Brane world effective actions from mixed amplitudes with general fluxes

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Foreword

This talk is based on a work in progress:

- M. Bertolini, M. B., A. Lerda, J.F. Morales and R. Russo, "Brane world effective actions for D-branes with fluxes", to appear (soon).

Plan of the talk

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1. Brane-worlds scenarios
2. D9 branes with general fluxes
3. Effective supersymmetric actions
4. The Kähler metric from strings
5. Relation to the Yukawa couplings

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Brane-worlds scenarios

Intersecting brane worlds

- Four-dimensional field theories with many “realistic” features arise from type IIA or B superstring models on suitable configurations of D-branes

[Bachas, 1995, Berkooz et al., 1996],...

- In particular, intersecting brane worlds have received much attention recently:

see, e.g., [Uranga, 2003]

  - Type IIA on $\mathbb{R}^{1,3} \times \mathcal{T}_6$ (or, more generally, on a CY - Not discussed here)
  - D6 branes wrapping intersecting 3-cycles in $\mathcal{T}_6$ support, on their non-compact world-volume, gauge groups and chiral matter (the latter are localized at the intersection points in the internal space)

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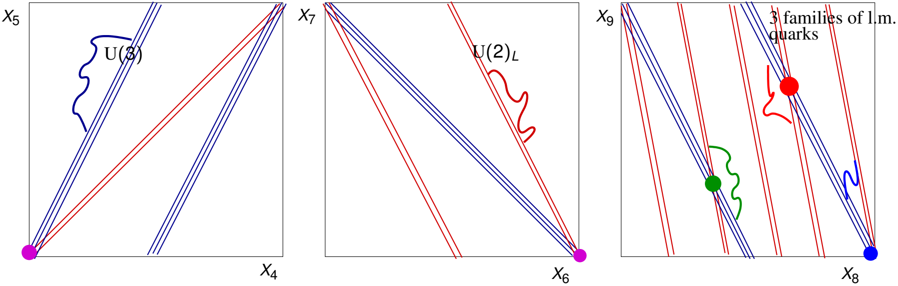
Gauge groups and chiral matter from branes

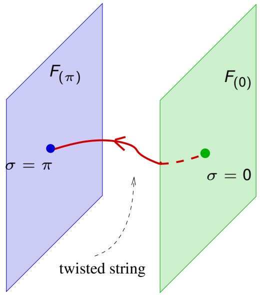
- Gauge groups from multiple branes, bifundamental chiral matter from “twisted” strings, replicas from multiple intersections



- **N.B.** The torus $\mathcal{T}_6$ is assumed to be factorized as $\mathcal{T}_2 \times \mathcal{T}_2 \times \mathcal{T}_2$.

T-duality and magnetized branes

- Upon T-duality (along one direction in each torus), IIA $\rightarrow$ IIB, and D6-branes intersecting on 3-cycles $\rightarrow$ D9 with magnetic fluxes



- Strings connecting two D9 with different fluxes feel different b.c.'s at their two end-points. They are twisted.
- The twists θ_i are determined from the quantized values of the fluxes

$$F^{(\sigma)}_{MN} = \frac{1}{2\pi}\,\frac{p_{MN}}{q_{MN}}$$

p_{MN} = Chern class, q_{MN} = wrapping of the D brane around the cycle $dX^M \wedge dX^N$.

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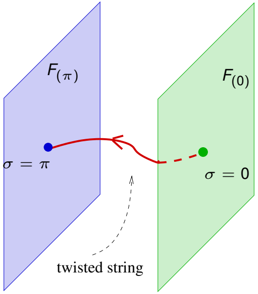

- If the torus is factorized as $\mathcal{T}_2 \times \mathcal{T}_2 \times \mathcal{T}_2$, fluxes respecting this factorization are matrices in $\mathrm{so}(2) \oplus \mathrm{so}(2) \oplus \mathrm{so}(2)$. Abelian situation: fluxes on different branes commute.
- General situation: fluxes on $\mathcal{T}_6$ represented by $\mathrm{so}(6)$ matrices. Oblique case: fluxes on different branes do not commute.
  - This generalization is important for in the context of the moduli stabilization problem [Antoniadis-maillard, 2004, Bianchi-Trevigne, 2005]

