Stringy instanton calculus

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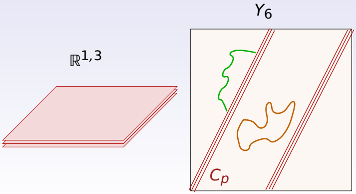
Introduction and motivations

D-brane worlds

- SM-like sector from open strings on stacks of D(3+p) branes wrapped on some internal p-cycles C_p
- Gravitational sector from closed strings in the bulk



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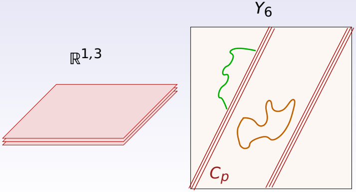

- Gauge and gravitational couplings depend on different volumes (expressed in units of $\sqrt{\alpha'}$):

$$\kappa_4^2 \sim g_s^2 \alpha' / V(Y_6) \,, \quad g_{YM}^2 \sim g_s / V(C_p)$$

  - String mass scale α' can be much lower than 4-d M_{Pl}

Arkani-Hamed et al., '98

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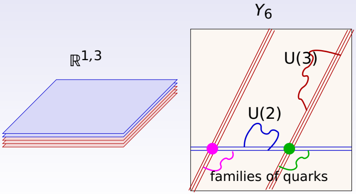


- Gauge groups from multiple branes, bifundamental chiral matter from “twisted” strings, replicas from multiple intersections

see, e.g., [Uranga, 2003, Kiritsis, 2004, Lust, 2004, Blumenhagen et al., 2005]

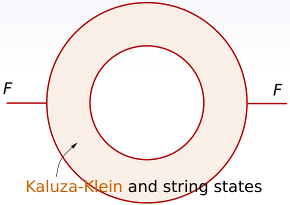
- (String) topology of the internal space + choice of branes (subject to tadpole cancellation): a rich model building scenario (using intersecting/magnetized branes of various dimensions)

Perturbative effects

of extra-dimension

- The higher-dimensional, stringy origin of a given D-brane world model bears also on the quantum properties of its low-energy effective action

- For instance, the perturbative corrections are affected by the extra states in the theory, resulting in threshold corrections



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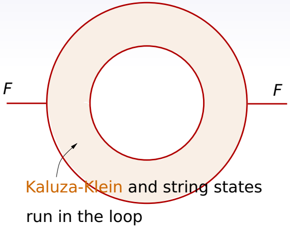
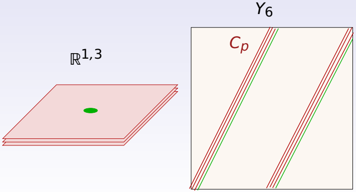


- Also non-perturbative corrections can be influenced

Non-perturbative corrections

Gauge instantons & D-brane instantons

- Non-perturbative sectors: partially wrapped E(uclidean)-branes
- Pointlike in $\mathbb{R}^{1,3}$: instanton configurations



- E-branes identical to a given D-brane stack in the internal directions: instantons for that gauge theory
  - ADHM from strings attached to the instantonic branes
    Witten, 1995; Douglas, 1995-1996; ...
  - non-trivial instanton profile of the gauge field
    Billo et al, 2001
  - Rules and techniques to embed the instanton calculus in string theory have been constructed
    Polchinksi, 1994; Green-Gutperle, 2000, ...; Turin/Rome/Münich/UPenn/Madrid,...

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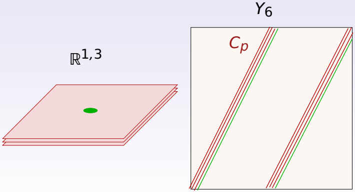

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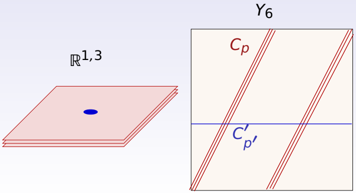
More non-perturbative corrections

“Stringy” or “exotic” instantons

- E-branes wrapped on a different internal cycle $C'_{p'}$, yield exotic (a.k.a. stringy) non-perturbative corrections



- Ordinary gauge instanton effects suppressed by $e^{-\frac{8\pi^2}{g_{YM}^2}}$
- Exotic instanton effects suppressed by $e^{-\frac{8\pi^2}{g_{YM}^2}\frac{V(C'_{p'})}{V(C_p)}}$
  - they would be ordinary instanton for the gauge theory of branes wrapped on $C'_{p'}$

More non-perturbative corrections

"Stringy" or "exotic" instantons

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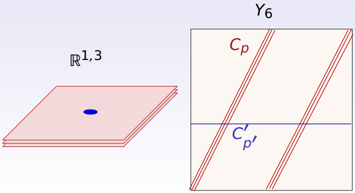


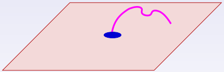
- Exotic instantons may lead to interactions that would be perturbatively forbidden in these models
- Such effects could be of great phenomenological relevance (Neutrino Majorana masses, Yukawas in certain GUT models,. . .)

Blumenhagen et al '06; Ibanez and Uranga, '06; Haack et al, '06; Blumenhagen et al, 2008; ...

- Need to understand their status in the gauge theory and to construct precise rules for the "exotic" instanton calculus

Exotic features

from the world-sheet point of view



- Consider the strings stretching between the gauge D-branes and the E-branes

- NS sector physicity condition:

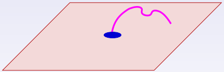
$$L_0 - \frac{1}{2} = N_X + N_\psi + \sum_{i=1}^{3} \frac{\theta_i}{2} = 0 \, ,$$

  - Ordinary case: internal twists $\theta_i = 0$. There are bosonic moduli $w_{\dot\alpha}$ typical of ADHM construction, related to the size
  - Exotic case: $\theta_i > 0$, i.e., there are "more than 4 ND directions" . The moduli $w_{\dot\alpha}$ are absent. Hints at zero-size limit of some gauge field configuration.

