Gauge instantons from perturbative open strings

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Abstract

The k-instanton sector of non-abelian (supersymmetric) $\mathrm{SU}(N)$ gauge theories in 4 dimensions can be described by means of open strings in presence of N D3 branes and k D-instantons. This description is more than just as a book-keeping device to keep track of the ADHM constraints describing the moduli space and its measure. The profile of the classical solution itself arises naturally from disks with mixed boundary conditions. So does the prescription to compute correlation functions in the instanton background, including the correct measure.

Based on: M. B., M. Frau, F. Fucito, I. Pesando, A. Lerda, A. Liccardo, `hep-th/0211250` (and on large previous literature).

– Utrecht, 24/01/2003 –

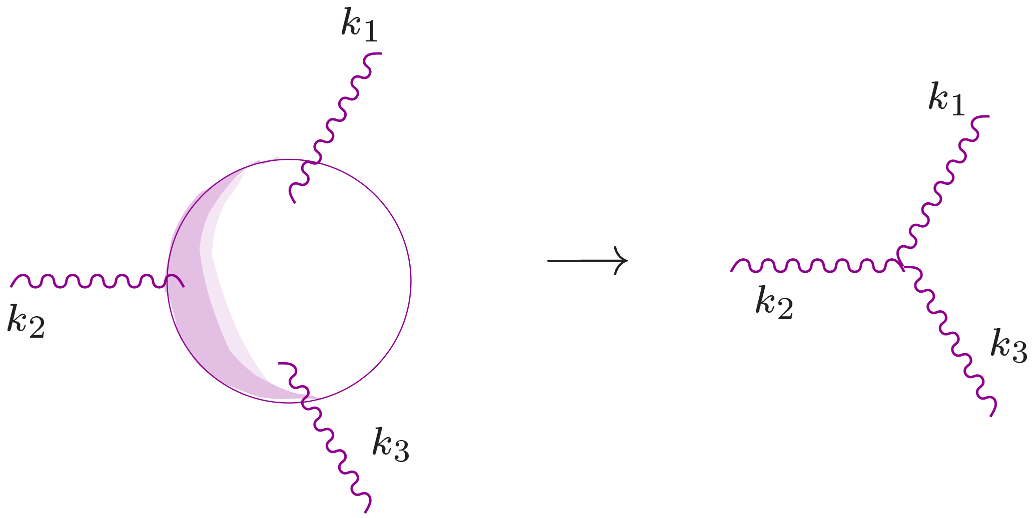
Introduction, main ideas and results

Usually, string theory S-matrix elements $\rightarrow$ effective vertices in field theory. E.g.,

- Closed strings:

$$\widehat{C}\ \langle V_h(z_1; k_1, \epsilon_1) V_h(z_2; k_2, \epsilon_2) V_h(z_3; k_3, \epsilon_3) \rangle_{S^2}$$

  - $\widehat{C}$: sphere normaliz. factor fixed by factorization
  - V_h: graviton vertex
  - All gravitons on-shell: $k^2 = 0$, $k_\mu \epsilon^{\mu\nu} = 0$.



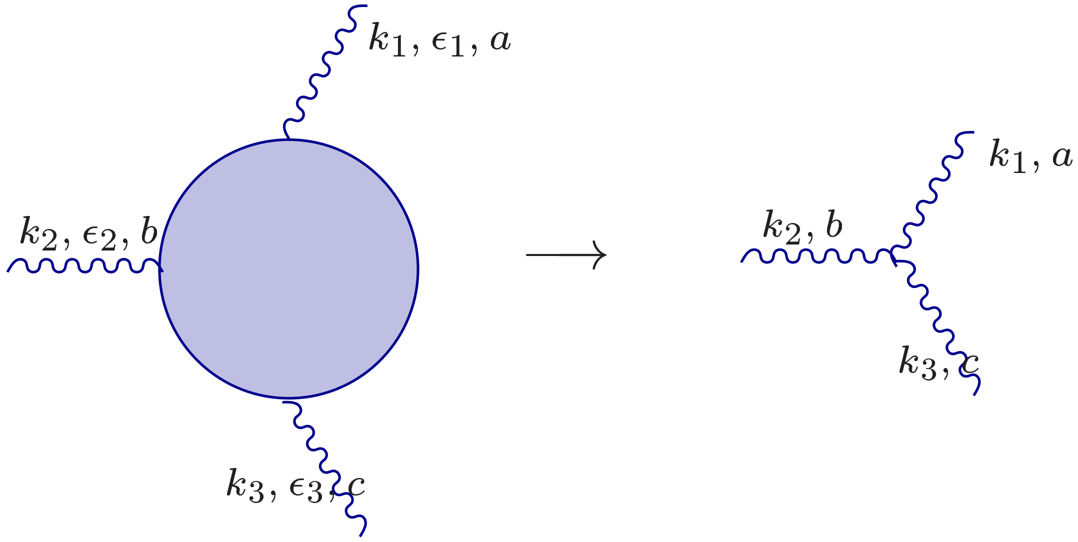
$\rightarrow$ effective 3-graviton vertex in SUGRA

- Open strings:

$$C_{p+1}\left(\langle V_A(z_1;k_1,\epsilon_1)V_A(z_2;k_2,\epsilon_2)V_A(z_3;k_3,\epsilon_3)\rangle_{\rm disk}\right.$$

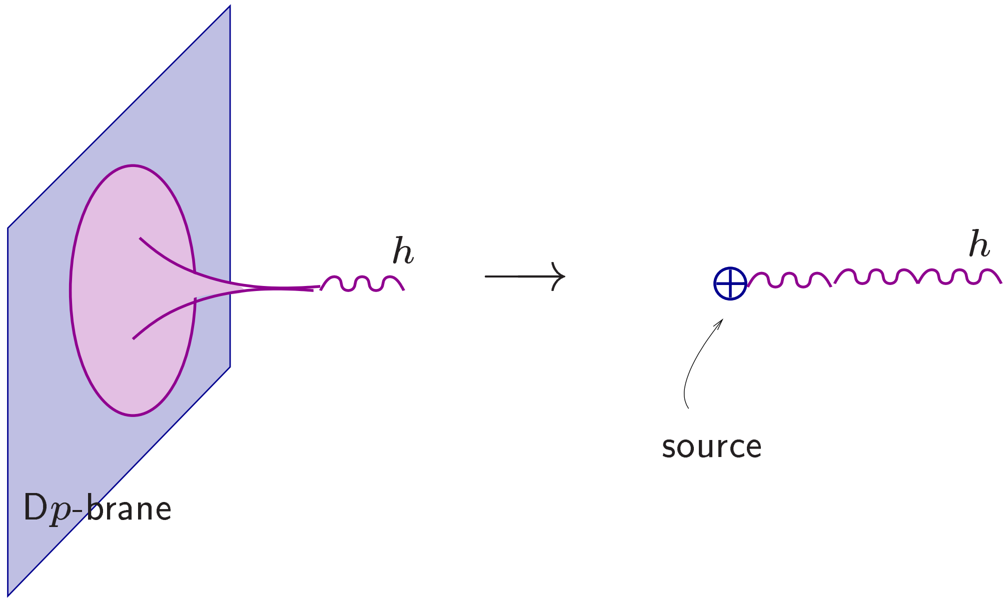
$$\left.\times\,\mathrm{tr}(t^a t^b t^c) + \mathrm{perm.s}\right)$$

  - C_{p+1}: disk normaliz. (if $p+1$ Neumann directions, $9-p$ Dirichlet)
  - V_A: gluon vertex
  - All gluons on-shell: $k^2 = k\cdot\epsilon = 0$



$\rightarrow$ 3-gluon vertex in SYM theory in $p+1$ dimensions

Nowadays, also the "solitonic" black p-brane solutions of SUGRA have a perturbative string interpretation:



- The Dp-brane allows boundaries on the world-sheet
- On the disk attached to the Dp, tadpoles no longer zero:

$$\langle \mathcal{V}_h \rangle_{\mathrm{disk},p}(\mathbf{k}_\perp) = \langle h(\mathbf{k}) | \mathrm{D}p \rangle \neq 0$$

($\mathcal{V}_h$ = graviton vertex without polarization: $V_h = h\, \mathcal{V}_h$)

- Insertion of propagator + Fourier transform $\rightarrow$ long-distance behaviour of the classical p-brane solution:

$$h(\mathbf{x}_\perp) = \int d\mathbf{k}_\perp \, \frac{1}{\mathbf{k}_\perp^2} \, \langle \mathcal{V}_h \rangle_{\mathrm{disk},p}(\mathbf{k}_\perp)$$

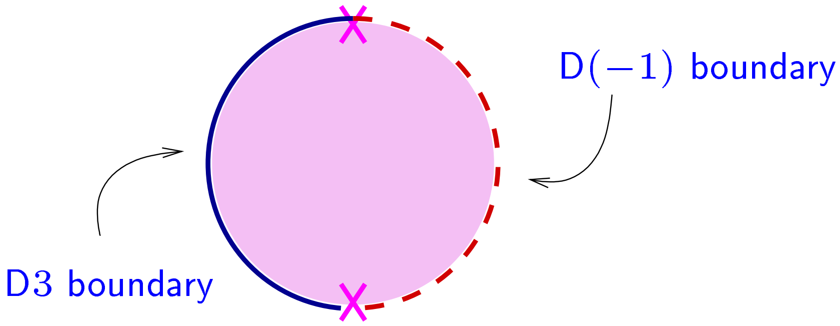
- More source terms (more boundaries) $\rightarrow$ subleading terms in the large-distance expansion. In any case, from SUGRA bulk equations + source terms $\rightarrow$ full p-brane solution.

Question: Can this be extended to the open string sector?

- Low energy theory of open strings (with $p+1$ Neumann directions) = super Yang-Mills in $p+1$ dimensions
- So the question is: $\exists$ a perturbative open strings description of the classical instantonic solutions of SYM?

