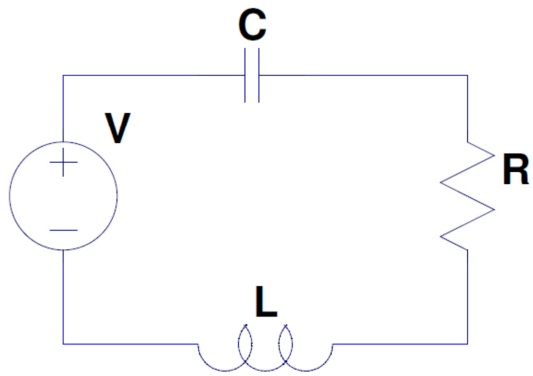
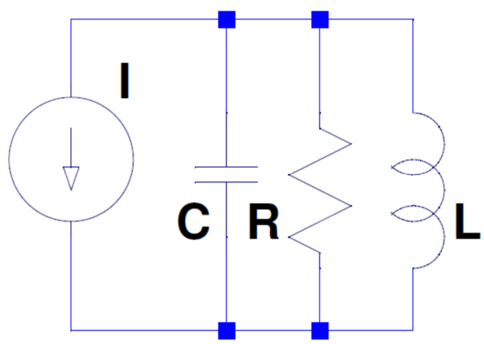
Circuiti risonanti: Due casi tipici

a) Risonanza serie



b) Risonanza parallelo



$$V = Z_T i$$

$$Z_T = R + j\omega L + \frac{1}{j\omega C} = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$T(\omega) = \frac{i}{V} = \frac{1}{Z_T} = \frac{1}{R + j\left(\omega L - \frac{1}{\omega C}\right)} = \frac{R - j\left(\omega L - \frac{1}{\omega C}\right)}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \qquad [T] = R^{-1}$$

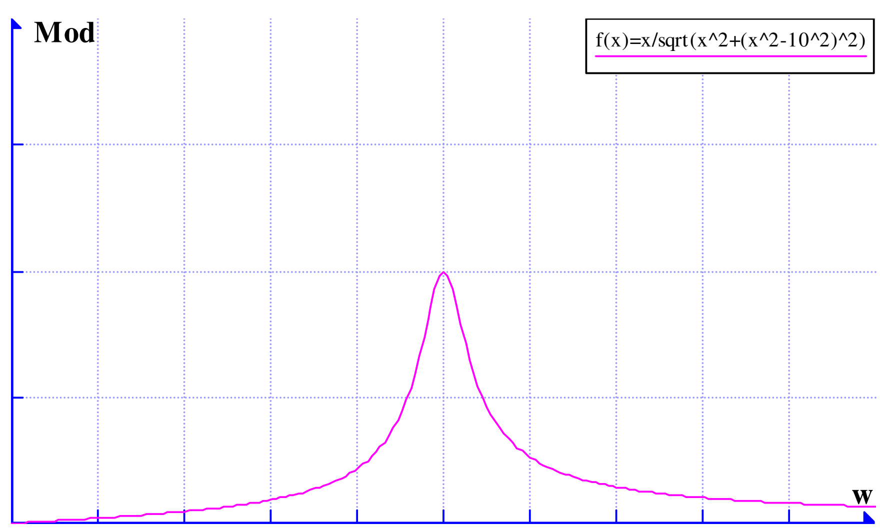
$$\rightarrow \left|T(\omega)\right|^2 = \frac{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^2} = \frac{1}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \frac{\omega^2 C^2}{\omega^2 C^2 R^2 + \left(\omega^2 LC - 1\right)^2}$$

$$\rightarrow \left|T(\omega)\right| = \frac{\omega C}{\sqrt{\omega^2 C^2 R^2 + \left(\omega^2 LC - 1\right)^2}}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \rightarrow \omega^2 C^2 R^2 + \left(\omega^2 LC - 1\right)^2 = \omega^2 C^2 R^2 + \left(\frac{\omega^2}{\omega_o^2} - 1\right)^2$$

