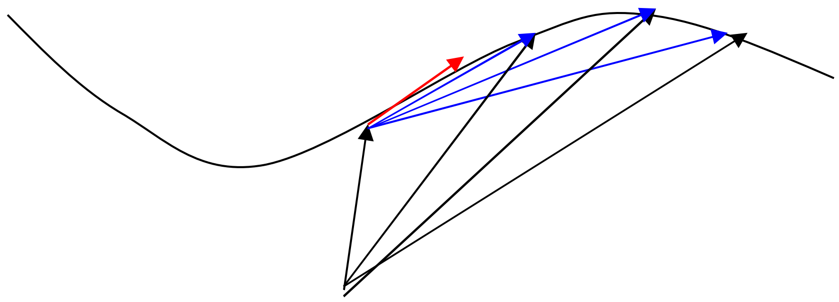
Proprieta' intrinseche di velocita' e accelerazione
Posizione del punto: se la traiettoria e' nota, si puo' dare anche con ascissa curvilinea lungo la traiettoria

$s = s(t)$

$v = \frac{ds}{dt}$ vel. scalare

Esaminando la figura:



$d\boldsymbol{r} = ds\hat{\boldsymbol{u}}_T$

$\hat{\boldsymbol{u}}_T$ versore tangente alla traiettoria ($\leftarrow non$ costante)

$\rightarrow \boldsymbol{v} = \frac{d\boldsymbol{r}}{dt} = \frac{ds}{dt}\hat{\boldsymbol{u}}_T = v\hat{\boldsymbol{u}}_T$

Scritta in forma indipendente dalle componenti

Piu' formalmente:

$$\boldsymbol{v} = \lim_{\Delta t\to 0}\frac{\Delta \boldsymbol{r}}{\Delta t} = \lim_{\substack{\Delta t\to 0\\ \Delta s\to 0}}\frac{\Delta \boldsymbol{r}}{\Delta s}\frac{\Delta s}{\Delta t}$$

$$\lim_{\Delta s\to 0}\frac{\Delta \boldsymbol{r}}{\Delta s} = \hat{\boldsymbol{u}}_T$$

$$\rightarrow \boldsymbol{v} = \frac{ds}{dt}\hat{\boldsymbol{u}}_T$$

Accelerazione

$$\boldsymbol{v} = \frac{ds}{dt}\hat{\boldsymbol{u}}_T = v\hat{\boldsymbol{u}}_T$$

$$\rightarrow \boldsymbol{a} = \frac{d\boldsymbol{v}}{dt} = \frac{d}{dt}\left(v\hat{\boldsymbol{u}}_T\right) = \frac{dv}{dt}\hat{\boldsymbol{u}}_T + v\frac{d\hat{\boldsymbol{u}}_T}{dt}$$

Usando componenti cartesiane di $\hat{\boldsymbol{u}}_T$:

φ angolo rispetto ad asse x

$$\hat{\boldsymbol{u}}_T = \cos\varphi\hat{\boldsymbol{u}}_x + \sin\varphi\hat{\boldsymbol{u}}_y$$

$$\rightarrow \frac{d\hat{\boldsymbol{u}}_T}{dt} = -\sin\varphi\frac{d\varphi}{dt}\hat{\boldsymbol{u}}_x + \cos\varphi\frac{d\varphi}{dt}\hat{\boldsymbol{u}}_y$$

$$\hat{\boldsymbol{u}}_N = -\sin\varphi\hat{\boldsymbol{u}}_x + \cos\varphi\hat{\boldsymbol{u}}_y$$

$\hat{\boldsymbol{u}}_N \cdot \hat{\boldsymbol{u}}_T = 0 \rightarrow \hat{\boldsymbol{u}}_N \perp \hat{\boldsymbol{u}}_T$ vers. normale

$\hat{\boldsymbol{u}}_N$ diretto verso il centro istantaneo di curvatura

$$\rightarrow \frac{d\hat{\boldsymbol{u}}_T}{dt} = \frac{d\varphi}{dt}\hat{\boldsymbol{u}}_N$$

$$\rightarrow \boldsymbol{a} = \frac{dv}{dt}\hat{\boldsymbol{u}}_T + v\frac{d\varphi}{dt}\hat{\boldsymbol{u}}_N$$

