

## 17 - BJT : Modello a $\pi$ ibrido

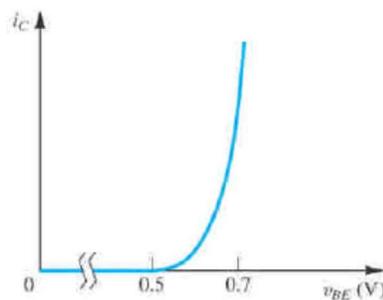
Convenzione, per tensioni e correnti:

$a_x$  val. incrementale

$a_X$  val. totale

$A_X$  val. statico

Trascaratteristica a emettitore comune:  $i_C$  vs  $v_{BE}$



$I_C = I_S e^{V_{BE}/V_T}$  simile a diodo (ma  $I_C$  non e' la corrente della giunzione)

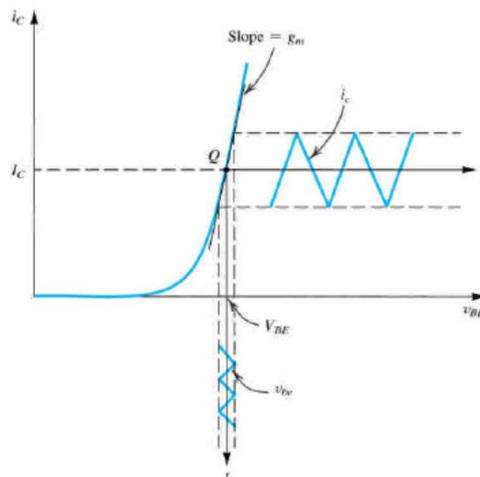
$v_{BE} = V_{BE} + v_{be}$  separazione di tensione statica e dinamica

$$\rightarrow i_C = I_S e^{v_{BE}/V_T} = I_S e^{(V_{BE} + v_{be})/V_T} = I_C e^{v_{be}/V_T}$$

Piccoli segnali:

$$v_{be} \ll V_T \rightarrow i_C \approx I_C \left( 1 + \frac{v_{be}}{V_T} \right) = I_C + \frac{I_C}{V_T} v_{be} \rightarrow i_c \approx \frac{I_C}{V_T} v_{be}$$

$$\frac{I_C}{V_T} \equiv g_m \text{ transconduttanza, } \frac{V_T}{I_C} \equiv r_e \text{ res. equivalente di emettitore } \rightarrow r_e = \frac{1}{g_m}$$

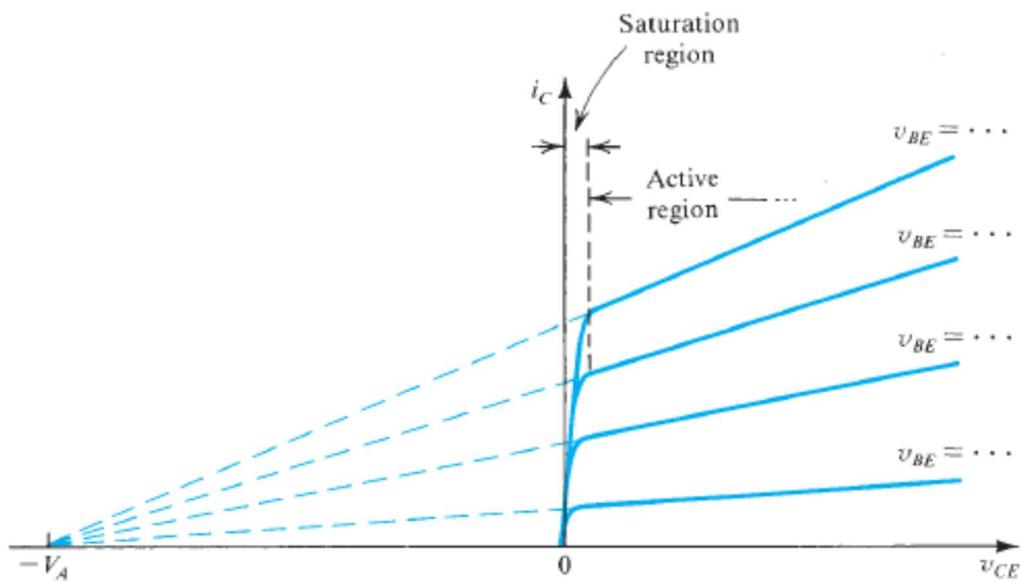


Non idealita' delle caratteristiche di uscita:

