Brane world effective actions for D-brane with fluxes

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D.F.T., Univ. Torino

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Foreword

- This talk is based on a work in progress:
  - M. Bertolini (SISSA), M. B., A. Lerda (UPO), J.F. Morales (CERN) and R. Russo (CERN - Queen Mary), “Brane world effective actions for D-branes with fluxes”, to appear (soon!).

We also thank L. Gallot (Annecy) for collaboration at the initial stage

- Direct stringy derivation of (some parts of) the $\mathcal{N} = 1$ effective action for the chiral matter in magnetized (or intersecting) D-brane models.
  - Computation of the Kähler metric in the completely non-factorized (or oblique) case
  - Conjecture about the correlators of non-abelian twist fields which enter the stringy Yukawa couplings in such oblique situations.

Disclaimer

- There is by now a very large literature about intersecting and magnetized brane worlds. The few references scattered on the slides are by no means meant to be exhaustive. I apologize for the many relevant ones which will be missing. (The reference list in the paper will be much longer)

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1. Brane-worlds scenarios
2. D9 branes with general fluxes
3. Effective supersymmetric actions
4. The Kähler metric from strings
5. Relation to the Yukawa couplings
6. FI susy breaking from string diagrams
7. Conclusions and outlook

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Brane-worlds scenarios

Intersecting brane worlds

- Four-dimensional field theories with many "realistic" features arise from type IIA or B superstring models on suitable configurations of D-branes (and orientifolds)

  [Bachas, 1995, Berkooz et al., 1996, Rabadan, 2001], ...

- In particular, intersecting brane worlds have received much attention recently:

  see, e.g., [Uranga, 2003, Kiritsis, 2004, Lust, 2004, Blumenhagen et al., 2005]

  - Type IIA on $\mathbb{R}^{1,3} \times \mathcal{T}_6$ (more generally on a CY - not discussed here)
  - D6 branes wrapping intersecting 3-cycles in $\mathcal{T}_6$ support, on their non-compact world-volume, gauge groups and chiral matter (the latter are localized at the intersection points in the internal space)
  - Consistency requirement: cancellation of RR tadpoles constrains the choice of 3-cycles.

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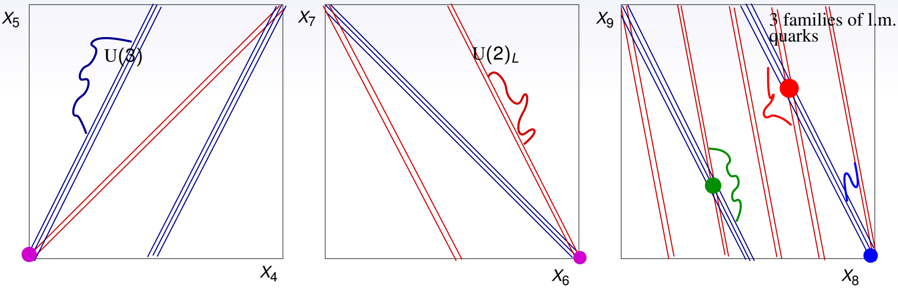
Gauge groups and chiral matter from branes

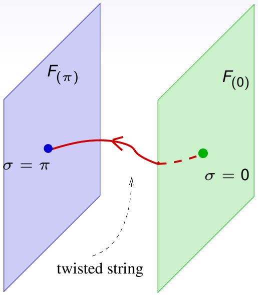
- Gauge groups from multiple branes, bifundamental chiral matter from "twisted" strings, replicas from multiple intersections



- **N.B.** The torus $\mathcal{T}_6$ is assumed to be factorized as $\mathcal{T}_2 \times \mathcal{T}_2 \times \mathcal{T}_2$.

T-duality and magnetized branes

- Upon T-duality (along one direction in each torus), IIA $\rightarrow$ IIB, and D6-branes intersecting on 3-cycles $\rightarrow$ D9 with magnetic fluxes



- Strings connecting two D9 with different fluxes feel different b.c.'s at their two end-points. They are twisted.
- The twists θ_i are determined from the quantized values of the fluxes

$$F^{(\sigma)}_{MN} = \frac{1}{2\pi} \frac{p_{MN}}{q_{MN}}$$

p_{MN} = Chern class, q_{MN} = wrapping of the D brane around the cycle $dX^M \wedge dX^N$.