D9 branes with general fluxes

Boundary conditions on magnetized branes

- Bosonic part of the open string action: ▸ Back
(x^M in the $\mathcal{T}_6$ directions, $\sigma = 0, \pi$ denotes the end-point)

$$S_{\rm bos} = -\frac{1}{4\pi\alpha'}\int d^2\xi \left[\partial^\alpha x^M \partial_\alpha x^N G_{MN} + {\rm i}\epsilon^{\alpha\beta}\partial_\alpha x^M \partial_\beta x^N B_{MN}\right] - {\rm i}\sum_\sigma q_\sigma \int_{C_\sigma} dx^M A^\sigma_M$$

- In presence of constant G, B and field-strengths F_σ, the boundary conditions read

$$\bar\partial x^M\Big|_{\sigma=0,\pi} = (R_\sigma)^M_{\ N}\,\partial x^N\Big|_{\sigma=0,\pi}$$

where the reflection matrix R_σ is givenb by

$$R_\sigma = \left(G - \mathcal{F}_\sigma\right)^{-1}\left(G + \mathcal{F}_\sigma\right), \quad \mathcal{F}_\sigma = B + 2\pi\alpha' F_\sigma$$

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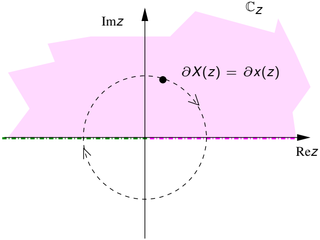
Twisted world-sheet fields

- The above b.c.'s can be solved in terms of a holomorphic, multivalued field $X^M(z)$ defined all over the complex z plane (*doubling trick*):

$$X^M(\mathrm{e}^{2\pi\mathrm{i}}z) = R^M_{\ N}\, X^N(z)\,, \quad R = R_\pi^{-1}\, R_0$$



- Both R_0 and R_π, and hence R, preserve the metric: ${}^tR\, G\, R = G$
- We can go to a complex basis $\mathcal{Z} = \left(Z^i\,,\ \bar{Z}^i\right) = \mathcal{E} X$, where

$$\mathcal{R} \equiv \mathcal{E}\, R\, \mathcal{E}^{-1} = \mathrm{diag}\left(\mathrm{e}^{2\mathrm{i}\pi\theta_1}, \cdots, \mathrm{e}^{2\mathrm{i}\pi\theta_d}, \mathrm{e}^{-2\mathrm{i}\pi\theta_1}, \cdots, \mathrm{e}^{-2\mathrm{i}\pi\theta_d}\right)$$

for $0 \le \theta_i < 1$ ($d = 3$ in our case). Back

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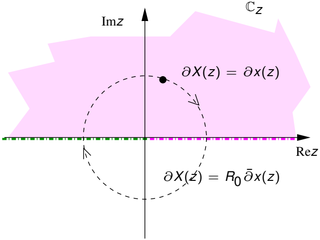

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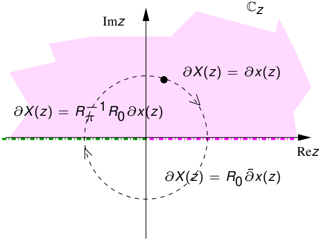

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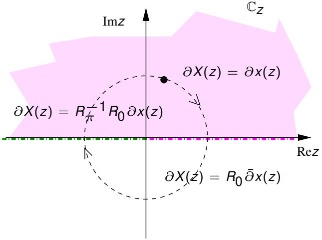

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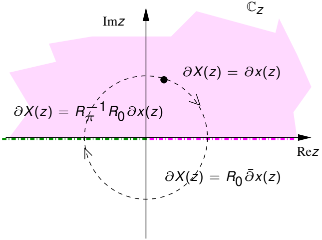