Corrente di collettore *non* indipendente dalla tensione CE

→ Il collettore *non* e' un generatore ideale di corrente

→ Equivalente a generatore ideale con in parallelo resistenza interna  $r_0$



Effetto Early:  $V_A$  tensione di Early

$$i_B = \frac{i_C}{\beta} = \frac{I_C}{\beta} + \frac{i_c}{\beta} = \frac{I_C}{\beta} + \frac{1}{\beta} \frac{I_C}{V_T} v_{be} = I_B + \frac{1}{\beta} \frac{I_C}{V_T} v_{be}$$

$$i_B = I_B + i_b$$

$$\rightarrow i_b = \frac{1}{\beta} \frac{I_C}{V_T} v_{be} = \frac{g_m}{\beta} v_{be} \equiv \frac{1}{r_\pi} v_{be}$$

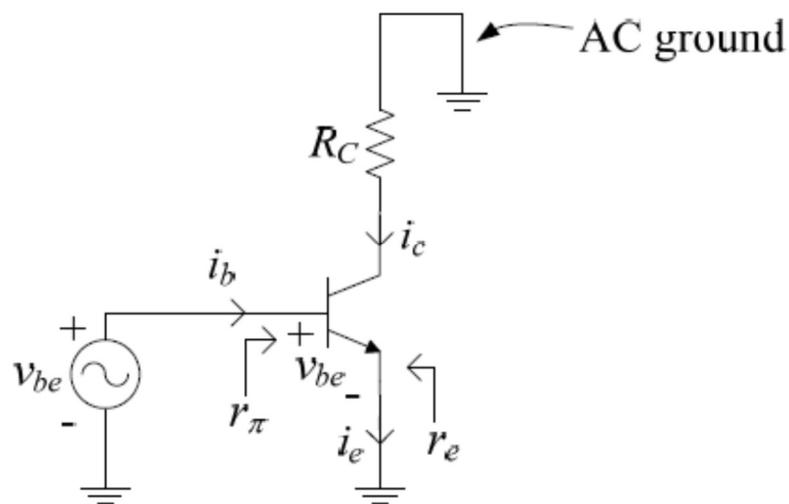
$$\rightarrow r_\pi = \beta \frac{V_T}{I_C} = \frac{V_T}{I_B}$$

$$i_E = I_E + i_e = \frac{i_C}{\alpha} = \frac{I_C}{\alpha} + \frac{i_c}{\alpha}$$

$$r_e \equiv \frac{v_{be}}{i_e} = \frac{i_c V_T}{i_e I_C} = \alpha \frac{V_T}{I_C} = \frac{V_T}{I_E} = \frac{\alpha}{g_m} \sim \frac{1}{g_m} \text{ res. equivalente di emettitore}$$

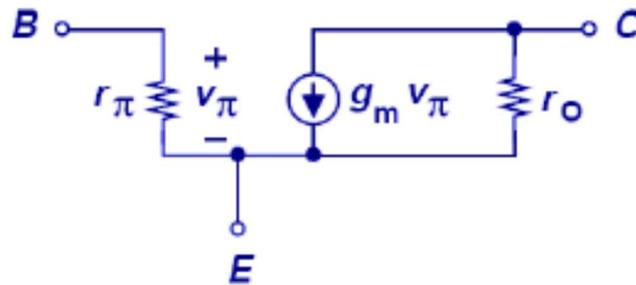
$$v_{be} = i_b r_\pi = i_e r_e \rightarrow r_\pi = \frac{i_e}{i_b} r_e \rightarrow r_\pi = (\beta + 1) r_e$$

Schema *in AC* della configurazione a emettitore comune: tensioni continue azzerate

