Holographic non-perturbative corrections to gauge couplings

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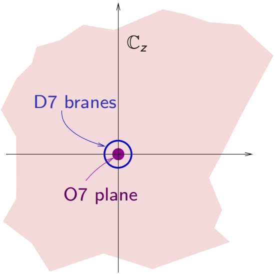
- In this example, the gauge/gravity relation explains an intriguing relation between the exact gauge coupling and the dynamics of the flavour sector, that might generalize to other cases.
- *Based mostly on M.B., Frau, Giacone, Lerda, arXiv:1105.1869.*

Local type I' model

- We consider 4 D7 branes at an O7 plane in flat space.
  - Local limit near an O7 plane of Type I', the T-dual of type I on a T_2. Type I' has 4 O7 fixed planes and 16 D7 branes.



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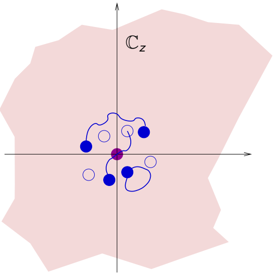


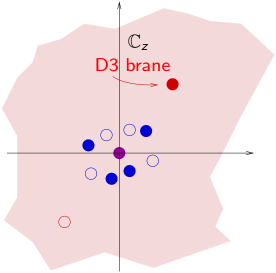
- Consider the D7/D7 open string sector.
  - The massless d.o.f. describe a 8d gauge theory.
  - Orientifold projection $\to$ SO(8) group.
  - Massless d.o.f. form a chiral superfield: (8 θ's)

$$M = m + \theta\psi + \frac{1}{2}\theta\gamma^{\mu\nu}\theta F_{\mu\nu} + \cdots$$

in the adjoint of SO(8).

Gauge theory on a D3 probe

- Add a D3-brane (plus its image).



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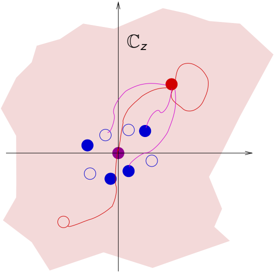
- Add a D3-brane (plus its image).
- We get an Sp(1)∼ SU(2), $\mathcal{N} = 2$ theory with $N_f = 4$ hypers from the D3/D7 strings, with flavor group $\mathrm{SO}(8)$ and vanishing β-function.

Banks et al., 1996



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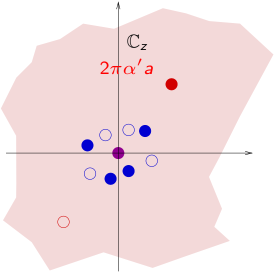

- Placing the D3 in z (and its image in $-z$) $\Leftrightarrow$ giving a v.e.v.

$$\phi_{cl} = (a, -a) \ , \quad a = z/(2\pi\alpha')$$

to the SU(2) complex adjoint scalar: $\mathbb{C}_z \Leftrightarrow$ Coulomb branch.

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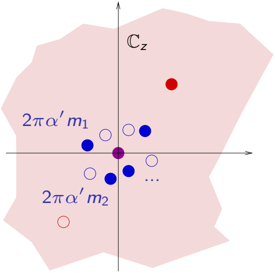

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- Displacing the D7's in z_i $(i = 1, \ldots, 4) \Leftrightarrow$ giving a v.e.v.

$$m_{cl} = (m_1, m_2, \ldots, -m_1, \ldots) \ , \quad m_i = z_i/(2\pi\alpha')$$

to the SO(8) complex adjoint scalar.

Parameters and coupling of the gauge theory

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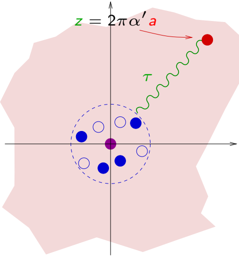

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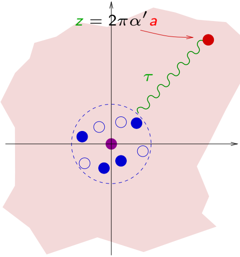
- We now focus on the latter.



