Instantonic effects in N=1 local models from magnetized branes

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Foreword

This talk is mostly based on

- M. Billo, M. Frau, I. Pesando, P. Di Vecchia, A. Lerda and R. Marotta, "Instanton effects in N=1 brane models and the Kahler metric of twisted matter," arXiv:0709.0245 [hep-th].
- M. Billo, M. Frau, I. Pesando, P. Di Vecchia, A. Lerda and R. Marotta, "Instantons in N=2 magnetized D-brane worlds," arXiv:0708.3806 [hep-th].

It, of course, builds over a vast literature

- The few references scattered on the slides are of course not exhaustive. I apologize for the missing ones.

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Some very recent work dealing with very similar issues

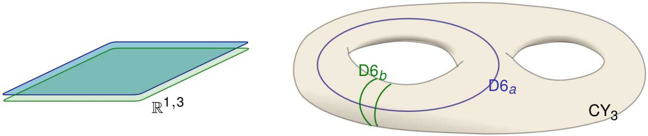
- R. Blumenhagen and M. Schmidt-Sommerfeld, "Gauge Thresholds and Kaehler Metrics for Rigid Intersecting D-brane Models," arXiv:0711.0866 [hep-th].

Plan of the talk

Introduction

Wrapped brane scenarios

- Type IIB: magnetized D9 branes
- Type IIA (T-dual): intersecting D6 (easier to visualize)



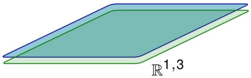
Supersymmetric gauge theories on $\mathbf{R}^{1,3}$ with chiral matter and interesting phenomenology

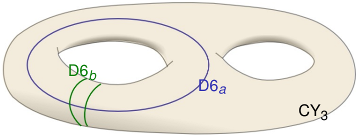
[review: Blumenhagen et al, Phys. Rept. **445** (2007)]

- families from multiple intersections, tuning different coupling constants, . . .

Wrapped brane scenarios

- Type IIB: magnetized D9 branes
- Type IIA (T-dual): intersecting D6 (easier to visualize)



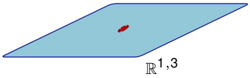


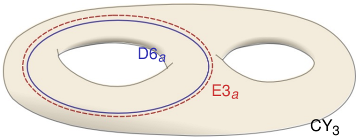
- low energy described by SUGRA with vector and matter multiplets
- can be derived directly from string amplitudes (with different field normaliz.s)
- novel stringy effects (pert. and non-pert.) in the eff. action?

Euclidean branes and instantons

Ordinary instantons

E2 branes wrapped on the same cycle as some D6 branes are point-like in $\mathbf{R}^{1,3}$ and correspond to instantonic config.s of the gauge theory on the D6





Analogous to the D3/D(-1) system:

- ADHM from strings attached to the instantonic branes

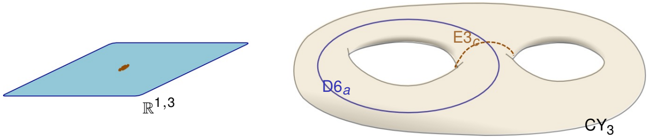
  Witten, 1995; Douglas, 1995-1996; ...
- non-trivial instanton profile of the gauge field — Billo et al, 2001

N.B. In type IIB, use D9/E5 branes

Euclidean branes and instantons

Exotic instantons

E2 branes wrapped differently from the D6 branes are still point-like in $\mathbf{R}^{1,3}$ but do not correspond to ordinary instantons config.s.



Still they can,in certain cases, give important non-pert, stringy contributions to the effective action, .e.g. Majorana masses for neutrinos, moduli stabilizing terms, . . .

Blumenhagen et al 0609191; Ibanez and Uranga, 0609213; (long list)... ; Petersson 0711.1837

- Potentially crucial for string phenomenology

Perspective of this work

Clarify some aspects of the "stringy instanton calculus", i.e., of computing the contributions of Euclidean branes

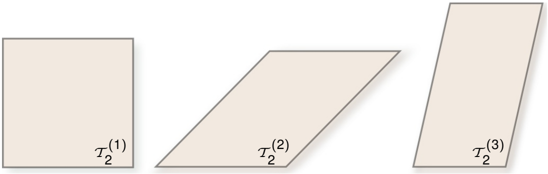
- Focus on ordinary instantons, but should be useful for exotic instantons as well
- Choose a toroidal compactification where string theory is calculable.
- Realize (locally) $\mathcal{N} = 1$ gauge SQCD in type IIB on a system of D9-branes and discuss contributions of E5 branes to the superpotential
- Analyze the rôle of annuli bounded by E5 and D9 branes in giving these terms suitable holomorphicity properties

The set-up

The background geometry

Internal space:

$$\frac{\mathcal{T}_2^{(1)} \times \mathcal{T}_2^{(2)} \times \mathcal{T}_2^{(3)}}{\mathbb{Z}_2 \times \mathbb{Z}_2}$$



- The Kähler param.s and complex structures determine the string frame metric and the B field:

$$G^{(i)} = \frac{T_2^{(i)}}{U_2^{(i)}} \begin{pmatrix} 1 & U_1^{(i)} \\ U_1^{(i)} & |U^{(i)}|^2 \end{pmatrix} \quad \text{and} \quad B^{(i)} = \begin{pmatrix} 0 & -T_1^{(i)} \\ T_1^{(i)} & 0 \end{pmatrix} .$$