Exotic features

from the world-sheet point of view

- Consider the strings stretching between the gauge D-branes and the E-branes



- In the R sector, fermionic anti-chiral moduli $\lambda_{\dot\alpha}$ always present
  - Ordinary case: Lagrange multipl. of fermionic ADHM constraints
  - Exotic case: the the abelian component of the λ's is a true fermionic zero-mode since the abelian part of ADHM constraint vanishes (it would cointain the $w_{\dot\alpha}$). Must be removed to get non-zero correlators:
    - ★ orientifold projections Argurio et al, 2007; ...
    - ★ closed string fluxes Blumenhagen et al, 2007; Billo et al, 2008; ...
    - ★ other mechanisms Petersson, 2007; ...

Strategy

- Select a simple example: D(-1)/D7 in type I' theory, sharing many features of stringy instantons
- Investigate the field-theory interpretation of D(-1)'s in this 8d gauge theory Billo et al, 2009a;
- Compute the non-perturbative effective action on the D7's extending the rules of stringy instanton calculus to this "exotic" case.
- Check against the results in the dual Heterotic $SO(8)^4$ theory. Impressive quantitative check of this string duality. Billo et al, 2009b
- Apply the technology to tractable examples leading to 4d models

Work in progress, Turin + Tor Vergata

Disclaimer

- This talk builds over a vast literature - some scattered references are given in the slides
  - I apologize for missing ones...
- Results presented here mostly from
  - M. Billo, M. Frau, L. Gallot, A. Lerda and I. Pesando, “Classical solutions for exotic instantons?,”, JHEP **03** (2009) 056, arXiv:0901.1666 [hep-th]
  - M. Billo, L. Ferro, M. Frau, L. Gallot, A. Lerda and I. Pesando, “Exotic instanton counting and heterotic/type I’ duality,” JHEP **0907** (2009) 092, arXiv:0905.4586 [hep-th]
  - M. Billo, M. Frau, F. Fucito, A. Lerda, F. Morales and R. Poghossyan, work in progress

Plan of the talk

An 8-dimensional example

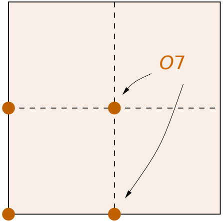
A D(-1)/D7 system in type I'

- Type I' is type IIB on a two-torus $\mathcal{T}_2$ modded out by

$$\Omega = \omega\,(-1)^{F_L}\,I_2$$

ω = w.s. parity, F_L = left-moving fermion #, I_2 = inversion on $\mathcal{T}_2$

- Ω has four fixed-points on $\mathcal{T}_2$ where four O7-planes are placed
- Admits D(-1), D3 and D7's transverse to $\mathcal{T}_2$
- Local RR tadpole cancellation requires 4 D7-branes at each fix point



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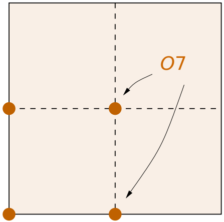

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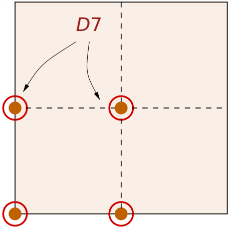

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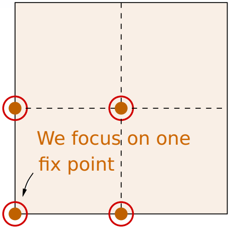

The gauge theory on the D7's

- From the D7/D7 strings we get $\mathcal{N} = 1$ vector multiplet in $d = 8$ in the adjoint of SO(8):

$$\left\{A_\mu, \Lambda^\alpha, \phi_m\right\}, \quad \mu = 1, \ldots 8, \quad m = 8, 9$$

- Can be assembled into a “chiral” superfield

$$\Phi(x, \theta) = \phi(x) + \sqrt{2}\,\theta\Lambda(x) + \frac{1}{2}\,\theta\gamma^{\mu\nu}\theta\, F_{\mu\nu}(x) + \ldots$$

  where $\phi = (\phi_8 + i\phi_9)/\sqrt{2}$.
  - Formally very similar to $\mathcal{N} = 2$ in $d = 4$

Effective action on the D7

(tree level)

- Effective action in $F_{\mu\nu}$ and its derivatives: NABI

$$
\begin{aligned}
S &= S_{(2)} + S_{(4)} + S_{(5)} + \cdots \\
&= \frac{1}{8\pi g_s} \int d^8x \Big[\frac{\mathrm{Tr}\big(F^2\big)}{(2\pi)^4 \alpha'^2} - \frac{t_8 \mathrm{Tr}\big(F^4\big)}{3\,(2\pi)^2} + \alpha' \mathcal{L}_{(5)}\big(F, DF\big) + \cdots \Big]
\end{aligned}
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- The quadratic Yang-Mills term $S_{(2)}$ has a dimensionful coupling $g^2_{\mathrm{YM}} \equiv 4\pi g_s (2\pi\sqrt{\alpha'})^4$

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- Contributions of higher order in α'

