Stringy instanton corrections to $\mathcal{N} = 2$ gauge couplings

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Disclaimer

- This talk builds over a vast literature - some scattered references are given in the slides
  - I apologize for missing ones...
- The results presented here come mostly from
  - M. Billo, M. Frau, F. Fucito, A. Lerda, F. Morales and R. Poghossyan, "Stringy instanton corrections to $\mathcal{N} = 2$ gauge couplings", to appear on JHEP, arXiv:1002.4322 [hep-th]
- Previous computation in an eigth-dimensional setting:
  - M. Billo, L. Ferro, M. Frau, L. Gallot, A. Lerda and I. Pesando, "Exotic instanton counting and heterotic/type I' duality," JHEP **0907** (2009) 092, arXiv:0905.4586 [hep-th]

Plan of the talk

Introduction and motivations

The set-up

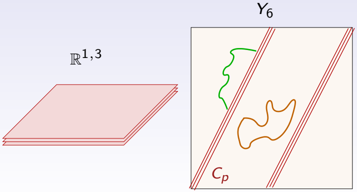
D-instanton effects

Explicit computation by localization

Conclusions

D-brane worlds

- SM-like sector from open strings on stacks of D(3+p) branes wrapped on some internal p-cycles C_p
- Gravitational sector from closed strings in the bulk



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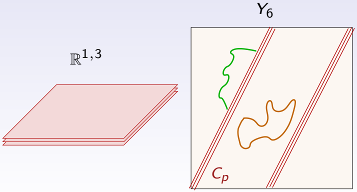

- Gauge and gravitational couplings depend on different volumes (expressed in units of $\sqrt{\alpha'}$):

$$\kappa_4^2 \sim g_s^2 \alpha' / V(Y_6) \,, \quad g_{YM}^2 \sim g_s / V(C_p)$$

  - String mass scale α' can be much lower than 4-d M_{Pl}

Arkani-Hamed et al., '98

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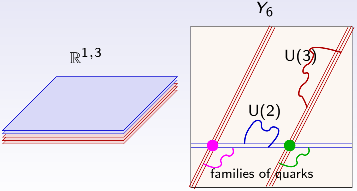


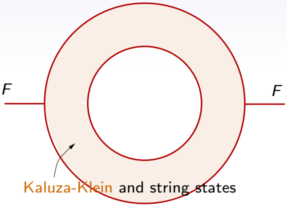
- Gauge groups from multiple branes, bifundamental chiral matter from "twisted" strings, replicas from multiple intersections
  see, e.g., [Uranga, 2003, Kiritsis, 2004, Lust, 2004, Blumenhagen et al., 2005]
- (String) topology of the internal space + choice of branes (subject to tadpole cancellation): a rich model building scenario (using intersecting/magnetized branes of various dimensions)

Perturbative effects

of extra-dimension

- The higher-dimensional, stringy origin of a given D-brane world model bears also on the quantum properties of its low-energy effective action

- Perturbative corrections are affected by the extra states in the theory, resulting in threshold corrections



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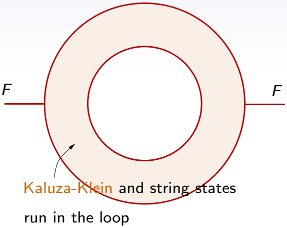
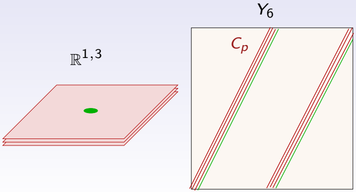


- Also non-perturbative corrections can be influenced

Non-perturbative corrections

Gauge instantons & D-brane instantons

- Non-perturbative sectors: partially wrapped E(uclidean)-branes
- Pointlike in $\mathbb{R}^{1,3}$: instanton configurations



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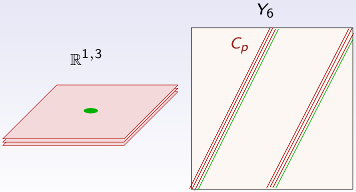
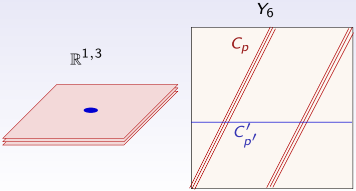


- E-branes identical to a given D-brane stack in the internal directions: instantons for that gauge theory
  - ADHM from strings attached to the instantonic branes
    Witten, 1995; Douglas, 1995-1996; ...
  - non-trivial instanton profile of the gauge field
    Billo et al, 2001
  - Rules and techniques to embed the instanton calculus in string theory have been constructed
    Polchinksi, 1994; Green-Gutperle, 2000, ...; Turin/Rome/München/UPenn/Madrid,...