The background geometry

Complex coordinates

- String fields: $X^M \to (X^\mu, Z^i)$ and $\psi^M \to (\psi^\mu, \Psi^i)$, with

$$Z^i = \sqrt{\frac{T_2^{(i)}}{2U_2^{(i)}}}(X^{2i+2} + U^{(i)} X^{2i+3})$$

- 10d spin fields decompose into space-time and internal parts:

$$S^{\dot{A}} \to (S_\alpha S_{---}, S_\alpha S_{-++}, \ldots, S^{\dot{\alpha}} S^{+++}, \ldots)$$

- Action of the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold group elements:

$$
\begin{aligned}
h_1 &: \ (Z^1, Z^2, Z^3) \to (Z^1, -Z^2, -Z^3) \ , \\
h_2 &: \ (Z^1, Z^2, Z^3) \to (-Z^1, Z^2, -Z^3) \ , \\
h_3 &: \ (Z^1, Z^2, Z^3) \to (-Z^1, -Z^2, Z^3) \ ,
\end{aligned}
$$

- The group has 4 irreducible representations:

$$R_0 \text{ (trivial)}, R_1, R_2, R_3$$

.

The geometric moduli

Supergravity basis- tree level

- Supergravity basis: $s, t^{(i)}, u^{(i)}$, with Back

Lüst et al, 0404134; ...

$$\mathrm{Im}(s) \equiv s_2 = \frac{1}{4\pi}\, \mathrm{e}^{-\phi_{10}}\, T_2^{(1)} T_2^{(2)} T_2^{(3)}\,,$$

$$\mathrm{Im}(t^{(i)}) \equiv t_2^{(i)} = \mathrm{e}^{-\phi_{10}}\, T_2^{(i)}\,, \quad u^{(i)} = u_1^{(i)} + \mathrm{i}\, u_2^{(i)} = U^{(i)}\,,$$

(real parts from suitable RR or B fields). N.B. $s_2 \sim 1/g_s$.

- $\mathcal{N} = 1$ bulk Kähler potential:

$$K = -\log(s_2) - \sum_{i=1} \log(t_2^{(i)}) - \sum_{i=1} \log(u_2^{(i)})$$

Antoniadis et al, 9608012

The geometric moduli

Supergravity basis - corrections

At one-loop level, there are corrections to the bulk Kähler potential (and to the Einstein term)

Antoniadis et al, 9608012; ... ; Berg et al, 0508043

- These lead to non-holomorphic redefinitions of the supergravity fields s andt_i w.r.t. the their tree-level expressions. In particular

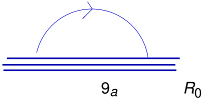
$$s_2^{(0)} = s_2 + \frac{\delta}{8\pi^2}$$

- Differently from corresponding Heterotic constructions, δ in these models appears to be of order g_s rather than 1.
- It would be interesting [see later!] to clarify if any other mechanism can induce, in the models we consider, a shift $\delta^{(0)}$ of order 1.

$\mathcal{N} = 1$ from magnetized branes

The gauge sector

Place a stack of N_a fractional D9 branes ("color branes" $9a$).



- Massless spectrum of $9a/9a$ strings gives rise, in $\mathbf{R}^{1,3}$, to the $\mathcal{N} = 1$ vector multiplets for the gauge group $U(N_a)$

- The gauge coupling constant is given (at tree level) by

$$\frac{1}{g_a^2} = \frac{1}{4\pi}\, \mathrm{e}^{-\phi_{10}}\, T_2^{(1)} T_2^{(2)} T_2^{(3)} = s_2^{(0)}$$

$\mathcal{N} = 1$ from magnetized branes

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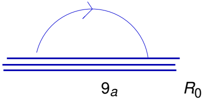


- Massless spectrum of $9a/9a$ strings gives rise, in $\mathbf{R}^{1,3}$, to the $\mathcal{N} = 1$ vector multiplets for the gauge group $U(N_a)$
- The Wilsonian coupling $1/\tilde{g}_a^2$ must correspond to (the imaginary part of) a chiral field, so it is corrected w.r.t. to the tree level: Back

$$\frac{1}{g_a^2} = \frac{1}{\tilde{g}_a^2} + \frac{\delta}{8\pi^2}$$

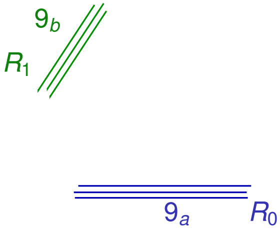
$\mathcal{N} = 1$ from magnetized branes

Adding flavors

Add D9-branes ("flavor branes" $9b$) with quantized magnetic fluxes

$$f_b^{(i)} = \frac{m_b^{(i)}}{n_b^{(i)}}$$

and in a different orbifold rep.



- (Bulk) susy requires $\nu_b^{(1)} - \nu_b^{(2)} - \nu_b^{(3)} = 0$, where

$$f_b^{(i)} / T_2^{(i)} = \tan \pi \nu_b^{(i)} \quad \text{with} \quad 0 \leq \nu_b^{(i)} < 1 \,,$$

(other possibilities by sign changes)

Marino et al, 9911206

$\mathcal{N} = 1$ from magnetized branes

Adding flavors

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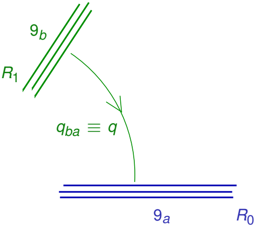

- 9a/9b strings are twisted by the relative angles ▸ Back

$$\nu_{ba}^{(i)} = \nu_b^{(i)} - \nu_a^{(i)}$$

- If $\nu_{ba}^{(1)} - \nu_{ba}^{(2)} - \nu_{ba}^{(3)} = 0$, this sector is supersymmetric: massless modes fill up a chiral multiplet q_{ba} in the anti-fundamental rep $\bar{N}_a$ of the color group