Answer: Yes

- (Consider the case $p = 3$) A D-instanton on the $D3$ world-volume allows for disk diagrams with mixed boundary conditions:



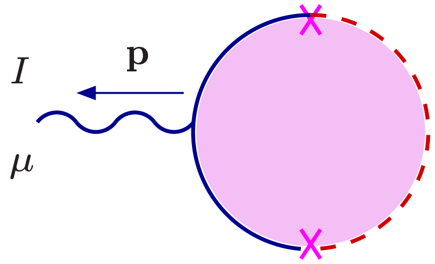
- Let $V_A = A\,\mathcal{V}_A$ = vertex in the D3/D3 string for the gauge field. While $\langle \mathcal{V}_A \rangle_{\mathrm{disk},D3} = 0$, the tadpole on a mixed disk does not vanish:

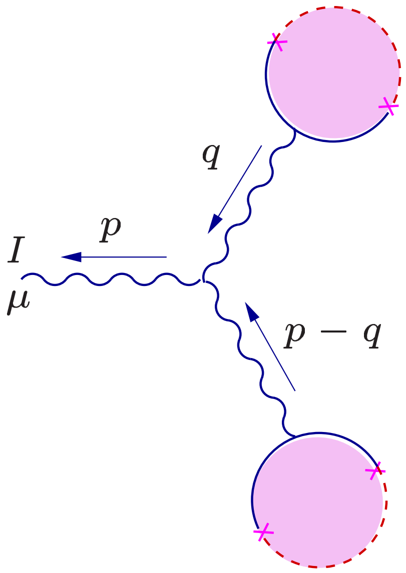
$$\langle \mathcal{V}_A \rangle_{\text{mixed disk}} \neq 0$$

- Insertion of propagator + Fourier transform $\rightarrow$ long-distance behaviour of the classical instanton solution in the singular gauge

$$A^I_\mu(\mathbf{x}) = \int d\mathbf{p}\ \frac{1}{\mathbf{p}^2}\,\langle \mathcal{V}_{A^I_\mu} \rangle_{\text{mixed disk}}(\mathbf{p})$$

- More mixed disk diagrams acting as "sources" $\rightarrow$ subleading terms in large distance:





Summarizing:

- Disks with mixed D3/D(-1) b.c.s $\leftrightarrow$ sources of the classical instanton conf.
- Analogous of Dp brane $\leftrightarrow$ source of classical p-brane conf.

Comments

- D3/D(-1) system in flat $10-d$ space $\rightarrow$ $\mathcal{N}=4$ SYM in 4 dims. The open string description leads to the $\mathcal{N}=4$ superinstanton Instantonic corrections to correl. functions severely limited by fermionic 0-modes; for instance, no non-perturbative corrections to gauge coupling.
- We're working on set-ups with lower SUSY (first of all $\mathcal{N}=2$). Here instantons do correct the l.e.e.a (resummed by Seiberg-Witten).
- Possible development: $\mathcal{N}=2$ gauge/gravity correspondece.
  - Avaliable SUGRA (wrapped branes) and string set-ups (fractional D3-branes on $\mathbb{C}^2/\Gamma$) $\rightarrow$ perturbative part of effective coupling only.
  - Mixed disks in the closed string context $\rightarrow$ recover instantonic contrib.s in the gravitational dual?

Some literature...

... about the stringy description of instantons and of their effects

- Basic references about D-instantons
  - J. Polchinski, Phys. Rev. D **50** (1994) 6041 [9407031].
  - M.B. Green and M. Gutperle, Phys. B **498** (1997) 195, [9701093]; ...
- ADHM construction in supersymmetric case, realiz. in brane set-up
  - E. Witten, Nucl. Phys. B **460** (1996) 335 [9510135]; M. R. Douglas, J. Geom. Phys. **28**, 255 (1998) [9604198] (main ideas of the brane realization)
  - A.V. Belitsky, S. Vandoren and P. van Nieuwenhuizen, Class. Quant. Grav. **17** (2000) 3521 [0004186]; N. Dorey, T. J. Hollowood, V. V. Khoze and M. P. Mattis, [0206063] (reviews) and references therein, e.g.:
  - N. Dorey, V.V. Khoze, M.P. Mattis and S. Vandoren, Phys. Lett. B **442** (1998) 145, [9808157]; N. Dorey, T.J. Hollowood, V.V. Khoze, M.P. Mattis and S. Vandoren, JHEP **9906** (1999) 023, [9810243]; N. Dorey, T. J. Hollowood, V. V. Khoze, M. P. Mattis and S. Vandoren, Nucl. Phys. B **552** (1999) 88 [9901128]; N. Dorey, T. J. Hollowood and V. V. Khoze, [0010015].
- Closely related discussion:
  - M.B. Green and M. Gutperle, JHEP **0002** (2000) 014 [0002011] However, focuses on D-instanton-induced modifications of the $\mathcal{N}=4$ action $\rightarrow$ only F^4 terms or gravitational interactions, and on the abelian case
- . . . and many others . . .

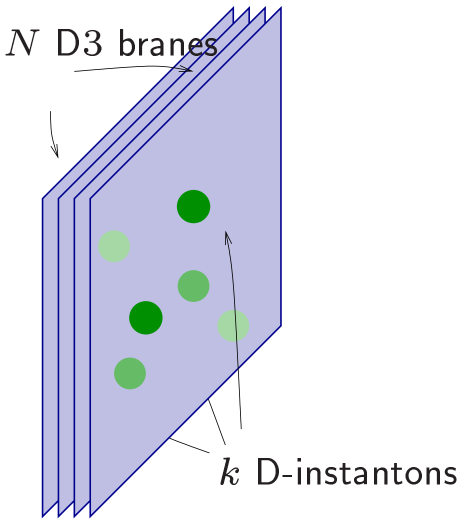
Gauge instantons and D-instantons

Consider the world-volume action of system of Dp-branes:

$$\text{non-ab. B.I.(F)} + \int_{\mathrm{D}p} \left(C_{p+1} + \frac{1}{2} C_{p-3} \wedge \mathrm{Tr} F \wedge F + \ldots \right)$$

(F = gauge field on the Dp brane. C_m = RR form fields)

- Instantonic conf. $\mathrm{Tr}F \wedge F \neq 0$ (localized) $\rightarrow$ localized charge for C_{p-3}, i.e., D$(p-4)$ charge
- Instanton on the Dp $\leftrightarrow$ D$(p-4)$ localized on the Dp, smeared with characteristic size = char. scale ρ of the instanton

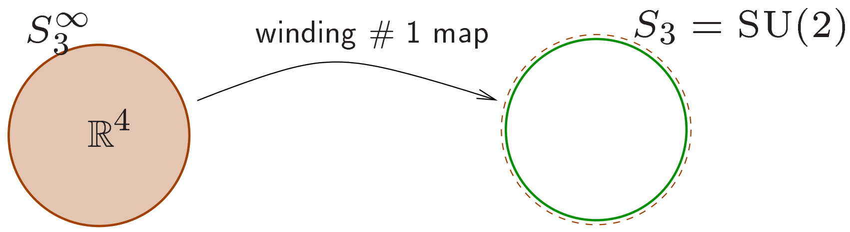


Stringy description of instanton number k sector of the $\mathrm{SU}(N)$ gauge theory in 4 dimensions (case $p = 3$). Largely used in literature to describe moduli space

Instantons & and their moduli (flashing review)

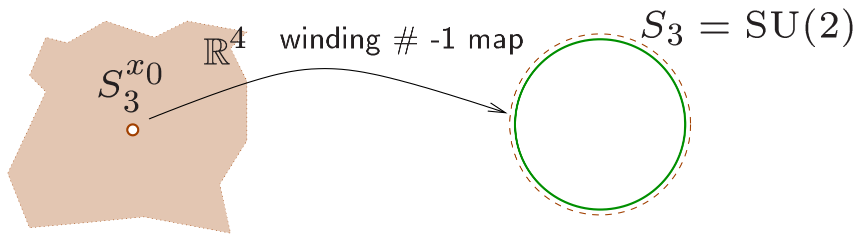
- Consider the $k = 1$ instanton of $\mathrm{SU}(2)$ theory

$$A_\mu^c(x) = 2\frac{\eta^c_{\mu\nu}(x-x_0)^\nu}{(x-x_0)^2+\rho^2}$$



With a singular gauge transf. $\longrightarrow$ so-called singular gauge:

$$\begin{aligned} A_\mu^c(x) &= 2\rho^2\,\bar\eta^c_{\mu\nu}\frac{(x-x_0)^\nu}{(x-x_0)^2\Big[(x-x_0)^2+\rho^2\Big]} \\ &\simeq 2\rho^2\,\bar\eta^c_{\mu\nu}\frac{(x-x_0)^\nu}{(x-x_0)^4}\left(1-\frac{\rho^2}{(x-x_0)^2}+\dots\right) \end{aligned}$$



- $\eta^c_{\mu\nu}$, $\bar\eta^c_{\mu\nu}$, self-dual (resp. anti-self-dual) 't Hooft symbols.
- A^c_μ in singular gauge is self-dual despite containing $\bar\eta^c_{\mu\nu}$

- Parameters (moduli) of $k = 1$ sol. in $\mathrm{SU}(2)$ theory:

moduli	meaning	#
x_0^μ	center	4
ρ	size	1
$\vec{\theta}$	orientation$^{(*)}$	3

$^{(*)}$ from "large" gauge transf.s $A \to U(\theta) A U^\dagger(\theta)$

- For an $\mathrm{SU}(N)$ theory, embed $\mathrm{SU}(2)$ instanton in $\mathrm{SU}(N)$:

$$A_\mu = U \begin{pmatrix} \mathbf{0}_{N-2 \times N-2} & \mathbf{0} \\ \mathbf{0} & A_\mu^{\mathrm{SU}(2)} \end{pmatrix} U^\dagger$$

Thus there are $4N - 5$ moduli parametrizing

$$\frac{\mathrm{SU}(N)}{\mathrm{SU}(N-2) \times \mathrm{U}(1)}$$

$\to$ total #: $4N$

- For instanton # k in $\mathrm{SU}(N)$: total # of moduli: $4Nk$, described by ADHM construction
  - Realized by the N D3 branes, k D-instantons set-up as described later

- We deal in fact with super YM, $\rightarrow$ super-instantons.
  - Semiclassical quantization in an instantonic sector:
    - the one-loop determinants (formally) cancel between bosons and fermions
    - we are left with integrals over moduli only
  - Fermionic 0-modes (Λ = Weyl fermion in rep R)