More non-perturbative corrections

“Stringy” or “exotic” instantons

- E-branes wrapped on a different internal cycle $C'_{p'}$ yield exotic (a.k.a. stringy) non-perturbative corrections



- Ordinary gauge instanton effects suppressed by $e^{-\frac{8\pi^2}{g^2_{YM}}}$
- Exotic instanton effects suppressed by $e^{-\frac{8\pi^2}{g^2_{YM}}\frac{V(C'_{p'})}{V(C_p)}}$
  - they would be ordinary instanton for the gauge theory of branes wrapped on $C'_{p'}$

More non-perturbative corrections

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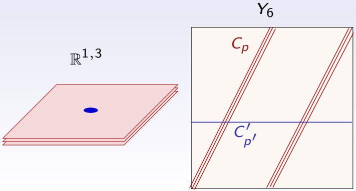


- Exotic instantons may lead to interactions that would be perturbatively forbidden in these models
- Such effects could be of great phenomenological relevance (Neutrino Majorana masses, Yukawas in certain GUT models, . . .)

Blumenhagen et al '06; Ibanez and Uranga, '06; Haack et al, '06; Blumenhagen et al, 2008; ...

- Need to understand their status in the gauge theory and to construct precise rules for the "exotic" instanton calculus

Computing stringy instanton corrections

- Stringy computational techniques for ordinary instantonic branes reproduce gauge theory instanton calculus
- Same kind of techniques techniques should extend to exotic instantonic branes, even if these conf.s have no field theory analogues
- Our strategy to test this assumption: select a set-up such that
  - exotic instantonic branes can contribute to the gauge effective action (not killed by fermionic zero-modes)
  - there are couplings to which all instanton numbers contribute (as it happens for ordinary gauge instantons in $\mathcal{N} = 2$ SYM)
  - the theory possesses a computable heterotic dual, so that the results of the exotic calculus can be tested against it

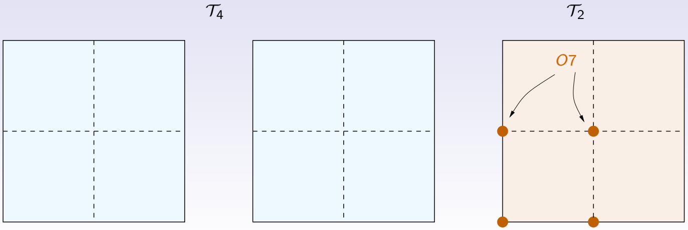
A 4-dimensional example

- We start from Type I', namely type IIB on a two-torus $\mathcal{T}_2$ modded out by

$$\Omega = \omega\,(-1)^{F_L}\, I_2$$

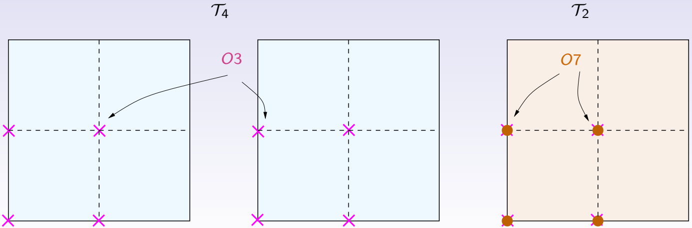
  ω = w.s. parity, F_L = left-moving fermion #, I_2 = inversion on $\mathcal{T}_2$
  - A D7/D(-1) system in this theory provides an example of exotic corrections to an 8d gauge theory Billo et al, 2009
- We compactify it on $\mathcal{T}_4/\mathbb{Z}_2$
- Can be seen as the BS-GP model Bianchi-Sagnotti 1991; Gimon-Polchinski, 1996 compactified on $\mathcal{T}_2$ and T-dualized
- The 4d gauge theory we will consider is a conformal $\mathcal{N} = 2$ theory, but it exhibits a series of exotic non-perturbative corrections to its quadratic prepotential