$\mathcal{N} = 1$ from magnetized branes

Add D9-branes (“flavor branes” $9b$) with quantized magnetic fluxes

$$f_b^{(i)} = \frac{m_b^{(i)}}{n_b^{(i)}}$$

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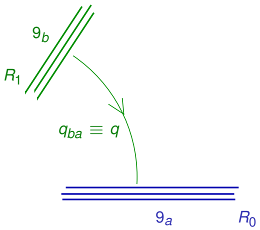


- The degeneracy of this chiral multiplet is $N_b|I_{ab}|$, where I_{ab} is the # of Landau levels for the (a, b) “intersection”

$$I_{ab} = \prod_{i=1} (m_a^{(i)} n_b^{(i)} - m_b^{(i)} n_a^{(i)})$$

Local vs global realization

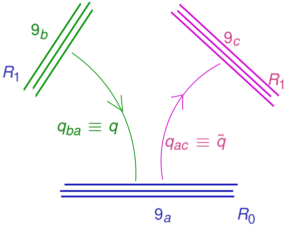
- Introducing branes in a compact space requires the cancellation of the associated tadpoles. This can be achieved by a suitable orientifold projection in the string description, and severely constrains the set-up.
- We take a “local” approach, and do not discuss the “global” requirement of tadpole cancellation (which is however to be assumed) and the contribution of orientifolds in these models:
  - our goal is to understand certain mechanisms of the stringy instanton calculus rather than provide phenomenological models
  - these aspects can be taken into accout, and the picture goes through

see Akerblom et al, 0705.2366; Blumenhagen et al, 0711.0866

$\mathcal{N} = 1$ from magnetized branes

Introduce a third stack of $9c$ branes such that we get a chiral mult. q_{ac} in the fundamental rep N_a and that

$$N_b|I_{ab}| = N_c|I_{ac}| \equiv N_F$$



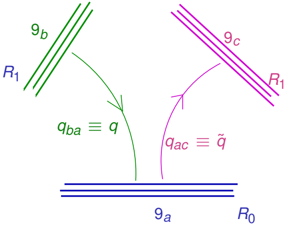
- This gives a (local) realization of $\mathcal{N} = 1$ SQCD: same number N_F of fundamental and anti-fundamental chiral multiplets, resp. denoted by q_f and $\tilde{q}^f$

$\mathcal{N} = 1$ from magnetized branes

Engineering $\mathcal{N} = 1$ SQCD

Introduce a third stack of $9c$ branes such that we get a chiral mult. q_{ac} in the fundamental rep N_a and that

$$N_b |I_{ab}| = N_c |I_{ac}| \equiv N_F$$



Kinetic terms of chiral mult. scalars from disks

$$\sum_{f=1}^{N_F} \left\{ D_\mu q^{\dagger f} D^\mu q_f + D_\mu \tilde{q}^f D^\mu \tilde{q}_f^\dagger \right\}$$

Sugra Lagrangian: different field normaliz. s

$$\sum_{f=1}^{N_F} \left\{ K_Q D_\mu Q^{\dagger f} D^\mu Q_f + K_{\tilde{Q}} D_\mu \tilde{Q}^f D^\mu \tilde{Q}_f^\dagger \right\}$$

- Related via the Kähler metrics: $q = \sqrt{K_Q}\, Q\,, \ \tilde{q} = \sqrt{K_{\tilde{Q}}}\, \tilde{Q}$

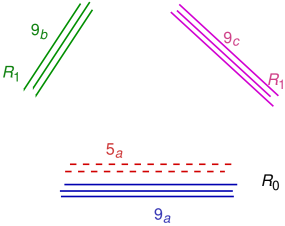
Back   Back'

Non-perturbative sectors from *E*5

Adding “ordinary” instantons

Add a stack of *k* E5 branes whose internal part coincides with the D9*a*:

- ordinary instantons for the D9*a* gauge theory
- would be exotic for the D9*b*, *c* gauge theories



- New types of open strings: $E5_a/E5_a$ (neutral sector), $D9_a/E5_a$ (charged sector), $D9_b/E5_a$ or $E5_a/D9_c$ (flavored sectors, twisted)
- These states carry no momentum in space-time: moduli, not fields. [Collective name: $\mathcal{M}_k$]
- charged or neutral moduli can have KK momentum

Non-perturbative sectors from $E5$

The spectrum of moduli

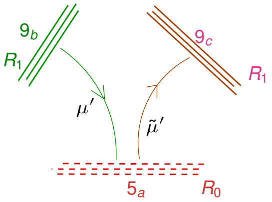
Sector		ADHM	Meaning	Chan-Paton	Dimension
$5_a/5_a$	NS	a_μ	centers	adj. $U(k)$	(length)
		D_c	Lagrange mult.	⋮	$(\text{length})^{-2}$
	R	M^α	partners	⋮	$(\text{length})^{\frac{1}{2}}$
		$\lambda_{\dot\alpha}$	Lagrange mult.	⋮	$(\text{length})^{-\frac{3}{2}}$
$9_a/5_a$	NS	$w_{\dot\alpha}$	sizes	$N_a \times \overline{k}$	(length)
$5_a/9_a$		$\bar{w}_{\dot\alpha}$	⋮	$k \times \overline{N}_a$	⋮
$9_a/5_a$	R	μ	partners	$N_a \times \overline{k}$	$(\text{length})^{\frac{1}{2}}$
$5_a/9_a$		$\bar\mu$	⋮	$k \times \overline{N}_a$	⋮
$9_b/5_a$	R	μ'	flavored	$N_F \times \overline{k}$	$(\text{length})^{\frac{1}{2}}$
$5_a/9_c$		$\tilde\mu'$	⋮	$k \times \overline{N}_F$	⋮