Introduciamo:

$R =$ raggio di curvatura punto per punto

$$\rightarrow ds = R d\varphi$$

$$\rightarrow \frac{d\varphi}{ds} = \frac{1}{R}$$

$$\frac{d\varphi}{dt} = \frac{d\varphi}{ds}\frac{ds}{dt} = \frac{1}{R}v$$

$$\rightarrow \boldsymbol{a} = \frac{dv}{dt}\hat{\boldsymbol{u}}_T + v\frac{d\varphi}{dt}\hat{\boldsymbol{u}}_N = \frac{dv}{dt}\hat{\boldsymbol{u}}_T + \frac{v^2}{R}\hat{\boldsymbol{u}}_N$$

Acc. tangenziale e normale/centripeta

$$\rightarrow a = \sqrt{\left(\frac{dv}{dt}\right)^2 + \frac{v^4}{R^2}}$$

Riassumendo: Derivate dei versori T e N rispetto a un parametro della traiettoria

$$\frac{d\hat{\boldsymbol{u}}_T}{dt} = \frac{d\hat{\boldsymbol{u}}_T}{ds}\frac{ds}{dt} = \frac{d\varphi}{ds}\frac{ds}{dt}\hat{\boldsymbol{u}}_N$$

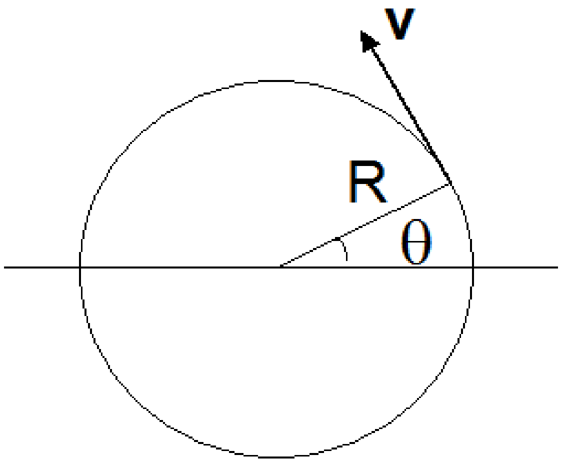
$$\rightarrow \begin{cases} \dfrac{d\hat{\boldsymbol{u}}_T}{ds} = \dfrac{d\varphi}{ds}\hat{\boldsymbol{u}}_N = \dfrac{1}{R}\hat{\boldsymbol{u}}_N \\ \dfrac{d\hat{\boldsymbol{u}}_N}{ds} = -\dfrac{1}{R}\hat{\boldsymbol{u}}_T \end{cases}$$

Formule di Frenet-Serret

$\frac{1}{R}$ curvatura ($rad \ m^{-1}$) della traiettoria

Moto circolare

Traiettoria: circonferenza
Nel piano del moto:



$$\boldsymbol{v} = \frac{d\boldsymbol{r}}{dt} = \frac{ds}{dt}\hat{\boldsymbol{u}}_T$$