The set-up



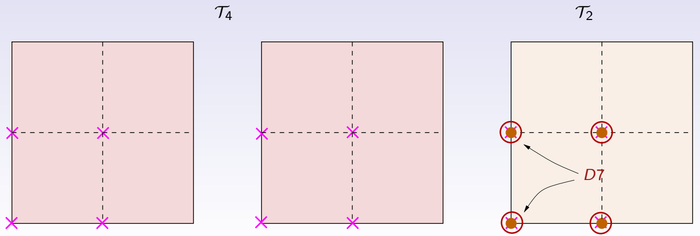
- In Type I', Ω has 4 fixed points on $\mathcal{T}_2$, where 4 O7 planes are located

The set-up



- Take an orbifold of $\mathcal{T}_4$ by $\mathbb{Z}_2$ generated by g
- There are 64 O3 planes fixed by Ωg

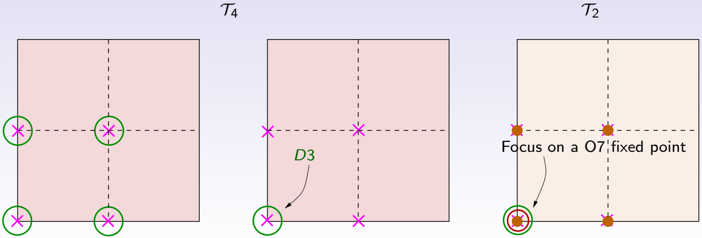
The set-up



- (Local) tadpole cancellation requires 4 D7's at each O7 f.p.
- The action of Ω and Ωg on the C.P. factors implies that the gauge group on the D7 is $U(4) \hookrightarrow SO(8)$ for each stack
- The gauge theory is compactified on $\mathcal{T}_4$, so it is 4-dimensional with a gauge coupling Back

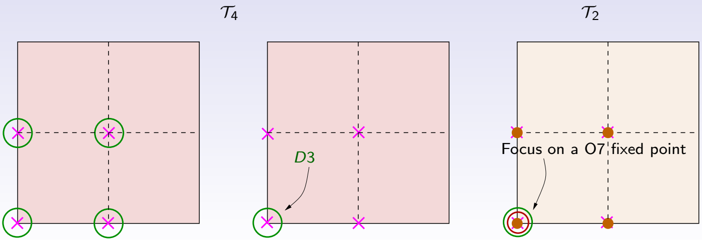
$$t_2 \equiv \frac{4\pi}{g_{YM}^2} \sim \frac{Vol(\mathcal{T}_4)}{g_s}$$

The set-up



- Tadpole cancellation also requires 8 dynamical D3's, to be distributed in the various fixed points.
- Place 4 half-D3's at 4 distinct $\mathcal{T}_4$ fixed points on top of the chosen D7 stack

The set-up



- The U(4) $\mathcal{N} = 2$ gauge theory on the D7 world-volume contains
  - adjoint vector mult. + 2 antisymm hypers (from D7/D7 strings)
  - 4 fundamental hypers (from D7/D3 strings)
- The theory is conformal: for the SU(4) part,

$$b_1 \propto 4 - m \quad \text{with } m \text{ fundam. hypers}$$

Effective action on the D7

- With $\mathcal{N} \geq 1$ susy, the quadratic effective action in the gauge fields involves holomorphic couplings f_{ab} (functions of the "moduli" scalar fields):

$$S = \int d^4x \left\{ (\mathrm{Re}\, f)_{ab} F^a_{\mu\nu} F^{b\mu\nu} + i (\mathrm{Im}\ f)_{ab} F^a_{\mu\nu}{}^* F^{b\mu\nu} \right\}$$

- In terms of the $\mathcal{N} = 2$ multiplet encoding our U(4) gauge d.o.f:

$$\Phi(x,\theta) = \phi(x) + \theta^\alpha \Lambda_\alpha(x) + (\theta\gamma^{\mu\nu}\theta)\, F_{\mu\nu}(x) \ ,$$

we will have, distinguishing the two colour structures, Back

$$S = \int d^4x d^4\theta \left\{ f\, \mathrm{Tr}\Phi^2 + f'(\mathrm{Tr}\Phi)^2 \right\} + \mathrm{c.c}$$