Non-perturbative sectors from $E5$

Some observations

- Among the neutral moduli we have the center of mass position x_0^μ and its fermionic partner θ^α (related to susy broken by the E5a): Back

$$a^\mu = x_0^\mu \, \mathbb{1}_{k\times k} + y_c^\mu \, T^c \quad , \quad M^\alpha = \theta^\alpha \, \mathbb{1}_{k\times k} + \zeta_c^\alpha \, T^c \, ,$$



- In the flavored sectors only fermionic zero-modes:
  - μ'_f (D9$_b$/E5$_a$ sector)
  - $\tilde{\mu}'^f$ (E5$_a$/D9$_c$ sector)

The stringy instanton calculus

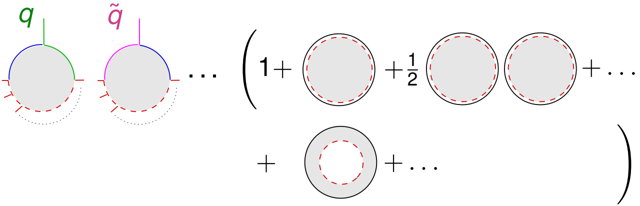
Instantonic correlators

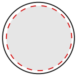
The stringy way

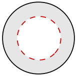
In presence of Euclidean branes, dominant contribution to correlators of gauge/matter fields from one-point functions.

Polchinski, 1994; Green and Gutperle, 1997-2000; Billo et al, 2002; Blumenhagen et al, 2006

E.g., a correlator of chiral fields $\langle q\tilde{q}\ldots\rangle$ is given by



Disks:  $= -\frac{8\pi^2}{g_a^2}k + S_{mod}(\mathcal{M}_k)$ (with moduli insertions)

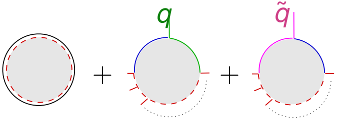
Annuli:  $\equiv \mathcal{A}_{5_a}$ (no moduli insert.s, otherwise suppressed)

The effective action

in an instantonic sector

The various instantonic correlators can be obtained shifting the moduli action by terms dependent on the gauge/matter fields. In the case at hand,

$$S_{mod}(q, \tilde{q}; \mathcal{M}_k) =$$



$$+ \dots$$

$$= \mathrm{tr}_k \Big\{ \mathrm{i} D_c \Big(\bar{w}_{\dot\alpha} (\tau^c)^{\dot\alpha}{}_{\dot\beta} w^{\dot\beta} + \mathrm{i} \bar{\eta}^c_{\mu\nu} \left[a^\mu, a^\nu \right] \Big)$$

$$- \mathrm{i} \lambda^{\dot\alpha} \Big(\bar{\mu} w_{\dot\alpha} + \bar{w}_{\dot\alpha} \mu + \left[a_\mu, M^\alpha \right] \sigma^\mu_{\alpha\dot\alpha} \Big) \Big\}$$

$$+ \mathrm{tr}_k \sum_f \Big\{ \bar{w}_{\dot\alpha} \left[q^{\dagger f} q_f + \tilde{q}^f \tilde{q}^\dagger_f \right] w^{\dot\alpha} - \frac{\mathrm{i}}{2} \bar{\mu}\, q^{\dagger f} \mu'_f + \frac{\mathrm{i}}{2} \tilde{\mu}'^f \tilde{q}^\dagger_f \, \mu \Big\} .$$

The effective action

in an instantonic sector

- There are other relevant diagrams involving the superpartners of q and $\tilde{q}$, related to the above by susy Ward identities. Complete result:

$$q(x_0)\,,\ \tilde{q}(x_0)\ \longrightarrow\ q(x_0,\theta)\,,\ \tilde{q}(x_0,\theta)$$

in $S_{mod}(q,\tilde{q};\mathcal{M}_k)$.

- The moduli have to be integrated over

The instanton partition function

as an integral over moduli space

Summarizing, the effective action has the form (Higgs branch)

$$S_k = \mathcal{C}_k \, \mathrm{e}^{-\frac{8\pi^2}{g_a^2} k} \mathrm{e}^{\mathcal{A}'_{5a}} \int d\mathcal{M}_k \, \mathrm{e}^{-S_{mod}(q,\tilde{q};\mathcal{M}_k)}$$

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- In $\mathcal{A}'_{5_a}$ the contribution of zero-modes running in the loop is suppressed because they're already explicitly integrated over

Blumenhagen et al, 2006

The instanton partition function

as an integral over moduli space

Summarizing, the effective action has the form (Higgs branch)

$$S_k = \mathcal{C}_k \, \mathrm{e}^{-\frac{8\pi^2}{g_a^2} k} \, \mathrm{e}^{\mathcal{A}'_{5a}} \int d\mathcal{M}_k \, \mathrm{e}^{-S_{mod}(q,\tilde{q};\mathcal{M}_k)}$$

- $\mathcal{C}_k$ is a normalization factor, determined (up to numerical constants) counting the dimensions of the moduli $\mathcal{M}_k$:

  Back

$$\mathcal{C}_k = (\sqrt{\alpha'})^{-(3N_a - N_F)k} \, (g_a)^{-2N_a k} \; .$$

Notice the appearing of the β-function coeff. b_1

Instanton induced superpotential

In $S_{mod}(q, \tilde{q}; \mathcal{M}_k)$, the superspace coordinates x_0^μ and θ^α appear only through superfields $q(x_0, \theta), \tilde{q}(x_0, \theta), \ldots$ ▸ Recall

