg 1\rightarrow$ Fermi-Dirac $\sim$ Maxwell-Boltzmann

$$\rightarrow n\approx\int_{E_C}^{\infty}\frac{\left(2m_n^*\right)^{3/2}}{2\pi^2\hbar^3}\sqrt{E-E_C}\,e^{-\frac{E-E_F}{kT}}\,dE=\frac{\left(2m_n^*\right)^{3/2}}{2\pi^2\hbar^3}e^{-\frac{E_C-E_F}{kT}}\int_{E_C}^{\infty}\sqrt{\underbrace{E-E_C}_{xkT}}\,e^{-\frac{E-E_C}{kT}}\,dE$$

$$x=\frac{E-E_C}{kT}\rightarrow dE=kTdx$$

$$\rightarrow n\approx\frac{\left(2m_n^*kT\right)^{3/2}}{2\pi^2\hbar^3}e^{-\frac{E_C-E_F}{kT}}\int_0^{\infty}\sqrt{x}e^{-x}dx$$

$$y=\sqrt{x}\rightarrow x=y^2\rightarrow dx=2ydy$$

$$\int_0^{\infty}\sqrt{x}e^{-x}dx=\int_0^{\infty}ye^{-y^2}dy^2=-\int_0^{\infty}yd\left(e^{-y^2}\right)=-y\,e^{-y^2}\Big|_0^{\infty}+\int_0^{\infty}e^{-y^2}dy=\frac{\sqrt{\pi}}{2}$$

$$\rightarrow n\approx\frac{\left(2m_n^*kT\right)^{3/2}}{2\pi^2\hbar^3}e^{-\frac{E_C-E_F}{kT}}\frac{\sqrt{\pi}}{2}=\frac{1}{4\hbar^3}\left(\frac{2m_n^*kT}{\pi}\right)^{3/2}e^{-\frac{E_C-E_F}{kT}}=N_Ce^{-\frac{E_C-E_F}{kT}}$$

$N_C=$ densita' efficace di portatori in BC

$$p=\int_{-\infty}^{E_V} g(E)\left[1-f(E)\right]dE=\int_{-\infty}^{E_V}\frac{\left(2m_p^*\right)^{3/2}}{2\pi^2\hbar^3}\sqrt{E_V-E}\left[1-\frac{1}{e^{\frac{E-E_F}{kT}}+1}\right]dE$$

$$1-\frac{1}{e^{\frac{E-E_F}{kT}}+1}=\frac{e^{\frac{E-E_F}{kT}}}{e^{\frac{E-E_F}{kT}}+1}=\frac{1}{e^{-\frac{E-E_F}{kT}}+1}$$

$$\rightarrow p=\int_{-\infty}^{E_V}\frac{\left(2m_p^*\right)^{3/2}}{2\pi^2\hbar^2}\sqrt{E_V-E}\,\frac{1}{e^{-\frac{E-E_F}{kT}}+1}dE$$

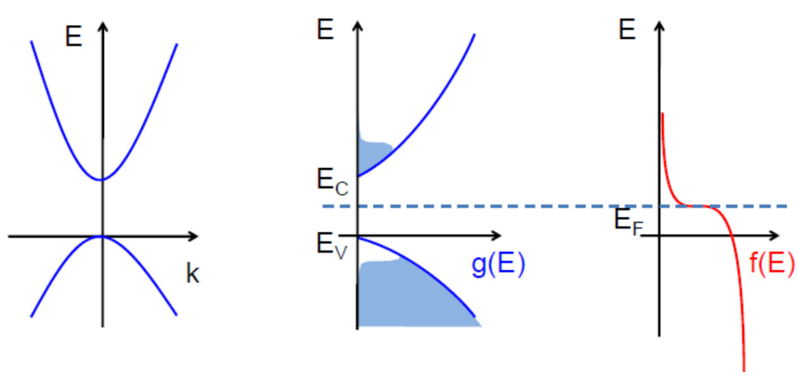
T basse $\rightarrow e^{\frac{E_F-E}{kT}}\gg 1\rightarrow$ Fermi-Dirac $\sim$ Maxwell-Boltzmann

$$\rightarrow n\approx\int_{E_C}^{\infty}\frac{\left(2m_p^*\right)^{3/2}}{2\pi^2\hbar^3}\sqrt{E_V-E}\,e^{\frac{E-E_F}{kT}}dE=\frac{\left(2m_p^*\right)^{3/2}}{2\pi^2\hbar^3}e^{\frac{E_V-E_F}{kT}}\int_{E_C}^{\infty}\sqrt{E_V-E}\,e^{\frac{E-E_V}{kT}}dE$$