Effective action on the D7

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- The quartic term has a dimensionless coupling:

$$S_{(4)} = -\frac{1}{96\pi^3 g_s} \int d^8x\, t_8 \mathrm{Tr}\left(F^4\right)$$

Effective action on the D7

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\end{aligned}
$$

- Adding the WZ term, we can write

$$
S_{(4)} = -\frac{1}{4!\,4\pi^3 g_s} \int d^8x\, t_8 \mathrm{Tr}\left(F^4\right) - 2\pi i\, C_0\, c_{(4)}
$$

where $c_{(4)}$ is the fourth Chern number

$$
c_{(4)} = \frac{1}{4!(2\pi)^4} \int \mathrm{Tr}\left(F \wedge F \wedge F \wedge F\right)
$$

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- Effective action in $F_{\mu\nu}$ and its derivatives: NABI

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\end{aligned}
$$

- Adding the fermionic terms, can be written using the superfield $\Phi(x, \theta)$ as

$$
S_{(4)} = \frac{1}{(2\pi)^4} \int d^8x\, d^8\theta\, \mathrm{Tr}\left[\frac{i\pi}{12}\, \tau\, \Phi^4 \right] + \text{c.c.}
$$

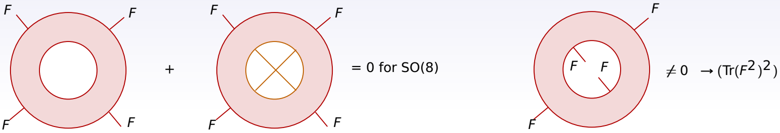
where $\tau = C_0 + \frac{i}{g_s}$ is the axion-dilaton.

- Receives one-loop and non-perturbative corrections

Effective action

1-loop effective action

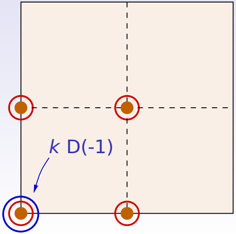
- At 1-loop we get contributions from annuli and Möbius diagrams. At the quartic level,



$$S^{1-\text{loop}}_{(4)} = \frac{1}{256\pi^4}\int d^8x \log\left(\text{Im}\tau\,\text{Im}U\,|\eta(U)|^4\right) t_8\left(\text{Tr}F^2\right)^2\Big]$$
$$= \frac{1}{(2\pi)^4}\int d^8x\, d^8\theta\,\left[\frac{1}{32}\log\left(\text{Im}\tau\,\text{Im}U\,|\eta(U)|^4\right)\left(\text{Tr}\Phi^2\right)^2\right] + \text{c.c.}$$

(U is the complex structure of the 2-torus $\mathcal{T}_2$)

Adding D-instantons



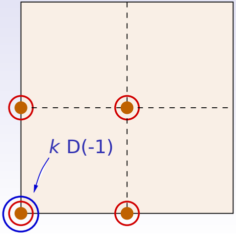
- Add k D-instantons.
- D7/D(-1) form a 1/2 BPS system with 8 ND directions
- D(-1) classical action

$$\mathcal{S}_{cl} = k\left(\frac{2\pi}{g_s} - 2\pi i C_0\right) \equiv -2\pi i k \tau \ ,$$

- Coincides with the quartic action on the D7 for gauge fields F with $c_{(4)} = k$ and

$$\int d^8x \, Tr\left(t_8 F^4\right) = -\frac{1}{2}\int d^8x \, Tr\left(\epsilon_8 F^4\right) = -\frac{4!}{2}(2\pi)^4 c_{(4)}$$

Adding D-instantons



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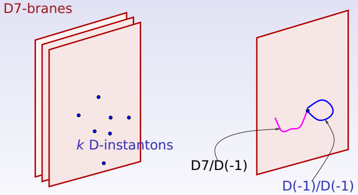
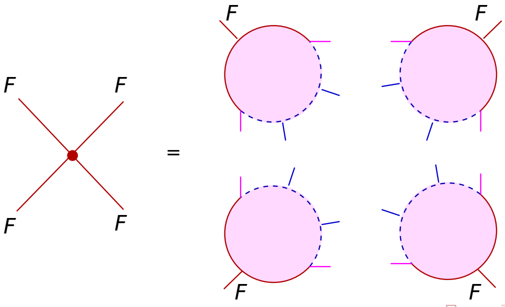
- Analogous to relation with self-dual YM config.s in D3/D(-1)
- Suggests relation to some 8d instanton of the quartic action

Effective action from D-instantons



- Open strings with at least one end on a D(–1) carry no momentum: they are moduli rather than dynamical fields.

- Effective interactions between gauge fields can be mediated by D-instanton moduli through mixed disks



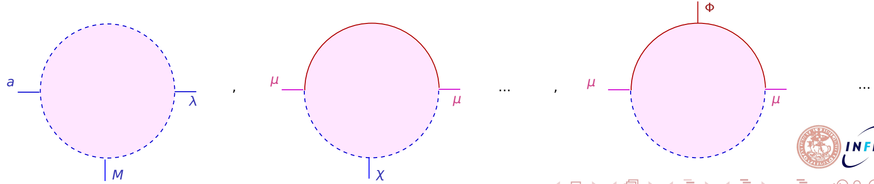
Effective action from D-instantons

Moduli integral

- Non-perturbative contributions to the effective action of the gauge degrees of freedom Φ arise integrating over the instanton moduli $\mathcal{M}_{(k)}$ and summing over all instanton numbers k

$$\mathcal{S}_{\mathrm{n.p.}}(\Phi) = \sum_k e^{2\pi i \tau k} \int d\mathcal{M}_{(k)}\, e^{-\mathcal{S}(\mathcal{M}_{(k)},\Phi)}$$

- $2\pi i \tau k$ is the classical value of the instanton action
- $\mathcal{S}(\mathcal{M}_{(k)},\Phi)$ arises from (mixed) disk diagrams describing interactions of the moduli among themselves and with the gauge fields



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- This procedure is by now well-established for instantonic brane systems corresponding to gauge instantons

  Polchinksi, 1994; Green-Gutperle, 2000, ...; Turin/Rome/Münich/UPenn/Madrid,...
- We want to apply it explicitly in our "exotic" instanton set-up
- This is a very complicated matrix integral ...

