

Scattering

Cinematica Relativistica

VI – Conservazione del 4-impulso, decadimento in 2 corpi

Conservazione del 4-impulso

Conseguenza delle proprietà di invarianza per traslazione nello spazio e nel tempo per un sistema isolato:

Conservazione del 4-impulso totale in ogni processo fisico

$$\sum_{i=1}^{N_{in}} p_i^{in} = \sum_{i=1}^{N_{fin}} p_i^{fin}$$

Numero e tipo di particelle nello stato iniziale sono in generale diversi da quelli nello stato finale:

Creazione e distruzione di particelle

Decadimento in 2 corpi - I

Decadimento di una particella 'madre' in 2 particelle 'figlie':
Conservazione del 4-impulso totale

$$P_m = p_{f1} + p_{f2}$$

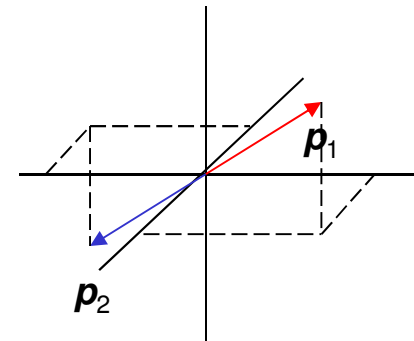
Scegliamo come riferimento il CM (=sistema di quiete)
della particella madre:

$$(M, 0, 0, 0) = (E_1, \mathbf{p}_1) + (E_2, \mathbf{p}_2)$$

$$M = E_1 + E_2 = \sqrt{m_1^2 + p_1^2} + \sqrt{m_2^2 + p_2^2}$$

$$\mathbf{p}_1 + \mathbf{p}_2 = 0 \rightarrow \mathbf{p}_1 = -\mathbf{p}_2 \rightarrow |\mathbf{p}_1| = |\mathbf{p}_2| = p$$

Le particelle figlie escono back-to-back nel CM



Decadimento in 2 corpi - II

L'impulso delle particelle figlie e' uguale in modulo nel CM:

$$\rightarrow M^2 = \left(\sqrt{m_1^2 + p^2} + \sqrt{m_2^2 + p^2} \right)^2$$

$$p = |\mathbf{p}| = \frac{1}{2M} \sqrt{\left[M^2 - (m_1 - m_2)^2 \right] \left[M^2 - (m_1 + m_2)^2 \right]}$$

L'energia delle particelle figlie e', in generale, diversa:

$$\left. \begin{aligned} E_1 &= \sqrt{m_1^2 + p^2} = \frac{1}{2M} \left(M^2 + m_1^2 - m_2^2 \right) \\ E_2 &= \sqrt{m_2^2 + p^2} = \frac{1}{2M} \left(M^2 + m_2^2 - m_1^2 \right) \end{aligned} \right\} \rightarrow E_1 + E_2 = M \quad \text{OK}$$

Osservazioni:

1) Impulsi (in modulo) ed energie hanno un valore fisso nel CM:

Caratteristica dei processi a due corpi

2) Perche' l'impulso abbia valori reali deve essere $M \geq m_1 + m_2$
come ci si attende intuitivamente

Decadimento in 2 corpi - III

Qual e' l'impulso delle particelle figlie nel riferimento del LAB, nel quale la particella madre viaggia con velocita' $\beta=p/E$?

Applicazione della relazione impulso-angolo trovata prima:

$$p_1 = \frac{(M^2 + m_1^2 - m_2^2) p \cos \theta_1 \pm 2E \sqrt{M^2 p^{*2} - m_1^2 p^2 \sin^2 \theta_1}}{2(M^2 + p^2 \sin^2 \theta_1)}$$

Come atteso, due situazioni possibili:

$$\frac{Mp^*}{m_1 p} > 1 \rightarrow \text{Nessun limite su } \theta_1$$

$$\frac{Mp^*}{m_1 p} < 1 \rightarrow \text{Deve essere } \sin \theta_1 < \sin \theta_1^{\max} = \frac{Mp^*}{m_1 p}$$

Decadimento $\pi^0 \rightarrow \gamma\gamma$ - I

Consideriamo il decadimento $\pi^0 \rightarrow \gamma\gamma$ nel CM

$$M_{\pi^0} \simeq 135 \text{ MeV}$$

$$M_{\pi^0} = E_{\gamma 1} + E_{\gamma 2}$$

$$0 = \mathbf{p}_{\gamma 1} + \mathbf{p}_{\gamma 2}$$

$$\rightarrow \mathbf{p}_{\gamma 1} = -\mathbf{p}_{\gamma 2} \rightarrow |\mathbf{p}_{\gamma 1}| = |\mathbf{p}_{\gamma 2}| \equiv |\mathbf{p}_{\gamma}|$$

$$\left. \begin{array}{l} \text{massa fotone} = 0 \rightarrow E_{\gamma 1} = \sqrt{m_{\gamma}^2 + |\mathbf{p}_{\gamma}|^2} = |\mathbf{p}_{\gamma}| \\ E_{\gamma 2} = |\mathbf{p}_{\gamma}| \end{array} \right\}$$

$$\rightarrow M_{\pi^0} = p_{\gamma} + p_{\gamma} \rightarrow p_{\gamma} = \frac{M_{\pi^0}}{2}$$

Quindi:

Nel CM del π^0 escono back-to-back 2 fotoni monocromatici di 67.5 MeV ciascuno

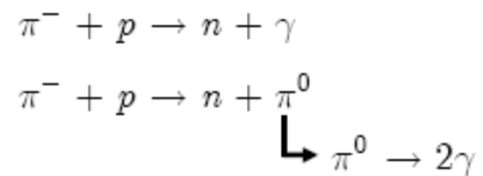
Decadimento $\pi^0 \rightarrow \gamma\gamma$ - II

Scoperta del pione neutro: Cattura di π in idrogeno liquido

1) Formazione di un atomo π -mesico (interazione e.m.)

$$M = m_p + m_{\pi^-}$$

2) Reazione $\pi + p$ (interazione forte e/o e.m.)



