Corso di Laurea in Fisica

A.A. 2014/15

Meccanica

Prova scritta – 24/03/2015

Problema 1

$$\mathbf{v}_P = \mathbf{v}_O + \mathbf{v}_{rot}$$

$$\mathbf{v}_O = -gt\hat{\mathbf{j}}$$

$$\mathbf{v}_{rot} = \boldsymbol{\omega} \times \mathbf{r}_P$$

$$\boldsymbol{\omega} = \omega\hat{\mathbf{k}}$$

$$\mathbf{r}_P = R\cos\left(\omega t + \frac{\pi}{2}\right)\hat{\mathbf{i}} + R\sin\left(\omega t + \frac{\pi}{2}\right)\hat{\mathbf{j}}$$

$$\rightarrow \mathbf{v}_{rot} = \omega\hat{\mathbf{k}} \times \left[-R\sin\omega t\hat{\mathbf{i}} + R\cos\omega t\hat{\mathbf{j}}\right] = \omega\left(-R\cos\omega t\hat{\mathbf{i}} - R\sin\omega t\hat{\mathbf{j}}\right)$$

$$\rightarrow \mathbf{v}_P = -\omega R\cos\omega t\hat{\mathbf{i}} - \left[\omega R\sin\omega t + gt\right]\hat{\mathbf{j}}$$

$$\rightarrow v_P = \sqrt{\omega^2 R^2\cos^2\omega t + \omega^2 R^2\sin^2\omega t + 2\omega Rgt\sin\omega t + g^2t^2}$$

$$\rightarrow v_P = \sqrt{\omega^2 R^2 + g^2t^2 + 2\omega Rgt\sin\omega t}$$

$$h = \frac{1}{2}gT^2 \rightarrow T = \sqrt{\frac{2h}{g}} \rightarrow \theta = \omega T = \omega\sqrt{\frac{2h}{g}} \rightarrow N = \frac{\omega T}{2\pi} = \frac{\omega}{2\pi}\sqrt{\frac{2h}{g}}$$

Problema 2

Vel. di regime: $F_{motrice} = F_{attvisc}$

$W = F_{mot} v$

$\left|F_{attvisc}\right| = bv$

$\rightarrow bv = \frac{W}{v} \rightarrow v = \sqrt{\frac{W}{b}}$

$\Delta F_{mot} = \frac{dp}{dt} = \frac{d(mv)}{dt} = m\cancel{\frac{dv}{dt}} + v\frac{dm}{dt} = \sqrt{\frac{W}{b}}\alpha$

$\rightarrow \Delta W = \Delta F_{mot} v = \frac{\alpha W}{b}$

Problema 3

Cons. mom. angolare:

$$-m_1 l_1 v_0 + m_2 l_2 v_0 = I\omega$$

$$I = m_1 l_1^2 + m_2 l_2^2 = 19 m_1 l_1^2$$

$$\rightarrow -m_1 l_1 v_0 + 2m_1 \cdot 3 l_1 v_0 = 19 m_1 l_1^2 \omega$$

$$\rightarrow 5 m_1 l_1 v_0 = 19 m_1 l_1^2 \omega$$

$$\rightarrow \omega = \frac{5 v_0}{19 l_1} \approx 3.16 \ rads^{-1}$$

Cons. energia:

$$\frac{1}{2} m_1 v_0^2 + \frac{1}{2} m_2 v_0^2 = \frac{1}{2} I \omega^2 + \Delta E$$

$$\rightarrow \Delta E = \frac{1}{2} m_1 v_0^2 + \frac{1}{2} m_2 v_0^2 - \frac{1}{2} I \omega^2$$

$$\rightarrow \Delta E = \frac{1}{2} m_1 3 v_0^2 - \frac{1}{2} 19 m_1 l_1^2 \left(\frac{5 v_0}{19 l_1} \right)^2$$

$$\rightarrow \Delta E = \frac{1}{2} m_1 v_0^2 \left(3 - \frac{25}{19} \right) \approx 7.5 \ J$$

$$I\alpha = -M_f \rightarrow \alpha = -\frac{M_f}{I}$$

$$\rightarrow \omega_f = \omega_i + \alpha t \rightarrow 0 = \omega - \frac{M_f}{I} t$$

$$\rightarrow t = \frac{\omega I}{M_f} \approx 9.4 \ s$$