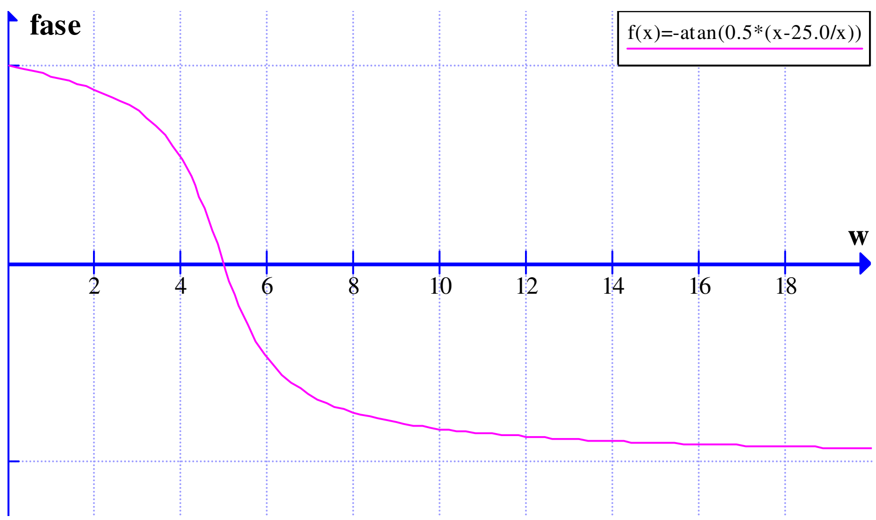
$$= \frac{\omega^2 C^2 L^2 R^2}{L^2} + \left(\frac{\omega^2 - \omega_o^2}{\omega_o^2}\right)^2 = \frac{\omega^2 \frac{R^2}{L^2}}{\omega_o^4} + \frac{\left(\omega^2 - \omega_o^2\right)^2}{\omega_o^4} = \frac{\omega^2 \frac{R^2}{L^2} + \left(\omega^2 - \omega_o^2\right)^2}{\omega_o^4}$$

$$\rightarrow \left|T(\omega)\right| = \frac{\omega C}{\sqrt{\omega^2 C^2 R^2 + \left(\omega^2 LC - 1\right)^2}} = \frac{\omega \omega_o^2 C}{\sqrt{\omega^2 \frac{R^2}{L^2} + \left(\omega^2 - \omega_o^2\right)^2}} = \frac{\omega}{R\sqrt{\omega^2 + \frac{L^2}{R^2}\left(\omega^2 - \omega_o^2\right)^2}}$$



$$T(\omega)=\frac{R-j\left(\omega L-\frac{1}{\omega C}\right)}{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}$$

$$\rightarrow \varphi_T=\arctan\left[-\frac{L}{R}\left(\omega-\frac{1}{\omega CL}\right)\right]=-\arctan\left[\frac{L}{R}\left(\omega-\frac{\omega_0^2}{\omega}\right)\right]$$



$$V = Z_T i$$

$$Y_T = \frac{1}{R} + \frac{1}{j\omega L} + j\omega C = \frac{R + j\omega L + j\omega C j\omega LR}{j\omega LR} = \frac{R - \omega^2 RLC + j\omega L}{j\omega LR}$$

$$\rightarrow Z_T = \frac{1}{Y_T} = \frac{j\omega L}{1 - \omega^2 LC + j\omega \frac{L}{R}}$$

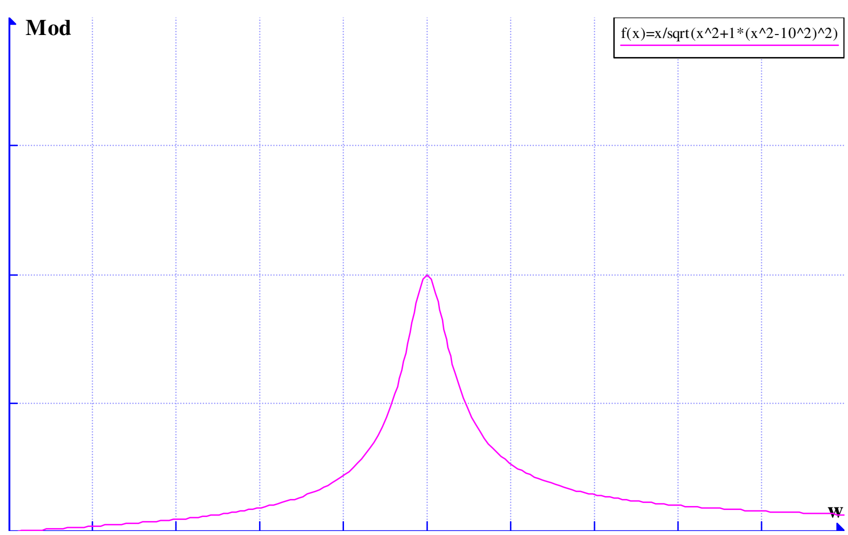
$$T(\omega) = \frac{V}{i} = Z_T = \frac{j\omega L}{1 - \omega^2 LC + j\omega \frac{L}{R}} \qquad [T] = R$$