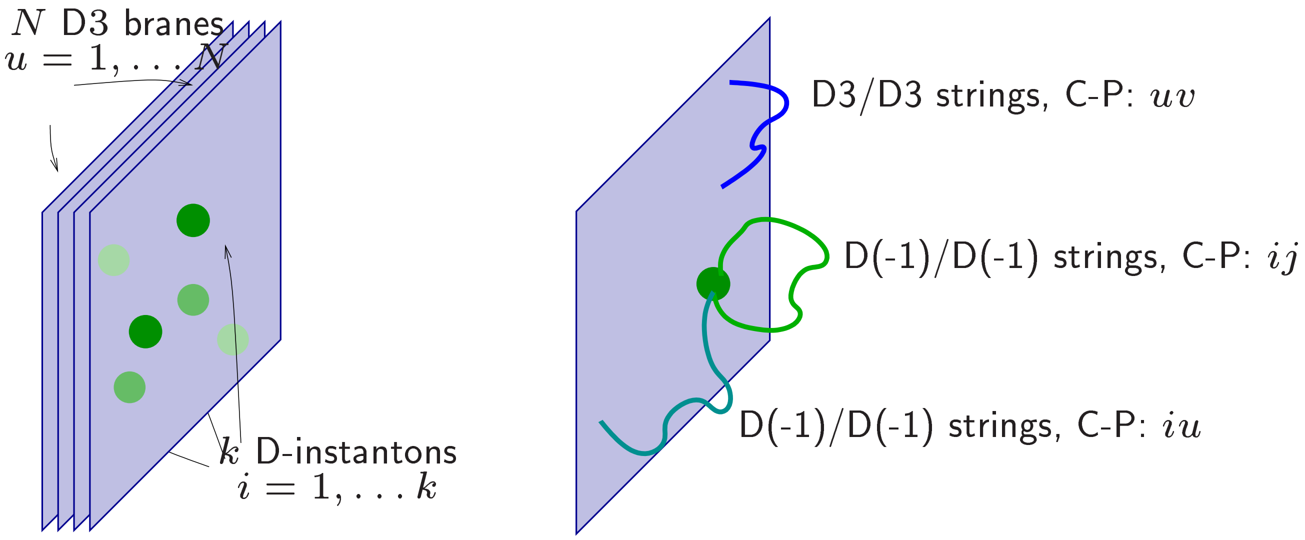
$$\not{D}(A_{\rm inst})\Lambda = 0$$

in an instanton background counted by index theorem

$$\#\text{0-modes} = \frac{1}{8\pi^2}\int \mathrm{Tr}_R F\wedge F = \frac{x_R}{x_{\rm fun}}\,k$$

    - R= fundam. $\rightarrow$ k zero-modes
    - R= adjoint $\rightarrow$ $2\,N\,k$ zero-modes
  - Some (but not all) 0-modes accounted for by broken susy and superconformal charges. E.g., in $\mathcal{N}=4$,
    - 4 adjoint gauginos $\rightarrow$ $8Nk$ ferm. zero-modes
    - 16 susy charges Q, 8 broken by instanton
    - 16 superconf. charges S , 8 broken by instanton
    - the remaining $8Nk-16$ modes $\rightarrow$ true supermoduli

The D3/D(-1) system



- Open string fields: X^μ, ψ^μ and $S^{\dot{\mathcal{A}}}$ (spin field).
  - Under $\mathrm{SO}(10) \to \mathrm{SO}(4) \times \mathrm{SO}(6)$,

$$\begin{array}{ccc} X^M & \to & X^\mu\,,\;\; X^a \\ \psi^M & \to & \psi^\mu\,,\;\; \psi^a \end{array} \qquad S^{\dot{\mathcal{A}}} \begin{array}{c} \nearrow \\ \searrow \end{array} \begin{array}{c} S_\alpha S_A \\ S^{\dot\alpha} S^A \end{array}$$

($\mu = 0, 1, 2, 3, a = 4, \ldots, 9$, $\alpha, \dot\alpha = \mathrm{SO}(4)$ spinor indices, $_A$ and ^{A} in the $\mathbf{4}$, resp. $\bar{\mathbf{4}}$ of $\mathrm{SU}(4) \sim \mathrm{SO}(6)$)

- Boundary conditions:

on a D(-1)	on a D3
X^M, ψ^M Dir.	X^μ, ψ^μ Neu., X^a, ψ^a Dir.
$S^{\dot{\mathcal{A}}}(z) = \tilde{S}^{\dot{\mathcal{A}}}(\bar z)\vert_{z=\bar z}$	$S^{\dot{\mathcal{A}}}(z) = \epsilon' \Gamma^{0123} \tilde{S}^{\dot{\mathcal{A}}}(\bar z)\vert_{z=\bar z}$

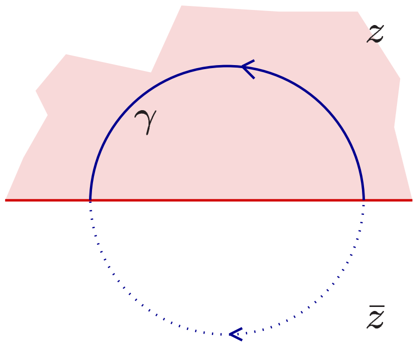
Broken symmetries

- Given an blueholomorphic current $j(z)$, the charge

$$q = Q - \widetilde{Q} = \frac{1}{2\pi \mathrm{i}} \left(\int_\gamma dz\; j(z) \;-\; \int_\gamma d\bar{z}\; \widetilde{j}(\bar{z}) \right)$$

is preserved by the b.c.

$$j(z) = \tilde{j}(\bar{z})|_{z=\bar{z}}$$



- The combination

$$q' = Q + \widetilde{Q} = \frac{1}{2\pi \mathrm{i}} \left(\int dz\; j(z) \;+\; \int d\bar{z}\; \widetilde{j}(\bar{z}) \right)$$

is instead broken by it. Deforming the contour in q' to the boundary gives

$$\int_{boundary} dz (j + \tilde{j})|_{z=\bar{z}}$$

with $(j + \tilde{j})(x)$ $=$ massless vertex for Goldstone field of broken symmetry generated by q'

- *Example* for $j^a(z) = \partial X^a(z)$, the transl. symmetry generated by q^a is broken by the Dirichlet b.c.

$$\partial X^a = -\bar{\partial} X|_{z=\bar{z}}$$

Goldstone fields = transverse scalars ϕ^a, vertex op.s:

$$(j^a - \tilde{j}^a)|_{z=\bar{z}} \propto \partial_\sigma X^a$$

Supersymmetries

- The supercurrent is

$$j^{\dot{\mathcal{A}}}(z) = S^{\dot{\mathcal{A}}}(z)\ \mathrm{e}^{-\frac{1}{2}\phi(z)}$$

Define the "bulk" l.m. and r.m. charges

$$Q^{\dot{\mathcal{A}}} = \frac{1}{2\pi\mathrm{i}} \int dz\ j^{\dot{\mathcal{A}}}(z)\ ,\ \tilde{Q}^{\dot{\mathcal{A}}} = \frac{1}{2\pi\mathrm{i}} \int d\bar{z}\ \tilde{j}^{\dot{\mathcal{A}}}(\bar{z})$$

- Boundary cond.s on spin fields can be written as follows:

on a D(-1)	on a D3
$S_\alpha S_A = \tilde{S}_\alpha \tilde{S}_A\vert_{z=\bar{z}}$	$S_\alpha S_A = \epsilon' \tilde{S}_\alpha \tilde{S}_A\vert_{z=\bar{z}}$
$S^{\dot{\alpha}} S^A = \tilde{S}^{\dot{\alpha}} \tilde{S}^A\vert_{z=\bar{z}}$	$S^{\dot{\alpha}} S^A = -\epsilon' \tilde{S}^{\dot{\alpha}} \tilde{S}^A\vert_{z=\bar{z}}$

$\Rightarrow$ preserved and broken supercharges for $\epsilon' = -1$:

charge	D(-1)	D3	parameter
$Q^{\dot{\alpha}A} - \widetilde{Q}^{\dot{\alpha}A}$	OK	OK	$\bar{\xi}_{\dot{\alpha}A}$
$Q^{\dot{\alpha}A} + \widetilde{Q}^{\dot{\alpha}A}$	broken	broken	$\rho_{\dot{\alpha}A}$
$Q_{\alpha A} - \widetilde{Q}_{\alpha A}$	OK	broken	$\xi^{\alpha A}$
$Q_{\alpha A} + \widetilde{Q}_{\alpha A}$	broken	OK	$\eta^{\alpha A}$

(For $\epsilon' = 1$ exchange chiralities $\leftrightarrow$ anti-instantons on the D3's)

Massless spectra

- **D3/D3 strings**
  - Massless modes $\rightarrow$ $\mathcal{N}=4$ gauge multiplet in $d=4$ (4-dim reduction of $\mathcal{N}=1$ SYM in $d=10$)
  - All modes: Chan-Paton indices uv $\rightarrow$ Chan-Paton factors $(T^I)_{uv}$ in the adjoint of $\mathrm{(S)U}(N)$ (not written below)
  - NS sector (space-time bosons): gauge field + 6 scalars

$$A^\mu \quad \leftrightarrow \quad V_A^{(-1)}(z) = A^\mu(p)\, \underbrace{\frac{1}{\sqrt{2}}\,\psi_\mu\, \mathrm{e}^{-\phi}\, \mathrm{e}^{\mathrm{i}p_\nu X^\nu}(z)}_{\mathcal{V}^{(-1)}_{A^\mu}(z;p)}$$

$$\varphi^a \quad \leftrightarrow \quad V_\varphi^{(-1)}(z) = \varphi^a(p)\, \underbrace{\frac{1}{\sqrt{2}}\,\psi_a\, \mathrm{e}^{-\phi}\, \mathrm{e}^{\mathrm{i}p_\nu X^\nu}(z)}_{\mathcal{V}^{(-1)}_{\varphi^a}(z;p)}$$

  - R sector (space-time fermions): gauginos, (4 + 4) Weyl

$$\Lambda^{\alpha A} \quad \leftrightarrow \quad V_\Lambda^{(-1/2)}(z) = \Lambda^{\alpha A}(p)\, \underbrace{S_\alpha\, S_A\, \mathrm{e}^{-\frac{1}{2}\phi}\, \mathrm{e}^{\mathrm{i}p_\nu X^\nu}(z)}_{\mathcal{V}^{(-1/2)}_{\Lambda^{\alpha A}}(z;p)}$$

$$\bar\Lambda_{\dot\alpha A} \quad \leftrightarrow \quad V_{\bar\Lambda}^{(-1/2)}(z) = \bar\Lambda_{\dot\alpha A}(p)\, \underbrace{S^{\dot\alpha}\, S^A\, \mathrm{e}^{-\frac{1}{2}\phi}\, \mathrm{e}^{\mathrm{i}p_\nu X^\nu}(z)}_{\mathcal{V}^{(-1/2)}_{\bar\Lambda_{\dot\alpha A}}(z;p)}$$

- $\mathcal{N} = 4$ fields related by 16 susies preserved by the D3's $(\bar{\xi}_{\dot{\alpha}A}, \eta^{\alpha a})$
- Also (the linear part of) these susies obtained by stringy manipulations.