Dilaton-axion background

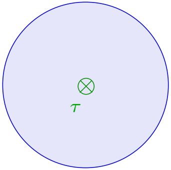
- Closed string massless d.o.f. of the can be organized in a chiral scalar superfield: Schwarz 1983, Howe-West 1983, de Haro et al. 2002

$$T = \tau + \theta\lambda + \cdots + 2\theta^8\left(\partial^4\bar{\tau} + \cdots\right)$$

where the ... terms contain the other d.o.f..

- D7's and O7 couple to it: δ-function sources for τ, localized in $\mathbb{C}_z$.

as expl. in T. Weigand's review



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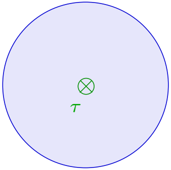

- The perturbative profile corresponding to displaced D7's is

$$2\pi \mathrm{i}\, \tau_{cl}(z) = 2\pi \mathrm{i}\, \tau_0 + \sum_{i=1}^{4}\left[\log\frac{z - z_i}{z} + \log\frac{z + z_i}{z}\right] .$$

It matches the 1-loop running of the dual gauge coupling.

Perturbative behaviour and beyond

- This background is naïvely (i.e., perturbatively) singular but can be promoted to a non-singular F-theory background, as done by Sen long ago using gauge/gravity:
  - the exact F-theory background must correspond to the exact effective gauge coupling of the SU(2), $N_f = 4$ theory.

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- We'll show explicitly that, on the gravity side, this arises by taking into account non-perturbative D-instanton corrections.
- To do so, we start by writing the perturbative profile, expanded for large z, in terms of the v.e.v of the D7 adjoint scalar m:

$$2\pi\mathrm{i}\,\tau_{cl}(z) = 2\pi\mathrm{i}\,\tau_0 - \sum_{\ell=1}^{\infty} \frac{(2\pi\alpha')^{2\ell}}{2\ell}\,\frac{\mathrm{tr}\,m_{cl}^{2\ell}}{z^{2\ell}}\ ,$$

where $m_i = z_i/(2\pi\alpha')$.

Equations of motion

- This profile solves the e.o.m. $\Box\tau = J_{cl}\,\delta^2(z)$ with

$$J_{cl} = -2\mathrm{i}\sum_{\ell=1}^{\infty}\frac{(2\pi\alpha')^{2\ell}\,\mathrm{tr}\,m_{cl}^{2\ell}}{(2\ell)!}\frac{\partial^{2\ell}}{\partial z^{2\ell}}\ .$$

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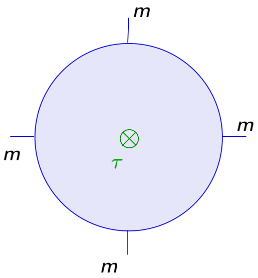
- J_{cl} arises from interactions on the D7 world-volume between the dilaton-axion field and the SO(8) adjoint scalar m that reduce to

$$S_{source} \sim -\int d^8x\, J_{cl}\,\bar{\tau}$$

when the scalar is frozen to its v.e.v..



8d prepotential and source terms

- Such interaction terms on the D7's can be written as

$$S_{pert} = \frac{1}{(2\pi)^4} \int d^8x\, d^8\theta\;\; F_{pert}(M, T)$$

using the open and closed superfields

$$M \sim m + \cdots \;\; , \quad T \sim \tau + \cdots + 2\theta^8 \partial^4 \bar{\bar{\tau}} + \cdots \; .$$

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- The prepotential F_{pert} contains an infinite series of terms:

$$F_{pert}(M, T) = 2\pi \mathrm{i} \sum_{\ell} \frac{(2\pi\alpha')^{2\ell-4}}{(2\ell)!} \, \mathrm{tr}\, M^{2\ell} \; \partial^{2\ell-4} \; T \; .$$

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- Comparing with the definition of the source action for the axio-dilaton we have *[trust the constants...]*

$$J_{cl} = -\frac{(2\pi\alpha')^4}{2\pi} \frac{\delta F_{pert}}{\delta(\theta^8 \bar{\tau})}\bigg|_{T=\tau_0, M=m_{cl}} \equiv -\frac{(2\pi\alpha')^4}{2\pi} \bar{\delta} F_{pert} \; .$$