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The open string basis

- The open string complex, multivalued, fields $\mathcal{Z}^i(z)$ (and the corresponding w.s fermions $\Psi^i(z)$ have mode expansions shifted by θ_i.
- The θ_i play *exactly* the same role as the angles between intersecting D6. They represent the 3 "open string moduli" which determine the open string CFT properties.
- The vacuum $|\theta\rangle$ is created by bosonic and fermionic twist fields

$$|\theta\rangle = \lim_{z\to 0} \prod_{i=1}^{d} \sigma_{\theta_i}(z)\, s_{\theta_i}(z)\, |0\rangle$$

- The physical vertices contain (excited) twist fields

Dependence of the twists on the closed moduli

- The d open string twists θ_i depend on the $4d^2$ closed string moduli G_{MN} and B_{MN} and on the quantized fluxes $F_{0,\pi}^{MN}$ (or on the wrapping numbers for the intersecting branes)

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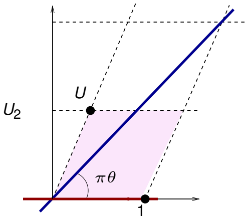

- For intersecting D-branes, the θ_i depend on the moduli describing the shape of the torus:

$$\tan(\pi\theta) = \frac{U_2\, n}{m + U_1\, n}$$

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- For general magnetized branes, from their definition as eigenvalues of the monodromy R we obtain Back

$$
\begin{aligned}
2\pi \mathrm{i} \, \frac{\partial \theta_i}{\partial m} = & \frac{1}{2} \left(\mathcal{E} \, G^{-1} \, \frac{\partial (G - B)}{\partial m} \, [R_\pi - R_0] \, \mathcal{E}^{-1} \right)_{ii} \\
& - \frac{1}{2} \left(\mathcal{E} \left[R_\pi^{-1} - R_0^{-1} \right] \, G^{-1} \, \frac{\partial (G + B)}{\partial m} \, \mathcal{E}^{-1} \right)_{ii}
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  where m is a generic closed string modulus, built out of G and B.
- Applies to general toroidal configurations with any G and B, and to generic (*i.e.* non-abelian) fluxes F_σ

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  where m is a generic closed string modulus, built out of G and B.
- *Crucial* formula to reconstruct the Kähler metric for the twisted scalars from mixed open/closed amplitudes

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  where m is a generic closed string modulus, built out of G and B.
- In the factorized case, and upon T-duality, reproduces the dependence of the angles just described

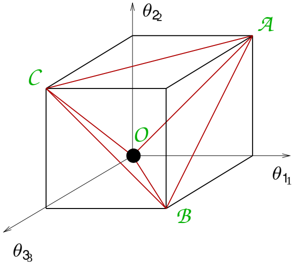
Effective supersymmetric actions

Supersymmetric brane-worlds?

- Simplest models with standard-model-like features break all susy.
- Preserving some susy requires some tuning, in the closed and in the open string sector.
- In the closed, bulk sector:
  - $\mathcal{T}_6$ compact $\longrightarrow$ cancel RR tadpoles
  - cancel NS-NS tadpoles for susy $\longrightarrow$ orientifolds;
- In the open sector, i.e. on the branes:
  - Susy generically broken for the open strings connecting two different D-branes: angles θ_i $\longrightarrow$ twists in the CFT $\longrightarrow$ mass split between *R* and *NS* spectrum
  - Susy (partially) preserved for particular values of the twists

Supersymmetric configurations

- The SUSY preserved on the twisted strings can be described in the space of the θ_i's, which we take in $[0, 1)$.