$$\rightarrow p\approx\frac{\left(2m_p^*kT\right)^{3/2}}{2\pi^2\hbar^3}e^{\frac{E_V-E_F}{kT}}\int_0^{\infty}\sqrt{x}e^{-x}dx=\frac{1}{4\hbar^3}\left(\frac{2m_p^*kT}{\pi}\right)^{3/2}e^{\frac{E_V-E_F}{kT}}=N_V e^{\frac{E_V-E_F}{kT}}$$

$N_V =$ densita' efficace di portatori in BV

$n(E)=f(E)\,g_C(E)$	coda dist. Fermi-Dirac * densita' stati in BC
$p(E)=\left[1-f(E)\right]g_V(E)$	coda $($1-dist. Fermi-Dirac$)$ * densita' stati in BV



Livello di Fermi nel semiconduttore intrinseco:

$$n = p = n_i \rightarrow N_C e^{-\frac{E_C - E_F}{kT}} = N_V e^{\frac{E_V - E_F}{kT}}$$

$$\rightarrow 1 = \frac{N_V e^{\frac{E_V - E_F}{kT}}}{N_C e^{-\frac{E_C - E_F}{kT}}} = \frac{N_V}{N_C} e^{\frac{E_V - E_F}{kT}} e^{\frac{E_C - E_F}{kT}} = \frac{N_V}{N_C} e^{-\frac{2E_F}{kT}} e^{\frac{E_C + E_V}{kT}}$$

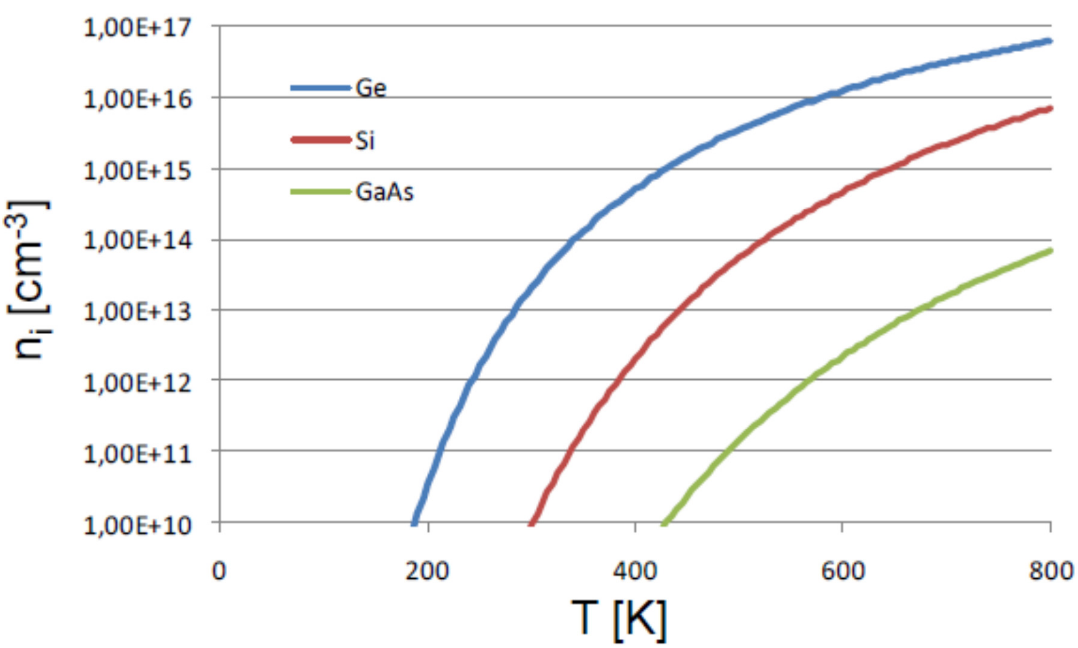
$$\rightarrow e^{\frac{2E_F}{kT}} = \frac{N_V}{N_C} e^{\frac{E_C + E_V}{kT}} \rightarrow E_F = \frac{kT}{2} \ln \frac{N_V}{N_C} + \frac{E_C + E_V}{2} \equiv E_i$$

$$\rightarrow E_i \approx \frac{E_G}{2} + \frac{kT}{2} \ln \frac{N_V}{N_C} = \frac{E_G}{2} + \frac{kT}{2} \ln \left(\frac{m_p^*}{m_n^*} \right)^{3/2} = \frac{E_G}{2} + \frac{3kT}{4} \ln \left(\frac{m_p^*}{m_n^*} \right)$$

