Aberration of light from binary stars: a paradox?

Introduction

It is believed that aberration depends on the relative velocity of source and observer.

If it were true:

- binary stars would look widely separated
- they would look rapidly rotating
- it would be contrary to Kepler’s third law

Aberration does not depend on the relative velocity of source and observer.

⇓

It depends only on the change in velocity of the observer between the times of aberration measurement.

Binary stars

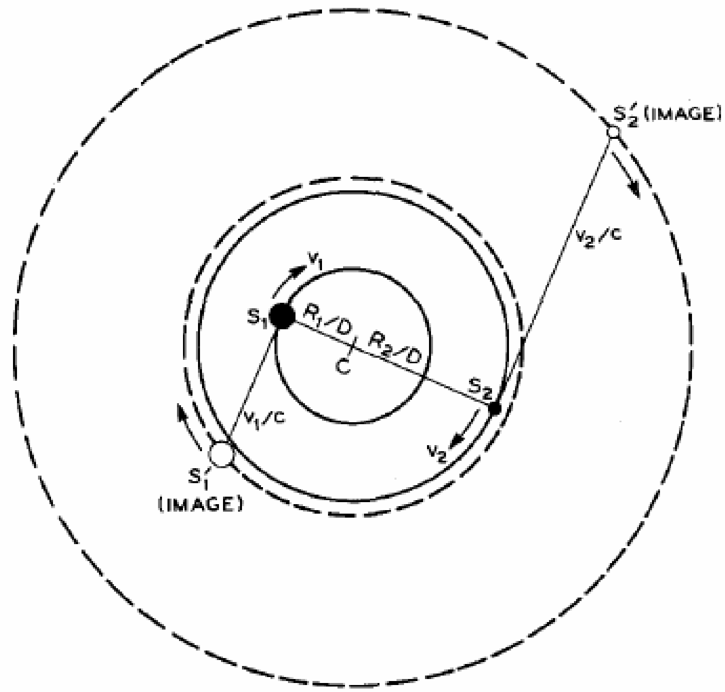


FIG. 1. Angular relations of the true and apparent positions and orbits of binary stars, if aberration depends on relative velocity of source and observer. ——: True orbits; — — —: apparent orbits.

v_1, v_2= velocities of the 2 stars at the moment of observation

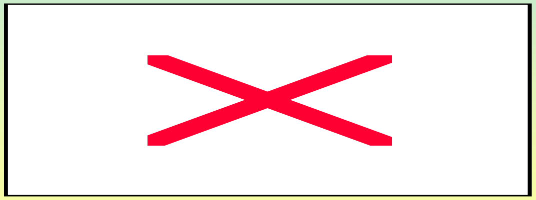
R_1,R_2= radii of their orbits

c= speed of light

D= distance of the observer from the center of mass

Let's assume the stars in uniform motion with respect to the observer O.

If the aberration depends only on the relative velocities of source and observer, the stars must appear to describe cirular orbits of angular radii ρ_1 and ρ_2:



If $\omega = \left(\frac{v_1}{c}\right)$, then $\frac{v_1}{c} = \frac{R_1\omega}{c} = \frac{R_1\omega}{c' 2\pi} = \frac{R_1}{c' T}$

So

$$\rho_1 = \sqrt{\left(\frac{R_1}{c' T}\right)^2 + \left(\frac{R_1}{D}\right)^2} = \frac{R_1}{c' T} \cdot \sqrt{1 + \left(\frac{1/D}{1/c' T}\right)^2} = \frac{R_1}{c' T} \cdot \sqrt{1 + \left(\frac{c' T}{D}\right)^2} = \frac{K \cdot m_2}{\sqrt[3]{T}} \cdot \sqrt{1 + \left(\frac{c' T}{D}\right)^2}$$

Similarly,

$$\rho_2 = \left[\left(\frac{v_2}{c}\right)^2 + \left(\frac{R_2}{D}\right)^2\right]^{1/2} = \frac{R_2}{c^{'}T} \cdot \sqrt{1+\left(\frac{c^{'}T}{D}\right)^2} = \frac{K \cdot m_1}{\sqrt[3]{T}} \cdot \sqrt{1+\left(\frac{c^{'}T}{D}\right)^2}$$

with $v_1 = |\mathbf{v}_1|$, $v_2 = |\mathbf{v}_2|$

T= period of revolution

$c' = c/2\pi$

m_1, m_2= masses of the stars

$$K = \frac{1}{c} \cdot \sqrt[3]{\frac{2\pi \cdot G}{(m_1 + m_2)^2}}$$

(G is the gravitational constant)

Paradoxical result!

- For a very distant observer, $\left(\frac{R_1}{D}\right) \to 0$; so angular radii would be

$\rho_1 = v_1/c$ and $\rho_2 = v_2/c$ independent of distance.

- v/c is commonly of order 10^{-4} (~ 20”)

about one star in 3 is a multiple system

→ skies should be filled with binary stars of apparent separation of order 40”.

- The apparent angular diameters of the orbits would increase with d decreasing period **contrary to Kepler's third law**!

IN FACT, there are VERY FEW PAIRS known of separation greater than 10", and these move very slowly!

Consider two sources of light moving relative to each other.

Pulse of light when they coincide: pulses are coincident to observer O at rest.

Consider light rays from the sources: in O's system they pass through a given point in space at the sources and through another given point at the observer.

If rays are straight, they coin~~cide~~ → direction independent on the relative velocity source-O.

Indeed, O sees a source in the point in space that it occupied when it emitted the light!