$$\rightarrow T(\omega) = \frac{j\omega L\left(1 - \omega^2 LC - j\omega \frac{L}{R}\right)}{\left(1-\omega^2 LC\right)^2 + \omega^2\left(\frac{L}{R}\right)^2} = \omega L \frac{\omega \frac{L}{R} + j\left(1 - \omega^2 LC\right)}{\left(1-\omega^2 LC\right)^2 + \omega^2\left(\frac{L}{R}\right)^2}$$

$$\rightarrow \left|T(\omega)\right|^2 = \omega^2 L^2 \frac{\omega^2\left(\frac{L}{R}\right)^2 + \left(1-\omega^2 LC\right)^2}{\left[\left(1-\omega^2 LC\right)^2 + \omega^2\left(\frac{L}{R}\right)^2\right]^2} = \frac{\omega^2 L^2}{\left(1-\omega^2 LC\right)^2 + \omega^2\left(\frac{L}{R}\right)^2}$$

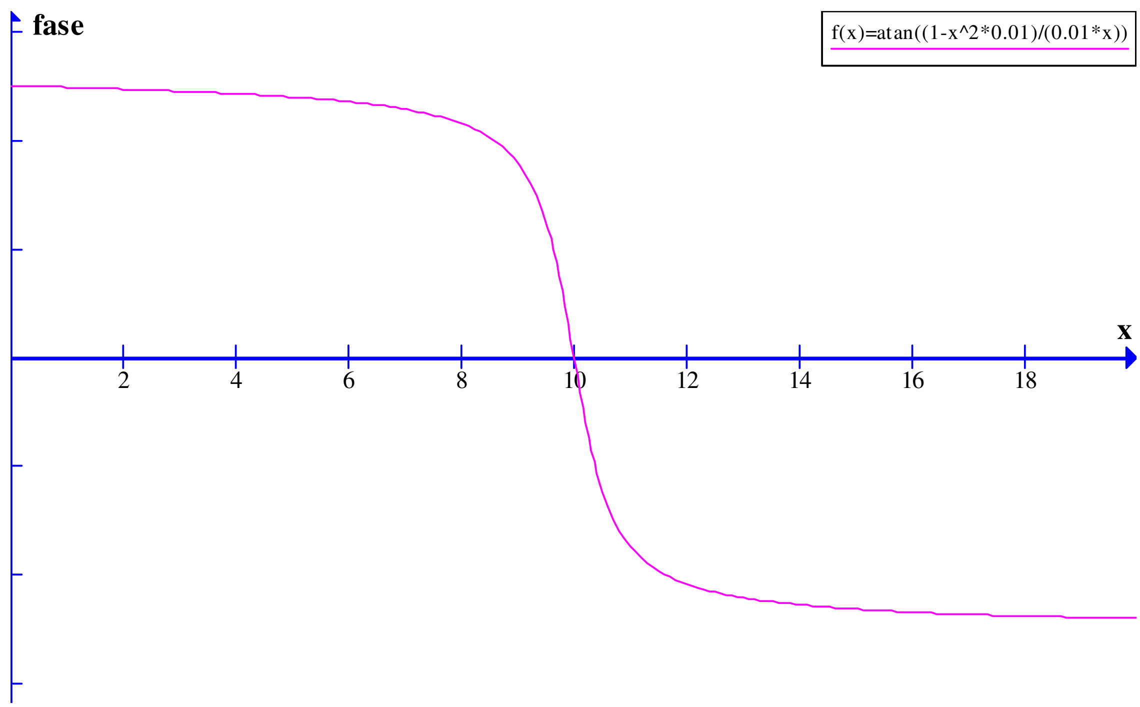
$$\rightarrow \left|T(\omega)\right| = \frac{\omega L}{\sqrt{\omega^2\left(\frac{L}{R}\right)^2 + \left(1 - \frac{\omega^2}{\omega_0^2}\right)^2}} = \frac{\omega L}{\sqrt{\frac{\omega^2\left(\frac{L}{R}\right)^2 \omega_0^4}{\omega_0^4} + \frac{\left(\omega_0^2 - \omega^2\right)^2}{\omega_0^4}}} = \frac{\omega L \omega_0^2}{\sqrt{\omega^2\left(\frac{L}{R}\right)^2 \omega_0^4 + \left(\omega_0^2 - \omega^2\right)^2}}$$

