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# Hybrid metamaterials combining pentamode lattices and phononic plates

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We propose a design strategy for hybrid metamaterials with alternating phononic plates and pentamode units that produce complete bandgaps for elastic waves. The wave control relies on the simultaneous activation of two scattering mechanisms in the constituent elements. The approach is illustrated by numerical results for a configuration comprising phononic plates with cross-like cavities. We report complete bandgaps of tunable width due to variations of geometric parameters. We show that the wave attenuation performance of the hybrid metamaterials can be further enhanced through implementation of lightweight multiphase material compositions. These give rise to efficient wave attenuation in challenging low-frequency regions. The proposed design strategy is not limited to the analyzed cases alone and can be applied to various designs of phononic plates with cavities, inclusions or slender elements. *Published by AIP Publishing.*
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Phononic and acoustic metamaterials demonstrate unusual mechanical properties[1,2] and the ability to control elastic waves by producing bandgaps[3–5] or negative group velocity.[6,7] They draw these remarkable functionalities from their engineered architectures, giving rise to unconventional dynamic characteristics in various frequency ranges. Numerous two-dimensional (2D) configurations with periodic patterns have been designed to activate wave manipulation mechanisms, resulting in omnidirectional, complete bandgaps for plane-polarized elastic waves (2D bandgaps).[8] Applications of such designs to three-dimensional (3D) geometries are usually characterized by poor attenuation of oblique or normally incident waves.[8,9] Common examples are phononic plates with voids[10] or internal resonators[11–13] that can manipulate waves in the plane of a plate, while waves with out-of-plane wavevector components can propagate freely.[9,13,14] This issue substantially limits the potential of 2D metamaterials for engineering applications, including seismic wave shielding,[15,16] vibration mitigation,[3,6,13] or wave focusing and splitting.[17,18]

Here, we propose a design strategy specially aimed at extending 2D bandgaps in phononic plates to a full 3D setting. We show that hybrid metamaterials, consisting of phononic plates interlayered by pentamode lattice units, exhibit complete 3D bandgaps due to the simultaneous activation of wave scattering in the plates and the hybrid structure.

Pentamode lattices belong to a class of "extremal materials" as introduced by Milton and Cherkaev.[19–21] These essentially 3D structures consist of periodic repetitions of four tapered bars meeting at point-like joints in a diamond-like lattice. Ideal pentamodes have zero shear modulus, and thus exhibit fluid-like dynamics, inhibiting the propagation of shear waves at any frequency.[19,22,23] Realistic structures are characterized by a finite, non-zero effective shear modulus. Typically, this modulus is much smaller than the effective bulk modulus.[23,24] Shear and compressional waves are thus weakly coupled. This leads to frequency intervals with a single compressional mode. As we shall show, the hybrid structures formed by a combination of pentamode lattices and phononic plates can be designed to produce 3D bandgaps. Such metastructures enable bandgap tuning by adjusting the geometrical parameters and maintain structural integrity due to incorporated spheres at the joints.

A typical phononic plate has an essentially 2D configuration if its cross-section is invariant along the thickness. This simplifies theoretical analysis of the plate dynamics, possible optimization procedures, and manufacturing processes. A 2D formulation of the related elastodynamic problem for the cross-sectional geometry (assuming an infinite thickness of the plate) enables the decoupling of motions into in-plane modes with displacements $\{u_x, u_y\}$ and out-of-plane (or transverse) modes with displacements $u_z$.[8,9] Scattering mechanisms for these mode families are governed by a 2D elasticity tensor and a shear modulus, respectively. This results in 2D bandgaps at different frequencies for different mode types.[9,11] In a 3D plate of finite thickness, the separation of modes is, in general, not possible. For waves in the cross-sectional plane, the band structures of in-plane and out-of-plane modes are superimposed, while for oblique incident waves, the two mode types are coupled, leading to the closing of bandgaps (see Figs. S1–S3 in the supplementary material). In order to induce complete 3D bandgaps, one needs to introduce a wave attenuation mechanism in the out-of-plane direction, suppressing the coupled modes. This is

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