Dipendenza da T delle concentrazioni intrinseche:

$$n_i^2 = N_C N_V e^{\frac{E_V - E_F}{kT}} e^{-\frac{E_C - E_F}{kT}} = N_C N_V e^{-\frac{E_G}{kT}} \rightarrow n_i = \left(N_C N_V \right)^{1/2} e^{-\frac{E_G}{2kT}}$$

	N_C [cm^{-3}]	N_V [cm^{-3}]	E_G [eV]	n_i [cm^{-3}]
Ge	1,03E+19	5,35E+18	0,66	2,15E+13
Si	3,22E+19	1,83E+19	1,12	9,68E+09
GaAs	4,21E+17	9,52E+18	1,42	2,42E+06



Legge di azione di massa:

$$\left.\begin{array}{l} n = N_C e^{-\frac{E_C - E_F}{kT}} \\ n_i = N_C e^{-\frac{E_C - E_i}{kT}} \\ p = N_V e^{\frac{E_V - E_F}{kT}} \\ p_i = N_V e^{\frac{E_V - E_i}{kT}} \equiv n_i \end{array}\right\} \rightarrow \begin{array}{l} n = N_C e^{-\frac{E_C - E_i}{kT}} e^{-\frac{E_i - E_F}{kT}} = n_i e^{-\frac{E_i - E_F}{kT}} \\ p = N_V e^{\frac{E_V - E_i}{kT}} e^{\frac{E_i - E_F}{kT}} = p_i e^{\frac{E_i - E_F}{kT}} \end{array}$$

$\rightarrow np = n_i p_i = n_i^2$ Legge di azione di massa

Ricordo di Chimica:

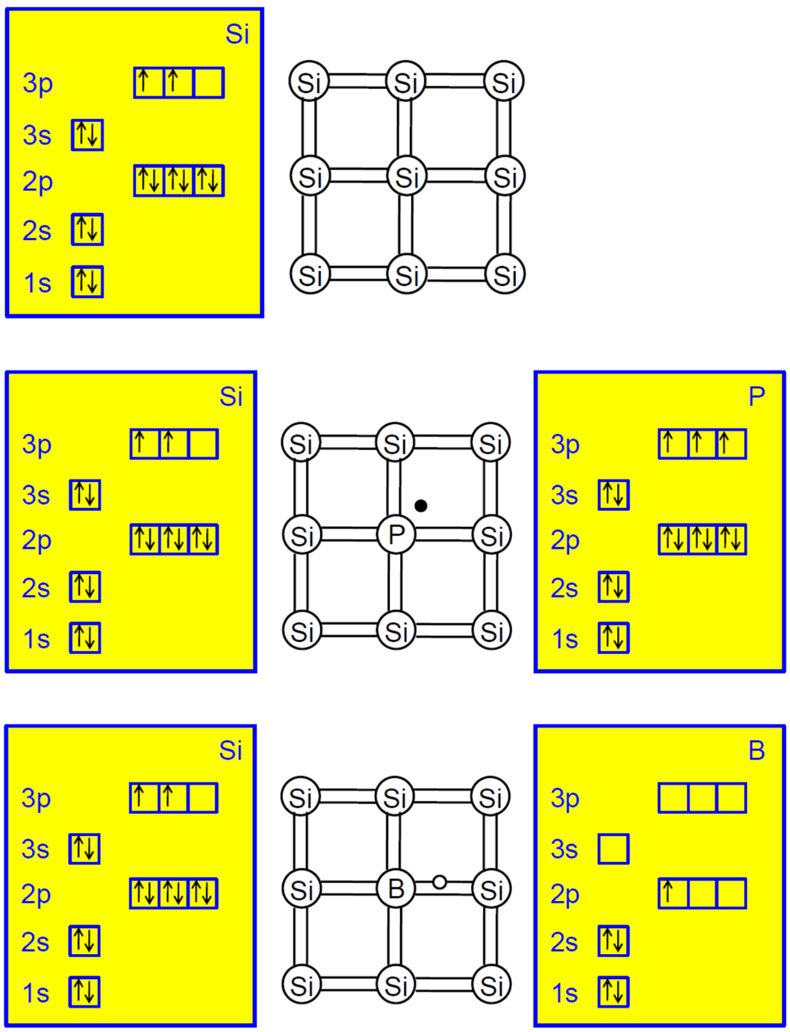
$$\frac{[H_2]^2 \cdot [O_2]}{[H_2O]^2} = K(T, p)$$