- For $\theta_1 = \theta_2 = \theta_3 = 0$, $\mathcal{N} = 4$ susy spectrum (like for strings between parallel branes in flat space) Back

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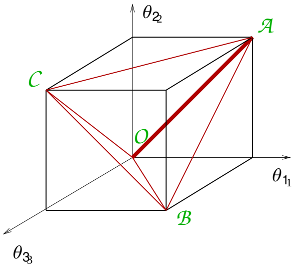

- When one θ vanishes, we get an $\mathcal{N} = 2$ hyper-multiplet:
  - two massless scalars from NS
  - two massless fermions from R sector

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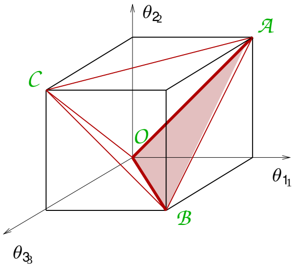

- On the faces, e.g., for $\sum_{j\neq i}\theta_j - \theta_i = 0$ (which we will write as $\sum_j \varepsilon_{j(i)}\theta_j = 0$) we have $\mathcal{N} = 1$ chiral multiplets Φ^i
  - one massless scalar ϕ^i from NS
  - a chiral fermion χ^i from R sector)

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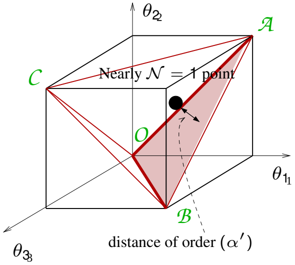

- We will consider softly broken $\mathcal{N} = 1$ by taking θ's *close* to a face:

$$\theta_i = \theta_i^{(0)} + 2\alpha' \epsilon_i \ , \qquad \sum_j \varepsilon_{j(i)} \theta_j^{(0)} = 0$$

with $\theta_i^{(0)}$ and ϵ_i fixed in the limit $\alpha' \to 0$.

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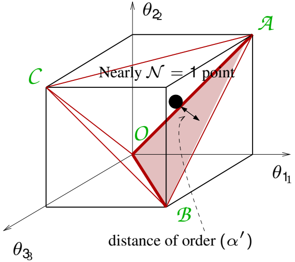

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- The scalar ϕ^i gets a mass $M^2 = \frac{1}{2\alpha'} = \sum_j \varepsilon_{j(i)} \theta_j = \sum_j \varepsilon_{j(i)} \epsilon_j$

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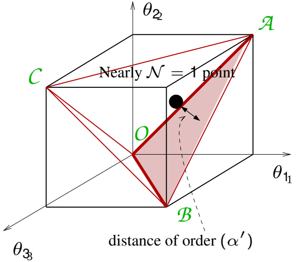

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- Amounts to soft susy breaking á la FI from v.e.v.'s of the auxiliary fields D. We're working on a direct description of this via string diagrams.

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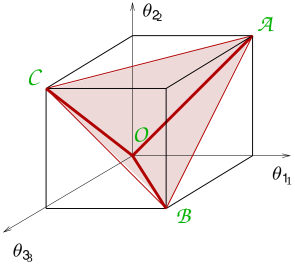

- In the interior of the tetrahedron, we still have a chiral massless fermion from R sector, but only massive scalars.

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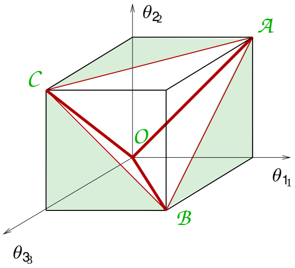

- Outside the tetrahedron, the scalars would become tachyonic.

Effective action in the N=1 case

- The l.e.e.a is an $\mathcal{N} = 1$ SUGRA coupled with gauged matter coming from diferent sectors:
- from the closed string sector, upon usual $\mathcal{T}_6$ compactification.
  - For instance, 6^2 moduli m from NS-NS bkg fields G_{MN}, B_{MN} describing the stringy shape of the $\mathcal{T}_6$.
- from the open string sector, gauge + matter fields living on the D-branes.
  - In particular, chiral multiplets Φ^i ("twisted" matter) from strings stretching between different D-branes (localized at their intersections)

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Effective N=1 action for twisted matter

- Regarding the moduli as fixed, the Kähler potential for the twisted chiral matter will be of the form

$$K = K_{\bar{\phi}^i \phi^i}(m)\bar{\phi}^i \phi^i + O(\phi^4)$$

(easy to check that there's no mixing between ϕ^i and ϕ^j with $i \neq j$ in our cases).