Non-perturbative corrections to the source

- The complete, non-perturbative source terms for the axio-dilaton are obtained by taking into account the D-instanton corrections to the 8d prepotential:

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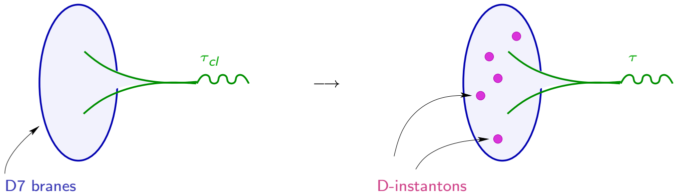
- Then we have

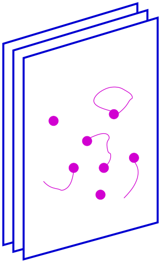
$$J = -\frac{(2\pi\alpha')^4}{2\pi}\,\bar{\delta}F$$

and solving the e.o.m. with the complete source we get the exact τ profile .



D-instanton moduli

- The moduli $\mathcal{M}_{(k)}$ (k is the instanton #) are the d.o.f. of the open strings with at least one boundary on a D-instanton.



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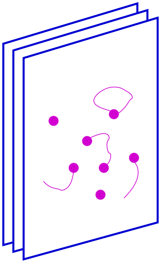

- The moduli action describes the interactions of the moduli, and is computed by string diagrams. One has

$$S_{inst}(\mathcal{M}_{(k)}, M, T) = S(\mathcal{M}_{(k)}) + S(\mathcal{M}_{(k)}, M) + S(\mathcal{M}_{(k)}, T) \ .$$

  - $S(\mathcal{M}_{(k)})$: pure moduli action $\Rightarrow$ measure on $\mathcal{M}_{(k)}$.
  - $S(\mathcal{M}_{(k)}, M)$: mixed moduli-gauge fields action .
  - $S(\mathcal{M}_{(k)}, T)$: mixed moduli-gravity fields action .

Non-perturbative prepotential

- The non-perturbative effective action on the D7's is obtained as an integral over the D-instanton moduli space:

$$S_{np} = \sum_k \int d\mathcal{M}_{(k)} \mathrm{e}^{-S_{inst}(\mathcal{M}_{(k)}, M, T)} = \int d^8x \, d^8\theta \, F_{np}(M, T)$$

so that the non-perturbative prepotential is

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- We need the explicit form of the moduli action. We focus here on its most relevant part for our goal, namely $S(\mathcal{M}_{(k)}, T)$.

Mixed moduli-gravity action

- To obtain $S(\mathcal{M}_{(k)}, T)$ we compute mixed open/closed disk diagrams with moduli and bulk fields.

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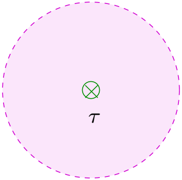
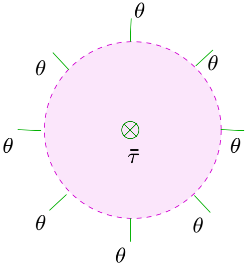
- Simplest diagrams yield the "classical" instanton action supersimmetrized by insertions of θ moduli:

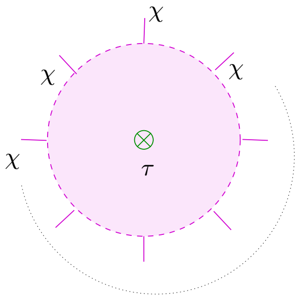




$$-2\pi\mathrm{i}\, k\, \tau \to -2\pi\mathrm{i}\, k\, T = -2\pi\mathrm{i}\, k\, (\tau + \ldots + \theta^8 \bar{p}^4 \bar{\tau}) \; .$$

Mixed moduli-gravity action (II)

- Other mixed diagrams involve the bosonic moduli χ (akin to the positions of the D(-1)'s transverse to the D7, but with anti-symmetric CP indices due to the orientifold).