  Example Acting with a preserved charge on a gaugino vertex gives schematically

$$\left[\bar{\xi}\, q, V_\Lambda\right] = V_{\delta_{\bar{\xi}} A}$$

In detail

$$\begin{aligned}
\left[\bar{\xi}_{\dot{\alpha}A}\, q^{\dot{\alpha}A}, V_\Lambda^{(-1/2)}(z)\right] &= \bar{\xi}_{\dot{\alpha}A} \oint_z \frac{dy}{2\pi \mathrm{i}} j^{\dot{\alpha}A}(y) V_\Lambda^{(-1/2)}(z) \\
= \quad & -\bar{\xi}_{\dot{\alpha}A} \Lambda^{\beta B} \oint_z \frac{dy}{2\pi \mathrm{i}} (S^{\dot{\alpha}} S^A \mathrm{e}^{-\frac{1}{2}\phi})(y)\, (S_\beta S_B \mathrm{e}^{-\frac{1}{2}\phi} \mathrm{e}^{\mathrm{i}p_\nu X^\nu})(z) \\
= \quad & \underbrace{\left(-\mathrm{i}\bar{\xi}_{\dot{\alpha}A} (\bar{\sigma}^\mu)^{\dot{\alpha}}{}_{\beta}\, \Lambda^{\beta A}\right)}_{\delta_{\bar{\xi}} A^\mu} \underbrace{\frac{1}{\sqrt{2}}\, \psi_\mu\, \mathrm{e}^{-\phi}\, \mathrm{e}^{\mathrm{i}p_\nu X^\nu}(z)}_{\mathcal{V}_{A^\mu}^{(-1)}(z;p)}
\end{aligned}$$

accounts for the term

$$\delta A^\mu = \mathrm{i}\, \bar{\xi}_{\dot{\alpha}A}\, (\bar{\sigma}^\mu)^{\dot{\alpha}\beta}\, \Lambda_\beta{}^A + \ldots$$

in the susy transf. rules of the $\mathcal{N} = 4$ gauge multiplet

- **D(-1)/D(-1) strings**
  - No momentum. Lowest lying "moduli" $\leftrightarrow$ 0-dimensional reduction of $\mathcal{N}=1$ SYM in $d=10$
  - Chan-Paton indices ij $\rightarrow$ CP factors t^U_{ij} in the adjoint of $\mathrm{U}(k)$
  - NS sector (bosonic moduli):

$$a^\mu \quad \leftrightarrow \quad V_a^{(-1)}(z) = a^\mu \frac{1}{\sqrt{2}} \psi_\mu \mathrm{e}^{-\phi}(z)$$

$$\chi^a \quad \leftrightarrow \quad V_\chi^{(-1)}(z) = \chi^a \frac{1}{\sqrt{2}} \psi_a \mathrm{e}^{-\phi}(z)$$

  - R sector (fermionic moduli):

$$M^{\alpha A} \quad \leftrightarrow \quad V_M^{(-1/2)}(z) = M^{\alpha a}\, S_\alpha S_A \mathrm{e}^{-\phi/2}(z)$$

$$\lambda_{\dot\alpha A} \quad \leftrightarrow \quad V_\lambda^{(-1/2)}(z) = \lambda_{\dot\alpha A}\, S^{\dot\alpha} S^A \mathrm{e}^{-\phi/2}(z)$$

  - Connected by 16 susies preserved by the D(-1) $(\xi_{\dot\alpha A}, \xi^{\alpha a})$
  - Again, (linear part of) susies can be retrieved by vertex manipulations e.g.:

$$[\bar\xi\, q, V_M] = V_{\delta_{\bar\xi} a}$$

- **D(-1)/D3 and D3/D(-1) strings**

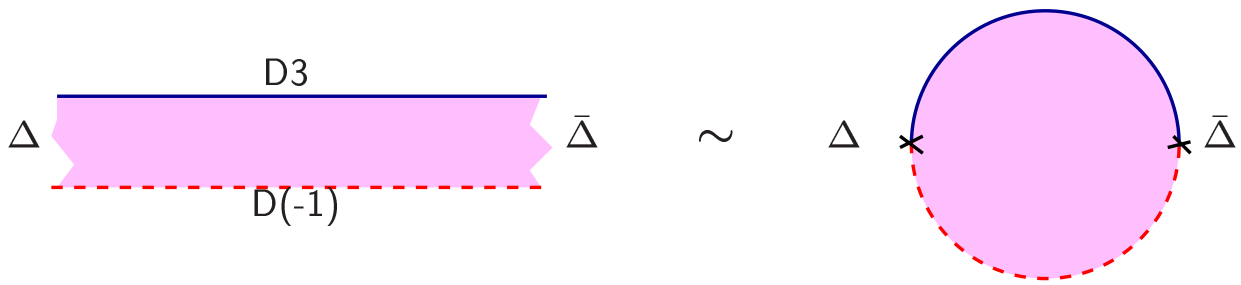
  – In the directions $\mu = 0, 1, 2, 3$ mixed D(irichlet)N(eumann) or ND boundary conditions:

	0	1	2	3	4	5	6	7	8	9
D3	—	—	—	—	·	·	·	·	·	·
D-1	·	·	·	·	·	·	·	·	·	·

  – On each $X^\mu(z)$, switch in b.c. $\leftrightarrow$ insertion of twist field $\Delta(z)$ of conformal weight $4 \times \frac{1}{16}$:

$$\Delta(z)\partial X^\mu(w) \sim \frac{\Delta'^\mu}{(z-w)^{1/2}}\,, \qquad \Delta(z)\bar{\Delta}(w) \sim (z-w)^{1/2}$$

  (the anti-twist D switches back the b.c.s)



  – Modings of X^μ, ψ^μ shifted by $1/2$ $\rightarrow$ peculiar spectrum
  – No momentum in DD directions nor in ND (or DN) $\rightarrow$ again, moduli rather than fields
  – Chan-Paton indices ui or iu, bifundamental of $\mathrm{SU}(N) \times \mathrm{U}(k)$

- NS sector: ψ^μ has 0-modes $\rightarrow$ (chiral) $\mathrm{SO}(4)$ spinors

$$w_{\dot\alpha} \quad \leftrightarrow \quad V_w^{(-1)}(z) = w_{\dot\alpha}\, \Delta S^{\dot\alpha} \mathrm{e}^{-\phi}(z)$$
$$\bar w_{\dot\alpha} \quad \leftrightarrow \quad V_{\bar w}^{(-1)}(z) = \bar w_{\dot\alpha}\, \bar\Delta S^{\dot\alpha} \mathrm{e}^{-\phi}(z)$$

  (Chirality choice: GSO + locality w.r.t. supercurrent $j^{\dot\alpha A}$ preserved on both boundaries. Would be reversed for anti-instantons)
- R sector: ψ^a has 0-modes $\rightarrow$ (chiral) $\mathrm{SO}(6) \sim \mathrm{SU}(4)$ spinors

$$\mu^A \quad \leftrightarrow \quad V_\mu^{(-1/2)}(z) = \mu^A\, \Delta S_A \mathrm{e}^{-\phi/2}(z)$$
$$\bar\mu^A \quad \leftrightarrow \quad V_{\bar\mu}^{(-1/2)}(z) = \bar\mu^A\, \bar\Delta S_A \mathrm{e}^{-\phi/2}(z)$$

  (Chirality choice appropriate to instantonic conf.s)
- Related by the 8 susies preserved on both D3 and D(-1), namely $\bar\xi_{\dot\alpha A}$
- (Linear part of) susies retrived by vertex manipulations, e.g.:

$$\left[\bar\xi\, q, V_\mu\right] = V_{\delta_{\bar\xi} w}$$

Effective action for massless modes

Tree level (=disk) string amplitudes
for massless vertex op.s

field theory limit "$\alpha' \to 0$"

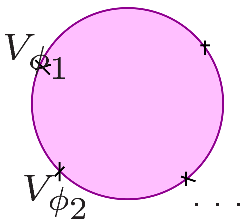
effective action for massless fields

classical inst.
from D3/D3 fiels
+ moduli

$\mathcal{N} = 4$ SYM action
for D3/D3 fields

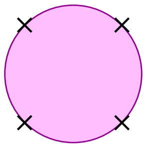
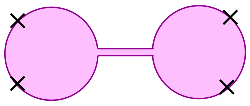
ADHM measure on
D(-1)/D(-1) and
D(-1)/D3 moduli

- **Disk amplitudes**



$$\langle\!\langle V_{\varphi_1}^{(s_1)} V_{\varphi_2}^{(s_2)} \ \ldots \rangle\!\rangle = \delta\Big(\sum s_i - 2\Big)$$

$$\times C_{p+1} \int \frac{\prod_i dz_i}{dV_{123}} \Big\langle V_{\varphi_1}^{(s_1)}(z_1) V_{\varphi_2}^{(s_2)}(z_2) \ldots \Big\rangle$$

- Above, $dV_{abc} = \frac{dz_a\, dz_b\, dz_c}{(z_a - z_b)(z_b - z_c)(z_c - z_a)}$ = proj. invariant volume (effect of ghosts)
- C_{p+1} = topological normalization of the disk
  - fixed by factorization:  $= \sum$ 
  - depends on the # of NN directions (= dimensionality of momentum), $p+1$
  - Explicitly,

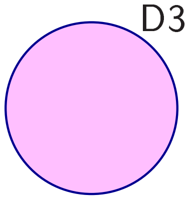
$$C_{p+1} = \frac{2\pi}{g_s} \frac{1}{(4\pi^2\alpha')^{\frac{p+1}{2}}} \frac{1}{x_{p+1}} = \frac{1}{2\pi^2\alpha'^2} \frac{1}{g_{p+1}^2\, x_{p+1}}$$

where

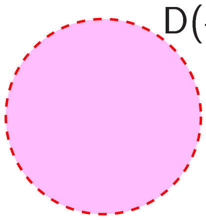
$$g_{p+1}^2 = 4\pi(4\pi^2\alpha')^{\frac{p-3}{2}}\, g_s$$

is the gauge coupling of the resulting SYM theory and x= index of the fundam. rep of the gauge group.