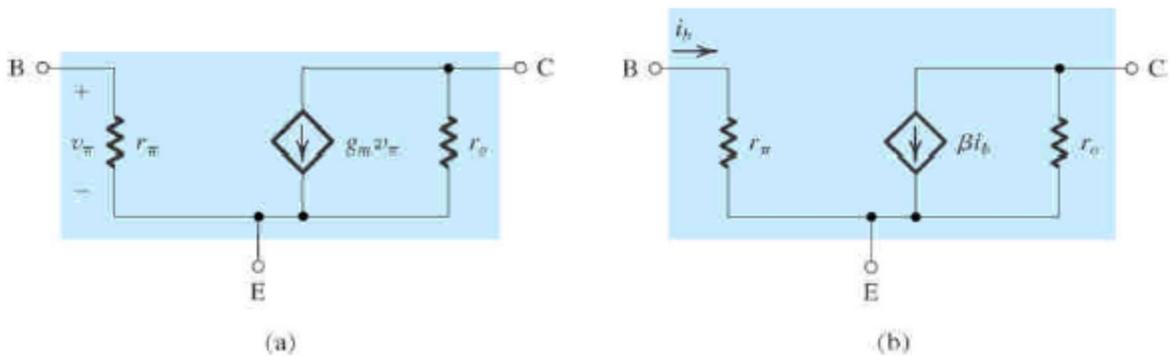


Conduce a un modello migliorato per piccoli segnali: Modello a  $\pi$  ibrido

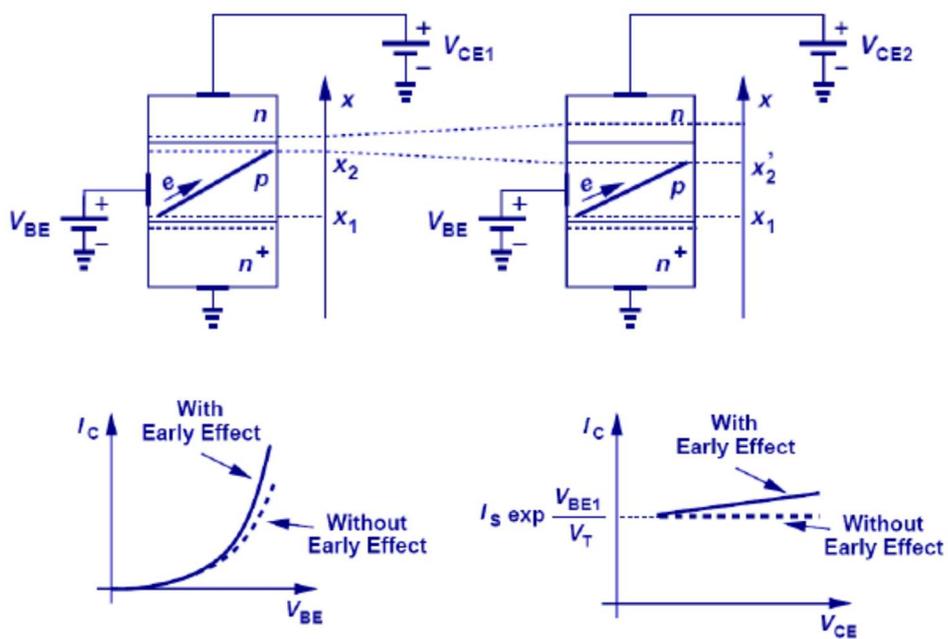
# Modello a $\pi$ ibrido: Versione a bassa frequenza

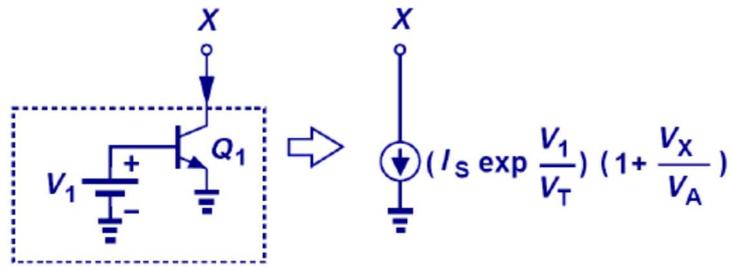


## Versioni equivalenti

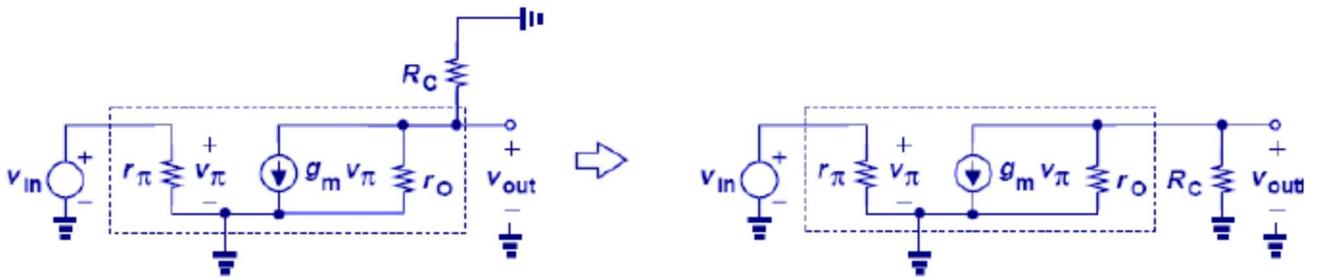
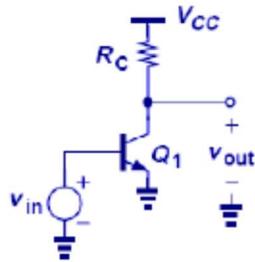