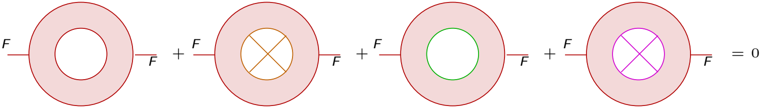
Perturbative results

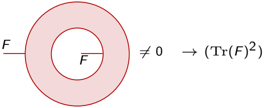
- In accordance with the general structure of holomorphic couplings derived from string computations DKL 1991; de Wit et al, 1995 we find tree level terms, one-loop threshold corrections and non-perturbative terms

$$\text{single trace:} \qquad \mathrm{Re}\, f = t_2 + f_{n.p} \ ,$$

$$\text{double trace:} \qquad \mathrm{Re}\, f' = -4\,|\eta(U)|^4 + f'_{n.p}$$

- One loop diagrams:





- Threshold correction $|\eta(U)|^4$ from massless states winding on $\mathcal{T}_2$
- U is the complex structure of $\mathcal{T}_2$

Non-perturbative corrections

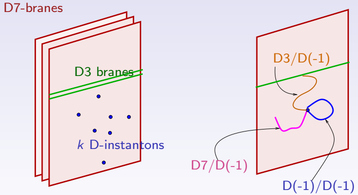
from D-instantons

- In this set-up there are BPS sectors including D(-1)'s or E3 branes along $\mathcal{T}_4/\mathbb{Z}_2$
- We focus on the D-instanton contributions Billo et al 2010.
  - Work in progress on the E3 sectors
- The D(-1)'s correspond to exotic instantons w.r.t. to the D7 gauge theory. Corrections weighted by ▸ Recall

$$\mathrm{e}^{-kS_{D(-1)}} \sim \mathrm{e}^{-\frac{2\pi k}{g_s}} \sim \mathrm{e}^{-\frac{8\pi^2 k}{g_{YM}^2 Vol(\mathcal{T}_4)}} \sim \mathrm{e}^{-2\pi k \frac{t_2}{Vol(\mathcal{T}_4)}}$$

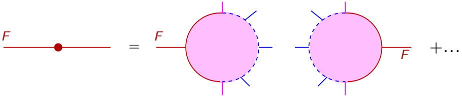
which is not the usual gauge istanton factor $\mathrm{e}^{-\frac{8\pi^2 k}{g_{YM}^2}}$

Effective action from D-instantons



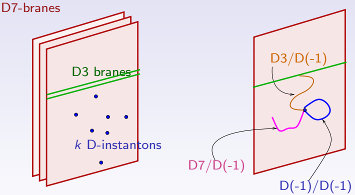
- Open strings with at least one end on a D(−1) carry no momentum: they are moduli rather than dynamical fields.

- Effective interactions between gauge fields (encoded in Φ) can be mediated by D-instanton moduli through mixed disks



connected by integration over the instanton moduli $\mathcal{M}_{(k)}$

Effective action from D-instantons



- Open strings with at least one end on a D(–1) carry no momentum: they are moduli rather than dynamical fields.

- We must sum over D(-1) conf.s and instanton # k and compute

$$\sum_{conf.s} \sum_{k} e^{2\pi i \tau k} \int d\mathcal{M}_{(k)} \, e^{-\mathcal{S}(\mathcal{M}_{(k)}, \Phi)}$$

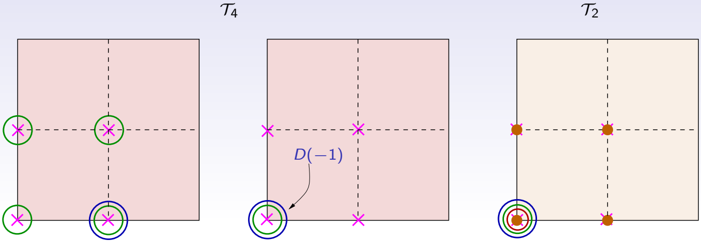
  - $2\pi i \tau k$ is the classical value of the instanton action
  - $\mathcal{S}(\mathcal{M}_{(k)}, \Phi)$ arises from (mixed) disk diagrams describing interactions of the moduli among themselves and with the gauge fields

- From this we should extract the n.p. effective action in the form

Back(1) Back(2)

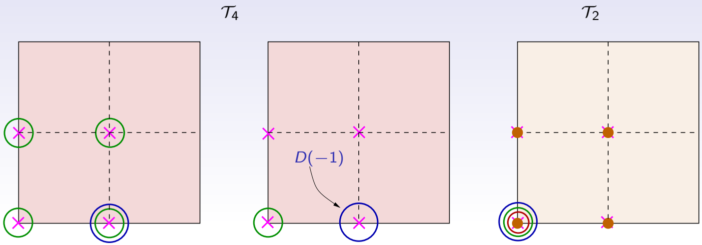
$$S_{n.p.}(\Phi) = \int d^4x \, d^4\theta \, \mathcal{F}_{n.p.}(\Phi)$$