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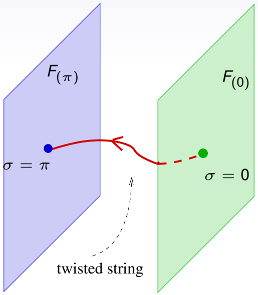

- If the torus is factorized as $\mathcal{T}_2 \times \mathcal{T}_2 \times \mathcal{T}_2$, fluxes respecting this factorization are matrices in $\mathrm{so}(2) \oplus \mathrm{so}(2) \oplus \mathrm{so}(2)$
  Abelian situation: fluxes on different branes commute.
- General situation: fluxes on $\mathcal{T}_6$ represented by $\mathrm{so}(6)$ matrices. Oblique case: fluxes on different branes do not commute.
  - Relevant in the context of the moduli stabilization problem
    [Antoniadis-Maillard, 2004, Bianchi-Trevigne, 2005, Villadoro-Zwirner], ...

D9 branes with general fluxes

Boundary conditions on magnetized branes

- Bosonic part of the open string action: Back
  (x^M in the $\mathcal{T}_6$ directions, $\sigma = 0, \pi$ denotes the end-point)

$$S_{\rm bos} = -\frac{1}{4\pi\alpha'}\int d^2\xi \left[\partial^\alpha x^M \partial_\alpha x^N G_{MN} + \mathrm{i}\epsilon^{\alpha\beta}\partial_\alpha x^M \partial_\beta x^N B_{MN}\right] - \mathrm{i}\sum_\sigma q_\sigma \int_{C_\sigma} dx^M A^\sigma_M$$

- In presence of constant G, B and field-strengths F_σ, the boundary conditions read

$$\bar\partial x^M\Big|_{\sigma=0,\pi} = (R_\sigma)^M{}_N\,\partial x^N\Big|_{\sigma=0,\pi}$$

where the reflection matrix R_σ is given by

$$R_\sigma = \left(G - \mathcal{F}_\sigma\right)^{-1}\left(G + \mathcal{F}_\sigma\right)\,, \quad \mathcal{F}_\sigma = B + 2\pi\alpha'\,F_\sigma$$

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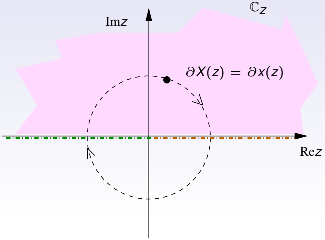
Twisted world-sheet fields

- The above b.c.'s can be solved in terms of a holomorphic, multivalued field $X^M(z)$ defined all over the complex z plane (*doubling trick*):

$$X^M(\mathrm{e}^{2\pi\mathrm{i}}z) = R^M_{\ N}\, X^N(z)\,, \quad R = R_\pi^{-1}\, R_0$$



- Both R_0 and R_π, and hence R, preserve the metric: ${}^tR\,G\,R = G$
- We can go to a complex basis $\mathcal{Z} = (Z^I, \bar{Z}^I) = \mathcal{E}X$, where

$$\mathcal{R} = \mathcal{E}\,R\,\mathcal{E}^{-1} = \mathrm{diag}\left(\mathrm{e}^{2\mathrm{i}\pi\theta_1}, \ldots, \mathrm{e}^{2\mathrm{i}\pi\theta_d}, \mathrm{e}^{-2\mathrm{i}\pi\theta_1}, \ldots, \mathrm{e}^{-2\mathrm{i}\pi\theta_d}\right)$$

for $0 \leq \theta_I < 1$ ($d = 3$ in our case).

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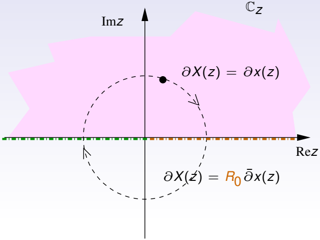

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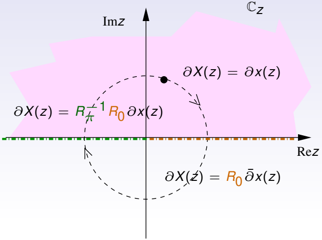

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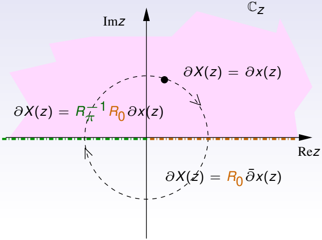

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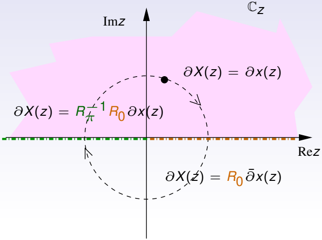

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The open string basis

- The open string complex, multivalued, fields $\mathcal{Z}^i(z)$, and the corresponding w.s fermions $\Psi^i(z)$, have mode expansions shifted by θ_i.
- The θ_i play *exactly* the same role as the angles between intersecting D6. They represent the 3 “open string moduli” which determine the open string CFT properties.
- The vacuum $|\theta\rangle$ is created by bosonic and fermionic twist fields

$$|\theta\rangle = \lim_{z\to 0} \prod_{i=1}^{d} \sigma_{\theta_i}(z)\, s_{\theta_i}(z)\, |0\rangle$$

- The physical vertices contain (excited) twist fields

Dependence of the twists on the closed moduli

- The d open string twists θ_i depend on the $4d^2$ closed string parameters G_{MN} and B_{MN} and on the quantized fluxes $F^{MN}_{0,\pi}$ (or on the wrapping numbers for the intersecting branes).