(source may even have disintegrated in the interval…)

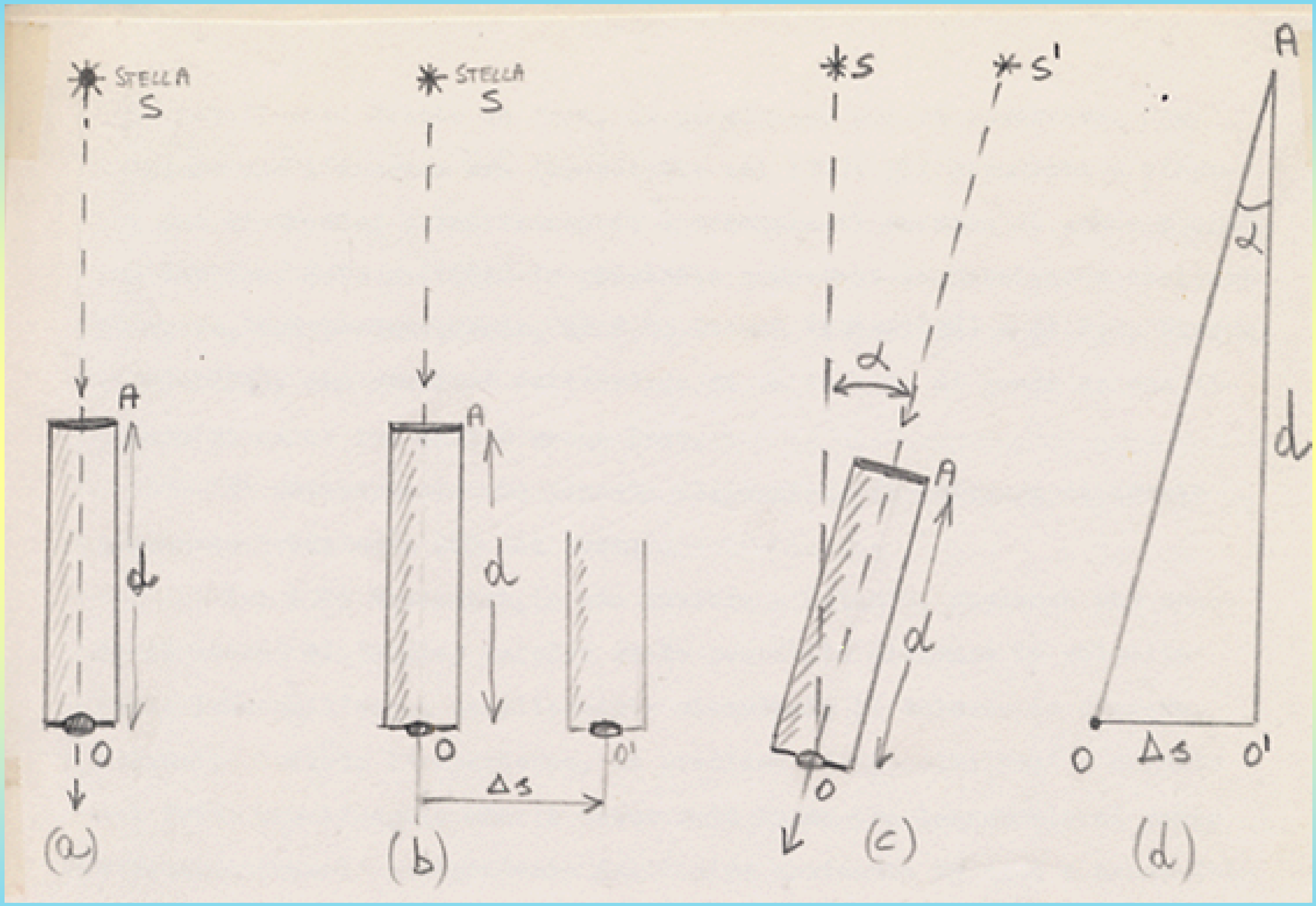
Meaning of aberration

2 systems (1 and 2); 2 moves with speed cβ relative to 1.

$$tg\,\alpha_2 = \frac{\sqrt{(1-\beta^2)}\,sen\,\alpha_1}{(\cos\alpha_1 + \beta)}$$

small β $\longrightarrow$ $\alpha_2 - \alpha_1 = -\beta \cdot sen\,\alpha_1 + \frac{\beta^2}{4} sen 2\alpha_1 + \ldots$

(α_2-α_1) is called **aberration**.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Distinction between secular and annual aberration

a) ANNUAL(or DIURNAL) ABERRATION:

1 and 2 at rest relative to centers of earth and sun

$c\beta$= orbital velocity of earth

b) SECULAR(or PLANETARY) ABERRATION:

1 and 2 at rest relative to sun and to a star

Defined as (α_3-α_1) = angle between the direction of a star and its true direction at the time of observation, rest relative to sun

True direction can't be measured until the light there emitted reaches sun

Different physical origin between (α_3-α_1) and (α_2-α_1).

Evident if the star does not move uniformly with respect to the sun; but even with uniform motion:

$$\tan \alpha_3 = \text{sen } \alpha_1 / (\cos \alpha_1 + \beta)$$

small β ⟶ $$\alpha_3 - \alpha_1 = -\beta \cdot sen\alpha_1 + \frac{\beta^2}{2} sen2\alpha_1 + ...$$

We believe that the source is no longer where it was when it emitted the light we observe.

It just says that we have a dynamical model of motion of the source with respect to the observer.

As it concerns binary stars, O sees the stars where they were when the light left. But

this has nothing to do with “aberration”!

Conclusion

"*Binary stars paradox*" is not a paradox at all.

Aberration should be treated as the transformation between the frames of references of two absservers, not a source and an observer.

Suggested replacement of "secular aberration" by the term

"light-time lag"