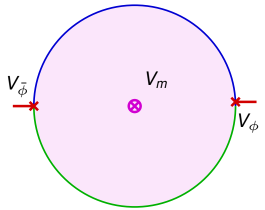
- This corresponds to a lagrangian kinetic term ▸ Back

$$\mathcal{L} = -K_{\bar{\phi}^i \phi^i}(m)(\partial_\mu \bar{\phi}^i \partial^\mu \phi^i + M^2 \bar{\phi}^i \phi^i)$$

- The dependence of the"metric" $K_{\bar{\phi}^i \phi^i}$ on the closed string moduli m can be determined from mixed open/closed amplitudes.

The Kähler metric from strings

Mixed amplitudes and the Kähler metric



- Let V_m be the closed string NS-NS vertex for the modulus m. The amplitude ▸ Back

$$\mathcal{A}_{\bar\phi^i \phi^i m} \sim \langle V_{\bar\phi^i}\, V_m\, V_{\phi^i} \rangle$$

is related to the derivative w.r.t. m of the scalar kinetic term.

- String amplitudes would give canonical kinetic terms, so ▸ Back

$$V_{\phi^i} \to \sqrt{K_{\bar\phi^i \phi^i}}\, V_{\phi^i} \;, \quad V_{\bar\phi^i} \to \sqrt{K_{\bar\phi^i \phi^i}}\, V_{\bar\phi^i}$$

- We have then ▸ Back

$$\mathcal{A}_{\bar\phi^i \phi^i m} = \mathrm{i}\, K^{-1}_{\bar\phi^i \phi^i} \frac{\partial}{\partial m} \frac{\partial}{\partial \phi^i} \frac{\partial}{\partial \bar\phi^i} \mathcal{L} = \mathrm{i}\, K^{-1}_{\bar\phi^i \phi^i} \frac{\partial}{\partial m} \left[K_{\bar\phi^i \phi^i} \left(k_1\, k_2 - M^2 \right) \right]$$

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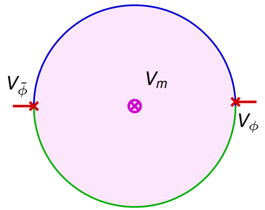

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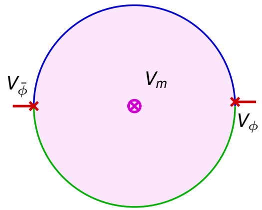

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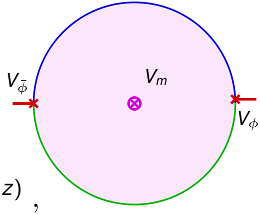
Closed string moduli vertices

- The vertex for the insertion of a generic modulus m reads ▸ Recall

$$V_m(z,\bar{z}) = \frac{\partial}{\partial m}(G-B)_{MN}\, V_L^M(z)\, V_R^N(\bar{z})$$

where

$$V_L^M(z) = \Big[\partial X_L^M(z) + \mathrm{i}(k_L\cdot\Psi_L)\Psi^M(z)\Big]\,\mathrm{e}^{\mathrm{i}\,k_L\cdot X_L(z)}\,,$$
$$V_R^N(\bar{z}) = \Big[\partial X_R^N(\bar{z}) + \mathrm{i}(k_R\cdot\Psi_R)\Psi^N(\bar{z})\Big]\,\mathrm{e}^{\mathrm{i}\,k_R\cdot X_R(\bar{z})}$$



The form of the amplitude

- The amplitude $\mathcal{A}_{\bar{\phi}^i \phi^i m}$ reads Back

$$\mathcal{A}_{\bar{\phi}^i \phi^i m} = \left[\frac{\partial}{\partial m} (G - B) \cdot R_0 \right]_{MN} \mathcal{E}^M{}_a \, \mathcal{E}^N{}_b \, \mathcal{A}^{ab}$$