D-instanton configurations



- There are different configurations of D(-1)'s, which have different spectra of moduli excitations from mixed strings ▸ Back
- The D(-1)/D7 mixed moduli are always present (only fermionic: typical of exotic instantons)
- In certain configurations (a) also D(-1)/D3 mixed moduli are present

D-instanton configurations



- There are also configurations (b) where no D(-1)/D3 are present: the ground states are massive, since the D(-1) and the D3 are separated in the internal space

From BPS to BRS

- We face a very complicated matrix integral:
  - the moduli spectrum contains bosonic and fermionic moduli with different transformation properties under the Chan-Paton groups

$$U(k) \times U(4) \times U(m)$$

    pertaining to strings ending on D(1), D7, D3
  - The moduli action S_{mod} contains many moduli interactions

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  - The "Lorentz" symmetry $SO(4) \times SO(4)$ is restricted to the $SU(2)^3$ subgroup that leaves Q invariant

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- Select a particular component of the susy charge as a BRS charge Q
  - The "Lorentz" symmetry $SO(4) \times SO(4)$ is restricted to the $SU(2)^3$ subgroup that leaves Q invariant
- This leads to an equivariant cohomological BRST structure and (upon suitable deformations) to the localization of the moduli integrals
  - Same type of techniques used by Nekrasov to check SW prepotential with instanton calculus Nekrasov, 2002

BRS structure: spectrum

Spectrum: ($m = 1$ for conf.s of type (a), $m = 0$ for type (b)) Back

sector	$(\mathcal{M}_0, \mathcal{M}_1)$	$U(k) \times U(4) \times U(m)$	$SU(2)^3$
D(-1)/D(-1)	(B_ℓ, M_ℓ)	$(\text{adj}, \mathbf{1}, \mathbf{1})$	$(\mathbf{2}, \mathbf{1}, \mathbf{2})$
	$(B_{\dot\ell}, M_{\dot\ell})$	$(\square\square, \mathbf{1}, \mathbf{1}) + \text{h.c.}$	$(\mathbf{1}, \mathbf{2}, \mathbf{2})$
	$(N_{\dot\alpha\dot a}, D_{\dot\alpha\dot a})$	$(\begin{smallmatrix}\square\\\square\end{smallmatrix}, \mathbf{1}, \mathbf{1}) + \text{h.c.}$	$(\mathbf{2}, \mathbf{2}, \mathbf{1})$
	(N_m, d_m)	$(\text{adj}, \mathbf{1}, \mathbf{1})$	$(\mathbf{1}, \mathbf{1}, \mathbf{3})$
	$(\bar\chi, \eta)$	$(\text{adj}, \mathbf{1}, \mathbf{1})$	$(\mathbf{1}, \mathbf{1}, \mathbf{1})$
	χ	$(\text{adj}, \mathbf{1}, \mathbf{1})$	$(\mathbf{1}, \mathbf{1}, \mathbf{1})$
D(-1)/D7	(μ', h')	$(\square, \overline{\square}, \mathbf{1}) + \text{h.c.}$	$(\mathbf{1}, \mathbf{1}, \mathbf{1})$
D(-1)/D3	(w_α, μ_α)	$(\square, \mathbf{1}, \overline{\square}) + \text{h.c.}$	$(\mathbf{1}, \mathbf{1}, \mathbf{2})$
	$(\mu_{\dot a}, h_{\dot a})$	$(\square, \mathbf{1}, \square) + \text{h.c.}$	$(\mathbf{1}, \mathbf{2}, \mathbf{1})$

- $B_\ell \sim$ positions of the D(-1)'s in spacetime; M_l superpartner
- Component along the identity $\sim$ Goldstone modes of broken (super)-translations $\sim$ supercoordinates (x, θ).