NB:

In questo caso, CM \sim LAB

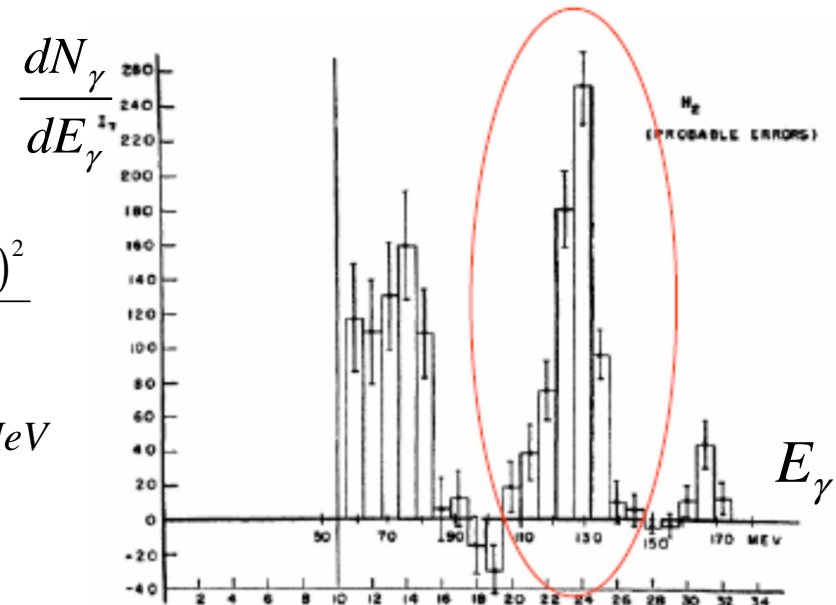
Decadimento $\pi^0 \rightarrow \gamma\gamma$ - III

Primo caso: Impulso di γ e n ha un unico valore

$$|\mathbf{p}| = \frac{1}{2M} \sqrt{[M^2 - (m_1 - m_2)^2][M^2 - (m_1 + m_2)^2]}$$

$$\rightarrow |\mathbf{p}| = \frac{1}{2M} \sqrt{[M^2 - (m_n)^2][M^2 - (m_n)^2]} = \frac{M^2 - (m_n)^2}{2M}$$

$$\rightarrow |\mathbf{p}| = \frac{(m_p + m_{\pi^-})^2 - (m_n)^2}{2(m_p + m_{\pi^-})} \approx \frac{1}{2} \frac{m_{\pi^-}^2 + 2m_p m_{\pi^-}}{m_p + m_{\pi^-}} \approx 129 \text{ MeV}$$



Fotone monocromatico, ben osservato nei dati

Decadimento $\pi^0 \rightarrow \gamma\gamma$ - IV

Secondo caso: Impulso del π^0 ha un unico valore

$$|\mathbf{p}| = \frac{1}{2M} \sqrt{\left[M^2 - (m_1 - m_2)^2 \right] \left[M^2 - (m_1 + m_2)^2 \right]}$$

$$\rightarrow |\mathbf{p}| = \frac{1}{2M} \sqrt{\left[M^2 - (m_n - m_{\pi^0})^2 \right] \left[M^2 - (m_n + m_{\pi^0})^2 \right]}$$

$$\rightarrow |\mathbf{p}| = \frac{\sqrt{\left[(m_p + m_{\pi^-})^2 - (m_n - m_{\pi^0})^2 \right] \left[(m_p + m_{\pi^-})^2 - (m_n + m_{\pi^0})^2 \right]}}{2(m_p + m_{\pi^-})} \approx 53.45 \text{ MeV}$$

Quello del γ no:

Il π^0 si muove nel LAB, quindi i fotoni sono *Doppler shifted*

Velocita' e fattore di Lorentz del π^0 nel LAB $\beta = \frac{p_{\pi^0}}{E_{\pi^0}}, \gamma = \frac{E_{\pi^0}}{m_{\pi^0}}$

Decadimento $\pi^0 \rightarrow \gamma\gamma$ - V

Trasformazione di Lorentz del 4-impulso del fotone n. 1

$$E_{\gamma 1} = \gamma(E_{\gamma 1}^* + \beta p_{\parallel \gamma 1}^*) \equiv p_{\gamma 1}$$

$$p_{\parallel \gamma 1} = \gamma(p_{\parallel \gamma 1}^* + \beta E_{\gamma 1}^*)$$

$$p_{\perp \gamma 1} = p_{\perp \gamma 1}^*$$

$$E_{\gamma 1}^* = \frac{M_{\pi^0}}{2} \equiv p_{\gamma 1}^*$$

$$p_{\parallel \gamma 1}^* = p^* \cos \theta_1^*$$

$$\cos \theta_1^* = -1 \rightarrow p_{\parallel \gamma 1}^* = -p^* = -\frac{M_{\pi^0}}{2} \rightarrow p_{\parallel \gamma 1} = \gamma \left(-\frac{M_{\pi^0}}{2} + \beta \frac{M_{\pi^0}}{2} \right) \left. \vphantom{\cos \theta_1^*} \right\} \text{Valori estremi}$$

$$\cos \theta_1^* = +1 \rightarrow p_{\parallel \gamma 1}^* = +p^* = +\frac{M_{\pi^0}}{2} \rightarrow p_{\parallel \gamma 1} = \gamma \left(+\frac{M_{\pi^0}}{2} + \beta \frac{M_{\pi^0}}{2} \right)$$

$$\rightarrow p_{\parallel \gamma 1} = \gamma \frac{M_{\pi^0}}{2} (\mp 1 + \beta) \rightarrow p_{\parallel \gamma 1} = \begin{cases} -\sqrt{\frac{1-\beta}{1+\beta}} \frac{M_{\pi^0}}{2} & \text{val. min} \rightarrow E_{\min} = \sqrt{\frac{1-\beta}{1+\beta}} \frac{M_{\pi^0}}{2} \\ +\sqrt{\frac{1+\beta}{1-\beta}} \frac{M_{\pi^0}}{2} & \text{val.max.} \rightarrow E_{\max} = \sqrt{\frac{1+\beta}{1-\beta}} \frac{M_{\pi^0}}{2} \end{cases}$$

Decadimento $\pi^0 \rightarrow \gamma\gamma$ - VI

Decadimento isotropo nel CM:

Distribuzione angolare del fotone n. 1

$$\frac{dP}{d\Omega^*} = \frac{1}{4\pi} \rightarrow \frac{dP}{d(\cos\theta^*)d\varphi^*} = \frac{1}{4\pi} \rightarrow \frac{dP}{d(\cos\theta^*)} = \frac{1}{2}$$

Distribuzione energetica del fotone n. 1 nel LAB:

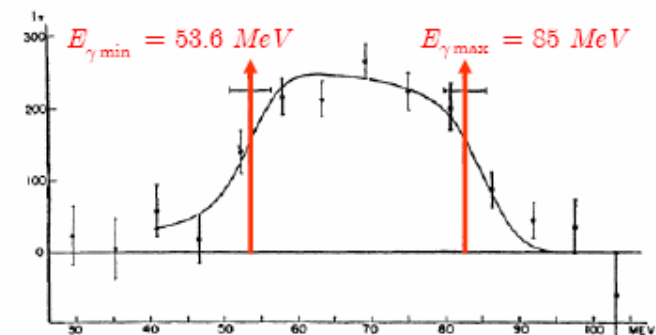
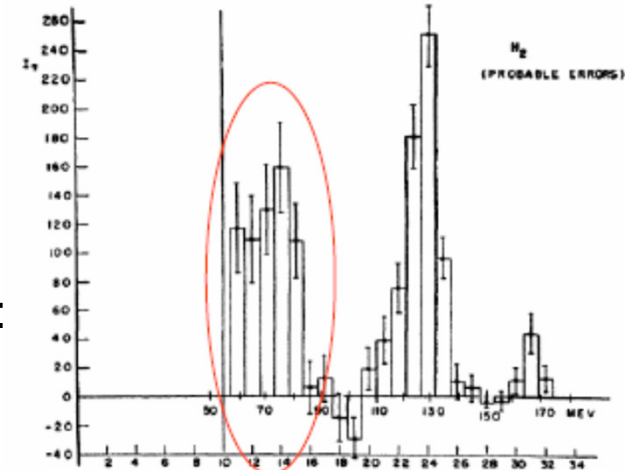
$$E_{\gamma 1} \equiv p_{\gamma 1} = \gamma(E_{\gamma 1}^* + \beta p_{\parallel \gamma 1}^*) = \gamma \left(\frac{M_{\pi^0}}{2} + \beta p^* \cos\theta^* \right)$$

$$= \gamma \left(\frac{M_{\pi^0}}{2} + \frac{M_{\pi^0}}{2} \beta \cos\theta^* \right) = \gamma \frac{M_{\pi^0}}{2} (1 + \beta \cos\theta^*)$$

$$\rightarrow dE_{\gamma 1} = \beta \gamma \frac{M_{\pi^0}}{2} d(\cos\theta^*)$$

$$\rightarrow \frac{dP}{dE_{\gamma 1}} = \frac{dP}{d(\cos\theta^*)} \frac{2}{\beta \gamma M_{\pi^0}} = \frac{1}{\beta \gamma M_{\pi^0}}$$

Distribuzione uniforme ('piatta') fra E_{\min} e E_{\max}



Decadimento $\pi^0 \rightarrow \gamma\gamma$ - VII

Nel LAB: angolo minimo fra i 2 fotoni provenienti dal decadimento di un π^0

$$p_{\parallel\gamma 1} = \gamma(p_{\parallel\gamma 1}^* + \beta E_{\gamma 1}^*), \quad p_{\parallel\gamma 2} = \gamma(p_{\parallel\gamma 2}^* + \beta E_{\gamma 2}^*)$$

$$p_{\perp\gamma 1} = p_{\perp\gamma 1}^*, \quad p_{\perp\gamma 2} = p_{\perp\gamma 2}^*$$

$$E_{\gamma 1}^* = E_{\gamma 2}^* = \frac{M_{\pi^0}}{2} \equiv p_{\gamma 1}^* = p_{\gamma 2}^*$$

$$p_{\parallel\gamma 1}^* = p^* \cos \theta_1^*, \quad p_{\parallel\gamma 2}^* = p^* \cos \theta_2^*$$

$$\cos \theta_1^* = -\cos \theta_2^*$$

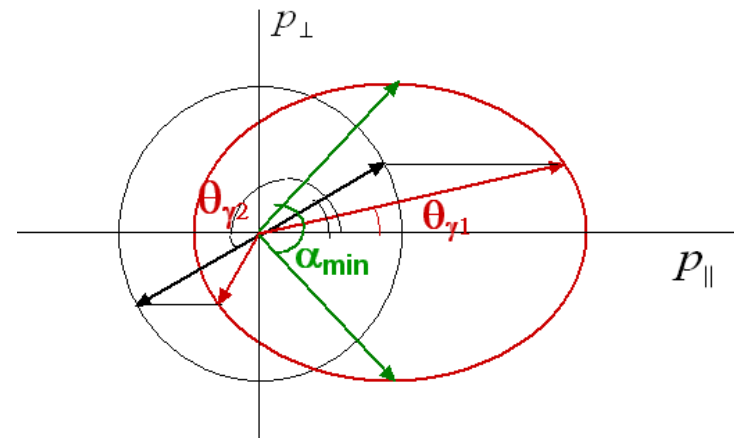
$$\sin \theta_1^* = \sin \theta_2^*$$

$$\rightarrow \tan \theta_1 = \frac{p_{\perp\gamma 1}}{p_{\parallel\gamma 1}} = \frac{p^* \sin \theta_1^*}{\gamma(p^* \cos \theta_1^* + \beta p^*)} = \frac{\sin \theta_1^*}{\gamma(\cos \theta_1^* + \beta)}$$

$$\rightarrow \tan \theta_2 = \frac{p_{\perp\gamma 2}}{p_{\parallel\gamma 2}} = \frac{\sin \theta_2^*}{\gamma(\cos \theta_2^* + \beta)} = \frac{\sin \theta_1^*}{\gamma(-\cos \theta_1^* + \beta)}$$

$$\rightarrow \theta_1^* = \theta_2^* = \frac{\pi}{2} \text{ da l'angolo minimo fra i due}$$

$$\alpha_{\min} = 2 \arctan\left(\frac{1}{\beta\gamma}\right), \quad \beta\gamma = \frac{p_{\pi^0}}{m_{\pi^0}} \rightarrow \alpha_{\min} = 2 \arctan\left(\frac{m_{\pi^0}}{p_{\pi^0}}\right)$$



Decadimenti in due corpi

$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$

$$K^+ \rightarrow \mu^+ + \nu_\mu$$

$$E_\mu^{(\pi)} = \sqrt{m_\mu^2 + p_{\mu,\pi}^2} = \frac{M_\pi^2 + m_\mu^2}{2M_\pi} \rightarrow E_\mu^{(\pi)} = \frac{(0.139)^2 + (0.106)^2}{2 * 0.139} \simeq 0.110 \text{ GeV}$$

$$E_\mu^{(K)} = \sqrt{m_\mu^2 + p_{\mu,K}^2} = \frac{M_K^2 + m_\mu^2}{2M_K} \rightarrow E_\mu^{(K)} = \frac{(0.494)^2 + (0.106)^2}{2 * 0.494} \simeq 0.258 \text{ GeV}$$