- We can separate x, θ from the other moduli $\widehat{\mathcal{M}}_k$ writing

$$S_k = \int d^4 x_0 \, d^2\theta \; W_k(q, \tilde{q}) \; ,$$

with the effective superpotential

$$W_k(q, \tilde{q}) = \mathcal{C}_k \, \mathrm{e}^{-\frac{8\pi^2}{g_a^2} k} \, \mathrm{e}^{\mathcal{A}'_{5a}} \int d\widehat{\mathcal{M}}_k \; \mathrm{e}^{-S_{mod}(q, \tilde{q}; \widehat{\mathcal{M}}_k)}$$

Issues of holomorphicity

A superpotential is expected to be holomorphic. We found

$$W_k(q,\tilde{q}) = \mathcal{C}_k \, \mathrm{e}^{-\frac{8\pi^2}{g_a^2}k} \, \mathrm{e}^{\mathcal{A}'_{5a}} \int d\widehat{\mathcal{M}}_k \, \mathrm{e}^{-S_{mod}(q,\tilde{q};\widehat{\mathcal{M}}_k)}$$

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- $S_{mod}(q,\tilde{q};\widehat{\mathcal{M}}_k)$ explicitly depends on $q^\dagger$ and $\tilde{q}^\dagger$. This dependence disappears upon integrating over $\widehat{\mathcal{M}}_k$ as a consequence of the cohomology properties of the integration measure.
- However, we have to re-express the result in terms of the SUGRA fields Q and $\tilde{Q}$ ▸ Recall

Issues of holomorphicity

A superpotential is expected to be holomorphic. We found

$$W_k(q,\tilde{q}) = \mathcal{C}_k \, \mathrm{e}^{-\frac{8\pi^2}{g_a^2} k} \, \mathrm{e}^{\mathcal{A}'_{5a}} \int d\widehat{\mathcal{M}}_k \, \mathrm{e}^{-S_{mod}(q,\tilde{q};\widehat{\mathcal{M}}_k)}$$

- The prefactors should combine into a dynamically generated holomorphic scale Λ_{hol}, obtained by integrating the Wilsonian β-function of the $\mathcal{N}=1$ SQCD

  Novikov et al, 1983; Dorey et al, 2002; ...

- To this effect, it is crucial that $\mathcal{A}'_{5_a}$ can introduce a non-holomorphic dependence on the complex and Kähler structure moduli of the compactification space. ▸ Back
- Our aim is to consider the interplay of all these observations. For this we need the explicit expression of the mixed annuli term $\mathcal{A}'_{5_a}$

The ADS/TVY superpotential

To be concrete, let's focus on the single instanton case, $k = 1$. In this case, the integral over the moduli can be carried out explicitly.

- Balancing the fermionic zero-modes requires $N_F = N_a - 1$
- The end result is

Dorey et al, 2002; Akerblom et al, 2006; Argurio et al, 2007

$$W_{k=1}(q, \tilde{q}) = \mathcal{C}_k \, \mathrm{e}^{-\frac{8\pi^2}{g_a^2} k} \, \mathrm{e}^{\mathcal{A}'_{5a}} \frac{1}{\det\left(\tilde{q} q\right)}$$

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- Same form as the ADS/TVY superpotential

Affleck et al, 1984; Taylor et al, 1983;

The ADS/TVY superpotential

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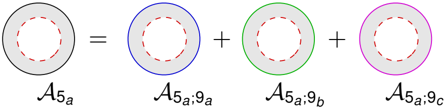
- We'll see how these factors conspire to give an holomorphic expression in the sugra variables Q and $\tilde{Q}$

Instanton annuli and threshold corrections

The mixed annuli

The amplitude $\mathcal{A}_{5_a}$ is a sum of cylinder amplitudes with a boundary on the E5a (both orientations)



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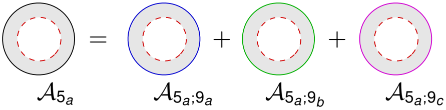
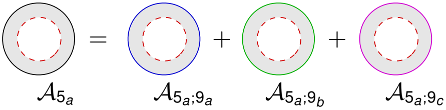


- Both UV and IR divergent. The UV divergences (IR in the closed string channel) cancel if tadpole cancellation assumed. Regulate the IR with a scale μ

The mixed annuli

The amplitude $\mathcal{A}_{5_a}$ is a sum of cylinder amplitudes with a boundary on the E5a (both orientations)



- There is a relation between these instantonic annuli and the running gauge coupling constant ▸ Back

Abel and Goodsell, 2006; Akerblom et al, 2006

$$\mathcal{A}_{5_a} = -\frac{8\pi^2 k}{g_a^2(\mu)|_{1-\mathrm{loop}}} .$$

- Indeed, in susy theories, mixed annuli compute the running coupling by expanding around the instanton bkg

Billo et al, 2007

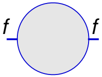
Computing the YM effective action

using different backgrounds

There are two gauge backgrounds on which string theory is computable and yields the effective action for the gauge fields

- Constant gauge field f (turned on a color D9-brane)
  - At tree level, the YM action $\frac{1}{g_a^2} \int d^4x \, \mathrm{Tr} \, \frac{1}{2} F_{\mu\nu}^2$ evaluates to

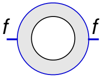
$$S(f) = \frac{\mathrm{Vol}_4 \, f^2}{2 \, g_a^2}$$