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- Impose the boundary identification $V_R^M(\bar{z}; k_R) = R_0{}^M{}_N \, V_L^N(\bar{z}; k_R)$

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- Switch to the open string complex basis $\mathcal{Z}^a = \mathcal{E}^a{}_M X^M$

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- The matrix $\mathcal{A}^{ab}$ is the CFT correlator

$$\mathcal{A}^{ab} = \frac{e^{-\pi i \alpha' s/2}}{8\pi\alpha'^2} \langle V_{\bar{\phi}^i} \, V_L^a V_L^b \, V_{\phi^i} \rangle$$

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  - Overall normalization

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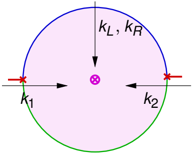

- ▸ Cocycle to put off-shell in a controlled way the closed string vertex

$$s = (k_1 + k_2)^2 = (k_L + k_R)^2$$
$$= 2(k_1 \cdot k_2 - M^2) = 2 k_L \cdot k_R$$

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  - Vertices in the open string complex basis $\mathcal{Z}^a$

The CFT correlator

- It is easy to see that the correlator $\mathcal{A}^{ab}$ has the matrix form

$$\mathcal{A} \equiv \begin{pmatrix} 0 & A_j\,\delta^{ij} \\ \bar{A}_j\,\delta^{ij} & 0 \end{pmatrix}\,, \quad \text{with} \quad A_j = \frac{e^{-\pi i \alpha' s/2}}{8\pi\alpha'^2}\,\langle V_{\bar\phi^i}\, V_L^j\, \overline{V}_L^j\, V_{\phi^i}\rangle$$

- Now we must:
  - insert the explicit form of the vertices $V_{\bar\phi^i}(x_1)$ and $V_{\phi^i}(x_2)$
  - integrate their positions $x_{1,2}$ over the real axis and the position z of the closed vertex $V_L^j(z)$ over the upper half plane, up to $\mathrm{SL}(2,\mathbb{R})$
- We get

$$\mathcal{A}_j = \frac{i\,\varepsilon_{j(i)}}{4\pi\alpha'}\,e^{i\pi\theta_j}\,\sin\left[\pi(\theta_j + \alpha' s/2)\right]\frac{\Gamma(\alpha' s+1)\Gamma(1-\theta_j-\alpha' s/2)}{\Gamma(1-\theta_j+\alpha' s/2)}$$
$$= \frac{i\,\varepsilon_{j(i)}}{4\pi\alpha'}\,e^{i\pi\theta_j}\,\sin(\pi\theta_j)(1-\frac{1}{2}\alpha'\,s\,\rho_j) + \mathcal{O}\left(\alpha' s^2\right)$$

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The result for the amplitude

- Altogether, one can write (up to 2-derivative terms, i.e. up to s^2) the correlator $\mathcal{A}^{ab}$ in matrix form as

$$\mathcal{A} = \frac{1}{2}\mathcal{G}^{-1}\left(\mathcal{R}^{-1} - 1\right)\mathcal{H}\,, \quad \mathcal{H} = i\begin{pmatrix} h_j & 0 \\ 0 & -h_j \end{pmatrix}$$

with

$$h_j = \frac{\varepsilon_{j(i)}}{4\pi\alpha'}\left(1 - \frac{1}{2}\alpha'\, s\, \rho_j\right) = \frac{1}{2\pi}K^{-1}_{\bar{\phi}^i\phi^i}\frac{\partial}{\partial\theta^j}K_{\bar{\phi}^i\phi^i}(k_1\cdot k_2 - M^2)$$

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$$K_{\bar{\phi}^i\phi^i} = \mathrm{e}^{2\gamma_E\,\alpha' M^2}\sqrt{\frac{\Gamma(1-\theta_i)}{\Gamma(\theta_i)}}\prod_{k\neq i}\sqrt{\frac{\Gamma(\theta_k)}{\Gamma(1-\theta_k)}}$$

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- We used the kinematics $s = 2(k_1\cdot k_2 - M^2)$, the dependence of M^2 on θ_j and the fact that $\psi(x) = \frac{d\ln\Gamma(x)}{dx}$.

