

Simmetria/Antisimmetria di singoletti e tripletti in $SU(2)$

Si considera valido un principio di Pauli generalizzato, dove particelle e antiparticelle sono considerate fermioni identici : per quanto apparentemente bizzarro, questo e' in realta' coerente con l'idea di ricercare le proprieta' di simmetria/antisimmetria degli stati come caratteristiche del gruppo, e non delle particelle

Particelle e antiparticelle stanno nella stessa rappresentazione

$$\mathbf{2} : \alpha = p, \quad \beta = n$$

$$\bar{\mathbf{2}} : \alpha = \bar{n}, \quad \beta = -\bar{p}$$

$$\mathbf{2} \otimes \mathbf{2} = \mathbf{1} \oplus \mathbf{3}$$

Triplet

$$\begin{matrix} (uu) & \rightarrow & (uu) & S \\ \alpha(1)\alpha(2) & & \alpha(1)\alpha(2) \end{matrix}$$

$$\begin{matrix} (dd) & \rightarrow & (dd) & S \\ \beta(1)\beta(2) & & \beta(1)\beta(2) \end{matrix}$$

$$\frac{1}{\sqrt{2}} \begin{matrix} (ud + du) \\ \alpha(1)\beta(2) + \beta(1)\alpha(2) \end{matrix} \rightarrow \frac{1}{\sqrt{2}} \begin{matrix} (du + ud) \\ \beta(1)\alpha(2) + \alpha(1)\beta(2) \end{matrix} \quad S$$

Singlet

$$\frac{1}{\sqrt{2}} \begin{matrix} (ud - du) \\ \alpha(1)\beta(2) - \beta(1)\alpha(2) \end{matrix} \rightarrow \frac{1}{\sqrt{2}} \begin{matrix} (du - ud) \\ \beta(1)\alpha(2) - \alpha(1)\beta(2) \end{matrix} \quad A$$

$$\mathbf{2} \otimes \bar{\mathbf{2}} = \mathbf{1} \oplus \mathbf{3}$$

Triplet

$$\begin{matrix} (u\bar{d}) & \rightarrow & (\bar{d}u) & S \\ \alpha(1)\alpha(2) & & \alpha(1)\alpha(2) \end{matrix}$$

$$\begin{matrix} ((-\bar{u})d) & \rightarrow & (d(-\bar{u})) & S \\ \beta(1)\beta(2) & & \beta(1)\beta(2) \end{matrix}$$

$$\frac{1}{\sqrt{2}} \begin{matrix} (u(-\bar{u}) + d\bar{d}) \\ \alpha(1)\beta(2) + \beta(1)\alpha(2) \end{matrix} \rightarrow \frac{1}{\sqrt{2}} \begin{matrix} ((-\bar{u})u + \bar{d}d) \\ \beta(1)\alpha(2) + \alpha(1)\beta(2) \end{matrix} \quad S$$

Singlet

$$\frac{1}{\sqrt{2}} \begin{matrix} (-u(-\bar{u}) + d\bar{d}) \\ -\alpha(1)\beta(2) + \beta(1)\alpha(2) \end{matrix} \rightarrow \frac{1}{\sqrt{2}} \begin{matrix} (-(-\bar{u})u + \bar{d}d) \\ -\beta(1)\alpha(2) + \alpha(1)\beta(2) \end{matrix} \quad A$$