- Exactly computable:

$$-2\pi\mathrm{i}\sum_{\ell=1}^{\infty}\frac{(2\pi\alpha')^{2\ell}}{(2\ell)!}\,\mathrm{tr}(\chi^{2\ell})\,\bar{p}^{2\ell}\,\tau\ .$$

- Susy-completed by extra θ-insertions

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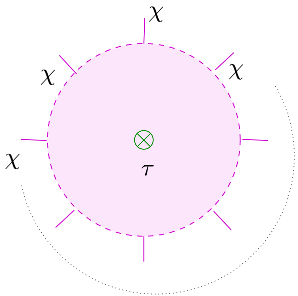

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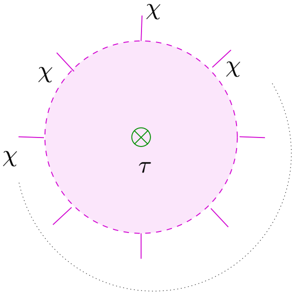

$$-2\pi \mathrm{i} \sum_{\ell=1}^{\infty} \frac{(2\pi\alpha')^{2\ell}}{(2\ell)!} \, \mathrm{tr}(\chi^{2\ell}) \, \bar{p}^{2\ell} \; T \; .$$

- Altogether, the mixed moduli-gravity action is

$$S(\mathcal{M}_{(k)}, T) = -2\pi \mathrm{i} \sum_{\ell=0}^{\infty} \frac{(2\pi\alpha')^{2\ell}}{(2\ell)!} \, \mathrm{tr}(\chi^{2\ell}) \, \bar{p}^{2\ell} \; T \; .$$

Explicit computation: overview

- The integration over the moduli space at generic k requires localization techniques.

  Nekrasov, 2002; Flume+Poghossian, 2002; Bruzzo et al, 2003; ...

  - A susy charge preserved by the D7/D(-1) system is selected as scalar BRST charge Q.
  - The moduli organize in BRST doublets ("topological twist").
  - The moduli action is BRST-exact: the integral reduces to the evaluation of determinants around the fixed points of Q.
  - To have isolated fixed points, the moduli action is deformed by suitable parameters (to be removed at the end). In our set-up, such parameters arise from a particular RR background.

    M.B., Ferro, Frau, Gallot, Lerda and Pesando, 2009

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- The concepts and basic techniques to study the coupling of D-instantons to closed string fields were studied long ago.

  Green-Gutperle, 1997-1999

  Localization techniques allow now to carry out the computation even when all instanton numbers contribute.

Ingredients of the result

- Setting $q = \exp(2\pi \mathrm{i} \tau_0)$, one writes $F_{np} = \sum_{k=1}^{\infty} q^k \, F_k$.

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where

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- The localization procedure expresses F_k's in terms of the (deformed) instanton partition functions at number k,

$$Z_k = \int d\mathcal{M}_{(k)}\,\mathrm{e}^{-S_{inst}(\mathcal{M}_{(k)}, M, T)}\;.$$

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where

$$\bar{\delta}\star = \frac{\delta\star}{\delta(\theta^8\bar{\tau})}\Big|_{T=\tau_0,M=m_{cl}}\ .$$

- The localization procedure expresses F_k's in terms of the (deformed) instanton partition functions at number k,

$$Z_k = \int d\mathcal{M}_{(k)}\, \mathrm{e}^{-S_{inst}(\mathcal{M}_{(k)},M,T)}\ .$$

- The variations $\bar{\delta}F_k$ can thus be expressed in terms of $\bar{\delta}Z_k$.

Ingredients of the result (II)

- Recall the explicit form of the the moduli action:

$$S_{inst} = -2\pi\mathrm{i}\, k\, \tau + \cdots - 4\pi\mathrm{i} \sum_{\ell=0}^{\infty} \frac{(2\pi\alpha')^{2\ell}}{(2\ell)!} \mathrm{tr}(\chi^{2\ell})\, \bar{p}^{2\ell+4}\, \theta^8 \bar{\tau} + \cdots .$$

- It follows that

$$\bar{\delta} Z_k = 4\pi\mathrm{i} \sum_{\ell=0}^{\infty} (2\pi\alpha')^{2\ell}\, \bar{p}^{2\ell+4}\, Z_k^{(2\ell)} ,$$

where we introduced the "correlators" of χ moduli in the instanton matrix theory

$$Z_k^{(2\ell)} = \frac{1}{(2\ell)!} \int d\mathcal{M}_{(k)}\, \mathrm{tr}(\chi^{2\ell})\, \mathrm{e}^{-S_{inst}} \Big|_{T=\tau_0\,,\, M=m_{\mathrm{cl}}} .$$