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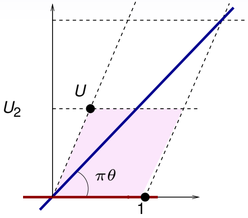

- For intersecting D-branes, the θ_i depend on the moduli describing the shape of the torus:

$$\tan(\pi\theta) = \frac{U_2\, n}{m + U_1\, n}$$

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- For general magnetized branes, from their definition as eigenvalues of the monodromy R we obtain Back Back (2)

$$2\pi \mathrm{i} \frac{\partial \theta_i}{\partial m} = \frac{1}{2} \left(\mathcal{E}\, G^{-1} \frac{\partial (G - B)}{\partial m} \left[R_\pi - R_0 \right] \mathcal{E}^{-1} \right)_{ii} - \frac{1}{2} \left(\mathcal{E} \left[R_\pi^{-1} - R_0^{-1} \right] G^{-1} \frac{\partial (G + B)}{\partial m} \mathcal{E}^{-1} \right)_{ii}$$

where m is a generic closed string modulus, built out of G and B.

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where m is a generic closed string modulus, built out of G and B.
- Applies to general toroidal configurations with any G and B, and to generic (*i.e.* non-abelian) fluxes F_σ

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- *Crucial* formula to reconstruct the Kähler metric for the twisted scalars from mixed open/closed amplitudes

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where m is a generic closed string modulus, built out of G and B.
- In the factorized case, and upon T-duality, reproduces the dependence of the angles just described

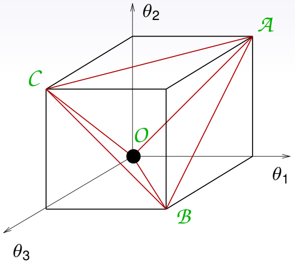
Effective supersymmetric actions

Supersymmetric brane-worlds?

- Simplest models with standard-model-like features break all susy.
- Preserving some susy requires some tuning, in the closed and in the open string sector.
- In the closed, bulk sector:
  - $\mathcal{T}_6$ compact $\longrightarrow$ cancel RR tadpoles
  - cancel NS-NS tadpoles for susy $\longrightarrow$ orientifolds;
- In the open sector, i.e. on the branes:
  - Susy generically broken for the open strings connecting two different D-branes: angles θ_i $\longrightarrow$ twists in the CFT $\longrightarrow$ mass split between *R* and *NS* spectrum
  - Susy (partially) preserved for particular values of the twists

Supersymmetric configurations

- The SUSY preserved on the twisted strings can be described in the space of the θ_i's, which we take in $[0, 1)$.



- For $\theta_1 = \theta_2 = \theta_3 = 0$, $\mathcal{N} = 4$ susy spectrum (like for strings between parallel branes in flat space) Back

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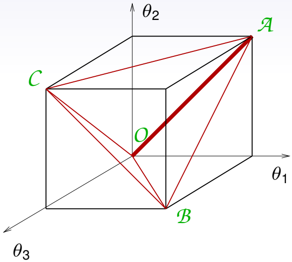

- When one θ vanishes, we get an $\mathcal{N} = 2$ hyper-multiplet:
  - two massless scalars from NS
  - two massless fermions from R sector

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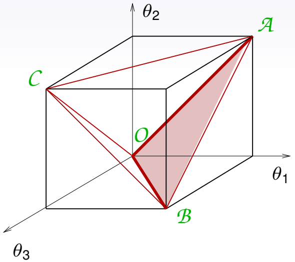

- On the faces, e.g., for $\sum_{j\neq i}\theta_j - \theta_i = 0$ (which we will write as $\sum_j \varepsilon_{j(i)}\theta_j = 0$) we have $\mathcal{N} = 1$ chiral multiplets Φ^i
  - one massless scalar ϕ^i from NS
  - one chiral fermion χ^i from R sector)
- Preserved susy charge on the w.s.:

$$Q_\alpha = \frac{1}{2\pi\mathrm{i}}\oint dz \mathrm{e}^{-\varphi/2} S_\alpha \mathrm{e}^{\frac{\mathrm{i}}{2}\sum_j \varepsilon_{j(i)}\varphi^j}(z)$$

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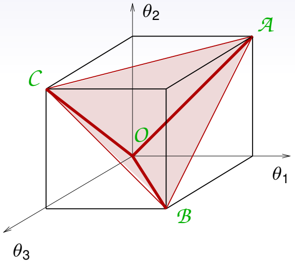

- In the interior of the tetrahedron, we still have a chiral massless fermion from R sector, but only massive scalars.