  - At loop level, we have (Δ are threshold corrections)

$$S(f)|_{1-\mathrm{loop}} = \left(\frac{b_1}{16\pi^2} \log \alpha' \mu^2 + \Delta \right) \mathrm{Vol}_4 \, f^2$$

$$= \frac{\mathrm{Vol}_4 \, f^2}{2 \, g_a^2(\mu)|_{1-\mathrm{loop}}}$$



Computing the YM effective action

using different backgrounds

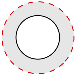
There are two gauge backgrounds on which string theory is computable and yields the effective action for the gauge fields

- Instanton background (realized by k E5 branes)
  - At tree level, the YM action evaluates to Back

$$S_{\text{inst}} = \frac{8\pi^2 k}{g_a^2}$$

  - At loop level, we have the analogous relation:

$$S_{\text{inst}}\big|_{1-\text{loop}} = \frac{8\pi^2 k}{g_a^2(\mu)\big|_{1-\text{loop}}} = \mathcal{A}_{5a}$$



With susy, the 1-loop determinants of the non-zero-modes cancel out: the only effect is the renormalization of the gauge coupling constant.

Dadda et al, 1977; ...

The explicit computation of the annuli confirms the relation of these annuli to the running coupling. Imposing the appropriate b.c.'s and GSO one starts from

$$\int_0^\infty \frac{d\tau}{2\tau} \left[\mathrm{Tr}_{\mathrm{NS}} \left(P_{\mathrm{GSO}}\, P_{\mathrm{orb.}}\, q^{L_0} \right) - \mathrm{Tr}_{\mathrm{R}} \left(P_{\mathrm{GSO}}\, P_{\mathrm{orb.}}\, q^{L_0} \right) \right] .$$

- For $\mathcal{A}_{5_a;9_a}$, KK copies of zero-modes on internal tori $\mathcal{T}_2^{(i)}$ give a (non-holomorphic) dependence on the Kähler and complex moduli

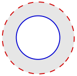
  Lüst and Stieberger, 2003

  .
- For $\mathcal{A}_{5_a;9_b}$ and $\mathcal{A}'_{5_a;9_c}$, the modes are twisted and the result depends from the angles $\nu_{ba}^{(i)}$ and $\nu_{ac}^{(i)}$ ▸ Recall

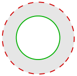
Expression of the annuli

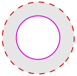
Explicit result

Back



$$\mathcal{A}_{5_a;9_a} = -8\pi^2 k \Big[\frac{3N_a}{16\pi^2} \log(\alpha' \mu^2) + \frac{N_a}{16\pi^2} \sum_i \log\left(U_2^{(i)} T_2^{(i)} (\eta(U^{(i)})^4) \right) \Big] ,$$



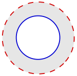


$$\mathcal{A}_{5_a;9_b} + \mathcal{A}_{5_a;9_c} = 8\pi^2 k \left(\frac{N_F}{16\pi^2} \log(\alpha' \mu^2) + \frac{N_F}{32\pi^2} \log\left(\boldsymbol{\Gamma}_{ba} \boldsymbol{\Gamma}_{ac} \right) \right) ,$$

Expression of the annuli

Explicit result

Back



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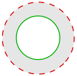

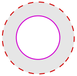

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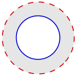
- β-function coefficient of SQCD: $3N_a - N_F$

Expression of the annuli

Explicit result

Back



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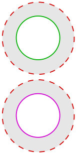

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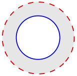
- Non-holomorphic threshold corrections

Expression of the annuli

Explicit result

▸ Back



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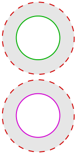

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- $\boldsymbol{\Gamma}_{ba} = \frac{\Gamma(1-\nu_{ba}^{(1)})}{\Gamma(\nu_{ba}^{(1)})} \frac{\Gamma(\nu_{ba}^{(2)})}{\Gamma(1-\nu_{ba}^{(2)})} \frac{\Gamma(\nu_{ba}^{(3)})}{\Gamma(1-\nu_{ba}^{(3)})}$

Lüst and Stieberger, 2003

Akerblom et al, 2007

Holomorphicity properties

The holomorphic gauge coupling

- Computing the pure instantonic disks and annuli yields the gauge coupling up to 1 loop in the form ▸ Recall

$$\mathcal{A}_{1-loop} = -\frac{8\pi^2 k}{g_a^2(\mu)} = -\frac{8\pi^2 k}{g_a^2} + \mathcal{A}_{5a}$$

- The very general Kaplunovsky-Louis formula expresses the one-loop gauge coupling in terms of the wilsonian coupling $1/\tilde{g}_a^2 = s_2$ and of other tree-level quantities in the effective action

Kaplunovsky-Louis relation

at one loop

Dixon et al, 1991; Kaplunovsky and Louis, 1994-95; ...

$$\frac{1}{g^2(\mu)} = \frac{1}{\tilde{g}^2} + \frac{1}{8\pi^2}\Big[\frac{b}{2}\log\frac{\mu^2}{M_P^2} - f^{(1)} - \frac{c}{2}K + T(G)\log\frac{1}{\tilde{g}^2} - \sum_r n_r T(r)\log K_r\Big]$$

- Here T_A = generators of the gauge group, n_r = # chiral mult. in rep. r and

$$T(r)\,\delta_{AB} = \mathrm{Tr}_r\big(T_A T_B\big) \quad , \quad T(G) = T(\mathrm{adj})$$

$$b = 3\,T(G) - \sum_r n_r\,T(r) \quad , \quad c = T(G) - \sum_r n_r\,T(r)\,,$$

Kaplunovsky-Louis relation

at one loop

Dixon et al, 1991; Kaplunovsky and Louis, 1994-95; ...