The moduli spectrum

Spectrum:

Sector		Name	Meaning	Chan-Paton	Dimension
$-1/-1$	NS	a_μ	centers	symm SO(k)	(length)
		$\chi, \bar{\chi}$		adj SO(k)	(length)$^{-1}$
		D_m	Lagr. mult.	adj SO(k)	(length)$^{-2}$
	R	M^α	partners	symm SO(k)	(length)$^{\frac{1}{2}}$
		$\lambda_{\dot\alpha}$	Lagr. mult.	adj SO(k)	(length)$^{-\frac{3}{2}}$
$-1/7$	R	μ		$\mathbf{8} \times \mathbf{k}$	(length)
	NS	w	(auxiliary)	$\mathbf{8} \times \mathbf{k}$	(length)^0

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- The SO(k) rep. is determined by the orientifold projection

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$-1/7$	R	μ		$\mathbf{8} \times \mathbf{k}$	(length)
	NS	w	(auxiliary)	$\mathbf{8} \times \mathbf{k}$	$(\text{length})^0$

- Abelian part of a_μ, M_α $\sim$ Goldstone modes of the (super)translations on the D7 broken by D(-1)'s. Identified with coordinates x_μ,θ_α

The moduli spectrum

Spectrum:

Sector		Name	Meaning	Chan-Paton	Dimension
$-1/-1$	NS	a_μ	centers	symm SO(k)	(length)
		$\chi, \bar{\chi}$		adj SO(k)	(length)$^{-1}$
		D_m	Lagr. mult.	adj SO(k)	(length)$^{-2}$
	R	M^α	partners	symm SO(k)	(length)$^{\frac{1}{2}}$
		$\lambda_{\dot{\alpha}}$	Lagr. mult.	adj SO(k)	(length)$^{-\frac{3}{2}}$
$-1/7$	R	μ		$\mathbf{8} \times \mathbf{k}$	(length)
	NS	w	(auxiliary)	$\mathbf{8} \times \mathbf{k}$	(length)^0

- For "mixed" strings, no bosonic moduli from the NS sector: characteristic of "exotic" instantons

The moduli action

- The action reads:

$$\mathcal{S}(\mathcal{M}_{(k)}, \Phi) = \mathrm{tr}\Big\{ i\lambda_{\dot\alpha}\gamma_\mu^{\dot\alpha\beta}[a^\mu, M_\beta] + \frac{1}{2g_0^2}\lambda_{\dot\alpha}[\chi, \lambda^{\dot\alpha}] + M^\alpha[\bar\chi, M_\alpha] + \frac{1}{2g_0^2}D_m D^m - \frac{1}{2}D_m(\tau^m)_{\mu\nu}\,[a^\mu, a^\nu] + \big[a_\mu, \bar\chi\big]\,[a^\mu, \chi] + \frac{1}{2g_0^2}\,[\bar\chi, \chi]^2\Big\} + \mathrm{tr}\Big\{\mu^T\,\mu\,\chi\Big\} + \mathrm{tr}\Big\{\mu^T\,\Phi(x, \theta)\,\mu\Big\} + \mathrm{tr}\Big\{\,w^T\,w\Big\}$$

- The “supercoordinate” moduli x, θ only appear through $\Phi(x, \theta)$. The remaining “centred” moduli are denoted as $\widehat{\mathcal{M}}_{(k)}$

All instanton numbers ...

... lead to quartic terms

- Effective action (using $q = e^{2\pi i\tau}$):

$$\mathcal{S}_{\rm n.p.}(\Phi) = \int d^8x\, d^8\theta \sum_k q^k \int d\widehat{\mathcal{M}}_{(k)}\, e^{-\mathcal{S}(\widehat{\mathcal{M}}_{(k)},\Phi(x,\theta))}$$

- In our "conformal" set-up, with with SO(8) gauge group on the D7, counting the dimensions of the moduli we get

$$\left[d\widehat{\mathcal{M}}_{(k)}\right] = (\text{length})^{-4}$$

- Thus $$\int d\widehat{\mathcal{M}}_{(k)}\, e^{-\mathcal{S}(\widehat{\mathcal{M}}_{(k)},\Phi(x,\theta))} = \text{quartic invariant in } \Phi(x,\theta)$$

- Integration over $d^8\theta$ leads to terms of the form "$t_8\, F^4$ "
- The "non-conformal" case of $N \neq 4$ D7's has been considered in

Fucito et al, 2009

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One-instanton case

- For $k = 1$ things are particularly simple
  - The spectrum of moduli is reduced to $\{x, \theta, \mu\}$
  - The moduli action is simply $\mathcal{S}_{\rm inst} = -2\pi i\tau + \mu^T\,\Phi(x,\theta)\,\mu$
- The instanton-induced interactions are thus

$$\int d^8x\, d^8\theta\; q \int d\mu\, e^{-\mu^T\,\Phi(x,\theta)\,\mu} \sim \int d^8x\, d^8\theta\; q\,\mathrm{Pf}\big(\Phi(x,\theta)\big)$$