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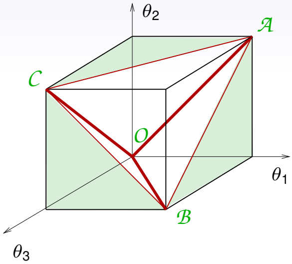

- Outside the tetrahedron, the scalars would become tachyonic.

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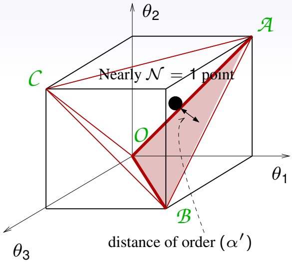

- We will consider spontaneously broken $\mathcal{N} = 1$ by taking θ's *close* to a face:

$$\theta_i = \theta_i^{(0)} + 2\alpha'\delta_i \ , \qquad \sum_j \varepsilon_{j(i)}\theta_j^{(0)} = 0$$

with $\theta_i^{(0)}$ and δ_i fixed in the limit $\alpha' \to 0$.

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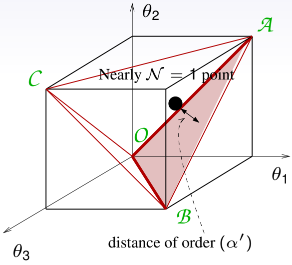

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- The scalar ϕ^i gets a mass $M^2 = \frac{1}{2\alpha'}\sum_j \varepsilon_{j(i)}\theta_j = \sum_j \varepsilon_{j(i)}\delta_j$

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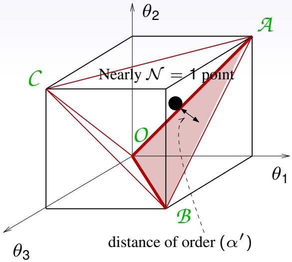

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- Amounts to spontaneous susy breaking á la FI from v.e.v.'s of the auxiliary fields *D*. We'll describe later it later at the string level.

Effective action in the N=1 case

- The l.e.e.a is an $\mathcal{N} = 1$ SUGRA coupled with gauged matter coming from diferent sectors:
- from the closed string sector, upon usual $\mathcal{T}_6$ compactification.
  - For instance, 6^2 moduli m from NS-NS bkg fields G_{MN}, B_{MN} describing the stringy shape of the $\mathcal{T}_6$.
- from the open string sector, gauge + matter fields living on the D-branes.
  - In particular, chiral multiplets Φ^i ("twisted" matter) from strings stretching between different D-branes (localized at their intersections)
- $\mathcal{N} = 1$ l.e.e.a for open string modes determined by moduli-dependent functions:
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Effective N=1 action for twisted matter

- Regarding the moduli as fixed, the Kähler potential for the twisted chiral matter will be of the form

$$K = K_{\bar{\phi}^i \phi^i}(m)\bar{\phi}^i \phi^i + O(\phi^4)$$

(easy to check that there's no mixing between ϕ^i and ϕ^j with $i \neq j$ in our cases).

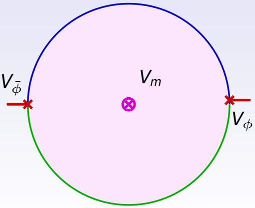
- This corresponds to a lagrangian kinetic term Back

$$\mathcal{L} = -K_{\bar{\phi}^i \phi^i}(m)(\partial_\mu \bar{\phi}^i \partial^\mu \phi^i + M^2 \bar{\phi}^i \phi^i)$$

- The dependence of the"metric" $K_{\bar{\phi}^i \phi^i}$ on the closed string moduli m can be determined from mixed open/closed amplitudes.