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	$(\mu_{\dot a}, h_{\dot a})$	$(\square, \mathbf{1}, \square) + \text{h.c.}$	$(\mathbf{1}, \mathbf{2}, \mathbf{1})$

- $(B_{\dot\ell}, M_{\dot\ell}) \sim$ posn.s of the D(-1)'s in $\mathcal{T}_4/\mathbb{Z}_2$

BRS structure: spectrum

Spectrum: ($m = 1$ for conf.s of type (a), $m = 0$ for type (b))

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- χ, $\bar\chi \sim$ posn.s on $\mathcal{T}_2$
- χ has a particular rôle and does not belong to a BRS doublet

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	(N_m, d_m)	$(\text{adj}, \mathbf{1}, \mathbf{1})$	$(\mathbf{1}, \mathbf{1}, \mathbf{3})$
	$(\bar\chi, \eta)$	$(\text{adj}, \mathbf{1}, \mathbf{1})$	$(\mathbf{1}, \mathbf{1}, \mathbf{1})$
	χ	$(\text{adj}, \mathbf{1}, \mathbf{1})$	$(\mathbf{1}, \mathbf{1}, \mathbf{1})$
D(-1)/D7	(μ', h')	$(\square, \overline{\square}, \mathbf{1}) + \text{h.c.}$	$(\mathbf{1}, \mathbf{1}, \mathbf{1})$
D(-1)/D3	(w_α, μ_α)	$(\square, \mathbf{1}, \overline{\square}) + \text{h.c.}$	$(\mathbf{1}, \mathbf{1}, \mathbf{2})$
	$(\mu_{\dot a}, h_{\dot a})$	$(\square, \mathbf{1}, \square) + \text{h.c.}$	$(\mathbf{1}, \mathbf{2}, \mathbf{1})$

- D(-1)/D7 moduli μ' fermionic only: typical of exotic instantons (h' are auxiliary)

BRS structure: spectrum

Spectrum: ($m = 1$ for conf.s of type (a), $m = 0$ for type (b))

sector	$(\mathcal{M}_0, \mathcal{M}_1)$	$U(k) \times U(4) \times U(m)$	$SU(2)^3$
D(-1)/D(-1)	(B_ℓ, M_ℓ)	$(\text{adj}, \mathbf{1}, \mathbf{1})$	$(\mathbf{2}, \mathbf{1}, \mathbf{2})$
	$(B_{\dot\ell}, M_{\dot\ell})$	$(\square\!\square, \mathbf{1}, \mathbf{1}) + \text{h.c.}$	$(\mathbf{1}, \mathbf{2}, \mathbf{2})$
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- All moduli (except χ) organize into BRS doublets $(\mathcal{M}_0, \mathcal{M}_1)$

BRS structure: transformations

- The moduli doublets are connected by BRS transformations

$$Q\mathcal{M}_0 = \mathcal{M}_1$$

such that Q is equivariantly closed: Back

$$Q^2\mathcal{M}_0 = T_{U(k)}(\chi)\mathcal{M}_0 + T_{U(4)}(\phi)\mathcal{M}_0 + T_{U(m)}(\varphi)\mathcal{M}_0 + T_{SU(2)^3}(\epsilon)\mathcal{M}_0$$

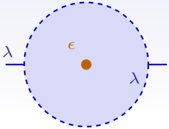
where

- $T_{U(k)}(\chi) =$ inf.mal U(k) rotation parametrized by χ
- $T_{U(4)}(\varphi) =$ inf.mal U(4) rotation param.d by ϕ (D7/D7 scalar)
- $T_{U(m)}(\varphi) =$ inf.mal U(m) rotation param.d by π (D3/D3 scalar)
- $T_{SU(2)^3}(\epsilon) =$ inf.mal SU(2)^3 rotation param.d by ϵ (RR backg.d)

BRS-exactness of the action

- The moduli action S_{mod} includes "deformation' terms describing the interaction of moduli with the D7/D7 $\mathcal{N} = 2$ multiplet Φ, its D3/D3 analogue Π and a suitable RR 3-form background ϵ

- These terms can all be consistently derived from disk diagrams
- In the following computation it will suffice to consider v.e.v.'s for Φ, Π, ϵ (but be careful!...)



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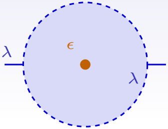

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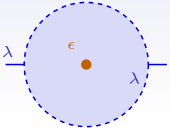

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- The (deformed) BRST structure allows to suitably rescale the integration variables and show that the semiclassical approximation is exact

Moore+Nekrasov+Shatashvili, 1998; ...; Nekrasov, 2002; Flume+Poghossian, 2002; Bruzzo et al, 2003; ...