$$s = R\theta$$

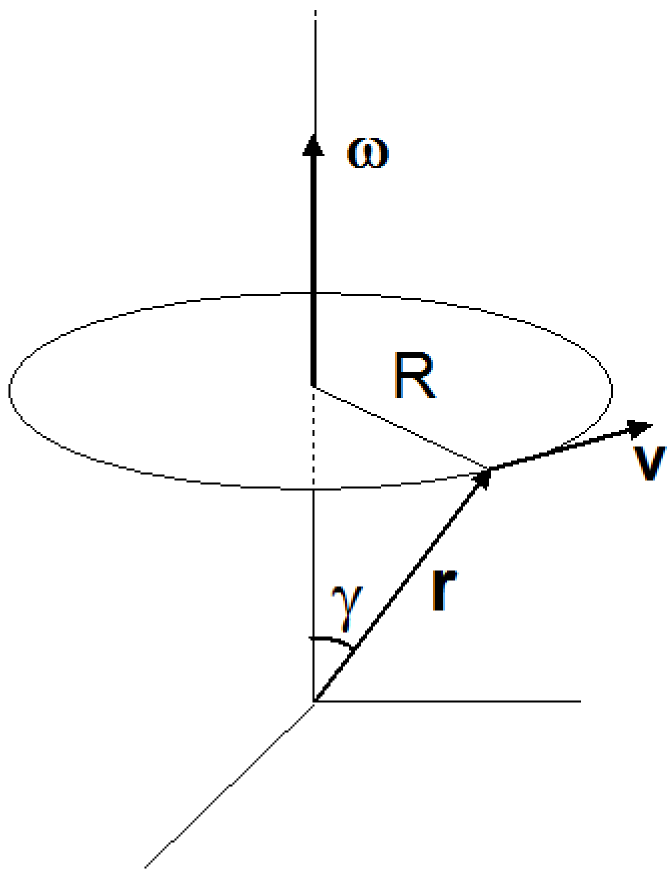
$$\rightarrow \boldsymbol{v} = \frac{ds}{dt}\hat{\boldsymbol{u}}_T = R\frac{d\theta}{dt}\hat{\boldsymbol{u}}_T$$

$$\omega = \frac{d\theta}{dt}$$ vel. angolare

$$\rightarrow v = \omega R$$

$$\boldsymbol{v} \perp \boldsymbol{r}$$

Misurando le posizioni rispetto a un'origine qualsiasi sull'asse di rotazione:



$$R = r\sin\gamma$$

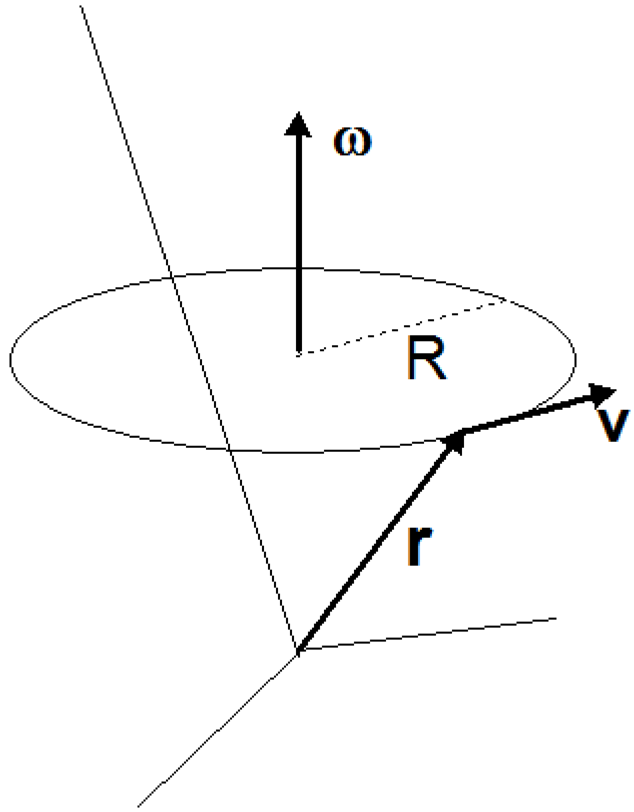
$$\rightarrow v = \omega R \sin\gamma$$

$$\rightarrow |\boldsymbol{v}| = |\boldsymbol{\omega} \times \boldsymbol{r}|$$

$$\boldsymbol{v} \perp \boldsymbol{r}, \boldsymbol{v} \perp \boldsymbol{\omega}$$

$$\rightarrow \boldsymbol{v} = \boldsymbol{\omega} \times \boldsymbol{r}$$

Relazione fra vettori → Indipendente da scelta assi
→Valida in generale



Acc. Angolare:

$$\boldsymbol{\alpha} = \frac{d\boldsymbol{\omega}}{dt}$$

Se:

$$\boldsymbol{\alpha} = \frac{d\boldsymbol{\omega}}{dt} \parallel \boldsymbol{\omega}:$$

