<div align="center">

Decadimenti/reazioni a due corpi:
Impulso nel CM

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Consideriamo la conservazione del 4-impulso totale in un decadimento a due corpi, scegliendo le componenti nel CM:

$$(M,0) = \left(E_1^*, \mathbf{p}^*\right) + \left(E_2^*, -\mathbf{p}^*\right)$$

Per semplicita' di scrittura eliminiamo gli asterischi:

$$M = E_1 + E_2 = \sqrt{m_1^2 + |\mathbf{p}|^2} + \sqrt{m_2^2 + |\mathbf{p}|^2}$$

$$\rightarrow M^2 = m_1^2 + |\mathbf{p}|^2 + m_2^2 + |\mathbf{p}|^2 + 2\sqrt{m_1^2 + |\mathbf{p}|^2}\sqrt{m_2^2 + |\mathbf{p}|^2}$$

$$\rightarrow M^2 - m_1^2 - m_2^2 = 2\left(|\mathbf{p}|^2 + \sqrt{m_1^2 + |\mathbf{p}|^2}\sqrt{m_2^2 + |\mathbf{p}|^2}\right) = 2\left(|\mathbf{p}|^2 + E_1 E_2\right)$$

$$\rightarrow \frac{M^2 - m_1^2 - m_2^2}{2} = |\mathbf{p}|^2 + E_1\left(M - E_1\right) = |\mathbf{p}|^2 + E_1 M - E_1^2 = -m_1^2 + E_1 M$$

Quindi:

$$E_1 = \frac{M^2 + m_1^2 - m_2^2}{2M}$$

$$\rightarrow E_1^2 = \frac{\left(M^2 + m_1^2 - m_2^2\right)^2}{\left(2M\right)^2}$$

$$\rightarrow |\mathbf{p}|^2 = \frac{\left(M^2 + m_1^2 - m_2^2\right)^2}{\left(2M\right)^2} - m_1^2 = \frac{\left(M^2 + m_1^2 - m_2^2\right)^2 - 4M^2 m_1^2}{\left(2M\right)^2}$$

$$\rightarrow |\mathbf{p}|^2 = \frac{\left[\left(M^2 + m_1^2 - m_2^2\right) - 2Mm_1\right]\left[\left(M^2 + m_1^2 - m_2^2\right) + 2Mm_1\right]}{\left(2M\right)^2}$$

Riarrangiando i termini nei due fattori:

$$|\mathbf{p}|^2 = \frac{\left[\left(M^2+m_1^2-2Mm_1\right)-m_2^2\right]\left[\left(M^2+m_1^2+2Mm_1\right)-m_2^2\right]}{\left(2M\right)^2}$$

$$\rightarrow |\mathbf{p}|^2 = \frac{\left[\left(M-m_1\right)^2-m_2^2\right]\left[\left(M^2+m_1\right)^2-m_2^2\right]}{\left(2M\right)^2}$$

$$\rightarrow |\mathbf{p}|^2 = \frac{\left[\left(M-m_1+m_2\right)\left(M-m_1-m_2\right)\right]\left[\left(M+m_1+m_2\right)-\left(M+m_1-m_2\right)\right]}{\left(2M\right)^2}$$

$$\rightarrow |\mathbf{p}|^2 = \frac{\left[\left(M^2-\left(m_1+m_2\right)^2\right)\left(M^2-\left(m_1-m_2\right)^2\right)\right]}{\left(2M\right)^2}$$

In conclusione, reintroducendo gli asterischi:

$$|\mathbf{p}|^* = \frac{\sqrt{\left[\left(M^2-\left(m_1+m_2\right)^2\right)\left(M^2-\left(m_1-m_2\right)^2\right)\right]}}{2M}$$

Commenti:

1. Se $m_1 = m_2$:

$$|\mathbf{p}|^* = \frac{\sqrt{\left[\left(M^2-\left(2m\right)^2\right)\left(M^2\right)\right]}}{2M} = \frac{\sqrt{M^2-4m^2}}{2}$$

2. Se $m_1 = m_2 = 0$:

$$|\mathbf{p}|^* = \frac{\sqrt{M^2-4m^2}}{2} = \frac{M}{2}$$