The chiral ring on the D7 branes

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- In fact one finds a very strict relation between the $\bar{\delta}$ variation of the prepotential and the non-perturbative chiral ring:

$$\bar{\delta} F_k = 4\pi\mathrm{i} \sum_{\ell=0}^{\infty} (2\pi\alpha')^{2\ell}\, \bar{p}^{2\ell+4} \frac{(-1)^\ell}{(2\ell+4)!} \langle \mathrm{tr}\, m^{2\ell+4} \rangle_k \ .$$

M.B., Frau, Giacone, Lerda, 2011

proved at all orders in Fucito, Morales, Pacifici, 2011

The non-perturbative axio-dilaton profile

- Altogether, we have obtained

$$J_{np} = -\frac{(2\pi\alpha')^4}{2\pi}\sum_{k=1}^{\infty} q^k\,\bar{\delta}F_k = -2\mathrm{i}\sum_{\ell=1}^{\infty}(-1)^{\ell}\,\frac{(2\pi\alpha')^{2\ell}\,\langle \mathrm{tr}\, m^{2\ell}\rangle_{np}}{(2\ell)!}\,\bar{p}^{2\ell}\ ,$$

where $\langle \mathrm{tr}\, m^{2\ell}\rangle_{n.p.} = \sum_k q^k\,\langle \mathrm{tr}\, m^{2\ell}\rangle_k$.

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  where $\langle \mathrm{tr}\, m^{2\ell}\rangle_{n.p.} = \sum_k q^k\,\langle \mathrm{tr}\, m^{2\ell}\rangle_k$.

- Solving the field equation $\Box\,\tau_{np} = J_{np}\,\delta^2(z)$ we get then the non-perturbative axio-dilaton profile:

$$2\pi\mathrm{i}\,\tau_{np}(z) = -\sum_{\ell=1}^{\infty}\frac{(2\pi\alpha')^{2\ell}}{2\ell}\,\frac{\langle \mathrm{tr}\, m^{2\ell}\rangle_{np}}{z^{2\ell}}\ .$$

The complete τ profile

Let us summarize our findings.

- At the perturbative level we had

$$2\pi\mathrm{i}\,\tau_{cl}(z) = 2\pi\mathrm{i}\,\tau_0 - \sum_{\ell=1}^{\infty} \frac{(2\pi\alpha')^{2\ell}}{2\ell}\,\frac{\mathrm{tr}\,m_{cl}^{2\ell}}{z^{2\ell}}\ .$$

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- The quantities $\mathrm{tr}\,m_{cl}^{2\ell}$ of the D7 theory source the axio-dilaton. The source, however, is non-perturbatively corrected.
- The exact result is thus obtained by replacing classical v.e.v.'s with quantum correlators in the D7-brane theory:

$$2\pi\mathrm{i}\,\tau(z) = 2\pi\mathrm{i}\,\tau_0 - \sum_{\ell=1}^{\infty} \frac{(2\pi\alpha')^{2\ell}}{2\ell}\,\frac{\langle \mathrm{tr}\,m^{2\ell}\rangle}{z^{2\ell}} \ .$$

Gauge/gravity relation

- Chiral ring elements $\langle \mathrm{tr} m^{2\ell} \rangle$ are computable via localization.
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- We get explicitly