$r_o$  legata all'effetto Early:  $I_C$  non indipendente da  $V_{CE}$

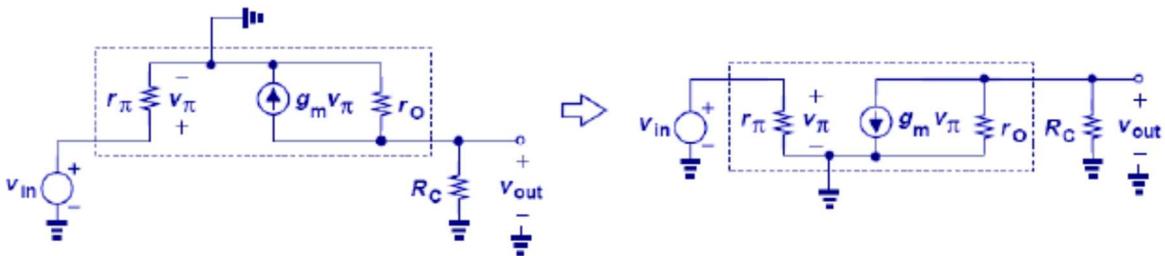
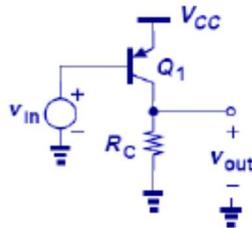




$C$  generatore non ideale di corrente  $\rightarrow r_0 < \infty$  in parallelo al generatore di corrente  
 Esempio: Applicazione a stadio semplice CE



Identico per PNP:



Impedenze intrinseche del BJT (in zona attiva!):

Base: Si 'guarda' dentro B

stato uscita irrilevante perche'  $r_\pi$  a ground

$$\rightarrow R_b = r_\pi$$

Collettore: Si 'guarda' dentro C

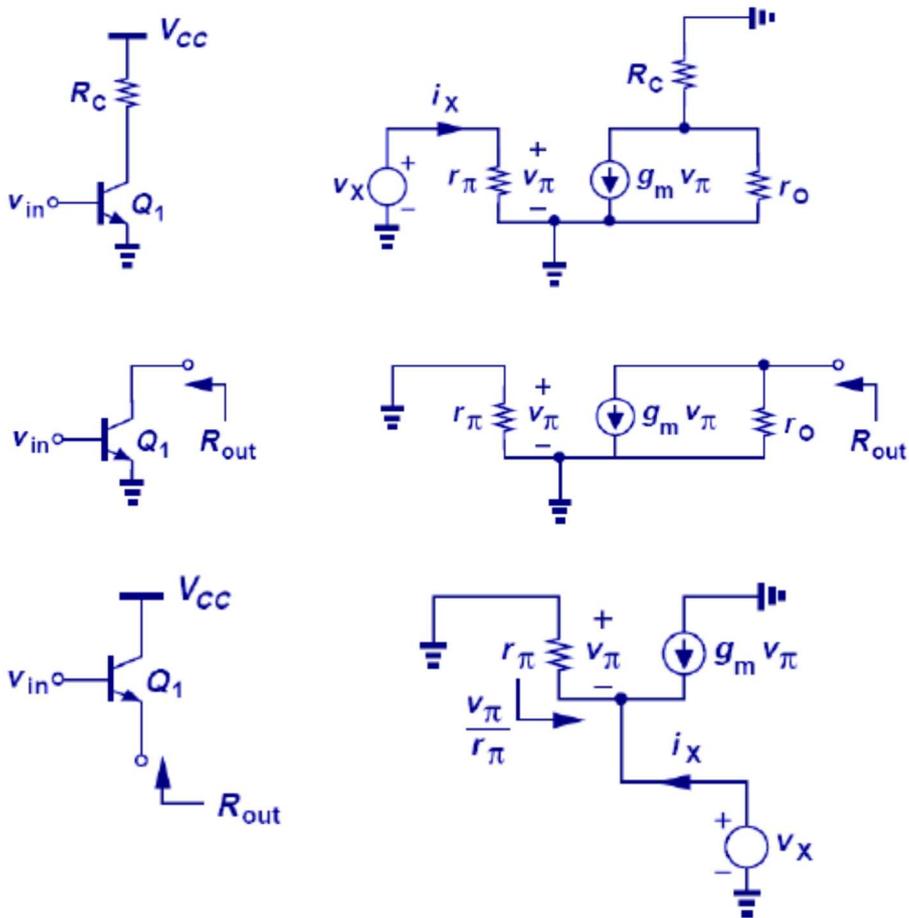
B (ed E) a ground  $\rightarrow v_\pi = 0 \rightarrow g_m v_\pi = 0$