Energie fissate → Range in emulsione nucleare fissati

Dal range delle tracce osservate: Riconoscimento π vs. K

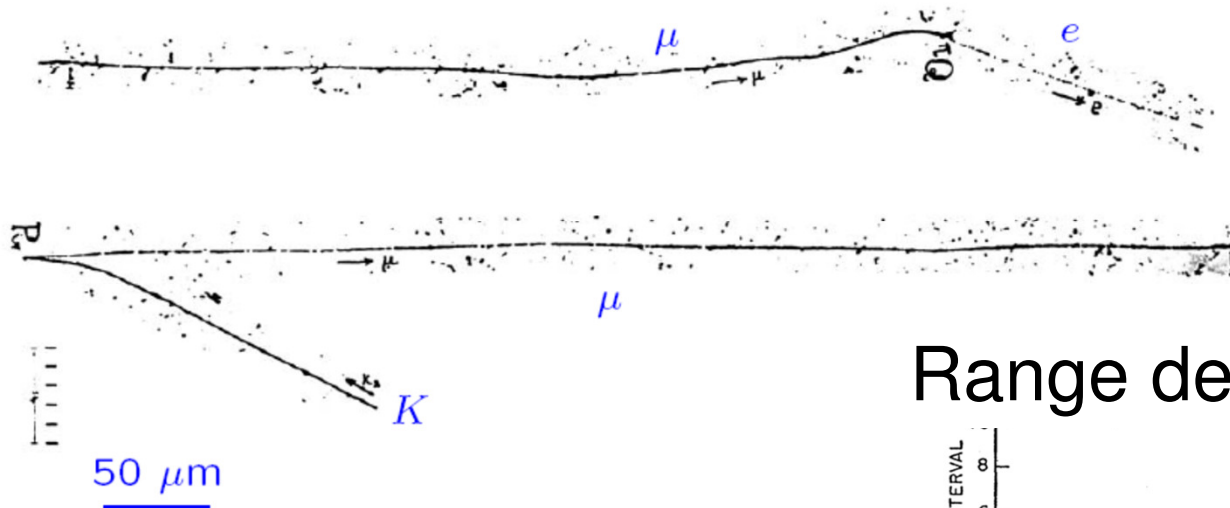
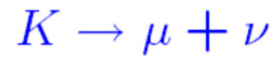
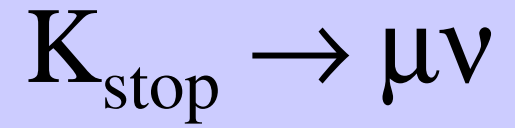
$$\pi_{stop} \rightarrow \mu\nu$$



Range dei μ osservati

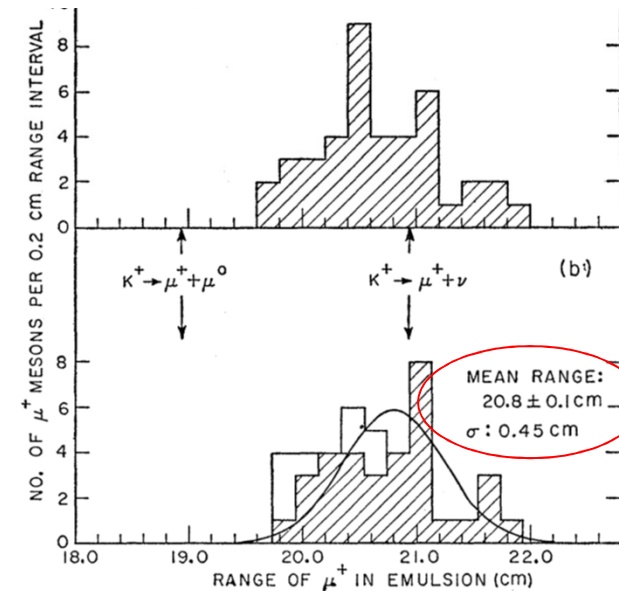
Event No.	Range in emulsion in microns of	
	Primary meson	Secondary meson
I	133	613
II	84	565
III	1040	621
IV	133	591
V	117	638
VI	49	595
VII	460	616
VIII	900	610
IX	239	666
X	256	637
XI	81	590

Mean range $614 \pm 8 \mu$. Straggling coefficient $\sqrt{\Sigma \Delta_i^2/n} = 4.3$ per cent, where $\Delta_i = R_i - \bar{R}$, R_i being the range of a secondary meson, and \bar{R} the mean value for n particles of this type.



Traccia totale molto lunga:
Solo due segmenti mostrati

Range dei μ osservati



$K, \pi \rightarrow \mu \nu$ in volo

Decadimento in volo

$$\tan \theta = \frac{\sin \theta^*}{\gamma(\cos \theta^* + \beta/\beta_\mu^*)}$$

$$\rightarrow \tan \theta = \frac{\beta_\mu^* \sin \theta^*}{\gamma(\beta_\mu^* \cos \theta^* + \beta)}$$

$$\rightarrow \tan \theta = \frac{p_\mu^* \sin \theta^*}{\gamma E_\mu^* \left(\frac{p_\mu^*}{E_\mu^*} \cos \theta^* + \beta \right)}$$

$$\rightarrow \tan \theta = \frac{p_\mu^* \sin \theta^*}{\gamma p_\mu^* \cos \theta^* + \beta \gamma E_\mu^*}$$

Dec. a 90° nel CM:

$$\rightarrow \tan \theta_{90} = \frac{p_\mu^* \sin \frac{\pi}{2}}{\gamma p_\mu^* \cos \frac{\pi}{2} + \beta \gamma E_\mu^*} = \frac{p_\mu^*}{\beta \gamma E_\mu^*} = \frac{1}{\beta \gamma} \frac{2M}{M^2 + m_\mu^2} \sqrt{\left(\frac{M^2 + m_\mu^2}{2M} \right)^2 - m_\mu^2} = \frac{1}{\beta \gamma} \frac{M^2 - m_\mu^2}{M^2 + m_\mu^2}$$

$$\rightarrow \tan \theta_{90} = \frac{M}{p} \frac{M^2 - m_\mu^2}{M^2 + m_\mu^2}$$

β, γ, M, p :

particella madre (K o π)

p^*, E^*, θ^* :