$$\frac{1}{g^2(\mu)} = \frac{1}{\tilde{g}^2} + \frac{1}{8\pi^2}\Big[\frac{b}{2}\log\frac{\mu^2}{M_P^2} - f^{(1)} - \frac{c}{2}K + T(G)\log\frac{1}{\tilde{g}^2} - \sum_r n_r T(r)\log K_r\Big]$$

- Holomorphic function

Kaplunovsky-Louis relation

at one loop

Dixon et al, 1991; Kaplunovsky and Louis, 1994-95; ...

$$\frac{1}{g^2(\mu)} = \frac{1}{\tilde{g}^2} + \frac{1}{8\pi^2}\Big[\frac{b}{2}\log\frac{\mu^2}{M_P^2} - f^{(1)} - \frac{c}{2}K + T(G)\log\frac{1}{\tilde{g}^2} - \sum_r n_r T(r)\log K_r\Big]$$

- Non-holomorphic corrections

Kaplunovsky-Louis relation

at one loop

Dixon et al, 1991; Kaplunovsky and Louis, 1994-95; ...

$$\frac{1}{g^2(\mu)} = \frac{1}{\tilde{g}^2} + \frac{1}{8\pi^2}\left[\frac{b}{2}\log\frac{\mu^2}{M_P^2} - f^{(1)} - \frac{c}{2}K + T(G)\log\frac{1}{\tilde{g}^2} - \sum_r n_r T(r)\log K_r\right]$$

- Inside the square bracket the bulk Kähler potential K and the Kähler metrics for the matter multiplets K_r are at tree level

Kaplunovsky-Louis relation

at one loop

Dixon et al, 1991; Kaplunovsky and Louis, 1994-95; ...

$$\frac{1}{g^2(\mu)} = \frac{1}{\tilde{g}^2} + \frac{1}{8\pi^2}\Big[\frac{b}{2}\log\frac{\mu^2}{M_P^2} - f^{(1)} - \frac{c}{2}K + T(G)\log\frac{1}{\tilde{g}^2} - \sum_r n_r T(r)\log K_r\Big]$$

- The only place where the shift δ ▸ Recall in the holomorphic coupling matters is the tree-level term. Moreover only a shift $\delta^{(0)}$ of order 1 in g_s is relevant at this level!

Instantonic annuli

in Kaplunovsky-Louis form

The result for the instantonic annuli ▸ Recall can be recast in the following form:

$$\mathcal{A}_{5_a} = -\frac{8\pi^2 k}{\tilde{g}_a^2} + k\Big[-\frac{3N_a - N_F}{2}\log\frac{\mu^2}{M_P^2} - N_a\sum_{i=1}^{3}\log\left(\eta(u^{(i)})^2\right) + \frac{N_a - N_F}{2}K + N_a \log g_a^2 - \delta^{(0)} + \frac{N_F}{2}\log(\mathcal{Z}_{ba}\mathcal{Z}_{ac})\Big]$$

with (similarly for $\mathcal{Z}_{ac}$)

$$\mathcal{Z}_{ba} = \left(4\pi\, s_2\right)^{-\frac{1}{4}} \left(t_2^{(1)} t_2^{(2)} t_2^{(3)}\right)^{-\frac{1}{4}} \left(u_2^{(1)} u_2^{(2)} u_2^{(3)}\right)^{-\frac{1}{2}} \left(\boldsymbol{\Gamma}_{ba}\right)^{\frac{1}{2}}$$

- If $\delta^{(0)} = 0$, $\mathcal{Z}_{ba}$ coincides with the Kähler metric K_{ab} of the twisted matter

Instantonic annuli

in Kaplunovsky-Louis form

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$$+\frac{N_a - N_F}{2}K + N_a \log g_a^2 - \delta^{(0)} + \frac{N_F}{2}\log(\mathcal{Z}_{ba}\mathcal{Z}_{ac})\Big]$$

- If there is some one-loop shift of s_2 of order 1, i.e., $\delta^{(0)} \neq 0$, then we have
$$K_{ab} = \chi_{ab}\,\mathcal{Z}_{ba}$$
with
$$\delta^{(0)} + \frac{N_F}{2}\log \chi_{ab}\chi_{bc} = 0$$

The Kähler metric for twisted matter

Thus, up to possible factors χ due to one-loop shifts $\delta^{(0)}$, the Kähler metric of chiral multiplets Q arising from twisted D9$_a$/D9$_b$ strings is given by Back

$$K_Q = (4\pi\, s_2)^{-\frac{1}{4}} \left(t_2^{(1)} t_2^{(2)} t_2^{(3)}\right)^{-\frac{1}{4}} \left(u_2^{(1)} u_2^{(2)} u_2^{(3)}\right)^{-\frac{1}{2}} \left(\mathbf{\Gamma}_{ba}\right)^{\frac{1}{2}}$$

with

$$\mathbf{\Gamma}_{ba} = \frac{\Gamma(1-\nu_{ba}^{(1)})}{\Gamma(\nu_{ba}^{(1)})} \frac{\Gamma(\nu_{ba}^{(2)})}{\Gamma(1-\nu_{ba}^{(2)})} \frac{\Gamma(\nu_{ba}^{(3)})}{\Gamma(1-\nu_{ba}^{(3)})}$$

This is very interesting because:

- for twisted fields, the Kähler metric cannot be derived from compactification of DBI

The Kähler metric for twisted matter

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This is very interesting because:

- the part dependent on the twists, namely Γ_{ba}, is reproduced by a direct string computation

  Lüst et al, 2004; Bertolini et al, 2005

- the prefactors, depending on the geometric moduli, are more difficult to get directly: the present suggestion is welcome!