The Kähler metric from strings

Mixed amplitudes and the Kähler metric



- Let V_m be the closed string NS-NS vertex for the modulus m. The amplitude Back

$$\mathcal{A}_{\bar\phi^i \phi^i m} \sim \langle V_{\bar\phi^i}\, V_m\, V_{\phi^i} \rangle$$

  is related to the derivative w.r.t. m of the scalar kinetic term. [Lust et al., 2004]

- String amplitudes would give canonical kinetic terms, so Back

$$V_{\phi^i} \to \sqrt{K_{\bar\phi^i \phi^i}}\, V_{\phi^i}\ , \quad V_{\bar\phi^i} \to \sqrt{K_{\bar\phi^i \phi^i}}\, V_{\bar\phi^i}$$

- We have then Back

$$\mathcal{A}_{\bar\phi^i \phi^i m} = \mathrm{i}\, K^{-1}_{\bar\phi^i \phi^i}\, \frac{\partial}{\partial m}\ \frac{\partial}{\partial \phi^i}\ \frac{\partial}{\partial \bar\phi^i}\, \mathcal{L} = \mathrm{i}\, K^{-1}_{\bar\phi^i \phi^i}\, \frac{\partial}{\partial m}\left[K_{\bar\phi^i \phi^i}\left(k_1\, k_2 - M^2\right)\right]$$

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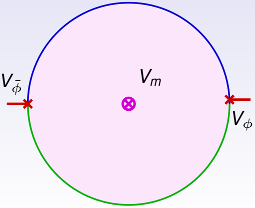

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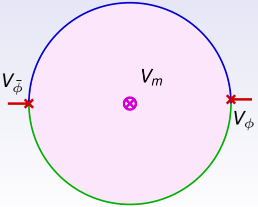

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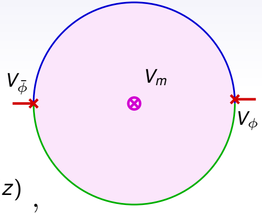
Closed string moduli vertices

- The vertex for the insertion of a generic modulus m reads ▸ Recall

$$V_m(z,\bar z) = \frac{\partial}{\partial m}(G-B)_{MN}\, V_L^M(z)\, V_R^N(\bar z)$$

where

$$V_L^M(z) = \left[\partial X_L^M(z) + \mathrm{i}(k_L\cdot\Psi_L)\Psi^M(z)\right]\mathrm{e}^{\mathrm{i}\,k_L\cdot X_L(z)}\,,$$
$$V_R^N(\bar z) = \left[\partial X_R^N(\bar z) + \mathrm{i}(k_R\cdot\Psi_R)\Psi^N(\bar z)\right]\mathrm{e}^{\mathrm{i}\,k_R\cdot X_R(\bar z)}$$



The form of the amplitude

- The amplitude $\mathcal{A}_{\bar{\phi}^i \phi^i m}$ reads Back

$$\mathcal{A}_{\bar{\phi}^i \phi^i m} = \left[\frac{\partial}{\partial m} (G - B) \cdot R_0 \right]_{MN} \mathcal{E}^M{}_a \, \mathcal{E}^N{}_b \, \mathcal{A}^{ab}$$

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- Impose the boundary identification $V_R^M(\bar{z}; k_R) = R_0{}^M{}_N \, V_L^N(\bar{z}; k_R)$

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  - Switch to the open string complex basis $\mathcal{Z}^a = \mathcal{E}^a{}_M X^M$

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  - Overall normalization

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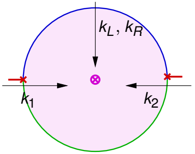
  - Cocycle to put off-shell in a controlled way the closed string vertex

$$s = (k_1 + k_2)^2 = (k_L + k_R)^2$$
$$= 2(k_1 \cdot k_2 - M^2) = 2k_L \cdot k_R$$



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  - Vertices in the open string complex basis $\mathcal{Z}^a$

The CFT correlator

- It is easy to see that the correlator $\mathcal{A}^{ab}$ has the matrix form

$$\mathcal{A} \equiv \begin{pmatrix} 0 & A_j\,\delta^{ij} \\ \bar{A}_j\,\delta^{ij} & 0 \end{pmatrix}\ , \quad \text{with} \ \ A_j = \frac{e^{-\pi i \alpha' s/2}}{8\pi\alpha'^2}\,\langle V_{\bar{\phi}^i}\, V_L^j\, \overline{V}_L^j\, V_{\phi^i}\rangle$$

- Now we must:
  - insert the explicit form of the vertices $V_{\bar{\phi}^i}(x_1)$ and $V_{\phi^i}(x_2)$
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- We get

$$\mathcal{A}_j = \frac{i\,\varepsilon_{j(i)}}{4\pi\alpha'}\,e^{i\pi\theta_j}\,\sin\left[\pi(\theta_j + \alpha' s/2)\right]\frac{\Gamma(\alpha' s+1)\Gamma(1-\theta_j-\alpha' s/2)}{\Gamma(1-\theta_j+\alpha' s/2)}$$

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The result for the amplitude