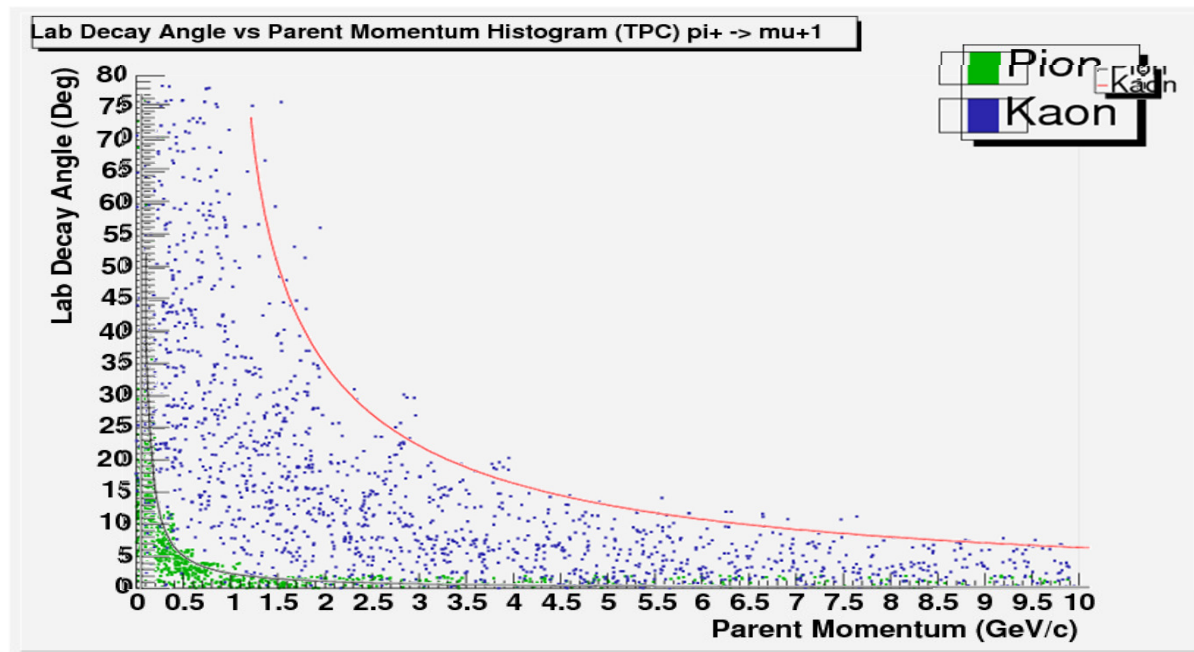
μ nel CM della part. madre

p, E, θ :

μ nel LAB

Identificazione di K ad ALICE

$$\tan \theta_{90^0} = \frac{M}{p} \frac{M^2 - m_\mu^2}{M^2 + m_\mu^2}$$



Misura dell'impulso della particella madre e dell'angolo di decadimento:
Identificazione di π e K con piccola ambiguita' residua

Fasci di neutrini - I

Da un fascio secondario di un acceleratore di protoni (es. SPS al CERN):
Fascio collimato di π e K , che decadono in 2 corpi

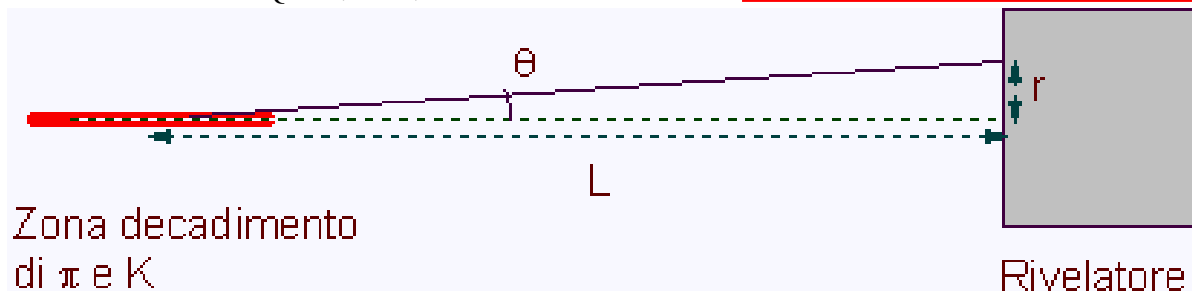
$$|\mathbf{p}|_{\nu}^2 = \frac{1}{2M_{\pi,K}} \sqrt{M_{\pi,K}^2 - m_{\mu}^2} \sqrt{M_{\pi,K}^2 + m_{\mu}^2} \simeq 248.2 \text{ MeV}$$

$$\frac{dP}{d(\cos\theta^*)} = \frac{1}{2} \rightarrow \frac{dP}{dE} = \frac{dP}{d(\cos\theta^*)} \frac{d(\cos\theta^*)}{dE}$$

$$E = \gamma(E^* + \beta p^* \cos\theta^*) \rightarrow dE = \gamma\beta p^* d(\cos\theta^*) \rightarrow d(\cos\theta^*) = \frac{dE}{\gamma\beta p^*}$$

$$\rightarrow \frac{dP}{dE} = \frac{1}{2\gamma\beta p^*} \begin{cases} \gamma(1+\beta)E^* & \text{max} \\ \gamma(1-\beta)E^* & \text{min} \end{cases}$$

Banda di impulso molto larga
→ E_{ν} poco nota



Fasci di neutrini - II

Misura dell'energia del neutrino:

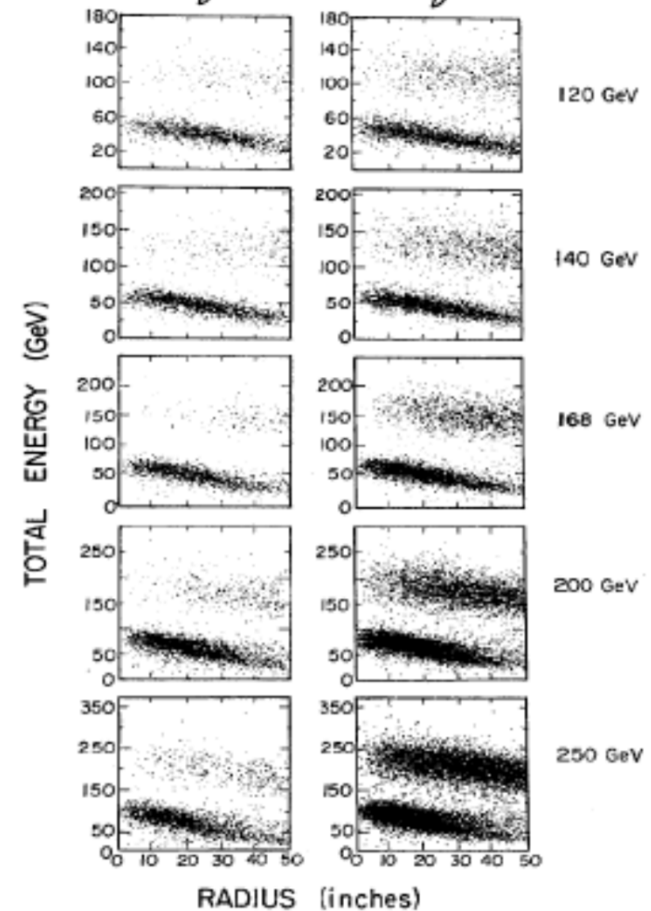
$$p_\pi = p_\mu + p_\nu \rightarrow p_\mu = p_\pi - p_\nu \rightarrow (p_\mu)^2 = (p_\pi - p_\nu)^2$$