Prodotto *np* indipendente dalla presenza di droganti

Estrinseco = impuro, drogato

Introduzione di impurezze: popolo la banda di conduzione o di valenza a seconda del tipo di drogante, donore o accettore

Il livello di Fermi si sposta dal livello intrinseco per assecondare lo sbilanciamento delle concentrazioni



Per confronto, i valori misurati in eV per donori (E_C-E_D) e accettori (E_A-E_V) in Si sono:

Li	Sb	P	As	B	Al	Ga	In
0,033	0,039	0,045	0,054	0,045	0,067	0,072	0,16

Donori e accettori $\sim$ Interamente ionizzati

$np = n_i^2$

$\rightarrow$ Spostamento livello di Fermi

Drogaggio *n*:

$$\left.\begin{aligned} & n \approx N_D \\ & p = \frac{n_i^2}{N_D} \ll n \end{aligned}\right\} \rightarrow n = n_i e^{-\frac{E_i - E_F}{kT}} \rightarrow E_F = kT \ln \frac{n}{n_i} + E_i \approx kT \ln \frac{N_D}{n_i} + E_i$$

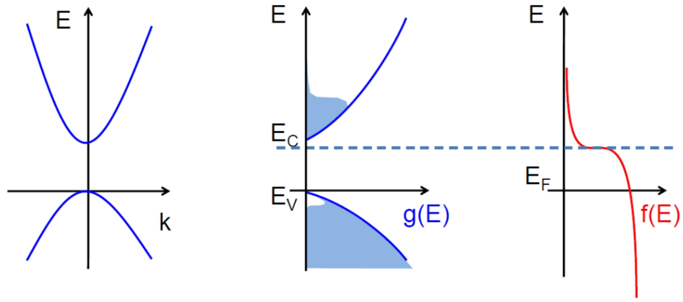
$E_F = kT \ln \frac{n_i}{p} + E_i$ da l. di azione di massa

Drogaggio p :

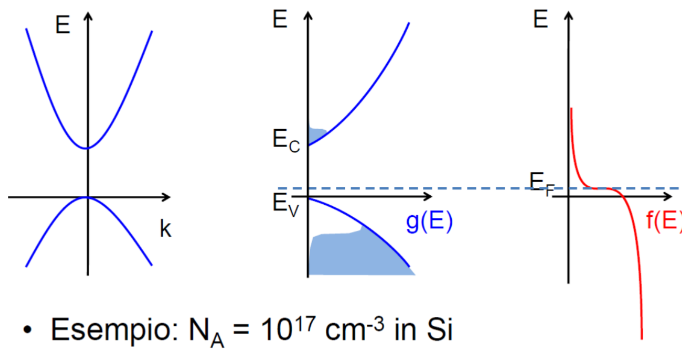
$$\left.\begin{aligned} & p \approx N_A \\ & n = \frac{n_i^2}{N_A} \ll p \end{aligned}\right\} \rightarrow p = n_i e^{\frac{E_i - E_F}{kT}} \rightarrow E_F = -kT \ln \frac{p}{n_i} + E_i \approx -kT \ln \frac{N_A}{n_i} + E_i$$

$E_F = -kT \ln \frac{n_i}{n} + E_i$ c.s.

$$\rightarrow \begin{cases} n = n_i e^{-\frac{\Delta E_F}{kT}} \\ p = p_i e^{\frac{\Delta E_F}{kT}} \end{cases}$$



- Esempio: $N_D = 10^{18}$ cm^{-3} in Si
- $p = n_i^2/N_D \approx 10^2$ cm^{-3}
- $E_F = E_i + kT\log N_D/n_i = E_i + 0.48$ eV ≈ 1.04 eV



- Esempio: $N_A = 10^{17}$ cm^{-3} in Si
- $n = n_i^2/N_A \approx 10^3$ cm^{-3}
- $E_F = E_i - kT\log N_A/n_i = E_i - 0.42$ eV ≈ 0.14 eV