The Kähler metric for twisted matter

Thus, up to possible factors χ due to one-loop shifts $\delta^{(0)}$, the Kähler metric of chiral multiplets Q arising from twisted $D9_a/D9_b$ strings is given by Back

with

$$\Gamma_{ba} = \frac{\Gamma(1-\nu_{ba}^{(1)})}{\Gamma(\nu_{ba}^{(1)})} \frac{\Gamma(\nu_{ba}^{(2)})}{\Gamma(1-\nu_{ba}^{(2)})} \frac{\Gamma(\nu_{ba}^{(3)})}{\Gamma(1-\nu_{ba}^{(3)})}$$

This is very interesting because:

- We have checked this expression against the known results for Yukawa couplings of magnetized branes: perfect consistency!

  Cremades et al, 2004

  N.B. This check also severely constrains the possible extra pre-factors $\chi_{ba}, \ldots$.

- Beside being related to the gauge thresholds, the instantonic annuli $\mathcal{A}_{5a}$ are relevant because they enter the stringy instanton calculus
- In particular, the form of the $\mathcal{A}_{5a}$ annuli is crucial for the holomorphicity properties of E5 non-perturbative contributions
- We consider for definiteness the ADS/TVY case.

Back to the ADS/TVY superpotential

Making it holomorphic

We found (recall that $N_F = N_a - 1$ in this case)

$$W_{k=1}(q,\tilde{q}) = \mathcal{C}_k \, \mathrm{e}^{-\frac{8\pi^2}{g_a^2} k} \, \mathrm{e}^{\mathcal{A}'_{5a}} \frac{1}{\det(\tilde{q}q)}$$

- Insert the expression of the annuli, from which we must subtract the contrib. of the zero-modes running in the loop, which are responsible for the IR divergences.
- Use the natural UV cut-off of the low-energy theory, the Planck mass $M_P^2 = \frac{1}{\alpha'} \mathrm{e}^{-\phi_{10}} s_2$ and write

$$\mathcal{A}_{5a} = -k \frac{b_1}{2} \log \frac{\mu^2}{M_P^2} + \mathcal{A}'_{5a}$$

Back to the ADS/TVY superpotential

Making it holomorphic

$$W_{k=1}(q,\tilde{q}) = \mathcal{C}_k \, e^{-\frac{8\pi^2}{g_a^2} k} \, e^{\mathcal{A}'_{5a}} \frac{1}{\det(\tilde{q}q)}$$

- Make explicit the prefactor $\mathcal{C}_k$ Recall
- Allow for a possible shift in the gauge coupling:

$$\frac{1}{g_a^2} = \frac{1}{\tilde{g}_a^2} + \frac{\delta}{8\pi^2}$$

Back to the ADS/TVY superpotential

Making it holomorphic

In this way we obtain

$$W_{k=1} = \mathrm{e}^{K/2} \prod_{i=1}^{3} \left(\eta(u^{(i)})^{-2N_a}\right) (\sqrt{\alpha'})^{-b_1}\, \mathrm{e}^{-\frac{8\pi^2}{\tilde{g}_a^2}}$$

$$\left(K_{\tilde{Q}} K_Q\right)^{\frac{N_a-1}{2}} \frac{1}{\det(\tilde{q}q)}$$

- Rescale the chiral multiplet to their sugra counterparts assuming $K_{\tilde{Q}}$, K_Q are the matter Kähler metrics ▸ Recall
- Introduce the invariant scale in the Wilsonian scheme

$$\Lambda_{\mathrm{hol}}^{b_1} = (\sqrt{\alpha'})^{-b_1}\, \mathrm{e}^{-\frac{8\pi^2}{\tilde{g}_a^2}}$$

Back to the ADS/TVY superpotential

Making it holomorphic

We get thus

$$W_{k=1} = \mathrm{e}^{K/2} \prod_{i=1}^{3} \left(\eta(u^{(i)})^{-2N_a} \right) \Lambda_{\mathrm{hol}}^{2N_a+1} \frac{1}{\det(\widetilde{Q}\, Q)}$$

$$\equiv \mathrm{e}^{K/2} \widehat{\Lambda}_{\mathrm{hol}}^{2N_a+1} \frac{1}{\det(\widetilde{Q} Q)}$$

- In the second step the moduli dependent factors of $\eta(u^{(i)})$ are readsorbed by a holomorphic redefinition of the scale
- A part from the prefactor $\mathrm{e}^{K/2}$, the final expression is holomorphic in the variables of the Wilsonian scheme

Back to the ADS/TVY superpotential

Making it holomorphic

We get thus

$$W_{k=1} = \mathrm{e}^{K/2} \prod_{i=1}^{3} \left(\eta(u^{(i)})^{-2N_a} \right) \Lambda_{\mathrm{hol}}^{2N_a+1} \frac{1}{\det(\widetilde{Q}\, Q)}$$

$$\equiv \mathrm{e}^{K/2} \widehat{\Lambda}_{\mathrm{hol}}^{2N_a+1} \frac{1}{\det(\widetilde{Q} Q)}$$

- The rôle of the annuli in these non-perturbative considerations leads to equivalent information on the Kähler metric of the twisted matter as the comparison with the perturbative KL formula

Remarks and conclusions

- Also in $\mathcal{N} = 2$ toroidal models the instanton-induced superpotential is in fact holomorphic in the appropriate sugra variables if one includes the mixed annuli in the stringy instanton calculus

Akerblom et al, 2007; Billo et al, 2007

- W.r.t. to the "color" $D9_a$ branes, the $E5_a$ branes are ordinary instantons. For the gauge theories on the $D9_b$ or the $D9_c$, they would be exotic (less clear from the field theory viewpoint)
- The study of the mixed annuli and their relation to holomorphicity can be relevant for exotic, new stringy effects as well.