  - In particular ($g_{\rm YM} \equiv g_4$) the pure D3 and D(-1) disks give:


D3

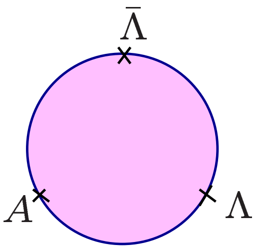
$$\longrightarrow \quad C_4 = \frac{1}{\pi\alpha'^2} \frac{1}{g_{\rm YM}^2}$$


D(-1)

$$\longrightarrow \quad C_0 = \frac{1}{2\pi^2\alpha'^2} \frac{1}{g_0^2} = \frac{8\pi^2}{g_{\rm YM}^2}$$

Effective action for D3/D3 fields

- All amplitudes computed on disks with D3 b.c.'s ($\to$ 4-dim. momentum)
- For instance,



$$\langle\!\langle V_{\bar\Lambda}^{(-1/2)}\, V_{A}^{(-1)}\, V_{\Lambda}^{(-1/2)} \rangle\!\rangle$$
$+$ ineq. color orderings

- Reinstate dim. factors of $2\pi\alpha'$ (previously set to 1). Rule:
  - $*$ NS sector $\sim$ bos. fields: $A_\mu \to (2\pi\alpha')^{\frac{1}{2}} A_\mu$ so that $[A_\mu] = l^{-1}$ (canonical)
  - $*$ R sector $\sim$ bos. fields: $\Lambda, \bar\Lambda \to (2\pi\alpha')^{\frac{3}{4}} \Lambda, \bar\Lambda$ so that $[\Lambda], [\bar\Lambda] = l^{-3/2}$ (canonical)
- In the end we get the effective vertex

$$-\frac{2\,\mathrm{i}}{g^2_{\rm YM}}\,\mathrm{Tr}\left(\bar\Lambda_{\dot\alpha A}\left[\bar{\not{A}}^{\dot\alpha\beta}\,,\,\Lambda_\beta{}^A\right]\right) \quad .$$

- Do the same for all other interactions surviving for

$$\alpha' \to 0 \;\text{ with } g^2_{\rm YM} \text{ fixed}$$

- From the 1PI part of the surviving amplitudes $\rightarrow$ $\mathcal{N} = 4$ SYM action

$$
\begin{aligned}
\mathcal{S}_{\mathrm{SYM}} &= \frac{1}{g_{\mathrm{YM}}^2} \int d^4x \; \mathrm{Tr} \Bigg\{ \frac{1}{2} F_{\mu\nu}^2 - 2\, \bar{\Lambda}_{\dot\alpha A}\, \bar{\not{\mathcal{D}}}^{\dot\alpha\beta}\, \Lambda_\beta{}^A \\
&+ \left(\mathcal{D}_\mu \varphi_a\right)^2 - \frac{1}{2} \left[\varphi_a, \varphi_b\right]^2 \\
&- \mathrm{i}\,(\Sigma^a)^{AB}\, \bar{\Lambda}_{\dot\alpha A} \Big[\varphi_a, \bar{\Lambda}^{\dot\alpha}{}_B\Big] - \mathrm{i}\,(\bar{\Sigma}^a)_{AB}\, \Lambda^{\alpha A} \Big[\varphi_a, \Lambda_\alpha{}^B\Big] \Bigg\}
\end{aligned}
$$

D(-1)/D(-1) moduli

- All amplitudes on disks with D(-1) b.c. ($\to$ no momentum)
- Same procedure as above: after rescaling to canonical dimensions, take

$$\alpha' \to 0 \quad \text{with } g_0^2 \text{ fixed}$$

- One remains with the "action"

$$\mathcal{S}_{(-1)} = \mathcal{S}_{\mathrm{cubic}} + \mathcal{S}_{\mathrm{quartic}}$$

where

$$\begin{aligned} \mathcal{S}_{\mathrm{cubic}} &= \frac{\mathrm{i}}{g_0^2}\,\mathrm{tr}\Big\{\lambda_{\dot\alpha A}\Big[\not{a}^{\dot\alpha\beta}, M_\beta{}^A\Big] - \frac{1}{2}\,(\Sigma^a)^{AB}\,\lambda_{\dot\alpha A}\Big[\chi_a, \lambda^{\dot\alpha}{}_B\Big] \\ &- \frac{1}{2}\,(\bar\Sigma^a)_{AB}\,M^{\alpha A}\Big[\chi_a, M_\alpha{}^B\Big]\Big\} \end{aligned}$$

$$\mathcal{S}_{\mathrm{quartic}} = -\frac{1}{g_0^2}\,\mathrm{tr}\Big\{\frac{1}{4}\,[a_\mu, a_\nu]^2 + \frac{1}{2}\,[a_\mu, \chi_a]^2 + \frac{1}{4}\,[\chi_a, \chi_b]^2\Big\}$$

- **Auxiliary fields** The quartic interactions can be decoupled by means of auxiliary fields:
  - From

$$\mathcal{S}' = \frac{1}{g_0^2}\mathrm{tr}\left\{\frac{1}{2}D_c^{\,2} + \frac{1}{2}D_c\bar{\eta}^c_{\mu\nu}\left[a^\mu, a^\nu\right] + \frac{1}{2}Y_{\mu a}^{\,2} + Y_{\mu a}\left[a^\mu, \chi^a\right] + \frac{1}{4}Z_{ab}^{\,2} + \frac{1}{2}Z_{ab}\left[\chi^a, \chi^b\right]\right\}$$

integrate out $D, Y, Z \to S_{\mathrm{quartic}}$.

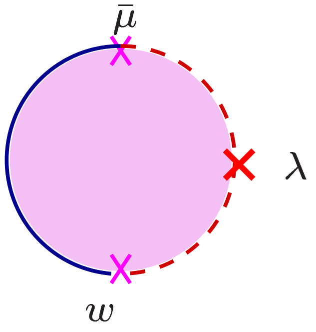
  - Notice: $D^{(-)}_{\mu\nu}$ antiselfdual sufficient $\to D_{\mu\nu} \equiv D_c\bar{\eta}^c_{\mu\nu}$
  - All the cubic interactions in S' obtained from disks using vertex operators for auxiliary fields (non BRST invariant):

$$\begin{aligned} V_D^{(0)}(z) &= \frac{1}{2}D_c\bar{\eta}^c_{\mu\nu}\,\psi^\nu\psi^\mu(z) \\ V_Y^{(0)}(z) &= Y_{\mu a}\,\psi^a\psi^\mu(z) \\ V_Z^{(0)}(z) &= \frac{1}{2}\,Z_{ab}\,\psi^b\psi^a(z) \quad . \end{aligned}$$

  - Auxiliary fields and vertices also linearize the SUSY tansf.s $\to$ susies completely derived via vertex manipulations

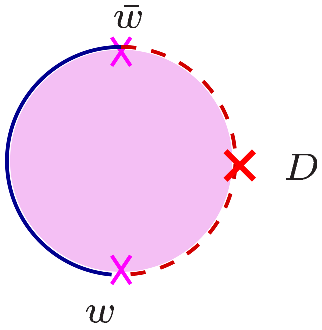
D(-1)/D3 and D3/D(-1) moduli

- Disks with mixed boundary cond.s. No momentum. Pairs of twist-antitwists Δ, $\bar{\Delta}$
- For instance, with usual procedure,



$$\left\langle\!\left\langle V_w^{(-1)} V_\lambda^{(-1/2)} V_{\bar{\mu}}^{(-1/2)} \right\rangle\!\right\rangle$$
$$\to \frac{2\,\mathrm{i}}{g_0^2} \mathrm{tr}\left(w_{\dot{\alpha}}{}^{u} \lambda^{\dot{\alpha}}{}_{A} \bar{\mu}^{A}{}_{u} \right)$$

- Also non-zero amplitudes with auxiliary fields D:



$$\left\langle\!\left\langle V_w^{(-1)} V_D^{(0)} V_{\bar{w}}^{(-1)} \right\rangle\!\right\rangle$$
$$\to \frac{1}{2g_0^2}\, \bar{\eta}^{c}_{\mu\nu}\, \mathrm{tr}\left(w_{\dot{\alpha}}{}^{u} D_c\, \bar{w}^{\dot{\beta}}{}_{u} \right) \left(\bar{\sigma}^{\mu\nu}\right)^{\dot{\alpha}}{}_{\dot{\beta}}$$
$$= \frac{2\mathrm{i}}{g_0^2} \mathrm{tr}\left(D_c\, W^c \right)$$

where we introduce the $k \times k$ matrices

$$\left(W^c\right)_j{}^{i} = w_{\dot{\alpha}}{}^{ui} (\tau^c)^{\dot{\alpha}}{}_{\dot{\beta}} \bar{w}^{\dot{\beta}}{}_{uj}$$

- Altogether one gets the “action”

$$
\begin{aligned}
\mathcal{S}'' \quad &= \quad \frac{2\mathrm{i}}{g_0^2}\mathrm{tr}\Bigg\{\Big(\bar{\mu}^A{}_u w_{\dot\alpha}{}^u + \bar{w}_{\dot\alpha u}\mu^{Au}\Big)\lambda^{\dot\alpha}_A - D_c\, W^c \\
&+ \quad \frac{1}{2}(\bar\Sigma^a)_{AB}\,\bar{\mu}^A{}_u\,\mu^{Bu}\chi_a - \mathrm{i}\,\chi_a\,\bar{w}_{\dot\alpha u}w^{\dot\alpha u}\chi^a\Bigg\}
\end{aligned}
$$

- Again, quartic terms can be disentangled introducing auxiliary fields that also linearize susy transf.s

Moduli space and ADHM measure

- For the moduli, both from D3/D3 and D3/D(-1) strings, we got $\mathcal{S}_{\text{moduli}} = \mathcal{S}_{\text{cubic}} + \mathcal{S}' + \mathcal{S}''$ in the limit

$$\alpha' \to 0 \ \text{ with } g_0^2 \text{ fixed}$$

- SYM action on the D3 however arises for

$$\alpha' \to 0 \ \text{ with } g_{\text{YM}}^2 \text{ fixed}$$

- Since
$$g_0 = \frac{g_{\text{YM}}}{4\pi^2\alpha'}$$
when $\alpha' \to 0$ with g_0^2 fixed we have $g_0 \to \infty$. Keeping fixed the moduli $a, \chi, \ldots$ with canonical dimensions, $\mathcal{S}_{\text{moduli}} \to 0$

- To retain the effect of the presence of the D(-1)'s:
  - rescale the moduli giving them non-canonical dimensions:

$$a = \sqrt{2}\, g_0 a' \ , \quad \chi = \chi' \ , \quad M = \frac{g_0}{\sqrt{2}} M' \ , \quad \lambda = \lambda' \ ,$$

$$D = D' \ , \quad Y = \sqrt{2}\, g_0\, Y' \ , \quad Z = g_0\, Z' \ ,$$

$$w = \frac{g_0}{\sqrt{2}} w' \ , \quad \bar{w} = \frac{g_0}{\sqrt{2}} \bar{w}' \ , \quad \mu = \frac{g_0}{\sqrt{2}} \mu' \ , \quad \bar{\mu} = \frac{g_0}{\sqrt{2}} \bar{\mu}' \ ,$$