$$m_\mu^2 = m_\pi^2 - 2p_\pi \cdot p_\nu \rightarrow m_\pi^2 - m_\mu^2 = 2(E_\pi E_\nu - \mathbf{p}_\pi \cdot \mathbf{p}_\nu)$$

$$E_\nu = \frac{m_\pi^2 - m_\mu^2}{2(E_\pi - p_\pi \cos \theta_\nu)} \approx \frac{m_\pi^2 - m_\mu^2}{2E_\pi (1 - \cos \theta_\nu)} \approx \frac{m_\pi^2 - m_\mu^2}{E_\pi \theta^2}$$

$$E_\nu \approx \left\{ \begin{array}{l} \frac{m_\pi^2 - m_\mu^2}{E_\pi \theta^2} = \frac{m_\pi^2 \left(1 - \frac{m_\mu^2}{m_\pi^2}\right) E_\pi}{E_\pi^2 \theta^2} = \frac{\left(1 - \frac{m_\mu^2}{m_\pi^2}\right)}{\gamma^2 \theta^2} E_\pi \\ \frac{m_K^2 - m_\mu^2}{E_K \theta^2} = \frac{\left(1 - \frac{m_\mu^2}{m_K^2}\right)}{\gamma^2 \theta^2} E_K \end{array} \right.$$

Energia vista nel detector
vs raggio



Fasci di neutrini - III

Migliore approssimazione

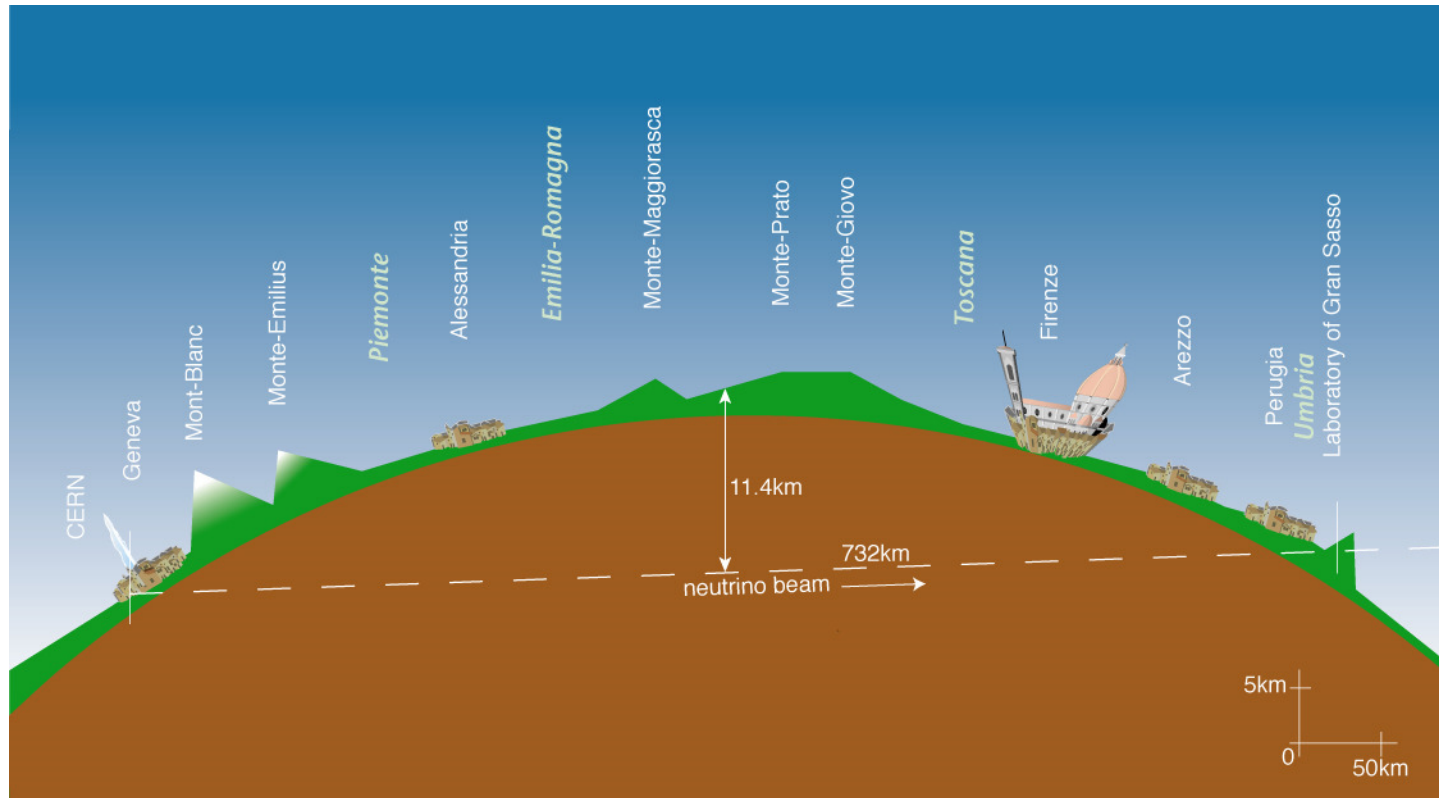
$$E_\nu = \frac{m_\pi^2 - m_\mu^2}{2E_\pi(1 - \beta \cos \theta)} = \frac{m_\pi^2 - m_\mu^2}{2E_\pi \left(1 - \beta \left(1 - \frac{\theta^2}{2}\right)\right)}$$

$$E_\nu = \frac{m_\pi^2 - m_\mu^2}{2E_\pi \left(1 - \beta + \frac{\theta^2}{2}\right)} = \frac{m_\pi^2 - m_\mu^2}{2E_\pi \left(\frac{1 - \beta^2}{1 + \beta} + \frac{\theta^2}{2}\right)} \simeq \frac{m_\pi^2 - m_\mu^2}{2E_\pi \left(\frac{1 - \beta^2}{2} + \frac{\theta^2}{2}\right)}$$