$$
\begin{aligned}
2\pi \mathrm{i}\tau &= 2\pi\mathrm{i}\tau_0 - \frac{1}{a^2}\sum_i m_i^2 \\
&+ \frac{1}{a^4}\left[-\frac{1}{2}\sum_i m_i^4 + 48\, m\, q + 24\sum_{i<j} m_i^2 m_j^2\, q^2 + 192\, m\, q^3 + \ldots\right] \\
&+ \frac{1}{a^6}\left[-\frac{1}{3}\sum_i m_i^6 - 240\sum_{i<j<k} m_i^2 m_j^2 m_k^2\, q^2 - 1280\, m \sum_i m_i^2\, q^3 + \ldots\right] \\
&+ \frac{1}{a^8}\left[-\frac{1}{4}\sum_i m_i^8 + 840\,(m)^2\, q^2 + 4480\, m\sum_{i<j} m_i^2 m_j^2\, q^3 + \ldots\right] + \ \ldots\ .
\end{aligned}
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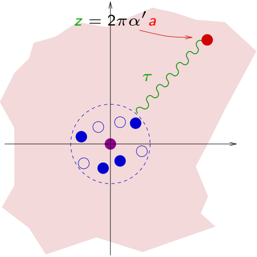
- If we parametrize the transverse directions with $a = z/(2\pi\alpha')$, all α' dependences are reabsorbed.

- By direct comparison with the large-a expansion of τ from the SW curve we check explicitly that

$$\tau(z) \Leftrightarrow \tau(a) \ ,$$

where $\tau(a)$ is the exact low-energy effective coupling of the SU(2), $N_f = 4$ gauge theory on a D3 probe, as expected.



Non-perturbative gauge/gravity relation: recap

- The exact effective coupling $\tau(a)$ is encoded in the SW curve.
  Seiberg and Witten, 1994

  Its large-a expansion is microscopically due to instanton contributions to the 4d prepotential. These are reproduced by D(-1) effects in our Type I' set-up. M.B, Gallot, Lerda, Pesando 2010.

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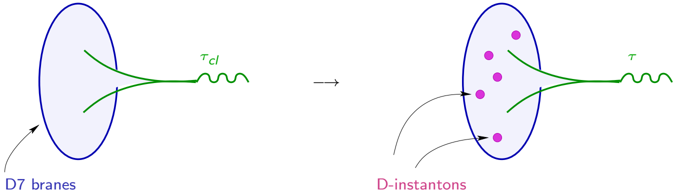
- We showed how, in the gravity dual approach, the large-z expansion of the field $\tau(z)$ emitted by the D7/O7 system is corrected by D-instanton effects that modify its source terms.



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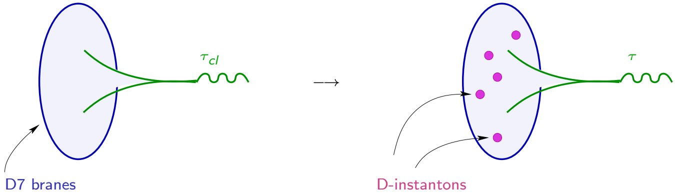

- The gravity result agrees with the gauge one.

An intriguing relation

- By studying its dual gravitational description in the type I' set-up we established a relation between
  - the 4d SU(2), $N_f = 4$ gauge theory (realized on the D3);
  - the 8d theory (realized on the D7's) which gauges its SO(8) flavor symmetry.

Already noticed in M.B, Gallot, Lerda, Pesando 2010

- This relation can be summarized as follows:

$$\tau(a) = \tau_0 + \frac{1}{2\pi \mathrm{i}} \langle \log \det \left(1 - \frac{m}{a}\right) \rangle \ ,$$

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where the expectation value on the r.h.s. is in the flavour theory.

- It would be nice to find other examples where the exact gauge coupling of a theory can be determined in terms of some subsector of it, or to its flavour sector.

A corollary

- Reconsider the 4d/8d relation described above, expanded or large a:

$$\tau(a) = \tau_0 - \frac{1}{2\pi \mathrm{i}} \sum_{\ell=1}^{\infty} \frac{1}{2\ell} \frac{\langle \mathrm{tr} m^{2\ell} \rangle}{a^{2\ell}} \ .$$

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- It is possible, developing recursion relations akin to Matone's relation in this set-up, to extract from the Seiberg-Witten curve the exact expression to all orders in q of any given coefficient of the expansion of τ in powers of $1/a$.

M. B., Frau, Gallot, Lerda, arXiv:1107.3691

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M. B., Frau, Gallot, Lerda, arXiv:1107.3691
- This amounts to finding the exact expression of the 8d chiral ring, previously computed to the first orders in its q expansion.
Fucito, Morales, Poghossian, 2009