  - keep fixed the rescaled moduli
    (we'll drop the primes, except for a', M' where are traditional)

- The result is

$$
\begin{aligned}
S_{\text{moduli}} \quad = \quad & \mathrm{tr}\Bigg\{ Y_{\mu a}^{\,2} + 2\, Y_{\mu a} \left[a'^{\mu}, \chi^{a} \right] + \frac{1}{4} Z_{ab}^{\,2} \\
& + \chi_a \bar{w}_{\dot{\alpha} u} w^{\dot{\alpha} u} \chi^{a} \\
& + \frac{\mathrm{i}}{2} (\bar{\Sigma}^{a})_{AB} \bar{\mu}^{A}{}_{u} \mu^{Bu} \chi_a - \frac{\mathrm{i}}{4} (\bar{\Sigma}^{a})_{AB} M'^{\alpha A} \left[\chi_a, M'{}_{\alpha}^{B} \right] \\
& + \mathrm{i} \left(\bar{\mu}^{A}{}_{u} w_{\dot{\alpha}}^{\ u} + \bar{w}_{\dot{\alpha} u} \mu^{Au} + \left[M'^{\beta A}, a'_{\beta \dot{\alpha}} \right] \right) \lambda^{\dot{\alpha}}{}_{A} \\
& - \mathrm{i} D_c \left(W^{c} + \mathrm{i} \bar{\eta}^{c}_{\mu\nu} \left[a'^{\mu}, a'^{\nu} \right] \right) \Bigg\}
\end{aligned}
$$

$\downarrow$

integrate out Y, Z

$\downarrow$

$\mathrm{e}^{-\mathcal{S}_{\text{moduli}}}$ = exp. measure on instanton moduli space as in ADHM construction

  - In particular, quadratic terms for D_c and $\lambda_A^{\dot{\alpha}}$ have been rescaled away $\longrightarrow$ multipliers of bosonic and fermionic constraints

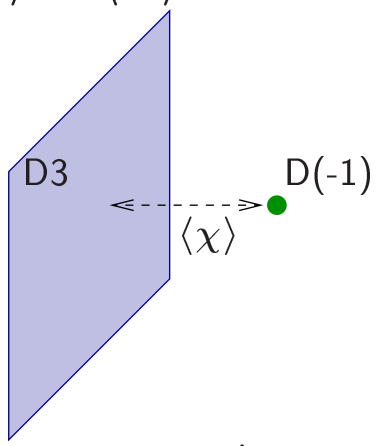
- Classical vacua (bosonic):

$$\mathrm{tr}\left[a'^{\mu}, \chi^{a}\right] = 0 \ , \quad \mathrm{tr}\left(\chi^{a} \bar{w}^{\dot{\alpha} u}\right) = 0$$

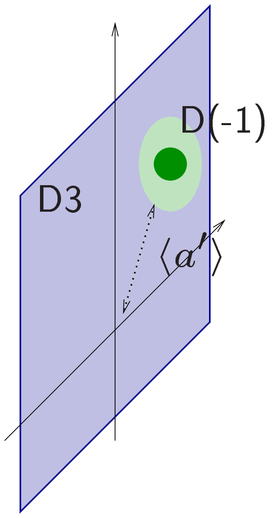
$$W^{c} + \mathrm{i}\bar{\eta}^{c}_{\mu\nu}\left[a'^{\mu}, a'^{\nu}\right] = 0$$

  – "Coulomb phase": $\langle \chi \rangle \neq 0 \Rightarrow \langle a' \rangle = \langle w \rangle = 0$

D(-1) separated from the D3



  – "Higgs phase": $\langle \chi \rangle = 0 \Rightarrow$ generically $\langle a' \rangle, \langle w \rangle \neq 0$, subject to the ADMH constraints $W^{c} + i\bar{\eta}^{c}_{\mu\nu}\left[a'^{\mu}, a'^{\nu}\right] = 0$



Phase of interest for us:
D(-1)$\leftrightarrow$ instantonic conf. localized on D3, centered at $\langle a' \rangle$, with spread related to $\langle w \rangle$

- Summarizing: in the Higgs phase, considering also fermionic moduli,

$$\mathcal{S}_{\rm moduli} \to \text{classical vacua given by ADHM constraints}$$

$$\begin{cases} W^c + \mathrm{i}\bar{\eta}^c_{\mu\nu}\left[a'^{\mu}, a'^{\nu}\right] = 0 & \text{(bosonic)} \\ \\ \bar{\mu}^A{}_u w_{\dot{\alpha}}{}^u + \bar{w}_{\dot{\alpha}u}\mu^{Au} + \left[M'^{\beta A},\, a'_{\beta\dot{\alpha}}\right] = 0 & \text{(fermionic)} \end{cases}$$

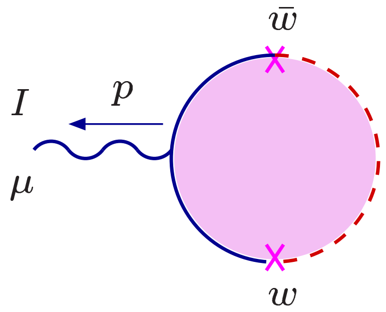
(Notice: $k \times k$ matrix constraints)

- Moduli space of k-instantons $=$ HyperKähler space, defined as a HyperKähler quotient by ADHM
  The quotient is realized in string theory by a brane set-up, as it often happens

The instanton solution from mixed disks

- A further piece of information can be extracted from the stringy description: the classical instanton profile.
- We concentrate on the one-instanton sector ($k = 1$). The method extends to the general case.
- Consider amplitudes with D3/D3 fields and moduli at the same time, on disks with part(s) of the boundary on a D3, part(s) on a D(-1)
- Simplest case: emission of a **gauge boson** from a mixed disk

$$A^I_\mu(p;\bar{w},w) = \langle\!\langle V^{(-1)}_{\bar{w}} \mathcal{V}^{(0)}_{A^I_\mu}(-p) V^{(-1)}_{w} \rangle\!\rangle$$



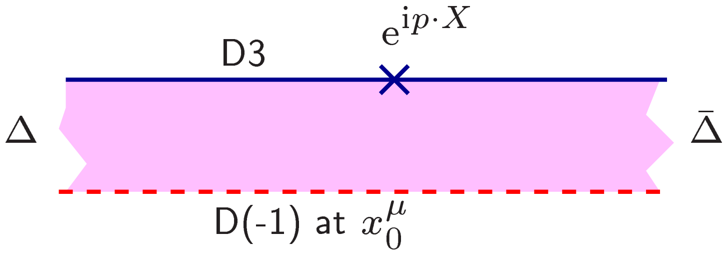
  - Gluon vertex op. in the picture 0:

$$\mathcal{V}^{(0)}_{A^I_\mu}(z;-p) = 2\mathrm{i}T^I \left(\partial X_\mu - \mathrm{i}p\cdot\psi\psi_\mu\right) \mathrm{e}^{-\mathrm{i}p\cdot X}(z)$$

    with *outgoing* momentum and *no polarization*
  - The amplitude $A^I_\mu(p;\bar{w},w)$ has Lorentz and quantum numbers of emitted gauge field

- Usual contractions of vertex op.s; in particular,

$$\left\langle \bar{\Delta}(z_1)\mathrm{e}^{-\mathrm{i}p\cdot X(z_2)}\Delta(z_3)\right\rangle = -\mathrm{e}^{-\mathrm{i}p\cdot x_0}(z_1-z_3)^{-1/2}$$



The D(-1) part of the boundary is fixed at x_0 $(=\langle a'\rangle)$

– this breaks translational invariace in the D3 world-volume

– we can have a tadpole $\propto \mathrm{e}^{\mathrm{i}p\cdot x_0}$

- The result is

$$A^I_\mu(p;\bar{w},w) = \mathrm{i}\,p^\nu\,J^I_{\nu\mu}(\bar{w},w)\,\mathrm{e}^{-\mathrm{i}p\cdot x_0}$$

where

$$J^I_{\nu\mu}(\bar{w},w) = (T^I)^v{}_u\,\bar{\eta}^c_{\nu\mu}\left(w_{\dot{\alpha}}{}^u\,(\tau_c)^{\dot{\alpha}}{}_{\dot{\beta}}\,\bar{w}^{\dot{\beta}}{}_v\right)$$

- To get the space-time profile of the field generated by the emission amplitude:

– attach a (massless) propagator $\delta_{\mu\nu}/p^2$

– Fourier transform

That is,

$$A^I_\mu(x) = J^I_{\nu\mu}(\bar{w},w)\int\frac{d^4p}{(2\pi)^2}\,\frac{\mathrm{i}p^\nu}{p^2}\,\mathrm{e}^{\mathrm{i}p\cdot(x-x_0)}$$

- We can write the field as

$$A^I_\mu(x) = J^I_{\nu\mu}(\bar{w}, w)\, \partial^\nu G(x - x_0)$$

where $G(x-x_0) = (x-x_0)^{-2}$ is the massless propagator in configuration space.

- The above solution contains as parameters:
  - the position x_0^μ of the D(-1)
  - the $4N$ moduli $w_u^{\dot\alpha}$, $\bar{w}_u^{\dot\alpha}$, up to an irrelevant phase rotation: $w \sim \mathrm{e}^{\mathrm{i}\theta} w$ and $\bar{w} \sim \mathrm{e}^{-\mathrm{i}\theta} \bar{w}$

  $\rightarrow$ it is defined on the $4N-3$-dim. *unconstrained* moduli space

- Upon enforcing the 3 bosonic ADHM constraints

$$W^c \equiv w^u_{\dot\alpha} (\tau^c)^{\dot\alpha}{}_{\dot\beta} \bar{w}^{\dot\beta}{}_v = 0$$

arising from the moduli action,

$$A^I_\mu(x) \ \rightarrow \ \text{instanton in singular gauge}$$

in the large-distance expansion, and

$$w_u^{\dot\alpha}\,,\ \bar{w}_u^{\dot\alpha} \begin{array}{l} \nearrow \quad \text{size } \rho \\ \searrow \quad \text{orientation of } \mathrm{SU}(2) \text{ inside } \mathrm{SU}(N) \end{array}$$

– Define

$$2\rho^2 \equiv \bar{w}^{\dot{\alpha}}_{\ u}$$

– The $N \times N$ matrices

$$(t_c)^u_{\ v} \equiv \frac{1}{2\rho^2}\left(w_{\dot{\alpha}}^{\ u}(\tau_c)^{\dot{\alpha}}_{\ \dot{\beta}}\bar{w}^{\dot{\beta}}_{\ v}\right)$$

satisfy an $\mathrm{su}(2)$ subalgebra: $[t_c, t_d] = \mathrm{i}\epsilon_{cde}\, t_e$ *iff* the constraints $W^c = 0$ hold.