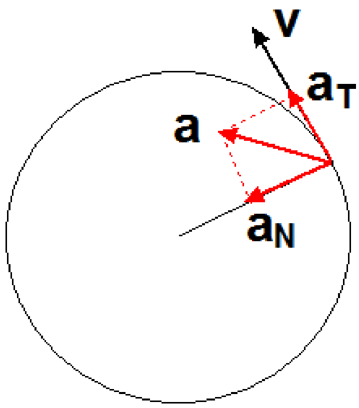
→Eq. oraria per θ, velocita'

$$\left.\begin{aligned} \theta(t) &= \theta_0 + \int_{t_0}^{t} \omega(t')dt' \\ \omega(t) &= \omega_0 + \int_{t_0}^{t} \alpha(t')dt' \end{aligned}\right\}$$ analogo a moto rettilineo

Acc. tangenziale e normale:

$$a_T = \frac{dv}{dt} = R\frac{d\omega}{dt} = R\alpha$$

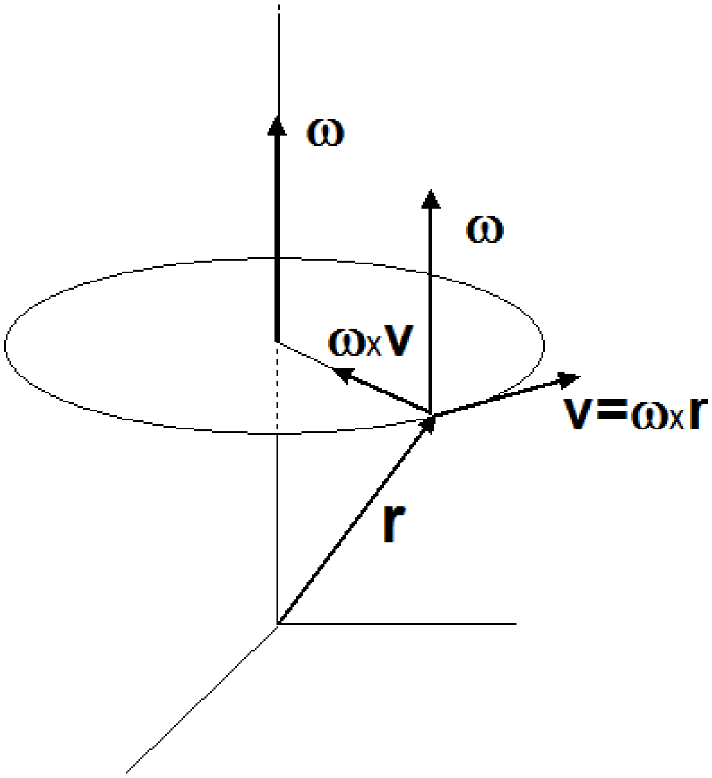
$$a_N = \frac{v^2}{R} = \omega^2 R$$



Moto circolare uniforme

$$\boldsymbol{a} = \frac{d\boldsymbol{v}}{dt} = \frac{d}{dt}(\boldsymbol{\omega} \times \boldsymbol{r})$$

$$\boldsymbol{\omega} = \text{costante} \rightarrow \boldsymbol{a} = \frac{d}{dt}(\boldsymbol{\omega} \times \boldsymbol{r}) = \boldsymbol{\omega} \times \underbrace{\frac{d\boldsymbol{r}}{dt}}_{\boldsymbol{v}} = \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r})$$



ω frequenza angolare = pulsazione $rads^{-1}$

$\nu = \frac{\omega}{2\pi}$ frequenza Hz

$T = \frac{1}{\nu} = \frac{2\pi}{\omega}$ periodo s

Velocita' e accelerazione su una ruota di bicicletta

$v = 25\ km\ h^{-1} \sim 7\ m\ s^{-1}$

$R = 40\ cm = 0.4\ m$

$\rightarrow \omega \sim \frac{v}{R} = \frac{7}{0.4}\ rad\ s^{-1} = 17.5 rad\ s^{-1}$