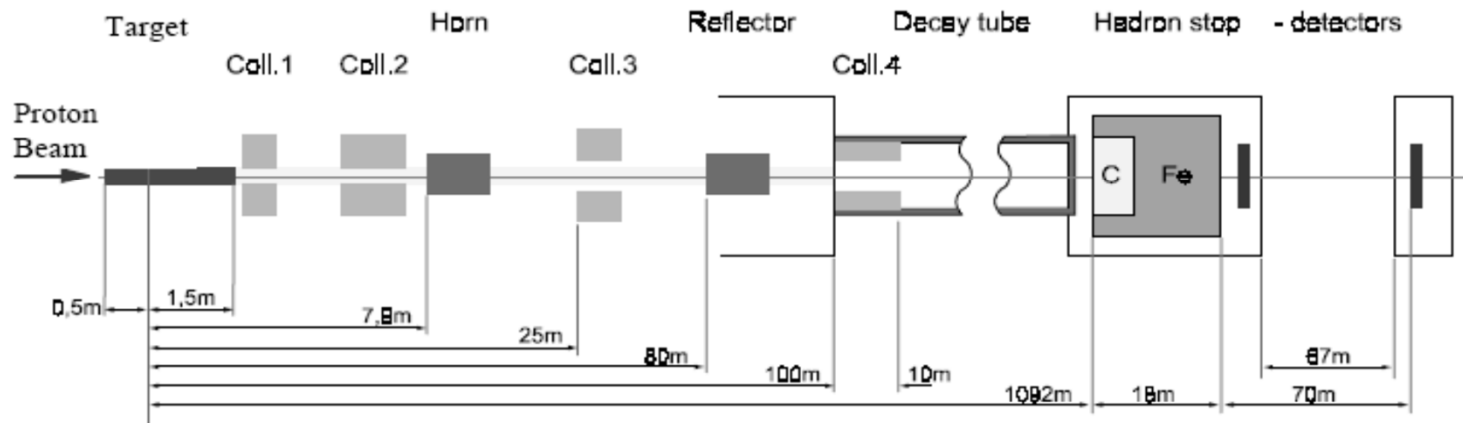
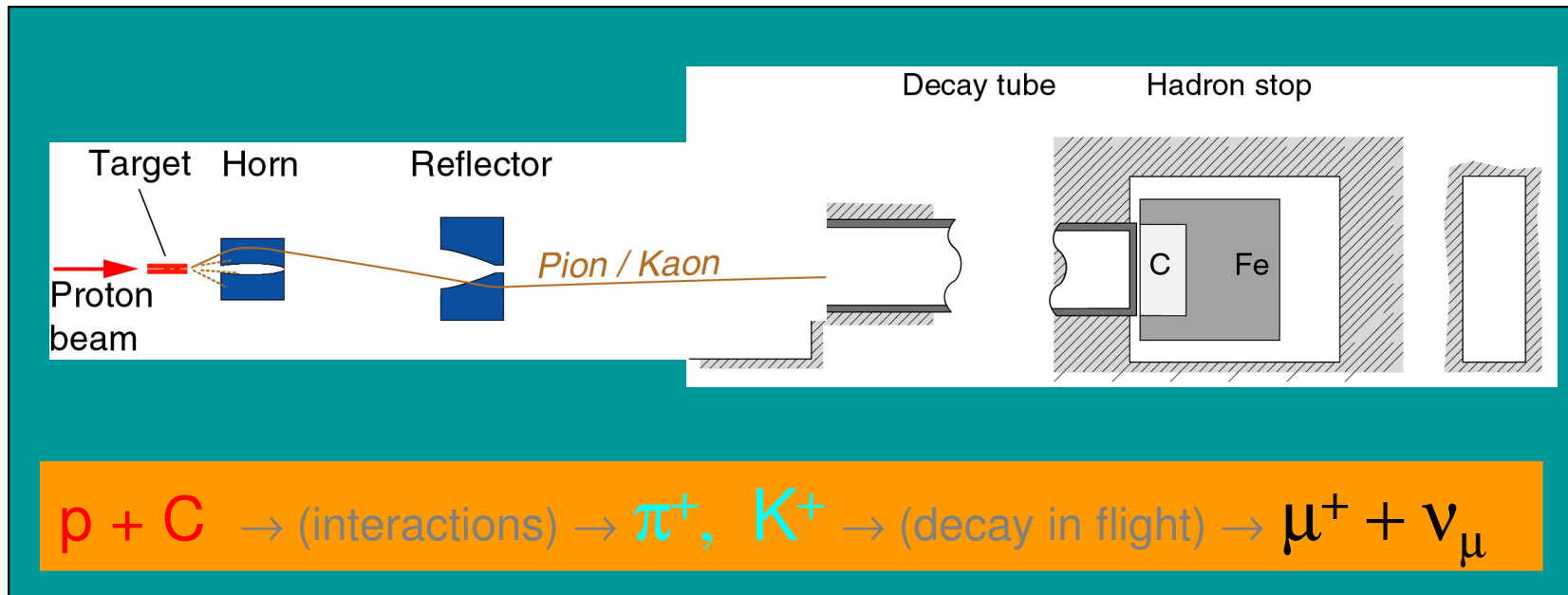
$$E_\nu = \frac{m_\pi^2 - m_\mu^2}{E_\pi \left(\frac{1}{\gamma^2} + \theta^2\right)} = \frac{m_\pi^2 \left(1 - \frac{m_\mu^2}{m_\pi^2}\right) E_\pi}{E_\pi^2 \left(\frac{1}{\gamma^2} + \theta^2\right)} = \frac{m_\pi^2 \left(1 - \frac{m_\mu^2}{m_\pi^2}\right) E_\pi}{\gamma^2 m_\pi^2 \left(\frac{1}{\gamma^2} + \theta^2\right)} = \frac{\left(1 - \frac{m_\mu^2}{m_\pi^2}\right)}{(1 + \gamma^2 \theta^2)} E_\pi$$