- The gauge vector profile can be written as

$$A^I_\mu(x) = 4\rho^2 \mathrm{Tr}(T^I\, t_c)\bar{\eta}^c_{\mu\nu}\frac{(x-x_0)^\nu}{(x-x_0)^4}$$

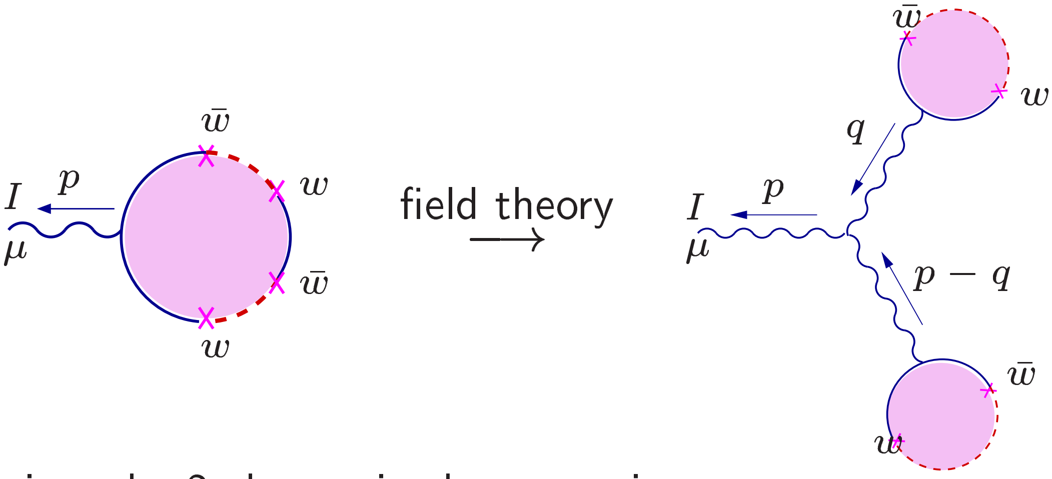
- For $N = 2$, we get the $\mathrm{SU}(2)$ connection

$$A^c_\mu(x) = 2\rho^2\bar{\eta}^c_{\mu\nu}\frac{(x-x_0)^\nu}{(x-x_0)^4}$$

$=$ leading term in $|x - x_0| \gg \rho$ expansion of $\mathrm{SU}(2)$ instanton in the *singular gauge*:

$$\begin{aligned} A^c_\mu(x) &= 2\rho^2\bar{\eta}^c_{\mu\nu}\frac{(x-x_0)^\nu}{(x-x_0)^2\Big[(x-x_0)^2+\rho^2\Big]} \\ &\simeq 2\rho^2\bar{\eta}^c_{\mu\nu}\frac{(x-x_0)^\nu}{(x-x_0)^4}\left(1 - \frac{\rho^2}{(x-x_0)^2} + \ldots\right) \end{aligned}$$

- Subleading terms can be recostructed from disks with more w, $\bar{w}$ insertions:



gives the 2nd term in the expansion:

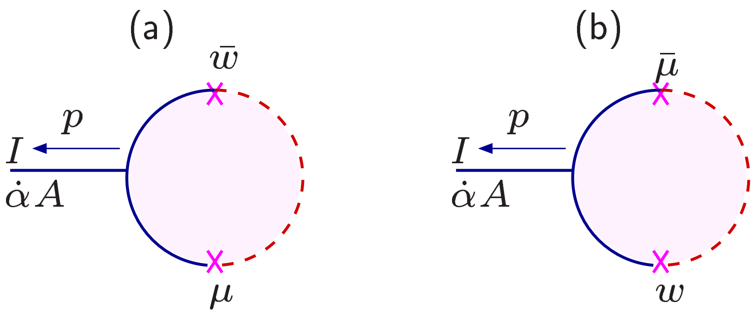
$$A^c_\mu(x)^{(2)} = -2\rho^4 \bar{\eta}^c_{\mu\nu} \frac{(x-x_0)^\nu}{(x-x_0)^6}$$

- **Question**: Why *singular* gauge?
  - Instanton produced by a point-like source, the D(-1), inside the D3 $\rightarrow$ singular at the location of the source
  - In the singular gauge, rapid fall-off of the fields $\rightarrow$ eq.s of motion reduce to *free* eq.s *at large distance* $\rightarrow$ "perturbative" solution in terms of the source term (leading term $A^I_\mu(x) = J^I_{\nu\mu}(\bar{w}, w)\partial^\nu G(x-x_0)$)
  - non-trivial properties of the instanton profile from the region near the singularity through the embedding

$$S_3^{x_0} \hookrightarrow \mathrm{SU}(2) \subset \mathrm{SU}(N)$$

The super-instanton profile

- We're dealing with $\mathcal{N} = 4$ SYM $\rightarrow$ we should recover the $\mathcal{N} = 4$ super-instanton.
- There are mixed disks that act as sources for the other fields in the multiplet
- For the **gauginos**, diagrams (a) and (b):



E.g., (a) corresponds to

$$\bar{\Lambda}^{\dot{\alpha}A,I}(p;\bar{w},\mu) = \langle\!\langle V_{\bar{w}}^{(-1)} \mathcal{V}_{\bar{\Lambda}^I_{\dot{\alpha}A}}^{(-1/2)}(-p) V_{\mu}^{(-1/2)} \rangle\!\rangle$$

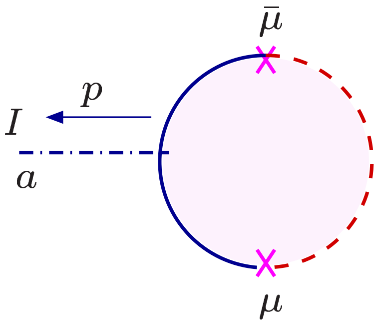
From (a) + (b), upon insertion of the fermionic massless propagator and Fourier transform, we get

$$\Lambda^{\alpha A,I}(x) = -2\mathrm{i}(T^I)^v{}_u \left(w_{\dot{\beta}}{}^u \bar{\mu}^A{}_v + \mu^{Au} \bar{w}_{\dot{\beta}v} \right) (\bar{\sigma}_\nu)^{\dot{\beta}\alpha} \frac{(x-x_0)^\nu}{(x-x_0)^4}$$

– Imposing ADHM fermionic constraints $\rightarrow$ leading order at large distance of the gaugino profile in the $\mathcal{N}=4$ instanton in the singular gauge:

$$(\widehat{\Lambda}^{\alpha A}(x))^{u}_{\ v} = \frac{(\sigma_\nu)^{\alpha}_{\ \dot\beta}\left(w^{\dot\beta u}\,\bar\mu^{A}_{\ v} + \mu^{Au}\,\bar w^{\dot\beta}_{\ v}\right)(x-x_0)^\nu}{\sqrt{(x-x_0)^2\Big[(x-x_0)^2+\rho^2\Big]^3}}$$

- For the 6 adjoint scalars φ^a (often rewritten as $\varphi^{AB} \equiv \frac{1}{2\sqrt{2}}(\Sigma^a)^{AB}\varphi^a$)



$$\varphi^I_a(p;\bar\mu,\mu)$$
$$= \left\langle\!\left\langle V_{\bar\mu}^{(-1/2)}\,\mathcal{V}^{(-1)}_{\varphi^I_a}(-p)\,V_{\mu}^{(-1/2)}\right\rangle\!\right\rangle$$

Inserting the massless propagator and Fourier transforming gives

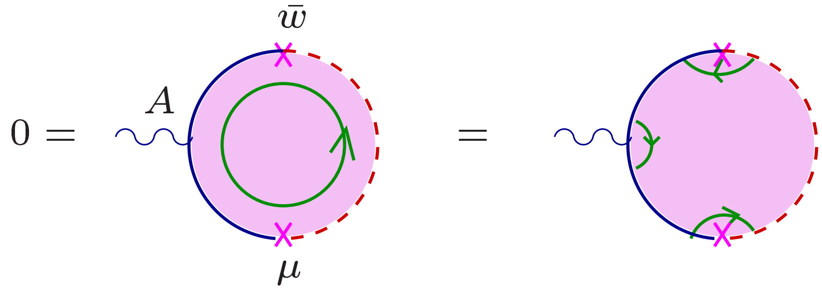
$$\varphi^{AB,I}(x) = -\frac{\mathrm{i}}{\sqrt{2}}(T^I)^{v}_{\ u}\,\mu^{[Au}\bar\mu^{B]}_{\ \ v}\frac{1}{(x-x_0)^2}$$

$=$ leading term at large distance of exact instanton solution

- The profiles for the gaugino and the scalars can be got via Ward identities on the disk amplitudes for the D3 susies preserved by the D(-1) (namely, $\bar{\xi}_{\dot{\alpha}A}$).

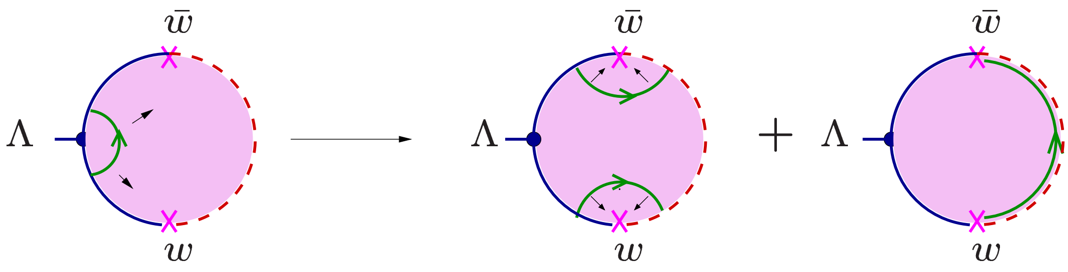
  These Ward identities relate the gaugino emission amplitude to the gauge boson one, and the scalar emission to the gaugino one

  Example:



$$\bar{\xi}_{\dot{\beta}A} p_\nu (\bar{\sigma}^{\nu\mu})^{\dot{\beta}}{}_{\dot{\alpha}} \left\langle\!\!\left\langle V_{\bar{w}} \mathcal{V}_{\bar{\Lambda}^I_{\dot{\alpha}A}}(-p) V_\mu \right\rangle\!\!\right\rangle = -\left\langle\!\!\left\langle V_{\bar{w}} \mathcal{V}_{A^I_\mu}(-p) V_{\delta_{\bar{\xi}} w} \right\rangle\!\!\right\rangle$$

- Acting with the D3 SUSY charges $q'_{\alpha A}$ *broken* by the D(-1) shifts the supermoduli
  - Move the integration contour in $q'_{\alpha A}$ to the D(-1) part of the boundary $\longrightarrow$ integrated vertex op. of the *goldstinos* $M'^{\alpha A}$
  - Ward identities relate emission diagrams with no D(-1)/D(-1) moduli to diagrams with extra insertions of M' moduli



– In the above example, one gets

$$\underbrace{\bar{\Lambda}^{\dot{\alpha}A,I}(p;\bar{w},w,M')}_{\text{4-point diagram}} = \underbrace{-\mathrm{i}\,M'^{\beta A}(\sigma^{\mu})_{\beta}{}^{\dot{\alpha}}\,A^{I}_{\mu}(p;\bar{w},w)}_{\text{alg. manipul. of 3-point diagram}}$$

From $\bar{\Lambda}^{\dot{\alpha}A,I}(p;\bar{w},w,M')$ $\rightarrow$ space-time profile

$$\Lambda^{\alpha A\,,\,I}(x) \stackrel{x\to\infty}{\simeq} \frac{\mathrm{i}}{2}M'^{\beta A}(\sigma^{\mu\nu})_{\beta}{}^{\alpha}F^{I}_{\mu\nu}(x)$$

$=$ chiral ferm. profile created acting by broken susies on the instanton background.