- A new structure, associated to the SO(8) invariant " $t_8\mathrm{Pf}(F)$ ", appears in the effective action at the one-instanton level after the $d^8\theta$ integration

Multi-instantons

- For $k > 1$ things are more complicated, but we can exploit the SUSY properties of the moduli action, which lead to:
  - an equivariant cohomological BRST structure
  - a localization of the moduli integrals (after suitable closed string deformations)
- Similar techniques have been successfully used to
  - compute the YM integrals in $d = 10, 6, 4$ and the D-instanton partition function — Moore+Nekrasov+Shatashvili, 1998
  - compute multi-instanton effects in $\mathcal{N} = 2$ SYM in $d = 4$ and compare with the Seiberg-Witten solution — Nekrasov, 2002; + ...
  - derive the multi-instanton calculus using D3/D(–1) brane systems — Fucito et al, 2004; Billò et al, 2006; ...

Deformations from RR background

- Suitable deformations that help to fully localize the integral arise from RR field-strengths 3-form with one index on $\mathcal{T}_2$

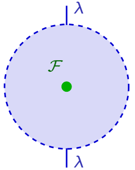
$$\mathcal{F}_{\mu\nu} \equiv F_{\mu\nu z}\ , \quad \bar{\mathcal{F}}_{\mu\nu} \equiv F_{\mu\nu z}$$

- The $\mathcal{F}_{\mu\nu}$ is taken in an SO(7)⊂ SO(8) (Lorentz) with spinorial embedding

- Disk diagrams with RR insertions modify the moduli action

$$\mathcal{S}(\widehat{\mathcal{M}}_{(k)}, \varphi) \to \mathcal{S}(\widehat{\mathcal{M}}_{(k)}, \varphi, \mathcal{F})$$

(here we introduced the v.e.v. $\varphi = \langle\Phi\rangle$)



BRST structure

Equivariance

- Single out one of the supercharges $Q_{\dot\alpha}$, say $Q = Q_8$. After relabeling some of the moduli:

$$M_\alpha \to M_\mu \equiv (M_m, -M_8)\ ,\quad \lambda_{\dot\alpha} \to (\lambda_m, \eta) \equiv (\lambda_m, \lambda_8)$$

one has

$$Qa^\mu = M^\mu\ ,\quad Q\lambda_m = -D_m\ ,\quad Q\bar\chi = -i\sqrt{2}\eta\ ,\quad Q\chi = 0\ ,\quad Q\mu = w$$

- Moreover, on any modulus,

$$Q^2 \bullet = T_{SO(k)}(\chi)\bullet + T_{SO(8)}(\varphi)\bullet + T_{SO(7)}(\mathcal{F})\bullet$$

where
  - $T_{SO(k)}(\chi)$ = inf.mal SO(k) rotation parametrized by χ
  - $T_{SO(8)}(\varphi)$ = inf.mal SO(8) rotation parametrized by φ
  - $T_{SO(7)}(\mathcal{F})$ = inf.mal SO(7) rotation parametrized by $\mathcal{F}$

Symmetries of the moduli

- The action of the BRS charge Q is thus determined by the symmetry properties of the moduli

	SO(k)	SO(7)	SO(8)
a^μ	symm	$\mathbf{8}_s$	$\mathbf{1}$
M^μ	symm	$\mathbf{8}_s$	$\mathbf{1}$
D_m	adj	$\mathbf{7}$	$\mathbf{1}$
λ_m	adj	$\mathbf{7}$	$\mathbf{1}$
$\bar{\chi}$	adj	$\mathbf{1}$	$\mathbf{1}$
η	adj	$\mathbf{1}$	$\mathbf{1}$
χ	adj	$\mathbf{1}$	$\mathbf{1}$
μ	**k**	$\mathbf{1}$	$\mathbf{8}_v$

BRST structure

Exactness

- The (deformed) action is BRST-exact:

$$\mathcal{S}(\widehat{\mathcal{M}}_{(k)}, \varphi, \mathcal{F}) = Q\Xi$$

- $\bar{\mathcal{F}}$ only appears in the "gauge fermion" Ξ: the final result does not depend on it

- The (deformed) BRST structure allows to suitably rescale the integration variables and show that the semiclassical approximation is exact

Moore+Nekrasov+Shatashvili, 1998; ...; Nekrasov, 2002; Flume+Poghossian, 2002; Bruzzo et al, 2003; ...

Scaling to localization

- Many integrations reduce to quadratic forms: Back

$$Z_k(\varphi,\mathcal{F}) \equiv \int d\mathcal{M}_{(k)}\, e^{-\mathcal{S}(\widehat{\mathcal{M}}_{(k)},\varphi,\mathcal{F})} \;=\; \ldots \;=\; \ldots$$

$$= \int \{da\, dM\, dD\, d\lambda\, d\mu\, d\chi\}\; e^{-\mathrm{tr}\{\frac{g}{2}D^2 - \frac{g}{2}\lambda\widetilde{Q}^2\lambda + \frac{t}{4}a\widetilde{Q}^2 a + \frac{t}{4}M^2 + {}^t\mu\widetilde{Q}^2\mu\}}$$

$$\sim \int \{d\chi\}\; \frac{\mathrm{Pf}_\lambda\left(\widetilde{Q}^2\right)\, \mathrm{Pf}_\mu\left(\widetilde{Q}^2\right)}{\det_a\left(\widetilde{Q}^2\right)^{1/2}}$$

- The χ integrals can be done as contour integrals and the final result for $Z_k(\varphi,\mathcal{F})$ comes from a sum over residues