$\rightarrow a = \omega^2 R = 122.5\ m\ s^{-2}$

c.s. sulla curva di un'autostrada

$v = 100\ km\ h^{-1} \sim 28\ m\ s^{-1}$

$R = 1 km\ \ = 10^3\ m$

$\rightarrow \omega \sim \frac{v}{R} = \frac{28}{10^3}\ rad\ s^{-1} = 2.8 10^{-2} rad\ s^{-1}$

$\rightarrow a = \omega^2 R = 7.8 10^{-4} 10^3 = 0.78\ m\ s^{-2}$

c.s. alla superficie della Terra (sull'Equatore)

$v = 40000 / 24\ km\ h^{-1} \sim 463\ m\ s^{-1}$

$R = 6400\ km = 6.4\ 10^6 m$

$\rightarrow \omega \sim \frac{v}{R} = \frac{463}{6.4 10^6}\ rad\ s^{-1} = 7.2 10^{-5} rad\ s^{-1}$

$\rightarrow a = \omega^2 R = 3.32 10^{-2}\ m\ s^{-2}$

c.s. per il moto orbitale della Terra

$v = 9.42 10^{11} / 3.15^7\ ms^{-1} \sim 2.99 10^4\ m\ s^{-1}$

$R = 1.5 10^8\ km = 1.5\ 10^{11} m$

$\rightarrow \omega \sim \frac{v}{R} = \frac{2.99 10^4}{1.5 10^{11}}\ rad\ s^{-1} = 1.98 10^{-7} rad\ s^{-1}$

$\rightarrow a = \omega^2 R = 4 10^{-14} 1.5 10^{11} = 6 10^{-3}\ m\ s^{-2}$

c.s. per il moto orbitale della Galassia

$$d_{Sole} \approx 3.268000 AL = 2.610^{4} AL$$

$$1AL = 310^{8} ms^{-1} 7.5710^{8} s = 2.2710^{17} m$$

$$2\pi d_{Sole} = 2\pi 5.910^{21} m = 3.7110^{22} m$$

$$T = 220\ 10^{6} anni = 220\ 7.5710^{14} s = 1.6610^{17} s$$

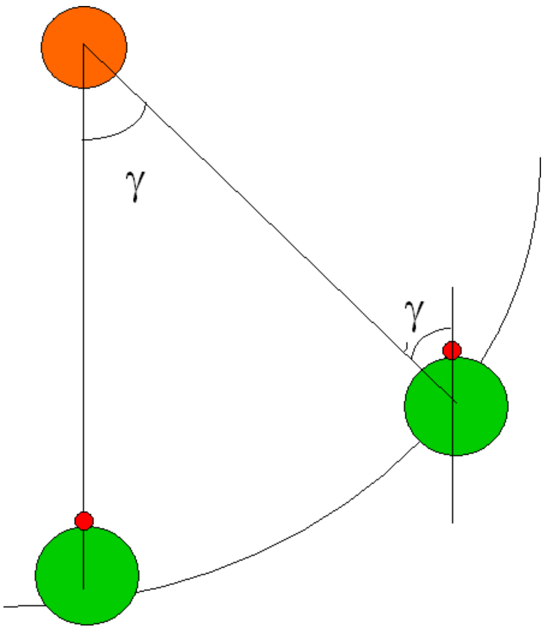
$$v = 3.7110^{22} m / 1.6610^{17} s\ \sim 2.2410^{5}\ m\ s^{-1}$$

$$R = 5.910^{21} m$$

$$\rightarrow \omega \sim \frac{v}{R} = \frac{2.2410^{5}}{5.910^{21}}\ rad\ s^{-1} = 0.3810^{-16} rad\ s^{-1}$$

$$\rightarrow a = \omega^{2} R = 0.1410^{-32} 5.910^{21} = 0.8310^{-11}\ m\ s^{-2}$$

Es: Vel. angolare della Terra, giorno solare, giorno sidereo



$$\omega_{rot} = \frac{2\pi}{T_{sid}} \approx \frac{2\pi}{T_{sol}} \approx \frac{2\pi}{86400} = 7.27210^{-5} rad\ s^{-1}$$

$$\Delta t = \frac{ang(1\ day)}{\omega_{rot}} \simeq \frac{1.710^{-2}}{7.27210^{-5}} \approx 234\ s$$