$$\rightarrow R_c = r_o \approx \infty$$

Emettitore : Si 'guarda' dentro E

B e C a ground  $\rightarrow i_x = \frac{v_x}{r_\pi} + g_m v_\pi, v_x = v_\pi$

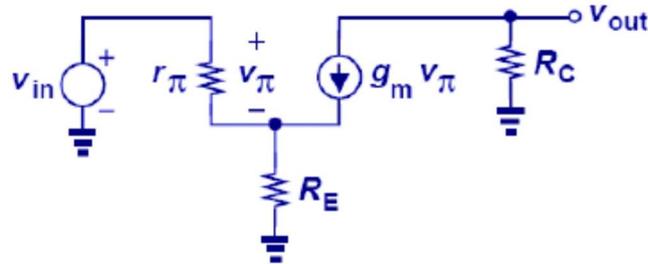
$$\rightarrow R_e = \frac{v_x}{i_x} = \frac{1}{g_m + \frac{1}{r_\pi}} \approx \frac{1}{g_m}, r_o \rightarrow \infty;$$



Applicazione: Stadio CE con resistenza su emettitore

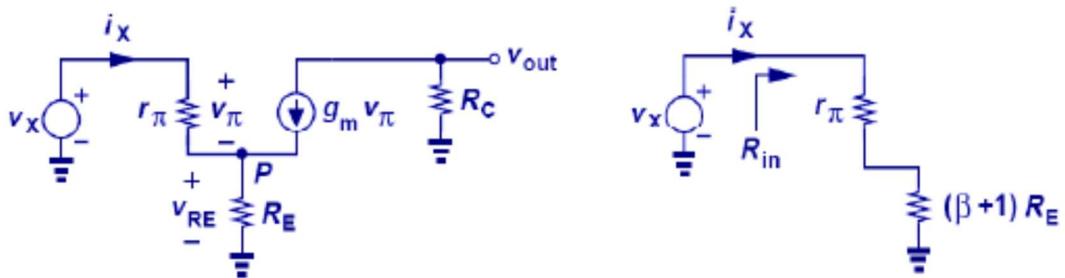
NB: Non considerate resistenze del partitore di bias

Non considerato effetto Early



$$A_v = -\frac{v_o}{v_i} = -\frac{g_m v_\pi R_C}{v_\pi + g_m v_\pi R_E} = -\frac{g_m R_C}{1 + g_m R_E} = -\frac{R_C}{\frac{1}{g_m} + R_E} \xrightarrow{R_E \rightarrow 0} -g_m R_C$$

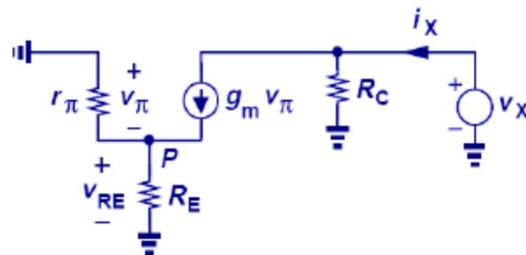
Impedenza di ingresso:



$$v_x = [r_\pi + R_E (1 + \beta)] i_x$$

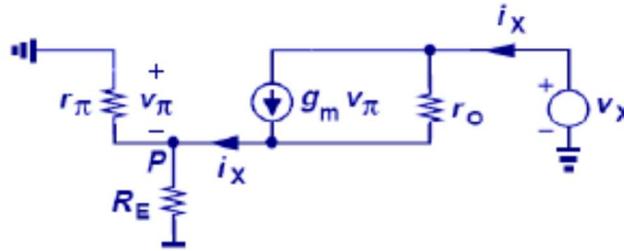
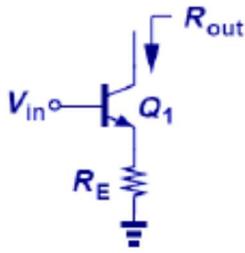
$$\rightarrow R_{in} = \frac{v_x}{i_x} = r_\pi + R_E (1 + \beta) \xrightarrow{R_E \rightarrow 0} r_\pi$$