- Altogether, one can write (up to 2-derivative terms, i.e. up to s^2) the correlator $\mathcal{A}^{ab}$ in matrix form as

$$\mathcal{A} = \frac{1}{2}\mathcal{G}^{-1}\left(\mathcal{R}^{-1} - 1\right)\mathcal{H}\,, \qquad \mathcal{H} = i\begin{pmatrix} h_j & 0 \\ 0 & -h_j \end{pmatrix}$$

with

$$h_j = \frac{\varepsilon_{j(i)}}{4\pi\alpha'}\left(1 - \frac{1}{2}\alpha'\, s\, \rho_j\right) = \frac{1}{2\pi} K^{-1}_{\bar{\phi}^i\phi^i}\frac{\partial}{\partial\theta^j} K_{\bar{\phi}^i\phi^i}(k_1\cdot k_2 - M^2)$$

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- We used the kinematics $s = 2(k_1\cdot k_2 - M^2)$, the dependence of M^2 on θ_j and the fact that $\psi(x) = \frac{d\ln\Gamma(x)}{dx}$.

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- The exponential term goes to 1 in the field theory limit

The magic of the result

- Substituting into the expression of the correlator $\mathcal{A}_{\bar{\phi}^i \phi^i m}$ ▸ Recall we get after some algebra

$$\mathcal{A}_{\bar{\phi}^i \phi^i m} = \frac{1}{2} \mathcal{E} \, G^{-1} \, \frac{\partial}{\partial m}(G - B) \, (R_\pi - R_0) \, \mathcal{E}^{-1}\Big|_j^j \, h_j - \text{h.c.}$$

- Comparing with the expression of the dependence of the twists θ_i from the moduli m ▸ Recall we can write

$$\mathcal{A}_{\bar{\phi}^i \phi^i m} = 2\pi \, \frac{\partial \theta_j}{\partial m} \, h_j = K^{-1}_{\bar{\phi}^i \phi^i} \frac{\partial \theta_j}{\partial m} \frac{\partial}{\partial \theta^j} K_{\bar{\phi}^i \phi^i} (k_1 \cdot k_2 - M^2)$$

- This is the expression we expected ▸ Recall if $K_{\bar{\phi}^i \phi^i}$ really is the Kähler metric

The field theory Kähler metric

- Summarizing, in the field theory limit the expression of the Kähler metric $K_{\bar{\phi}^i \phi^i}$ for the scalar ϕ^i depends on the moduli only through the open string twists
$$\theta_i^{(0)} = \lim_{\alpha' \to 0} \theta_i$$
in an $\mathcal{N} = 1$ configuration. Explicitly,
$$K_{\bar{\phi}^i \phi^i} = \sqrt{\frac{\Gamma(1 - \theta_i^{(0)})}{\Gamma(\theta_i^{(0)})}} \prod_{k \neq i} \sqrt{\frac{\Gamma(\theta_k^{(0)})}{\Gamma(1 - \theta_k^{(0)})}}$$
- This holds for a general toroidal compactification, and with arbitrary magnetic fluxes, also non-commuting

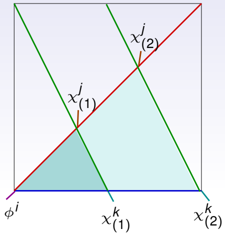
Generalizes [Lust et al., 2004]

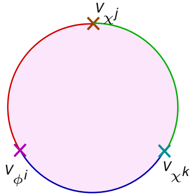
Relation to the Yukawa couplings

Stringy expression of the Yukawa couplings

In the stringy description, Yukawa couplings have the form $Y_{ijk} = \mathcal{A}_{ijk}\,\mathcal{W}_{ijk}$, where

- $\mathcal{W}_{ijk}$ = classical contribution from extended world-sheets bordered by the intersecting branes. [Cremades et al. 2003],[Abel-Owen, 2003],...
  - Multiple intersections → families
  - different minimal world-sheets → exponential hierarchy of couplings
  - have counterparts in magnetized brane worlds [Cremades et al., 2004]
- $\mathcal{A}_{ijk}$ = quantum fluctuations given by the correlator of the twisted vertices located at the intersections. [Cvetic-Papadimitriou, 2003],... ▸ Back





Yukawa couplings and N=1 superpotential

- In $\mathcal{N} = 1$ susy, the Yukawa couplings arise from the superpotential

$$\int d^2\theta\, W(\Phi^i) + \text{c.c} \to \ldots + \frac{\partial W}{\partial\phi^i\partial\phi^j}\chi^i\chi^j + \text{h.c.}\,.$$

  For $W = W_{ijk}\Phi^i\Phi^j\Phi^k$, the W_{ijk} are the Yukawa couplings in the basis where the kinetic terms are determined by the Kähler potential K Recall

- When realized in string compactifications, non-renormalization property: W gets no perturbative α' corrections

  [GSW, vol. 2], [Burgess et al, 2005], ...