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- The exponential term goes to 1 in the field theory limit

The magic of the result

- Substituting into the expression of the correlator $\mathcal{A}_{\bar{\phi}^i \phi^i m}$ ▸ Recall we get after some algebra

$$\mathcal{A}_{\bar{\phi}^i \phi^i m} = \frac{1}{2}\,\mathcal{E}\,G^{-1}\,\frac{\partial}{\partial m}(G - B)\,(R_\pi - R_0)\,\mathcal{E}^{-1}\big|_j^j\,h_j - \text{h.c.}$$

- Comparing with the expression of the dependence of the twists θ_i from the moduli m ▸ Recall we can write

$$\mathcal{A}_{\bar{\phi}^i \phi^i m} = 2\pi\,\frac{\partial\theta_j}{\partial m}\,h_j = K^{-1}_{\bar{\phi}^i \phi^i}\,\frac{\partial\theta_j}{\partial m}\,\frac{\partial}{\partial\theta^j}K_{\bar{\phi}^i \phi^i}(k_1 \cdot k_2 - M^2)$$

- This is the expression we expected ▸ Recall if $K_{\bar{\phi}^i \phi^i}$ really is the Kähler metric

The field theory Kähler metric

- Summarizing, in the field theory limit the expression of the Kähler metric $K_{\bar{\phi}^i \phi^i}$ for the scalar ϕ^i depends on the moduli only through the open string twists
$$\theta_i^{(0)} = \lim_{\alpha' \to 0} \theta_i$$
in an $\mathcal{N} = 1$ configuration. Explicitly,
$$K_{\bar{\phi}^i \phi^i} = \sqrt{\frac{\Gamma(1 - \theta_i^{(0)})}{\Gamma(\theta_i^{(0)})}} \prod_{k \neq i} \sqrt{\frac{\Gamma(\theta_k^{(0)})}{\Gamma(1 - \theta_k^{(0)})}}$$
- This holds for a general toroidal compactification, and with arbitrary magnetic fluxes, also non-commuting

Generalizes [Lust et al., 2004]

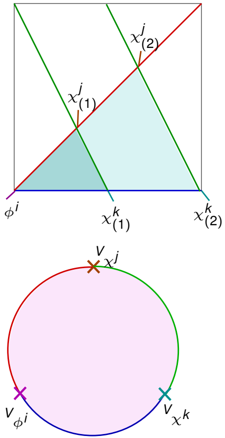
Relation to the Yukawa couplings

Stringy expression of the Yukawa couplings

In the stringy description, Yukawa couplings have the form $Y_{ijk} = \mathcal{A}_{ijk}\, W_{ijk}$, where

- W_{ijk} = classical contribution from extended world-sheets bordered by the intersecting branes.
  - Multiple intersections $\rightarrow$ families
  - different minimal world-sheets $\rightarrow$ exponential hierarchy of couplings

  (have counterparts in magnetized brane worlds [Cremades et al., 2004])
- $\mathcal{A}_{ijk}$ = quantum fluctuations given by the correlator of the twisted vertices located at the intersections. Back



Yukawa couplings and N=1 superpotential

- In $\mathcal{N} = 1$ susy, the Yukawa couplings arise from the superpotential

$$\int d^2\theta\, W(\Phi^i) + \mathrm{c.c} \to \ldots + \frac{\partial W}{\partial\phi^i\partial\phi^j}\chi^i\chi^j + \mathrm{h.c.}\ .$$

For $W = W_{ijk}\Phi^i\Phi^j\Phi^k$, the W_{ijk} are the Yukawa couplings in the basis where the kinetic terms are determined by the Kähler potential K Recall