Scaling to localization

- The integrals over all moduli except χ become quadratic and yield in the end

$$\prod_{\mathcal{M}_0} det_{\mathcal{M}_0}^{\pm\frac{1}{2}}(Q^2)$$

where $\mathcal{M}_0 =$ first components of BRS doublets in the spectrum

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- Then, we still have to integrate over the χ moduli

D-instanton partition function

- At instanton # k we get

$$Z_k^{(m)}(\phi,\pi,\epsilon) = \left(\frac{s_3}{\epsilon_1\epsilon_2}\right)^k \int \prod_{i=1}^{k} \frac{d\chi_i}{2\pi i} \prod_{i<j}^{k} (\chi_i-\chi_j)^2 \left((\chi_i-\chi_j)^2 - s_3^2\right)$$

$$\times \prod_{i<j}^{k} \prod_{\ell=1}^{2} \frac{\left((\chi_i+\chi_j)^2 - s_\ell^2\right)}{\left((\chi_i-\chi_j)^2-\epsilon_\ell^2\right)\left((\chi_i+\chi_j)^2-\epsilon_{\ell+2}^2\right)}$$

$$\times \prod_{i=1}^{k} \left[\prod_{\ell=1}^{2} \frac{1}{\left(4\chi_i^2 - \epsilon_{\ell+2}^2\right)} \prod_{r=1}^{m} \frac{\left((\chi_i+\pi_r)^2 - \frac{(\epsilon_3-\epsilon_4)^2}{4}\right)}{\left((\chi_i-\pi_r)^2 - \frac{(\epsilon_1+\epsilon_2)^2}{4}\right)} \prod_{u=1}^{4} \left(\chi_i - \phi_u\right) \right]$$

(here $\{\epsilon_A\}$ with $\sum_{A=1}^{4} \epsilon_A = 0$ are the Cartan param.s of $SU(2)^3$ embedded in $SO(4) \times SO(4)$ rot.s and $s_1 = \epsilon_2 + \epsilon_3$, $s_2 = \epsilon_1 + \epsilon_3$, $s_3 = \epsilon_1 + \epsilon_2$)

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- The χ integrals can be done as contour integrals and the final result for $Z_k(\phi,\pi,\epsilon)$ comes from a sum over residues

Moore+Nekrasov+Shatashvili, 1998

Taking the logarithm

- Once the integrals are done, we should be able to remove the ϵ-deformations and get the contributions to the eff. action. Recall
- In the deformed theory, at instanton number k, there are disconnected contributions from smaller instantons k_i (with $\sum_i k_i = k$).
- To isolate the connected components we have to take the log of the "grand-canonical" part. function:

$$Z^{(m)} \equiv \sum_k Z_k^{(m)} q^k \;\to\; \log Z^{(m)}$$

where $q = \exp(2\pi i\tau)$.

The 8-dimensional part

- $\log Z^{(m)}$ is still divergent as $1/(\epsilon_1\epsilon_2\epsilon_3\epsilon_4)$.
- With Φ, Π restricted to their v.e.v's and the def.s turned on, this factor arises from the integral over the moduli corresponding to the (super)coordinates in the first 8 directions
- Let us define

$$\mathcal{F}_{IV}(\phi) = \lim_{\epsilon\to 0} \epsilon_1\epsilon_2\epsilon_3\epsilon_4 \log Z^{(m)}(\phi, \pi, \epsilon)$$

  - It has an 8d interpretation as a quartic prepotential for Φ. It agrees with the one computed in the D7/D(-1) system in type I' Billo et al, 2009
  - It does not depend on the D3 d.o.f. π (hence not on m)

The 4d prepotential

- $\log Z^{(m)}$ has also subleading divergences in $1/(\epsilon_1\epsilon_2)$
- To isolate them we define

$$\mathcal{F}_{II}^{(m)}(\phi) = \lim_{\epsilon\to 0}\left(\epsilon_1\epsilon_2 \log Z^{(m)}(\phi,\pi,\epsilon) - \frac{1}{\epsilon_3\epsilon_4}\mathcal{F}_{IV}(\phi)\right)\Big|_{\pi=0}$$