Moore+Nekrasov+Shatashvili, 1998

The recipe

- From the explicit expression of $Z_k(\varphi, \mathcal{F})$, we can obtain the non-perturbative effective action. However:
  - At instanton number k, there are disconnected contributions from smaller instantons k_i (with $\sum_i k_i = k$). To isolate the connected components we have to take the log:

$$\mathcal{Z} = \sum_k Z_k(\varphi, \mathcal{F})\, q^k \;\rightarrow\; \log \mathcal{Z}$$

  - In obtaining $Z_k(\varphi, \mathcal{F})$ we integrated also over x and θ producing a factor of $\mathcal{E}^{-1} \sim \det(\mathcal{F})^{-1/2}$. To remove this contribution we have to multiply by $\mathcal{E}$

$$\log \mathcal{Z} \;\rightarrow\; \mathcal{E} \log \mathcal{Z}$$

  before turning off the RR deformation.

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The prepotential

- All in all, we obtain the non-perturbative part of the D7-brane effective action:

$$S_{(\text{n.p.})} = \frac{1}{(2\pi)^4} \int d^8x \, d^8\theta \; F_{(\text{n.p.})}\big(\Phi(x,\theta)\big)$$

- The "prepotential" $F_{(\text{n.p.})}(\Phi)$ is given by

$$F_{(\text{n.p.})}(\Phi) = \mathcal{E} \log \mathcal{Z}\Big|_{\varphi\to\Phi,\mathcal{F}\to 0}$$

with

$$\mathcal{Z} = \sum_k Z_k(\varphi,\mathcal{F})\, q^k \,, \qquad \mathcal{E} \sim \det(\mathcal{F})^{1/2}$$

- Notice: the prepotential F must be finite in the $\mathcal{E} \to 0$ limit. This requires very delicate (almost "miracolous") cancellations

Explicit results

- Expanding in instanton numbers, $F^{(\mathrm{n.p.})} = \sum_k q^k F_k$, we find in the end

$$
\begin{aligned}
F_1 &= 8\mathrm{P}f(\Phi)\ , \\
F_2 &= \frac{1}{2}\mathrm{Tr}\Phi^4 - \frac{1}{4}\left(\mathrm{Tr}\Phi^2\right)^2\ , \\
F_3 &= \frac{32}{3}\,\mathrm{P}f(\Phi)\ , \\
F_4 &= \frac{1}{4}\mathrm{Tr}\Phi^4 - \frac{1}{4}\left(\mathrm{Tr}\Phi^2\right)^2\ , \\
F_5 &= \frac{48}{5}\,\mathrm{P}f(\Phi)\ ,
\end{aligned}
$$

$\ldots\ldots$

Explicit results

- The D-instanton induced effective “prepotential” is

$$F^{(\mathrm{n.p.})}(\Phi) = 8\,\mathrm{Pf}(\Phi)\left(q + \frac{4}{3}q^3 + \frac{6}{5}q^5 + \ldots\right) + \mathrm{Tr}\Phi^4\left(\frac{1}{2}q^2 + \frac{1}{4}q^4 + \ldots\right)$$
$$+ \left(\mathrm{Tr}\Phi^2\right)^2\left(\frac{1}{4}q^2 + \frac{1}{4}q^4 + \ldots\right)$$

- These results corresponds to the first few orders in q of

$$F^{(\mathrm{n.p.})}(\Phi) = 8\,\mathrm{Pf}(\Phi)\sum_{k=1} d_{2k-1}\,q^{2k-1} + \frac{1}{2}\mathrm{Tr}\Phi^4\sum_{k=1}\left(d_k\,q^{2k} - d_k\,q^{4k}\right)$$
$$+ \frac{1}{8}\left(\mathrm{Tr}\Phi^2\right)^2\sum_{k=1}\left(d_k\,q^{4k} - 2d_k\,q^{2k}\right)$$

with $$d_k = \sum_{\ell|k}\frac{1}{\ell}$$ sum over the inverse divisors of k

Complete result

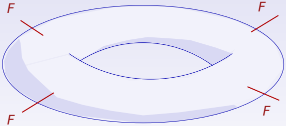
- Taking into account the contributions at tree-level for $\mathrm{Tr}F^4$ and at 1-loop for $\left(\mathrm{Tr}F^2\right)^2$, the full expression for the quartic terms in the effective action of the D7-branes reads

$$2\, t_8\, \mathrm{Pf}(F)\, \log\left|\frac{\eta(\tau+1/2)}{\eta(\tau)}\right|^4 + \frac{t_8\, \mathrm{Tr}F^4}{4}\, \log\left|\frac{\eta(4\tau)}{\eta(2\tau)}\right|^4$$
$$+\frac{t_8\, (\mathrm{Tr}F^2)^2}{16}\, \log\left(\mathrm{Im}\,\tau\, \mathrm{Im}\, U\, \frac{|\eta(2\tau)|^8\, |\eta(U)|^4}{|\eta(4\tau)|^4}\right)$$

with $q = e^{2\pi i\tau}$

Heterotic / Type I′ duality

- In the SO(8)^4 Heterotic String on $\mathcal{T}_2$ the BPS-saturated quartic terms in F arise at 1-loop:



$$\frac{t_8\,\mathrm{Tr}F^4}{4}\,\log\left|\frac{\eta(4T)}{\eta(2T)}\right|^4+\frac{t_8\,(\mathrm{Tr}F^2)^2}{16}\,\log\left(\mathrm{Im}\,T\,\mathrm{Im}\,U\,\frac{|\eta(2T)|^8\,|\eta(U)|^4}{|\eta(4T)|^4}\right)$$