- When realized in string compactifications, non-renormalization property: W gets no perturbative α' corrections
- In the brane-world context, we identify therefore the W_{ijk} as the classical world-sheet instanton contributions

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Kähler metric and quantum Yukawas

- The $\mathcal{N} = 1$ holomorphic couplings W_{ijk} are related to the physical ones, Y_{ijk} (the ones provided by the string computation) by rescaling the fields ϕ^i, χ^j, χ^k to give them canonical kinetic terms.
  Recall
- One has thus

$$Y_{ijk} = (K_{\bar{\phi}^i \phi^i} K_{\bar{\phi}^j \phi^j} K_{\bar{\phi}^k \phi^k})^{-1/2} \, W_{ijk}$$

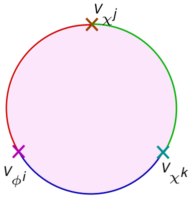
- We had already found Recall

$$Y_{ijk} = \mathcal{A}_{ijk} \, W_{ijk}$$

- Hence, the amplitude $\mathcal{A}_{ijk}$ for the three twisted vertices should be factorizable into

$$\mathcal{A}_{ijk} = (K_{\bar{\phi}^i \phi^i} K_{\bar{\phi}^j \phi^j} K_{\bar{\phi}^k \phi^k})^{-1/2}$$

The abelian case



- In the case of a factorized torus with commuting angles (or for D-branes at angles) the direct computation of the string amplitude $\mathcal{A}_{ijk}$ is possible
- It involves in particular the correlator of three bosonic twist fields on the torus which are simultaneously expressible in terms of twist angles $\{\theta_i\}$, $\{\nu_i\}$, $\{\lambda_i\}$
- This correlator is computable by factorization of the 4-twist amplitude, and its dependence on the three sets of angles factorizes
- In the end, one indeed finds

$$\mathcal{A}_{ijk} = (K_{\bar{\phi}^i \phi^i} K_{\bar{\phi}^j \phi^j} K_{\bar{\phi}^k \phi^k})^{-1/2}$$

in agreement with the non-renormalization theorem

[Cvetic-Papadimitriou, 2003, Lust et al., 2004]

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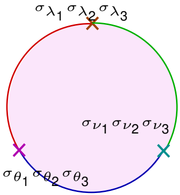

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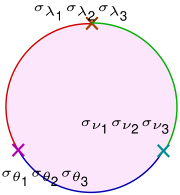

- In the case of a factorized torus with commuting angles (or for D-branes at angles) the direct computation of the string amplitude $\mathcal{A}_{ijk}$ is possible
- It involves in particular the correlator of three bosonic twist fields on the torus which are simultaneously expressible in terms of twist angles $\{\theta_i\}$, $\{\nu_i\}$, $\{\lambda_i\}$
- This correlator is computable by factorization of the 4-twist amplitude, and its dependence on the three sets of angles factorizes
- In the end, one indeed finds

$$\mathcal{A}_{ijk} = (K_{\bar{\phi}^i\phi^i}K_{\bar{\phi}^j\phi^j}K_{\bar{\phi}^k\phi^k})^{-1/2}$$

in agreement with the non-renormalization theorem

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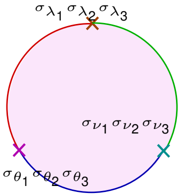

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The non-abelian case?

- We have considered the general case in which the reflection matrices at the various boundaries do not commute, and shown that the Kähler metric remains the same
- Hence the monodromy matrices $R_{\theta,\nu,\lambda}$ induced by the the three twist operators cannot, in general, be simultaneously diagonalized
- We have thus to deal with ("non-abelian twist fields"), whose 3-point CFT correlators are not known. Their computation represents a challenge.
- The non-renormalization theorem, however, suggests that the correlator still factorizes and depends on the three sets $\{\theta_i\}$, $\{\nu_i\}$, $\{\lambda_i\}$ of monodromy eigenvalues

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Some references

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