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- Explicit result, up to 3 instantons:

$$\mathcal{F}_{II}^{(m=0)}(\phi) = \Big(-\sum_{i<j}\phi_i\phi_j\Big)\,q + \Big(\sum_{i<j}\phi_i\phi_j - \frac{1}{4}\sum_i \phi_i^2\Big)\,q^2 + \Big(-\frac{4}{3}\sum_{i<j}\phi_i\phi_j\Big)\,q^3 + \cdots ,$$

$$\mathcal{F}_{II}^{(m=1)}(\phi) = \Big(3\sum_{i<j}\phi_i\phi_j\Big)\,q + \Big(\sum_{i<j}\phi_i\phi_j + \frac{7}{4}\sum_i \phi_i^2\Big)\,q^2 + \Big(4\sum_{i<j}\phi_i\phi_j\Big)\,q^3 + \cdots .$$

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$$\mathcal{F}_{II}^{(m=1)}(\phi) = \Big(3\sum_{i<j}\phi_i\phi_j\Big)\,q + \Big(\sum_{i<j}\phi_i\phi_j + \frac{7}{4}\sum_i \phi_i^2\Big)\,q^2 + \Big(4\sum_{i<j}\phi_i\phi_j\Big)\,q^3 + \cdots\ .$$

- We still have to sum over configurations of type (a) and (b) Recall. The correct combinatorial factors imply that we should consider

$$\mathcal{F}_{n.p.} = 12\,\mathcal{F}_{II}{}^{(m=0)} + 4\,\mathcal{F}_{II}{}^{(m=1)}$$

The 4d prepotential (continued)

- The $1/(\epsilon_1\epsilon_2)$ factor arose in our computation from the integration over the moduli x, θ which correspond to the 4d spacetime supercoordinates. We reinstate these integrals, and promote the v.e.v. ϕ to the multiplet $\Phi(x,\theta)$
- We obtain thus the following non-perturbative contributions to the effective action: Recall

$$S_{n.p.}(\Phi) = \int d^4x\, d^4\theta\, \mathcal{F}_{n.p.}(\Phi) \; ,$$

$$\mathcal{F}_{n.p.}(\Phi) = 4\left[-\mathrm{Tr}\Phi^2 + 2(\mathrm{Tr}\Phi)^2\right] q^2 + O(q^4)$$

- In other words, the n.p holomorphic couplings read Recall

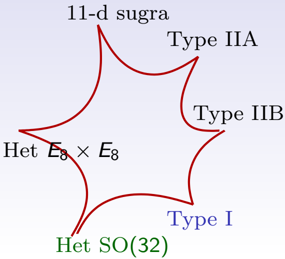
$$f_{n.p.} = \alpha\, q^2 + O(q^4) \; , \quad f'_{n.p.} = -2\alpha\, q^2 + O(q^4)$$

(α an overalll normaliz.)

  - No contrib.s in q and q^3 (as effect of sum over conf.s)
  - At order q^2, a fixed ratio between f and f'

Heterotic check

- The type I' on $\mathcal{T}_2 \times T_4/\mathbb{Z}_2$ has a computable heterotic dual: the U(16) compactification of the SO(32) heterotic string on $T_4/\mathbb{Z}_2$ plus Wilson lines on $\mathcal{T}_2$ breaking U(16) to $U(4)^4$



- The holomorphic gauge couplings for a U(4) factor are derived from a protected one-loop threshold computation
- Not present in the literature, so we carried it out finding precise agreement (under the heterotic/type I' duality map) with our D-instanton predictions
- This represents a very non-trivial duality check, but we mainly regard it as a test of the correctness of our procedure to tackle exotic instanton calculus

Conclusions

- We've considered a consistent string set-up where the 4d gauge theory living on a D-brane stack receives non-perturbative corrections from "exotic" brane instantons which do not correspond to usual gauge instantons
- We computed explicitly such corrections by integrating over exotic instanton moduli space by means of localization techniques
- We successfully checked the result against a dual heterotic computation

Perspectives

- In the set-up I described, there are other possible n.p. corrections from E3 branes wrapped on $\mathcal{T}_4/\mathbb{Z}_2$. They correspond to usual gauge instantons for the D7 theory, and would be n.p. on the heterotic side. We're investigating them.

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- The n.p. description of D7 bacgkrounds should be geometrized by F-theory. D7/D3/D(-1) systems are a testing ground to link directly F-theory curves to n.p. prepotentials both in 8d and 4d. Work in progress.

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- Most important, the exotic instanton calculus might be applied in different set-ups and to diferent kind of couplings, possibly of more direct (string)-phenomenological interest