$$\rightarrow \left|T(\omega)\right| = \frac{\omega L \frac{1}{LC}}{\sqrt{\omega^2\left(\frac{L}{R}\right)^2 \frac{1}{L^2C^2} + \left(\omega_0^2 - \omega^2\right)^2}} = \frac{\omega}{C\sqrt{\frac{\omega^2}{R^2C^2} + \left(\omega_0^2 - \omega^2\right)^2}} = \frac{\omega R}{\sqrt{\omega^2 + R^2C^2\left(\omega_0^2 - \omega^2\right)^2}}$$



$$T(\omega) = \omega L \frac{\omega \frac{L}{R} + j\left(1-\omega^2 LC\right)}{\left(1-\omega^2 LC\right)^2 + \omega^2 \left(\frac{L}{R}\right)^2}$$

$$\rightarrow \varphi_T = \arctan\left[\frac{1-\omega^2 LC}{\omega \frac{L}{R}}\right]$$



Enfasi su sorgenti di segnale sinusoidali: legata a *Teorema di Fourier*

Se $v(t)$ periodica con periodo T soddisfa alle *condizioni di Dirichlet*
(in breve: limitata, continua a tratti, n. max,min $<\infty$ e integrabile in modulo sul periodo)

$\omega = \frac{2\pi}{T}$ frequenza (pulsazione) fondamentale, $\omega_n = n\omega$ armoniche

$$\rightarrow v(t) = \sum_{n=0}^{\infty} a_n \cos(n\omega t) + b_n \sin(n\omega t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega t) + b_n \sin(n\omega t)$$

$$a_n = \frac{2}{T}\int_0^T v(t)\cos(n\omega t)\,dt, \quad a_0 = \frac{1}{T}\int_0^T v(t)\,dt$$

$$b_n = \frac{2}{T}\int_0^T v(t)\sin(n\omega t)\,dt$$

Sviluppo in serie di Fourier

Reti lineari:
Risposta a una somma di stimoli = Somma delle risposte (anche infinita)

→ Risposta a segnale periodico qualsiasi
= Somma delle risposte a singole componenti (sinusoidali) di Fourier

→ Sufficiente (o quasi) limitarsi alla risposta in regime sinusoidale

per segnali periodici

Es Onda quadra

$$v(t)=\begin{cases} V & 0<t<\dfrac{T}{2} \\ 0 & \dfrac{T}{2}<t<T \end{cases}$$

$$\rightarrow a_n=\frac{2}{T}\int_0^T v(t)\cos(n\omega t)\,dt=\frac{2}{T}\int_0^{\frac{T}{2}} V\cos(n\omega t)\,dt$$

$$=\frac{2V}{T}\frac{1}{n\omega}\sin\left(n\omega\frac{T}{2}\right)=\frac{2V}{T}\frac{T}{n2\pi}\sin\left(n\frac{2\pi}{T}\frac{T}{2}\right)=\frac{V}{n\pi}\sin(n\pi)=0, n>0$$

$$a_0=\frac{1}{T}\int_0^T v(t)dt=\frac{V}{T}\frac{T}{2}=\frac{V}{2}$$

$$\rightarrow b_n=\frac{2}{T}\int_0^T v(t)\sin(n\omega t)\,dt=\frac{2}{T}\int_0^{\frac{T}{2}} V\sin(n\omega t)\,dt$$

$$=\frac{2V}{T}\left[-\frac{1}{n\omega}\cos\left(n\omega\frac{T}{2}\right)+\cos(0)\right]=\frac{2V}{nT}\frac{T}{2\pi}\left[-\cos\left(n\frac{2\pi}{T}\frac{T}{2}\right)+1\right]$$

$$\rightarrow b_n=\frac{V}{n\pi}\left[1-\cos(n\pi)\right]=\frac{V}{n\pi}\left[1-(-1)^n\right]$$

$$\rightarrow v(t)=\frac{V}{2}+\frac{2V}{\pi}\sum_{n=0}^{\infty}\frac{\sin\left[(2n+1)\omega t\right]}{2n+1}$$