- In the brane-world context, we identify therefore the W_{ijk} as the classical world-sheet instanton contributions:

$$W_{ijk} = \mathcal{W}_{ijk}$$

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  For $W = W_{ijk}\Phi^i\Phi^j\Phi^k$, the W_{ijk} are the Yukawa couplings in the basis where the kinetic terms are determined by the Kähler potential K ▸ Recall

- When realized in string compactifications, non-renormalization property: W gets no perturbative α' corrections

  [GSW, vol. 2], [Burgess et al, 2005], ...

- In the brane-world context, we identify therefore the W_{ijk} as the classical world-sheet instanton contributions:

$$W_{ijk} = \mathcal{W}_{ijk}$$

Yukawa couplings and N=1 superpotential

- In $\mathcal{N} = 1$ susy, the Yukawa couplings arise from the superpotential

$$\int d^2\theta \, W(\Phi^i) + \mathrm{c.c} \to \ldots + \frac{\partial W}{\partial\phi^i \partial\phi^j} \chi^i \chi^j + \mathrm{h.c.}\ .$$

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Kähler metric and quantum Yukawas

- The $\mathcal{N} = 1$ holomorphic couplings W_{ijk} are related to the physical ones, Y_{ijk} (the ones provided by the string computation) by rescaling the fields ϕ^i, χ^j, χ^k to give them canonical kinetic terms.
  Recall
- One has thus

$$Y_{ijk} = (K_{\bar{\phi}^i \phi^i} K_{\bar{\phi}^j \phi^j} K_{\bar{\phi}^k \phi^k})^{-1/2} W_{ijk}$$

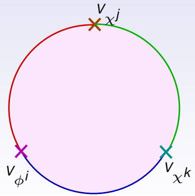
- We had already found Recall

$$Y_{ijk} = \mathcal{A}_{ijk} W_{ijk}$$

- Hence, the amplitude $\mathcal{A}_{ijk}$ for the three twisted vertices should be factorizable into

$$\mathcal{A}_{ijk} = (K_{\bar{\phi}^i \phi^i} K_{\bar{\phi}^j \phi^j} K_{\bar{\phi}^k \phi^k})^{-1/2}$$

The abelian case



- In the case of a factorized torus with commuting angles (or for D-branes at angles) the direct computation of the string amplitude $\mathcal{A}_{ijk}$ is possible
- It involves in particular the correlator of three bosonic twist fields on the torus which are simultaneously expressible in terms of twist angles $\{\theta_i\}$, $\{\nu_i\}$, $\{\lambda_i\}$
- This correlator is computable by factorization of the 4-twist amplitude, and its dependence on the three sets of angles factorizes
- In the end, one indeed finds

$$\mathcal{A}_{ijk} = (K_{\bar{\phi}^i \phi^i} K_{\bar{\phi}^j \phi^j} K_{\bar{\phi}^k \phi^k})^{-1/2}$$

in agreement with the non-renormalization theorem

[Cvetic-Papadimitriou, 2003, Lust et al., 2004]

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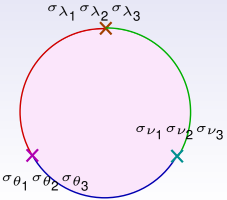

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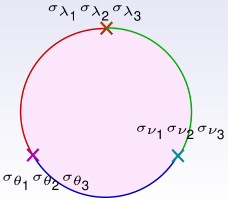

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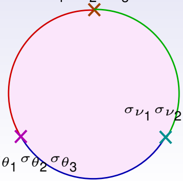

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The non-abelian case?

- We have considered the general case in which the reflection matrices at the various boundaries do not commute, and shown that the Kähler metric remains the same
- Hence the monodromy matrices $R_{\theta,\nu,\lambda}$ induced by the the three twist operators cannot, in general, be simultaneously diagonalized
- We have thus to deal with ("non-abelian twist fields"), whose 3-point CFT correlators are not known. Their computation represents a challenge.
- The non-renormalization theorem, however, suggests that the correlator still factorizes and depends on the three sets $\{\theta_i\}$, $\{\nu_i\}$, $\{\lambda_i\}$ of monodromy eigenvalues

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FI susy breaking from string diagrams

The mass of the scalars and the FI mechanism

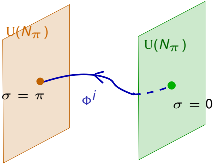
- When the θ_i are close to $\mathcal{N} = 1$ values: $\theta_i = \theta_i^0 + 2\pi\alpha'\delta_i$, with $\sum_j \varepsilon_{j(i)}\theta_i^0 = 0$, the twisted scalar ϕ^i acquires a mass

$$M^2 = \frac{1}{2\pi\alpha'} \sum_j \varepsilon_{j(i)}\theta_i = \sum_j \varepsilon_{j(i)}\delta_i$$