Lerche+Stieberger, 1998; Gutperle, 1999; Kiritsis et al, 2000; ...

$$+2\,t_8\,\mathrm{Pf}(F)\,\log\left|\frac{\eta(T+1/2)}{\eta(T)}\right|^4$$

Gava et al, 1999

- Agrees with our Type I′ result under the duality map

$$T:\text{Kähler structure of the 2-torus }\mathcal{T}_2 \longleftrightarrow \tau:\text{axion-dilaton}$$

$$\text{world-sheet instantons} \longleftrightarrow \text{D-instantons}$$

Remarks

- If we do not switch off the RR background $\mathcal{F}$ in the final expressions we get also non-perturbative gravitational corrections to $\mathrm{Tr}R^4$ and $\mathrm{Tr}R^2\mathrm{Tr}F^2$
- The result checks out perfectly against the dual Heterotic SO(8) theory:
  - Assuming the duality, confirms our procedure to deal with the stringy instantons
  - Assuming the correctness of our computation, yields very non-trivial check of this fundamental string duality

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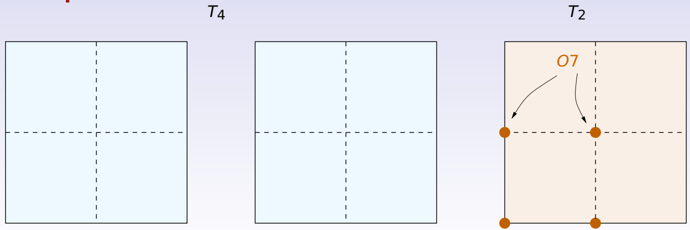
A 4-dimensional example

Going 4-dimensional

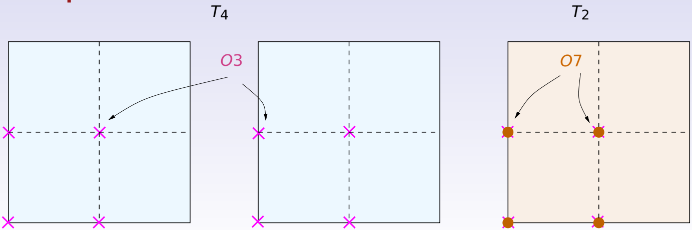
- Of course, we are interested in exotic instanton effects in 4d gauge theories
- We look for a 4d model sharing certain properties of the 8d system we described above:
  - To receive corrections at all instanton numbers
  - To be simple enough as to allow explicit computations
  - To possess a computable heterotic dual, allowing to check the result of the instanton calculus
- We focus on the compactification of the type I' theory on $T_4/\mathbb{Z}_2$
  - Can be seen as the BS-GP model Bianchi-Sagnotti 1991; Gimon-Polchinski, 1996 compactified on $\mathcal{T}_2$ and T-dualized
  - The 4d gauge theory we will consider is a conformal $\mathcal{N} = 2$ theory, but it exhibits a series of exotic non-perturbative corrections to its quadratic prepotential

The set-up



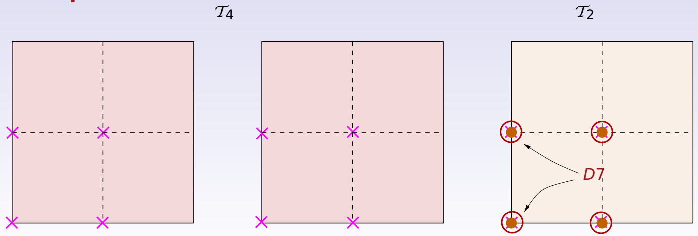
- Further compactify type I' on a T_4

The set-up



- Take an orbifold of T_4 by $\mathbb{Z}_2$ generated by g
- There are 64 O3 planes fixed by Ωg

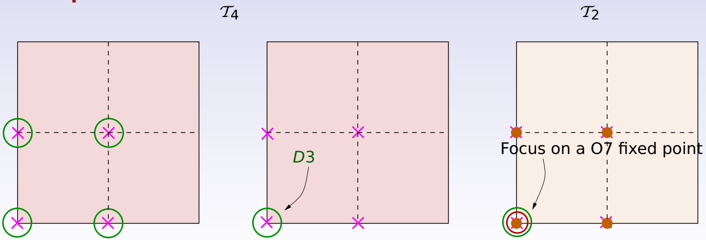
The set-up



- (Local) tadpole cancellation requires 4×4 D7's at each O7 f.p.
- The action of Ω and Ωg on the C.P. factors implies that the gauge group on the D7 is U(4)$\hookrightarrow$ SO(8) for each stack
- The gauge theory is compactified on T_4, so it is 4-dimensional with a gauge coupling

$$\frac{1}{g_{YM}^2} \sim \frac{Vol(T_4)}{4\pi g_s}$$

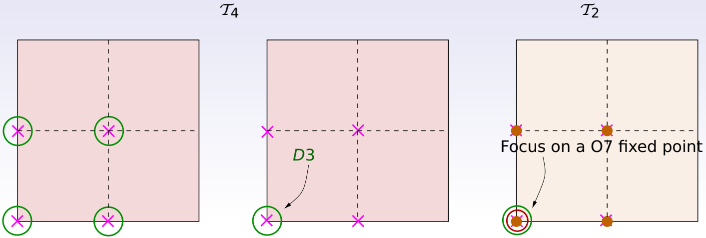
The set-up



- Tadpole cancellation also requires 8 dynamical D3's, to be distributed in the various fixed points.
- Place 4 half-D3's at 4 distinct T_4 fixed points on top of the chosen D7 stack
- The U(4) $\mathcal{N} = 2$ gauge theory on the D7 world-volume contains
  - adjoint vector mult. + 2 antisymm hypers (from D7/D7 strings)
  - 4 fundamental hypers (from D7/D7 strings)
- The theory is conformal:

$$b_1 \propto 4 - m \quad \text{with } m \text{ fundam. hypers}$$

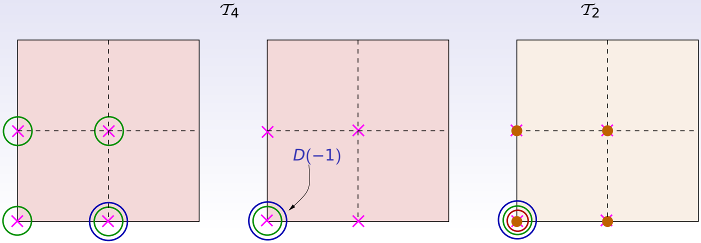
Heterotic dual



- This configuration has an heterotic dual, given by an orbifold of the Heterotic SO(8)^4 theory on $\mathcal{T}_2$.
- In this dual theory, the one-loop thresholds can be computed (work in progress).
- Under the duality map, these have the structure of one-loop + D-instanton contributions

In the GP model: Cámara-Dudas, 2008

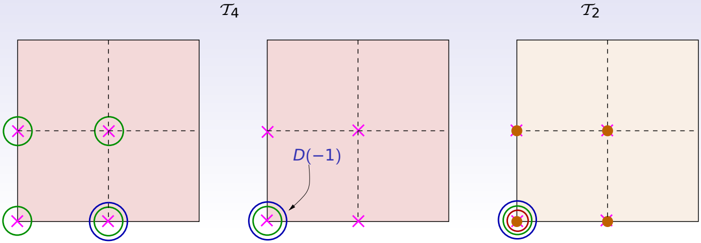
Non-perturbative corrections



- Indeed, on the type I' side there are non-perturbative corrections
- In particular, Exotic corrections from D(-1)'s
  - 8 ND directions, no bosonic mixed moduli w
  - Corrections weighted by

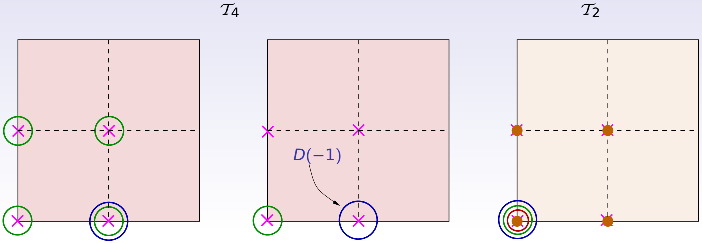
$$e^{-k S_{D(-1)}} \sim e^{-\frac{2\pi k}{g_s}} \sim e^{-\frac{8\pi^2 k}{g_{YM}^2 Vol(T_4)}}$$

Non-perturbative corrections



- The D(-1)/D7 fermionic mixed moduli are always present
- We must take into account configurations where also D(-1)/D3 mixed moduli are present
- These are both bosonic and fermionic: the D(-1) would be ordinary instantons for the D3 theory

Non-perturbative corrections



- There are also configurations where no D(-1)/D3 are present: the ground states are massive, since the D(-1) and the D3 are separated in the internal space
- In both cases, the moduli measure $d\mathcal{M}$ is dimensionless: all instanton numbers can contribute

Preliminary results

- Moduli spectrum and moduli action can be derived
- Moduli integration: BRS structure, RR deformations, localization
  - Expressed as contour integrals over χ moduli ▸ Recall
  - Need to take into account different types of D(-1)'s
  - Residue sum and log prescription algebrically very involved: done up to $k=3$, still problematic at $k=4$

- We do get contributions to the quadratic prepotential for the $\mathcal{N}=2$ gauge multiplet Φ:

$$\sum_k c_k q^k \mathrm{Tr}\Phi^2 \ , \quad \sum_k c'_k q^k (\mathrm{Tr}\Phi)^2 \ , \quad q = \mathrm{e}^{-\frac{8\pi^2}{g_{YM} Vol(T_4)}}$$

  - Example of exotic multi-instanton contributions in $\mathcal{N}=2$ theories (worth generalizing)
  - Seem to agree with twisted sector contrib.s in the heterotic dual

Preliminary results

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- There are also quartic contributions, corresponding to the dimensional reduction of the D(-1)/D7 type I' result
- The 4d interpretation is not yet totally clear. However
  - beside the exp suppression, they will appear with an $\alpha'^2 Vol(T_4)$
  - they seem to agree with the untwisted heterotic contributions

Conclusions and perspectives

Exotic instanton calculus

- SM-like theories obtained by D-brane constructions receive non-perturbative corrections from (wrapped) instantonic branes
- Exotic corrections do not correspond to usual field-theory instantons and may give rise to phenomenologically important perturbatively forbidden interactions
  - One-instanton effects in $\mathcal{N} = 1$ models have mostly been considered (such as those leading to ν_R Majorana mass)
  - In general, also exotic multi-instanton contributions may occur
- Using techniques such as deformation and localization, the string computation of exotic corrections can be explicitly performed
  - We showed this in an 8d type I' example
  - We are now working on a 4d example with $\mathcal{N} = 2$ susy
  - In both cases, nice check with an heterotic dual theory

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- Conclusion of the work in progress :-)
- Applications to phenomenologically relevant models and interactions
- Relation to F-theory models, where non-perturbative corrections are somehow incorporated in the geometry of the construction

Thanks for your attention!

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