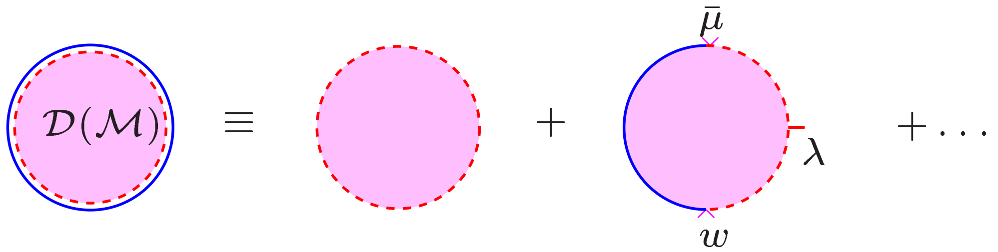
– Repeating the procedure $\rightarrow$ entire structure of fermionic 0-modes

Correlators in the instanton background

- Tree-level string amplitudes for D3/D3 states (without polarizations) $\xrightarrow{\alpha'\to 0}$ amputated Green functions:

$$\Big\langle \phi_1(p_1)\ldots\phi_n(p_n)\Big\rangle\Big|_{\text{amp.}} = \Big\langle\!\!\Big\langle \mathcal{V}_{\phi_1}(-p_1)\ldots\mathcal{V}_{\phi_n}(-p_n)\Big\rangle\!\!\Big\rangle\Big|^{\text{1PI}}_{\alpha'\to 0}$$

- What modifications from D-instantons?
  - Disk diagrams with only moduli ($\mathcal{M}$) insertions



These "vacuum" contrib.s (from the D3 point of view) give

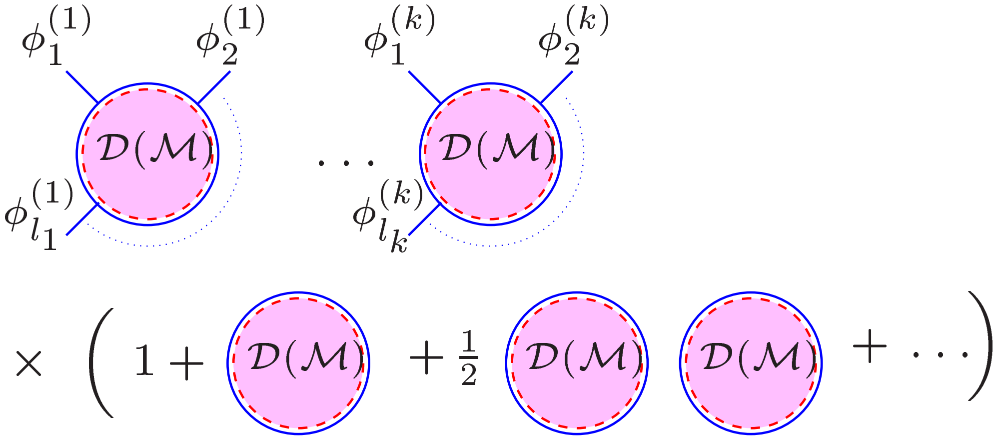
$$\langle\!\langle\, 1\, \rangle\!\rangle_{\mathcal{D}(\mathcal{M})} \overset{\alpha'\to 0}{\simeq} -S\,[\mathcal{M}] \equiv -\frac{8\pi^2 k}{g^2_{\text{YM}}} - S_{\text{moduli}}$$

(the "pure" D(-1) disk gives $C_0 = \frac{8\pi^2 k}{g^2_{\text{YM}}}$ [Polchinski])

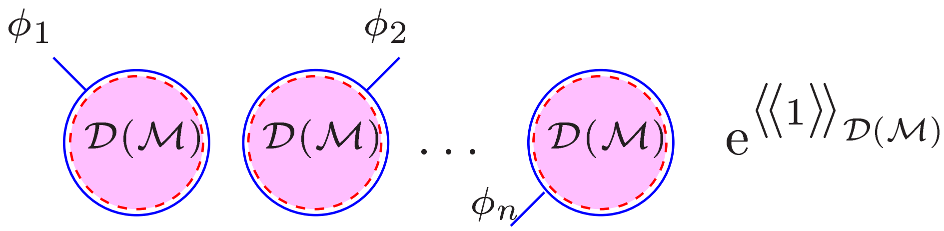
  - Correlators of D3/D3 fields on $\mathcal{D}(\mathcal{M})$, i.e. with mixed b.c.'s and insertion of moduli

$$\langle\!\langle \mathcal{V}_{\phi_1}(p_1)\ldots\mathcal{V}_{\phi_n}(p_n)\rangle\!\rangle_{\mathcal{D}(\mathcal{M})}$$

- The correlators on $\mathcal{D}(\mathcal{M})$ have to be *integrated on the moduli*. Several important consequences
  - Diagrams disconnected from the world-sheet point of view are connected from the point of view of the field theory on the D3 because of the moduli integration



    Notice: the combinatorics of boundaries (Polchinski) $\to$ the "vacuum" terms $\langle\!\langle 1 \rangle\!\rangle_{\mathcal{D}(\mathcal{M})}$ exponentiate
  - Every 2-d amplitude on $\mathcal{D}(\mathcal{M}) \propto C_0 \propto g_s^{-1}$, dominant contrib = most disconnected one = product of tadpoles (Green-Gutperle)



namely,

$$\left\langle\!\!\left\langle \mathcal{V}_{\phi_1}(p_1) \right\rangle\!\!\right\rangle_{\mathcal{D}(\mathcal{M})} \cdots \left\langle\!\!\left\langle \mathcal{V}_{\phi_n}(p_n) \right\rangle\!\!\right\rangle_{\mathcal{D}(\mathcal{M})} \mathrm{e}^{\langle\!\langle\, 1 \,\rangle\!\rangle_{\mathcal{D}(\mathcal{M})}}$$

- Altogether we have

$$\left\langle \phi_1(p_1) \ldots \phi_n(p_n) \right\rangle \Big|_{\mathrm{amput.}}^{\mathrm{D-inst.}} =$$

$$\int d\mathcal{M} \left\langle\!\!\left\langle \mathcal{V}_{\phi_1}(-p_1) \right\rangle\!\!\right\rangle_{\mathcal{D}(\mathcal{M})} \cdots \left\langle\!\!\left\langle \mathcal{V}_{\phi_n}(-p_n) \right\rangle\!\!\right\rangle_{\mathcal{D}(\mathcal{M})} \mathrm{e}^{\langle\!\langle 1 \rangle\!\rangle_{\mathcal{D}(\mathcal{M})}} \Big|_{\alpha' \to 0}$$

$$\downarrow$$

insert propagators + Fourier transform

$$\downarrow$$

Green function

$$\left\langle \phi_1(x_1) \ldots \phi_n(x_n) \right\rangle \Big|_{\mathrm{D-inst.}} =$$

$$\int d\mathcal{M}\ \phi_1^{\mathrm{disk}}(x_1; \mathcal{M}) \cdots\ \phi_n^{\mathrm{disk}}(x_n; \mathcal{M})\, \mathrm{e}^{-S[\mathcal{M}]}$$

where

$$\phi^{\mathrm{disk}}(x; \mathcal{M}) = \int \frac{d^4 p}{(2\pi)^2}\, \mathrm{e}^{\mathrm{i}p\cdot x}\, \frac{1}{p^2} \left\langle\!\!\left\langle \mathcal{V}_{\phi}(-p) \right\rangle\!\!\right\rangle_{\mathcal{D}(\mathcal{M})} \Big|_{\alpha' \to 0}$$

- As already argued (main point of the talk)

$$\phi(x;\mathcal{M})^{\rm disk} = \phi^{\rm cl}(x;\mathcal{M})$$

- Under this identification,

stringy prescription for corrl.s in presence of D-instantons

$$\updownarrow$$

standard field theory prescription of instanton calculus

- Effect of D-inst. $\rightarrow$ effective vertices for the D3/D3 fields
  - originate from one-point functions on $\mathcal{D}(\mathcal{M})$ $\rightarrow$

$$S_{(-1)/3} = -\sum_\phi \int \frac{d^4p}{(2\pi)^2}\,\phi(p)\,\Big\langle\!\Big\langle \mathcal{V}_\phi(p)\Big\rangle\!\Big\rangle_{\mathcal{D}(\mathcal{M})}\Big|_{\alpha'\to 0}$$

  - Since $\langle\!\langle \mathcal{V}_\phi(p)\rangle\!\rangle_{\mathcal{D}(\mathcal{M})} \sim J_\phi(\mathcal{M})\,\mathrm{e}^{\mathrm{i}p\cdot x_0}$,

$$S_{(-1)/3} = -\sum_\phi \phi(x_0)\,J_\phi(\mathcal{M})$$

  - Explicitly,

$$\begin{aligned} S_{(-1)/3} &= -\frac{1}{2}\,F^I_{\mu\nu}(x_0)\,J^{\mu\nu\,,\,I}(\mathcal{M}) \\ &- \bar\Lambda^I_{\dot\alpha A}(x_0)\,J^{\dot\alpha A\,,\,I}(\mathcal{M}) - \varphi^I_{AB}(x_0)\,J^{AB\,,\,I}(\mathcal{M}) \end{aligned}$$

(non-ab. extension of Green-Gutperle approach)