Simile per decadimenti dei K

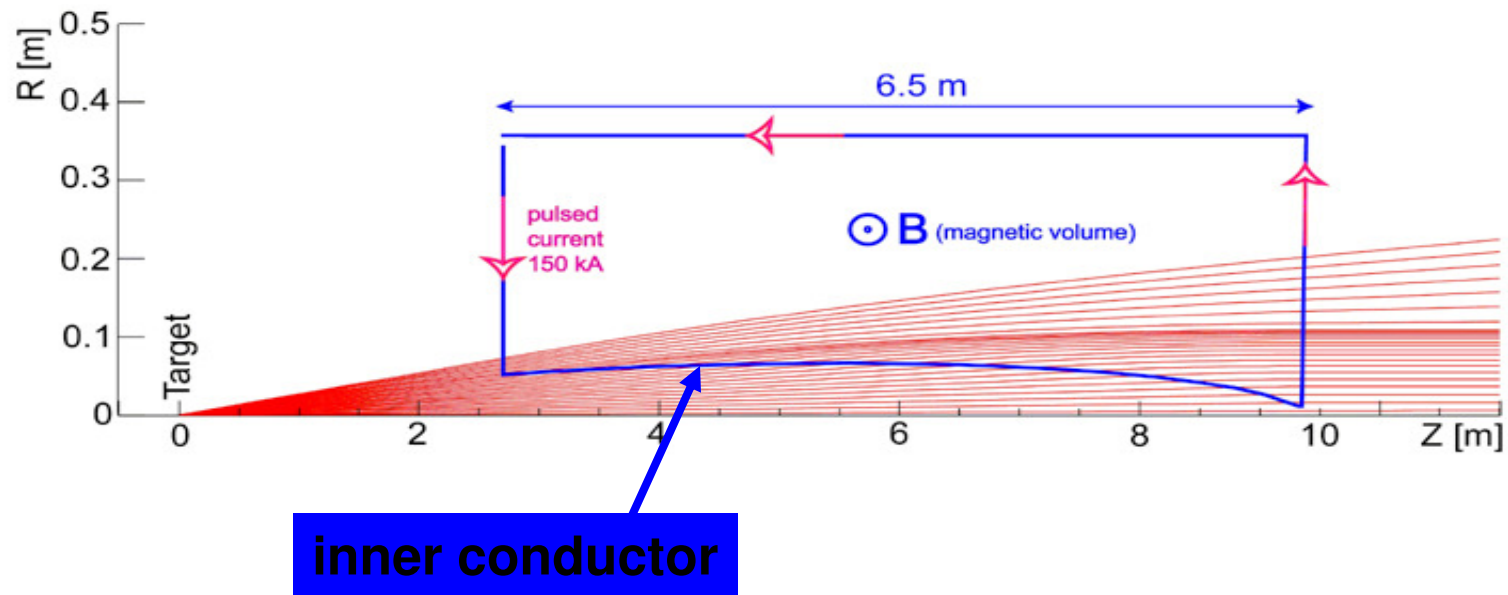
Neutrini dal CERN al Gran Sasso - I



Neutrini dal CERN al Gran Sasso - II

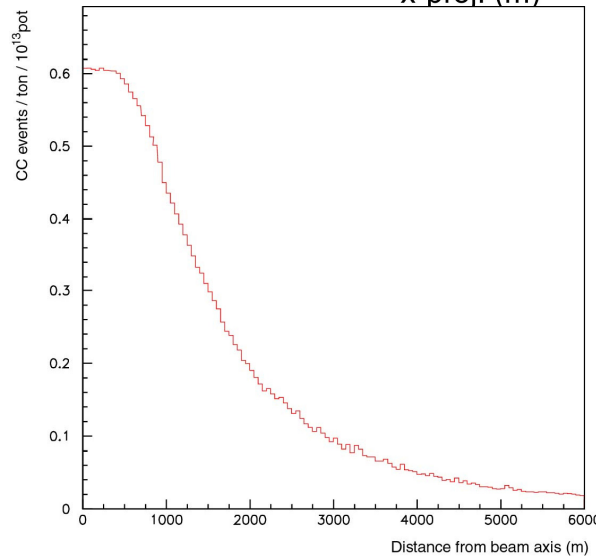
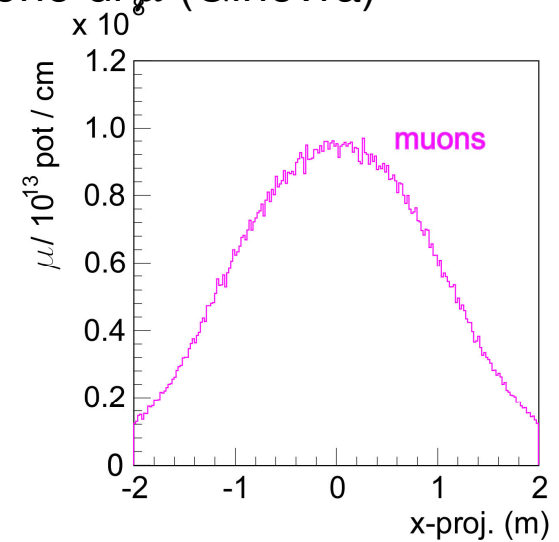
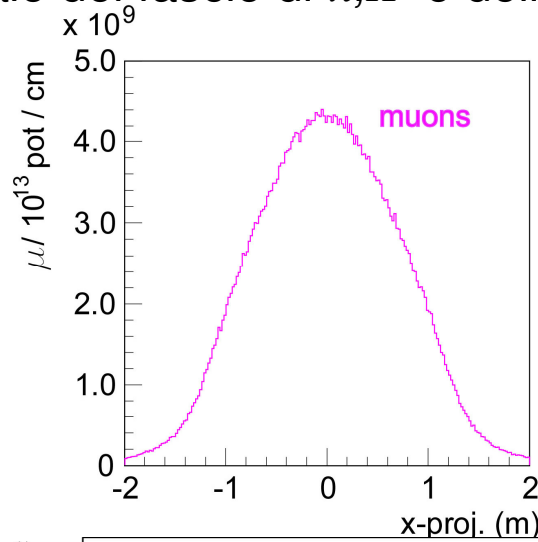
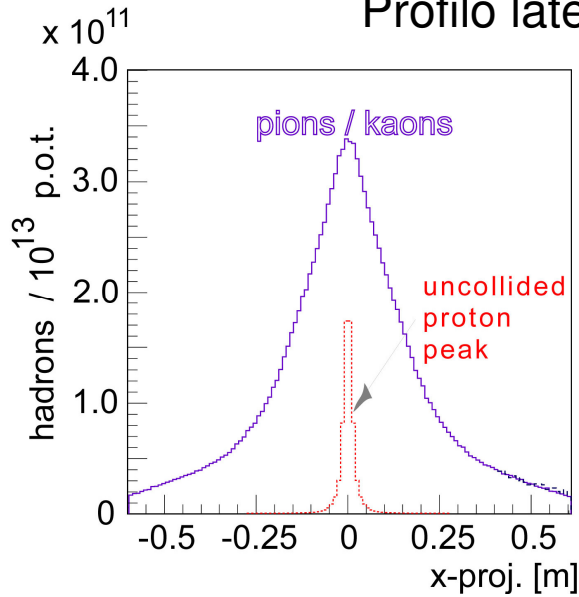


Neutrini dal CERN al Gran Sasso - III



Neutrini dal CERN al Gran Sasso - IV

Profilo laterale del fascio di π, K e dell'alone di μ (Ginevra)



Profilo laterale del fascio di ν (Gran Sasso)