Impedenza di uscita:



$$v_\pi + \left( \frac{v_\pi}{r_\pi} + g_m v_\pi \right) R_E = 0 \rightarrow v_\pi = 0 \rightarrow R_{out} = \frac{v_x}{i_x} = R_C$$

Tenendo conto dell'effetto Early:



$$i_x = \frac{v_x - v_\pi}{r_o} - g_m v_\pi \quad \text{-- perche' conf. invertente}$$

$$v_\pi = -i_x (R_E \parallel r_\pi) \quad \text{c.s.}$$

$$i_x = \frac{v_x}{r_o} + v_\pi \left( g_m + \frac{1}{r_o} \right)$$

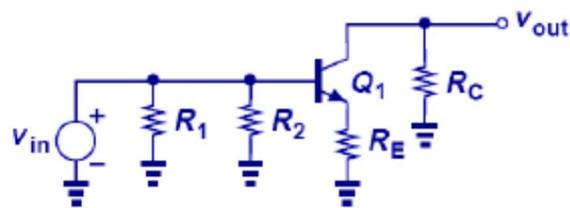
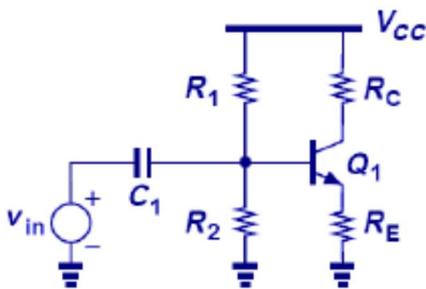
$$i_x \left[ 1 + (R_E \parallel r_\pi) \left( g_m + \frac{1}{r_o} \right) \right] = \frac{v_x}{r_o}$$

$$\rightarrow \frac{v_x}{i_x} = r_o \left[ 1 + (R_E \parallel r_\pi) \left( g_m + \frac{1}{r_o} \right) \right] \quad \text{imp. di uscita}$$

$$\rightarrow R_{out} \approx r_o \left[ 1 + (R_E \parallel r_\pi) g_m \right] \quad g_m r_o \gg 1$$

$R_{out}$  aumentata  $\rightarrow$  Migliorato generatore di corrente

Tenendo conto del partitore di bias:

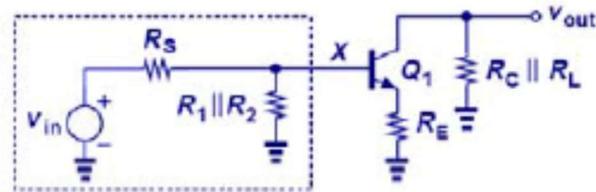
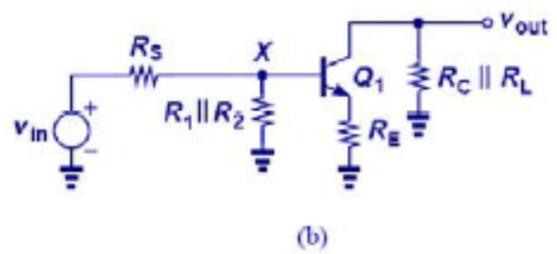
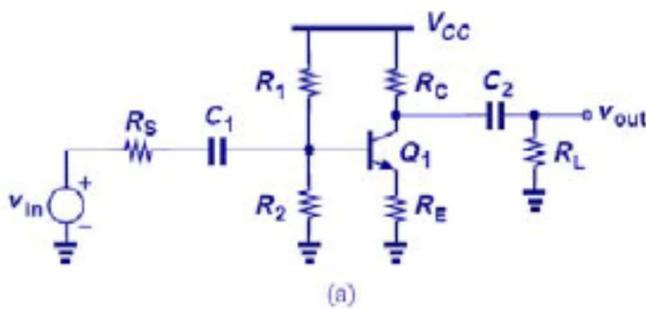


$$A_v = -\frac{R_C}{\frac{1}{g_m} + R_E}$$

$$R_{in} = [r_\pi + (\beta + 1)R_E] \parallel R_1 \parallel R_2$$

$$R_{out} = R_C$$

Al gran completo:



$$A_v = -\frac{R_C \parallel R_L}{\frac{1}{g_m} + R_E + \frac{R_S \parallel R_1 \parallel R_2}{\beta + 1}}$$