- This susy breaking arises as a FI process involving the auxiliary fields D in the (untwisted) gauge multiplets

- The twisted fields transform in the bi-fundamental
- We espect by susy a coupling to the auxiliary fields D_π, D_0 of the gauge multiplets:

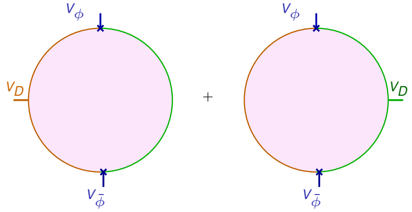
$$(D_\pi - D_0)\bar{\phi}^i\phi^i$$



Stringy description of auxiliary fields

- The vertex describing the auxiliary field D w.r.t. to the preserved susy ▸ Recall is ▸ Back

$$V_D \propto \sum_j \varepsilon_{j(i)} \bar{\Psi}^i \Psi^i$$

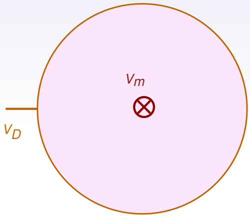


- These diagrams account for the interaction term

$$(D_\pi - D_0)\bar{\phi}^i \phi^i$$

The VEV of the auxiliary fields

- The auxiliary field D gets a vev $\langle D\rangle$ in presence of NS-NS background



- This diagram computes the derivative $\partial_m\langle D\rangle$ w.r.t. a generic NS-NS modulus m: Back

$$\partial_m\langle D\rangle = \langle V_m\, V_D\rangle = \frac{1}{4\pi\alpha'}\frac{\partial}{\partial m}(G-B)_{MN}\langle V_L^M\, V_R^N\, V_D\rangle$$

- Boundary reflection: $V_R^N = R^N_P\, V_L^N$. Go to the complex basis Ψ^i, get a simple correlator. Finally

$$\partial_m\langle D\rangle = -\frac{1}{4\pi\alpha'}\sum_{i=1}^{3}\mathcal{E}G^{-1}\frac{\partial(G-B)}{\partial m}R\mathcal{E}^{-1}\Big|_{ii} - \mathrm{h.c.}$$

The induced mass term for the twisted scalars

- The coupling to the D fields induces a mass term for ϕ^i

$$M^2 \bar{\phi}^i \phi^i = (\langle D_\pi \rangle - \langle D_0 \rangle) \bar{\phi}^i \phi^i$$

- From the above direct string computation we find

$$\begin{aligned} \frac{\partial M^2}{\partial m} &= \frac{\partial}{\partial m} \langle D_\pi - D_0 \rangle \\ &= -\frac{1}{4\pi\alpha'} \sum_i \mathcal{E} G^{-1} \frac{\partial (G - B)}{\partial m} (R_\pi - R_0) \mathcal{E}^{-1} \Big|_{ii} - \text{h.c.} \end{aligned} \quad (1)$$

We reconstruct the Jacobian $\partial \theta_i / \partial m$ ▸ Recall

- We get thus

$$\frac{\partial M^2}{\partial m} = \frac{1}{2\alpha'} \frac{\partial}{\partial m} \sum_j \varepsilon_{j(i)} \theta_j \quad (2)$$

What about the F auxiliary fields?

- The stringy vertex for the untwiwsted auxiliary fields F

$$V_{F_{(i)}} \propto \sum_j \epsilon_{ijk} \Psi^j \Psi^k$$

  - Notice the difference w.r.t. the D vertex ▸ Recall
- Gets a v.e.v. $\langle F_{(i)} \rangle$ from the interaction with the NS-NS moduli m similarly to the D field ▸ Recall
  - However, it is non-zero only when the reflection matrix R has (2,0) components in the complex basis Ψ^i
- The $F_{(i)}$ have no trilinear coupling to $\bar{\phi}^i$, ϕ^i so its v.e.v. does not give a mass to ϕ^i.

Conclusions and outlook

Summary

- We discuss the derivation of the $\mathcal{N} = 1$ effective action for the chiral matter arising from twisted open strings in magnetized/intersecting brane worlds directly from string diagrams
- We extend the derivation of the Kähler metric to the general case:
  - compactification on a non-factorized $\mathcal{T}_6$, with any G_{MN}, B_{MN};
  - oblique magnetic fluxes on the branes
  - susy breaking á la FI inducing a mass term for the scalars
- The connection to the Yukawa couplings provided by the non-renormalization of the superpotential leads to a conjecture about correlators of non-abelian twist fields.

Outlook

- The most pressing task:
  - ... finish the paper!
- Investigate the CFT of non-abelian twist fields
- The dependence from RR closed backgrounds

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