Osservazioni:

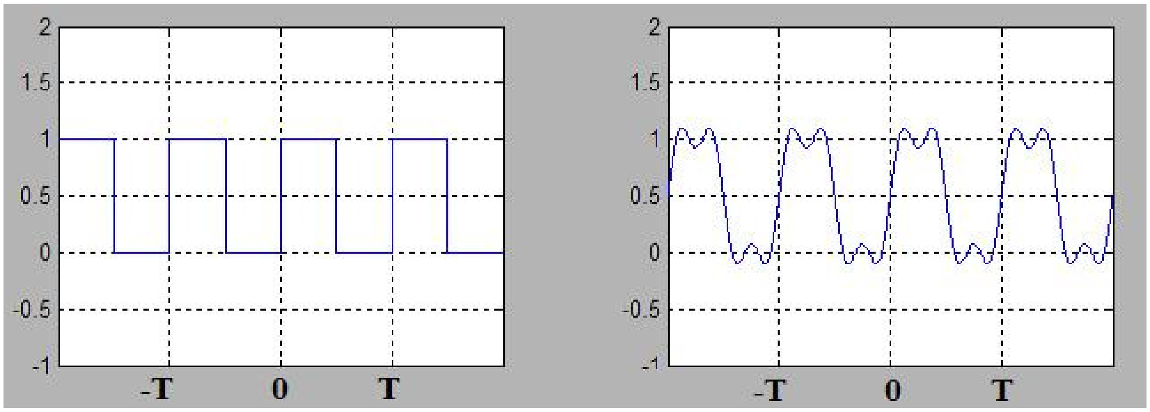
Solo seni $\rightarrow$ Funzione $v(t)$ dispari

Termine costante $=$ Val. medio di $v(t)$

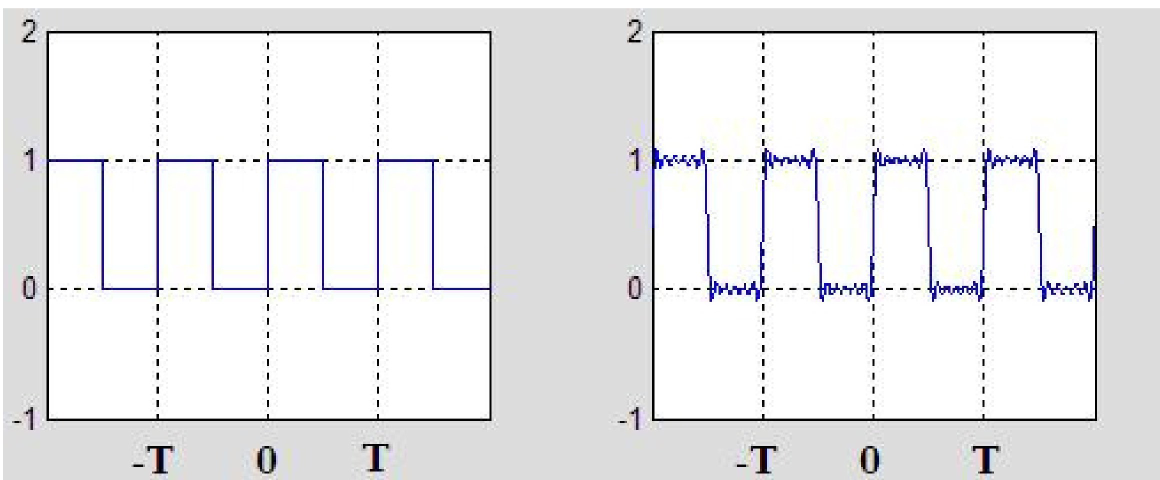
Peso decrescente $\propto \frac{1}{n}$ per armoniche di ordine crescente

Solo armoniche di ordine dispari

Primi 3 termini per onda quadra



Primi 7 termini per onda quadra:



Fenomeno di Gibbs

Es Sinusoide raddrizzata

$$v(t)=\begin{cases} V\sin\omega t & 0<t<\dfrac{T}{2} \\ 0 & \dfrac{T}{2}<t<T \end{cases}$$

$$\rightarrow a_n=\frac{2}{T}\int_0^T v(t)\cos(n\omega t)\,dt=\frac{2}{T}\int_0^{\frac{T}{2}} V\sin\omega t\cos(n\omega t)\,dt=$$

$$=\frac{2V}{T\omega}\int_0^{\frac{T}{2}}\frac{1}{2}\left\{\sin\left[(n+1)\omega t\right]+\sin\left[(n-1)\omega t\right]\right\}d(\omega t)$$

$$=-\frac{V}{2\pi}\left\{\frac{\cos\left[(n+1)\omega t\right]}{(n+1)}+\frac{\cos\left[(n-1)\omega t\right]}{(n-1)}\right\}_0^{T/2}$$

$$=-\frac{V}{2\pi}\left\{\frac{\cos\left[(n+1)\frac{\pi}{2}\right]-1}{(n+1)}+\frac{\cos\left[(n-1)\frac{\pi}{2}\right]-1}{(n-1)}\right\}=\begin{cases} 0 & n\ \text{pari} \\ -\dfrac{2V}{\pi\left(n^2-1\right)} & n\ \text{dispari} \end{cases}$$

$$a_0=\frac{1}{T}\int_0^T v(t)dt=\frac{V}{T}\int_0^{T/2}\sin\omega t dt=-\frac{V}{2\pi}\cos\omega t\Big|_0^{T/2}=-\frac{V}{2\pi}(\cos\pi-\cos 0)=\frac{V}{\pi}$$

$$\rightarrow b_n=\frac{2}{T}\int_0^T v(t)\sin(n\omega t)\,dt=\frac{2}{T}\int_0^{\frac{T}{2}} V\sin(\omega t)\sin(n\omega t)\,dt=\begin{cases} \dfrac{V}{2} & n=1 \\ 0 & n>1 \end{cases}$$

$$\rightarrow v(t)=\frac{V}{\pi}\left(1+\frac{\pi}{2}\sin\omega t-2\sum_{n=1}^{\infty}\frac{\cos 2n\omega t}{4n^2-1}\right)$$

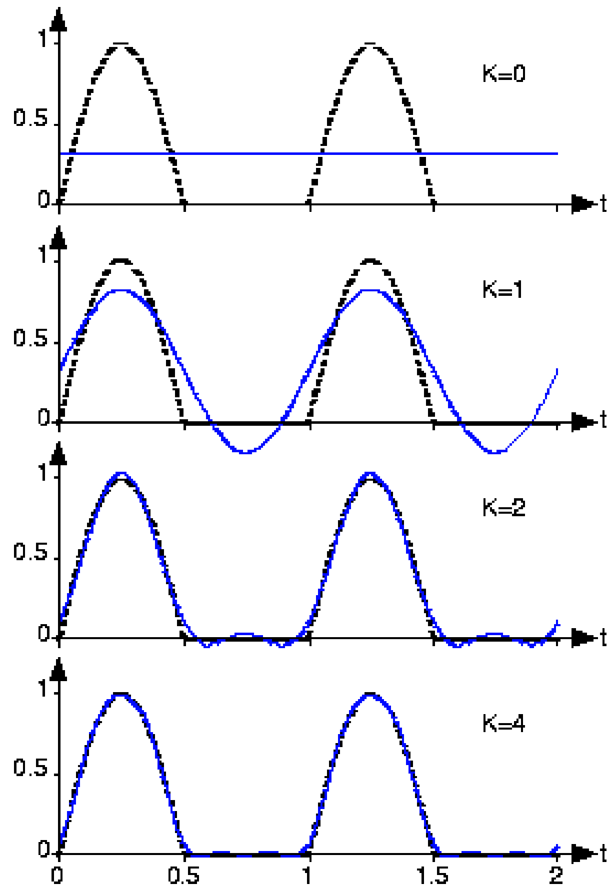
Osservazioni:

Seno + Coseni $\rightarrow$ Funzione $v(t)$ non ha parita' definita

Termine costante = Val. medio di $v(t)$

Peso fortemente decrescente $\propto\frac{1}{n^2}$ per armoniche di ordine crescente

Solo armoniche di ordine pari + fondamentale



K=0
K=1
K=2
K=4
1
0.5
0
t
0
0.5
1
1.5
2